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The economics and engineering of bus stops: Spacing, design and congestion.

By

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TITLE: **The economics and engineering of bus stops: Spacing, design and congestion.**

ABSTRACT: This paper re-considers the problem of choosing the number of bus stops along urban routes, first by estimating the probability of stopping in low demand markets, and second by analysing the interplay between bus stop size, bus running speed, spacing and congestion in high demand markets. A comprehensive review of the theory and practice on the location and spacing of bus stops is presented. Using empirical data from Sydney we show that the widely used Poisson model overestimates the probability of stopping in an on-call bus stopping regime, and consequently underestimates the optimal number of bus stops that should be designed. For fixed-stop services, we show that bus running speed, frequency and dwell time are crucial to determining the relationship between bus stop spacing and demand, with bus stop congestion in the form of queuing delays playing a key role. In particular, we find that bus stop spacing should be decreased if demand increases at a constant bus running speed; however, if both bus running speed and the speed of the passenger boarding process increase, then the distance between bus stops should be kept long even at high demand levels, a result that is consistent with the implementation of Bus Rapid Transit systems that feature high bus running speeds and long distances between stops relative to conventional bus services.

KEY WORDS: ***Bus stop, Poisson, congestion, queue, bus delay, bus rapid transit.***

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1. Introduction

Bus stops and train stations provide accessibility to public transport services at the expense of slowing down vehicles and increasing riding time. This simple fact makes the decision of what number of stops to provide on a network far from trivial. The purpose of this paper is two-fold: first, to review the theoretical approaches and common practices in bus stop location, spacing and design; and second, to provide an integrated approach for the analysis of bus stop placement in order to understand the relationships between bus stop spacing and demand, bus size, bus stop size, queuing delays, bus running speed and the probability of stopping. As argued by Wirasinghe and Groheim (1981), no optimisation is necessary to establish that bus stops should be located at hospitals, schools, universities, shopping centres and other points of high boarding and alighting demand, but because it is unclear where bus stops should be located in between major activity centres, an optimisation approach could be useful to gain an indication of the best average distance between stops.

Three typical stopping regimes are usually found in urban bus operations (Kikuchi and Vuchic, 1982): (i) demand stopping: buses stop at any location at which passengers wish to get on and off; (ii) on-call stopping: fixed stops are provided but buses stop only when required; (iii) fixed stopping: vehicles stop at all stops or stations. The implementation of one regime or the other is usually dictated by demand levels (Vuchic, 2005): when demand is very low it seems natural to provide demand-stopping, but as demand grows it eventually becomes more convenient to group passengers in a limited number of locations, providing on-call stops in close proximity to each other. Finally, when demand is high, it is more reasonable to locate stops further apart and stop at all of them.

This paper analyses on-call and fixed stopping patterns. From a modeller's perspective, the main difference between these two regimes is that in the former it is necessary to estimate the probability that a bus will stop, a problem that does not exist in the latter case. The only theoretical approach published for modelling the stopping probability in on-call regimes is the Poisson model proposed by Hauer (1971) and Mohring (1972) and subsequently applied by several authors (Wirasinghe and Ghoneim, 1981; Kikuchi and Vuchic, 1982; Kikuchi, 1985; Furth and Rahbee, 2000; Furth *et al.*, 2007; Li and Bertini, 2009; Chien *et al.*, 2010). Using empirical data collected in the outer suburbs of Sydney, we show that the Poisson model overestimates the number of stops actually made, and consequently underestimates the optimal number of bus stops that should be established.

On systems with a fixed stopping pattern, characteristic of high demand markets, we pay special attention to the relationship between bus stop spacing and demand. The existent literature is not conclusive in this regard, as some studies find that bus stop spacing should decrease with demand while others find that it should increase. The theoretical and numerical analyses presented in this paper demonstrate the conditions that lead to one result or the other. We highlight the importance of the bus operating speed and bus stop congestion in a total cost minimisation model that for the first time includes the choice of bus stop size as a decision variable.

The reminder of the paper is organised as follows: Section 2 presents an extensive review of the literature, including academic papers on the optimal spacing of bus stops and train stations (Section 2.1), guidelines and common practices (Section 2.2) and recommendations regarding the location of bus stops relative to intersections (Section 2.3). In Section 3 we provide a description of the Poisson model to estimate the probability of stopping at bus stops and empirically derive two alternative models using data from Sydney. Section 4 introduces a simple total cost minimisation model to analyse the relationship between bus stop spacing and demand. In Section 5 we estimate queuing delays at bus stops for different sizes of buses and bus stops. Optimal bus stop spacing and size are determined and discussed with an extended total cost model in Section 6 for the cases of fixed stopping (6.1) and on-call stopping (6.2). Section 7 summarises the findings of the paper.

2. The spacing and location of bus stops: theory and practice

2.1 Theoretical approaches and main results

The first studies that identify the trade-off between access and riding time that characterises the problem of locating boarding and alighting stations were published almost one hundred years ago, which makes this problem one of the oldest in the field of transport economics and engineering. Vuchic and Newell (1968) report that between 1913 and 1930 at least five studies on the subject were published by German authors, who were concerned with finding the optimal spacing of stations for urban and suburban railways, usually with the objective of minimising passengers' travel time, including both access and in-vehicle times. These studies assumed a uniform population distribution along the route and kept the interstation spacing constant. The next wave of works came in the 1960s when Vuchic and Newell (1968) and Vuchic (1969) analysed the problem of a population commuting to the Central Business District (CBD), and found that the station spacing is a function of the ratio between the number of passengers aboard a train and those waiting to board and alight; correspondingly, station spacing increases in the direction of passenger accumulation (towards the CBD during the morning peak).

After these early contributions, a large number of authors have worked on the analysis of stop location and spacing, either as a single decision variable or in combination with other factors such as network design, bus frequency, route density and bus size. The most common approach is the development of optimisation models for which several objective functions have been proposed and analysed, namely:

- Total cost (users plus operator) minimisation, e.g.: Mohring (1972), Wirasinghe and Ghoneim (1981), Kikuchi and Vuchic (1982), Kuah and Perl (1988), Chien and Qin (2004), dell'Olio *et al.* (2006), Ibeas *et al.* (2010), Tirachini and Hensher (2011).
- User cost minimisation subject to a supply-side constraint (frequency, fleet size or budget): Vuchic and Newell (1968), Kikuchi (1985), Furth and Rahbee (2000), van Nes and Bovy (2001), Li and Bertini (2009), Chien *et al.* (2010).
- Social welfare maximisation: van Nes and Bovy (2001), Basso and Silva (2010).
- Private profit maximisation: van Nes and Bovy (2001).

Mohring (1972) proposed the first microeconomic model to jointly optimise bus frequency and stop spacing, which was later extended by Kuah and Perl (1988) and Chien and Schonfeld (1998) who added route density as a decision variable for the analysis of a rail line with a feeder bus network. In general, the number of buses required for a service depends on the product of the frequency and the number of bus stops, and this multiplicative term prevents the problem from having a closed form solution when both elements are variables (see discussion in Section 4). By ignoring this term, Kuah and Perl (1988) find a closed form for the optimal bus stop spacing (the total route length divided by the number of stops), namely a square root formula that increases with the average trip length, walking speed, the delay due to stopping and the value of in-vehicle time savings, and decreases with the value of access time.

Regarding access, until the 1990s most models assumed a continuous or uniform distribution of demand along a route or over an urban area. As argued by Furth and Rahbee (2000), the analytical tractability of a continuous demand model is its main benefit, as it allows the analyst to investigate the sensitivity of the optimal stop spacing to the various parameters and variables that influence it. However, continuous demand models do not explicitly treat the application of results to an actual network with all its physical constraints. Thus, a discrete demand model assuming a finite set of possible locations for bus stops (given by road intersections and the geometric characteristics of a network) is more useful as a tool to determine the actual (optimal) location of stops (Furth and Rahbee, 2000; Chien and Qin, 2004; Furth *et al.*, 2007). With the increased availability of Geographic Information System tools, researchers have been able to identify more precise walking distances to bus stops[1], which can be embedded in discrete models to optimise bus stop location (Furth *et al.*, 2007; El-Geneidy *et al.*, 2010).

Access speed and the relationship between the values of access, waiting and in-vehicle time savings have been identified as key determinants of the optimal bus stop spacing, and several authors have performed sensitivity analyses on these input parameters (e.g., Kikuchi and Vuchic, 1982; Chien and Schonfeld, 1998; Chien and Qin, 2004). Optimal stop spacing decreases with the value of access time savings and increases with access speed, therefore stop spacing should increase if faster modes than walking are used to access the stop (e.g., bicycles, feeder buses, jitney services, park and ride). Differences in access speed introduced by motorised access may have a significant impact on optimal spacing (Vuchic and Newell, 1968; Vuchic, 1969), although the solution is quite robust if only walking speed differences are considered (Chien and Schonfeld, 1998; van Nes and Bovy, 2001). If the trade-off between access and riding time for users is considered along with the fact that operating cost also increases with the number of bus stops (for example, due to accelerating and decelerating delays and extra fuel consumption), it follows that optimal bus stop spacing is greater when operator costs are considered at par with user costs in the objective function, as opposed to an approach that only minimises user costs. Along these lines, van Nes and Bovy (2001) find that for a fixed frequency, stop spacing is greater when maximising bus operator profit than when maximising social welfare.

Other elements progressively introduced to the analysis of bus stop spacing include mode choice (Basso and Silva, 2010; Ibeas *et al.*, 2010; Alonso *et al.*, 2011), bus stop construction costs (dell'Olio *et al.*, 2006; Tirachini and Hensher, 2011), and a constraint on the maximum distance between stops given by, for example, twice the maximum distance that users are willing to walk to reach a bus stop (Saka, 2001)[2]. A bi-level cost minimisation approach to locate bus stops in an urban network has been proposed by dell'Ollio *et al.* (2006) and Ibeas *et al.* (2010), with an upper level consisting of the total cost (users plus operators) and a lower level where the behaviour of users and network equilibrium (bus route flows) is computed. The selection of nodes as bus stops has recently been included as part of the public transport network design problem (Estrada *et al.*, 2011; Bagloee and Ceder, in press).

To conclude this review of the academic literature, we discuss the relationship between stop spacing and total demand, one of the main focuses of this study. As presented in Section 1, three stopping patterns are used in urban public transport operation: demand stopping, on-call stopping and fixed stopping. When the fleet size is kept constant, the distance between stops should increase with demand, moving from demand to on-call and fixed stopping regimes (Vuchic, 2005), a result also found by Chien *et al.* (2010) in a model that optimises frequency and stop spacing to minimise user travel costs. Nevertheless, when frequency is optimally increased, most studies find that bus stop spacing decreases with demand (Kikuchi, 1985; Alonso *et al.*, 2011; Jara-Diaz and Tirachini, 2011). This result is challenged when the bus operating speed can be increased as demand grows (for example, through an investment in road infrastructure for buses), in which case the increased delay due to accelerating and decelerating, which is directly proportional to the cruising speed, increases the optimal distance between stops (Tirachini and Hensher, 2011). This outcome is aligned with the current practice of high

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[1] Instead of, for example, assuming an average access distance of one quarter the distance between two consecutive stops, as done in models with a uniform distribution of demand.

