# Investigating the Shortest Survey Route in a GNSS Traverse Network

Bülent BOSTANCI, Abdullah KARAAĞAÇ

**Abstract:** Upon the progress in the satellite positioning systems after 1990's particularly, Global Positioning System and Global Navigation Satellite System (GNSS) networks have been setup and used in scientific researches. Operational Research (OR) techniques have been used in the design and optimization of the surveying networks based on GNSS, as well. After 2000's, both developed and developing countries have launched and developed Continuously Operating Reference Stations (CORS) networks for surveying procedures, which require precision to the centimetre. Surveying the detail points by creating traverse networks in the field may be realized thanks to the accuracy of the position obtained. Although many researches in the field of the design and optimization of the GNSS networks exist, finding the shortest route in the survey of such networks remained limited. Objective of this study is to find the shortest survey route in a traverse network created in the CORS-Turkey (CORS-TR) system. Based on the optimum solution of Traveling Salesman Problem we form the best route and timing plan.

**Keywords:** Continuously Operating Reference Stations; Global Navigation Satellite System; Shortest survey route; Travelling Salesman Problem

## 1 INTRODUCTION

In order to conduct the operational activities in a more productive and efficient way, Operational Research (OR) has been applied as a scientific method to the complicated problems. OR was applied to the design and optimization problems of the geodetic networks, in the past, as well. Kuang [1] used the quadratic programming in the design and optimization of leveling networks while Cross and Thapa [2] preferred a linear programming approach. Benzao and Shaorong [3] used linear programming for designing the tracking of the deformation networks. Kuang [4] used a quadratic programming formulation in the design of three dimension geodetic networks. By using graph theory, for the optimization of the cost in the trilateration networks, Kortesis and Dermanis [5] developed a method based on the survey of a surveyor who visited multiple points a day.

Fixed positioned GNSS receivers [6] or Real Time Kinematic/CORS satellite positioning systems that track and correct the route of a moving antenna [7] are used in the scientific studies to determine the positions of the points. Today, depending on the increasing satellite positioning systems, GNSS observation period concept and geodetic network design optimization are still valid. Wells et al. [8] and Vanicek et al. [9] have developed algorithms that solve the quantity of the receivers required considering the fixed and moving receivers. Snay [10] and Janes et al. [11] have studied developing algorithms that form survey plans considering the different combinations of the receivers. A Second Order Design (SOD) in the optimization of the Global Positioning System (GPS) Networks through linear programming have been performed by Even-Tzur and Papo [12] while Mehrabi and Voosoghi [13] have used quadratic programming algorithm for the optimal survey plan of the GPS Networks.

Saleh and his colleagues have carried out several studies on using the OR techniques for designing the surveying network and planning the measuring procedures referring to the improved satellite positioning system as of 2000's. Dare and Saleh [14] have found a solution, by using Simulated Annealing (SA) technique, with optimum results in small GPS networks and with results closer to optimum in big networks. Saleh and Dare [15] have used SA and Tabu Search (TS) metaheuristics on Malta GPS surveying network. Saleh [16] has used artificial intelligence in the design of GPS surveying networks. Saleh and Dare [17] have performed the near optimal design of GPS network by using TabuSearch heuristics. Saleh and Chelouah [18] have used genetic algorithm in the design of GNSS surveying network. Fidanova and Saleh have studied Ant Colony Optimization (ACO) for planning the measuring methods of the satellite positioning networks [19].

Although numerous researches have been seen in the literature on Operational Research, design and optimization of the geodetic or GNSS network, studies to find the shortest route in the traverse network based on Continuously Operating Reference Stations (CORS) and planning surveying time remained limited. It is considered that the Traveling Salesman Problem (TSP) which is one of the OR techniques to find the shortest route may be used within the frame of the following researches.

TSP was formulated first by W.R. Hamilton, an Irish mathematician and Thomas Kirkman, an English mathematician in 1800's [20]. TSP, which became popular suddenly in 1950's and, soon after in1960's, has been a major method in solving many problems. G. Dantzig, R. Fulkerson and S. Johnson [21] in their article published in 1954 on solution of a large-scale TSP, solved the problem optimally among the 48 districts in Washington D.C.by using linear programming.

Upon the improvement in computer science and heuristic approaches, TSP has begun to be solved by using heuristic methods in large size problems. For example, Baraglia et al. [22] used genetic algorithm and Lin & Kernighan [23] method on a local search algorithm to find the shortest route among 13509 cities and compared both algorithms.

TSP has been used to solve the problems in the field of mapping, vehicle routing, tourism, Geographical Information System (GIS), and solving the applications with GPS support. Jung et al. [24] have studied on the optimization technologies: GPS and GIS integration for the effective control of the parcelling service. Wijesinghe et al.

[25] have used TSP in the travel planning system with GIS, while Keenan [26] used TSP for modelling the vehicle routing based on GIS. Objective of this study is to show that TSP is the core problem to solve the shortest route and timing problem of the surveyor(s) moving among traverse stations, which will be defined by CORS-TR system. In the study, firstly, the CORS-TR system is introduced, and then practicality of such system for measuring the traverse stations is discussed. The methodology of the TSP and an application in the CORS-TR traverse station network are given. Finally, our results are presented.

## 2 MATERIAL AND METHOD

After the satellite positioning systems and communication technologies have become widespread, measuring methods have undergone a significant change. Today, satellite-positioning systems are used in various areas such as civil engineering projects, deformation analysis, geolocation of the fixed station networks and relevant ground surveys, geographical information systems and mapping, general and private navigation applications [27]. The most common satellite positioning systems known today are as follows: The American Global Positioning System (GPS), Russian Global Navigation Satellite System (GLONASS), and European Satellite Navigation System (GALILEO) [28]. Geolocation by satellites is called GNSS.

Methods of relative geolocation by satellites are considered under static and kinematic measuring methods, particularly, Real Time Kinematic (RTK) measuring method is intensively used. Therefore, countries, local organizations and private sector set up real time fixed GNSS Networks called CORS [29].

The CORS system forms a network consisting of internet technology, data communication and computer usage with the support of various reference stations and data centers. The system uses satellite navigation and positioning technologies to enable the fixed GNSS reference stations operate continuously for a long term for a certain distance within a certain range. CORS is formed by four parts including reference station systems, data processing systems, data communication systems and user applications [30]. Many developed and developing countries continue to launch and improve the CORS networks based on the Satellite Positioning System. For example, USA has a CORS network consisting of more than 1900 fixed points and yields precise positions to the centimeter [31].

