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## LOAD NOISE INCREASE OF TRANSFORMERS BY LOAD HARMONICS

### SUMMARY

Harmonic components in the load-current have a larger impact to the increase of the load noise level of transformers than might be expected from their amplitude. The reasons are (a) the interaction of the harmonics with the large load-current at power frequency, (b) the rising sound radiation efficiency with frequency and (c) the A-weighting filter curve which suppresses sound components at lower frequencies. This paper presents a calculation scheme able to estimate the noise increase and the noise spectrum of electrical transformers for non-sinusoidal load conditions. The proposed calculation scheme is applied to three practical examples.

### 1. INTRODUCTION

Electrical power and distribution transformers in field operation regularly show significant deviation in the load noise level in comparison to the sound measurements in the test field of the manufacturer where controlled and standardised measurement conditions apply. The main reason therefore are (a) the changed acoustical ambient conditions on-site caused by sound reflection and diffraction effects of nearby obstacles; (b) changed structural and acoustical resonances, and (c) the different principal operation conditions on-site. Whereas at the factory tests a pure sinusoidal current with short-circuited secondary winding is applied for load noise determination, transformers on-site operate at changed load power factors (phase angle between voltage and load-current), loading beyond the rated power (overload, partial load) as well as harmonics in the load-current.

Harmonic components in the power grid are caused primarily by nonlinear loads and by the nonlinear transfer characteristic of the power grid. Any electrical device which shows a nonlinear consumption of active power or a nonlinear change of the source impedance draws a distorted current waveform even if the supply voltage is sinusoidal. With the growing use of electrical and electronic devices, distorted waveforms in power supply grid by harmonics have been largely increased within the past years. Devices like variable speed drives, six-pulse bridge rectifiers used in power electronics, and discharge lamps draw a non-sinusoidal but pulsating current. Whereas most electric devices generate solely odd harmonics, some devices with fluctuating power consumption generate odd, even and also interharmonic currents [1].

Load noise is generated by electromagnetic forces acting on transformer windings. The forces are proportional to the square of the load-current (Section 2.2). This quadratic response behaviour generates winding vibrations at frequencies beyond the power frequency. In case of harmonics in the load-current, the resulting magnetic force and noise components have harmonics as well- with a more complex frequency spectrum. The related increase of the load noise level must be considered in the estimation of the total noise level of transformers under real load conditions. The question arise how to

predict the resulting noise level at these loading conditions to guarantee the maximum allowed sound level at the transformer site as requested by legislative regulations or the owners specification.

Starting from basic principles, we describe and quantify the generation of harmonics in winding forces, winding vibrations and load noise of electrical transformers caused by load harmonics. A calculation scheme for a fast and practically accurate estimation of the increase in the load noise level for a given load spectrum presented. The effects of A-weighting and frequency-dependent sound radiation efficiency of transformers to the total noise level are discussed. Finally, three practical examples are given.

## 2. LOAD NOISE GENERATION WITH RESPECT TO LOAD HARMONICS

### 2.1. Description of harmonics in the load-current

Consider a transformer in operation with power supply at the operating frequency  $f_1$ , with load-current amplitude  $\hat{I}_1$ , and phase  $\varphi_1$ . In case of superimposed harmonics, the transient load-current has the general form

$$I(t) = \sum_{i=1}^N \hat{I}_i \sin(\omega_i t + \varphi_i). \quad (1)$$

In equation (1),  $N$  denotes the total number of harmonics, whereby a harmonic component of order  $n_i$  is completely described by its angular frequency  $\omega_i = 2\pi f_i$ , amplitude  $\hat{I}_i$ , and phase angle  $\varphi_i$ .

### 2.2. Generation of magnetic forces on windings

In a real transformer the magnetic coupling of the windings is imperfect so that a magnetic leakage flux field arise between the concentric windings if the transformer is at load. According to Ampere's law in the quasistatic case, the load-current  $I$  in the transformer windings generates a magnetic field which is proportional to the current density

$$\nabla \times \mathbf{H} = \mathbf{J}. \quad (2)$$

Hereby,  $\mathbf{H}$  denotes the vector of the magnetic field strength and  $\mathbf{J}$  the current density vector. Since the windings of liquid-immersed transformers are located in an oil-filled tank, the constant magnetic permeabilities  $\mu$  in this non-ferromagnetic material dominate the magnetic field properties in any closed contour of magnetic flux crossing the windings. Thus, the magnetic flux density  $B$  is proportional to the magnetic field strength

$$\mathbf{B} = \mu \mathbf{H}. \quad (3)$$

The magnitude of the leakage flux next to a winding can be estimated by

$$B \approx \mu_0 \frac{N I}{l}, \quad (4)$$

whereby  $\mu_0 = 4\pi \times 10^{-7} \text{ VsAm}$  denotes the magnetic constant,  $N$  the number of winding turns and  $l$  the winding length. It is an appropriate to assume this linear transfer function between  $B$  and  $I$  and consider the spatial field distribution of the leakage flux to be identical for all load harmonics. This assumption is valid for standard transformers with vector groups without phase-angle differences - but not valid for double-tier-transformer with a vector group where the phasing of harmonics is changing (e.g. at wye and delta connection). In the later case, the magnetic field and magnetic force distribution differ with changing harmonic components [2, 3].

The interaction of the magnetic leakage flux field generated by one current carrying winding with the currents in the conductors of other windings gives rise to magnetic volume forces

$$\mathbf{f}_m = \mathbf{J} \times \mathbf{B} \quad (5)$$

which act on the conductors of the windings. Hereby,  $\mathbf{f}_m$  denotes the magnetic force density vector. Equation (5) corresponds to the force relation defined by Lorentz in case that eddy currents in the windings are neglected. A winding section with volume  $V$  therefore experience the magnetic force

$$\mathbf{F}_m = \int \mathbf{f}_m dV. \quad (6)$$

The force density is proportional to the magnetic flux density and the electrical current density, both which are actually present in a certain winding section. It is a good approximation to assume that the magnetic flux and the load-current are there always in phase - even in a 3-phase transformer. Thus, the magnitude of the resulting winding forces are proportional to the square of the load-current and we can write

$$\begin{aligned} F_m(t) &\propto I(t)^2 \\ &= \left( \sum_{i=1}^N \hat{I}_i \sin(\omega_i t + \varphi_i) \right)^2 \\ &= \sum_{i=1}^N \sum_{j=1}^N \hat{I}_i \sin(\omega_i t + \varphi_i) \hat{I}_j \sin(\omega_j t + \varphi_j). \end{aligned} \quad (7)$$

This quadratic response behaviour gets obvious if we remind that for any load-current component at frequency  $f_i$ , the direction of the magnetic field vector alternates simultaneously with the direction of the current density vector. Thus, for each half-wave the vector product of the magnetic volume force (5) is oriented in the same direction (Fig. 1).

