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Public Transport Timetables and Vehicle Scheduling with Balanced Passenger Loads

By

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ABSTRACT: This work attempts to combine the creation of public transport timetables and vehicle scheduling so as to improve the correspondence of vehicle departure times with passenger demand while minimising the resources (the fleet size required). The methods presented for handling the two components simultaneously can be applied for both single and interlining transit routes, and can be carried out in an automated manner. With the growing problems of transit reliability, and advance in the technology of passenger information systems, the importance of even and clock headways is reduced. This allows for the possibility to create more efficient schedules from both the passenger and operator perspectives. The methodology framework contains a developed algorithm for the derivation of vehicle departure times (timetable) with even average loads and smoothing consideration in the transition between time periods. It is done while ensuring that the derived timetables will be carried out by the minimum number of vehicles. The procedures presented are accompanied by examples and clear graphical explanations. It is emphasised that the public timetable is one of the predominant bridges between the operator (and community) and the passengers.

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1. Introduction

There is a saying that "A stitch in time would have confused Einstein". Along this line one can say that many stitches in a public transport timetables would confuse the passengers. No doubt that the public timetable is one of the predominant bridges between the operator (and/or the community) and the passengers. Therefore more attention should be provided for the construction of timetable in order to improve its correspondence with the fluctuated passenger demand.

In general terms, the public transport operational planning process includes four basic components performed in sequence: (1) network route design, (2) setting timetables, (3) scheduling vehicles to trips, and (4) assignment of drivers (crew). It is desirable for all the four components to be planned simultaneously to exploit the system's capability to the greatest extent and maximize the system's productivity and efficiency. However this planning process is extremely cumbersome and complex, and therefore seems to require separate treatment of each component, with the outcome of one fed as an input to the next component. In the last twenty years, a considerable amount of effort has been invested in the computerization of the four components mentioned above, in order to provide more efficient controllable and responsive schedules. The best summary as well as the accumulative knowledge of this effort was presented in the second through the seventh International Conferences on Transit Scheduling, and appear in the book edited by Wren (1981), Rousseau (1985), Daduna and Wren (1988), Desrochers and Rousseau (1992), Daduna, Branco, and Paixao (1995), and Wilson (1999).

This work attempts to combine the two components of creating timetables and vehicle scheduling so as to improve the correspondence of vehicle departure times with passenger demand while minimizing the resources (the fleet size required). While the vehicle scheduling problem is treated extensively in the transit scheduling conference proceedings, only small attention is given in these proceedings to the problem of efficiently constructing vehicle frequencies and timetables.

Mathematical programming methods for determining frequencies and timetables have been proposed by Furth and Wilson (1981), Koutsopoulos, Odoni, and Wilson (1985), Ceder and Stern (1984), and Ceder and Tal (1999). The objective in Furth and Wilson is to maximize the net social benefit, consisting of ridership benefit and wait time saving, subject to constraints on total subsidy, fleet size and passenger loading levels. Koutsopoulos *et al* extended this formulation by incorporating crowding discomfort costs in the objective function and treating the time dependent character of transit demand and performance. Their initial problem comprises a non-linear optimization program relaxed by linear approximations. Ceder and Stern addressed the problem with an integer programming formulation and heuristic person-computer interactive procedure. The latter approach focuses on reconstructing timetables when the available vehicle fleet is restricted. Finally Ceder and Tal used mixed integer programming and heuristic procedures for constructing timetables with maximum synchronization. That is maximization of the number of simultaneous arrivals of vehicles to connection stops.

Other methods for frequency and timetable determination are related to the type and adequacy of the input passenger count data. These methods aimed at practicability appear in Ceder (1984, 1986) and are briefly described in the following (Background) **section 2**. In **section 3** the scope and framework of this study are outlined. In **section 4** an algorithm is proposed for the derivation of vehicle departure times with even average loads and smoothing techniques in the transition between time periods. In **section 5** the proposed algorithm is interpreted and implemented graphically. In **section 6** the timetable construction procedure is integrated with the creation of chains of trips (Blocks), and **sections 7** and **8** provide an example and concluding remarks. Finally is worth mentioning that with the growing problems of transit reliability, and advance in the technology of passenger information systems, the importance of even headways and clock headways (see definitions next section) is reduced. This allows for the possibility to create more efficient schedules from both the passenger and the operator perspectives.

2. Background

2.1 Optimal Timetables

Public transport timetable is commonly constructed for given sets of derived frequencies. The basic criteria for the determination of frequencies are: (a) to provide adequate vehicle's space to meet passenger demand, and (b) to assure a minimum frequency (maximum-policy headway) of service. Ceder (1984) described four different methods for calculating the frequencies. Two are based on point-check (counting the passengers on-board the transit vehicle at certain point(s)), and two – on ride-check (counting the passengers along the entire transit route). In the point-check methods the frequency is the division between passenger load at the maximum (max) load point (either the one across the day or in each hour) and the desired occupancy or load factor. In the ride-check methods the frequency is the division between and average or restricted-average passenger load and the desired occupancy. The average load is determined by the area under the load profile (in passenger-km) divided by the route length (km), and the restricted average is a higher value than the average one, in order to assure that in certain percentage of the route length the load does not exceed the desired occupancy. This desired occupancy (or load factor) is the desired level of passenger load on each vehicle, in each time period (e.g. number of seats).

In a follow-up study Ceder (1986) analyzed optional ways for generating public timetables. This analysis allows for establishing a spectrum of alternative timetables, based on three categories of options: (a) selection of type of headway, (b) selection of frequency determination method for each period, and (c) selection of special requests. In category (a) the headway (time interval between adjacent departures) can be equal or balanced. Equal headway refers to the case of evenly spaced headways and balanced headway – to the case of unevenly spaced headways but with even average passenger load at the hourly maximum load point. These cases are being extended in this work. In category (b) it is possible to select for each time period one of the four frequency determination methods (two point-check, and two ride check) mentioned above, or a given frequency by the scheduler. In category (c) it is possible to request clock headways (departure times that repeat themselves in each hour, easy-to-memorize) and/or certain number of departures (usually for cases with limited resources).

The outcome of these analyses is a set of optional timetables in terms of vehicle's departure times at all specified timepoints, using passenger load data. Each timetable is accompanied by two comparison measure which are used as an evaluation indicator in conjunction with resource saving. The first measure is the total required vehicle runs (departures) and the second is an estimate for the minimum required fleet size at the route level only.

2.2 Deficit Function

Following is a description of the deficit function approach described by Ceder and Stern (1981), for assigning the minimum number of vehicles to allocate for a given timetable. A deficit function is simply a step function that increases by one at the time of each trip departure and decreases by one at the time of each trip arrival. Such a function may be constructed for each terminal in a multiterminal transit system. To construct a set of deficit functions, the only information needed is a timetable of required trips. The main advantage of the deficit function is its visual nature. Let $d(k,t,S)$ denote the deficit function for the terminal k at the time t for the schedule S. The value of $d(k,t,S)$ represents the total number of departures minus the total number of trip arrivals at terminal k, up to and including time t. The maximal value of $d(k,t,S)$ over the schedule horizon $[T_1,T_2]$ is designated $D(k,S)$.

Let t_s^i and t_e^i denote the start and end times of trip i, $i \in S$. It is possible to partition the schedule horizon of $d(k,t,S)$ into sequence of alternating hollow and maximal intervals. The maximal intervals $[s_i^k, e_i^k], i = 1,\ldots,n(k)$ define the interval of time over which $d(k,t)$ takes on its maximum value. Note that the S will be deleted when it is clear which underlying schedule is being considered. Index i represents the *i*th maximal intervals from the left and $n(k)$ represents the total number of maximal intervals in $d(k,t)$. A hollow interval is defined as the interval between two maximal intervals. Hollows may consist of only one point, and if this case is not on the schedule horizon boundaries $(T_1$ or $T_2)$, the graphical representation of $d(k,t)$ is emphasized by clear dot.

If the set of all terminals is denoted as *T*, the sum of $D(k)$ for all $k \in T$ is equal to the minimum number of vehicles required to service the set *T*. This is known as the fleet size formula. Mathematically, for a given fixed schedule *S*:

$$D(S) = \sum_{k \in T} D(k) = \sum_{k \in T} \max_{t \in [T_1, T_2]} d(k,t) \qquad (1)$$

Where $D(S)$ is the minimum number of buses to service the set *T*.

When Deadheadings (DH) trips are allowed, the fleet size may be reduced below the level described in Equation 1. Ceder and Stern (1981) described a procedure based on the construction of a unit reduction DH chain (URDHC), which, when inserted into the schedule, allows a unit reduction in the fleet size. The procedure continues inserting URDHCs until no more can be included or a lower boundary on the minimum fleet in reached. The lower boundary $D_m(S)$ is determined from the overall deficit function defined as $g(t,S) = \sum_{k \in T} d(k,t,S)$ where $D_m(S) = \max_{t \in [T_1,T_2]} g(t,s)$. This function represents the number of trips simultaneously in operation. Initially, the lower bound was determined to be the maximum number of trips in a given timetable that are in simultaneous operation over the schedule horizon. Stern and Ceder (1983) improved this lower bound, to $D_m(S') > D_m(S)$ based on the construction of a temporary timetable, *S'*, in which each trip's arrival time is extended to the time of the first trip that may feasibly follow it in *S*.

The deficit function theory was extended by Ceder and Stern (1982) to include possible shifting in departure times within bounded tolerances. Basically, the shifting criteria is based on a defined tolerance time $\left[t_s^i - \Delta_a^i, t_s^i + \Delta_d^i\right]$ where Δ_a^i is the maximum advance of the trip scheduled departure time (early departure), and Δ_d^i is the maximum delay allowed (late departure). The maximum interval is then compared with the appropriate tolerance time elements for establishing conditions in which it is possible to reduce the fleet size by one via certain shifts.

The algorithms of the deficit function theory are described in detail by Ceder and Stern (1981, 1982). However, it is worth mentioning the next terminal (NT) selection rule and the URDHC routines. The selection of the NT in attempting to reduce its maximal deficit function may rely on the basis of garage capacity violation, or on a terminal whose first hollow is the longest. The rationale here is to try to open up the greatest opportunity for the insertion of the DH trip.

Once a terminal k is selected, the algorithm searches to reduce $D(k)$ by shifting departure times (if allowed). Then all of the $d(k,t)$ values are updated and the NT rule is again applied. When no more shiftings are possible, the algorithm searches for a URDHC from the selected terminal while considering possible blending between DH insertion and shiftings in departure times. In the URDHC routines there are four rules: R=0 for inserting the DH trip manually in a conversational mode, R=1 for inserting the candidate DH trip that has the minimum travel time, R=2 for inserting a candidate DH trip whose hollow starts farthest to the right, and R=3 for inserting a candidate DH trip whose hollow ends farthest to the right. In the automatic mode (R=1,2,3), if a DH trip cannot be inserted and the completion of a URDHC is blocked, the algorithm backs up to a DH candidate list and selects the next DH candidate on that list.

