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Consumer Surplus based Method for Quantifying and Improving the Material Flow Supply Chain Network Robustness

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TITLE: **Consumer Surplus based Method for Quantifying and Improving the Material Flow Supply Chain Network Robustness**

ABSTRACT: Recent advances in network science has encouraged researchers to adopt a topological view when characterising the robustness of supply chain networks (SCNs). However, topology based characterisations, without considering the heterogeneity among the supply chains which form the SCN, can only provide a partial understanding of robustness. Hitherto, focus of robustness studies have been on cyclic SCNs, with unweighted and undirected links representing general inter-firm interactions. Here, we consider the specific case of a material flow SCN with multi-sourcing, which is characterised by a tiered structure with directed and weighted links. The proposed method uses the multinomial logit model to estimate the utility levels of supply chains within the SCN, as perceived by a focal firm which is indicative of the SCN consumers. The robustness of the SCN is characterised by considering the degree to which supply chains overlap with each other as a cost in the logit formulation. Finally, using a randomisation scheme to generate ensembles of SCN configurations which preserve the number of connections at each firm, the configuration which maximises the consumer surplus for the focal firm is identified. The proposed method is implemented on a real world SCN to identify the optimal configuration in terms of robustness.

KEY WORDS: ***supply chain network; supply network topology; supply network robustness; logit model; consumer surplus***

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1.0 INTRODUCTION

The traditional view of supply chains as linear processes has, in recent years, evolved towards a network based view representing the complexity of these systems. Multi sourcing and outsourcing practices due to product specialisation, have transformed simple independent supply chains towards supply chain networks (SCNs) comprising of multiple entangled supply chains. Interdependencies of businesses in the globalised economy has enabled supply chain firms to no longer be isolated by geography or industry. Despite the seemingly efficient and cost effective operations under regular conditions, these interdependencies have created unprecedented risks to SCN operations when firms are disrupted due to unforeseen events (Manuj and Mentzer, 2008). This 'fragility of interdependence' (Vespignani, 2010) has steered the attention of the industry towards risk management of supply chain systems. Therefore, it is not surprising that a large part of contemporary research in the area of supply chain management has focussed on understanding and quantifying the robustness of SCNs. In particular, recent advances in network science has encouraged researchers to adopt a topological lens in characterising the robustness of SCNs (Choi et al., 2001; Surana et al., 2005; Pathak et al., 2007; Hearnshaw and Wilson, 2013).

Due to difficulty in obtaining large scale datasets on supplier-customer relationships, which are often proprietary and confidential, early studies have relied on computer simulations to generate network topologies (through various growth mechanisms) supposedly representative of real world SCNs (Thadakamalla et al., 2004; Xuan et al., 2011). Recently however, a number of data driven studies have appeared in literature, which used Bloomberg database to obtain SCN data for publicly listed firms (Brintrup et al., 2015; Orenstein, 2016). In general, these studies have considered the topological robustness of SCNs, which have been determined either analytically (through various network science metrics) or via simulations (by recording the change in a pre-defined topological performance metric as nodes, which represent individual firms, are sequentially removed from the network).

While the above methods provide general insights into the topological robustness of SCNs, the lack of specificity due to high level of abstraction leads to limited real world applicability. Additionally, consideration of the topological structure without incorporating the heterogeneity of various supply chains that form the SCN, can only account for a part of the full picture. Therefore, there exists a need for a more specific method for assessing the robustness of SCNs. In particular, this method should consider the topological structure of the SCN while at the same time capturing the heterogeneity in utilities of each supply chain as perceived by the consumers. In this work, we seek to fill this gap by developing a consumer surplus based SCN robustness assessment method which is capable of accounting for both the topology of the SCN and the heterogeneity in supply chain utility levels. In particular, by adopting the point of view of the consumer (represented by a focal firm), we seek to answer the question, 'how can a focal firm, quantify the robustness of its SCN and subsequently configure the SCN to achieve highest possible robustness?'.

In most cases, flow data for supply chains are either not available or only partially available. Therefore, the proposed method relies on the multinomial logit model to estimate the utility levels of each supply chain within the SCN, as perceived by the focal firm, by considering the cost added by each firm to the final product along each supply chain. The robustness of the system is characterised by considering the degree to which supply chains overlap with each other on the premise that more distinct/independent the supply chains are, better it is in terms of robustness. In particular, the level of overlap between the links of a given supply chain with other supply chains in the SCN is reflected as a cost (or a disutility) in the logit formulation. The robustness of a given SCN configuration can therefore be characterised by its consumer surplus (i.e. the maximum utility derived by the focal firm from a given SCN configuration). Finally, using a randomisation scheme, recommendations can be provided to improve the robustness of the overall SCN, while preserving the number of supply chain connections at each firm.

The remainder of this manuscript is structured as follows. Section two provides the background to this study and introduces key theoretical concepts. Section three describes the proposed methodology using a toy example. Section four demonstrates the results obtained for a large scale real-world SCN. Section five provides a discussion of the results obtained and Section six concludes the paper.

2.0 BACKGROUND

2.1 Network Science based Robustness Assessment of Supply Chains

Recent advances in network science has encouraged researchers to adopt a topological perspective in robustness assessment of SCNs. A typical SCN model includes nodes and links which represent firms and various relationships between firms, respectively. The inter-firm relationships in a SCN can be broadly categorised into three classes, namely; (1) material flows, (2) financial flows, and (3) information exchanges. Material flows are usually unidirectional from suppliers to retailers, while financial flows are unidirectional in the opposite direction. Both material and financial flows mostly occur vertically, across the functional tiers of a SCN (however, in some cases, two firms within the same tier, such as two suppliers, could also exchange material and finances) (Lazzarini et al., 2001). In contrast, information exchanges are bidirectional (i.e. undirected) and include both vertical and horizontal connections (i.e. between firms across tiers and between firms within the same tier). Therefore, the same SCN can include different topologies based on the specific type of relationship denoted by the links in the model.

There exists a large body of literature investigating the robustness of SCNs where the links represent the undirected general relationships between firms. These studies have gained insights into SCN robustness based on either or both of the following avenues (Perera et al., 2017a):

(1) Analytically determining the topological metrics of the networks, such as the degree distribution, average path length, clustering coefficient, nestedness and assortativity. These metrics reveal the structural features of the SCN which have direct or indirect robustness implications (Kim et al., 2011; Brintrup et al., 2012; Büttner et al., 2013; Kito et al., 2014; Brintrup et al., 2015; Gang et al., 2015; Orenstein, 2016; Perera et al., 2017b); and

(2) Using generic network science based simulation techniques, which involve sequential removal of nodes (randomly or targeted by degree or some other topological attribute) and recording at each time step, the size of the largest connected component and/or the average/maximum shortest path length in the largest connected component. By creating profiles of these metrics across the percentage of nodes removed, one could compare the robustness character for various SCN topologies (Thadakamalla et al., 2004; Zhao et al., 2011a; Zhao et al., 2011b; Wen and Guo., 2012; Xu et al., 2014; Kim et al., 2015; Perera et al., 2016; Li and Du, 2016).

By adopting a system wide perspective, the above methods capture the topological robustness of various SCNs. However, they do not consider the heterogeneity between various supply chains which form the SCN. Also, contemporary studies have mainly focussed on cyclic SCNs where links are unweighted and represent various undirected generic relationships between firms rather than acyclic material flows towards the consumer(s). A key feature of material flow SCNs, that set them apart from transport networks or other types of networks considered in social network analysis, is their acyclic/tiered structure with an output to a defined consumer base. In particular, the firms within a material flow SCN tend to be separated into functional tiers with the consumer base forming the bottom-most tier (Willems, 2008). Review of literature reveals that little effort has been made to quantify and improve the robustness of material flow SCNs.

2.2 Proposed consumer surplus based view of robustness

In this study, we consider the case of a directed material flow SCN – as illustrated in **Figure 1**. Our SCN model is considered to have the following features;

- It is strictly partite (i.e. flow links can only occur between firms in adjacent tiers, not within the same tier). Therefore, our SCN model is acyclic.
- The focal firm represents the consumer(s) of the SCN and is interested in obtaining a single product from various suppliers in Tier 1. This multiple sourcing practice (as described in **Section 2.2.1**) is assumed to be present across all tiers of the SCN.
- Firms within each tier are assumed to be homogeneous in their functional capabilities and are substitutable without capacity constraints.
- The focal firm has the knowledge on which firms are connected with other firms in the SCN (i.e. the adjacency matrix of the SCN), as well as the cost added to the final product by each firm.
- Substituting the connections between firms in adjacent tiers is assumed to be costless.