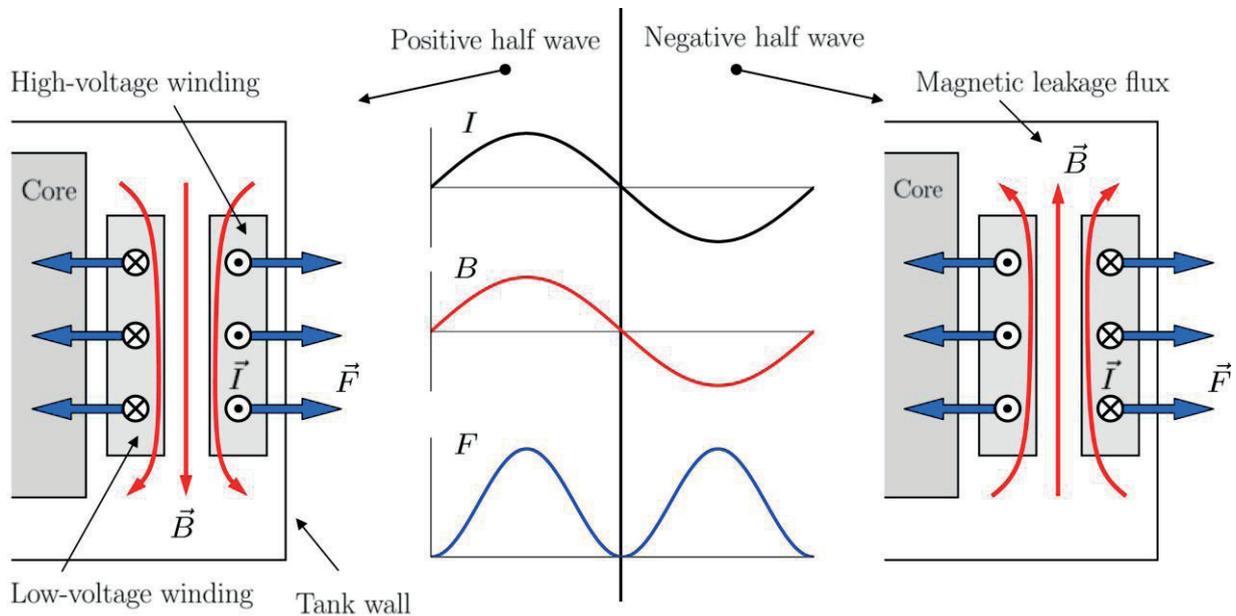


Figure 1: Quadratic response of the magnetic forces acting on the concentric windings of a transformer with respect to load-current.

The double sum in equation (7) can be split up into a sum of quadratic terms ( $i = j$ ) and a sum of product terms ( $i \neq j$ )

$$F_m(t) \propto \underbrace{\sum_{i=1}^N \hat{I}_i^2 \sin^2(\omega_i t + \varphi_i)}_{\text{quadratic terms}} + 2 \underbrace{\sum_{i=1}^{N-1} \sum_{j=i+1}^N \hat{I}_i \sin(\omega_i t + \varphi_i) \hat{I}_j \sin(\omega_j t + \varphi_j)}_{\text{product terms}} \quad (8)$$

Applying trigonometric identities, the two terms of equation (8) can be rewritten to

$$F_m(t) \propto \frac{1}{2} \sum_{i=1}^N \hat{I}_i^2 + \frac{1}{2} \sum_{i=1}^N \hat{I}_i^2 \sin \left[ 2\omega_i t + 2\varphi_i - \frac{\pi}{2} \right] + \sum_{i=1}^{N-1} \sum_{j=i+1}^N \hat{I}_i \hat{I}_j \sin \left[ (\omega_i - \omega_j)t + \varphi_i - \varphi_j + \frac{\pi}{2} \right] + \sum_{i=1}^{N-1} \sum_{j=i+1}^N \hat{I}_i \hat{I}_j \sin \left[ (\omega_i + \omega_j)t + \varphi_i + \varphi_j - \frac{\pi}{2} \right]. \quad (9)$$

Now, for each line in (9) we can deduce frequency components of the magnetic force:

#### Line 1: Constant component generation

Each of the  $N$  load-current harmonics contributes a force component at  $f = 0\text{Hz}$ . The resulting constant force component cause a static deformation of the winding and will add no dynamic vibrations. Therefore no sound pressure fluctuations are generated and the constant force component does not contribute to load noise generation (Fig. 2).

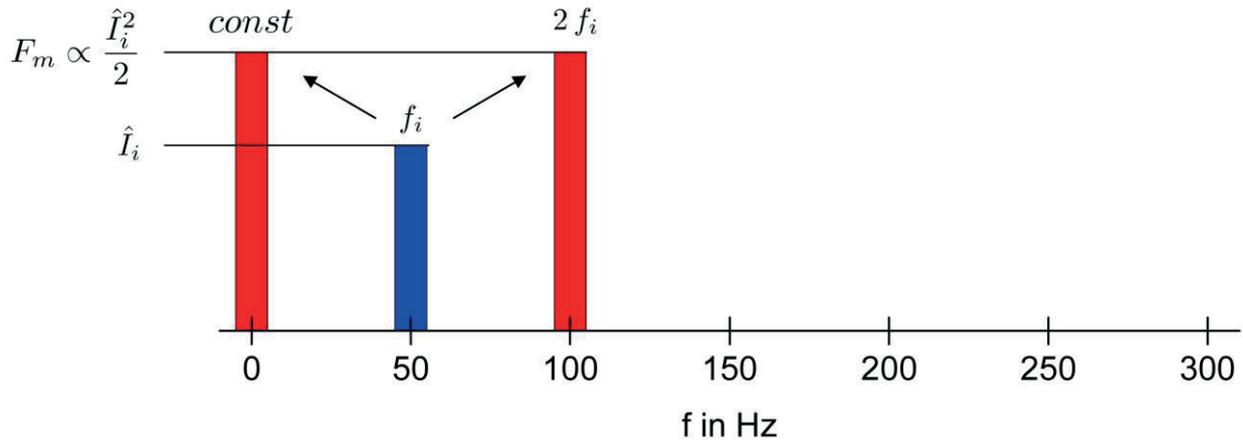


Figure 2: Generation of second harmonic and constant force component by quadratic terms

#### Line 2: Second harmonic generation

Each of the  $N$  load-current components generates further a force component at doubled frequency  $2f_i$ , in total again  $N$  magnetic force components. The amplitude of each component is proportional to the square of the load-current amplitude  $I_i$  and the phase  $2\varphi_i - \pi/2$  (Fig. 2).

### Line 3: Difference frequency generation

For each pair of load-current components  $n_i \neq n_j$ , the product terms in line 3 give rise to a magnetic force component at difference frequency  $|f_i - f_j|$  and phase  $\varphi_i - \varphi_j + \pi/2$  (Fig. 3). The amplitude of the magnetic force components is proportional to the product of the corresponding load-current harmonics  $I_i I_j$ . The product terms deliver  $N(N-1)/2$  force components at difference frequency.

### Line 4: Sum frequency generation

As in line 3, each pair of load-current components  $n_i \neq n_j$  generates further a magnetic force component at sum frequency  $f_i + f_j$  with same amplitude but differing phase  $\varphi_i + \varphi_j - \pi/2$  (Fig. 3). Again,  $N(N-1)/2$  magnetic force components are generated.

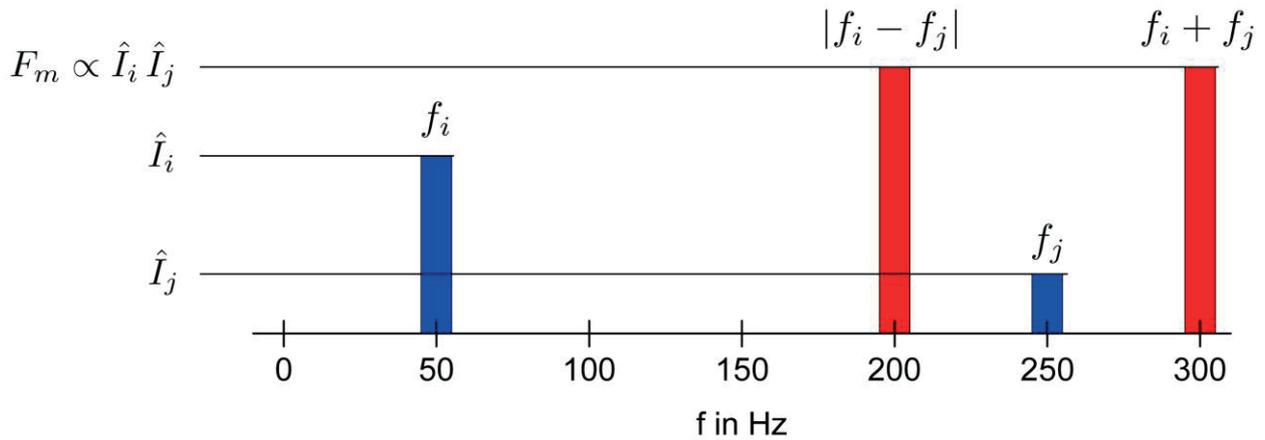


Figure 3: Generation of force components at sum and difference frequencies by product terms