In the fixed schedule problem, the algorithm also terminates when $D(S)$ is equal to the improved lower bound. In the variable schedule problem (when shifting are allowed), the algorithm also uses this comparison, and if $D(S)$ is equal to the improved lower bound, the URDHC procedure (with shiftings) ceases and the shifting-only mode applied. If the latter results in reducing $D(S)$, the URDHC procedure is again activated. The process terminates when $D(S)$ cannot be further reduced.

Finally, all of the trips, including those that were shifted and the DH trips, are chained together for constructing the vehicle schedule (blocks). Two rules can be applied for creating the chains: first in-first out (FIFO), and a chain-extraction procedure described by Gertsbach and Gurevich (1977). The FIFO rule simply links the arrival time of a trip to the nearest departure time of another trip (at the same location), and continues to create a schedule until no connection can be made. The trips considered are deleted and the process continues. The chain-extraction procedure allows an arrival-departure connection for any pair within a given hollow (on each deficit function). The pairs considered are deleted and the procedure continues. Both methods end with the minimum derived number of vehicles (blocks).

3. Scope and Framework

3.1 Scope

Transit public timetable is perhaps the main reference for defining unreliable transit service. The assumption that passengers will adjust themselves to given timetables (with headways of, say, longer that 10 minutes) instead of adjusting the timetables to

the passenger demand is one of the largest sources of unreliable service. When passenger demand is not met, the transit vehicles are slowing down (increased dwell time), behind the schedule and entering the inevitable process of further slow down. This will eventually lead to the known bunching phenomenon with the vehicles behind. Opposite to that is the situation of overestimating the demand which may result in transit vehicles running ahead of time. Both situations are not observed when the service is highly frequent and characterized by low variance of the headway distribution.
It is the purpose of this work to establish a method for better matching the passenger demand with a given timetable while attempting to minimize the fleet size (one of the main resources). This will result in a more reliable and comfortable service. Fig. 1 illustrates the research progress regarding the construction of public timetables. Level 1 is related to the studies of Ceder (1984, 1986), where the average passenger load is counted at the max load point on an hourly basis, and the division of this load by the desired occupancy results in the frequency unless the minimum required frequency is not reached. The non-integer value of the frequency is then kept and based on the accumulative frequency curve (adding the frequency at each hour with respect to time), the departure times (Timetable 1) at the hourly max load point are determined with even headways. Level 2 is related to the study of Ceder (1986), where the average passenger loads are counted at the max load point of the route for each vehicle separately. These loads are accumulated with respect to time and based on the desired occupancy values the departure times (Timetable 2) at the route maximum load point are determined with uneven headways and even average loads only at this route point. Level 3 represents this work where the average passenger loads are counted at each vehicles max load point as opposed to the route max load point at level 2. In order to derive the departure times (Timetable 3) with even load at the critical max load point of each vehicle an algorithm is developed in the next section. This algorithm is then applied graphically in section 5. All the Timetables (1,2,3) in Fig.1 are also based on a smoothing procedure between the time periods such that there is no need to round any number, and the desired occupancy is kept in these transition periods.

3.2 Framework

The study presented in this work is shown in a flowchart format in Fig. 2. This flowchart has three columns: input, component, and output. The output of the components is also served as an input to a next component. Basically Fig. 2 is the framework of the study with the following input: Network of transit routes; Set of time periods; Average loads on the transit vehicles at their max load points; Average trip travel times; Average trip layover times; Average DH travel times; Tolerances for the departure time shifting; and Tolerances for the desired occupancies (load factors). The overall study process starts with the derivation of vehicle departure times with even average loads and smoothing consideration in the transition between time periods. Then the initial timetable is constructed and a new set of possible departure time shifting is determined. The next step is the construction of what is described in the background section and known as Deficit Functions (DF) where in every departure (at a certain terminal or major stop) the DF is moving up by one and every arrival the DF is moving down by one. Then the DFs are going through both shifting and dead-heading trip insertion procedures, and the timetable is adjusted while complying with the tolerance constraints. The final step is the establishment of vehicle schedules (blocks).

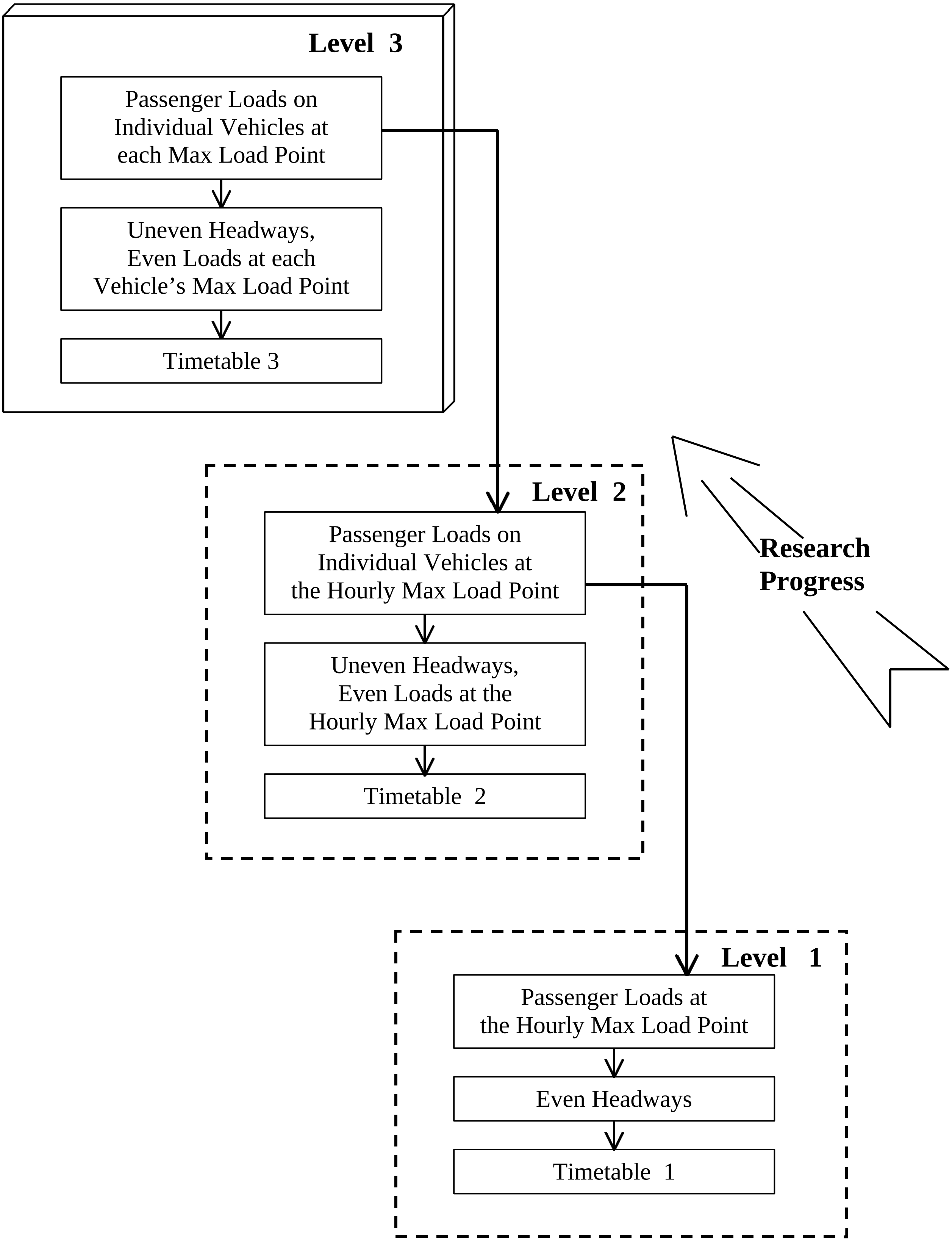


Fig. 1 Optional Timetables from a Balanced Passenger Load Perspective

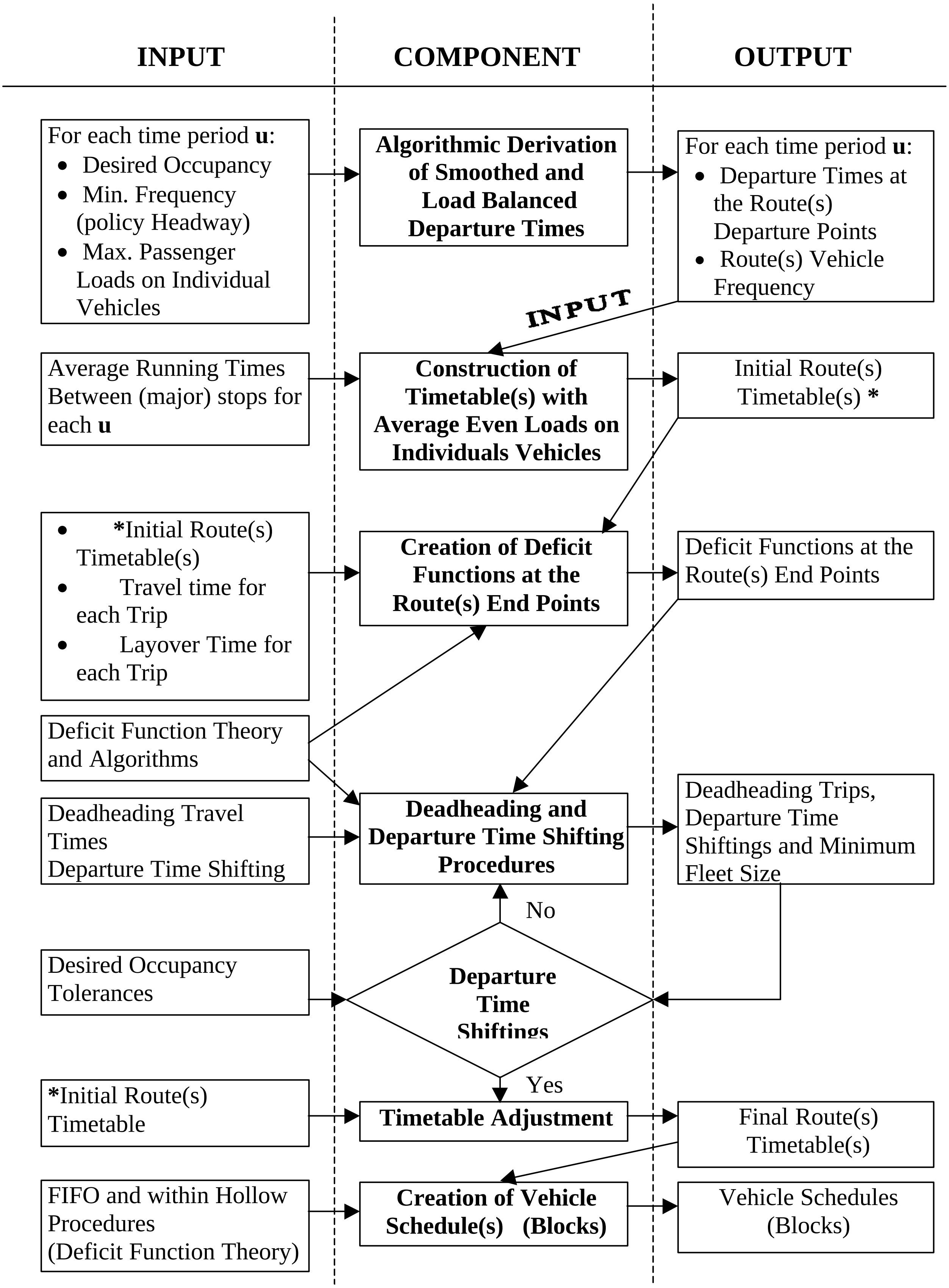


Fig. 2 Framework of study

4. An Algorithm for Balancing the Critical Loads

4.1 Definitions and Objective

Let: $L_i(t)$ = accumulative load (# of passengers) curve at stop i w.r.t. time t, where changes in the slope are made at the departure times of the transit vehicles at i, i = 1, 2, …, n, and the last slope is extrapolated to the schedule (timetable) horizon.

t_{ij} = the derived j-th candidate departure time at stop i, i = 1, 2, …, n, j = 1, 2, …, m.

