The Capture of Spring

Hooke's "Vibrative Pulse Communicated"

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Abstract: In 1678, Robert Hooke published a treatise on his metaphysics of vibration. *Lectures de Potentia Restitutiva or Of Spring* contains not only experimental and geometrical demonstrations of the spring law (which mutated into Hooke's law after his time), but also a principle at the heart of his dynamic matter theory – Congruity and Incongruity. Namely, that harmonious and discordant forces unify, shape and separate vibrating matter. This thesis reconstructs Hooke's production of congruity and incongruity, and the spring law, analysing the inversions, reversals and paradoxes moulding his knowledge-making practices. I argue that artificial instruments and apparatuses capable of magnifying and measuring never-before-seen minute bodies and motions also made the creation of a novel geometry necessary. I attempt to show how Hooke addressed these challenges by reassessing and reconfiguring the role of traditional Euclidean geometry, and reformulating practical-geometrical definitions to create a geometry that could demonstrate the spring law. Specifically, I focus on Hooke's studies of vibrating bodies and vibrations, and his practical geometry. By investigating Hooke's studies within the context of his matter theory, I show that, in an epistemological inversion, Hooke used optical instruments to shift frames of reference from the microscopic to the celestial and vice versa for his knowledge production. Further, Hooke's work is a cohesive whole centred on his studies of the similitudes between vibrating phenomena. Finally, his knowledge-making practices are a conflation of his predominant careers as an experimentalist and geometer. By constructing natural laws from physical reality, thereby implying that nature, artificial instruments, and laws such as the spring law are related, Hooke legitimised the application of instruments and mathematics to the study of nature. This process was far from straightforward or self-evident.

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INTRODUCTION

"A SUPERSTRUCTURE OF CONCLUSIONS"

"*Saturday April* the 10*th* 1697. I began this Day to write the History of my own Life, wherein I will comprize as many remarkable Passages, as I can now remember or collect out of such Memorials as I have kept in Writing," Hooke confided to his little pocket diary, "or are in the Registers of the ROYAL SOCIETY: together with all my Inventions, Experiments, Discoveries, Discourses etc. …"[1] Six years later, he would be dead, and his autobiography nowhere to be found.[2] Whether he ever penned it remains a mystery. But to get to know Robert Hooke a touch more intimately, one might begin by asking his mistresses. According to his biographer Richard Waller, admittedly sometimes a narrator of questionable reliability, but not in this instance, Hooke "first made himself Master of *Euclide's* Elements" while at the Westminster School, before his time at Oxford, "and thence proceeded orderly from that sure Basis to the other parts of the Mathematicks, and after thereof to Mechanicks, his first and last Mistress."[3] Here are Hooke's mistresses, then: mechanics and mathematics. And in late November of 1678, an excellent year for Hooke, he published a treatise on his metaphysics of vibration. *Lectures de Potentia Restitutiva* or *Of Spring* contains not only experimental and geometrical demonstrations of the spring law (which mutated into Hooke's law after his time), but also a principle at the very heart of his dynamic matter theory – the principle of "Congruity and Incongruity". Namely, the concept that congruous, harmonious and discordant forces unify, shape and separate vibrating matter.

Now, although Hooke enjoyed flouting his mistresses around rather flexibly,

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1 Robert Hooke cited by Richard Waller, *The Life of Dr.* Robert Hooke, in Robert Hooke, *The Posthumous Works*, ed. Richard Waller (London: Sam Smith and Benjamin Walford, 1705), i.

2 Felicity Henderson, "Unpublished Material from the Memorandum Book of Robert Hooke, Guildhall Library MS 1758", *Notes and Records of the Royal Society* 61, no. 2 (2007): 129-175, 131.

3 Waller, *The Life of Dr.* Robert Hooke, in *Posthumous Works*, iii.

he kept the heart of his metaphysics cloistered for years, confiding mostly in his diary ("Wrote theory of springs"[4]), and in his good friend, fellow Westminster Schoolboy and Wadham circle affiliate Sir Christopher Wren. "I told and Demonstrated to him the Theory of Springs and vibrations," Hooke almost whispers to his diary at the close of 1677, "none but he and I there."[5] But the following year, in the months leading up to the publication of the treatise that Hooke referred to simply and tellingly as "Spring", cumulative diary entries reveal his growing, palpable excitement. For example, July 21st: "wrote theory of springs, cleerd head";[6] August 4th: "Thought further of Springs"; August 10th: "told him [Wren, of course] … my equation of springs."[7] As the dates of Hooke's diary entries show, he saw Wren a few times a week and thought and conversed of very nearly nothing but 'spring'. August 20th: "Met Sir Chr. Wren on the water … discoursd about equation of Springs, etc." August 21st: "To Sir Chr. Wren with him at Mans. Discoursd much about Demonstration of spring motion."[8]

Indeed, even though Hooke had "read [his] Theory of Springs and shewd the experiments to illustrate it" to the Society on August 1st, and even though "all were pleasd" – not an observation that Hooke indulges in often – he was anxiously seeking Wren's approval, as an August 28th entry attests. "Dined with Sir Christopher Wren. Could not procure his judgement of springs" (there were competing spring hypotheses in the air).[9] He finally got it on Friday, September 13th: "Sir Chr. Wren approved ….. spring theory."[10] Wren's approval seems to have helped to set the John Martyn printing press in motion, and at the end of October Hooke proclaimed *Of Spring* "almost printed".[11] It was winter, as he notes in a

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4 Robert Hooke, *The Diary of Robert Hooke* 1672–1680 [henceforth *Diary*], eds H.W. Robinson and W. Adams (London: Wykeham Publications, Ltd., 1968), 214.
5 Hooke, *Diary*, 334.
6 Hooke, *Diary*, 367.
7 Hooke, *Diary*, 370.
8 Hooke, *Diary*, 372.
9 Hooke, *Diary*, 374, 379.
10 Hooke, *Diary*, 376.
11 Hooke, *Diary*, 380.

December 1st entry, describing a day with "cold cleer air, great frost, snow on the houses", but Hooke burned, handing out marbled and gilt copies of "Spring" to friends and colleagues, gifting his last one to Denis Papin on January 5th the following year.[12] Hooke had every reason to be excited; his path to a geometrical demonstration of the spring law had started as early as 1661 with his first mistress – mechanics, and capillary action experiments with Robert Boyle and his famous air-pump. The reasons the spring law was and is so important are manifold. For Hooke, reducing spring to an "equation" was crucial because of his ambition to develop a clock for marine navigation – that is, for solving the longitude problem.[13] He also needed to pin down and formulate a spring law because his entire cosmology depended on it. The latter reason is my priority here.

In this work, I reconstruct Hooke's production of congruity and incongruity, and the spring law, analysing the inversions, reversals, compromises and paradoxes shaping his knowledge-making practices. I argue that artificial instruments and apparatuses capable of magnifying and measuring never-before-seen minute bodies, their pores and motions, also made it necessary for the creation of a new geometry, capable of handling the new objects created by the New Science, and I attempt to show how Hooke addressed these challenges by reassessing and reconfiguring the role of traditional Euclidean geometry and, more importantly, reformulating practical-geometrical definitions in order to create a geometry that could demonstrate the spring law.

I mentioned that Hooke enjoys flouting mechanics and mathematic *flexibly*. Although experimental practices and mathematical analysis are emblems of the New Science, Hooke's work marks an exceptional approach to the legitimation of mathematics for studying nature – an effect of the conflation of his sometime simultaneous careers as the Curator of Experiments for the Royal Society and the

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[12] Hooke, *Diary*, 386, 391.

[13] Ofer Gal, *Meanest Foundations and Nobler Superstructures* (Dordrecht: Kluwer Academic, 2002), Chapter Two.

Gresham Professor of Geometry. Throughout, I underscore his unique way of working a problem, moving between the practical and the theoretical with little friction and without preamble, often employing one for the other in his knowledge-making practices, a conflation particular to Hooke, which Ofer Gal described as a "hybrid status between theory and instrumentation".[14] Consequently, to elucidate the relations between the practical and theoretical aspects of Hooke's way of working a problem, this thesis has two inter-related parts.

The overall structure moves from Hooke's experimental practices to his practical geometry chronologically, with each part and chapter also broken down chronologically. Part I, 'Congruity and Incongruity', focuses on his experimental procedures, and the practical origins of the creation and development of the dual concept of congruity and incongruity as Hooke's primary theoretical tool, as well as its harmonising effect on his work. Owing to Hooke's multiple commitments and his insatiable interest in all things, several scholars have characterised him as a man who stretched himself too thin, leaving behind, for the most part, bits and pieces of haphazard work.[15] That Hooke and Waller[16] corroborated both accounts of this persona in what remains of his papers only adds to his image as a man always in a rush, out of time, promising to explain the rest later. But when viewed as a whole from the perspective of congruity and incongruity, Hooke's career can be restated as a tireless attempt to understand and explain the harmonies and discords of the universe from several facets such as surface tension, acoustics, optics and gravity, expressed in phenomena like consonant vs. dissonant vibrating musical strings, reflection and refraction and so on. In this respect, I attempt to show how what started off as a way to explain capillary action, itself an explanation of several natural phenomena at the time, matured into a generalised theory of matter as

[14] Gal, *Meanest Foundations and Nobler Superstructures*, 59.

[15] Michael Hunter and Simon Schaffer (eds), *Robert Hooke: New Studies* (Woodbridge, England: Boydell Press, 1989), 1–2.

[16] Robert Hooke, *The Posthumous Works*, ed. Richard Waller (London: Sam Smith and Benjamin Walford, 1705).

inseperable from motion, a theory in which vibrations and "all manner of sonorous or springing Bodies",[17] are either congruous or incongruous based on vibrational frequency, the backbeat of which is 'congruity and incongruity'. Part I also serves as a necessary foundation for the longer Part II, on Hooke's novel 'Practical Geometry'.

The way that Hooke taught and practiced mathematics shows that for him Euclidean geometry was a tool, created before the invention of optical instruments, and like naked eye astronomy was limited by the human senses – by our incapacity to sense and resolve constituents. Nevertheless, it remained an important tool for making concepts more comprehensible to the senses, specifically because it was limited by them. In Part II, I will examine Hooke's development of the spring law from the perspective of mathematical bodies and motions by focusing predominantly on his practical geometry – the mediator between sensible and insensible physical reality and abstractions in his work. One example, which I only mention here, is Hooke's light strings and sound rays, or his use of ray optics to depict and think about insensible sound. Hooke was adamant that mathematics should spring from physical operations, so it became necessary for him to reformulate the fundamental definitions of geometry in a way that would complement his epistemology; that is, to fashion physicalised definitions for a mathematics contingent on the configurations of matter. For example, a point became a body, and instead of assuming a mathematical skeleton, geometry respected the material. He had already undertaken the task in the *Micrographia*, where he studied nature "as a geometer", structuring his observations according to this framework – starting with the simplest bodies and building up to the most complex.[18] Indeed, the *Micrographia's* structure reveals the forethought of an epistemological ladder, and I try to outline how Hooke is able to reduce globular bodies to points that possess either geometrical congruence or similarity, creating a chain of proportionality based on similitude, which allows him to move between

[17] Robert Hooke, *Of Spring* (London: John Martyn, 1678), 7.
[18] Hooke, *Micrographia*.

sensible and insensible realms. I attempt to show how he forms a relation between microscopic and celestial points with the claim that the difference between pinpoints and planets is a matter of magnification or diminishment, or proportionality. To achieve this epistemological inversion of the microscopic and celestial, Hooke adopts Galileo's argument that one should replace the senses with instruments,[19] which can magnify, diminish, quantify and measure points; and he employs Galileo's maculate moon as a trope with which to communicate the inversion. Yet the observations show that instead of reducing nature to its essentials, Hooke's lenses resolved seemingly immeasurable complexity.[20]

To the best of my knowledge, a work of this nature and scope on congruity and incongruity, as well as the invention of Hooke's law of springing bodies, has never been attempted before (see Literature Review below).

CHAPTERS

Chapter One, 'Pressure', sketches the germination of Hooke's matter theory by introducing his popular explanation for the cause of capillary action – the rise of liquid in thin tubes. Boyle's praise of the Hooke's theory of capillarity prompted the latter to publish for the first time, penning *An Attempt for the Explication of the Phaenomena, Observable in an Experiment Published by the Honourable Robert Boyle* (1661). In the *Attempt*, Hooke forges congruity and incongruity as a pair of theoretical tools to explain the phenomenon of capillarity, which he claims is caused by a difference in air pressure. However, Hooke's explanation for the *causes* of congruity and incongruity, in turn, would have to wait for his acoustical experiments in the *Micrographia* (1665), a book that kept the diarist Samuel Pepys up till 2 o'clock ("the most ingenious book that I ever read in my life"[21]).

[19] Ofer Gal and Raz Chen-Morris, *Baroque Science (*Chicago: The University of Chicago Press, 2013), esp. Chapter Three.
[20] Gal and Chen-Morris, *Baroque Science.*
[21] Samuel Pepys, *The Diary of Samuel Pepys Volume VI*. Edited by Robert Latham and William G.

Chapter Two, 'Causes', begins by analysing how and why Hooke moved from the *Attempt* to its reworked second edition: the *Micrographia's* sixth observation. In Observ. *VI Of Small Glass Canes*, Hooke first ties congruity and incongruity to stretched musical strings, vibrations, sound, creating a crucial argument from analogy that he would rely on again in *Of Spring* and indeed throughout his career. By studying Hooke's experimental trials on capillarity for their content on what Hooke would eventually come to call a 'chime of motions', I try to show that it is clear even this early on that congruity and incongruity are the hammer and file not only of his theory of capillarity, but of his cosmology. Further, the chapter is a study of the development of Hooke's string similitudes, during the long duration of time from the *Micrographia* onwards, and reveals three things. First, that strings remained a reliable constant as a material model capable of accounting for all the fundamental properties of matter around which Hooke could build his theory. Second, that music provided a way for him to work a problem with instruments designed for human senses in order to gain knowledge outside sense limits. An example is Hooke's sound wheels, invented to demonstrate that sounds are aggregates of pulses which continue beyond the limits of human hearing, leading Hooke to turn his back on the senses, paradoxically replacing both the eye and the ear with countable pulses and musical ratios.[22] Third, although mathematics is only touched upon here, I try to show, as a prelude to Part II, how music was also a means for Hooke to construct a physical matter theory from which mathematics follows.

Chapter Three, 'Vibrations', examines the claim made by Hooke at the start of *Of Spring* that he had already "hinted the principle" of congruity and incongruity in the *Attempt*. It highlights his developing notions on particles as well as his developing and contradictory notions on the aether. I follow Hooke's changing early notions on particles and the aether to show that the employment of various experiments and observations for the development of his matter theory illuminates

Matthews (California: University of California Press, 2000), 18.

22 For the optical part of this paradox, also see Gal and Chen-Morris, *Baroque Science*.

how blurred the line between practice and theory is in his work. I argue that Hooke's ontology is contingent upon his experimental practices, which alter how he imagines what insensibles like particles and the aether are, and subsequently also his representations of these new objects in his work and to his peers – for example, Boyle and Christiaan Huygens. In this way, I try to explain how and why Hooke was, on the one hand, able to conclude that particles are vibrating globular bodies even though, on the other hand, his concept of the aether remained contradictory and ultimately oxymoronic. Nevertheless, after reinforcing his description of the aether as a dynamic propagator of forces in *Of Spring*, Hooke could then commit himself more to the various vibrations crisscrossing through it, and to his geometrical demonstration of the spring law, which is contingent upon the material world and its parts.

Chapter Four, 'Points,' is the first chapter in Part II: 'Practical Geometry', and thus lays some necessary groundwork. First, it outlines Hooke's Gresham geometry programme to explain Euclidean or speculative vs. practical geometry; the former, Hooke teaches, is useful for grounding concepts and making insensibles intelligible to the senses; the latter, as far as Hooke is concerned, is the geometry of the New Science. From the perspective of Hooke's practical geometry, starting with a theoretical or speculative point when representing nature is to start with an instrument as fallible as the bare human eye. I further attempt to show that the *Micrographia* challenges the Jesuit astronomer Christopher Clavius's claim that geometers should avoid meddling in matters of physics by examining Hooke's two reasons for making his first observation on the point of a needle. His first reason is to erase the divide between art and nature, thereby making the resolved sights and textures allowed by his new geometrical tools, the microscope and telescope, as ordinary as those experienced by human senses. His second reason is to fashion an analogy about points, and here Hooke employs the Galileo trope mentioned above. His use of optical instruments to smudge the edges between microscopic and macroscopic worlds both frames the *Micrographia* and becomes a leitmotif as the ordered observations develop in scope and complexity from the smallest artificial point to the moon. I will deconstruct two diagrams of points in parabolic motion, in an attempt to further explain the epistemological inversion and its importance in Hooke's knowledge-making process. This chapter also introduces Hooke's crucial tool of proportionality, which was glanced at in Part I when discussing music in his work, and sketches how Hooke creates a chain of proportionality based on physical *hence* mathematical similitude.

Chapter Five, 'Lines', analyses the mathematisation of Hooke's stretched musical strings (discussed from an experimental and instrumental perspective in Part I). Specifically, Hooke's subversion of light, which is visually sensible, with sound, which is visually insensible, as a means to represent all vibratory phenomena with a mixture of practical and abstract 'ray' optics. I attempt to show that Hooke's strings reverse the epistemological role of the Pythagorean monochord; that is, a reversal of the idea of harmony as an underlying skeleton of ratios in a perfect monochord – since sound, like Hooke's geometry, is contingent on *physical* causes. For similar reasons, Hooke's rejection of Isaac Newton's theory of light and colours, during which Hooke stresses how a ray of light is like a stretched string – physically hence mathematically, is a defence of his metaphysics of vibration. Further, to underscore the uniqueness of Hooke's geometry, and how essential it is to his work on insensibles and infinitesimals, I also compare Hooke and Leon Battista Alberti (1404–1472). While Hooke's reformulation of the fundamental definitions of geometry resembles Alberti's, the latter did not consider invisible entities as necessary subjects for painters who represent only what they 'see'; moreover, Alberti's overall concept of geometry is closer to Johannes Kepler's metaphysical assumptions about mathematics. Finally, I consider the differences between a speculative vs. a practical simple line to show why practical geometry is a better representation of nature according to Hooke.

Chapter Six, 'Superficies', examines Hooke's use of scale and proportion in detail with a study of how Hooke lifts practical geometry off the faces of crystals, and of his novel use of a scale bar – another new instrument in his practical geometry toolbox. I identify Hooke's appropriation of Kepler's semi-thought-experiment on close-packed lattices, and show how Hooke physicalises the revamped experiment by merging crystals with mechanical models from which geometry follows, implying that all are integrally related. Hooke's experiment re-involves the senses, which according to him is a crucial step in forming a link between insensibles and sensibles, and without which nothing can be understood or utilised. Thus, contrary to Kepler's metaphysics, Hooke demonstrates that geometry is not "coeternal with God",[23] but a cultural product. At last, I examine Hooke's artificial sections of cork in the *Micrographia* as resources for the construction of his springing particles representation in *Of Spring,* and I reconstruct his geometrical proof of the spring law to analyse how his practical and speculative geometry form a new mixed geometry. Although Hooke borrows the term 'mixed' from his mentor John Wilkins, and although he is indebted to Wilkins technically and philosophically, I argue that Hooke's new mixed geometry is radically different. Hooke's practice of mixed geometry grounds his mathematics, and his graph of the spring law also exhibits his attempt at a solution to the question of infinitesimals.

Consequently, in Chapter Seven, 'Solids', I focus on how real, material lenses shift frames of reference, and how it is that a solid, the moon, for example, can be a pockmarked superficies like the point of a needle; a smooth globular body; and a mathematical point. By constructing natural laws from physical reality, thereby implying that nature, artificial instruments, and laws such as the spring law are related, Hooke legitimised the application of instruments and mathematics to the study of nature. The twisty turns explicated throughout attest that the process was far from straightforward or self-evident.

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[23] Johannes Kepler, *Harmonices Mundi* in *Gesammelte Werke* 3, Axiom 7, 6:104. Also see Kepler, *The Harmony of the World*, trans. E.J. Aiton, A.M. Duncan, and J.V. Field (Philadelphia: American Philosophical Society, 1997), 146–147.

LITERATURE REVIEW

Classics such as E. Williams's "Hooke's law and the concept of the elastic limit" and Mary Hesse's "Hooke's Vibration Theory and the Isochrony of Springs" suffer from anachronism in their quests to uncover why Hooke himself left the law that is named after him allegedly incomplete, instead of closely following Hooke's practical and theoretical procedures to ascertain what the "Theory of Springs" is.[24] Williams was more interested in the Young modulus, but failed to notice that Hooke was aware of an elastic limit, which was not important to him because it had little impact on his metaphysics. Meanwhile, Hesse missed the point about the relation between the spring law and Boyle's law, dismissing Hooke's supportive arguments as "confused".[25] Nevertheless, Hesse's work remains a valuable contribution, and parts of chapter sections in my work aim to clarify and explain these problems of interpretation with reappraisals of relevant material. Later, more contextual papers such as Albert E. Moyer's "Robert Hooke's Ambiguous Presentation of 'Hooke's Law'," still fall into the ambiguity trap by focusing only on small parts of Hooke's extensive material.

Several scholars have understandably approached Hooke's vibrations from the vantage point of music. A classic here is Penelope Gouk's "The Role of Acoustics and Music Theory in the Scientific Work of Robert Hooke".[26] Gouk studied the role of music in Hooke's cosmology, but her main concern was from whom Hooke acquired his intellectual tools rather than the more interesting question of how he applied these tools to create a matter theory. Moreover, she was confused by Hooke's concept of the aether, though this is not unwarranted, as I will attempt to show in Chapter Three, for Hooke tries to maintain two positions at once. Similarly,

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[24] Robert Hooke, *Lectures De Potentia Restitutiva* or *Of Spring*. London: John Martyn, 1678, 1.

[25] E. Williams, "Hooke's Law and the Concept of the Elastic Limit", *Annals of Science* 12, no. 1 (1956): 74-83; Mary Hesse, "Hooke's Vibration Theory and the Isochrony of Springs", *ISIS* 57, no. 4 (1966): 433; Albert E. Moyer, "Robert Hooke's Ambiguous Presentation of 'Hooke's Law,' ISIS 68, no. 2 (1977): 266.

[26] Penelope Gouk, "The Role of Acoustics and Music Theory in the Scientific Work of Robert Hooke", 585, in *Annals of Science* 37, no. 5 (1980): 573–605.

Jamie Kassler and David Oldroyd's "Robert Hooke's Trinity College 'Musick Scripts', his Music Theory and the Role of Music in his Cosmology" is a comprehensive account of Hooke's knowledge of music, and how he applied it to his matter studies; the 'Musick Scripts' transcribed and interpreted by them are valuable primary sources.[27] Finally, an interpretation of congruity and incongruity, which Michael Cooper and Michael Hunter described as an 'overstated case', is John Henry's "Robert Hooke, the Incongruous Mechanist." Henry argues that the terms congruity and incongruity are synonymous with sympathy and antipathy in natural magic.[28] I argue against this thesis.[29]

In their recent work *Baroque Science*, Ofer Gal and Raz Chen-Morris designated "Baroque" as a "particular set of tensions, anxieties, and paradoxes" identifiable in early modern science practices, arguing that the new ways of producing knowledge were inextricably a part of Baroque culture, which is usually perceived as the antithesis of rigour, order, logic.[30] Gal and Chen-Morris concentrate on three inter-related paradoxes embedded in early modern observation, mathematisation and the passions, examining what they call radical empirical instrumentalism and the rejection of the senses for instrument-mediated knowledge; constructed natural laws enforced upon nature; and objective passions, to examine the implications of "instrument-mediated empiricism", study the "paradoxical compromises" involved in the mathematisation of nature, and question the consequences of reconfiguring

[27] Kassler and Oldroyd, "Robert Hooke's Trinity College 'Musick Scripts', his Music Theory and the Role of Music in his Cosmology," *Annals of Science* 40, no. 6 (1983): 559–595.

[28] John Henry, "Robert Hooke, the Incongruous Mechanist," in Michael Hunter and Simon Schaffer (eds), *Robert Hooke: New Studies* (Woodbridge, England: Boydell Press, 1989). Michael Cooper and Michael Hunter, *Robert Hooke: Tercentennial Studies* (Aldershot, England: Ashgate, 2006), xviii.

[29] For an alternative thesis against Henry's, see also M. E. Ehrlich's "Mechanism and Activity in the Scientific Revolution: The Case of Robert Hooke," *Annals of Science* 52 (1995): 127–151. In the historiography, it is difficult to avoid at least mentioning congruity and incongruity when discussing aspects of Hooke's work such as optics, celestial mechanics and so on. For example, A.I. Sabra's, *Theories of Light* (Cambridge: Cambridge University Press, 1981) contains an attempt at summarising Hooke's concept of waves when discussing his theory of light and colours. I will reference relevant and notable summaries in the footnotes throughout.

[30] Ofer Gal and Raz-Chen Morris, *Baroque Science*, 10, 9.

"reason and the senses".[31] I explore similar themes in an attempt to strip away the answers, resolutions and self-affirming narratives in Hooke's 'completed' work, and follow his contingent, often convoluted and imaginative object- and knowledge-making practices.

I: CONGRUITY AND INCONGRUITY

1. PRESSURE

From 1658 to 1659, Hooke designed a machine for Robert Boyle that could pump air out of a "receiver", in order to stick all manner of things into a big vessel of thick glass, such as capillary tubes in a reservoir of red wine, and pump out as much air as possible to observe the various effects. The air-pump needs no introduction.[32] Boyle documented the experimental results in his 1660 *New Experiments Physico-Mechanical*, calling Hooke's conjecture regarding the cause of capillary action in experiment XXXV "ingenious":

> The cause of this ascension of the water appeared to all that were present so difficult … Wherefore, in favour of his [Hooke's] ingenious conjecture, who ascribed the phaenomenon under consideration to the greater pressure made upon the water by the air without the pipe, than by that within it…[33]

As mentioned, Hooke's explanation for the cause of capillary action is a difference of air pressure. That is, the greater pressure pressing down upon the cistern of water in the vessel outside the pipe than on the water within causes the water inside the capillary to rise. But why was the cause of capillary action important in and of itself, without any of the bells and whistles that Hooke attaches to it, to seventeenth-century savants? According to Alice Stroup in *A Company of Scientists* "Capillary action seemed to seventeenth-century scientists to explain several natural

[31] Ofer Gal and Raz-Chen Morris, *Baroque Science*, 11–12.

[32] For a thorough account, see Steven Shapin and Simon Schaffer's *Leviathan and the Air-Pump* (Princeton, NJ: Princeton University Press, 1985).

[33] Robert Boyle, *The Works of the Honourable Robert Boyle* (London: Printed for J and F Rivington, L Davis, W Johnston, S Crowder et al., 1772), 81.

phenomena".[34] As Stroup has also noticed, Hooke provides a brief answer in list form:

> … the rising of Liquors in a Filtre, the rising of Spirit of Wine, Oyl, melted Tallow … in the Week of a Lamp … the rising of Liquors in a Spunge, piece of Bread, Sand … perhaps also the ascending of the Sap in Trees and Plants, through their small, and some of them imperceptible pores …[35]

Viewed in this light, the little pipes or canes that Hooke makes by melting glass "in the flame of a Lamp, and then very suddenly Draw[ing it] out into great length",[36] some of them cobweb-thin filaments and yet still perforated with pores or channels when viewed under a microscope, turn into artificial filters, wicks, sponges and stems. Thus a simple experiment has the potential to account for a wide range of phenomena, and elevates Hooke's artificial tubes to the same status as nature's capillaries. Acting as if there has been no line drawn between art and nature, Hooke replaces "imperceptible pores" with artificial capillary tubes – objects of his making, 'perceptible' enough for his microscopes.

A crucial part of any experiment is the move from a local, specific laboratory setting to a global or universal generalisation. That is, from turning a material thinking tool, an explanatory model, into a theory.[37] Although Boyle proclaimed Hooke's theory "ingenious", it was not uncontested: nevertheless, it enjoyed a long life, which underscores its success in accounting for the phenomenon of capillarity. For example, the teacher and textbook compiler Alexander Jamieson, in his 1837 *Dictionary of Mechanical Sciences*, cites "Dr Hook" on three occasions: once under "Capillary Action", where he mentions Hooke's measurements of maximum liquid height in capillary tubes, and "the diminished pressure of the air on the fluids in the tubes" as one of the "various hypotheses".[38]

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[34] Alice Stroup, *A Company of Scientists: Botany, Patronage, and Community at the Seventeenth-Century Parisian Royal Academy of Sciences* (Berkeley: University of California Press, 1990), 139.

[35] Robert Hooke, *Micrographia* (London: Jo. Martin and Jo. Allestry, 1665), 21.

[36] Hooke, *Micrographia*, 10.

[37] David Gooding, Trevor Pinch and Simon Schaffer (eds), *The uses of experiment* (Cambridge: Cambridge University Press, 1989).

[38] Alexander Jamieson, *A Dictionary of Mechanical Sciences, Arts, Manufactures and Miscellaneous Knowledge*

Hooke demonstrates this "diminished pressure" by attempting to prove two propositions true with arguments from experiment designed to convince his audience that his theory has the power to account for all the observable effects of capillarity:

1) "The first of which is, That an unequal pressure of the incumbent Air, will cause an unequal height in the waters Surfaces".[39]
2) "And the Second is, That in this Experiment there is such an unequal pressure".[40]

It makes sense that Hooke should attribute the cause of the "unequal height in the waters Surfaces" to "an unequal pressure", considering that he designed Boyle's air-pump, and that the pair conducted capillarity experiments inside it. It is then not surprising that Hooke should try to create a difference in air pressure, without the luxury of an air-pump, in his own experiments on capillary action. Thus, following the above two propositions, Hooke breezes through an experiment elegant in its simplicity, turning an inverted glass syphon and some water into a kind of crude air-pump that also isolates one artificial capillary tube ("Fig: 6" in my Fig. 1.1). That the experiment *is* simple, a material Ockham's razor, and not spectacular like Boyle's hard-to-operate, expensive air-pump,[41] has its advantages.

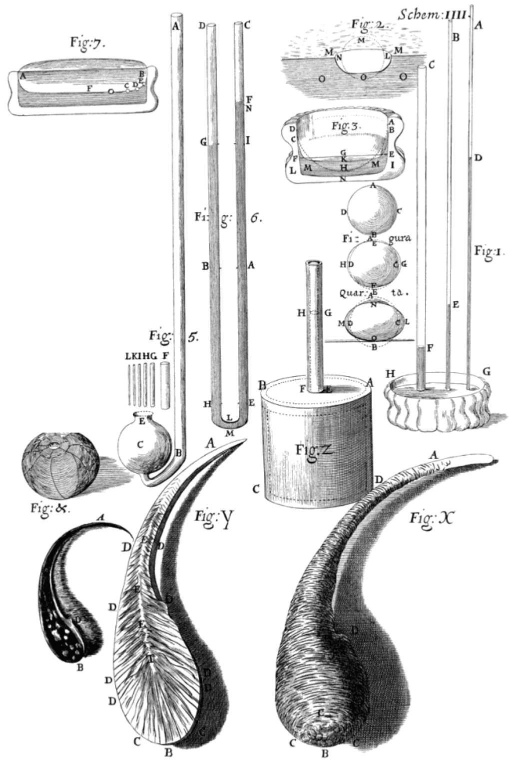

Figure 1.1 Schem. 4 (*Micrographia*). Hooke's accompanying illustrations for the experiments in Observ. VI Of Small Glass Canes.

According to Hooke's friend

(London: H. Fisher, 1829), 146.

39 Hooke, *Micrographia*, 11.

40 Hooke, *Micrographia*, 11.

41 Steven Shapin and Simon Schaffer, *Leviathan and the Air-Pump*.

Christopher Wren, simplicity in experimental and instrumental design is equivalent to the seventeenth century definition of 'ingenuity': "the misapprehending World measures the Excellence of things by their Rarity, or Difficulty of Framing", whereas "a Master" works with "a far smaller number of Peeces, & those perhaps of more trivial Materials, but compos'd with more Brain & less ostentation, frames the same thing in a little Volume".[42] What Wren means is that if one can demonstrate a concept using plain words, and a few simple parts and materials, then one has mastered the subject; whereas ostentation masks a misapprehension of the subject at hand. When Boyle refers to Hooke's theory of capillarity as "ingenious", he grants it the same assessment as Wren does regarding ingenious instruments and experiments.

The syphon in Hooke's design mimics the air-pump's glass receiver and a capillary tube simultaneously, and Hooke's lungs take on the role of the pump itself by alternatively blowing and "gently sucking"[43] on one end of the syphon to pump air into the 'receiver' (creating compression and an increase in air pressure) as well as 'pumping' air out (causing rarefaction and a drop in pressure inside the syphon-receiver). Before blowing or sucking on the syphon, the height of the water in the two vertical sections is equal and at equilibrium (AB). Blowing at D depresses the water on the same side as the compression at B and elevates it to A; sucking the air out gently produces "clean *contrary* effects".[44] In both the *Attempt* and its expanded second edition "Observ. VI" in the *Micrographia*, Hooke leaves the wording of this experiment unchanged, which indicates his satisfaction with the material model's power to function as an explanatory tool for thinking, and to prove the first proposition true. Namely, "That an unequal pressure of the incumbent Air, will cause an unequal height in the waters Surfaces". Further, Hooke's experiment

[42] Christopher Wren, cited in Jim Bennett, "Instruments and Ingenuity", in Michael Hunter and Michael Cooper (eds), *Robert Hooke Tercentennial Studies*, 71. For a detailed explanation on the early modern definition and use of the word 'ingenious' in experimental philosophy, see Bennett, "Instruments and Ingenuity", 65–76.

[43] Hooke, *Micrographia*, 11.

[44] Hooke, *Micrographia*, 11.

doubles as a practical demonstration for one of his later comments: “we were able to *separate* the *Air* from the *Aether* by *glass* …”[45] – an obvious reference to his air-pump work with Boyle, as well as being a remark on the ability of glass, whether receiver or tube, to manipulate pressure by filtering the solute of air out of the solvent aether.[46]

The second experiment, designed to buttress the second proposition, “That in this Experiment there is such an unequal pressure”, involves consecutively thinner and thinner capillary tubes, cemented in turn to the open end of a hollow glass bulb attached to the bottom of a glass cane, and filled with water (“Fig: 5” in *Figure 1.1*).[47] When the vertical cane section (AB) is filled with water, the water’s weight presses down on the air in the bulb, increasing its pressure and compressing it into the attached capillary tube. Hooke conjectures that the particles of air expend and store force to squeeze into the capillary tubes, which have diameters smaller than those of the air particles themselves. Because of this squeezing, the air particles have less force or pressure to exert against the water inside the tubes, which is then free to rise against gravity due to a lack of atmospheric pressure weighing it down. That is, the proportionally diminishing air pressure on the water inside the consecutively thinning tubes, relative to the atmospheric pressure on the reservoir, causes an increase in fluid height.[48] This is the type of mathematical analysis that Hooke will continue to favor throughout his career (see Part II) – in this case the proportions between forces.

Thus, it appears that Hooke has two models in support of the one phenomenon. The first, separating “the *Air* from the *Aether* by *glass*” which acts as a

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[45] Hooke, *Micrographia*, 14.

[46] For a typical Hooke hint on how he perceived both the spring and pressure laws as being two phenomena of springing bodies early on in his career, see Hooke, *Micrographia*, 40–41. This is, to the best of my knowledge, the only instance where Hooke shifts the frame of reference to the solvent aether rather than the solute air to describe what he would later refer to as “the same proportions one to the other” (Hooke, *Of Spring*, 3).

[47] Hooke, *Micrographia*, 19–20.

[48] Hooke, *Micrographia*, 19–20.

sieve. The second, the force theory just described, where air particles lose power by exerting it against the walls of the capillary tubes, and by storing it in their compressed springy parts, as they are pushed and squeezed in. However, together the experiments are complementary, and disclose essential details about Hooke's matter theory. The inverted syphon experiment shows the pressure law or inverse proportion law (nowadays Boyle's law[49]), and, according to Hooke proves the first proposition of his theory of capillary action true. The experiment using consecutively thinner tubes shows an intuitive understanding of the spring law (Hooke's law), and according to Hooke proves the second proposition true. Together the capillary action experiments display an inverse relation because of Hooke's notion of *springy air* particles (see Chapter 3, *Particle*). That is, when "separat[ing] the *Air* from the *Aether* by *glas*s", as in the inverted syphon experiment, the springy air particles are initially compressed in the syphon until the pressure upon them is relieved when they are released. In contrast, when air is forced into consecutively thinner tubes, the air particles contract to fit inside tubes with diameters smaller than their own ("an *Hole* less in *Diameter* then it self"), losing force to exert against the water by storing power in their compressed parts. "What degrees of force are requisite to reduce them into longer and longer *Ovals*, or to press them into less and less *holes*, I have not yet experimentally calculated," Hooke confesses.[50] Over two centuries later, Jamieson, referencing Hooke and his capillary tube observations, gives a visual explanation of the "reciprocal proportion" law or Boyle's law. The liquid

> will immediately rise in the tubes to a considerable height above the surface of that into which they are immersed; these heights varying nearly in reciprocal proportion of the diameters; the greatest heights, according to Dr. Hooke, being about 21 inches.[51]

Now, I mentioned that Hooke coins his theoretical tools 'congruity' and

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[49] For example, given a fixed amount of air at a constant temperature, there is an inverse proportion between air pressure and volume.