In summary, for a load-current having  $N$  frequency components, the quadratic response behaviour results in a magnetic force having  $N^2$  frequency components at maximum (including the constant component at  $f = 0\text{Hz}$ ). Hereby, the quadratic terms in (8) are responsible for the generation of static force components and force components at doubled frequencies, whereas the product terms in (8) contribute force components at difference frequencies  $f_i - f_j$  and sum frequencies  $f_i + f_j$ . Since the arising force terms can have identical frequencies, the total number of arising magnetic force harmonics is at real conditions much smaller than the theoretical maximal number of force harmonics ( $N^2$ ) and typically in the range of  $2N$ . In case that force components have identical frequencies, vectorial addition of mono-frequent forces is performed to achieve the total magnetic force amplitude and phase angle at the corresponding frequency. Hereby the amplitude of the resulting magnetic force is determined by the phase angle relations of the involved force components. Since the superpositions can be constructive or destructive, the resulting total force component can show large variations in amplitude and phase.

### 2.3. Determination of the winding vibrations

A transformer winding typically consists of isolated conductors, spacers and pre-compressed boards which can be considered as a passive system of distributed mass, stiffness and damping. Thus, the winding assembly is a linear and lightly damped mechanical system able to vibrate which undergoes a forced vibration by distributed magnetic forces. For dynamic analyses, we start from the equation of motion

$$M\{\ddot{d}\} + C\{\dot{d}\} + K\{d\} = \{F_m\}, \quad (10)$$

whereby the mechanical multi-degree-of-freedom system is described by the mass matrix  $M$ , the damping matrix  $C$  and the stiffness matrix  $K$ . The vectors of acceleration, velocity, displacement, and magnetic force are denoted by  $\{\ddot{d}\}$ ,  $\{\dot{d}\}$ ,  $\{d\}$  and  $\{F_m\}$  respectively. The displacements are with respect to a rest position at which no dynamic forces act on the windings. For a single-degree-of-freedom subsystem of (10) with mass  $m$ , damping  $c$ , and stiffness  $k$ , the steady-state solution for forced vibration with a harmonic magnetic force  $F_m = \hat{F}_m \sin(\omega t)$  is (see, e.g. [4])

$$d = R \frac{\hat{F}_m}{k} \sin(\omega t - \theta). \quad (11)$$

The winding displacement is at the forcing frequency  $\omega$  and for a non-zero damping coefficient  $c$ , the phase  $\theta$  in equation (11) between the applied force and the resulting displacement is different than zero and given by

$$\theta = \tan^{-1} \left( \frac{2\xi\omega / \omega_n}{1 - \omega^2 / \omega_n^2} \right) \quad (12)$$

$$\text{with natural frequency } \omega_n = \sqrt{\frac{k}{m}}$$

$$\text{and fraction of critical damping } \xi = \frac{c}{2m\omega_n}.$$

In equation (11), the resulting winding displacement in radial or in axial direction is a linear function of the applied magnetic force and the effective stiffness in the considered force direction (radial or axial). This corresponds to Hooke's law in the linear-elastic range for a statically applied force. The dimensionless response factor  $R$  in (11) writes

$$R = \frac{1}{\sqrt{(1 - \omega^2 / \omega_n^2)^2 + (2\xi\omega / \omega_n)^2}} \quad (13)$$

and represents the amplitude ratio of the dynamic displacement to the displacement in case the force  $F_m$  were applied statically. In the stiffness-controlled frequency range at low frequencies ( $\omega \ll \omega_n$ ), the response factor  $R$  is about 1. It rises with higher frequencies and has its maximum in the damping-controlled range near the natural frequency ( $\omega = \omega_n$ ), and approaches zero in the mass-controlled range for high frequencies ( $\omega \gg \omega_n$ ), see Figure 4. The first natural frequency of a transformer winding assembly is typically below the first mechanical excitation frequency (100Hz/120Hz). Thus, the winding vibration can be considered as mass-controlled the steady-state solution of the equation of motion can be rewritten to

$$d \simeq \frac{\hat{F}_m}{k} \left( \frac{\omega_n}{\omega} \right)^2 \sin \omega t = \frac{F_m}{m\omega^2} \sin \omega t. \quad (14)$$

The homogeneous composition of the transformer windings (i.e. with periodic repeated substructures like conductor loops) allows us to assume an uniform viscous damping [4]. In this case, each mode in a damped multi-degree-of-freedom system responds as a simple damped oscillator as in equation (11).

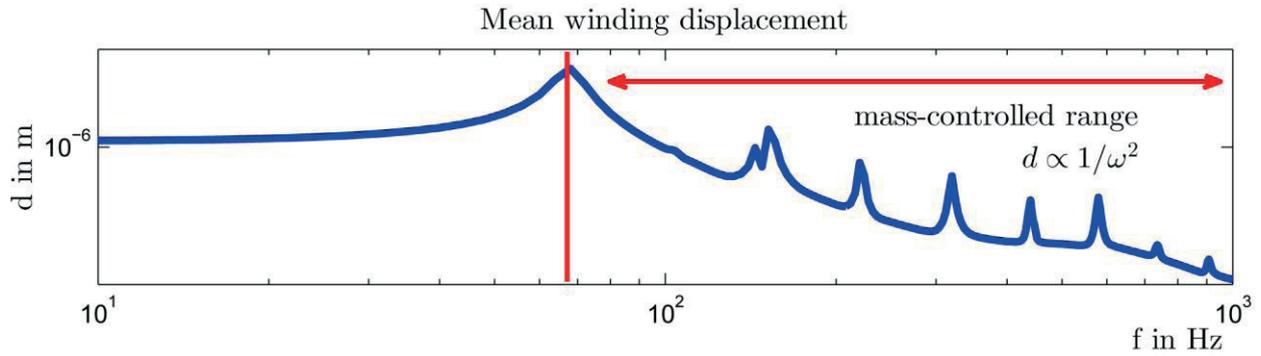


Figure 4: Frequency response characteristics of the mean displacement of a typical oil-immersed transformer winding representing a mechanical multi-degree-of-freedom system.

At nominal operation conditions of the transformer, the dynamic magnetic forces acting on the windings result in a linear-elastic stress-strain response within all materials of transformer winding assembly. Therefore we assume a mechanical linear system and apply a linear superposition of all frequency components of magnetic forces arising from load-current harmonics.

#### 2.4. Calculation of the radiated sound

The vibrating solid surfaces of the windings are in contact with the surrounding fluid (transformer oil) and displace the fluid volume at the structure-fluid-interface. As the fluid which completely surrounding the vibrational sources is essentially incompressible and there is no significant sound absorption in the fluid, the sound power transmitted through the fluid is conserved. In the low frequency range, where the distances of the windings to the tank are relatively small compared to an acoustic wavelength in the fluid, the net volume velocity of the windings will be translated to a net volume velocity of the tank. Interestingly, in this configuration the transmitted sound power is very insensitive to any details of the spatial distribution of the normal surface velocity [5].

