Influence of Masing damping on 1D site response using equivalent linear and non-linear methods

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ABSTRACT: Despite significant developments in one-dimensional (1D) site response methods using non-linear (NL) analysis, the Equivalent Linear (EL) analysis remains the most widely used approach by practitioners to investigate the total stress response of a soil deposit. While the EL method is implemented using both shear modulus reduction curves and damping curves from laboratory test results, NL Cyclic Stress (CS) models are often computed so that they are consistent with the backbone stress-strain characteristics, but accept some inconsistencies in the energy dissipated by the hysteresis loops that are represented using a Masing's rule approach. As such, EL analysis often provides a conservative site amplification factor and ground response spectrum, while NL models can lead to an over-damping of the ground response spectrum. This paper discusses the impact of discrepancies in these damping formulations on 1D site response using the EL model and a non-linear CS model computed in OpenSees. Traditional EL analysis is performed, as well as a modified EL approach that changes the original formulation in order to capture frequency-dependent soil stiffness and soil damping parameters that are consistent with the frequency content of the vertically propagating shear waves. While it is largely recognized that Masing behaviour introduces a bias in the site response at large strains, it has been found that a moderate strain pattern within the soil profile can also lead to a substantial under-prediction of the ground response.

1 INTRODUCTION

Theoretical 1D site response was first implemented in the frequency domain by the use of equivalent linear (EL) analysis (Schnabel et al. 1972), assuming linear soil behaviour along with viscous material damping (Kramer 1996). There is general agreement that EL analysis is a reliable method for performing total stress analysis on sites that exhibit low levels of nonlinearity during shaking. However, in certain conditions, the EL ground response implemented with 'equivalent' dynamic soil properties can produce unconservative outcomes, especially for sites subject to strong variation in soil properties during the ground motion record. In this case, nonlinear (NL) analysis based on direct time integration algorithms is preferred as soil properties are updated at each time step. Independent of soil shaking direction considerations, two families of NL models exist: (1) cyclic shear-strain (CSS) models also referred as hysteretic models, and (2) advanced constitutive models.

CSS models rely on the use of backbone curves obtained from shear strain-stress relationships, namely hyperbolic models, which handle the initial stress loading path (e.g. Kondner & Zelasko 1963, Hardin & Drnevich 1972, Darendeli 2001). Combined with backbone curves, reversal shape functions initially defined with the Masing rules (Masing 1926) are used to control the unloading-reloading path during the shaking as shown in Figure 1(a). While the EL model uses soil degradation curves that characterise soil stiffness and material damping independently, in the CSS model, the material damping is defined implicitly from the backbone curve by means of hysteric damping (area within the hysteretic loops in Figure 1). Accordingly, establishing a consistent CSS model that can effectively characterise both the modulus reduction and the material damping curves is a challenge. To address this issue, curve-fitting parameters have been introduced in hyperbolic models along with hysteretic shape functions, some of which are implemented in specific nonlinear codes such as TESS (Pyke 2000), DMOD_2 (Matasovic 2006) or DEEPSOIL (Hashash and Park 2001).

Among the different strategies employed for soil modelling, many studies prefer the use of consistent modulus reduction curves in order to predict nonlinear soil effects, by considering that the nonlinear response is governed by the soil modulus degradation and also assuming that the misfit of energy dissipation introduced by Masing hysteresis loops remains acceptable when low to moderate strain deformation occurs. This approach is of practical interest as recently developed soil models are generally well constrained from laboratory measurement databases for low to moderate shear strains that remain generally below 0.1–0.3% (Darendeli 2001), covering the strain range induced by many ground motion records. This study emphasizes the effects of Masing behaviour in nonlinear soil response especially when low to moderate strains occur during shaking.

