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# COMPARISON OF ROBOT TRAJECTORY TRACKING EFFICIENCY USING VARIOUS NONLINEAR CONTROL METHODS[3]

## ABSTRACT

*The Denavit-Hartenberg algorithm and the Lagrange-Euler method are used to derive realistic kinematics and dynamic models of a three-axis electric driven articulated planar robot with viscous, dynamic and static frictions. These robot models are further used for testing the following presented nonlinear robot control methods: fuzzy control, variable-structure control and model-reference variable-structure control. In the fuzzy-logic control method seven fuzzy sets are defined for two input variables. Triangular input membership functions and the 7x7 fuzzy rule table are chosen. The fuzzy controller output value is calculated according to the centre of gravity principle. The same fuzzy control algorithm is used in all robot servo control loops with a proper scaling of the linguistic variables. To eliminate the chattering of the variable-structure control signal and to reduce energy consumption, sign function in the original variable-structure control law is replaced with the following functions: a continuous, saturation and exponential function, all of them with a very thin boundary layer. The same modifications are also made in the original model-reference variable-structure control method. In all presented control methods controller parameters are chosen according to the principle of maximal allowed tracking error and a minimum of energy consumption. These control methods are tested by computer simulations in C programming language in the case of moving the tool of the chosen robot arm. The simulation results proved similar efficiencies of all mentioned modified nonlinear robot control methods, although modified variable-structure control algorithms are the most suitable because of their simplicity and lower number of controller parameters.*

***Key words:*** *robot, fuzzy control, variable-structure control, model-reference control, chattering*

## 1. INTRODUCTION

Industrial robots are very complex kinematics and dynamic systems with a lot of nonlinearities (Kurfess, 2005), (Kovačić et al., 2002), (Kreith, Goswami, 2005), (Lewis et al., 2006), (Nwokah, Hurmuzlu, 2002), (Selig, 1992), (Shell, Hall, 2000), (Schilling, 1990), (Stadler, 1995). It is very

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important to achieve a precise robot trajectory tracking and robustness of the complex closed-loop robot control system (Cheng et al., 2010), (Xi, Hesketh, 2010), (Xu et al., 2003a and 2003b). Some of the nonlinear control methods which are suitable according to these criteria for the use in robotics are fuzzy-logic control, variable-structure control with a sliding mode and model-reference variable-structure control. The goal of this paper is to explore which of these nonlinear control methods is the most suitable for efficient and simple robot control. It is done by comparing energy consumptions of all these control methods if the maximal allowed tracking error along the desired robot trajectory is set.

The application of the fuzzy-logic control in robotics is useful for achieving fast and precise robot tool tip TCP trajectory tracking by using fuzzy sets and fuzzy rules to control robot motor shaft positions and speeds (Klir, Yuan, 1995), (Kovačić, Bogdan, 2000), (Kovačić, Bogdan, 2005).

Variable-structure sliding-mode control is also often used for industrial robot motion control (Cavallo, Natale, 2004), (Chang, 2009), (Hwang, Wu, 2013), (Islam, Liu, 2011), (Šabanović, 2011) because of its robustness and simplicity of the control algorithm (Kardoš, 2007), (Kurfess, 2005), (Zeinali, Notash, 2010). If any deviation in robotic system variables occurs, this control method immediately pushes it back to the constraint by using sliding mode (Chen et al., 1990), (Edwards, Spurgeon, 1998), (Hirschorn, 2007), (Kurfess, 2005), (Morgan, Özguner, 1985), (Temel, Ashrafiuon, 2012). After reaching the sliding surface, the system behaves like a linear-time invariant robust system with a reduced order (Kardoš, 2007), (Perruquetti, Barbot, 2002). For this very fast and powerful control reaction, variable-structure controller needs a lot of energy which leads to very dangerous high-frequency vibrations of the controlled system, i.e. chattering effect (Levant, 2010), (Utkin et al., 1999), (Vukić et al., 2003). To avoid discontinuity in control signal and high heat and energy losses which cause this chattering, various modified variable-structure control methods with a continuous signal (Hashimoto et al., 1987) or saturation function (Bastidas, Vinante, 1997), (Mujanović, 1997), (Myszkorowski, 1990), (Perruquetti, Barbot, 2002), (Yu et al., 2005) are proposed. As a drawback, the ultimate accuracy and robustness of the sliding mode are partially lost (Perruquetti, Barbot, 2002).