T_{iu} = the average service travel time between the departure terminal i = 1 and stop i, i = 1 2, 3, …, n $(T_{1u} = 0)$ during time interval u = 1, 2, …, v.

d_u = desired occupancy (load factor) on each vehicle for each time interval u, u = 0, 1, 2, …, v.

t_u = transition time between time intervals, u and $u+1$, u = 1, 2, …, v.

F_{mu} = minimum required frequency (# of vehicles) in interval u, u=1, 2, …, v.

N_u = number of derived departures for $t_{u-1} \le t < t_u$, u = 1, 2, …, v.

d_{um} = desired occupancy for situations when F_{mu} is applied $d_{um} < d_u$.

The purpose of the algorithm presented below is to derive the transit timetable provided that in an average sense all vehicles will have even load (equal to the desired occupancy) at the max load stops of each vehicle. That is, for a given time period each vehicle may have a different max load point across the entire transit route with a different observed average load. The objective set forth is to change the departure times such that all observed average max loads will be same and equal to d_u during all u. The curve $L_i(t)$, which is the basic input of the analysis and has the same slope (straight line) between each two adjacent departures, represents uniform arrival rate of passengers.

Certainly the adjustments in the timetable are not intended for highly frequent urban services where the headway is less than say, 10 minutes, on an hourly frequency of about 6 vehicles or more. Behind this algorithm is the notion that passenger overcrowding situations (loads greater than d_u) should be avoided. Since the max load point is commonly defined for a time period of one hour or more, there are situations in which the critical points for individual vehicles do not coincide with this max load point. These critical points (max load points for individual vehicles) are treated by the following Algorithm *T*.

4.2 Algorithm *T*

Initialization : Set $j = u = 1$ and determine t_{i1} by calculating $L(t_{i1}) = d_1$ for all $i = 1, 2, \ldots, n$

Step 1 : Determine the $j-th$ departure time (during time interval u) at $t*_{1j}$ such that $t*_{1j} = \min\left[t_{1j}, (t_{2j} - T_{2u}), \ldots, (t_{nj} - T_{nu})\right]$

Step 2 : Determine $t_{i,j+1}$ by calculating $L_i\left(t_{i,j+1}\right) = L_i\left(t^*_{1j} + T_{iu}\right) + d_u$, for all i = 1, 2,…, n, where $t_{i,j+1}$ belongs to u. When $t_{i,j+1}$ crosses the schedule (timetable) horizon STOP. Otherwise set $j + 1 := j$ and go to Step 1.

Note: This algorithm also incorporates the criterion of minimum frequency through a certain check and adjustment described below in this section.

Theorem 1

Algorithm T produces departure times for individual transit vehicles such that their maximum average load equals d_u.

Proof and Explanation

The basic assumptions of Theorem 1 are: (a) the change of departure times will not affect the arrival pattern of passengers, and (b) the change of departure times (with same frequency) will not affect the passenger demand.

Given these two assumptions, the new (balanced load) departure times are constructed on an accumulative load curve, by coordinating the appropriate cumulative desired occupancy value, d_u, and the time axis. That is, if $i = q$ is the hourly max load point, then $L_q(t)$ is the curve of accumulated max loads (at *q*) vs. time, with a straight line (on this curve) between each two adjacent departures. Each increase of $L_q(t)$ by d_u is coordinated with a time point to be the resultant departure time (Ceder, 1986). This is the case for a single max load point in a given time period (usually one hour). However, Algorithm T refers to the case of different max load points, each is associated with a different transit vehicle. If t_{ij} will be derived by $L_q(t)$, $i \neq q$, rather than by $L_i(t)$ as in Algorithm *T*, then $t_{ij} - T_{iu} \leq t_{qj} - T_{qu}$. That is, according to the algorithm $t^*_{1j} = t_{ij} - T_{iu} = \min(\ldots, t_{ij} - T_{iu}, \ldots, T_{qj} - T_{qu}, \ldots)$ in the case that stop i comes before stop *q* (similarly is the other case when stop q comes before *i*). Since $L_i(t)$ and $L_q(t)$ are two accumulative passenger load curves vs. time, the fact that $t_{ij} - T_{iu}$ is less than or equal to $t_{qj} - T_{qu}$ means that $L_i\left(t_{qj}\right) \geq L_q\left(t_{qj}\right)$. In others words, if the departure time of a trip will be set at $t_{qj} - T_{qu}$ (at the route dep. point), the average observed load at i will be equal or greater than d_u (and exactly d_u at q), and hence may create overcroadwing at the critical point *i*. Algorithm *T* assures that this cannot be the case, and that the max average load at any stop will be d_u.

4.3 Minimum Frequency Criterion

Algorithm *T* or the balanced load procedure does not guarantee that the minimum (min) frequency criterion (inverse of the policy headway), F_{mu}, for each time interval u, will be met. Usually F_{mu} is needed in the beginning and end of the day. Therefore, during the process of Algorithm *T*, in the transition between time intervals, *i.e.*, whenever a new t^*_{1j} becomes ahead of the next t_u, F_{mu} needs to be checked. Considering N_u which is the number of derived departures by Algorithm *T* during $t_{u-1} \le t < t_u$, F_{mu} is checked and attained in the following manner:

(a) check for each *u*, $N_u \ge F_{mu}$?, if yes –END, otherwise continue $u = 1, 2, \dots$, v.

(b) calculate the new desired occupancy, d_{um}, for the min frequency situation: $d_{um} = \dfrac{\max_{i=1,2,\dots,n}\left[L_i(t_u) - L_i(t_{u-1})\right]}{F_{mu} + 1}$ and return to Algorithm T at t_{u-1}, where in the Initialization $d_1 = d_{um}$.

(c) change d_u to d_{um} in Step 2 of Algorithm T and continue until $t^*_{1j} \ge t_u$, then END.

The procedure described by (a), (b) and (c) assures that the min frequency criterion will always be met. If Algorithm T results in time interval *u* with a frequency less than F_{mu} then the maximum difference in passenger loads on $L_i(t)$ between t_u and t_{u-1} determined a new desired occupancy, d_{um}. This d_{um} replaces then d_u, until the derivation of the first departure time in interval *u* + 1.

There are two more notes worth mentioning about Algorithm *T* and the min frequency case:

(1) if for a given departure to be determined, a different type of vehicle (than the one for which d_u is set) is considered d_u can be changed in Step 2 of Algorithm *T*. This may be the case for excessive load which may result in a too short headway or large amount of empty seat-km resulting in F_{mu}. Both cases can be observed by the accumulative load curves or load profiles;

(2) if it is preferred to use a policy headway criterion (inverse of F_{mu}, for each *u*) then in the procedure for F_{mu}, in (c), for each derived departure time (starting with t'_{ij}) there will be an extra check: if $t_{ij} - t_{i,j-1} > \dfrac{1}{F_{mu}}$, set $t_{ij} = t_{i,j-1} + \dfrac{1}{F_{mu}}$ and continue with Algorithm *T*, otherwise – END.

5. Graphical Interpretation of Algorithm T

The principal of Algorithm *T* can be demonstrated using the accumulative passenger load curve at each stop *i*, *i*=1,2,…,n, in a transit line with *i*=1 as the departure point and *i*=n+1 as the arrival point. Fig. 3 illustrates a simple example of the algorithm.

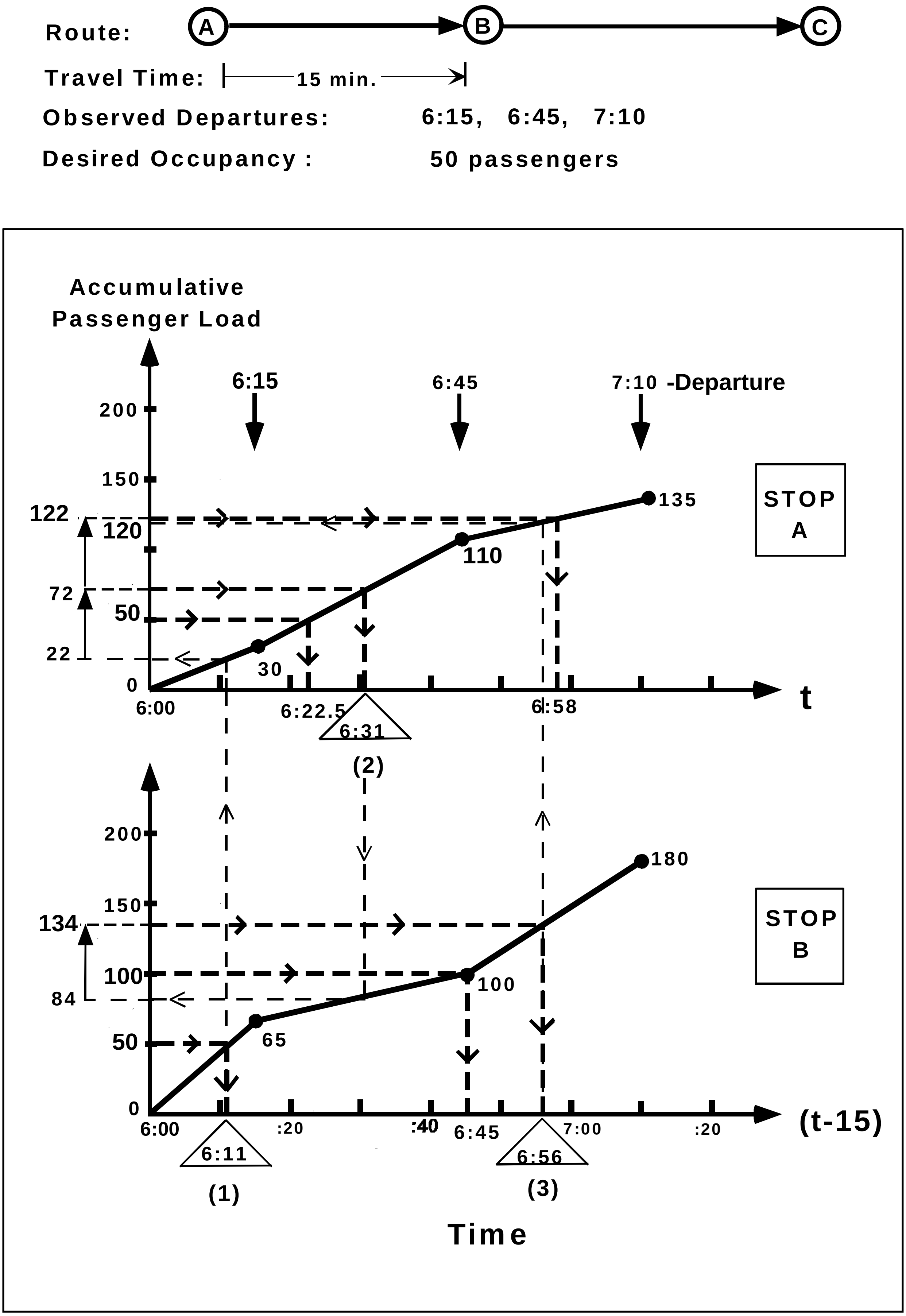


Fig. 3 Interpretation of Algorithm *T*

Given a transit line A→B→C with average travel time of 15 minutes between A and B, with three departures at 6:15, 6:45 and 7:10, and a desired occupancy of 50 passengers. The average observed on-board loads on the 6:15 vehicle are 30 passengers at stop A, and 65 at stop B. On the 6:45 vehicle: 80 and 35 passengers at A and B, and on the 7:10 vehicle: 25 and 80 passengers at A and B, respectively. Fig. 3 shows $L_A(t)$ and $L_B(t-15)$ as the accumulative load curves of the three vehicles where the curve at B is shifted by 15 minutes to allow for an equal time basis (at the route's departure point) in the analysis. In fact this shift in the time scale complies with ($t_{ij} - T_{iu}$) in step 1 of Algorithm *T*.

