Teachers' conceptions of mathematical fluency

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This is to certify that to the best of my knowledge; the content of this thesis is my own work.

This thesis has not been submitted for any degree or other purposes.

I certify that the intellectual content of this thesis is the product of my own work and that all the assistance received in preparing this thesis and sources have been acknowledged.

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Abstract

Fluency in mathematics is defined in various forms, such as computational fluency, procedural fluency, and mathematical fluency (Keiser, 2012; Kilpatrick, Swafford, & Findell, 2001; Sullivan, 2011b). Terms like 'procedural' and 'computational' often leave teachers interpreting fluency as simply being able to follow a set formula or to quickly compute mathematics. This narrow belief may affect the way teachers teach mathematics and what they expect their students to be able to do to be fluent.

This study explored practicing primary teachers' conceptions of mathematical fluency; including how they define mathematical fluency, what features they associate with the term and what role, if any, understanding plays in mathematical fluency. Exploration of teacher conceptions is essential to gain insight into what teachers think and how what they know and believe affects their teaching.

A qualitative approach was taken and data were collected from primary teachers via an on-line questionnaire (n = 42) and semi-structured interviews (n= 17). Thematic analysis of the questionnaire and interview data focused on the words teachers used to define and explain mathematical fluency and how they described fluency in their students. A theoretical framework of teacher conceptions was applied, highlighting internal and external factors that influence teachers' conceptions of mathematical fluency. Rich descriptions of fluency shared by teachers was captured through the analysis process.

Findings revealed that teachers held mainly contemporary views of mathematics and of how students learn mathematics. Teachers believed that students did not truly have mathematical fluency if they could not also apply, and demonstrate or communicate their understanding of concepts. Teachers spoke of students having 'fluidity' and 'flexibility' in their ways of thinking, that students were able to move beyond errors that caused others to be 'stuck'. A case is made for the reframing of mathematical fluency based on these findings. Adopting a view of fluency as an amalgamation of conceptual understanding and strategic competence, making it synonymous with mathematical proficiency.

Chapter 1 Introduction

One important aspect of students' abilities to work mathematically is having fluency. Not only do students have to develop knowledge of mathematical concepts *(the what)*, they also need to display this knowledge through working mathematically processes *(the how)*. For a number of decades, the face of mathematics education has been changing. The focus has been shifting from simply viewing mathematics as a set of skills to memorise, to a reform-orientated approach to teaching (Anderson & Bobis, 2005). Contemporary teaching of mathematics is where problem solving comes to the forefront and the mathematical processes students use in learning mathematics are paramount (Rittle-Johnson & Alibali, 1999). The importance of holding both skills and knowledge of subject content is highlighted along with the processes needed to *use* the skills and knowledge in a range of situations.

Fluency is a commonly used word in the subject of English in describing a student's ability to read. However, its use in the mathematics curriculum is fairly new, only specifically being included in the Australian Curriculum in the last few years (ACARA, 2010). Mathematics is often thought of as having a language all of its own. If mathematics is a language, it makes sense that teachers want students to be fluent and be able to communicate what they understand.

Mathematical fluency can be defined and interpreted in many ways. The literature surrounding mathematics generally defines fluency as procedural or computational fluency (Kilpatrick et al., 2001; McClure, 2014; National Council of Teachers of Mathematics, 2014; Russell, 2000). The majority of research studies conducted about fluency in mathematics use a definition of procedural fluency similar to the definition in Kilpatrick et al.'s (2001) conceptualisation of mathematical proficiency (Bass, 2003; Graven, Stott, Nieuwoudt, Laubscher, & Dreyer, 2012; Ramos-Christian, Schleser, & Varn, 2008; Stott, 2013; Thomas, 2012). When looking at the Australian context and conceptualisation of fluency, Watson and Sullivan's (2008) definition as "mathematical" fluency is a broader term and is used as the definition of fluency described in the Australian Curriculum (ACARA, 2010).

The focus of this present study is on exploring teachers' conceptions of fluency. For this research, the term 'conceptions' is inclusive of both teachers' beliefs and knowledge that they hold of the concept (Beswick, 2012; Holm & Kajander, 2012; Thompson, 1992). Teachers' conceptions are highly dependent on their personal beliefs formed through life and educational experiences. Conceptions are also influenced by teachers' knowledge of mathematics, and of how mathematics is learned (Borg, 2003; Melketo, 2012).

This study pertains to primary teachers' descriptions and definitions of mathematical fluency as a starting point in exploring fluency. Teachers' conceptions of how to teach or assess mathematical fluency of students is not specifically explored within this study. Fluency, as stated in the NSW mathematics syllabus (NSW, 2012), is one of five components of working mathematically. This study explored one aspect of working mathematically in depth, fluency. Although the other components of working mathematically: understanding, communicating, reasoning and problem solving are addressed in parts of the research, they are not the main topic under investigation. Primary school teachers are the focal group, the inclusion of students' and parents' conceptions of fluency are not part of the scope for this study.

Despite more contemporary views of mathematics teaching being encouraged both through research and the curriculum, there still exists a substantial focus on procedural fluency and a perpetuation of traditional methods of teaching mathematics (Handal & Herrington, 2003; Yates, 2006). Students often develop processes for undertaking mathematics tasks with little or no knowledge of why the process works, or how it could be adapted in other contexts when procedural fluency is the focus (Hiebert, 1999; McClure, 2014). Debate still exists regarding “teaching for speed and teaching for meaning” (Thomas, 2012, p. 328) and what relationship, if any, they hold. An overemphasis on the end point of quick recall or immediate knowledge of facts affects the ways teachers teach mathematics, often teaching for speed by speed testing. Not only does this result in a lack of understanding but has the potential to cause mathematics anxiety in students from a young age (Ashcraft & Krause, 2007; Boaler, 2015). Effective, flexible and accurate use of procedures are essential in becoming mathematically fluent (Kilpatrick et al., 2001). Given the limitations associated with viewing fluency as solely about memorising procedures and having quick recall of facts, further research is required regarding the way teachers describe fluency.

Little research exists about practicing primary teachers’ conceptions of mathematical fluency and how they describe mathematically fluent students. Research mainly centres on students and generally focuses on procedural fluency and its relationship to conceptual knowledge or on testing and improving their procedural fluency (Arroyo, Royer, & Woolf, 2011; Bauer, 2013; Ramos-Christian et al., 2008; Rittle-Johnson & Alibali, 1999; Russell, 2000).

This study aims to explore teachers’ views of mathematical fluency and devise a more robust conception of *mathematical* fluency that emphasises aspects beyond procedures. Even though the term *procedural* fluency may describe other features of fluency in its definition, the use of the term ‘procedural’ to describe fluency results in teachers interpreting procedural fluency at face value. By exploring teachers’ conceptions of mathematical fluency, a shared view of fluency is needed that reflects the current trends in mathematics education where understanding with fluency is what is valued by all.

The findings of this study will assist in discovering whether teachers have a shared conception of mathematical fluency and assist in identifying features that could, through further research, be observed in students to identify, monitor and assess mathematical fluency. Teachers' response data will hopefully provide a common set of key features of mathematical fluency that can be observed in students to identify points of need for future learning.

This study will explore teachers' conceptions of mathematical fluency and what relationship if any, understanding plays within mathematical fluency. In establishing whether teachers share similar conceptions of what it means to be mathematically fluent and what fluency looks like in students, the following research questions were devised for this study:

What are primary teachers' conceptions of mathematical fluency?

I. How do primary teachers define the term 'mathematical fluency'?

II. What knowledge and beliefs do primary teachers have about mathematical fluency?

- As it relates to their students
- As it relates to the other working mathematically processes (understanding, communicating, reasoning and problem solving)

This study presents features of fluency as described by teachers, considers the similarities or differences in these features between participating teachers and compared to current research definitions.

The thesis is divided into five chapters: Chapter 1 provides an introduction, Chapter 2 presents the literature review related to both the concept of fluency and what constitutes teacher conceptions; Chapter 3 discusses in more detail the methodology utilised for this qualitative study along with the data collection tools and forms of analysis; Chapter 4 outlines the findings of the research, and Chapter 5 provides a conclusion to the research discussing the findings and future directions for the research.

Chapter 2 Literature Review

Considerable research exists surrounding students' fluency in mathematics and how to best teach or assess fluency of mathematics skills (Finnane, 2004; Foster, 2013; Graven et al., 2012; Russell, 2000; Stott, 2013). The research findings generally centre on the student, and are often focused on fluency as a skill of speed or automaticity to be practiced and mastered without attention to how students then apply or transfer this knowledge (Arroyo et al., 2011; Greenwood, 1984; Miller & Heward, 1992; Poncy, Skinner, & Jaspers, 2007). Therefore, mathematics may be interpreted as facts to memorise not concepts to be understood. Although past research covers a variety of aspects of fluency such as; the teaching of fluency, the assessment of fluency as a proficiency, and the practice of fluency-based tasks, studies into classroom teachers' understanding or conceptions of fluency are not so numerous. This present study focuses on exploring teacher conceptions of fluency as a necessary precursor to studying what it looks like in students or how it is addressed in the classroom for students. In this chapter, the focus will be on how fluency is defined in the literature *and* what research exists on teacher conceptions of mathematics in general (encompassing beliefs and knowledge). A theoretical framework for studying teacher conceptions in mathematics will be discussed and how it may be applied to mathematical fluency. The framework is inclusive of factors that may influence how teachers define and conceptualise mathematical fluency, these will be explored when answering the research questions.

This chapter is structured around answering questions regarding the why, how and what that underpin the research questions being explored. For example, what is fluency?, why is mathematical fluency important and how is mathematical fluency described in the research literature.

What is fluency, and specifically, in mathematics?

Fluency takes on many forms and for most primary teachers, fluency in reading is the first context to come to mind. According to Kuhn and Stahl (2003) reading fluency is the '*ungluing from print*' stage of literacy learning. Students develop fluency, or automaticity with print in reading after establishing their accuracy in decoding words. This means that there is less halting in their reading and children are able to establish a level of comfort with print. Once students have an ease with print, understanding comes easier than "when they are still struggling with word identification" (Kuhn & Stahl, 2003, p. 6). Reading fluency is described as the bridge between decoding skills and reading comprehension by Pikulski and Chard (2005). Accuracy is an important goal, but fluency is not about who can read the fastest. Structure and fluency are also areas in first language development, where the learner's capacity to "communicate meanings in real time" describes their fluency (Skehan, 1996, p. 46). In learning a second language such as French, fluency reflects the operation of underlying cognitive processes (Préfontaine, Kormos, & Johnson, 2016). Lennon (2000) defines language fluency as the "rapid, smooth, accurate, lucid, and efficient translation of thought or communicative intention" (p. 26). Fluency as it relates to the learning of a language, is where the development of 'flow' of speech is as important for the listener as the receiver, as it is for the speaker as the producer (Fulcher, 2013). It is fluid and smooth, "language is motion" as imagined by Koponen and Riggenbach (2000, p. 7).

Fluency in reading and in language both have decoding and comprehension as core activities, 'cogs' that need to be in motion at the same time. Students are unable to focus attention on both processes at once and therefore need to have some automaticity with one activity, decoding. When all of a student's focus is on decoding words "little or no capacity is available for the attention-demanding process of comprehending" (Pikulski & Chard, 2005, p. 511). In the primary curriculum, fluency has long been related to aspects of literacy and language acquisition, therefore reading fluency is a common term used by teachers. In other areas, fluency is defined and expressed as; performing with ease, speaking smoothly or easily without effort, fluid, easily changed or adapted (Fluency, 2017).

The abovementioned definitions of fluency may not always be directly transferable to mathematics. None the less, the definitions provide a wider view of fluency as something that requires both an ease of flow or delivery, as well as a harmonious relationship between working memory and cognitive processing to understand and communicate knowledge.

How is fluency in mathematics defined and described?

What about fluency in mathematics? As mentioned in the beginning of this chapter, in mathematics, fluency is often synonymous with speed or quick recall (Ramos-Christian et al., 2008; Wong & Evans, 2007). Definitions of fluency outside of mathematics generally do not relate fluency with speed. In describing fluency, Boaler (2015) takes a more holistic view of fluency in mathematics as incorporating the 'doing' not just the 'knowing' of mathematics. She quotes Parish (2014) who draws from Fosnot & Dolk (2001) "who define fluency as 'knowing how a number can be composed and decomposed and using that information to be flexible and efficient with solving problems'"(p. 3).

In a similar description to Boaler's 'doing' and 'knowing', The NSW Education Standards Authority (NESA) Mathematics K-10 syllabus (2012) component of fluency, and the Australian Curriculum: Mathematics proficiency of fluency, are described as the thinking and doing of mathematics (ACARA, 2010). Fluency is a relatively new term for NSW primary teachers and is an addition to the syllabus' working mathematically components that before

2012, made no clear delineation of fluency as one of the five processes of working mathematically. Teachers are now required to embed fluency in their mathematics teaching, assess it through aspects of the content, and decide whether students are 'fluent' or not. It therefore begs the question, what is 'mathematical' fluency, and what does it look like?

Fluency in both the NSW mathematics syllabus and the Australian Curriculum: Mathematics is one part of an interrelated framework designed to focus teachers on teaching mathematical content through the mathematical processes, not just teaching procedures (Sullivan, 2012). These processes are adapted from Kilpatrick et al.'s (2001) framework for the conceptualisation of mathematical proficiency. This framework, shown pictorially in Figure 2.1, depicts 'procedural fluency' as one of the five intertwined strands needed for students to be proficient in mathematics.

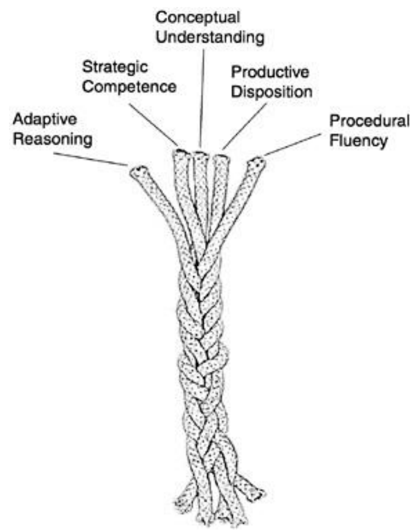

Figure 2.1 Intertwined strands of proficiency (Kilpatrick et al., 2001)

Being able to carry our procedures flexibly, accurately, efficiently and appropriately (Kilpatrick et al., 2001) although clearly described as intertwined, is presented as a discrete skill from conceptual understanding and the other strands. Hence fluency is often interpreted by teachers as automaticity with the mechanics of procedures involving memorising and recall (Thomas, 2012) without considering its interrelatedness to the other strands. Mathematics is then seen as separate skills with a fixed set of facts and procedures for determining answers (Smith, 1996). This view of fluency as being only procedural, can lead to a disconnect between the teaching of the procedure (the what), and the understanding of the concept (the why), of mathematics which need to be learned in unison (McClure, 2014). As stated by Hiebert (1999, p. 15) "if students over-practice procedures before they understand them, they have more difficulty making sense of them later." In agreement with these sentiments around procedures, Kilpatrick et al. (2001) held the idea that "once students have learned procedures without understanding, it can be difficult to get them to engage in activities to help them understand the reasons underlying the procedure" (p. 122). Kilpatrick et al. also referenced numerous studies that compare and explore procedural fluency and conceptual understanding (Brownell, 1935; Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Hatano, 1988; Mack, 1995; Rittle-Johnson & Alibali, 1999; Wearne & Hiebert, 1988). These provide further insight into the relationship between procedures and mathematical understanding.

A broader definition of fluency as 'mathematical fluency' was constructed by Watson and Sullivan (2008). Fluency involves carrying out procedures flexibly, accurately, efficiently and appropriately *as well as having* "factual knowledge and concepts that come to mind readily" (p. 112). Their definition combines both the ability to perform the mechanics of mathematics (procedural) and the understanding of the mathematics being learned (conceptual) thus providing a wider scope to focus on various aspects of fluency. *Mathematical* fluency will be the operational definition of fluency utilised for this present study, emphasising the importance of understanding as an integral part of fluency.

Why is mathematical fluency important?