[50] Hooke, *Micrographia*, 19.

[51] Jamieson, *A Dictionary of Mechanical Sciences, Arts, Manufactures and Miscellaneous Knowledge*, 146.

'incongruity' during these studies on capillarity and pressure. Having practically demonstrated his two propositions, Hooke next needs to account for the differences between pressures inside and outside his glass capillary tubes. He argues that these differences can be inferred from the "Congruity or Incongruity of Liquids one with another". That is, "That there is such an unequal pressure, I shall prove from this, That there is a much greater inconformity or incongruity (call it what you please) of Air to Glass, and some other Bodies, than there is of Water to the same". [52] Although in the *Attempt* the terms 'congruity' and 'incongruity' designate little more than "visible effects" – for example, the miscibility and immiscibility of various fluids – I have chosen to cite this from the *Attempt* instead of the *Micrographia* because it is Hooke's first published statement of what Penelope Gouk claims is "the most original part of Hooke's theory … determin[ing] the way that bodies in motion are united or divided from one another".[53] While I agree with Gouk on this point, it is impossible to comprehend the unique aspects of Hooke's matter theory without analysing its construction and development – especially because it was a lifelong preoccupation of his, woven through all his work.

2. CAUSES

In the *Attempt*, while Hooke confidently attributes the cause of capillary action to a difference in air pressure, and is more than happy to attribute the cause of air pressure and other phenomena to 'congruity and incongruity', when it comes to providing causes for congruity and incongruity in turn, he shirks from the challenge, and weasel-words his way out of an explanation with an excuse about "[it] being an enquiry more proper to be followed and explained among the general Principles of

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[52] Hooke, *Attempt*, 7, 9. To compare the wording in the *Attempt* with Observ. VI, see also, Hooke, *Micrographia*, 11.

[53] Hooke, *Attempt*, 10. Gouk, "The Role of Acoustics and Music Theory in the Scientific Work of Robert Hooke", 585.

Philosophy".[54] In other words, he does not know yet. Nevertheless, he does provide a list of possible causes:

> … whether from the Figure of their constituent Particles, or interspersed pores, or from the differing motions of the parts of the one and the other, as whether circular, undulating, progressive, *etc.*, whether … from one or more of these enumerated causes …[55]

Here, Hooke has an idea that is still taking shape with the help of experiments and observations – for example, the "interspersed pores" are material pores that he studies when making microscopic observations – though he will refer back to each one of these possible causes in *Of Spring* seventeen years later, integrating them into and expounding his matter theory. However, the main developments concerning congruity and incongruity occur in the middle ground between the *Attempt* and *Of Spring* – the *Micrographia*. In the *Attempt*, Hooke lays the foundations for his matter theory, as previously discussed, with definitions and observations of effects of what is today called surface tension. Four years later, in the *Micrographia*, he relies on the same definitions and observations in Observ. VI, but is also comfortable and confident enough to discuss causes. If the addenda of acoustical experiments and arguments from analogy are anything to go by, then the core of this new knowledge is motion. Specifically, studies of consonant and dissonant mechanical sound wave vibrations.

MUSIC

Music was not an illustrative analogy for Hooke, but a way for him to work a problem with instruments designed for human senses in order to gain knowledge outside sense limits. It was also a means to construct a physical matter theory from which mathematics follow (see *Part II: Practical Geometry*). Although Hooke began to investigate sound at an early age, according to Jamie Kassler and David Oldroyd,

[54] Hooke, *Attempt*, 10.
[55] Hooke, *Attempt*, 10.

this excluded music theory until "sometime in the 1660s",[56] which coincides with the start of his experiments on musical strings, and other musical instruments, at Royal Society meetings and in the *Micrographia*. Hooke played the organ while at Westminster School, teaching himself twenty lessons, and entered Christ Church, Oxford, as a chorister in 1650.[57] Boyle and Hooke were two of few people in the world to have witnessed the sound of a ticking watch fading as air was pumped out of the receiver only for the ticking to return again gradually as the air was allowed back in, which demonstrated that sound needs a medium through which to travel. Moreover, Hooke showed that sound travels faster through denser media, and hypothesised that condensing the air in the receiver would amplify the sound.[58] During his time as Curator of Experiments, he turned Francis Bacon's Sound House from *New Atlantis* into a reality by conducting "many investigations into sound generation, transmission and reception".[59] He discoursed regularly with a select few friends and colleagues, such as Wren, who were also interested in music theory, and developed a unique proportional tuning system, and several systems of music notation.[60] In this vein, his trials with monochords, which allowed him to manipulate stretched musical strings and their vibrational frequencies to better comprehend consonance, dissonance and tone would prove most crucial to the development of his metaphysics of vibration by giving him a means to account for all the fundamental properties of matter.

As mentioned, Hooke conducted trials on the speed of sound in various

[56] Kassler and Oldroyd, "Robert Hooke's Trinity College 'Musick Scripts', his Music Theory and the Role of Music in his Cosmology". *Annals of Science* 40, no. 6 (1983): 559-595, 574.

[57] Gouk, "The Role of Acoustics and Music Theory in the Scientific Work of Robert Hooke", 575. Kassler and Oldroyd, "Robert Hooke's Trinity College 'Musick Scripts', his Music Theory and the Role of Music in his Cosmology", 590.

[58] Hooke, *Micrographia*, Preface. Gouk, "The Role of Acoustics and Music Theory in the Scientific Work of Robert Hooke", 576.

[59] Kassler and Oldroyd, "Robert Hooke's Trinity College 'Musick Scripts', his Music Theory and the Role of Music in his Cosmology", 583.

[60] Robert Hooke, *The Diary of Robert Hooke, 1672–1680* [henceforth *Diary*], eds Henry W Robinson and Walter Adams (London: Wykeham Pub., 1968), 152. Kassler and Oldroyd, "Robert Hooke's Trinity College 'Musick Scripts', his Music Theory and the Role of Music in his Cosmology".

media. In the *Micrographia's* Preface, he describes inflecting sound around corners through wires, sending it through rods and walls, and attempts to compare it with the speed of light. Thomas Birch and Richard Waller, the latter witnessing at least some of the experiments, provide more detailed descriptions. Birch recounts that Hooke's primary interest was a "way of conveying force to a great distance, which he conceived would best be done with some stiff and inflexible rod";[61] Waller testifies that "the sound conveyed by the Air [came] a considerable time after that by the Wire";[62] Birch and Waller separately describe an experiment with a great monochord, designed to exhibit that each pitch has a unique vibrational frequency. According to Waller,

> In *July* [6] 1664 [Hooke] produced an Experiment to shew the number of Vibrations of an extended String, made in determinate time, requisite to give a certain Tone or Note, by which it was found that a Wire making two hundred seventy two Vibrations in one Second of Time, sounded *G Sol Re Ut* [middle G] in the Scale of all Musick.[63]

Birch adds more, explaining that a 136 foot wire (with a diameter of 1/32 inches), was stretched by weights (with a total constant tension of roughly 4 3/4 pounds), and that although "the velocity of the vibration of a string tuned to *G. Sol. Re. Ut.* [was] two hundred seventy-two times in a second [when stopped to 1 foot]", the musical note was "ghessed", but confirmed at the next meeting by comparing the tone "with a pipe".[64]

In the *Micrographia's* Observ. VI, Hooke employs a vibrating strings similitude to argue for the causes of congruity and incongruity, because

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[61] Birch, *The History of the Royal Society*, Vol. IV, 545.

[62] Waller, "The Life of Dr. Robert Hooke," in Hooke, *Posthumous Works*, xxiv; "Hooke Folio Online", livesandletters.ac.uk, 2017, http://www.livesandletters.ac.uk/cell/Hooke/Hooke.html., 29.

[63] Waller, "The Life of Dr. Robert Hooke," in Hooke, *Posthumous Works*, x.

[64] Birch, *History of the Royal Society, Vol. I*, 446–7, 449; "Hooke Folio Online", livesandletters.ac.uk, 2017, <http://www.livesandletters.ac.uk/cell/Hooke/Hooke.html>, 29. 272 Hz corresponds approximately to a middle C♯/D♭ in today's equal-tempered scale. According to Theo. Baker's *Dictionary of Musical Terms*, 8th ed. (New York: G Schirmer, 1904), 182, *G sol re ut* was the solmisation term for middle G and its octave; for low G, *Gamma-ut* was favoured. For an alternative interpretation, see Benjamin Wardhaugh, "Mathematics, Music and Experiment in Late Seventeenth-Century England", in Eleanor Robson and Jacqueline A. Stedall (eds), *The Oxford Handbook of the History of Mathematics* (Oxford: OUP, 2009) 639–61.

> particles that are *similar,* will, like so many *equal musical strings equally stretcht*, vibrate together in a kind of *Harmony* or *unison*; whereas others that are *dissimilar* … like so many *strings out of tune* to those unisons, though they have the same agitating *pulse*, yet make quite *differing* kinds of *vibrations* [so that] they *cannot agree* together, but *fly back* from each other to their similar particles.[65]

Thus, although all matter, such as stretched musical strings, has "the same agitating *pulse*" (which I will examine in the subsequent section), "unison" strings that are congruent, and harmonious strings (for example, an octave) that are "similar", represent and mechanically demonstrate Hooke's concept of congruity – a sympathetic resonance that causes particles to "vibrate together" and cohere. And dissonant or "dissimilar" strings represent incongruity.

Not satisfied with relying solely on a strings similitude to serve as a model for vibrating particles, Hooke employs a drum and sand in an experiment designed to make his abstract notion of an "agitating *pulse*" concrete and visible to the sense of sight. The experiment emulates particles transitioning from a solid to a fluid state. Hooke suspends a dish of sand over a drum, and beats the drumhead with "a *quick* and *strong vibrating motion*" to show "how a body actually divided into small parts becomes a *fluid*"; the result is that the agitated sand, which can be imagined as magnified particles, displays all the properties of a fluid.[66] Moreover, mixing sands of various grain sizes produces the same effect as playing dissonant strings together: instead of mixing harmoniously and homogenously, the finer sand tosses out the coarser sand, which congregates into a congruent pile. Immediately preceding his strings similitude, Hooke connects his percussion and string models thus:

> I suppose the *pulse* of heat to *agitate* the small parcels of matter, and those that are of a *like bigness*, and *figure*, and *matter*, will *hold*, or *dance* together, and those which are of a *differing* kind will be *thrust* and *shov'd* out from between them …[67]

As Ofer Gal and Raz-Chen Morris explain, "For Hooke this comes to mean that the very structure of matter is produced by motion … [M]atter is in constant motion, and it is this motion and its 'harmonies' that create clusters of particles that become

[65] Hooke, *Micrographia*, 15.
[66] Hooke, *Micrographia*, 12.
[67] Hooke, *Micrographia*, 15.

substances".[68] Further connecting his vibrating sand to his strings, Hooke adds, "To which three properties in strings, will correspond three properties also in sand, or the particles of bodies, their Matter or Substance, their Figure or Shape, their Body or Bulk".[69] The substance, shape and bulk of all objects determine their natural frequency of oscillation. Thus, by playing with various combinations of these three properties, on which the vibrations of all objects depend, one can make just as many "harmonies and discords"[70] as are possible with musical strings.

Later in life, from his *Lectures of Light* to his *Lectures concerning Navigation and Astronomy* in the 1680s, Hooke would also employ resonating bells.

> I have already, I think, fully proved in Light and Colour, the Object of Sight, that the Motion which is produced in the Eye, proceeds from an internal Motion made in the Sun … I could also as easily prove, that Sound in the Ear, which is a real Motion in some part thereof, is produced by the internal Motion of the Parts of the Bell some Miles perhaps distant.[71]

The bell is a good example, he explains, "because both the Motion in the Bell, and the Motion in the Ear, or some other Body there placed, is discovered by other Senses, namely, by the Sight and Touch, as well as by the Ear". Therefore, it is "evident first to the Sense of Seeing, that the Bigger the Body is, the slower its Vibrations, and the smaller the quicker." This is true of "all pendulous Motions", for example, "in the Recursions and Vibrations of Pieces of Timber, which the longer and bigger they are, the more slow are the Vibrations made by them; and the smaller and shorter, the quicker."[72] But when the vibrations are so fast that they blur before the eye, the ear proves to be the more sensitive natural instrument: "when the eye is unable to assist us any further in distinguishing the swiftness of Vibrations, there the Ear comes in with its assistance, and carries us much further".[73]

> As I shewed in the Vibrations of Strings, so now I instance further in Bells, where we find by the Tone, that the smaller the Bell, the sharper and more shrill its

[68] Ofer Gal and Raz Chen-Morris, *Baroque Science*, 156.
[69] Hooke, *Micrographia*, 15.
[70] Hooke, *Micrographia*, 15–16.
[71] Hooke, *Of Comets and Gravity*, in *Posthumous Works*, 184.
[72] Hooke, *Lectures of Light*, in *Posthumous Works*, 135.
[73] Hooke, *Lectures of Light*, in *Posthumous Works*, 135.

> Sound; and this carries us on to a Sound so sharp, that we can only call it screeking, and at length it becomes offensive to the Ear, because beyond that it cannot endure the Sense of a shriller note or quicker Vibration …[74]

In this way, bells are an important explanatory musical instrument for Hooke, because although they still work within the sensible realm, they operate a step higher than the sense of sight, showing sensibly and thus imaginably that vibrations continue with "insensible velocities" into the realm beyond the senses.

Further, Hooke claims that he could "more largely explain by particular Experiments … that the Motions of several Bodies at a distance, are caused by the internal Motion of the sounding Body; and that this Power of moving is every way propagated [according to the inverse square law[75]] by the ambient *Medium*, which excites in solid Bodies at a distance, a similar Motion."[76] But how these different "Motions of several Bodies at a distance" pass like pond ripples through one another is a line of thought leading Hooke to a problem that he never seems to solve to his satisfaction. How different vibrations crisscross, "confound[ing] the regular propagation of each others Rings", he admits, "does much confound the Imagination".[77] Yet, Hooke argues, "'tis enough for a Principle to build upon, that we are assured it is so, and that such and such are the Effects that flow from it".[78] And he allows himself to build a solution to the problem by using a "Chime of Impulses" – his term for how various point sources communicate "every one of their impressions distinct and successively within [the] Period" of a least-sensible moment.[79] Notice that Hooke instantiates the abstract word "Impulses", which is equivalent with "Motion" here, by connecting it to the word "Chime", thereby forming an experiment-based metaphor already in the wind owing to his studies of bells. Growing up his metaphysics of vibration a notch, he imagines that, at the particle level, "there might be found distinct Parts enough, within the orb of this

[74] Hooke, *Lectures of Light*, in *Posthumous Works*, 135.
[75] Hooke, *Of Comets and Gravity*, in *Posthumous Works*, 185.
[76] Hooke, *Of Comets and Gravity*, in *Posthumous Works*, 184.
[77] Hooke, *Lectures of Light*, in *Posthumous Works*, 133.
[78] Hooke, *Lectures of Light,* in *Posthumous Works*, 133.
[79] Hooke, *Lectures of Light,* in *Posthumous Works*, 136.

least sensible Point, to propagate every one of those Motions distinct."[80] That is, via sympathetic resonance, which, in the case of light, Hooke describes as a "harmonious Chime, as it were, of the Pulsations of several Luminous Points or Bodies". In the case of sound, he explains "that the Motions of several Bodies at a distance, are caused by the internal Motion of the sounding Body; and that this Power of moving is every way propagated" and "excites in solid Bodies at a distance, a similar Motion."[81] Moreover, the behaviour of drops and pond ripples, coupled with a conflation of light and sound studies, and thinking about the propagation of powers in terms of chiming bells, leads Hooke to consider the earth as such a vibrating point source.

> Suppose that there is in the Ball of the Earth such a Motion, as I, for distinction sake, will call a Globular Motion, whereby all the Parts thereof have a Vibration towards and fromwards to the Centre, or of Expansion and Contraction.[82]

And that the cause of gravity works correspondingly. "*For this Power* [gravity] *propagated … does continually diminish according as the* Orb *of Propagation does continually increase*", namely, "*always reciprocal to the* Area *or Superficies of the Orb of Propagation, that is duplicate of the Distance*", or the inverse square law – "*as we find the Propagation of the* Media *of Light and Sound also to do; as also the Propagation of Undulation upon the Superficies of Water.*"[83] Thus resonating bells also help Hooke to imagine how light, sound, gravity and so on can be described by the same natural law, *because of* his theory of congruity and incongruity, which he consciously weaves into all of his work.

But stretched sounding strings remained Hooke's favourite tools for congruity and incongruity, repeated and expounded upon both in *Of Spring*, where he would strip them of their qualitative properties (see Chapter 5*: Lines*), and in *Lectures of Light*, where he would use "a long String [stretched] out between two Pins" to show

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80 Hooke, *Lectures of Light,* in *Posthumous Works*, 136.

81 Hooke, *Lectures of Light*, in *Posthumous Works*, 176; Hooke, *Of Comets and Gravity*, in *Posthumous Works*, 184.

82 Hooke, *Of Comets and Gravity*, in *Posthumous Works*, 184.

83 Hooke, *Of Comets and Gravity*, in *Posthumous Works*, 185.

that the sense of hearing operates in a region of time that is a rung up from the sense of sight. On the one hand, Hooke explains again, "if [the string] be long and but slack, we are able to distinguish it [with our eyes], as it moves from one side to the other … because it makes its Vibrations within the compass of several human Moments of time".[84] A 'human moment' is, according to Hooke, a unit of measure quick as a human thought – the least-sensible moment imaginable.[85] On the other hand, a tense, plucked string blurs before one's eyes even as one's ears take over, registering the tone.

> But if [the string] be strain'd yet straighter, so as to make its whole Vibration within one human Moment, we see it as if it were in all parts of its space and in the two *Termini* at once, about which time, and not before, it begins to sound. [86]

Owing to these studies of strings, Hooke takes music theory and vibrations a step further, breaking free of the limits of the human senses. After lamenting the limits of the eye to detect vibrations, he rejects the ear as well, which detects only a narrow spectrum of frequencies, and so replaces the listener with quantifiable vibrations and musical ratios. Again, Hooke reasons that if there are motions that the eye cannot detect, and if sounds are nothing but vibrational frequencies, that is, motions, then there must necessarily be harmonious and dissonant frequencies in the sonorous silence above human hearing.

> For that the Shrillness of the Note depends upon the quickness of the Vibration, I think I need not instance. Hence I conceive that there may be yet beyond the reach of our Ears infinite shriller and shriller Notes.[87]

Thus strings are not only a model for congruity and incongruity, but for Hooke's epistemology. That is, material thinking tools – instruments, apparatuses and experiments – constructed to work at the level of the human senses can be used to gain reliable knowledge in the realms beyond the senses. Owing to this universality, early on in the *Micrographia*, where Hooke first introduces his musical

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[84] Hooke, *Lectures of Light*, in *Posthumous Works*, 134.
[85] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 551.
[86] Hooke, *Lectures of Light*, in *Posthumous Works*, 134.
[87] Hooke, *Lectures of Light*, in *Posthumous Works*, 135.

strings model, he also introduces his metaphysics of vibration:

> Now that the parts of all bodies, though never So solid, do yet vibrate, I think we need go no further for proof, then that all bodies have some degree of heat in them, and that there has not been yet found any thing perfectly cold: Nor can I believe indeed that there is any such thing in Nature, as a body whose particles are at *rest*, or lazy and unactive in the great Theatre of the World, it being quite contrary to the grand Oeconomy of the Universe.[88]

PULSE

In the *Micrographia* stage of his matter theory, Hooke still clings to qualitative descriptions and "Relative propert[ies]" gained from experimental trials and observations.[89] Congruity is "a property of a fluid Body, whereby any part of it is readily united with any other part, either of itself, or of any similar, fluid, or solid body"; and its highest property is "a Cohesion of the parts of the fluid together, or a kind of attraction and tenacity". Incongruity is "a property of a fluid, by which it is hindered from uniting with any dissimilar, fluid, or solid Body".[90] For the latter, he recycles his examples from the *Attempt*, listing raindrops in air, bubbles of air in water, drops of oil in water and so on. From microscopic observations, Hooke speculates that all smaller parcels of matter with a globular form seem to have been in a fluid state first, and applies congruity and incongruity to explain their globularity: a congruous body is "forc't into as little space as it can possibly be contained in, namely, into a Round Globule", against the surrounding incongruous fluid.[91] But he is quick to point out that if one wishes to understand the cause of 'congruity and incongruity', then one must first ask what is the cause of fluidness. This, like the *Micrographia's* percussive and string instruments experiments and analogies, is a new development, and Hooke answers immediately that the cause of fluidness is "nothing else but a certain *pulse* or *shake* of *heat*". Further, heat itself is "nothing else but a very *brisk* and *vehement* agitation of the parts of a body … [The]

[88] Hooke, *Micrographia*, 16.
[89] Hooke, *Micrographia*, 15.
[90] Hooke, *Micrographia*, 12, 15.
[91] Hooke, *Micrographia*, 12.

parts of a body are thereby made so *loose* from one another, that they easily *move any way*, and become *fluid*" – such as the grains of sand vibrating in the dish above the drumroll.[92]

In a second experiment, which also demonstrates how a body transitions from a solid to a fluid state, Hooke sets up an iron block with a pin in it. The pin cannot be unscrewed by hand. Hooke explains that grating the iron block with a file creates vibrations and heat, "by which means the agitation of heat so easily *loosens* and *unties* parts of *solid* and *firm* bodies", allowing him to unscrew the pin with his fingers.[93] Since "there has not been yet found any thing perfectly cold", and Hooke supposes that "the *pulse* of heat [will] *agitate* the small parcels of matter", he concludes that "all bodies have some degree of heat in them". It follows that all bodies have a vibrational frequency that changes, becoming faster or slower – though never "at rest" – depending on how much heat is applied or removed.[94] In this way, Hooke's concept of "pulse" is interwoven with his studies of vibrations and his metaphysics.

Yet, Hooke's use of the word 'pulse' depends upon whether the context is practical or theoretical, qualitative or quantitative. Moreover, like his amalgamation of careers as Curator and geometry professor, he often moves between practical and theoretical knowledge with little display. As such, 'pulse' represents a physical, mechanical striking, a quantifiable vibration and aggregates of pulses. In a diary entry dated Saturday 15 January 1676, Hooke succinctly sets the scene: at "Sir Christopher Wrens", discussing "my notion of sound" with him and "Dr. Holder". Remaining faithful to the results and observations of his 1664 monochord trials, and his *Micrographia* strings similitude, Hooke defines sound as "nothing but strokes within a Determinate degree of velocity."[95]

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[92] Hooke, *Micrographia*, 12.
[93] Hooke, *Micrographia*, 13.
[94] Hooke, *Micrographia*, 12–13.
[95] Hooke, *Diary*, 211.

> I told [Wren and Holder] how I would make all tunes by the stroke of a hammer … that there was no vibration in a puls of sound, that twas a puls propagated forward, that the sound in all bodys was the striking of parts one against the other and not the vibration of the whole.[96]

Hooke reiterates to Wren and Holder that sound needs a medium through which to propagate, "that twas a puls propagated forward," because "sound in all bodys was the striking of parts one against the other and not the vibration of the whole".[97] In other words, the wave or pulse propagates from part to part, and the speed of sound is not instantaneous. Further, "the vibrations of a string [are] not Isocrone" but "the vibrations of particals [are]",[98] meaning that the string's amplitude is independent of its vibrational frequency. If this were not the case, there would be no music, only noise. Hooke first demonstrates the isochrony of springing bodies experimentally and geometrically in *Of Spring*. Almost a decade later, in a lecture on navigation and astronomy, he would remind his audience that

> … when the Vibrations are Isocrone, as I have formerly here proved those of strained or extended strings to be, which act upon the principle of Spring … they are Musical sounds; but when they are not Isocrone they are not Musical.[99]

The same year that Hooke discussed sound with Wren and Holder, in a successful attempt to mimic, control and manipulate 'pulse' or "tunes by the stroke of a hammer", he invented brass ratchet wheels, or "sound wheels", which could emit tones of various frequencies depending on the number of teeth and how fast they struck a metal plate. A 1676 diary entry reads: "Directed Thompion about sound wheels".[100] Today, this invention is known as Savart's wheel, but over a century before Felix Savart (1791–1841), Hooke had showcased sound wheels, in between unveiling a telescope aperture and a helioscope, before the Royal Society in 1681.[101]

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96 Hooke, *Diary*, 211.
97 Hooke, *Diary*, 211.
98 Hooke, *Diary*, 211.
99 Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 551. The lecture is dated 29 June 1687.
100 Hooke, *Diary*, 223.
101 Thomas Birch, *The History of the Royal Society, Vol. IV* (London: Millar, 1756), 96–97. Gouk, "The Role

> Mr. Hooke shewed an experiment of making musical and other sounds by the help of teeth of brass wheels; which teeth were made of equal bigness for musical sounds, but of unequal for vocal sounds.[102]

Kassler and Oldroyd attest that Hooke "devised brass wheels fixed to clockwork for an experiment on the frequency of vibration – an experiment devised to demonstrate the theory of Francis North"[103] who plotted pulses along an axis of time, showing frequencies and the rates at which they meet in consonance. However, Hooke's sound wheels preceded North's pulse plots by about a year.[104] As mentioned, the sound wheels worked by having a certain number of teeth per wheel, which could be spun at swift speeds to produce measurable periodic vibrations. Played together, the wheels could produce consonants such as the perfect fifth. In a biography about Francis North, authored by his brother Roger, Hooke's wheels are praised: "The ingenious Mr. Hook, made an engine of wheels that made pulses in any musical proportion, as 2, 3, 4, 5, or 6 to 1 and so 3 to 2 and the like".[105]

As a simple explanation of how Hooke's sound wheels work fundamentally, imagine a system of two gears. The first gear, cranked by hand, has a big diameter and 360 ratchets or teeth; the second gear, connected by ratchets to the first, has a small diameter and 36 teeth; a 10:1 ratio. With a 10:1 ratio, if the larger gear is turned by hand at 1 revolution per second, then the smaller gear in turn spins at 10 revolutions per second (that is, ten times faster). So, with this simple two-gear system, if Hooke had wanted to spin a sound wheel at a frequency of 360 pulses per second, he would have had to turn the larger wheel one time per second for the smaller wheel to sound 360 pulses as its ratchets strike a vibrating, sounding strip of metal. Moreover, by adding wheels with different ratios of teeth to the end of a common axle, it would be possible to make consonances or chords by striking more

of Acoustics and Music in the Scientific Work of Robert Hooke", 583.

[102] Birch, *The History of the Royal Society, Vol. IV*, 96.

[103] Kassler and Oldroyd, "Robert Hooke's Trinity College 'Musick Scripts', his Music Theory and the Role of Music in his Cosmology", 584.

[104] Wardhaugh, "Mathematics, Music and Experiment in Late Seventeenth-Century England", 650.

[105] From Roger North's "The World", quoted in Benjamin Wardhaugh, "Mathematics, Music and Experiment in Late Seventeenth-Century England", in Robson, Eleanor, and Jacqueline A Stedall. *The Oxford Handbook of the History of Mathematics* (Oxford: Oxford University Press, 2011), 650.

than one wheel's ratchets simultaneously with a sounding metal strip. For example, by adding a second wheel with 18 teeth, one would hear an octave (1/2); or, as Roger North testifies, with a second wheel Hooke could sound a perfect fifth.

> [Hooke] would begin to turne slow, and so long the pulses were distinct, and he could discern them, as smiths at anvill, without any other idea; but then coming to a might swiftness, the consonance called fifth (for instance), which is 3/2 [would sound].[106]

According to Benjamin Wardhaugh's interpretation, North describes how Hooke's sound wheels "illustrated that a continuous sensation in general resulted from a series of separate events too frequent to be distinguished".[107] Recall that Hooke rejects both the eye and the ear for this very reason, replacing the senses with countable pulses and musical ratios that the sound wheels embody.[108] Thus, even when the distinct pulses blur into a continuous aggregate of sound, Hooke can use experiences and knowledge gained in the human-sized realm to scale up the ladder of consonances into the realm of insensible bodies and motions. Late in life, he would explain that sense knowledge is "of the first and inferior Region, wherein we distinguish the parts of Time by Monades or Unites … not considering [moments] singly, but together" – just as the sound wheels exhibit. Yet this knowledge "brings us to another Region, where we find another prospect of Time, and Partitions thereof far differing from that of the first and inferior Region".[109] Finally, this concept also works in reverse; that is, from a continuous aggregate of sound to distinct pulses, which means that all pulses – however swift – are calculable, so long as one has a sensible starting point. But it was not enough for Hooke to exhibit only the motion part of his matter theory with sounding strings and wheels. He needed to show that

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106 From Roger North's "The World", quoted in Wardhaugh, "Mathematics, Music and Experiment in Late Seventeenth-Century England", in Robson and Stedall, *The Oxford Handbook of the History of Mathematics*, 649. Also see Floris H. Cohen, *Quantifying Music* (Dordrecht: D. Reidel Pub. Co., 1984), 96–97, for an explanation of the 'coincidence theory of consonance'.

107 Wardhaugh, "Mathematics, Music and Experiment in Late Seventeenth-Century England", 649.

108 Also see Gal, "Empiricism without the Senses: How the Instrument Replaced the Eye," in Ofer Gal and Charles Wolfe (eds), *The Body as Object and Instrument of Knowledge* (Netherlands: Springer, 2010), 121–48.

109 Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 551.

isochronous vibrations can and do shape and change material structures in an orderly manner. To do this, he relied once again on musical proportions.

In July 1680, Hooke and Wren continued their studies on sound and musical proportions by considerably improving the ninth experiment in Francis Bacon's *Experiments in consort, touching Motion of bodies upon their pressure* in a supportive move for Hooke's matter theory. "Take a glass," Bacon instructs, "and put water into it, and wet your finger, and draw it round about the lip of the glass, pressing it somewhat hard; and after you have drawn it some few times about, it will make the water frisk and sprinkle up in a fine dew." Although Bacon's primary concern is to demonstrate "the force of compression in a solid body" owing to "an inward tumult in the parts thereof, seeking to deliver themselves from the compression",[110] a little later on in the thirteenth experiment, he divulges that an effect of rubbing one's wet finger over the rim of a glass is the production of sound by "subtile percussion of the minute parts".[111] Today, this effect is attributed to the phenomenon of slip-stick friction. In Hooke and Wren's version,

> Mr. Hooke related, that he had observed, that the motion of the glass was vibrative perpendicular to the surface of the glass, and that the circular figure [of the water inside] changed into an oval one way, and the reciprocation presently changed it into an oval the other way; which he discovered by the motion of undulation of the rising water in the glass … in four places of the surface, in a square posture.[112]

Not taking Hooke's word for it, the experiment was "tried before the Society", and

> Wren coming in said, that the glass would vibrate much stronger, being struck on the edge with a violin-bow. This was also tried, and then the square undulation was extremely plain.[113]

It was shown "upon further trials" that the shape and number of undulations corresponds to a specific musical ratio:

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[110] Francis Bacon, *Sylva Sylvarum: or, a Naturall History*, seventh ed. (London: printed by A.M. for William Lee, 1658), 2–3.
[111] Bacon, *Sylva Sylvarum*, 3.
[112] Birch, *The History of the Royal Society*, Vol. IV, 46.
[113] Birch, *The History of the Royal Society*, Vol. IV, 46.

> But there was likewise discovered another undulation, by which the water was observed to rise in six places like a hexagon … also in eight places like an octagon. Each of these gave their particular and distinct sounds: the 4 and 8 were octaves, and the 6 and 4 were fifths. [114]

Notice that the octave and perfect fifth are the same harmonious proportions that Hooke favoured when demonstrating his sound wheels, thus forging a relation not only between sound and forms of matter, but also between insensible sounds and matter. For if "the 4 and 8 were octaves", then it is reasonable for Hooke to assume that some multiple of four well beyond the limits of human hearing will also shape matter. At another Society meeting, the experiment was repeated on a "large glass holding about three quarts, almost filled with water", and it was concluded by the number and shape of undulations formed inside the rim of the glass that "some were confounded and broad, which seemed to participate of two sounds",[115] capable of more complicated and congruous formations and manipulations. That the medium of choice was water would have only helped to corroborate Hooke's claims on the congruity and incongruity of *fluids* during his capillarity trials.

Hooke's sounding strings, discussed previously, reveal again that he already had the epistemological notions in the *Micrographia* upon which his sound wheels are based, and that the motivating questions of his 'observations' revolved around vibrations. One interesting instance of Hooke applying insensible pulses to a human-sized endeavour is Observ. XXXVIII *Of the Structure and Motion of Wings of* Flies. In this observation, Hooke wants to find out the frequency of a fly's beating wings in order to calculate their velocity, because he supposes that "by the sound, the wing seem'd to be mov'd forwards and backwards with an equal velocity", or isochronous motion.[116] Listening to the hum a fly's wing strokes, Hooke conjectures that

[114] Birch, *The History of the Royal Society*, Vol. IV, 46.
[115] Birch, *The History of the Royal Society*, Vol. IV, 48.
[116] Hooke, *Micrographia*, 172.

> (from the sound [the fly] affords, if it be compar'd with the vibration of a musical string, tun'd unison to it) it makes many hundreds, if not some thousands of vibrations in a second minute of time.[117]

And comparing the sound to that of a bee's, Hooke concludes that the bee's wing-strokes are swifter on account of their higher pitch: "if we may be allowed to ghes by the sound," he says, "the wing of a Bee is yet more swift; for the tone is much more acute, and that, in all likelihood proceeds from the exceeding swift beating of the air by the small wing".[118]

It may seem an obvious comparison – matching the tone of a fly's vibrating wings with the sound made by an artificial instrument – "the vibration of a musical string, tun'd unison to it" in order to "ghes by the sound" and hence the number of pulses, but a candid conversation recorded by Pepys reveals scepticism on his part, and that Hooke's blurring and conflation of the natural and artificial was not exactly a commonplace. On 8 August 1666, having run into Hooke on the street, Pepys records Hooke's claims "about the nature of sounds" and "how many strokes a fly makes with her wings":

> … and [Hooke] did make me understand the nature of musicall sounds made by strings, might prettily; and told me that having come to a certain number of vibrations proper to make any tone, he is able to tell how many strokes a fly makes with her wings (those flies that hum in their flying) by the note it answers to in musique during their flying.[119]

Yet Pepys ends his diary entry with the remark that Hooke's relation between the humming of strings and the humming of a fly's wings "is a little too much refined", even if his "discourse in general of sound was mighty fine".[120]

SPRING

Hooke's strings analogies, far from being merely illustrative, demonstrate his dynamic matter theory. His work on springy bodies and vibrations culminated into

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117 Hooke, *Micrographia*, 173.
118 Hooke, *Micrographia*, 173.
119 Samuel Pepys, *The Diary of Samuel Pepys. Volume VII*, ed. Henry B. Wheatley (New York: George E. Croscup, 1894), 239.
120 Samuel Pepys, *The Diary of Samuel Pepys. Volume VII*, 239.

a treatise on congruity and incongruity: *Of Spring*. Here, Hooke translates pulses to powers, and uses the fundamental hypothesis from the *Micrographia* – again, *all* matter vibrates, *all* forces are the effects of congruous and incongruous vibrations[121] – to account for the sensible and insensible physical properties of matter. Taking a leaf out of Descartes's *Principles of Philosophy*, in preparation for his geometrical demonstration of the spring law (see Chapter 6, *Mixt*), Hooke replaces the qualitative and sensual descriptions favoured in the *Micrographia* with matter, motion, and proportion.[122] Following this, Hooke's definitions for congruity and incongruity, as well as his strings similitude, undergo the same reduction. Remembering his promise in the *Attempt*, over a decade before, to further explain "what I thereby meant on some other occasion", Hooke states: "By Congruity and Incongruity then I understand nothing else but an agreement or disagreement of Bodys as to their Magnitudes and motions."[123] Expounding each in turn, he specifies that

> Bodies then I suppose congruous whose particles have the same Magnitude, and the same degree of Velocity, or else an harmonical proportion of Magnitude, and harmonical degree of Velocity. And those I suppose incongruous which have neither the same Magnitude, nor the same degree of Velocity, nor an harmonical proportion of Magnitude nor of Velocity.[124]

To explain, Hooke relies again on his strings similitude, presented as a thought experiment that can nevertheless be replicated empirically. In a separate though interrelated move, he abandons the drum-and-sand experiment from the *Micrographia* – which was designed to demonstrate, first, how solids become fluids and, second, incongruity – and instead focuses his audience's attention onto a single springing particle. In this new particle analogy, Hooke instructs the reader to imagine a plate of iron with dimensions of 1 foot squared. When this plate is knocked into a "Vibrative motion forwards and backwards the flat ways", it

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[121] Hooke, *Micrographia*, 16.
[122] Hooke, *Of Spring*.
[123] Hooke, *Of Spring*, 6.
[124] Hooke, *Of Spring*, 7.

occupies a volume of 1 cubic foot of what Hooke calls a "sensible body", which he defines as "a determinate Space or Extension defended from being penetrated by another, by a power from within".[125] This notion of defence 'from being penetrated" is taken directly from Hooke's observations of drops and surface tension;[126] however, instead of 'heat', as in the drum-and-sand experiment, incongruous vibrations now account for the physical aspects of incongruous fluids. Thus "a determinate Space or Extension" results from vibrations; and no two vibrating particles can occupy the same space at the same time. That objects appear motionless to the human eye is merely an effect caused by the inability of the sense organ to detect vibrations outside a narrow band of frequencies – for recall Hooke's remark that it is possible to observe the vibrations in a slack string, but not in a taut one.

