Swarm Precise Orbit Determination and ionospheric signatures

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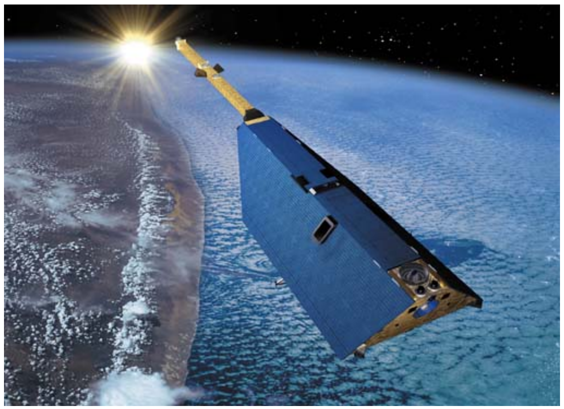
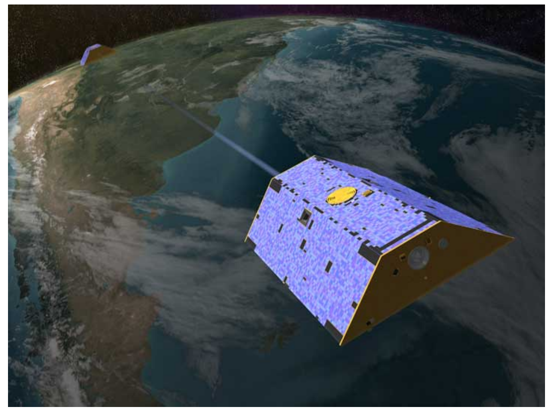
Low Earth Orbiters (LEOs)

CHAMP



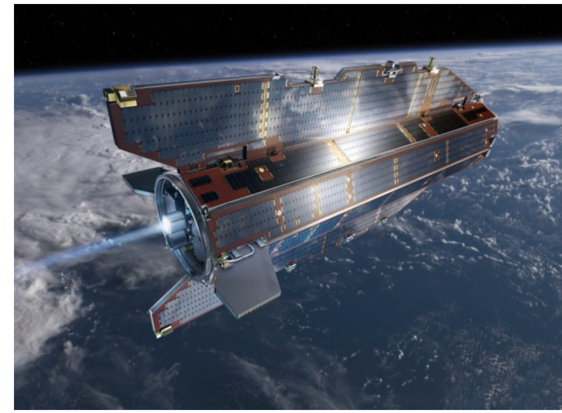
CHAllenging **M**inisatellite **P**ayload