Within mathematics, proficiency results from being both procedurally and conceptually fluent. Students need to understanding why they are using specific strategies and "know when it is appropriate to use different methods" (McClure, 2014, p. 10). According to McClure (2014) students who "engage in a lot of practice without understanding what they are doing often forget, or remember incorrectly, those procedures" (p. 12). Mathematical fluency is important as it enables students to be able to communicate easily their understanding of concepts and share ideas and strategies used to solve problems. Fluency is ongoing, changing and increasing, it is not something you can 'tick off' your list of skills stating, 'I'm fluent'. As the complexity of the mathematics increases, your ability to be fluent and flexible may plateau until a point where it becomes 'comfortable' again and you are able to recall the necessary facts and understand the processes needed to solve more complex tasks.

Why is fluency being explored?

Procedural fluency dominates many of the resources developed for students and is often the sole focus of mathematics homework. This overemphasis on procedural fluency is the reason for the present study's focus on understanding the features of *mathematical* fluency that go beyond learning procedures alone. Teachers and researchers know what they are looking for in reading fluency, decoding *and* comprehension (Rasinski, 2004), but what about in mathematics? Do we as teachers consistently look for efficiency, accuracy and flexibility (McClure, 2014) to indicate fluency? Are these the only characteristics of mathematical fluency? Are teachers aware of what to look for beyond the ability to recall facts?

Fluency in calculations is an essential part of mathematical thinking. When too great a significance is placed on automaticity with the mechanics of mathematics, this overshadows the equal importance of the role understanding plays in student development. Excessive time spent on the mechanics of calculations may be detrimental to the student's ability to deepen their understanding of mathematical ideas (Kilpatrick et al., 2001). A deep understanding of mathematical ideas requires students to communicate solutions to problems and moreover find alternate solutions and strategies for the problem. The role of more challenging tasks that require working beyond procedures alone, was also explored as part of Russo and Hopkin's (2017) study into task-first approaches versus teach-first approaches in mathematics. They indicated that cognitively demanding tasks must be "solvable through multiple means (i.e. have multiple solution pathways) and may have multiple solutions" (p. 290). When writing about understanding in multiplication, Hurst and Hurrell (2016) make the point that “the development of genuine multiplicative thinking (beyond recalling number facts) has been hindered through the teaching of procedures at the expense of conceptual understanding” (p. 34).

What are the issues?

Procedural fluency is important. Teachers need to develop in their students fluency in calculation “as a way of reducing the load on working memory, so allowing more capacity for other mathematical actions” (Sullivan, 2011a, p. 7). When students have knowledge of number facts readily recallable from memory they no longer need to concentrate on processing the calculations, it becomes second nature. Instead of focusing all their energy on the mechanics of the calculation, an ease with numbers allows students to focus on solving more complex tasks shifting the effort to choosing and using appropriate methods for problem solving. This shift from the ‘‘nitty-gritty’’ mechanics allows students to see the big picture, selecting which strategies to use and being able to explain why they chose them (Foster, 2014, p. 6).

Findings from previous studies regarding procedural fluency generally focus on fluency as computation of basic skills (Bass, 2003). Students’ understanding of the concept or their strategies used to solve the tasks are not taken into consideration. A singular focus on fact recall results in many children being *trained* to do mathematical calculations rather than being *educated* to think mathematically (Noyes, 2007). Research involving mathematicians conducted by Dowker (1992) adds weight to the argument that students need to be able to think mathematically. The study’s findings indicated that when mathematicians are making use of learned facts, “their knowledge of number… and their use of strategies based on this knowledge, seemed to involve an enjoyment of thinking and playing with numbers, rather than rote memorisation” (p. 52). A similar observation is made by Boaler (2015) who claims that there is also a narrow focus in other countries on fluency in mathematics, emphasising speed and memorisation.

When procedures are learned without understanding, more complex tasks take a considerable amount of time to solve. This is often due to a "compartmentalization of procedures … so that students believe that even slightly different problems require different procedures" (Kilpatrick et al., 2001, p. 123). More energy is expended on the calculation processes, taking up "working memory resources" (Pegg, 2010, p. 37), when it may be more beneficial to be using working memory to trial possible solutions. Russell's research (2000) into computational fluency with whole numbers found students who were using procedures without understanding were not "looking at the whole problem or using what they know to reason about the answer" (p. 3). Students do have knowledge "but do not think to draw on it", they try to simply remember the steps instead of trying to make sense of the problem through building on what they already know (Russell, 2000, p. 3). Students who display procedural fluency without understanding are unable to choose the appropriate tools, only being able to use the procedure in the original context they were taught. Mathematics presents an increased challenge to students when there is a lack of understanding:

> When students learn a procedure without understanding, they need extensive practice so as not to forget the steps… when skills are learned without understanding, they are learned as isolated bits of knowledge. Learning new topics then becomes harder since there is no network of previously learned concepts and skills to link a new topic to (Kilpatrick et al., 2001, p. 123).

Equating fluency with quick recall is another unwanted result of teachers and the wider community focusing solely on procedural fluency. Studies depicting fluency as fast 'fact fluency'(Bauer, 2013) encourage teachers and parents to value quick recall above all else. The rise of commercial computer-based skills practice software also places emphasis on tests for checking students' procedural fluency above students' conceptual understanding (Duhon, House, & Stinnett, 2012; Foster, Anthony, Clements, Sarama, & Williams, 2016; Kanive, Nelson, Burns, & Ysseldyke, 2014). Aligning fluency with quick recall results in teaching for fluency through timed or speed related 'drill and practice' tasks in isolation from rich tasks that focus on understanding (Ramos-Christian et al., 2008; Steele, 2013; Wong & Evans, 2007). 'Drill and practice' tasks focus on the content not the student where fluency seems to be something *done to* students not something *developed in* students. Findings from studies of fluency as quick recall often showed improvement in speed of students in answering questions, but made no connection to teachers' observations or opinions in the decision of whether or not the student was fluent (Miller & Heward, 1992; Wong & Evans, 2007). As the majority of 'fact recall' research explores fluency based on student task or test results, not on the teacher's perspective of fluency, there has been little focus on how to shift practices to include other features of fluency, including conceptual understanding.

Issues of fluency without understanding and fluency as quick recall indicate a need to first explore fluency beyond procedures. The goal being to identify a range of specific features of mathematical fluency, including necessary procedures, but not procedures alone. This present study will focus on understanding fluency from a teacher's perspective, as the majority of past studies of fluency in mathematics only focus on students. By exploring teachers' conceptions of mathematical fluency, beliefs that currently equate fluency to speed and recall may be further discussed. It would be beneficial to explore teacher conceptions further as there may be a relationship between the conceptions teachers' hold regarding the teaching and learning of mathematics and their view of fluency.

What are teacher conceptions?

Teacher conceptions encompass a teacher's beliefs, concepts, meanings, rules, mental images and knowledge that bear on their experiences (Thompson, 1992). Thompson uses the construct of teacher conceptions to address both beliefs and knowledge together, stating that; "to research and only look at beliefs and not knowledge, would give an incomplete picture" (Thompson, 1992, p. 131). There is a need to identify both teacher beliefs about students' mathematical thinking *and* teachers' knowledge of a concept. It is justifiable to relate these aspects because beliefs are held in varying degrees, "what is knowledge for one person may be a belief for another, depending upon whether one holds the conception as beyond question" (Thompson, 1992, p. 259). This idea of knowledge as belief beyond question leads many researchers to equate beliefs and knowledge, as teachers' knowledge is subjective, and therefore is much like beliefs (Richardson, 1996). The important question when studying teachers' conceptions is not whether they are actually true, but *how* the teacher views the conception as true or not (Philipp, 2007). Teachers' beliefs are their own theories consisting of sets of interrelated conceptual frameworks (Cochran-Smith & Lytle, 1990) that connect self-knowledge and the act of teaching as a kind of "knowledge-in-action" (Philippou & Constantinos, 2002, p. 212). The view of teacher conceptions as involving both teacher beliefs and teacher knowledge equally (Thompson, 1992) is the operational definition of teacher conceptions used for this present study.

Why focus on teacher conceptions?

Teacher conceptions are “important considerations in understanding teachers’ thought processes, classroom practices, change, and learning to teach” (Richardson, 1996, p. 1). There is considerable research conducted with pre-service and in-service teachers regarding beliefs about mathematics (Maasepp & Bobis, 2015; Nisbet & Warren, 2000; Pehkonen, Törner, & Leder, 2002; Richardson, 1996). These studies focus on teachers’ conceptions of mathematics content and pedagogical knowledge, though not specifically mentioning mathematical proficiencies or working mathematically. Specific studies on practicing teachers’ conceptions of what mathematical fluency is and how they identify this in their students is scant. Research is needed to explore teachers’ knowledge of fluency and to discover how teachers observe fluency. This knowledge can then be utilised to plan for future teacher professional learning and student instruction.

Concerning working mathematically, Charalambous and Philippou (2010) explored teachers’ efficacy beliefs specifically related to problem solving. Compare this to mathematical fluency, where little to no research on teachers’ conceptions exist. Although there is mention of teacher beliefs in some of the research of fluency in mathematics (Thomas, 2012), the main focus is on teaching fluency and classroom practice. In studying reflection as a key tool in teacher learning, Sullivan (2003) connected the themes of knowledge and beliefs about mathematics as the focus of his research.

Research into measuring beliefs (Chrysostomou & Philippou, 2010; Jang, 2010) and on understanding existing beliefs of teachers’ own self efficacy in mathematics has also been undertaken (Holm & Kajander, 2012; Talaga, 2015; Vacc & Bright, 1999). Beliefs are a ‘practical indicator’ providing a good estimation of teachers’ experiences and therefore should be explored to gain insight into teaching and learning practices (Pehkonen & Törner, 1999, November). More in-depth studies are needed though, as Thompson (1992) suggested, research on beliefs tended to measure beliefs in isolation to knowledge. Research on teachers' beliefs and knowledge also indicate that "there seems to be some link between teaching grade level and association with traditional and contemporary views" (Anderson, White, & Sullivan, 2005, p. 27). If Kindergarten teachers' perspectives alone were examined in this research, the common descriptors found may differ from those of teachers of another grade, e.g. Year 6. Therefore, in light of Anderson et al.'s findings this present study will involve teachers from a range of primary grades to address the following research question and sub-questions:

What are primary teachers' conceptions of mathematical fluency?

I. How do primary teachers define the term 'mathematical fluency'?

II. What knowledge and beliefs do primary teachers have about mathematical fluency?

- As it relates to their students
- As it relates to the other working mathematically processes (understanding, communicating, reasoning and problem solving)

Theoretical framework

Exploration of teacher conceptions fits within a broader social constructivist framework. Just as holding a constructivist view of *student* learning sees students using prior knowledge to construct new meaning, reflecting on current knowledge (von Glasersfeld, 2008), and learning from social interaction (Cobb, Wood, & Yackel, 1990), so it should be with *teacher* learning (Ball, 1988).

There is a wealth of existing research into teacher beliefs regarding mathematics and how these beliefs affect teaching practices (Bobis, Way, Anderson, & Martin, 2016; Fennema, Carpenter, & Loef, 1990; Grouws, 1992; Smith, 1996; Thompson, 1984a). How teachers organise classroom lessons is very much dependent on “what they know and believe about mathematics and on what they understand about mathematics teaching and learning” (Anthony & Walshaw, 2009, p. 25). Many studies explore the effects of teacher beliefs on their teaching of mathematics, discussing both internal and external factors that affect what teachers know and believe, which in turn affects how they teach (Barkatsas & Malone, 2005; Handal & Herrington, 2003; Holm & Kajander, 2012; Li, Moschkovich, Schoenfeld, & Törner, 2013; Nisbet & Warren, 2000; Richardson, 1996; Stipek, Givvin, Salmon, & MacGyvers, 2001; Thompson, 1984b; Wilkins, 2008; Yates, 2006; Zakaria & Maat, 2012).

A number of factors were commonly addressed in these studies: teachers’ experience with schooling, teachers’ professional experiences and teachers’ social experiences. As teachers’ knowledge and beliefs of mathematical fluency may be influenced by similar experiences, specific factors have been adopted for this present study. These factors include: mathematical content knowledge, knowledge of research to support teaching mathematics, their own schooling experience and the individual teacher’s perspectives and experiences in and out of the classroom (Bryman, 2016). Examining influential factors provides a useful framework for thinking about mathematics teaching and learning, but it does not tell us how to teach mathematics (Simon, 1995). Nevertheless, we must acknowledge that the complex web of factors that influences teacher conceptions, may also affect how teachers teach mathematics and therefore effect how students learn (Ball, 1988).

For the present study, the data gathered may shed some light on how and why teachers hold certain beliefs and knowledge regarding mathematical fluency. The conceptual framework, represented in Figure 2.2, is a synthesis of frameworks from Borg (2003), who addressed teacher cognition in language teaching, and Melketo (2012) who explored the relationship between teacher beliefs and practices in English writing. This framework will be the lens through which data in the present study is analysed.

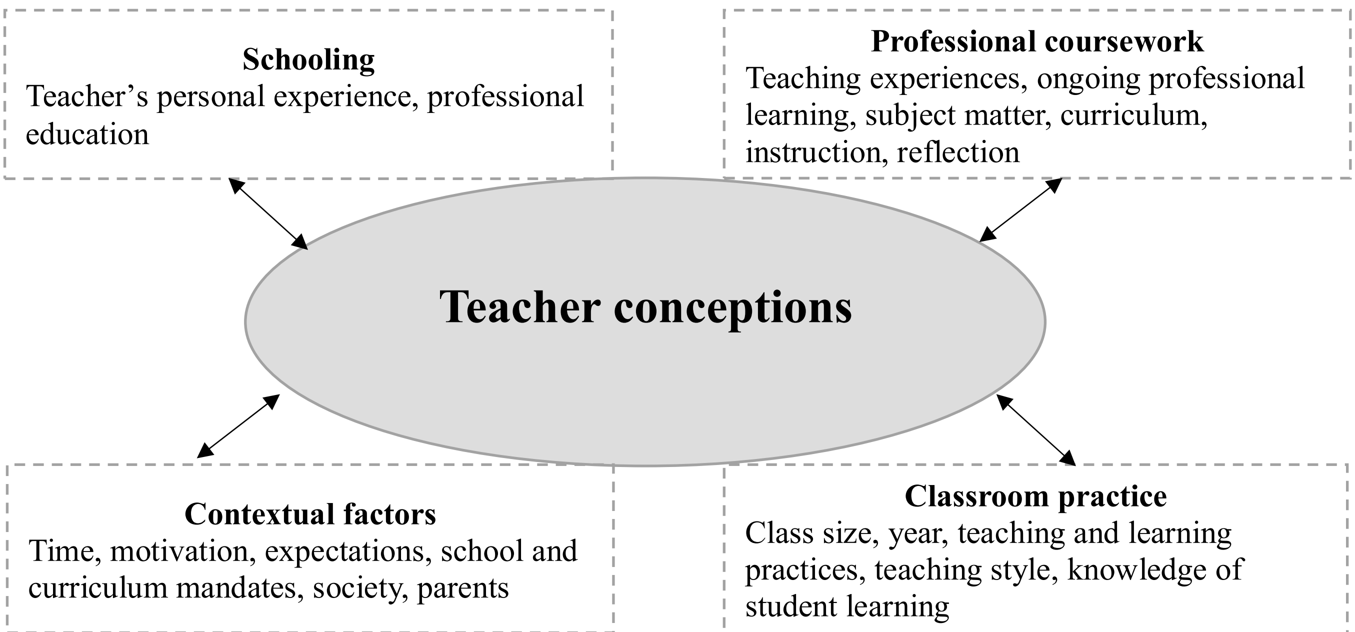

Figure 2.2 A model of teachers' conceptions: factors and influences, synthesised from (Borg, 2003) and (Melketo, 2012)

Although this framework is being utilised within a mathematical context, the framework illustrates the complex web of factors that can be applied to any learning area.

The research places teachers' conceptions of mathematical fluency at the centre and addresses key factors such as teacher knowledge of mathematical fluency and beliefs on student learning, acknowledging this is influenced by where they are now in relation to a number of contextual factors. For example, these beliefs may be affected by a teacher's own personal school experience when learning mathematics. In her research into teacher beliefs, Hart (2002) quotes Richardson (1996) "life experiences are a major contributor to the formation of beliefs" (p. 167). Other factors include teachers' pedagogical knowledge of how students learn, or the kinds of professional learning they have participated in. Teachers' knowledge of mathematical fluency may also be influenced by their subject content knowledge, their experience of and reflections on classroom practice and student learning, or the expectations and culture set by their current school environment. As identified by Hamilton and Richardson (1995) "school culture connotes the beliefs and expectations apparent in a school's daily routine …it is manifest in the norms or beliefs shared by participants—students, teachers, administrators, parents, and other workers within a school." (p. 369).