In this study, we adopt the point of view of the consumer, represented by a focal firm. We use the consumer surplus as a benchmark to characterise the SCN robustness. It represents the expected maximum utility, from a given configuration of the SCN, by the focal firm. In particular, we are interested in the change in consumer surplus between the original SCN configuration and each new configuration. The configuration which provides the highest positive change in the consumer surplus, when compared against the original SCN configuration, can be recommended to the focal firm.

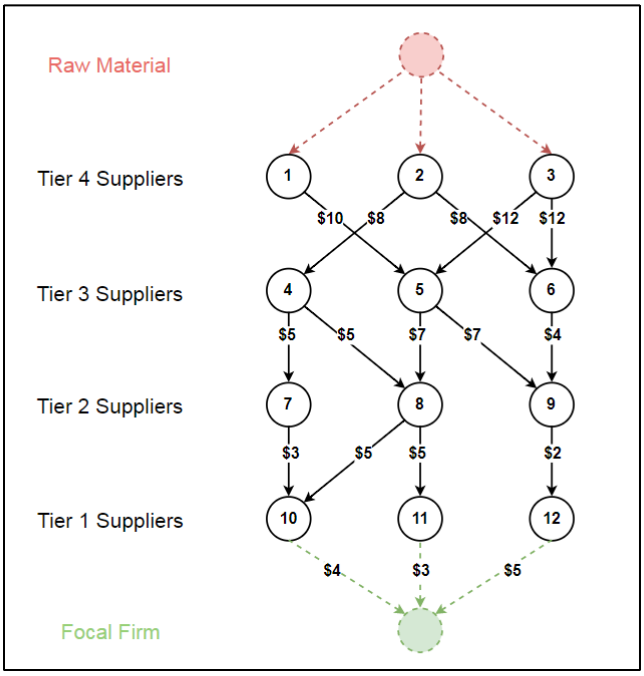


Figure 1: Example SCN Scenario

2.2.1 Multiple sourcing

Multiple sourcing relates to a single buyer securing multiple supply sources to obtain a single product or a service. In this model, the buyer generally has a range of prequalified suppliers to source various amounts of a certain product from and as a result there exists high levels of competition between suppliers to maximise their output (Cousins and Lamming, 2008). Therefore, in addition to providing the buyer with redundancy in terms of substitutable suppliers and robustness in terms of being able to manage unexpected demand fluctuations, this model also guarantees the buyer receives the most competitive price, quality and delivery times from each supplier (Najafi et al., 2014).

Chopra and Sodhi (2004) present a motivating example which highlights the benefits of multiple sourcing. In 2000, a lightning strike induced fire damaged millions of microchips at a local plant owned by Royal Philips Electronics, N.V. in Albuquerque, New Mexico. Following the incident, the Finnish mobile phone manufacturer Nokia Corp., a major customer of the plant, immediately began switching its chip orders to other Philips plants as well as to other Japanese and American suppliers. Thanks to the multiple sourcing strategy, Nokia managed to retain its production levels with little impact due to the disrupted supplier. In contrast, Telefon AB L.M. Ericsson, another customer of the disrupted Philips plant who employed a single-sourcing strategy, failed to maintain their production levels for months following the incident. As a result, Ericsson lost $400 million in sales (Eglin, 2003).

2.3 Stochastic path choice

When considering the choice of the supply chains (also referred to as paths hereinafter) by a focal firm, not only the least cost path will be used, due to the multiple sourcing practices. Therefore, a better behavioural description is obtained by specifying a smooth response surface where the probability of selecting a specific path from a set of alternative paths is a function of its cost relative to the costs of the alternative paths. In particular, the response surface should possess the following two features to account for reasonable behaviours: (1) the probability of selecting any path should change smoothly with its cost (i.e. decreasing as the cost is increased and vice versa) and (2) alternative paths with same cost should have the same probability of being selected (Bell and Iida, 1997).

Random utility theory is perhaps the most well established method for capturing the above discussed behavioural aspects. In the random utility model, the utility perceived by a decision maker includes a random component which reflects the misperceptions or variations in taste (Ben-Akiva and Lerman, 1985).

Assume that all possible paths between an origin o and a destination d have been enumerated. Since path costs are assumed to be the sum of the costs of its constituent links, the cost c_{pod} of a path p which connects the origin o with the destination d is established as:

$$c_{pod} = \sum_k a_{kpod} c_k \qquad \textbf{(1)}$$

Where a_{kpod} is an element of the link-path incidence matrix and c_k is the link cost.

Based on the random utility argument, the perceived utility of a path p consists of its cost and a random error term as follows;

$$U_{pod} = -c_{pod} + \varepsilon_p \qquad \textbf{(2)}$$

Where U_{pod} is the perceived utility of a path p which connects the origin o with the destination d, c_{pod} is the cost of path p (expressed as a negative) and ε_p is the random error term.

The probability that path p is chosen from a set of alternative paths P_{od} which connect o with d is given as;

$$\Pr(p \mid o,d) = \Pr(U_{pod} > U_{p'od}, p' \in \mathrm{P}_{od}, p' \neq p) \qquad \textbf{(3)}$$

When the random error terms ε_p are identically and independently distributed following Gumbel distributions, the probability that path p is chosen for traversing between the origin o to the destination d is given by the logit path choice model as follows (McFadden, 1973);

$$\Pr(p \mid o,d) = \frac{\exp(-\alpha c_{pod})}{\sum_p \exp(-\alpha c_{pod})} \qquad \textbf{(4)}$$

Where α is termed the dispersion parameter and it dictates the sensitivity of the choice to the cost. When $\alpha = 0$, the choice is insensitive to cost all paths are chosen with equal probability. As $\alpha \to \infty$, the choice tends to be concentrated on the least cost path. It is noted that, based on the above formulation, for finite cost differences, every path must include a positive share of flow (between the specified origin and destination), reflecting the fact that every path in the path set will be used.

The above discussed path choice logit model can also be assigned to links by substituting Eq (1) into Eq (4) as follows (Bell and Iida, 1997);

$$\Pr(p \mid o,d) = \beta \exp\left(-\alpha \sum_k a_{kpod} c_k \right) \qquad \textbf{(5)}$$

Where

$$\beta = \frac{1}{\sum_p \exp\left(-\alpha \sum_k a_{kpod} c_k \right)}$$

Therefore,

$$\Pr(p \mid o,d) = \beta \exp(-\alpha a_{1pod} c_1).\exp(-\alpha a_{2pod} c_2).....\exp(-\alpha a_{1Kpod} c_K) \qquad \textbf{(6)}$$

Where K is the number of links in the network.

Eq (6) implies that one could traverse the network assigning each link k a weight equal to $\exp(-\alpha c_k)$. On this basis, the probability of choosing a path is proportional to the product of the above assigned weights along the constituent links of that path. The constant of proportionality β is calculated to ensure that the sum of products of the link weights across all feasible paths is one (i.e. path choice probabilities between a given OD pair adds up to 1). Note that a-priori path enumeration is not required for this procedure.

2.3.1 Fixed cost logit assignment

The first step in fixed cost logit assignment involves construction of the Weights matrix (referred to as the W matrix) (Bell and Iida, 1997).

Let

$$w_{ij} = \begin{cases} \exp(-\alpha c_{ij}) & \text{if a link connects node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases} \quad \textbf{(7)}$$

Where c_{ij} is the cost of the link between node *i* and node *j*, if there is a link, and infinity otherwise. Bell (1995) has shown that the probability of travelling between node *i* and node *j* by a path with only two links is proportional to $\sum_{h=1 \text{ to } N} w_{ih} w_{hj}$. Similarly, the probability of travelling between node *i* and node *j* by a path with only three links is proportional to $\sum_{g=1 \text{ to } N} \sum_{h=1 \text{ to } N} w_{ig} w_{gh} w_{hj}$. Therefore, the probability of travelling between any node *i* and any node *j* by any path irrespective of the number of links, is proportional to element v_{ij} of the following matrix – referred to as the **N** matrix (Bell, 1995);

$$N = \text{W+W}^2\text{+W}^3\text{+...=(I-W)}^{-1} - \text{I} \quad \textbf{(8)}$$

In the above formulation, no paths are excluded as per the logit model, although with declining probability as the cost of the path increases. It is important to note that for networks with cycles, the number of paths in the path set will be infinite. However, this issue is not related to our context since the material flow SCNs are directed and acyclic in nature.

2.4 Penalising overlapping paths

In the multinomial logit model, the choice probability for a given path *p* in the choice set ***P*** is determined directly from the path utilities and do not depend on the topological structure of the underlying network. As a result, correlations due to overlap are not taken into account in the MNL model (Bliemer and Bovy, 2008).

