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Imposing Symmetry On A Complete Matrix Of Commuter Travel Elasticities

by

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ABSTRACT: Travel price and time elasticities are increasingly being derived from discrete choice models of the multinomial or nested logit form. These elasticities are then applied to obtain predictions of changes in travel demand consequent on a policy change in prices and travel times. The majority of the choice elasticities are estimated within the behavioural setting of modal choice, holding total travel fixed. A few mode choice models have recently relaxed the strong assumption of constant variance in the random components of the indirect utility function to enable the derivation of behaviourally meaningful cross choice elasticities. Under constant variance, only the direct choice elasticities have behavioural meaning. While this advance in discrete choice modelling is to be applauded, the procedures used derive share elasticities conditional on a fixed total demand, and in addition make no corrections for two important conditions required to 'convert' the choice elasticity matrix into a demand elasticity matrix - namely symmetry and share weighted column sums. This paper takes a set of empirical choice elasticities and shows the procedures required to adjust these elasticities to arrive at a matrix of demand elasticities. We draw on a recent data set collected in Sydney which utilises revealed preference and stated choice data to estimate a joint model of ticket choice conditional on mode and choice of mode for commuter travel.

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Introduction

When analysts depended primarily on time series for price and travel data, it was difficult to estimate cross-price demand parameters for any type of travel, particularly urban travel where the evaluation set of market alternatives might include new alternatives and/or an extended set of attribute levels outside of the range observed in the market. This difficulty has been largely overcome by the fusion of revealed and stated choice data (Morikawa, 1989, Hensher 1996). In addition, recent studies by Bhat (1995, 1996) and Hensher (1996, 1997) have enriched the standard methods of deriving direct elasticities (ie multinomial and nested logit) to capture the behavioural richness required to produce estimates of cross choice elasticities, derived by the relaxation of the constant variance assumption of the random component of the indirect utility expression associated with each alternative. This method was used in the 1995 inquiry into transit fare levels and mixes for Sydney (Hensher and Raimond 1996). The objective was to determine the sensitivity of Sydney residents to changes in public transport fares and to establish a full matrix of own and cross price elasticities for each transport mode and ticket type. To test potential pricing policies, the ordinary demand elasticities are needed but it is shown that, for the case of commuters only, choice and ordinary elasticities are approximately equal.

It is well established theoretically and empirically that ordinary elasticities conform to the symmetry condition (Brown and Deaton, 1972; Barten, 1977). This is a matter of internal and mutual consistency between the fare elasticities. Reliable evaluations of urban transit policies, as discussed by Glaister and Lewis (1978) and De Borger, Mayers, Proost and Wouters (1996), can only be made on the basis of mutually consistent estimates of ordinary demand elasticities. The work reported in this paper was done to optimally adjust the elasticity matrix, estimated by combined revealed and stated choice methods, so that it satisfies the symmetry condition.

The paper is organised as follows. We begin with a formalisation of the relationship between choice and ordinary demand elasticites, which is followed by a discussion of the essential constraints which have to be imposed on an elasticity matrix to enable conversion of choice elasticities to demand elasticities. Next we present the full matrix of corrected demand elasticities based on choice elasticities and the optimisation method required to minimise derive empirical demand elasticites conditioned on the imposed theoretical constraints. The empirical setting is briefly presented followed by the presentation of the matrices of choice and demand elasticities. We summarise the main findings in the conclusion.

Choice and Ordinary Elasticities

The relationship between an ordinary elasticity e_{ij} and the corresponding choice elasticity m_{ij} is:

$$e_{ij} = m_{ij} + \frac{\P Q}{\P P_j} \cdot \frac{P_j}{Q}$$

where $\frac{\P Q}{\P P_j} \cdot \frac{P_j}{Q} = h_j$ is the generation or second stage elasticity (Taplin 1982). The change in aggregate traffic volume $\P Q$ is in response to a price change $\P P_j$ on travel alternative j. In many cases, it is difficult to estimate the generation/suppression elasticity but a simplifying assumption can be made for the commuter market. It is assumed that the number of commuter trips is fixed in the short run, meaning that the aggregate traffic volume will remain constant regardless of a price variation for any mode or ticket type. This means that the generation/suppression elasticity is taken to be zero for all transport modes. Such an assumption could not be made for non-commuter trips[1]. In the commuter case, the assumption of constant aggregate travel yields $m_{ij} - e_{ij} = 0$, so that matrix **M** (the matrix of choice elasticities) and matrix **B** (the matrix of ordinary elasticities) are identical.

