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A cost-based maritime container assignment model and port choice.

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ABSTRACT: A recently proposed frequency-based maritime container assignment model (Bell et al, 2011) seeks an assignment of full and empty containers to paths that minimises expected container travel time, whereas containers are in practice more likely to be assigned to minimise expected cost. There are significant economies of scale in the maritime transport of containers; the cost per container per unit time falls with increasing ship occupancy and larger ships when full cost less per container per unit time than smaller ships. A cost-based container assignment model is proposed here. The objective is to assign containers to maritime routes to minimize sailing costs plus expected dwell costs at the ports of origin and transhipment. The constraints in the model are extended to include route as well as port capacity constraints. Although the cost per container per unit time depends on ship occupancy, it is shown that the problem remains a linear program. A small numerical example is presented to illustrate the properties of the model. The paper concludes by considering the many applications of the proposed maritime container assignment model.

KEY WORDS: ***Logistics; container assignment model; maritime network.***

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1. Introduction

The status and function of the port has been evolving rapidly with the development of world trade. Economies of scale in shipping are driving the emergence of large seaports performing mainly the function of container transhipment hubs in maritime hub-and-spoke networks. Additionally, empty container management has grown in significance as a result of trade imbalances between regions of production and consumption. Typically it is assumed that full containers take priority over empty containers when capacity constraints bind on maritime routes, however the model proposed here allows this to be relaxed when otherwise there would be a shortage of empty containers.

According to the Anyport model (Bird, 1980), *setting*, *expansion* and *specialization* are the three main stages in the evolution of a port. Notteboom and Rodrigue (2005) added a fourth stage, *regionalization*, based on the relationship between the port and its hinterland. With the progressive integration of ports in supply chains, it has become clear that shippers are no longer choosing a port per se but rather a supply chain in which a port is an important element. The shipper's influence on port choice is diminishing, particularly now that a single shipping line, a third party logistics providers or a supply chain integrator may control the freight from origin to destination (Magala, 2008; Joyce et al., 2008). These changes point to port choice being the outcome of a container assignment process.

This paper presents a maritime container assignment model that takes into account the economies of scale that is driving the evolution of maritime hub-and-spoke networks as well as the conditional priority given to full containers when repositioning empty containers. This model will be of use to shipping lines, port authorities, terminal operators, shippers, national and regional planning authorities as well as marine insurers. The paper has been structured into four sections. The first section reviews the relevant literature including freight flow assignment, empty container repositioning and the primary cost factors in container shipping. Section two introduces the methodology and the key assumptions upon which the proposed model is based. A numerical example is presented in section three to highlight the proposed model properties. Finally, conclusions are drawn in the final section.

2. Previous freight models

Chisholm and O'Sullivan (1973) applied the gravity model for trip distribution to freight in an early attempt to model freight flows. Subsequently, Land Use-Transport Interaction (LUTI) and Input-Output (I/O) models were extended to explain the generation and the distribution of freight flows. The first mode choice models for freight were introduced as behavioural modelling gained popularity for passenger transport. In the 1980s, there was increasing interest in explaining the generation, distribution, mode split and assignment of freight flows simultaneously by applying general equilibrium principles to freight networks (Harker and Friesz, 1986a,b). In the 1990s, freight flow models evolved further by adding commodity differentiation, improved probabilistic choice models, and inventory considerations.

Most freight flow models applied in practice are based on the four-stage approach, with generation, distribution, modal split and assignment sub-models run sequentially and iteratively. Usually all steps are managed at the aggregate (zonal) level. First, production-consumption matrices are estimated by multi-regional or regionalised national input-output models (Marzano and Papola, 2008). Then, value-to-weight transformations and vehicle load factors allow production-consumption matrices to be assigned to transport networks. De Jong and Ben Akiva (2007) describe two freight models, NEMO (Norway) and SAMGODS (Sweden), which are typical of this approach.

More recently, there has been interest in the use of agent-based simulation and the application of game theory. The Container World project (reviewed in Newton, 2008) treated every ship, port, shipping line, service, trucking company and rail operator as an agent. Following the construction of a global network based on the actual routes of the shipping lines at the time of the study, an origin-destination (OD) matrix of containers movements were assigned to routes consisting of a combination of transport modes. Although the Container World project managed to construct a container assignment framework at a global level based on microscopic (or container level) simulation, it proved to be too data intensive in an industry where companies in competition with each other are reluctant to share data. A macroscopic approach therefore looks more feasible than microscopic simulation.

Perrin et al. (2008) presented a macroscopic container assignment model. A global network is constructed where the nodes correspond to origins, destinations, ports and maritime waypoints while maritime legs represent services operated by shipping lines and land legs represent services between inland origins or destinations and ports. An OD matrix of container flows sourced from UNCTAD and Eurostat is assigned to this network. For each OD pair, a *k*-best path algorithm is applied to generate a set of routes. After creating sets of routes, a logit route choice model is implemented. It is planned to replace a simple multinomial logit model by a path-size logit model to allow for the correlations created by overlapping paths. To calibrate the model, port shadow prices are calculated to reproduce given port throughput data as closely as possible. However, interpretation of these shadow prices is unclear.

