NEURAL NETWORKS-BASED ROBUST ADAPTIVE FLIGHT PATH TRACKING CONTROL OF LARGE TRANSPORT

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Abstract:

For the ultralow altitude airdrop decline stage, many factors such as actuator nonlinearity, the uncertain atmospheric disturbances, and model unknown nonlinearity affect the precision of trajectory tracking. A robust adaptive neural network dynamic surface control method is proposed. The neural network is used to approximate unknown nonlinear continuous functions of the model, and a nonlinear robust term is introduced to eliminate the actuator's nonlinear modeling error and external disturbances. From Lyapunov stability theorem, it is rigorously proved that all the signals in the closed-loop system are bounded. Simulation results confirm the perfect tracking performance and strong robustness of the proposed method.

1 Introduction

Ultra-low-altitude airdrop, mainly used for delivering heavyweight equipment and supplies to the desired region precisely, is one of the essential functions of a large transport aircraft and it is critical to the success of many military tasks [1,2]. The process of ultra-low-altitude airdrop includes five stages: preparation, fallin, flat, tracking and pull.. After the stage of falling, heavyweight equipment and supplies are dropped to the desired place accurately. During the airdrop process, uncertainty is inevitable, so it is very likely that the model function is unknown. Besides, the ground effect [3,4,5], a sensor measurement error and the low altitude airflow and other uncertain factors seriously disturb trajectory control and threaten the flight safety and mission performance [6,7,8].What's more, the aircraft with a low-speed flying state has a poor anti-interference performance, which is highly susceptible to low-altitude atmospheric disturbances. Over the recent years, quite a few meaningful achievements have been reported in developing advanced aircraft controllers to ensure the accuracy and aircraft safety of the airdrop [1,9-16]. For example, a strong robustness of double ring mixed iterative sliding mode controller is proposed to reject constant uncertainties and uncertain atmospheric disturbance [1]. In addition, on the basis of decoupled and linearized aircraft model achieved by using the input-output feedback linearization approach, an iterative SM(sliding-mode) flight controller is presented. This method establishes a global dynamic switching function in the first-level for the sake of eliminating the reaching phase of the sliding motion, meanwhile, a nonlinear function in the second-level is designed to constitute an integral sliding manifold, weakening the over-compensation of the integral term to big errors effectively [9]. Moreover, on the basis of the feedback linearization aircraft-cargo model, a SMC(sliding-mode control) approach has been developed based on a projection adaptive function approximation, in which an adaptation strategy is designed to acquire robustness against the uncertainties of a model, besides, the prior knowledge of the complicated uncertainties is not

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demanded[10]. However, it is noted that when designing the controller, the above references are not considering actuator nonlinearities' input, such as dead zone and backlash, ignoring the actuator dynamic characteristics and nonlinear factors, and considering that the actual deflection angle is equal to the rudder angle instruction[11]. But because the actual steering control rudder deflection actuator includes mechanical link and hydraulic transmission device, which will inevitably lead to dead zone or backlash nonlinearity in the steering gear, weakening of the stability of the system occures, even resulting in the system divergence [16].

At present, controllers with actuator input dead zone or backlash of transport have not been reported, but the control methods for nonlinear system with dead zone or backlash have been already done a lot of researches[17-23]. For example, a novel adaptive inverse method is proposed to cope with nonlinearities, which constructs the adaptive inverse of the dead-zone [17]and backlash [18] that are cascaded with the control object to eliminate the adverse effects. Recently, a novel robust adaptive controller is designed. The proposed method constructs a global linear model regarding dead-zone as a linear input and bounded of nonlinear perturbation, and effectively overcomes the influence of dead-zone on the system[19] Moreover, an adaptive tracking scheme has been proposed to use a global linear model to establish a dead-zone nonlinear model, which relaxes the condition that the slopes of dead-zone must be time invariant[20]. More recently, an adaptive neural method is proposed for a class of nonlinear system by using Nussbaum gain technique, where the dead-zones are entirely unknown [21]. In addition, for a class of unknown backlash nonlinear uncertain system, on the basis of the backstepping method, the adaptive compensation term of backlash nonlinearity is introduced to suppress the modeling error of backlash. However, in the controller design process, the differential of the virtual control variables needs to be repeated, which greatly increases the complexity of the algorithm [22].

Inspired by the discussion above, in the execution of the input nonlinearity of airdrop decline phase of flight path angle tracking control problem, this paper proposes an adaptive neural network dynamic surface control method, of which a first-order low-pass filter is introduced in the traditional backstepping technique to avoid explosion of differential problems[23]. The adaptive law is used to estimate the unknown model error and external disturbance, and the robust compensation term and neural network are introduced to achieve the closed-loop system stability control, which effectively eliminates the effect of actuator nonlinearity on the system. Moreover, It is proven that the designed controller is able to guarantee that all signals are the(semi-global) uniform ultimate bounded. Finally, simulation verifies the feasibility and effectiveness of the obtained theoretical results.