Constraints and Model Specification

Spending on commuter travel represents a small portion of total household expenditure and can be considered separable from all other expenditure items. In this closed demand system, symmetry and substitutability are important.

[1] An approximate method of estimating generation elasticities, based on income elasticities, is indicated for the general travel case in Taplin (forthcoming).

Symmetry

Symmetry between cross-elasticities is derived from the basic theoretical equality (Green 1976, p.312), x_i and x_j being quantities consumed and p_i and p_j prices:

$$\left[\frac{\partial x_i}{\partial p_j}\right]_{compensated} = \left[\frac{\partial x_j}{\partial p_i}\right]_{compensated}$$

The "compensated" subscript indicates that income compensation keeps the household at a constant level of utility as price changes. The equality follows from the fact that the second derivatives in the utility maximisation are equal (Deaton and Muellbauer 1980 p.44, Theil 1975 p.3). The Slutsky relationship without compensated cross-price effects is obtained by adding an income compensating term to each side:

$$\frac{\partial x_i}{\partial p_j} + x_j \frac{\partial x_i}{\partial \Psi} = \frac{\partial x_j}{\partial p_i} + x_i \frac{\partial x_j}{\partial \Psi}$$

If ε_{ij} is elasticity of demand for x_i with respect to p_j and ε_{iy} is elasticity of demand for x_i with respect to income Y, the relationship becomes:

$$\varepsilon_{ij} \frac{x_i}{p_j} + x_j \; \varepsilon_{iy} \frac{x_i}{Y} = \varepsilon_{ji} \frac{x_j}{p_i} + x_i \; \varepsilon_{jy} \frac{x_j}{Y}$$

Multiplying by $\frac{Y}{x_i x_j}$ and substituting shares of total expenditure, $R_i = \frac{p_i x_i}{Y}$ and $R_j = \frac{p_j x_j}{Y}$

$$\frac{1}{R_j} \varepsilon_{ij} + \varepsilon_{iy} = \frac{1}{R_i} \varepsilon_{ji} + \varepsilon_{jy}$$

$$\varepsilon_{ij} = \frac{R_j}{R_i} \varepsilon_{ji} + R_j \left(\varepsilon_{jy} - \varepsilon_{iy}\right)$$

This symmetry equation reflects consumers' consistency of preferences. Tests of the relationship have been influenced by model specification and have mainly been performed on complete consumer demand systems, often using the Rotterdam model. Symmetry has been generally verified by such empirical tests (Deaton and Muellbauer 1980, p.69, Blundell 1988, Theil 1975, p.197) and prevails most clearly between close substitutes.

In the travel case, e_{ij} is the elasticity of demand for mode-ticket i with respect to fare j. Here, the Slutsky income correction, the second term on the right hand side, can be omitted as R_i the expenditure proportion is very small and the income elasticity difference $\left(e_{jy} - e_{iy}\right)$ is also small. The ratio R_j/R_i represents the ratio of expenditures on the two mode-ticket choices. In the absence of the income effect, symmetry requires equalisation of expenditure weighted elasticities.

Gross substitutes

Because some urban commuters use more than one mode on each work trip, there are some complementarities between modes of transport. For example, where a bus feeds to a railway station, a reduction in bus fare would tend to attract passengers to the train. In far more cases, however, bus and train are substitutes and commuters who have a choice will tend to change from train to bus if the bus fare is reduced. The same applies to substitution between ticket types. Similarly, car and public transport are complementary in the cases of park-and-ride and kiss-and-ride but cases in which car and public transport are substitutes are much more common. On these grounds, it has been assumed that the complementarities are outweighed by the substitution effects resulting from any fare or cost change. This means that the ticket types and the car also are gross substitutes, so that the cross-price elasticities are constrained to be non-negative:

$$e_{ij} \geq 0, \quad i \neq j$$

Share weighted column sum

If s_k is the share of choices going to k, a complete matrix of choice elasticities has the following property.

$$\sum_{k=1}^{n} s_k m_{kj} = 0$$

This means that, in a mode-choice system, patronage diverted from the j^{th} mode due to a rise in price must be allocated to the other choices.