Model-Reference Variable-Structure Control (MRVSC) (Ben Azza et al., 2014), (Mujanović, 1997), (Stefanello, Gründling, 2011) is a combination of Model Reference Adaptive Control (MRAC) ((Åström, Wittenmark, 1995), (Ban, 1999), (Bishop, 2002), (Feng, Lozano, 1999), (Ioannou, Sun, 1996)) and previously mentioned Variable-Structure Control (VSC). A reference model in MRAC is used to specify ideal response of an adaptive control system to the input signal, while the aim of an adaptation mechanism is to keep the difference between the model and the plant states as small as possible (Åström, Wittenmark, 1995), (Bishop, 2002), (Ioannou, Sun, 1996). Previously mentioned very fast and computationally simple variable-structure control method with a sliding mode can be used to deal with robot parameters variations and unmodelled robot dynamics for obtaining good tracking control (Kurfess, 2005). MRVSC is often used for controlling slower robot motions and it also has to be modified to avoid chattering (Mujanović, 1997).

## 2. KINEMATICS AND DYNAMIC ROBOT MODELS

Before presenting previously mentioned robot control methods it is necessary to derive realistic kinematics and dynamic robot models which are exact as much as possible. Therefore, the Denavit-Hartenberg algorithm of assigning coordinate frames to each link (Kurfess, 2005), (Kovačić et al., 2002), (Schilling, 1990) of a three-axis electric driven articulated planar robot (Kovačić et al., 2002), (Schilling, 1990) shown in Figure 1 is used for deriving robot kinematic parameters shown in Table 1.

Figure 1. A three-axis articulated planar robot.

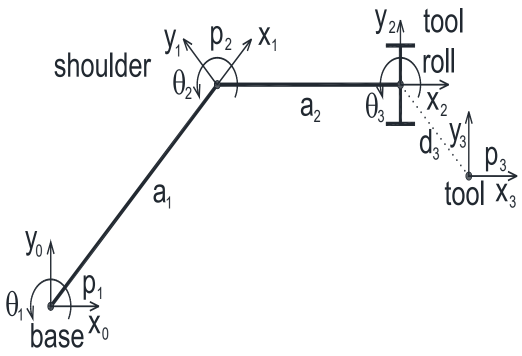

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Table 1. Kinematic parameters of a three-axis articulated planar robot.

| **Axis** | **θ** | **d** | **a** | **α** |
|---|---|---|---|---|
| 1 | $q_1$ | 0 | $a_1$ | 0 |
| 2 | $q_2$ | 0 | $a_2$ | 0 |
| 3 | $q_3$ | $d_3$ | 0 | 0 |

Source: (Schilling, 1990) and authors

For solving the chosen robot dynamics problem, the Lagrange-Euler (Kovačić et al., 2002), (Kurfess, 2005), (Perruquetti, Barbot, 2002), (Schilling, 1990), or the Newton-Euler (Kovačić et al., 2002), (Kurfess, 2005), (Schilling, 1990) method can be used because both methods give the same robot dynamic model:

$$\begin{aligned}\tau_1 = {} & \left[\left(\frac{m_1}{3}+m_2+m_3\right)\cdot a_1^2+\left(\frac{m_2}{3}+m_3\right)\cdot a_2^2+(m_2+2\cdot m_3)\cdot a_1\cdot a_2\cdot\cos(q_2)+J_{m1}\cdot N_{r1}^2\right]\cdot\ddot{q}_1+ \\ & +\left[\left(\frac{m_2}{3}+m_3\right)\cdot a_2^2+\left(\frac{m_2}{2}+m_3\right)\cdot a_1\cdot a_2\cdot\cos(q_2)\right]\cdot\ddot{q}_2- \\ & -(m_2+2\cdot m_3)\cdot a_1\cdot a_2\cdot\sin(q_2)\cdot\left(\dot{q}_1\cdot\dot{q}_2+\frac{\dot{q}_2^2}{2}\right)+ \\ & +g_0\cdot\left[\left(\frac{m_1}{2}+m_2+m_3\right)\cdot a_1\cdot\cos(q_1)+\left(\frac{m_2}{2}+m_3\right)\cdot a_2\cdot\cos(q_1+q_2)\right]+b_1(\dot{q}_1),\end{aligned} \tag{1}$$