When cyclic shear stress loading develops through a nonlinear soil element based on Masing's rules, the shape of each stress reversal is deduced from the initial loading path enlarged by a double factor and the tangent shear moduli at each reversal stress is identical to the initial shear modulus G_{max}. Acknowledging that the energy dissipation based on Masing behaviour along with shear modulus degradation ratios G/G_{max} is a poor trade-off when compared with laboratory test data, Darendeli among others proposed a corrected damping model using scaling factors and curve fitting parameters. Figure 1 depicts the misfit of Masing-based damping ratio curves across different hyperbolic models, herein referred to as the Kondner and Zelasko model (KZ), the modified Kondner and Zelasko model (MKZ) and the Darendeli model presenting both the Masing-based damping without adjustment and then the consistent damping curve adjusted to fit with laboratory test data.

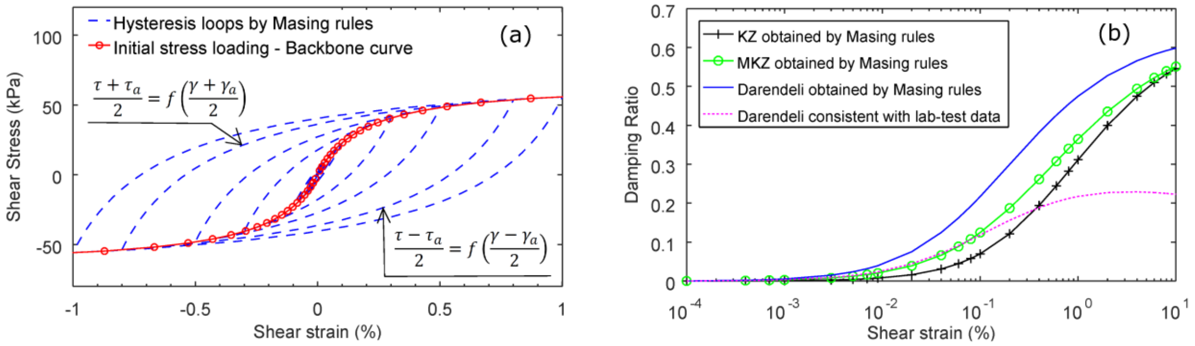

Figure 1. (a) Hysteretic loops by Masing rules obtained from incremental CSS tests in OpenSees, (b) Masing-based damping ratio curves from hyperbolic shear modulus degradation models compared to a consistent damping ratio curve with laboratory test data using the model of Darendeli (2001).

2 METHODOLOGY

A subset of soil moduli reduction curves from the model of Darendeli (2001) were used to investigate the effects of Masing-based damping by comparing the site response predictions between a NL code implemented with the Masing rules, herein OpenSees (Parra 1996, Yang 2000), and EL methods implemented with both (1) Masing-type damping curves consistent with OpenSees and (2) target damping curves from laboratory test results. These are compared against the NL model to assess the salient features of each and the impact of the inconsistencies in the damping formulation across these models.

A suite of 10 records consistent with a rock site class B condition as defined in the NZ1170.5 (2004) loadings standard were selected and used as control motions. Selection was made across range of source mechanisms for crustal earthquake scenarios along with a set of motion intensities suitable for performing a site response analysis. One half of the records are found in the Pacific Earthquake Engineering Research Center ground motion database (PEER, NGA-West2) and the other half are from the New Zealand database GeoNet developed by GNS Science. These records comprise recent seismic events in NZ from the 2010-2011 Canterbury earthquake sequence and the Kaikoura earthquake (2016), all recorded from stations that exhibit rock site class B condition. A scaling factor was applied to the time-history records to match the PGA of the Uniform Hazard Spectrum (UHS) defined in the NZS1170.5 standard (Z=0.25, R=1, N=1). Figure 2 shows the pseudo-response spectra of the 10 records and the geometric mean response spectrum compared to the UHS spectrum.