The Model

When the assumptions and constraints are taken into account, the matrix of ordinary demand elasticities for commuters takes the following form:

$$E = \begin{bmatrix} \boldsymbol{e}_{11} = \frac{1}{-s_1}\left(\sum_{\substack{i=1 \\ i \neq 1}}^{n} s_i \boldsymbol{e}_{i1}\right) + \boldsymbol{h}_1 & \boldsymbol{e}_{12} = \frac{R_2}{R_1}\boldsymbol{e}_{21} + \boldsymbol{h}_2 & \boldsymbol{e}_{13} = \frac{R_3}{R_1}\boldsymbol{e}_{31} + \boldsymbol{h}_3 & \boldsymbol{e}_{1n} = \frac{R_n}{R_1}\boldsymbol{e}_{n1} + \boldsymbol{h}_n \\ \boldsymbol{e}_{21} + \boldsymbol{h}_1 & \boldsymbol{e}_{22} = \frac{1}{-s_2}\left(\sum_{\substack{i=1 \\ i \neq 2}}^{n} s_i \boldsymbol{e}_{i2}\right) + \boldsymbol{h}_2 & \boldsymbol{e}_{23} = \frac{R_3}{R_2}\boldsymbol{e}_{32} + \boldsymbol{h}_3 & \boldsymbol{e}_{2n} = \frac{R_n}{R_2}\boldsymbol{e}_{n2} + \boldsymbol{h}_n \\ \boldsymbol{e}_{31} + \boldsymbol{h}_1 & \boldsymbol{e}_{32} + \boldsymbol{h}_2 & \boldsymbol{e}_{33} = \frac{1}{-s_3}\left(\sum_{\substack{i=1 \\ i \neq 3}}^{n} s_i \boldsymbol{e}_{i3}\right) + \boldsymbol{h}_3 & \\ & & & \\ \boldsymbol{e}_{n1} + \boldsymbol{h}_1 & \boldsymbol{e}_{n2} + \boldsymbol{h}_2 & \boldsymbol{e}_{n3} + \boldsymbol{h}_3 & \boldsymbol{e}_{nn} = \frac{1}{-s_n}\left(\sum_{\substack{i=1 \\ i \neq n}}^{n} s_i \boldsymbol{e}_{in}\right) + \boldsymbol{h}_n \end{bmatrix}$$

Where: s_i is the trip share of mode or fare type i;

R_i is the expenditure share of mode or fare type I ;

$R_j/R_i\ e_{ij}$ represents the symmetry effect;

$\frac{1}{-s_i}\left(\sum_{\substack{i=1 \\ i \neq 2}}^{n} s_i e_{ii}\right) + h_i$ results from the share weighted column sum constraint;

the generation/suppression elasticity for column j, $h_j = 0$ for all j; and

$e_{ij} \geq 0,\ \ i \neq j$ acts as a bounded constraint for the variable estimates.

Matrix E is a complete set of elasticities that satisfy symmetry and zero weighted column sum. The above diagonal elasticities have the functional form of below diagonal elasticity parameters and the diagonal elasticities, divided by their negative travel share, are equal and opposite to the sum of all other elasticities weighted by their travel shares. An important feature of matrix E is that both trip shares (s_i) and expenditure shares (R_i) enter the calculations.