The initial works on the Turkish National Fixed GNSS Stations Network (TUSAGA-Active/ CORS -TR), a geocentric positioning system formed by fixed GNSS stations and control centers for real time positioning data, started on 8 May 2006 with a project support of TÜBİTAK (The Scientific and Technological Research Council ofTurkey) and it was fully operational on May 2009. The system is active with its 146 stations dispersed homogenously in a distance of about 70 to 90 km, and monitored by 2 control centers [32]. The CORS network formed by GNSS stations, which perform continuous surveillance in Turkey and Turkish Republic of Northern Cyprus is consisted of four different components; a system that beams real time position correction data, fixed earth stations receiving signals from GNSS positioning satellite system monitoring centers, fixed communication units for data transfer, single or double frequency receivers capable to receive the phase/code satellite data having the feature of differential GPS and RTK on the users side. In the system, fixed earth stations receive data from global positioning system satellites such as GPS of the USA, GLONASS of Russia under the scope of the GNSS [33].

It has been aimed to obtain real time precise (centimetric) position data for engineering applications with the data to be received from TUSAGA-Active /CORS-TR project [34]. On the other hand, it has also been aimed to define the transformation parameters between the ED50 (European Datum 1950) and ITRF96 (International Terrrestrial Reference Frame) data precisely in order to obtain the cadastral and the topographic map based on the international datum as well as to present data for the scientific geodetic studies. [35].

According to the Large Scale Map and Map Data Production Regulation that issues standards for the mapping activities in Turkey: "Traverse station coordinates may be defined by any of the methods namely static, fast static, kinematic or real time kinematic based on high order national GNSS control points (called as C1, C2, C3)". Digital production and control of cadastral maps are under the reponsibility of General Directorate of Land Registry and Cadastre. Therefore, a Circular No: 11 is issued in 2010 by the organization which introduces standards for the production and control of the cadastral maps [36]. According to the circular: "Coordinates of the traverse stations may be defined by the Network RTK method under the TUSAGA-Active/CORS-TR system. Definition of the coordinates of the traverse stations in this respect shall be performed at least in two GNSS sessions in different times to meet the requirements shown in Tab. 1. Difference between coordinates and ellipsoidal heights obtained by such sessions shall not be greater than 7 cm."

**Table 1** CORS-TR traverse network survey requirements

| Measuring feature | Requirement |
|---|---|
| Technique to be used to receive correction data | VRS, FKP and MAC |
| Ambiguity Solution | Fixed |
| Number of satellite | Min. 5 |
| Cut- off angle | Min.10° |
| Data collection interval | 1 (one) second |
| Observation span | Min. 5 times at each point |
| Period between sessions | Min. 1 hour |

As presented above, upon the examination of Turkish legislation, CORS-TR system may obviously be used in order to form traverse networks for mapping purposes.

### 2.1 Finding the Shortest Survey Route

TSP was initially a problem of a salesman who has a list of cities and the distances between each pair of cities to find the shortest possible route that visits each city exactly once and returns to the origin city [22].

TSP to be used for the GNSS traverse stations may be explained as follows: A surveyor desires to go to a number (m) of different traverse stations to define the positions and back to the point started. Total route length should be minimized provided going to a station is not performed

twice. Although the TSP seems simple as an idea, since there are $m!$ number of route alternatives finding the optimal solution is hard. When we define $x_{ij} \in \{0, 1\}$ as the variable of the tour of a surveyor that goes from station i to j, integer linear programming equations of TSP may be as follows [37]:

$$Min.\ z = \sum_{i=1}^{m} \sum_{j=1}^{m} d_{ij} \cdot x_{ij} \tag{1}$$

$$\sum_{i=1}^{m} x_{ij} = 1 \quad j = 1, 2, .., m;\ i \neq j \tag{2}$$

$$\sum_{j=1}^{m} x_{ij} = 1 \quad i = 1, 2, .., m;\ i \neq j \tag{3}$$

$$u_i - u_j + m \cdot x_{ij} \leq m - 1 \quad i, j = 2, 3, .., m;\ i \neq j \tag{4}$$

All $x_{ij} = 0$ or 1, all $u_i \geq 0$ and set of integers $x_{ij}$ is a parameter, which is valued 1 when the traveler goes from $i$ to $j$ point. Otherwise, it would be 0. The u parameter is an artificial variable added to the model to provide conditions for a complete cycle.

Objective of the TSP model $d_{ij}$ and $x_{ij}$ is to minimize the multiplying totals.

Parameters used in the model are:
$d_{ij}$: Distance between $i$ and $j$ stations $\forall i, j = 1, 2, …, m,\ i \neq j$.
$m$: Number of stations

Decision variables used in the model:

$x_{ij}$: in case going from i to j it is 1 and 0 in the other cases as decision variable.

$$\forall i, j = 1, 2, ……. m,\ i \neq j$$

$u_i$: is an artificial decision variable that shows the turn of station $i$ to be visited.

$$\forall i = 2, ……. m$$

Objective function shown in Eq. (1) aims to minimize the total travel length during the tour of the surveyor team to define the positions of the traverse stations. Constraint set shown in Eq. (2) defines the surveyors to go to each station once. Constraint set shown in Eq. (3) enables the surveyors to leave each station once. Constraint set shown in Eq. (4) is formed to eliminate the sub tours that divide the tour of the surveyors. Miller et al. [35] defined constraints in Eq. (4) in order to eliminate the sub tours.

## 3 STUDY AREA

Kayseri is a city of culture, industry and trade located in the Central Anatolia, close to Capadoccia with its great historic value and population close to 1 million. Erciyes University campusis chosen as the study area.It is located on an area for $4.940.000\ m^2$, $590.000\ m^2$ of which is indoor area. Traverse network in the university campus formed by CORS-TR system covers an area of about $835.000\ m^2$ (Fig. 1).
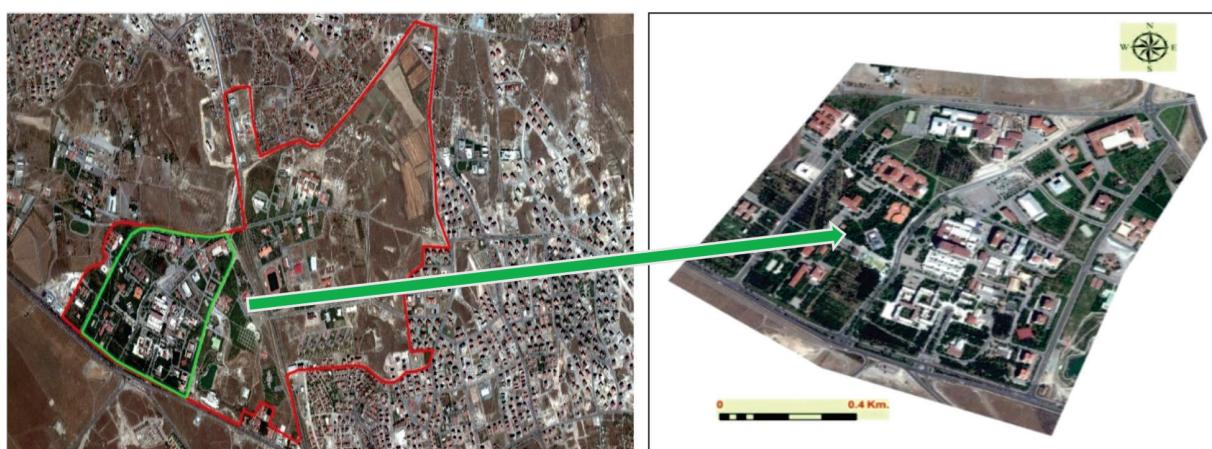