The purpose of this research is to address the research gap surrounding both fluency in mathematics beyond procedural fluency, and teachers' conceptions of mathematical fluency inclusive of the factors influencing their beliefs and knowledge. It aims to build on the current body of research that concentrates on students' learning of fluency by providing further exploration that focuses on teachers' understanding of fluency.

Chapter 3 Methodology

3.1 Introduction

The previous chapter addressed the literature concerning mathematical fluency. It explored both the definition of fluency in general and more specifically, in mathematics. The chapter delved into the differing views of 'mathematical' fluency and how these ideas often create issues for understanding fluency. It described a framework for how both internal and external factors influence teacher conceptions and how these may affect teachers' beliefs and knowledge of fluency. Aspects of this framework guided the categorizing of themes that emerged in teachers' responses to the research questions: What are primary teachers' conceptions of mathematical fluency? How do teachers define the term 'mathematical fluency'? and What knowledge and beliefs do teachers have about mathematical fluency?

In this chapter, the research approach and design will be outlined, as will the interpretive approach chosen for this exploratory study. The purpose is to demonstrate how the methodological approach, research design, data collection instruments, process and analysis techniques assist in answering the research questions posed.

3.2 Interpretive approach

An interpretive approach aims to understand individual experiences, with the belief that reality is subjective and constructed by the individual (Lather, 2006). Lather follows on from this description with a metaphor, if interpretivism were an event, it would be a community picnic. This metaphor provides a clear picture of interpretivism as a cooperative, interactive and humanistic endeavour (Lather, 2006, p. 5). An interpretive approach to research focuses on exploring a social idea, concept or phenomenon to gain understanding. This exploration is founded on the belief that when interpreting a concept, people's beliefs, values and perceptions provide meaning and influence knowledge. Qualitative research "enhances our understandings and insights into a situation or phenomenon, and these conditions are grounded in meaning" (Shank, 2006, p. 347). Interpretivism sits within this definition and is consistent with Denzin and Lincoln's statement on qualitative research as an interpretive approach, "qualitative researchers study things in their natural settings, attempting to make sense of, or to interpret, phenomena in terms of the meanings people bring to them" (Denzin & Lincoln, 2000, p. 3).

Taking the view that understanding is often defined by the meanings people bring to a phenomenon, justifies this study's use of an interpretive approach. Interpretivism is listed as one of Wolcott's (1992) 'big' theories, (see Table 3.1). 'Big' theories, more commonly referred to as paradigms, are frameworks that function as maps or guides that define methods and techniques used to solve problems (Usher, Scott, & Usher, 1996). One of the strengths of using an interpretive approach is its emphasis on examining texts, such as written words, or conversations (Neuman, 2003). Interpretive researchers 'read' these texts to absorb viewpoints to develop a deep understanding of the concept and to make connections among the messages and parts of the text (Neuman, 2003).

Table 3.1 Interpretivist paradigm and related theoretical perspectives (O'Donoghue, 2007)

‘Big’ theories	*Four major theoretical perspectives within the interpretivist ‘big’ theory*
Positivism	
Interpretivism	• Hermeneutics • Ethnomethodology • Phenomenology • Symbolic interactionism
Critical theory	
Postmodernism	

Adopting an interpretive paradigm therefore guides the methodology and methods of data collection selected. Given that this study focuses on exploring conceptions teachers hold, it fits within the epistemological assumption that evidence is constructed and based on individual views and experiences (Creswell, 2013). Holding this perspective of mathematics where the meaning resides within the individual, “positions mathematical truth as a socially constructed reality located in collective and agreed meaning making” (Thornton, Kinnear, & Walshaw, 2016, p. 36). Conceptions are how individuals understand and make meaning of the world, they are personal and are influenced by numerous internal and external factors based on teachers’ own educational experiences, social experiences, their students in their classroom and their knowledge of content and pedagogy formed through professional learning (See Figure 2.2 A model of Teachers’ Conceptions Framework in chapter 2, Literature Review). This interpretive framework based on pragmatism focuses on the outcomes of the research having practical implications for teachers in the classroom (Creswell, 2013).

3.3 Research design

Research design is the 'glue' between the research questions and the data, showing how the research questions will be connected to the data (Punch, 2014). An interpretive paradigm, applied to educational research, enables researchers to build rich local understandings of the life-world experiences of teachers and of the cultures of classrooms (Taylor & Medina, 2013). The current study is exploratory in nature, aiming to gain a deeper understanding of teacher knowledge and beliefs by studying real-world settings inductively to generate rich narrative descriptions (Patton, 2005).

The study was divided into two research stages (see Figure 3.1). Stage one consisted of an online questionnaire collecting background information and teachers' overall conceptions of how students learn mathematics and the role of fluency. Stage one engaged a larger population of teachers across NSW Department of Education (DoE) schools. Stage two focused on interviewing a smaller number of teachers gaining a detailed view of their conceptions, specifically answering the second research question elements of; how mathematical fluency relates to their students, and; how mathematical fluency relates to the other working mathematically processes.

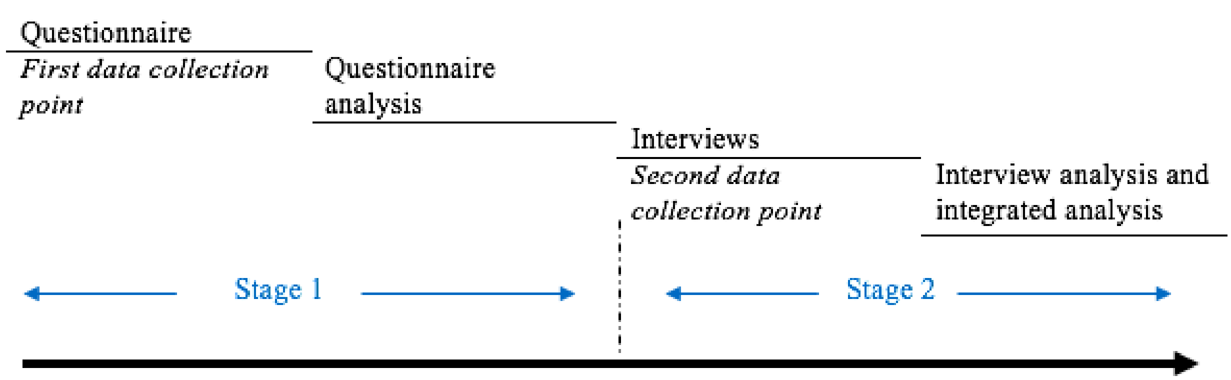

Figure 3.1 Sequence of research data collection (Halcomb & Andrew, 2009)

3.4 Participants

Participants of this study were primary school teachers currently teaching grades Kindergarten (K) through to Year 6 in NSW DoE schools. Participants were chosen as all NSW DoE schools address the Australian Curriculum through the implementation of the NSW Educational Standards Authority (NESA) Mathematics K-10 syllabus (NSW, 2012) that embeds fluency as one of the five working mathematically components.

The target population for this study was a cross section of K-6 teachers, from schools varying in context, size and location. The sample population was randomly selected for the initial questionnaire, interviews were then conducted with a smaller number of teachers from the questionnaire participants. These processes will be further explained in the following sub sections. See Table 3.2 for the overview of the identification processes used throughout this study.

Table 3.2 Participant identification processes

Sampling	Sample size	Methods
Step 1: Identify population, NSW DoE primary teachers	N=27 000 teachers	Identified through NSW DoE schools list
Step 2: Randomly select schools to invite to participate in questionnaire	300 schools	Random sampling formula used in excel to select schools.
Step 3: Questionnaire	n=42 teachers	Structured online questionnaire gathering background information, responses to a set of Likert statements and open response to questions.
Step 4: Conduct 1:1 semi-structured interview	n=17 teachers	Identify a range of teachers from each K-6 grade level. One on one interviews (Video Conference for rural)

3.4.1 Participants: Questionnaire sampling process

Using a list of NSW DoE schools, NSW Department of Education (2015) as a sampling frame, stratified random sampling identified 300 schools to approach, 190 of these schools were metropolitan while 110 were non-metropolitan or rural and remote. Schools were emailed information about participating in the study and included the Participant Information Statement and a link to the online questionnaire. Interested teachers completed the questionnaire and indicated their agreement to be contacted via email for interview as part of Stage two of the study, or could chose to remain anonymous. See Table 3.3 for demographics of participating teachers.

3.4.2 Participants: Interview selection process

Through the participant identification processes, background information in Part A of the questionnaire was analysed to ascertain what grades were being taught by the interested participants. Seventeen teachers indicated their interest in further participation in the study. These teachers were representative of all grades and were therefore selected for interview. See Table 3.3 for the range of grades participants selected were teaching. Of the 17 teachers, 12 were from metropolitan schools and 5 were from rural or remote locations. The majority of teachers were from mid-sized schools with 300-700 students, one teacher was from a large (>700 students) school and three were from smaller schools (<300 students). Considering length of service, 11 of the teachers had >15 years of experience, 4 teachers had between 6-15 years of experience and 2 teachers were in their first 3 years of teaching. There were 16 females and one male teacher. This indicates a high gender imbalance, even compared to the overall NSW Department of Education statistics where 16% of all primary teachers are male (NSW Department of Education, 2015).

Table 3.3 Demographics and details of respondents to the online questionnaire (n=42)

Category	Details	No.
School type	Primary	39
	Infants	3
School location	Metropolitan	35
	Rural or Remote	7
School size (no. of students)	>700	5
	300-700	32
	<300	5
Highest level of teacher education	Bachelor Degree	24
	Diploma of Education	8
	Masters Degree	9
	2-3 years teacher's diploma	1
Teacher classification	Classroom teacher	22
	Assistant principal	11
	Deputy principal	7
	Principal	2
Gender	Female	39
	Male	3
Teaching experience (years)	1-5 years	9
	6-15 years	10
	>15 years	23
Teaching years at current school	<12 months	4
	1-3 years	12
	4-5 years	5
	6-10 years	8
	11-15 years	6
	>15 years	7
Current grade teaching	Kindergarten*	6
	K-1	1
	K-2*	4
	Year 1	1
	Year 1-2*	3
	Year 2*	5
	Year 2-3	1
	Year 3*	1
	Year 3-4*	1
	Year 4*	3
	Year 5*	2
	Year 5-6*	4
	Year 6	4
	K-6*	2
	Not specified*	4

* indicates grades participants in the interviews were teaching

3.5 Data collection tool and procedures

This section outlines the instruments selected for Stage one and Stage two of the study and the procedures for their use, the data collection timeframe and plan.

3.5.1 Instruments

Questionnaires and semi-structured interviews were selected to collect data in this study. These two methods were chosen as both are important means of obtaining direct responses from participants about their understandings, conceptions, beliefs, and attitudes (Harris & Brown, 2010a).

Questionnaires are used extensively by researchers when gathering data of teachers’ beliefs and knowledge concerning mathematics (Anderson & Bobis, 2005; Barkatsas & Malone, 2005; Beswick, 2012; Thompson, 1984a). As the questionnaire is a self-report survey there are mixed findings in research surrounding the accuracy of what teachers espouse compared to what happens in the classroom (Ross, McDougall, Hogaboam-Gray, & LeSage, 2003). Therefore, a second data source was selected to build a more complete picture of teacher conceptions. Online questionnaires were chosen as they help to create a nonthreatening and comfortable environment with anonymity (Nicholas et al., 2010). They provide greater ease for participants to complete the survey with time and space flexibility that allows them more time to consider and respond at their leisure (Creswell, 2013).

Semi-structured interviews provide detailed understandings of a phenomenon in relation to the classroom and to students that can only be established through talking directly with teachers (Creswell, 2013). Interviews are one of the most utilised forms of data collection sources for qualitative research. Rubin and Rubin (2004) write that qualitative social researchers “gather information either by *observing* or by *talking with and listening* carefully to the people who are being researched” (p. 2). Interviews capture rich and realistic detail of the experiences and perspectives of those being studied (Lincoln & Guba, 1985) that can then be conveyed to others. The specific use of semi-structured interviews allowed space to create questions linked directly to the research questions. It also allowed for flexibility as additional questions were included based on general patterns of responses from the questionnaire data analysed. Enabling prompts were also included for individual interviewees on specific questions where participants mentioned an explicit aspect of fluency in their questionnaire response.

In their discussion on practical problems arising when attempting to align data, Harris and Brown (2010b) noted several recommendations. To maximise both questionnaire and interview data, “ensure interview prompts and questionnaire items are structured and highly similar” and “present the object of interest in a highly concrete and specific way” (2010a, p. 11). These recommendations for ensuring consistency of the questioning and asking specific, practical questions, were considered within the design of the two instruments. The interview questions aligned to aspects of the questionnaire and were seen as an extension of the conversation started through the questionnaire.

Questionnaire

The questionnaire’s role aligns to Johnson and Onwuegbuzie’s (2004) comments on mixed methods research where “a closed-ended instrument to systematically measure certain factors considered important in the relevant research literature” (p. 7) may be beneficial to supplement other qualitative methods. The questionnaire contained three parts, Part A: 10 closed-ended questions, Part B: 10 Likert scale items and Part C: two open-ended questions. All questions were developed in RedCap survey software. The questionnaire served as a method to gauge the general view of mathematical fluency prior to conducting interviews. Table 3.4 includes the mapping of the questionnaire to the research questions.

Part A was designed to collect descriptive information about the teacher's school; type of school, size of school and location to determine representation of these variables from the population. Teachers also provided personal demographic details about themselves and their teaching careers. This information was utilised as part of the selection process for interviews described in section 3.4.2 Participants: Interview selection process. A copy of the questionnaire is attached as Appendix A.

Part B contained 10 Likert-type items with a 5-point response scale ranging from *strongly disagree* (1) to *strongly agree* (5). The items were adapted from a survey designed by Ross et al. (2003) that set out to gauge teachers' beliefs and practices surrounding standards-based mathematics teaching in the USA by questioning teachers around nine important dimensions of teaching mathematics. Of the nine dimensions in the research undertaken by Ross et al. (2003), this present study predominantly focuses on three dimensions. The items were designed to elicit teacher beliefs pertaining to; Dimension 8: conceptions of mathematics, do teachers see mathematics as a dynamic subject (contemporary) rather than a fixed body of knowledge (traditional) regarding students' development of mathematics and fluency (5 items), Dimension 6: student to student interaction: how students communicate mathematical ideas (2 items), and Dimension 9: mindsets and confidence and how these affect student learning in mathematics (3 items). There were both positively and negatively worded items. During data analysis the coding was inverted for the negatively worded items so the data could more easily be compared.

Part C contained two open, free-text response questions on mathematical fluency. Question 1: What three words best describe how you would define mathematical fluency? and

Question 2: What key features are observable/ present in students that have mathematical fluency? The purpose of the two questions was to assist in identifying emerging themes or patterns that could be explored further through interview. They also offered a snapshot of the words teachers used to describe mathematical fluency.

Semi-structured Interviews

The semi-structured interviews during Stage two of the study enhanced and built upon the picture of teachers' conceptions of mathematical fluency (suggested by the questionnaire data), making sense of the phenomena in terms of the meanings they [the teachers] brought to it (Denzin & Lincoln, 2003). Interviewing places value on the personal language teachers use to describe situations and experiences as data (Newton, 2010). Conducting the interviews face-to-face allowed respondents time to answer direct questions in a more conversational manner, often adding extra information, sharing anecdotes or pondering further questions themselves, all of which contributed to the overall narrative.

The interview structure reflected the research questions and was categorised into key concepts: definition, features, examples and connections ensuring that different aspects of the research questions were addressed explicitly via direct questions. See Table 3.4 for mapping of these concepts. Note that the superscript numbering links the research questions to the key concepts in the right-hand column. The interview schedule is attached as Appendix B.