2 Problem statement

2.1 Aircraft model with an actuator nonlinearity

At the airdrop decline stage, the pilot tracks the reference flight path angle instruction mainly through the frequent manipulation servo drive the rudder deflection to ensure an aircraft quickly and accurately. .In this process, considering only the aircraft pitch movement, including steering nonlinear aircraft longitudinal model it can be expressed as follows[1]:

$$\begin{cases} \dot{\gamma} = -f_6 - f_5\gamma - f_4 \cos\gamma + \\ + f_5\theta + \Delta d_w(C,\gamma,\theta) \\ \dot{\theta} = q \\ \dot{q} = f_0 + f_1 q + f_2\theta - f_2\gamma + \\ + f_3\delta(u) + \Delta d_n(C,\gamma,\theta,q,\delta) \\ \delta(u) = f_\delta(u) \end{cases} \tag{1}$$

where γ is the flight path angle; $\theta = \gamma + \alpha$ with θ being the pitch angle; q is the pitch rate; u is rudder angle instruction and δ is the servo actuator driving actual rudder angle. $f_\delta(\cdot)$ is theservoactuator nonlinearities; $f_0 = \bar{q}Sc_A C_{m0}/I_y$, $f_1 = \bar{q}Sc_A C_{mq}/I_y$, $f_2 = \bar{q}Sc_A C_{m\alpha}/I_y$, $f_3 = \bar{q}Sc_A C_{m\delta_e}/I_y$, $f_4 = g/V$, c_A is the mean aerodynamic chord; $f_5 = (\bar{q}SC_{L\alpha} + T)/(mV)$, $f_6 = -\bar{q}SC_{L0}/(mV)$, S is the wing area; I_y is the pitch moment of inertia; m is the mass of the aircraft; V is the airspeed; T is the engine thrust, $\bar{q} = \rho V^2/2$ is the dynamic pressure; ρ is the air mass density; C_{m*} is the pitch moment coefficients and C_{L*} is the lift coefficients.

Assumption 1: There are uncertain functions $\Delta d_w(C,\gamma,\theta)$ and $\Delta d_n(C,\gamma,\theta,q,\delta)$ satisfy $|\Delta d_w(C,\gamma,\theta)| \le \Delta D_w$, $|\Delta d_n(C,\gamma,\theta,q,\delta_e)| \le \Delta D_n$,where $\Delta D_w > 0$ and $\Delta D_n > 0$ are unknown constants.

2.2 The actuator input dead-zone or backlash nonlinearity model

In order for the actual aircraft actuator to perform with dead-zone and backlash, a class of nonlinearities can be represented by a generalized model as follows:

$$f_\delta(u) = k(u,t)\cdot u + \varepsilon_\delta(u) \tag{2}$$

Where $k(u,t) > 0$ is an unknown continuous function, $\varepsilon_\delta(u)$ is bounded modeling error which satisfies $|\varepsilon_\delta(u)| \le \varepsilon_\delta^*$ with ε_δ^* being an unknown constant.

Assumption 2: $k(u,t)$ is bounded and there exist unknown constants $k_{\min}$ and $k_{\max}$ satisfy $0 < k_{\min} \le k(u,t) \le k_{\max}$.

Case1: When considering the dead-zone nonlinearity, $\delta(u)$ can be described as

$$\delta(u) = \begin{cases} k(u,t)(u-b_r), & u \ge b_r \\ 0, & -b_l < u < b_r \\ k(u,t)(u+b_l), & u \le -b_l \end{cases} \tag{3}$$

where $k(u,t)$ stands for the slope of the dead-zone characteristic, b_r and b_l represent the breakpoints of the dead-zone nonlinearity, b_r and b_l are unknown positive constants, the $\varepsilon_\delta(u)$ function is chosen as

$$\varepsilon_\delta(u) = \begin{cases} -k(u,t)b_r, & u \ge b_r \\ -k(u,t)u, & -b_l < u < b_r \\ k(u,t)b_l, & u \le -b_l \end{cases} \tag{4}$$

According to the eq.(4) and the assumption 2, it can be known that $|\varepsilon_\delta(u)| \le \varepsilon_\delta^* = k_{\max}\cdot\max\{|b_r|,|b_l|\}$.

Case 2: When considering backlash nonlinearity, the analytical expression of $\delta(u)$ can be delivered as

$$\delta(u) = \begin{cases} k(u,t)(u-B_r), \dot{u} > 0 \\ and\ \delta = k(u,t)(u-B_r) \\ k(u,t)(u-B_l), \dot{u} < 0 \\ and\ \delta = k(u,t)(u-B_l) \\ \delta(t_-), \text{others} \end{cases} \tag{5}$$

where $k(u,t) > 0$ is the slope of the backlash, $B_r > 0$ and $B_l < 0$ are relative positions and they are constant parameters. The function $\varepsilon_\delta(u)$ in model (2) is chosen as

$$\varepsilon_\delta(u) = \begin{cases} -k(u,t)B_r, \dot{u} > 0\ and\ \delta = k(u,t)(u-B_r) \\ -k(u,t)B_l, \dot{u} < 0\ and\ \delta = k(u,t)(u-B_l) \\ \delta(t_-) - ku, \text{others} \end{cases} \tag{6}$$

Then we have that assumption 2 is satisfied and $|\varepsilon_\delta(u)| \le \varepsilon_\delta^* = k_{\max}\cdot\max\{|b_r|,|b_l|\}$. Thus, it can be seen from the above discussion that dead-zone and backlash nonlinearities can be viewed as the particular cases of the input nonlinearity in our note.

Assumption 3: The reference flight path angle instruction y_d 、 $\dot{y}_d$ and $\ddot{y}_d$ are smooth and bounded, and they are included in the compact set Ω_1

$$\Omega_1 = \left\{(\gamma_d, \dot{\gamma}_d, \ddot{\gamma}_d) \mid \gamma_d^2 + \dot{\gamma}_d^2 + \ddot{\gamma}_d^2 \le K_0\right\} \tag{7}$$

Objective: For the aircraft longitudinal model with the actuator nonlinearity, uncertain external atmospheric disturbance, and unknown model function, the Nussbaum-gain technique will be used in this paper to design controller so that the flight path angle y can track the reference flight path angle y_d instruction quickly and accurately.