De Jong et al. (2004) reviewed a number of regional, national, and international freight flow models. De Jong and Ben Akiva (2007) noted that the conventional four-stage model lacks important logistical elements, such as shipment size, use of consolidation and distribution centres, mode and vehicle type, and loading unit. Consequently, a logistics model that takes commodity flows generated by a conventional four-stage approach as input is specified. Empty container repositioning is identified as an integral factor of an overall efficient maritime logistics system. There are four geographical repositioning levels; local, regional, interregional and global. The local and regional levels mainly involve drayage operations; interregional and global levels involve maritime repositioning.

The existing literature mainly focuses on optimising the management of the container fleet. Early work applied linear programming to the problem (White, 1972; Dejax and Crainic, 1987). During the 1990s, the stochastic aspects of the empty container allocation problem attracted attention (Crainic et al., 1993). More recently, Cheang and Lim (2005) present a minimum cost flow model to optimise empty container positioning and leasing decisions between ports. Olivo et al. (2005) considered empty container management in ports and depots with multiple transport modes using an integer programming model. A number of studies have focused on empty container repositioning between ports (Shen and Khoong, 1995; Lai et al., 1995; Cheung and Chen, 1998; Choong et al., 2002) and empty container allocation in a single port or a multi-port system (Li et al., 2004; Song, 2007; Li et al., 2007; Song and Dong, 2008).

Collaboration among carriers in order to achieve more efficient container operations and cost reduction has been observed. Song (2007) presents a theoretical analysis to quantify the cost savings of a collaborative strategy between two shipping lines operating a shuttle service subject to uncertain customer demand. Factors such as the fleet size, the demand pattern and the variance of demand impact the performance of the collaborative strategy significantly. In recent years, some researchers focus on the technological and strategic aspects of empty container repositioning. For example, IT support systems such as SynchroNet and InterBox can serve as a neutral platform to facilitate container sharing among shipping lines, shippers and freight forwarders. The traffic and emission impacts of the reuse of empty containers, off-dock empty return depots, and IT support systems are explored in a number of works (Tioga Group, 2002; Hanh, 2003; Lopez, 2003; Jula et al., 2006).

3. Assumptions and methodology

The classic frequency-based transit assignment approach of Spiess and Florian (1989) has been applied to the assignment of containers, both full and empty, in Bell et al. (2011). In this paper, the model is modified to assign full and empty containers on the basis of expected cost rather than expected travel time. While the strategy of container assignment to the first attractive sailing remains in the modified model, the definition of attractiveness is changed: A sailing is attractive with respect to a given destination if by including it in the choice set, the expected cost of container shipment to that destination is reduced. Link capacity constraints have been added to port capacity constraints.

With respect to the costs in the container liner shipping, the cost of running a container liner shipping service depends on three factors. First, the *ship*, consisting of its fuel consumption, the number of crew to operate it and its physical condition, which in turn dictates the requirement for maintenance and repairs. Second, *bought-in items*, the prices of which are subject to economic trends out of the ship owner's control, particularly bunkers, crew wages, consumables, and maintenance costs. Third, *liner management*, the cost of which depends on operational efficiency and administrative overheads. Although the liner industry has no universally accepted classification, costs can be classified into five categories; operating costs, periodic maintenance, capital costs, voyage costs and cargo-handling costs. In practice, the sum of operating costs, periodic maintenance and capital costs determine the *charter rate* while the sum of fuel consumption and port costs determine the *voyage costs*. There are two types of cargo-handling cost, loading/unloading and transhipment, with a huge difference between them.

The virtual network approach described in Jourquin et al. (2008) is adopted but in this case the real network is added in order to include link capacity constraints. The basic concepts adopted in the network design are therefore route, link, leg and path defined as follows:

- Route – A scheduled sequence of port calls
- Link – An adjacent pair of port calls served by a route
- Leg – A transport task executed by a given route
- Path – A chain of transport tasks or legs

Note that multi-legs are possible whereby more than one route connects a given pair of ports. Parallel legs cannot be aggregated as they may have different attributes, like sailing time or cost. In Figure 1, an example real network with two routes is presented in order to clarify the concepts.