When different path alternatives are overlapping, the MNL model is known to yield biased predictions which result in too high probabilities on paths with a high overlap with other paths in the choice set. In order to correct the choice probabilities when there are overlaps, several models such as the C-Logit (Cascetta et al., 1996) and Path-Size Logit (Ben-Akiva and Bierlaire, 1999) have been proposed in literature. These models aims to capture, in a crude way, the impact of overlap of a route on predicted choice probabilities by including a 'commonality factor' which accounts for the level of overlap in the path alternatives. In particular, higher the overlap of a given path with other paths, higher is the commonality factor and the utility of this path is negatively affected by this factor.

For instance, the 'commonality factor' CF_i^{od} of path *i* (between nodes *o* and *d*) is given in the C-Logit model as;

$$\text{CF}_i^{od} = \sum_{j \in C^{od}} \left(\frac{L_{ij}^{od}}{\sqrt{L_i^{od} L_j^{od}}} \right) \quad \textbf{(9)}$$

Where L_i^{od} and L_j^{od} are the lengths of paths i and j, respectively. L_{ij}^{od} is the common length of paths i and j.

The path choice probabilities of this model are then calculated using;

$$\Pr(p \mid o,d) = \frac{\exp(-\alpha(c_{pod} + \beta \ln \mathrm{CF}_p^{od}))}{\sum_p \exp(-\alpha(c_{pod} + \beta \ln \mathrm{CF}_p^{od}))} \quad \textbf{(10)}$$

Where the parameter β governs the impact of the path overlap on the path cost and is generally estimated through sensitivity analysis.

Inspired by the idea of commonality factor in the C-Logit model, in this study we develop a customised commonality factor metric applicable to SCNs. In contrast to the application to transport networks which relies on the geographical lengths of path overlaps, in the SCN context we consider the topological overlaps of supply chains. We develop a customised algorithm to calculate the link betweenness (LBW) values for each link in the SCN, which represents the number of supply chains traversing through that link. The commonality factor for each supply chain is then calculated as the product of the LBW values of the links along it.

We interpret the commonality factor values as indicative of robustness of the system. Robustness of a networked system is defined as its ability to maintain basic functionality even when some of its nodes and/or links may be missing. One way to characterise the robustness of a network is to measure the level of redundant components present within the system, which can be relied upon under removal of various nodes and/or links (Barabasi, 2016). Redundancy in our context, implies presence of multiple independent supply chains between the topmost tier suppliers and the focal firm. Since LBW values indicate the number of supply chains reliant on a given link, by considering it as a cost (or a disutility), supply chains with high levels of overlap with others are avoided in the final SCN configuration. This idea can be viewed as a continuous approximation of the concept of distinct paths which is defined as the number of paths connecting a given origin and destination pair, where all paths do not share any links (Kurauchi, et al., 2003).

2.5 Consumer Surplus

In the random utility model, the expected minimum cost of a set of paths ***P***, is given by the expected value of the maximum utility, which has the following functional form under the logit model;

$$E\{\mathrm{Max}_{p\in P} U_p\} = \frac{1}{\alpha} \ln\left(\sum_{p\in P} \exp(-\alpha c_p) \right) \quad \textbf{(11)}$$

Note that the above term for expected maximum utility can be further simplified as $\frac{1}{\alpha}\ln(v_{ij})$, where v_{ij} is the element in row i and column j of the **N** matrix described in Eq (8). The element V and consumer surplus are proportionally related as discussed below.

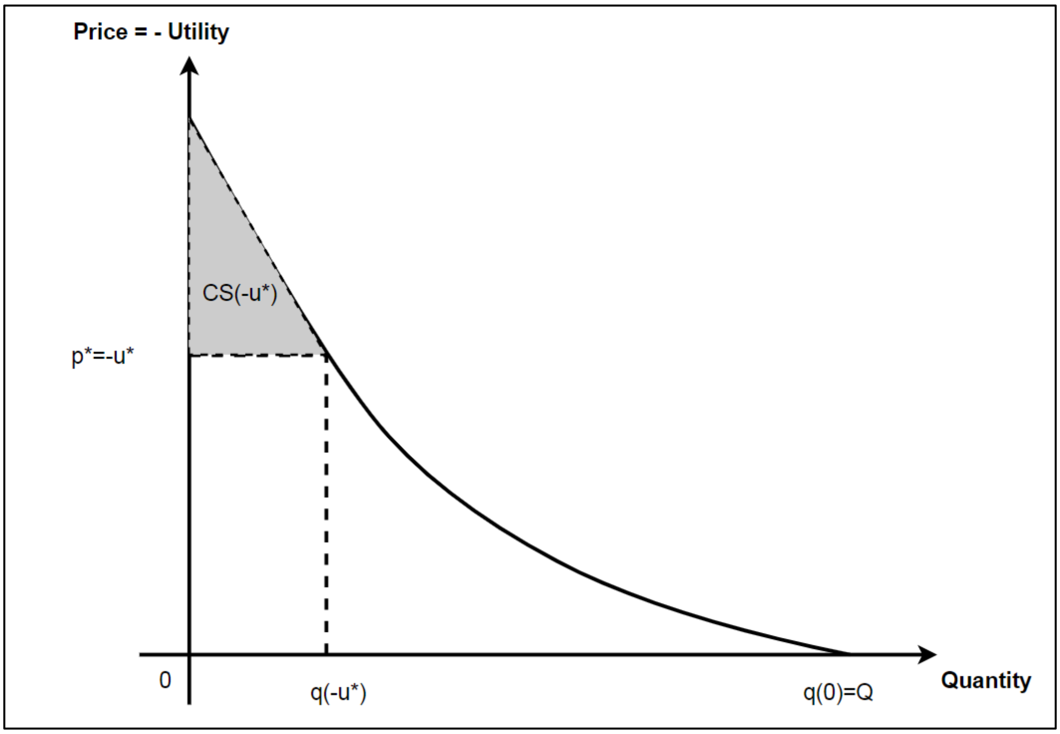


Figure 2: Relationship between expected maximum utility and consumer surplus

$$q(-u*) = Qe^{\alpha u*} = Qe^{-\alpha p*}$$

$$CS(-u*) = \int_{p*=-u*}^{\infty} q(p)dp = Q\left[-\frac{e^{-\alpha p}}{\alpha}\right]_{p*}^{\infty} = Q\frac{e^{-\alpha p*}}{\alpha} = \frac{q(-u*)}{\alpha}$$

Note that $u* = E\{\text{Max } U\} = \frac{1}{\alpha}\ln(\sum e^{\alpha u}) = \frac{1}{\alpha}\ln(V*)$

Where $V*$ is the corresponding element of **N** matrix.

Therefore, $CS(-u*) = \frac{q(-u*)}{\alpha} = Q\frac{e^{\alpha u*}}{\alpha} = Q\frac{e^{\ln V*}}{\alpha} = Q\frac{V*}{\alpha}$

In general, one would be interested in the change in consumer surplus, which is calculated under the conditions before and after the change (Winkler, 2016). Therefore, the change in consumer surplus between two SCN configurations can be calculated as follows (which causes the constant of proportionality to drop out);

$$\Delta CS = \frac{CS(-u**)}{CS(-u*)} = \frac{V**}{V*}$$

The above term is >1 if the new SCN configuration (denoted by **) offers a better consumer surplus than the original SCN configuration (denoted by *). Throughout this paper, we use V as an indicative consumer surplus measure since V is in fact proportional to consumer surplus as shown above. We use this framework in our analysis to compare the consumer surplus between various topological configurations of a given base SCN, obtained through a randomisation process.

2.6 Degree Preserving Randomisation

An important question when testing hypothesis related to network topologies is whether the degree distribution on its own is sufficient to describe the structure of a network, i.e. whether the topological features observed in the network are explained by the ensembles of networks generated by its degree distribution while preserving the degree vector. In this regard, degree preserving randomisation (DPR) plays an important role in generation of null models.

DPR involves rewiring the original network, to generate an ensemble of null models, while preserving the degree vector (Maslov and Sneppen, 2002; Noldus and Van, 2015). At each time step, the DRP process randomly picks two connected node pairs and switches their link targets. This switching is repeatedly applied to the entire network until each link is rewired at least once. The resulting network represents a null model where each node still has the same degree, yet the paths through the network have been randomised. Comparison of properties of a given network, with the properties of an ensemble of networks generated by DPR, allows one to identify if the properties observed in the real network are unique and meaningful or whether they are common to all networks with that degree sequence (Fosdick et al., 2016).

