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WATER PRODUCTION FUNCTION OF SORGHUM FOR NORTHEAST BRAZIL

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ABSTRACT

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+ The results of an experiment to determine the water production function of sorghum for Northeast Brazilian conditions are reported. The experiment was designed in two random blocks consisting of four growth stages for irrigation deficit and four levels of nitrogen.

The crop response to water was found to be of quadratic nature. Regression coefficients were developed for each nitrogen and growth stage treatment between water use and grain yields. The most critical stage was found to be the vegetative stage followed by flowering and grain formation stages.

The yield response factors as according to Doorenbos and Kassam were found to vary drastically not only with different nitrogen levels and crop growth stages but also with different irrigation levels. A modified linear equation has been suggested. The coefficients of this modified equation have been determined for all four stages and for the four nitrogen levels for sorghum.

The highest average water use efficiency of 109.3 kg ha^{-1} cm^{-1} of water and highest obtainable grain yield of 4.92 t/ha was found to be at 425 mm of water use and 45 kg/ha of nitrogen.

The crop coefficients (K_c) to compute crop water use from pan evaporation vary between 0.4 and 0.81 with an average for the whole growing period of 0.75.

INTRODUCTION

Northeast Brazil is climatically one of the most erratic regions of the world. Supplemental irrigation is being proposed to improve crop production. Such irrigation projects usually involve high expenditures. Often, in the past, projects have been planned without adequate knowledge of water production functions. To fill this gap in information a research project for determining water production functions of major Northeast Brazilian dry land crops was started in 1983. This paper reports the results for sorghum.

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MATERIALS AND METHODS

The experiment was conducted on an Oxisol. The field was cleared of native forest 2 years before the experiment. After clearing, irrigated tomatoes were grown for 1 year, and then the site was left fallow for a year. Some physical characteristics of the soil on the experimental site are given in Table 1. Chemical analysis of the soil is reported in Table 2.

The experiment was designed in two randomized blocks consisting of four growth stages and four levels of nitrogen. The growth stages were with irrigation deficits at vegetative, flowering and grain formation stages, and no deficit. The levels of nitrogen applied were 0, 45, 90, and 135 kg/ha. All fields obtained 30 kg/ha K_2O and 90 kg/ha of P_2O_5. The nitrogen was applied in two parts, one half before sowing and the other half 3 weeks after germination. The method of a line source sprinkler as described by Hanks et al. (1976) was used. Plots of 15 m × 4.5 m for each of the treat-

TABLE 1

Some physical properties of the Oxisol of the experimental site (Choudhury and Millar, 1981)

Characteristics	Depth interval (cm)			
	0—30	30—60	60—90	90—120
Texture:				
Gross sand (%)	4	5	3	3
Fine sand (%)	87	81	79	76
Silt (%)	4	5	6	8
Clay (%)	5	9	12	13
Textural classification (USDA)	Sandy	Sandy loam	Loamy sand	Loamy sand
Apparent density (g/cm³)	1.62	1.68	1.64	1.62
Real density (g/cm³)	2.72	2.74	2.74	2.82
Field capacity (%)	8.94	9.00	9.20	9.00
Permanent wilting point (15 atm) (%)	1.84	2.52	3.07	3.22

TABLE 2

Chemical properties of the experimental site

pH H_2O (1 : 2.5)	Electrical conductivity (25°C), Sat. Ext. ($S\ m^{-1}$)	Exchangeable cations (meq/100 g of soil)					Al^{+3} (meq)	P (ppm)
		Ca^{2+}	Mg^{2+}	Na^{+}	K^{+}	S		
4.8	0.025	0.8	0.4	0.01	0.16	1.4	0.36	6.5

ments were laid out to give two replications per block by locating one replication on either side of the sprinkler line. Wind velocities often exceeded 300 km/day, therefore the upper wind replication of each block had to be rejected since this replication could only be partially wetted. Figure 1 shows a typical irrigation distribution pattern. Thus the analysis presented here is actually for one replication. The layout for one of the growth stage treatments is shown in Fig. 2. Other growth stage treatments were similar.