Remark 1: For the convenience of the expression,definedvariables $[x_1, x_2, x_3]^T = [\gamma, \theta, q]^T$; $\Delta d_w(\cdot)$ 、 $\Delta d_n(\cdot)$ 、 $\Delta D_w(\cdot)$ and $\Delta D_n(\cdot)$ are replaced by Δd_w 、 Δd_n 、 ΔD_w and ΔD_n , thus, the system model(1) can be rewritten as

$$\begin{cases} \dot{x}_1 = f_5 x_2 + g_1(\bar{x}_1) + \Delta d_w \\ \dot{x}_2 = x_3 + g_2(\bar{x}_2) \\ \dot{x}_3 = f_3\delta(u) + g_3(\bar{x}_3) + \Delta d_n \\ \delta(u) = k(u,t)\cdot u + \varepsilon_\delta(u) \\ y = x_1 \end{cases} \tag{8}$$

where $g_1(\bar{x}_1) = -f_6 - f_5 x_1 - f_4\cos x_1$, $g_2(\bar{x}_2) = 0$ are the known model functions and $g_3(\bar{x}_3) = f_0 + f_1 x_3 + f_2 x_2 - f_2 x_1$ is an unknown smooth function.

2.3 The Nussbaum type gain

Since the Nussbaum-gain technique will be used in this paper, some results for the Nussbaum-gain are given as following:.

A function $N(\cdot)$ is called a Nussbaum-type function if it is even smooth and has the following properties[24]:

$$\lim_{p\to\infty}\sup\frac{1}{p}\int_0^p N(\zeta)d\zeta=+\infty$$
$$\lim_{p\to\infty}\inf\frac{1}{p}\int_0^p N(\zeta)d\zeta=-\infty \tag{9}$$

Lemma 1 [25]**:** Let $V(\cdot)$ and $\zeta(\cdot)$ be smooth functions defined on $[0,t_f)$ with $V(t)\geq 0$, $\forall t\in[0,t_f)$, where $t_f\in[0,\infty]$. $N(\zeta)$ is an even smooth Nussbaum function. If the following inequality holds:

$$V(t)\leq \kappa_1+e^{-\kappa_2 t}\int_0^t\left[g(x(\tau))N(\zeta(\tau))+1\right]\dot{\zeta}e^{\kappa_2\tau}d\tau,\quad \forall t\in[0,t_f) \tag{10}$$

where κ_1 represents a suitable constant, κ_2 is a positive constant, and $g(x(\tau))$ is a time-varying parameter, which takes values in the unknown closed intervals $I=\left[l^-,l^+\right]$, with $0\notin I$, then $V(t)$, $\zeta(t)$ and $\int_0^t N(\zeta(\tau))\dot{\zeta}d\tau$ must be bounded on $[0,t_f)$.

Lemma 2 [26]**:** The hyperbolic tangent function $\tanh(\cdot)$ will be used in this note, and it is well known that it is continuous, differentiable, and monotonic, and it fulfills that for any $\upsilon>0$ and $q\in R$.

$$\begin{cases}0\leq |q|-q\tanh\left(\dfrac{q}{\upsilon}\right)\leq 0.2785\upsilon\\ 0\leq q\tanh\left(\dfrac{q}{\upsilon}\right)\end{cases} \tag{11}$$

3 The adaptive neural flight control law design

3.1 NN basics

Considering the unknown nonlinear function of model (8), this paper uses the RBF neural network to approximate the unknown function $g_i(\bar{x}_i)$

$$g_i(\bar{x}_i)=W_i^{*T}\Phi_i(\bar{x}_i)+\varepsilon_i(\bar{x}_i) \tag{12}$$

where $W_i^*=\arg\min_{\hat{W}_i^T\in R^{s_i}}\left\{\sup_{\bar{x}_i\in R^i}\left\|g_i(\bar{x}_i)-\hat{W}_i^T\Phi_i(\bar{x}_i)\right\|\right\}$ is optimal weight vector and is an unknown constant parameter, $\Phi_i(\bar{x}_i):R^i\to R^{s_i}$ is the basis function vector, s_i is the number of nodes in the neural network i. $\hat{W}_i$ is the estimate of W_i^*, define the estimate error $\tilde{W}_i=W_i^*-\hat{W}_i$. Φ_i and ε_i indicate the $\Phi_i(\bar{x}_i)$ and $\varepsilon_i(\bar{x}_i)$ respectively, ε_i is neural network approximation error, and satisfies $|\varepsilon_i|\leq\varepsilon_i^*$, ε_i^* is an unknown positive constant.

3.2 Controller design

According to the backstepping progressive controller design method, the adaptive law is introduced to estimate the system unknown parameters. The design steps of the adaptive dynamic surface controller are as following:
To begin with this work, define the first tracking error variable $e_1=x_1-\gamma_d$,and the time derivative of e_1 is

$$\dot{e}_1=f_5x_2+g_1(\bar{x}_1)+\Delta d_w-\dot{\gamma}_d \tag{13}$$

a combination Eq.(11), we can rewrite (13) as

$$\dot{e}_1=f_5x_2+W_1^{*T}\Phi_1+\varepsilon_1+\Delta d_w-\dot{\gamma}_d \tag{14}$$

The virtual control law and adaptive law of parameters are designed as following:

$$\alpha_1=-\frac{1}{f_5}\left(k_1e_1+\hat{W}_1^T\Phi_1+\hat{\delta}_1\tanh\left(\frac{e_1}{\upsilon_1}\right)-\dot{\gamma}_d\right) \tag{15}$$

$$\begin{cases}\dot{\hat{W}}_1=\Gamma_1\left(\Phi_1e_1-\sigma_1\hat{W}_1\right)\\ \dot{\hat{\delta}}_1=\mu_1\left(e_1\tanh\left(\dfrac{e_1}{\upsilon_1}\right)-\sigma_2\hat{\delta}_1\right)\end{cases} \tag{16}$$

Where $\delta_1^*=\varepsilon_1^*+\Delta D_w$, $\hat{W}_1$ and $\hat{\delta}_1$ are the estimates of W_1^* and δ_1^* respectively, $k_1>0,\upsilon_1>0,\mu_1>0,\sigma_1>0,\sigma_2>0$ are design parameters. $\Gamma_1=\Gamma_1^{-1}$ is the adaptive gain matrix. The term $\dot{\hat{\delta}}_1$ in (16) is viewed as a robust compensator which can reject the influence of modeling approximation error and external disturbance.