In this study, we modify the above discussed DPR process, so that randomised ensembles not only preserve the degree of the nodes but also preserves the tiered structure of SCNs. We refer to this process as the Tier Constrained - Degree Preserving Randomisation (TC-DPR). By generating randomised ensembles which preserve the key features of the original SCN, our aim is to identify the SCN configuration which maximises the consumer surplus measure for the focal firm.

3.0 METHODOLOGY

In this section, the adopted methodology is described using a small numerical example.

3.1 Proposed Procedure

Consider the small SCN illustrated in **Figure 1**. The following step by step procedure is proposed to identify the optimal SCN configuration in terms of consumer surplus.

Step 1: Process the cost matrix

The cost matrix is processed by weighting the adjacency matrix by the cost added by each firm to the final product which is sourced by the focal firm. **Figure 1** includes a pseudo node representing raw materials for convenience in calculations.

Step 2: Establish the link betweenness value matrix

In order to capture the supply chain overlaps, similar to the C-Logit model, here we propose a simple algorithm. In general, it is a time and computationally expensive task to find the paths of minimum cost (or distance), especially in networks where plural paths will be found due to cycles. However, since the SCN structure considered here is tiered and is acyclic in nature, the procedure becomes simple. In this regard, the following algorithm can be applied to establish the link betweenness (LBW) values which represent the number of supply chains reliant on a given link.

a. Set $t = 1$ (t represents the tier level, '$t = 1$' means 'Raw Material' level in Figure 1).
b. Update node level 'b_{it}', as follows;

   If $t = 1$, set b_{it} as 1. Otherwise, $b_{it} = \sum_{(i,j)\in \mathbf{A}} b_{j,t-1}$. (It means that if node i at tier t is connected with the node j at tier $t-1$, add the values of $b_{j,t-1}$ to b_{it}.)
   If $t = T$ ($T = 6$ on the example), go to c. Otherwise, $t = t + 1$ and return to b.
c. Set LBW_a, the link betweenness centrality as follows;
   - $LBW_a = \min\left(b_{i(a),t}, b_{j(a),t-1}\right)$

Figure 2 represents the calculation result of bs (in the nodes) and LBWs (beside the links) for the SCN in **Figure 1**. Note that the node level for the node representing the focal firm represents the total number of supply chains in the SCN.

Using the above framework, we can define the commonality factor (CF) for each path as the product of the LBWs of links along it as follows;

$$CF_p = \prod_{a\in \mathbf{A}} \delta_{ap} LBW_a \qquad \textbf{(12)}$$

where, $\delta_{ap} = 1$, if link a is lying on path p.

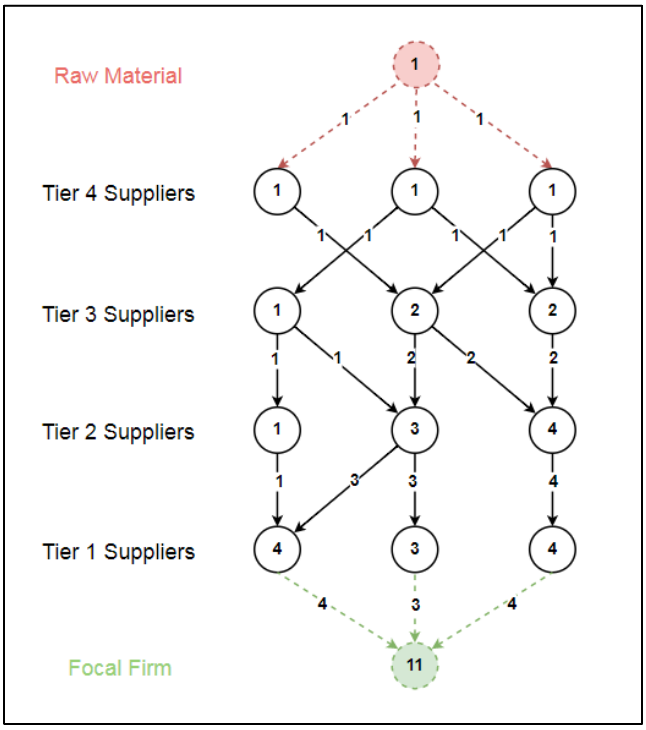


Figure 2: Link Betweenness Values for the SCN

Step 3: Calculate the W matrix

The Weights (W) matrix is generally determined through Eq. (7) which uses link costs to calculate the exponentially weighted link weights. However, in this study, we have modified Eq. (7) to include the betweenness of the links (LBWs) as shown in Eq. (13).

$$w_{mn} = \begin{cases} \exp(-\alpha(c_{mn} + \beta lnLBW_{mn})) & \text{if a link connects node } m \text{ to node } n \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

Inclusions of the LBW values as a cost represents the fact that links with higher LBWs are undesirable on the basis of robustness as the failure of a single link with a high LBW will incapacitate multiple supply chains. The β patameter controls the relative cost of LBW against the link cost. The multiplicative nature of LBWs to produce the path CF is reflected by the log transformation.

Step 4: Calculate the N matrix

Using Eq. (8) we calculate the N matrix. **Figure 3** illustrates the results of the matrix calculations for the SCN configuration presented in **Figure 1**.

	RM	1	2	3	4	5	6	7	8	9	10	11	12	FF
RM	1E+9	0.00	0.00	0.00	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9
1	1E+9	1E+9	1E+9	1E+9	1E+9	10.00	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9
2	1E+9	1E+9	1E+9	1E+9	8.00	1E+9	8.00	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9
3	1E+9	1E+9	1E+9	1E+9	1E+9	12.00	12.00	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9
4	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	5.00	5.00	1E+9	1E+9	1E+9	1E+9	1E+9
5	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	7.00	7.00	1E+9	1E+9	1E+9	1E+9
6	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	4.00	1E+9	1E+9	1E+9	1E+9
7	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	3.00	1E+9	1E+9	1E+9
8	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	5.00	5.00	1E+9	1E+9
9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	2.00	1E+9
10	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	4.00
11	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	3.00
12	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	5.00
FF	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9	1E+9

(a) Cost Matrix

	RM	1	2	3	4	5	6	7	8	9	10	11	12	FF
RM	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	2	2	1	1	1	1
6	1	1	1	1	1	1	1	1	1	2	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1	3	3	1	1
9	1	1	1	1	1	1	1	1	1	1	1	1	4	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1	4
11	1	1	1	1	1	1	1	1	1	1	1	1	1	3
12	1	1	1	1	1	1	1	1	1	1	1	1	1	4
FF	1	1	1	1	1	1	1	1	1	1	1	1	1	1

(b) Link Betweenness Matrix

	RM	1	2	3	4	5	6	7	8	9	10	11	12	FF
RM	0.000	1.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	0.000	0.000	0.000	0.000	0.000	0.607	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.670	0.000	0.670	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	0.549	0.549	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.779	0.779	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.681	0.681	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.791	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.861	0.000	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.737	0.737	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.844	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.764
11	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.815
12	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.727
FF	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

(c) Weights (W) Matrix

	RM	1	2	3	4	5	6	7	8	9	10	11	12	FF
RM	0.000	1.000	1.000	1.000	0.670	1.155	1.219	0.522	1.308	1.751	1.414	0.965	1.478	2.940
1	0.000	0.000	0.000	0.000	0.000	0.607	0.000	0.000	0.413	0.413	0.304	0.304	0.349	0.734
2	0.000	0.000	0.000	0.000	0.670	0.000	0.670	0.522	0.522	0.530	0.834	0.385	0.448	1.276
3	0.000	0.000	0.000	0.000	0.000	0.549	0.549	0.000	0.374	0.808	0.275	0.275	0.682	0.930
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.779	0.779	0.000	1.244	0.574	0.000	1.418
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.681	0.681	0.502	0.502	0.575	1.210
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.791	0.000	0.000	0.668	0.485
7	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.861	0.000	0.000	0.657
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.737	0.737	0.000	1.164
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.844	0.613
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.764
11	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.815
12	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.727
FF	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

(d) N Matrix

Figure 3: Matrix Calculations

Step 5: Determine the Indicative Consumer Surplus

When considering the path commonality factors, the consumer surplus term presented in Eq. (11) needs to be modified as below;

$$E\{\mathrm{Max}_{p\in P} U_p\} = \frac{1}{\alpha}\ln\left(\sum_{p\in P}\exp - \alpha(c_p + \beta \ln CF_p)\right) \qquad \textbf{(14)}$$

The above term can be simplified as $\frac{1}{\alpha}\ln(v_{ij})$, where v_{ij} is the element in row *i* and column *j* of the **N** matrix. In particular, we are interested in the element of **N** matrix in Raw Material (RM) row and Focal Firm (FF) column (i.e. $V_{RM\text{-}FF}$, which represents the maximum expected utility derived by the focal firm from the supply chains which connect it with the upper-most level suppliers). Therefore, the indicative consumer surplus, V_{RM-FF}^{Base} offered by the original SCN configuration illustrated in **Figure 1** (using α =0.05 and β =1) is 2.940.