The experiment was carried out using the IPA 7301011 granifero variety of sorghum because this variety is one of the highest grain yielding varieties of the region. The plant population was maintained at 100 000 plants per ha. Each plot consisted of six rows spaced at 75 cm. Two of these six rows on the sides were borders. Six levels of irrigation and soil moisture to 120 cm soil depth were monitored: 1.25, 3.75, 6.25, 8.75, 11.25, and 13.75 m, from the line source. Wind velocities, rainfall, daily evaporation rates and mean relative humidity were obtained from the nearby meteorological station of the irrigation research centre and used for irrigation scheduling.

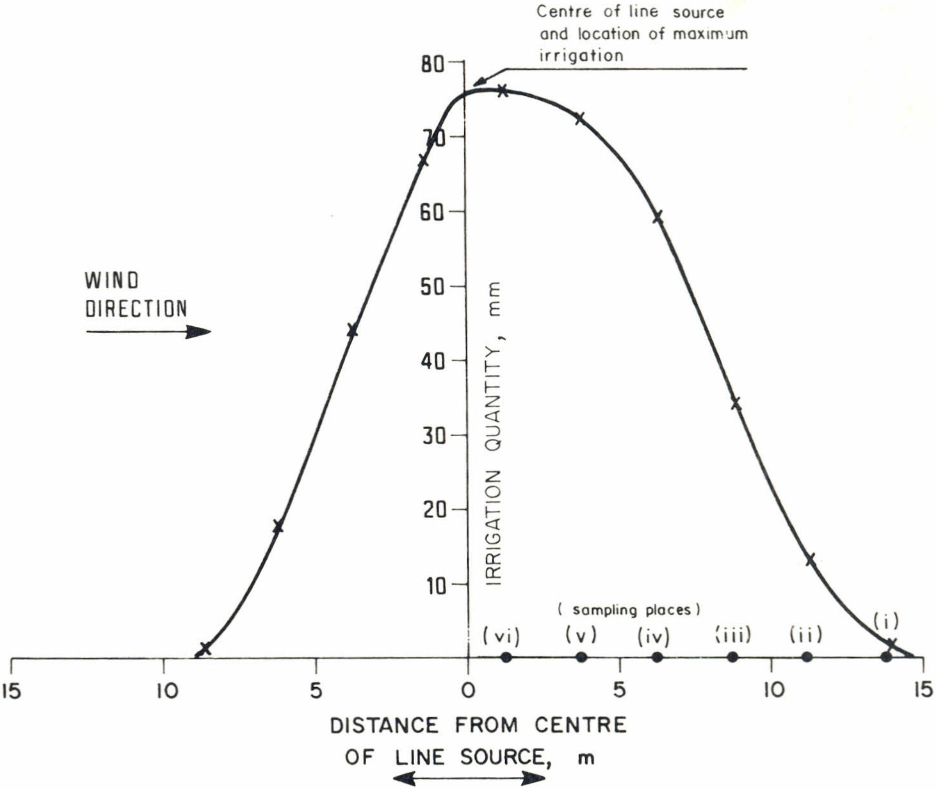

Fig. 1. Typical irrigation water distribution pattern from centre of line source (irrigation of 31 October 1983).

In total eight irrigations were given. The first three irrigations totalling 84.2 mm were given uniformly on 23 August, 30 August and 9 September 1983 for establishing the crop. The other five irrigations continuously varied with distance from the centre of the line source and were given on 22 September, 7 and 19 October, 1 and 19 November 1983, respectively. All irrigations were scheduled at 50% moisture depletion level in the first