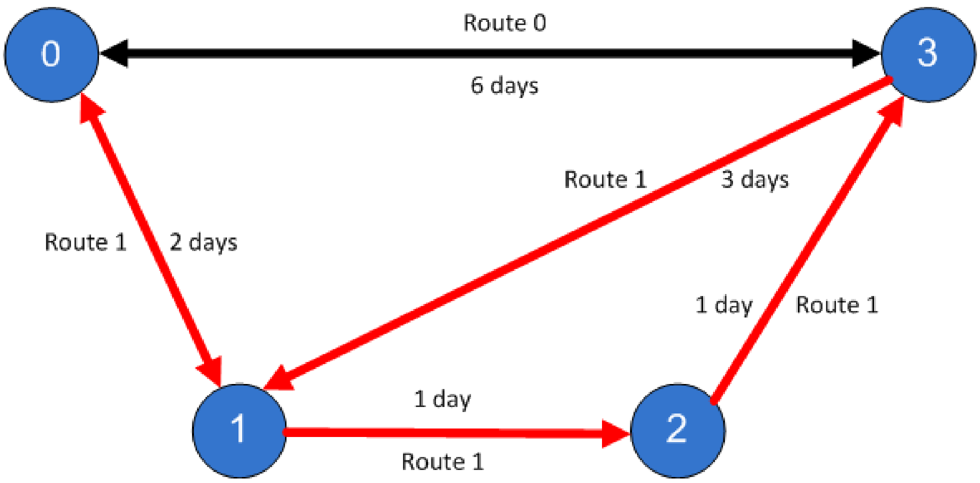

Figure 1: Real network with two routes

The rotations for Route 0 and Route 1 in Figure 1 are:

- Route 0: Port 0 (Mon/Tue), Port 3 (Mon/Tue), Port 0 (Mon/Tue)
- Route 1: Port 0 (Sat/Sun), Port 1 (Tue/Wed), Port 2 (Thu/Fri), Port 3 (Sat/Sun), Port 1 (Wed/Thu), Port 0 (Sat/Sun)

The black and red lines in Figure 1 represent the links on Route 0 and Route 1 respectively. The conversion of the real network to the corresponding virtual network is illustrated in Figure 2. A leg corresponds to a task, so for example Leg 1 corresponds to a container loaded at Port 0 on a given route and unloaded at Port 2. A path consists of a series of legs. Hence in Figure 2 there are two paths from Port 0 to Port 2, one is direct (Leg 1) and the other involves transhipment at Port 1 (Leg 2 followed by Leg 3). Transhipment would only make sense, however, if Legs 2 and 3 were associated with different routes, since transhipping on to the same route would be an unnecessary operation.

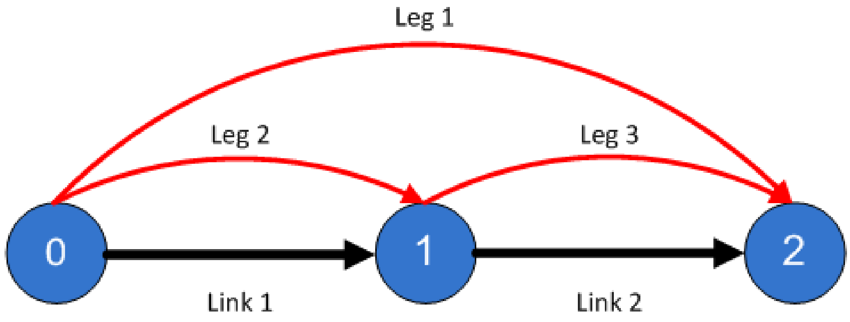

Figure 2: Task oriented legs

There are potentially two cargo handling modes at each port as the cost of loading or unloading depends on whether the container is being transhipped or not. The cost of transhipment at an intermediate port is generally much lower than the cost of loading or unloading at the first or final port in the journey.

In order to represent the route capacity constraint, it is necessary to associate each leg with the relevant links. The flow on the relevant legs can be summed to obtain the flow on each link, which in turn can be compared with the route capacity constraint. For example, Figure 2 shows that the flow on Legs 1 and 2 may be summed to give the flow on Link 1, which can be compared to the relevant route capacity constraint provided Legs 1 and 2 relate to the same route.

In order to simplify the problem and demonstrate the basic theory behind the proposed cost-based maritime container assignment model, the following key assumptions are made:

- Only one type of container is considered. Containers are assumed to be interchangeable and are assigned to minimise an objective function measured in cost units.
- Containers are carried by shipping lines operating routes. Each route has a given service frequency, but the arrival and departure time at each port of ships on any route is random and uncoordinated with the arrival and departure time of ships on any other route. When the demand for a route exceeds the available capacity, the shipping line, being a profit maximising agent, levies a surcharge sufficient to bring demand into line with capacity.
- Similarly container terminals are profit maximising agents, so if demand threatens to exceed the capacity of the terminal, a surcharge sufficient to reduce demand to the available capacity is levied on each loading or unloading movement.
- Full and empty containers are considered separately. Constraints ensure that a net outflow of full containers from any port is balanced by a net inflow of empty containers, and vice versa.
- Costs considered include container handling costs, container rental and depreciation of container contents. It is shown that ship charter cost, fuel cost and port cost do not influence the assignment.

The objective function consists of the sum of the full and empty container sailing costs at sea and expected container dwell costs in ports. Full and empty containers are managed separately subject to a condition that a net outflow of full containers from any port is balanced by a net inflow of empty containers and vice versa. Under the strategy of assigning a container to the next available service on an attractive leg, a set of paths known as a *hyperpath* in the transit assignment literature (Nguyen and Pallotino, 1988) is generated. In the case of binding port or route capacity constraints, we infer a surcharge sufficient to align demand with port or route capacity. The randomisation assumption together with the strategy of assigning a container to the first available attractive service leads to an assignment of containers among alternative attractive routes departing a port that is proportional to their service frequencies. Consequently, the inverse of the sum of these service frequencies is equal to the expected dwell time of a container at that port.

