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ANALYSIS OF GEOPHYSICAL SIGNALS BY USING HILBERT SPACE GEOMETRY

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The contribution presents some results achieved by the authors during several years of research of the process of rotary drilling of rocks by using metric spaces. The authors successfully apply this approach in the solution of specific problems in geophysics and also metallurgy. The authors use abstract structures – so called Hilbert spaces - for the implementation of process signals as algebraic vectors. The geometric structure of these spaces enables mutual metric comparisons of geophysical signals in relation to the rock type and the mode of drilling. Some space is also given to the visualization of the degree of divergence between geophysical or process signals being analyzed.

Key words: geophysical signal, rocks disintegration, rotary drilling, the Hilbert space, inner product operation, visualization of process

INTRODUCTION

Many technological processes in the area of mining and processing of raw materials are problematic from the viewpoint of their direct control. Complications are often related to the impossibility to directly measure key process quantities. Into this category belong e.g. the processes of separation of rocks in mining and tunneling, metallurgical processes in heating machinery, but also some methods of testing the quality of casts. The solution can be reading the vibro-acoustic expressions of the process. Such a suitably read physical signal can represent an integrated information source which, with a suitable method of processing, enables classification of the process and subsequently the control of the process with state space methods.

Functional analysis is a significant part of so-called modern mathematics the beginnings of which date back to the early 20th century. The theory of functional analysis is based upon the abstraction and generalization of the classical Euclidean physical 3-dimensional space E_3 [1-5]. The elements of the space E_3 are the points that are uniquely identified by an ordered triple of real coordinates relating to the corresponding mutually orthonormal unit vectors $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$.

The elements of abstract spaces in functional analysis are functions (so-called functional spaces). The coordinates of functions in such functional space are directly their functional values from the definition interval as relative coordinates corresponding to the chosen orthonormal or orthogonal base of the space.

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Functional spaces have a defined set, topological, algebraic, and geometric structure. These structures were obtained by generalization and abstraction of corresponding structures of the Euclidean space E_3.

In practice it means that the function $f \equiv (f\ (t);\ t : a \mapsto b)$ as an element of the functional space can be viewed as an algebraic vector with corresponding algebraic operations of vector addition and multiplication of vector with a scalar.

HILBERT SPACE

The highest degree of generalization and abstraction of the physical space E_3 represent the classes of functional spaces called Hilbert spaces. The definition of Hilbert space is the following: *Hilbert space is a complete space with inner product. It can be infinite-dimensional and complex.* This definition contains notions whose detailed explanation at this point is not possible. For a significant class of functional spaces (so-called L^2 spaces) it is a necessary condition for the functions as elements of the space to be integrable. That is because the definition of the inner product in these spaces has the form of an integral. The explicitely stated condition of the existence of inner product in the definition of Hilbert space shows its significance for these spaces since the inner product generates the norm of the space and the norm in turn generates its metric. Consequently Hilbert space is a normed linear vector space with metric.

HILBERT SPACE AS SIGNAL SPACE AND ITS GEOMETRY

The properties of functional spaces, especially Hilbert spaces, yield many application possibilities. A condition for using the theory of Hilbert functional spaces in signal processing is that the signal could be interpreted as a function that satisfies conditions for being an element of Hilbert space of the corresponding class. Therefore it is useful to study general properties of each physical signal.

Each electrical signal of the sensor has the following properties:

- Signal is a physical manifestation of physical quantity of the process,
- Signal can take on nonzero values only in finite time interval (physical signal is finite),
- Signal values – amplitudes are bounded (finite),
- No physical signal has complex character,
- Signal can be described with a unique continuous real function of time.

Only signals with these properties can be technically realized.

Computer implementation of methods based on Hilbert L^2-space requires digitization of the process signal. The result of this digitization is an n-component algebraic vector $x \equiv (x_1, x_2, ..., x_n) \in C^n$ as an ordered sequence of n real samples of the signal. With this algebraic vector x we substitute the continuous analog signal $x \equiv x\,(t) \in L^2$ and Hilbert space of class L^2 we substitute with unitary space C^n. It is also a Hilbert space whose points are all possible n-tuples of complex scalars. The operation of inner product here has the form of a sum.

From the mathematical viewpoint, this process of digitization is an n-point approximation of the continuous function x.