This simple example can be subjected to Level 1 and Level 2 analyses in Fig. 1 for comparison purposes. The load profile of these 3 vehicles is the histogram of 135 passengers between A – B, and 180 - between B – C. It means that the max load point for the vehicles is point B. Dividing 180 by the desired occupancy 50, one obtains 3.6 vehicles during the considered time period, say, 6:00 to 7:10, with about 17 minutes even headways. This is for Level 1. For Level 2 $L_B(t-15)$ in Fig. 3 is the necessary curve, and by coordinating the desired occupancy value of 50, 100 and 150 with the time axis, one obtains the departure at 6:11, 6:45, 7:01.

Coming back to Algorithm *T* (Level 3 in Fig. 1) then at the initialization the value of 50 is coordinated with $L_A(t)$ and $L_B(t-15)$ to obtain: 6:11 at B and 6:22.5 at A. Step 1 of the Algorithm *T* selects the minimum time between the two to be the first departure at 6:11 (emphasized in Fig. 3). It means that the first vehicle will be shifted backward by 4 minutes to have at B, in an average sense, 50 instead of 65 passengers. Algorithm T continues in Step 2 by adding $d_u = 50$ to 50 at $L_B(t-15)$, and to $L_A(6{:}11)$=22 at $L_A(t)$. This results in $t_{A2} = 6{:}31$ and $t_{B2} = 6{:}45$ (for *j*=2). Step 2 then selects 6:31 as the next departure, and the algorithm continues and results in 6:56 as the last departure at the period [6:00 - 7:10]. Adding $d_u = 50$ to 122 (at A) or to 134 (at B) will results in departures beyond 7:10.

The comparison between the observed data and the results of Levels 1, 2, 3 is summarized in Table 1 below where the associated average max load its corresponding stop appear in brackets.

Table 1. Departure times associated with Levels 1, 2, 3 (1st example)

Departure	1st	2nd	3rd	Characteristic
Observed	6:15 (65, B)	6:45 (80, A)	7:10 (80, B)	Observed
Level 1	6:17 (67, B)*	6:34 (46, A)	6:51** (35, A)	Even Headway
Level 2	6:11 (50,B)	6:45 (88, A)	7:01 (51, B)	Uneven Headway, Combined Max Load Point
Level 3	6:11 (50, B)	6:31 (50, A)	6:56 (50, B)	Uneven Headway, Individual Max Load Point

* average max load on each vehicle at its specified stop

**Level 1 results also in a 4th dep. at 7:08 with max load of 74 at B.

For example the max load at dep. 6:34 (Level 1) is determined from the difference in average loads between the 6:34 and the 6:17 departures. At the 6:17 dep. 67 passengers will be observed at stop B based on $L_B(t-15)$ in Fig. 3, and 35 passengers at A (intersect between 6:17 and $L_A(t)$ in Fig. 3). Then the 6:34 dep. intersects $L_A(t)$ at 81, and results in 46=81-35 passengers. Similarly the 6:34 dep. intersects $L_B(t-15)$ at 87, and results in 20=87-67 passengers at B. The max load, therefore, of the 6:34 dep. is max. (46,20)=46 at A.

Consequently the balanced load along the entire route is attained only at Level 3 (Algorithm *T*) whereas at Level 2 this balanced is attained only at stop B, the combined max load point of the three vehicles. Certainly due to the different derived departure times for the 3rd departure (much below the observed 7:10), the sum of the max loads is neither same for each Level nor for the observed data.

Another example for the interpretation of Algorithm *T* can be based on the example appeared in Ceder (1986). Fig. 4 exhibits, from this example, the initial accumulate passenger load curve at the hourly max load points (Level 2). Algorithm *T* is then applied for this example given the loading levels of each of the four vehicles at each stop along the entire 10-km transit route. This interpretation of Algorithm *T* is shown in Fig. 5 in which two departures are associated with the time interval 6-7 a.m. and two (out of six that appear in Ceder (1986)) between 7-8 a.m. The load profile of each of the four vehicles (dep.) along the 10-km route is illustrated on the right hand side of Fig. 5 with an emphasize on each individual vehicle max load. On the left hand side there are four accumulation curves associated with 4 stops: Dep, 1, 2, and 3. When applying Level 2 analysis (Fig.1), the hourly max load point between 6-7a.m. is stop 1 with 23+67=90 passengers, and stop 2 between 7-8a.m. (the max loads observed are 56, 63 passengers and those of 4 more departures not in the example).

Algorithm *T* starts with $d_u = 50$ to determine the minimum dep. time associated with d_u at 6:17 (stop 2). Then $d_u = 50$ is added to $L_i(t - T_{i1})$, *i*=Dep, 1, 3, $T_{i1} = 0$, 7, 18 respectively. That is, 27+50, 43+50, and 43+50 for *i*=Dep, 1, 3, respectively. The minimum second departure is then determined at 6:46, and the other two at 7:05 and 7:14. Similarly to the previous example, the comparison between the observed four departures and the results of the analyses at Levels 1,2, and 3 is shown in Table 2:

Table 2. Departure times associated with Levels 1, 2, 3 (2nd example)

Departure	1st	2nd	3rd	4th	Characteristic
Observed	6:25 (35, 2)*	6:45 (67, 1)	7:05 (70, 1)	7:15 (72,Dep)	Observed
Level 1	6:23 (32, 2)	6:46 (72, 1)	7:03 (63, 1)	7:12 (56, 1)	Even Headway
Level 2	6:33 (55, 2)	6:49 (54, 1)	7:08 (71, 1)	7:18** (70,Dep)	Uneven Headway, Combined Max Load Point
Level 3	6:31 (50, 2)	6:46 (50, 1)	7:05 (65, 1)	6:56 (65,Dep)	Uneven Headway, Individual Max Load Point

* <u>average</u> max load on each vehicle at its specified Stop (Dep, 1, 2, 3).
**with extrapolation over 7:15 at the Dep Stop.

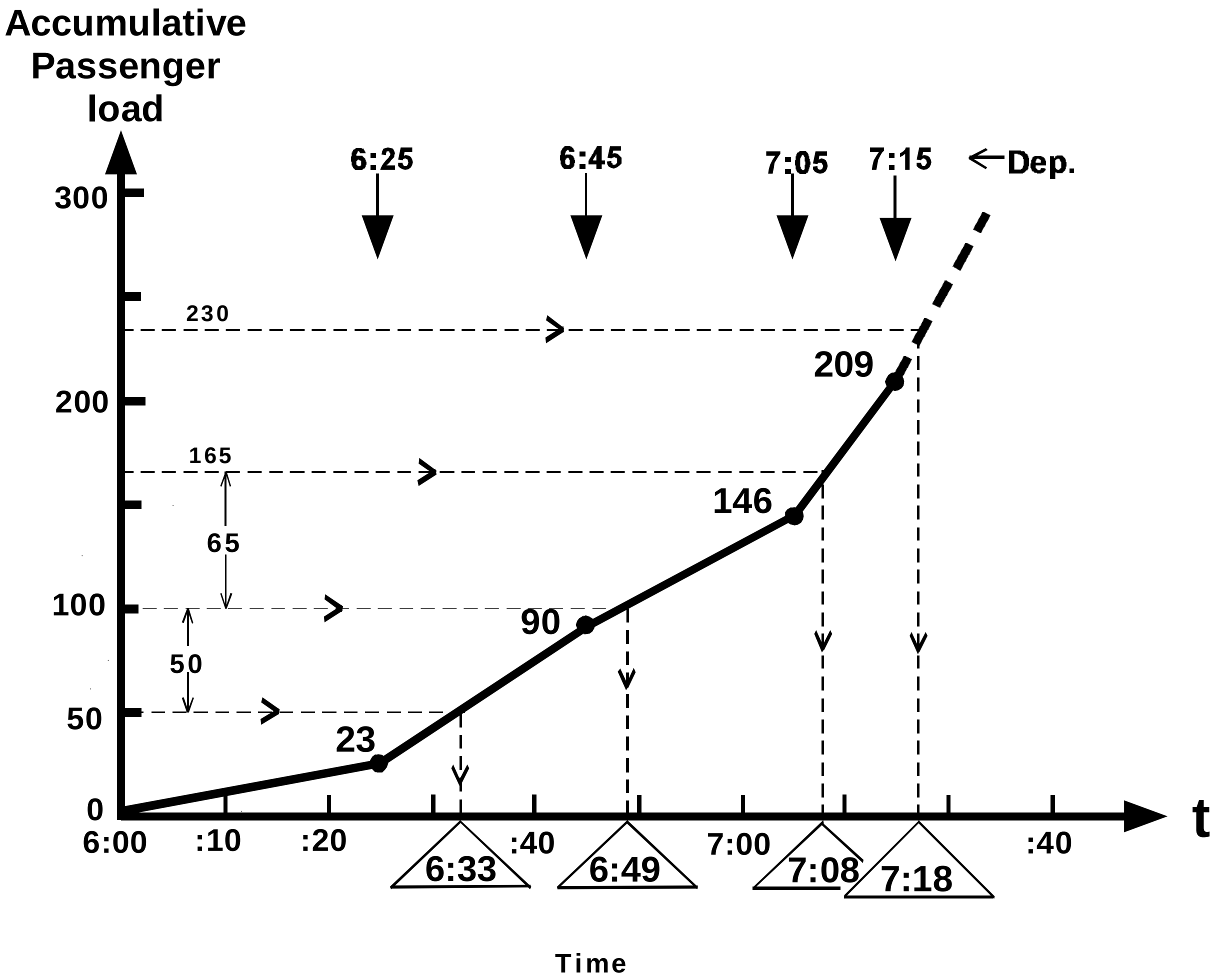


Fig. 4 Accumulative Passenger Load Curve using Level 2 Analysis

Vehicle Departure Time	Passenger Load at the Hourly Max. Load Point	Desired Occupancy	Hourly Max. Load Point
6:25 a.m.	23	50	Stop 1
6:45	67		
7:05	56	65	Stop 2*
7:15	63		

Average Travel Time between Dep. Stop and each Stop (Dep. to Dep.)	
Dep. → Stop 1	7 min.
Dep. → Stop 2	12 min.
Dep. → Stop 3	18 min.