For arbitrary shaped vibrating surfaces of finite size, the sound power  $P$  that is radiated into free field at a certain frequency can be estimated by the fundamental equation of vibroacoustics (see, e.g. [6]),

$$P = \sigma \rho c S \langle \overline{v^2} \rangle. \quad (15)$$

Hereby,  $\sigma$  denotes the frequency-dependent radiation factor (see below),  $\rho$  the density of air,  $c$  the speed of sound in air,  $S$  the total surface area of the vibrating body (here: the tank), and  $\langle \overline{v^2} \rangle$  the time-and-space-mean of the square normal surface velocity. Applying the mentioned net volume velocity balance between winding and tank wall surfaces, the mean square velocity of the tank can be calculated from the volume velocity of the windings. Hereby, the surface velocity at any winding location can be derived from the surface vibration amplitude  $d$  in (14). Using  $v = \dot{d} = \omega d$ , the time-mean square surface velocity at a location writes in the harmonic case

$$\overline{v^2} = \frac{1}{T} \int_0^T v^2(t) dt = \frac{\omega^2 \hat{d}^2}{2}. \quad (16)$$

The most uncertain parameter in the estimation of the radiated sound power is the radiation factor  $\sigma$ , which is a function of the frequency, geometry dimensions, material properties, and of the presence of any structural discontinuities (e.g. tank stiffeners, attached structures to the tank). The radiation factor is a measure for the sound radiation effectiveness of a vibrating structure. The transformation of structure-borne-sound (normal tank surface vibration) into air-borne sound changes with frequency due to the dispersive character of bending waves in the thin-walled tank. This leads to a frequency-dependent ratio of the structural bending wavelength and the wavelength of the radiated sound [5, 6].

Maximal conversion of sound energy occurs at and above the coincidence frequency  $f_c$ . At this frequency, the bending wavelength of the structure is identical to the wavelength of the sound wave in the surrounding fluid (total sound radiation). Below the coincidence frequency, the sound radiation efficiency decreases with frequency. In this frequency range, acoustic near field effects have to be considered. These are mainly the local compensation of sound pressure variation (acoustic short-circuit) and evanescent wave components that don't propagate to the far-field [5].

#### 2.4.1. The radiation factor of transformer tanks

All dominating sound components of transformer load noise are below the coincidence frequency of the transformer tank. Therefore we need a sufficient accurate determination of the frequency response curve of the radiation factor to estimate the contribution of each frequency component to the total load noise level. The radiation factor  $\sigma$  can be determined by measurements or by numerical calculations. In an analytical approach, the sides of a transformer tank can be considered as rib-stiffened, flat plates with a added mass by the one-sided heavy fluid load of the oil [5]. The radiation factor of this configuration shows the typical frequency-response characteristics of Fig. 5. The radiation factor approaches unity at the coincidence frequency  $f_c$  within the frequency band of transformer noise. Larger transformer tanks have a coincidence frequency  $f_c$  at lower frequencies, so that they show a better radiation efficiency for higher load noise harmonics. In case that no measurement or numerical data are available, it is a good starting point for a typical power transformer to set  $f_c \approx 600\text{Hz}$  and set the rise of the radiation factor to be 7dB per octave for frequencies below  $f_c$ .

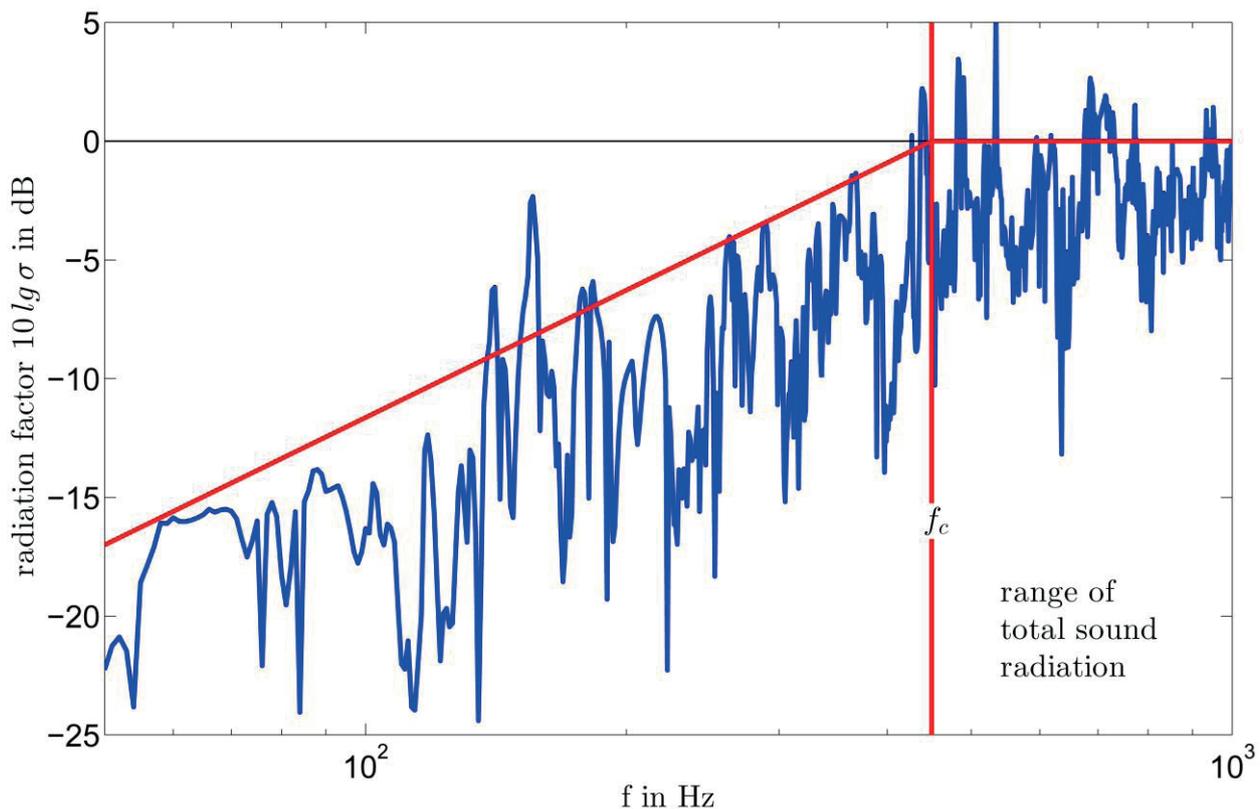


Figure 5: Typical sound radiation factors of a oil-filled 150MVA transformer. Data based on Finite-Element analyses including the fluid-structure-coupling at the surfaces oil-tank-air.

For a first approximation of the radiated sound power, any further changes of the sound radiation characteristics arising from the modal response of the tank or internal fluid-structure-interactions like acousto-structural mode coupling or cavity modes are not considered in this investigations. Hence the modal averaged radiation factor of a transformer tank as shown by the red line in Fig. 5 is used in equation (15).

### 2.4.2. The determination of a load noise component

By using the magneto-mechanical coupling term (9), the solution of the equation of motion (14), the fluid-structure-coupling term (15) and equation (16), the sound power generated by a magnetic force component at any frequency can be written to

$$P(f) = C \sigma \hat{F}_m(f)^2 . \quad (17)$$

The added constant  $C$  unites all non-frequency-dependent parameters which arise in the underlying equations mentioned above. As we will see later, the actual value of the constant  $C$  is of no interest for the calculation of sound level differences due to load harmonics.

The related sound power level generated by a magnetic force component acting on the windings can be finally written as

$$\begin{aligned} L_w &= 10 \lg \frac{P}{P_0} \text{ dB} \\ &= 10 \lg (C \hat{F}_m^2) + L_\sigma \quad \text{with} \quad L_\sigma = 10 \lg \sigma , \end{aligned} \quad (18)$$

whereby  $P_0 = 2 \times 10^{-12} \text{ W}$  denotes the reference sound power at the auditory threshold and  $L_\sigma$  the level of radiation efficiency.