> Suppose a number of musical strings, as A B C D E, &c. tuned to certain tones, and a like number of other strings *a,b,c,d,e*, &c. tuned to the same sounds respectively, A shall be receptive of the motion of *a*, but not of that of *b*, *c*, nor *d*; in like manner B shall be receptive of the motion of *b* … And so of the rest.[127]

In other words, Hooke's model of the universe operates on the principle of a dynamic balance of consonance and dissonance in sympathetic resonance; not only harmony, but a constant collaboration of opposing forces to maintain equilibrium within 'normal' range. Finally, more than a decade after having first introduced the terms, Hooke proclaims: "This is that which I call *Congruity* and *Incongruity*."[128]

Continuing the analogical relation between strings and other springy bodies, Hooke repeats his notions on congruous particles from the *Micrographia*, here reduced to matter, motion and proportion.

> Now as we find that musical strings will be moved by Unisons or Eighths [octaves], and other harmonious chords, though not in the same degree; so do I suppose that the *particles of matter* will be moved principally by such motions as

[125] Hooke, *Of Spring*, 7.
[126] Hooke, *Micrographia*, 12.
[127] Hooke, *Of Spring*, 8.
[128] Hooke, *Of Spring*, 8.

> are Unisons, as I may call them, or of equal Velocity with their motions, and by other harmonious motions in a less degree.[129]

Some scholars have suggested that Hooke's 'congruity and incongruity' is synonymous with the principle of sympathy and antipathy, and therefore a thoroughly anti-Cartesian move not only because of Descartes's dismissal of invisible correspondences and powers between things, but also on account of the principle's component of action at a distance.[130] But this interpretation of congruity and incongruity ignores an important moment of historical change: how natural philosophers like Hooke used Descartes as an intellectual resource in different ways, accepting the challenge to explain sympathy and antipathy with a mechanistic (and in Hooke's case also mechanical) account that complemented their own notions on matter and motion.[131] Descartes argues in Part IV of his *Principles* that one can provide a causal account of sympathy and antipathy – for example, the "various attractions", "such as are in amber and in the magnet" – "from the figure, magnitude, situation, and motion of particles of matter".[132] As I attempt to show throughout, this is what Hooke does, albeit with his own version of the mechanical philosophy.

Although Hooke uses 'sympathy' and 'antipathy' in conjunction with 'congruity' and 'incongruity' once in the *Micrographia*, it is not to tie his terms to an Aristotelian concept rejected by the mechanical philosophy, but rather to redefine what sympathy and antipathy are according to *his* new theory of elastic matter-in-vibration. Thus, only after explaining "Congruity and Incongruity" with his first

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[129] Hooke, *Of Spring*, 9.

[130] Recently, in the introduction to *Sympathy*, Eric Schliesser remarked that although Descartes dismissed sympathy and antipathy as occult, and although these notions were not part of mainstream Cartesian natural philosophy, Hooke and a handful of other early moderns did not reject action at a distance (Eric Schliesser, "Introduction: On Sympathy", 4–5, 13 – in Schliesser, Eric (ed.), *Sympathy: A History* (Oxford: OUP, 2015). See also Henry, "Robert Hooke, the Incongruous Mechanist," in Michael Hunter and Simon Schaffer (eds), *Robert Hooke: New Studies* (Woodbridge, England: Boydell Press, 1989), for an alternative interpretation which paints Hooke as a magician.

[131] See also Domenico Bertoloni Meli, *Thinking with Objects* (Baltimore: Johns Hopkins University Press, 2009) for the various ways that early modern savants used Descartes as a resource in general.

[132] Descartes, *Principles of Philosophy*, IV.187.

stretched strings similitude does Hooke allow himself to claim, "We see therefore *what is the reason* of the *sympathy* or uniting of some bodies together, and of the *antipathy* or flight of others from each other".[133] As Domenico Bertoloni Meli explained in *Thinking with Objects*, Hooke's congruity and incongruity reframes the occult notion of action at a distance in "mechanical terms by means of analogies with simple objects such as springs and strings."[134] Nevertheless, Hooke criticises weaknesses in Descartes's work, such as his non-experimental hypothesis about whether a spark from a fire striker is made from a bit of the flint or steel, his account of colours from refraction, and the infamous *conatus ad motum*, which Hooke argues is "not so properly a motion, as an action or propension to motion".[135] Almost two decades later, in one of his *Lectures of Light*, Hooke reveals that he is still bothered by *conatus*: a "bare Propension to Motion, is not Motion," he reiterates, "and consequently cannot propagate Motion … for the Propagation of Motion, Motion is necessary."[136] He goes so far as to accuse Descartes (and Thomas Hobbes) of incoherence:

> we may assign to every Propagation of Light through the least sensible space, a real temporary local Motion. And if Mons. D*es Cartes* by his Propension to Motion, and Mr. *Hobbs* by his *Conatus* or Endeavour to Motion, do not mean … a real local Motion, their Notions are neither of them intelligible to others, nor did they really understand them themselves.[137]

Hooke attempts to counter Descartes's *conatus* by constructing "four Considerations" to convince his audience that "we may assign to every Propagation of Light through the least sensible Space, a real temporary local Motion".[138] That is, as rephrased

[133] Hooke, *Micrographia*, 16. Italics added.
[134] Domenico Bertoloni Meli, *Thinking with Objects*, 245.
[135] Hooke, *Micrographia*, 46, 54, 60. For an explanation of *conatus* or striving employed by Descartes in his famous and problematic stone-and-sling argument from analogy, see Descartes, *Principles of Philosophy*, III.57.
[136] Hooke, *Lectures of Light*, in: *Posthumous Works*, 136. Hooke was not the only English natural philosopher to find Descartes's *conatus ad motum* problematic. For example, Roger North also rejects Descartes's theory of the propagation of light, which he finds "not apt" because of Descartes's use of *conatus* as an explanatory device – in Jamie Croy Kassler, *Seeking Truth* (London: Routledge, 2016), 93–94.
[137] Hooke, *Lectures of Light*, in *Posthumous Works*, 136.
[138] Hooke, *Lectures of Light*, in *Posthumous Works*, 136.

succinctly by his biographer Waller, "Every sensible Moment of Time, as well as every Sensible Particle of Matter, [is] composed of infinite lesser".[139] To summarise, Hooke's four considerations are that 1) insensible points, separated by 2) insensible spaces, allow for 3) insensible motions or impulses to traverse these spaces in 3) insensible time with 4) insensible velocities.[140] Here Hooke's tools of similitude and proportionality prove pivotal for his argument on insensible motions, because

> if therefore I can understand, comprehend, and imagine one Local Motion that falls under the reach of my Senses, I can by Similitude comprehend and understand another that is ten thousand Degrees below the reach of them, they having both the same Properties, and differing only in the Spaces of the times.[141]

These considerations were already crucial for Hooke when he demonstrated the spring law geometrically as well as with sensible similitudes – various mechanical springs – six years before. Yet the most striking difference between Hooke and Descartes is the former's paramount claim that matter and motion "may be one and the same", because it means that matter and power or force are inseparable, complementing Hooke's theory that there is not a single body in the universe whose particles are at rest.[142]

"By Motion," Hooke states in *Of Spring*, "I understand nothing but a power or tendency progressive of Body according to several degrees of Velocity".[143] And this power is related to the amount of matter making up a body, "for a little body with great motion is equivalent to a great body with little motion as to all its sensible effects in Nature." Because of this inverse

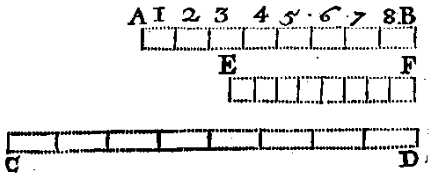

Figure 2.1 Hooke's demonstration of an idealised springy body of 8 vibrating particles, captured in moments of equilibrium, compression and extension (*Of Spring*).

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139 Hooke, *Lectures of Light*, in *Posthumous Works*, 129.

140 Hooke, *Lectures of Light*, in *Posthumous Works*, 134–136.

141 Hooke, *Lectures of Light*, in *Posthumous Works*, 131.

142 Hooke, *Of Spring*, 7; Hooke, *Micrographia*, 16. One clear example where Hooke and Descartes differ dramatically, in this respect, is in their explanations of what a solid is. In the *Principles*, Descartes states that the particles of solids are "all contiguous and at rest" because "no other mode can be more opposed to the movement which would separate these particles other than is their own rest" (II.54, II.55).

143 Hooke, *Of Spring*, 6.

relation, matter and motion "do always counterbalance each other in all effects, appearances and operations of Nature, and therefore," Hooke concludes, "it is not impossible that they may be one and the same".[144] In his *Lectures of Light* again a few years later, he would summarise this compounding of matter and motion with the observation that "neither can Matter without Motion, nor Motion without Matter, produce any Effect".[145] As Gal and Chen-Morris elucidated in *Baroque Science*, for Hooke this means "that order is created, rather than demolished, by motion".[146]

For example, in *Of Spring* Hooke captures changes in internal motions when an idealised springy body made of eight particles, which represents "solid bodies, as Steel, Glass, Wood *etc*., which have a Spring both inwards and outwards", is compressed and dilated from equilibrium (*Figure 2.1*; see also Chapter 6, *Sections,* for an analysis of the practical geometry in the figure).[147] The particles in the springy body at equilibrium "perform a million single Vibrations, and consequently of occursions with each other in a second minute of time"; the particles in the stretched body perform at a slower 666,666 vibrations per second relative to its natural frequency at equilibrium; and the particles in the compressed body perform at a faster 1,500,000 vibrations per second. Relying on the congruity of ordered musical harmonies again, Hooke deliberately makes the frequency ratios a perfect fifth in each direction. The progression E, B and F# of these insensible vibrations, with B as the tonic or springy body at equilibrium, is pulled from the Pythagorean circle of fifths, and Hooke uses it to scale up the ladder of perfect fifths into regions of sound beyond the range of the human auditory spectrum.[148]

That Hooke extended this enforced order into the insensible realm is paradoxical because to 'hear' consonance and dissonance it became necessary for

144 Hooke, *Of Spring*, 7.
145 Hooke, *Lectures of Light*, in *Posthumous Works*, 172.
146 Gal and Chen-Morris, *Baroque Science*, 156.
147 Hooke, *Of Spring*, 13.
148 Hooke, *Of Spring*, 13. Erich Neuwirth, *Musical Temperaments* (Wien: Springer, 1997).

him to replace the senses with strings and sound wheels and proportion, turning his back on the ear altogether by reducing tones to pulses that could be ideally counted, halved, doubled, compared and so on for the development of his metaphysics of vibration.[149] He remained committed to developing this metaphysics throughout his career. Hooke's string similitudes, during the long duration of time from the *Micrographia* onwards, remained a reliable constant, a material model capable of accounting for all the fundamental properties of matter, such as solidity and fluidity, around which Hooke could build his theory.

3. Vibrations

In *Of Spring*, although Hooke claims to have already "hinted the principle" of 'congruity and incongruity' in the *Attempt*, specifically "in the 31 page thereof in the English Edition", he is exaggerating.[150] Hooke's excuse for failing to elaborate on his vibrating matter theory in 1661 is that he was loath to disclose it due to anxiety over attempting to procure a patent for his watch balance spring.[151] Page 31 consists of typical Hooke "hints", which are often hypotheses that he promises to test in detail in future; and indeed, most hints in the *Attempt* become the *Micrographia's* observations. Nevertheless, comparing a couple of Hooke's hints, specifically the fourth and fifth, with a letter penned by him the following year on his initial concept of springy particles reveals that he did have some notion of congruous and incongruous pulsations or "strokes" in mind – although he had not yet conducted trials on the sounds and vibrations of giant monochords. By examining the evolution of Hooke's springy particles, as well as his concept of the aether (different from Descartes's) through which vibrations propagate, I will show how his ontology

149 Gal and Chen-Morris, in Part I of *Baroque Science,* analyse a similar move in early modern optics, which they call 'the optical paradox': namely, a rejection of the observer, and empirical science turning its back on sense knowledge, in preference for manmade artificial instruments, and thus instrument-mediated empiricism and knowledge.

150 Hooke, *Of Spring*, 6; Hooke, *Attempt*, 31.

151 Hooke, *Of Spring*, 6.

was contingent upon his experimental practices, and as a consequence, how his understanding of bodies and motion altered accordingly.

PARTICLE

In the fourth hint of the *Attempt*, Hooke wants to know whether sparks "struck out of a flint" are bits of molten flint, steel, or a compounding of both. Examining the cooled "parcels" of spark residue microscopically, he notes the globularity of the bodies, and although he "cannot here stay to examine the particular Reasons of it", he imagines that the flint-and-steel residue is first "made so glowing hot, 'tis melted into a *Vitrium*" "by the violence of the stroke". Recall from the previous section that Hooke uses 'stroke' and 'pulse' synonymously. The spark is then "driven into a round Globul" by "the ambient Air" with which it is incongruous.[152] This hypothesis resembles Hooke's *Micrographia* explanation for a water droplet's globularity, for recall also that a drop of water assumes its shape when a congruous body is "forc't into as little space as it can possibly be contained in, namely, into a Round Globule", against the surrounding incongruous fluid.[153] Moreover, Hooke deliberately connects "the violence of the stroke" to his fifth hint in the *Attempt*: "A Fifth thing which I thought worth Examination was, Whether the motion of all kind of Springs might not be reduc'd to the Principle whereby the included heterogeneous fluid seems to be moved", which I will discuss later in the section on aether.[154] Important here is that Hooke has a springing motion in mind as early as 1661 – not only for artificial, mechanical springs, but *all* springy bodies in general.

In "Hooke's Vibration Theory and the Isochrony of Springs", Mary Hesse disclosed Boyle's adoption of Hooke's particle hypothesis by shedding light on a paper trail between Christiaan Huygens, Robert Moray, Boyle and Hooke in July 1662. To summarise the convoluted correspondence, after reading Boyle's *Defence of*

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[152] Hooke, *Attempt*, 31.
[153] Hooke, *Micrographia*, 12.
[154] Hooke, *Attempt*, 31.

the Doctrine Touching the Spring and Weight of Air, Huygens attacked Boyle's description of air, Boyle admitted that it was Hooke's hypothesis, and then Huygens read Hooke's "more copious explanation" via Boyle and retreated satisfied.[155] An examination of body and motion in Hooke's letter shows how his theory of matter-as-vibrations was contingent upon his experimental interests and practices, as well as how his notions on vibrations changed with his monochord trials and musical strings, which gave him the practical and theoretical tools he needed to provide a causal account for congruity and incongruity in the *Micrographia*.

According to Hooke's reply to Huygens, the "difficulty lyes" in his "first hypothesis … being Epicurean", wherein he supposes "an internall motion in the particles of bodyes … which therefore though [the motion] may be retarded by the occursion [strokes] of other bodys … yet those impediments are noe sooner remo[v]'d, then the freed particles begin again their natural and congenite [innate] motion".[156] Hooke further supposes that the motion is "circular", because of "the parts [themselves] being suppos'd much of the shape of a watch-spring, or coyle of wire". Providing an illustrative analogy for his particles, Hooke further explains that because they possess a circular motion "like that of ye meridian of a Globe upon it's poles" they "thereby become potentiall sphaeres or globules … that is, they defend a sphaericall space from being entred into by any other of the like globules, [u]nless they be thrust on with a sufficient strength" – namely, unless they are bent.[157]

In contrast, three years later in the *Micrographia*, having conducted countless

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[155] Hesse, "Hooke's Vibration Theory and the Isochrony of Springs", *ISIS* 57, no. 4, 1966: 433. doi: 10.1086/350160; Huygens, *Oeuvres Completes De Christiaan Huygens Correspondence*, Vol. IV, 1662–1663.

[156] Huygens, *Oeuvres Completes De Christiaan Huygens Correspondence*, Vol. IV, 1662–1663, 221. Hooke changes his mind about "Epicureanism" over the course of his career, shifting from the Epicurean notion of an indivisible physical parcel, such as those famously poeticised by Lucretius in his *De Rerum Natura*, to a definition favoured, for example, by early modern chymists such as Daniel Sennert who, according to William R. Newman, did not define atoms as indivisible (see William R. Newman, *Atoms and Alchemy: Chymistry and the Experimental Origins in the Scientific Revolution*, Chicago, London: UCP, 2006, xi–xii).

[157] Huygens, *Oeuvres Completes De Christiaan Huygens Correspondence*, Vol. IV, 1662–1663, 221.

and various trials as Curator of Experiments, Hooke would be in a position to compare motions other than those of watch springs, and to change his mind about the motion of particles being "circular".[158] Instead, having experimented upon a variety of motions such as "*turbinated*" and "any other *irregular* motion of the parts", Hooke eliminates them all as "improbable" because of their irregularity. "It must therefore be a *Vibrating* motion," he concludes, because if particles are globular, then a vibrating motion is the only one that accounts for all observable effects.[159] Moreover, this indicates that a particle no longer spins to create the volume of its sensible body, but rather that it vibrates in and out periodically like the plate of iron becoming a sensible cube of iron in *Of Spring*. Indeed, some years after, in a lecture *Of Comets and Gravity*, Hooke solidifies his notion of vibrating bodies, reformulating the definition "Globular Motion", for recall he supposes that the earth has such a motion, "*whereby all the Parts thereof have a Vibration towards and fromwards the Center, or of Expansion and Contraction*."[160]

Yet in the *Micrographia*, even if globular bodies have an "orbicular pulse" like light from a point source, or round pond ripples, Hooke remains reluctant to abandon the image of a globular air particle "resemble[ing] a *round Spring*" when describing how it contracts into a capillary tube with a diameter less than its own, for "as in a *round Spring* there is required an additional *pressure* against two opposite sides", "an *extraordinary* and *adventitious force*".[161] He resurrects the old analogy not for the sake of justification, but provisionally, because it has simplicity, is easier to imagine and thus has greater explanatory power.

The above illustrates that Hooke's employment of various experiments and observations for the development of his matter theory underscores the contingency of the relation between practice and theory, and highlights just how blurred the boundary is between the two in his work. Nonetheless, amongst Hooke's revisions

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158 Hooke, *Micrographia*, 55–57.
159 Hooke , *Micrographia*, 56.
160 Hooke, *Of Comets and Gravity*, in *Posthumous Works*, 184.
161 Hooke, *Micrographia,* 19, 61.

and developments, his notion of particles as globular bodies that powerful enough external forces can squeeze into spheroids and ovoids remains constant. For example, in the *Micrographia*, Hooke insists that particles can be nothing but globular ("neither can it be imagined, how it should otherwise be any other Figure then *Globular*") because whether parts or bodies, whether drops or planets, their incongruity with the surrounding aether accounts for their sphericity.[162] He remains committed to developing this physical principle of globularity for the rest of his career.[163]

Having explained why particles assume a globular form, and how it is that they are bent into ovals by "additional *pressure*", Hooke next has to account for how congruous and similar particles join to form the variety of sensible objects in the world. For this, he relies on pores and the aether, because the aether "passes between the Particles, that is, through the Pores of bodies".[164] Namely, all bodies, no matter how close-packed their globular particles, are "perforated with innumerable pores, which are nothing else but the interstitia between those multitudes of minute globular particles".[165] In the *Micrographia*, Hooke uses his experiments on capillary action as well as congruous and incongruous fluids to explain changes to a body's superficies. Using a wine glass to represent an enlarged pore, or an enlarged capillary tube, he pours water into it, and observes that "the surface of the water" is "all the way concave, till it rise even with the top, when you shall find it (if you gently and carefully pour in more) to grow very protuberant and convex".[166] Hooke argues with the support of his numerous fluid experiments that the meniscus inverts because of incongruity: once the water passes the rim of the glass vessel, which it is congruous with relative to the air, the water inverts against the air, which it is incongruous with. Hooke claims that the flattened shape of the

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[162] Hooke, *Micrographia*, 19.

[163] As previously discussed, see, for example, Hooke, *Of Spring*, 12–13; and Hooke, *Of Comets and Gravity*, in: *Posthumous Works*, 184–185, for clear lines of development.

[164] Hooke, *Micrographia*, 32.

[165] Hooke, *Micrographia*, 95.

[166] Hooke, *Micrographia*, 19.

inverted meniscus is a result of varying proportions of congruity and incongruity.[167] Because the wine-glass is an enlarged capillary tube, one can infer that at the microscopic level it is possible to reduce any superficies into a series of concave and convex curves. Now, since the aether pervades all bodies on account of their pores, these changes occur not only on outside surfaces, but inside bodies as well; thus, this dynamic physical process accounts for the variety of bodies in the world.

In *Of Spring*, Hooke reiterates that the aether "incompasseth and pervades all other bodies"; that there are perforations even in solid bodies these "perforations", "which are not defended by the motion of the particles from being pervaded by the Heterogeneous fluid menstruum"; and that these spaces "we call the insensible pores of bodies".[168] If the aether could only surround and not pervade other bodies, then all bodies would be globular like drops; if the aether could pervade all other bodies enough to make their congruous particles separate, then all things would be fluid.[169] Thus, Hooke explains again that bodies and their particles have "peculiar and appropriate motions which are kept together by the differing or dissonant Vibrations of the ambient bodies or fluid [aether]".[170] Moreover, "[a]ccording to the difference of these Vibrative motions … [a]ll bodies are more or less powerful in preserving their peculiar shapes."[171] Finally, the "smaller the particles of bodies are, the nearer do they approach to the nature of the general fluid [aether], and the more easily do they mix and participate *of its motion*".[172] In other words, the smaller a particle, the closer it is to a particle of aether.

AETHER

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167 Hooke, *Micrographia*, 18–19.
168 Hooke, *Of Spring*, 9, 10.
169 Hooke, *Of Spring*, 9–10.
170 Hooke, *Of Spring*, 9.
171 Hooke, *Of Spring*, 10.
172 Hooke, *Of Spring*, 10–11. Italics added.

> A Fifth thing which I thought worth Examination was, Whether the motion of all kind of Springs might not be reduc'd to the Principle whereby the included heterogeneous fluid seems to be moved.[173]

Recall that the above citation is from the *Attempt*. By "included heterogeneous fluid", Hooke means the aether mixed with other fluids like air, through which all unison and harmonious – that is, congruous – vibrations are propagated. The particles of the aether itself, Hooke claims as early as the *Attempt*, are responsible for incongruity, elaborating in the *Micrographia* that the cause is dissonant or incongruous vibrations. In Observ. XV., concerned again with the "porousness" of bodies just as he was in his capillarity studies, Hooke lays down a series of axiomatic statements about all properties of the aether save for its particles. He refuses to 'examine' the aether's particles in the *Micrographia*, instead giving the excuse that what he *is* willing to hypothesise on is "sufficient to solve all the Phaenomena".[174]

> Nor do I much concern my self, to determine what the Figure of the particles of this exceedingly subtile fluid *medium* must be, nor whether it have any interstitiated pores or vacuities, it being sufficient to solve all the *Phaenomena* to suppose it an exceedingly fluid, or the most fluid body in the world, and as yet impossible to determine the other difficulties.[175]

Focusing on the aether's fluidity and the motions it propagates instead, "[propounding his] conjectures and Hypothesis about the medium and conveyance of light" Hooke supposes that

> the greatest part of the *Interstitia* of the world, that lies between the bodies of the Sun and Starrs, and the Planets, and the Earth, to be an exceeding fluid body, very apt and ready to be mov'd, and to communicate the motion of any one part to any other part.[176]

Because the aether is "so exceeding fluid a body," Hooke reiterates, "it easily gives passage to all other bodies to move to and fro in it." 'To and fro' is meant to convey a rocking, rhythmic, periodic motion. Next, contrary to Descartes, he states that no motions pulsating through the aether are instantaneous, even if the motion propagated is "with an unimaginable celerity and vigour", because the aether

173 Hooke, *Attempt*, 31.
174 Hooke, *Micrographia*, 96.
175 Hooke, *Micrographia*, 96.
176 Hooke, *Micrographia*, 96.

neither receives nor communicates "any impulse, or motion in a direct line, that is not of a determinate quickness".[177] But where the aether becomes heterogeneous on account of mixing with other bodies, such as air in the earth's atmosphere, Hooke falls back on his studies of miscible and immiscible fluids for an apt similitude to explain the observable effects on light.[178] The air is "much like those … very deep tinging bodies, where by a very small parcel of matter is able to tinge and diffuse it self over a very great quantity of the fluid dissolvent" aether.[179] Applying his newly minted chymical similitude to explain the observable effects on a "propagated pulse of light" in the atmosphere, Hooke explains that these "solutions and tinctures" alter the "aptness to propagate a motion or impulse through them [like] the particles of the Air, Water, and other fluid bodies … which are commixt with this bulk of the *Aether*".[180] Yet Hooke grapples with the aether, employing both the notion of it as an infinitely divisible vibrating menstruum and as indivisible particles vibrating in a vacuum, because he needs to carry both positions. On account of his aetheral problems, he refuses to elaborate on the aether's particles until *Of Spring.*

In *Of Spring*, after reiterating his definitions for congruity and incongruity, Hooke follows with the aether: "I do further suppose, A subtil matter that incompasseth and pervades all other bodies, which is the Menstruum in which they swim … and which is the medium that conveys all Homogenious or Harmonical motions from body to body". Next, he is finally ready to introduce "the ambient bodies" of this "subtil matter" to explain that their vibrations are incongruous with the vibrations of other bodies, which is how congruous and harmonious bodies are "kept together":

> All bulky and sensible bodies whatsoever I suppose to be made up or composed of such particles which have their peculiar and appropriate motions which are

[177] Hooke, *Micrographia*, 96.

[178] Two decades later, in a lecture on a similar topic concerning light and the aether, read in 1685, Hooke also resorts to describing the aether as "stagnant" – although this position deviates from the descriptions and explanations that he usually favours (Hooke, *Of Light*, in *Posthumous Works*, 197).

[179] Hooke, *Micrographia*, 96–97.

[180] Hooke, *Micrographia*, 97.

> kept together by the differing or dissonant Vibrations of the ambient bodies or fluid.[181]

Further, "all such particles of matter as are of a like nature … strengthen the common Vibrations of them all against the differing Vibrations of the ambient bodies".[182] Note that, except for his singling out of the "ambient bodies", Hooke has been repeating this line under one guise or another since at least 1661. But then he makes a new move, informing the reader about the size of the aether's *particles* relative to all others. Recall from the previous section that "the smaller the particles of [other] bodies are, the nearer do they approach the nature of the general fluid [aether]"; the "Air then is a body consisting of particles so small as to be almost equal to the particles of the [aether]".[183] Finally, Hooke explains how a body's size affects its vibrational frequency: "According to the bigness of the bodies [in general] the motions are, but in reciprocal proportion: that is, the bigger or more powerful the body is, the slower is its motion with which it compounds the particles".[184] If the aether's particles are the smallest, then their vibrations are also the swiftest, and since these particles "pervade" other bodies, it makes sense that they should be the smallest. This inverse relationship also translates across to the fluidity and solidity of bodies, for the more incongruous a body's vibrations are with the aether's, the more solid it is.[185] Moreover, Hooke explains the latter inverse relation by relying on the pressure law or 'Boyle's law' again, this time to account for the inverse proportion between congruent or harmonious and incongruous vibrations:

> The parts of all springy bodies would recede and fly from each other were they not kept together by the Heterogeneous compressing motions of the ambient whether fluid or solid.[186]

Yet this "compressing motion" is not new, for recall also that Hooke explained the globularity of bodies such as drops and planets in the *Micrographia* by stating that a

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[181] Hooke, *Of Spring*, 9.
[182] Hooke, *Of Spring*, 9.
[183] Hooke, *Of Spring*, 10, 15.
[184] Hooke, *Of Spring*, 10.
[185] Hooke, *Of Spring*, 10, 12.
[186] Hooke, *Of Spring*, 13.

"Round Globule" is the result of a body that is incongruous with the surrounding aether being "forc't into as little space as it can possibly be contained in" by it.[187] What is new in *Of Spring* is the relation between a body's internal vibrations vs. its relation with the incongruous vibrations of the aether and how these shifts in the frame of reference incorporate both the pressure law (Boyle's law) and the spring law (Hooke's law).

In *Meanest Foundations and Nobler Superstructures*, Ofer Gal explains the difference between solids and the spring law vs. fluids and the pressure law in Hooke's natural philosophy. Solids have a fundamental state because their particles touch, so it follows that they can be compressed or dilated beyond this state of equilibrium.[188] Since the fundamental state is created by the congruous or harmonious vibrations of a body's particles, "which is a strictly internal property", although solids are better able to resist displacement, their compression or extension from equilibrium "not only disrupts the balance of internal and external vibrations, but also the internal harmony of the natural state". This tension created by distance removed from equilibrium follows Hooke's law.[189] The particles of fluids, on the other hand, are under constant tension like a balloon underwater, owing to the surrounding pressure of the pervading menstruum, and this tension between a fluid's congruous vibrations and the aether's incongruous vibrations works according to Boyle's law.[190] Thus, a fluid's spring is predicated on total volume, while the spring of a solid is predicated on how far it is compressed or dilated from its fundamental state.[191] To summarise, the aether is responsible for incongruous vibrations, for the solidity and fluidity of bodies, and consists of the smallest and most agile particles.

Moreover, because the aether is fluid, and can mix with the air in the earth's

[187] Hooke, *Micrographia*, 12.
[188] Gal, *Meanest Foundations and Nobler Superstructures*, 94–95.
[189] Gal, *Meanest Foundations and Nobler Superstructures*, 94–95.
[190] Gal, *Meanest Foundations and Nobler Superstructures*, 94–95.
[191] Gal, *Meanest Foundations and Nobler Superstructures,* 95; Hooke, *Of Spring*, 4. Quoted in Gal, *Meanest Foundations and Nobler Superstructures*, 95.

atmosphere, Hooke is quick to argue that it allows for 'action at a distance'. [192]

> Fluid bodies do not immediately touch each other, but permit the mixture of the other Heterogeneous fluid near the Earth, which serves to communicate the motion from particle to particle without the immediate contact of the Vibrations of the Particles.[193]

After *Of Spring*, he would also read this notion back into his earlier work with Wren by reminding his audience about their trials on the effects of vibrations on water in a glass vessel: "for the Water it self," says Hooke, "by means of a vibrative Motion in the Parts of the Glass, acquired a Motion towards the vibrating Parts." "Nor is this way of working at a distance, by means of the internal Motion of the Particles of the Body; so strange a thing in Nature," Hooke adds.[194] Recall from his use of bells to explain sympathetic resonance that he uses the vibratory phenomena of light and sound propagated by "the ambient *Medium*" as examples of action at a distance. Both light and sound stimulate the sense of sight and hearing respectively by causing some part of the organ to vibrate by a "Motion made in the Sun" or a "Bell some Miles perhaps distant".[195]

But does all this imply that the aether's density is more 'menstruum' or void? It seems that Hooke is leaning on a plenum crutch, but a few years later in Section VI of his *Lectures of Light*, he forces a compromise between two notions on the aether when he provides an argument for its fluidity. First, Hooke argues that if vibrations "move the whole *Expansum* of the Ethereal Matter", then this "make[s] and preserve[s] the perfect Fluidity of the *Aether*". Moreover, the aether must necessarily consist of the smallest, swiftest particles because every point of matter vibrates in every direction like light spreading from a point source "with incredible Velocity … to and fro". Thus, the aether "must necessarily have its Parts indefinitely divided,

[192] For an alternative interpretation, see Gouk, "The Role of Acoustics and Music Theory in the Scientific Work of Robert Hooke", 585, in *Annals of Science* 37, no. 5 (1980): 573–605. Gouk appears more concerned with Hooke's resources rather than how he applied them as intellectual tools. Further, Gouk claims that a "reason for [Hooke] adopting his own concept of the ether was his belief in the existence of a vacuum", but this is a conflation of reason and conclusion.

[193] Hooke, *Of Spring*, 12.

[194] Hooke, *Of Comets and Gravity*, in *Posthumous Works*, 183–184.

[195] Hooke, *Of Comets and Gravity*, in *Posthumous Works*, 184.

and loose from one another"; that is, fluid.[196] Notice that Hooke has switched from the "Epicurean" atoms that he favoured in the early 1660s to "indefinitely divided" parts – a consequence of his reformulation of the definition of 'point' as a solid body in his practical geometry (see Part II). The compromise comes in Hooke's conclusion. Instead of a vibrating plenum or 'menstruum', his 'indefinitely divisible parts' are so small and rarefied that they are very nearly a vacuum or "almost nothing":

> and consequently being thus fluid … and these Motions being proportionably swifter than the swiftest Motion of the more bulky Mass: It follows, I say, that the Impediment to any bulky Bodies moving through it, must be inconsiderable, or almost nothing.[197]

The above is what Hooke calls a vacuum, just as what remains in the evacuated receiver of the air-pump is what Boyle defines as a vacuum.[198] But although Hooke remains committed to this construct of the aether hereafter, it suffers from the same inconsistency that led him, twelve years earlier, to discard one of two possible causes for why the planets move around in curved trajectories. In a 23 May 1666 *Address* to the Royal Society, Hooke presented a paper on orbits as effects. A "paper … concerning the inflection of a direct motion into a curve by a supervening attractive principle"; an "introduction to an experiment to shew, that circular motion is compounded of an endeavour by a direct motion by the tangent, and of another endeavour tending to the center".[199]

Hooke provides two possible causes for "inflection".[200] But the first, which

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[196] Hooke, *Lectures of Light,* in *Posthumous Works*, 136.

[197] Hooke, *Lectures of Light,* in *Posthumous Works*, 136.

[198] For Boyle's vacuum, see Steven Shapin and Simon Schaffer, *Leviathan and the Air-Pump.*

[199] Birch, *The History of the Royal Society of London*, Vol. I, 90, 92; Gunther, *Early Science in Oxford*, Vol. 6, 265, 267.

[200] Birch, *The History of the Royal Society of London, Vol. I*, 90, 92; Gunther, *Early Science in Oxford*, Vol. 6, 265, 267. For a meticulous analysis of how Hooke modifies and employs "inflection" from the *Micrographia's* studies of multiple atmospheric refractions of light to the curved and compounded trajectories of orbits, see Gal, *Meanest Foundations and Nobler Superstructures*. Other scholars, such as F.F. Centore in *Hooke's Contribution to Mechanics* (The Hague: Nijhoff, 1970), have attempted this analysis and failed to notice that Hooke repeatedly underscores the deficiencies in his mechanical model, which uses a conical pendulum to represent a planet's inflected motion. In fact, Hooke's accompanying geometrical demonstration is *designed* to illustrate these deficiencies, which he also

"may be from an unequal density of the medium, thro' which the planetary body is to be moved", he discards as problematic immediately after introducing it, because of "improbabilities".[201] One can infer what the "improbabilities" are, since the 1666 concept is inconsistent with Hooke's developing notions on the aether as a propagator of vibrations, because a requirement of this concept of inflection is that the aether has a density gradient on account of being rarefied by the heat of the sun. Thus, "the direct motion [of the planet] will be always deflected inwards [towards the sun], by the easier yielding of the inward, and the greater resistance of the outward [condensing] part of that medium [the aether]".[202] Here is the inconsistency again: on the one hand, Hooke proposes an "unequal density of the medium" to push planetary bodies into areas of less resistance where the aether is rarefied; on the other hand, he needs "an almost nothing" with no "Impediment to any Bulky bodies" which can propagate vibrations at a distance. Hooke's compromise in *Lectures of Light*, discussed above, attempts to solve the "improbabilities" by assigning to the aether a material state of "almost nothing" so that "the Impediment to any bulky Bodies moving through it, must be inconsiderable".

It could be that even in 1666 Hooke was attempting to enforce a compromise between the menstruum and the void, but it is more likely that he was simply doing what he does best – modifying a flexible tool to suit his purposes at the time. When the aether proved to be the wrong tool in 1666, he put it aside. Thus, whether Hooke's aether compromise coheres is somewhat beside the point. After he had reinforced his description of the aether as the propagator of vibrations in *Of Spring*, he could enforce oxymoronic descriptions such as "*radiating Vibration of this exceeding Fluid, and yet exceeding dense Matter*", and could then commit himself more to the various vibrations crisscrossing through it, or the "continued Chime of motions"

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points out, as well as his new use of "inflection".