$$\begin{aligned}\tau_2 = {} & \left[\left(\frac{m_2}{3}+m_3\right)a_2^2+\left(\frac{m_2}{2}+m_3\right)a_1 a_2\cos(q_2)\right]\ddot{q}_1+\left[\left(\frac{m_2}{3}+m_3\right)\cdot a_2^2+J_{m2}\cdot N_{r2}^2\right]\cdot\ddot{q}_2+ \\ & +\left(\frac{m_2}{2}+m_3\right)\cdot a_1\cdot a_2\cdot\sin(q_2)\cdot\dot{q}_1^2+g_0\left(\frac{m_2}{2}+m_3\right)a_2\cos(q_1+q_2)+b_2(\dot{q}_2),\end{aligned} \tag{2}$$

$$\tau_3 = J_{m3}\cdot N_{r3}^2\cdot\ddot{q}_3+b_3(\dot{q}_3), \tag{3}$$

where $q_i$, $\dot{q}_i$ and $\ddot{q}_i$ are the ith joint variable, velocity and acceleration respectively ($1\leqslant i\leqslant 3$), $\tau_i$ is the ith actuator torque, $a_i$ and $m_i$ are length and mass of the ith robot segment, $J_{mi}$ is moment of inertia for the ith motor, $N_{ri}$ is the ith gear ratio, $g_0$ is gravitational constant and $b_i()$ denotes friction opposing the motion of the ith joint. This realistic dynamic robot model contents the following viscous, dynamic and static joint and motor frictional forces (Kovačić et al., 2002), (Schilling, 1990):

$$b_i(\dot{q}_i) = b_{v_i}\dot{q}_i+\left[b_{d_i}+\left(b_{s_i}-b_{d_i}\right)\cdot e^{\frac{-|\dot{q}_i|}{\varepsilon_i}}\right]\cdot\operatorname{sgn}(\dot{q}_i), \tag{4}$$

where $\dot{q}_i$ is the velocity for joint i ($1\leqslant i\leqslant 3$); $b_{v_i}$, $b_{d_i}$, $b_{s_i}$ are the coefficients of viscous, dynamic and static friction, respectively, for joint i; $e_i$ is a small positive parameter.

In computer simulations, robot construction and actuator limits are considered and the following total energy of all chosen robot motors is calculated:

$$E = \sum_{i=1}^{3}\left(\int_0^{T_s} U_{a_i}\cdot I_{a_i}\cdot dt,\quad U_{a_i}\cdot I_{a_i}>0\right), \tag{5}$$

where $T_s$ denotes the whole robot trajectory traverse time, $U_{ai}$ is armature voltage and $I_{ai}$ is armature current for the ith robot motor ($1\leqslant i\leqslant 3$).

## 3. FUZZY CONTROL METHOD

The discrete inputs of the fuzzy-logic controller used in the ith robot joint servo control loop are motor position error signal $e_i(k)$ and the change of motor position error signal $de_i(k)$:

$$e_i(k) = \theta_{r_i}(k) - \theta_i(k), \quad 1 \le i \le n, \tag{6}$$

$$de_i(k) = e_i(k) - e_i(k-1), \quad 1 \le i \le n. \tag{7}$$

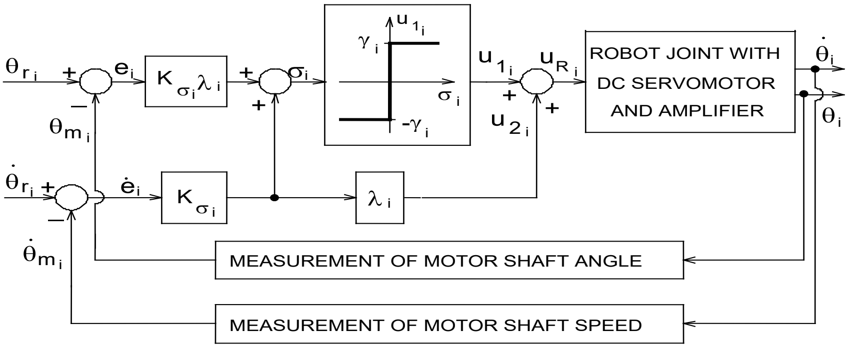
For these fuzzy-logic controller input variables seven fuzzy sets are defined: large negative, medium negative, small negative, zero, small positive, medium positive and large positive (Kovačić, Bogdan, 2000), (Kovačić, Bogdan, 2005) to achieve a sufficiently good robot control method. The fuzzy controller input membership functions with a triangular form are chosen, where only two adjacent functions overlap (Kovačić, Bogdan, 2000), (Kovačić, Bogdan, 2005). The limits of robot motor torques are taken into account during the creation of the 7x7 fuzzy rule table (Kovačić, Bogdan, 2000), (Kovačić, Bogdan, 2005). The fuzzy controller output value is calculated according to the centre of gravity principle (Kovačić, Bogdan, 2000), (Kovačić, Bogdan, 2005) which simplifies the calculation procedure.