To avoid repeatedly differentiating α_1, which results in the explosion of complexity, let α_1 pass through a first-order filter with time constant $\tau_2>0$ to acquire $\alpha_{2,f}$ as

$$\tau_2\dot{\alpha}_{2,f}(t)+\alpha_{2,f}(t)=\alpha_1(t),\alpha_{2,f}(0)=\alpha_1(0) \tag{17}$$

Then, define the second tracking error variable $e_2 = x_2 - \alpha_{2,f}$, and the time derivative of e_2 is

$$\dot{e}_2 = x_3 - \dot{\alpha}_{2,f} \quad (18)$$

Similarly, design the virtual law and the parameter adaptive law

$$\alpha_2 = -\left(k_2 e_2 + \hat{W}_2^T \Phi_2 + \hat{\delta}_2 \tanh\left(\frac{e_2}{\upsilon_2}\right) - \dot{\alpha}_{2,f} \right) \quad (19)$$

$$\begin{cases} \dot{\hat{W}}_2 = \Gamma_2 \left(e_2 \Phi_2 - \sigma_3 \hat{W}_2 \right) \\ \dot{\hat{\delta}}_2 = \mu_2 \left(e_2 \tanh\left(\frac{e_2}{\upsilon_2}\right) - \sigma_4 \hat{\delta}_2 \right) \end{cases} \quad (20)$$

where $\delta_2^* = \varepsilon_2^*$, $\hat{W}_2$ and $\hat{\delta}_2$ are the estimates of W_2^* and δ_2^* respectively, $k_2 > 0$, $\upsilon_2 > 0$ $\mu_2 > 0$, $\sigma_3 > 0$ and $\sigma_4 > 0$ are design parameters. $\Gamma_2 = \Gamma_2^{-1}$ is the adaptive gain matrix. let α_2 pass through a first-order filter with time constant $\tau_3 > 0$ to achieve $\alpha_{3,f}$ as

$$\tau_3 \dot{\alpha}_{3,f}(t) + \alpha_{3,f}(t) = \alpha_2(t), \alpha_{3,f}(0) = \alpha_2(0) \quad (21)$$

Design the third tracking error variable $e_3 = x_3 - \alpha_{3,f}$, noting(8) and (12), and the time derivative of e_3 is

$$\begin{aligned} \dot{e}_3 = {} & f_3(k(u,t) \cdot u + \varepsilon_\delta(u)) + \\ & + W_3^{*T} \Phi_3 + \varepsilon_3 + \Delta d_n - \dot{\alpha}_{3,f} \end{aligned} \quad (22)$$

Finally, The virtual control law and adaptive law of parameters are designed as following:

$$u = N(\zeta) \left[k_3 e_3 + \hat{W}_3^T \Phi_3 + \hat{\delta}_3 \tanh\left(\frac{e_3}{\upsilon_3}\right) - \dot{\alpha}_{3,f} \right] \quad (23)$$

$$\dot{\zeta} = k_3 e_3^2 + e_3 \hat{W}_3^T \Phi_3 + \hat{\delta}_3 e_3 \tanh\left(\frac{e_3}{\upsilon_3}\right) - e_3 \dot{\alpha}_{3,f} \quad (24)$$

$$\begin{cases} \dot{\hat{W}}_3 = \Gamma_3 \left(e_3 \Phi_3 - \sigma_5 \hat{W}_3 \right) \\ \dot{\hat{\delta}}_3 = \mu_3 \left(e_3 \tanh\left(\frac{e_3}{\upsilon_3}\right) - \sigma_6 \hat{\delta}_3 \right) \end{cases} \quad (25)$$

where $\delta_3^* = |f_3| \varepsilon_\delta^* + \varepsilon_3^* + \Delta D_n$, $k_3 > 0$, $\upsilon_3 > 0$, $\mu_3 > 0$, $\sigma_5 > 0$ and $\sigma_6 > 0$ are design parameters. $\hat{W}_3$ and $\hat{\delta}_3$ are the estimates of W_3^* and δ_3^* respectively, $\Gamma_3 = \Gamma_3^{-1}$ is the adaptive gain matrix.

4 Stability and tracking performance analysis

Theorem 1: According to the control system (8), for the closed-loop system composed of control law(14) (19) (23) (24), and the adaptive law of parameters(15) (20) (25), if assumptions 1~4 are satisfied, and the initial states of the system are bounded, control parameters $\sigma_i (i = 1 \cdots 6)$ 、 k_i 、 υ_i and $\tau_i (i = 2,3)$ exist there. Make all the variables of the closed-loop system semi-globally uniformly ultimately stable and the tracking error can converge to a point of origin that can be made arbitrarily small..

Define the third order subsystem Lyapunov function V_3

$$V_3 = \frac{1}{2} e_3^2 + \frac{1}{2} \tilde{W}_3^T \Gamma_3^{-1} \tilde{W}_3 + \frac{1}{2\mu_3} \tilde{\delta}_3^2 \quad (26)$$

a combination Eq.(22) (23) and (25), the time derivative of (26) is

$$\begin{aligned} \dot{V}_3 \le {} & f_3 k(u,t) N(\zeta) \dot{\zeta} + \\ & + |e_3| \left(|f_3| \varepsilon_\delta^* + \varepsilon_3^* + \Delta D_n \right) + (e_3 y_3)/\tau_3 \\ & + W_3^{*T} \Phi_3 - \tilde{W}_3^T \Gamma_3^{-1} \dot{\hat{W}}_3 - \frac{1}{\mu_3} \tilde{\delta}_3 \dot{\hat{\delta}}_3 \end{aligned} \quad (27)$$