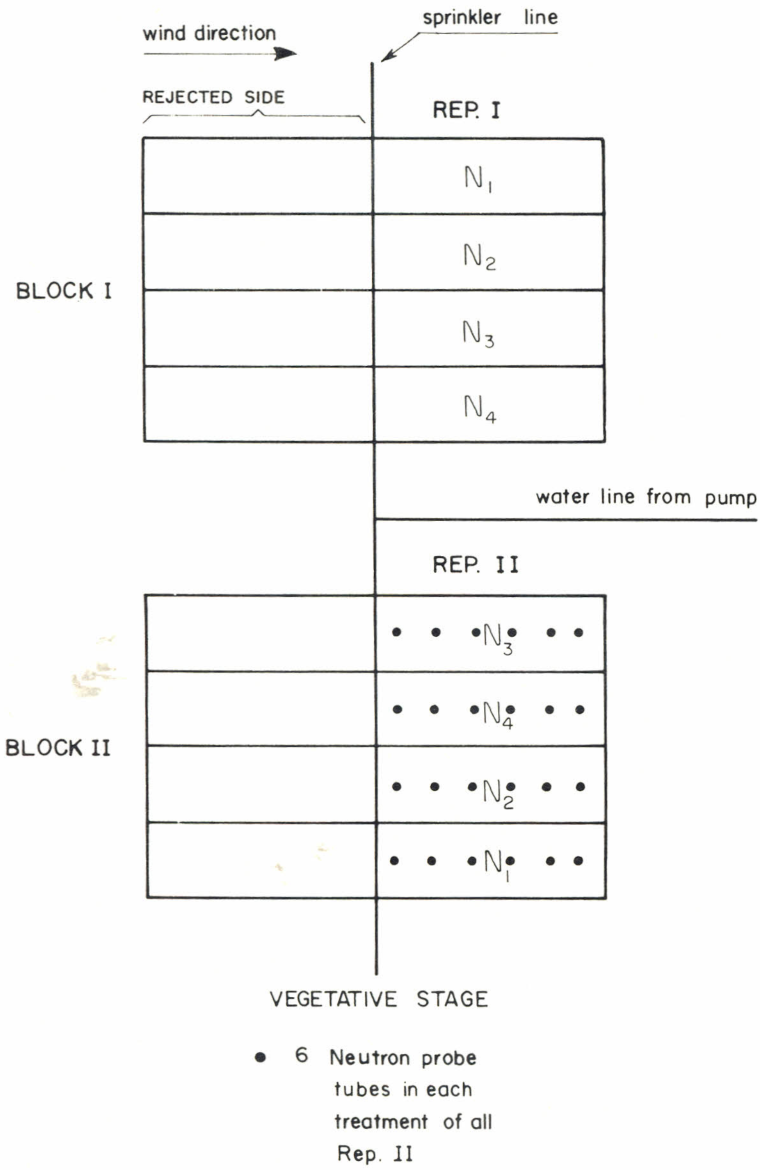

Fig. 2. Lay-out sketch of the vegetative stage of the experiment.

100 cm of soil, at the point of maximum water application which is at the centre of the line source. The pan and crop coefficients used were those recommended by Doorenbos and Kassam (1979). Deficits at a particular stage were obtained by skipping the irrigation on that stage while all other stages were irrigated. The 4th irrigation (30 days after planting) was skipped for a deficit in the vegetative stage. The 6th (56 days after planting) and 7th (68 days after planting) were skipped to give the deficits in the flowering and grain formation stages. The no deficit stage got all eight irrigations. The crop was harvested after 100 days.

Irrigation quantity was monitored by cans and soil moisture was monitored in one replicate of each of the four growth stages for each nitrogen treatment by neutron probes at six locations.

The water use was calculated by summing the irrigation quantity applied at the six locations, soil moisture contribution and rainfall. Deep percolation was monitored by taking soil moisture readings after an irrigation between 90 and 120 cm depth. Because soil water at this depth never reached field capacity there were no deep percolation losses.

A total of 61.9 mm rainfall was recorded. Except for one event of 39 mm in the last week of the experiment, all rainfall was in small quantities. There was no runoff loss from any rainfall event. Crop yield samples taken from plots of 3 m $\times$ 1 m for grain and fodder were collected at the same six places of each treatment.