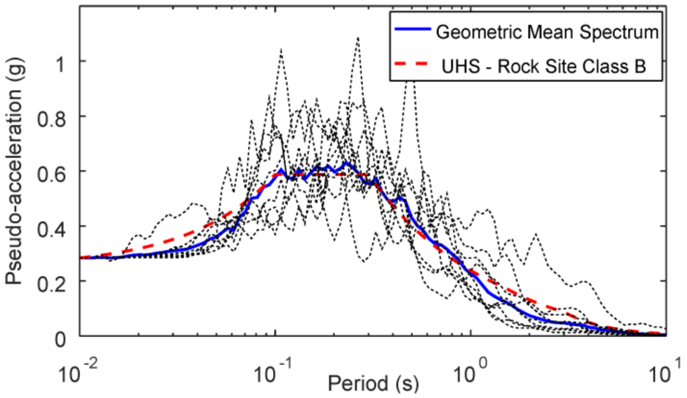

Figure 2. Soil response spectra with PGA scaled on normalized UHS rock spectrum, site class A/B, according to NZ1170.5 (2004) standard – Spectral shape factors: Z=0.25, R=1, N=1.

The 1D site response analysis was carried out considering a local site condition that would be classified as site class D according to NZ1170.5 (2004), which characterizes a deep soil deposit exhibiting a site period greater than 0.6 s. The shear wave velocity profile was set up for a multi-layered soil system consisting of loose to dense sand in a drained condition suitable for performing a total stress analysis. Details of the soil parameters are summarised in Table 1.

Table 1. Characterization of site profile used for 1D site response analysis

Site Period (s)	$V_{s,Eq}$** (m/s)	$V_{s,30}$* (m/s)	Description	Layer Depth (m)	Layer Thickness (m)	Density (kg/m^3)	V_s (m/s)	Friction Angle (°)
0.88	206	188	Loose Sand - Dr=30%	0.0	7.50	1.7E+03	165	28
			Medium Sand - Dr=50%	7.5	7.50	1.8E+03	181	33
			Medium Sand - Dr=50%	15.0	7.50	1.8E+03	197	33
			Medium-Dense Sand - Dr=70%	22.5	7.50	2.0E+03	219	39
			Medium-Dense Sand - Dr=80%	30.0	7.50	2.0E+03	241	41
			Dense Sand - Dr=90%	37.5	7.50	2.1E+03	263	44

Note: * $V_{s,30}$ denotes the time-averaged shear wave velocity to 30 m depth
** $V_{s,Eq}$ denotes the time-averaged shear wave velocity of the overall soil profile above bedrock

The Darendeli model was used to predict nonlinear soil effects when low to moderate strain deformations are expected to occur during the shaking, i.e. with a threshold strain value between 0.1–0.3%. The Darendeli shear modulus degradation model relies on the hyperbolic relationship:

$$\frac{G}{G_{max}} = \frac{1}{1+\left(\frac{\gamma}{\gamma_r}\right)^a} \tag{1}$$

where γ_r is a reference strain and a is a curvature coefficient. In particular, these two parameters are defined as a function of the soil type and plasticity index (PI), the over-consolidation ratio (OCR), the confining pressure $(\sigma_0^{'})$ and also a series of model calibration parameters $(\varnothing_i)$ obtained from a regression analysis conducted over range of experimental data measurements. Finally the reference strain is expressed as following:

$$\gamma_r = \left(\varnothing_1 + \varnothing_2 * PI * OCR^{\varnothing_3}\right) * \sigma_0^{'\varnothing_4} \tag{2}$$

The lab-consistent damping model proposed by Darendeli consists in adjusting the Masing-based damping prediction D_{Masing} by the use of a scaling coefficient b and a shape function of G/G_{max} so that the formula becomes:

$$D_{Adjusted} = b * \left(\frac{G}{G_{max}} \right)^{0.1} * D_{Masing} \tag{3}$$

The parameters used for the hyperbolic model for sand were the following:

- $PI = 0$; $OCR = 1$
- $\varnothing_1 = 0.0352$; $\varnothing_2 = 0.0010$; $\varnothing_3 = 0.3246$; $\varnothing_4 = 0.3483$; $a = \varnothing_5 = 0.9190$
- $b = 0.6329 - 0.0057 * \ln(100) = 0.6066$