For a given destination, a leg is attractive if the objective function is reduced by its inclusion in the choice set at a given node. In the proposed model, the objective function and therefore attractiveness is based on cost. When a port or route capacity constraint is binding, a surcharge sufficient to eliminate queuing is levied. Transhipped containers are charged twice since the container is handled both on arrival and on departure.

At the heart of the cost-based container assignment model is a linear program, so efficient solvers can be applied and the dual variables allow inference of the surcharges arising from bottlenecks. The expected dwell cost for full containers is based on the expected container dwell time, the container rent per unit time and the rate of depreciation of the cargo within the container; for empty containers only the rental cost of container is considered, giving full containers greater urgency in the assignment model subject to the container conservation constraints.

4. Notation and model

The notation for the container assignment model is as follows:

c_a	Sailing time on leg a, including loading and unloading time at the ends
Y_n	Cost per container per unit time on route n, including the cost of loading and unloading
t_{rs}^f	Flow of full containers from origin r to destination s
x_{as}^f	Flow of full containers on leg a en route to destination s
x_a^e	Flow of empty containers on leg a
f_a	Frequency of sailing on leg a
k_i	Maximum throughput of port i
w_{is}^f	Expected dwell time at port i for a full container en route to destination s
w_i^e	Expected dwell time at port i for an empty container
q_{as}^f	Dual variable for a full container on leg a en route to destination s
q_a^e	Dual variable for an empty container on leg a
u_{is}^f	Expected sailing, dwell and surcharge cost for a full container from port i en route to destination s
u_i^e	Expected repositioning cost for an empty container at port i
v_i	Surcharge for loading or unloading a container at port i
v_{nl}	Surcharge for a container on link l of route n
A	Set of legs
T	Set of leg types
N	Set of routes

A_n	Set of legs on route n
A_n^t	Set of legs of type t on route n
O	Set of origin ports
D	Set of destination ports
I	Set of ports
L	Set of links
L_n	Set of links on route n
A_i^+	Set of legs entering port i
A_i^-	Set of legs leaving port i
RC_n	Capacity of route n
δ_{aln}	1 if leg a uses link l on route n, and 0 otherwise
CC_n	Charter cost per unit time on route n
FC_n	Fuel cost per unit time on route n
PC_n	Port cost per unit time on route n
CHC^t	Cargo handling cost per container for a leg of type t
DV	Depreciation rate of the cargo in a container
CR	Rental cost per unit time per container

There are four types of leg; legs which connect the origin port to the destination port, legs which connect the origin port to a transhipment port, legs which connect a transhipment port to the destination port, and finally legs which connect two transhipment ports. The container handling cost is different for each leg type; basically transhipment costs less than loading or unloading at the origin or destination port because the paperwork is less and the gate system is not involved.

In the following, the following conventions are used to simplify the notation:

$x_{a+}^f = \sum_{s \in D} x_{as}^f$, $w_{++}^f = \sum_{i \in I} \sum_{s \in D} w_{is}^f$, etc.

The container assignment model can now be expressed as the following linear program:

P[0]:

$$\min_{x,w} \left(\sum_{n \in N} \sum_{a \in A_n} (x_{a+}^f + x_a^e) Y_n c_a + \left(\sum_{a \in A} x_{a+}^f c_a + w_{++}^f\right)(CR + DV) + \left(\sum_{a \in A} x_a^e c_a + w_+^e\right) CR \right)$$

Subject to

(1) $\sum_{a\in A_i^+} x_{as}^f - \sum_{a\in A_i^-} x_{as}^f = b_{is}^f$ for all $i \in I, s \in D$

(2) $\sum_{a\in A_i^+} x_a^e - \sum_{a\in A_i^-} x_a^e = -b_i^e$ for all $i \in I$

(3) $x_{as}^f \leq w_{is}^f f_a$ for all $a \in A_i^-, i \neq s \in I, s \in D$

(4) $x_a^e \leq w_i^e f_a$ for all $a \in A_i^-, i \in I$

(5) $k_i \geq \sum_{a\in A_i^-}(x_{a+}^f + x_a^e) + \sum_{a\in A_i^+}(x_{a+}^f + x_a^e)$ for all $i \in I$

(6) $RC_n \geq \sum_{a\in A}(x_{a+}^f + x_a^e)\delta_{aln}$ for $l \in L_n, n \in N$

(7) $x_{as}^f \geq 0$ for all $a \in A, s \in D$

(8) $x_a^e \geq 0$ for all $a \in A$

(9) $$b_{is}^f = \begin{cases} -t_{rs}^f \text{ if } i = r \in O \\ t_{+s}^f \text{ if } i = s \in D \\ 0 \text{ otherwise} \end{cases}$$

(10) $$b_i^e = \begin{cases} t_{+i}^f - t_{i+}^f \text{ if } i = r \in O \text{ or } i = s \in D \\ 0 \text{ otherwise} \end{cases}$$