Matrix E is a set of ordinary demand elasticities conditioned by theoretical constraints, which are not all satisfied by the estimated matrix of discrete-choice elasticities. In order to generate matrix E we have to violate the elasticities obtained by discrete-choice estimation. To make these violations as small as possible and particularly to avoid large changes, the elasticities have been adjusted so that sum of the squared deviations of the constrained ordinary elasticities in E from the corresponding elasticities in M is minimised.

$$f = \min\Big(\underset{\text{for } r=c}{\sum_i \left(m_{rc} + \frac{1}{si} \sum_{\substack{j \\ j \neq i}} s_i \boldsymbol{e}_{ij} \right)^2} + \underset{\text{for } r>c}{\sum_j \left(m_{rc} - \boldsymbol{e}_{ij} \right)^2} + \underset{\text{for } r<c}{\sum_j \left(m_{rc} - \frac{R_i}{R_j} \boldsymbol{e}_{ij} \right)^2} \Big)$$

Where m_{rc} is the discrete choice estimate of the elasticity on mode r with respect to price on c, belonging to matrix **M**:

$$\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} & . & . & . & M_{1N} \\ M_{21} & M_{22} & & & & M_{2N} \\ . & & . & & & . \\ . & & & . & & . \\ . & & & & . & . \\ M_{N1} & . & . & . & . & M_{NN} \end{bmatrix}$$

A Newton method was used to find the changes which minimise the sum of squared deviations. For each e_{ij}, the coefficient of the squared term is positive:

$$\left(\frac{R_j S_j}{R_i S_i} \right)^2 + \left(\frac{S_i}{S_j} \right)^2 + \left(\frac{R_i}{R_j} \right)^2 + 1$$

Thus, the surface is simply a quadratic in more than one variable and has a unique minimum. Nevertheless, convergence was also tested by reversing the functional dependency between the elements above and below the diagonal and by using various starting values. Each specification converged on the same set of values.

The Empirical Context

In a survey of 324 Sydney commuters, each respondent was asked to reveal characteristics of their current transport behaviour and state their preferred method of transport under a selection of price scenarios (Hensher and Raimond 1996). Survey respondents were asked to think about the last trip they made, where they went, how they travelled, how much it cost etc., and then were asked to describe an alternative way of making that trip if their current mode was not available. The current behaviour provided the revealed preference data. The stated preference component of the survey involved a series of different pricing scenarios for current and alternative methods of travel.

The choice of mode and ticket type is estimated using a mixture of revealed preference (RP) and stated preference (SP) data. The RP data's strengths lie in reflecting the current state of market behaviour, whereas the SP data's strengths are that it mirrors a more robust and less restricted decision environment and presents a well-conditioned design matrix. RP data provides information on the current market equilibrium for the behaviour of interest and is useful for short term forecasting of departures from the current equilibrium. In contrast SP data is especially rich in attribute trade-off information, but is to some extent affected by the degree of 'contextual realism' that we can establish for the respondents (Hensher 1994). In deriving estimates of elasticities, the set of choice probabilities must reflect observed market behaviour (ie market shares), and hence we use the RP model enriched by the parameter estimates produced from the SP data appropriately re-scaled for each alternative when transferred to the RP model.

In order to offer realistic scenarios to all respondents, there was a range of showcards with different modal combinations and different travel distances. They covered every combination of main mode (car, train, bus and ferry) with short trips (less than 15 minutes), medium trips (15-30 minutes) and long trips (over 30 minutes). Ticket prices were varied 50% above and below prevailing levels. An illustrative showcard in presented in Table 1. Each respondent was presented with four different scenarios and different respondents are presented with different combinations of scenarios.

Table 1. Illustrative Set of Show Cards for the SP Experiment 1: Bus or Train for a Short Trip

You have told us that you could either use a Bus or a Train as the main form of transport to travel to the destination that we have discussed. If public transport fares changed and were priced as below, would you have used Bus or Train as the main form of transport for your trip? Which ticket type would you choose?

BUS FARES		TRAIN FARES	
Single	$0.60	Single	$0.80
TravelTen	$4.00	Off Peak Return	$0.90
(10 single trips)		*(purchase after 9am)*	
TravelPass	$8.60	Weekly	$6.80
(7 days bus/ferry)		*(7 days train only)*	
TravelPass	$10.00	TravelPass	$10.00
(7 days bus/ferry/train)		*(7 days bus/ferry/train)*	

A fractional factorial design was used, each respondent being presented with four scenarios. Different respondents were presented with different combinations of scenarios. Responses to the different scenarios were recorded in terms of which mode and which fare type would be used, these individual travel responses providing the data for the derivation of aggregate mode choice elasticities with respect to fare prices. See Hensher and Raimond (1996) for more details.

