

## Calculations of lightning-induced voltages in distribution lines with LSA

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### SUMMARY

In this paper the results of the calculation of the effect of the surge arresters, placed in one of the conductors of a distribution line, are shown.

The calculations were made using a new computer program, written in MatLab environment, which is based in a new coupling model in order to take into account the presence of a lossy ground under the line. Other important feature of the new program is that, after a discretization only in space, the resulting system of ordinary differential equations (ODE) is solved using the powerful ODE solvers actually existing.

It is shown that a surge arrester protects the points behind it (seen from the strike point of the lightning discharge), but, it will only protect the points in front of it within a certain "effective distance", which depends on the risetime of the induced voltages.

It is also shown that, in general, the line span in front of the strike location is not protected, and, the protection afforded to the line span in front of the strike location, when it exists, is dependent on the risetime of the induced voltages.

### KEYWORDS

Surge arresters, induced voltages, lightning protection, modeling.

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## INTRODUCTION

Most of the calculations of lightning induced voltages, in lines with discontinuities (such as surge arresters), presented in the literature assume a perfectly conducting ground [1],[2]. One important exception is Reference [3] that presents a computer program to calculate induced voltages in lines with discontinuities, over a lossy ground. That program is based on the coupling model of Agrawal et al. [4], to take into account the lossy ground, and on the point-centered finite difference technique in the time domain, to integrate the resulting equations.

In this paper, the results of a new computer program are presented. The program takes into account the lossy ground using a new coupling model [5], and, to integrate the resulting equations, after a discretization only in space, the resulting system of ordinary differential equations (ODE) is solved using the powerful ODE solvers actually existing [6].

## COUPLING MODEL

The problem of the coupling of an electromagnetic field to a transmission line consists of describing the voltages and currents induced in the conductors of the line in terms of the inducing fields.

The current  $i(x,t)$  at a point along a line conductor is defined, as usual, as the charge flow across the cross section of the conductor at that point.

Considering that the voltages are quantities that are not associated with a point but with a trajectory in space; in the case of a horizontal line, the induced voltage  $u(x,t)$  at a point of the line is defined as the integral of the electric field along a vertical trajectory from that point on the conductor to some reference point in the ground, usually ground level:

$$u^j(x,t) = \int_{h_j}^0 \vec{E} \cdot d\vec{l} = - \int_0^{h_j} E_z dz \quad (1)$$

The model used in this paper is based on the application, for all conductors, of a formulation of Faraday's law in terms of the magnetic potential vector field  $A(x, y, z, t)$  and the scalar potential field  $\phi(x, y, z, t)$ :

$$\oint (\vec{E} + \frac{\partial \vec{A}}{\partial t}) \cdot d\vec{l} = 0 \Rightarrow \vec{E} = -\nabla \phi - \frac{\partial \vec{A}}{\partial t} \quad (2)$$

The application of Faraday's law applied to a conductor placed along the "x" direction can be written as:

$$-\frac{\partial \phi_j}{\partial x} - \frac{\partial A_{jx}}{\partial t} = E_x(x) \quad (3)$$

Assuming, as usual, that it is possible to define "inductances" and resistances" per unit length as:

$$A_{jx}^s(x,t) = \sum_k L_{jk} i^k(x,t) \quad (4)$$

$$E_x(x,t) = R_j i^j(x,t) \quad (5)$$

Where the suffix "s" denotes the potential due only to the currents in the line (scattered).

Introducing equations (4) and (5) into equation (3) a relation among scalar potentials and currents is obtained, that can be written as:

$$\frac{\partial i^k}{\partial t} = -L_{jk}^{-1} \left\{ R_j i^j + \frac{\partial A_{jx}^i}{\partial t} + \frac{\partial \phi_j}{\partial x} \right\} \quad (6)$$

Where the suffix "i" denotes the potential due to the currents external to the line (inducing).