201 Birch, *The History of the Royal Society of London*, Vol. I, 91.

202 Birch, *The History of the Royal Society of London*, Vol. I, 91.

discussed in Chapter 2.[203] As mentioned, *Of Spring* is also where Hooke provided a geometrical demonstration of the spring law, contingent upon the material world and its parts, and it is to the development of his practical and "Mixt" geometry that I now turn.

II: PRACTICAL GEOMETRY

In the *Micrographia*, Hooke employed a musical strings similitude to explain his matter-as-vibrations theory, "Congruity and Incongruity". I analysed the construction of this theory predominantly from the perspective of experiments, apparatuses and instruments in Part I. I showed that Hooke's strings analogies – far from being merely illustrative – demonstrate his dynamic matter theory. Namely, strings were arguments for and models of Hooke's cosmology, constructed to support his claim that matter and motion are "one and the same".[204] His work on springy bodies and vibrations culminated in the treatise *Of Spring*, where he provided a refined version of the strings model, rejecting the qualitative and sensual descriptions favoured in the *Micrographia* for "Heterogeneous motions" and "proportion".[205] This move underscores Hooke's epistemological preference for pulling mathematics from physics, and hints that it was not enough for him to provide a causal account which was just a mechanical model alone. To produce a "Theory of Springs", a "Rule or Law of Nature", which could be used to improve upon nature, he needed to reduce vibrating strings, his matter model, to geometrical demonstrations.[206] Hooke took this a step further, fashioning a new geometry, with new artificial instruments such as the microscope and telescope, capable of describing the reality of objects created by the New Science. Here, as promised in the introduction, I will examine Hooke's development of the spring law from the

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203 Hooke, *Of Comets and Gravity*, and *Lectures of Light*, in *Posthumous Works*, 184, 137.
204 Hooke, *Of Spring*, 7.
205 Hooke, *Of Spring*, 12.
206 Hooke, *Of Spring*, 1, 3, 4, 1–6.

perspective of mathematical bodies and motions by focusing predominantly on his practical geometry – the mediator between sensible and insensible physical reality and abstractions in his work. Points, lines, superficies and solids in Hooke's practical geometry are all three-dimensional, and I will try to explain why and how this is his attempt at the question of infinitesimals.

Studying nature as a geometer presents Hooke with two challenges. One challenge is to create familiarity in this newfound variety. Hooke's solution is to draw attention to two patterns standing out amongst the Baroque plethora of details, which lend themselves to analysis via the language of geometry: globular bodies or points in particular, and motions or lines. Together, points and lines enforce a mathematically workable order upon Hooke's observations – an analogy to geometry in nature. A second challenge is to achieve this physicalisation of Euclidean definitions without stepping on the toes of well-established views about traditional geometry. Hooke tackles this difficulty with the claim that, on the one hand, Euclidean or speculative geometry, albeit with motion, is necessary for understanding concepts and for assisting the imagination. On the other hand, practical geometry, with Hooke's stipulative definitions, is a better representation of nature, because mathematics is an approximation of the world's workings and is dependent on the power, accuracy and precision of artificial instruments. Moreover, unlike traditional geometry, Hooke's practical geometry is also capable of exhibiting nature's magnified, fundamental, rough surfaces. Again, traditional geometry is a tool, created before the invention of optical instruments, and like naked eye astronomy, is limited by human senses – by our incapacity to sense parts. For example, abstractions such as a straight line between two points fail to model what experiments and instruments capture, even though our senses 'see' and 'touch' straight lines. But this does not signify a problem with the material. Lines are a limit of the senses, and Hooke's reversal reveals that Euclidean geometry is an idealised expression of the senses, bounded by them; it is not the language of nature. Thus Hooke often replaces the ruler and compasses with a scale bar and microscope for his practical geometry, and interweaves speculative and practical into a novel "Mixt geometry", capable of simultaneously representing discrete and continuous operations, as well as both sensible and insensible phenomena; speculative and practical respectively. The master-class for this new geometry, which exhibits Hooke's infinitesimals, is his geometrical proof of the spring law.[207]

4. POINTS

Although experimental philosophy and mathematical analysis were a hallmark of the New Science, Hooke's practical geometry is a unique amalgamation of the two; displaying supreme indifference between concrete and abstract, it reflects Hooke's dual role as both the Curator of Experiments for the Royal Society and the Gresham Geometry Professor. Gresham College (est. 1597) implemented radical educational reforms, eschewing scholasticism in favour of educating each student with purpose and for a specific vocation rather than education for its own sake – though legislators prevented most of the proposed reforms from being put into effect.[208] That is, the College exemplified the push to a more practical education instead of the scholastic studies still prevalent in the universities at the time. But this is not to say that the Gresham professors concerned themselves only with practical matters as a technical college might today; rather, it is a move that is considered to be one of the great achievements of 17th century mathematics, for it stimulated new mathematical (amongst other) interests, pursuits and practices.[209] For example, the

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[207] Hooke, *Of Spring*; Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 523.

[208] Jamie C. Kassler, "The Science of Music to 1830", in: *Music, Science, Philosophy* (UK: Routledge, 2001), 184.

[209] Mordechai Feingold, "Gresham College and London practitioners: the nature of the English mathematical community," in Francis Ames-Lewis (ed.*), Sir Thomas Gresham and Gresham College* (Hampshire: Ashgate Publishing Ltd., 1999), 174–188; see especially p. 179–180. Feingold provides a detailed account of the shift from scholastic to early modern mathematics with Gresham College as the epicentre of change, which I lack space to indulge in here. Wilson, "Who invented the calculus?– and other 17th century topics", online Gresham College lecture recording, 1:03:01, 16th November 2005. <http://www.gresham.ac.uk/lectures-and-events/who-invented-the-calculus-and-other-17th-century-topics>.

musician Thomas Ravenscroft (c. 1588–1635) claims in his discourse on *Measurable Musicke* published in 1614 that the College stimulated studies in "especially the Mathematicks, which were somewhat neglected euen in the Universities".[210] Ravenscroft's remark is personified by Thomas Hobbes, who, according to the story told by his biographer John Aubrey, studied no mathematics at Oxford, but encountered geometry for the first time at age 40 by stumbling upon a copy of Euclid's *Elements* open at the Pythagorean theorem.[211] Jamie Kassler adds that Gresham College established the first endowed chairs for music and mathematics.[212] The statute for geometry reads as follows:

> The solemn lectures of astronomy and geometry are to be read … either of the said lectures twice every week, on Friday astronomy, on Thursday geometry, between the hours of eight and nine in the forenoon, and two and three in the afternoon; whereof the lectures in the forenoon to be in Latin, and the lectures in the afternoon to be in English. Touching the matter of said solemn lectures, the geometrician is to read as followeth, viz. every Trinity term arithmetique, in Michaelmas and Hilary terms theorical geometry, in Easter term practical geometry.[213]

'Theoretical geometry' (also 'classical', 'Euclidean' or 'traditional'), which in the seventeenth century was semi-official like the curricula, was more commonly known as "Speculative Geometry" amongst Hooke and his peers. According to Hooke's way of understanding and teaching its worth as a tool for natural philosophy, speculative geometry should serve "practical geometry". Late in life, during a lecture on "*Navigation and Astronomy*", Hooke attempts to explain to his audience that the

> business of Speculative Geometry being only to demonstrate the propriety of such quantities, as Lines, Superficies and Solids from their Definitions or Descriptions; it is sufficient to have only a right Conception of what is to be understood by those Appellations, and they are things possible to be done, or conceiv'd so to be, for grounding the Demonstrations thereupon, and that the actual drawing and

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210 Thomas Ravenscroft, *A Brief Discovrse of the True (but Neglected) Use of Charact'ring the Degrees of their Perfection, Imperfection, and Diminution in Measurable Musicke, against the Common Practise and Custome of these Times …*, cited in Kassler, "The Science of Music to 1830", in *Music, Science, Philosophy*, 185.

211 John Aubrey, *Brief Lives, Vol. 1*, (Oxford: Clarendon Press, 1898), 332.

212 Kassler, "The Science of Music to 1830," in: *Music, Science, Philosophy*, 184.

213 John Ward, *The lives of the professors of Gresham College* (London: John Moore, 1740), Preface, viii.

> delineating of them there, is only to help the Imagination to conceive the notion of them aright.[214]

That is, traditional Euclidean geometry is "sufficient" for teaching rules and concepts "for grounding the Demonstrations thereupon", and hence speculative-geometrical drawings serve only "to help the Imagination" to understand these concepts, "and thereby to exhibit the thing done to Sense, which is one of the ends and uses of Speculation".[215] In contrast, practical geometry is not only demonstrated, but "experimentally verify'd and exhibited".[216] Yet one can reduce practical points to speculative ones. From the same lecture as previously cited, referring back to his opening observation on the point of a needle in the *Microraphia*, Hooke explains that by

> Point then I do not here understand an imaginary nothing, which, in speculative Geometry, is defin'd to be a Negation of Quantity, or an Entity that hath no Part or Quantity; but I understand such a Point as hath Quantity and Extension, but yet so small and minute, as that the sense cannot distinguish that it hath any Parts; such as the Point of a very sharp Needle, or the Point of a very curious pair of Compasses; or such a mark with Ink as is made with a very sharp nibb'd Pen upon fine smooth Paper, which tho' it may be easily enough prov'd, either by Microscopes and other Glasses and by Reasons too to have breadth, and so both Longitude and Latitude, nay, and Profundity too or thickness, yet as to be use, for which it is here design'd, it is sufficient, and may pass for a true Mathematical Point, if at least we will but suppose the middle of it to be that which is aimed at in our Operation.[217]

In practical geometry, the tools of which are the microscope, scale bar and so on, a point is the smallest body observable by the senses and thus capable of being imagined – a point made of insensible, infinitesimal parts.[218] In Observ. 1. *Of the Point of a sharp small Needle*, Hooke opens his first observation in the *Micrographia* by claiming that one should study nature as a geometer. That is, one ought to begin with a body "of the most simple nature first", the point, and then progress to more "compounded" structures.[219] This is not exactly a novel concept, but Hooke's

[214] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 523.
[215] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 523.
[216] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 525.
[217] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 520.
[218] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 520.
[219] Hooke, *Micrographia*, 1.

physical frame of reference is. For example, in 1612, the Jesuit astronomer and mathematician Christopher Clavius, whom Hooke read,[220] remarked on Euclid's definition of the point that

> [n]o example of this [point] can be found in material things, unless you mean that the extremity of the sharpest needle expresses some similitude to a point; which nevertheless is wholly untrue, since this extremity can be divided and cut to infinity, but a point must be supposed altogether indivisible [individuum porsus].[221]

Taken in this context, Hooke's point, indeed his entire *Micrographia*, is a direct challenge to Clavius's claim that geometers avoid meddling with matters of physics. And Hooke's challenge did not fall on deaf ears. For example, almost three decades after the *Micrographia*, the naturalist John Ray (1627–1705) would reference "Mr. *Hook*" on points when discussing "Animalcules".[222] In defence against objections to the "Doctrine" that "the Ovaries of one Female should actually include and contain the innumerable myriads of Animals", for who "can conceive such a small portion of matter to be capable of such division, and to contain such an infinity of parts", Ray answers by paraphrasing Observ. I, that "our sight doth not give us the just magnitude of things, but only their proportion, and what appears to the Eye as a Point, may be magnified so, even by Glasses, as to discover an incredible multitude of parts".[223] This idea, that "Glasses" are tools of practical geometry, which allow for shifts in scale or "proportion", was a crucial methodological maxim for Hooke.

Before Ray, Hooke, and Clavius, Leonardo conceived of the point as a resonating structure between nothing and a line. It was a paradoxical idea that he attempted to realise in his art by creating the technique of *sfumato*, which builds a picture up from translucent layers of thinned oil colours, the edges hazy, the

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[220] Felicity Henderson, Yelda Nasifoglue and Will Poole (eds), "Hooke's Books Database | Robert Hooke's Books", hookesbooks.com, 2017, <http://www.hookesbooks.com/hookes-books-database/>.

[221] Christopher Clavius, Christophori Clavii Bambergensis E Societate Jesu Opera Mathematica V Tomis distribute, 1:13, cited in Jesseph, *Squaring the Circle*, 78.

[222] John Ray, Three *Physico-Theological* Discourses, 2nd. ed. (London: Printed for *Sam. Smith*, at the *Princes Arms* in St. *Paul's* Church-yard, 1693), 52, 51.

[223] Ray, Three *Physico-Theological* Discourses, 52.

painting constructed of points caught somewhere between nothings and lines.[224] Perhaps the most renowned painting displaying this ambitious attempt to capture an in-between structure is the *Mona Lisa*; with vaporous gradations of light briefly caught in between the paint particles in the oil, *Mona Lisa* appears ethereal and ephemeral, a moment of transition captured. Gal and Chen-Morris observed a similarity between Leonardo's ideas about the point and his hydraulic studies, which express nature's continuous transitions from order to destruction as a chaotic process of creation, and his frustrating attempt to capture these motions with geometry.[225] Closer to Hooke on the timeline, Kepler also argued for beginning with a mathematical point, which would transform into a physical body by expanding into a sphere via lines: "… a geometrical figure constructed through constant [insensible] motion from the centre toward the surface".[226]

Yet both Leonardo and Kepler's points originate from "speculative geometry"[227] or Euclidian, where the definition of "*a Point is that which hath no part*",[228] and thus can only be described by a negative. It is not like the point of a needle, or the Earth, which are reducible to infinitesimal parts.[229] During this swathe of time, Euclidean definitions were routine, and remained so in the seventeenth century. The Greek Neo-Platonist Proclus (410–485 CE), a commentator of Euclid, and an important resource for scholastic mathematicians, argued that separating geometric forms from matter increases precision: "the ideas of the boundaries exist in themselves and not in the things bounded … Matter muddies

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224 Frank Fehrenbach, "Leonardo's Point", in Alina Payne (ed.), *Vision and its Instruments*, 1st ed. (University Park, Pennsylvania: Pennsylvania State University Press, 2015), 72.

225 Gal and Chen-Morris, *Baroque Science*, Part II: Mathematization.

226 Raz Chen-Morris, *Measuring Shadows* (University Park, PA: The Pennsylvania State University Press, 2016), 51. John Dee's *Monas Hieroglyphica* also uses the notion of a point expanding to become a physical space. (I am indebted to Ofer Gal for this insight.)

227 Hooke, *The Method of Improving* Natural Philosophy, in *Posthumous Works*, 69; Hooke, *Lectures concerning Navigation and Astronomy*, in: *Posthumous Works*, 520–525, 531.

228 Hooke, *The Method of Improving* Natural Philosophy, in *Posthumous Works*, 65.

229 Hooke, *The Method of Improving* Natural Philosophy, in *Posthumous Works*, 65.

their precision …"[230] Hooke, as noted earlier, with Clavius as his unnamed interlocutor, challenges this view vehemently. Whereas for Leonardo "the very regularity of geometrical figures generated transformation and change,"[231] and for Kepler mathematics was necessary, ideal, "coeternal with God",[232] for Hooke it is contingent on physical reality. That is, mathematics follows from physics, the abstract dependent on physical processes, lest we should "quickly lose *Nature* our Guide, and *our selves* too … left to wander in the *labyrinth* of groundless opinions".[233] In other words, Hooke believes that beginning with a "true Mathematical Point" instead of a physical one risks building a faulty model on which to force physical data. That is, if imaginable points are sensible, and a mathematical point is unimaginable, then a mathematical point is speculative and can only be made sense of with a physical representation. Even the geometrical representation of nothing needs a point. But it is this very demarcation between abstract and concrete that allows Hooke to conflate mathematics and physics. If the practical point produces the speculative one, and the speculative point describes actual physical processes derived from empirical observations, assisting the imagination to make sense of the physical because the speculative point captures the limits of the senses, then Hooke can trust the physical-mathematical model to provide causal explanations. Moreover, this is why it is important that a practical point "may pass for a true Mathematical Point, if at least we will but suppose the middle of it to be that which is aimed at in our Operation". Practical geometry is, as Hooke states in his *Lectures of Light*, "Physicks Geometrically handled", and as such, "as in pure Geometry nothing is to be let pass for a Truth, whose Cause and Principles are not so clearly shown by the Progress of Reasoning, and the Process of

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230 Proclus, *A Commentary on the First Book of Euclid's Elements*, 87, cited in Douglas Michael Jesseph, *Squaring the Circle* (Chicago, IL: University of Chicago Press, 1999), 78.
231 Gal and Chen-Morris, *Baroque Science*, 140.
232 Kepler, *Harmonices Mundi* in *Gesammelte Werke* 3, Axiom 7, 6:104.
233 Hooke, *Micrographia*, 1. See also *Lectures of Light*, in *Posthumous Works*, 84, for Hooke's later development of his take on the early modern labyrinth metaphor.

Demonstration".[234] Understanding concepts is obviously necessary for grounding demonstrations.[235] But by beginning with practical geometry grounded by traditional geometry's constraints – "by the help of the Instruments and Methods that are hitherto us'd to make Observation on which to ground Calculation"[236] – Hooke paradoxically frees himself from the worry of fallacies committed by weak human senses, which "cannot distinguish that it [a point] hath any Parts" although "it may be easily enough prov'd, either by Microscopes and other Glasses". From the perspective of Hooke's practical geometry, starting with a speculative point "which hath no part" when representing the material is to start with an instrument as fallible as the human eye.

This mingling of mathematics with microscopes and other instruments was not wholeheartedly embraced by all. On the one hand, Descartes defends and practices the use of instruments other than the traditional ruler and compasses. In Book 2 of his *Geometry*, Descartes argues that if one were to call complex curves, like those drawn by his proportional compass, "mechanical rather than geometrical", "because it is necessary to use a certain instrument to describe them", then it would be "necessary to reject, for the same reason, circles and straight lines, seeing that they can only be described on paper with a compass and ruler, which we can also call instruments."[237] He accuses the "ancients" of this fallacy, speculating that

> since as yet they knew only a few things about conic sections, and there was even much that they did not know about what could be done with the ruler and compass – they believed they should not approach more difficult material.[238]

Like Hooke after him, for Descartes so-called "mechanical" curves are the "more difficult material". That the instruments "used to trace" complex curves are themselves "more complex than the ruler and compass" means not that they are in

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234 Hooke, *Lectures of Light*, in *Posthumous Works*, 73.

235 Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 523.

236 Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 520.

237 René Descartes, *Discourse on Method, Optics, Geometry, and Meteorology,* trans. and ed. Paul J. Olscamp (Indianapolis, Cambridge: Hackett, 2001), 190.

238 Descartes, *Discourse on Method, Optics, Geometry, and Meteorology*, 191.

any way inferior or "not as exact" – quite the contrary, and for practical reasons of practical origin. If this were "the case", Descartes argues, then "it would be necessary to exclude them from mechanics, where exactness of works made by hand is desired, rather than from Geometry, where one seeks only exactitude in reasoning".[239] On the other hand, Newton's teacher Isaac Barrow, in his *Euclide's Elements compendiously demonstrated*, relies unsurprisingly on a ruler and compasses for his constructions; but when demonstrating proposition XVI from the fourth book of the *Elements*, Barrow betrays an aversion to the use of non-traditional instruments:

> Any other way of dividing the circumference into any parts given, is as yet unknown, wherefore in the construction of ordinate figures, *we are forced to have recourse to mechanick artifices, concerning which you may consult the Writers of practical Geometry*.[240]

Hooke seems to take the middle ground, for he is careful not to enforce his predilection for the practical upon his speculative geometry lectures, where he teaches "right Conception[s]" of "things possible to be done, or conceiv'd so to be", because it is necessary to learn the concepts first.[241] The worst crime that he commits in the only extant lecture on the subject, which Waller was kind enough to provide a sample of in Hooke's *Posthumous Works*, is to

> show that innumerable Points do make a Mathematical Line, innumerable Lines do make a Mathematical Superficies, innumerable Superficies do make a Mathematical Body, innumerable Moments make a Velocity, innumerable Instants make a Mathematical Time, by supposing Motion joyn'd to them … and contrary Motion reduce them back again, which is exprest, or perform'd by Multiplication and Division.[242]

By "supposing Motion joyn'd to" Euclidean geometry, Hooke eschews Euclid's definition of line – "A line is a breadthless length"[243] – for an interpretation favoured by, for example, Aristotle, Clavius, Descartes, Hobbes, and most practical

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[239] Descartes, *Discourse on Method, Optics, Geometry, and Meteorology*, 190.

[240] Isaac Barrow, *Euclide's Elements compendiously demonstrated* (London: R. Daniel, 1660), 90. Italics added.

[241] Hooke, *The Method of Improving* Natural Philosophy, in *Posthumous Works*, 69.

[242] Hooke, *The Method of Improving* Natural Philosophy, in *Posthumous Works*, 66–67.

[243] Euclid, *Elements*, Book I, 1. The first six books are on plane geometry.

geometers.[244] "For a Point moved," says Hooke, "makes a Line in a Mathematical Sense".[245] This allows him to consider locations, or the motion or trace of a body rather than the body itself; or to consider a line constructed of physical points, again without having to consider bodies; and is fitting for a new science where order is in motion, not rest.[246] According to Douglas Jesseph citing Proclus, Proclus rejected this interpretation because it "appears to explain [the line] in terms of its generative cause and sets before us not line in general, but the material line"[247]. Although "line in general" is ambiguous, Proclus's rejection of this definition of line is the reason why Hooke adopts it for his lectures on speculative geometry. Hooke does not employ the definition "a Point moved makes a Line in a Mathematical Sense" for practical geometry, unlike other "Writers of practical Geometry" before, during, and well after his time; to the best of my knowledge at this time, most practical geometers were content to begin their textbooks and manuals with Euclid's definitions – albeit with Hooke's preferred speculative definition of line – or to avoid them altogether as self-evident.[248] More in the style of Gresham College, although Barrow had taught there too for a spell, Hooke uniquely embraces the "mechanic artifices" disdained by Barrow, fusing them with his physicalised definitions to serve as tools of his radical instrumental empiricism for a geometry that does not

[244] Descartes, *Discourse on Method, Optics, Geometry, and Meteorology*, 191. See also David Marshall Miller, *Representing Space in the Scientific Revolution* (Cambridge: Cambridge University Press, 2016), 162: since Descartes's proportional compass draws curves by intersecting two of its rectilinear linkages, Miller explains that it "constructed [curves] by the motion of points along presupposed straight lines. The mechanical connections of the compass ensured the commensurability of these motions." Thomas Hobbes, Elements of Philosophy the *First Section*, Concerning Body [On Matter] (London: Andrew Crocke [Crooke], at the Green Dragon in Paul's Churchyard, 1652), 52. See also Jesseph, *Squaring the Circle*; and Katherine Neal, *From Discrete to Continuous: the broadening of number concepts in early modern England* (Dordrecht/Boston/London: Kluwer Academic Publishers, 2002).

[245] Hooke, *The Method of Improving* Natural Philosophy, in *Posthumous Works*, 67.

[246] For this mathematical paradox, see Gal and Chen-Morris, *Baroque Science*, Part II: Mathematization.

[247] Proclus, A Commentary on the First Book of Euclid's "Elements," 79, cited in Jesseph, *Squaring the Circle*, 80.

[248] For a small sample of these types of books, which Hooke had in his personal library in one edition or another, see Thomas Rudd and C.K. Ogden, *Practicall Geometry, In Two Parts* (London: Robert Leybourn, 1650); John Wilkins, *Mathematical Magick*, in Wilkins, *The Mathematical and Philosophical Works of the Right Reverend John Wilkins* ... (London: Printed for J. Nicholson, 1707 [1648]; and Sébastien Le Clerc, *Practical Geometry*, 3rd English ed. (London, 1727).

structure matter, but whose structure *is* matter.

INTERWEAVING

Hooke begins his *Micrographia* observations not with nature, but with the point of a needle. This is a significant move for two reasons. First, it collapses the walls between art and nature, naturalising the microscope by making the magnified and resolved sights and textures as ordinary as those experienced by human senses. In this way, Hooke replaces the senses of sight and touch with the instrument: for example, "the roughness and smoothness of a Body is made much more sensible by the help of a Microscope then by the most tender and delicate Hand".[249] Second, beginning with an artificial point creates a new set of mores for instruments, shifting worth from natural organs such as the human eye to the microscope, telescope and other instruments of vision. Catherine Wilson points out that Hooke's drawings in the *Micrographia*, and genre paintings like Vermeer's, which cast light on the beauty of mundane moments in seventeenth century Dutch life, mirror this change in values.[250] As discussed in Part I, like his friend Wren, Hooke prefers simplicity in experimental design, instruments and explanations. This preference carries over to the objects under his microscope lenses, such as common and ordinary needles and flies, which his hand-drawn micrographs depict as intricately beautiful and complex. Yet, and perhaps similarly to Vermeer's use of a camera obscura, Hooke could not reveal the often surprising complexity in the minute details of the everyday without lenses. Thus, by beginning with the point of a needle in his first major publication, and his most famous one, Hooke picks up Galileo's mantle of radical instrumentalism, replacing natural organs with artificial ones.[251]

Within the limits of the human senses, the needle point is "made so sharp, that

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[249] Hooke, *Micrographia*, Preface, 114.

[250] Catherine Wilson, "Aesthetic Appreciation of Nature in Science", in Payne (ed.), *Vision and its Instruments*, 49–58.

[251] Hooke, *Micrographia*, 1. See also Gal, "Empiricism without the Senses: How the Instrument Replaced the Eye," in Ofer Gal and Charles Wolfe (eds), *The Body as Object and Instrument of Knowledge* (Netherlands: Springer, 2010), 121–48; Gal and Chen-Morris, *Baroque Science*.

the naked eye cannot distinguish any parts of it … But if view'd with a very good *Microscope*, we may find that the top of a Needle (though as to the sense very *sharp*) appears a *broad, blunt*, and very *irregular* end; not resembling a Cone, as imagin'd, but onely a piece of a tapering body, with a great part of the top remov'd, or deficient."[252] Yet even though Hooke's "very good *Microscope*" is good enough to reduce a seemingly sharp needle to a "*blunt* and very *irregular* end", Hooke sees that his lenses lack the power to reduce nature's points to blunt ends; that is, the microscope is not good enough to distinguish the parts of nature as it does the parts of art. Instead of revealing nature's fundamentals, his lenses magnify seemingly infinite complexity. "[B]ristles" and "claws" remain sharp even under the microscope, and insensible points such as the "hairs of leaves"[253] become visible. Hooke's problem is that the needle fails to live up to his "imagin'd" expectations. As a crude instrument of art, designed to work at the level of the senses – which it does well as anyone who has ever been pricked by a needle will attest – the needle nevertheless shows how far instruments of art have to go before they can be like nature's needles – its 'bristles' and 'claws' – under the microscope. Hooke is also aware that even his best optical instruments allow only for both qualitative and quantitative approximations. Yet he never doubts that "were we able *practically* to make *Microscopes* according to the theory of them", we would be able to reduce even nature's sharpest points to "*broad, blunt* and very *irregular*" needles.[254] Also similarly to Wren, though more radically perhaps, Hooke is aware that instruments used to be the characteristic tools of mathematics and that the *new* instruments of practical geometry are optical ones.[255]

Later, in a preface To the Reader of his *Attempt to Prove the Motion of the Earth by Observations*, Hooke would claim that nature and art are interwoven in a fabric

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252 Hooke, *Micrographia*, 1–2.
253 Hooke, *Micrographia*, 2.
254 Hooke, *Micrographia*, 2.
255 Bennett, "Christopher Wren's Greshamite history of astronomy and geometry", in Francis Ames-Lewis (ed.), *Sir Thomas Gresham and Gresham College*, 193.

that clothes natural philosophy. "I design always to make them [art and nature] follow each other by turns, and as 'twere to interweave them, being apart but like the Warp or Woof before contexture, unfit either to Cloth, or adorn the Body of Philosophy".[256] Later still, when arguing for the benefits of practical over speculative geometry, Hooke would state similarly to the above quote, although less poetically, "that tho' Science can easily suppose and conceive things as possible to be done, yet Art doth find many difficulties in the actual performance of them, and both ought to be call'd in for assistants in the prosecution of experimental Philosophy".[257] So, representing natural points with artificial ones renders art and nature as parts of a whole. Moreover, Hooke is quick to point out the advantages of his 'practical or mechanical geometry'[258] over the abstractions of ruler and compasses. "The Points of Pins are yet more blunt, and the Points of the most curious Mathemati[c]al Instruments do very seldome arrive at so great a sharpness; how much therefore can be built upon demonstrations made onely by the productions of the Ruler and Compasses, he will be better able to consider that shall but view those *points* and *lines* with a *Microscope*".[259] The crucial difference is that technological advancements can improve microscopes, which as optical instruments embody nature, manipulating points and 'lines' of light that follow natural laws; whereas the ruler and compasses are instruments of art, only as nice as the human realm for which they were created. Again, rulers and compasses are instruments made to measure only at the level of the naked human eye. Nevertheless, because of this instrumental limitation, traditional or speculative geometry remains an important tool for drawing concepts that can be comprehended at the level of the sense of sight. Thus, the limitation allows for the integration of speculative geometry and Hooke's new practical geometry.

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[256] Robert Hooke, *An Attempt To prove the Motion of the Earth from Observations*, in *Lectiones Cutlerian* [Cutlerian Lectures] (London: Printed for John Martyn ..., 1679), page preceding 1.

[257] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 532.

[258] Hooke, *The Method of Improving* Natural Philosophy, in *Posthumous Works*, 69.

[259] Hooke, *Micrographia*, 2.

The shift in mores created by elevating the status of artificial instruments over natural ones is exemplified by Hooke's integration of nature and art in his experimental philosophy. First, as Gal and Chen-Morris argue, "Hooke's instruments embody Kepler's optics: they manipulate light. They have no recourse to visual rays or *species* because they do not defer to the human observer … in Hooke's "Scheme" there is no eye. His instruments are not meant as aides to a weak human organ, they are meant to replace it".[260] Further, instruments that succeed in fully achieving this status are automatic, and "interweave" art and nature. Hooke's wheel barometer, created to "*shew all the minute variations in the pressure of the Air*" needs no human intervention to convert those "minute variations" into readable measurements. But the barometer is not only an instrument for reading measurements off a dial – it is the first step in creating an artificial organ for detecting "*all those steams, which seem to issue out of the Earth, and mix with the Air (and so to precipitate some* aqueous *Exhalations, wherewith 'tis impregnated … before they produce the effect*".[261] In contrast, an example of an instrument that does not embody nature is Hooke's refractometer: although it is based on Kepler's optics, it merely traces light with linkages that must be manipulated. The refractometer legitimises the use of instruments in natural philosophy by demonstrating that mathematics "can vouch for the ability of optical constructs to represent physical reality accurately";[262] but the barometer is a perfect tool for natural philosophy, better than nature, better than human senses, and better than instruments of art that require constant fiddling because it "interweave[s] them". Finally, examining instruments of art with a microscope, "an organ more acute th[a]n that by which they were made",[263] reveals "the rudeness and bungling of Art"[264] compared with nature's creations, which are "able to include as great a variety of parts and contrivances in

[260] Gal and Chen-Morris, *Baroque Science*, 104.
[261] Hooke, *Micrographia*, Preface.
[262] Chen-Morris, *Measuring Shadows*, 23.
[263] Hooke, *Micrographia*, 2.
[264] Hooke, *Micrographia*, 2.

the yet smallest Discernible Point, as in the vaster bodies (which comparatively are called also Points) such as the *Earth, Sun,* or *Planets*".[265] Yet, although nature seems to possess complexity all the way down, Hooke yearns to build an instrument capable of reducing nature to the rudeness of art, and superseding it with art.[266] Hooke designs artificial instruments, compounded of art and nature, to replace natural ones like the eye, which makes it possible for him to make sensible experiments and predictions about insensible phenomena. In his *Lectures of Light*, Hooke repeats his microscope dream:

> Now we are sensibly informed by the Microscope, that the least visible Space (which is that which appears under an Angle of half a Minute of a Degree) may be actually distinguished into a thousand sensible Spaces: And could we yet further improve Microscopes, 'tis possible we might distinguish even a thousand more Spaces in every one of those we can now see by the help of those Microscopes we have already.[267]

TROPE

Hooke's second reason for beginning with the point of a needle is to fashion an analogy about points. The analogy demonstrates that the microscopic implies the macroscopic.

> Nor need it seem strange that the Earth it self may be by an *Analogie* call'd a Physical Point: For as its body, though now so near us as to fill our eyes and fancies with a sense of the vastness of it, may by a little Distance, and some convenient *Diminishing* Glasses, be made vanish into a scarce visible Speck, or Point (as I have often try'd on the *Moon*, and (when not too bright) on the *Sun* itself.) So, could a Mechanical contrivance successfully answer our *Theory*, we might see the least spot as big as the Earth it self; and Discover, as *Des Cartes* also conjectures [*Diop.* ch. 10. § 9.], as great a variety of bodies in the *Moon*, or *Planets*, as in the *Earth*.[268]

This use of instruments to smudge the edges between microscopic and macroscopic worlds both frames the *Micrographia* and becomes a leitmotif as Hooke's observations develop in scope and complexity. Just as his lenses show the parts of a

[265] Hooke, *Micrographia*, 2.
[266] Gal and Chen-Morris argue that the tension between "irreducible complexity" and simplicity is an important phenomenon of Baroque culture (Gal and Chen Morris, *Baroque Science*, 177).
[267] Hooke, *Lectures of Light*, in *Posthumous Works*, 134.
[268] Hooke, *Micrographia*, 2–3.

pin point, and just as the telescope shows a moon that "*Diminishing* Glasses" turn into a "scarce visible Speck", if one *could* construct microscopes "according to the theory of them", then

> we might find hills, and dales, and pores, and a sufficient bredth, or expansion, to give all those parts elbow-room, even in the blunt top of the very Point of any of these so very sharp [natural] bodies. For certainly the *quantity* or extension of any body may be *Divisible in infinitum*, though perhaps not the *matter*.[269]

This telling metaphor of "hills, and dales, and pores" plays a significant explanatory role in Hooke's final observation, Observ. LX. *Of the Moon*,[270] where he takes the metaphor literally, converting his fancies about the surface of a pin point into descriptions of the superficies of a celestial body. That Hooke chooses to expand the analogy by comparing the pin point with the moon in Observ. I, and then the moon with the earth as a final observation in the *Micrographia* is no coincidence. It is a clever way to employ a trope created by Galileo in his *Sidereus Nuncius* (1610), a description of the maculate superficies of the moon which had some forty-five years later become a commonplace conceptual idiom.[271] Hooke uses the trope to shift the point of reference. Galileo blazons that

> we have been led to the conclusion that we certainly see the surface of the Moon to be not smooth, even, and perfectly spherical … but on the contrary, to be uneven, rough, and crowded with depressions and bulges. And it is like the face of the Earth itself, which is marked here and there with chains of mountains and depths of valleys.[272]

Hooke applies Galileo's word-painting interchangeably for a needle point and a planet. Focusing a "thirty foot Glass" on "a small spot" of the moon, he describes the spot as "a very spacious Vale, incompassed with a ridge of Hills … the Vale may have Vegetables *analogus* to our Grass, Shrubs, and Trees".[273] This creates a relation between micro- and macroscopic points, easing Hooke's struggle to communicate

[269] Hooke, *Micrographia*, 2.
[270] Hooke, *Micrographia*, 242.
[271] Eileen Reeves, *Painting the Heavens* (New Jersey: Princeton University Press, 1999).
[272] Galileo Galilei, *The Sidereal Messenger of Galileo Galilei and a Part of the Preface to Kepler's Dioptrics Containing the Original Account of Galileo's Astronomical Discoveries*, trans. Edward Stafford Carlos (London: Rivingtons, 1880), 40.
[273] Hooke, *Micrographia*, 242–243.

his claim that the difference between points and planets is a matter of magnification, hence observations of one may be used to gain knowledge about the other. This further legitimises his use of the microscope to produce macroscopic and celestial knowledge. The trope mirrors this epistemological inversion, since the hills, valleys and pores create physical patterns in Hooke's observations. These patterns are toeholds of order and familiarity in nature's variety, whether microscopic, bare eye, or telescopic. They range from a simple point, the smallest imaginable one, to planets. Instruments extend Hooke's imagination in both directions, and mathematics derived from physics prevents flights of fancy.

GRANADOES

Hooke's architecture of matter presents particles as springy globular bodies (points). Here I will examine two figures that illustrate Hooke's points in application, in an attempt to further explain the epistemological inversion and its importance in Hooke's knowledge-making process. The figures, and Hooke's wording, disclose his indebtedness to Galileo;[274] yet Hooke's figures are dynamic. The first is an inconspicuous little diagram from *Of Spring* (1678), labelled *"Fig 5."* (*Figure 4.1*) squeezed into the upper right-hand corner of a plate of realistic engravings of various spring scales conflated with accompanying practical-geometrical demonstrations. *"Fig 5."* falls into the latter category: it is both a theoretical explanation of projectile motion (gravity compounded with a projectile's "oblique motion"[275]), and a "Scale".[276] The scale is bipartite, separated by a

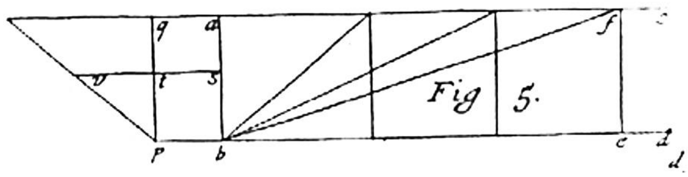

Figure 4.1 "Fig 5." or Hooke's "Scale" for projectile motion *(Of Spring).*

[274] Galileo Galilei, *Dialogues Concerning the Two New Sciences*, Day 4, trans. Henry Crew and Alfonso de Silvio (New York: Dover Publications, 1954 [1638]). Hooke, *Of Spring*, 22 in *Cutlerian Lectures*; Hooke, *Lampas*, 33, in *Cutlerian Lectures*.