[2] The maximum interstop distance considered in Saka (2001) is 1600 m, assuming a maximum walking distance to bus stops of 800 m. This maximum distance does not need to be constant along a route or network to accommodate local topographical features such as the existence of parks (Furth and Rahbee 2000).

standard Bus Rapid Transit (BRT) systems in which buses circulate at high speeds with greater spacing between stops than conventional mixed-traffic bus services. All in all, the extant literature does not provide a clear answer on whether bus stop spacing should be increased or reduced with demand. Therefore, in Sections 4 and 6 we establish a microeconomic framework to determine the conditions that lead to these divergent results.

2.2 Guidelines and current practice

A number of transport engineering textbooks, guidelines and manuals address the issue of bus stop design and spacing (EBTU, 1982; TRB, 1996, 2003; Vuchic, 2005; Transport for London, 2006; Wright and Hook, 2007). These works are mainly concerned with the design of shelters and benches, platform sizing, height, lighting, the station-vehicle interface, interactions with passing traffic, passenger safety and security, wheelchair accessibility and placement (curb-side or on a bus bay). The spacing recommendations in these guidelines is location specific; the usual advice for city centre areas is that bus stops should be placed no further than 300 metres apart, whilst for residential areas outside the CBD distances between 300 and 500 metres are recommended in the United Kingdom (IHT, 1997) and Brazil (EBTU, 1982, as cited by Valencia, 2007), with shorter distances advised for the United States (e.g., between 150 and 365 metres in TRB, 1996).

In practice, an average spacing between 300 and 450 metres is common in European cities like Paris, London, Rotterdam and Zurich (van Nes and Bovy, 2001). In Sydney, the average spacing is around 300 metres in the CBD and 350-400 metres in the suburbs. The average spacing in the United States is shorter, commonly between 200 and 270 metres, with even shorter spacing allowed in CBDs (Furth and Rahbee, 2000)[3]. Extremely short spacing such as placing stops at every corner (a practice inherited from the times of horse-drawn forms of transport as noted by Vuchic, 2005) does not seem to be supported by any formal analysis.

Systems with circulation on dedicated busways are usually characterised by greater distances between stations to provide higher operating speeds than conventional bus routes. A survey of 37 BRT systems around the world reveals an average stop spacing of 758 metres (roughly double the usual stop spacing in conventional urban bus services) over a wide range of values from 300 to 1,800 metres, as shown in Figure 1.

[3] According to Furth and Rahbee (2000), political considerations may govern the close bus stop spacing characteristic of US cities, as the benefits of having many stops for nearby residents are easier to observe than the costs for other riders and for the bus operator.

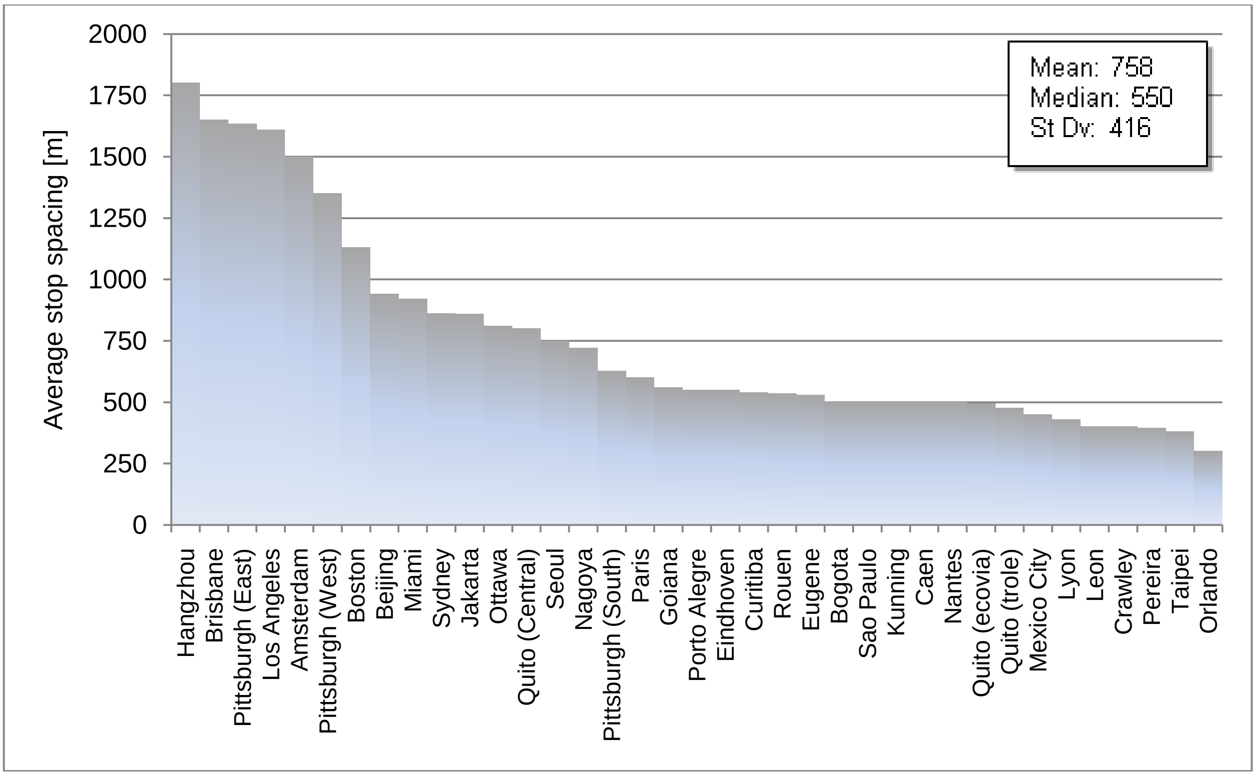

Figure 1: Average stop spacing of 37 Bus Rapid Transit systems from Latin America (11), the US and Canada (9), Europe (8), Asia (7) and Australia (2). Own compilation based on Wright and Hook (2007)[4].

2.3 The location of bus stops and interactions with other modes

Most of the studies on the optimal number of bus stops reviewed in Section 2.1 are not concerned with the actual stop locations (an exception is Furth and Rahbee, 2000). The analysis of location is relevant because the performance of a bus stop in terms of capacity and delays encountered upon entering or leaving the stop depends on several factors, such as the amount and composition of traffic circulating on the road and the proximity to traffic lights and the road grade (Kraft and Boardman, 1972; TRB, 2000; Fernández and Planzer, 2002; Furth and SanClemente, 2006), which are normally not accounted for in economic models of bus operation.

Bus stop locations in urban networks are usually classified into three groups: (i) before an intersection or *nearside*, (ii) after an intersection or *farside*, and (iii) isolated from intersections or *midblock*. Each location has advantages and disadvantages that make it impossible to give general recommendations without taking into account myriad local considerations, such as the programming of signalised intersections, the number of vehicles turning left or right at intersections, the geometry of bus access to the curb, the size of the bus stop, the distance between the bus stop and the nearest intersection, traffic safety[5], and pedestrian interference with bus movements at bus stops and with general traffic at intersections[6]. However, when bus stops are analysed in isolation from other bus stops upstream or downstream, authors tend to agree that farside stops are generally preferable to midblock or nearside stops (TRB, 1996;

[4] This figure excludes Adelaide's guided busway O-Bahn, which has an average station spacing of 5 km.

[5] Buses at bus stops may interfere with the visibility of car drivers, cyclists and pedestrians attempting to turn or cross at intersections (TRB, 1996).

[6] A detailed discussion of these and other factors regarding bus stop location is provided in Chapter 3 of TRB (1996) and Part 4 of TRB (2003).

Furth and SanClemente, 2006). When comparing bus delays at bus stops, Kraft and Boardman (1972) found that farside stops are superior when the intersection features high right turn flow and pedestrian crossing volume, whereas Furth and SanClemente (2006) found that farside stops are superior to nearside stops in most cases except for those where buses circulate in exclusive bus lanes, in which case no cars interfere with the bus' approach to the intersection. TRB (1996) further adds that midblock bus stops should be avoided because they increase walking distance for passengers crossing at intersections and encourage unauthorised midblock crossings. For these reasons, midblock bus stops are only desirable in major activity centres. On the other hand, when traffic signals are synchronised to facilitate car flow, buses can reduce overall delays by alternating between nearside and farside bus stop locations (TRB,2003; Vuchic, 2005).

Buses affect traffic streams when entering and leaving bus stops, and a few models have addressed the effects of bus stops on reducing road capacity and increasing the travel time of other modes that share the right-of-way with buses. Koshy and Arasan (2005) analyse the influence of two types of bus stops - curbside and bus bays - on the running speed of other modes that share the road with buses in India (cars, trucks, motorised two-wheelers, autorickshaws and bicycles). Their study finds that curbside bus stops cause more congestion for other modes than bus bays, and the impact increases with the dwell time of buses. Zhao *et al.* (2007) obtain that road capacity reductions due to the operation of a bus stop depend on the location of the stop with respect to a signalised intersection (nearside versus farside as well as the stop-intersection distance).

3. The probability of stopping in low demand markets

3.1 The poisson model

This section analyses the case of an on-call stopping regime, in which buses may skip a stop if no one desires to board or alight. The following analysis is due to Hauer (1971), who presented the first formal study of the probability that a bus will stop to load and unload passengers, as a function of the passenger volume N [pax/h] and the number of designated bus stops S. If f is the service frequency, the average number of passengers per vehicles is N/f and the total number of boardings and alightings is $2N/f$, which represents an upper bound for the number of stops on a bus ride. This upper bound is reachable if (i) passengers travel independently and wish to board and alight from vehicles at different places and (ii) S is sufficiently large (in fact S could be considered infinite if the service is on-demand, i.e., buses stop anywhere that passengers want to board or alight). On the other hand, if S is small relative to $2N/f$, it is unlikely that any bus stops will be skipped. Then, if S_a is the number of times that a bus actually stops along the route, then there are two extreme cases:

$$S \to 0 \Rightarrow S_a \to S$$

$$S \to \infty \Rightarrow S_a \to 2N/f$$

Thus, the number of actual stops S_a would depend on the number of designated stops S as shown in Figure 2.

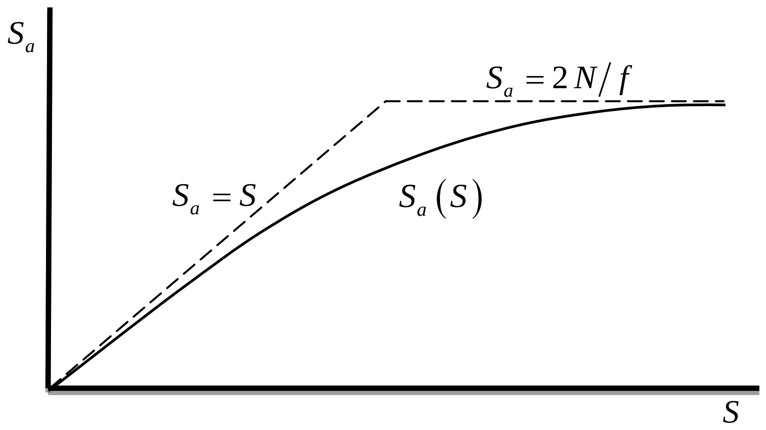

Figure 2: Number of actual stops S_a as a function of the number of designated stops S

(adapted from Hauer, 1971).