On the other hand, the charge conservation law (or continuity equation) can be written as:

$$\frac{\partial \lambda^j(x,t)}{\partial t} = - \frac{\partial i^j(x,t)}{\partial x} \quad (7)$$

Where " $\lambda$ " is the linear charge density along the conductor "j".

Assuming again that it is possible to define "capacitances" per unit length as:

$$\lambda^j(x, t) = C_{jk} \phi_k^s(x, t) = C_{jk} (\phi_k - \phi_k^i) \quad (8)$$

Introducing equation (8) into equation (7) a second relation among scalar potentials and currents is obtained, that can be written as:

$$\frac{\partial \phi_j}{\partial t} = \frac{\partial \phi_j^i}{\partial t} - C_{jk}^{-1} \frac{\partial i^k}{\partial x} \quad (9)$$

After a discretization in space of equations (6) and (9), the potential at potential nodes and the current at current nodes were considered as time evolving states of the system. The resulting system of ordinary differential equations (ODE) was then solved using the powerful ODE solvers actually existing [6].

This form of considering the vertical conductors permits, for example, to represent the grounding wire of a shielding wire or of a surge arrester not as a connection to ground, but, as a connection to the ground electrodes whose potential is then considered as more one state of the system.

## INDUCED VOLTAGE CALCULATIONS

The induced voltage calculations presented in this paper will refer to a line with the geometry shown in Figure 1. In order to avoid the effect of imperfectly matched line terminations, the induced voltages calculated during the first 20  $\mu$ s, for a 9 km line, will be presented.

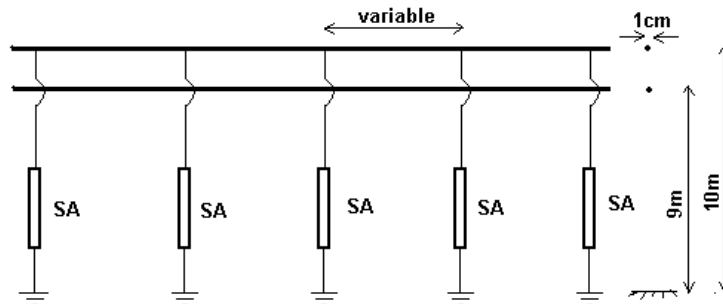


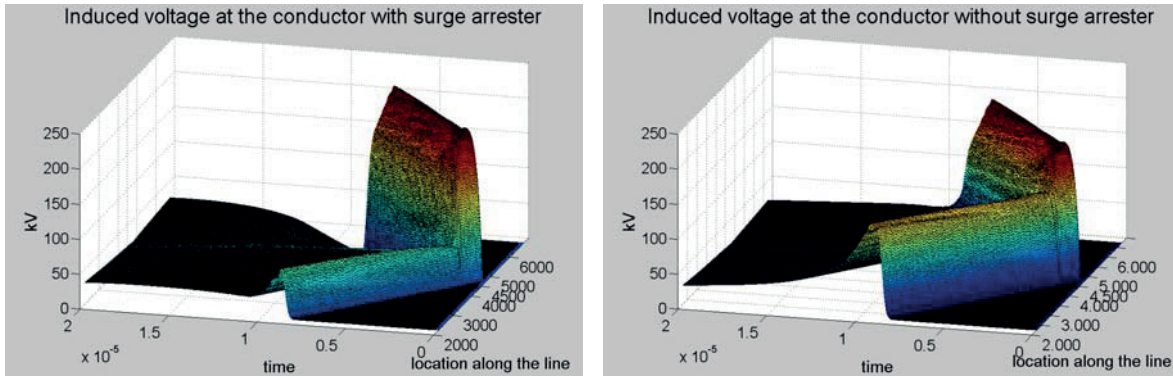
Figure 1 – Line geometry adopted in the calculations.