* Includes 4 more departures between 7- 8 a.m.

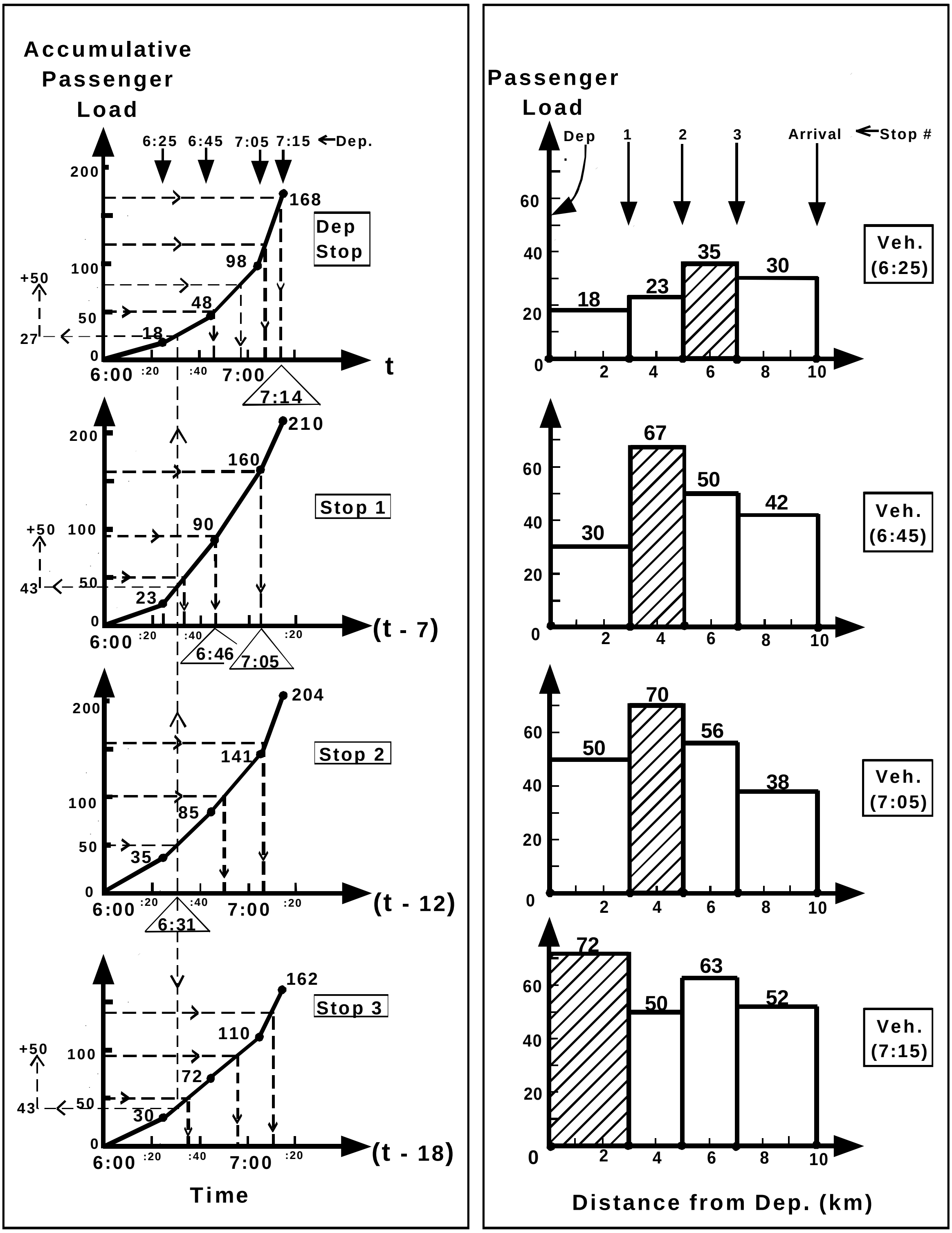


Fig. 5 Interpretation of Algorithm *T* for the example in Fig. 4

The departures at Level 1 are based on even headways of 23, and 9 minutes, for 6-7 and 7-8 a.m., respectively. That is, the accumulative load curve in Fig. 4 attains 132 at 7:00, and dividing this load by $d_u = 50$ loads to the frequency of 2.64 (veh/hr) with the inverse of 23 minutes. The 9 minutes headway between 7-8 a.m. is based on 4 more departures. The average max load, appears underneath the dep. time, is calculated from the previous dep.. For example at 6:23 the loads on $L_i(t)$ are 17, 21, 32, 28 passengers for *i*=Dep, 1, 2, 3, and the loads at 6:46 are 51, 93, 88, 46 for the four stops, respectively. The loads at 6:23 are associated with individual vehicles and, hence, the max load is 32 at *i*=2. However the loads at 6:46 are being accumulative, end hence the max load is max (51-17, 93-21, 88-32, 74-28)=72 at stop *i*=1.
The departures at Level 2 are based on the accumulative curve in Fig. 4, with extrapolation to 230 (2x50+2x65) for the 4th dep.. The calculations of the max loads are done in the same manner as for Level 1. Finally the departures at Level 3 are carried out by Algorithm *T* with the initialization phase shown on the four accumulative curves in Fig. 5. Once again only at the Level 3 analysis the average max loads attains the desired occupancy at each time interval ($d_u = 50$, 6-7 a.m. and $d_u = 65$, 7-8 a.m.) whereas at the other analyses the undesirable imbalanced load exists at the critical points.

6. Integration with Vehicle Scheduling

Once the timetables are constructed to meet best the fluctuated passenger demand, consideration should be given on how to execute them in an efficient manner. Each trip in the timetable becomes one element of a daily chain of trips to be carried out by a single vehicle. The problem is then to find the minimum number of chains (vehicle schedules, blocks) that contain all the trips in all timetables. Usually this minimum is fully required during peak hours and therefore represents the fleet size. The less is the fleet size, the higher is the saving in capital resources.
Section 2.2 outlines one efficient way to handle the problem of minimizing the fleet size required. There is no doubt that interlinings (vehicles are allowed to switch from one route to another) can further reduced the fleet size. The deficit function (DF) theory in section 2.2 provides the procedures for determining the minimum fleet size with interlinings. In transit systems without interlinings the fleet size can be optimized by the short-turning strategies described by Ceder (1990, 1991).
This section presents a procedure to integrate the derived timetables and vehicle scheduling with interlinings using some fine tuning. For this procedure let:

Δ_{eu} = a given positive tolerance (in minutes) for maximum shifting t_{1j}^* to the left (early departure), for each interval *u*, *j*=1, 2,…, m; *u*=1, 2,…, v

Δ_{lu} = a given positive tolerance (in minutes) for maximum shifting t_{1j}^* to the right (late departure), for each interval *u*, *j*=1, 2,…, m; *u*=1, 2,…, v

$SL_i(t)$ = the slope of $L_i(t)$ at *t*, *i*=1, 2, …,n

Δd_u = a given positive tolerance (in passengers)of the desired occupancy at time interval u, u=1, 2,…, v.

6.1 Tolerances of Departure Times

The DF procedures include possible shifting in departure times within bounded tolerances. It is based on a defined tolerance time $\left[t^*_{1j} - \Delta_{eu}, t^*_{1j} + \Delta_{lu}\right]$ where $\Delta_{eu} > 0$ is the maximum advance (early dep.) of the trip scheduled departure time, t^*_{1j}, and $\Delta_{lu} > 0$ is the maximum delay allowed (late dep.). These tolerances are used wherever shifting in dep. times is allowed, and can be a necessary element in saving vehicles.
There are three ways to use Δ_{eu} and Δ_{lu}. First, to shift once t^*_{1j} to the left (early dep.) or to the right (late dep.) by value of time less than or equal Δ_{eu}, and Δ_{lu} respectively, in order to reduce a given DF by one and save a vehicle. Second, to shift two departures in opposite directions (left and right) within the bounded tolerances, in order to save one vehicle. Third to shift t^*_{1j} to left or right in order to allow for an insertion of a deadheading (DH) trip that will save one vehicle. Further explanation appears in the example in section 7.

6.2 Tolerances of Desired Occupancies and Their Analysis

Once a shift has been made in a trip departure time it violates the balanced load criterion of this trip and the one to follow. That is, the attained desired occupancy at the trip's max load point is dependent on this trip's dep. time, and by changing it the trip's average max load will increase or decrease as well as for the next trip.
Let $t^*_{1j} = t_{qj} - T_{qu}$ be a departure time determined by Algorithm T at stop q. That is by adding d_u the minimum time that intersects one of the accumulative load curves is at q. Hence,

$$d_u = L_q\left(t^*_{1j}\right) - L_q\left(t^*_{1,j-1}\right) \tag{2}$$

The shifts in departure times made in the DF procedure are defined as:

$$\begin{aligned} t^-_{1j} &= t^*_{1j} - \Delta_{eu} \\ t^+_{1j} &= t^*_{1j} + \Delta_{lu} \end{aligned} \tag{3}$$

for all j=1,2, …, m, and relevant u for t^-_{1j} and t^+_{1j}.
In order to avoid excess average load, beyond $d_u + \Delta_{du}$, at each trip's critical point two criteria are established bellow for early and late departures.

<u>**Early Departure Criteria**</u>
Based on the above definitions:

$$L_i\left(t^-_{1j}\right) = L_i\left(t^*_{1j}\right) - \Delta_{eu} SL\left(t^*_{1j}\right), \quad \text{for } SL_i\left(t^*_{1j}\right) = SL_i\left(t^-_{1j}\right), \quad i=1, 2, \ldots, \text{n}, \tag{4}$$

t^-_{1j} belongs to u.

Note: in case that the slope is charged within Δ_{eu} shift for any stop *i*, Eq.(4) should consider two (or more) decreased portions, each is related to a different slope and its associated part of Δ_{eu}.