The magnitudes of these shares are shown in Table 2. The single fares account for much larger shares of expenditure than of trips. Car travel accounts for a slightly smaller share of expenditure than of trips.

Table 2 Sydney Commuters: Trip and Spending Shares by Mode and Fare Type

	Trip Share %	Spending Share %
Train Single	3.4	6.4
Train Off-Peak Return	0.3	0.3
Train Weekly	10.1	16.8
Train Travelpass (BFT)	1.7	2.7
Bus Single	4.2	4.3
Bus Travel Ten	7.0	4.6
Bus Travelpass (BF)	3.2	2.0
Bus Travelpass (BFT)	3.9	2.3
Ferry Single	0.4	0.8
Jet Cat Single	0.1	0.3
Ferry Ten	0.4	0.4
Jet Cat Ten	0.1	0.2
Ferry Travelpass (BF)	0.3	0.3
Ferry Travelpass (BFT)	0.2	0.2
Car	64.8	58.2
Total	100.0	100.0

Source: Hensher and Raimond 1996

Results

Table 3 shows the results of the adjustment process for the own-price elasticities. Some of the changes in elasticities are substantial. In the Bus Single case, where the originally estimated elasticity is small in absolute value, the percentage change is large.

The complete original and adjusted matrices of own and cross-elasticities are shown in the Appendix. Although there have been some large percentage changes in cross-elasticities, because the original values were small, the differences in actual magnitude have more significance for forecasting the effects of fare changes. The largest decrease is in elasticity of demand for Ferry Single tickets with respect to the price of a Ferry Travelpass (BFT) from 0.217 to 0.011. The largest increase is in Bus Travelpass (BFT) with respect to Bus Travel Ten from 0.116 to 0.267.

Table 3 Deviations of Optimally Adjusted Own-Price Elasticities from Original Estimates: Sydney Commuters

	Own-Price Elasticity			
	Original	Adjusted	Change	% Change
Train Single	-0.080	-0.122	-0.042	-53
Train Off-Peak Return	-0.123	-0.186	-0.063	-51
Train Weekly	-0.250	-0.225	0.025	10
Train Travelpass (BFT)	-0.529	-0.521	0.008	1
Bus Single	-0.078	-0.189	-0.111	-143
Bus Travel Ten	-0.383	-0.336	0.047	12
Bus Travelpass (BF)	-0.813	-0.696	0.117	14
Bus Travelpass (BFT)	-0.822	-0.665	0.157	19
Ferry Single	-0.183	-0.211	-0.028	-15
Jet Cat Single	-0.268	-0.313	-0.045	-17
Ferry Ten	-0.344	-0.343	0.001	0
Jet Cat Ten	-1.943	-1.941	0.002	0
Ferry Travelpass (BF)	-0.347	-0.340	0.007	2
Ferry Travelpass (BFT)	-0.308	-0.306	0.002	1
Car	-0.014	-0.024	-0.010	-72

Indicative Comparisons with Other Estimates

In demand studies based on household consumption data, the level of aggregation is varied by forming composite commodities before estimation, as desired by the analyst (Green 1976, Deaton and Muellbauer 1980). This is not feasible in the present study which was designed to analyse behaviour at the highly disaggregated level of ticket types. The only point of aggregating is to make indicative comparisons with previous estimates and to consider the broad relationships between whole modes. In this situation, an approximate method of condensing the fare-type elasticities into a matrix of modal elasticities is used:

The own-price elasticity for train $$\boldsymbol{e}^{TT} = \sum_{i \in T} \sum_{j \in T} R_i^T \boldsymbol{e}_{\mathbf{ij}}$$

The cross-price elasticity for train with respect to bus fare $$\boldsymbol{e}^{TB} = \sum_{i \in T} \sum_{j \in B} R_i^T \boldsymbol{e}_{\mathbf{ij}}$$

The cross-price elasticity for bus with respect to train fare $\boldsymbol{e}^{BT} = \sum_{i \in B} \sum_{j \in T} R_i^B \boldsymbol{e}_{\mathbf{ij}}$

and so on, where the train and bus the weights are:

$R_i^T = \frac{R_i}{\sum_i R_i}$ **where** $i \in T$, **The set of demand functions for ticket types on the train mode**

$R_i^B = \frac{R_i}{\sum_i R_i}$ **where** $i \in B$, **The set of demand functions for ticket types on the bus mode**

Similar weighted sums are calculated for all modal own-price and cross elasticities (Table 4). The resulting approximations can be applied to a uniform percentage fare change within a mode. The method preserves the theoretical properties of the elasticity matrix at all levels, including symmetry.