**Figure 1** Erciyes University campus and study area

## 4 APPLICATION

A network containing 26 traverse stations in the field to survey the detail points has been planned. Approximate positions of stations have been formed as point layer with coordinates on the geocorrected satellite image by means of ArcGIS program. Considering the pavements of the main/sub routes, walkways and crosswalks within the campus site, walkways among the stations have been defined as line features with coordinates on the satellite image (Fig. 2). Traverse network points defined on the satellite map have been monumanted on the ground permanently within the framework of Large Scaled Map and Map Data Production Regulation by searching the view opportunities in the first stage of the field operations.

Second stage of the field operations shall enable the definition of the 3D coordinates ($X$, $Y$, $Z$) in the ITRF96 datum of traverse stations. Due to the requirement of measuring a station twice in different times (by the regulation), 2 CORS-TR adaptive Astech GNSS receivers are used by two surveying teams. In order to finish the survey as early as possible, it is necessary to define the shortest route that visits all of the stations just for once. Before the second phase of the study, the shortest route and time planning has been realized by formulating a TSP integer linear programming model.

In order to solve the TSP model, it is necessary to form the Origin-Destination (OD) length matrix. As this problem requires to find the shortest route, it is necessary to describe the OD length matrix among stations through

to the walkways. For that reason, traverse station points layer and walkway lines layer formed in the satellite image are evaluated in the ArcGIS Network Analysis module and the OD length matrix has been calculated as shown in the Fig. 3.



**Figure 2** CORS traverse stations and walkways

It is required to describe the objective function of the TSP model. Any solver will try to find the best minimum for that objective function subject to constraints of the model. Objective function, which will be used during the practice, may be written as follows:

$$Min\,Z = 159.45X_{1-2} + 268.70X_{1-3} + 423.86X_{1-4} + 557.16X_{1-5} + 687.43X_{1-6} + 911.91X_{1-7} + 1086.62X_{1-8} + 1322.84X_{1-9} + 1534.70X_{1-10} + 1675.66X_{1-11} + 1684.62X_{1-12} + 1484.14X_{1-13} + 1329.22X_{1-14} + 1319.93X_{1-15} + 1139.42X_{1-16} + 965.14X_{1-17} + 624.48X_{1-18} + 346.27X_{1-19} + 622.90X_{1-20} + 881.96X_{1-21} + 1121.87X_{1-22} + 1383.91X_{1-23} + 1547.79X_{1-24} + 1506.99X_{1-25} + 832.46X_{1-26} + 159.45X_{2-1} + 109.25X_{2-3} + 264.41X_{2-4} + .$$
$$............................ + 840.27X_{25-26} + 832.46X_{26-1} + 819.51X_{26-2} + 710.26X_{26-3} + 738.08X_{26-4} + 871.39X_{26-5} + 972.27X_{26-6} + 969.78X_{26-7} + 1014.48X_{26-8} + 1111.08X_{26-9} + 1172.86X_{26-10} + 1031.90X_{26-11} + 852.16X_{26-12} + 651.68X_{26-13} + 496.76X_{26-14} + 487.47X_{26-15} + 390.51X_{26-16} + 548.65X_{26-17} + 207.98X_{26-18} + 486.19X_{26-19} + 302.85X_{26-20} + 561.92X_{26-21} + 424.66X_{26-22} + 717.19X_{26-23} + 801.10X_{26-24} + 840.27X_{26-25}$$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 159.45 | 268.70 | 423.86 | 557.16 | 687.43 | 911.91 | 1086.62 | 1322.84 | 1534.70 | 1675.66 | 1684.62 | 1484.14 | 1329.22 | 1319.93 | 1139.42 | 965.14 | 624.48 | 346.27 | 622.90 | 881.96 | 1121.87 | 1383.91 | 1547.79 | 1506.99 | 832.46 |
| 2 | 159.45 | 0.00 | 109.25 | 264.41 | 397.71 | 527.98 | 752.46 | 927.17 | 1163.39 | 1375.25 | 1516.21 | 1658.19 | 1457.71 | 1302.79 | 1306.98 | 1210.02 | 1124.59 | 783.93 | 505.72 | 516.66 | 763.37 | 1003.28 | 1224.46 | 1388.34 | 1347.54 | 819.51 |
| 3 | 268.70 | 109.25 | 0.00 | 155.16 | 288.46 | 418.73 | 643.21 | 817.92 | 1054.14 | 1266.00 | 1406.96 | 1548.95 | 1348.46 | 1193.54 | 1197.73 | 1100.77 | 1233.84 | 893.17 | 614.96 | 407.41 | 654.12 | 894.03 | 1115.21 | 1279.10 | 1238.29 | 710.26 |
| 4 | 423.86 | 264.41 | 155.16 | 0.00 | 133.30 | 263.57 | 488.05 | 662.76 | 898.98 | 1110.84 | 1251.80 | 1393.79 | 1193.31 | 1038.39 | 1173.75 | 1128.59 | 1286.73 | 946.07 | 711.86 | 435.23 | 498.96 | 738.87 | 960.05 | 1123.94 | 1083.13 | 738.08 |
| 5 | 557.16 | 397.71 | 288.46 | 133.30 | 0.00 | 130.27 | 354.75 | 529.46 | 765.68 | 977.54 | 1118.50 | 1260.49 | 1060.00 | 905.08 | 1040.44 | 1236.44 | 1410.73 | 1079.37 | 845.16 | 568.53 | 365.66 | 605.57 | 826.75 | 990.63 | 949.83 | 871.39 |
| 6 | 687.43 | 527.98 | 418.73 | 263.57 | 130.27 | 0.00 | 224.48 | 399.19 | 635.41 | 847.27 | 988.23 | 1167.97 | 1059.62 | 949.78 | 1085.14 | 1281.14 | 1455.42 | 1180.25 | 946.05 | 669.42 | 410.35 | 650.26 | 696.48 | 860.37 | 819.56 | 972.27 |
| 7 | 911.91 | 752.46 | 643.21 | 488.05 | 354.75 | 224.48 | 0.00 | 174.71 | 410.93 | 622.79 | 763.75 | 943.48 | 835.14 | 940.22 | 1075.59 | 1271.58 | 1445.87 | 1177.77 | 943.56 | 666.93 | 407.87 | 647.78 | 472.00 | 635.88 | 595.08 | 969.78 |
| 8 | 1086.62 | 927.17 | 817.92 | 662.76 | 529.46 | 399.19 | 174.71 | 0.00 | 236.22 | 448.08 | 589.04 | 768.78 | 660.43 | 765.52 | 900.88 | 1096.88 | 1271.16 | 1222.47 | 1118.27 | 841.64 | 582.57 | 589.83 | 297.29 | 461.18 | 420.37 | 1014.48 |
| 9 | 1322.84 | 1163.39 | 1054.14 | 898.98 | 765.68 | 635.41 | 410.93 | 236.22 | 0.00 | 211.86 | 352.82 | 532.56 | 733.04 | 862.11 | 997.47 | 1193.47 | 1367.75 | 1319.06 | 1354.49 | 1077.86 | 818.79 | 686.42 | 393.89 | 557.77 | 270.81 | 1111.08 |
| 10 | 1534.70 | 1375.25 | 1266.00 | 1110.84 | 977.54 | 847.27 | 622.79 | 448.08 | 211.86 | 0.00 | 140.96 | 320.70 | 521.18 | 676.10 | 811.46 | 1007.46 | 1181.74 | 1380.84 | 1566.35 | 1289.72 | 1030.65 | 898.28 | 605.75 | 657.63 | 482.67 | 1172.86 |
| 11 | 1675.66 | 1516.21 | 1406.96 | 1251.80 | 1118.50 | 988.23 | 763.75 | 589.00 | 352.82 | 140.96 | 0.00 | 179.73 | 380.22 | 535.14 | 670.50 | 866.50 | 1040.78 | 1239.88 | 1518.09 | 1332.52 | 1073.45 | 833.54 | 541.01 | 516.67 | 417.93 | 1031.90 |
| 12 | 1684.62 | 1658.19 | 1548.95 | 1393.79 | 1260.49 | 1167.97 | 943.48 | 768.78 | 532.56 | 320.70 | 179.73 | 0.00 | 200.48 | 355.40 | 490.76 | 686.76 | 861.05 | 1060.15 | 1338.35 | 1153.89 | 894.83 | 654.91 | 478.46 | 336.94 | 355.38 | 852.16 |
| 13 | 1484.14 | 1457.71 | 1348.46 | 1193.31 | 1060.00 | 1059.62 | 835.14 | 660.43 | 733.04 | 521.18 | 380.22 | 200.48 | 0.00 | 154.92 | 290.28 | 486.28 | 660.57 | 859.66 | 1137.87 | 953.41 | 694.34 | 454.43 | 363.13 | 199.25 | 486.21 | 651.68 |
| 14 | 1329.22 | 1302.79 | 1193.54 | 1038.39 | 905.08 | 949.78 | 940.22 | 765.52 | 862.11 | 676.10 | 535.14 | 355.40 | 154.92 | 0.00 | 135.36 | 331.36 | 505.65 | 704.74 | 982.95 | 798.49 | 539.42 | 299.51 | 468.22 | 304.34 | 591.30 | 496.76 |
| 15 | 1319.93 | 1306.98 | 1197.73 | 1173.75 | 1040.44 | 1085.14 | 1075.59 | 900.88 | 997.47 | 811.46 | 670.50 | 490.76 | 290.28 | 135.36 | 0.00 | 196.00 | 370.28 | 695.45 | 973.66 | 790.32 | 674.79 | 434.87 | 603.59 | 439.70 | 726.67 | 487.47 |
| 16 | 1139.42 | 1210.02 | 1100.77 | 1128.59 | 1236.44 | 1281.14 | 1271.58 | 1096.88 | 1193.47 | 1007.46 | 866.50 | 686.76 | 486.28 | 331.36 | 196.00 | 0.00 | 174.29 | 514.95 | 793.16 | 693.36 | 870.78 | 630.87 | 799.58 | 635.70 | 922.66 | 390.51 |
| 17 | 965.14 | 1124.59 | 1233.84 | 1286.73 | 1410.73 | 1455.42 | 1445.87 | 1271.16 | 1367.75 | 1181.74 | 1040.78 | 861.05 | 660.57 | 505.65 | 370.28 | 174.29 | 0.00 | 340.66 | 618.87 | 851.50 | 1045.07 | 805.16 | 973.87 | 809.99 | 1096.95 | 548.65 |
| 18 | 624.48 | 783.93 | 893.17 | 946.07 | 1079.37 | 1180.25 | 1177.77 | 1222.47 | 1319.06 | 1380.84 | 1239.88 | 1060.15 | 859.66 | 704.74 | 695.45 | 514.95 | 340.66 | 0.00 | 278.21 | 510.84 | 769.90 | 632.64 | 925.18 | 1009.08 | 1048.26 | 207.98 |
| 19 | 346.27 | 505.72 | 614.96 | 711.86 | 845.16 | 946.05 | 943.56 | 1118.27 | 1354.49 | 1566.35 | 1518.09 | 1338.35 | 1137.87 | 982.95 | 973.66 | 793.16 | 618.87 | 278.21 | 0.00 | 276.63 | 535.69 | 775.61 | 1068.14 | 1232.02 | 1191.22 | 486.19 |
| 20 | 622.90 | 516.66 | 407.41 | 435.23 | 568.53 | 669.42 | 666.93 | 841.64 | 1077.86 | 1289.72 | 1332.52 | 1153.89 | 953.41 | 798.49 | 790.32 | 693.36 | 851.50 | 510.84 | 276.63 | 0.00 | 259.06 | 498.98 | 791.51 | 955.39 | 914.59 | 302.85 |
| 21 | 881.96 | 763.37 | 654.12 | 498.96 | 365.66 | 410.35 | 407.87 | 582.57 | 818.79 | 1030.65 | 1073.45 | 894.83 | 694.34 | 539.42 | 674.79 | 870.78 | 1045.07 | 769.90 | 535.69 | 259.06 | 0.00 | 239.91 | 532.45 | 696.33 | 655.53 | 561.92 |
| 22 | 1121.87 | 1003.28 | 894.03 | 738.87 | 605.57 | 650.26 | 647.78 | 589.83 | 686.42 | 898.28 | 833.54 | 654.91 | 454.43 | 299.51 | 434.87 | 630.87 | 805.16 | 632.64 | 775.61 | 498.98 | 239.91 | 0.00 | 292.53 | 456.42 | 415.61 | 424.66 |
| 23 | 1383.91 | 1224.46 | 1115.21 | 960.05 | 826.75 | 696.48 | 472.00 | 297.29 | 393.89 | 605.75 | 541.01 | 478.46 | 363.13 | 468.22 | 603.59 | 799.58 | 973.87 | 925.18 | 1068.14 | 791.51 | 532.45 | 292.53 | 0.00 | 163.88 | 123.08 | 717.19 |
| 24 | 1547.79 | 1388.34 | 1279.10 | 1123.94 | 990.63 | 860.37 | 635.88 | 461.18 | 557.77 | 657.63 | 516.67 | 336.94 | 199.25 | 304.34 | 439.70 | 635.70 | 809.99 | 1009.08 | 1232.02 | 955.39 | 696.33 | 456.42 | 163.88 | 0.00 | 286.96 | 801.10 |
| 25 | 1506.99 | 1347.54 | 1238.29 | 1083.13 | 949.83 | 819.56 | 595.08 | 420.37 | 270.81 | 482.67 | 417.93 | 355.38 | 486.21 | 591.30 | 726.67 | 922.66 | 1096.95 | 1048.26 | 1191.22 | 914.59 | 655.53 | 415.61 | 123.08 | 286.96 | 0.00 | 840.27 |
| 26 | 832.46 | 819.51 | 710.26 | 738.08 | 871.39 | 972.27 | 969.78 | 1014.48 | 1111.08 | 1172.86 | 1031.90 | 852.16 | 651.68 | 496.76 | 487.47 | 390.51 | 548.65 | 207.98 | 486.19 | 302.85 | 561.92 | 424.66 | 717.19 | 801.10 | 840.27 | 0.00 |