### 2.5. Total load noise and A-weighting of noise harmonics

The international standard for the determination of the sound levels of transformers [7] specifies the use of the A-weighting filter for transformer sound measurements (Fig. 6). This is to suppress sound signals at low and high frequencies and thus account for the lower sensitivity of the human ear in this frequency regions.

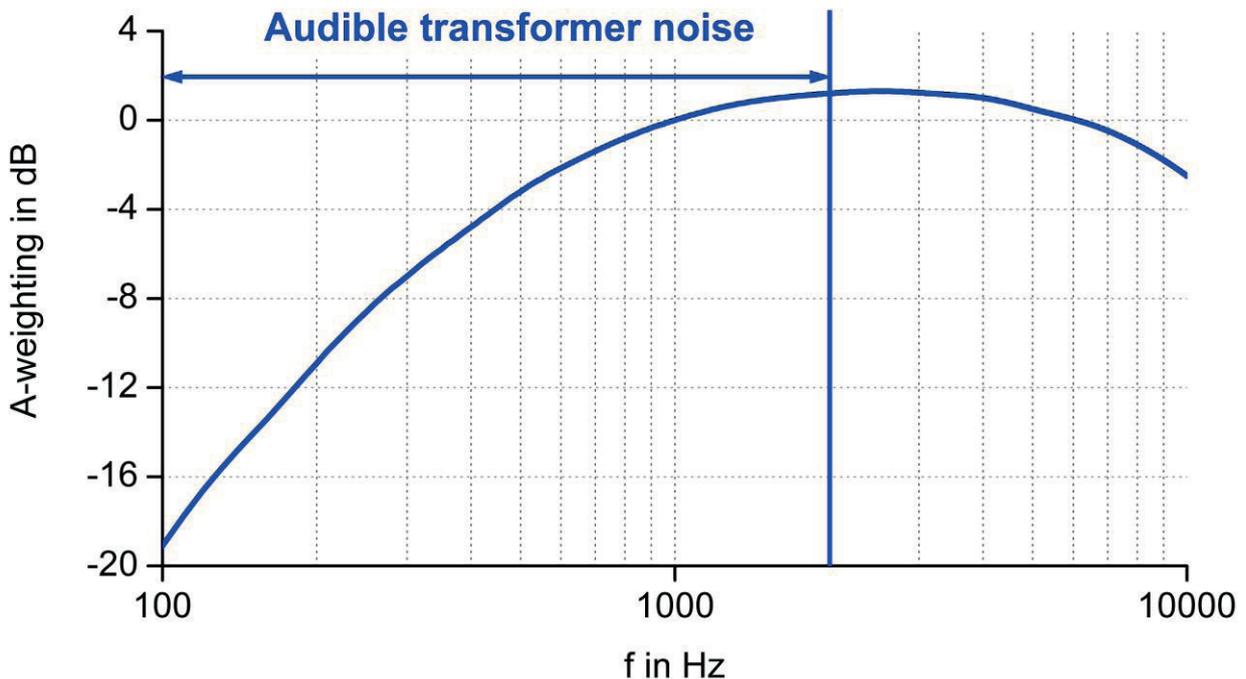


Figure 6: The A-weighting curve suppresses the lower frequency components of audible transformer noise.

The A-weighted sound power level of a sound frequency components reads

$$L_{WA} = (L_W + A) dB(A) \quad (19)$$

whereby  $A$  denotes the correction term due to the frequency-dependent A-weighting filter. The total A-weighted load noise sound power level of a transformer having load-current harmonics  $L_h$  can be achieved by logarithmic addition of all sound components whose levels  $L_i$  are calculated according to equation (19):

$$L_h = 10 \lg \left( \sum_i^N 10^{L_i/10} \right) dB(A) \quad (20)$$

In absence of higher harmonics in the load-current, the load noise of a transformer has solely one fundamental sound component

$$L_n = \left( 10 \lg \left( C \hat{F}_n^2 \right) + R_1 + A_1 \right) dB(A) \quad (21)$$

Hereby  $A_1$  denotes the A-weighting of the fundamental sound component which is generated by the fundamental magnetic force component at twice the line frequency

$$F_n(t) \propto \frac{\hat{I}_1^2}{2} \cos 2(\omega_1 t + \varphi_1). \quad (22)$$

Note that the sound level of the fundamental sound component  $L_n(f_1)$  in the absence of harmonics can differ from the corresponding sound level  $L_1(f_1)$  in the presence of harmonics. This change originates in additional magnetic force components which are superposed to the fundamental force component in the presence of harmonics by the difference frequency generation according to line 3 of equation (9).

Eventually, the load noise increase due to harmonics in the load-current corresponds to the level difference

$$\begin{aligned} \Delta L &= L_h - L_n \\ &= 10 \lg \left( \frac{\sum_i^N \left( \hat{F}_i^2 10^{(R_i + A_i)/10} \right)}{\hat{F}_n^2 10^{(R_1 + A_1)/10}} \right) dB(A). \end{aligned} \quad (23)$$

By taking the difference of logarithms, the (unknown) non-frequency dependent parameter  $C$  cancels out.