GRACE



Gravity **R**ecovery **A**nd **C**limate **E**xperiment

GOCE



Gravity and steady-state **O**cean **C**irculation **E**xplorer

But there are many more missions equipped with GPS receivers …

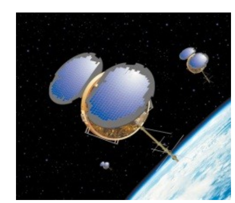
Jason

Jason-2

MetOp-A

Icesat

COSMIC



LEO Constellations

TanDEM-X

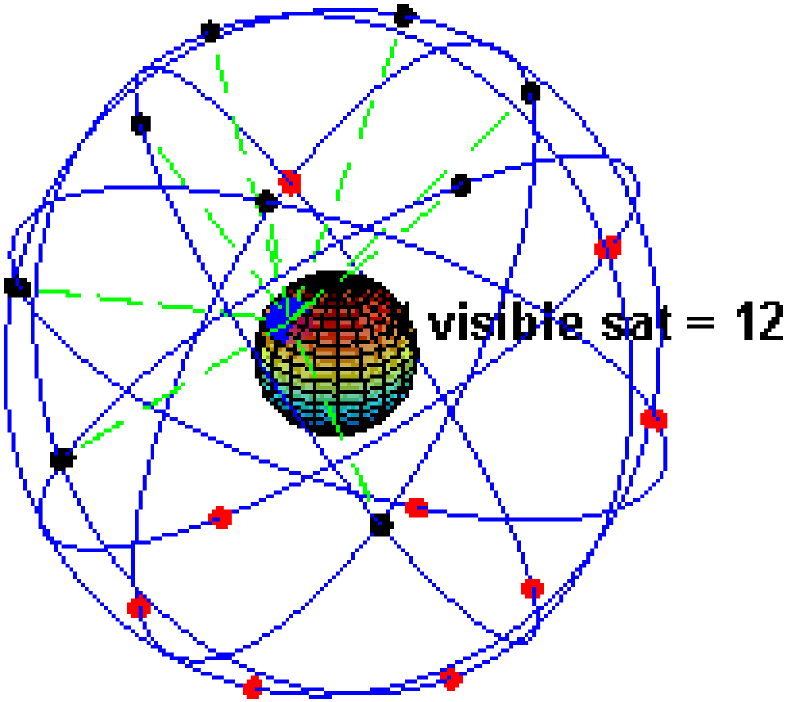
Swarm

Sentinel

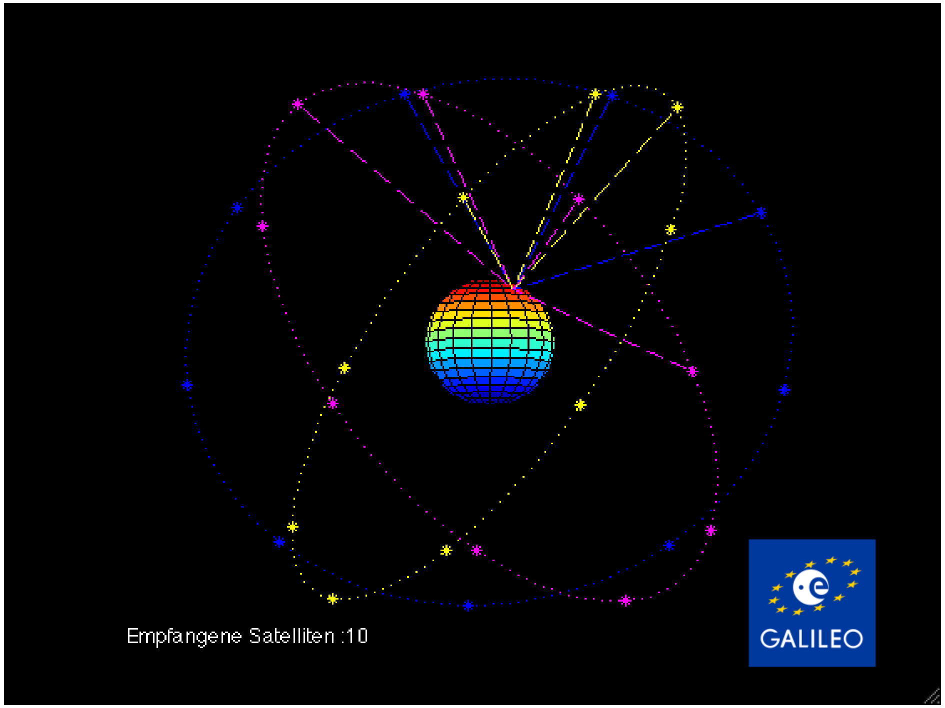
and of course, in the future

Global Navigation Satellite Systems (GNSS)