Adding and subtracting $\dot{\zeta}$ on the right-hand side of (27), one has

$$\begin{aligned} \dot{V}_3 \le {} & \left[f_3 k(u,t) N(\zeta) + 1 \right] \dot{\zeta} + \\ & + \delta_3^* \left(|e_3| - e_3 \tanh\left(e_3/\upsilon_3 \right) \right) - \\ & - \tilde{W}_3^T \Gamma_3^{-1} \left(\dot{\hat{W}}_3 - e_3 \Gamma_3 \Phi_3 \right) - \\ & - e_3 \delta_3^* \tanh\left(e_3/\upsilon_3 \right) - \\ & - \tilde{\delta}_3 / \mu_3 \left(\dot{\hat{\delta}}_3 - e_3 \mu_3 \tanh\left(e_3/\upsilon_3 \right) \right) - \\ & - k_3 e_3^2 \end{aligned} \quad (28)$$

Substituting Eq.(25) into Eq.(28) and according to the lemma 2 leads to

$$\dot{V}_3 \le \left[f_3 k(u,t) N(\zeta) + 1 \right] \dot{\zeta} + 0.2785 \delta_3^* \upsilon_3 - k_3 e_3^2 + \sigma_5 \tilde{W}_3^T \hat{W}_3 + \sigma_6 \tilde{\delta}_3 \hat{\delta}_3 \quad (29)$$

using the following inequalities

$$\sigma_5 \tilde{W}_3^T \hat{W}_3 \le -\frac{\sigma_5}{2} \|\tilde{W}_3\|^2 + \frac{\sigma_5}{2} \|W_3^*\|^2$$
$$\sigma_6 \tilde{\delta}_3 \hat{\delta}_3 \le -\frac{\sigma_6}{2} \tilde{\delta}_3^2 + \frac{\sigma_6}{2} \delta_3^{*2} \quad (30)$$

Noting (30), one can obtain

$$\dot{V}_3 \le -\beta V + \left[f_3 k(u,t) N(\zeta) + 1 \right] \dot{\zeta}_3 + a_0 \quad (31)$$

where $\beta = \min\left\{ 2k_3, \mu_3 \sigma_6, \sigma_5 / \lambda_{\max}(\Gamma_3^{-1}) \right\}$, $a_0 = \frac{\sigma_6}{2} \delta_3^{*2} + 0.2785 \delta_3^* \upsilon_3 + \frac{\sigma_5}{2} \|W_3^*\|^2$.

Multiply (31) by $e^{\beta t}$, and then integrate (31) over $[0,t]$. Thus

$$\dot{V}_3 \le \int_0^t \left[f_3 k(u,\tau) N(\zeta) + 1 \right] \dot{\zeta} e^{-\beta(t-\tau)} d\tau + + \frac{a_0}{\beta} + V_3(0) \quad (32)$$

According to the assumption 2 and lemma 1, hence, the $V_3(t)$ with $\zeta(t)$ and the term $\int_0^t f_3 k(u,\tau) N(\zeta) \dot{\zeta} d\tau$ are bounded.

Define the upper bound Q as follows

$$\int_0^t \left| f_3 k(u,\tau) N(\zeta) + 1 \right| \dot{\zeta} e^{-\beta(t-\tau)} d\tau \le Q \quad (33)$$

From (32) and (33), we can get

$$V_3 \le \frac{a_0}{\beta} + V_3(0) + Q \quad (34)$$

Notice (26) and (34), the $V_3(t)$ is bounded and

$$|e_3| \le \sqrt{2V_3(t)} \le \sqrt{2(a_0/\beta + V_3(0) + Q)} = M$$
$$|\tilde{\delta}_3| \le \sqrt{2V_3(t)} = M \ and \ |\tilde{W}_3| \le \sqrt{2V_3(t)} = M \quad (35)$$

where $Q > 0$ and $M > 0$ are unknown constants.

Define the output error $y_2 = \alpha_{2,f} - \alpha_1$ and $y_3 = \alpha_{3,f} - \alpha_2$. From (13)~(16)、(18)~(21) and (23)~(26) we can know that continuous functions $B_2(\cdot)$ and $B_3(\cdot)$ exist there

$$|\dot{y}_2 + y_2/\tau_2| \le B_2(e_1, e_2, y_2, \tilde{W}_1, \tilde{\delta}_1, \gamma_d, \dot{\gamma}_d, \ddot{\gamma}_d)$$
$$|\dot{y}_3 + y_3/\tau_3| \le B_3(e_1, e_2, e_3, \tilde{W}_1, \tilde{W}_2, \tilde{\delta}_1, \tilde{\delta}_2, y_2, y_3, \gamma_d, \dot{\gamma}_d, \ddot{\gamma}_d) \quad (36)$$

From (36), we can get the following inequalities

$$y_2 \dot{y}_2 \le |y_2| B_2 - \frac{y_2^2}{\tau_2}, y_3 \dot{y}_3 \le |y_3| B_3 - \frac{y_3^2}{\tau_3} \quad (37)$$

To be same as (26), define the first order subsystem of the Lyapunov function

$$V_1 = \frac{1}{2} e_1^2 + \frac{1}{2} \tilde{W}_1^T \Gamma_1^{-1} \tilde{W}_1 + \frac{1}{2\mu_1} \tilde{\delta}_1^2 + \frac{1}{2} y_2^2 \quad (38)$$