To make the problem mathematically tractable, Hauer (1971) and Mohring (1972) assume that passengers' arrivals at bus stops are Poisson distributed (i.e., users' travel decisions are independent of each other), therefore the probability of *n* passengers boarding or alighting at a particular bus stop is given by $P[n] = e^{-\lambda}\, \lambda^n / n!$, where $\lambda = 2N/f\,S$ is the average number of passengers per bus stop, and consequently, the probability of a bus stopping is $1 - P[0] = 1 - e^{-\lambda}$. Therefore, the expected number of stops is:

$$S_a = S\left(1 - e^{-2N/f\,S}\right) \tag{1}$$

which looks like $S_a(S)$ in Figure 2. After this model was proposed by Hauer (1971) and Mohring (1972), equation (1) has become the standard procedure to model the actual number of bus stops along a route. To the best of our knowledge, all subsequent bus stop optimisation models that have been used to analyse the case of stop skipping if no one wants to board or alight have assumed that the boarding and alighting distribution follows a Poisson distribution (Wirasinghe and Ghoneim, 1981; Kikuchi and Vuchic, 1982; Kikuchi, 1985; Furth and Rahbee, 2000; Furth *et al.*, 2007; Li and Bertini, 2009; Chien *et al.*, 2010). This approach has been applied without a thorough examination of its suitability to represent an observed distribution of actual bus stops along a route. Therefore, this work will test the accuracy of expression (1) using empirical data collected in Sydney, Australia.

3.2 Empirical versus probabilistic estimation of the number of stops

The number of stops S_a was estimated through on-board travel time surveys collected on weekdays from November 2007 to March 2009 in the Blacktown area in the western suburbs of Sydney, approximately 25 km from the city centre. This is a low density residential area with a relatively low demand for public transport (2.1 pax/bus-km on average) and an on-call stopping pattern. The data comprise 348 travel time surveys spread over 20 bus routes; the surveys were manually collected by a single observer aboard buses on either one-way or round trips. The numbers of passengers boarding and alighting per ride were recorded along with the number of actual stops S_a. This information is combined with the distance between scheduled bus stops to estimate S_a as a function of the number of scheduled stops S and the average demand per bus per kilometre P/L where *P* is the total number of passengers that board a bus ($= N/f$) and *L* is the route length. Appendix A summarises statistics including the sample size per route, average demand, and the scheduled and actual number of stops. Two functions are estimated:

(a) A power regression between the actual number of stops, demand and the scheduled number of stops (equation 2), which was found to present the best fit to the observed data.

(b) A modified version of the Poisson model (1), in which the factor k_5 that multiplies the demand per stop is estimated (expression 3).

$$\frac{S_a}{L} = \min\left\{k_o + k_1\left(\frac{P}{L}\right)^{k_2} + k_3\cdot\left(\frac{S}{L}\right)^{k_4}, \frac{S}{L}\right\} \quad \text{[stops/km]} \tag{2}$$

$$S_a = S\left(1 - e^{-k_5 N/fS}\right) \quad \text{[stops]} \tag{3}$$

The estimation of parameters k_i is presented in Table 1.

Table 1: Parameter estimates

Parameter	Estimate	Std. Error
k_0	-1.364	0.633
k_1	1.825	0.647
k_2	0.230	0.073
k_3	0.049	0.058
k_4	1.873	0.754
k_5	0.677	0.140
R^2 model a (eq. 2)	0.765	
R^2 model b (eq. 3)	0.613	
Sample size	348	

Figure 3 compares the probability of stopping (S_a/S) estimated by the three different models, the Poisson model (equation 1), the power regression (equation 2) and the modified Poisson model (equation 3), over the whole sample as a function of the average number of passengers per scheduled bus stop. The Poisson model overestimates the number of stops by a large margin, likely because the travel decisions of people are not always independent (e.g., passengers travelling together) and because in some cases a large number of boardings and alightings are concentrated in only a few stops (e.g., schools, shops). Therefore, care should be taken before using the Poisson model to estimate the probability of stopping, for example, by only modelling the spacing of bus stops in between major generators and attractors of demand, as done by Wirasinghe and Groneim (1981). Furthermore, the parameter estimated for the modified Poisson model can be interpreted as follows: the number of stops actually made by a bus is better estimated by assuming that only 33.9 percent of users ($0.5k_5$) board and alight buses according to a Poisson distribution. The Mean Absolute Percentage Error is 80 percent for the Poisson model, 26 percent for the modified Poisson model and 20 percent for the power model.

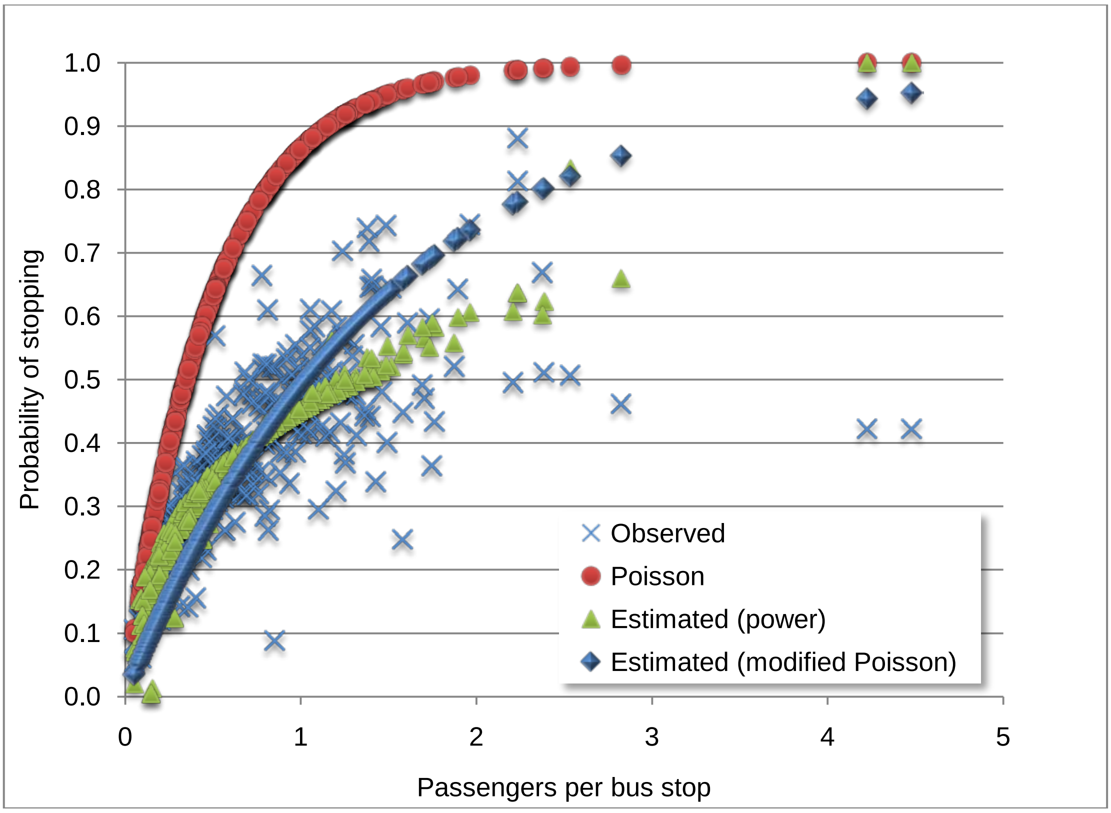

Figure 3: Probability of stopping, comparison of observed proportion vs. Poisson and empirical models.

4. Exposing the relationship between optimal bus stop spacing and demand—A simple analytical approach

This section presents a total cost minimisation model with the objective of investigating the relationship between the optimal number of bus stops and passenger demand in an effort to explain the differences found in the literature regarding this issue, as described in Section 2.1. This analysis will uncover the influence of elements such as frequency and bus running speed. Despite its simplicity, the model is able to unambiguously explain the divergent conclusions that are reached in the literature (last paragraph of Section 2.1), and it can also be linked to current practice on urban bus stop spacing policy.

We assume a fixed stopping regime. The total cost consists of operator and user costs, where the operator cost C_o is expressed as (4):

$$C_o = c(v_0) \cdot f \cdot t_c(v_0, f, N, S) \quad , \qquad (4)$$

where $c(v_0)$ is the cost of operating one bus [$/bus-h], which is a function of the running speed v_0 [km/h], f [bus/h] is the bus frequency and t_c [h] is the bus cycle time, which depends on v_0, f, passenger demand N [pax/h] and the number of stops S along the route. As is standard, the user cost C_u consists of access and egress (C_a), waiting (C_w) and in-vehicle (C_v) time costs:

$$C_u = C_a + C_w + C_v = P_a \frac{L}{2v_w S} N + P_w \frac{1}{2f} N + P_v \frac{l}{L} t_c N \tag{5}$$

P_a, P_w and P_v are the values of access, waiting and in-vehicle time savings [$/h], v_w is the walking speed [km/h], l is the average travel distance [km] and L is the route length [km]. Expression (5) is obtained by assuming that demand is uniformly distributed along the bus route[7] and taking the average waiting time as half of the bus headway. A detailed description of this functional form is provided in Jara-Díaz and Gschwender (2003).

The cycle time t_c is the summation of two components: the non-stop running time L/v_0 plus the delays due to bus stops, given by (i) the total boarding and alighting time $\beta \cdot N/f$, where β [s/pax] is the average boarding and alighting time per passenger and N/f is the number of passengers per bus, and (ii) the stopping delay t_s (apart from the transfer of passengers):

$$t_c = \frac{L}{v_0} + \beta \frac{N}{f} + S\, t_s \tag{6}$$

The stopping delay t_s comprises delays due to decelerating and accelerating t_{da}, waiting in queue t_q (in the event of congested operation) and due to the door opening and closing t_{oc}:

$$t_s = t_{da} + t_q + t_{oc} \tag{7}$$

The delay due to deceleration and acceleration is directly proportional to the running speed and inversely proportional to the bus acceleration rate a_0 and deceleration rate a_1:

$$t_{da} = \frac{v_0}{2}\left(\frac{1}{a_0} + \frac{1}{a_1}\right) \tag{8}$$

Finally, combining equations (4)-(7), we can write the total cost C_t as:

$$C_t = c \cdot f \cdot \left(\frac{L}{v_0} + \beta \frac{N}{f} + S\, t_s\right) + P_a \frac{L}{2v_w S} N + P_w \frac{1}{2f} N + P_v \frac{l}{L}\left(\frac{L}{v_0} + \beta \frac{N}{f} + S\, t_s\right) N \tag{9}$$

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[7] The average walking distance is one quarter the distance between stops, $L/4S$, and therefore the total access plus egress distance is $L/2S$.