Table 3.4 Data instruments mapping to research questions

Instrument	Mapping to research questions	Key concepts
Questionnaire	*How do primary teachers define mathematical fluency?*[1] *What knowledge and beliefs do primary teachers have about mathematical fluency?*[2] ▪ *As it relates to their students*[3]	[1]**Definition** Fluency words [2] **Beliefs** of fluency – traditional or contemporary view of teaching and learning mathematics [3]Observable **features** of fluency
Semi-structured interviews	*How do primary teachers define mathematical fluency?*[1]	[1]**Definition** mathematical fluency [1]**Features** of mathematical fluency identified by teachers
	What knowledge and beliefs do primary teachers have about mathematical fluency? ▪ *As it relates to their students*[2]	[2]**Examples** of mathematical fluency in students
	As it relates to the other working mathematically processes[3] *(understanding, communicating, reasoning and problem solving)*	[3]**Connections** of fluency with understanding and other WM processes

3.5.2 Data collection plan

Schools were contacted in December 2016 via email and given an open time frame to complete the questionnaire. In February 2017, schools were sent a reminder email regarding their participation in the research and given until the beginning of April to access and complete the questionnaire. The questionnaire data was then used to select the interviewees and further develop the interview questions. Teachers who indicated an interest in Stage two of the study were contacted during April. Interviews were conducted during May 2017. Each interview took between 15 and 25 minutes and was audio recorded and later transcribed. An interview code was used for each teacher in the de-identification process. The interviews were conducted face-to-face on location at each participant's school (14 different sites) except for two cases where interviews were conducted via the video conferencing application Blue Jeans.

3.6 Data analysis

The data analysis techniques applied in this study included descriptive statistics and thematic analysis that provided an illustrative and exploratory orientation to the study (Guest, MacQueen, & Namey, 2012). Using both inductive and deductive coding as different layers of analysis allow codes to flow from the principles that underpin the research, and the specific questions one seeks to answer (Joffe & Yardley, 2004). Figure 3.2 outlines the steps involved in the analysis process.

Statistical analysis

Descriptive statistics were used with Part A and Part B of the questionnaire to familiarise the researcher with the data and summarise the demographic information. Standard deviation was also applied to describe and measure the variance of teachers' beliefs evident in the Likert scale items across all teachers for each question.

Thematic analysis

Thematic analysis was undertaken on Part C of the questionnaire and on the interview data transcripts. This involved both deductive methods elaborating on the major themes emerging from the questionnaire, and inductive methods considering new themes that arose from the interviews. For Part B, deductive coding was used via the Likert items' three main dimensions; conceptions of mathematics, student to student interaction and mindsets and confidence. These dimensions later provided one lens through which the interview data was analysed. Part C of the questionnaire provided initial themes for the definition of fluency that were then explored and mapped to the interview data. The teacher conceptions framework, see Figure 2.2 in the literature review, provided another lens through which to analyse and code the teacher interview data. The framework highlighted influencing factors such as: teachers' own educational experiences, social experiences, knowledge of their students in their classroom and their knowledge of content and pedagogy.

Inductive coding also developed from the interview data where themes emerged from the teacher responses relating to their definitions of mathematical fluency and their descriptions of specific features students demonstrate when mathematically fluent.

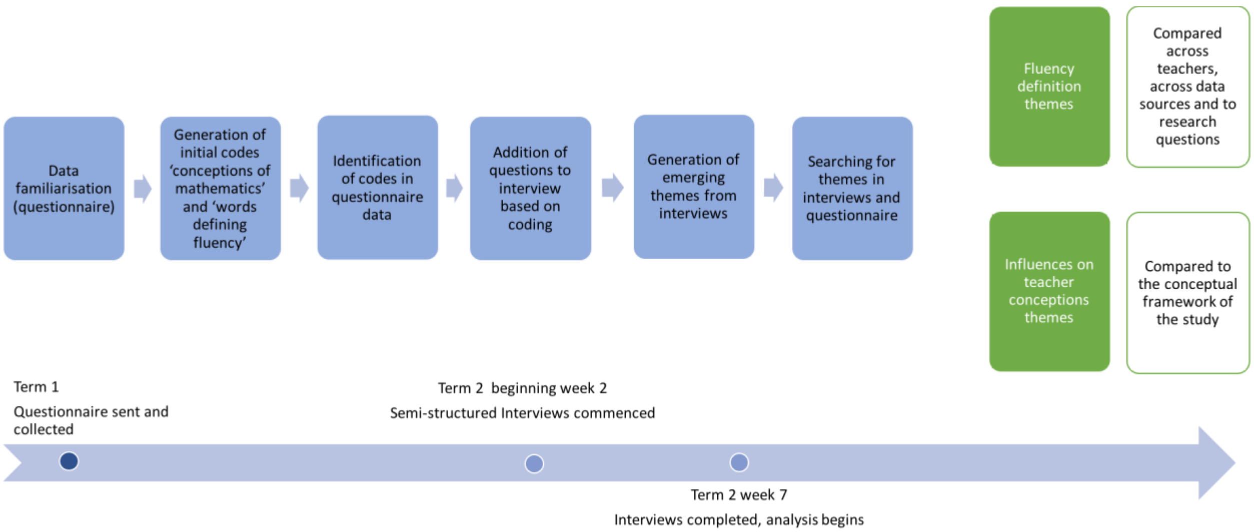

Figure 3.2 Process of analysis based on a Thematic Analysis (Clarke & Braun, 2017)

Combining questionnaire and interview data allowed for a fuller picture of teachers' beliefs about and examples of fluency. These types of data collection were applied to build patterns, categories and themes used to work back and forth between the data sets (Creswell, 2013) to create a comprehensive view of mathematical fluency. During and after the two stages of data collection, multiple opportunities emerged to analyse the data gathered. Similar to Clarke & Braun's (2017) thematic process phases, the analysis was undertaken in 6 steps: 1) questionnaire data summarised, 2) questionnaire data analysed, 3) identification of codes from questionnaire data, 4) interview questions refined based on questionnaire data, 5) interview data analysed for emerging themes, 6) searching for these themes in the questionnaire and interview data, mapped to the research questions, the Likert dimensions, and the teacher conceptions framework’s influencing factors. These phases were applied flexibly as analysis is not a linear process of simply moving from one phase to the next. Thematic analysis allows for cutting between data sets using a range of layered themes which clarify meaning by moving back and forth between the whole data set and its parts (Holloway & Todres, 2003).

3.7 Reliability and validity

In the interpretation of qualitative data the processes implemented need to be credible, authentic and fair. Guba & Lincoln’s (1989) standards of trustworthiness and authenticity provide criteria by which to gauge the quality of the research. As a primary teacher and an advisor of mathematics for the NSW DoE, the researcher has established credibility in the field of study. The teachers’ explanations of mathematical fluency and accounts of their students, provided a rich description for the reader to compare to their own social and school context, highlighting the transferability of the research. As a cross section of K-6 teachers was involved in the study, this is a fair representation of a typical primary school setting. The use of the questionnaire and semi-structured questions in the interviews allowed for consistency of procedures and dependability of the instruments and of the collection of data (Bryman, 2016).

Analysing both data types separately and in conjunction was not an attempt to validate or seek confirmatory results from one data to the other but as Kendall (2008) states to gather more in-depth insights on participant attitudes, thoughts and actions. According to Smith (2006) “the results from these two methods (i.e., survey questionnaire and semi-structured, qualitative interview) should be considered not so much as confirmatory or divergent, but rather as complementary.”

3.8 Ethical considerations

This research complies with the University of Sydney Human Research Ethics Committee’s primary objectives. Protecting “the mental and physical welfare, rights, dignity and safety of participants of research involving humans, their data or human tissue” The University of Sydney (2016, p. 1) through maintaining participant’s anonymity and de-identifying individual’s data. All material related to ethics including the approval letter, participant information statement and the participant consent form are attached as Appendix C. Information consent forms were signed by participants, outlining the steps to be taken in safeguarding their identities and indicating that they could withdraw at any time during the research period. The researcher is an employee of the department in an advisory position. As this may have been seen as a position of power over the participants, it was made clear in the consent forms and during the interviews that the researcher was representing the university, not the department for this present study. During the interviews, it was also explained to the participants that they would not be identifiable by name or school and that their interview data would be stored appropriately. As the research was exploratory, and the questions were not specifically related to working in a departmental school or the departmental school system, the teachers were comfortable in answering freely. This is in line with The National Statement on Ethical Conduct in Human Research, National Health and Medical Research Council (2015). Another set of ethical issues is benefits, costs and reciprocity (Huberman & Miles, 1994). Participants were informed of the value of the research and what they would potentially gain professionally from taking part in the research. One of the aims of this study is to contribute new knowledge in the education field, that the findings will have educational benefit. Communication of this goal was imperative in meeting the guidelines criteria from the State Education Research Applications Process (SERAP) (NSW DoE, 2015). The approved SERAP application is attached as Appendix D.

3.9 Summary

In this chapter, the research design and methodology for the study of primary teachers' conceptions of mathematical fluency was presented. It stated the purpose of the study as explorative and provided reasons for the choice of an interpretivist paradigm.

An overview of the stages of the study was provided and information regarding the qualitative methods of data collection in the form of a questionnaire and semi-structured interviews was outlined. These methods were chosen to collect the knowledge and beliefs of the participants concerning mathematical fluency that would provide background information regarding their pedagogical beliefs and experiential data regarding their students.

The development of the data collection instruments was outlined as was the plan for data collection. The process of thematic analysis, based on the work of Clarke and Braun (2017), was also outlined highlighting the back and forth nature of analysis to compare and reanalyse the data collections to obtain rich descriptions by layering themes.

This chapter has provided justification for the design and methods used and explained the reasons for the choices to best answer the research questions. Ensuring the methods of data collection and analysis are trustworthy, fair and reliable.

In chapter 4, the findings of the data analysis will be outlined and presented for each of the data then combined as an integrated analysis of both data sets. The emerging themes will be discussed and the data will be explored through numerous lenses, layering the data based on the research questions, the Likert dimensions and the teacher conceptions framework's influencing factors.

Chapter 4 Results

This chapter comprises an overview of the results, showing the different aspects of analysis that were conducted on the data. The chapter is structured according to the analysis process stated in Chapter 3 Methodology: descriptive statistical analysis of questionnaire data, identification of codes in both data sets, and analysis of data mapped to the research questions, the dimensions from the questionnaire and the teacher conceptions framework.

The data generated by the questionnaire helps explain conceptions that were explored further through the interviews. The interview data and developing themes elaborate findings from the questionnaire, adding the 'meat to the bones' (Bryman, 2016) of the initial findings. Finally, combining both data sets makes the survey data more robust, serving as a checking method for the validity of the questionnaire data. Bringing the evidence together by analysing the questionnaire data and open questions with the interview data for coding themes provides a better understanding of the phenomenon being explored (Teddlie & Tashakkori, 2012).

Descriptive statistics

The background data that included demographic information of participants collected as Part A of the questionnaire can be found as Table 3.3 in Chapter 3. Descriptive statistics were used to analyse questionnaire data Part B: Conceptions of Fluency. Table 4.1 includes descriptions of the dimensions that were the focus of the Likert items based on dimensions developed by Ross et al. (2003) and Table 4.2 contains the Likert items and the dimension they were mapped against.

Table 4.1 Dimensions (D) of Conceptions of Fluency

Description of dimensions, adapted from (Ross et al., 2003)
D6: Student to student interaction How students communicate mathematical ideas, the opportunity for discussion and the promotion of student talk
D8: Conceptions of mathematics Do teachers see mathematics as a dynamic subject (contemporary) rather than a fixed body of knowledge (traditional) regarding students' development of mathematics and fluency
D9: Mindsets and confidence Teachers' mindsets and how these affect student learning in mathematics and a focus on raising student self-confidence in mathematics

Table 4.2 Likert scale items mapped to dimensions

Item No.	**Item**	**Dimension**
1*	A lot of things in mathematics must simply be accepted as true and remembered	D9
2	Mathematical ideas are something that students can discover for themselves	D9
3*	Mathematical fluency relies on students' capacities to remember procedures	D8
4	Students can be mathematically fluent but still not understand the concepts	D8
5*	It is more important for students to be able to get to the answer quickly in mathematics than to be able to reason and explain their answers	D8
6	Students need to be able to communicate with others what they know to be fluent in mathematics	D6
7*	I like my students to master basic mathematical operations before they tackle complex problems	D8
8	I encourage students to explain their strategies	D6
9*	When students are working on mathematics problems, I put more emphasis on getting the correct answer than on the process followed	D8
10*	Fluency is something that develops naturally, it doesn't need to be taught specifically	D9

* Denotes negatively worded item

The participants' scores for each item were added together then averaged for a final score. Negatively worded items were reverse coded for ease of comparison. The results for all 42 participants can be seen in Table 4.3. A low score (<3) indicates a more traditional view of teaching and learning mathematics, and a higher score (>3) indicates a more contemporary view of teaching and reflecting constructivist views of how students learn mathematics. All participating teachers' scores showed a more contemporary view of teaching mathematics and a belief in a constructivist view of how students learn. Of the few teachers that did score an average of 3 or closer to 3, there were some beliefs that indicate a traditional view of mathematics with an emphasis on correct answers and little value placed on communicating knowledge. For example, participant 9 strongly disagreed to both dimension 6 item statements related to student to student interaction (item 6 and item 8). Participant 9 also agreed with the statement that 'a lot of things in mathematics must simply be accepted as true and remembered', this is consistent with their strong agreement to the statement 'When students are working mathematically on mathematics problems, I put more emphasis on getting the correct answer than on the process followed'. However, there are inconsistencies within the responses of participant 9 as on item 5 they strongly disagreed that 'It is more important for students to be able to get to the answer quickly in mathematics than to be able to reason and explain their answer'. This view is more contemporary and is in direct opposition to their response regarding students explaining their thinking in item 8 where they strongly disagreed with the statement 'I encourage students to explain their strategies.' For participant number 28, who also scored 3, responses indicated a more traditional view of teaching mathematics and of how students learn mathematics. On item 3 they agreed that 'Mathematical fluency relies on students' capacity to remember procedures' and on item 9 they agreed that 'When students are working on mathematics problems, I put more emphasis on getting the correct answer than on the process followed.' For participants who scored highly (>4) there was agreement on the growth mindset they held of how students learn mathematics, all participants who scored >4 agreed or strongly agreed with the statement 'Mathematical ideas are something students can discover for themselves' (item 2). These participants' constructivist views of how students learn mathematics is also clear, with all participants strongly disagreeing that 'It is more important for students to be able to get the answer quickly in mathematics than to be able to reason and explain their answers' (item 5). This focus on identifying that there is something more important to mathematics and fluency than quick recall matched the initial themes identified in Part C of the questionnaire and was also recognised in the interview data.

Standard deviation (SD) was also calculated for each item across all participants and is included at the end of Table 4.3. The greatest agreement, with the least variation in responses was on item 5 (SD 0.63) and item 10 (SD 0.58). Item 5 poses that learning mathematics is more importantly about speed of answering questions than explaining. Participants' disagreement with this statement suggests an overall belief that knowledge of skills (procedural fluency) is not as important as understanding of concepts and being able to explain and reason with solutions. Item 10 highlights participants' mindset around mathematical fluency as either a fixed set of skills you may be born with, or something that can to be learned and therefore taught. The majority of participants believed that fluency is something that needs to be learned and taught, linking their responses to a growth mindset in mathematics (Boaler, 2016). Items with the greatest variance in SD provided insight into both participants' differing beliefs (between each other), and the complexity of teachers' conceptions (within themselves) where teachers may hold beliefs that match differing ends of the traditional vs contemporary spectrum.