[275] Hooke, *Of Spring*, 22 in *Cutlerian Lectures.*

[276] Hooke, *Of Spring*, 23 in *Cutlerian* Lectures.

common vertical line ab, and the two parts on either side allow for the calculation of velocities and distances of projectiles.

Having shown "how the Velocity of a Spring may be computed" in "*Fig 4.*" (see *Figure 4.2*), Hooke's demonstration of the spring law (which I will return to later), Hooke claims "it will be easie to calculate to what distance it will be able to shoot or throw any body that is moved by it."[277] The part to the right of line *ab* in "*Fig 5.*" is for the calculation of the distance travelled by a projectile shot from the ground at a given angle of inclination and with a known velocity, and the time taken to cover that distance, or "the length of the *Tactus* or shot, and the time it will spend in passing that length".[278] 'Tactus' is a conductor's tempo, or a 16th century term for a beat or pulse of one second[279] – the fundamental unit of time in "*Fig 5.*" – and seems to be a deliberate word choice meant to form a relation between the pulse of the spring which causes the "shot" and Hooke's other musical analogies for springy bodies and vibrations, since Hooke thinks of bows, cannon fire and so on as springs.[280] Suppose a spring of air shoots the heavy body upwards, for "of all springy bodies there is none comparable to the Air for the vastness of its power of extension and contraction".[281] Since "the *Tactus*

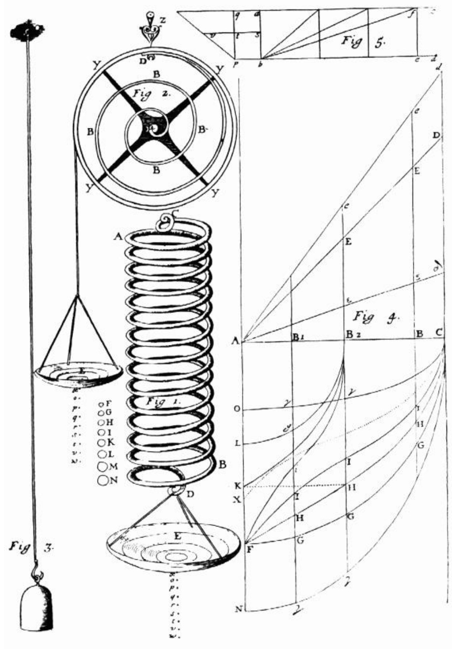

Figure 4.2 The main plate from Hooke's *Of Spring* (1678).

277 Hooke, *Of Spring*, 21 in *Cutlerian Lectures.*
278 Hooke, *Of Spring*, 22 in *Cutlerian Lectures.*
279 Willi Apel, *The Harvard Dictionary of Music* (Cambridge, Ma.: Belknap Press of Harvard University Press, 2000), 832. See also Sebastian de Brossard and James Grassineau, *A Musical Dictionary* (London: Printed for L. Wilcox, at *Virgil's Head* opposite the *New Church* up in the *Strand*, 1740), 268, 127.
280 Hooke, *Of Spring*, 21–23; Hooke, *Lampas*, 33, in *Cutlerian Lectures*.
281 Hooke, *Of Spring*, 23 in *Cutlerian Lectures*. Hooke further remarks upon a fountain made by his mentor John Wilkins in the latter's gardens at Wadham College, Oxford, which works on the "the Principle" of spring: "a Fountain so contrived, as by the Spring of the included Air to throw up to a great height a large and lasting stream of water" (*Of Spring*, 23–24). A year before Hooke's *Of Spring*, the naturalist Robert Plot had praised Wilkins's "*Water-works* of Pleasure" in his *The Natural History of Oxford-shire*, 235. But where Hooke was enamoured by the "Leaden Cisterns" and "two force Pumps"

given by this Scheme or Scale [is] appropriated to the particular [known] Velocity" of a projectile, the distance travelled by the projectile "is found by comparing the time of its ascent with the [known] time of descent of heavy bodies" or the "true Velocity of a falling body".[282] The "time of descent of heavy bodies", according to Hooke's examples, is 16 feet per second, and is represented by line *ab* and thus other equal parallel lines in the diagram. "The ascent of any body is easily known by comparing its Velocity with the [known] Angle of Inclination", the latter represented by the three diagonal lines to the right of *ab*, such as line *bf*.

The part to the left of line *ab* is for the calculation of "the whole Velocity of the ascent of a body by an equal motion [that is, uniform velocity, as if the body does not decelerate]" and "the whole Velocity of the accelerated descending motion", as well as "space ascended" and "space descended".[283] Subtracting lines of descending velocity from lines of ascending velocity along *stu* (that is, *st* minus *tu*) with respect to time, gives the mixed motion at each moment as the point labelled '*t*' moves along the line *pq*. By the proportionality of lines, when *st* equals *tu*, point *t* is 'at rest'. Subtracting the resultant areas descended from the areas ascended, in aggregates of slices that represent equal units of time as *stu* rises from *pb* to *rqa*, gives the projectile's altitude, plotting "the points it passeth through in all the intermediate spaces".[284] In other words, the left-hand side of Hooke's range and velocity finder is a coordinate system.

The area *pbqa* represents constant ascending velocity because *pb*, and so *st*, remains unchanged; the area *pqr* represents the effect of gravity upon the projectile, and by the similarity of triangles, the ratio of descending space subtracted from the ratio of ascending space gives the altitude. By beginning with a known angle of inclination, and a velocity of spring expressed in feet per second, the left-hand side

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of the "Engine", Plot marvelled at the rainbows in the mist, with a nod at Descartes, but "what kind of *Instrument* it was that forced the *water*, I dare not venture to relate" (Robert Plot, *The Natural History of Oxford-shire* [Oxford and London, 1677], 235).

282 Hooke, *Of Spring*, 23, 22, in *Cutlerian Lectures.*

283 Hooke, *Of Spring*, 22-3, in *Cutlerian Lectures.*

284 Hooke, *Of Spring,* 22, in *Cutlerian Lectures.*

of Hooke's scale plots a parabola with height on the vertical axis and time in seconds on the horizontal axis, and the projectile's velocity at any point. However, the height vs. time graph that results, while tracing a parabola in time, traces only the motion of point *t* ascending (and descending) along the line *pq* in space. It is like observing the projectile's motion from the perspective of a bombardier with no depth perception standing directly behind the flight path. Nevertheless, Hooke has already explained how to find the range, with reference to the spring's tactus. Moreover,

> having the tactus given by this Scheme, or Scale, appropriated to the particular Velocity, wherewith any body is moved in this or that line of Inclination, it will be easie to find what Velocity in any Inclination will throw it any length; for in any Inclination as the square of the Velocity thus found in this Scale for any inclination is to the square of any other Velocity, so is the distance found by this Scale to the distance answering to the second Velocity.[285]

This is because power, as Hooke has shown with a section of "*Fig 4.*" on the same plate, is proportional to velocity squared. Indeed, "*Fig 5.*" may be viewed as a slice of "*Fig 4.*" made particular for projectiles propelled by spring. Thus, just as with a section of "*Fig 4.*", which is for calculating infinitesimal points of a spring's "power", because "every point of the flexure hath a peculiar power, consequently there being infinite points of the space, there must be infinite degrees of power",[286] according to Hooke, one can approximate the velocity and displacement of any point of an ideal projectile's path in "*Fig 5.*" Recall that Hooke's explanation for this has grounds in the simple points of Observ. I in the *Micrographia*: "For certainly the *quantity* or extension of any body may be *Divisible in infinitum*, though perhaps not the *matter*". Indeed, what makes this kind of geometry useful is that Hooke's points and lines are infinitesimally divisible. Thus, perhaps the most interesting point in "*Fig 5.*" is *t*: it ascends along the vertical line *pq* to trace the projectile's trajectory, "allowance being made for the Resistance and impediment of the medium through which it passes".[287] In this way, and typical of Hooke's preferred way of working, *t*

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[285] Hooke, *Of Spring*, 23, in *Cutlerian Lectures.*
[286] Hooke, *Of Spring*, 17.
[287] Hooke, *Of Spring*, 21.

represents both the physical point of, for example, a stone shot from a sling, and its mathematical trace, the parabola.[288] That is, Hooke's practical geometry stems from his insistence on constructing mathematics from the observable physical properties and processes of natural phenomena. Again, Hooke's diagram also functions to make the insensible sensible.

Published in the same year as *Of Spring*, "*Fig. 22*" in Hooke's *Cometa* illustrates perhaps his most imaginative application of points in parabolic motion and their traces. The plate of observations in *Cometa* (*Figure 4.3*) shares the deliberate layout of the *Of Spring* plate. That is, proceeding from left to right as though reading, Hooke breaks down his observations from a naked eye drawing of the comet of April 1677, to a drawing of its appearance through the telescope, and then finally to physical-mathematical analysis. "*Fig. 22*" on the far right represents a three-dimensional section of the comet, "a solid parabolical conoeid",[289] "the *Nucleus* or Ball in the middle of the head"[290] and its upper body, constructed with a reticulation of parabolic traces. The "*Nucleus* or Ball", Hooke "conceive[s] to be dissolved equally on all sides,

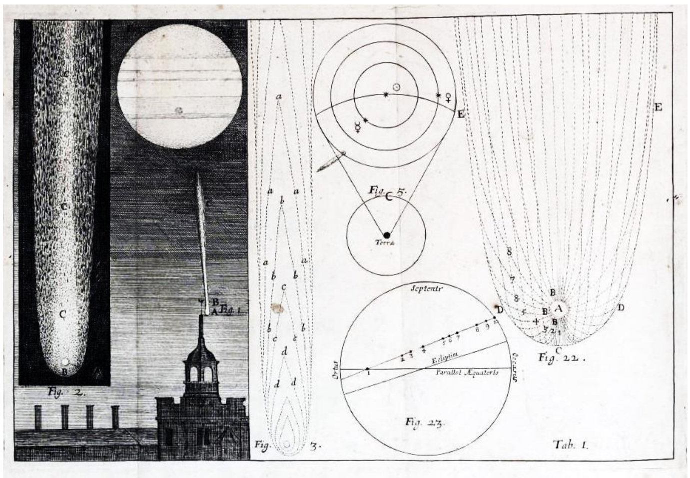

Figure 4.3 The main plate from Hooke's *Cometa* (1678).

> And the parts which are dissolved or separated from it … fly every way from the center of it, with pretty near equal celerity or power, like so many blazing Granadoes or Fire-balls, they continue their motion so far toward the way they are shot, till … the Sun deflect them upwards, or in opposition to the Sun into a Parabolick curve, in which Parabolick curve, every single particle continues its motion till it be wholly burnt out, or dissolved into the Aether.[291]

Hooke claims that he has

[288] For a comprehensive account of Hooke's way of working, see Ofer Gal, *Meanest Foundations and Nobler Superstructures*.

[289] Hooke, *Cometa*, 48, in *Cutlerian Lectures*.

[290] Hooke, *Cometa*, 48, in *Cutlerian Lectures*.

[291] Hooke, *Cometa*, 48, in *Cutlerian Lectures*.

compared the points to "blazing granadoes"[292] simply for explication's sake, though he thinks that they are not "of any large bulk" for he sees "no necessity to suppose them bigger than the Atoms of smoke, or the particles of any other steaming body, or than the parts of Air …"[293] On the one hand, this motion picture of "fire-balls" flowing spherically from the nucleus before the sun pushes them into parabolic paths serves to make Hooke's explanation sensible and imaginable, that is, to pull it within the limits of the human senses. On the other hand, Hooke's atomic description functions similarly to his projectile motion scale, where the angle of inclination, combined with the projectile's initial velocity, change the shape of the parabola, but the natural laws describing its trajectory remain consistent irrespective of size. Factoring in the flights of granadoes by formulating them as points in motion – in other words, as lines – and adjusting the scale's parameters produces their trajectories. Thus the behaviour of granadoes forms a bridge of knowledge between the insensibly small and the insensibly big. Four years later, in his *Lectures of Light*, Hooke would explain it thus:

> I cannot have an imagination of a Space, but the thousandth Part of the breadth of a Hair, yet, by my Reason, I can be certainly informed that such a Space there is, and even by Microscopes we can make such a Space visible, and yet our fancy will diminish no farther than the least sensible Point to the naked Eye; as the point of a sharp Needle or the like: But we are not less certain of it, though we cannot imagine it, that is, make an Image or Representation of it to the Mind.[294]

It is the danger of making a "Representation of it to the Mind" not grounded in physical reality that Hooke attempts to avoid with practical followed by speculative geometry. Only by beginning with simple points can one then "draw *single* strokes"[295] such as parabolic paths, and only after this should one consider more complicated bodies.

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[292] For more details on granadoes, see: CHAP. XVIII. "How to make Hand-Granadoes to be Hove by Hand", in John Seller's *The Sea Gunner* (London: H Clark, 1691).

[293] Hooke, *Cometa*, 49, in *Cutlerian Lectures.*

[294] Hooke, *Lectures of Light*, in *Posthumous Works*, 131.

[295] Hooke, *Micrographia*, 1.

5. LINES

"The sharpest *Edge* hath the same kind of affinity to the sharpest *Point* in Physicks, as a *line* hath to a *point* in Mathematicks; and therefore the Treaty concerning this, may very properly be annexed to the former."[296] It should come as no surprise that Hooke opens his second *Micrographia* observation, *Of the Edge of a Razor*, with this passage. Hooke explains that the line of a razor's edge follows the same fate as a point under the microscope: it appears rough and wide, not sharp. But, "since as we have just now shew'd that a *point* appear'd a *circle*, 'tis rational a *line* should be a *parallelogram*".[297] Hooke reminds the reader that his observations of pins, needles and razors apply also to nature's points and lines, the smoothness and sharpness of which could be reduced to ruggedness with powerful enough instruments. For "perhaps future observators may discover even these [fluid bodies, which appear smooth] also rugged; it being very probable, as I elsewhere shew [in a later observation on pigment particles], that fluid bodies are made up of small solid particles variously and strongly moved …"[298]

Since light and sound share observable physical similitudes, Hooke expects that his geometrical optics will describe the mechanism of all vibratory phenomena. In Observ. *VI Of Small Glass Canes*, to which Hooke's remark on how "solid bodies" are "strongly moved" to fluid states refers, he fuses his studies of sound and light to describe with ray optics how it is possible to learn from refraction-traces whether media are congruous or incongruous, as well as what happens when rays of vibrations, or the "Chime of Impulses" discussed in Part I, in general interact with points of matter at interfaces. This learning is crucial for the construction of Hooke's matter theory, 'congruity and incongruity'.

> Upon consideration of the *congruity* and *incongruity* of Bodies, as to touch, I found also the like *congruity* and *incongruity* … as to the *Transmitting* of the *Raies* of Light … whence an *oblique Ray* out of *Glass*, will pass into *water* with very little *refraction*

[296] Hooke, *Micrographia*, 4.
[297] Hooke, *Micrographia*, 4.
[298] Hooke, *Micrographia*, 5.

> from the *perpendicular*, but none [of the rays] out of *Glass* into *Air*, excepting a *direct*, will pass without a very great refraction from the perpendicular, nay any oblique Ray under thirty degrees, will not be admitted into the Air at all … So also as to the property of cohesion or congruity, Water seems to keep the same order, being more congruous to Glass th[a]n Air.[299]

The term 'congruity and incongruity' is an explanatory tool fashioned to explain how and why congruent and similar bodies attract and stay together while dissimilar bodies repel. Equivalently, congruent geometrical figures share the same shape and size, and similar figures share the same shape or angles, but not the same size – like Hooke's concept of globular bodies, or particles. This concept enables Hooke to make his microscopic and macroscopic inversions, since points and planets become a matter of scale, and some of Hooke's scale bars in the *Micrographia* are designed with this in mind. To put it another way, since "particles that are *similar*, will, like so many *equal musical strings equally stretcht*, vibrate together in a kind of *Harmony* or *unison",*[300] as discussed in Part I, one can infer that particles vibrating together in unison are congruent, whereas particles that vibrate together "in a kind of *Harmony*" are "*similar*". This enforces physical patterns of order within nature's variety, and these patterns are necessary for sympathetic resonance to occur, because particles that are congruent share the same vibrational frequencies, whereas the vibrational frequencies of similar particles are in ratios such as the perfect fifth – Hooke's favourite. Moreover, the similar points and planets are proportional, forming a chain of ratios from the microscopic to the macroscopic, which creates a constant of proportionality that becomes important for Hooke's formulation of the spring law later on. This is important because the spring law is perhaps the best demonstration of how his concept of geometry works in practice as well as why he needs it for his physics. Indeed, in *Of Spring*, Hooke provides a refined version of the above citation, rejecting descriptions of quality such as "Light", "Glass" and "Air" in favour of "Heterogeneous motions" and "proportion":

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[299] Hooke, *Micrographia*, 21–2.
[300] Hooke, *Micrographia*, 21.

> Heterogeneous motions from without are propagated within the solid in a direct line if they hit perpendicular to the superficies or bounds, but if obliquely in ways not direct, but different and deflected according to the particular inclination of the body striking, and according to the proportion of the Particles striking and being struck.[301]

Here Hooke generalizes particular substances to "Particles striking and being struck", and replaces visible light with insensible sound vibrations, that is, of frequencies well above the limits of human hearing, which travel in straight lines. A reader familiar with the earlier *Micrographia* paragraph may expect a reference to the refraction of light again owing to his diction, but in *Of Spring,* Hooke subverts light with "motions", employing the same practical geometry for both light and sound.

This adaption of optics to acoustics maps onto Hooke's ambition to extract knowledge beyond sense limits. Light rays are sensible: they can be manipulated, traced and measured – but Hooke lacks this luxury with sound. Where in the *Micrographia* Hooke treated light as sound, here in *Of Spring* his solution is to treat sound as though it were light geometrically to 'see' constructions of sound which can thus be manipulated like light. It is a way to construct a theoretical framework that breaks the boundaries between sensibles and insensibles by mathematising the interactions of vibrational frequencies with matter. Thus a line, in addition to allowing Hooke to analyse a point in motion, such as a flaming granado – for recall that "a Point moved makes a Line in the Mathematical Sense" – also enables him to geometrically describe points strung together into physical lines in nature and art, and motions (the propagation of "power" or "force") from one point to another.

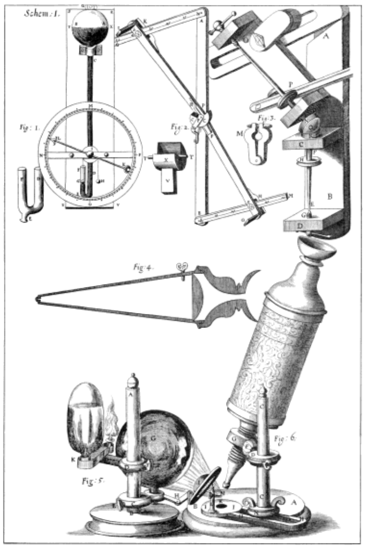

Figure 5.2 A plate of Hooke's optical instruments; "Fig. 2" depicts the refractometer *(Micrographia).*

301 Hooke, *Of Spring*, 12.

RAYS

Drawing lines as representations of physical motions and forces is not novel, but Hooke's matter-as-vibrations aspirations are. By putting a line or ray under the microscope, turning it into a "*parallelogram*", Hooke can describe and discuss these motions in two dimensions. He can then make conjectures on refraction and motion in the magnified area between a pair of parallel lines. When it comes to further developing his metaphysics of vibration, especially mathematically, this is beneficial because applying ray optics and the laws of reflection and refraction to his theory of congruity and incongruity allows Hooke to analyse more complicated matters such as how the attractive power of congruity diminishes with distance, the reflection of incongruous vibrations, the refraction of similar ones, and so on.

For example, in the *Micrographia's* ninth observation, *Of the Colours observable in Muscovy Glass …*, Hooke claims that an "exceeding *quick*" and "very *short vibrating motion*" is necessary "to produce the effect call'd Light in the Object". A "Diamond [which shines more when struck] being the hardest body we yet know in the World, and consequently the least apt to yield or bend, must consequently also have its vibrations exceeding short."[302] Hooke will shift reference frames for his explanation of what light is. Here, his first frame of reference is vibrations; his second is the "Object", which is compounded of bodies or points "*susceptible*" to this kind of motion. When this vibration propagates from susceptible point to point, "through the interpos'd pellucid body to the eye", it produces the effect of light in these objects, which is observed as a ray. The "motion is propagated every way through an *Homogeneous medium* by *direct* or *straight* lines extended every way like Rays from the centre of a Sphere … with

The observed angle in the liquor.	The observed angle in the air.	The sine of the observed angle in the liquor.	The sine of the observed angle in the air.	The hypothetical proportion of the sines of the angles of inclination in the air.	The angle answering the hypothetical sine.
		Common water.			
5.	6.39	8716	11407	11609	6.40
10.	13.19	17365	23033	23352	13.30
15.	20. 5	25882	34339	34474	20.10
20.	27. 4	34202	45503	45557	27. 6
25.	34.15	42262	56280	56294	34.15+
30.	41.45	50000	66588	66666	41.46−
35.	50.00	57358	76604	76400	49.50
40.	58.45	64279	85491	85419	58.40
45.	70.30	70711	94264	94387	70.42
		Brine.			
10.	13.35	17365	23486	23759	13.44+
20.	27.43	34202	46510	46756	27.52+
30.	43.10	50000	68412	68412	43.10
		Oil of turpentine.			
10.	14.45	17365	25460	25900	15.00+
20.	30. 5	34202	50126	50298	30.11+
30.	47.20	50000	73531	73531	47.20

Figure 5.1 Hooke's refractive index, enclosed in a letter sent to Robert Boyle (Gunther, *Early Science in Oxford, Vol. 6*, 1930 [1664]).

302 Hooke, *Micrographia*, 55, 56.

equal velocity," and similarly to water ripples, "all parts of these Spheres undulated through an *Homogeneous medium* cut the Rays at right angles" to the direction of propagation. He would later reiterate this concept in *Lampas*.[303]

"But because all transparent mediums are not *Homogeneous* to one another," Hooke wants to better explain "how this pulse or motion will be propagated through differingly transparent *mediums*." He states 'Descartes's law' of refraction as a preliminary,[304] supposing "the sign [sine] of the angle of incidence in the first medium to be to the sign of refraction in the second."[305] But Hooke does not simply take the sine law on authority. Rather, in the *Micrographia's* preface, he agrees with "the Laws of refraction", again, "that the lines of the angles of Incidence are proportionate to the lines of the angles of Refraction", because he has *experimentally verified* them – finding that the angles and proportions of the "hypothetical sines" correspond closely to the respective angles and proportions found by experiment.[306] In a 1664 letter to Boyle, Hooke encloses his tabulated results (*Figure 5.1*), which show that empirical trials support the "hypothesis of sines".[307] Consequently, Hooke describes in painstaking detail how to build and use a refractometer of his own invention (*Figure 5.2*, "Fig: 2").[308] Thus, with the "hypothesis of sines" as his theoretical foundation, he constructs 'parallelograms' to represent a ray "refracted towards the perpendicular" of a "plain surface NO" as it moves from a "Homogeneous transparent *medium* LLL" into "the *medium* MMM" (*Figure 5.3*, "Fig:

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303 Hooke, *Micrographia*, 56. Later, in *Lampas*, Hooke reiterates that this "motion we suppose to be propagated by a Pulse or Wave in all uncoloured Rays at Right Angles with the Line of Direction" (Hooke, *Lampas*, in *Cutlerian Lectures*, 39).

304 Although the law of refraction was first published by Descartes, Willebrord Snellius (Snell) worked on the same law simultaneously, and it is possible that Descartes saw Snell's papers. Though there is evidence that Thomas Harriot had established the same law, and so on. See A.I. Sabra, *Theories of Light*, especially pp. 99–100; and for a thorough account of Harriot's work on refraction, see Amir R. Alexander, *Geometrical Landscapes* (Stanford, California: Stanford University Press, 2002), 128, 112–125.

305 Hooke, *Micrographia*, 57.

306 Hooke, *Micrographia*, Preface. Hooke, in R.T. Gunther, *Early Science in Oxford,* Vol. 6, (Oxford: Oxford University Press, 1930), 211–212.

307 Hooke, in R.T. Gunther, *Early Science in Oxford,* Vol. 6, 212.

308 Hooke, *Micrographia*, Preface.

I").[309]

Suppose, Hooke says,

> AFCD to be the physical Ray, or ABC and DEF to be two Mathematical Rays, *trajected* from a very remote point of a luminous body through an *Homogeneous* transparent *medium* LLL, and DA, EB, FC, to be small portions of the orbicular impulses which must therefore cut the Rays at right angles[.][310]

The ray is either a "physical Ray" "of some Latitude", *or* "two Mathematical Rays", and these two types of geometry – practical and speculative – serve different though inter-related explanatory functions.[311] Because Hooke has magnified a ray segment, the physical ray is not a line, but a parallelogram – a superficies, one level up in geometrical complexity; and it is cut at "right angles" by "small portions of the orbicular impulses", the lines DA, EB and FC, like the wave-fronts in Hooke's water ripples analogy. In *Lampas*, Hooke again clarifies that "the stroke of the Pulse [is] the length of the space between" the lines cutting the ray at right angles.[312] Or the ray is "two Mathematical Rays", namely, the bounding parallel lines of the physical ray, which are abstractions of it, and which therefore make the concept of refraction more intelligible to the senses according to Hooke's use of speculative geometry. Before refraction, the segments of these lines are equal, representing "*equal velocity*", but after refraction, the line segments on the left-hand side mathematical ray, which strikes the interface first, change in length to reflect a change in velocity. According to Hooke, "the *medium* MMM" "is more easily trajected then the former by a third", so when the incident ray strikes the interface "obliquely" first with "point C of the orbicular pulse FC", it "will be mov'd to H four spaces in the same time that F the other end of it is mov'd to G three spaces". Note that, according to Hooke, if the ray moves "more easily" through the medium MMM, then that medium is the denser, not the rarer, one. That is, he believes light travels faster in a denser medium – like sound; this is an interesting error for it traces, again, Hooke's developing

309 Hooke, *Micrographia*, 57.
310 Hooke, *Micrographia*, 57.
311 Hooke, *Micrographia*, 57; Hooke, *Lampas*, in *Cutlerian Lectures*, 39.
312 Hooke, *Lampas*, in *Cutlerian Lectures*, 39.

metaphysics of vibration to his sound studies, and I will return to it in the subsequent section. Now, that the ray of light is propagated by "*orbicular* pulses" like water ripples is crucial because it determines how Hooke constructs the refracted ray.[313]

The smaller pricked circle arc reveals Hooke's construction lines for obtaining the pricked tangent line GT. GT is perpendicular to the refracted 'mathematical lines', and gives the direction of the refracted ray according to 'Descartes's law', which proposes an inverse relation between the velocities and sines to allow for the notion of light speeding up in a denser medium.[314] Thus, FG/CT = sin(i)/sin(r) = 4/3 = v_r/v_i, where v_r represents the velocity of the refracted ray, and v_i, the velocity of the incident ray, using the radial distances already specified by Hooke.[315] After obtaining the radius of the smaller pricked line by taking ¾ of the line segment FG with his compasses – the radius of the incident 'orbicular pulses' – Hooke uses point C as his new centre, and draws the smaller pricked arc,

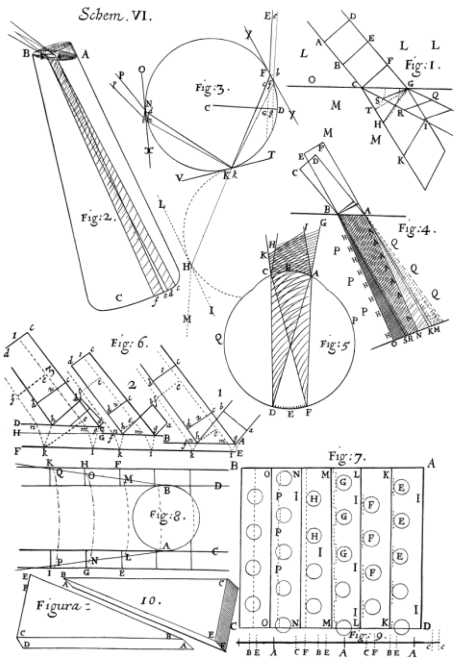

Figure 5.3 "Fig: 1" represents Hooke's concept of optical refraction; "Fig: 4", his hypothesis on colour formation. "Fig: 7" illustrates Hooke's notion of particles in a quincunx formation *(Micrographia).*

> for the sign [sine] of the inclination is to be the sign of refraction as GF to TC the distance between the point C and the perpendicular from G on CK, which being as four to three, HC being longer then GF is longer also then TC, therefore the angle GHC is less than GTC.[316]

The larger pricked circle arc, around centre C, reveals how Hooke obtains the length of what he imagines to be the refracted ray's faster

[313] Hooke, *Micrographia*, 57.
[314] Sabra, *Theories of Light.*
[315] See Sabra, *Theories of Light*, 194, 195 (fn. 30) for a more technical as well as hypothetical analysis that compares Hooke and Huygens's treatment of waves; however, note that Sabra is confused about Hooke's statement concerning the direction of the pulses relative to the propagation of light in a homogeneous medium.
[316] Hooke, *Micrographia*, 57.

"orbicular pulse". With the line segment FG as the radius of the incident ray's "orbicular pulse", and knowing that medium MMM "is more easily trajected then the former by a third", Hooke gets the line CH: he takes the length or distance FG, the incident pulse's velocity multiplied by the time taken for it to traverse from F to G, and then further opens his compasses by one third of FG's length to represent the increase in "velocity" of the refracted impulse line CH. With this new radius, he draws part of "orbicular pulse" CH, which represents the velocity multiplied by the time taken for the refracted pulse to spread from C to H. This makes sense according to Hooke's description of the changes in velocity that occur from medium LLL to medium MMM upon refraction.[317]

Focusing now on the 'physical ray' and "the pulses themselves", which "by refraction acquire another propriety," Hooke draws a tangent line from point G to H, and concludes that "the whole refracted pulse GH shall be *oblique* to the refracted Rays CHK and GI".[318] That is, according to Hooke, the refracted pulses represented by the parallel lines GH and IK are not perpendicular to the refracted ray's direction of propagation, as in the incident ray, but are "*oblique*". "So that henceforth the parts of the pulses GH and IK are mov'd ascew, or cut the Rays at *oblique* angles."[319] This obliquity occurs whether the ray refracts into a denser or rarer medium, as the second refracted ray with pulses GS and QR illustrates – by moving away from the normal or perpendicular. Hooke is aware that obliquity is a strange idea, an "odd propriety" "of a refracted Ray";[320] he is also proud of it, claiming that it also "conduces to the production of colours".[321] Why *should* refraction change the direction of the pulses so that they are no longer perpendicular to the ray's direction of propagation, but are instead "*oblique*" to it? Or to borrow from Hooke, why should "the pulse [be] made oblique to the progressive, and that so much more, by

[317] Hooke, *Micrographia*, 57.
[318] Hooke, *Micrographia*, 57.
[319] Hooke, *Micrographia*, 57.
[320] Hooke, *Micrographia*, 58.
[321] Hooke, *Micrographia*, 62.

how much greater the refraction is"?[322] One answer is that the pulses are oblique because angle GTH is ninety degrees;[323] this is a purely mathematical explanation, which works well at the level of the senses to make the concept more comprehensible and imaginable. But Hooke's mathematics stems from the physical world, not the other way around, just as his use of 'Descartes's law' stems from results obtained during his refractometer trials. Hooke building the instrument, filling the box with liquid, adjusting the rulers to trace the ray, looking through the sights and measuring angles of "inclination" and "refraction" with "cross threads" is what makes the geometry taken from this process meaningful to him, and what gives the natural law its power.[324] Hooke's hypothesis on "the production of colours", which uses this notion of oblique or deflected pulses as its foundation, provides a physical answer for how and why vibrating lines of light display this "odd propriety" when moving through transparent media with different refractive indices.

STRINGS

A few years after the *Micrographia*, in his 1672 critique of Isaac Newton's "*New Theory About* Light *and* Colors" (henceforth *New Theory*), Hooke takes care to explain his light strings and sound rays:

> [the] string (by the way) is a pretty representation of the shape of a refracted [light] ray to the eye; and the manner of it may be somewhat imagined by the similitude thereof: for the ray is like the string, strained between the luminous object and the eye, and the stop or fingers is like the refracting surface …[325]

This loaded aside, typical of Hooke, gives away his real concerns. Hooke's critique is more than a defence of his own hypothesis of light and colours; it is a defence of his matter theory, for which he constructed an ontological and epistemological

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[322] Hooke, *Micrographia,* 62.
[323] Sabra, *Theories of Light*, 194.
[324] Hooke, *Micrographia*, Preface. Gooding, Pinch and Schaffer (eds). *The uses of experiment.*
[325] Hooke, "Hooke's Critique of Newton's Theory", in Isaac Newton and I. Bernard Cohen, *Isaac Newton's Papers and Letters on Natural Philosophy, and Related Documents,* ed. I. Bernard Cohen, assist. Robert E. Schofield (Cambridge: Cambridge University Press, 1958), 111.

framework in the *Micrographia* with a musical strings analogy, and from which natural laws, such as the inverse square law, and Hooke's law, follow. In his critique of Newton's *New Theory*, Hooke defends the ideas on light and colour that he developed in the *Micrographia*: he maintains that light and colour are effects, and attempts to explain how these effects are produced from two different frames of reference: vibrations and susceptible vibrating bodies.[326] Most importantly, Hooke begins and ends his critique with the "supposition" that light

> is nothing but a pulse or motion, propagated through an homogeneous, uniform and transparent medium: and … colour is nothing but the disturbance of that light, by the communication of that pulse to other transparent mediums, that is, by the refraction thereof.[327]

And "so long as those motions remain distinct in the same part of the medium or propagated ray, so long they produce the same effect, but when blended by other motions, they produce other effects [that is, colours]". A "direct contrary motion" 'destroys' and 'reduces' a colour "to the first simple motion [white light]".[328] This is his particular yet general version of the 'modification hypothesis' of light and colour, popular in the seventeenth century,[329] and to explain, Hooke refers back to his work in the *Micrographia*. For example, "Fig: 4" (see *Figure 5.3*) is an attempt to capture congruous and incongruous vibrations, their parts represented by lines as refracted rays superimpose and blend to produce colours. In this way colours, similarly to visibly refracted rays, are a visibly sensible representation of congruity and incongruity. And in his analysis of colours Hooke expects that the reader takes his previous statements on optical refraction, examined in the last section, as axioms – especially the "odd" idea that "the pulse is made *oblique* to the progressive" line of direction of a refracted ray, which contributes "to the production of colours".

As before, thinking of magnified lines – or more appropriately rays – as

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326 Hooke, "Hooke's Critique of Newton's Theory", 114.
327 Hooke, "Hooke's Critique of Newton's Theory", 111 (also see p. 114).
328 Hooke, "Hooke's Critique of Newton's Theory", 114.
329 Sabra, *Theories of Light*. Simon Schaffer, "Glass works", in, Gooding, Pinch and Schaffer (eds), *The uses of experiment*, 67–104.

parallelograms, Hooke reasons that if the ray strikes a refractive superficies obliquely, then "that part or end of the pulse which precedes the other" must be "impeded by the resistance of the transparent [denser] *medium*, then the other part or end of it which is subsequent, whose way is, as it were, prepared by the other".[330] In addition, the preceding end of the ray will be "especially [impeded] if the adjacent *medium*" is not "agitated" "in the same manner" – in other words, if the vibrations of its particles are incongruous with the ray's pulses. And colours are not produced because of innate properties that compound to create white light; rather, colours result when an "infinite" number of refracted "Rays collateral" superimpose with their pulses deflected from the perpendicular[331] – Hooke's "odd propriety". Thus, for colours to form in accordance with Hooke's matter theory there needs to be a relation between the refracted rays with their oblique pulses, which are analogous to stretched musical strings "strained between the luminous object and the eye" and bent by refracting "fingers", and the physical, "agitated" or vibrating medium through which these rays transmit. To put it more in terms of 'congruity and incongruity', colour is produced amongst the more or less incongruous vibrations of the light-refracting medium when the oblique pulses of refracted rays superimpose. Geometrically (Fig*ure 5.3*, Fig: 4"),

> the Ray AAAHB will have its side HH more deadned by the resistance of the dark or quiet *medium* PPP, whence there will be a kind of deadness superinduc'd on the side HHH, which will continually increase from B, and strike deeper and deeper into the Ray by the [hatched] line BR[.][332]

To explicate with blue, "all the parts of the triangle, RBHO will be of a dead *Blue* colour, and so much the deeper [blue] by how much nearer they lie to the [mathematical] line BHH". Recall from Hooke's refraction diagram (Figure 5.3, "Fig: 1") that the mathematical line on the left-hand side, labeled here as BHH, strikes the refractive surface first; and thus it is the faster yet weaker part of the pulse, or the part "which is most deaded or impeded". Consequently, the deep blue, formed on

[330] Hooke, *Micrographia*, 63.
[331] Hooke, *Micrographia*, 63.
[332] Hooke, *Micrographia*, 63.

the extremity of this line where its vibrations are most 'in contact' with the incongruous vibrations of the surrounding and penetrating medium, become "so much the more *dilute*, by how much nearer it [the medium] approaches the line BR", where blue finally overlaps into the oblique pulses that produce the hues of green. Examining the diagram from left to right, the greens run into yellows and oranges; the latter are 'dilutions' of red, or the stronger yet slower extremity of the ray AAN.