The unit cost per unit time on route n is:

(11) $$Y_n = \frac{CC_n + FC_n + PC_n}{\sum_{a\in A_n}\left(x_{a+}^f + x_a^e\right)c_a} + \frac{\sum_{t\in T}\sum_{a\in A_n^t} CHC^t\left(x_{a+}^f + x_a^e\right)}{\sum_{a\in A_n}\left(x_{a+}^f + x_a^e\right)c_a} \text{ for } n \in N$$

Note that substituting (11) into the P_0 objective function yields:

(12) $$\min_{x,w}\left(\sum_{n\in N}\sum_{t\in T}\sum_{a\in A_n^t} CHC^t\left(x_{a+}^f + x_a^e\right) + \left(\sum_{a\in A} x_{a+}^f c_a + w_{++}^f\right)(CR + DV) + \left(\sum_{a\in A} x_a^e c_a + w_+^e\right)CR\right)$$

The objective function is therefore the sum of the loading and unloading costs for full and empty containers, the container rental and depreciation for full containers and the container rental for empty containers. Note that the objective function remains linear, so the problem remains a linear program. The route-related costs, CC_n, FC_n and PC_n, do not appear in the objective function as they are unrelated to the decision variables, namely the container flows and container dwell times. The assumption is that the routes are operated at given frequencies with ships of given capacity irrespective of container flows or container dwell times.

Constraints (1) and (2) enforce container conservation. For transhipment ports, flow in must be equal to flow out, for full containers distinguished by destination and for empty containers without differentiation. Constraints (3) and (4) ensure that for the legs that are attractive at port i, the delay per full or empty container is at least as large as the inverse of the combined service frequency, differentiating by destination s in the case of full containers. Given the random arrivals and departures assumption, the expected delay is equal to the inverse of the combined service frequency. Constraints (5) ensure that the port capacities are not exceeded. In practice, this capacity will be determined by the length of quay, the number of ship-to-shore cranes, the extent and nature of the horizontal transport, and the container stacking arrangements. Constraints (6) and (7) ensure that the combined full and empty container flow of each link does not exceed the corresponding route capacity. Constraints (8) and (9) are non-negativity constraints. Origin and destination constraints are given in (10) and (11). Remarkably the objective function remains linear despite the economies of scale inherent in Y_n.

The Lagrangian equation for P_0 is:

(12) $L_{x,w,q,u,v} =$
$\sum_{n\in N}\sum_{t\in T}\sum_{a\in A_n^t} CHC^t\left(x_{a+}^f + x_a^e\right) + \left(\sum_{a\in A} x_{a+}^f c_a + w_{++}^f\right)(CR+DV) +$
$\left(\sum_{a\in A} x_a^e c_a + w_+^e\right)CR -$
$\sum_{i\in I}\sum_{s\neq i\in D} u_{is}^f\left(b_{is}^f + \sum_{a\in A_i^-} x_{as}^f - \sum_{a\in A_i^+} x_{as}^f\right) -$
$\sum_{i\in I} u_i^e\left(-b_i^e + \sum_{a\in A_i^-} x_a^e - \sum_{a\in A_i^+} x_a^e\right) -$
$\sum_{i\in I}\sum_{a\in A_i^-}\sum_{s\neq i\in D} q_{as}^f\left(w_{is}^f f_a - x_{as}^f\right) -$
$\sum_{i\in I}\sum_{a\in A_i^-} q_a^e\left(w_i^e f_a - x_a^e\right) -$
$\sum_{i\in I} v_i\left(k_i - \sum_{a\in A_i^-}\left(x_{a+}^f + x_a^e\right) - \sum_{a\in A_i^+}\left(x_{a+}^f + x_a^e\right)\right) -$
$\sum_{n\in N}\sum_{l\in L_n} v_{nl}\left(RC_n - \sum_{a\in A}\left(x_{a+}^f + x_a^e\right)\delta_{aln}\right)$

At the optimum, the Lagrangian equation is minimised with respect to the decision variables *x* and *w* but maximised with respect to the dual variables *q*, *u* and *v*. Without loss of generality, we can set:

(13) $u_{ss}^f = 0$ for all $s \in D$

The Lagrangian equation can be rearranged using a more compact notation to yield:

(14) $L_{x,w,q,u,v} =$
$\sum_{rs} t_{rs}^f u_{rs}^f + \sum_s\left(t_{+s}^f - t_{s+}^f\right)u_s^e - \sum_i v_i k_i - \sum_{nl} v_{nl} RC_n + \sum_{nas} x_{as}^f\Big(CHC^{t(a)} +$
$c_a(CR+DV) + \frac{q_{as}^f}{f_a} - u_{is}^f + u_{js}^f + v_i + v_j + v_{nl}\delta_{aln}\Big) + \sum_{nal} x_a^e\Big(CHC^{t(a)} + c_a CR +$
$\frac{q_a^e}{f_a} - u_i^e + u_j^e + v_i + v_j + v_{nl}\delta_{aln}\Big) + \sum_{as} w_{is}^f\left((CC+DV) - q_{as}^f\right) + \sum_a w_i^e\left(CC - q_a^e\right)$

where implicitly nodes *i*, *j* represent the entrance and the exit port to leg *a* respectively. The possibility of more than one leg connecting nodes *i*, *j* is included in this notation. Thus, the dual problem can be formulated as follow:

P_1: $\max_{q,u,v}\left(\sum_{rs} t_{rs}^f u_{rs}^f + \sum_s\left(t_{+s}^f - t_{s+}^f\right)u_s^e - \sum_i v_i k_i - \sum_{nl} v_{nl} RC_n\right)$

Subject to:

(15) $CHC^{t(a)} + c_a(CR+DV) + \frac{q_{as}^f}{f_a} - u_{is}^f + u_{js}^f + v_i + v_j + \sum_{ln}\delta_{aln} v_{nl} \geq 0$ for $a \in A_n, s \in D$

(16) $CHC^{t(a)} + c_a CR + \frac{q_a^e}{f_a} - u_i^e + u_j^e + v_i + v_j + \sum_{ln}\delta_{aln} v_{nl} \geq 0$ for $a \in A_n$

(17) $(CC+DV) - \sum_{j\in A_i^-} q_{as}^f \geq 0$ for all $i \neq s \in I, s \in D$

(18) $CC - \sum_{j\in A_i^-} q_a^e \geq 0$ for all $i \in I$

(19) $q_{as}^f \geq 0$ for all $a \in A, s \in D$

(20) $q_a^e \geq 0$ for all $a \in A$

(21) $v_i \geq 0$ for all $i \in I$

(22) $v_{nl} \geq 0$ for all $l \in L_n, n \in N$

The following proposition shows that after appropriate initialisation u_{rs}^f is the cost of shipping a container from *r* to *s* including the expected dwell cost and any port and link surcharges and u_s^e is the average cost of repositioning an empty container from *s* including the expected dwell cost and any port and link surcharges.

Proposition 1: $\sum_{rs} t_{rs}^f u_{rs}^f$ is the total cost of shipping full containers and $\sum_s (t_{+s}^f - t_{s+}^f) u_s^e$ is the total cost of repositioning empty containers.

Proof: If $x_{as}^f > 0$ then from complementary slackness and (16)

(23) $$u_{is}^f - u_{js}^f = CHC^{t(a)} + c_a(CR + DV) + \frac{q_{as}^f}{f_a} + v_i + v_j + \sum_{ln} \delta_{aln} v_{nl}$$

where as described above ports *i* and *j* represent the entrance and the exit to leg *a* respectively. To obtain the total cost of shipping full containers, we multiply (24) by x_{as}^f and sum the result with respect to *a* and *s*. Noting that if $q_{as}^f > 0$ then $x_{as}^f = w_{is}^f f_a$ and if $w_{is}^f > 0$ then $\sum_{j \in A_i^-} q_{as}^f = (RC + DV)$, and using (18) and (9), we obtain

(24) $$\sum_{as} x_{as}^f (u_{is}^f - u_{js}^f) = \sum_{rs} t_{rs}^f u_{rs}^f - \sum_s t_{+s}^f u_{ss}^f = w_{++}^f (RC + DV) + \sum_{an} x_{a+}^f (CHC^{t(a)} + c_a(CR + DV) + v_i + v_j + \sum_{ln} \delta_{aln} v_{nl})$$

Without loss of generality, set $u_{ss}^f = 0$ for all $s \in D$. Hence

(25) $$\sum_{rs} t_{rs}^f u_{rs}^f = w_{++}^f (RC + DV) + \sum_{an} x_{a+}^f (CHC^{t(a)} + c_a(CR + DV) + v_i + v_j + \sum_{ln} \delta_{aln} v_{nl})$$

Similarly, if $x_a^e > 0$ then from complementary slackness and (17)

(26) $$u_i^e - u_j^e = CHC^{t(a)} + c_a CR + \frac{q_a^e}{f_a} + v_i + v_j + \sum_{ln} \delta_{aln} v_{nl}$$

To obtain the total cost of shipping empty containers, we multiply (27) by x_a^e and sum the result with respect to *i* and *j*. Noting that if $q_a^e > 0$ then $x_a^e = w_i^e f_a$ and if $w_i^e > 0$ then $\sum_{j \in A_i^-} q_a^e = RC$, and using (19) and (10), we obtain

(27) $$\sum_a x_a^e (u_i^e - u_j^e) = \sum_s (t_{+s}^f - t_{s+}^f) u_s^e = w_+^e RC + \sum_{an} x_a^e (CHC^{t(a)} + c_a CR + v_i + v_j + \sum_{ln} \delta_{aln} v_{nl})$$

It is clear from (26) that $\sum_{rs} t_{rs}^f u_{rs}^f$ is the total cost of shipping full containers and from (28) that $\sum_s (t_{+s}^f - t_{s+}^f)) u_s^e$ is the total cost of repositioning empty containers.

QED

5. Numerical example

In order to illustrate the properties of the proposed model, the following small numerical example is presented. The network is shown in Figure 3.