The same fuzzy control algorithm is used in all robot servo control loops, but a proper scaling of the linguistic variables is done (Kovačić, Bogdan, 2000), (Kovačić, Bogdan, 2005). These input and output scaling parameters of fuzzy controllers are chosen according to the principle of maximal allowed tracking error along the desired robot tool tip TCP trajectory with a minimum of energy consumption.

## 4. VARIABLE-STRUCTURE CONTROL METHOD

The original variable-structure robot control scheme (Chen et al., 1990), (Morgan, Özguner, 1985) is shown in Figure 2, where robot variables are error of each ith robot motor shaft angle $e_i(t)$ and speed $de_i(t)/dt$.

Figure 2. The original variable-structure robot control scheme.



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As can be seen in Figure 2 the part of the control law is a signum function (the 1st VSC method):

$$u_{1_i}(t) = \gamma_i \cdot \mathrm{sgn}[\sigma_i(t)] = \gamma_i \cdot \frac{\sigma_i(t)}{|\sigma_i(t)|}, \quad 1 \le i \le n, \tag{8}$$

with very frequent and fast switching which causes high-frequency oscillations of the control signal, i.e. chattering, as mentioned before. To reduce or eliminate unwanted chattering of the control signal, a signum function in the first part of the control law (8) can be replaced with a continuous signal given in (Hashimoto et al., 1987) (the 2nd VSC method):

$$u_{1_i}(t) = \gamma_i \cdot \frac{\sigma_i(t)}{|\sigma_i(t)| + \delta_i}, \quad 1 \le i \le n, \tag{9}$$

or by a saturation function suggested in (Bastidas, Vinante, 1997), (Mujanović, 1997), (Myszkorowski, 1990), (Perruquetti, Barbot, 2002), (Yu et al., 2005), (the 3rd VSC method):

$$u_{1_i}(t) = \gamma_i \cdot sat[\sigma_i(t)] = \begin{Bmatrix} -\gamma_i, & \sigma_i < -\delta_i \\ \gamma_i \cdot \dfrac{\sigma_i}{\delta_i}, & -\delta_i < \sigma_i < \delta_i \\ \gamma_i, & \sigma_i > \delta_i \end{Bmatrix}, \quad 1 \le i \le n, \tag{10}$$

or with modified exponential function (the 4th variable-structure control method):

$$u_{1_i}(t) = \gamma_i \cdot \mathrm{sgn}[\sigma_i(t)] \cdot \left(1 - e^{-\frac{|\sigma_i|}{\delta_i}}\right) = \gamma_i \cdot \frac{\sigma_i(t)}{|\sigma_i(t)|} \cdot \left(1 - e^{-\frac{|\sigma_i|}{\delta_i}}\right), \quad 1 \le i \le n, \tag{11}$$

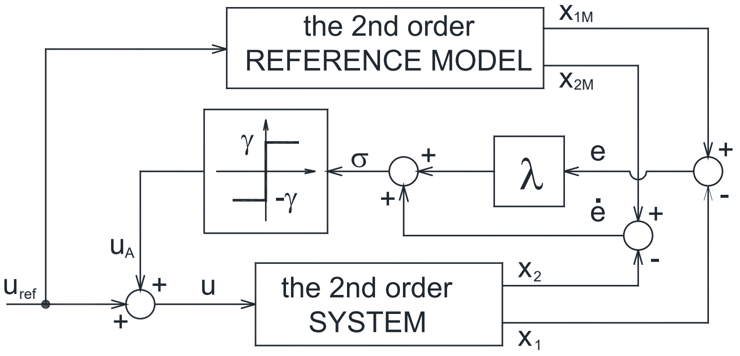
where $\delta_i$ is a thickness of the boundary layer for the ith robot motor, $1 \leqslant i \leqslant$ n.