VISUALIZATION OF SIGNAL DISTRIBUTION ON SIGNAL SPACE BASED ON THE MEASURE OF THEIR MUTUAL SIMILARITY

Map F_3: $C^n \rightarrow E_3$

This map has the best capability to distinguish individual signals. In this case three measures of differentiation of signal location are used at the same time, namely their L^2 - norm and a couple of angles subtended with two reference signals. The map F_3 part of the space $E3 \equiv \mathbf{R}^3$. The procedure is as follows (Figure 1).

In the signal space C^n we define a suitable reference signal x_{ref} and to the signal we construct an orthogonal vector $x^{\perp}_{ref}$. We then have for their mutual inner product: determines to each signal a location in a bounded

$$\left\langle x_{ref}, x^{\perp}_{ref} \right\rangle = 0. \qquad (1)$$

Further let us choose in the space E_3 a Cartesian system of coordinates (0; $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$) with a triple of coordinate axes (X, Y, Z). This way the Euclidean space is divided into eight octants.

Assume that for the location vectors of images of both reference signals in the mapping F_3 we have:

$$\mathbf{r}_{x_{ref}} = \left(\left\| x_{ref} \right\|_2 \mathbf{e}_X, 0\mathbf{e}_Y, 0\mathbf{e}_Z \right), \qquad (2)$$

$$\mathbf{r}^{\perp}_{x_{ref}} = \left(0\mathbf{e}_X, 0\mathbf{e}_Y, \left\| x^{\perp}_{ref} \right\|_2 \mathbf{e}_Z \right). \qquad (3)$$

This way we located the couple of images of mutually orthogonal reference vectors onto the axes X and Z. Let us Figure 1 further introduce in the space E_3 a spherical system of coordinates (0; r, φ, ϑ), associated with the defined Cartesian coordinate system according to Figure 1. Then there are well-known transformation relations between Cartesian coordinates and spherical coordinates of the same point in the space E_3.

The location of the signal point x in this Euclidean space E_3 is then defined in the spherical coordinates by its location vector $\mathbf{r}_x = \left(\|x\|_2, \varphi, \vartheta \right)$ with the starting point at point 0. The length of this location vector is equal to the norm $\|x\|_2$ of the vector x in space C^n and represents the distance of point x form pole 0 of the spherical system of the space E_3. The coordinate φ represents the magnitude of the oriented angle subtended by axis X with the vertical projection of the location vector $\mathbf{r}_x$ into the plane (X, Y). It is determined by the magnitude of the oriented angle $\varphi = \varphi\,(x_{ref}, x)$ in the signal space C^n. The coordinate ϑ represents the magnitude of the oriented angle subtended by axis Z with the location vector $\mathbf{r}_x$. It is determined by the magnitude of the oriented angle $\vartheta = \vartheta\,(x^{\perp}_{ref}, x)$ in the signal space C^n.

APPLICATION OF MAP F_3

An experiment was realized with the signal of concurrent vibrations of the process of separation of rock massif by rotary drilling on a special drilling stand. The map F_3 was applied to this concurren vibration signal.