If both *f* and *S* are optimisation variables, then the multiplicative term $c \cdot t_s \cdot f \cdot S$ is what prevents this problem from having an analytical solution, as mentioned in Section 2.1. However, when the bus frequency is fixed and the number of stops is the only variable, the optimal solution can be found by applying first order conditions as follows:

$$S^* = \sqrt{\frac{P_a L N}{2 v_w t_s \left(c f + P_v \frac{l}{L} N \right)}} \tag{10}$$

which resembles the square root form found by Kuah and Perl (1988). Expression (10) shows the influence of various relevant parameters on the optimal number of stops, specifically, that the optimal number of bus stops increases with the value of access time savings P_a but decreases with the value of in-vehicle time savings P_v, the access speed v_w, the stopping delay t_s and the average trip distance l. In this section, we further explore the relationship between the number of stops, demand, running speed and congestion.

If f is held constant, according to equation (10) *S* should increase if demand *N* increases, which is because the influence of *S* on reducing C_a and increasing C_v is given by a linear dependence on *N*, but the effect of an extra delay due to stopping (t_s) on increasing C_o is insensitive to demand; therefore, increasing the number of stops as demand grows implies a proportional benefit for access and a less than proportional extra cost (in-vehicle time plus operator cost). Note that

$$N \to \infty \Rightarrow S^* \to \sqrt{\frac{P_a L}{2 v_w t_s P_v \frac{l}{L}}} \tag{11}$$

which is equivalent to minimising access plus in-vehicle times only, i.e., user costs (waiting time does not depend on the number of stops). Therefore, in the hypothetical case of large demand and fixed capacity (with a sufficiently large bus frequency and size to accommodate demand), only the trade-off between access and in-vehicle times should be considered in estimating optimal stop spacing.

We now analyse the case in which frequency is adjusted according to demand. When the number of stops is fixed, the result of minimising the total cost (9) is the well-known square root formula first derived by Mohring (1972). More generally, under a total cost minimisation approach an increase in demand should be met by a less than proportional increase in frequency when the capacity constraint of vehicles is not binding, and a proportional increase to demand if buses are full. Following the same argument that in the case of fixed frequency, a less than proportional frequency increase would be coupled with an increase in the number of stops, whilst if buses are full, frequency takes the form $f = a N$ with constant a, and therefore the optimal number of stops is insensitive to demand variations:

$$S^* = \sqrt{\frac{P_a L}{2 v_w t_s \left(c a + P_v \frac{l}{L} \right)}} \tag{12}$$

This analysis does not consider the fact that an increase in demand may also be met by speeding up buses on the road, for example, through the provision of bus lanes or segregated busways. An increase in the bus running speed v_0 reduces non-stop travel time L/v_0 but increases the acceleration and deceleration delay t_{da} (equation 8), with the result that the effect on the optimal number of bus stops is not straightforward. In (10), if both v_0 and f increase with N and the combined effect of N on the cost due to stopping is more than proportional, i.e., $f \cdot v_0 \propto N^b$ with $b > 1$, the relationship between stop spacing and demand is reversed, i.e., the optimal number of stops decreases with demand. This matches the numerical findings of Tirachini and Hensher (2011) in a model that allows increases in v_0 through the investment in road infrastructure for buses. In other words, we have shown that a total cost minimisation approach that is able to accommodate optimal changes in bus running speed is in line with the observed practice of large bus stop spacing on Bus Rapid Transit systems (Figure 1), which are generally characterised by greater demand and higher running speeds than conventional bus services that operate with shorter distances between stops. This conclusion cannot be obtained with standard models in which bus running speed is fixed regardless of the level of demand.

5. Bus stop design, bus size and congestion

Bus stops have the lowest capacity amongst the components bus route, and therefore, are the first elements subject to congestion (Fernández and Planzer, 2002)[8]. When frequency is high and/or dwell times are long, buses may arrive at a bus stop when all berths are in use by preceding vehicles loading and unloading passengers, making the system subject to congestion amongst buses in the form of queuing delays. Most studies on stop spacing do not consider how congestion may affect the optimal number of stops along a route, with the exception of Tirachini and Hensher (2011) who find that queuing delays behind high-demand bus stops can be reduced to a large extent by speeding up the passenger boarding process (through quicker fare collection systems) and by increasing the number of designated bus stops (to reduce the number of passengers per stop). No change in bus stop design was considered, although design is relevant because the capacity of a bus stop depends on its size, in particular on the number and length of berths or stopping bays (Fernández and Planzer, 2002). Basso and Silva (2010) propose a car-bus equilibrium model and find that increasing the number of berths at bus stops to reduce bus congestion yields large social welfare gains. However, this analysis does not consider the capital costs of bus stops, which is necessary to determine optimal bus stop size. In Section 6 we consider the higher capital cost of larger bus stops (Table C.1, Appendix C) to identify the optimal bus stop size in terms of the number of berths to be provided at bus stops.

The literature contains a few efforts to estimate queuing delays at bus stops. Fernández *et al.* (2000) use the bus stop simulator IRENE (Gibson *et al.*, 1989; Fernández and Planzer, 2002) to model queuing delays t_q for different bus stop sizes (one, two and three linear berths) and a single bus size and fare payment system. Lu *et al.* (2010) apply a Cellular Automaton model to simulate t_q for bus stops with multiple berths and multiple bus routes arriving. Finally, Tirachini and Hensher (2011) include four fare collection systems but keep the size of the bus stop fixed (two berths). This paper extends these previous works by estimating a queuing delay function that depends on the design of the bus stop (number and length of berths), bus length, bus frequency and average dwell time (with the latter controlled by the number of passengers getting on and off, the fare collection system and the number of doors to board and alight). We use the bus stop simulator IRENE to estimate the average queuing delay with the objective of embedding this function into a model of optimal bus stop spacing that is sensitive to the design

[8] This is particularly so on segregated bus corridors; on shared roads buses may experience severe congestion at intersections as well due to heavy traffic flow.

of a bus stop and bus size (Section 6), considering linear bus stops with one, two or three berths and four possible bus sizes: mini (8 m), standard (12 m), rigid long (15 m) and articulated (18 m). Appendix B presents a description of the bus stop simulator and assumptions regarding bus stop location, berth length and bus saturation flow for the simulations.

The estimated model for the queuing delay t_q [sec/bus] as a function of the bus length L_b [m], dwell time t_d [s/bus], frequency f [veh/h] and number of berths per bus stop is shown in expression (13).

$$t_q = 0.001\left[b_0 + b_{l1}L_b + \left(b_{d1} + b_{d2}Y_2 + b_{d3}Y_3\right)t_d\right]e^{0.001f\left[b_f + b_{l2}L_b + \left(b_{d4} + b_{d5}Y_2 + b_{d6}Y_3\right)t_d\right]} \quad (13)$$

where b_0, b_{l1}, b_{l2}, b_{d1}, b_{d2}, b_{d3}, b_{d4}, b_{d5}, b_{d6} and b_f are parameters, and Y_2 and Y_3 are dummy variables defined as follows:

$$Y_2 = \begin{cases} 1 & \text{if bus stop has two berths} \\ 0 & \text{otherwise} \end{cases}$$

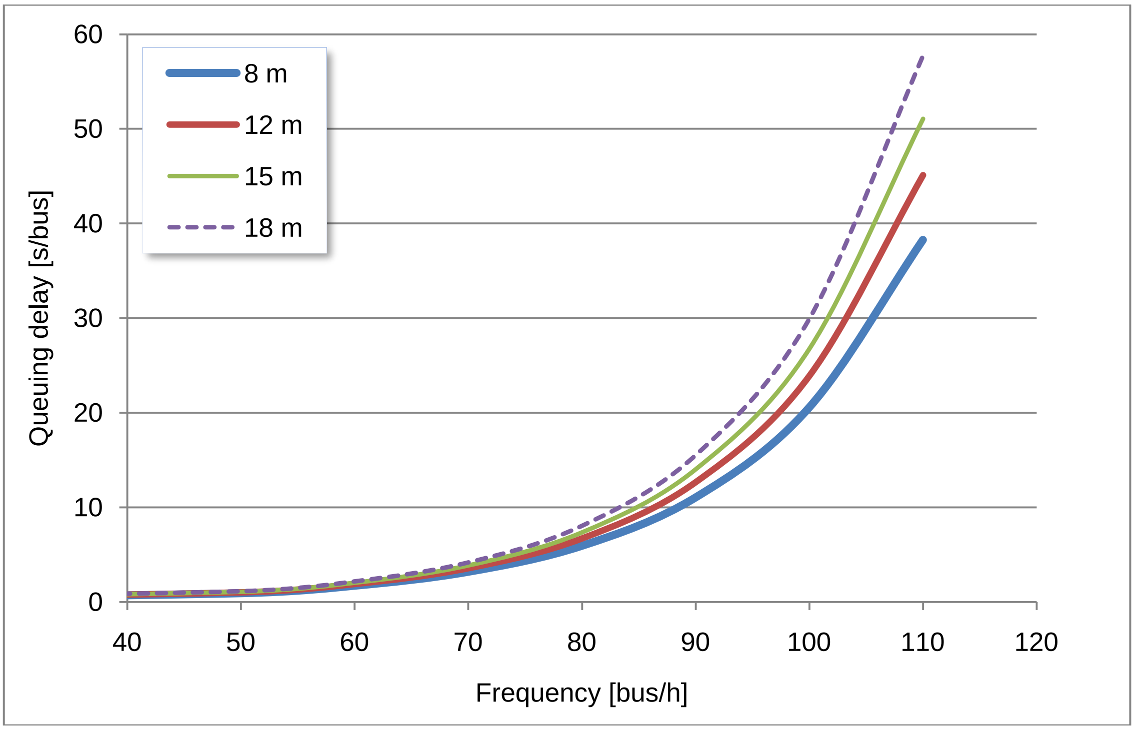
$$Y_3 = \begin{cases} 1 & \text{if bus stop has three berths} \\ 0 & \text{otherwise} \end{cases}$$

Equation (13) is a generalisation of the models developed by Fernandez *et al.* (2000) and Tirachini and Hensher (2011) and can be used to assess the optimal bus size and bus stop design under congested bus stop operations. The case of split bus stops (a large stopping area consisting of two subgroups with one, two, or three berths each) can be accommodated by establishing a rule for the assignment of buses to the stopping areas (e.g., 50 percent of buses assigned to each stopping area). The parameters estimated for equation (13) are presented in Table 2.