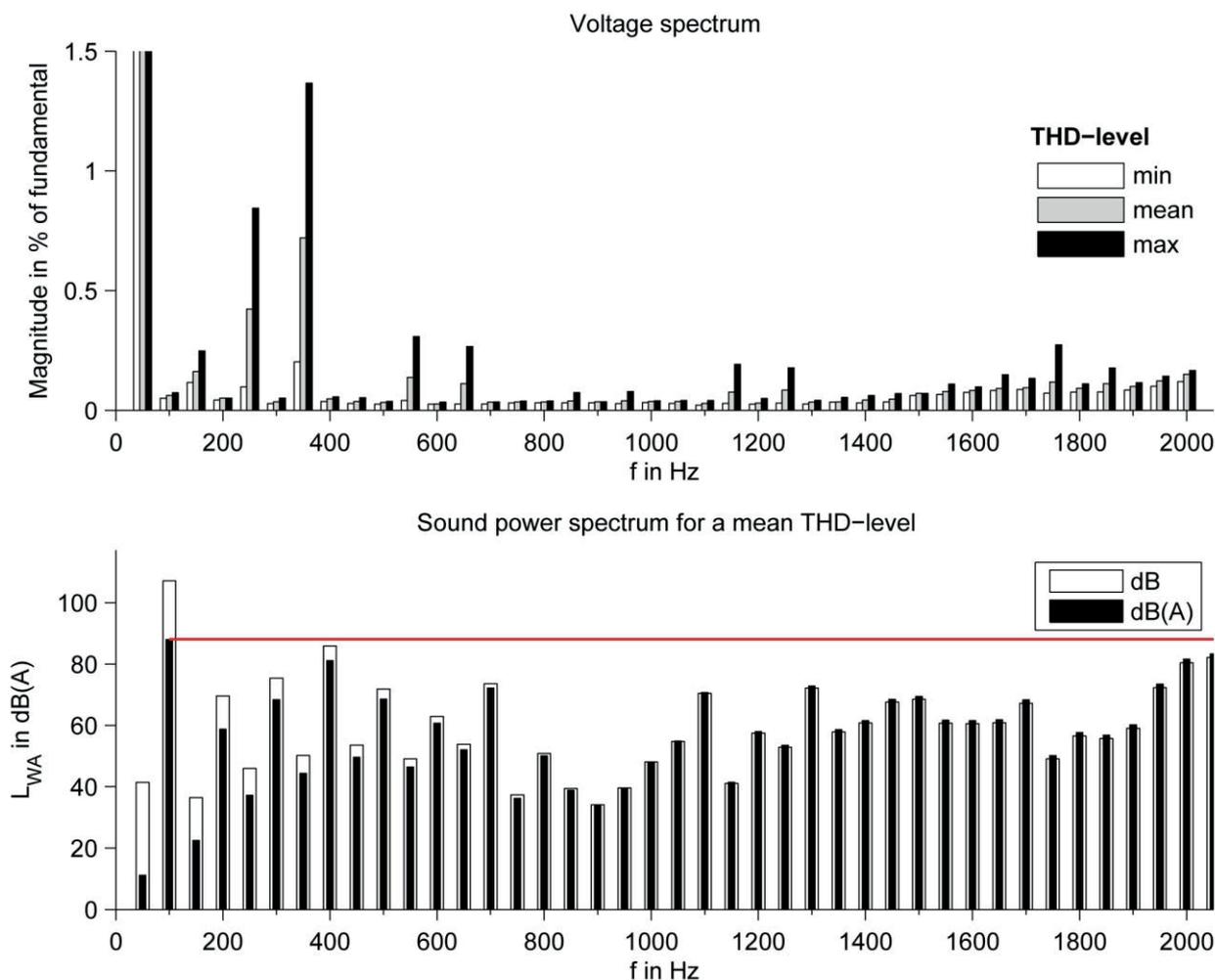


Figure 7: Example 1: Sound spectra for voltage harmonics in a 400kV power line at different harmonic distortion levels

### 3. PRACTICAL EXAMPLES OF LOAD NOISE INCREASE AT LOAD HARMONICS

The presented calculation scheme for load noise increase at power grid harmonics is applied to two practical examples for further analyses of the generation process of load noise harmonics.

#### 3.1. Example 1: Voltage harmonics in a 400kV power line

The first example is related to a step-down oil-immersed power transformer with a rated power of 300MVA and a nominal load noise level of  $L_{WA} = 88\text{dB(A)}$ . To estimate the increase in load noise due to load harmonics, we consider the voltage spectra in a German 50Hz/400kV power grid for three different total-harmonic-distortion levels (THD) as shown in Figure 7.

The THD is defined as the amplitude ratio

$$THD = \frac{\sqrt{\sum_{i=2}^N \hat{I}_i^2}}{\hat{I}_1} \quad (24)$$

The standard EN 50160 [8] defines the voltage characteristic of electricity supplied by public supply networks and limits the THD of the supply voltage including all harmonics up to the order 40 to a

maximum of 8%. The related load noise increase due to load-current harmonics in this example is summarised in Table I. Already at a mean total-harmonic-distortion-level of 1% - which is far below the allowed tolerance level of EN 50160 - the psycho-acoustical relevant A-weighted sound power level increases by about 3dB. This sound level increase is clearly noticeable by the human perception of noise and corresponds formally to a doubling of the sound energy (twice the number of transformers).

Table I: Load noise increase by power grid harmonics without and with A-weighting

Distortion level	THD in %	$\Delta L$ in dB	$\Delta L(A)$ in dB
min	0.4	0.0	1.6
mean	1.0	0.0	2.9
max	1.8	0.0	5.8

### 3.2. Example 2: Load harmonics at distribution transformers in a computer centre

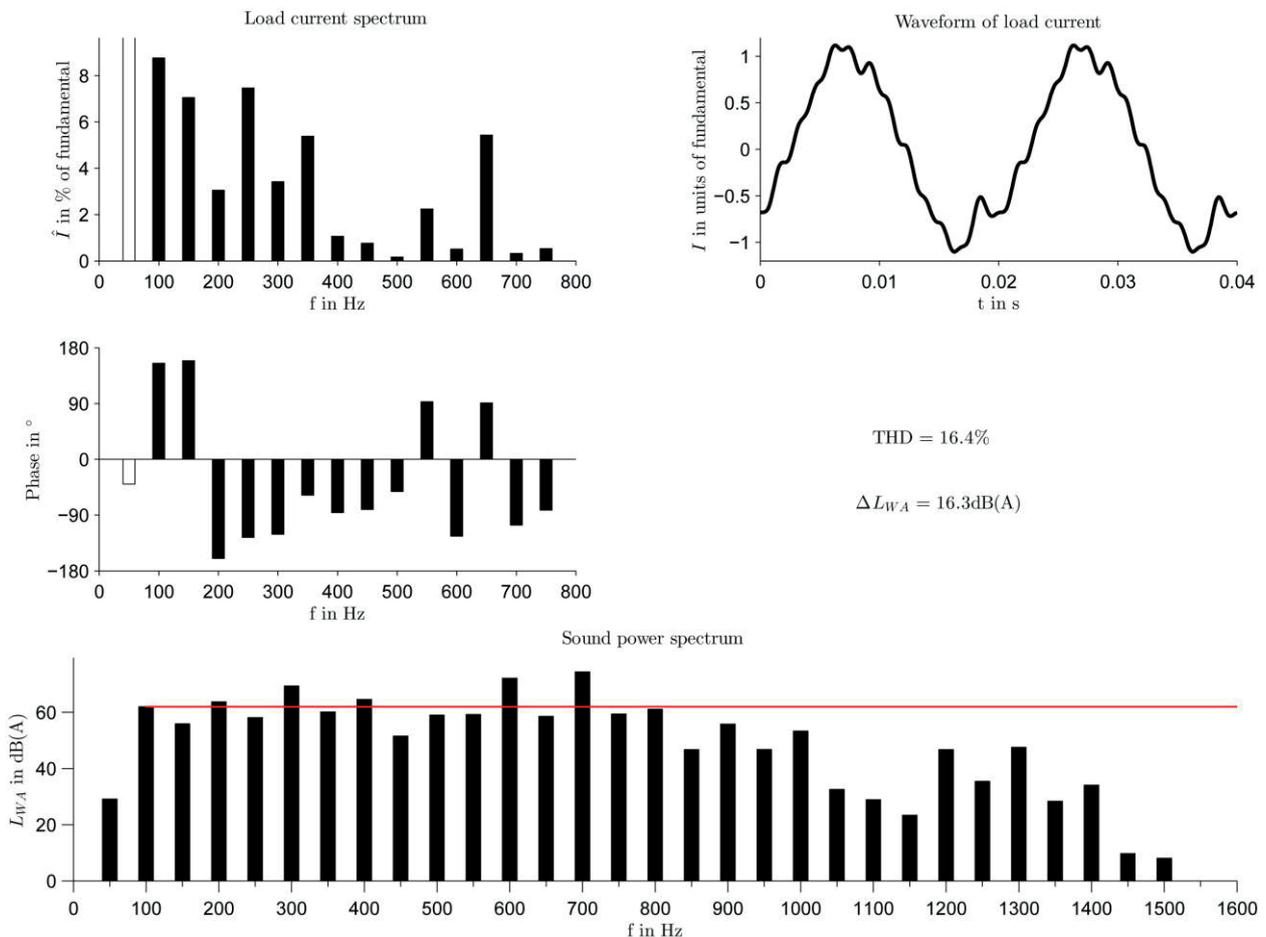


Figure 8: Example 2: Load noise sound spectrum of a distribution transformer with extraordinary high harmonic distortion

The second example is related to a 150kVA distribution transformer in a computer centre which was reported to be extraordinarily noisy. The transformer operates at a load factor of about 0.7 and the distinctive non-linear power consumption of converters in electronic equipment draws high distortion in the current waveform. The phase relations of the load-current components were also measured (Figure 8).

The presented calculation scheme shows magnetic force components with amplitudes up to 20% of the fundamental force component (Table II). The increasing sound radiation efficiency and A-weighting with rising frequency results in sound harmonics that largely exceed the fundamental sound component at frequencies below 700Hz. The corresponding increase of the total load noise level is 16.3dB(A) for a harmonic distortion of THD=16.4%.