**Figure 3** OD matrix formed through to the walkways among the traverse stations

As the whole 676 constraint equations formed to solve the application have not been shown in the article, only six of them are presented below, others have been configured similarly.

Constraints $1: 1.0X_{1-2} + 1.0X_{1-3} + 1.0\,X_{1-4} + 1.0X_{1-5} + 1.0X_{1-6} + 1.0X_{1-7} + 1.0X_{1-8} + 1.0X_{1-9} + 1.0X_{1-10} + 1.0X_{1-11} + 1.0X_{1-12} + 1.0X_{1-13} + 1.0X_{1-14} + 1.0X_{1-15} + 1.0X_{1-16} + 1.0X_{1-17} + 1.0X_{1-18} + 1.0X_{1-19} + 1.0X_{1-20} + 1.0X_{1-21} + 1.0X_{1-22} + 1.0X_{1-23} + 1.0X_{1-24} + 1.0X_{1-25} + 1.0X_{1-26} = 1$

Constraints $51: 1.0X_{26-1} + 1.0X_{26-2} + 1.0X_{26-3} + 1.0X_{26-4} + 1.0X_{26-5} + 1.0X_{26-6} + 1.0X_{26-7} + 1.0X_{26-8} + 1.0X_{26-9} + 1.0X_{26-10} + 1.0X_{26-11} + 1.0X_{26-12} + 1.0X_{26-13} + 1.0X_{26-14} + 1.0X_{26-15} + 1.0X_{26-16} + 1.0X_{26-17} + 1.0X_{26-18} + 1.0X_{26-19} + 1.0X_{26-20} + 1.0X_{26-21} + 1.0X_{26-22} + 1.0X_{26-23} + 1.0X_{26-24} + 1.0X_{26-25} = 1$

Constraints 52: $u_2 - u_3 + 25X_{2-3} \le 24$

Constraints 65l: $u_{26} - u_{25} + 25X_{26-25} \le 24$

Constraints 652: $u_2 \le 25$

Constraints 676: $u_{26} \le 25$

The OD length matrix formed, objective function and constraint equations have been evaluated by the CPLEX Optimizer 12.1 program. In the optimum solution, values of variables that give the shortest route among 676 variables are 1, while values of other variables are 0 (see Tab. 2). As a result, shortest survey route in the traverse net is found as 5452.72 m.

Surveying order of traverse stations according to the shortest route solution set is: P.1-P.2-P.3-P.4-P.5-P.6-P.7-P.8-P.9-P.10-P.11-P.12-P.25-P.23-P.24-P.13-P.14-P.15-

P.16-P.17-P.18-P.26-P.22-P.21-P.20-P.19 and P.1 shortest route map is shown in Fig. 4.

Some tests have been performed in the field in order to plan the shortest survey time. Therefore, it has been defined that a surveyor who carries a GPS surveying instrument for about 2 kg moves with an average speed of 4 km/h (66.67 m/minute), searching for traverse station monument in the field and the operation of defining the coordinates by using CORS-TR takes 1.30 minutes for each point. Time data beginning from the starting point P.1 and the time passed to reach to each station together with the total surveying time is given in Tab.3.