3 NONLINEAR 1D SITE RESPONSE IMPLEMENTED IN OPENSEES

3.1 Soil column model

The 1D soil column model implemented in OpenSees to model nonlinear site response is a finite element continuum model constrained to convey horizontal stresses when a horizontal base motion is applied. The model consists of quadrilateral elements with two degrees-of-freedom and a plane strain formulation. As is recommended when using outcropping records as control motions, part of the strain energy transmitted within the bedrock interface should be dissipated, herein using a viscous dashpot at the base of the soil column (Lysmer-Kuhlemeyer 1969).

The use of hyperbolic model for shear modulus degradation along with Masing behaviour tends to produce zero energy dissipation at low strains. To overcome this drawback, it is common to introduce a minimal viscous damping component into the model based on the Rayleigh damping formulation extended to two target frequencies calibrated from the elastic soil response. However, this protocol can introduce a bias in the energy dissipated across harmonics when nonlinearities occur, affecting the transfer functions within elements when the predominant modes of vibration shift to longer periods. In order to avoid any misinterpretation when comparing the NL response with the EL procedures, Rayleigh damping was not utilised in this study. In lieu of this process, the time-history records obtained from NL responses in OpenSees were filtered using a low-pass 4th order Butterworth filter with a cut-off frequency of 15Hz so as to remove the undesirable noise in the records. Furthermore, to be consistent with the idea that underpins the use of Rayleigh damping, while preventing the potential for bias introduced by this method, a better alternative could be to introduce this additional damping directly into the oscillator parameter considered in the development of the pseudo-response spectrum.

3.2 Pressure Independent Multi-Yield soil model (PIMY)

Total stress analysis was performed using the Pressure Independent Multi-Yield (PIMY) model built-in OpenSees (Mazzoni et al. 2010) to simulate the deviatoric stress-strain soil response under a monotonic or cyclic loading. The plastic stress behaviour implemented is based on the multi-yield surfaces framework (Iwan 1967, Prevost 1985) and further modified (Parra 1996, Yang 2000), as illustrated in Figure 3. The PIMY model is suitable for soils whose confining pressure is insensitive to cyclic rate, typically undrained response of clays or sands in drained condition. Detailed documentation on OpenSees soil materials can be found in Parra (1996), Elgamal et al. (2003) and Yang et al. (2008).

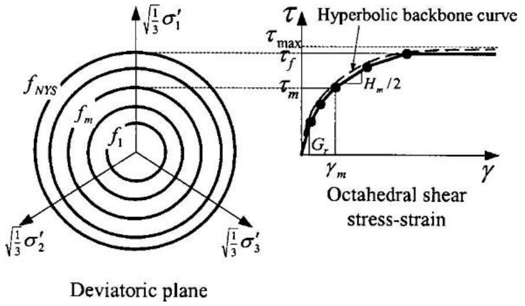

Figure 3. Schematic representation of the backbone curve into multi-yield surfaces through its piecewise-linear transformation (after Prevost 1985 and Parra 1996).

The hyperbolic backbone curve by Konder and Zelasko (1963) can be generated automatically in the PIMY material model using a built-in hyperbolic pattern between the shear strain and the shear stress. Alternatively, the model allows the implementation of user-specified modulus reduction ratios over a finite number of yield surfaces. In this study, the aforementioned shear modulus degradation model (Darendeli 2001) was generated for each soil element considering the confining pressure at each depth.

4 EQUIVALENT LINEAR PROCEDURES

4.1 The original equivalent linear method

The EL site response was computed in a GNU Octave code program based on the theoretical approach using transfer functions (Kramer 1996) along with the Kelvin-Voigt soil model formulation. Then the EL iterative procedure (Schnabel et al. 1972) was reproduced, with an effective shear strain γ_{eff} within elements defined as:

$$\gamma_{eff} = \alpha * \gamma_{max} \tag{4}$$

where γ_{max} is the maximum shear strain, scaled by a factor α set to 0.65 in this study.