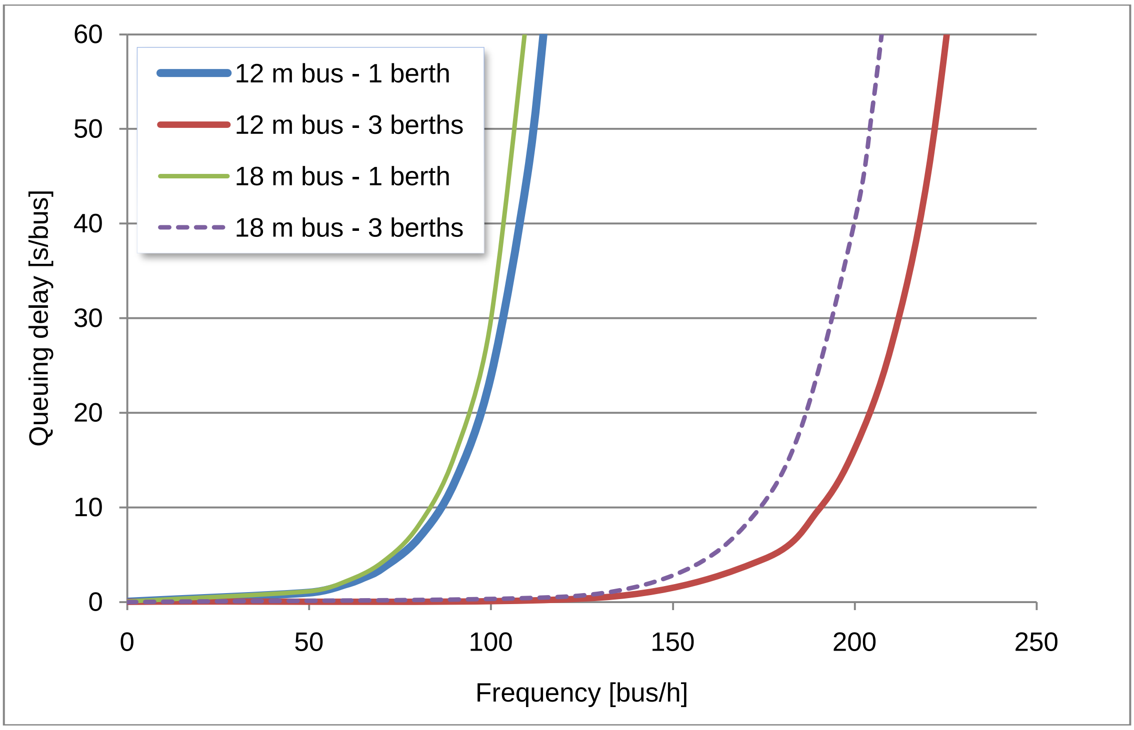
Table 2: Queuing delay parameters

Parameter	Estimate	Std. Error
b_0	-2.952	0.887
b_{l1}	0.061	0.020
b_{d1}	2.185	0.530
b_{d2}	-1.903	0.495
b_{d3}	-2.044	0.510
b_f	23.089	0.723
b_{l2}	0.361	0.046
b_{d4}	1.807	0.091
b_{d5}	-0.374	0.093
b_{d6}	-0.627	0.087
R^2	0.921	
Sample size	265	

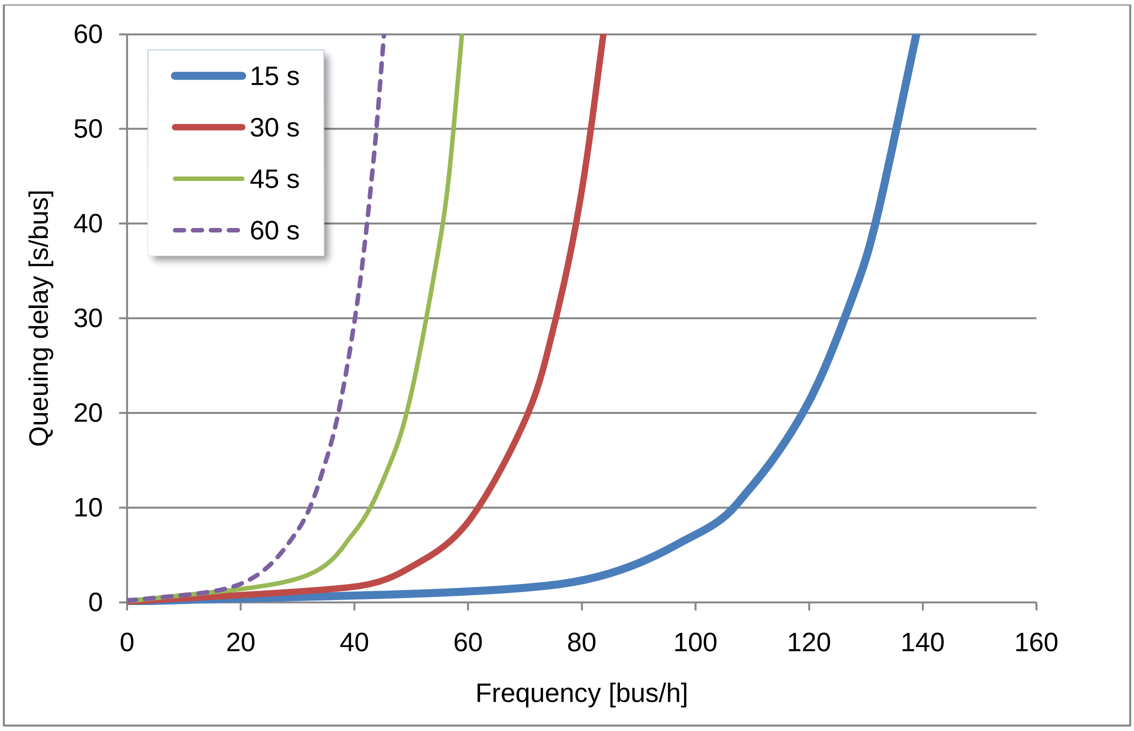
Figure 4 presents examples of the estimated queuing delay as a function of bus frequency. For a given frequency, t_q increases with bus size (Figure 4a), a difference that is amplified as more berths are provided on the bus stop (Figure 4b). Finally, dwell time has a sizeable influence on queuing delays (Figure 4c).



(a) Average queuing delay for buses of 8, 12, 15 and 18 metres, 1 berth, dwell time = 20 s



(b) Average queuing delay for buses of 12 and 18 metres, 1 and 3 berths, dwell time = 20 s



(c) Average queuing delay for 12 metre long buses, 1 berth, dwell time between 15 and 60 seconds

Figure 4: Bus stop queuing delay in different configurations

6. An extended optimisation model

6.1 CBD - inner suburbs route

First, we analyse the case of a high-demand fixed stopping route that links the CBD with its adjacent inner suburbs. We develop a total cost minimisation model that, aside from the usual variables – frequency, bus size and stop spacing – includes the bus stop size as a decision factor in the design of a bus route. Bus stops can have one, two or three berths and can also be split in two subgroups with two berths each to enable the assignment of buses to two stopping areas, reducing bus stop congestion (equation 13). We consider a linear bi-directional route of length L (therefore the total length is $2L$) and P time periods. The route is divided into zones 1 (CBD) and 2 (inner suburbs) of lengths L_1 and L_2, such that $L = L_1 + L_2$, and demand is uniformly distributed within each zone. Let S_1 and S_2 be the number of stops per zone. Directions are denoted as e (to the suburbs) and w (to the CBD).

Demand per period is denoted as follows (see Figure 5):

N_{12}^{p}: demand from zone 1 to zone 2

N_{11-i}^{p}: intra-demand zone 1, direction i

N_{21}^{p}: demand from zone 2 to zone 1

N_{22-i}^{p}: intra-demand zone 2, direction i

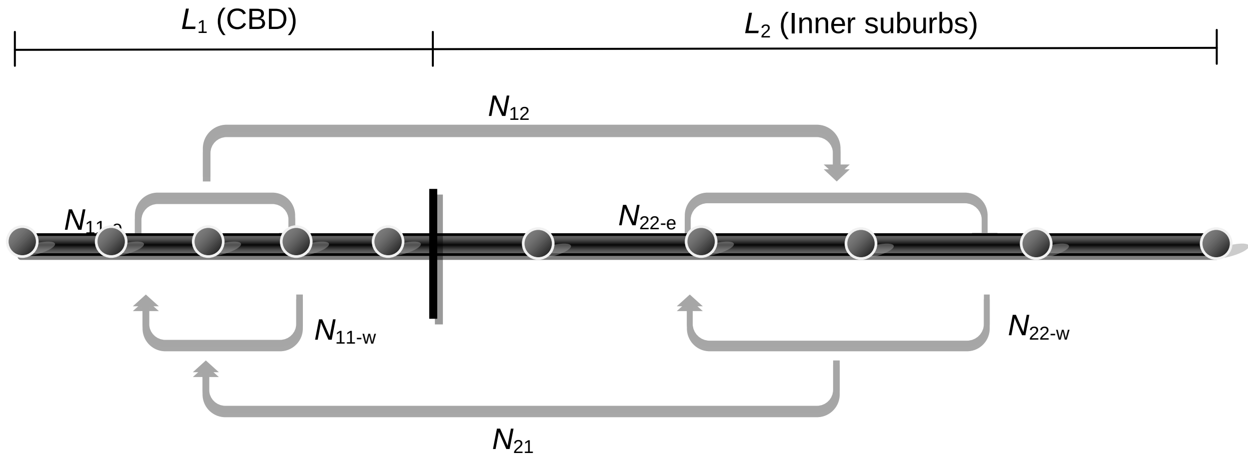


Figure 5: Route and demand configuration

User cost encompasses access, waiting and in-vehicle times, whereas operator cost consists of busway infrastructure investment, bus stop cost, bus capital (fleet size) cost, crew costs and running costs. The model, which is developed in Appendix C, is a modified version of Tirachini and Hensher (2011) after introducing two demand zones along the route and including time-of-day differences in demand embedded into a multiperiod framework.

Consider a hypothetical bus corridor that runs from the city centre in Sydney (zone 1, L_1 =5 km) to the eastern suburbs (zone 2, L_2 =11 km). The demand per zone, direction and period is based on the Sydney transport simulator TRESIS (Hensher, 2008) as presented in Table D1 (Appendix D), with uniform decreases and increases introduced to analyse the optimal stop spacing over a wide demand range from 8,000 to 340,000 pax/day (the base scenario has a total demand of 114,450 pax/day). Three scenarios are compared:

i. Base case with no bus infrastructure investment. Bus running speed is given by traffic conditions; assumed bus running speed per time period is given in Table D1. For example, in the CBD buses run at 25 km/h in peak periods (between 7 and 9 AM and between 3 and 6 PM), at 30 km/h between 9 AM and 3 PM, and at 35 km/h before 7 AM and after 6 PM. Boarding time is 5.7 s/pax, which is the average boarding time in Sydney considering both passengers that pay cash to drivers and those with prepaid tickets that are validated with a magnetic strip inside buses.