RESULTS AND DISCUSSION

The observed sorghum grain yield (y) at different water use (Q) levels for the four stages (T) and for the four nitrogen levels (N) have been reported in detail elsewhere (Sharma, 1985). For brevity a summery of the observed data is given here in Table 3. Table 3 also shows the range of variation of the FAO yield response factor (K) and the water use efficiency within the given water use and yield data range.

Yield response to various variables

The nature of crop response (yield) to water use was found to be of quadratic nature. Regression coefficients for the quadratic equations developed for different stages, T (days) at different levels of nitrogen, N (kg/ha) between grain yield, y (kg/ha, dependent variable) and water use, Q (mm, independent variable), and the values of R^2 and standard error (σ) for each regression equation are given in Table 4. The coefficients which have an asterisk ($\star$) on N give negative values of y at very low water use hence should not be used at such low values of Q.

When N (kg/ha) is introduced as another independent variable, the multiple regression analysis gives the following equation (significant at 1% level) for different stages:

Vegetative stage (T = 30 days):
$y = -83.24 + 2.06N - 0.11N^2 - 0.75Q + 0.01Q^2 - 0.005NQ$
$R^2 = 0.86, \sigma = 182.8$

Flowering stage (T = 56 days):
$y = -1778.82 + 0.21N + 0.046N^2 + 9.61Q - 0.003Q^2 - 0.21NQ$
$R^2 = 0.76, \sigma = 320.74$

Grain formation stage (T = 68 days):
$y = -4855.47 - 16.20N + 0.14N^2 + 35.51Q - 0.033Q^2 - 0.037NQ$
$R^2 = 0.80, \sigma = 507.01$

No deficit (T = 100 days):
$y = -4677.61 + 3.92N - 0.026N^2 + 30.38Q - 0.025Q^2 - 0.015NQ$
$R^2 = 0.77, \sigma = 792.11$

TABLE 3

Summary of the observed water use and grain yield data, and variation of FAO yield response factor and water use efficiency within the observed data range

Water deficit stage, T (days)	Nitrogen level, N (kg/ha)	Range of observed values (rounded)							
		Water, use, Q (mm)		Grain yield, y (kg/ha)		FAO yield response factor, K		Water use efficiency (kg ha^{-1} cm^{-1} of water)	
		from	to	from	to	from	to	from	to
Vegetative (30)	0	131	409	93	1187	1.4	21.2	7.1	29
	45	152	401	17	1067	1.6	13.9	1.1	26.6
	90	151	400	47	1593	1.5	11.7	3.1	39.8
	135	144	388	37	823	1.5	9.7	2.5	21.2
Flowering (56)	0	213	413	0	1903	2.0	21.5	0.0	46.1
	45	220	423	0	1687	2.1	139.5	0.0	39.9
	90	177	419	0	1880	1.7	44.5	0.0	44.9
	135	169	422	0	1460	1.7	114.9	0.0	34.6
Grain formation (68)	0	172	333	60	3333	0.85	1.5	3.5	100.2
	45	171	369	0	2083	1.7	4.47	0.0	70.1
	90	232	369	50	2720	1.7	3.39	2.2	73.8
	135	189	338	136	2037	1.75	2.87	7.23	60.24
No deficit (100)	0	205	427	10	3903	1.9	39.4	0.5	91.46
	45	200	462	666	3453	-26.1	∞	33.0	115.9
	90	177	442	87	3000	1.7	-9.68	4.9	67.92
	135	161	449	17	3067	1.6	-7.89	1.0	82.39

Finally the growth stage represented by time of the beginning of deficit was also introduced as an independent variable along with nitrogen and water use. The quadratic multiple regression analysis nature gives the following equation (significant at 1% level):

$$y = -1526.55 - 26.71T - 0.03T^2 - 2.66N + 0.041N^2 + 14.17Q - 0.028Q^2 - 0.05TN + 0.17TQ - 0.008TNQ$$

$R^2 = 0.75$, $\sigma = 604.91$

These multiple regressions in general are valid within the range of data. However, sometimes these equations give negative values of y for the lowest values of water use and should be used with caution.