> Next on the other side of the [mathematical] Ray AAN, the end of A of the [physical] pulse AH will be promoted, or made stronger, having its passage already prepar'd as 'twere by the other parts preceding [that is, line BHH], and so its impression will be stronger …[333]

In other words, red results from the stronger yet slower portion of the pulse succeeding the part BHH, the vibrations of which penetrate into the medium. Hooke explains it thus:

> because of its *obliquity* to the Ray, there will be propagated a kind of faint motion into [the medium] QQ … which faint motion will spread further … into QQ as the Ray is propagated further … from A, namely, as far as the line MA, whence all the triangle MAN will be ting'd with a *Red*, [which] will be the deeper the nearer it approaches the line MA, and the *paler* or *yellower* the nearer it is the line NA.[334]

It is evident, from Hooke's attempt to capture parts of these spherically spreading motions geometrically, that his theory of congruity and incongruity is woven inextricably into his hypothesis on light and colours. And that his constructions of 'mathematical and physical rays', with lines cutting across parallelograms to represent parts of "orbicular pulses", are a means for him to make these "parts" and "portions" of the natural world more comprehensible both to the senses and the imagination. Defining white light as an undisturbed "pulse" propagated through a homogenous … transparent medium", and colour as "the disturbance of that light", accordingly defines the epistemological significance of light for Hooke. He rejects Newton's "connate [innate] properties"[335] of light because *if* all sounds are already

[333] Hooke, *Micrographia*, 63.
[334] Hooke, *Micrographia*, 63.
[335] Newton, "Newton's Theory about Light and Colors", in Isaac Newton and I. Bernard Cohen, *Isaac*

in any string,[336] whether "strained" like a ray "between the luminous object and the eye" or like a monochord distended by weights – as Newton claims about all colours in any 'string' or ray of white light – *then* all matter, irrespective of size, shape, density and tension, can be congruent or incongruent. Moreover, any "Object", body or point could propagate light. But according to Hooke's notion, when explaining to Newton how the effects of light and colour are produced from the reference frame of "susceptible" bodies, "as many colours as degrees thereof as there may be, so many sorts of bodies there may be", though he doubts that all the bodies in the world compounded would make white light.[337] Hooke also grants that "all luminous bodies are compounded of such substances condensed";[338] recall from Part I that this supposition forms part of his explanation for what he would later dub a 'chime of motions' in his *Lectures of Light*; that is, how different waves can cross. What Hooke refuses to allow is that white light is made of "connate properties".

Forced to summarise the main points from several of the *Micrographia's* observations in his 1672 critique of Newton's hypothesis, such as the structural colours observable in Muscovy glass as well as peacock feathers and butterfly wings, Hooke reiterates that

> The motion of light in an uniform medium, in which it is generated, is propagated by simple and uniform pulses or waves, which are at right angles with the line of direction; but falling obliquely on the refracting medium, it receives another impression or motion, which disturbs the former motion, somewhat like the vibration of a string … [339]

Now, according to Newton's *New Theory*,

> Light is not similar, or homogeneal, but consists of difform Rays, some of which are more refrangible than others: So that of those, which are alike incident on the

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Newton's Papers and Letters on Natural Philosophy, 53.

[336] Hooke, "Hooke's Critique of Newton's Theory", in Isaac Newton and I. Bernard Cohen, *Isaac Newton's Papers and Letters on Natural Philosophy*, 111.

[337] Hooke, "Hooke's Critique of Newton's Theory", in Cohen and Schofield, *Isaac Newton's Papers & Letters on Natural Philosophy*, 112.

[338] Hooke, "Hooke's Critique of Newton's Theory", in Cohen and Schofield, *Isaac Newton's Papers & Letters on Natural Philosophy*, 112.

[339] Hooke, "Hooke's Critique of Newton's Theory", in Cohen and Schofield, *Isaac Newton's Papers & Letters on Natural Philosophy*, 112.

> same medium, some shall be more refracted than others, and that not by an virtue of the glass, or other external cause, but from a predisposition, which every particular Ray hath to suffer a particular degree of Refraction.[340]

Based on prism experiments begun in 1666, Newton argues that seven of these 'particular degrees of Refraction' are crucially not altered by a second refraction, leading him to conclude that these angles produce primary colours or rays.[341] Therefore, colours are "not *Qualifications of Light*, derived from Refractions, or Reflections … (as 'tis generally believed,) but *Original* and *connate properties*".[342] Newton's concept of light "*not* similar, or homogeneal", *not* refracted by an "external cause", but with "a predisposition" where some rays "shall be more refracted than others" is the antithesis of all seventeenth century modification hypotheses of light and colours, including Hooke's wave hypothesis.[343] Acceptance of Newton's ontology of light, namely, that "Light it self is a Heterogeneous mixture of differently refrangible Rays", "a confused aggregate … indued with all sorts of Colours"[344], would destroy Hooke's continuous efforts to develop and establish his cosmology, because it is incommensurable with his epistemological need to show that matter is either congruous or incongruous based on vibrations, the latter represented geometrically with parallelograms and lines.

After reading Hooke's critique of his *New Theory*, Newton seems to have picked up on Hooke's primary anxiety – his protectiveness of his metaphysics of vibration, for Newton attempts not only to subvert Hooke's stretched musical strings analogy, but to convince his readers that Hooke's 'congruity and incongruity' is "impossible". Newton had pored over Hooke's *Micrographia* in 1665,[345] and possessed intimate knowledge of its contents, as is further evidenced when he seemingly (at first) tries to convince Hooke that their competing hypotheses cohere

340 Newton, "*Newton's Theory about* Light *and* Colors", 53.
341 Newton, "*Newton's Theory about* Light *and* Colors", 47–59.
342 Newton, "*Newton's Theory about* Light *and* Colors", 53.
343 Sabra, *Theories of Light*. Schaffer, "Glass works", in Gooding, Pinch and Schaffer (eds), *The uses of experiment*, 67–104. Italics added.
344 Newton, "*Newton's Theory about* Light *and* Colors", 50, 55.
345 Schaffer, "Glass works", in Gooding, Pinch and Schaffer (eds), *The uses of experiment*, 74.

by showing him that it is possible to wed the theory of congruity and incongruity, as concerns light (and sound), to the "New Theory" on light and colours. The "Objectors [Hooke's] *Hypothesis*," Newton claims, "as to the fundamental part of it, is not against me."[346] According to Newton's interpretation, the "Fundamental Supposition" of Hooke's work on light in the *Micrographia* consists of what happens to it before it is incident upon the surface of a denser medium and then afterwards when it strikes the retina:

> That the parts of bodies, when briskly agitated, do excite Vibrations in the Aether, which are propagated every way from those in straight lines, and cause a Sensation of Light by beating and dashing, against the bottom of the Eye, something after the manner that Vibrations in the Air cause a Sensation of Sound by beating against the Organ of Hearing.[347]

This summary leaves out Hooke's ideas on refraction and colour so that Newton can replace them with his "New Theory", carefully modified to mesh with obviously paraphrased parts of the *Micrographia*.

For example, Newton begins his first statement by borrowing from Hooke's stretched musical strings analogy for congruity and incongruity. In his rendition, he breaks apart the strings argument from analogy by leaving the metaphor behind, taking only its meaning as concerns sizes of particles vs. their vibrations: "That the agitated parts of bodies, according to their several sizes, figures, and motions, do excite Vibrations in the aether of various depths or bignesses," Newton claims, sounding like Hooke, before adding his own twist, "which being promiscuously propogated through that Medium to our Eyes, effect in us a Sensation of Light of a *White* colour".[348]

> But if by any means those of unequal bigness be separated from one another, the largest beget a Sensation of *Red* colour, the least or shortest, of a deep *Violet,* and the intermediates, of intermediate colors[.][349]

Newton ends by qualifying the insertion of his own ideas amongst Hooke's – in this

[346] Newton, "*Newton's Theory about* Light *and* Colors", 120.
[347] Newton, "*Newton's Theory about* Light *and* Colors", 120.
[348] Newton, "*Newton's Theory about* Light *and* Colors", 120.
[349] Newton, "*Newton's Theory about* Light *and* Colors", 120.

case by returning to a musical analogy. These bodies of light, separated, act "much after the manner that bodies, according to their several sizes, shapes, and motions, excite vibrations in the Air of various bignesses, which, according to those bignesses, make several Tones in Sound".[350] He employs this rhetorical device of separation (of Hooke's text), replacement and insertion (of his own text) throughout his reply, progressively adding more of his own ideas and less of Hooke's, first, to convince his readers that his "New Theory" improves Hooke's "*insufficient*" wave "hypothesis", which is "in some respects to me (at least) *un-intelligible*", and second, to argue that his abstracted presentation of light needs no hypothesis.[351] It becomes clear as the text proceeds that Newton makes use of musical analogies specifically in response to Hooke's comparison of a ray of light with a tense musical string in the latter's critique, which Newton attempts to subvert by abstracting the string into several "false" strings: "For if light be consider'd abstractedly without respect to any *Hypothesis*, I can as easily conceive, that the several parts of a shining body may emit rays of differing colours … [like] the several parts of a false or uneven string",[352]

> since (even by the *Animadversor's* concessions) there are bodies apt to *reflect* rays of one colour, and stifle or *transmit* those of another; I can easily conceive, that those bodies, when illuminated by a mixture of all colours, must appear of that colour only which they reflect.[353]

In this way, Newton performs a hostile reformulation and abstraction of Hooke's concept of congruity and incongruity – for example, "bodies apt to reflect" are incongruous with the reflected "rays" – with his own "Doctrine" of seven primary rays of colour. Thus, according to him, Hooke's single-string analogy is untenable in the case of light because "when the *Objector* would insinuate a difficulty in these things, by alluding to Sounds in the string of a Musical instrument before percussion … I must confess, I understand it as little, as if one had spoken of Light in a piece of

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[350] Newton, "*Newton's Theory about* Light *and* Colors", 120.
[351] Newton, "*Newton's Theory about* Light *and* Colors", 122.
[352] Newton, "*Newton's Theory about* Light *and* Colors", 123.
[353] Newton, "*Newton's Theory about* Light *and* Colors", 123.

Wood before it be set on fire …"[354] Here, instead of separating the analogy from its metaphor as before when he pulled apart Hooke's vibrating strings model of congruity and incongruity, Newton takes the single string metaphor literally to make it seem ridiculous, and in doing so purposely misses Hooke's point. But recall that for Hooke white light is not corpuscles, but vibrations, and colour is caused when this vibration is modified upon refraction, and "mixt" or "blended" with at least one other colour-producing wave, which is what Hooke's single-string analogy describes. Yet Hooke's fusion of light and sound *does* come with an intellectual cost, if not exactly the one envisioned by Newton.

Although Hooke's conclusions on refraction result from experiments that manipulate light in a controlled, artificial environment, and the angles and ratios taken with his refractometer, Hooke's initial observations stem from studying the transmittance of light through porous and pellucid bodies with his microscopes. Like the "*Interstitia* of the world, that lies between the bodies of the Sun and Starrs, and the Planets, and the Earth",[355] close-packed microscopic points form "pores" through which 'lines' of vibrations may transmit. For example, during Hooke's experiments on "kettering stone" (a globular body composed of microscopic globules, known today as "Ketton stone"[356]), he claims that "the smaller those pores are, the weaker is the *Impulse* of light communicated through them, though the more quick be the progress".[357] As mentioned earlier, Hooke's error regarding the quickening of light in a denser medium results from a confusion of light and sound – that is, a mistake of similitude.

In the *Micrographia*'s preface, Hooke recounts acoustical experiments where he bends sound around corners with wires, sends sound through thick walls, and very roughly compares the speed of sound with the speed of light. Birch and Waller

[354] Newton, "*Newton's Theory about* Light *and* Colors", 123.
[355] Hooke, *Micrographia*, 96.
[356] Derek Hull, "Robert Hooke: A Fractographic Study of Kettering-Stone," *Notes and Records of the Royal Society*, 1997, 45–55.
[357] Hooke, *Micrographia*, 96.

provide better descriptions of some of these experiments. According to Birch,

> Mr Hooke read a discourse concerning the way of conveying force to a great distance, which he conceived would best be done by some stiff and inflexible rod [in Hooke's words, a material line], as a Wire, or long pole, or the like and shewed the experiment communicating a force given in the inner hall of Gresham-college across the quadrangle by means of a packthread, which was found to perform to satisfaction.[358]

Waller adds his own summary in the *Posthumous Works*, stating, "it was observed, that the sound was propagated instantaneously, even as quick as the motion of Light, *the sound conveyed by the Air coming a considerable time after that by the Wire*".[359] Thus sound moves faster in a denser medium, and because of similitude, Hooke concludes that light does too. Hooke never changes his mind about this, reiterating and arguing the point in various phases of his career.[360] Yet the properties shared by light and sound allow Hooke to apply practical-geometrical optics to the study of acoustics for the development of his dynamic concept of matter-as-vibrations, which sits on foundations of resonance, as demonstrated well by the musical strings analogy.

MONOCHORD

The power of Hooke's practical geometry is uniquely demonstrated in reversing the epistemological role of the Pythagorean monochord. As I have shown, Hooke's strings account for all the fundamental properties of matter, such as the differences between solids and fluids; and his mathematics is contingent upon physical reality. As a consequence, he reverses the epistemological role of the string by exhibiting that all sounds, including consonant chords, are dependent not on an underlining structures of simple ratios but on physical causes.

The monochord is an instrument as old as Pythagoras, a device designed to inspire awe over the simple, perfect, harmonic ratios of nature, and to study these

358 Birch, *The History of the Royal Society*, Vol. IV, 545.
359 Waller in Hooke, *Posthumous Works*, xxiv. Italics added.
360 See, for example, *Lampas* and *Lectures of Light*, in Hooke, *Posthumous Works.*

proportions by manipulating the length of a string.[361] Abstracted and idealised, the string as a line allowed Platonist mathematicians to enforce the idea that mathematical harmonies precede, underlie and are independent of nature's material infrastructure. But recall that Hooke's concept of sound is that it is "nothing but strokes within a determinate degree of velocity",[362] and that sound is musical when these strokes – vibrations – are isochronous,[363] because if they were not isochronous then the pitch would change. Thus for Hooke, consonance and dissonance depends not on abstractions, but on the material structure of the string.

This order of discussion in Hooke's text – that is, starting with strings – creates an appearance of order. Namely, each tone has a *unique* vibrational frequency based on the size, shape, density and tension of its parts, independent of all harmonies.[364] Hence the physical string, an elastic body, provides ontological illustration, whereas the string abstracted multiplies entities without necessity; and harmonies such as the perfect fifth are not pleasing because of a mathematical substrate, but are simply pleasing to human ears, because their vibrations are congruous. Indeed, Hooke's sound wheels, examined in Part I, demonstrate the isochrony principle even better than his strings. But Hooke is aware that the sound wheel is a new instrument, and so lacks the mathematical history necessary to role of the monochord, which he takes full advantage of with his strings analogies, employing consonances such as the octave and the perfect fifth as explanatory tools for congruity.

As discussed, in the second half of the 17th century and a little into the 18th, Hooke lectured arithmetic, theoretical and practical geometry for 37 year while simultaneously ensuring that Gresham College became an epicentre of experiments

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361 Sigalia Dostrovsky, "Early Vibration Theory: Physics and Music in the Seventeenth Century," *Arch. Rational Mech.* 14, no. 3 (1975), 169 – 218. See also Paolo Mancosu, "Acoustic and Optics", in Lorraine Daston, and Katharine Park (eds), *The Cambridge History of Science* (New York: Cambridge University Press, 2006), 596–631.

362 Hooke, *Diary*, 211.

363 Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 551.

364 Hooke, *Of Spring*, 8–9.

because of his position in the Royal Society.[365] Yet the type of teaching and practice of practical geometry that Hooke lectures, where a vibrating point is defined as having extension and a line is defined as "Physical … of some Latitude", is to the best of my knowledge uniquely his.[366] The Renaissance humanist Leon Battista Alberti made a similar attempt at physicalising practical geometry. In his trailblazing *De Pictura*, Alberti writes: "[the mathematicians], in fact, measure figures and shapes of things with the mind only, without considering *the materiality of the object*"[367]. Alberti's aim was to present a new materialised geometry for a particular vocation; as such, at first glance it may appear as if Alberti and Hooke stand on common ground where practical geometry is concerned; that is, the physicalisation of abstract definitions. Yet the fundamental difference is that Alberti's geometry, designed to instruct painters in the art of linear perspective, is static – the vanishing point and the viewer are always at rest. Moreover, it is necessary for Alberti to stress the "materiality" of his geometry because he does not consider parts of nature insensible to the naked eye as *objects* hence as subjects of painting.[368] Although this permits the painter to "represent the dead to the living many centuries later",[369] it still allows *only* sensible representation, not the materialisation of imaginary "invisible elements"[370] such as particles, for instance. Hooke's practical geometry is designed for natural philosophising, and is dynamic because rest is "*quite* contrary *to*

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365 Wilson, "Who invented the calculus?–and other 17th century topics", online Gresham College lecture recording, 1:03:01, 16th November 2005. <http://www.gresham.ac.uk/lectures-and-events/who-invented-the-calculus-and-other-17th-century-topics>.

366 As discussed in Chapter 4, works on practical geometry were written primarily for the education of artisans such as architects, engineers and surveyors. Unlike Hooke's stipulative, physicalised definitions, these textbooks rely on the Euclidian definition for the fundamentals: that is, point, line, superficies and solid. This creates an unresolved contradiction, a crack between theory and practice. Hooke resolves this problem by treating practical and theoretical geometry as discrete subjects dealing with different matters of natural philosophy: the former extracts mathematics from physics; the latter assists the imagination in moving beyond sense limits and generalising the practical to natural laws.

367 Leon Battista Alberti, *On Painting*, trans. Rocco Sinisgalli (Cambridge/New York: Cambridge University Press, 2011), l. 446. Italics added.

368 Alberti, *On Painting*, l. 446.

369 Alberti, *On Painting*, l. 925.

370 Chen-Morris, *Measuring Shadows*, 105.

the grand Oeconomy of the Universe";[371] his points, which represent all globular bodies from particles to planets, belong to this physical reality. The seeming geometrical comradeship between Alberti and Hooke cracks with the former's definitions for point and line:

> Before anything else, therefore, one must have understood that the point is a sign, so to speak, *that in no way can be divided into parts*. By sign, here I mean anything that rests on a surface so that it can be observed by the eye … The points will certainly make a line if they are joined without interruption, according to a sequence. Consequently, for us [painters], the line will be a sign, the length of which is certainly possible to divide into parts, *but [its] width will be so thin that it [the width] can never be divided*.[372]

Thus, although Alberti's attempt is similar to Hooke's, the latter's practical geometry is crucially different at a fundamental level: Hooke's points and lines *are* infinitesimally divisible. And although Alberti and Hooke agree on the notion that observation of nature ought to precede imagining nature, in his Basel version of *De Pictura* (the final 1540 Latin draft),[373] Alberti states in his prologue to Filippo Brunelleschi that the first book is "entirely on mathematics, [and] causes this pleasant and most noble art [painting] to spring from its roots in Nature".[374] This declaration, that the art of painting has "roots in Nature" owing to the *underlying* mathematics shared by both, has more in common with Kepler's harmonies of the world than Hooke's approximations, pulled from physics and dependent on the power, accuracy and precision of artificial instruments.

Indeed, Hooke's practical geometry creates a tense dichotomy that is both caused and resolved by approximation. To recapitulate, on the one hand, Hooke is disappointed by the limitations imposed upon him by technology, and by extension by his understanding that both practical and theoretical tools are approximations; on the other hand, Hooke is also excited at the prospect of building a microscope powerful enough to reduce nature to the rudeness of art, to reveal that its sharp lines

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[371] Hooke, *Micrographia*, 16.
[372] Alberti, *On Painting*, l. 446. Italics added.
[373] Rocco Sinisgalli, "Introduction", in Alberti, *On Painting*, l.182.
[374] Alberti, *On Painting*, l. 426.

are rugged as razors; additionally, translating insensible knowledge back to human-sized limits is necessary if it is to have utility. In other words, knowledge of nature must necessarily remain a human approximation if it is to be of practical and intellectual value to us. For example, and as discussed in Part I, enforcing an ordered musical scale onto the relatively novel idea of vibrational frequencies creates a ladder of pitches and consonances capable of descending into infrasound and ascending into ultrasound silences while at the same time remaining grounded by the limited keyboard of a harpsichord and its twelve notes.

Hooke teaches from as early as 1665 that not only should the practical precede the speculative, but also that practical geometry is a better representation of art and nature than speculative geometry. In his 1685 *Lectures concerning Navigation and Astronomy*, Hooke explains this by asking his audience to consider the line, which he defines as "not a length without bredth, as in pure and speculative Geometry, but a length that hath the least sensible bredth that can be describ'd, such as a Line drawn with the point of a very sharp Needle"[375] – such as the very sharp Needle in the beginning of the *Micrographia*. Hooke next instructs his audience to consider the simplest line: "the shortest that can be drawn between two Points",[376] to pronounce how straight lines are "taken for granted" in speculative geometry, exemplifying the ease of defining them theoretically vs. the difficulty of drawing straight lines practically in art and in nature. He complains that

> in speculative Geometry, 'tis put for a *Postulatum*, that such a Line may be suppos'd drawn, or is easy to be drawn; but in practical Geometry we must consider of the means how to draw it actually, which in some cases is not so easily perform'd, if extraordinary truth and exactness be requir'd.[377]

Rulers bend, needles blunt, the human hand often fails to follow a perfectly straight line. And it is next-to-impossible to find a perfect plane to, for example, construct an 'ideal' ruler. Gravity and the air also bend lines into curves: evoking an image of Galileo's chain line drawing, a catenary curve which Galileo incorrectly devised as a

[375] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 521.
[376] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 521.
[377] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 521.

way to draw parabolas for studies of projectile motion, and "(which has of late Years much exercised the Speculative Geometers to contemplate, and they have given it the name of the *catenaria*)",[378] Hooke shows that "streigning a fine Wire, or Hair, or fine Silken Clew between two Points" significantly also does not describe a straight line, because even though a "Hair, or fine Silken Clew" may be considered virtually weightless, on account of its own physicality

> such a Line can never be streign'd strait whatever strength it be streign'd withal; for its own weight shall make it bend down in the middle, as has been sufficiently demonstrated by the ingenious *Galileo*, and *Mersennus*, and divers others; especially if there be any considerable distance between the two Points.[379]

Finally, Hooke generalises the particular scenario of the silk line pinned between two points by applying his arguments concerning it to the 'straightest' line observable in nature: light.

> [F]or not again to mention the bending of Rulers or Line, which 'tis impossible to prevent, even the sight itself, that is the Ray of Light, passing from Point to Point through the Air, is not a strait Line as to its Position, by reason of the differing Refraction which is in the Medium of the Air, which I my self have very often prov'd by Observation, finding the same three Points [of position used to measure changes over time,] which appear at one time in a straight Line, at another time, sometimes within half an hour, have appear'd out of it very considerably, which I have very often diligently remark'd.[380]

Light has no truly homogeneous medium through which to propagate;[381] thus, just

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[378] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 531. Gal and Chen-Morris, *Baroque Science*, 128; Tito M Tonietti, *And yet it is heard, Vol. 2* (Basel: Birkhäuser, 2014), 215–216.

[379] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 521.

[380] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 522. For an illustration of Hooke's points of position, see "Fig: 1" in the 37th scheme of the *Micrographia*, which pertains to Observ. LVIII "*Of a new Property in the* Air, *and several other transparent* Mediums *nam'd* Inflection …" In Observ. LVIII, Hooke superimposes imaginary points and lines over a drawing of an experimental apparatus – a glass tank filled with a supersaturated and gradational solution of brine – in order to refract a sunbeam and create an artificial environment of atmospheric refraction. The points in "Fig: 1", similar to the points of position here, serve as locations for comparison between the inflected physical ray and an imaginary rectilinear pricked line above it, which represents how the ray would have appeared without being refracted in the atmosphere. Relative to the imaginary line of the incident ray, the inflected ray proves bent, satisfying the requirements of Hooke's "ocular demonstration" that "the parts of the *medium* being continually more dense the neerer they were to the bottom, the Ray … was continually more and more deflected downwards from the streight line" (Hooke, *Micrographia*, 220).

[381] See also, for example, Hooke, *Micrographia*, 56–7, 220 and 228.

as lines of art prove bent or rough when viewed with a powerful enough instrument, and just as this can be mimicked macroscopically with chain lines, there are no straight lines in nature; and speculative geometry is an abstraction and hence cannot represent physical reality more precisely, which recall was Proclus's claim. This conclusion relieves Hooke of some of his anxieties about the technological limits of artificial instruments as mediators between nature and 'human approximations'[382] enforced upon it, because nature is not in lines drawn between two points; nature is approximations.

6. SUPERFICIES

A point is a body with latitude and longitude. A line is "a length that hath the least sensible bredth that can be describ'd". In addition, "innumerable Lines do make a Mathematical Superficies"[383] in Hooke's speculative geometry, so lines with breadth and depth make a physical surface in his practical geometry. The first superficies that Hooke presents as his third observation in the *Micrographia* is "fine Lawn, or Linnen Cloth" – "another product of Art".[384] He notes that "the threads were *scarce discernible by the naked eye*", but more importantly how "an *ordinary Microscope*" exposes the proportionality of the threads in the lawn's warp and weft, which when magnified look like ropes: "what *proportionable* cords each of its threads are, being not unlike, both in shape and size, the bigger and coarser kind of single Rope-yarn, wherewith they usually make Cables."[385] A second feature that Hooke confirms is the cause of the lawn's diaphanous appearance: a "multitude of square holes which are left between the threads," which appear "to have much more hole in respect of the intercurrent parts" – like "a *lattice-window*, which it does a little

[382] Also see Gal and Chen-Morris, *Baroque Science*, Part II.
[383] Hooke, *The Method of Improving* Natural Philosophy, in *Posthumous Works*, 66–67.
[384] Hooke, *Micrographia*, 5.
[385] Hooke, *Micrographia*, 5. Italics added.

resemble, onely the crossing parts are round and not flat."[386] These two details, the "proportionable cords" and the "holes" in the weaving, preview the various surfaces studied in the *Micrographia*; namely, bodies and their parts and pores. When Hooke compares lawn flax with natural silk one page later, he declares that he can probably find a way to make artificial silk that is even better than nature's,[387] despite his disappointment with artificial superficies, which "when view'd with a Microscope, there is little else observable, but their deformity".[388] This frustration with things that are "design'd for no higher a use, then what we [are] able to view with our naked eye"[389] further fuels Hooke's "radical instrumentalism",[390] for recall that if one *could* build microscopes "according to the theory of them", then one would perhaps be able to – in this case – reduce natural silk to the 'rudeness' of lawn. Better instruments also allow for a more accurate and precise practical geometry, and the proportionality that Hooke notices and pays attention to when viewing the magnified warp and weft and comparing it to "Rope-yarn", the structure of which is easily noticeable with the naked eye, further helps him to develop his geometry. First, it shows him that one 'rope' may be substituted for the other – so long as the proportionality holds. Although Hooke has no need to do that here, the idea serves him well in a related, later observation, which I will analyse in the subsequent section, when he is forced to swap 'flint' for larger 'Cornish Diamants' because the former are too tiny to be viewed with ease under any of his microscopes. Second, the comparison to a "*lattice-window*" is no throwaway simile. Not only does the warp and weft "resemble" the crisscrossing of leadlight cames (the lead frames that hold panes together), but the points of crossing form a geometric quincunx pattern (four points forming a quadrilateral with a fifth point in its centre), which Hooke employs for particles in his studies of refraction and colour (*Figure 5.3*, "Fig: 7"):

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386 Hooke, *Micrographia*, 5.
387 Hooke, *Micrographia*, 7.
388 Hooke, *Micrographia*, 8.
389 Hooke, *Micrographia*, 8.
390 Gal and Chen-Morris, *Baroque Science*, 203.

> [The particles] (whether round, or some other determinate Figure is little to our purpose) are first of a determinate and equal bulk … [and] are rang'd into the form of *Quincunx*, or *Equilaterotriangular* order, which that probably they are so, and why they are so, I shall elsewhere endeavour to shew.[391]

By "elsewhere", Hooke means 'Observ. XIII", ten levels up in complexity from lawn cloth, where he examines the structure of the 'flints' and 'Cornish diamants' I mentioned above, and puts his interest in proportionality to intellectually profitable use. Indeed, examining the *Micrographia* through lenses of geometry reveals the forethought of an epistemological ladder, the meticulousness with which the observations are structured, and the consistency in their variety, owing to Hooke linking everything to his matter theory.

SCALE

Several observations after the 'lattice windows' of lawn, Hooke moves from examining discrete points to figures composed of the coagulation of several congruous points. Namely, substances. This lifts his observations up a metaphysical level to the inanimate natural bodies in "Observ. XIII *Of the small* Diamants, *or* Sparks *in* Flints", and further illustrates some basic mathematical operations embedded in his practices. The association between lawn and crystals is intentional: the ordered motions traced by the warp and weft of weaving in linen or silk surfaces share a physical hence geometrical similitude with the order in the "Diamants" and "Flints" – both are "rang'd into the form of [a] *Quincunx*, or *Equilaterotriangular* order".[392] In between observations on lawn and Diamants, Hooke explicated a theory for how the point or globular body "proceeded from a propriety of fluid bodies, which I have call'd *Congruity*, or *Incongruity*". I analysed this in detail in Part I; however, it is worth repeating that a "body encompast with a *Heterogeneous* fluid must be protruded into a *spherule* or *Globe*"[393] because of the incongruous "fluid

[391] Hooke, *Micrographia*, 68.
[392] Hooke. *Micrographia*, 68.
[393] Hooke, *Micrographia*, 85.

forcing equally against every side of it"[394]. Here, in Observ. XIII, Hooke examines the lattices or "texture" of quartz crystals, specifically "the regularity of their *Figure*", which he claims "is the most worthy, and next in order to be considered after the contemplation of the *Globular Figure*". So, next to the globular figure or point, "the most simple principle that any kind of form can come from", arise 'flints' and 'diamants', and

> only from three or four several positions or postures of *Globular* particles, and those the most plain, obvious, and necessary conjunctions of such figur'd particles that are possible …[395]

In other words, Hooke claims that all crystals of this kind can be built up from several of "the most plain" lattices of globular bodies. He will support this mechanically and geometrically.

Now, the title of this observation is somewhat misleading, as Hooke admits in his third introductory paragraph. The observation is not exactly of flint, because the crystals are too tiny even for his compound microscope, so he substitutes them with bigger "*Cornish Diamants* [Cornwall quartz]:"

> these being very pellucid, and growing in a hollow cavity of a Rock … much after the same manner as these do in the Flint; and having besides their outward surface very regularly shap'd, retaining very near the same Figures with some of those I observ'd in the other, became a convenient help to me for the Examination of the proprieties of those kinds of bodies.[396]

Again, Hooke's explanation for the substitution implies the imperative presence of proportionality: that is, that "Flint" crystals and "Cornish diamants" possess geometrical similarity, "having besides their outward surface very regularly shap'd, retaining very near the same Figures with some of those I observ'd in the other". This similarity also allows Hooke to replace the *particles* of flint or quartz with "bullets" in a mechanical model designed to demonstrate the claim that these particular forms "arise only from three or four several positions or postures of *Globular* particles, and those the most plain". To make the mechanical model, Hooke

[394] Hooke, *Micrographia*, 17.
[395] Hooke, *Micrographia*, 85.
[396] Hooke, *Micrographia*, 82.

appropriates and adapts parts of a semi-thought-experiment suggested by Kepler in *Strena, seu De nive sexangula*, translated as *The Six-Cornered Snowflake* (1611).[397]

In the *Snowflake*, Kepler studies, amongst other things, beehives, pomegranate seeds, mineral crystals and close-packed spheres. He composed the treatise as "a most desirable New Year's gift for the lover of Nothing" – his Epicurean[398] friend and patron Wacker von Wackenfels, and devotes most of it trying to convince Wackenfels, also his interlocutor, that "the material is certainly not a factor" of the cause of a snowflake's form.[399] Likewise, upon noting that small pomegranate seeds are round, and become rhombi only when squashed together for lack of room, Kepler argues that this shape-shift is due to "material necessity" and not a "formal property" hence it cannot be the "real cause of the shape", which according to him must necessarily be a formal property.[400] Similarly, with his studies of beehives, even though "the [hexagonal] archetype was imprinted upon it [the bee] by the creator", the hive and its individual cells owe their structure to utility, "because straight frames are stronger" and so on.[401] Although Kepler's work is another way to put mathematics into physics, from his claim about the material it is obvious that Kepler and Hooke have differing worldviews, and thus they represent different approaches to the relations between mathematics and physics. Indeed, Kepler's explanation for the regularly repeating patterns of quartz crystals is their "plan", "formative faculty", or "archetype"; Kepler insists, just as he would eight years later in *Harmonices Mundi*, that geometry is "coeternal with God".

> In the first place, the entire category of souls is kindred to the regular geometric figures from which the universe is constructed, as can be shown by many examples. For since souls are, one might say, likenesses of God the Creator,

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[397] Johannes Kepler, *The Six-Cornered Snowflake*, trans. Jacques Bromberg (Philadelphia: Paul Dry Books, 2010 [1611]); Johannes Kepler, *The Six-Cornered Snowflake*, trans. Colin Hardie (Oxford: The Clarendon Press, 1966 [1611]). I cross-reference both translations, though I cite the Paul Dry Books first edition.
[398] I am indebted to Raz Chen-Morris for this insight.
[399] Kepler, *The Six-Cornered Snowflake*, 33, 49.
[400] Kepler, *The Six-Cornered Snowflake*, 53.
[401] Kepler, *The Six-Cornered Snowflake*, 61, 63.

> assuredly the truth of these figures exists in the mind of God the Creator and is coeternal with Him.[402]

He rejects that *physical* microstructure causes the naked eye appearance of snowflakes and so on, and explains away the problem of "material necessity" by claiming that a drop, a seed, or a crystal's "purpose" must have been taken into account when deciding its geometry. In other words, a predetermined "plan".[403] Kepler ends his snowflake ruminations on a humorously melancholic note. After first mentioning the element of earth in the beginning, he diverts to fire, air and water in turn (with an interlude on animals), expanding on earth and mineral crystals in closing. This is because "Rock crystal [quartz], for example, is always hexagonal … But the formative faculty of the earth does not embrace one figure: It is practiced and well-versed in the whole of geometry."[404] By concluding with "the whole of geometry", Kepler risks making everything of nothing; that is, by ending on "the whole of geometry", "[he has] very nearly recreated the entire universe, which contains everything!", and skirts on the edge of gifting Wacker von Wackenfels with everything instead of nothing.[405]

In his own meditations on frozen figures, Hooke marvels at the "infinite variety" of snowflakes, and states "that it would be as impossible to draw the Figure and shape of every one of them, as to imitate exactly the curious and Geometrical *Mechanisme* of Nature in any one";[406] that is, material causes, and mechanical 'rules'. Unlike Kepler, for whom geometry is "coeternal with God", Hooke's instruments reveal that geometry is a tool, a cultural product created before the invention of optical instruments, limited by the senses. In Hooke's metaphysics, geometry is not 'coeternal with God', but is akin to naked eye astronomy, or observations restricted by the eye's incapacity to resolve parts. Thus, in the *Snowflake*, physics depends upon geometry; in the *Micrographia*, geometry depends upon physics. Furthermore,

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402 Kepler, *The Six-Cornered Snowflake*, 95.
403 Kepler, *The Six-Cornered Snowflake*, 61, 89.
404 Kepler, *The Six-Cornered Snowflake*, 112.
405 Kepler, *The Six-Cornered Snowflake*, 99.
406 Hooke, *Micrographia*, 91.

the former employs what Hooke and his peers call 'speculative geometry', and the latter, a geometry with practical origins. Finally, Kepler's worry – to show that mathematics can be trusted to provide causal explanations – is not a worry shared by Hooke.[407] Thus, by recasting Kepler's experiment in a flipped mould of mathematics lifted off the surfaces of physics, Hooke forces his readers to see the world anew again; yet in drawing the mathematical always from the physical, he nevertheless legitimises the application of mathematics to the study of nature by interweaving crystals with artificial models and geometry, implying that all are fundamentally related.