4.2 Frequency-dependent equivalent linear method

As the original EL method has been shown to attenuate the higher frequencies, resulting in a flat response spectrum at smaller periods, a modified EL method using both frequency-dependent soil moduli and damping ratios was proposed in order to better fit with the frequency content observed in the NL site response. This procedure is herein referred to as the Frequency-Dependent Equivalent Linear (FDEL) method.

Two broad classes of FDEL formulation were proposed, the original approach proposed by Sugito et al (1994), and the alternative methods expressed by Kausel and Assimaki (2002) and Yoshida et al (2002) intended to better fit with the observations. It appears that the site response predictions inherent to these procedures are strongly dependent on the set of calibration parameters used to set up a smoothed frequency-dependent effective shear strain. In the previous studies, the definition of these calibration parameters relied on authors' judgments and limited benchmarking studies assessing the FDEL model capabilities against nonlinear methods and vertical arrays over a range of ground motion intensities and site conditions. As a result, the implementation of the FDEL methods have not found use by practitioners.

In this study, a new FDEL approach is proposed and tested to generalize the numerical scheme over a range of ground motion intensities and frequency content and to simplify its computation through the EL iterative procedure. Previous FDEL formulations available in the literature are based on the Fourier response of the shear strain along with calibrated parameters to adjust the prediction over frequencies of interest. The proposed FDEL method relies on the Fourier Transformation (FT) of the instantaneous elastic power stress P_E scaled on the maximal shear strain as a proxy for the frequency-dependent effective shear strain expressed as followed:

$$\gamma_{eff}(\omega) = \gamma_{max} \frac{FT[P_E]}{\max\{FT[P_E]\}} = \gamma_{max} \frac{FT\left[G(t)\gamma(t)\dot{\gamma}(t)\right]}{\max\left\{FT\left[G(t)\gamma(t)\dot{\gamma}(t)\right]\right\}} \tag{5}$$

where the component $G(t)\gamma(t)$ represents the elastic shear stress and $\dot{\gamma}(t)$ denotes the strain rate.

The advantage of this approach is that it is readily computable in a routine implemented within the EL iterative procedure and no additional calibration parameters were used for the model prediction. Instead, the Fourier spectra was resampled directly to smooth the spectral effective strain and ease the convergence into the EL iterative scheme. Figure 4 presents the additional steps inherent to the FDEL procedure computed for each soil element. An effective shear strain $\gamma_{eff}(\omega)$ is expressed in the frequency-domain using Eq. (5) as shown in Figure 4(a), and two additional soil parameters must be computed, the shear wave velocity spectrum of Figure 4(b) obtained by fitting each strain amplitude $\gamma_{eff}(\omega)$ with the modulus degradation curve, and the damping ratio spectrum of Figure 4(c) obtained by similar means.

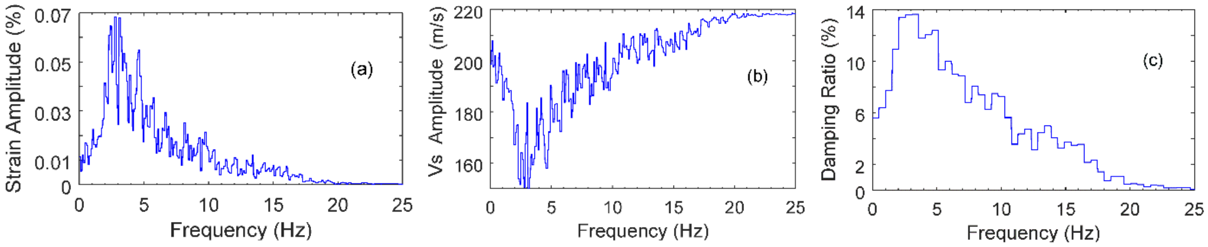

Figure 4. Frequency-dependent soil parameters implemented into the FDEL procedure: (a) effective shear strain spectrum ; (b) shear wave velocity spectrum; (c) damping ratio spectrum.