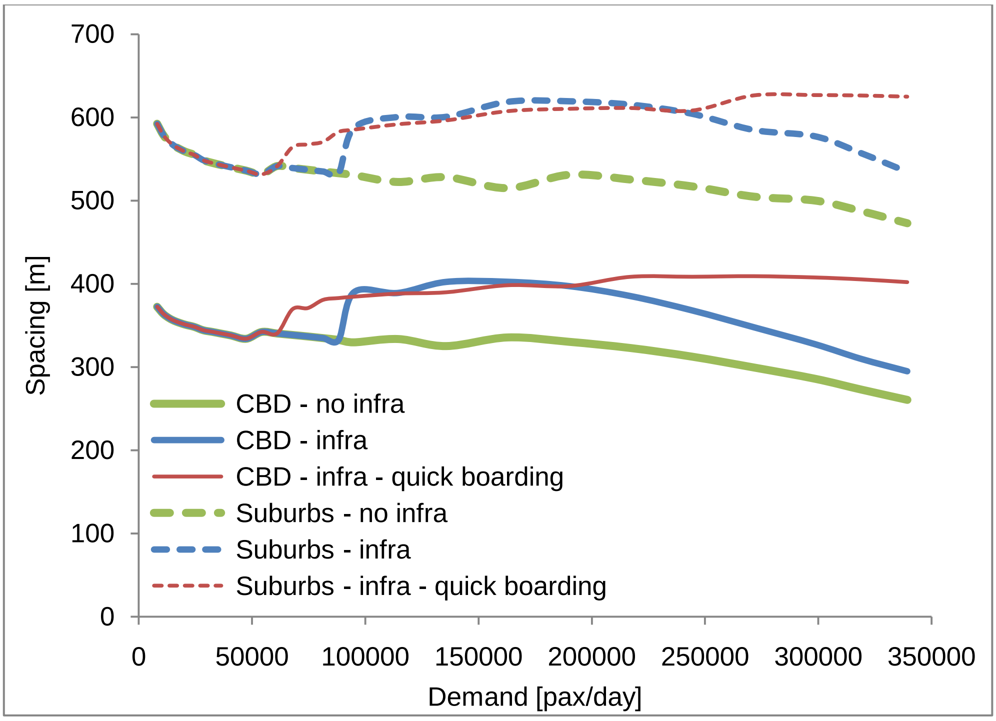
ii. Base case plus infrastructure investment in segregated busways. Following Tirachini and Hensher (2011), we assume a linear relationship between infrastructure cost and running speed (see Appendix C).

iii. Base case plus infrastructure investment in segregated busways, with the elimination of cash transactions onboard buses. Bus fare is paid by means of a magnetic strip only, which decreases average boarding time to 2.9 s/pax (Tirachini, 2011).

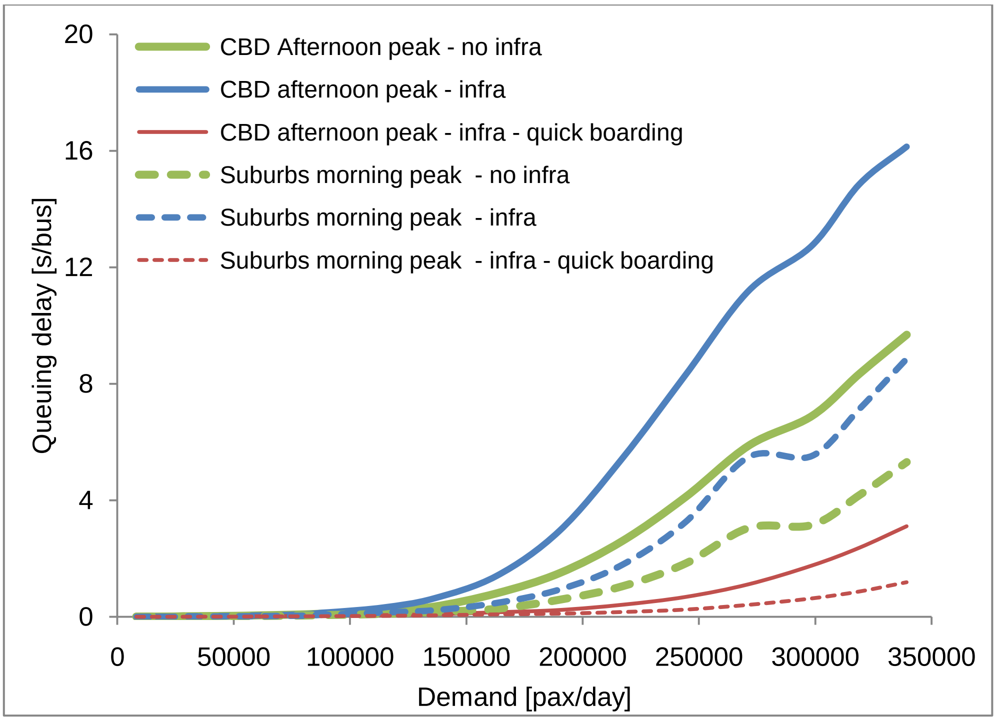
A discussion of results follows. Figure 6a presents optimal bus stop spacing in both the CBD area and the adjacent eastern suburbs. First, the higher boarding and alighting demand density in the city centre supports a shorter distance between stops relative to the suburbs. In the base case (labelled “no infra” in Figure 6) optimal spacing decreases with demand from 600 to 470 metres in the suburbs across the demand range analysed here and from 370 to 260 metres in the CBD. However, if it is possible to build busways to increase bus speed (labelled “infra”), this construction is justified if demand is higher than 85,000 pax/day, at which level optimal spacing is increased from 330 to 380 metres in the CBD and from 530 to 590 metres in the suburbs due to the increased bus running speed. Providing segregated busways then implies wider distances between stops; optimal spacing increases as patronage grows in the intermediate demand range (from 85,000 to 200,000 pax/day), but decreases for demand over 200,000 pax/day (blue curves in Figure 6). However, if both buses and the boarding process are sped up (labelled "infra – quick boarding"), then optimal demand spacing increases slightly as a function of demand for any demand level beyond 65,000 pax/day (with the optimal level at around 600 metres in the suburbs and around 400 metres in the CBD), a numerical corroboration of the discussion in Section 4.

To understand the differences in optimal stop spacing it is necessary to observe how bus stop congestion builds up in the various scenarios. In terms of queuing delay at bus stops (equation 13), the congested periods are the morning peak (7-9 AM) in the suburbs and the afternoon peak (4–6 PM) in the CBD, as shown in Figure 6b. The scenario with a prepaid fare (quick boarding) features little congestion even at the highest demand levels, allowing a greater distance between stops. On the other hand, the scenarios with slow boarding are associated with long dwell times, which in turn trigger queuing delays when bus frequency is high, and consequently reduce the distance between stops to spread passengers among more bus stops, even if segregated busways are introduced ("infra" scenario).

In summary, this framework clearly identifies the relevance of both bus running speed and dwell time (the latter influenced by the bus boarding time); furthermore, the result of Figure 6a is in line with current practice, since spacing is kept high for all demand ranges if a bus road infrastructure is built and a cashless fare collection system is introduced (compared to the case with no infrastructure investment and slow boarding), as is the case for several high-standard Bus Rapid Transit systems with longer average distances between stops relative to conventional bus services.



(a) Optimal bus stop spacing



(b) Queuing delay in peak periods

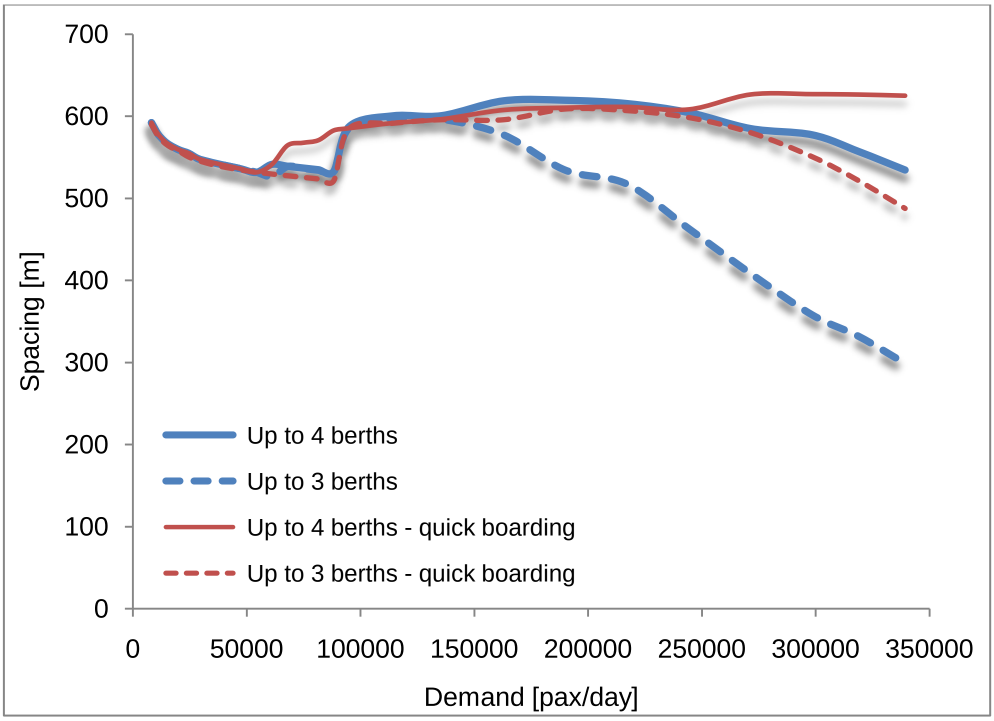
Figure 6: Results, fixed stopping pattern

Table 3 summarises other important outcomes of the optimisation model; the base case is chosen for illustration. Bus size and frequency are the usual variables in microeconomic public transport models, the novelty of Table 4 is that we can obtain the optimal bus stop size that results from the trade-off between capacity (lower congestion) and construction cost (Table C.1). The third and fourth columns show the evolution of bus stop size, from 1 to 4 linear berths per bus stop in both the CBD and the suburbs. Stops with one, two and three berths are simple linear stops, whilst a 4-berth bus stop is a group of two stopping areas with two stopping berths each. It is interesting to note that for some demand levels, bus stops in the CBD are bigger than those in the suburbs, because of the higher concentration of passengers in the CBD. Optimal bus size increases from mini buses (8 metres long) with less than 40,000 pax/day, to articulated 18 metre long buses when the daily demand is around 300,000 passengers. Medium size buses (12 and 15 metres long) are more cost-effective for demand levels between 40,000 and 290,000 pax/day. Optimal frequency is presented for three periods, the morning peak (7-9 AM), the afternoon peak (4-6 PM) and the evening off-peak (after 6 PM).