Table II: Parameter related to the generation of load noise harmonics in Example 2. Magnetic force components and the resulting sound level differences are relative to the fundamental component at 100Hz.

Harmonic component $f$ in Hz	Magnetic force $F / F_0$ in %	Radiation factor $L_\sigma$ in dB	Sound level difference $\Delta L$ in dB
50	14.7	-22.1	-32.9
100	100.5	-17.0	0.0
150	19.5	-14.0	-6.0
200	26.2	-11.9	1.8
250	8.8	-10.3	-3.9
300	23.0	-8.9	7.4
350	6.0	-7.8	-1.8
400	8.0	-6.8	2.6
450	1.5	-5.9	-10.5
500	2.9	-5.2	-2.9
550	2.6	-4.5	-2.7
600	10.0	-3.8	10.2
650	1.9	-3.2	-3.4
700	10.4	-2.7	12.5

### 3.3. Example 3: Load harmonics at a HVDC-transformer

The interconnecting of a high-voltage, direct current (HVDC) electric power transmission line within an AC system requires the conversion from AC to DC and inversion from DC to AC. In both cases, the converter operation in HVDC substations results in AC current harmonics. By the half wave symmetry of pulse rectifiers, no even harmonics and no triplen (multiples of 3rd) harmonics are generated [9]. The stepwise change in load-current during the commutation from one valve to another results in the characteristic distribution of mainly odd harmonics from the 5th upwards (Fig. 9). This results in a very high harmonic distortion of THD=29% .

The sound power of a transformer in a HVDC substation was measured on-site using the sound pressure method according to the international standard [7]. The measured total sound increase of the transformer was  $\Delta L_{WA} = 21.9$  dB(A) compared with the nominal sound level of the transformer at the acceptance test without load harmonics. Beside the load noise, the emitted noise comprises also the no-load noise caused by magnetostriction and magnetic forces in the transformer core. The valve operation will give rise to a DC-magnetisation in the core, which can be identified by the existence of odd harmonics in the sound spectrum. Unfortunately, the sound signals of both sound sources can not be separated in the operation of a HVDC transformer. Thus, the calculated sound increase of 16.8dB(A) due to load-current harmonics is smaller than the measured total sound increase due to load harmonics and DC-magnetisation.

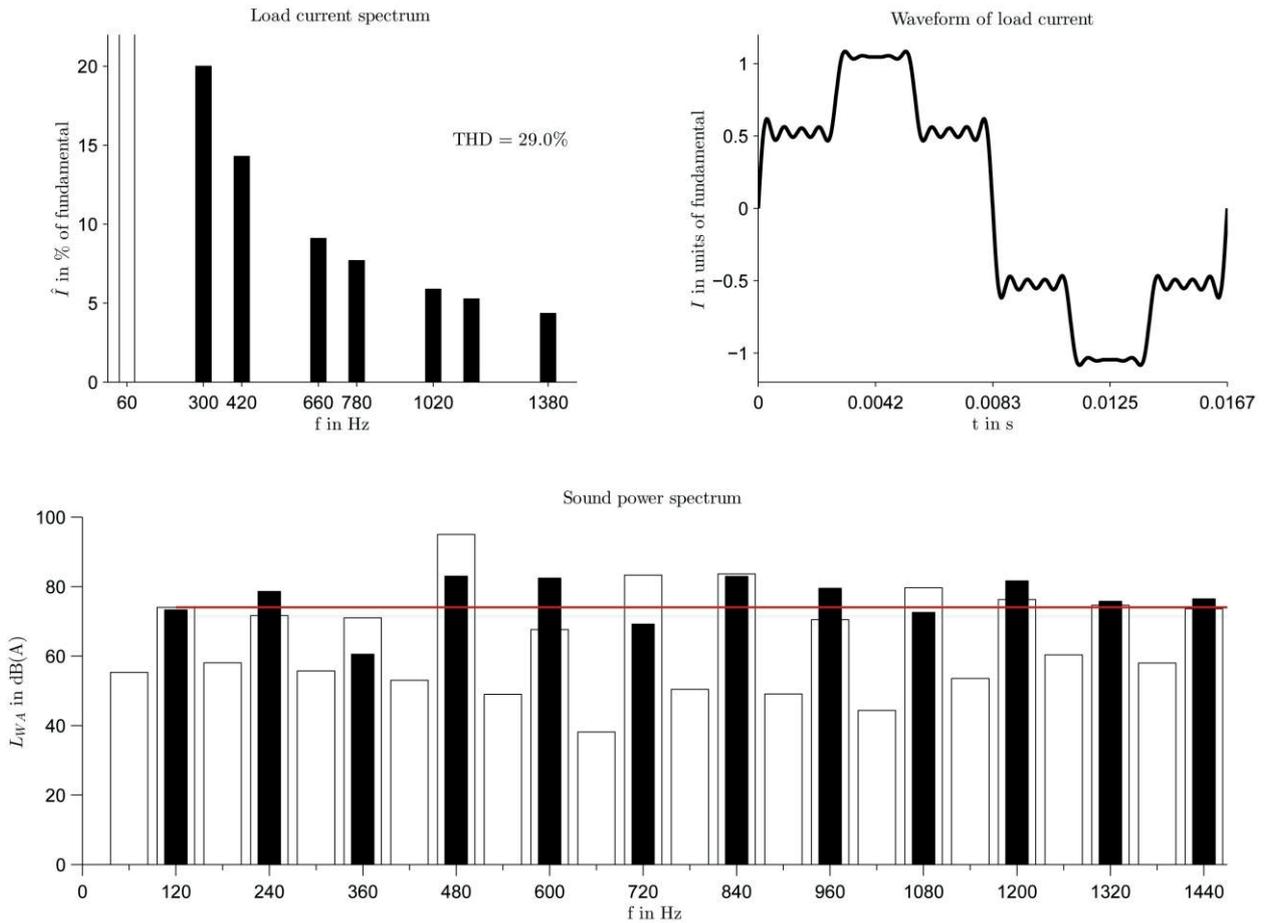


Figure 9: Example 3: Comparison of the calculated load noise spectrum and measured total sound spectrum (load noise and no-load noise) for a HVDC-transformer in operation. The measured odd harmonics are due to DC-magnetisation of the core.

In this example, we can study further the generation of sound component by the pronounced odd harmonics of the load-current. By the quadratic response of magnetic forces to load-current, the arising sound components are dominated by even harmonics as a result of the second harmonic, difference and sum frequency generation as described in equation (9).

#### 4. DISCUSSION AND RESULTS

##### 4.1. Effect of the A-weighting to the increase of the load noise level

Higher harmonics in the load-current have a disproportionately high impact to the total load noise level of the transformer. The main reason for the increase of the total noise level is the large rise of harmonic sound components by the A-weighting filter relative to the A-weighted fundamental sound component (Table III).

Table III: Rise of higher sound harmonics by the A-weighting filter.

f in Hz	200	300	400	500	600	700	800
$\Delta L$ in dB(A) re 100Hz	8.3	12.1	14.4	15.9	17	17.8	18.4

The A-weighting-effect can be clearly seen in the sound power spectrum of example 1 (see Fig. 7 and Table 1). Here, the load noise increase without A-weighting is negligible whereas the increase of the relevant A-weighted sound level is of practical interest.

## 4.2. Effect of the phase relation of load-current harmonics

Next, the effect of the phase relations in load-current components to the total load noise level is investigated. The load-current amplitude spectrum of example 2 is taken and a total variation of the phase relations of all load-current components is performed in the range  $\varphi = [0; 2\pi]$ . For each phase relation, the sound spectrum and the total load noise level are calculated.