4.3 Capabilities of equivalent linear procedures

This section aims to discuss the performance of the implemented EL procedures in capturing the salient effects of soil nonlinearities in 1D site response through comparison with the NL response from OpenSees. For consistency between models, 1D site response analyses were conducted for both EL procedures, the original EL and the FDEL methods, using Masing-based damping ratio curves for each element within the multi-layer soil system for comparison with the NL site response computed in OpenSees. The model predictions in terms of pseudo-spectral acceleration (pSA) at the ground surface are presented in Figure 5(a). As expected, it appears that the FDEL procedure provides a better agreement with the NL response than the original EL prediction does, especially when the ground motion exhibits a higher frequency content. Indeed one shortcoming in the original EL method is that higher harmonics are attenuated as the degree of nonlinearities increases. As a result, the soil response intensities, e.g. acceleration time-history response or pseudo-response spectra, used for further structural analysis are often provided with a filtered frequency content illustrated by a flat shape in the response spectrum beyond a threshold frequency herein over 4 Hz which is unconservative for further structural analysis. Clearly, the FDEL method by its formulation overcomes this shortcoming. Moreover, it has been found that both EL methods tends to under-predict the degree of nonlinearity experienced at lower frequencies, which in turn results in a higher peak observed at the fundamental mode in the ground response spectra compared with the NL response. This bias at lower frequencies is more pronounced in the FDEL prediction, providing a more conservative spectral amplification. These observations are in line with previous studies comparing FDEL and EL methods (e.g. Hartzell 2004).

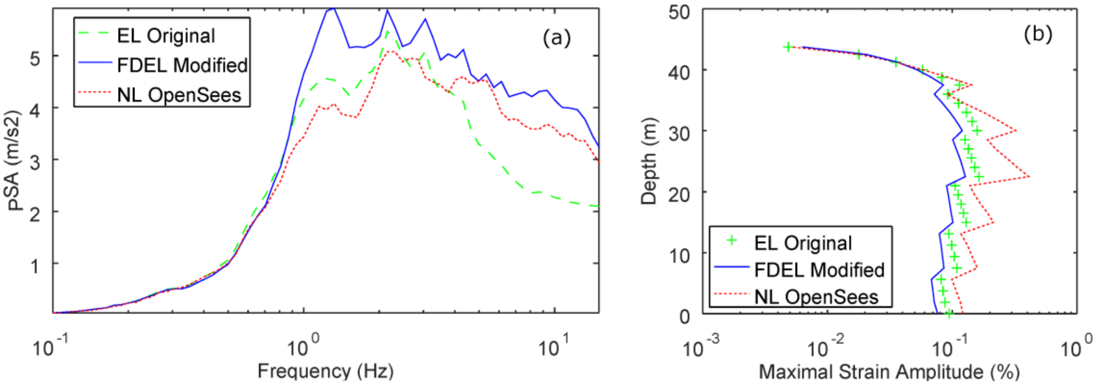

Figure 5. (a) Comparison between the geometric mean response spectra from EL methods and NL soil column model in OpenSees and (b) Maximal shear strain profile within soil layers over depth.

Moreover, larger strains developed within the soil column model in OpenSees with higher fluctuation in maximal amplitudes over depth. Finally, one should keep in mind when considering the whole procedure that the model predictions result in a balance between the competing effects of soil softening, which tends to amplify the site response, and the material damping effects that attenuate the motion intensities, as illustrated by the original EL formulation.

5 EFFECTS OF MASING BEHAVIOUR IN SITE RESPONSE PREDICTION

While the previous section used the Masing-based damping ratio curves, this section presents 1D EL site response analyses conducted using more representative damping ratio curves consistent with laboratory data (Darendeli 2001). As shown in Figure 6(a), the NL response in OpenSees tends to under-predict the ground response spectrum, with spectral amplitudes clearly below the spectral amplitudes obtained when consistent EL procedures are considered, with an attenuation observed from 0.7 Hz and extended up to the cut-off frequency at 15 Hz.