Step 6: Carry out randomisation to determine the optimal SCN configuration

In this study we use a modified DPR technique to generate ensembles of SCN configurations. Since the material flow SCNs are strictly tiered, in addition to the degree of each node we also conserve the numbers of links present between all pairs of tiers. Therefore, we refer to this process as the Tier Constrained-Degree Preserving Randomisation (TC-DPR). At each time step, TC-DPR process picks a pair of links which lie across the same two tiers and swaps their target nodes. This process is applied to the original network until at least one link pair is rewired – then, steps 1-5 are carried out to determine if there is an improvement in the solution.

Note that preservation of degree of each node in each realised network configuration implies that we maintain the same 'contractual load' by each firm compared to the original SCN configuration. Similarly, preservation of the number of links between each pair of tiers implies that we maintain the same number of 'exchange relationships' between each tier pair compared to the original SCN configuration. Therefore, the resulting network is a null model whose degree distribution and the tier structure is identical to the original SCN.

In particular, the following algorithm has been used to identify the optimal SCN configuration;

Algorithm 1: Algorithm for optimising the consumer surplus using tier constrained degree preserving randomisation

Process the cost matrix;
Establish the link betweenness value matrix;
Calculate the weights (W) matrix;
Calculate the (N) matrix;
Determine the $V_{RM-FF(Base)}$ value;
$V_{RM-FF} := V_{RM-FF(Base)}$;
while V_{RM-FF} *is not converged* **do**
    Randomly pick two links (a, b) and (c, d) between T_i, T_{i+1}, where T_i and T_{i+1} are two adjancet tiers;
    `/* The following condition ensures the uniqueness of the two links selected, in terms of source and destination nodes */`
    if $a = c$ *or* $c = d$ **then**
        Continue to pick another link pair;
    `/* The following condition prevents creation of multi-links between node pairs */`
    if *Link* (a, d) *or link* (c, b) *already exists* **then**
        Continue to pick another link pair;
    Remove links (a, b) and (c, d), and create links (a, d) and (c, b);
    Calculate $V_{RM-FF(New)}$;
    if $\frac{V_{RM-FF(New)}}{V_{RM-FF(Base)}} < 1$ **then**
        Remove links (a, d) and (c, b), and add links (a, b) and (c, d);
    else
        $V_{RM-FF} := V_{RM-FF(New)}$

Figure 4: Modified TC-DPR Algorithm for Finding the Optimal SCN Configuration

For the example presented in **Figure 1**, the above algorithm converges within a few iterations (see **Figure 5**). **Figure 6** illustrates the optimal SCN configuration obtained, along with one intermediate configuration.

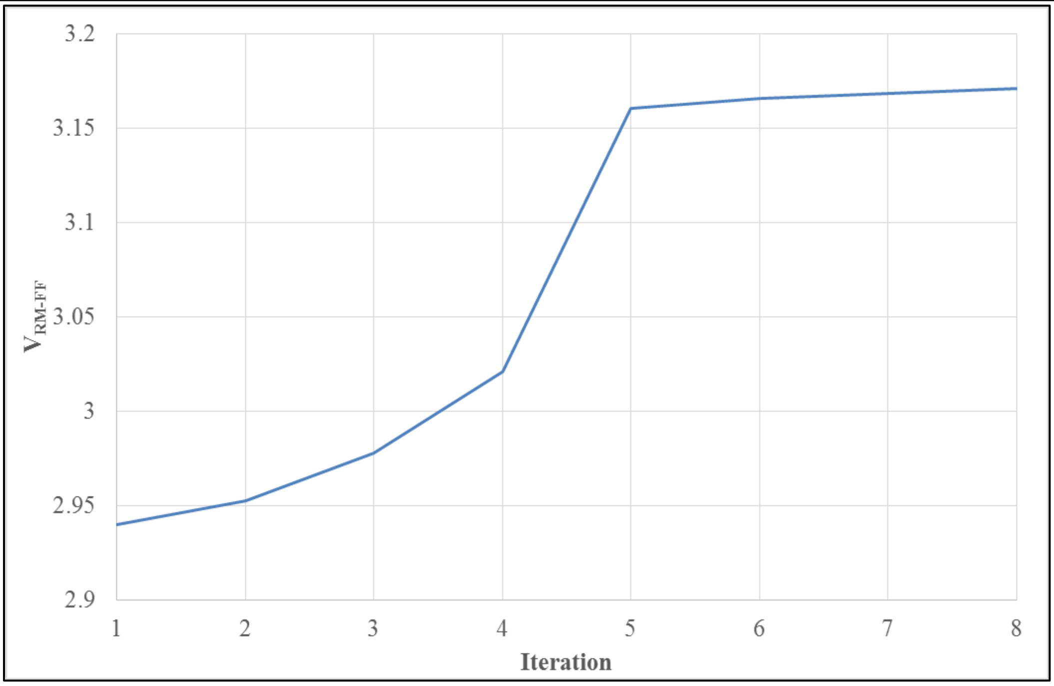
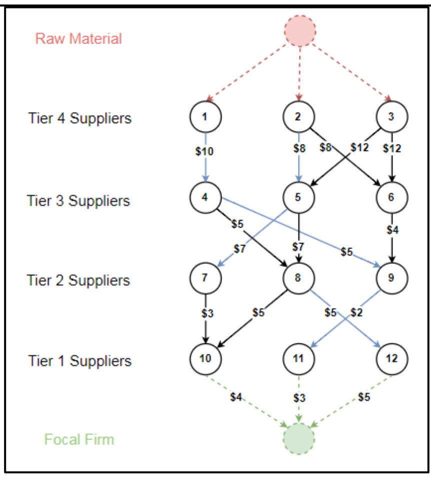
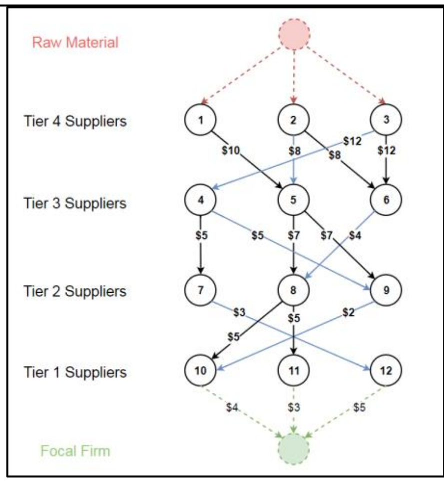
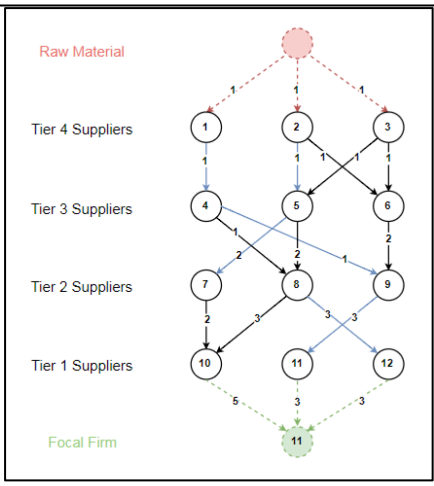
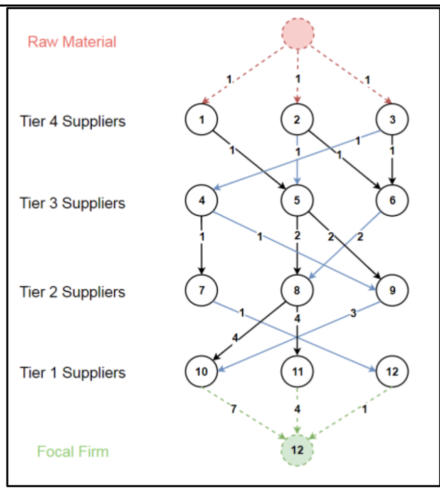


Figure 5: Convergence of VRM-FF for the toy example

TC-DPR SCN Configuration		
Link BWC Values		

Weights (W) Matrix

Left (Intermediate realisation):