**Table 2** Shortest survey route set found by using CPLEX Solution program

| Survey route name | Value | Distance (m) | Survey route name | Value | Distance (m) | Survey route name | Value | Distance (m) |
|---|---|---|---|---|---|---|---|---|
| $X_{1\text{-}2}$ | 1 | 159.45 | $X_{10\text{-}11}$ | 1 | 140.96 | $X_{16\text{-}17}$ | 1 | 174.29 |
| $X_{1\text{-}3}$ | 0 | 268.70 | $X_{10\text{-}12}$ | 0 | 320.70 | $X_{16\text{-}26}$ | 0 | 390.51 |
| … | … | … | … | … | … | … | … | … |
| $X_{2\text{-}3}$ | 1 | 109.25 | $X_{11\text{-}12}$ | 1 | 179.73 | $X_{17\text{-}18}$ | 1 | 340.66 |
| $X_{2\text{-}4}$ | 0 | 264.41 | $X_{11\text{-}13}$ | 0 | 320.70 | $X_{17\text{-}19}$ | 0 | 618.87 |
| … | … | … | … | … | … | … | … | … |
| $X_{3\text{-}4}$ | 1 | 155.16 | $X_{12\text{-}25}$ | 1 | 355.38 | $X_{18\text{-}26}$ | 1 | 207.98 |
| $X_{3\text{-}5}$ | 0 | 288.46 | $X_{12\text{-}13}$ | 0 | 200.48 | $X_{18\text{-}19}$ | 0 | 278.21 |
| … | … | … | … | … | … | … | … | … |
| $X_{4\text{-}5}$ | 1 | 133.30 | $X_{25\text{-}23}$ | 1 | 123.08 | $X_{26\text{-}22}$ | 1 | 424.66 |
| $X_{4\text{-}6}$ | 0 | 263.57 | $X_{25\text{-}26}$ | 0 | 840.27 | $X_{26\text{-}20}$ | 0 | 302.85 |
| … | … | … | … | … | … | … | … | … |
| $X_{5\text{-}6}$ | 1 | 130.27 | $X_{23\text{-}24}$ | 1 | 163.88 | $X_{22\text{-}21}$ | 1 | 239.91 |
| $X_{5\text{-}7}$ | 0 | 354.75 | $X_{23\text{-}22}$ | 0 | 292.53 | $X_{22\text{-}23}$ | 0 | 292.53 |
| … | … | … | … | … | … | … | … | … |
| $X_{6\text{-}7}$ | 1 | 224.48 | $X_{24\text{-}13}$ | 1 | 199.25 | $X_{21\text{-}20}$ | 1 | 259.06 |
| $X_{6\text{-}8}$ | 0 | 399.19 | $X_{24\text{-}22}$ | 0 | 456.42 | $X_{21\text{-}19}$ | 0 | 535.69 |
| … | … | … | … | … | … | … | … | … |
| $X_{7\text{-}8}$ | 1 | 174.71 | $X_{13\text{-}14}$ | 1 | 154.92 | $X_{20\text{-}19}$ | 1 | 276.63 |
| $X_{7\text{-}9}$ | 0 | 410.93 | $X_{13\text{-}15}$ | 0 | 290.28 | $X_{20\text{-}3}$ | 0 | 407.41 |
| … | … | … | … | … | … | … | … | … |
| $X_{8\text{-}9}$ | 1 | 236.22 | $X_{14\text{-}15}$ | 1 | 135.36 | $X_{19\text{-}1}$ | 1 | 346.27 |
| $X_{8\text{-}10}$ | 0 | 448.08 | $X_{14\text{-}22}$ | 0 | 299.51 | $X_{19\text{-}26}$ | 0 | 486.19 |
| … | … | … | … | … | … | … | … | … |
| $X_{9\text{-}10}$ | 1 | 211.86 | $X_{15\text{-}16}$ | 1 | 196.00 | Total Minimal Route Distance | | 5452.72 |
| $X_{9\text{-}11}$ | 0 | 352.82 | $X_{15\text{-}26}$ | 0 | 487.47 | | | |
| … | … | … | … | … | … | | | |



**Figure 4** Shortest survey route in the satellite map

According to Tab.3, in case only one team works at the traverse network and performs two sessions through the shortest survey route, the shortest time needed to complete the survey is:

$n$ = number of session (2)

$Total\ Z = 2^{\text{h}}\ 00^{\text{m}}\ 37^{\text{s}} \times n$

$Total\ Z = 4^{\text{h}}\ 01^{\text{m}}\ 14^{\text{s}}$

(Approx. 4 hours)

On such a survey net with 26 traverse stations, instead of the shortest route, if a walkway of P.1-P.2-P.3-P.4-P.5-P.6-P.7-P.8-P.9-P.10-P.11-P.12-P.13-P.14-P.15-P.16-P.17-P.18-P.19-P.20-P.21-P.22-P.23-P.24-P.25-P.26 and P.1 is preferred, it will have the total length of 6527.01 m. In this case, the period to complete the survey shall be as follows:

$$Total\ Z = \left(26 \times 1.30 + \frac{6527.01}{66.67}\right) \times n$$

$$Total\ Z = 4^{\text{h}}\ 33^{\text{m}}\ 48^{\text{s}}$$

According to the above calculation, the time loss of about 34 minutes for a total survey tour will occur.

**Table 3** Time required measuring the positions of the points in the traverse station according to the shortest survey route

| Point No | Survey route name | $K^*$ (minute) | $L^{**}$ (minute) | Cumulative time (h) $T = K + L$ | Point No | Survey route name | $K^*$ (minute) | $L^{**}$ (minute) | Cumulative time (h) $T = K + L$ |
|---|---|---|---|---|---|---|---|---|---|
| P.1 | | 1.30 | | 0.01.30 | P.23 | | 1.30 | | 0.55.55 |
| | $X_{1\text{-}2}$ | | 2.23 | 0.03.53 | | $X_{23\text{-}24}$ | | 2.27 | 0.58.22 |
| P.2 | | 1.30 | | 0.05.23 | P.24 | | 1.30 | | 0.59.52 |
| | $X_{2\text{-}3}$ | | 1.38 | 0.07.01 | | $X_{24\text{-}13}$ | | 2.59 | 1.02.51 |
| P.3 | | 1.30 | | 0.08.31 | P.13 | | 1.30 | | 1.04.21 |
| | $X_{3\text{-}4}$ | | 2.19 | 0.10.50 | | $X_{13\text{-}14}$ | | 2.19 | 1.06.40 |
| P.4 | | 1.30 | | 0.12.20 | P.14 | | 1.30 | | 1.08.10 |
| | $X_{4\text{-}5}$ | | 1.60 | 0.14.20 | | $X_{14\text{-}15}$ | | 2.01 | 1.10.11 |
| P.5 | | 1.30 | | 0.15.50 | P.15 | | 1.30 | | 1.11.41 |
| | $X_{5\text{-}6}$ | | 1.57 | 0.17.47 | | $X_{15\text{-}16}$ | | 2.56 | 1.14.37 |
| P.6 | | 1.30 | | 0.19.17 | P.16 | | 1.30 | | 1.16.07 |
| | $X_{6\text{-}7}$ | | 3.22 | 0.22.39 | | $X_{16\text{-}17}$ | | 2.36 | 1.18.43 |
| P.7 | | 1.30 | | 0.24.09 | P.17 | | 1.30 | | 1.20.13 |
| | $X_{7\text{-}8}$ | | 2.37 | 0.26.46 | | $X_{17\text{-}18}$ | | 5.06 | 1.25.19 |
| P.8 | | 1.30 | | 0.28.16 | P.18 | | 1.30 | | 1.26.49 |
| | $X_{8\text{-}9}$ | | 3.32 | 0.31.48 | | $X_{18\text{-}26}$ | | 3.07 | 1.29.56 |
| P.9 | | 1.30 | | 0.33.18 | P.26 | | 1.30 | | 1.31.26 |
| | $X_{9\text{-}10}$ | | 3.10 | 0.36.28 | | $X_{26\text{-}22}$ | | 6.22 | 1.37.48 |
| P.10 | | 1.30 | | 0.37.58 | P.22 | | 1.30 | | 1.39.18 |
| | $X_{10\text{-}11}$ | | 2.07 | 0.40.05 | | $X_{22\text{-}21}$ | | 3.36 | 1.42.54 |
| P.11 | | 1.30 | | 0.41.35 | P.21 | | 1.30 | | 1.44.24 |
| | $X_{11\text{-}12}$ | | 2.41 | 0.44.16 | | $X_{21\text{-}20}$ | | 3.53 | 1.48.17 |
| P.12 | | 1.30 | | 0.45.46 | P.20 | | 1.30 | | 1.49.47 |
| | $X_{12\text{-}25}$ | | 5.19 | 0.51.05 | | $X_{20\text{-}19}$ | | 4.09 | 1.53.56 |
| P.25 | | 1.30 | | 0.52.35 | P.19 | | 1.30 | | 1.55.26 |
| | $X_{25\text{-}23}$ | | 1.50 | 0.54.25 | | $X_{19\text{-}1}$ | | 5.11 | 2.00.37 |