GPS

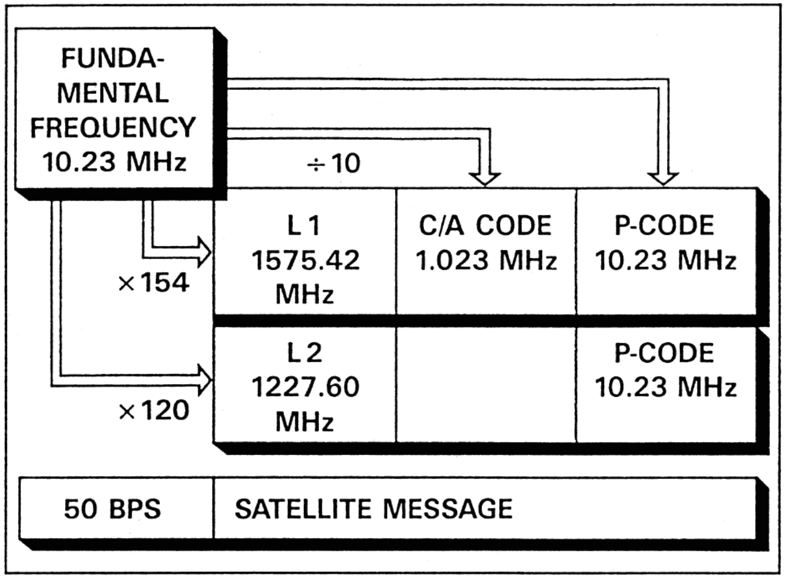


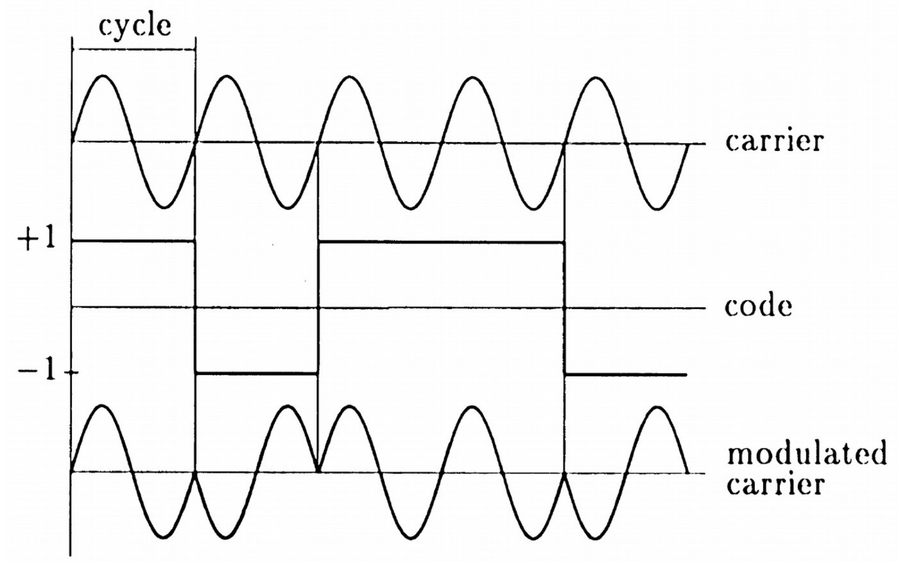
Galileo



Other GNSS are already existing (GLONASS) or being built up (Galileo, Beidou), but there are no multi-GNSS spaceborne receivers (with open data policy) in LEO orbit yet.

GPS Signals





Signals driven by an **atomic clock**

Two **carrier signals** (sine waves):

- **L_1**: f = 1575.43 MHz, λ = 19 cm
- **L_2**: f = 1227.60 MHz, λ = 24 cm

Bits encoded on carrier by phase modulation:

- **C/A-code** (Clear Access / Coarse Acquisition)
- **P-code** (Protected / Precise)
- **Broadcast/Navigation Message**

(e.g. Blewitt, 1997)

Improved Observation Equation

$$L_i^k = \rho_i^k - c \cdot \Delta t^k + c \cdot \Delta t_i + \cancel{T_i^k} + \cancel{I_i^k} + \lambda \cdot N_i^k$$
$$+ \Delta_{rel} - c \cdot b^k + c \cdot b_i + m_i^k + \epsilon_i^k$$