Notice $x_2 = e_2 + \alpha_1 + y_2$, according to the lemma 2、Eq.(15) and (16) and the young's inequalities $f_5 e_1 e_2 \le (f_5^2 e_1^2)/2 + e_2^2/2$, $f_5 e_1 y_2 \le y_2^2/2 + (f_5^2 e_1^2)/2$, the time derivative of V_1 is

$$\dot{V}_1 \le \left(f_5^2 - k_1 \right) e_1^2 + \frac{e_2^2}{2} - \frac{\sigma_1}{2} \|\tilde{W}_1\|^2 - \frac{\sigma_2}{2} \tilde{\delta}_1^2 + \frac{1}{2} y_2^2 + y_2 \dot{y}_2 + a_1 \quad (39)$$

where $a_1 = \frac{\sigma_2}{2} \delta_1^{*2} + \frac{\sigma_1}{2} \|W_1^*\|^2 + 0.2785 \upsilon_1 \delta_1^*$. Then, define the second order subsystem of the Lyapunov function

$$V_2 = \frac{1}{2} e_2^2 + \frac{1}{2} \tilde{W}_2^T \Gamma_2^{-1} \tilde{W}_2 + \frac{1}{2\mu_2} \tilde{\delta}_2^2 + \frac{1}{2} y_3^2 \quad (40)$$

Using lemma 2, the time derivative of V_2 is

$$\dot{V}_2 \le (1 - k_2) e_2^2 + \frac{e_3^2}{2} - \frac{\sigma_3}{2} \|\tilde{W}_2\|^2 - - \frac{\sigma_4}{2} \tilde{\delta}_2^2 + \frac{1}{2} y_3^2 + y_3 \dot{y}_3 + a_2 \quad (41)$$

where $a_2 = \sigma_4 \delta_2^{*2}/2 + \sigma_3 \|W_2^*\|^2 / 2 + 0.2785 \upsilon_2 \delta_2^*$. According to Eqs.(34) and (35), V_3 is bounded and

e_3、$\hat{W}_3$ and $\hat{\delta}_3$ are semi globally uniformly ultimately stable and bounded.
Define the following Lyapunov function:

$$V = V_1 + V_2 \tag{42}$$

Note (37) (39) and (41). The time derivative of V is

$$\begin{aligned}\dot{V} \le & \sum_{i=2}^{3}\left(|y_i|B_i\right) + \sum_{i=2}^{3}\left[\left(-1/\tau_i + 1/2\right)y_i^2\right] + a_3 \\ & + \left(f_5^2 - k_1\right)e_1^2 + (3/2 - k_2)e_2^2 - \sigma_2\tilde{\delta}_1^2/2 \\ & - \sigma_4\tilde{\delta}_2^2/2 - \frac{\sigma_1}{2}\|\tilde{W}_1\|^2 - \frac{\sigma_3}{2}\|\tilde{W}_2\|^2\end{aligned} \tag{1}$$

where $a_3 = a_1 + a_2 + M^2/2$, define following set $\Omega_2 = \left\{(e_1, e_2, e_3, y_2, y_3, \tilde{W}_1, \tilde{W}_2, \tilde{\delta}_1, \tilde{\delta}_2) \middle| V \le P, |e_3| \le M\right\}$, where $P > 0$ is a constant, since the Ω_1 is a compact set, $\Omega_1 \times \Omega_2$ is also compact, it is easy to see from (36) and (37) that all the variables of the continuous functions $B_2(\cdot)$ 、 $B_3(\cdot)$ are in the set $\Omega_1 \times \Omega_2$. Therefore, $B_2(\cdot)$ and $B_3(\cdot)$ have maximums, $D_2(\cdot)$ and $D_3(\cdot)$ on $\Omega_1 \times \Omega_2$ respectively. Using the inequalities
$|y_2||D_2| \le y_2^2 D_2^2/(2c_2) + c_2/2, |y_3||D_3| \le y_3^2 D_3^2/(2c_3) + c_3/2$, where $c_2 > 0, c_3 > 0$ are design parameters. So, Eq.(43) can be written as:

$$\begin{aligned}\dot{V} \le & -\frac{\sigma_1}{2}\|\tilde{W}_1\|^2 - \frac{\sigma_3}{2}\|\tilde{W}_2\|^2 + \\ & + \sum_{i=2}^{3}\left[\left(D_i^2/(2c_i) - 1/\tau_i + 1/2\right)y_i^2\right] \\ & + \left(f_5^2 - k_1\right)e_1^2 + (\frac{3}{2} - k_2)e_2^2 - \\ & - \frac{\sigma_2}{2}\tilde{\delta}_1^2 - \frac{\sigma_4}{2}\tilde{\delta}_2^2 + a_4\end{aligned} \tag{44}$$

where $a_4 = a_3 + \left(c_2 + c_3\right)/2$, then, let $1/\tau_i > D_i^2/(2c_i) + 1/2 + \lambda (i = 2,3)$ and $k_1 > f_5^2 + k_0$, $k_2 > 3/2 + k_0$, where $k_0 > 0$ *and* $\lambda > 0$ are design parameters, we arrive at

$$\dot{V} \le -2\mu V + a_4 \tag{2}$$

Solving the inequality (45), we can achieve $V \le a_4/(2\mu) + \left[V(0) - a_4/(2\mu)\right]e^{-2ut}$, obviously, all the signals of the closed loop system are bounded and we can get

$$\lim_{t\to\infty} V(t) \le \frac{a_4}{2\mu} \tag{3}$$

By increasing the design parameters k_0、λ、μ_i and $\sigma_i (i=1,\cdots,6)$, and meanwhile reducing the $\lambda_{\max}(\Gamma_1^{-1})$ 、 $\lambda_{\max}(\Gamma_2^{-1})$, which makes $\mu > a_4/(2P)$. When $V \ge P$, $\dot{V} < 0$, so $V \le P$ is an invariant set. If $V(0) \le P$, then $V(t) \le P$ is for $\forall t > 0$. The tracking error can converge to a sphere with a radius of $a_4/(2\mu)$, choose $\mu > \max\{a_4/(2P), a_4/\varepsilon\}$, thus $e_1^2 \le 2V \le \varepsilon$, by adjusting the design parameters, the ε can be arbitrarily small, and the tracking error can converge to any small area of the origin.