	RM	1	2	3	4	5	6	7	8	9	10	11	12	FF
RM	0.000	1.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	0.000	0.000	0.000	0.000	0.607	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000	0.670	0.670	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	0.549	0.549	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.779	0.779	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.681	0.681	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.791	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.831	0.000	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.737	0.000	0.737	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.856	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.755
11	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.815
12	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.737
FF	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Right (Optimal configuration):

	RM	1	2	3	4	5	6	7	8	9	10	11	12	FF
RM	0.000	1.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	0.000	0.000	0.000	0.000	0.000	0.607	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000	0.670	0.670	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.549	0.000	0.549	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.779	0.000	0.779	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.681	0.681	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.791	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.861	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.727	0.727	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.856	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.743
11	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.803
12	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.779
FF	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

N Matrix

Left (Intermediate realisation):

	RM	1	2	3	4	5	6	7	8	9	10	11	12	FF
RM	0.000	1.000	1.000	1.000	0.607	1.219	1.219	0.830	1.302	1.437	1.650	1.230	0.960	2.956
1	0.000	0.000	0.000	0.000	0.607	0.000	0.000	0.000	0.472	0.472	0.348	0.405	0.348	0.849
2	0.000	0.000	0.000	0.000	0.000	0.670	0.670	0.456	0.456	0.530	0.716	0.454	0.336	1.159
3	0.000	0.000	0.000	0.000	0.000	0.549	0.549	0.374	0.374	0.434	0.586	0.372	0.275	0.949
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.779	0.779	0.574	0.667	0.574	1.400
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.681	0.681	0.000	1.068	0.000	0.502	1.176
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.791	0.000	0.677	0.000	0.552
7	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.831	0.000	0.000	0.628
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.737	0.000	0.737	1.100
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.856	0.000	0.698
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.755
11	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.815
12	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.737
FF	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Right (Optimal configuration):

	RM	1	2	3	4	5	6	7	8	9	10	11	12	FF
RM	0.000	1.000	1.000	1.000	0.549	1.277	1.219	0.427	1.833	1.297	2.443	1.332	0.368	3.171
1	0.000	0.000	0.000	0.000	0.000	0.607	0.000	0.000	0.413	0.413	0.654	0.300	0.000	0.726
2	0.000	0.000	0.000	0.000	0.000	0.670	0.670	0.000	0.986	0.456	1.108	0.717	0.000	1.398
3	0.000	0.000	0.000	0.000	0.549	0.000	0.549	0.427	0.434	0.427	0.681	0.315	0.368	1.046
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.779	0.000	0.779	0.667	0.000	0.670	1.018
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.681	0.681	1.078	0.495	0.000	1.198
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.791	0.000	0.575	0.575	0.000	0.888
7	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.861	0.670
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.727	0.727	0.000	1.123
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.856	0.000	0.000	0.636
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.743
11	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.803
12	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.779
FF	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Figure 6: Two realisations of the SCN (Left: Intermediate realisation, Right: Optimal configuration)

The change in consumer surplus, against the original SCN configuration, for each realisation presented in **Figure 6**, can be calculated as follows;

Intermediate realisation

$$\Delta CS = V_{RM-FF}^{Intermediate} / V_{RM-FF}^{Base} = 2.956 / 2.940 = 1.005$$

Optimal configuration

$$\Delta CS = V_{RM-FF}^{Optimal} / V_{RM-FF}^{Base} = 3.171 / 2.940 = 1.079$$

As per the above, the optimal SCN configuration provides 7.9% more consumer surplus compared to that offered by the original SCN configuration.

3.2 Parameter sensitivity and model validation

The consumer surplus measure presented in Eq (14) includes two parameters, α and β. The α parameter is referred to as the dispersion parameter and it controls the sensitivity of supply chain choice to the cost as perceived by the focal firm. When α is set to zero, the focal firm is insensitive to cost and as a result all supply chains will be chosen with equal probabilities. As α tends to infinity, the focal firm becomes increasingly sensitive to cost, resulting in low cost supply chains being chosen with higher probabilities compared to the higher cost ones. In the example calculation, we set α =0.05.

The β parameter controls the relative cost of the log of supply chain commonality factor against the supply chain cost. It can be viewed as the monetary cost of robustness, i.e. how much the focal firm is willing to spend to have a more robust SCN? When β is zero, the overlapping supply chains are not penalised in the consumer surplus calculation for the focal firm. As a result, the supply chain choice will only depend on the supply chain cost. When β is very large, the commonality factor of each supply chain will dominate the consumer surplus and supply chain choice will mostly be based on the level of overlap of each supply chain with others. In the example calculation, we set β at unity, since the supply chain path costs and commonalities are comparable.

The proposed method is computationally efficient as it does not require a-priori path enumerations for each network configuration. In order to validate the results obtained from the matrix calculations in our example, the paths were enumerated for the original SCN in **Figure 1** along with the two configurations shown in **Figure 6**. **Tables 1-3** below illustrate the results obtained for each SCN configuration (with and without considering the path commonality). As can be seen, the summation of logit probabilities considering the path commonalities of each supply chain represents the element of **N** matrix in Raw Material (RM) row and Focal Firm (FF) column in each SCN configuration. It is interesting to note that, in addition to commonalities, the total number of supply chains available for the focal firm has also changed (in the optimal configuration).

Table 1: Path enumerated logit calculation for the original SCN configuration

Path ID (Node IDs)	Link Cost ($)				Total Path Cost ($)	Exp(-α(Cost))	Link BWCs				Path Commonality Factor (CFp)	Exp(-α(Cost+β*LN(CFp)))
	Tier 4 - Tier 3 Suppliers	Tier 3 - Tier 2 Suppliers	Tier 2 - Tier 1 Suppliers	Tier 1 Suppliers - Focal Firm			Tier 4 - Tier 3 Suppliers	Tier 3 - Tier 2 Suppliers	Tier 2 - Tier 1 Suppliers	Tier 1 Suppliers - Focal Firm		
1 (RM-1-5-8-10-FF)	10	7	5	4	26	0.273	1	2	3	4	24	0.232
2 (RM-1-5-8-11-FF)	10	7	5	3	25	0.287	1	2	3	3	18	0.248
3 (RM-1-5-9-12-FF)	10	7	2	5	24	0.301	1	2	4	4	32	0.253
4 (RM-2-4-7-10-FF)	8	5	3	4	20	0.368	1	1	1	4	4	0.343
5 (RM-2-4-8-10-FF)	8	5	5	4	22	0.333	1	1	3	4	12	0.294
6 (RM-2-4-8-11-FF)	8	5	5	3	21	0.350	1	1	3	3	9	0.314
7 (RM-2-6-9-12-FF)	8	4	2	5	19	0.387	1	2	4	4	32	0.325
8 (RM-3-5-8-10-FF)	12	7	5	4	28	0.247	1	2	3	4	24	0.210
9 (RM-3-5-8-11-FF)	12	7	5	3	27	0.259	1	2	3	3	18	0.224
10 (RM-3-5-9-12-FF)	12	7	2	5	26	0.273	1	2	4	4	32	0.229
11 (RM-3-6-9-12-FF)	12	4	2	5	23	0.317	1	2	4	4	32	0.266
					Sum	**3.393**					**Sum**	**2.940**

Table 2: Path enumerated logit calculation for Realisation 1

Path ID (Node IDs)	Link Cost ($)				Total Path Cost ($)	Exp(-α(Cost))	Link BWCs				Path Commonality Factor (CFp)	Exp(-α(Cost+β*LN(CFp)))
	Tier 4 - Tier 3 Suppliers	Tier 3 - Tier 2 Suppliers	Tier 2 - Tier 1 Suppliers	Tier 1 Suppliers - Focal Firm			Tier 4 - Tier 3 Suppliers	Tier 3 - Tier 2 Suppliers	Tier 2 - Tier 1 Suppliers	Tier 1 Suppliers - Focal Firm		
1 (RM-1-4-8-10-FF)	10	5	5	4	24	0.301	1	1	3	5	15	0.263
2 (RM-1-4-8-12-FF)	10	5	5	5	25	0.287	1	1	3	3	9	0.257
3 (RM-1-4-9-11-FF)	10	5	2	3	20	0.368	1	1	3	3	9	0.330
4 (RM-2-5-7-10-FF)	8	7	3	4	22	0.333	1	2	2	5	20	0.287
5 (RM-2-5-8-10-FF)	8	7	5	4	24	0.301	1	2	3	5	30	0.254
6 (RM-2-5-8-12-FF)	8	7	5	5	25	0.287	1	2	3	3	18	0.248
7 (RM-2-6-9-11-FF)	8	4	2	3	17	0.427	1	2	3	3	18	0.370
8 (RM-3-5-7-10-FF)	12	7	3	4	26	0.273	1	2	2	5	20	0.235
9 (RM-3-5-8-10-FF)	12	7	5	4	28	0.247	1	2	3	5	30	0.208
10 (RM-3-5-8-12-FF)	12	7	5	5	29	0.235	1	2	3	3	18	0.203
11 (RM-3-6-9-11-FF)	12	4	2	3	21	0.350	1	2	3	3	18	0.303
					Sum	**3.407**					**Sum**	**2.956**