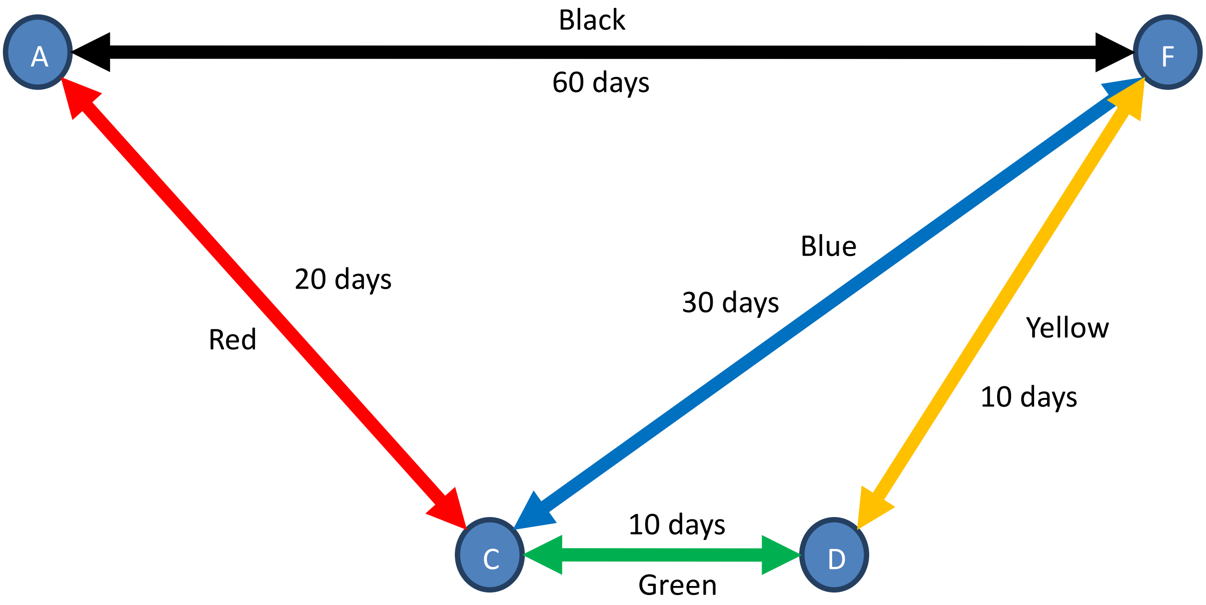

Figure 3: Example network

There are five colour-coded routes, which are operated at given frequencies by ships of a given size. Note that in this simple network legs and links are synonymous. For full containers, there is one origin port (port A) and one destination port (port F). Empty containers flow in the reverse direction. There are thus three paths in either direction involving respectively no, one or two transhipments. Since in this model the frequencies and ship sizes are given and not affected by the decision variables, ship charter rates, fuel (bunkers) consumption and port dues play no role in container assignment. We assume that the cost of loading a container at port A or F or unloading it at port F or A costs AUD 200. The cost of loading or unloading a container at a transhipment port, however, is less at AUD 150 because there is less paperwork and the gate system is not involved. We assume that container rental is AUD 4.50 per day and the loss of value of the contents of one container is AUD 20 per day. Finally, we assume that 1000 full containers a day leave port A. Table 1 shows the assignment for the above parameters and the sailing frequencies shown.

Table 1: Container assignment in the absence of constraints

Route	Direction	Container	Sailing time (days)	Frequency (sailings per day)	Flow (boxes per day)
Black	A to F	Full	60.00	0.14	122.81
Red	A to C	Full	20.00	1.00	877.19
Blue	C to F	Full	30.00	1.00	877.19
Green	C to D	Full	10.00	1.00	0.00
Yellow	D to F	Full	10.00	1.00	0.00
Black	F to A	Empty	60.00	0.14	1000.00
Red	C to A	Empty	20.00	1.00	0.00
Blue	F to C	Empty	30.00	1.00	0.00
Green	D to C	Empty	10.00	1.00	0.00
Yellow	F to D	Empty	10.00	1.00	0.00

For full containers, two routes are used and they are used in proportion to their respective frequencies. The transhipment cost, consisting of the container handling cost plus the cost of the container dwell time, is sufficiently compensated by the value of the reduced sailing time to make the path involving one transhipment attractive, but not the path involving two transhipments. However, the direct route still remains attractive as well, so the full containers are split between these two routes. The empty containers return by the direct route only, as the value of the reduced sailing time is insufficient for empty containers to make a transhipment attractive. There are no active route or port capacity constraints, so there are no route or port surcharges.

Suppose now that the direct route from port F to port A only has sufficient capacity for 500 containers per day. The assignment then changes to that given in Table 2.

