METHOD OF MATERIAL QUALITY ESTIMATION WITH USAGE OF MULTIFRACTAL FORMALISM

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Abstract: Feasibility of application of multifractal theory for evaluation of materials mechanical properties, cast iron in particular, has been considered. The proposed method enables evaluation of mechanical properties of materials based on determination of their sensitivity to dimensions of structure elements from the multifractal Renyi spectrum. Sensitivity of cast iron ultimate strength to informational dimension of carbides, ultimate bending strength to fractal dimension of carbides, impact strength to correlation dimension of carbides and hardness to fractal dimension of graphite have been determined. Fractal prediction models of quality characteristics of cast iron based on the analysis of the following its structure elements (carbides and graphite) have been received.

Keywords: cast iron; graphite; mechanical properties; sensitivity coefficient; spectrum of Renyi dimensions; theory of multifractals

1 INTRODUCTION

Establishing the connection between composition, structure and properties of materials is determined with application of different methods including mathematic simulation. For example, in works [1, 2], about the use of multifactor modeling, the residual load-bearing capacity of the damaged reinforced concrete structures is calculated and ways to strengthen them are considered; in [3, 4] the properties of concretes on lightweight porous fillers for thin-walled structures are evaluated.

Many elements of materials structure at different scale levels have a complex geometrical shape configuration which cannot always can be appropriately described by the use of traditional methods. For example, based on analysis of quantitative and qualitative assessment of many iron-carbon alloys of real structures, there are some differences between the results of direct experiments on determination of their qualitative characteristics using predictive methods [5] that must be observed for usage in practical purposes.

This fact represents insufficiency of formal axiomatic which arises during approximation of metal structure elements using the traditional figures of Euclid's geometry; this emphasizes the necessity of usage of the other prospective approaches to its assessment.

Fractal approach is these days successfully used for the structure analysis [6-9]. Particularly, multifractal theory is used for estimation of heterogeneous structures of metals, concretes, polymers and other materials. Having that in mind, here is the proposed method of estimation of quantitative characteristics of different materials based on multifractal analysis of their structure.

2 MATERIALS AND METHODS

Cast iron for manufacture of casting roller with spherical form of graphite was used as test material (0.5…2.5%); its chemical composition is as follows: 3.00…3.40% C, 1.45…1.80% Si, 0.45…0.65% Mn, 0.040…0.200% P, 0.004…0.010% S, 0.60…0.90% Cr, 0.70…1.30% Ni, 0.035…0.056% Mg. Quantity of carbides (cementite, Fe_3C) is 20-30%. Figs. 1 and 2 shows cast iron structure.
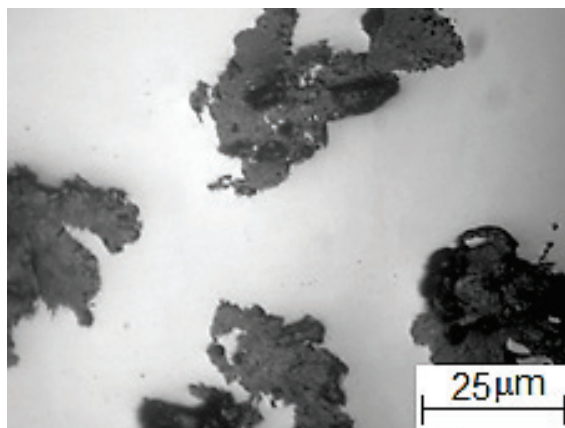

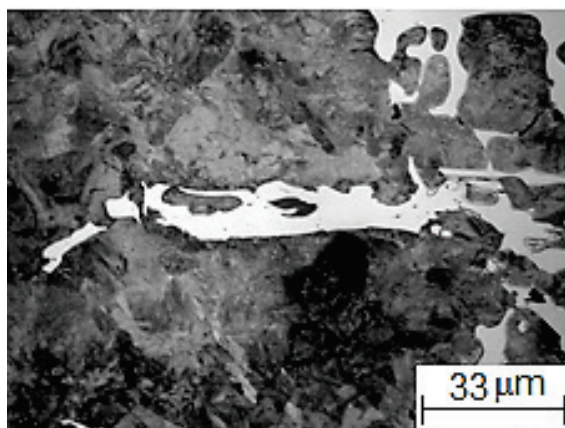**Figure 1** Structure of the working layer of cast iron roll: graphite eutectic

Figure 2 Colonies of Ledeburite, Graphite Eutectic, Pearlite Matrix, HNO3 Etching in working layer of cast iron roll