Table 4 Optimally Adjusted Commuter Elasticities Condensed to Modes

	Elasticity w.r.t. Fare or Cost of Trips by:			
Travel Mode	Train	Bus	Ferry	Car
Train	-0.156	0.032	0.003	0.037
Bus	0.063	-0.070	0.006	0.046
Ferry	0.039	0.037	-0.195	0.003
Car	0.016	0.011	0.000	-0.024

The calculated own-price elasticities for train and bus of -0.156 and -0.07 are appreciably less elastic than the London peak travel elasticities[2] of -0.30 and -0.35 estimated by Glaister and Lewis (1978) and our cross-elasticities between train and bus are also smaller. However, our cross-elasticities with respect to car operating cost (Table 4) of 0.037 for train and 0.046 for bus are comparable to the Glaister and Lewis estimates of 0.056 and 0.025.

[2] The Glaister and Lewis (1978) estimates are expressed as compensated elasticities but they do not differ appreciably from the ordinary elasticities.

Table 4 provides two indicators that price is not particularly persuasive on commuter mode choices. First, the own-price elasticity for each mode is smaller than the own-price elasticities for ticket types (Table 2) because the cross-elasticities between tickets within the mode group are substantial. A price increase on a ticket type mainly diverts travellers to another ticket type on the same mode. The second indicator is more direct: cross-elasticities between modes (Table 4) are small, indicating that price is a minor factor in commuter mode choices. This is consistent with discrete choice studies in the Eindhoven area of the Netherlands (Richards and Ben-Akiva, 1975) which found price to have little influence on commuter mode choice. However, among these small elasticities, the elasticity of demand for bus travel with respect to train fares of 0.063 is relatively large.

A further step was to condense the three public transport modes into one (Table 5). The resulting own-price elasticity of -0.082 can be compared to the transit elasticities in the range -0.09 to -0.52 recorded in a review of aggregate studies by Oum, Waters and Yong (1992). Not all of those studies were for peak travel; commuters lie at the inelastic end of the range.

Table 5 Commuter Elasticities Condensed to Public Transport and Car

	Elasticity w.r.t. Fare or Cost of Trips by:	
Travel Mode	Public Transport	Car
Public Transport	-0.082	0.038
Car	0.027	-0.024

Conclusions

In most cases, travel choice elasticity estimates are inadequate for analysing the effects of pricing policies. The travel generation/suppression responses, which are embodied in ordinary demand elasticities, are also needed. Consequently, choice elasticities alone are insufficient to forecast responses to price changes in non-commuter travel markets.

In the case of commuter travel, however, it can be assumed that the number of trips is approximately fixed, at least in the short run so that the only responses to a fare or price change are shifts between modes and ticket types. In the long run price changes may induce shifts in total demand. This means that there are virtually no generation/suppression elasticities and the choice elasticities are approximately the same as the ordinary elasticities. This was assumed in Sydney where a choice analysis was based on a survey of 324 commuters who were asked to reveal characteristics of their current transport behaviour and state their preferred method of transport under a selection of price scenarios. From the responses, a complete matrix of choice elasticities was derived for 14 ticket types used for trips by train, bus and ferry, as well as car trips.

Although the choice elasticities can be treated as approximations to the ordinary elasticities, the estimated matrix will be reliable for pricing policy analysis only if it conforms to the symmetry condition for ordinary demand systems. To achieve this, each upper diagonal element (cross-elasticity) of the matrix was expressed as a symmetric function of the corresponding lower diagonal element. Then, each own-price elasticity was expressed as an exact function of the cross-elasticities in its column, using the choice condition that the trip-weighted elasticities in each column sum to zero. The lower diagonal elements were then adjusted, using a Newton procedure, to minimise the sum of the squared deviations from all of the original values. In effect, all elements of the matrix were subject to change. The condition that all cross-elasticities must be non-negative was also imposed, meaning that the modes and ticket types were assumed to be gross substitutes.