Table 3: Path enumerated logit calculation for Realisation 2

Path ID (Node IDs)	Link Cost ($)				Total Path Cost ($)	Exp(-α(Cost))	Link BWCs				Path Commonality Factor (CFp)	Exp(-α(Cost+β*LN(CFp)))
	Tier 4 - Tier 3 Suppliers	Tier 3 - Tier 2 Suppliers	Tier 2 - Tier 1 Suppliers	Tier 1 Suppliers - Focal Firm			Tier 4 - Tier 3 Suppliers	Tier 3 - Tier 2 Suppliers	Tier 2 - Tier 1 Suppliers	Tier 1 Suppliers - Focal Firm		
1 (RM-1-5-8-10-FF)	10	7	5	4	26	0.273	1	2	4	7	56	0.223
2 (RM-1-5-8-11-FF)	10	7	5	3	25	0.287	1	2	4	4	32	0.241
3 (RM-1-5-9-10-FF)	10	7	2	4	23	0.317	1	2	3	7	42	0.263
4 (RM-2-5-8-10-FF)	8	7	5	4	24	0.301	1	2	4	7	56	0.246
5 (RM-2-5-8-11-FF)	8	7	5	3	23	0.317	1	2	4	4	32	0.266
6 (RM-2-5-9-10-FF)	8	7	2	4	21	0.350	1	2	3	7	42	0.290
7 (RM-2-6-8-10-FF)	8	4	5	4	21	0.350	1	2	4	7	56	0.286
8 (RM-2-6-8-11-FF)	8	4	5	3	20	0.368	1	2	4	4	32	0.309
9 (RM-3-4-7-12-FF)	12	5	3	5	25	0.287	1	1	1	1	1	0.287
10 (RM-3-4-9-10-FF)	12	5	2	4	23	0.317	1	1	3	7	21	0.272
11 (RM-3-6-8-10-FF)	12	4	5	4	25	0.287	1	2	4	7	56	0.234
12 (RM-3-6-8-11-FF)	12	4	5	3	24	0.301	1	2	4	4	32	0.253
				Sum	**254**	**3.480**				**Sum**	**402**	**3.171**

4.0 RESULTS

This section presents the results obtained from applying the above derived methodology to a real world large scale SCN.

Willems (2008) provides a dataset of real world multi echelon (i.e. multi-tiered) SCNs, used for inventory optimization purposes. The overall dataset includes a total of 38 multi echelon SCNs, from various industries. The SCNs described in this paper comprise actual supply chain maps created by either company analysts or consultants. Since these maps have been implemented in practice, they demonstrate the structure of actual SCNs.

The above-mentioned dataset from Willems (2008) includes the following key information;

- The industry sector of each SCN;
- For each SCN;
  - The stages (nodes) representing each firm involved; and
  - The arcs (links) representing precedence relationship between stages.
- For each stage (node);
  - Its classification and tier based on its function within the overall supply chain;
  - The direct cost added at the stage (stage cost); and
  - The average processing time at the stage (stage time).

We have used the SCN #29 (primary batteries, dry and wet) in the above dataset to implement the proposed method. The original SCN 29 included a total of 617 firms. However, this SCN was subsequently modified to include only the complete supply chains which span across all tiers of the network. In this regard, 49 firms which represented partial supply chains were removed. The following table outlines the tier structure and cost range information for the firms in the processed SCN.

Table 4: Number of firms and cost ranges within each tier of the SCN

Tier Level	No. of Firms	Cost Range	No. of Links
Tier 5 Suppliers	79	\$0.01 - \$3.01	
			111
Tier 4 Suppliers	16	\$1.37 - \$4.06	
			55
Tier 3 Suppliers	54	\$0.01 - \$2.65	
			54
Tier 2 Suppliers	54	\$0.97 - \$3.44	
			365
Tier 1 Suppliers	365	\$0.01 - \$2.15	
Total	**568**		**585**

It is noted that this SCN is characterised by a highly heterogeneous degree distribution (see **Figure 6**), which follows power law (in line with other SCNs reported in literature, see Perera et al., 2017). **Figure 7** provides a visualisation of this SCN with pseudo nodes added to the top and bottom to represent the raw materials and the focal firm, respectively.