First, the effect of an isolated surge arrester placed at the middle of the line ( $z = 4500$  m), grounded with a grounding resistance  $R_g = 10$  Ohms, will be shown. The calculations will be done for the case of a 45 kA return stroke propagating with a velocity  $v = 0.3 c$ , with a lightning current of triangular waveshape  $2 \times 40 \mu$ s, that strikes at a distance of 70 m from the line, at the location along the line  $z = 4800$  m.

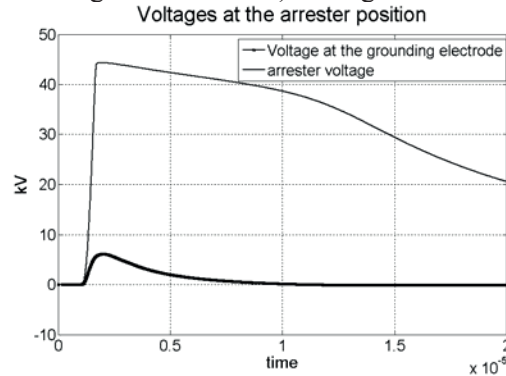
Figure 2 shows the profile of the induced voltage both in the conductor with surge arrester and in the conductor without surge arrester and, also, the voltage at the grounding electrode (that is proportional to the grounding current) and the arrester voltage.

From figures 2a and 2b it can be clearly seen that the effect of the arrester operation is perceived by the other points of the line with a certain time delay. The mitigating effect of the arrester, both on the conductor with surge arrester and on the conductor without surge arrester, is clearly seen for the points that are behind it (seen from the strike point of the lightning discharge), but, for the points in front of it the mitigating effect only exist within a certain "effective distance" that depends on the risetime of the induced voltages. Beyond that distance, the maximum value of the induced voltage is of the same value than for the case with no arrester, because the effect of the operation of the arrester arrives after the maximum of the induced voltage had already occurred.

Next, the effect of multiple surge arresters, placed at a certain interval along the line and each of them grounded with a grounding resistance  $R_g = 10$  Ohms, for the same lightning discharge previously considered will be shown. Figure 3 shows the profile of the induced voltage both in the conductor with surge arresters and in the conductor without surge arresters and, also, the voltage at the grounding electrode (that is proportional to the grounding current) and the arrester voltage at the same pole of the previous case.

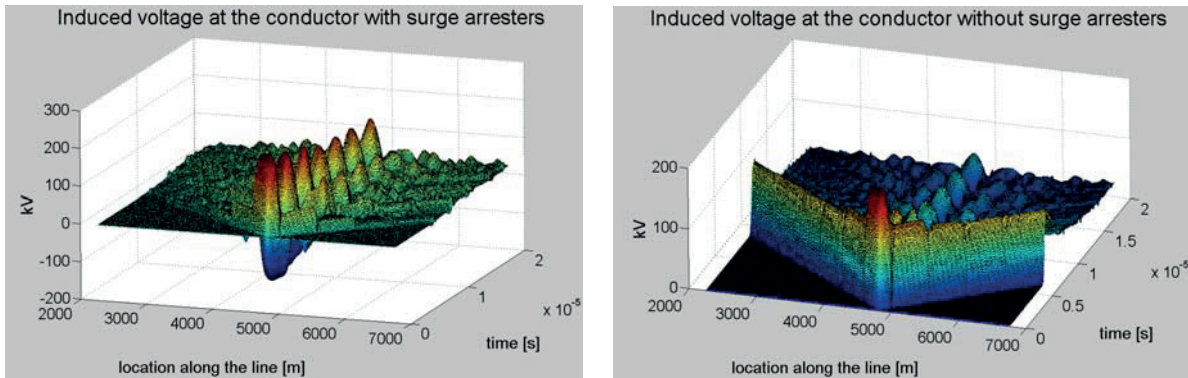


a) Voltage at the conductor with surge arrester. b) Voltage at the conductor without surge arrester.