The results include some substantial deviations of the elasticities in the adjusted symmetric matrix from the original values. Although most of the own-price elasticities do not change a great deal, the Bus Single fare own-elasticity changed from -0.078 to -0.189. A number of the cross-elasticities have changed by large percentages because the original values were very small. The absolute magnitude of changes is of more interest, the largest decrease in a cross-elasticity being in demand for Ferry Single tickets with respect to the price of a Ferry Travelpass (BFT) from 0.217 to 0.011. The largest increase was in Bus Travelpass (BFT) with respect to Bus Travel Ten from 0.116 to 0.267.

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APPENDIX 1
The Stated Choice Experiment Fare Categories and Levels

Train: Single (Off Peak Return)	*Low Fare*	*Current Fare*	*High Fare*
Short Medium Long	\$0.80 (\$0.90) \$1.30 (\$1.40) \$1.80 (\$2.00)	\$1.60 (\$1.80) \$2.60 (\$2.80) \$3.60 (\$4.00)	\$2.40 (\$2.60) \$3.90 (\$4.20) \$5.40 (\$6.00)
Train: Weekly	*Low Fare*	*Current Fare*	*High Fare*
Short Medium Long	\$6.80 \$9.70 \$13.20	\$11.50 \$19.40 \$26.00	\$18.30 \$29.00 \$40.00
Train: TravelPass	*Low Fare*	*Current Fare*	*High Fare*
Short Medium Long	\$10.00 \$14.00 \$20.00	\$20.00 \$28.00 \$39.00	\$30.00 \$42.00 \$59.00
Bus: Single	*Low Fare*	*Current Fare*	*High Fare*
Short Medium Long	\$0.60 \$1.30 \$2.00	\$1.20 \$2.50 \$3.90	\$1.80 \$3.80 \$5.90
Bus: TravelTen	*Low Fare*	*Current Fare*	*High Fare*
Short Medium Long	\$4.00 \$8.00 \$16.00	\$8.00 \$16.00 \$32.00	\$12.00 \$24.00 \$48.00
Bus:TravelPass (Bus/Ferry)	*Low Fare*	*Current Fare*	*High Fare*
Short Medium Long	\$8.60 \$11.70 \$17.20	\$17.10 \$23.00 \$34.00	\$26.00 \$35.00 \$52.00
Bus: TravelPass (Bus/Ferry/Train)	*Low Fare*	*Current Fare*	*High Fare*
Short Medium Long	\$10.00 \$14.00 \$19.50	\$20.00 \$28.00 \$39.00	\$30.00 \$42.00 \$59.00