The loads at t_{1j}^{-}, can be expressed as:

$$L_i\left(t_{1j}^{-}\right) - L_i\left(t_{1,j-1}^{*}\right) < d_u, \quad i=1, 2, \ldots, n, \quad t_{1j}^{-} \text{ belongs to } u. \tag{5}$$

These loads of the new t_{1j}^{-} departure across all stops are based on Algorithm *T* in which only at *q* the desired occupancy d_u, is attained for t_{1j}^{*} and in all other stops the loads is less than d_u. Using Δ_{eu} shift to the left (early dep.) will further reduce these loads. However, the loads at each stop *i*, for the adjacent dep. to t_{1j}^{-} at $t_{1,j+1}^{*}$, will increase the loads by the Δ_{eu} shift. This increase is $\Delta_{eu} SL_i\left(t_{1j}^{*}\right)$, for all *i* and relevant *u*, or it is the sum of portions according to the note beneath Eq.(4).The increased new loads at $t_{1,j+1}^{*}$ need to be checked against $d_u+\Delta d_u$ across all stops. Certainly this check is applied for the maximum increase of load, and hence the early dep. criterion for accepting Δ_{eu} is:

$$\max_{i,1,2,\ldots,n}\left[L_i\left(t_{1,j+1}^{*}\right) - L_i\left(t_{1j}^{-}\right)\right] \le d_u + \Delta d_u \tag{6}$$

where $t_{1,j+1}^{*}$ belongs to interval u, and $L_i\left(t_{1j}^{-}\right)$ is obtained by Eq.(4) while considering Eq.(4)'s note.

Fig. 6 presents the example of Fig. 3 with an Δ_{eu} shift in part (a), and Δ_{lu} shift in part (b) and Δd_u = 10 passengers for both parts. Considering part (a), the shift is for the early departure of 6:31 by $\Delta_{eu} = 3$ minutes. The solid lines show graphically how to determine the new loads on the (new) 6:28, and the 6:56 departures at both stops A and B. The slope at A between 6:28 and 6:31 is 8/3 and times 3 minutes it results in an average load of 8 more passengers on the 6:56 dep., and 8 less – on the 6:28 departure. At stop B the slope is 7/6 and change is 3.5 passengers. The maximum change is determined by Eq.(6) where the load at A is 120-72+8=56, and at B 134-84+3.5=53.5 passengers for the 6:56 departure. The max load is 56 and since Δd_u = 10this max load can increase up to 60 passengers. Therefore the $\Delta_{eu} = 3$ minutes shift is accepted.

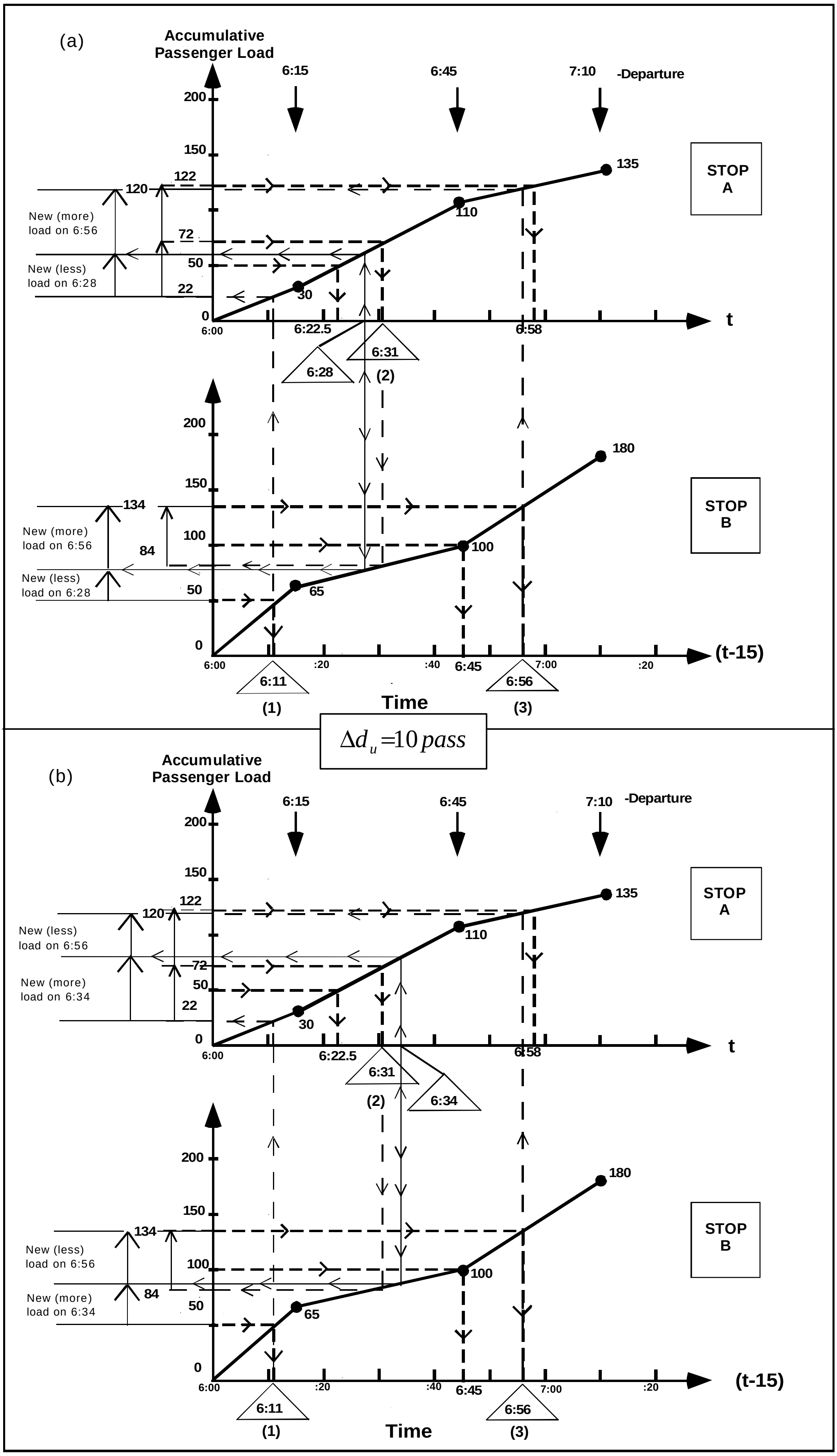


Fig. 6 Interpretation of Algorithm *T* with early (part a) and late (part b) shifts in a Departure Time

Late Departure Criterion

Following similar explanation to the Δ_{eu} criterion in Eq.(6), one can derive the Δ_{lu} criterion. That is,

$$L_i\left(t^+_{1j}\right) = L_i\left(t^*_{1j}\right) + \Delta_{lu} SL\left(t^*_{1j}\right), \quad \text{for } SL\left(t^*_{1j}\right) = SL\left(t^+_{1j}\right), \quad i=1, 2, \ldots, \text{n}, \qquad (7)$$
$$t^*_{1j} \text{ belongs to } u.$$

Note: in case that the slope is changed within the Δ_{lu} shift for any *i*, Eq.(7) should consider two (or more) increased portions each is related to a different slope and its associated part of Δ_{lu}.

The loads at $t^*_{1,j+1}$ can be expressed as:

$$L_i\left(t^*_{1,j+1}\right) - L_i\left(t^+_{1j}\right) < d_u, \quad i=1, 2, \ldots, \text{n}, \quad t^*_{1,j+1} \text{ belongs to } u. \qquad (8)$$

In this case of late departure all the average loads on the $t^*_{1,j+1}$ dep. will be decreased. However the average loads on the t^+_{1j} dep. will be increased in comparison with the average loads of the t^*_{1j} dep. across all stops. This increase is $\Delta_{lu} \cdot SL_i\left(t^*_{1j}\right)$ for all *i* and relevant *u*, or it is by the sum of some portions according to the note beneath Eq.(7).
The late departure criterion for accepting Δ_{lu} is therefore:

$$\max_{i,1,2,\ldots,n}\left[L_i\left(t^+_{1j}\right) - L_i\left(t^*_{1,j-1}\right)\right] \le d_u + \Delta d_u \qquad (9)$$

where t^+_{1j} belongs to interval u and is obtained by Eq.(7) while considering its note.
Fig. 6 part (b) presents an example of Δ_{lu}=3 minutes for the 6:31 departure. The solid lines show graphically how to determine the new loads on the (new) 6:34, and 6:56 departures at both A and B. The slope at A is same as for part (a) of Fig. 6 and results in the difference of 8 passengers, while the slope at B leads to the difference of 3.5. The increased load at A is 50+8=58 and at B is 84+3.5-50=37.5 passengers for the 6:34 new departure. Since Δd_u= 10, the max load can be up to 60 passengers, and, therefore, the Δ_{lu}=3 minutes shift is accepted.

7. Example

The examples in this works are used as an explanatory device for the developed procedures. Prior to wrapping up the presented methodology further clarity can be obtained by at a complete example of what it is illustrated in a flow-chart form in Fig. 2.
Table 3 contains the necessary information and data for a 3-hour example of a transit line from A to B and B to A. Point B can be perceived as the CBD that attracts the majority of the demand between 6-9 a.m.. There are 14 and 8 departures for A to B and B to A, respectively. The average observed max load on each trip, service and DH

travel times, desired occupancies, minimum frequency (not policy headway), and the tolerances for early and late departures and desired occupancy, are all shown in Table 3.