Water use efficiency (WUE)

The water use efficiency, calculated by dividing the grain yield by the quantity of water (kg ha^{-1} cm^{-1} of water), give the highest values for no

TABLE 4

Regression coefficients of the quadratic equation ($Y = a + bQ + cQ^2$) of yield response to water (a, b and c are the regression coefficients)

Water deficit stage, T (days)	Nitrogen level, N (kg/ha)	Regression coefficients a	b	c	R^2	Standard error (σ)
Vegetative (30)	0	267.36	-3.73	-0.016	0.94	131.83
	45	-943.72	7.18	0.006	0.86	188.49
	90	892.67	-9.68	0.03	0.91	182.55
	135	-538.87	4.00	-0.001	0.9	132.69
Flowering (56)	0	-1126.83	5.06	0.004	0.84	296.21
	45	-1534.86	7.85	-0.003	0.75	335.60
	90	151.18	-3.67	0.016	0.69	444.22
	135	-2403.02	18.12	-0.02	0.94	136.17
Grain, formation (68)	0	-7721.99	58.16	-0.076	0.87	548.41
	45	-2515.14	15.32	-0.005	0.93	278.95
	90	-760.48	45.09	-0.05	0.65[a]	651.18
	135	-4362.19	30.40	-0.035	0.71[a]	518.49
No deficit (100)	0	-8265.87	55.02	-0.065	0.73	867.44
	45	-9919.42	66.00	0.079	0.76	950.71
	90*	-1834.3	8.77	0.006	0.96	282.23
	135*	-2013.1	10.26	0.004	0.92[b]	612.07

[a] Significant at 5% level.
[b] Significant at 10% level. All others significant at less than 1% level.
*Coefficient give negative values of y for first data set (lowest water level in Table 3).

deficit followed by a deficit at the grain filling stage as shown in Table 3. The WUE for both vegetative and flowering stage were low. The highest average (of the two replicates) water use efficiency was observed to be 109.3 kg ha^{-1} cm^{-1} of water at 425 mm water use for 45 kg/ha applied nitrogen and the no deficit stage. The average (of the two replicates) highest grain yield was also obtained at the same point. From the values of WUE obtained it can be concluded that if available water is limited, the deficit should not be allowed to occur during vegetative and flowering stages.

Yield response factors (K)

Doorenbos and Kassam (1979) used the following equation for predicting relative yield decrease for relative evapotranspiration deficit:

$$(1 - y/y_m) = K(1 - Q/Q_m) \qquad (1)$$

where y is actual yield (kg/ha) and Q is the corresponding water use or actual evapotranspiration (mm), y_m is maximum obtainable yield (kg/ha), and Q_m is the corresponding maximum evapotranspiration (mm). The coefficient K is called the yield response factor.

It is to be understood that the maximum yield, y_m is defined (quoted from Doorenbos and Kassam, 1979) as: " The harvested yield of a high producing variety, well adapted to the given growing environment, including the time available to reach maturity, under conditions where water, nutrients and pests and diseases do not limit yield."

Thus y_m is the global maximum yield having only one value. It should not be confused with the highest yield at different fertility or growth treatments. Hence the term $(1 - y/y_m)$ is the relative yield decrease and $(1 - Q/Q_m)$ is the relative evapotranspiration deficit. The factor K relates the two. Doorenbos and Kassam (1979) consider equation (1) to be valid up to 50% relative evapotranspiration deficit.

Taking a closer look at equation (1) it was found that it is not valid when fertility and growth stages are variable. For example, it is obvious that even if relative evapotranspiration deficit is zero but the fertility level is below (or above) the optimum fertility required, there will be a substantial yield decrease. Similarly when water is not available at a certain critical growth stage but overall there is no relative evapotranspiration deficit, there shall be a substantial yield decrease. Equation (1) does not take account of this.

The maximum obtainable yield was taken as 4.92 t/ha at 425 mm of water use from our data. The research station that developed the variety gives for the genetic yield potential 5 t/ha, which is very close to our maximum. When yield response factors are calculated by equation (1), they are found to vary within various stages, nitrogen and water levels, (this is demonstrated in Table 3); according to equation (1) there should be a fixed value or at most a narrow range for each stage and nitrogen level.