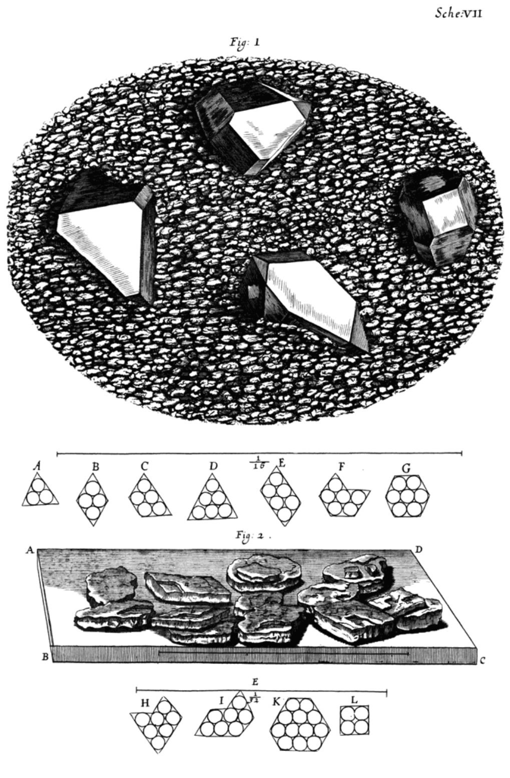

Figure 7.1 Hooke's drawings of microscopic crystals and their mean forms and lattice structures under scale bars *(Micrographia).*

The way that Hooke uses Kepler's experiment as a resource is telling because he modifies it to suit his different approach to the relationship between mathematics and physics. Following his beehive studies, Kepler begins to experiment with soft beads, which for him represent the smallest part of a substance, its "element", "just as water has a smallest natural part, which is the drop".[408]

> And if someone were to take many round little beads of equal size and of the same soft material, put them in a round vessel, and begin to compress it from all sides with bronze rings, many of the beads would be squeezed into a rhombic shape, especially if by carefully shaking the container you first allowed them to settle into narrower spaces by their own free rotation.[409]

In addition, Kepler finds that "spheres of equal size will arrange themselves in one of two ways when placed in a container, corresponding to the two ways [triangular

[407] For a thorough analysis, and reappraisal, of Kepler's optical geometry and metaphysics, see Chen-Morris, *Measuring Shadows*.
[408] Kepler, *The Six-Cornered Snowflake*, 77.
[409] Kepler, *The Six-Cornered Snowflake*, 53. Italics added.

and square] in which they can be arranged on a plane".[410] That is, his beads illustrate the differences between cubic and hexagonal packing, and that hexagonal packing provides the tightest arrangement possible – even though he makes 'nothing' of the matter.[411] Hooke, taking Kepler's remark on "someone" to heart, but neglecting to cite him, sets up a revamped reconstruction, rolling congruent bullets down the inside of "a round vessel" to model how congruous particles attract to form shapes imitating the "outward surface[s]" or faces of flints and Cornish diamants. He wants to see whether he will obtain the same faces and interfacial angles. "I have *ad oculum* demonstrated with a company of bullets …"[412]

> so that there was not any regular Figure, which I have hitherto met withal, of any of those bodies that I have above named, that I could not with the composition of bullets or globules … imitate, even almost by shaking them together."[413]

'Shaking them together' provides another clue that Hooke's bullets were rolled inside "a round vessel", even though he neglects to say, and that he picked this tip up from Kepler who advises "carefully shaking the container" after first allowing the bullets "to settle into narrower spaces by their own free rotation". "And thus for instance we may find," observes Hooke,

> that the *Globular* bullets will of themselves, if put on an inclining plane, so that they may run together, naturally run into a *triangular* order, composing all the variety of figures that can be imagin'd to be made out of æ*quilateral triangles* …[414]

The results of Hooke's experiment with "*Globular* bullets" confirm Kepler's conjecture, not by 'compressing' the "round vessel" "from all sides with bronze rings", but from a "company of bullets" "*naturally*" running into "all the variety of figures that can be imagin'd to be made out of æ*quilateral triangles*".

The first lattice is the "*æquilatero-triangular* form" labelled "A" in Hooke's drawing (*Figure 7.1*), represented by three equal circles or globular bodies inscribed

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[410] Kepler, *The Six-Cornered Snowflake*, 55.
[411] Kepler, *The Six-Cornered Snowflake,* 45, 57. This observation on the density of packed equal spheres is nowadays known as the 'Kepler conjecture'.
[412] Hooke, *Micrographia*, 85.
[413] Hooke, *Micrographia*, 85.
[414] Hooke, *Micrographia*, 85.

in an equilateral triangle. Hooke provides descriptive geometrical details for all seven lattices. For example,

> [i]f a fifth [globule] be joyn'd to them on either side in as close a position as it can, which is the propriety of the *Texture*, it makes a *Trapezium*, or four-sided Figure, two of whose angles are 120, and two 60. degrees, as C. If a sixth be added, as before, either it makes an *aequilateral triangle*, as D, or a Rhomboeid, as E, or an *Hex-angular Figure*, as F, which is compos'd of two *primary Rhombes*.[415]

Moreover, he has been taking for granted his new theoretical tool of 'congruity and incongruity' since Observ. VI, and he reinforces it here in this controlled trial: both "shaking [the bullets] together" and their 'running' together represent congruity. This, as well as rolling balls down an inclined plane – a practice immortalised by Galileo during his studies of free-falling bodies – is not surprising since Hooke's second query towards the end of Observ. VI was whether gravity might not be explained by congruity.[416] Finally, because the bullets 'attract' in imitation of the "outward surface[s]" of quartz, by visually representing his results, Hooke can measure them with a new device in microscopy – a scale bar.

He provides hand-drawn realistic micrographs of the 'diamants',[417] followed by an innovative abstraction: seven close-packed lattices or superficies under a scale bar, the mean of multiple "trials".[418] By adding a scale bar to his hand drawn micrographs and lattices, Hooke again employs the geometrical concepts of congruence and similarity in a powerful way. He uses similar figures to forge links of proportionality between the micrographs and scale bar, as well as the scale model of rolling bullets, to theorise how congruent crystal 'parts' compound. Like Hooke's

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[415] Hooke, *Micrographia*, 85–6.

[416] Hooke, *Micrographia*, 22.

[417] Scheme VII (*Figure 7.1*) is typical of what would become Hooke's preferred layout of conveying visual information in the next decade, during his *Cutlerian Lectures*, which I previously described with a comparison of his main plates from *Cometa* and *Of Spring*. That is, beginning with naked eye observations, followed by replacing the eye with instruments, ending with a reduction to physico-geometrical abstraction. However, here, owing to the small size of the 'Cornish diamants', a naked eye representation would be unnecessary.

[418] See Figure 0-2 in the Appendix for the results from my reconstruction of this experiment. For an alternative interpretation, with a very similar reconstruction, see also Matthew C. Hunter, "Experiment, Theory, Representation: Robert Hooke's Material Models." In: R. Frigg and M. Hunter (eds), *Beyond Mimesis and Convention (*Dordrecht: Springer-Verlag, 2010).

mechanical model, the scale bar builds a relation from the microscopic to the macroscopic that re-involves the senses: it allows for the measurement of microscopic crystal shapes drawn around ‘parts’ found by rolling bullets down an ‘inclined plane’. More important, it turns Hooke’s diagrams, of how many equal circles or spheres are necessary to construct the equilateral triangle “D” and so on, into forms with physicality, which can be measured as not only wholes, but also divisible and multipliable aggregates.

For example, in the interrelated observation preceding “Cornish diamants”, upon examining “for the most part flat” urine crystals (*Figure 7.1*, “Fig: 2”), Hooke geometrically reconstructs their four mean forms under a scale bar (“line E”) and states: “The line E which was the measure of the Microscope [the diameter of the microscope’s field of view], is 1/32 part of an English Inch, so that the greatest bredth of any of them [the urine crystals], exceeded not 1/128 part of an Inch.”[419] Apart from providing measurements of the crystals, left unsaid is that the scale bar allows one to measure the radius of the hypothetical bodies too. To obtain the mean measurement of a urine crystal’s width, or a Cornish diamant’s face, one would simply divide the length of the scale bar by the number of crystals represented under it. The scale bar for the seven “diamants” in *Figure 7.1* measures 1/16 of an inch across, and thus the “greatest bredth” of any diamant face is roughly 1/112 of an inch (approximately 200 microns). Using this estimate, one could work out the radius of a point in a diamant lattice by knowing the properties of equilateral triangles, of 30-60-90 triangles, and the Pythagorean theorem.[420] But the most important part of these considerations is that by first measuring the dimensions of a quartz face, and then working out the fewest number of ‘points’ necessary for close-packing according to the dimensions and face shape, Hooke’s conception of practical geometry allows him to employ proportionality to scale down into the insensible

[419] Hooke, *Micrographia*, 81.

[420] For one way to approach this, reverse engineer the construction of equal circles drawn inside an equilateral triangle in “Quest. 45” of Rudd, *Practical Geometry*, 63–67.

realm of a particle or up into the sensible realm of a round bullet or even a planet.

To recapitulate, for his observations of "Cornish diamants" (stand-ins for "flints"), taking 'congruity and incongruity' as a self-evident explanation, and replacing his ruler and compasses with a microscope and a scale bar, Hooke attempts to show that the practical geometry lifted off these particular crystal facets is contingent upon the microstructure of vibrating matter, which produces these particular shapes of substances. Furthermore, because of a chain of proportionality, and the constancy of interfacial angles (Steno's law nowadays) represented by the bounding lines of "angles of which will be either 60. Degrees, or 120", Hooke can make predictions about minute bodies with mechanical models on a macro scale.[421] Hooke explains it thus:

> And though there be never so many [globules] placed together, they may be range'd into some of these lately mentiond Figures, all the angles of which will be either 60. Degrees, or 120. as the figure K. [*Figure 7.1*, "Fig: 2"] which is an *aequiangular hexagonal* Figure is compounded of 12. *Globules*, or may be of 25, or 27, or 36, or 42 *&c.* and by these kinds of texture, or position of globular bodies, may you find out all the variety of regular shapes …[422]

That is, "all the angles" between the faces "will be either 60 Degrees or 120" degrees, because of "the position of the globular bodies", or how the "Globules" are "compounded" into a "texture" (lattice), irrespective of size, which might be "25, or 27, or 36" or greater globules; and since this is constancy of angles is a defining characteristic of crystals of this kind regardless of size and growth, whether tiny "flints" or bigger "Cornish diamants" and so on, one may "find out all the variety of regular shapes".

Further, like Thomas Harriot's cannon balls and Kepler's beads, Hooke concludes "it's obvious" that stacking globular bodies with respect to the angles mentioned above turns a superficies into a solid:

[421] For a comparison with Steno, see Drake, "The Geological Observations of Robert Hooke (1635–1703) on the Isle of Wight", *Geological Society, London, Special Publications*, 287, 19-30, 1 January 2007, 26; for Hooke and crystallography in general, see, Hammond, *The Basics of Crystallography and Diffraction*, 3rd ed. (Oxford: Oxford University Press, 2009).

[422] Hooke, *Micrographia*, 85–6.

> nor does it hold only in superficies, but in solidity also, for it's obvious that a fourth Globule laid upon the third in this texture, composes a regular Tetrahedron, which is a very usual Figure of the *Crystals* of *Alum*. And (to hasten) there is no one Figure into which *Alum* is observ'd to be crystallized, but may by this texture of Globules be imitated, and by no other".[423]

Finally, just as Kepler comes close to creating everything from nothing, Hooke boasts that had he enough leisure time on hand for further observations and experiments, he

> could instance also in the Figure of *Sea-salt*, and *Sal-gem*, that it is compos'd of a texture of *Globules*, placed in a *cubical* form, as L, and that all the Figures of those Salts may be imitated by this texture of *Globules* and by no other whatsoever. And that the forms of *Vitriol* and of *Salt-Peter*, as also of *Crystal*, *Hore-frost*, &c. are compounded of these two textures, but modulated by certain proprieties …[424]

SECTIONS

Hooke's studies of superficies in the *Micrographia* are a crucial step in his geometrical representation of springy bodies and his demonstration of the spring law in *Of Spring* – in particular, and perhaps surprisingly, his observations on cork. Not content with nature's surface appearances, and wanting to understand the function of various "pores" as intimately as bodies – for example, to explain how pores play an important role in the internal motions of the parts of bodies – Hooke turns to creating new artificial surfaces of nature by sectioning and fracturing stuff such as petrified wood, charcoal, fossils and cork.

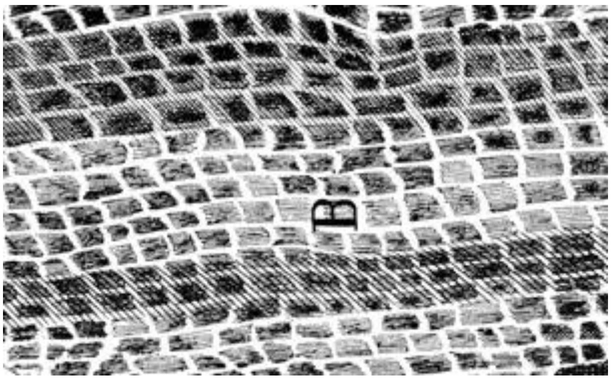

Figure 7.2 A detail of Hooke's drawing of cork cells turned anticlockwise to show how closely his drawing of cork *(Micrographia)* resembles his diagram of an eight-particle springy body captured in stages of equilibrium, compression and rarefaction *(Figure 7.3)* in *Of Spring*

> I took a good clear piece of Cork, and with a Pen-knife sharpen'd as keen as a Razor, I cut a piece of it off, and thereby left the surface of it exceeding smooth …

[423] Hooke, *Micrographia*, 85–6.
[424] Hooke, *Micrographia*, 86.

> and casting the light on it with a deep *plano-convex Glass*, I could exceeding plainly perceive it to be all perforated and porous …[425]

"Observ. XVIII. *Of the* Schematisme *or* Texture *of* Cork, *and of the Cells and Pores of some other such frothy Bodies*" is one of the most famous observations in the *Micrographia*, mostly because it is where Hooke allegedly coins the word "cell", though it is not his first mention of cells. Since the observations are ordered by growing physical hence geometrical complexity rather than chronologically, though Hooke mentions "cells or Boxes"[426] in the seventh observation, and compares a petrified shell's (fossil) "diaphrams or partitions" to "a multitude of very proportionate regular cells or caverns" in the seventeenth observation,[427] in Observ. XVIII he states that the cells or "pores" of cork were his first: "I no sooner discern'd these … which were indeed the first *microscopical* pores I ever saw, and perhaps, that were ever seen …"[428] Hooke also notes that cork cells are similar to honeycomb cells in structure,

> in that these pores, or cells, were not very deep, but consisted of a great many little Boxes, separated out of one continued long pore, by certain *Diaphragms,* as is visible by the Figure B [*Figure 7.2*], which represents a sight of those pores split the long-ways.[429]

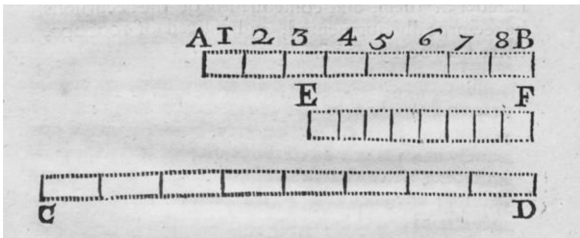

Figure 7.3 Hooke's diagram of an eight-particle springy body captured in stages of equilibrium, compression and rarefaction *(Of Spring).*

Examining his overall use of the word 'cell' provides a definition of what he means by it. A cell according to Hooke is any compartmentalisation that segments "one continued long pore" into proportional spaces. An example of a "long pore" is Hooke's microscopic glass capillary tubes from Observ. VI (discussed in Part I). Today's use of "cell" in histology has little in common with Hooke's meaning, or his observations of "*the*

425 Hooke, *Micrographia*, 112–3.
426 Hooke, *Micrographia,* 46.
427 Hooke, *Micrographia,* 111.
428 Hooke, *Micrographia,* 113.
429 Hooke, *Micrographia,* 113.

Schematisme *or* Texture *of* Cork", and thus only serves to mask his actual meaning. Recall that he uses the word "texture" for lattice, such as the crystal lattices examined in the previous section; that is, a particular geometrical arrangement of the structure of cork, which is contingent upon how the particles of cork compound. Moreover, the microscope "easily informs us" that the material structure of cork "consists of *an infinite company of small Boxes*" of springy air, which Hooke confirms by mechanically manipulating a piece of cork to test its elasticity, finding that with his "hands" alone he can compress it "into less then a twentieth part of its usual dimensions neer the Earth".[430]

> *Our Microscope will easily inform us*, that the whole mass consists of *an infinite company of small Boxes* or Bladders of Air, which is a substance of a springy nature, and that will suffer a considerable condensation (as I have several times found by divers trials, by which I have most evidently condens'd it into less then a twentieth part of its usual dimensions neer the Earth …) …[431]

Hooke's use of quasi-mathematical language in these descriptions is deliberate – for example, "proportionate regular cells", "proportional spaces", "*infinite company of small Boxes*". Cork is a natural spring of microscopic boxes, and sectioning it – creating new artificial surfaces of nature for study – gives Hooke a way to represent springy bodies geometrically, and a way to think about how and why to section a geometrical spring of 'infinite boxes' to calculate either discrete boxes or aggregates of boxes of power and so on.

By way of visual comparison, let us examine a detail from Hooke's hand-drawn micrograph of a longitudinal section of cork in the *Micrographia* (*Figure 7.2*) with his *Of Spring* constructions of an eight-particle springing body – "a line of such a body compounded of eight Vibrating particles" (*Figure 7.3*).[432] To summarise from Part I, in *Figure 7.3*, the line AB represents the body at equilibrium, vibrating 1,000,000 times per second; the line EF represents the body compressed, vibrating 1,500,000 times per second; the line CD represents the body extended, vibrating

[430] Hooke, *Micrographia,* 114.
[431] Hooke, *Micrographia,* 113–114.
[432] Hooke, *Micrographia*, Scheme XI between pages 114 and 115; Hooke, *Of Spring*, 13.

666,666 times per second. The vibrational frequencies are in intervals of a perfect fifth in each direction – 2/3 moving up a fifth in insensible pitch from AB to EF and 3/2 dropping down a fifth in insensible pitch from AB to CD – and the line lengths illustrate this harmonious ratio. In addition, Hooke designs *Figure 7.3* with two types of lines used in practical geometry: 1) finite apparent lines for the edges or interfaces A and B, E and F, and C and D (which are the same interfaces), and for the partitions between the boxes or springy particles; and 2) finite occult or pricked lines for the changing length of the eight-particle spring, which he ignores when explaining how the incongruous vibrations of the aether affect the vibrating solid. He ignores the long sides because the pricked lines indicate that the vibrating solid is an extracted part of a whole, as if he sliced out a long pore of cork from *Figure 7.2*, and so the longer sides are not 'real' physical edges.[433]

Thus, from the detail of the longitudinal section combined with Hooke's experiments on the spring of cork in the *Micrographia*, and his choice of lines when drawing the eight-particle diagrams in *Of Spring,* one can infer that the latter are "experimentally verify'd and exhibited"[434] representations of a springing strip of cork generalised to represent any vibrating solid captured in moments of equilibrium, compression, and extension. The ability of Hooke's geometry to capture nature in action with slices and sections is epitomised in his demonstration of the spring law, which I will expound upon next.

MIXT

Analysing Hooke's diagram of "a Body moved by a Spring" earlier (*Figure 7.5*, "*Fig 5"*), I showed how Hooke slices areas into infinitesimal sections to make better approximations, the trace of a projectile's trajectory becoming smoother and smoother with each section. Here I will explicate *"Fig 4"* from the same, main plate

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[433] For an explanation of the different types of lines used in practical geometry, see, for example, Le Clerc, *Practical Geometry*.

[434] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 525.

of *Of Spring* to study how and why Hooke delineates the spring law with his "Mixt geometry" – a term and technique that he picked up from his mentor John Wilkins, and then developed and refined. Hooke himself personified 'mixed' during his dual roles as the Society's Curator of Experiments and Gresham's geometry professor, and mixed geometry is an indispensible tool for his unique way of practicing natural philosophy. But before delving into Hooke's graph of the spring law, a slight detour on his technical and philosophical indebtedness to Wilkins is necessary to emphasise Hooke's improvements, for although Hooke's eight-particle springing body diagrams are in the style of Wilkins, his geometrical generalisation of the spring law is radically different.

Wilkins was Warden of Wadham College in Oxford. He recruited Hooke into "Warden Wilkins's club", a philosophical circle whose members would in the start of the 1660s begin to meet in Gresham College, forming the Royal Society.[435] The title page of Wilkins's *Mathematical Magick: or the Wonders That may be perform'd by Mechanical Geometry* boasts that it is on "mixed Mathematicks", "Being one of the most Easy, Pleasant, Useful (and yet most Neglected) Part of the Mathematicks", and "Not before treated of in this Language". By 'Magick' Wilkins means 'wonder', and *Magick* is comprised of two books: the first is on mechanical powers; the second, on mechanical motions. Wilkins further subdivides the subject matter into two kinds of mixed mathematics: "Rationall", "which treats of those principles, and fundamentall notions, which may concern these Mechanicall practices"; and "Cheirurgicall", or "the making of these instruments, and the exercising of such particular experiments". *Magick* concerns the "Rationall" kind of mixed mathematics – that is, the

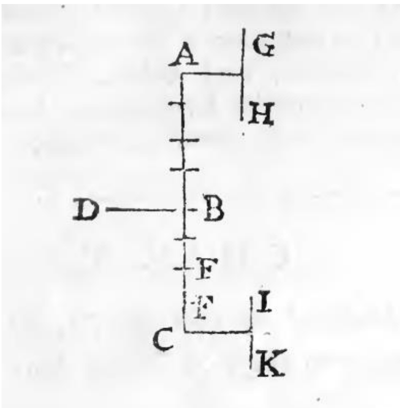

Figure 7.4 Wilkins's representation explaining the law of the lever for even weight distribution on a carriage *(Mathematical Magick).*

[435] A. Chapman, "Fly Me to the Moon", *Astronomy & Geophysics* 55, no. 1 (2014): 1.26-1.31, 1.31.

principles of mechanics.[436]

The fundamental distinction between Wilkins and Hooke is that the "Rationall" kind of mixed mathematics taught and practiced by Wilkins is concerned only with making already established natural laws intelligible via examples of everyday applications, whereas Hooke uses mixed geometry, both "Cheirurgicall" and "Rationall" to *create* laws of nature. Wilkins's work differs from other practical geometry manuals and 'textbooks' in that instead of beginning with the essentials of geometry, he first illustrates mechanisms, reduces them to geometrical constructions, and then shows how the geometry can be modified to accommodate a variety of similar practical problems: for example, using the law of the lever for even weight distribution when fastening horses or oxen to a carriage with a heavy burden (*Figure 7.4*).

> Let the line DB, represent the Pole or Carriage on which the burden is sustained, and the line AC, the crosse barre; at each of its extremities, there is a severall spring-tree GH, and IK, to which either horses or oxen may be fastned. Now because A, and C, are equally distant from the middle B, therefore in this case the strength must be equall on both sides … Whence it is easie to conceive how a husbandman … may proportion the labour of drawing according to the severall strength of his oxen.[437]

Hooke's indebtedness to Wilkins here is clear, and is also obvious when the latter launches into a diatribe against abstract mathematics, claiming that

> these Mechanicall disciplines, which in this respect are by so much to be preferred … by how much their end and power is more excellent. Nor are they therefore to bee esteemed lesse noble, because more practicall, since our best and most divine knowledge is intended for action, and those may justly be counted barren studies, which doe not conduce to practise as their proper end.[438]

Wilkins points an admonitory finger at "the ancient Mathematicians [who] did place all their learning in abstracted speculations, refusing to debase the principles of that noble profession unto Mechanicall experiments", and states that because of this obsession with "abstracted speculations" divorced from reality, "it came to passe

[436] John Wilkins, *Mathematical Magick*, in John Wilkins, *The Mathematical and Philosophical Works of the Right Reverend John Wilkins ...* (London: Printed for J. Nicholson, 1707 [1648]), 5.
[437] Wilkins, *Mathematical Magick*, Book I, in Wilkins, *The Mathematical and Philosophical Works*, 15.
[438] Wilkins, *Mathematical Magick*, Book I, in Wilkins, *The Mathematical and Philosophical Works*, 2.

that the science of Geometry was, so universally neglected, receiving little or no addition for many hundred years …"[439] Hooke's reversal, that speculative geometry is subservient to practical geometry, with his instruments that interweave art and nature into a fabric fit to clothe natural philosophy, is a radical reparation of this 'neglect' and more. Although Hooke learnt how to represent instruments and apparatuses geometrically from Wilkins, a crucial skill for Hooke's way of working a problem, this is where his indebtedness to his mentor ends.

Wilkins admits that *Magick* contains nothing new on mechanics or natural philosophy; he turns reductions of stick figure mechanisms into geometrical representations with manipulable variables – such as "how a husbandman … may proportion the labour of drawing according to the severall strength of his oxen" – to teach artificers and artisans geometrical reasoning and mathematical problem-solving.[440] Similarly, when Wilkins applies his geometry to natural-philosophical questions, it is not so much to develop something new as it is to support existing work. For instance, in his *The Discovery of a World in the Moone,* Wilkins represents suppositions about the moon along the lines of Kepler in *Somnium* and Galileo in *Sidereus Nuncius* with quasi-geometrical explanatory drawings, but makes few original observations and experiments.[441] By comparison, Hooke's "Mixt geometry" is a conflation of realistic drawings illustrating his observations and experiments, accompanied by practical and speculative geometry within the same diagram; and it is designed for the creation and dissemination of new natural knowledge; and, in *Of Spring*, a natural law.

PRELIMINARIES

[439] Wilkins, *Mathematical Magick*, Book I, in Wilkins, *The Mathematical and Philosophical Works*, 2–3.
[440] Wilkins, *Mathematical Magick*, Book I, in Wilkins, *The Mathematical and Philosophical Works*, epistle To the Reader, 15.
[441] Wilkins, *The Discovery of a World in the Moone*, in Wilkins, *The Mathematical and Philosophical Works.*

> It now remains that I shew how the constitutions of springy bodies being such, the Vibrations of a Spring, or a Body moved by a Spring, equally and uniformly shall be of equal duration whether they be greater or less.[442]

After constructing a framework on "the constitutions of springy bodies" such as cork – that is, Hooke's 'principles of congruity and incongruity' with harmony, dissonance and resonance as key explanatory devices[443] – his main aim here is to prove that any spring has vibrations of equal time independent of amplitude; and he does so with a geometrical demonstration, "*Fig 4*" (*Figure 7.5*). But for the readers to be able to comprehend "*Fig 4*", Hooke assumes that they are familiar with what experiments the illustrations on the left-hand side of the graph help to explain (*Fig's 1, 2* and *3*); the detailed accounts of experimental procedures preceding the demonstration, which "have here already shewed … that the power of all Springs is proportionate to the degree of flexure" or the spring constant; as well as a few mathematical preliminaries. In Hooke's diction, "degree of flexure" is synonymous with "space bended". In other words, all bodies bend, compress and extend proportionally to the applied external force. Thus "one degree of flexure, or one space bended hath one power, two hath two, and three hath three, and so forward".[444]

> From which it is very evident that the Rule or Law of Nature in every springing body is, that the force or power thereof to restore it self to its natural position is always proportionate to the Distance or space it is removed therefrom … Respect being had to the particular figures of the bodies bended, and that advantagious or disadvantagious ways of bending them.[445]

The relations that Hooke forms between experimental records, realistic drawings of sensible, mechanical springs, a practical-geometrical construct of an eight-particle vibrating line and so on under the theme of 'spring', are designed to support his "Rule or Law of Nature", priming his reader for the "Mixt" geometry

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[442] Hooke, *Of Spring*, 16.
[443] Hooke, *Of Spring*, 6.
[444] Hooke, *Of Spring*, 16.
[445] Hooke, *Of Spring*, 4. Scholars in the past have neglected to notice the final line in the citation, where Hooke mentions (because it is not important for his metaphysics) his awareness of an elastic limit. See, for example, E. Williams, "Hooke's Law and the Concept of the Elastic Limit", *Annals of Science* 12, no. 1 (1956): 74-83.

generalisation. He further expects that the reader accepts the critical principle that "every point of the space of flexure hath a peculiar power, and consequently there being infinite points of space, there must be infinite degrees of power". Notice that Hooke is referring to *any* springing body, so by "point" he means not only a location, but a physical 'part' of the springing body, such as the parts of the solid eight-particle line (*Figure 7.3*). Again, Hooke's mathematics is contingent upon the material world and its "infinite" parts.

Thus his first declaration based on the preliminaries is that

> all those powers beginning from nought, and ending at the last degree of tension or bending, added together into one sum, or aggregate, will be in duplicate proportion to the space bended or degree of flexure …[446]

That is, let P = "power", and s = space or length.

$P_0 + P_1 + P_2 + \ldots + P_n \propto s^2$.

Notice that power comes from an external source, whereas tension is the spring's strain. To expound, the total tension in a spring at whatever point of extension or length

> is equal, or in the same proportion to the square of one (supposing the said space infinitely divisible into the fractions of one;) to two, is equal, or in the same proportion to the square of two, that is four … and so forward …[447]

Just as he did in the *Micrographia* with an observation of a single point closely followed by observations of crystals compounded of an aggregate of points or globular bodies, Hooke builds up his explanation from a simple point of power to a more complex springing body with an infinite aggregate of points and their corresponding powers. Moreover, since he uses 'space' and 'length' interchangeably, by space Hooke means the physical space or length taken up by the spring. Therefore, "the sum of the first space will be one, of the second space, three, of the third space will be five … in Arithmetical proportion, being the degrees or excesses by which these aggregates exceed one another". So the sums of the spaces

[446] Hooke, *Of Spring*, 17.
[447] Hooke, *Of Spring*, 17.

follow the odd number rule made famous by the Merton calculatores from Hooke's alma mater Oxford, Nicole Oresme, and in particular Galileo after them. Briefly, about three centuries before Galileo, the Merton calculatores were interested in explaining any change itself, and applied this as a query to studies of the uniform and 'nonuniform' motion of bodies. The geometrical proof of the odd number rule, which is attributed to Oresme, bears striking similarity to Galileo's proof for his law of free-falling bodies on the Third Day in his *Two New Sciences* – though according to the literature, the Merton scholars never attributed uniform acceleration as a property of free-falling bodies.[448] Hooke refers to this "traditional Merton-style formula for the accumulation of 'degrees of motion'"[449] repeatedly as "the General Rule of Mechanicks"; that is, "the proportion of the strength or power of moving any Body is always in a duplicate proportion of the Velocity it receives from it;[450] however, the more important point to notice is that for Hooke it is "General" because it is contingent on physical reality and hence the mathematics stems from a physical hypothesis.

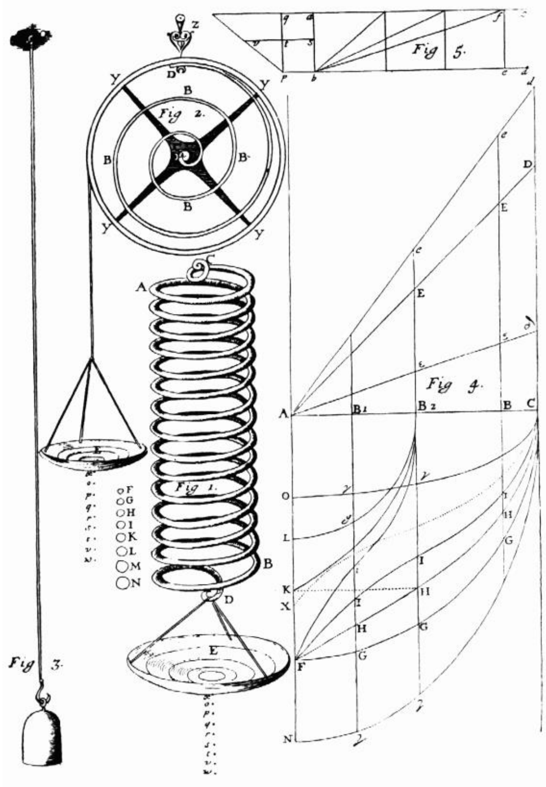

Figure 7.5 Hooke's demonstration of the spring law *(Of Spring).*

Hooke's second declaration based on the preliminaries concerns the "degrees of impulse" that the spring expends in its return from "any degree of flexure" to which it was "bent by any power". Again, Hooke begins with a single point, changing his diction from the pulses of "power" put in during stretching to

448 Galileo, *Dialogues Concerning the Two New Sciences*, 203–208; Michael J Crowe, *Mechanics from Aristotle to Einstein* (Santa Fe, NM: Green Lion Press, 2007), 12–14.
449 Gal, Meanest Foundations and Nobler Superstructures, 97, citing Hooke in Of Spring, 18–19.
450 See, for example, Hooke, *Lampas*, 32–33, in *Cutlerian Lectures*.

"impulse" in order to reflect the opposite motion of the spring released from tension, though the impulse at any point of the spring is equal to "the power of the Spring in that [physical] point of Tension". Next, considering the springing body as an aggregate of compounded parts or points, such as the vibrating eight-particle line restrained by the surrounding pressure of the incongruent aether, Hooke explains that the whole spring receives "the whole aggregate of all the forces belonging to the greatest degree of Tension from which it returned". Finally, combining impulses and spaces, Hooke explains that

> a Spring bent two spaces in its return receiveth four degrees of impulse, that is, three in the first space returning, and one in the second … So bent ten spaces it receives in its whole return one hundred degrees of impulse, to wit, nineteen in the first, seventeen in the second [etc.].[451]

This allows him to transform external power into internal tension released as impulse, and claim that the springing body's tension is proportional to its power, which he had published two years before in an encrypted anagram at the end of his Cutlerian lecture *Helioscopes* (1676).

> The true Theory of Elasticity or Springiness, and a particular Explication thereof in several Subjects in which it is to be found: And the way of computing the velocity of Bodies moved by them. ceiiinosssttuu.[452]

The unencrypted anagram spells out "ut tensio sic uis": as the tension so the force, and Hooke will fashion a force vs. distance graph, where distance equals the length of a stretched spring, to illustrate this concept.

To complete the necessary preliminaries before explaining the geometrical generalisation proper, Hooke turns his attention from power to velocity and isochronous vibrations, stating that

> the comparative Velocities of any body moved are in subduplicate proportion to the aggregates of sums of the powers by which it is moved, therefore the Velocities of the whole spaces returned are always in the same proportions with those spaces, they being both subduplicate to the powers, and consequently all the times shall be equal.[453]

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[451] Hooke, *Of Spring*, 17–8.
[452] Hooke, *Of Spring*, 1; Hooke, *Helioscopes*, 31, in *Cutlerian Lectures.*
[453] Hooke, *Of Spring*, 18.

That is, the power of the vibrating spring is proportional to velocity squared; hence, the square root of power is proportional to velocity; and the velocity of "the whole spaces returned" from ten spaces is $\sqrt{(19 + 17 + 15 + 13 + 11 + 9 + 7 + 5 + 3 + 1)} = 10$.[454] Reversing from wholes to parts, Hooke continues with his example of a spring "bent ten spaces", explaining that "the Velocities of the parts of the space returned … will always be proportionate to the roots of the aggregates of the powers impressed in every of these spaces;" for example, in the 9th space of pulsation, the velocity would equal $\sqrt{(19 + 17)} = \sqrt{36} = 6$.

> Now since the Velocity is in the same proportion to the root of the space, as the root of the space is to the time, it is easie to determine the particular time in which every one of these spaces are passed for dividing the spaces by the Velocities corresponding the quotients [proportions] give the particular times.[455]

In other words, an increase in stretch or displacement is proportional to an increase in the spring's velocity when released from strain, thus the spring's vibrations are isochronous and independent of amplitude. "[P]articular time" concludes the preliminaries necessary to produce a generalised "Theory of Springs". The fact that Hooke inextricably interweaves 'general rules of mechanics' with his new spring law reinforces his epistemological conviction that mathematics should be pulled from physics, since the 'rules' are related because of matter's uniformity. To "explain this more intelligibly", he unleashes "*Fig 4*" – his unique "Mixt" geometrical proof.