5 The simulation analysis

We simulate a 25,000 kg transport aircraft with a 5,000 kg cargo for example. Design of an adaptive dynamic surface control law guarantees the aircraft flight path angle γ tracking of the desired trajectory $\gamma_d = 3^\circ \sin(t)$ accurately, assuming the atmospheric disturbance $\Delta d_w = 0.02\sin(2t)$ and $\Delta d_n = 0.05\cos(t)$. The initial state of the system are $x_1(0) = x_2(0) = x_3(0) = 0$, the estimation initial values of adaptive parameters are set $\hat{\delta}_1(0) = \hat{\delta}_2(0) = \hat{\delta}_3(0) = 0$, and $\zeta(0) = 1$. Adaptive gain matrix is $\Gamma_1 = \Gamma_3 = diag[0.5]$, the controller design parameters are chosen as $\sigma_1 = \sigma_2 = \sigma_5 = \sigma_6 = 0.5, k_0 = 2, k_1 = 2.5, k_2 = 1.5, k_3 = 3,$ $\tau_1 = \tau_2 = \tau_3 = 0.2$ and $c_2 = c_3 = 1, \lambda = 0.5$ after experimental tuning. The Nussbaum function $N(\zeta) = e^{\zeta^2}\cos(\pi\zeta/2)$ is used. The Gauss function is selected as the basis function of radial basis neural network, so

$$\Phi(\boldsymbol{x}) = \mathrm{e}^{-(x-\mu_i)^T(x-\mu_i)/\upsilon_i^2}, i = 1, 2, ..., l \tag{4}$$

where $\hat{W}^T\Phi(\boldsymbol{x})$ contains $l = 27$ nodes with centers evenly spaced in $[-4,4]\times[-4,4]\times[-4,4]$ and width $\upsilon_i = 2$, the initial values of the neural network weights $\hat{\boldsymbol{W}}(0)$ are set to 0.

5.1 Control performance analysis with considering dead zone nonlinearity

In order to investigate the influence of the dead-zone on airdrop control performance, the scheme proposed in this paper (scheme 1) is compared with the traditional adaptive dynamic surface controller (scheme 2).

Firstly, adopting of scheme 2 to merely investigate the effect of dead zone on closed loop system without taking the external disturbances into consideration, the dead-zone model is shown as Eq.(48). The simulation result is shown as in Figure 1.

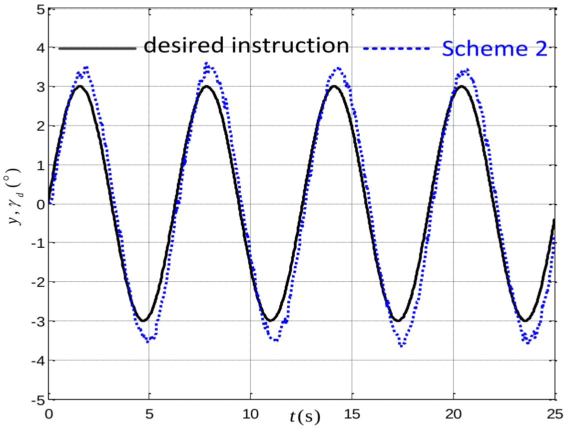

Figure 1. Flight path angle tracking curves with considered dead zone nonlinearities.

It is easy to see from fig.1 that the dead-zone leads to reduction of performance of the control system, resulting in the aircraft unable to track the desired trajectory command accurately.

Then, coupled with the outside atmosphere disturbance influence of Δd_w and Δd_n on the aircraft control performance, the simulation result is shown as in figure 2.

It can be seen from fig.2 that flight path angle tracking performance declines seriously, which leads to the instability of the closed-loop system and seriously affects the accuracy and safety of airdrop.

5.2 Tracking control analysis with considered dead-zone or backlash

Example 1: When dead-zone nonlinearity happens to be present in the system (8), choose the expression of $\delta(u)$ as following:

$$\delta(u)=\begin{cases}1.2(u-0.35), & u\geq 0.35\\ 0, & -0.35<u<0.35\\ 1.2(u+0.35), & u\leq -0.35\end{cases} \tag{5}$$

The initial conditions and disturbance expressions remain unchanged. The simulation results are shown as in the figures 3-5.

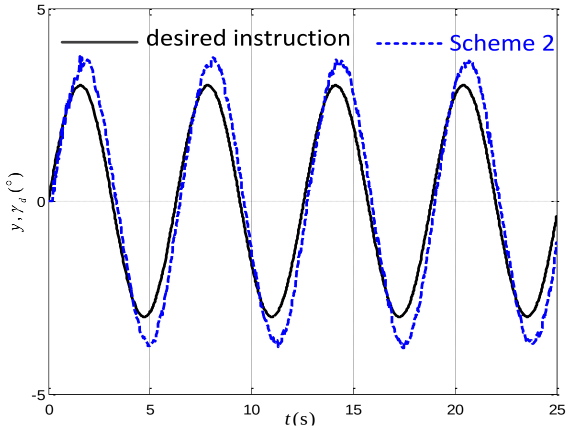

Figure 2. Flight path angle tracking curves with considering dead zone and disturbances.

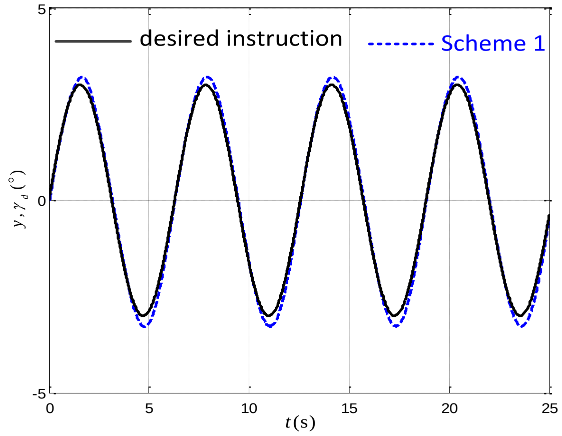

Figure 3. Flight path angle tracking curves with scheme 1.