To compare the different procedures and damping ratio representation, residual predictions in terms of pSA ratios are shown in Figure 6(b). Comparison of the ratio of the prediction from NL response and the consistent FDEL response (in solid blue line) shows that the misfit inherent to the Masing model is a maximum at the fundamental frequency of the soil profile (1.1 Hz) with a 40% reduction in the pSA. Beyond the fundamental frequency, the residual fluctuates at an approximately 25% reduction, before stabilizing at 15 Hz. Bearing in mind the aforementioned trend that the FDEL procedure over-predicts the amplification at fundamental modes, this 40% reduction is obviously biased. That is, the inherent bias in model prediction across procedures can be bypassed when the residual prediction is scrutinized by comparing a FDEL using a Masing-based damping model to a FDEL with a lab-consistent damping model. In this case, when the soil profile exhibits moderate strains over depth (i.e. up to 0.1%), it appears that the Masing-based damping response is under-predicted by more than 18% beyond 2 Hz, ie. for periods less than 0.5 s.

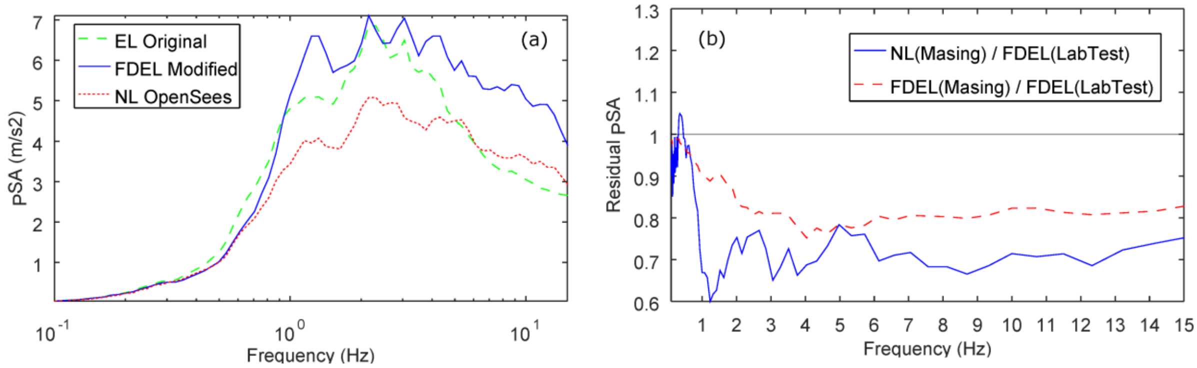

Figure 6. Site response predictions between NL model with Masing behaviour and EL procedures using consistent soil damping with laboratory test data (Darendeli, 2001) showing: (a) Ground response spectra (pSA) and (b) Residual pSA ratios between NL and EL predictions.

6 CONCLUSIONS

This study found that when modulus reduction curves consistent with laboratory test measurements, such as the Darendeli model, are used along with Masing behaviour to represent damping, the 1D site response in terms of pSA is clearly attenuated even when moderate nonlinearities develop within the soil profile, with an upper bound of strain deformation between [0.1–0.3]%. Thus, the widely accepted view that the over-damping effects of Masing damping mainly manifest when large strain deformation occurs in the model, i.e beyond 1% strain deformation, was not verified in this case study. Indeed the nonlinear soil response predicted by the Darendeli model provides non-negligible discrepancies even at low to moderate strain deformation. Furthermore, an experimental frequency-dependent EL formulation was proposed to better capture the nonlinear soil effects at lower periods in an attempt to ease its computation within a numerical routine compatible with the EL iterative procedure. It has been found again that the FDEL procedure substantially improves the EL response in terms of frequency content at lower periods, i.e. beyond 4 Hz, but tends to over-predict the spectral response at the fundamental period of the soil profile.

7 ACKNOWLEDGEMENTS

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