Table 4.3 Questionnaire results Part B

Dimension	**D9**	**D9**	**D8**	**D8**	**D8**	**D6**	**D8**	**D6**	**D8**	**D9**	
Participant No.	*Item 1**	*Item 2*	*Item 3**	*Item 4*	*Item 5**	*Item 6*	*Item 7**	*Item 8*	*Item 9**	*Item 10**	*MEAN*
1	4	5	2	4	5	4	3	5	5	5	4.2
2	4	4	2	4	5	5	1	5	5	4	3.9
3	4	2	4	4	4	4	4	5	4	4	3.9
4	5	5	5	3	5	5	1	5	5	5	4.4
5	4	4	3	2	4	5	2	5	5	4	3.8
6	2	2	2	4	5	5	2	5	5	5	3.7
7	2	3	3	4	2	4	2	4	4	4	3.2
8	4	2	2	4	5	4	4	5	5	4	3.9
9	2	5	2	5	5	1	5	1	1	3	3
10	2	4	2	4	4	4	2	5	4	4	3.5
11	4	2	5	2	5	4	3	5	4	4	3.8
12	5	2	3	1	5	2	1	5	5	5	3.4
13	2	2	4	5	4	4	1	5	5	5	3.7
14	2	5	4	2	5	5	2	5	5	4	3.9
15	5	5	4	2	5	4	4	5	4	4	4.2
16	4	4	2	4	4	4	1	4	4	4	3.5
17	5	4	2	2	5	2	5	1	5	4	3.5
18	4	4	4	3	4	3	1	5	5	5	3.8
19	4	4	2	3	4	4	2	4	4	4	3.5
20	4	4	4	2	4	5	2	5	4	4	3.8
21	5	2	4	4	5	1	4	4	5	5	3.9
22	4	2	4	4	4	5	4	5	4	4	4
23	4	4	2	2	4	4	2	5	4	4	3.5

Dimension	D9	D9	D8	D8	D8	D6	D8	D6	D8	D9	
Participant No.	*Item 1**	*Item 2*	*Item 3**	*Item 4*	*Item 5**	*Item 6*	*Item 7**	*Item 8*	*Item 9**	*Item 10**	*MEAN*
24	5	4	5	1	5	4	2	4	5	5	4
25	4	4	5	1	5	4	3	4	4	4	3.8
26	4	5	5	1	5	5	5	4	5	5	4.4
27	4	4	4	2	4	2	4	5	4	4	3.7
28	4	4	2	2	4	2	4	2	2	4	3
29	4	3	4	4	5	2	1	5	5	4	3.7
30	4	3	4	4	5	3	2	5	5	2	3.7
31	5	1	5	1	5	5	4	5	5	4	4
32	4	4	4	1	5	5	5	5	5	4	4.2
33	2	4	2	2	4	4	3	5	4	4	3.4
34	4	5	4	4	5	5	5	5	4	5	4.6
35	4	5	4	2	5	4	2	5	5	4	4
36	5	5	4	2	5	2	4	4	5	4	4
37	4	4	3	4	5	4	2	4	5	4	3.9
38	4	4	2	5	5	3	4	5	5	4	4.1
39	4	2	4	4	4	4	3	4	4	4	3.7
40	3	5	3	3	4	5	2	5	4	4	3.8
41	2	2	2	4	4	4	2	4	4	4	3.2
42	4	2	2	2	4	5	1	5	4	4	3.3
Standard Deviation	0.98	1.19	1.12	1.26	0.63	1.17	1.34	0.99	0.83	0.58	

* Denotes negatively worded items that have been reverse coded

Table 4.4 shows the items grouped by their dimensions and contains each dimension's mean for each question. This allowed for comparison across dimensions regarding teachers' traditional or contemporary views. The mean for dimension 6: student to student interaction, was high and indicated more consistent contemporary views of mathematics learning by teachers. The mode has also been included and uses the Likert scale levels to indicate if teachers agreed or disagreed with the statements. Items that were negatively worded (identified by an asterisk) should have been disagreed with if teachers held more contemporary views, however this was not always true. Teachers who scored >4 overall, were identified as holding contemporary views of mathematics teaching, except in their responses to items 7 and 4 about some conceptions of mathematics. For example, item 7 with a SD of 1.34 (see Table 4.3) highlights the existence of differing beliefs by teachers about *when* and *where* explicit teaching of skills in mathematics should occur. The item 7 statement 'I like my students to master basic mathematical operations before they tackle complex problems' resulted in responses from all five of the Likert scale levels indicating it was not a universal belief and is worthy of further discussion and exploration in future research studies. The mode for item 7 indicated most teachers agreed with the statement, which would generally indicate a more traditional view of mathematics. However, some of the teachers who were identified as holding contemporary views, also agreed with the statement.

Table 4.4 Questionnaire Part B organised and analysed by Dimensions

Dimension	*Item*	*Mode (using Likert scale wording)*	*Mean for combined dimension questions*
Dimension 6: Student to student interaction	Item 6: Students need to be able to communicate with others what they know to be fluent in mathematics	Agree	
	Item 8: I encourage students to explain their strategies	Strongly agree	4.1
Dimension 8: Conceptions of mathematics	Item 3: Mathematical fluency relies on students' capacities to remember procedures*	Disagree	
	Item 4: Students can be mathematically fluent but still not understand the concepts	Agree	
	Item 5: It is more important for students to be able to get to the answer quickly in mathematics than to be able to reason and explain their answers*	Strongly disagree	
	Item 7: I like my students to master basic mathematical operations before they tackle complex problems*	Agree	
	Item 9: When students are working on mathematics problems, I put more emphasis on getting the correct answer than on the process followed*	Strongly disagree	3.6
Dimension 9: Mindsets and confidence	Item 1: A lot of things in mathematics must simply be accepted as true and remembered*	Disagree	
	Item 2: Mathematical ideas are something that students can discover for themselves	Agree	
	Item 10: Fluency is something that develops naturally, it doesn't need to be taught specifically*	Disagree	3.8

* Denotes negatively worded items

Item 4 also presented greater variance in teacher responses, its SD of 1.26 made this a focus item for analysis (see Table 4.3). The item stated that 'Students can be mathematically fluent but still not understand the concepts.' The mode for this item was 'agree' however there was an equal number of teachers who chose strongly agree/agree and strongly disagree/disagree (n=19 each) with 4 teachers responding 'unsure'. This signifies that the item caused great division among the participants. There are a number of potential reasons why this is the case. The wording of the item may have resulted in ambiguity, as the word *mathematical* not *procedural*, is used to describe fluency. Mathematical fluency may suggest more than procedural knowledge is needed, therefore participants may have disagreed with the statement believing it is not possible for students to be considered as fluent without understanding. Whereas interpreting mathematical fluency in the item as procedural, participants may have agreed with the statement believing it is possible for students to have only procedural knowledge without understanding. When comparing the responses of this item to other Dimension 8: Conceptions of mathematics items, there was no consistency of score between item 4 and the other items. This item connects to the crux of this present study, if fluency is more than procedural, what role does understanding play in mathematical fluency? Item 4 highlighted an area that required further questioning and assisted in refining the semi-structured interview questions that were posed during stage two of data collection.

Thematic analysis of questionnaire Part C

The two questions in Part C were analysed for diversity of words used to describe mathematical fluency, providing broader perspectives and conceptions of the phenomena and initial features of fluency to explore during the interviews. Table 4.5 shows occurrence of high frequency words. 'Efficient' was the most prevalent word used to describe fluency, 'flexible' and 'understanding' were also used frequently. These terms align to Watson and Sullivan's (2008) definition of mathematical fluency that involves students being able to carry out procedures flexibly, accurately, efficiently and appropriately *with* understanding.

Table 4.5 High frequency words used to describe fluency

Word	No. of times mentioned
Efficient / Efficiency	15
Flexible	11
Understanding	10
Strategies	9
Explaining (strategies)	8
Confidence	8
Recall	7
Accuracy	7
Transfer	7

Of the 15 teachers who mentioned efficiency, 11 also mentioned 'flexible', or 'strategies', or both. Table 4.6 lists the three words these participants chose to explain fluency. The belief in the importance of flexibility along with efficiency in describing fluency suggests that teachers hold similar beliefs adhered to by both Kilpatrick et al. (2001) and Watson and Sullivan (2008). 'Recall' [of facts] was far lower down the list of words utilised by teachers in describing fluency, and 'speed' was not mentioned at all. The place of recall in fluency, and other features teachers listed were initial codes to search for in the interview data. It was interesting to note the inclusion of 'confidence', which links more closely to Kilpatrick's (2001) framework thread of productive disposition, more so than procedural fluency.

Table 4.6 Three words when efficient was mentioned

Participant number	First word	Second word	Third word
4	flexible	accurate	efficient
6	accuracy	efficiency	flexibility
7	recall	efficiently	strategies
8	flexible	efficient	appropriate
14	understanding	competence	efficiency
16	efficient	strategic	solution
18	strategy	transfer	efficient
20	efficient	flexible	strategies
21	efficient	flexible	effective
25	flexible	efficient	strategic
27	efficient	choices	strategies
29	efficient	confident	choice
30	understanding	confident	efficient
32	efficient	accurate	flexible
35	confidence	efficiency	accuracy

* Grey highlights where participant mentioned efficient/ efficiency

When analysing the longer description of fluency participants provided in Part C, there were a number of noteworthy statements. For instance, participant no.6 stated: "able to apply skills to new learning… and… recognise different pathways" as an important feature, indicating the importance of transferability. Participant no.9 mentioned the "ability to communicate reasoning" and participant no.23 mentioned "peer tutoring" suggesting that sharing knowledge was an important feature of fluency. For participant no.13 a "willingness to risk being wrong" was highlighted, aligning to beliefs concerning the significance of productive struggle. There was also mention of students needing to make connections by having "the ability to link concepts" (participant no.35). These aspects of fluency mentioned are additional to current research and curricula definitions of procedural fluency and describe mathematical fluency beyond recall and procedural strategies. These aspects were used as initial codes by which the interview data was explored.

Thematic analysis of interviews mapped to research questions

The 17 interviews were audio recorded and transcribed. Initial analysis was conducted by highlighting key features of fluency that emerged within and across participant responses. Statements and quotes that directly related to the research questions were then highlighted and added to a spreadsheet for further analysis. Table 4.7 provides examples of interview quotes that directly matched the research questions. Codes were created for each interviewee as part of the de-identification process. The teacher codes consist of a two-digit interview schedule number (i.e. the order in which the interviews were conducted), a two-digit code for gender (female 01, male 02) and a two-digit/letter code for grade teaching (e.g. 0K indicates a Kindergarten teacher, 01 indicates a year 1 teacher and so on. One teacher did not indicate the year level they taught, they are represented by 00).

From reading the teachers’ responses, it is clear that mathematical fluency is a complex concept. Teachers have strong beliefs about what fluency is: “it’s about having a strategy that you can use to answer a question efficiently” [16_0100] and what fluency isn’t: “some teachers say it's speed, and it isn't speed to me” [08_0102]. Teachers also described fluency in relation to their students: “she gets it… if we're doing multiplication, she'll make a connection with area” [08_0102] and to other working mathematically processes: “I guess too, part of the fluency is to be able to then reason out why they did that” [16_0100].

When teachers responded to the questions regarding working mathematically, and specifically understanding, the responses suggested that teachers believed understanding was needed to be fluent– understanding may be present without fluency, but not vice versa. Responses included:

> Fluency for me is about understanding and about being able to take the fluency and adapt it from one situation to the next quite easily. They may have a process for getting there but they don't really understand why that process works [07_02K6].

> Sometimes I think my best kids that I've taught that have really deep understanding, lack fluency [05_01K2].

> I think they can have understanding without fluency. But I don't think it can work the other way around [10_0104].

Table 4.7 Sample of interview responses mapped to research questions

What are primary teachers' conceptions of mathematical fluency?	*How do primary teachers define the term 'mathematical fluency'?*	*What knowledge and beliefs do primary teachers have about mathematical fluency?*	
		As it relates to their students	*As it relates to the other working mathematically processes*
"I think it's about having a strategy that you can use to answer a question efficiently." 16_0100	"One of the big markers is the ability to be able to articulate why you did a certain thing, how you got to your answer… a lot of that fluency for me is about understanding and about being able to take the fluency and adapt it from one situation to the next quite easily. That's what I would consider mathematical fluency." 07_02K6 "I've worked with some teachers where they say it's speed, and it isn't speed to me. With me, it's about going sideways… I always like to say, it's from the known to the unknown." 08_0102	"…because if they can't articulate the answer they didn't really understand how they got there. They may have a process for getting there but they don't really understand why that process works." 07_02K6 "It's great that she can make connections… she gets it… if we're doing multiplication, she'll make a connection with area, and she'll make a connection with arrays and - oh, yeah, she's amazing." 08_0102	"Sometimes I think my best kids that I've taught, that have really deep understanding, lack fluency. Because they take the time to really understand each component; it's not a race. Sometimes it's easier to have kids who struggle a little bit, because they have to think about the process, rather than the product." 05_01K2 "A mathematician to me is someone who sees those patterns, understands the usefulness of a particular strategy… I think they can have understanding without fluency. But I don't think it can work the way around. I think if you're fluent in maths you're going to have the understanding with it." 10_0104 "I think there's a level of logic as well. That they need to be able to logically look at it… knowing which strategy to apply in which situation. I guess too, part of the fluency is to be able to then reason out why they did that." 16_0100

Thematic analysis of interviews mapped to teacher conceptions framework

Data was also mapped against the Teacher Conceptions Framework's (Figure 2.2) influencing factors of: teachers' own educational experiences, social experiences, knowledge of their students in their classroom and their knowledge of content and pedagogy. Many teachers talked in terms of their own learning when defining and providing examples of mathematical fluency. One teacher commented:

> When I went through high school - Year 7, Year 8 and 9 were a breeze. They were really easy. Then when I found I got to Year 10, the concepts got harder. But I still had that fluency with the number because I knew how to use numbers and I knew how to use maths. But then I had to change a bit because those concepts were harder. So, it made me work a little bit harder. 11_0104

There was also mention of social factors that affect fluency and other social experiences were used as a parallel. For instance:

> For whatever job they're going to do, which might not even have happened yet, created, then they would need to have that fluency in that area, in life outside of school. 15_0134

> It all depends on what task they're actually attempting. I can be a fluent reader, but if you give me a medical text book I'm going to slow down and my fluency disappears. I think it's just the challenge of the text that's in front of you, whether it's mathematical text or literacy text. 13_01K2

When describing examples of fluency in students, some teachers used their classroom or student learning experiences to frame their responses. One teacher related fluency to student thinking stating:

> The power of being able to model to the kids what it looks like to be fluent … and how powerful it can be in expressing it as well as understanding it. I say to the teachers, if you can record for the student what they're thinking is, you're showing what fluency looks like in black and white. 13_01K2

Teachers' knowledge of content and pedagogy came to the fore in their responses where a strong focus was placed on syllabus knowledge and the positive effects of professional learning they had experienced regarding mathematics. Some teachers associating beliefs to their own learning:

> Early intervention programs have that ability for the students to learn how to reflect on their learning [which] has huge power. Because it doesn't just increase their fluency, and their accuracy. It gives them the ability to go, I made a mistake and this is where I think I made the mistake, and this is where I think I need to correct it. 05_01K2

> I think as an educator, if you ask the right questions, then you can tease our where their head's at. If you just keep asking the yes, no questions, then you're not going to understand. You need to ask the why and the how. 08_0102

The influencing factors represented in the teacher conceptions framework, used as themes for analysis, were evident across all interviewee data. There were many similarities in teacher responses once the data were organised according to these factors. Examples of these similarities included: a personal passion for teaching mathematics (teachers' own experiences), making connections between fluency in reading or writing and fluency in mathematics (social contexts), identification of fluency as important in mathematics (content and pedagogy knowledge), and fluency as a way of expressing and sharing mathematical knowledge (student/ classroom experiences).

Thematic analysis of interviews for emerging themes

Numerous themes emerged from the interview data including: fluency as fluidity, no need for speed, not being stuck, curriculum parallels, language barriers, and working through errors.

Fluency as fluidity

Whether it was by hand gesture or verbal response, teachers defined fluency as fluid, having a "smoothness in terms of how did they get to that answer" 12_0112 and flowing "so, it all flows together" 08_0102. Indicating its ongoing nature and the need for fluency to eventually come naturally, to be second nature. There is less stopping and starting.

> So, if you don't have a strong understanding of number, then that's going to throw you out in most maths concepts. I think that once you understand how to use numbers, then everything sort of just flows. 11_0104

No need for speed

Teachers mentioned words like 'recall', 'automaticity', 'with ease', 'mechanics' and 'accuracy' when defining fluency. Some mentioned 'quickly knowing' however the emphasis was always on the need for understanding as well as 'quick recall', such as: "it's not a matter of being really fast. Like five plus five is 10. But it's just being able to understand that concept and not needing to have to slow down." 11_0104. Interestingly, when teachers did mention speed, the speed was in reference to the student knowing what to do (the choice of efficient strategy) not speed in solving the problem (getting an answer quickly).