It can be seen from Fig. 3-4 that scheme 1 can effectively overcome the influence of the dead zone and disturbances on the system and ensure the fast track of the desired flight path angle instruction. Tracking error converges to zero rapidly. From fig.5, it can be seen that the scheme 1 can effectively overcome the problem of the control input flutter caused by dead-zone nonlinearity.

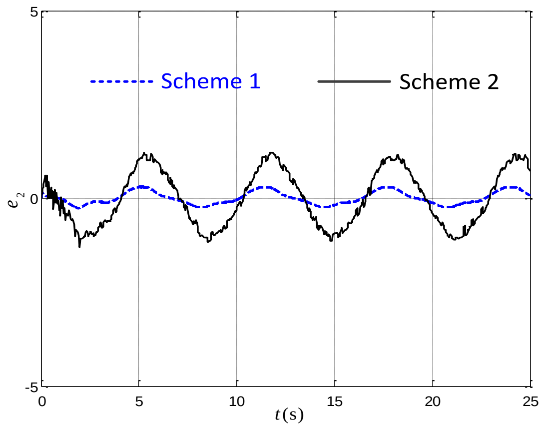

Figure 4. Comparison of tracking error curves.

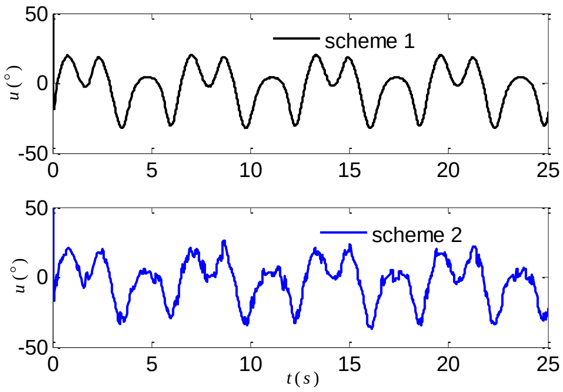

Figure 5. Comparison of control input curves.

Example 2: When the backlash nonlinearity is concerned, choose the expression of $\delta(u)$ as following:

$$\delta(u)=\begin{cases}1.2(u-1/57.3),\dot{u}>0\\ and\ \delta=1.2(u-1/57.3)\\ 1.2(u+1/57.3),\dot{u}<0\\ and\ \delta=1.2(u+1/57.3)\\ \delta(t\_),\text{others}\end{cases} \tag{6}$$

The controller is the same as in example 1, without changing the control parameters, initial conditions, and the Nussbaum functions. The simulation results are as shown in Fig.6 and 7.

From Fig.6, it can be seen that when considering actuator dead-zone nonlinearity, the scheme 1 can achieve the same good tracking control performance as that of the dead zone nonlinearity. It effectively overcomes the bad influence of backlash nonlinearity on the system and has strong robustness characteristics. According to the Fig.7, the estimation of the unknown parameters values are gradually approaching the actual values, with good approximation effect.

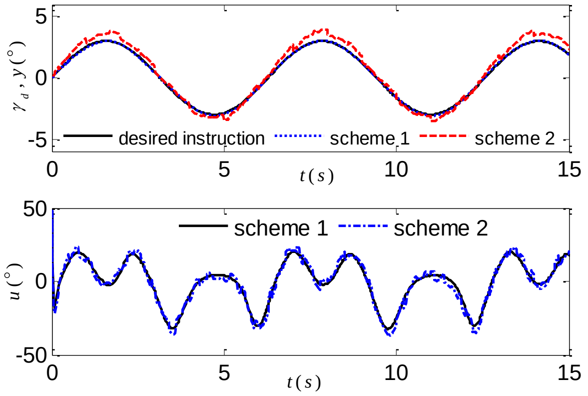

Figure 6. Comparison of flight path angle and control input curves.

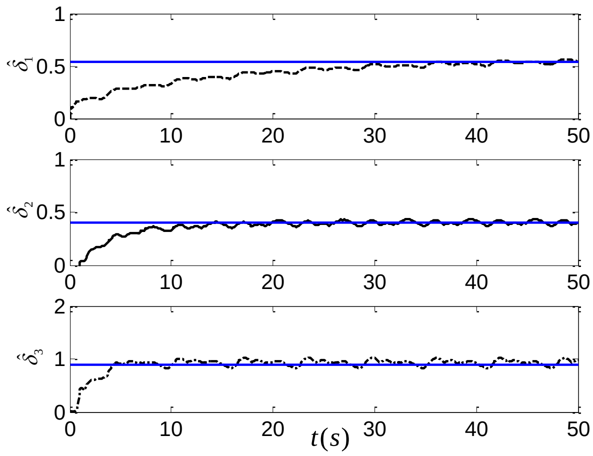

Figure 7. The curves of adaptive parameter estimation.

6 Conclusion

The method has the following advantages. Firstly, the scheme is not only applicable to the aircraft actuator dead-zone nonlinearity, but also it is suitable for the backlash nonlinearity. Secondly, the approach can accurately estimate the unknown model parameters, using the neural networks to approximate the unknown system function. The assumption that model function must be known has been canceled. Thirdly, a robust adaptive compensation term is introduced to eliminate the adverse influence of the external atmospheric disturbances, neural network approximation error, and actuator nonlinear modeling error. The method has a certain reference value for solving the tracking control problem for a class of uncertain nonlinear systems with actuator nonlinearity, which is similar to the structure.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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