Similar variations were also found for maize by the first author who carried out similar experiments at the same site in the year after the sorghum experiment reported here (Sharma and Pereira, 1985). This problem can be taken care of if equation (1) is modified as follows:

$$(1 - y/y_m) = K_1 + K_2\,(Q/Q_m) \tag{2}$$

where K_1 and K_2 are modified yield response factors which are to be determined by experimentation. Linear regression analysis of the data was carried out to give values of K_1 and K_2. These values are reported in Table 5. Equation (2) should also be valid up to about 50% water deficit.

TABLE 5

Modified yield response factors for sorghum at different nitrogen levels and different growth stages

Water deficit stage, T (days)	Nitrogen level, N (kg/ha)	New yield response factors[a]		R^2	Standard error (σ)	
		K_1	K_2			$(K_1 + 1/2\,K_2)$
Vegetative (30)	0	0.75	0.34	0.76	0.05	0.92
	45	0.74	0.37	0.56	0.08	0.93
	90	0.67	0.55	0.75	0.08	0.95
	135	0.81	0.30	0.83	0.03	0.96
Flowering (56)	0	0.71	0.44	0.53	0.09	0.93
	45	0.73	0.49	0.80	0.06	0.98
	90	0.71	0.52	0.67	0.09	0.97
	135	0.72	0.42	0.85	0.04	0.93
Grain formation (68)	0	0.0	1.68	0.79	0.14	0.84
	45	0.37	1.07	0.89	0.07	0.91
	90	0.39	1.03	0.69	0.11	0.91
	135	0.32	1.10	0.66	0.14	0.87
No deficit (100)	0	0.33	1.13	0.56	0.22	0.90
	45	0.33	0.98	0.51	0.23	0.82
	90	0.41	1.02	0.97	0.05	0.92
	135	0.33	1.10	0.9	0.09	0.88

[a]Significant at 1% level.

The new yield response factor K_1 gives the minimum relative yield decrease which takes place when the nitrogen level is different from the optimum or when one of the critical growth stages suffers from water deficit. The factor K_2 is a multiplier to the relative yield decrease due to overall change in water level and is similar to the K factor of Doorenbos and Kassam (1979). The higher the values of K_1 the more critical is the growth stage for irrigation. Inversely the lower the values of K_1, the less the relative yield decreases. From the experimental results the importance of deficits in the growth stages can be ranked as:

vegetative $\geqq$ flowering $>$ grain filling $>$ no deficit stage

where the vegetative and flowering stages are almost equally critical followed by the grain filling stage.

In the grain filling stage and 0 kg/ha nitrogen the value of K_1 is zero, but K_2 has the highest value (= 1.68). As equation (2) is valid only up to 50% ET deficit, the combined effect of K_1 and K_2 can have a maximum value of (K_1 + 1/2 K_2) only, which gives the maximum yield response factor of 1.0. In Table 5 the value (K_1 + 1/2 K_2) is also shown. The ranking of critical stages by this factor is the same as for K_1 as already discussed. The lowest value of (K_1 + 1/2 K_2) is for 45 kg/ha nitrogen and no deficit (also K_1 has one of the lowest values for this treatment). This is the optimum combination of nitrogen and irrigation because it will have the lowest yield deficit at various evapotranspiration deficit levels.

Theoretically, K_1 for the optimum combination should be zero. However in practice even with the best treatments this theoretical value can hardly be achieved. The maximum potential yield is very rarely obtainable. For this reason many researchers in irrigation systems management have questioned the validity of theoretical water production functions (Levine, 1980).

From Table 5 all possible combinations can be made. For example if shortage of water is expected it will be best to allow that shortage in the grain filling stage and not apply any nitrogen since it has the next to lowest value of (K_1 + 1/2 K_2). This is not surprising since many researchers in Northeast brazil have also found that many dry land crops under dry conditions do not respond to nitrogen (Richardson and Van Vught, 1965; Poultrey, 1968).