Impact strength of cast iron KC was determined with groove of notch dimensions of 10×10×55 mm. For

estimating the strength level during bending, σ bars of 10×10×90 mm were used, and during tensile tests σ_B bars of 25 mm diameter were used with 50 mm gauge length. Hardness of cast iron bars, *HSD*, was defined according to Shore. For every structural component (cementite of ledeburite eutectic, pearlite, graphite) spectrum of statistical dimensions was counted as per Renyi formula [10]:

$$D(q) = \frac{1}{q-1} \cdot \lim_{\delta \to \infty} \frac{\ln \sum_{i=1}^{N} p_i^q}{\ln \delta} \qquad (1)$$

where p_i is hit probability of point that is located on the test specimen to the i cell of square mesh with size δ. Dimensions of spectrum D_0, D_1, D_2, D_∞, $D_{-\infty}$ were calculated: D_0 is homogeneous fractal with $q=0$ (Hausdorff-Besikovitch dimension); D_1 is an informational dimension with $q=1$ (informational entropy) characterizing growth rate of the amount of information and showing growth of information, necessary for determining the point location on the test specimen as cell size δ tends to zero; D_2 is a correlation dimension with $q=2$, characterizing probability of occurrence in the same mesh cell of two points on the test specimen; D_∞ is a dimension characterizing the most vacuum space in the test item (structure light segments); $D_{-\infty}$ is a dimension characterizing the most concentrated space (structure dark segments).

Fig. 3 shows the example of calculated spectrum of dimensionalities $D(q)$ of the structure, depending on interval of their variation q. In this case changes of index q were specified within limits from −100 to 100.
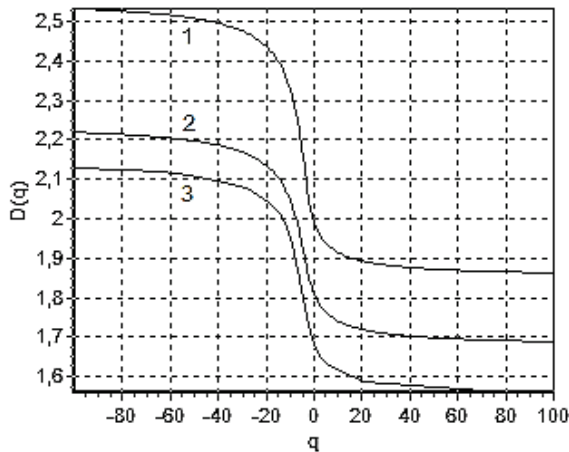

Figure 3 Spectrum of Dimensions of Cementite (1), Pearlite (2), Graphite (3)

3   RESULTS AND DISCUSSION

Method of material quality estimation with usage of multifractal theory comprises the following stages:
- Calculation of statistical dimensions' spectrum of structure elements according to Renyi formula.
- Determination of qualitative characteristics of the test material.

- Calculation of sensitivity of qualitative characteristics to statistical dimensions of structure elements.
- Description of relation between the most sensitive dimensions of the material structure elements and its qualitative characteristics.

For the materials science issues it is useful to study sensitivity of fractal or any other dimension of the material structure elements to changes of their qualitative characteristics through calculation of sensitivity coefficient K_i (2) according to the method [11]:

$$K_i = \frac{|Y_i - Y_{i+1}|}{|X_i - X_{i+1}|} \qquad (2)$$

The sensitivity will be maximal at those points of the structure where the maximum change in the metal properties is observed. It is shown in use [12] that sensitivity of the dimension's spectrum of the elements of ferrite and perlite structure of low-carbon low-alloy steel to mechanical properties is relatively high. This indicates that dimensional evaluation may be applied as an indicator of the material's quality characteristics.

Figs. 4-6 show the calculation results of sensitivity coefficients of cast iron mechanical properties K_i to dimensional evaluation D_0, D_1, D_2, D_{100}, D_{-100} of the structure calculated according to the formula (2).
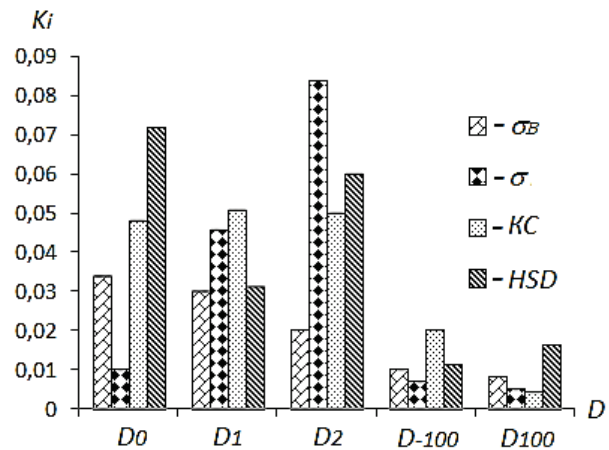

Figure 4 Sensitivity of Cast Iron Mechanical Properties to Dimensions of Graphite