Controller parameters for all presented VSC methods can be adjusted according to the following principles: maximum allowed robot trajectory tracking error, no chattering and minimum of energy.

## 5. MODEL-REFERENCE VARIABLE-STRUCTURE CONTROL METHOD

One of the simplest MRVS robot control methods (Ben Azza et al., 2014), (Mujanović, 1997), (Stefanello, Gründling, 2011) is given in Figure 3. A reference model is a part of this control system which consists of two loops: the inner and the outer control loop.

Figure 3. The original model-reference variable-structure robot control scheme.



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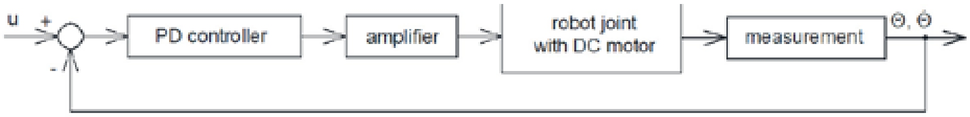
The inner loop is an ordinary control loop composed of the PD-controller and the process, i.e. amplifier, robot joint with motor and sensors for measurement of motor shaft angle $x_1$ and velocity $x_2$, as shown in Figure 4, which act together as the 2nd order system. The coefficients of PD-controller are set according to demand to eliminate the smaller motor time constant and to have no overshooting.

Figure 4. The scheme of the 2nd order system of the inner control of robot joint with motor.



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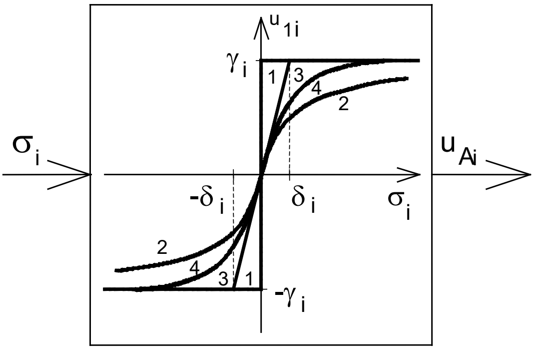
The input signal to the system is adjusted by the outer loop with a 2nd order reference model and a variable-structure controller, in such a way that the errors between the model states $x_{1M}$ and $x_{2M}$ and the system states $x_1$ and $x_2$ become zero. This original variable-structure controller has the same chattering problem as previously explained in equation (8), so the same modifications of the variable-structure control law $u_{Ai}(t)$ as for $u_{1i}(t)$ in equations (9)-(11) have to be made, as shown in Figure 5.

Figure 5. The modifications in MRVS control.



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## 6. SIMULATION RESULTS

All explained and modified nonlinear robot control methods are tested by computer simulations in C programming language in the case of moving the tool of a three-axis electric driven articulated planar robot shown in Figure 1. The lengths of robot segments are: $a_1$=0.3 [m], $a_2$=0.2 [m], and the distance between the tool tip TCP and the working plane is $d_3$=0.1 [m] (i.e. length of the third segment). The masses of the segments are: $m_1$=1 [kg], $m_2$=0.7 [kg], $m_3$=0.3 [kg]. The following friction coefficients are used in simulations: viscous motor friction coefficients $b_{vm1}=b_{vm2}=b_{vm3}$=0.0000385534 [kgžm$^2$/s]; viscous joint friction coefficients $b_{v1}=b_{v2}=b_{v3}$=0.2 [kgžm$^2$/s]; dynamic joint friction coefficients $b_{d1}=b_{d2}=b_{d3}$=0.1 [kgžm$^2$]; static joint friction coefficients $b_{s1}=b_{s2}=b_{s3}$=0.3 [kgžm$^2$]; small constants $e_1=e_2=e_3$=0.1.