Crop coefficients (K_c)

The crop coefficients K_c are used for irrigation scheduling together with pan evaporation data. A sample calculation is given in Table 6 for conditions which give maximum WUE and yield and hence optimum water use. The first four growth periods are each after an interval of two successive irrigations while the last growth period (harvest) is between the 8th irrigation and date of harvest. These calculated crop coefficients K_c are about the same as those given by Doorenbos and Kassam (1979) but the mid-season value is lower than the FAO value.

Table 7 gives the values of K_c for different nitrogen levels, calculated in the same way for all other values of N. Crop coefficients for N = 0 kg/ha and N = 45 kg/ha for different growth periods are about equal. The K_c values for N = 90 kg/ha and N = 135 kg/ha are initially the same as for the other N levels, but are lower between 18 and 85 days and higher from

TABLE 6

Crop coefficients K_c for sorghum at N = 45 kg/ha, no deficit stage and maximum average water utilization efficiency and yield

Crop development periods	Initial	Crop development	Mid-season	Late season	Harvest	Total
Duration from planting (days)	0—17	18—44	45—68	69—85	86—106	0—106
Irrigation (mm)	59	96	123	69	55	402
Soil moisture contribution to Q (N = 45 kg/ha)	-17	10	10	-3	-48	-39
Rainfall (mm)	—	—	2.6	11.4	47.9	61.9
Actual evapotranspiration, Q (mm)[a] (N = 45 kg/ha)	41.7	106.3	135.8	77.5	54.5	424.6
Pan evaporation, EV (mm)	158.9	221.3	256.8	162.4	168.6	868.1
Pan coefficient, K_p^b	0.65	0.65	0.65	0.65	0.65	0.65
Ref. crop ET, $ET_0 = K_p$ EV (mm)	103.3	143.9	167	105.5	109.6	564.3
$K_c = Q/ET_0$ (calculated)	0.40	0.74	0.81	0.73	0.5	0.75
FAO K_c values	0.3	0.7	1.05	0.75	0.5	0.75

[a]Q = (irrigation + soil moisture contribution + rainfall); there was no deep percolation and no runoff.
[b]For moderate wind velocities (175—425 km/day) and high mean relative humidity (RH_{mean} ⩾70%), FAO source, Doorenbos and Kassam (1979).
The pan is located in the green grass cover (< 1 m).

TABLE 7

Values of crop coefficients K_c for sorghum at different nitrogen levels

Nitrogen level, N (kg/ha)	Value of K_c at different growth periods					
	Initial (0—17 days)	Crop development (18—44 days)	Mid season (45—68 days)	Late season (69—85 days)	Harvest (86—106 days)	Total period
0	0.4	0.74	0.86	0.74	0.45	0.76
45	0.4	0.74	0.81	0.73	0.5	0.75
90	0.4	0.59	0.81	0.64	0.58	0.75
135	0.4	0.65	0.76	0.65	0.68	0.69
FAO values	0.3	0.7	1.05	0.75	0.5	0.75

86 to 106 days. This indicates that for all treatments the plant growth starts equally but at higher N levels (i.e. $N = 90$ and 135 kg/ha) plant growth is restrained for some unknown reason. Finally plant growth improves which is reflected by higher K_c values between 86 and 106 days; this was also observed in the field. The higher nitrogen levels gave greener plants at harvest time.

CONCLUSIONS

The multiple regression equations developed can be utilized for economic analysis of irrigation projects. The new yield response factors (K_1 and K_2) developed here give a better representation of the crop response to water than those suggested earlier by Doorenbos and Kassam (1979). The highest average water use efficiency of 109.3 kg ha^{-1} cm^{-1} of water and highest grain yield of 4.92 t/ha was obtained at 425 mm of water use at 45 kg/ha of applied nitrogen for the no water deficit case for which the value of modified crop response factors K_1 and K_2 are 0.33 and 0.98, and the value of K_c for the total period is 0.75.

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