Relatively high sensitivity indexes among those under consideration demonstrate strength characteristics (σ_B and σ) and hardness *HSD* to dimensions D_0, D_1, D_2 of cementite (Fig. 5) and graphite of spherical form (Fig. 4). This sensitivity results from influence of cementite Fe_3C in cast iron alloy on increase in its hardness and strength, stabilization of pearlite, expansion in number of basic strengthening phase, wear resistance increase and reduce of plastic and antifriction properties. Upon the increase of graphite quantity in the cast iron structure, its strength, hardness and wear resistance reduce. Influence of graphite with spherical form on the mechanical properties is proven by sensitivity of these characteristics to dimensional

evaluation (Fig. 4). For the ultimate strength sensitivity index to fractal dimension D_0 of cementite is 0.06; to informational dimension D_1 – 0.07 (Fig. 5). Sensitivity indexes for the ultimate bending strength σ to cementite dimension D_0 and D_2 are 0.09 and 0.08 respectively (Fig. 5), and to graphite dimension D_2 is 0.084 (Fig. 4). Sensitivity coefficients between impact strength indexes KC, cementite dimensions D_0, D_2 (Fig. 5) and pearlite matrix D_2 (Fig. 6) are defined within 0.07, which also confirms the influence of these structure elements on plastic properties of cast iron.
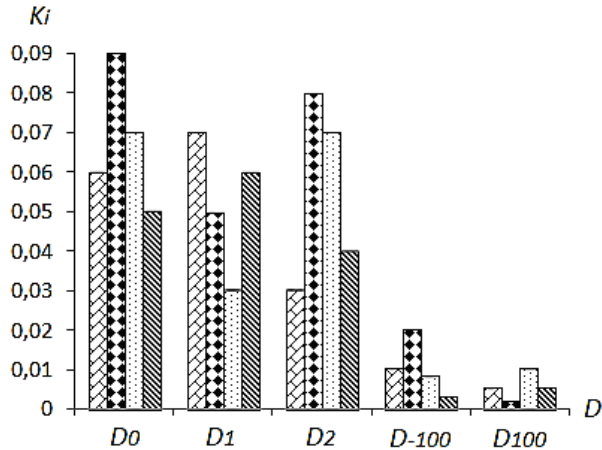

Figure 5 Sensitivity of Cast Iron Mechanical Properties to Dimensions of Cementite
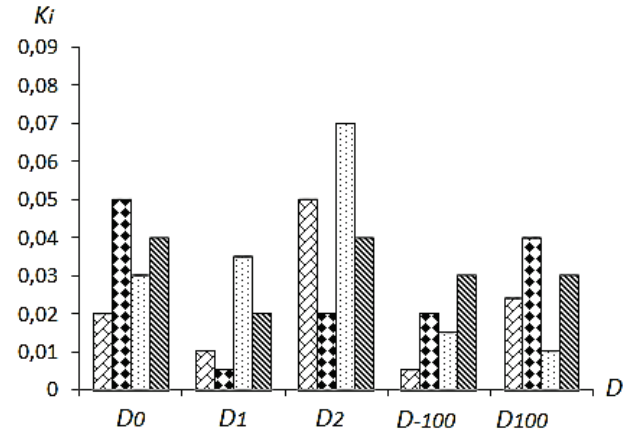

Figure 6 Sensitivity of Cast Iron Mechanical Properties to Dimensions of Pearlite

The above results indicate the influence of the structural elements form, expressed in fractal dimension, and their distribution in volume, expressed in the information and correlation dimensions on the mechanical properties of the metal.

Sensitivity indexes σ_B, σ, KC and HSD to dimensions D_{-100} and D_{100} of cementite and graphite are relatively lower compared to dimensions D_0, D_1, D_2 (Figs. 4-6), which indicates small correlation between them.