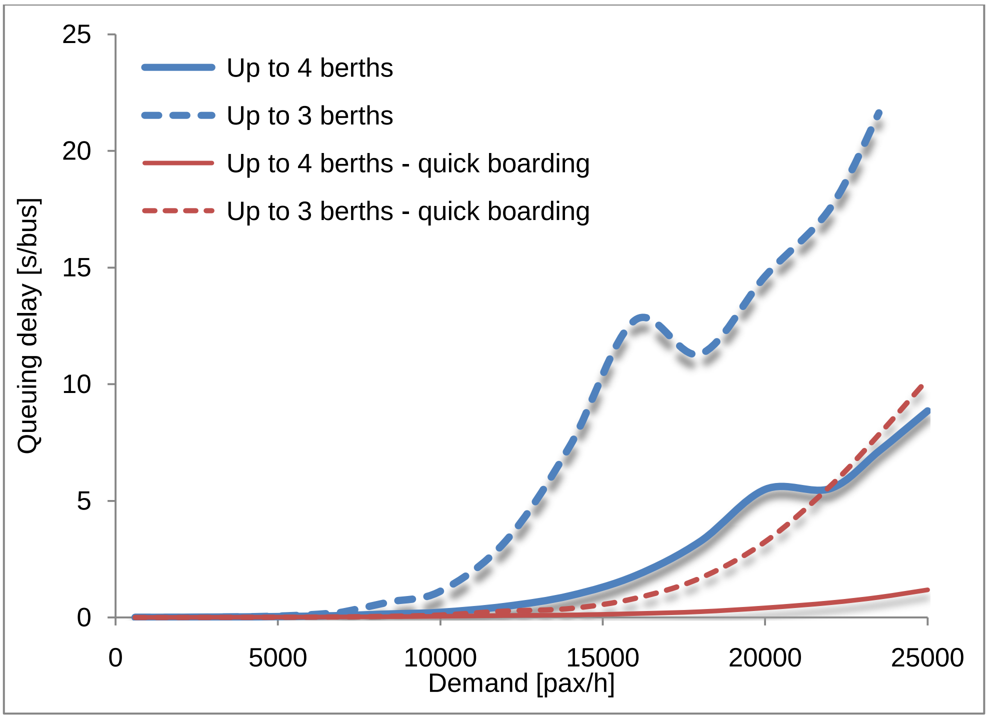
Table 3: Number of berths, bus size, frequency and average cost.

Demand [pax/day]	Bus length [m]	Berths CBD	Berths Suburbs	Frequency [bus/h]			Average cost [$/pax]				
				7-9 AM	4-6 PM	6 PM+	C_a	C_w	C_v	C_o	C_{tot}
8,142	8	1	1	11	11	5	0.74	0.90	3.40	2.99	6.62
24,426	8	1	1	34	28	10	0.70	0.41	3.50	2.06	5.90
40,710	12	1	1	32	32	13	0.68	0.33	3.63	1.40	5.69
61,066	12	2	1	48	45	17	0.67	0.24	3.67	1.28	5.59
81,421	12	2	2	64	58	21	0.67	0.19	3.70	1.21	5.53
114,450	15	3	2	70	71	26	0.66	0.15	3.78	1.03	5.49
135,701	15	3	3	84	83	29	0.65	0.13	3.80	1.00	5.47
162,842	15	4	3	100	101	34	0.65	0.11	3.81	0.99	5.47
189,982	15	4	4	117	114	39	0.66	0.10	3.82	0.97	5.46
244,262	15	4	4	150	135	47	0.64	0.08	3.89	0.94	5.47
298,543	18	4	4	138	136	54	0.60	0.07	4.03	0.82	5.52
339,253	18	4	4	157	138	60	0.56	0.07	4.12	0.82	5.56

To further analyse the interplay between bus stop size, spacing and congestion, we compare the optimal bus stop spacing and resulting queuing delay for the case in which split bus stops with two berths per boarding group (four berths in total) are allowed against a scenario in which only simple bus stops with no more than three berths are possible. Figure 7 presents results on the optimal spacing for the suburbs during morning peak congestion. Beyond a demand of 150,000 pax/day, bus stops with three berths are spaced more closely than split bus stops with four berths because three-berth stops have a smaller capacity and are more prone to queuing delays.



(a) Optimal bus stop spacing



(b) Queuing delay during the AM peak period.

Figure 7: Influence of bus stop size on optimal spacing, suburbs.

6.2 Outer suburbs routes

Finally, we analyse bus stop spacing in a low demand market set up with an on-call stopping regime. Four routes are chosen from Sydney's outer suburb Blacktown (25 km away from the CBD), which has a much lower demand compared to the CBD and the eastern suburbs (Table D2, Appendix D, average demand is obtained from the bus operator). The objective of this section is to illustrate differences in the estimation of optimal bus stop spacing between the Poisson and empirically estimated models for the calculation of the probability of stopping (equations 1, 2 and 3). We apply a modified version of the model developed in Appendix C with no zonal differentiation of demand since demand is fairly low and uniformly distributed in the Blacktown area. Table 4 shows optimal bus stop spacing in five scenarios including the current situation. As the Poisson model overestimates the probability of stopping, the optimal number of stops is underestimated relative to the empirical power regression model (equation 2). In fact, the optimal stop spacing is overestimated by 15 to 23 percent when a Poisson distribution is assumed for passenger boarding and alighting. The modified Poisson model yields a spacing between 6 and 19.5 percent lower than the more accurate power regression. On the other hand, if fixed stopping is provided, the optimal spacing is 31 to 55 percent greater than with on-call spacing. Finally, the last row shows that the current average bus stop spacing is slightly shorter than the optimal values.

Table 4: Optimal bus stop spacing

	Spacing [m]				Difference wrt on-call empirical			
Route	**5**	**6**	**11**	**12**	**5**	**6**	**11**	**12**
Empirical power model	332	340	364	336				
Empirical modified Poisson model	268	278	342	275	-19.5%	-18.1%	-6.1%	-18.1%
Poisson model	408	412	419	414	22.7%	21.2%	15.0%	23.3%
Fixed stopping	514	521	480	517	54.7%	53.4%	31.8%	53.9%
Current situation	279	288	313	320	-16.2%	-15.2%	-14.1%	-4.7%

7. Conclusions

This paper has re-examined the problem of deciding the optimal spacing of bus stops in urban routes by re-considering the method used to calculate the probability of stopping in low demand markets (e.g., outer suburbs) and by analysing the interplay between bus stop size, bus running speed, spacing and congestion in high demand markets (e.g., the CBD and inner suburbs). First, using empirical data from Sydney we show that the Poisson model commonly used to estimate the actual number of stops made by buses in on-call bus stopping patterns overestimates the probability of stopping, and consequently underestimates the optimal number of bus stops to be designed. Second, on services with a fixed-stopping regime, we show both analytically and numerically that the bus running speed, frequency and dwell time are crucial determinants of the relationship between bus stop spacing and demand, with bus stop congestion in the form of queuing delays playing a relevant role. In particular, we show that if the bus running speed is kept constant, bus stop spacing should be decreased with demand, but if both bus running speed and the boarding process of passengers are sped up then bus stop spacing is kept high even for high demand levels. This result is in line with the implementations of several Bus Rapid Transit systems around the world, which feature high bus speeds and long distances between stops as one of their key characteristics. This work also analysed the optimal size of bus stops by estimating average queuing delays as a function of vehicle size, berth size and number, and bus frequency and dwell time.

Several elements have not been considered in this paper. The possibility of establishing express services may affect the optimal location of bus stops, as, for example, in peak periods it may be more convenient to provide limited stop services in addition to all-stop traditional operations instead of reducing the number of bus stops on a single all-stop service (TRB, 2003). The sizing and design of platforms and waiting areas was not treated either, which should account for the peak flow of passengers walking into bus stops and alighting from buses (TRB, 2003). Finally, in mixed-traffic operations the delays introduced by bus stops for cars and other modes of transport is expected to reduce the optimal number of stops on heavily congested roads and intersections (Valencia, 2007).

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Appendices

Appendix A: Sample size per route, average demand, average stop spacing and observed probability of stopping

Route	Obser-vations	Route length [km]	Average demand [Pax/ bus-km]	Average demand [Pax/ bus-stop]	Average stop spacing (scheduled) [m]	Average stop spacing (actual) [m]	Probability of stopping
1	3	13.14	3.25	0.84	258	597	0.43
2	18	16.36	2.87	0.77	268	718	0.37
3	22	25.07	2.63	0.72	275	728	0.38
4	6	6.52	0.96	0.27	278	1069	0.26
5	32	12.51	3.17	0.88	279	637	0.44
6	25	12.46	2.09	0.60	288	833	0.35
7	14	27.68	1.76	0.53	298	900	0.33
8	26	18.89	2.27	0.68	301	763	0.39
9	5	27.93	2.19	0.67	305	849	0.36
10	12	9.33	0.85	0.26	309	1433	0.22
11	10	8.55	2.93	0.92	313	613	0.51
12	24	14.00	3.30	1.06	320	849	0.38
13	19	12.91	1.60	0.53	333	1033	0.32
14	15	24.20	0.56	0.19	344	1830	0.19
15	4	5.97	2.47	0.87	351	884	0.40
16	23	10.16	2.31	0.87	377	843	0.45
17	26	17.24	1.33	0.51	383	1274	0.30
18	28	24.65	1.22	0.47	384	1506	0.25
19	20	18.21	2.06	0.83	403	942	0.43
20	6	11.20	2.75	1.95	708	2168	0.33
21	10	19.36	0.75	0.58	779	1995	0.39
Total	348						
Mean	16.57	16.02	2.06	0.71	360	1070	0.36
Median	18.00	14.00	2.19	0.68	313	884	0.37
Min	3.00	5.97	0.56	0.19	258	597	0.19
Max	32.00	27.93	3.30	1.95	779	2168	0.51

Appendix B: Estimation of the queuing delay function

To estimate the queuing delay of buses we use the bus stop simulator IRENE, which can determine the capacity, queuing delay, dwell time, berth usage and other indicators of the performance of a bus stop as a function of a number of inputs such as the boarding and alighting demand, number of berths, bus size and frequency. For a more detailed description of the program see Fernández and Planzer (2002).

Regarding inputs, the following assumptions are made:

- Bus size: Four different bus sizes are considered in accordance with standard commercial vehicle sizes: 8-, 12-, 15- and 18-metre long buses.
- Number of berths: Three configurations are simulated, with one, two and three contiguous berths. For a split bus stop with two stopping areas with two berths each, we assume that half of the buses are assigned to each stopping area.
- Berth length: Each berth is assumed to be 1.5 times the bus length, which is the minimum distance necessary for buses to manoeuvre and overtake a preceding bus if necessary (Wright and Hook, 2007).
- Bus saturation flow: This parameter depends on the length of the bus and influences the queuing delay. We assume a basic saturation flow of s = 2086 passenger cars per hour per lane (Akcelik and Besley, 2002) and apply the following equivalency factors depending on the size of the bus (Basso and Silva, 2010): 1.65 (8 m), 2.19 (12 m), 2.60 (15 m) and 3.00 (18 m), yielding estimated saturation flows of 1262, 951, 823 and 694 bus/h for 8, 12, 15 and 18-metre buses, respectively.

A total of 265 simulations were run encompassing all bus sizes and bus stop designs previously described for a range of frequencies from 20 to 220 bus/h and dwell times between 10 and 65 seconds. Buses are assumed to arrive at a constant rate at stops (no bus bunching) and bus stops are isolated from traffic lights.