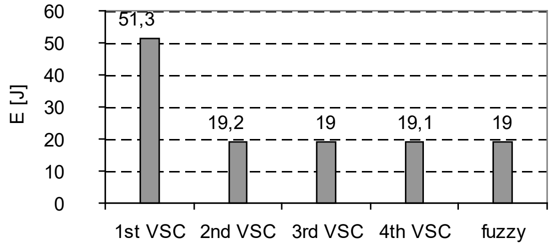
Other robot motor parameters are: armature winding gains $K_{a1}=K_{a2}=K_{a3}$=0.12195 [W$^{-1}$], armature time constants $T_{a1}=T_{a2}=T_{a3}$=2.012195 [ms], torque constants $K_1=K_2=K_3$=0.0394 [Nžm/A], moments of inertia $J_{m1}=J_{m2}=J_{m3}$=0.00000268 [kgžm$^2$], maximal armature currents $I_{am1}=I_{am2}=I_{am3}$=0.745 [A], maximal output controller voltage $U_{Rm1}=U_{Rm2}=U_{Rm3}$=10 [V], amplifier coefficients $K_{AM1}=K_{AM2}=K_{AM3}$=2.4, gear ratios $N_{r1}$=291, $N_{r2}$=388, $N_{r3}$=582.

The presented fuzzy-logic and variable-structure sliding-mode control methods are tested in the case of moving the tool tip TCP of the chosen robot along the straight-line vertical trajectory from the starting point (0.2 [m], 0 [m]) to the ending point (0.2 [m], 0.4 [m]). During robot trajectory planning, the robot trajectory traverse time $T_S$ is set to the value $T_S$=4.44 [s], in order to get as fast robot movement as possible regarding robot acceleration and velocity limits. The maximal allowed whole trajectory tracking error is set to 0.5 [mm]. The goal is to adjust the parameters of all proposed robot controllers to achieve the desired error with the minimum of energy consumption.

Due to simplicity and good performance, the same fuzzy controllers are used for all three robot control loops, but different scaling of the fuzzy controller outputs is necessary. Figure 6 shows that energy consumption E=19 [J] is achieved along the robot trajectory. The fuzzy-logic controller reacts very fast, so it is convenient for fast robot tool tip movements.

The 1st variable-structure controller parameters are set according to the following procedure: the value of parameter $K_\sigma$ is set to 0.01 to enable trajectory tracking, the value of parameter $\lambda$ defines the amount of energy consumption and by changing parameter $\gamma$ the maximal allowed tracking error is reached. As can be seen in Figure 6 in simulations with controller parameters $\lambda$=13.74 [$s^{-1}$] and $\gamma$=3.837, 2.7 times more energy is needed for all robot motors in comparison with the fuzzy-controller because of chattering of the control signal. This chattering can be eliminated in all modified variable-structure control methods with a proper thickness of the boundary layer of $\delta$=0.05. After that, as can be seen in Figure 6, the amounts of consumed energy in all modified VSC methods are very similar to the fuzzy controller consumed energy. However, the modified variable-structure controllers are easier to use because they have fewer parameters to adjust than the fuzzy controllers.

Figure 6. Energy consumptions for different variable-structure and fuzzy control methods.



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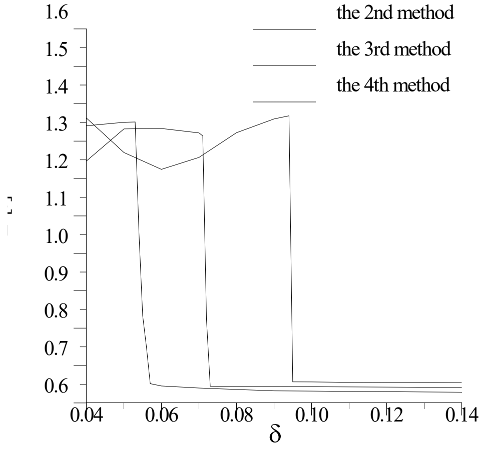
The proposed model-reference variable-structure robot control methods are usually used for slower robot tool tip TCP trajectories. All four MRVSC methods explained earlier are tested in the case of the same step input signal $u_{ref}(t)=1*S(t)$ acting on each motor of the chosen robot shown in Figure 1. The PD-controllers gain coefficients are set as: $P_1=P_2=P_3=0.512$ [V/rad] and $D_1=D_2=D_3=0.001256$ [V·s/rad]. The parameters of MRAC are set to be equal to parameters of the 2nd order system: $K_{Mi}=K_{si}$, $\xi_{Mi}=\xi_{si}$, $T_{Mi}=T_{si}$, $1\leqslant i\leqslant 3$. The parameters of VSC are defined according to the demands of sufficient velocity of transient responses: $\lambda_1=\lambda_2=\lambda_3=100$ [$s^{-1}$] and maximal allowed tracking error $e_{max}=0.04*u_{ref}$ for each robot motor: $\gamma_1=14.11$, $\gamma_2=1.029$, $\gamma_3=0.1221$. The signum function in the 1st MRVSC method causes chattering of the control signal $u_A(t)$ and energy consumption of 1.43 [J]. Therefore, the 1st MRVSC method is modified in three presented ways, but thickness of boundary layer $\delta$ has to be as small as possible because of the existence of a steady-state error. As can be seen in Figure 7 the thickness of the boundary layer for the first robot motor $\delta_1=0.057$ in the 2nd MRVSC method, $\delta_1=0.095$ in the 3rd method and $\delta_1=0.073$ in the 4th MRVSC method reduce total energy consumption by one half.