$K^*$: Time passes for being and surveying at the station $L^{**}$: Time passes for walking between two stations

In case of having two surveyors' teams to define the positions of the points at the traverse network each of whom has one GNSS survey instrument, CORS-TR traverse station survey requirements are shown in Tab. 1 in the section Material and Methodology. Since the time between the survey sessions performed by different teams is one hour at the least, and when the first survey team starts surveying at point P.1, it is required to put the second team to the point P.13 with timing for 1.02.51 hour forward and continue the surveying tour from here (Tab. 3).

However, as the time chart given in Tab. 3 includes the experimental average values, it is required that two teams should inform each other regarding the survey time and enable the time coordination for 1 hour at the least among the sessions. As seen in Tab. 3, the period required to complete the survey for one team is 1.55.26 h. at average and the period to return to the starting point of P.1 is calculated as 2.00.37 h. Therefore, a survey operation to be performed on the shortest route with a session difference for 1 hour shall be completed by the two teams in about 2 hours.

## 5 CONCLUSION AND RECOMMENDATIONS

The problem of finding the shortest route on networks with GIS is related to a well-known optimization problem TSP. This problem can be modeled as an integer linear programming problem and could be solved exactly (up to some number of nodes) with special OR techniques. Solvers such as CPLEX use these special techniques to optimize mathematical models. On the other hand, a heuristic technique, such as ant colony, genetic algorithm etc. could be used for large instances (large number of nodes for example $n > 1000$). However, these heuristics do not guarantee to give the optimal solution. They just give good solutions.

Integer linear programming method used for practicing the TSP at a traverse station gives mathematically the shortest survey route as an exact result. It is not probable to find the exact result every time by using heuristic methods such as Genetic Algorithm and Ant Colony Algorithms. Even using only one team shall be helpful to find the shortest route and planning the survey at the earliest.

The route to be found will enable us to complete the definition of the traverse station coordinates by using CORS-TR system at the earliest. As two teams are used

during the practice, it is observed the measuring became half against one team. In this study, if the number of points is less than 15 and survey route is shorter than 2500 m, it is determined that using two surveyor teams is not economical. In fact, as the surveying of one team will take one hour on average, the same team will go on the second session after completing the first one.

The recent generalization of the well-known traveling salesman problem is the multiple TSP. The proposed solution has two sessions and the time interval between sessions should be one hour. Therefore, different conditions are added in the problem. In this case the optimal solution is difficult with multiple TSP. Future operations related to finding the survey route that requires less energy considering the land slopes on the traverse station route may be performed. Hence, CORS-TR traverse station surveys may then be performed effortlessly.

## 6 REFERENCES

[1] Kuang, S. (1993a). On optimal design of levelling networks. *Australian surveyor, 38*(4), 257-273 https://doi.org/10.1080/00050326.1993.10438874

[2] Cross, P. A. & Thapa, K. (1979). The optimal design of levelling networks. *Survey Review, 25*(192), 68-79. https://doi.org/10.1179/sre.1979.25.192.68

[3] Benzao, T. & Shaorong, Z. (1995). Optimal design of monitoring networks with prior deformation information. *Survey Review, 33*(258), 231-246. https://doi.org/10.1179/003962695791624632

[4] Kuang, S. (1993b). On optimal design of three-dimensional engineering networks. *Manuscripta Geodaetica,* 18, 33-33.

[5] Kortesis, S. and Dermanis, A. (1987). An application of graph theory to the optimization of costs in trilateration networks. *Manuscripta Geodaetica, 12*(4), 296-308.

[6] Chu, C. M., Ning, F. S., Hung, P. S., & Chu, S. (1997). The Study of Orthometric Heighting by GPS. *Geomatics Research Australasıa*, 55-76.

[7] Kutoglu, H. S. Toker, M., and Mekik, C. (2015). The 3-D Strain patterns in Turkey using Geodetic velocity fields from the RTK-CORS (TR) Network. *Journal of African Earth Sciences*. https://doi.org/10.1016/j.jafrearsci.2015.12.002

[8] Wells, D. E., Lindlohr, W., Schaffrin, B., & Grafarend, E. (1987). GPS Design: Undifferenced Carrier Beat Phase Observations and the Fundamental Differencing Theorm. *Technical Report No. 116*, Department of Surveying Engineering, Universityof New Brunswick, Fredericton, N. B.

[9] Vanicek, P., Beutler, G., Kleusberg, A., Langley, R. B, Santerre, R., & Wells, D. E. (1985). DIPOP: Differential Positioning Program Package for the Global Positioning System. *Geodetic Survey of Canada, Report No. 85-005*, Ottawa.

[10] Snay, R. A. (1986). Network design strategies applicable to GPS surveys using three or four receivers. *Bulletin Géodésique, 60*(1), 37-50. https://doi.org/10.1007/BF02519353

[11] Janes, H., Doucet, K., Roy, B., Wells, D. E., Langley, R., Vanicek, P., & Craymer, M. (1986). GPSNET: A program for the interactive design of geodetic networks. *Geodetic Survey of Canada CR*, 86-003.