Table 3. Given Data for the Example Problem

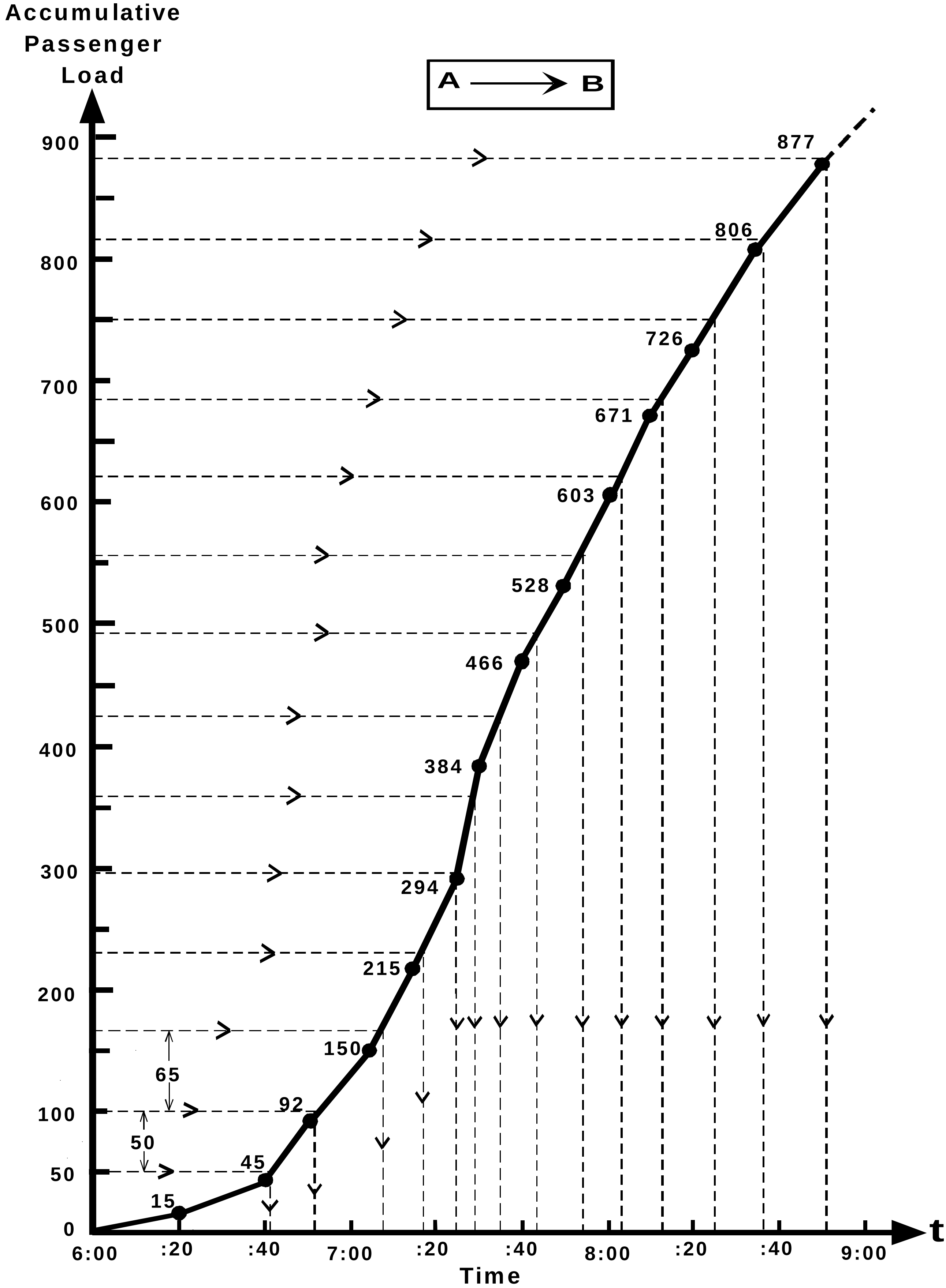
Time	Departure Time at the Route Dep. Point		Average Observed Max Number of Passengers on - board the vehicles		Travel Time including Layover Time (min.)				Desired Occupancy (Pass.)	
					Service		Deadheading			
	A→B	B→A	A→B	B→A	A→B	B→A	A→B	B→A	A→B	B→A
6 - 7 a.m.	6:20 6:40 6:50	6:30 6:45	15 30 47	22 38	60	50	40	35	50	50
7 - 8 a.m.	7:05 7:15 7:25 7:30 7:40 7:50	7:10 7:25 7:45	58 65 79 90 82 62	52 43 59	75	60	45	40	65	50
8 - 9 a.m.	8:00 8:10 8:20 8:35 8:50	8:25 8:40 8:55	75 68 55 80 71	23 51 28	70	60	45	40	65	50
Minimum Frequency : 2 Vehicles per hour, for all hours both directions										
Early Departure Tolerance: 2minutes, for all hours, both directions										
Late Departure Tolerance: 3 minutes, for all hours, both directions										
Desired Occupancy Tolerance: 8 passengers, for all hours, both directions										

In order to construct the balanced load timetable, these Table 3 data are used for running Algorithm *T*. Assuming that the max load is observed at the same stop for each direction, Algorithm *T* determines the new departure times shown in Figs. 7 and 8. There are 14 new departures for direction A to B, in Fig. 7, that are based on desired occupancies of 50 and 65 passengers. There are 7 new departures for direction B to A in Fig 8. For the A-B direction, in Fig. 7, d_u=50, u=1(6-7 a.m.), and d_u=65, u=2, 3

(7-9 a.m.). The third dep. check for $L_p(t-T_{m1})=150$, where p is the single max load point (both for A-B, and B-A directions) and T_{m1} is the average travel time from A to m, results in a departure over 7:00 a.m.. Therefore d_u is changed to 65 and the third dep. is coordinated with $L_p(t-T_{m1})=165$ to be 7:07 a.m.
For the B-A direction, in Fig.8, at first $d_{u=1}=50$ is set and results in one departure between 6-7 a.m. whereas $F_m=2$ in Table 3. That is, the second dep. at $L_p\left(t-T'_{m1}\right)=100$ is beyond 7:00 a.m. (7:04 a.m.). Consequently there is only one departure between 6-7 a.m. which doesn't comply with $F_m=2$, and hence the minimum frequency complementary component of Algorithm *T*, is applied. Step (b) of this complementary component provides the formula for the new desired occupancy: $d_{1m}=\dfrac{L_p(7{:}00)}{2+1}=\dfrac{91}{3}$. That is, the first two departures are at load levels of 30 and 60, and the third at level of 60+50=110 passengers. Another observation in Fig. 8 is related to the policy headways. Since $F_m=2$ is the only requirement, there is a large headway between 7:44 and 8:32 departures. However, if there is a policy (max) headway criterion then the note beneath the description of the minimum frequency component of Algorithm *T* (section 4.2) can be used.

Once the departure times are set at both route end points, the vehicle scheduling component can be integrated into the two-direction timetables. First, two deficit functions (DF) are constructed at A and B as it is shown in Fig. 9. These DFs are based on the schedule of 21 trips (14 of A-B, 7 of B-A), presented with respect to their travel times in the upper part of Fig. 9. Second, the DF theory leads to save one vehicle at *d*(A, *t*) through a shifting of trip # 17 by one minute forward (late dep.), and inserting a DH trip from B to A (7:52 to 8:32 a.m). The total fleet required is then 8+4=12 vehicles.

Since $\Delta_{lu}=3$ minutes for all *u* in Table 3, the shifting of 1 minute is allowed of trip #17 from A (dep. 8:26) to B (arrival 9:36). However the feasibility of this shift must be checked against the allowed tolerance for the desired occupancy change. That is, the check based on Eq.(9): $L_A(8{:}26)-L_A(8{:}13)\le 65+8$
where $L_A(8{:}26)=758$ and $L_A(8{:}13)=687.5$ are derived from Fig. 7 and Table 3. Thus 758-687.5=70.5<73 complies with $\Delta d_3=8$. Another way to fond this compliance is to look at the relevant slope of $L_A(8{:}25)$ in Fig. 7 between $L_A(8{:}20)=726$ and $L_A(8{:}35)=806$. This $SL_A(8{:}25)=\dfrac{806-726}{15}=5.3$ pass/minute will increase the average load on the 8:25 dep. by 5.3 since the shift is one minute. It means that the balanced max load of 65 will change to 70.3 (this is not exactly 70.5 like in Eq.(9) due to the rounding of departure times to integer minutes).



Dep. Time : 6:41, 6:52, 7:07, 7:17, 7:25, 7:29, 7:35,
(determined) 7:44, 7:54, 8:03, 8:13, 8:25, 8:37, 8:51

Fig. 7 Determination of Balanced Load Departure Times for the Example Problem, direction A-B.

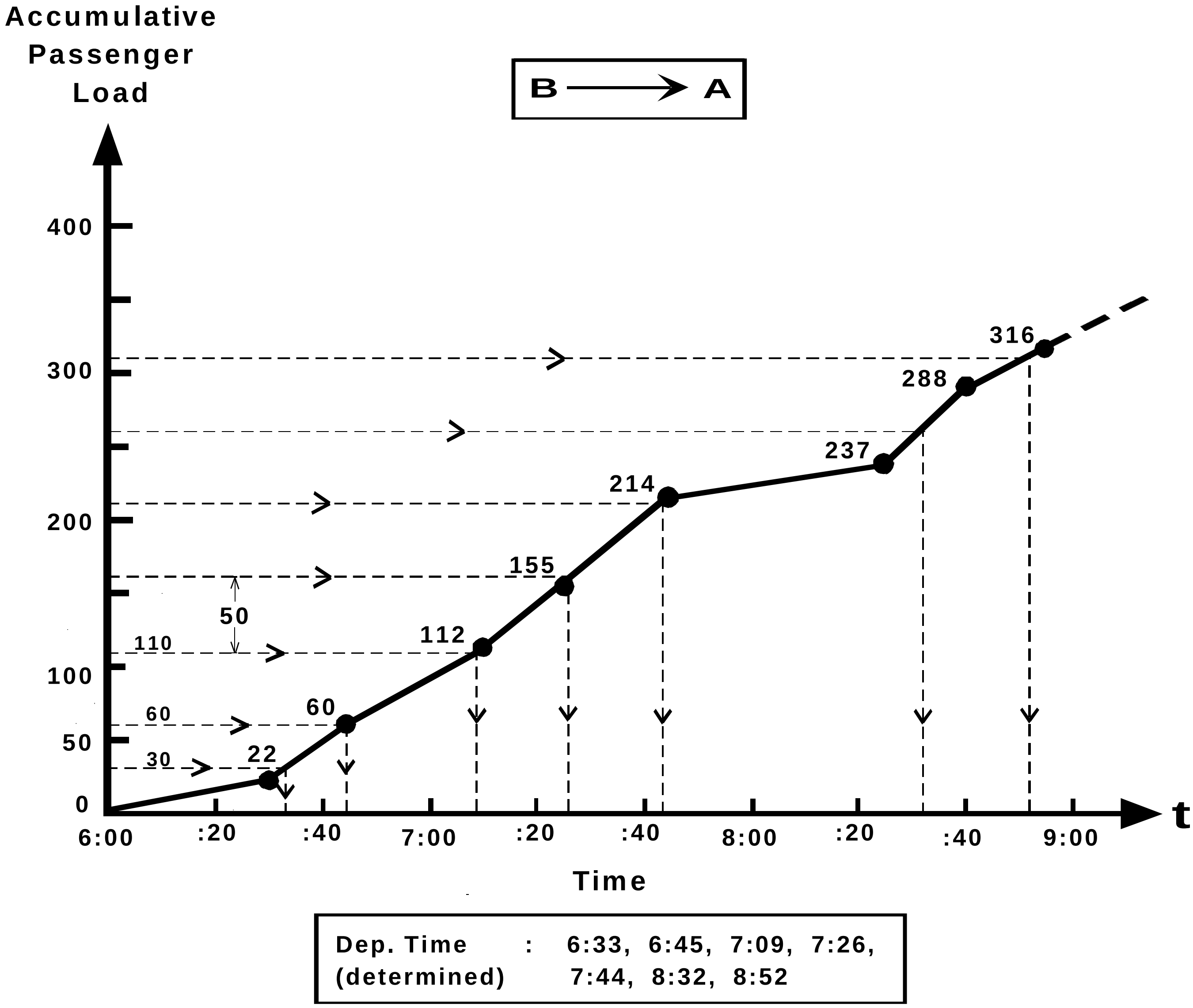


Fig. 8 Determination of Balanced Load Departure Times for the Example Problem, direction B-A.