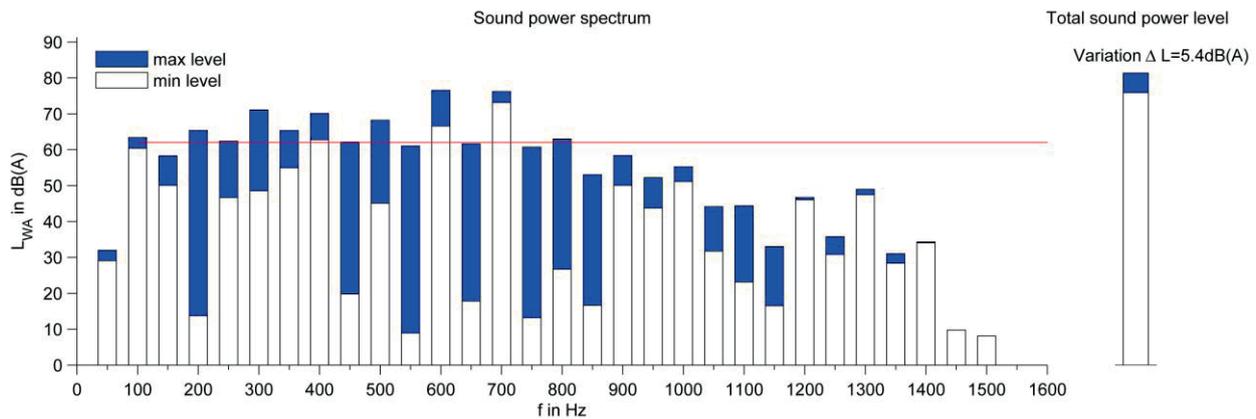


Figure 10: Maximal variation of load-noise components and the total load noise level by variation of the phase relations

Figure 10 shows the maximal variation of each individual sound component and the total load noise level caused by variations in the phasing of load-current harmonics. Depending if magnetic force components are added constructively or destructively, sound components vary up to 50dB by changed phase relations. The variation of the total noise level of 5.4dB due to changed phase relation is typical for transformers operating at a high harmonic distortion level. Thus we conclude, that for a useful estimation of the load noise increase due to load harmonics, the knowledge of the phase relations is required - at least for the dominant harmonic components.

## 4.3. Evaluation of the impact of a single harmonic load component

A graphical representation of the load noise increase due to a single harmonic load component is given in Figure 11 for the first 22 load-current harmonics at a fundamental frequency of 50Hz. Hereby the amplitudes of all other load-current harmonics are set to be zero. Further, the phase angle between the investigated single harmonic component and the fundamental component is also set to be zero.

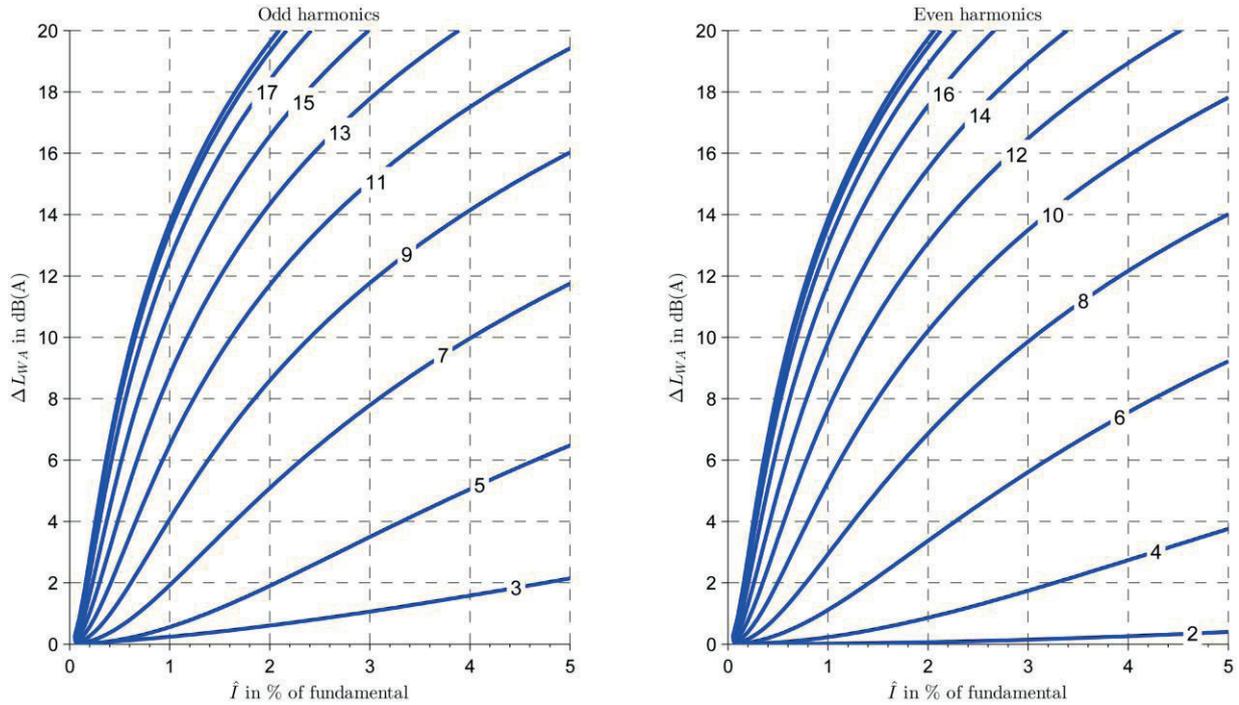


Figure 11: Load noise increase by a single harmonic component of the load-current for a typical 300 MVA power transformer.

Now we compare the increase in the load noise level by the existence of a single load-current component at a constant relative amplitude (vertical line in Fig. 11). Whereas the impact of low order harmonics is negligible due to the suppressing effect of the A-weighting, the higher harmonics will dominate the increase of the total load noise. Thus, the graph reflects again the dominating effect of the A-weighting.

Figure 11 can be used also for a rough graphical determination of the load noise increase by load harmonics. Hereby, the load noise increase  $\Delta L_i$  due to a single harmonic component is taken for each significant component in the load-current spectrum. The increase in the total load noise level  $\Delta L_h$  can be calculated then by logarithmic addition of all  $N$  components of  $\Delta L_i$  according to <sup>1</sup>

$$\Delta L_h = 10 \lg \left( 1 - N + \sum_i^N 10^{\Delta L_i / 10} \right) \text{dB(A)}. \quad (25)$$

## 5. CONCLUSIONS

The presented calculation scheme describes the change of the acoustic source strength of transformers at load due to load harmonics and offers a fast and sufficiently precise prediction of the increase in the load noise level. The disproportionately high impact of load harmonics to the total load noise level is attributed to the rising sound radiation efficiency of the transformer tank with frequency as well as the higher rating of harmonic sound components by the A-weighting filter.

By the analysis of the generation process of load noise harmonics we can further conclude:

- 1) To estimate the total load noise increase, it is sufficient to consider only force components where the fundamental load-current component is involved.
- 2) For a given current amplitude, high order load harmonics have the largest impact to the load noise increase.
- 3) Information of the phasing of load-current harmonics is required for a reliable estimation of load noise components and the total load noise level.

<sup>1</sup>The term  $1 - N$  originates in the fact that each sound level increase  $\Delta L_i$  due to a single harmonic component includes also the fundamental sound component.

The presented calculation scheme represents the minimum level in load noise increase we can expect. In measurements, the increase in the load noise levels by load-current harmonics tends to be higher. The main reason therefore is that the higher number of magnetic force components increases the probability of excitation of resonant vibrations in structural parts of the transformer (core, windings, tank) and the excitation of acoustical resonances (cavity modes inside the transformer tanks).

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