POWER

The horizontal line AC, both a trace of the vibrating motion of the end of a physical spring and the displacement of the stretched spring itself, cuts the graph in half; power is represented or "exhibited" (Hooke's term for practical-geometrical constructions) by the top half; the bottom half represents velocity. Beginning with power as before, to show "an Image to represent the flexure and the powers, so as to

[454] Note that in *Of Spring*, '10' is written as '100', though this typo is corrected in the errata at the end of the book.
[455] Hooke, *Of Spring*, 18.

plainly solve and answer all Questions and Problems concerning them",[456]

> let A in the fourth figure represent the end of a Spring not bent, or at least counterpoised in that posture by a power fixed to it, and movable with it, draw a line ABC, and let it represent the way in which the end of the Spring by additional powers is to be moved …"[457]

Again, starting with a single point just as in the *Micrographia*, the point A in "*Fig 4*",[458] Hooke represents "the end of a Spring not" compressed, extended or "bent", or a spring "counterpoised in that posture by a power fixed to it". A readily available example of "a power fixed to it" is a weight, such as the proportional weights represented by the circles F, G, H, I, K, L, M and N in the mechanical spring and balance of "*Fig 1*". Although Hooke does not refer to weights in his geometrical explanation of power, it is obvious by comparing *Fig.'s 1* and *4* that weights extend the mechanical springs in *Fig.'s 1, 2* and *3*, and as such that these schemes illustrate "Ut pondus sic tensio" (as the weight so the tension) whereas *Fig. 4* represents "Ut tensio sic vis". Most scholars ignore the former, focusing on the latter, but "Ut pondus sic tensio" is metaphysically as important as "Ut tensio sic vis", since it is both a mechanism and a theory for describing how congruent or similar parts added to a springing body increase its magnitude and slow its vibrations, but that is beyond the scope of this section.

Next, moving from a point to a line, Hooke draws the line AC, segmented by points B_1, B_2 and B respectively, which represent the spring stretched or "moved" "by additional powers".[459] He draws an ordinate CD orthogonal to AC, and lets it "represent the power that is sufficient to bend or move [stretch] the end of the Spring A to C".[460] Drawing a third line from point A to point D represents what would today be called the slope of the spring constant or linear relationship of force

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456 Hooke, *Of Spring*, 19.
457 Hooke, *Of Spring*, 18–19.
458 For alternative analyses of Fig. 4, see Gal, *Meanest Foundations and Nobler Superstructures*; and Bertoloni Meli, *Thinking with Objects*.
459 Hooke, *Of Spring*, 18–19. See Figure 0-1 in the Appendix for my reconstruction of "Fig 4", which shows possible construction lines not in Hooke's graph.
460 Hooke, *Of Spring*, 19.

CD vs. displacement AC, and forms the right-angled triangle ACD – a superficies. Hooke drops three ordinates from AD (EB_1, EB_2, and EB) that are parallel to CD and segment AC, explaining that

> from any point of the Line AC [the stretched spring] … the lines BE … represent the respective powers requisite to bend the end of the Spring A to B, which lines BE … CD will be in the same proportion with the length of the bent of the Spring AB … AC.[461]

In modern terminology, stress is proportional to the strain of the spring, and this is further demonstrated by the geometrical similarity of what Hooke calls the "lesser triangles" ABE, AB_2E, and so on, which are parts of the triangle ACD. Likewise, when the spring is "let go" from "any point [B] of the Line AC", it "will exert in its return to [point] A all those powers which are equal to the respective ordinates BE … the sum of all which make up the Triangles ABE". Moreover,

> the aggregate of the powers with which it returns from any point, as from C to any point of the space CA as to BB, is equal to the Trapezium [of whichever] CDEB … or the excesses of the greater Triangles above the less.[462]

That is, the areas of the right-angled triangles represent "the aggregate of all the Powers of the Spring bent from A to [whichever] B". So, sectioning the area of the triangle ACD with vertical ordinates provides power at any particular point of a spring's stretch or release from strain; adding the ordinates together into areas of "lesser triangles" provides an aggregate of power when the spring is stretched; and solving for the "Trapezium … or the excesses of the greater Triangles above the less" provides a sum of impulses when the spring is released from strain at point C to some particular point B.

This may sound like a switch from practical to speculative geometry within the same construction, because Hooke moves from the line AC, a real springing body, to abstractions of its power represented by perpendiculars for particular points and integrations of perpendiculars as triangles and trapeziums for aggregates of points or lines. But it is in fact a fine example of Hooke's mixed geometry, and his erection

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[461] Hooke, *Of Spring*, 19.
[462] Hooke, *Of Spring*, 19.

of theoretical entities from the structures and motions of matter with combinations of arithmetic and geometry, where mathematical lines drawn from infinite physical points can paint surfaces, allowing for the calculation of discrete or continuous powers and velocities. Hooke takes care to explain that "because the Spring hath in every Point of the line of bending AC, a particular power, therefore imagining infinite Lines drawn from every point of AC parallel to CD till they touch the Line AD, they will all of them fill and compose the triangle ACD".[463] First, the triangle ACD, a representation of the spring's total accumulated power at point C, is a mathematical surface. Secondly, Hooke's description of "infinite lines [which] fill and compose the triangle ACD" with vertical ordinates to calculate finer approximations of points of power along the vibrating spring resembles his definition of a speculative-geometrical 'superficies', for recall that "innumerable Lines do make a Mathematical Superficies … by supposing Motion joyn'd to them". Yet the triangle is erected from the springing body.

To summarise, Hooke's geometrical proof of spring power arises from the physical structuring of matter. Hooke begins with practical geometry: the point A, which represents the end of a physical spring, and then constructs a horizontal line of the stretched spring with a second point C; point C represents the end of the spring at a new point of strain, and therefore the line AC is both the stretched spring itself as well as a trace of the oscillating end of the physical spring upon release from tension. Following this, Hooke forms a right-angled triangle superficies ACD to represent the stretched spring's aggregate of powers at point C – fashioning a new mixed geometry that allows for both magnifying and resolving its infinitesimal points with each ordinate as if it were a section of cork; the ordinates originate perpendicularly from the points to "make a Mathematical Superficies" – the triangle ACD, the similar or "lesser triangles" and their excesses the trapeziums. This process of always beginning with physical reality and lifting geometry from nature,

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[463] Hooke, *Of Spring*, 19.

like a stone rubbing, grounds Hooke's mathematics.

VELOCITY

> [I]n the next place I come to represent the Velocities appropriated to the several powers.[464]

I explained that the horizontal line AC is both a trace of the motion of the end of a physical spring returning to equilibrium from flexure and a representation of the displacement of the stretched spring itself. I also mentioned that AC cuts the graph in half, and that the lower half represents velocity. Here I will analyse the lower half of the graph "*Fig 4*": Hooke's construction of a springing body's velocities and isochronous times. Nothing in Hooke's graph is arbitrary, and his geometry in the lower half is particularly interesting because of an idiosyncratic use of curves for the proportional segmentation of the velocity and time ordinates: BG and BI respectively. Moreover, the lower half of "*Fig 4*" reinforces Hooke's epistemological predilection for drawing the abstract from the concrete, and why his particular brand of mixed geometry is appropriate and necessary for the way he plows a problem. Namely, because traditional or speculative geometry relies on the ruler and compasses, instruments made to measure at the level of the bare eye, Hooke refuses to count on it to represent natural knowledge accurately; nevertheless, because of its very limitations, traditional geometry remains an indispensible tool for drawing concepts that the senses can comprehend.

Hooke begins with point A again, the end of the spring at rest, followed by the strained spring line AC. In the lower half of the graph, he moves from a line to a surface by taking a pair of compasses and drawing a quarter arc of a circle from point C to point F with a radius equal to the length of the stretched spring, that is, the line AC. This creates the new superficies – the circle sector ACGGGF. And now AC = AF = CD. Next he pulls perpendiculars down from points B segmenting AC to points G segmenting the arc. These perpendiculars BG represent velocity because

464 Hooke, *Of Spring*, 19.

they are proportional to the square "Root of the powers impressed"[465] – any square root of BE or square root of CD – Hooke's "General Rule of Mechanicks", reworked to solve for velocity instead of power.[466] Hooke switches to describing the relationship between power and velocity by imagining the physical spring line AC released from its end point of strain at C, accelerating as it flies through "infinite" points B back to equilibrium at A. Again, points "CG[G]GF" describe the quarter arc of a circle; and dropping perpendiculars from whichever point B of the spring line radius AC to the arc, Hooke segments the arc between C and F with infinite mathematical points G. Since the springing body is composed of infinite physical points B, one could colour in the sector ACGGF, forming aggregates with the lengths of the "Lines BG", which as mentioned represent the spring's velocity as it returns to equilibrium at A.

> The said ordinates [BG] being always in the same proportion with the Roots of the Trapeziums CDEB, CDEB …[467]

This is yet another example of Hooke drawing the abstract from the concrete, and is also an excellent example of mixed geometry. The reasons why the lines BG represent velocity are both physical and mathematical: Hooke's "General Rule of Mechanicks" on the one hand, which is

> true of the motion of … Slings; of Pendulums moved by Gravity or Weights; of Musical Strings; of Springs, and all other vibrating Bodies … and in a word, of all other Mechanical and Local motions, allowance only being made for the impediment of Air or other Fluid Medium through which the Body is moved[;][468]

and on the other, Pythagoras's theorem.

In his preliminaries, recall Hooke states that the spring line AC equals 10 spaces of distance, so one can use this arbitrary number to find all the unknowns in the graph, which is helpful for analysing its construction. If the line AC = 10 spaces, then the segment AB_2 is 5 spaces, and the equal segments AB_1 and B_1B_2 represent 2 ½

[465] Hooke, *Of Spring*, 20.
[466] Hooke, *Of Spring*, 20.
[467] Hooke, *Of Spring*, 20.
[468] Hooke, *Lampas*, 32–3, in *Cutlerian Lectures.*

spaces each respectively, because point B_2 bisects the line AC, and point B_1 bisects the segment AB_2. Further, to find the space represented by the line segment B_2B, and hence the line segment AB, Hooke bisects the right angle of triangle AB_2E to obtain half the hypotenuse, which equals B_2B.

By Pythagoras's theorem and using the values obtained from Hooke's preliminary AC equals 10 spaces, the hypotenuse of AB_2E equals $\sqrt{50}$, and so half of this value is B_2B. That is, let the hypotenuse of $AB_2E = c$, and let $AB_2 = a = 5$, and let $B_2E = b = 5$ according to the spring law (and the construction), which states that AB_2 is proportional to B_2E – the other leg of the right angled triangle AB_2E. Hence, $c = \sqrt{50}$, and $\sqrt{50}/2$ gives half the hypotenuse or the line segment B_2B. It follows that the segment AB equals $(5 + [\sqrt{50}/2])$ or 8 ½ spaces. By rearrangement of Pythagoras's theorem, Hooke can calculate the spring's velocity lines BG at any point B in its return to equilibrium from flexure at C, by "putting AC = a, and [any] AB = b, [any] BG will always be equal to $[\sqrt{(a^2 - b^2)}]$, the square of the ordinate being always equal to the Rectangle of the intercepted parts of the Diameter [radius]".[469]

Hooke's instrument of mixed geometry imagined as if in motion works because of his ruler and compasses construction; thus it shows why speculative geometry is necessary albeit compliant to practical geometry approximated from the physical formations and motions of matter. Later in *Lectures of Light*, Hooke would explain that "because nothing is so well understood or apprehended, as when it is represented under some sensible Form, I would, to make my Notion the more conceivable, make a mechanical and sensible Figure and Picture thereof".[470] Moreover, in building a mixed geometry for the spring law, Hooke also succeeds in strengthening the relations between vibratory phenomena that were previously held together only by similitudes, or what he referred to back in the *Micrographia's* observations of cork as groping around in the dark:

[469] Hooke, *Of Spring*, 20.
[470] Hooke, *Lectures of Light*, in *Posthumous Works*, 141.

> but till such time as our *Microscope*, or some other means, enable us to discover the true *Schematisme* or *Texture* of all kinds of bodies, we must grope, as it were, in the dark, and onely ghess at the true reasons of things by similitudes and comparisons.[471]

Using a microscope and a scale bar together with a ruler and compasses as tools for his new mixed practical-speculative geometry gives Hooke "other means" alongside the "similitudes and comparisons" that he nevertheless remains fond of, committed to and defensive of throughout his career. "But I know it may be said, that *Omne simile non est idem* …" Hooke complains, "[but] in a subject where we cannot obtain such sufficient Proofs as we can desire, we must be contented with what we can obtain".[472] Comparison, as I will show in detail in the subsequent section, remains an important reasoning tool for Hooke. The microscope and scale bar make it possible for him to zoom in on and quantify congruous and harmonious, or geometrically congruent and similar, parts; the ruler and compasses, instruments fashioned to function within the limits of the naked eye, allow him to exhibit this knowledge in a way comprehensible to human senses.

TIME

"Having thus found the Velocities" of the springing body, Hooke moves on to the corresponding times by instructing his readers to "draw a Parabola CHF whose Vertex is C, and which passeth through the point F," followed by dropping ordinates from the springing body AC to the parabolic section.

> The Ordinates of this Parabola BH, BH, AF, are in the same proportion with the Roots of the spaces CB, CB, CA …[473]

What Hooke means is that by definition of a parabola,[474] point H is the square of CB_2: using point B_2 for example, line segment B_2H is "in the same proportion with" the square root of the line segment CB_2, because B_2H squared equals CB_2 (Hooke draws the pricked line segment KH, forming the rectangle KHB_2A, to show this

[471] Hooke, *Micrographia*, 114.
[472] Hooke, *Of Comets and Gravity*, in *Posthumous Works*, 167.
[473] Hooke, *Of Spring*, 20.
[474] The equation of a parabola contains a squared term.

relation.) Next, for any point B, the velocity ordinate GB is proportional to the square root of the corresponding power line (by the spring law) CB.

> [T]hen making GB to HB as HB to IB [GB is proportional to HB as HB is proportional to IB, or GB:HB::HB:IB] … and through the points CIIF drawing the curve CIIIF, the respective ordinates of this curve shall represent the proportionate time that the Spring spends in returning the spaces CB, CB, CA.[475]

The key word here is "proportionate". Because of proportionality, GB is proportional to the square root of CB, as the square root of CB is proportional to IB (or GB:√CB::√CB:IB), and equivalently, via the vertical line segments or "Ordinates of this Parabola BH" in the lower half of the graph, GB:HB::HB:IB. Hence, by rearrangement, the line segment GB (velocity) multiplied by the line segment IB (time) is proportional to the line segment HB squared; and by further rearrangement $HB^2/GB \propto IB$; and IB gives the "proportionate time".

Hooke takes it as self-evident that the reader comprehends his curves, such as "the S-like Line" of "proportionate time" CIIIF, in the lower half of the graph. He seems to construct CIIIF by shifting or reflecting segments of the quarter circle arc up into the parabolic section, thereby maintaining the proportionality of the vertical ordinates; because the circle's radius AC is the parabola's axis of symmetry, and since by definition of a circle the arc maintains constant curvature at all points, this translation (of the circle segment into the curve CIIIF) creates a constant line of times for the respective powers and velocities. One can confirm that this is so because if GB:HB::HB:IB, as Hooke claims, then the ordinate segments GH and HI, which Hooke neglects to mention, are in proportion, which they are.[476] In general terms, the proportionality agrees with Hooke's claim in his preliminaries: "the Velocities of the whole spaces returned are always in the same proportions with those spaces, they being both subduplicate to the powers, and consequently all the times shall be equal". This is represented by "the S-like Line" CIIIF, whose "respective ordinates [IB] represent the proportionate time that the Spring spends in returning from the

[475] Hooke, *Of Spring*, 20.

[476] Except for the line BG, where the ordinate segments GH and HI are not in proportion, on account of what appears to be an engraver's error.

spaces CB, CB, CA".[477]

Hooke's graph works because of the similarity of right-angled triangles and quadrilaterals in its construction. But this is not a construction choice with Euclidean or speculative foundations; rather, the geometrical similarity in the graph originates from practical geometry founded on an intimate scrutiny and quantification of nature's parts, pores and motions with artificial instruments. For example, Hooke's studies of the 'texture' or lattice structure of quartz and his observations of cork. Moreover, since the spring law is the keystone of Hooke's metaphysical principle of matter-as-vibrations, *Fig. 4* exhibits that his idiosyncratic mixed geometry is a novel, necessary and appropriate tool 'to make this the more intelligible' (as Hooke is fond of saying). That is, for plaiting a proof that can make sense of a body's insensible vibrations by describing them with tools made to work within the limits of the human eye – compasses and a ruler, points and lines and so on. Recall that this is 'the business of Speculative Geometry' – to represent a concept in a form that is comprehensible to the senses. Hooke makes an "Image to represent the flexure"[478] of matter by literally drawing the abstract from the practical parts of the graph – the springing body. Just as he creates instruments that "interweave" art and nature, Hooke interweaves different geometries into a new one with new instruments like microscopes and scale bars, telescopes, sextants with telescopic sights and micrometers. Indeed, later on in life during a lecture on practical geometry for the art of navigation, Hooke differentiates between tools of speculative and practical geometry, stating that

> by the help of Ruler and Compasses, [a problem can] be truly protracted and measur'd upon a Plain, with as great exactness as 'tis possible, *by the help of the Instruments and Methods that are hitherto us'd to make Observation on which to ground the Calculation*.[479]

Paradoxically, by creating a point "such as [with] … the Point of a very curious pair of Compasses", Hooke turns the point into a practical one, in that it has "Quantity

[477] Hooke, *Of Spring*, 20–21.
[478] Hooke, *Of Spring*, 19.
[479] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 520. Italics added.

and Extension, but yet so small and minute, as that the sense cannot distinguish that it hath any Parts".[480] Although this once again reinforces Hooke's view that the theoretical stems from the practical, his practical geometry creates the supposed conundrum that a body can be once a point, once a solid.

7. SOLIDS

Solid figures, composed fundamentally of points, represent the highest level of complexity in Hooke's physical reality and geometry.[481] In *Of Spring*, Hooke imagines how a thin iron plate of one square foot, a superficies in practical geometry, becomes a solid, occupying a volume of one cubic foot with the addition of a "Vibrative motions forwards and backwards the flat ways"[482] (Part I). Because of Hooke's metaphysics of vibration, which supposes motion joined inseparably to matter just as in his geometry, points, lines, superficies and solids are all three-dimensional bodies in his practical geometry. So, all parts of a solid possess the same dimensionality as the solid – the difference is not the number of dimensions but the level of magnification or diminishment coupled with the frame of reference. Later, in *Lectures concerning Navigation and Astronomy*, he magnifies a "Prism of the Air" to hypothesise how one might calculate the power of the wind reflecting off an incongruous body, and describes the air prism's parts, explaining that it is

> to be consider'd as made up of an indefinite number of small Cylinders, Prisms, Wires or Strings lying close together …[483]

It may seem strange that Hooke imagines the air as a prism, but it is a choice with practical origins – his observations and experiments on light and atmospheric refraction, the air bending rays of light like so many lenses or prisms.[484] The remaining solids that Hooke lists – "Cylinders", "Wires or Strings" – are lines,

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[480] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 520.
[481] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 451.
[482] Hooke, *Of Spring*, 8.
[483] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 565.
[484] Hooke, *Micrographia*, 233.

similar to the sound-carrying rods previously discussed. Thus the prism of air is composed of clusters of compounded solids reducible to superficies that are made up of lines "lying close together", which are in turn reducible to their constituent points or globular bodies:

> each of these small Prisms or Wires may be suppos'd as made up of an indefinite number of small Beads or Dies lying one behind another, and so following each other immediately in the same Line, and with the same Velocity of motion, and every one of these compounding Beads or Dies coming to beat or strike against the Body that lieth in the way …[485]

Moreover, "Beads" or points, as has been shown, are also infinitesimally divisible. When it comes to navigating the earth, this concept further allows Hooke to contemplate how one might "find out some means to distinguish every Point or Part of the Surface of this Globular Body [the earth] in respect of any other",[486] but the same applies to any solid and its points. Therefore, just as a line representing a stretched springing body such as in *Fig. 4* of the spring law is divisible into its parts, so too is a planet.

And like *Fig. 4*, which depicts both discrete points and continuous aggregates of power and velocity because a spring line is infinitesimally divisible into compoundable parts, Hooke's representation of practical-geometrical bodies as points, circles, or spherical or other solid figures depends upon magnification and diminishment. That is, on shifts in the frame of reference. Recall that from as far back as the *Micrographia*, Hooke explains that he uses magnifying and "Diminishing Glasses"[487] as a means to manipulate these perspective shifts, that is, as intellectual instruments that show, in a radical epistemological inversion of the microscopic and the celestial, how the moon can be a pockmarked superficies like the point of a needle; a smooth globular body; and a point.

Carrying over into Hooke's diagrams, magnification and diminishment by real, material lenses are used together as tools that represent shifts in perspective

[485] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 565.
[486] Hooke, *Lectures concerning Navigation and Astronomy*, in *Posthumous Works*, 468.
[487] Hooke, *Micrographia*, 2–3.

based on whether a body is the primary subject of enquiry, and on whether a self-similar part vs. a compounded whole is under scrutiny. In this way, Hooke's practical geometry is also capable of exhibiting nature's rough surfaces as magnification, as well as the smoothness of diminishment.

GLASSES

To recapitulate, the *Micrographia's* 'observations' open with points and conclude with solids. Hooke could have chosen to end with any solid compounded of points, but because of their sphericity, the stars in Observ. LIX and the moon in the final Observ. LX complement the magnified point of a needle and the fullstop or period in Observ. I, and frame the text with Hooke's epistemological inversion. The frame controls how the text as a whole is perceived. According to Hooke, the point of a needle is usually taken to be the smallest imaginable point; and the moon, a planet, is one of the largest imaginable points. Applying Galileo's trope of the maculate moon to describe the surfaces of both a needle point and a planet increases the communicability of Hooke's inversion and forges an analogical relation between bodies at the very limits of the human senses. Moreover, the recognition of globular bodies as repeating and compounded patterns enforces order upon nature's variety.

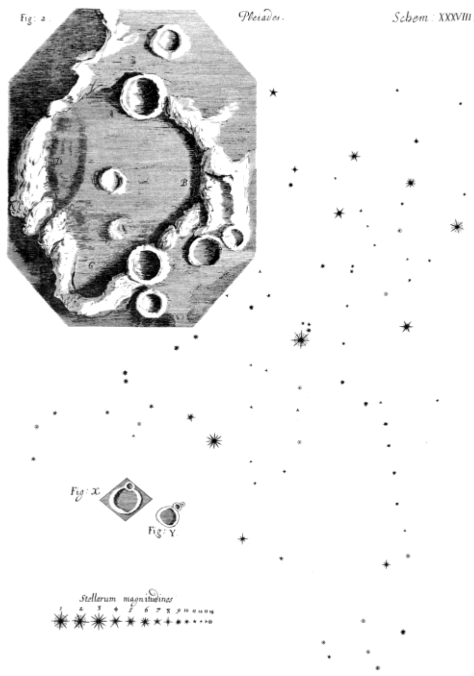

Figure 7.1 Hooke's representation of a part of the moon ("Fig: 2"), illustrating the superior level of detail revealed by his

Hooke's microscopes, telescopes, scale bars and micrometers are the necessary instruments of his practical geometry. They become his senses, and expand the edges of the imagination by, for example, revealing that the surface of the moon – a crucially huge hence already magnified and *natural* globular solid – is rough, not smooth. Indeed, just as Hooke often refers to pores between microscopic globular bodies as 'interstitia', he recycles the term for "the *Interstitia* of the world", that is, the spaces of aether "between the Bodies of the Sun and Starrs, and the Planets, and the Earth".[488] The earth, a bigger planet than the moon and even more 'naturally' magnified by virtue of Hooke being on its surface, further supports his concept that nature's fundamentals are physically rough as instruments of art are under the microscope. Taking pages out of both Kepler and Wilkins's works,[489] Hooke imagines looking at the earth from the moon, and postulates

> that could we look upon the Earth from the Moon, with a good *Telescope*, we might easily enough perceive its surface to be very much like that of the Moon.[490]

A "good *Telescope*" is what allows Hooke to reach this conclusion, as he illustrates by comparing his drawing of "one small *Specimen* of the appearance of the parts of the Moon" (*Fig. 2* or "Z"*)* with renditions attributed to "Hevelius" (*Fig. X*) and "Ricciolus" (*Fig. Y*) of the same area of the moon, but with a smoother, less detailed surface (*Figure 7.1*). Hooke complains that

> though taken notice of, both by the Excellent *Hevelius* … and also by the Learn'd *Ricciolus* … yet how far short both of them come of the truth, may be somewhat perceiv'd by the draught, which I have here added of it, in the Figure Z …[491]

Because of their inferior instruments, both Johannes Hevelius and the Jesuit astronomer Giovanni Battista Riccioli's 'specimens' lack several hemispherical pits in the 'vale' and surrounding 'pear-shaped' elevation of Hooke's drawing,

> (which I drew by a thirty foot Glass, in October 1664. just before the Moon was half inlightned) but much better by the Reader's diligently observing it himself, at a convenient time, with a Glass of that length, and much better yet with one of threescore foot long[.][492]

Nine years later, Hooke would publish *Animadversions on the first part of the Machina coelestis* …, openly attacking Hevelius's preference for making measurements with big instruments and plain (naked eye) sights after Tycho Brahe

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488 Hooke, *Micrographia*, 95–6.
489 Kepler, *Somnium: the Dream, or Posthumous Work on Lunar Astronomy*, trans. Edward Rosen (New York: Dover, 2003). Wilkins, *The Discovery of a World in the Moone.*
490 Hooke, *Micrographia*, 245.
491 Hooke, *Micrographia*, 242.
492 Hooke, *Micrographia*, 242.

over instruments mounted with telescopic sights and reticules – an attack that was already brewing in the *Micrographia*.[493] For example, early on in the Preface, Hooke explains that "*some parts of [Nature] are* too *large to be comprehended, and some* too *little to be perceived. And from thence it must follow, that not having a full sensation of the Object, we must be very lame and imperfect in our conceptions about it, and in all the propositions which we build upon it*". This is because of "*the* disproportion of the Object to the [sense] Organ[s], *whereby an infinite number of things can never enter in to them*", but "artificial Instruments *and* methods" provide "*an* inlargement of the dominion, *of the Senses*", expanding the limits of the imagination and hence our ability to reason.[494] Hooke's intolerance towards naked eye observations further underscores his preference for practical geometry as a theoretical tool, and the above manoeuvres allow Hooke to make the bold claim quoted at the start of Part II: if one could build microscopes practically according to the theory of them, then one would be able to reduce all of nature's sharpest points to the rudeness of art – that is, to the rough and approximate point of a needle.

Hooke's ontology and epistemology depend upon these shifts in perspective, and knowledge gained telescopically is applicable to both the macroscopic and the microscopic realms, and vice versa. Again, the difference between a point and a solid is not a difference in the number of dimensions, for both are three-dimensional, but in the *physical* complexity of the body. The countless experimental and theoretical tools and techniques, observations, analogies and hypotheses involving discrete points or compounded solids form a complicated argument as Hooke's studies of what there is develop in complexity. Here, from inception, Hooke's distinction between 'point' and 'solid' is that points are the most "simple and uncompounded bodies", whereas solids are "bodies of a more complicated

493 For a comprehensive account of the dispute between Hooke and Hevelius, and Hooke's radical instrumentalism, see Gal and Chen-Morris, *Baroque Science*. For further details on the ensuing controversy, and on Hooke's telescopic sights and micrometers, see Nakajima, "Robert Hooke as an Astronomer: Hooke's Optical Research and Instruments in their Historical Context", in Hunter, *Robert Hooke: Tercentennial Studies, 49–62*.

494 Hooke, *Micrographia*, Preface.

nature".[495] The optical-instrumental relations between magnifying and "Diminishing Glasses" translate mathematically to congruent solids, similar solids and proportionality. Later in his career, Hooke further employs this concept of practical-geometrical similarity to explain how what is sensed and imagined affects one's ability to reason:

> For neither can we form a simple Idea of any thing that is a Million of Millions of times less than the Idea of the least visible Point; nor can we form an Idea of a *Maximum* which is Millions of Millions bigger than the imaginary bigness of the Heavens we see; but by Composition, and Comparisons, and Proportion, we make the compounded Ideas, which suffice for a Material to be made use of in Reasoning.[496]

Here is Hooke's novel yet situated, physicalised, practical-geometrical attempt at the problematic question of infinitesimals, which engaged his contemporaries. In Hooke's rendition, infinitely small practical-geometrical parts possess the same dimensionality as solids, thus they can be treated as small as one wishes yet still finite. By choosing to order the *Micrographia's* observations according to their level of geometrical complexity, a practical geometry lifted off nature's parts with new instruments, Hooke dictates the relations between the observations, and gives his arguments greater authority. By reformulating the common definitions of point, line, superficies and solid for practical geometry, he creates a new language capable of describing the "texture" of physical reality beyond the senses, because it is contingent upon it. Euclidean or speculative geometry, fashioned long before the invention of artificial instruments such as the telescope, is incapable of describing the reality of objects created by the New Science, though it is nevertheless a necessary tool for grounding concepts by making the insensible sensible. Thus the moon represents not the smooth perfection of speculative geometry, but rough approximations; the agent of this enforced order is Hooke, making the approximations human-sized with his new instruments and the geometry that they allow. This justifies calling both Hooke's practical and mixed geometry new.

[495] Hooke, *Micrographia*, 1.
[496] Hooke, *Of Comets and Gravity*, in *Posthumous Works*, 176.

CONCLUSION

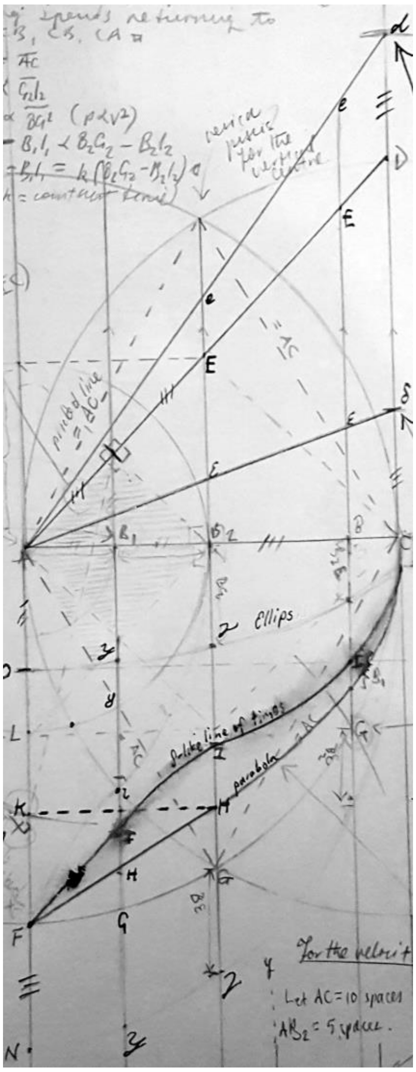
Hooke's unique way of working a problem reflects the synergy of his sometimes simultaneous careers as the Curator of Experiments for the Royal Society and the Gresham Professor of Geometry. To produce a generalised "Theory of Springs", a "Rule or Law of Nature", which could be used to improve upon nature, Hooke needed to reduce his vibrating strings similitudes to a geometrical demonstration. He also needed to formulate a spring law because his entire matter theory, Congruity and Incongruity, present in all his work, hinged on it. Artificial instruments and apparatuses capable of magnifying and measuring never-before-seen minute bodies, their pores and motions, not only allowed for the creation of a new geometry *from* nature, but made it necessary. Because the process of magnification is indefinite in principle, there are no dimensionless points or perfectly straight lines. Rather, these are fictions of an imagination circumscribed by human senses. Thus, a necessary and useful geometry is one that is capable of sliding back and forth from physical points to circles and spheres, from lines to parallelograms, and so on. As a result of these challenges, Hooke had to reassess and reconfigure the role of traditional Euclidean geometry. The inversions, reversals and the subversion explicated throughout attest that the process was far from straightforward or self-evident. Moreover, Hooke and his contemporaries had to justify and legitimise newly minted mathematical practices, showing them to be capable of handling the constellation of natural laws governing the objects produced by the instruments of the New Science. Hooke's reformulation of practical geometry, that for him is the mediator between sensible and insensible physical reality, originated from his epistemological preference of lifting mathematics off the many facets of nature's points, lines, surfaces and solids. It is a mathematics pulled from physics and dependent on the power, accuracy and precision of artificial instruments. And the process of always beginning with physical reality also grounded Hooke's novel *mixed* geometry, as shown in the analysis of "*Fig 4*" – Hooke's exhibition of the spring law.

As I have also shown, Hooke achieves the above in three artful moves. Reversing the traditional roles of speculative and practical geometry, he teaches that the former is subservient to the latter. Giving examples of how and why traditional geometry is an ancient instrument as fallible as the human eye, Hooke experimentally supports his claim that it is incapable of analysing the objects detected and measured by the new instruments of the New Science. For example, there are no 'ideal' rulers or needle points with which to inscribe a 'straight' line. Even rays of light, the straightest lines in nature, are bent by refraction because no medium is truly homogeneous. Speculative geometry is an abstraction and, contrary to Proclus's claim, cannot represent physical reality more precisely than practical geometry. Because of its primacy of the physical over the mathematical, Hooke concludes that his practical geometry is a better representation of art and nature than speculative geometry; this relieves him of some of his anxieties about the technological limitations of artificial instruments as mediators between nature and human approximations enforced upon it. Thus, Hooke's reformulation of practical-geometrical definitions constructs a theoretical tool from physical reality. Replacing the ruler and compasses with instruments such as microscopes and telescopes allows him to make his microscopic and macroscopic epistemological inversions by shifting frames of reference. When the difference between points and planets becomes a matter of scale, Hooke can reduce globular bodies to points or circles that possess either geometrical congruence or similarity, welding links into a chain of proportionality from the microscopic to the celestial realms. The changing scales, made possible by new instruments, create the need for a more flexible geometry; and the instruments are embodiments of geometrical scaling. Nevertheless, speculative geometry and its ancient instruments remain necessary: they are theoretical and practical tools for grounding concepts, for making insensibles intelligible to the human senses.

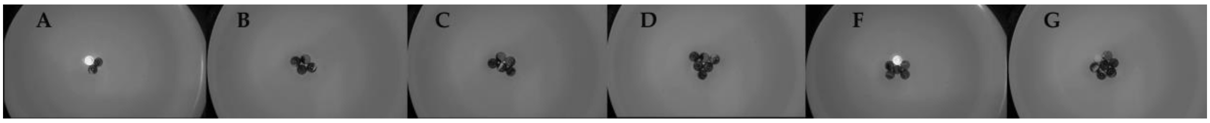
To ease the communicability of his inversion, Hooke employs Galileo's trope of the maculate moon, forming an analogical relation between bodies of art and bodies of nature at the opposite limits of the human senses: the point of a needle and the moon. By using the needle point as a gauge, Hooke turns the *Micrographia* into a direct challenge against Clavius's claim that geometers ought not to meddle in matters of physics. Moreover, just like Hooke's instruments, which he designs by interweaving art and nature, his argument from analogy smudges the line between art and nature, bolstering his conjecture that with powerful enough instruments, one could reveal nature to be like art – rough and approximate. The surface of the moon is as rough as the surface of the point of a needle. It also enforces an order of points as repeating and compounded patterns upon nature's variety. Thus, Hooke's practical geometry creates a tense distinction between sensibles and insensibles that is both caused and resolved by approximation, since knowledge gained in insensible realms must necessarily be pulled back within sensible limits if it is to have utility.

Finally, practical geometry demonstrates its power with Hooke's theory of matter-as-vibrations. Hooke's replacement of the epistemological status of light, which is visually sensible, with sound, which is visually insensible, paves the way for the mathematisation of his vibrating strings model. Because the sense of hearing registers the isochronous vibrations of a musical string as a particular pitch, and because these same motions prove too fast for the sense of sight, which registers only a blur of movement or none at all, Hooke chooses to study sound instead of light as a route to developing a general law of vibratory motion. Yet because pond ripples, light and sound share observable physical similitudes, Hooke expects that representing his matter-as-vibrations theory of 'congruity and incongruity' with geometrical optics will describe the mechanism of all vibratory phenomena. That is, by drawing both 'mathematical lines' and magnified 'physical lines' (parallelograms) to demonstrate the rectilinear propagation of "orbicular pulses" from faraway point sources, Hooke can describe the reflection of a ray against an incongruous surface, the refraction of rays and the obliquity of their pulses through incongruous or similar media, and the congruity of an undisturbed ray in a homogeneous medium. Hooke's conflation of light strings and sound rays is viable because congruity and harmony, transmission and refraction, and congruence and similarity offer different ways of representing the same characteristics of 'congruity'. Consequently, Hooke replaces all qualitative descriptions of light and sound with bodies and motions in *Of Spring*, and succeeds in constructing a geometrical demonstration of the spring law with his new mixed geometry in the graph, "*Fig 4*". Like his geometry, Hooke's metaphysics of vibration supposes motion joined inseparably to its objects. Thus, the graph also exhibits his solution to the question of infinitesimals: one-dimensional lines with length and no breadth cannot aggregate into a surface; but by origin, definition and use, points, lines, superficies and solids are all physically three-dimensional in Hooke's practical geometry.

APPENDIX



*0-1*A reconstruction of "Fig 4", Hooke's demonstration of the spring law (*Of Spring),* to show possible ruler and compasses construction lines.



0-2 A reconstruction of Hooke's rolling bullets experiment (*Micrographia*, "Observ. XIII Of the small Diamants, or Sparks in Flints", "Fig: 2") using marbles and a parabolic bowl. The labels correspond to Hooke's in his "Fig: 2'; "E" is missing from my figure because I did not obtain it after repeating the experiment four times. "D" was obtained during the second attempt. All other shapes were obtained during all four attempts.

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