> It's the idea of how to do it that has to be the fast thing. It may take a little while for them to solve the question because they've got to put this part together and that part together and this part together and that part together. 04_0104

Seeing speed more as an 'ease with mechanics' and with less cognitive load. Seeing fluency as "more that ability to be able to apply because you're not bogged down in the mechanics of trying to work out what this is versus that is" 07_02K6.

Not being stuck

All 17 teachers' responses included a statement that emphasised that at some point in learning mathematics, without fluency, students will struggle or "get stuck". These sticking points then become barriers for student development and achievement.

> Where they get stuck it's that question of, what do I actually have to do here? If they can't do that they can't apply which means they don't have fluency. 07_02K6

> They can rattle off their times tables in two seconds flat. I put it into real-world problems. Yeah, that's where it comes unstuck. 08_0102

> That's when they get unstuck it's not that they can't do it, but they can't really tell you the why. 05_01K2

> I can tell that they're not confident or comfortable with what they're doing. So straight away, I know there's a little bit of a struggle. 11_0104

> A lot of kids in my class struggle with place value and so they're stuck because that's really a key concept. 10_0104

> Whereas without that fluency, they're kind of stuck on that mechanical stage. 09_0112

Curriculum parallels

Teachers also drew parallels to reading fluency when defining fluency and making the distinction between the mechanics of mathematics and mathematical fluency. For instance:

> The parallel to reading: barking at print, as opposed to saying yeah, I know four and six is 10, but what does that mean? What does that look like and how can I use that. 12_0112

> It's the same as if you have to sound out when you're reading. You know the A-N-D goes together and makes ‘and’ but if you have to chunk it apart first and then read it, well then that's not fluency. 17_0103

> Because if it wasn't so much I'm bogged down in the process of decoding what's on the page I'm now actually able to focus on the meaning of what's going on. 07_02K6

Language barriers

Being literate in English and understanding the language of mathematics, plays a vital role in a student’s ability to read, comprehend, interpret and solve mathematical problems (Newman, 1977). For students from a non-English speaking background, this may cause a potential barrier in developing mathematical fluency, as was highlighted by teachers in their responses. For example, when students are solving word-based problems “you have to make sure that you're not allowing their English acquisition to be the barrier to demonstrating their mathematical skills.” 10_0104

Working through errors

Learning through making mistakes and being able to try alternative strategies was another theme that emerged from the teacher interviews. Teachers highlighted the importance of communicating where “most kids find their errors through talking about it. If they don't talk about it, they often just think they're right” 14_0156. Teachers also valued students eventually finding a correct solution through perseverance as "I think it's good for them to know that they can make a mistake, but work out what the mistake is and then fix it" 15_0134. An emphasis was also placed on learning from mistakes across tasks, for example one teacher shared that their student "was able to notice errors in his other work" 05_01K2.

Thematic analysis by combining data

Interviews mapped to questionnaire dimensions

The three dimensions from Part B of the questionnaire were used as a lens to examine the interview data. The narratives teachers used to describe their students were categorised into the three dimensions of: teacher conceptions of mathematics, student to student interaction, and mindsets and confidence. Illustrations of teacher responses mapped to the Likert dimensions are provided in Table 4.8 and Table 4.9. Many of the teachers' responses to the interview questions could be mapped directly to one or more of the questionnaire dimensions. A greater number of examples were found for dimensions 8 and 9 whereas a limited number of examples could be mapped to dimension 6: student interaction. The analysis of each teacher's interview responses to the dimensions from the questionnaire was for the purpose of exploring whether teachers' traditional or contemporary views of mathematics were evident in their interview responses. Regarding contemporary views, interviewee 03_010K had a mean score of 4 overall, suggesting a contemporary view of teaching mathematics. This view is consistent with the same teacher's responses as can be seen by the examples in Table 4.8. Whereas for interviewee 10_0104 who had a mean score of 3.8 overall suggesting a more traditional view, their interview responses were more contemporary (see Table 4.8). There was also some evidence of traditional views of mathematics within each dimension. However, these views were mostly evident when mapped to dimension 9: mindsets and confidence (see Table 4.9) where there was evidence of a fixed mindset of mathematics by some teachers. Interestingly though, teachers who scored <4 overall in the questionnaire, suggesting a more traditional view of mathematics, still had a mixture of responses during the interviews that linked to both contemporary and traditional views. This indicated that for some teachers it is not one view or the other, some teachers held more traditional views of mathematics as a subject of study but contemporary views of learning mathematics.

Table 4.8 Interview responses (contemporary view) mapped to questionnaire dimensions

	Interviewee 03_010K	**Interviewee 10_0104**
D6: Student to student interaction	"I think conversations are very important, and listening to what children say when they're working, [doing] that little wonder and listen to what they're - their conversations with each other."	"She can peer tutor other students and adjust her language so she can help them to understand. She can also draw her ideas which can be very useful if she's doing some peer tutoring."
D8: Conceptions of mathematics	"I'm a person who believes in not teaching formulas to children. They have to understand what it means, and you don't have to remember a formula."	"In some ways, you are looking for their ability to recognise the patterns and to make the connections. You want them to make connections to themselves and to the world and to other things that they've seen."
D9: Mindsets and confidence	"Sometimes we will kill it by making it by rote. It's not like you can go out the front and say C-A-T, cat, C-A-T - oh okay, got that. You can't do it by rote- mathematics is understanding. You can't learn mathematics by rote… I think - that the children that I've met - I think it's more about their confidence."	"I think it's a change in your mindset as well. I think that those students who can make those connections and recognise those patterns and communicate their ideas clearly just have more confidence and get more engaged in the learning process than the ones that get stuck."

Table 4.9 Interview responses (traditional view) mapped to questionnaire dimensions

D8: Conceptions of mathematics	"I've always known, you have to learn your tables, you have to know what to do, but I've never really thought about it in terms of having fluency." 06_0105	"They have to understand the rules, then they can manipulate them" 03_010K	"I've taught for many, many years and it used to be just pages of sums on a piece of paper, and a lot of teaching from textbooks. But with thinking mathematically, there's a lot more involved" 02_010K
D9: Mindsets and confidence	"Some children have a brain wired to literacy and other kids just have brains that are just wired to maths. Maths makes sense to them and the - it just happens. They just get it. So, they're very, very happy to work that way." 17_0103	"I don't think you can necessarily help all of them." 10_0104	"…because there's ability as well. There's some kids who will always struggle. But if they have processing systems that can help them tackle the struggle, then they can still be successful." 05_01K2

Interviews mapped to initial definition codes

The definition codes identified from Part C of the questionnaire were also evident in interviews when teachers described students from their class who had mathematical fluency. Figures 4.1 and 4.2 are excerpts from two of the interviews with the definition codes identified within the text. Emerging themes from the interview data have also been coded in the text.

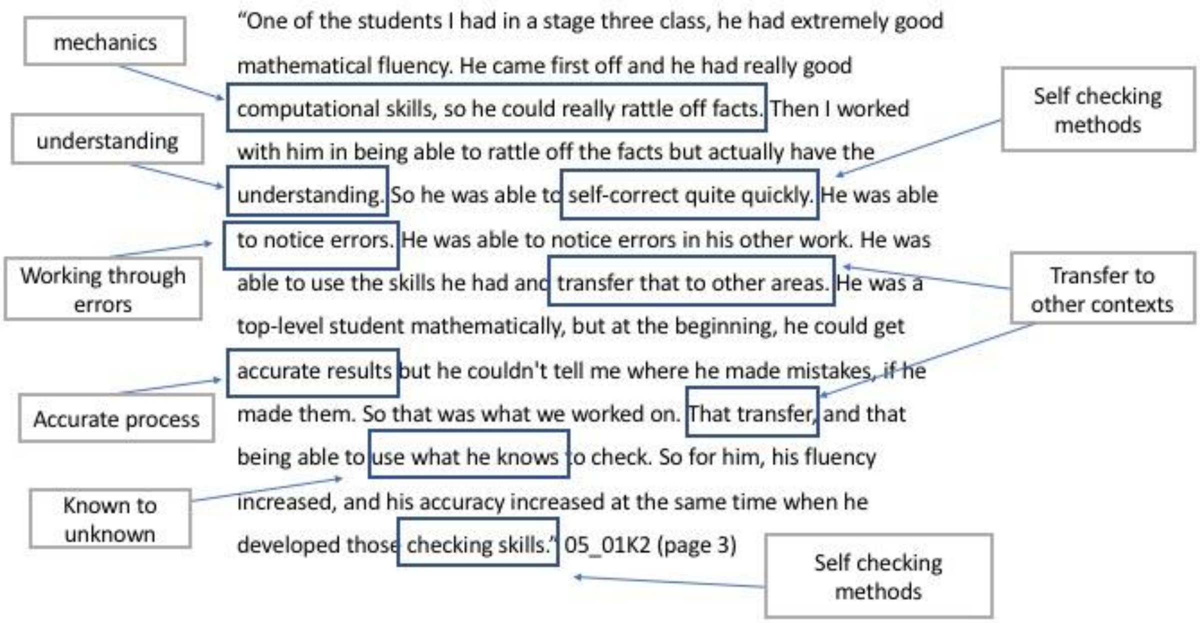

Figure 4.1 Excerpt from 05_01K2 interview mapped to definition codes

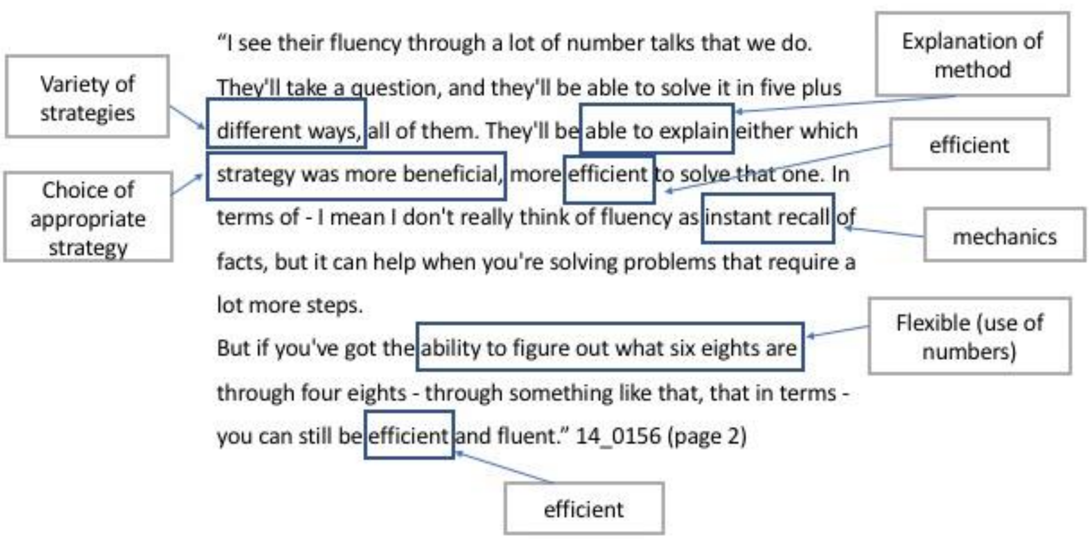

Figure 4.2 Excerpt from 14_0156 interview mapped to definition codes

In summary, the results within this chapter have shown teachers in both the questionnaire and interviews defined fluency as inclusive of an extensive list of characteristics. Often citing similar words related to the strategic competence students display (efficient, different, choice), the conceptual understanding they communicate (understanding, making connections, explaining) and the adaptive reasoning they use (transferring, working through errors, self-correcting). These characteristics have been aligned (in Table 4.10) to three of the five strands in Kilpatrick et al.'s (2001) intertwined strands of proficiency (Figure 2.1).

Table 4.10 Fluency characteristics chart

Strategic competence	Multiple strategies Variety of strategies/ ways Choice of/ identification of appropriate strategy Accurate process (articulation) (Ease of) mechanics- *automaticity* Fluidly (switch between strategies)
Conceptual understanding	Comprehension Making connections between concepts (known to unknown) Flexible use of numbers and their relationships Explanation of method (the how) Sharing strategies [with peers] (communicate)
Adaptive reasoning	Justifying strategy or method (the why) Transfer to other contexts or problems (application into new situations) Self-checking method (reasonableness) Working through errors

Teachers' conceptions of fluency are influenced by both internal factors (own educational experiences and personal knowledge and beliefs of content and pedagogy) and external factors (knowledge of students in their classroom, their social experiences and school context) that in turn become a means by which they frame their definitions and describe students' abilities. By using these factors as a lens to analyse data each teacher's past and present experiences were highlighted, often justifying *the why* in regard to their knowledge and beliefs about fluency.

The use of both questionnaire and semi-structured interviews allowed for the collection of rich, thick, descriptive data (Lincoln & Guba, 1985) analysed to show interconnected ideas and themes using teacher quotes. This provides robust findings that can be transferred into other contexts and settings in future research studies.

Chapter 5 Discussion and Conclusion

Within this chapter the key findings will be discussed and in the context of previous research studies. Future research will be suggested and limitations of the present study will be addressed.

Meeting the goals of the research questions

The current study provides evidence that primary teachers *are* speaking the same language when talking about mathematical fluency. Many of the participants in the questionnaire used similar words and provided similar characteristics when answering the research question, *how do primary teachers define the term 'mathematical fluency'?* Terms used were consistent with research definitions (Kilpatrick et al., 2001; Watson & Sullivan, 2008) of fluency (efficiently, flexibly, appropriately) and some shed new light on what fluency encompasses (risking being wrong, peer tutoring, making connections).

There were also common themes from the interviews regarding the research question, *what knowledge and beliefs do teachers have of fluency as it relates to their students?* The examples of student behaviours shared during the interviews indicated the complex nature of fluency that stretched far beyond efficiency with procedural knowledge.

In the case of the research question, *what knowledge and beliefs do teachers have of fluency related to other working mathematically processes?* the notion of fluency needing understanding was prevalent in all of the interviewee's responses. It is clear that fluency, from teachers' perspectives, is determined by a student's ability to apply, and demonstrate or transfer knowledge, for example, in problem solving tasks.

5.1 Discussion

Fluency as proficiency

Previous studies regarding reading fluency indicated that "language researchers have offered countless different aspects that contribute to defining fluency as an overall oral proficiency in speech" (Götz, 2013). Why has fluency as an *overall proficiency* not been adapted to mathematics? The findings in this present study suggest that teachers believed fluency was an overall proficiency of mathematics. This is supported by the range of characteristics teachers used to describe mathematical fluency and the multiple ways in which they saw fluency enacted by their students. Fluency *is* mathematical proficiency. This belief moves beyond a definition pertaining solely to procedures, beyond compartmentalising fluency as one aspect of mathematical proficiency. Being mathematically fluent is the *result* of, to adapt a phrase, 'having all your "mathematical" ducks in a row'. This thinking is not in opposition to Kilpatrick et al.'s (2001) intertwined strands of proficiency (see Figure 2.1), but an adjustment to the point of view from which those strands are viewed, see Figure 5.1 for a reframing of fluency.

Reframing fluency

Viewing fluency as the *product* of multiple characteristics, results in the other strands of Kilpatrick et al.'s framework becoming the necessary skills and processes for *acquiring* fluency. Once fluency is shifted from being one strand in the rope, to being the rope, many of the responses teachers made regarding fluency and understanding fall into place.