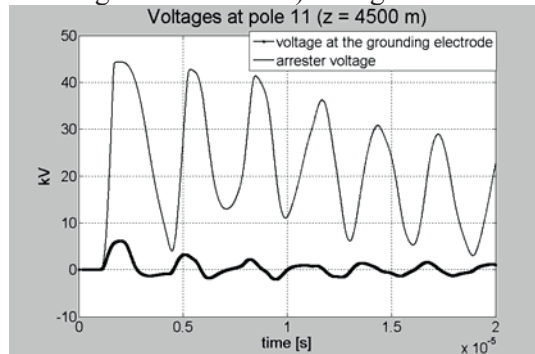


c) Voltages at the pole where the arrester is located ( $z = 4500$  m).

Figure 2– Voltages along a line with one surge arrester ( $R_g = 10$  Ohms), placed at  $z = 4500$  m, produced by a 45 kA lightning discharge ( $2 \times 40 \mu s$ ), propagating with a velocity  $v = 0.3 c$ , that strikes at  $z = 4800$  m, at a distance of 70 m from the line.



a) Voltage at the conductor with surge arresters. b) Voltage at the conductor without surge arresters.



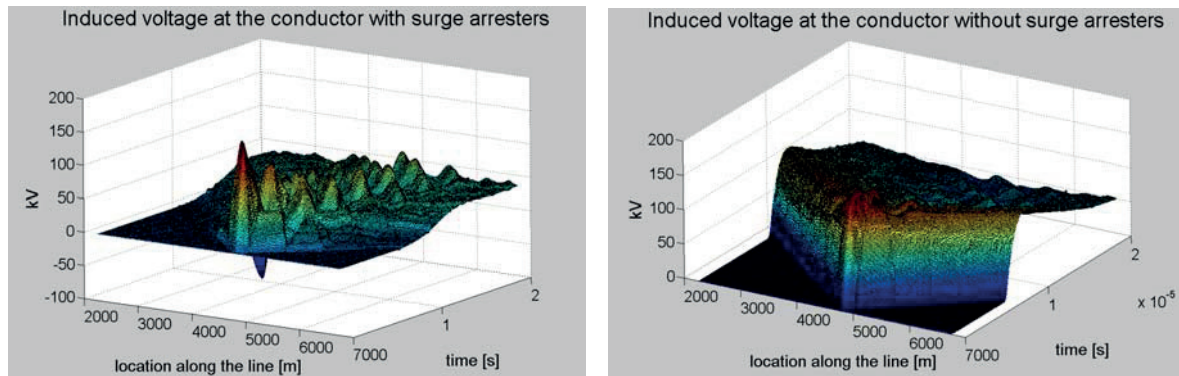
c) Voltages at the pole No.11 ( $z = 4500$  m).

Figure 3– Voltages along a line with surge arresters each 450 m ( $R_g = 10$  Ohms), produced by a 45 kA lightning discharge ( $2 \times 40 \mu s$ ), propagating with a velocity  $v = 0.3 c$ , that strikes at  $z = 4800$  m, at a distance of 70 m from the line.

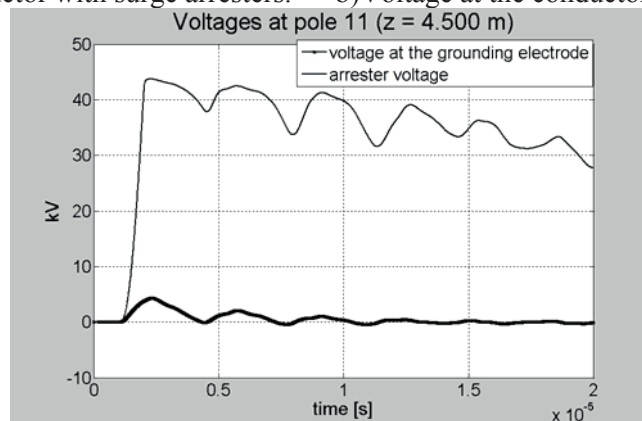
Comparing figure 3c with figure 2c, the effect of the operation of the arresters, which are placed in the neighboring poles, on the current through the arrester and also on the grounding current can be clearly seen.