Table 2: Container assignment with a capacity constraint on the black route

Route	Direction	Container	Sailing time (days)	Frequency (sailings per day)	Flow (boxes per day)
Black	A to F	Full	60.00	0.14	122.81
Red	A to C	Full	20.00	1.00	877.19
Blue	C to F	Full	30.00	1.00	877.19
Green	C to D	Full	10.00	1.00	0.00
Yellow	D to F	Full	10.00	1.00	0.00
Black	F to A	Empty	60.00	0.14	500.00
Red	C to A	Empty	20.00	1.00	500.00
Blue	F to C	Empty	30.00	1.00	500.00
Green	D to C	Empty	10.00	1.00	0.00
Yellow	F to D	Empty	10.00	1.00	0.00

Each empty container that shifts from the direct route to the path with one transhipment causes the objective function to increase by AUD 227.36. This is therefore the shadow price, or equivalent surcharge, per empty container on the black route.

If instead port C has a capacity constraint of 1000 movements (loading plus unloading) per day, the assignment as shown in Table 3 is obtained.

Table 3: Container assignment with a capacity constraint at port C

Route	Direction	Container	Sailing time (days)	Frequency (sailings per day)	Flow (boxes per day)
Black	A to F	Full	60.00	0.14	500.00
Red	A to C	Full	20.00	1.00	500.00
Blue	C to F	Full	30.00	1.00	500.00
Green	C to D	Full	10.00	1.00	0.00
Yellow	D to F	Full	10.00	1.00	0.00
Black	F to A	Empty	60.00	0.14	1000.00
Red	C to A	Empty	20.00	1.00	0.00
Blue	F to C	Empty	30.00	1.00	0.00
Green	D to C	Empty	10.00	1.00	0.00
Yellow	F to D	Empty	10.00	1.00	0.00

A flow of 377.19 full containers per day has returned to the black route increasing the objective function by AUD 95.50 per full container that shifts path. This is equivalent to a surcharge of AUD 47.75 per movement at port C.

The previous cases have assumed that the black service operates weekly while all other services are daily. When the red service is also weekly, the flow of full containers captured by the path via port C is reduced to 50%, as shown by Table 4.

Table 4: Container assignment when sailings from port A are weekly

Route	Direction	Container	Sailing time (days)	Frequency (sailings per day)	Flow (boxes per day)
Black	A to F	Full	60.00	0.14	500.00
Red	A to C	Full	20.00	0.14	500.00
Blue	C to F	Full	30.00	1.00	500.00
Green	C to D	Full	10.00	1.00	0.00
Yellow	D to F	Full	10.00	1.00	0.00
Black	F to A	Empty	60.00	1.00	1000.00
Red	C to A	Empty	20.00	0.14	0.00
Blue	F to C	Empty	30.00	1.00	0.00
Green	D to C	Empty	10.00	1.00	0.00
Yellow	F to D	Empty	10.00	1.00	0.00

It is worth recalling the basis of this model. It is assumed that the ships of given size are running with given frequencies anyway, so route costs play no role in the assignment. The only costs that play a role are the container handling costs, *CHC*, the container rental, *CR*, and the freight depreciation, *DV*. This is the assignment problem as viewed from the perspective of a shipping line, which is sensitive to the costs incurred by the client who rents the container, possibly from the shipping line, and owns its contents.

There are of course other perspectives, for example the shipper, who would wish to take the rate charged by the shipping line for carrying the container into account. The costs of the shipping line include the charter cost of the ships, the fuel (bunkers) consumed and the port costs. When these costs are spread across the containers carried, the economies of scale offered by larger ships become important. In a highly competitive market, as at present, the shipping lines with the lowest cost per container per unit time will survive, and these will in general be the shipping lines with the largest ships. Other perspectives would lead to other assignment models, but the perspective of the shipping line is the more important for operational planning.

6. Conclusions

This paper presents a cost-based version of the frequency-based container assignment model described in Bell et al. (2011). Unlike the earlier time-based model, the cost-based model can represent the impact of economies of scale offered by higher ship occupancy and larger ships. In addition, the model presented here can take port and route capacity constraints into account. The model therefore can represent the effects of sailing time, service frequency, port capacity, link capacity and a range of cost factors. To implement this model the following are required:

- A global maritime network with sailing times, frequencies and ship sizes. Sailing times and frequencies could be constructed from the published timetables for container liners. Container ship sizes mainly affect the link capacity constraint and can be sourced from the Containerization International Yearbook.
- OD matrices of container flows, differentiating by shipping line (or possibly shipping alliance) and container type as the containers within each OD matrix need to be interchangeable. The assignment of the different OD matrices is linked only by the port capacity constraints, so each shipping line (if each matrix corresponds to a shipping line) is minimizing its own objective function, linked only by common port surcharges.

It is shown that, even after the addition of endogenous route-specific unit costs, P_0 remains a liner program. This enables the model to be applied to large networks efficiently. Furthermore, the dual variables provide additional information about the solution, like the presence of any port or link surcharges.

The maritime container assignment model, once fully developed, will provide shipping lines in particular, but also port authorities, shippers and third party service providers with valuable information with which to inform decision-making. It will provide a tool to enable shipping lines plan new routes or predict the consequences of changing service frequencies and/or ship sizes. It will allow port authorities to predict the consequences of alternative master plans by relating investment decisions to revenue projections. It could also provide shippers with the basis for container journey planning.

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