Figure 7. Influence of boundary layer thickness on energy consumption in MRVSC methods.



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## 7. CONCLUSION

The results of computer simulations proved previously assumed similar efficiencies in robot tool tip TCP trajectory tracking, chattering elimination and energy consumption minimization of all presented modified nonlinear robot control methods. The modified variable-structure control algorithms are the most suitable for robot control because of its simplicity and a lower number of controller parameters. The fuzzy-logic control method is very convenient for controlling fast robot tool tip movements, but it represents a more complex robot control method because of a lot of fuzzy-controller parameters which have to be determined. The model-reference variable-structure robot control method is the most useful for controlling slow robot tool tip TCP motions.

The next research goal can be further efficiency improvement of the presented modified nonlinear robot control methods by optimizing their controller parameters according to different optimization criteria and by analyzing their robustness.

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# USPOREDBA EFIKASNOSTI SLIJEĐENJA TRAJEKTORIJE ROBOTA PRI UPOTREBI RAZLIČITIH METODA NELINEARNOG UPRAVLJANJA[3]

## SAŽETAK

*Denavit-Hartenbergov algoritam i Lagrange-Eulerova metoda upotrijebljeni su za izradu realnog kinematičkog i dinamičkog modela troosnog rotacijskog ravninskog robota s električnim motorima i viskoznim, dinamičkim i statičkim trenjem. Ti su modeli robota kasnije korišteni za provjeru sljedećih predstavljenih nelinearnih postupaka upravljanja robotom: neizrazitog upravljanja, upravljanja s promjenjivom strukturom te upravljanja s referentnim modelom i promjenjivom strukturom. U metodi upravljanja s neizrazitom logikom definirano je sedam neizrazitih skupova za dvije ulazne varijable. Izabrane su trokutaste ulazne funkcije pripadnosti i tablica neizrazitih pravila veličine 7 x 7. Vrijednost izlaza neizrazitog regulatora izračunata je po principu težišta neizrazitog skupa. Isti neizraziti upravljački algoritam upotrijebljen je u svim petljama slijednog upravljanja robotom, uz odgovarajuće skaliranje jezičnih varijabli. Za uklanjanje trešnje iz upravljačkog signala s promjenjivom strukturom i zbog smanjenja potrošnje energije, funkcija predznaka je u prvobitnom zakonu upravljanja s promjenjivom strukturom zamijenjena sljedećim funkcijama: neprekidnom, funkcijom zasićenja i eksponencijalnom funkcijom, s vrlo tankim graničnim slojem u svima. Iste su promjene također napravljene i u originalnoj metodi upravljanja s referentnim modelom i promjenjivom strukturom. U svim su predstavljenim postupcima upravljanja parametri regulatora izabrani po principu najveće dozvoljene pogreške slijeđenja i najmanje potrošnje energije. Ove su metode upravljanja provjerene računalnim simulacijama u programskom jeziku C na primjeru kretanja alata izabrane robotske ruke. Rezultati simulacija dokazali su sličnu efikasnost svih spomenutih promijenjenih nelinearnih postupaka upravljanja robotom, iako su modificirani upravljački algoritmi s promjenjivom strukturom najprimjenjiviji zbog svoje jednostavnosti i manjeg broja parametara regulatora.*

**Ključne riječi:** *robot, neizrazito upravljanje, upravljanje s promjenjivom strukturom, upravljanje s referentnim modelom, trešnja*

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