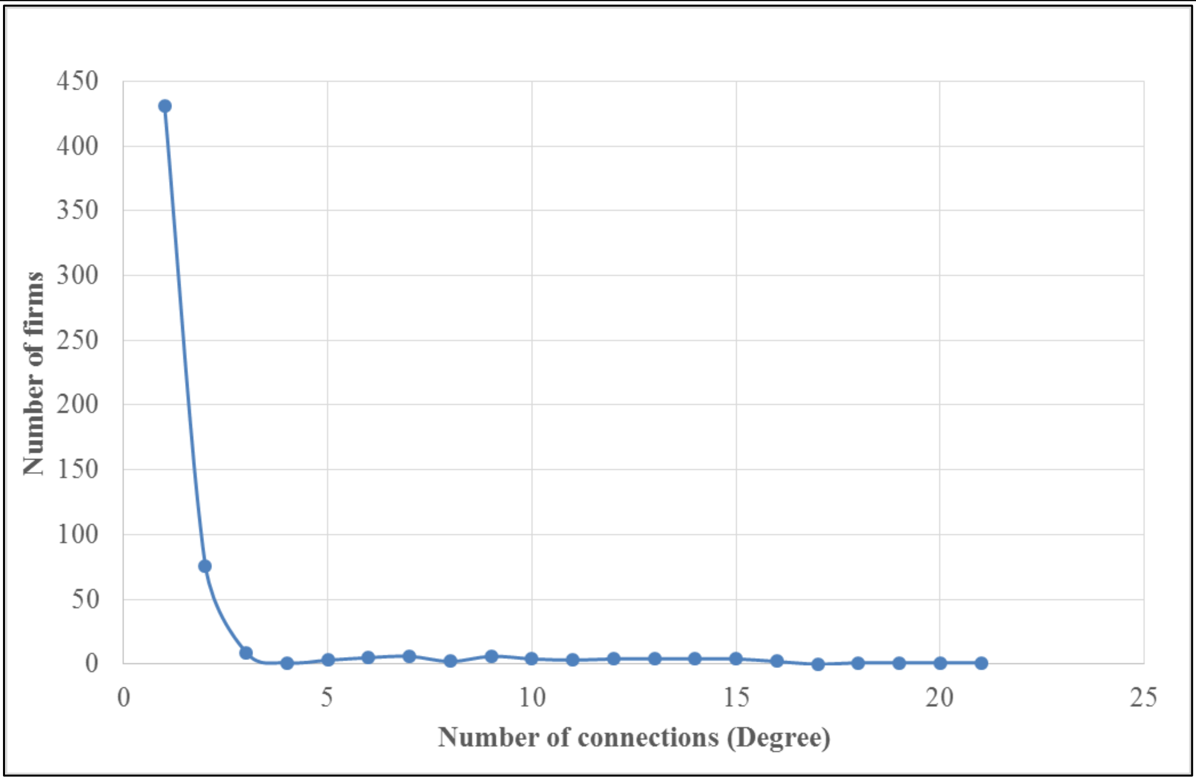


Figure 6: Degree distribution of the SCN considered for analysis

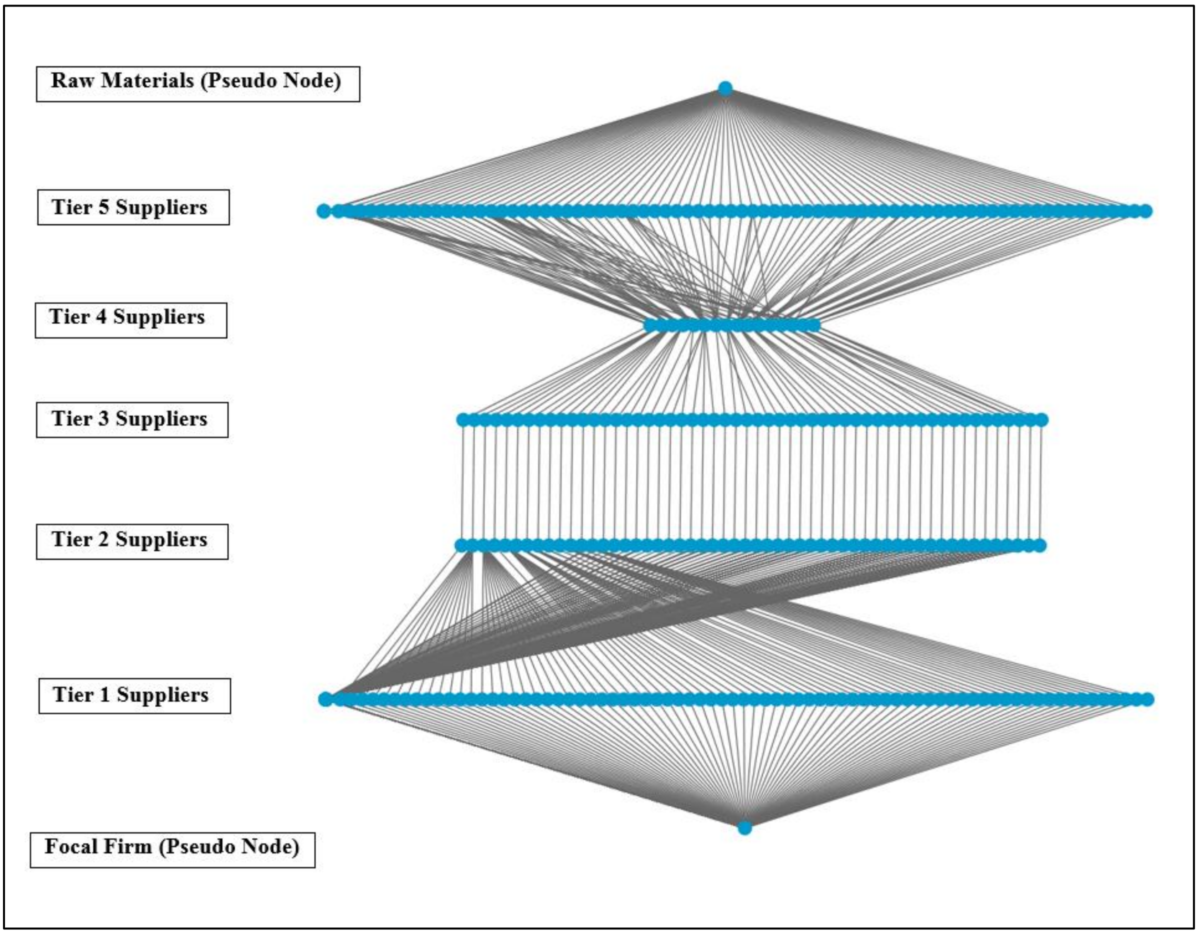


Figure 7: Structure of the SCN analysed

Applying the algorithm presented in **Figure 4** (with α =0.05 and β =1) to the above mentioned SCN gives the following result (see **Figure 8**). As can be seen, the base SCN includes a V_{RM-FF} of 846.00 which improves up to 1005.76 and plateaus at iteration number 992. Therefore, the reconfigured optimal SCN provides 19% higher consumer surplus compared to the base SCN configuration.

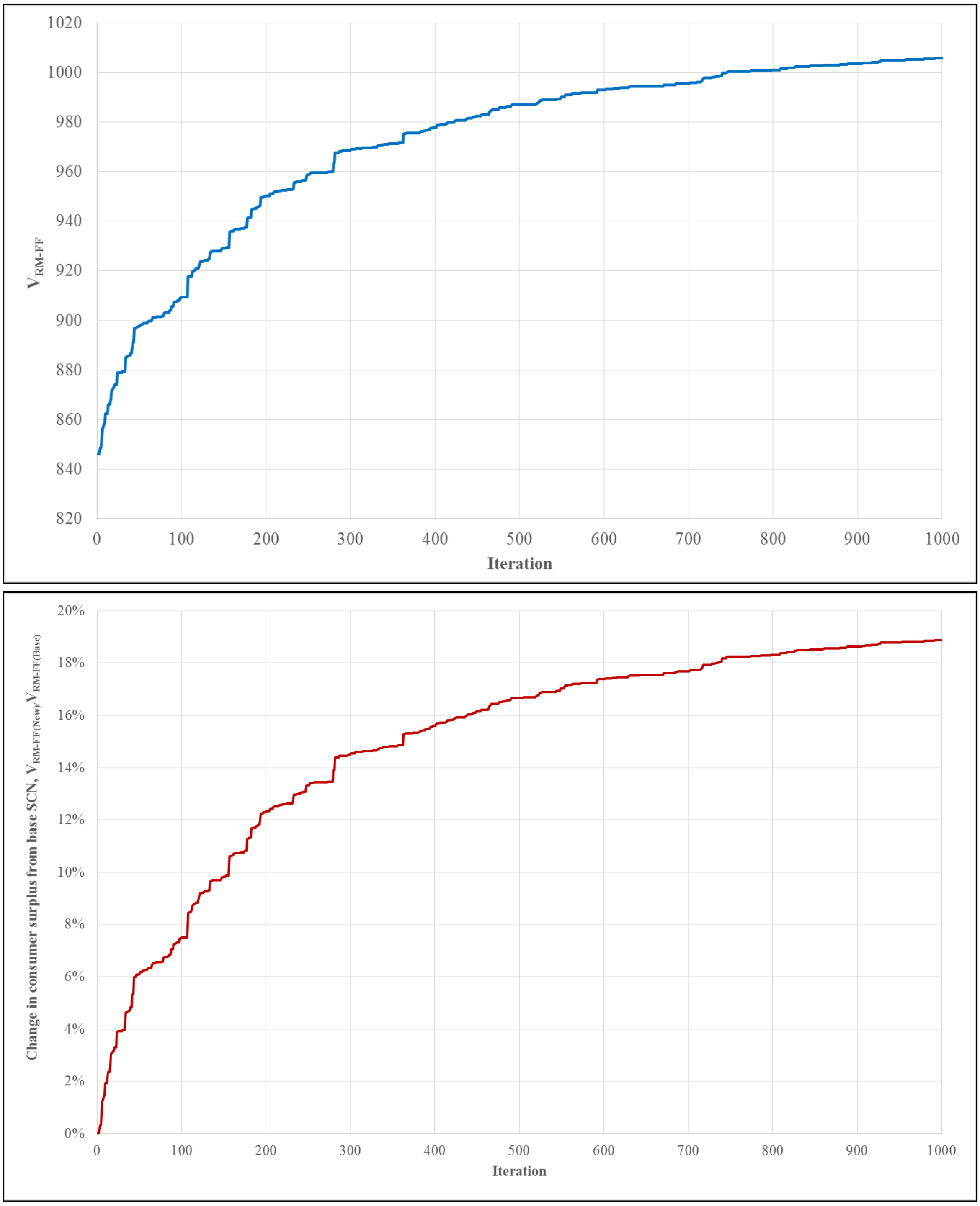


Figure 8: Convergence of maximum achievable consumer surplus (top) and change in consumer surplus from the base SCN configuration (bottom)

5.0 DISCUSSION

Robustness of SCNs have been modelled in literature using network science metrics or simulations which sequentially remove nodes (randomly or targeted by degree or some other topological attribute) and record at each time step, the size of the largest connected component and/or the average shortest path length in the largest connected component. Such topology based methods assume homogeneity in SCN components, in terms of importance, and are generally illustrated for cyclic SCNs where the links indicate inter-firm relationships. Additionally, the high level of abstraction in these models limit their real world applicability.

By adopting the perspective of the consumer(s) of the SCN, this paper has presented a novel methodology to quantify and improve the robustness of tiered material flow SCNs. We assume that while the consumer(s) (indicated by the focal firm) are trying to minimise the cost of products, due to multi sourcing requirements, they may have to use products of higher costs. This phenomenon is captured by assuming the random perturbation of cost of products. By assuming the Gumbel distributions for perturbation terms, the probability of getting products from upstream firms can be expressed by the multinomial logit model.

The final SCN configuration is robust in the sense that it includes the minimal supply chain overlaps. Sheffi and Rice (2005) note the importance of building flexibility and redundancies into SCNs as a way of improving the robustness of these systems. In this regard, parallel supply paths with minimal dependencies could be incorporated into SCNs, so that a disruption in one firm does not impact the operations of the other.

6.0 CONCLUSIONS AND FUTURE WORK

This paper has presented a novel methodology to assess and improve the robustness of material flow SCNs. The proposed model incorporates information beyond the topology of the SCN as is a useful tool for decision making when access to operational data is limited. Future work could include a budget in relation to supply link rewiring between firms during TC-DPR process.

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