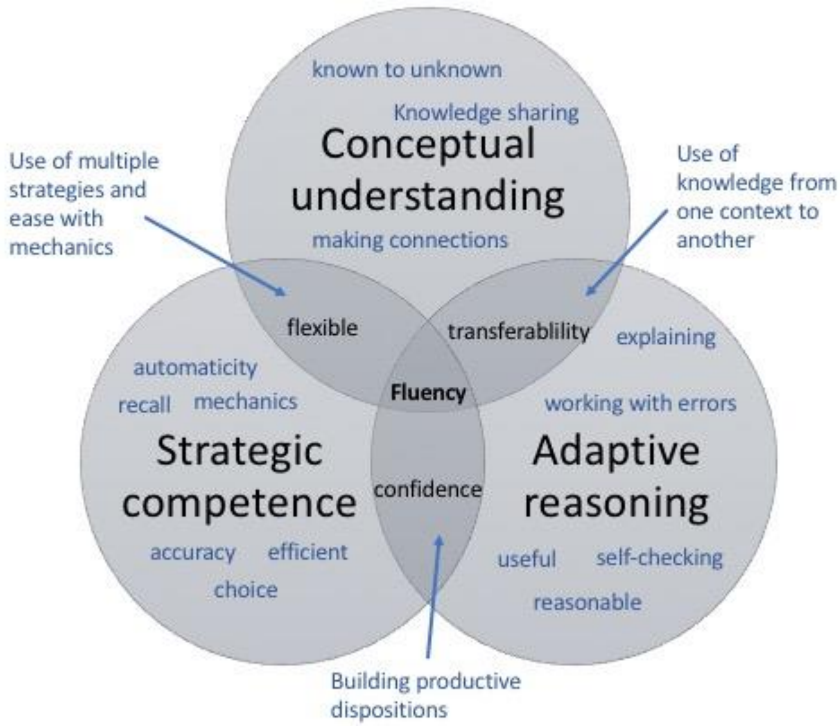

Figure 5.1 Reframing fluency

Teachers' descriptions of fluency mapped easily to the other strands of strategic competence, conceptual understanding and adaptive reasoning. A representation of these characteristics was shown in Table 4.10 where themes that emerged from the questionnaire and interview data were aligned to the three strands utilised in Figure 5.1 Reframing Fluency based on Kilpatrick et al.'s (2001) intertwined strands of proficiency. Thematic analysis in this present study has enabled the identification of specific characteristics of fluency that potentially could be referenced by teachers when assessing student conversations, work samples or collaborative group situations. Potentially assisting teachers in identifying aspects of fluency students possess and aspects of fluency yet to be developed.

Key fluency features

The findings showed that teachers made a clear distinction between the students they thought had fluency, compared with students who did not have fluency. Stating that students who often possess procedural fluency or have learned content by rote (e.g. times tables) are not truly fluent. These students often became 'stuck' when questions were asked differently or when the students were required to adapt or transfer these skills to new situations. Whereas students that teachers felt were fluent, had the procedural knowledge as well as understanding, knowing what to do with the processes and knowing which strategies were more efficient in different situations. This is consistent with Watson and Sullivan's description of mathematical fluency (Watson & Sullivan, 2008).

It is therefore suggested that the term *procedural fluency* may give a false impression that knowing procedures equates to being mathematically fluent. As mentioned in the Literature Review, this results in many children being *trained* to do mathematical calculations rather than being *educated* to think mathematically (Noyes, 2007). More appropriate terms to use when referring to procedures could be 'procedural ease' or 'mechanical ease' or 'automaticity' as procedural literacy (Shellard & Moyer, 2005). These terms still emphasise the importance of developing procedural knowledge which is necessary to solve more complex tasks and problems. However, what *is* needed is a shift from fluency being a descriptive term of how students follow the procedure itself, to fluency describing the thinking in choosing and using the procedure. This shift was highlighted in Chapter 4's theme of *No need for speed* where interviewee 04_0104 is quoted "it's the idea of **how to do it** that has to be the fast thing."

The findings from this present study contribute to current research that is moving the focus away from speed equaling fluency (or vice versa), "fluency comes about when students develop number sense, when they are mathematically confident because they understand numbers… speed and memorization are two directions that we urgently need to move away from, not towards" (Boaler, 2015, p. 5).

When focusing on the specific students the teachers thought had fluency (students who were not getting stuck) the word *flow* was used several times. The alternative to 'stuck' seems to be 'flow' or 'fluidity'. It was surprising to hear the word flow used by teachers as it is not a term generally used in the research literature regarding mathematical fluency. Teachers used the term in both their definition of fluency and when describing fluent students. Teachers emphasised that mathematical fluency is fluid and has motion, yet this flow may be stopped, if or when errors occur. This view sees mathematical fluency as changeable, adaptable and flexible, relying on students' abilities to move through errors to continue learning and developing fluency.

Understanding for fluency

It was clear from the responses in the interviews that it was possible for students to have understanding without fluency but not the other way around. When looking at the Venn diagram representation of fluency in Figure 5.1 this belief can hold true. Students may have the mechanics of mathematics as part of strategic competence, but be lacking the knowledge of when to use the procedures, as part of conceptual understanding. Kilpatrick et al.'s (2001) description of procedural fluency echoes this belief that separating procedures (skills) from understanding can have dire results, "students who learn procedures without understanding can typically do no more than apply the learned procedures, whereas students who learn with understanding can modify or adapt procedures to make them easier to use" (p. 124).

Teachers' conceptions: personal and professional

Another strength of the thematic analysis was the use of the Teacher Conceptions Framework (Figure 2.2) as a lens to explore internal and external factors that might have influenced teachers’ knowledge and beliefs concerning fluency. Teachers generally made links to their own learning, social influences, their classroom practice and their students to justify their beliefs. It was interesting to note the shift in thinking regarding both mathematics teaching and mathematics learning that the teachers went through. Some indicated their shift in beliefs were based on their own, often negative, recollections of school mathematics learning (internal factor). Whereas several mentioned the shift in their beliefs had happened over the course of their teaching career, describing differences between their earlier teaching styles compared to now (external factors). Often teachers’ justification of their conceptions of mathematical fluency were based on examples of students experiencing success in mathematics, the way fluency developed in their students over time or by making connections to other ‘fluencies’ such as writing or reading. Observing the data through the teacher conceptions framework’s internal and external factors exposes a complexity to teachers’ conceptions. It therefore cannot simply be stated that teachers held to one fixed belief about fluency. However, what can be noted is that as teachers’ experiences change and evolve over time in their careers, within their personal lives and about their understanding of students, so too do their beliefs and knowledge. This is consistent with Thompson’s review of research into teachers’ conceptions where she quotes Carpenter, Fennema, Peterson, Chiang and Loef’s (1989) findings that stated “teachers’ beliefs and practices underwent large changes when teachers learned about children’s mathematical thinking” (Thompson, 1992, p. 261).

Influences of traditional or contemporary views

Although the results from the questionnaire indicated that most teachers held contemporary beliefs about mathematics, a few items divided the group that may impact on their conceptions of mathematical fluency. One item of note from Part B of the questionnaire that requires further analysis involved traditional lesson structures compared with contemporary teaching approaches, *item 7: I like my students to master basic mathematical operations before they tackle complex problems.* Teachers held opposing beliefs regarding this concept. These results possibly indicated that those who agreed with the statement felt that students needed some understanding, through explicit teaching, prior to applying knowledge. Whereas those who disagreed with the statement may believe student understanding can be achieved by teaching through problem solving where students construct their own knowledge (von Glasersfeld, 2008). There exist contrasting views on whether beginning a lesson with a more cognitive task is valuable for student learning (Kirschner, Sweller, & Clark, 2006; Stein, Engle, Smith, & Hughes, 2008). Multiple factors play a role for teachers in deciding whether explicit teaching is required for students prior to exposure to problem solving, such as the belief that the work may be too hard and students disengage (Leikin, Levav-Waynberg, Gurevich, & Mednikov, 2006). Russo and Hopkin's (2017) research explored the notion of task-first versus teach-first in lesson structure. The results of the study focused more on student motivation and preferences for either of the lesson structure. The study also debated that lesson structure may have an effect on student learning experiences and outcomes. The idea that explicit instruction does not always have to proceed challenging problems is one worth further study particularly as it may be influenced by teachers' views of teaching and learning regarding fluency and other working mathematically processes. When using this same item, Ross et al. (2003) observed that teachers who agreed with the item statement had classroom lessons that were teacher-directed and focused primarily on finding solutions with limited discussion of reasons. This conclusion is not being suggested for the teachers in this present study, it only highlights the need for further research involving classroom practice to confirm ideas. Discussion with, and observation of practicing teachers would be beneficial, as there are still questions to be answered regarding whether agreement or disagreement with the statement in item 7 is reflective of traditional or contemporary classroom practice.

5.2 Limitations and future research

There are a few limitations to this present study. The study was limited by the participant sample size, with only a small number of teachers (n=42 for the questionnaire and n=17 for the interviews) therefore findings may not be representative of the beliefs of the wider teaching population. Although this is a small sample size, effort was made to ensure teachers from every grade in the primary school structure were included in the interview data collection. The teachers self-nominated to be part of the study leading to a possible skewed representation of the general teaching population. Teachers who agreed to be interviewed may have had a passion for teaching mathematics and felt confident about their mathematical beliefs and knowledge enough to be interviewed. It emerged that mainly teachers with contemporary views of mathematics were willing to participate in follow up interviews. It is speculated that this may have been due to working mathematically, and therefore fluency, being a newer term that may not have been familiar or important to teachers with more traditional views. It may be overstating the confluence of evidence when the teachers' responses in the interviews were interpreted as confirmatory of their traditional or contemporary views of mathematics as indicated in the questionnaire. Some of the items in the Part B of the questionnaire could have been worded more clearly as it depended on how teachers interpreted the question as to how they responded, for example, *item 4: Students can be mathematically fluent but still not understand the concepts* is slightly ambiguous. It could be interpreted as a statement of fact rather than seeking whether teachers agree or disagree that it is acceptable to have fluency without understanding. It could be interpreted as yes, I agree that students can have fluency without understanding, and is therefore acceptable to only focus on fluency. Or it could be interpreted as yes, some students seem fluent, but do not really understand the concepts, and is therefore not acceptable. Whereas, the question as it is stated was seeking to discover if it is *possible* to have fluency without understanding. Some teachers indicated in the interview that they disagreed with the statement as they didn't consider students fluent if they didn't also have understanding, causing inconsistency in the item data. Also, although the interview data may be seen as a validation of their conceptions shared from the questionnaire, a direct correlational of these conceptions to their classroom practice cannot be made. Previous research concludes that teachers' espoused traditional or contemporary beliefs may differ from their classroom practices. Philipp (2007) summarises Thompson's comments on the relationship between teachers' conceptions and their instructional practices:

> Thompson cautioned that inconsistencies between professed beliefs and instructional practice raise a methodological concern related to how beliefs or conceptions are measured, and she suggested that researchers must go beyond teachers' professed beliefs and at least examine teachers' verbal data along with observational data of their instructional practice or mathematical behaviour. (A. G. Thompson, 1992, p.135)

Another potential limitation is confirmability of students' fluency. As no student observations or interviews were undertaken, validating the descriptions teachers provided of students who they believed were fluent is difficult. Therefore, questions could still be explored to confirm that the characteristics of fluency stated by the teachers are actually present in the students. It should be noted that the descriptions teachers provided were detailed and provided numerous examples of varied aspects of fluency regarding a specific student and may be generally reliable data.

There is further research that can be undertaken in exploring mathematical fluency that can address the identified limitations. As the findings of this present study pertain only to teachers, a logical next step in the research of mathematical fluency would be to study *fluency in action* with students. Utilising the fluency characteristics chart (Table 4.10) to research collaboratively with teachers to observe students working on tasks, discuss with students their reasoning and strategies, and analyse student work samples for characteristics of fluency. This would assist in confirming teachers’ conceptions of mathematical fluency with student behaviour and potentially confirm the reframing of mathematics proposed in Figure 5.1 of this present study.

Further research could also be conducted surrounding the concept of flow or fluidly in mathematics and its associated role as a feature of mathematical fluency. Teachers indicated the importance of working fluidly in mathematics and not becoming ‘stuck’. When students are not fluent, or are not displaying flow of thought when working through mathematics tasks, it may be an indicator of stumbling blocks or misconceptions students possess. These ‘pauses’ in a student’s flow may also be caused by other factors beyond mathematical content knowledge. Students may become ‘stuck’ due to their beliefs or attitudes towards mathematics or from anxiety. Mathematics anxiety due to limited experiences of success, or boredom in class due to lack of challenge in tasks, are also areas of interest in current research (Ashcraft & Krause, 2007; Frenzel, Pekrun, & Goetz, 2007) and may be factors that have an effect on a student’s mathematical fluency.

Suggesting future research on fluency as fluidity and its influencing factors, segues into teacher and student mindsets of mathematics learning and how this may in turn affect students’ fluency. This is another area of research that would enrich the findings of this present study by further probing into teachers’ conceptions. Specific focus could be placed on dimension 9: confidence and mindsets from Part B of the questionnaire, and could be broadened to be inclusive of student, parent and community as well as teacher mindsets of mathematics and how these mindsets foster mathematical fluency.

5.3 Conclusion

The results of this present study, and further research into mathematical fluency provide an opportunity to change how fluency is perceived, by educators, students and the wider community. The purpose of this study was to explore primary teachers' conceptions of mathematical fluency. The depth to which the teachers explained their thinking and justified their ideas through student examples provided a rich tapestry representative of the complex nature of fluency, and mathematics itself.

There existed a gap in the research between researcher definitions of fluency and evidence of how teachers viewed and interpreted the characteristics of these definitions. This present study gave a voice to teachers in unfolding mathematical fluency and how, or if, their ideas aligned to the current research definitions. It was apparent that much of what teachers said matched various definitions of fluency and gave strength to the broader definition of fluency as pertaining to carrying out procedures flexibly, accurately, efficiently and appropriately (Kilpatrick et al., 2001). These characteristics however, were just one small aspect of teachers' conceptions of fluency.

Any notion that fluency was merely about procedural knowledge was ruled out by all interviewees within the first few questions asked. Their descriptions of mathematically fluent students as self-reliant, fluid transformers of knowledge showed not only the teachers' confidence in their student's skills, but also the teachers' strong belief in fluency with understanding. This belief supports Watson and Sullivan's definition (2008).

Along with these characteristics, newer ideas emerged. Teachers placed importance on self-checking methods, working through errors, switching between strategies, explaining the *how* with the *why* and sharing strategies with peers. Of note was teachers' view of being able to do things quickly, with automaticity. At face value this could have been aligned to traditional teaching styles of rote learning or the wider communities' view that mathematics is all about knowing facts fast, but this wasn't the focus. Teachers spoke of the quickness in *finding* appropriate ways and methods to solve the problem, not speed with the solution itself. This belief is associated more closely with what mathematicians do, using strategies quickly, with ease, without having to resort to basic structures while possibly taking years to solve one real-world problem.

Other parallels were made by teachers that added to a move away from *procedural* fluency to encompass understanding, for example links to reading. Relating mathematical fluency to fluency in reading emphasised fluency beyond decoding. If someone knew every word in a book but could not make connections between the words to make meaning, then it's not really reading. Mathematics is about making sense, sense of numbers, sense of patterns, sense of problems, to be able to do this more than procedures are required. This strengthens the notion that true fluency is a result of understanding and therefore a new perspective of fluency would be beneficial for teachers' understanding of what is important in mathematics learning, and the nature of how understanding and fluency interact.

It was clear that teachers regarded mathematical fluency as proficiency in mathematics. This has implications for teachers, students and curriculum developers. Teachers need a consistent way of viewing fluency with a set of characteristics that can be identified, assessed and developed as conceptual understanding progresses. Students need clear advice on what is valued in the mathematics classroom, that it is understanding and flexibility, not speed and memorisation. Curriculum developers need to place a greater focus on fluency as the outcome not the output. It is the outcome of having conceptual understanding, strategic competence and adaptive reasoning not the output of processes and procedures alone.

Many teachers used analogies to explain fluency such as learning to drive, reading medical text, building a house and shopping, for me it's swimming. To be a successful Olympic swimmer is to obtain the fastest time, but being the fastest, or most efficient, does not equate to singularly practicing for speed or practicing at speed. Watching squads train, they train each element of each stroke slowly first, maybe it is just freestyle kick, or backstroke arm techniques. They then start making connections between elements of stroke, say for breaststroke combining hand positioning and arm placement. Then, as they become more confident in each of these elements, and as the elements are combined together, then the speed and accuracy develop, naturally as the result. To me this is just one example that mirrors how mathematical fluency should be developed in students. Other analogies would tell similar stories, fluency in playing piano, in learning how to ride a bike or in becoming fluent in French. Why then has it been so different for so long in mathematics? Different to the point where the term mathematics anxiety is common place in our primary schools, places where learning should be most exploratory, most creative and most enjoyable. Changing the perspective of fluency in mathematics and highlighting how learning mathematics with understanding produces fluency is a start. For this shift to occur it needs to be in conjunction with a shift in the way we assess mathematical fluency and what about fluency we assess. If reading fluency was only assessed by a student's ability to recall spelling words how would teachers and parents react?