Relations between the most sensitive dimensions of the cast iron structure elements and its mechanical properties (Fig. 7) as well as the relevant regression Eqs. (3)-(6) are shown below.
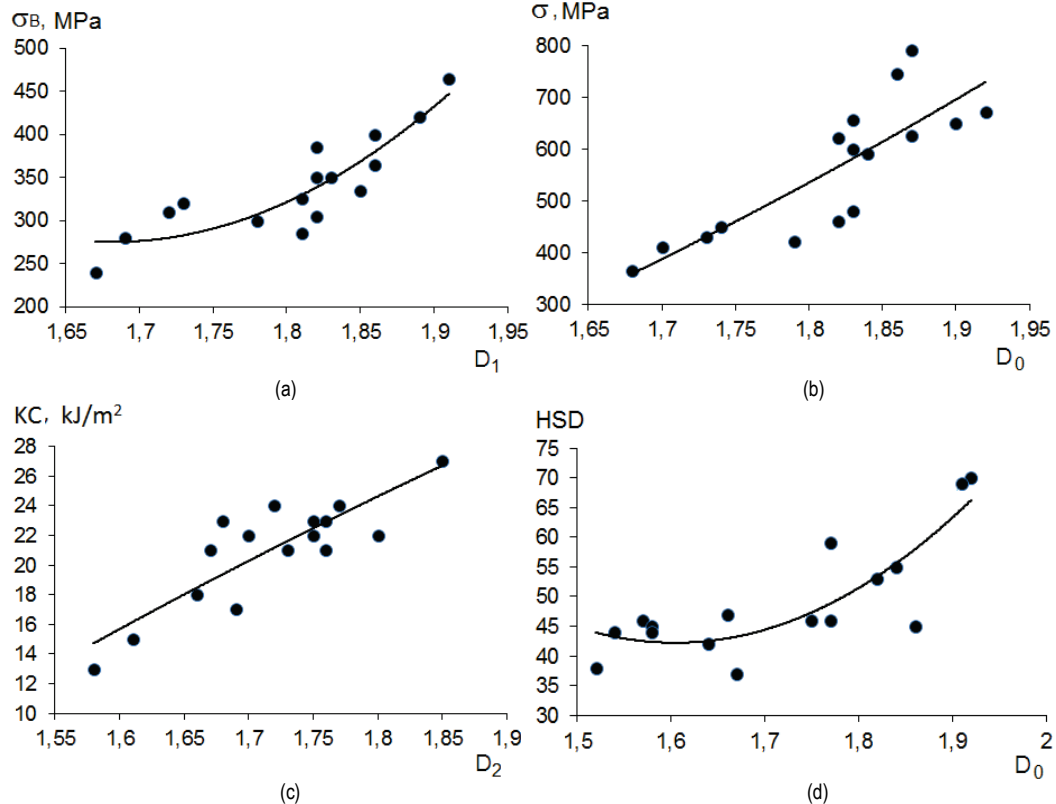

(a)

(b)

(c)

(d)

Figure 7 Dependence of Mechanical Properties on Cementite (a-c) and Graphite (d) dimensions

$$\sigma_{\text{B}} = 3284,30 \cdot D_1{}^2 - 11043 \cdot D_1 + 9558,40 \quad R^2 = 0,77 \quad (3)$$

$$\sigma = 1535,90 \cdot D_0 - 2226,80 \quad R^2 = 0,68 \quad (4)$$

$$KC = 44,20 \cdot D_2 - 54,91 \quad R^2 = 0,72 \quad (5)$$

$$HSD = 241,76 \cdot D_0{}^2 - 775,92 \cdot D_0 + 664,79 \quad R^2 = 0,70 \quad (6)$$

Improvement of the cast iron mechanical properties σ_{B}, σ, KC (Figs. 7a-7c) is caused by influence of such technological factors as chemical composition, heat removal rate during cooling down of cast iron rolls. They influence not only the relation of structure elements but also their geometrical configuration, which is registered by the multifractal spectrum of Renyi dimensions.

Limits of graphite inclusions serve as microstress concentrators and increase of its contents weaken metal matrix of cast iron, which is indicative of influence on mechanical properties of cast iron either composition, sizes, distribution of graphite or form of such inclusions. The increase of hardness HSD from 35 to 70 units is connected with the increase of carbon content from 3.00 to 3.40%. And growth of fractal dimension of graphite to 1.93 (Fig 6d) is caused by the change of its form (Fig. 1) to a spherical one because of increase of Mg inclusions from 0.035 to 0.056%.

4 CONCLUSION

As a result of implementation of the cast iron quality estimation method, the following results have been obtained:

1) For the prediction of mechanical properties of roll-foundry iron, it is effectually to use dimensional evaluations D_0, D_1, D_2 of graphite and cementite, which is proven by the determined sensitivity between them.
2) Relatively high sensitivity among the indexes under consideration – ultimate strength, ultimate bending strength and impact strength – is recorded for dimensions D_0, D_1, D_2 of cementite, hardness sensitivity to fractal dimension D_0 of graphite.
3) Statistical dimensions of graphite and cementite D_{-100}, D_{100} have relatively low sensitivity to the mechanical properties in the analyzed scale ($\times 300$, $\times 400$).
4) The results indicate the influence of the structural elements form, expressed in fractal dimension, and their distribution in volume, expressed in the information and correlation dimensions on the mechanical properties of the metal.

The proposed method allows to evaluate the quality of cast iron on the basis of the dimensional values analysis characterizing the most sensitive elements of its structure and enables evaluation of mechanical properties of materials based on determination of their sensitivity to dimensions of structure elements from the multifractal Renyi spectrum.

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