[12] Even-Tzur, G. & Papo, H. B. (1996). Optimization of GPS networks by linear programming. *Survey Review, 33*(262), 537-545. https://doi.org/10.1179/sre.1996.33.262.537

[13] Mehrabi, H. & Voosoghi, B. (2014). Optimal observational planning of local GPS networks: assessing an analytical method. *Journal of Geodetic Science, 4*(1). https://doi.org/10.2478/jogs-2014-0005

[14] Dare, P. & Saleh, H. (2000). GPS network design: logistics solution using optimal and near-optimal methods. *Journal of Geodesy, 74*(6), 467-478. https://doi.org/10.1007/s001900000104

[15] Saleh, H. A. & Dare, P. (2001). Effective heuristics for the GPS survey network of Malta: simulated annealing and tabu search techniques. *Journal of Heuristics, 7*(6), 533-549. https://doi.org/10.1007/s001900000104

[16] Saleh, H. A. (2003). An artificial intelligent design for GPS surveying networks. *GPS Solutions, 7*(2), 101-108. https://doi.org/10.1007/s10291-003-0056-4

[17] Saleh, H. A., & Dare, P. 2003. Near-optimal design of Global Positioning System (GPS) networks using the Tabu Search technique. *Journal of Global Optimization, 25*(2), 183-208, https://doi.org/10.1023/A:102195371

[18] Saleh, H. A. & Chelouah, R. (2004). The design of the global navigation satellite system surveying networks using genetic algorithms. *Engineering Applications of Artificial Intelligence, 17*(1), 111-122. https://doi.org/10.1016/j.engappai.2003.11.001

[19] ACO-GPS_sofiahttp://parallel.bas.bg/~stefka/ACO-GPS_sofia.doc (Accessed Date 01.01.2016)

[20] Biggs, N., Lloyd, E. K., and Wilson, R. J. (1976). *Graph Theory*, 1736-1936. Oxford University Press.

[21] Dantzig, G. B., Fulkerson, D. R., & Johnson S. M. (1954). The solution of a large-scale traveling salesman problem. *Oper. Res., 2*(4), 393-410. https://doi.org/10.1287/opre.2.4.393

[22] Baraglia, R., Hidalgo, J. I., & Perego, R. (2001). A hybrid heuristic for the traveling salesman problem. *Evolutionary Computation, IEEE Transactions on, 5*(6), 613-622, https://doi.org/10.1109/4235.974843

[23] Lin, S. & Kernighan B. W. (1973). An Effective Heuristic Algorithm for the Traveling Salesman Problem. *Operations Research,* 21, 498-516. https://doi.org/10.1287/opre.21.2.498

[24] Jung, H., Lee, K., & Chun, W. (2006). Integration of GIS, GPS, and optimization technologies for the effective control of parcel delivery service. *Computers & Industrial Engineering, 51*(1), 154-162. https://doi.org/10.1016/j.cie.2006.07.007

[25] Wijesinghe, K. D. K. D., Delpachithra, C. N., Perera, L. K. C. T., Wijetilake, S. H., Rajapaksha, S., & Fernando, N. (2013). GIS Enabled Travel Planner System with TSP Implementation. *PNCTM;* Vol 2, Jan 2013.

[26] Keenan, P. (2008). Modelling vehicle routing in GIS. *Operational Research, 8*(3), 201. https://doi.org/10.1007/s12351-008-0021-7

[27] Elliott, D. (editör) (1996). *Understanding GPS Principles and Applications,* Mass Artech House, Boston, USA.

[28] Fidanova, S. (2006). Hybrid Heuristic Algorithm for GPS Surveying Problem, Numerical Methods and Applications, *6th International Conference*, August 20-24, Borovets, Bulgaria, 239-246.

[29] Kahveci, M. & Yıldız, F. (2009). *GPS/GNSS Uydularla Konum Belirleme Sistemleri*. Nobel Yayın Dağıtım, Ankara.

[30] Wang, L. & Hu, W. S. (2013). Study and Application in Road Survey on CORS Technique. *Procedia-Social and Behavioral Sciences, 96*, 1707-1711. https://doi.org/0.1016/j.sbspro.2013.08.193

[31] Continuously Operating Reference Station (CORS). http://www.ngs.noaa.gov/CORS/ (Accessed Date 10.08.2015)

[32] Cingöz, A., Erkan, Y., Kurt, A. İ., & Peker, S. (2013). Türkiye Ulusal Sabit GNSS İstasyonları Ağı (TUSAGA-Aktif) Sistemi. TMMOB Harita ve Kadastro Mühendisleri Odası. *14. Türkiye Harita Bilimsel ve Teknik Kurultayı*, 14-17 Mayıs, Ankara.

[33] Tusaga Aktif Genel Bilgi İnternet Notu http://www.hgk.msb.gov.tr/images/projeler/ce0bf019443e0a2.pdf. (Accessed Date 05.10.2015)

[34] Alkan, R. M., Ozulu, İ. M., İlçi, V., Tombuş, F. E., & Şahin, M. TUSAGA-Aktif and WEB-based online PPP Services: A case study in ÇORUM. http://mycoordinates.org/tusaga-aktif-and-web-based-online-ppp-services / (Accessed Date 22.01.2018)

[35] Ulukavak, M., Yaçınkaya, M., & Tanır, E. (2011). Tusaga-Aktif (CORS-TR) Ağ Noktalarının Tektonik Araştırmalarda Kullanılması Üzerine bir Çalışma: Orta ve Doğu Karadeniz Bölgesi Örneği.TMMOB Harita ve Kadastro Mühendisleri Odası. *13. Türkiye Harita Bilimsel ve Teknik Kurultayı*, 18-22 Nisan, Ankara.

[36] Mevzuat: Tapu Kadastro Genel Müdürlüğü https://www.tkgm.gov.tr/sites/default/files/mevzuat/2010_11_0.pdf (Accessed Date 22.01.2018)

[37] Miller, C. E., Tucker, A. W., & Zemlin, R. A. (1960). Integer programming formulation of traveling salesman problems. *Journal of the ACM (JACM)*, *7*(4), 326-329. https://doi.org/10.1145/321043.321046

**Contact information:**

**Bülent BOSTANCI**, Assist. Prof. Dr.
(Corresponding author)
Erciyes University, Department of Geomatics Eng.
38039 Kayseri, Turkey
bbostanci@erciyes.edu.tr

**Abdullah KARAAĞAÇ**, Research Assist.
Erciyes University, Department of Geomatics Eng.
38039 Kayseri, Turkey
akaraagac@erciyes.edu.tr