Exploring teachers' conceptions provided a window into their classrooms and into their beliefs and knowledge not just about fluency but about mathematics learning and mathematics itself.

Sometimes I think my best kids that I've taught, that have really deep understanding, lack fluency. Because they [the students who lack procedural fluency] take the time to really understand each component; it's not a race. Sometimes it's easier to have kids who struggle a little bit, because they have to think about the process, rather than the product. 05_01K2

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Appendices

APPENDIX A: Questionnaire

Mathematics questionniare

Page 1 of 2

Please complete the survey below.

Thank you!

Thank you for your interest in participating in this questionnaire. Please read the Participant impact statement prior to completing the questionnaire.

School type	☐ primary ☐ infants ☐ central ☐ SSP
School location	☐ metropolitan ☐ rural or remote
School size	☐ P1 ☐ P2 ☐ P3 ☐ P4 ☐ P5 ☐ P6
Highest level of teacher education qualifications	☐ bachelor ☐ diploma of education ☐ masters ☐ PHD ☐ 2 or 3 year teacher's diploma ☐ other
What other qualifications do you have?	__________________________________
Teacher classification level	☐ classroom teacher ☐ assistant principal ☐ deputy principal ☐ principal
Gender	☐ female ☐ male
How long have you been teaching?	☐ first year ☐ 1-3 years ☐ 4-5 years ☐ 6-10 years ☐ 11-15 years ☐ > 15 years
How long have you been teaching at your current school?	☐ < 12 months ☐ 1-3 years ☐ 4- 5 years ☐ 6- 10 years ☐ 11- 15 years ☐ > 15 years
What grade do you currently teach?	☐ Kindergarten ☐ Year 1 ☐ Year 2 ☐ Year 3 ☐ Year 4 ☐ Year 5 ☐ Year 6

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Conceptions of mathematics

	Strongly disagree	Disagree	Unsure	Agree	Strongly agree
A lot of things in mathematics must simply be accepted as true and remembered	☐	☐	☐	☐	☐
Mathematical ideas are something that students can discover for themselves	☐	☐	☐	☐	☐
Mathematical fluency relies on students' capacities to remember procedures	☐	☐	☐	☐	☐
Students can be mathematically fluent but still not understand the concepts	☐	☐	☐	☐	☐
It is more important for students to be able to get to the answer quickly in mathematics than to be able to reason and explain their answers	☐	☐	☐	☐	☐
Students need to be able to communicate with others what they know to be fluent in mathematics	☐	☐	☐	☐	☐
I like my students to master basic mathematical operations before they tackle complex problems	☐	☐	☐	☐	☐
I encourage students to explain their strategies	☐	☐	☐	☐	☐
When students are working on mathematics problems, I put more emphasis on getting the correct answer than on the process followed	☐	☐	☐	☐	☐
Fluency is something that develops naturally, it doesn't need to be taught specifically	☐	☐	☐	☐	☐

What three words best describe how you would define 'mathematical fluency'? ____________________

What key features are observable/ present in students that have mathematical fluency? ____________________

Do you agree to provide your email address to be contacted for a follow up interview?

☐ yes I am willing to be contacted
☐ no I would prefer to remain annoymous

Please provide your contact email address ____________________

APPENDIX B: Interview schedule

Questions to address key concept: Definition

1. WHAT SUBJECT/ LEARNING AREA DO YOU THINK OF WHEN YOU THINK ABOUT FLUENCY?

2. HOW WOULD YOU DEFINE *MATHEMATICAL* FLUENCY?

Question to address key concept: Features

3. WHAT FEATURES WOULD YOU LOOK FOR IN STUDENTS TO DECIDE THEY HAVE MATHEMATICAL FLUENCY?

Questions to address key concept: Examples

4. CAN YOU GIVE ME AN EXAMPLE OF A STUDENT IN YOUR CLASS THAT YOU THINK HAS MATHEMATICAL FLUENCY?

-DESCRIBE WHAT CHARACTERISTICS THIS STUDENT DISPLAYS

5. DO YOU THINK THESE FEATURES (OF MATHEMATICAL FLUENCY) ARE THE SAME FOR EVERY GRADE?

-CAN YOU PROVIDE SOME REASONS WHY/ WHY NOY?

6. AT WHAT POINT DO YOU THINK A STUDENT IS MATHEMATICALLY FLUENT?

Questions to address key concept: Connections

7. IS MATHEMATICAL FLUENCY ONLY SEEN WHEN STUDENTS TALK OR DO YOU THINK IT CAN BE SEE IN THEIR WRITING?

8. HOW DO YOU THINK FLUENCY RELATES TO THE OTHER WORKING MATHEMATICALLY PROCESSES IN THE NSW MATHEMATICS SYLLABUS, REASONING, COMMUNICATING, PROBLEM SOLVING AND UNDERSTANDING?

9. DO YOU THINK STUDENTS CAN HAVE FLUENCY WITHOUT UNDERSTANDING? OR VISE VERSA?

10. HOW DOES A STUDENT'S FLUENCY IN ENGLISH IMPACT ON THEIR MATHEMATICAL FLUENCY?

11. WHAT ROLE DOES FLUENCY PLAY IN HOW WE COMMUNICATE OUR KNOWLEDGE OF MATHEMATICS?

12. IN THE QUESTIONNAIRE SOME RESPONDANTS TALKED ABOUT FLUENCY IN RELATION TO NUMBER CONCEPTS. IS THIS THE ONLY PLACE FLUENCY IS SEEN/ NEEDED IN MATHEMATICS? WHY? WHY NOT?

APPENDIX C: Participant forms

THE UNIVERSITY OF SYDNEY

Faculty of Education and Social Work

ABN 15 211 513 464

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Pro Dean Research, Professor Mathematics Education

Room 335
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Exploring primary teachers' conceptions of mathematical fluency

PARTICIPANT INFORMATION STATEMENT

(1) What is this study about?

You are invited to take part in a research study about mathematical fluency as one of the working mathematically processes in the NSW Mathematics K-10 syllabus. The purpose of the study is to discover primary teachers' beliefs and knowledge of what mathematical fluency means and what it may look like in students. The aim is to find a number of key features that teachers identify as important to being mathematically fluent that could then be observed during student learning in future studies.

You have been invited to participate in this study because you are a primary school teacher currently teaching in grade K-6 in a NSW Departmental school. This Participant Information Statement tells you about the research study. Knowing what is involved will help you decide if you want to take part in the research. Please read this sheet carefully and ask questions about anything that you don't understand or want to know more about.

Participation in this research study is voluntary.

By giving your consent to take part in this study you are telling us that you:
- ✓ Understand what you have read.
- ✓ Agree to take part in the research study as outlined below.
- ✓ Agree to the use of your personal information as described.

You will be given a copy of this Participant Information Statement to keep.

(2) Who is running the study?

The study is being carried out by the following researchers:
- Katherin Cartwright, Mathematics Advisor K-6 for the NSW Department of Education in collaboration with Professor Janette Bobis from the University of Sydney.

Katherin Cartwright is conducting this study as the basis for the degree of Masters of Education by Research at The University of Sydney. This will take place under the supervision of Professor Janette Bobis.

While the researcher is an employee of the NSW Department of Education (DoE), this research is being carried out independently of the NSW DoE. The study is being undertaken to inform the researcher's practice as a teacher and educator. The report of findings from this research will not contain names, details or specific information about any individual participants. General information in regards to the size of the school or metropolitan or rural location of the school participants have come from may be identified in the results.

(3) What will the study involve for me?

This study will involve you completing an online questionnaire that should take approximately 20 minutes. There are three sections to the questionnaire:

Section 1: Background information

You will be asked to provide background information in regards to, for example, how long you have been teaching, what grade you are currently teaching, your level of teaching qualifications and the size and location of your school.

Section 2: Conceptions of Mathematics

You will be asked to rate a number of questions using a 5 point Likert scale (strongly disagree to strongly agree). The ten questions are in relation to how students learn mathematics and how you teach mathematics.

Section 3: Open-ended questions

You will be asked two open-ended questions in relation to mathematical fluency where you will write your own responses.

At the end of the questionnaire you will be asked if you consent to be contacted for a possible follow up interview.

If you tick yes, you will be asked for your email address to be able to contact you.

If you tick no, your participation in the research is complete and your questionnaire will remain anonymous.

If you would like to participate in this research, the online questionnaire can be accessed here https://surveys.sydney.edu.au/surveys/?s=XLD2aWVyYC

Follow up interview

If you consent to the follow up interview, you may be contacted to participate in a 30-minute interview.

The selection process for the interviews will take into account the researcher wanting a variety of metropolitan and rural schools involved in the interviews, as well as a variety of grades represented in the interviews. Also, selection to be interviewed may be based on your answers to sections 2 and 3 of the questionnaire.

These interviews will happen face-to-face at your school or possibly via video conference for rural teachers. The interview involves open questions in regards to mathematical fluency, building on the questions asked of you in the questionnaire.

The interviews will be voice recorded so they can be easily transcribed later. Although you will have provided your name/ email to be contacted for the interview, your personal details will remain confidential and will not be published as part of the research.

(4) How much of my time will the study take?

The study will take place during Term 4 2016 and Term 1 2017. The online questionnaire will take you approximately 20 minutes to complete and can be completed at a time suitable to you. If you are selected for a follow up interview, these may take place before school, after school or during school hours. If the interview needs to be conducted during school hours, your school will be reimbursed for any casual relief costs. The interview should take approximately 30 minutes.

(5) Who can take part in the study?

Any NSW DoE Kindergarten to Year 6 teacher can participate in this research. Teachers of composite and multi-stage classes are included in this definition of K-6 teachers. The researcher is also hoping to have wide representation by metropolitan and rural and remote school teachers.

(6) Do I have to be in the study? Can I withdraw from the study once I've started?

Being in this study is completely voluntary and you do not have to take part. Your decision whether to participate will not affect your current or future relationship with the researchers or anyone else at the University of Sydney or the NSW Department of Education.

If you decide to take part in the study and then change your mind later, you are free to withdraw at any time. You can do this by email or phone call to the researcher.

Submitting your completed questionnaire is an indication of your consent to participate in the study. You can withdraw your responses if you change your mind about having them included in the study, up to the point that we have analysed and published the results. However, if you completed the questionnaire anonymously, once you have submitted it, your responses cannot be withdrawn because they are anonymous and therefore we will not be able to tell which one is yours.

In regards to the follow up interview, you are free to stop the interview at any time. Unless you say that you want us to keep them, any recordings will be erased and the information you have provided will not be included in the study results. You may also refuse to answer any questions that you do not wish to answer during the interview.

(7) Are there any risks or costs associated with being in the study?

Aside from giving up your time, we do not expect that there will be any risks or costs associated with taking part in this study.

(8) Are there any benefits associated with being in the study?

We cannot guarantee that you will receive any direct benefits from being in the study.

(9) What will happen to information about me that is collected during the study?

By providing your consent, you are agreeing to us collecting personal information about you for the purposes of this research study. Your information will only be used for the purposes outlined in this Participant Information Statement, unless you consent otherwise.

Your information will be stored securely and your identity/ information will be kept strictly confidential, except as required by law. Study findings may be published, but you will not be individually identifiable in these publications.

(10) Can I tell other people about the study?

Yes, you are welcome to tell other people about the study.

(11) What if I would like further information about the study?

When you have read this information, Katherin Cartwright or Janette Bobis will be available to discuss it with you further and answer any questions you may have. If you would like to know more at any stage during the study, please feel free to contact:

- Katherin Cartwright kcar8223@uni.sydney.edu.au
- Janette Bobis, University of Sydney janette.bobis@sydney.edu.au

(12) Will I be told the results of the study?

You have a right to receive feedback about the overall results of this study. You can tell us that you wish to receive feedback by answering the relevant question in the online questionnaire. This feedback will be in the form of a summary of the findings. You will receive this feedback after the study is finished.

(13) What if I have a complaint or any concerns about the study?

Research involving humans in Australia is reviewed by an independent group of people called a Human Research Ethics Committee (HREC). The ethical aspects of this study have been approved by the HREC of the University of Sydney *[INSERT* protocol number once approval is obtained*]*. As part of this process, we have agreed to carry out the study according to the *National Statement on Ethical Conduct in Human Research (2007)*. This statement has been developed to protect people who agree to take part in research studies.

If you are concerned about the way this study is being conducted or you wish to make a complaint to someone independent from the study, please contact the university using the details outlined below. Please quote the study title and protocol number.

The Manager, Ethics Administration, University of Sydney:

- **Telephone:** +61 2 8627 8176
- **Email:** human.ethics@sydney.edu.au
- **Fax:** +61 2 8627 8177 (Facsimile)

This information sheet is for you to keep

THE UNIVERSITY OF SYDNEY

Faculty of Education and Social Work

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Pro Dean Research, Professor Mathematics Education

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Exploring primary teachers' conceptions of mathematical fluency

PARTICIPANT CONSENT FORM

I, ... [PRINT NAME], agree to take part in this research study.

In giving my consent I state that:

- ✓ I understand the purpose of the study, what I will be asked to do, and any risks/benefits involved.
- ✓ I have read the Participant Information Statement and have been able to discuss my involvement in the study with the researchers if I wished to do so.
- ✓ The researchers have answered any questions that I had about the study and I am happy with the answers.
- ✓ I understand that being in this study is completely voluntary and I do not have to take part. My decision whether to be in the study will not affect my relationship with the researchers or anyone else at the University of Sydney or the NSW Department of Education now or in the future.
- ✓ I understand that I can withdraw from the study at any time.
- ✓ I understand that I may stop the interview at any time if I do not wish to continue, and that unless I indicate otherwise any recordings will then be erased and the information provided will not be included in the study. I also understand that I may refuse to answer any questions I don't wish to answer.
- ✓ I understand that personal information about me that is collected over the course of this project will be stored securely and will only be used for purposes that I have agreed to. I understand that information about me will only be told to others with my permission, except as required by law.
- ✓ I understand that the results of this study may be published, and that publications will not contain my name or any identifiable information about me.

I consent to:

- **Audio-recording** YES ☐ NO ☐

Would you like to receive feedback about the overall results of this study?

YES ☐ **NO** ☐

If you answered **YES**, please indicate your preferred form of feedback and address:

☐ Postal: ____

____

☐ Email: ____

....

Signature

....

PRINT name

....

Date

APPENDIX D: SERAP approval form

Education
Public Schools

Mrs Katherin Cartwright
26 Edward st
Carlton
NSW - 2218

DOC16/1172859
CORP16/41723
SERAP 2016578

Dear Mrs Cartwright

I refer to your application to conduct a research project in NSW government schools entitled *Exploring primary teachers' conceptions of mathematical fluency*. I am pleased to inform you that your application has been approved.

You may contact principals of the nominated schools to seek their participation. **You should include a copy of this letter with the documents you send to principals.**

This approval will remain valid until 24-Nov-2017.

The following researchers or research assistants have fulfilled the Working with Children screening requirements to interact with or observe children for the purposes of this research for the period indicated:

Researcher name	WWCC	WWCC expires
Katherin Cartwright	WWC0757808E	11-Jul-2020

I draw your attention to the following requirements for all researchers in NSW government schools:

- The privacy of participants is to be protected as per the NSW Privacy and Personal Information Protection Act 1998.
- School principals have the right to withdraw the school from the study at any time. The approval of the principal for the specific method of gathering information must also be sought.
- The privacy of the school and the students is to be protected.
- The participation of teachers and students must be voluntary and must be at the school's convenience.
- Any proposal to publish the outcomes of the study should be discussed with the research approvals officer before publication proceeds.
- All conditions attached to the approval must be complied with.

When your study is completed please email your report to: serap@det.nsw.edu.au
You may also be asked to present on the findings of your research.

I wish you every success with your research.

Yours sincerely

Dr Robert Stevens
Manager, Research
Wednesday, 23 November 2016

School Policy and Information Management
NSW Department of Education
Level 1, 1 Oxford Street, Darlinghurst NSW 2010 – Locked Bag 53, Darlinghurst NSW 1300
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