From figures 3a and 3b it can be seen that the span in front of the strike location, in this case, is not protected by the arresters, in the sense that there are points within that span where the maximum value of the induced voltage is practically the same value than for the case with no arresters. It must be emphasized that all points within this span are in front of the arresters (seen from the strike point of the lightning discharge).

To show that the protection effectiveness of a certain interval between arresters depends on the risetime of the induced voltages, a new calculation was done with the same parameters of the previous calculation (the same discharge, the same interval between arresters, the same grounding resistance, etc.) except that now the propagating velocity of the return stroke is  $v = 0.1 c$ . Figure 4 shows the results in this case.



a) Voltage at the conductor with surge arresters. b) Voltage at the conductor without surge arresters.



c) Voltages at the pole No.11 ( $z = 4500$  m).

Figure 4– Voltages along a line with surge arresters each 450 m ( $R_g = 10$  Ohms), produced by a 45 kA lightning discharge ( $2 \times 40 \mu s$ ), propagating with a velocity  $v = 0.1 c$ , that strikes at  $z = 4800$  m, at a distance of 70 m from the line.

From figures 4a and 4b it can be clearly seen that now the span in front of the strike location is protected by the arresters, in the sense that there the maximum value of the induced voltage is less than the maximum value for the case with no arresters.

## CONCLUSIONS

In this paper the results of a new computer program to calculate lightning induced voltages on distribution lines, placed over a lossy ground, are presented. Concerning the conclusions that can be obtained from these results, referent to the protective effect of surge arresters against lightning induced overvoltages, we can mention the following ones (some of these facts are very well known others not so well known):

- The surge arresters act as a local protective device and the effect of its operation is perceived by the other points of the line with a certain time delay. So, there is a clear protective effect on the points behind it (seen from the strike point of the lightning discharge), but, for the points in front of it the mitigating effect only exist within a certain "effective distance" that depends on the risetime of the induced voltages.
- Multiple surge arresters, placed at a certain interval along the line, protect the region of the line outside the span in front of the strike location. The protection afforded to the line span in front of the strike location depends on the risetime of the induced voltages. Consequently, any statement about the effectiveness of a certain interval between arresters should be qualified by a certain risetime for which it is valid or by a probability that such risetime occurs.

## BIBLIOGRAPHY

- [1] S. Rusck, "Protection of distribution lines" (Lightning, London: Academic Press, Vol.2, cap. 23, pp.747-771, Ed. R.H. Golde, 1977)
- [2] S. Yokoyama, "Calculation of lightning-induced voltages on overhead multiconductor systems" (*IEEE Trans. on Power App. Syst.*, Vol. PAS-103, pp.100-108, Jan. 1984)
- [3] M.Paolone, C.A.Nucci, E.Petrache & F.Rachidi, "Mitigation of lightning-induced overvoltages in medium voltage distribution lines by means of periodical grounding of shielding wires and of surge arresters: Modeling and experimental validation" (*IEEE Trans. on Power Delivery*, Vol.19, No.1, pp. 423-431, Jan. 2004)
- [4] A.K. Agrawal, H.J. Price and S.H. Gurbaxani, "Transient response of a multi-conductor transmission line excited by a nonuniform electromagnetic field" (*IEEE Trans. on Electromagn. Compat.*, Vol. EMC-22, No.2, pp. 119-129, May 1980)
- [5] P.E. Munhoz Rojas & C.L. da S. Pinto, "Calculations of Lightning-Induced Voltages in Medium Voltage Distribution Lines" (Proceedings of IX SIPDA, Paper VI.2, pp.317-322, Nov. 2007)
- [6] L.F.Shampine, "Numerical Solution of Ordinary Differential Equations" (Chapman & Hall, New York, 1994).