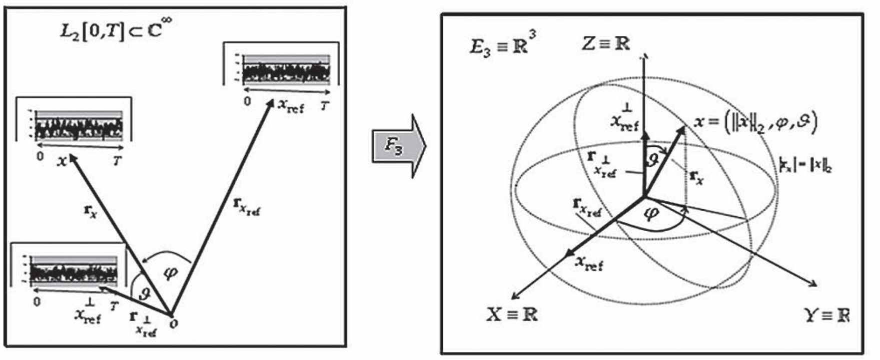

Figure 1 Map F_3 of Hilbert signal space $L^2[0, T] \approx C^n$ into space E_3

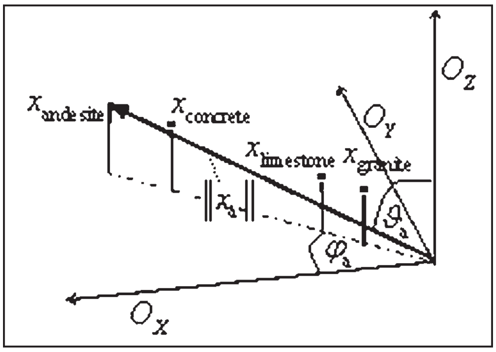

Figure 2 3D visualization of four separated rocks represented by concurrent vibrations as vectors in Hilbert space (*n*=1024)

The process of rotary drilling of rock massif generates concurrent vibro-acoustic emissions that correspond to the character of the rock separation. The sensor of these vibrations (accelerometer) generates at its output random analog signal with the shape of continuous random fluctuations [6-8].

The above method was experimentally used in the solution of the task of effective control of the process of rotary drilling of rock massif [9-11]. From realizations of concurrent vibrations of the process of separation of four types of rock (A – andesite, V – limestone, Z – granite, B – concrete) were calculated the power spectra. They represent, in the sense of above considerations, the functions – vectors of Hilbert space. In view of random, but stationary character, these power spectra were averaged for each rock from 30 realizations of the signal. The mode of drilling (pressure, revolutions) was stabilized during the measurement. On the basis of the above procedure, calculation of the triple of digital positional characteristics was performed for the centroid of each rock. Those characteristics then determined a unique location in 3-D space (Figure 2) for the centroid of each rock.

CONCLUSIONS

The above methods of processing physical signals using the so-called new mathematics enable solution of many complex problems. Functional analysis, namely Hilbert spaces, makes it possible to analyze signals as functions and generalize geometric relations among them. The proposed method of unique map of vectors of Hilbert space into the 3D space was applied on geophysical vibration signals from the process of rotary drilling of rock. Visualized was the differentiability of four kinds of rock on the basis of vibrations and also the sensitivity of the location of vibration signal in Hilbert space on the mode of drilling.

REFERENCES

[1] I. N. Bronshtein, K. A. Semendyayev, G. Musiol, H. Muehlig, Handbook of mathematics. Springer-Verlag Berlin Heidelberg, 2007, 5th Ed., pp. 596-670.

[2] G. James, Advantech Modern Engineering Mathematics. Pearson, 2015, 5th Ed., pp. 181-254.

[3] R. A. Martínez, Avandano, P. Rosenthal, An Introduction to Operators on the Hardy – Hilbert Space. Springer, 2007, pp. 1-160.

[4] A.W. Naylor, G. R. Sell, Theory of linear operators in engineering and natural sciences, Alfa, 1981, pp. 320-341.

[5] V. L. Hansen, Functional analysis – Entering Hilbert space, World Scientific, Singapore, 2006, pp. 1- 422.

[6] Z. Kaláb, P. Štemon, Influence of seismic events on shallow geotechnical structures, In: Acta Montanistica Slovaca, 22(2017)4, pp. 212-221, ISSN 1335-1788.

[7] B. Pandula: J. Kondela, K. Holub, Investigation on seismic signals for blasting in quarries. In: EGRSE, 1(2012)19, p. 41-59, ISSN 1803-1447.

[8] I. Leššo, P. Flegner, J. Futó, Z. Sabová, Utilization of signal spaces for improvement of efficiency of metallurgical process, In: Metallurgy, 53(2014)1, p. 75-77, ISSN 0543-5846.

[9] P. Flegner, J. Kačur, M. Durdán, I. Leššo, M. Laciak, Measurement and processing of vibro-acoustic signal from the process of rock disintegration by rotary drilling, In: Measurement, 56 (2014)1, p. 178-193, ISSN 0263-2241.

[10] S. Shreedharan, C. Hegde, S. Sharma, H. Vardhan, Acoustic fingerprinting for rock identification during drilling. In: International Journal of Mining and Mineral Engineering, 5(2014)2, pp. 89-105.

[11] Z. Li, K. Itakura, Y. Ma, Survey of measurement-while-drilling technology for small-diameter drilling machines. In: Electronic Journal of Geotechnical Engineering, 19(2014)2, pp. 10267-10282.

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