Appendix C: Multiperiod total cost minimisation model

The circulation of buses is separated into three components: links, bus stops and signalised intersections. We consider a linear bi-directional route of length L (therefore the total length is $2L$) and P time periods. The cycle time T_c^p in period p is defined as the total travel time during one cycle or round-trip, given both the service time and slack time at termini. Let T_r^p be the running time, T_i^p the delay due to traffic lights, T_s^p the time lost at bus stops and T_k^p the layover time at the end of the route; then the cycle time is

$$T_c^p = T_r^p + T_i^p + T_s^p + T_k^p \tag{C.1}$$

Travel time stages are defined as follows:

(i) Running time (without any stopping delay)

$$T_r^p = \frac{2L}{v_{0p}} \tag{C.2}$$

v_{0p}: running speed in period p.

(ii) Delay due to intersections (Tirachini and Hensher, 2011)

$$T_i^p = \left[\frac{0.5 C_T \left(1-u\right)^2}{1-ux^p} + \frac{v_0^p}{2}\left(\frac{1}{a_0}+\frac{1}{a_1}\right)\frac{1-u}{1-ux^p} \right] I \qquad \text{(C.3)}$$

I: total number of signalised intersections along the route

C_T : traffic light cycle time [s]

$u = g/C_T$: ratio of effective green time g [s] to the cycle time C_T (g and C_T are fixed on all periods)

$x^p = f^p/C$: degree of saturation, given the capacity of the intersection $C = s_b \cdot u$ [veh/h], where s_b is the saturation flow rate [veh/h] and f^p [veh/h] is the bus frequency in period p

a_0 and a_1: bus acceleration and deceleration rates [m/s^2]

(iii) Delay due to bus stops

Bus stop delays comprise acceleration and deceleration delays (second term inside the bracket in equation C.3), queuing delays (equation 13) and dwell time, which is derived next.

We assume that boarding and alighting are simultaneous, with boarding at the front door and alighting at the back doors (with average boarding and alighting times denoted as β_b and β_a [s/pax], respectively). Therefore, if t_{oc} is the time required to open and close doors, the average dwell time per stop for each zone and direction is:

Zone 1, direction e: $t_{d1-e}^p = t_{oc} + \max\left\{ \beta_b \frac{N_{12}^p + N_{11-e}^p}{f^p S_1}, \beta_a \frac{N_{11-e}^p}{f^p S_1} \right\}$

Zone 2, direction e: $t_{d2-e}^p = t_{oc} + \max\left\{ \beta_b \frac{N_{22-e}^p}{f^p S_2}, \beta_a \frac{N_{12}^p + N_{22-e}^p}{f^p S_2} \right\}$

Zone 2, direction w: $t_{d2-w}^p = t_{oc} + \max\left\{ \beta_b \frac{N_{21}^p + N_{22-w}^p}{f^p S_2}, \beta_a \frac{N_{22-w}^p}{f^p S_2} \right\}$

Zone 1, direction w: $t_{d1-w}^p = t_{oc} + \max\left\{ \beta_b \frac{N_{11-w}^p}{f^p S_1}, \beta_a \frac{N_{21}^p + N_{11-w}^p}{f^p S_1} \right\}$

For the sake of simplicity, we do not take into account the peculiarity of the spacing between the stops that are next to the border of each zone (e.g., for a person in zone 2 close to zone 1, it may be shorter to walk to the last stop in zone 1).

(iv) Layover time

T_k^p is assumed to be constant and exogenous at 5 min. The optimisation of T_k^p is treated in Zhao *et al.* (2006).

Operator cost is divided into five components:

(v) Busway infrastructure and land costs (C_1)

If segregated busways are provided, we assume a linear relationship between the investment in road infrastructure per kilometre and the bus running speed (Tirachini and Hensher, 2011):

$$C_1 = \left(c_{10} + c_{11} v_0\right) L \quad \text{(C.4)}$$

(vi) Bus stop cost (C_2)

Let S be the number of stops and c_2 the stop infrastructure cost [$/stop-day], then the total infrastructure cost associated with bus stops is $C_2 = 2c_2 S$, assuming directional stops in cases with no dedicated busways and bi-directional stations in the case with dedicated busways, the cost of which is double the cost of simple one-directional stops. Based on guidelines from Wright and Hook (2007) and FTA (2009), we estimate the following bus stop costs for bus size given berth number and length, which are then allocated assuming 10 years of asset life and that a year of operation is equivalent to 294 working days.

Table C.1: Bus stop infrastructure cost

Bus length [m]	Berth Length [m]	Infrastructure cost [$]			
		1 berth	2 berths	3 berths	2+2 berths
8	12	20,000	30,000	40,000	50,000
12	18	40,000	60,000	80,000	100,000
15	23	60,000	90,000	120,000	150,000
18	27	100,000	150,000	200,000	250,000

(vii) Fleet size (C_3)

Vehicle capital cost is given by the period with the highest fleet size requirement $B^p = f^p \cdot T_c^p$. If $\eta > 1$ captures the need for a reserve fleet to deal with unexpected breakdowns and maintenance (e.g., η = 1.05 means that 5 percent of vehicles are not used and kept at depots), the fleet size cost is given by

$$C_3 = c_3\left(L_b\right) \eta \max_p \left\{ f^p T_c^p \right\} \quad \text{(C.5)}$$

where $c_3\left(L_b\right)$ is the discounted cost of a bus [$/bus-day] of length L_b.

(viii) Crew costs (C_4)

Define D^p as the duration of period p [h] and c_4 as the driving cost [$/h-bus], then

$$C_4 = c_4 \sum_{p=1}^{P} D^p f^p T_c^p \quad \text{(C.6)}$$

(ix) Running costs (C_5)

Running costs consist of fuel consumption, lubricants, tires, maintenance, etc. If $c_5^p\left(L_b, v_0^p\right)$ is the running cost per kilometre [$/bus-km] of a bus of length L_b running at speed v_0^p, then the total running cost is

$$C_5 = 2L\sum_{p=1}^{P} c_5^p D^p f^p \tag{C.7}$$

Values for the bus capital cost, crew and running costs are given in Tirachini and Hensher (2011). After deriving cost components (v) to (ix), the total operator cost is

$$C_o = \left(c_{10} + c_{11}v_0\right)L + 2c_2S + c_3\eta \max_p\left\{f^pT_c^p\right\} + c_4\sum_{p=1}^{P} D^p f^p T_c^p + 2L\sum_{p=1}^{P} c_5^p D^p f^p \tag{C.8}$$

User cost is divided into access and egress (C_a), waiting (C_w) and in-vehicle (C_v) time costs. For the access and egress time cost, we distinguish between the different demand groups as follows

$$C_a = P_a\frac{1}{4v_w}\sum_{p=1}^{P} D^p\left[\frac{2L_1}{S_1}\left(N_{11-e}^p + N_{11-w}^p\right) + \frac{2L_2}{S_2}\left(N_{22-e}^p + N_{22-w}^p\right) + \left(\frac{L_1}{S_1} + \frac{L_2}{S_2}\right)\left(N_{21}^p + N_{12}^p\right)\right] \tag{C.9}$$

For the waiting time cost we distinguish between low and high frequency cases. When the frequency is high, passengers usually arrive at the stations randomly at a constant rate, but when the frequency is low most passengers arrive at stations according to a published timetable. The two cases can be formulated as a single expression (Tirachini *et al.*, 2010):

$$C_w\left(f\right) = P_w\sum_{p=1}^{P} D^p N^p\left(t_0^p + \frac{t_1^p}{2f^p}\right) \tag{C.10}$$

with

$$t_o^p = \begin{cases} 0 & if\ f^p \geq 5veh/h \\ t_w & if\ 0 < f^p < 5veh/h \end{cases} \qquad t_1^p = \begin{cases} 1 & if\ f^p \geq 5veh/h \\ \mu & if\ 0 < f^p < 5veh/h \end{cases}$$

P_w is the value of waiting time savings [$/h]. For the low frequency case $\left(0 < f^p < 5veh/h\right)$, t_w is a fixed 'safety threshold' time that passengers spend waiting at stations before the expected arrival of the next vehicle, and $\mu = P_h/P_w$ is the ratio of the value of *home waiting time* savings P_h to the value of *station waiting time* savings P_w (for example, $\mu = 0.33$) to account for the schedule delay caused by the fact that bus departures do not occur at the times desired by users.

In-vehicle time is modelled as

$$C_v = P_v \sum_{p=1}^{P} \frac{D^p}{2} \left[T_{c1-e}^p N_{11-e}^p + \left(T_{c1-e}^p + T_{c2-e}^p \right) N_{12}^p + T_{c2-e}^p N_{22-e}^p + T_{c2-w}^p N_{22-w}^p + \left(T_{c1-w}^p + T_{c2-w}^p \right) N_{21}^p + T_{c1-w}^p N_{11-w}^p \right]$$

(C.11)

where P_v is the value of in-vehicle time savings and T_{c1-j}^p and T_{c1-j}^p are the times required to traverse zones 1 and 2 in direction *j*.

Total cost (the summation of equations C.8 to C.11) is minimised with respect to frequencies f^p, bus size L_b, number of stops S_1 and S_2, and the number of berths per stop b_1 and b_2 subject to a frequency constraint (C.12) and a capacity constraint (C.13). By the construction of the problem (uniformly distributed boarding and alighting at each zone), these parameters are given by the maximum of the inter-zonal demands:

$$f_{min} \le f^p \le f_{max} \quad \text{(C.12)}$$

$$\max\left\{N_{12}^p, N_{21}^p\right\} \le \kappa K\left(L_b\right) f^p \quad \text{(C.13)}$$

Bus capacity K [pax/bus] is given by the size L_b and κ is a safety factor introduced to allow for spare capacity to absorb random variations in demand (for example, $\kappa = 0.9$). The constrained optimisation is solved using the optimisation toolbox in Matlab.

Appendix D: Demand and bus running speed

Table D1: High demand market, CBD (zone 1) and inner suburbs (zone 2)

Period	Duration [h]	Demand [pax/h]						Running speed [km/h]	
		11-e	12	22-e	22-w	21	11-w	Zone 1	Zone 2
06:00-07:00	1	143	224	38	38	1713	143	35	40
07:00-09:00	2	934	1256	459	860	5705	1869	25	30
09:00-15:00	6	645	2127	655	655	2146	645	30	35
15:00-16:00	1	965	4048	1064	709	2901	965	25	30
16:00-18:00	2	1530	4720	847	565	2525	765	25	30
18:00-00:00	6	197	1469	142	142	551	197	35	40

Table D2: Low demand market, outer suburbs example

Period	Duration [h]	Demand [pax/h]				Running speed [km/h]
		Route 5	Route 6	Route 11	Route 12	
04:00-07:00	3	26	24	3	37	40
07:00-09:00	2	135	132	46	152	30
09:00-15:00	6	78	68	21	71	35
15:00-17:00	2	142	116	44	117	30
17:00-19:00	2	70	68	15	69	30
19:00-21:00	2	22	18	0	14	40