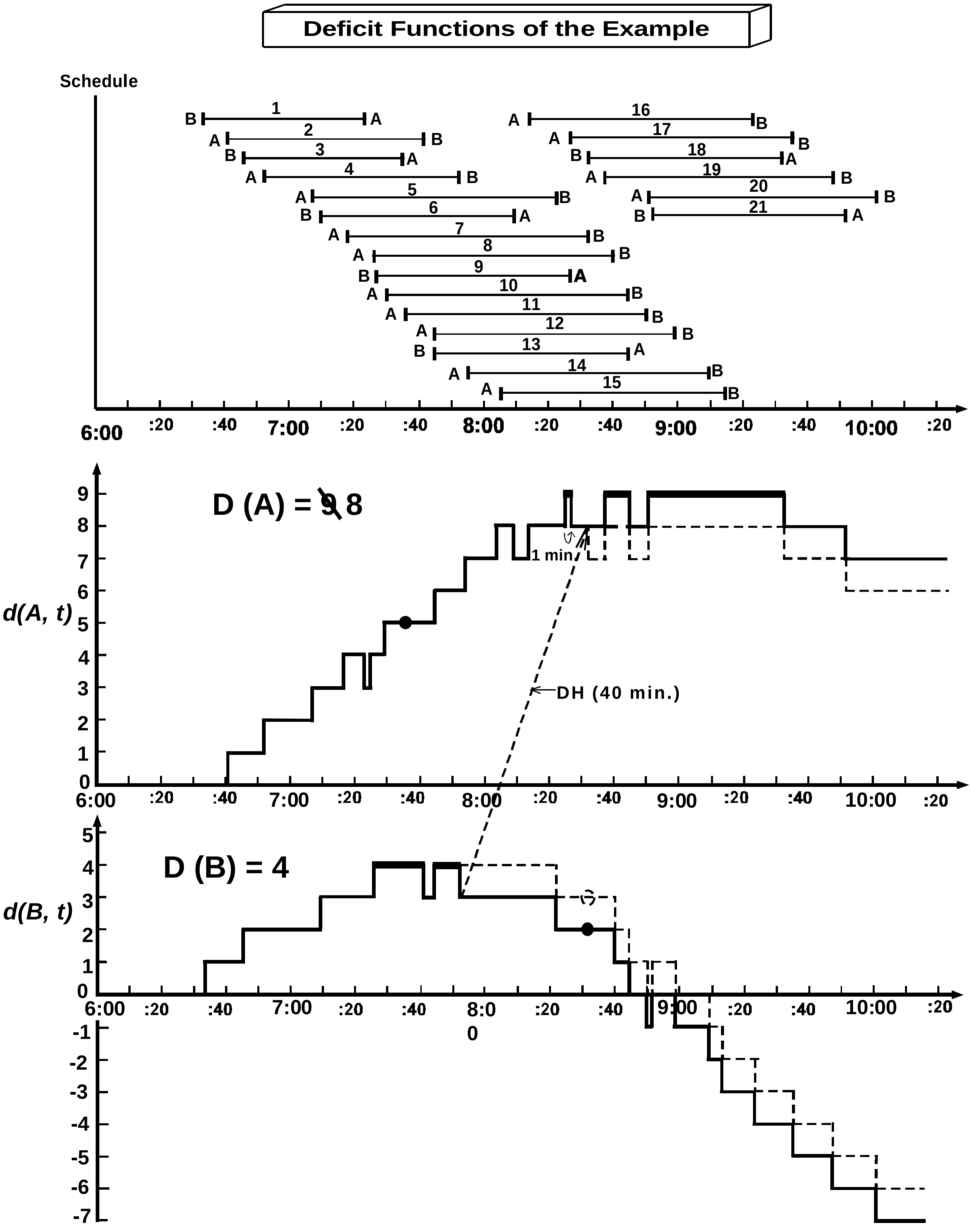


Fig. 9 Deficit Function Analysis for the Example Problem

Having done the check for the desired occupancy tolerance criterion, the final efficient schedule can be set for both balancing the passenger loads at each trip's critical point, and for the minimum fleet size required. The timetables at the route's end points appear in the upper part of Table 4, and the blocks in its lower part. The blocks (vehicle schedules) are contracted from the timetable using the FIFO rule. The first block for example, starts with trip #1 which linked with its first feasible connection at A, trip #8 (7:23 links to 7:25), and the trip #21 at B (8:40 links to 8:52). This FIFO rule can be replaced by the chain-extraction procedure that allows an arrival-departure connection for any pair within a given hollow on the deficit function. Fig. 10 illustrates for clarity one hollow (between two peaks of the deficit function) with arrivals of trips 1, 2, 3 and departures of trips 4, 5, 6. Below the figure there is the FIFO chain (within this hollow) as well as other alternatives, where in all- the minimum fleet size is maintained.

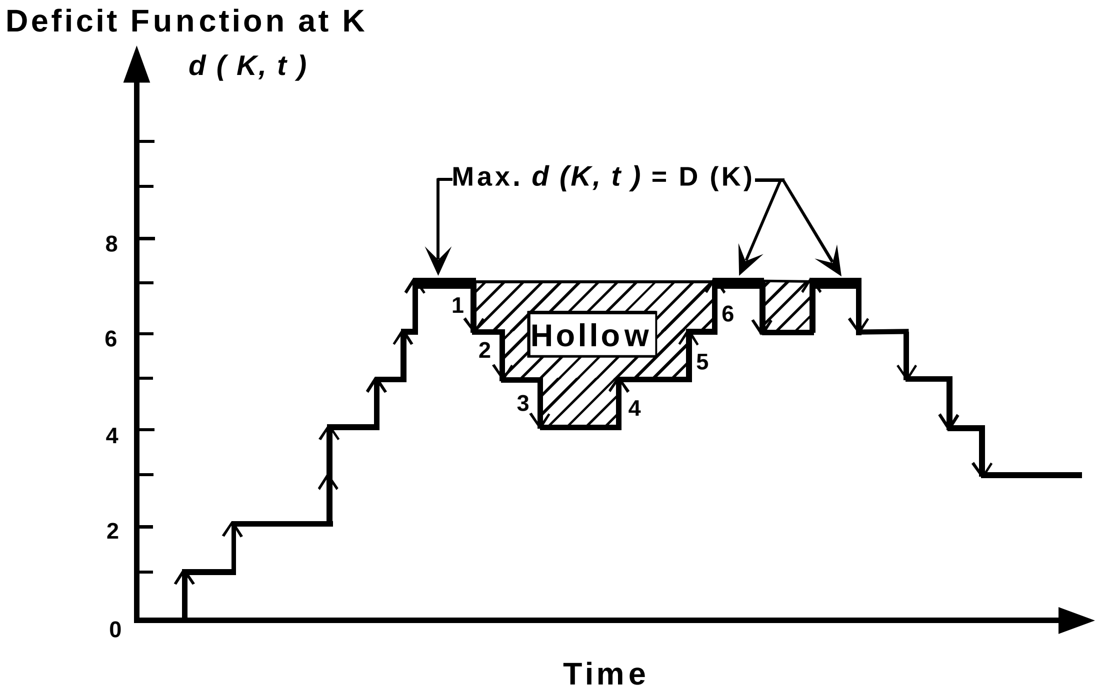
Table 4. Timetable and Vehicle Schedule (Blocks) of the Example Problem

A→B			B→A		
Trip N°	**Departure Time**	**Arrival Time**	**Trip N°**	**Departure Time**	**Arrival Time**
2	**6:41**	**7:41**	**1**	**6:33**	**7:23**
4	**6:52**	**7:52**	**3**	**6:45**	**7:35**
5	**7:07**	**8:22**	**6**	**7:09**	**8:09**
7	**7:17**	**8:32**	**9**	**7:26**	**8:26**
8	**7:25**	**8:40**	**13**	**7:44**	**8:44**
10	**7:29**	**8:44**	**DH***	**7:52**	**8:32**
11	**7:35**	**8:50**	**18**	**8:32**	**9:32**
12	**7:44**	**8:59**	**21**	**8:52**	**9:52**
14	**7:54**	**9:09**			
15	**8:03**	**9:13**			
16	**8:13**	**9:23**			
17	**** 8:26**	**9:36**			
19	**8:37**	**9:47**			
20	**8:51**	**10:01**			

*** Inserted Deadheading Trip**
**** Trip 17 was shifted by 1 minute**

(see Fig. 9)

Block Number	**Trips in Block (in sequence, Via FIFO)**
1	**1-8-21**
2	**2-13-20**
3	**3-11**
4	**4-DH-19**
5	**5-18**
6	**6-16**
7	**7**
8	**9-17**
9	**10**
10	**12**
11	**14**
12	**15**



FIFO Set of Chains : [(1 - 4) , (2 - 5) , (3 - 6)]

Other Sets of Chains : [(1 - 4) , (2 - 6) , (3 - 5)] ,
[(1 - 5) , (2 - 6) , (3 - 4)] , [(1 - 6) , (2 - 4) , (3 - 5)] ,
[(1 - 5) , (2 - 4) , (3 - 6)] , [(1 - 6) , (2 - 5) , (3 - 4)]

Fig. 10 An Example of creating Chains of Trip within a Hollow using FIFO Rule and all other possibilities

8. Discussion and Concluding Remarks

Different public transport agencies use different scheduling strategies based primarily on their own schedulers' experience, and secondarily on their scheduling software (if any). As the result, it is unlikely that two independent public transport agencies will use exactly the same scheduling procedures, at the detailed level. In addition, even at the same public transport agency, the schedulers may use different scheduling procedures for different groups of routes. Consequently, there is a need when developing computerized procedures to supply the schedulers with alternative schedule options along with interpretation and explanation of each alternative. One such alternative is developed in this work. Also, undoubtedly, it is desirable that one of the alternatives will coincide with the scheduler manual procedure. In this way, the scheduler will be in a position not only to expedite manual tasks but also to compare methods with others regarding the trade-off between passenger comfort and operating cost.

This work presents two main procedures: (*i*) creation of public transport timetables with even average passenger loads on individual vehicles, and (*ii*) integration of public transport timetables and vehicle scheduling for attaining the minimum fleet size.

Average even loads on individual vehicles can be approached by relaxing the evenly spaced headways pattern (rearrangement of departure times). It is known that passenger demand varies even within one hour, reflecting the business, industrial, educational, cultural, social and recreational public transport needs of the community. This dynamic behavior can be detected through passenger load counts, and information provided by road supervisors. The adjustments of departure times, made in this work by Algorithm T, form the basis to improve the correspondence of vehicle departure times with the fluctuated passenger demand. These adjustments resulting in a balanced load timetables are based on a given vehicle desired occupancy at the maximum load point of each vehicle. The load input for each vehicle is an average load profile that provides the measures of both passenger-km and empty-seat (space) km. In cases of large empty-seat (space)km the desired occupancy can be lifted while controlling, in an average sense, the maximum load at the critical point. The keyword here is to be able to control the loading instead of being exposed repeatedly to an unreliable service resulted from imbalance loading situations.

With the growing problems of public transport reliability, and advance in the technology of passenger information system the importance of even and clock headways is reduced. This allows for introducing optional timetables with the consideration of even average loads on individual vehicles. The construction of such timetables takes into account, in essence, the passenger perspective. A complementary measure to that is to consider the minimum number of public transport vehicles that are needed for the execution of the timetables.

This work provides a procedure to integrate these two components (timetabling and vehicle scheduling) based on the deficit function theory and given tolerances. The outcome is a set of efficient schedules from both the passenger and operator perspectives. The stepwise and graphical procedures to attain it allow some man-machine intervention and dialogue. The controlled procedures, especially for adjusting the timetable, will eventually reduce one of the major sources of unreliable service, resulting also in the reduction of wait and travel times. Theophrastus (300 B.C.) already said that: “Time is the most valuable thing one can spend”, and attempts must be made to avoid that passengers will spend unnecessary time when using public transport.

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