APPENDIX 2

Original And Adjusted Matrices Of Commuter Demand Elasticities

Travel By Mode and Fare Type:	**ELASTICITY OF DEMAND WITH RESPECT TO FARE OR TRAVEL COST BY:**														
	Train				**Bus**				**Ferry**						**Car**
	Single	Off-Peak Return	Week-ly	Travel pass (BFT)	Single	Travel Ten	Travel pass (BF)	Travel pass (BFT)	Single	JetCat Single	Ferry Ten	JetCat Ten	Travel pass (BF)	Travel pass (BFT)	
ORIGINAL															
Train Single	-0.080	0.033	0.123	0.141	0.002	0.010	0.008	0.009	0.002	0	0.010	0	0.010	0.011	0.014
Off-Peak Ret	0.048	-0.123	0.161	0.200	0.003	0.011	0.010	0.011	0.003	0	0.008	0	0.008	0.008	0.016
Weekly	0.010	0.012	-0.250	0.059	0.001	0.004	0.003	0.003	0	0	0.006	0	0.007	0.008	0.009
Travelpass (BFT)	0.014	0.013	0.084	-0.529	0.001	0.008	0.006	0.007	0.001	0	0.010	0	0.010	0.010	0.011
Bus Single	0.002	0.002	0.011	0.013	-0.078	0.165	0.171	0.182	0.001	0.001	0.005	0.001	0.005	0.005	0.006
TravelTen	0.001	0.001	0.009	0.010	0.025	-0.383	0.087	0.098	0.001	0.001	0.007	0.001	0.006	0.007	0.005
Travelpass (BF)	0.002	0.002	0.011	0.013	0.040	0.133	-0.813	0.147	0.001	0.001	0.006	0.002	0.005	0.006	0.005
Travelpass (BFT)	0.002	0.001	0.009	0.010	0.033	0.116	0.113	-0.822	0.001	0.001	0.006	0.001	0.006	0.006	0.005
Ferry Single	0.011	0.010	0.056	0.057	0.005	0.027	0.022	0.025	-0.183	0.034	0.209	0.041	0.212	0.217	0.004
JetCat Single	0.002	0.002	0.012	0.015	0.009	0.045	0.039	0.046	0.062	-0.268	0.194	0.092	0.188	0.196	0.003
Ferry Ten	0.002	0.001	0.022	0.018	0.001	0.008	0.004	0.006	0.011	0.006	-0.344	0.008	0.071	0.085	0.002
JetCat Ten	0.002	0.001	0.011	0.015	0.007	0.036	0.032	0.038	0.078	0.095	0.145	-1.943	0.194	0.213	0.004
Travelpass (BF)	0.002	0.001	0.021	0.017	0.001	0.008	0.005	0.006	0.010	0.006	0.068	0.007	-0.347	0.082	0.002
Travelpass (BFT)	0.001	0.001	0.018	0.013	0.001	0.006	0.004	0.005	0.008	0.005	0.054	0.006	0.054	-0.308	0.002
Car	0.001	0.001	0.015	0.013	0.001	0.007	0.004	0.006	0.008	0.006	0.004	0.005	0.004	0.005	-0.014

ADJUSTED

Train Single	-0.122	0.000	0.076	0.022	0	0	0.001	0.002	0	0	0	0.000	0.000	0	0
Off-Peak Ret	0.012	-0.186	0.123	0.160	0	0	0	0	0	0	0	0	0	0	0
Weekly	0.029	0.002	-0.225	0.017	0	0.012	0.007	0.010	0.001	0	0.001	0.000	0.000	0.000	0.053
Travelpass (BFT)	0.050	0.015	0.103	-0.521	0	0.033	0.037	0.046	0.007	0	0.003	0.001	0.003	0.001	0.025
Bus Single	0	0	0	0	-0.189	0.024	0.040	0.051	0	0	0	0.000	0	0	0
TravelTen	0	0	0.043	0.019	0.022	-0.336	0.110	0.134	0	0	0.001	0.002	0.001	0.000	0.039
Travelpass (BF)	0.002	0	0.062	0.051	0.087	0.254	-0.696	0.285	0	0	0.004	0.004	0.004	0.001	0.086
Travelpass (BFT)	0.006	0	0.075	0.054	0.094	0.267	0.245	-0.665	0	0	0.004	0.004	0.004	0.001	0.110
Ferry Single	0	0	0.025	0.023	0	0	0	0	-0.211	0	0.041	0.019	0.024	0.011	0
JetCat Single	0	0	0	0	0	0	0	0	0	-0.313	0.048	0.050	0.020	0.015	0
Ferry Ten	0	0	0.024	0.021	0	0.013	0.020	0.023	0.077	0.037	-0.343	0.055	0.059	0.036	0.003
JetCat Ten	0.004	0	0.015	0.019	0.011	0.045	0.044	0.050	0.082	0.090	0.130	-1.941	0.152	0.122	0.011
Travelpass (BF)	0.002	0	0.026	0.023	0	0.020	0.028	0.031	0.063	0.022	0.083	0.091	-0.340	0.052	0.009
Travelpass (BFT)	0	0	0.020	0.015	0	0.010	0.015	0.016	0.046	0.025	0.078	0.112	0.080	-0.306	0.004
Car	0	0	0.015	0.001	0	0.003	0.003	0.004	0	0	0	0	0	0	-0.024

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