ρ_i^k	Distance between satellite and receiver ←	Satellite positions and clocks
Δt^k	Satellite clock offset wrt GPS time ←	are known from ACs or IGS
Δt_i	Receiver clock offset wrt GPS time	
~~T_i^k~~	~~Tropospheric delay~~ ←	Not existent for LEOs
~~I_i^k~~	~~Ionospheric delay~~ ←	Cancels out (first order only) when forming the ionosphere-free linear combination:
N_i^k	Phase ambiguity	
Δ_{rel}	Relativistic corrections	
b^k	Delays in satellite (cables, electronics)	
b_i	Delays in receiver and antenna	
m_i^k	Multipath, scattering, bending effects	
ϵ_i^k	Measurement error	

$$L_c = \frac{f_1^2}{f_1^2 - f_2^2} L_1 - \frac{f_2^2}{f_1^2 - f_2^2} L_2$$

Geometric Distance

Geometric distance ρ_{leo}^{k} is given by:

$$\rho_{leo}^{k} = \left|\boldsymbol{r}_{leo}(t_{leo}) - \boldsymbol{r}^{k}(t_{leo} - \tau_{leo}^{k})\right|$$

$\boldsymbol{r}_{leo}$	Inertial position of LEO antenna phase center at reception time
$\boldsymbol{r}^{k}$	Inertial position of GPS antenna phase center of satellite k at emission time
τ_{leo}^{k}	Signal traveling time between the two phase center positions

Different ways to represent $\boldsymbol{r}_{leo}$:

- **Kinematic** orbit representation
- **Dynamic** or **reduced-dynamic** orbit representation

Kinematic Orbit Representation (1)

Satellite position $\boldsymbol{r}_{leo}(t_{leo})$ (in inertial frame) is given by:

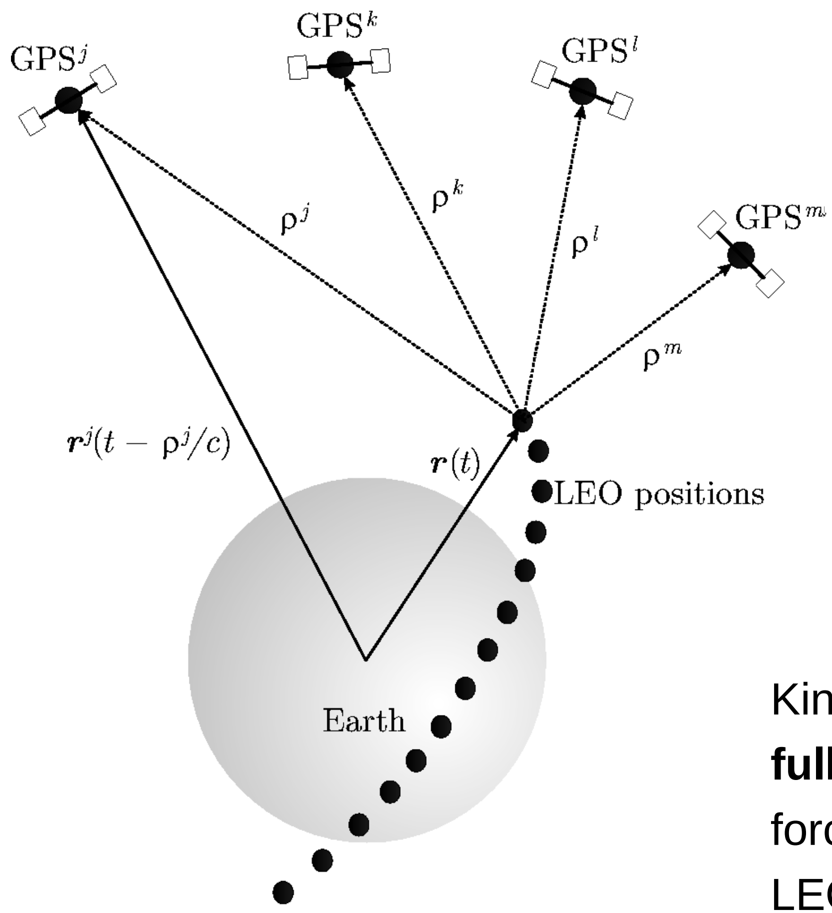
$$\boldsymbol{r}_{leo}(t_{leo}) = \boldsymbol{R}(t_{leo}) \cdot (\boldsymbol{r}_{leo,e,0}(t_{leo}) + \delta\boldsymbol{r}_{leo,e,ant}(t_{leo}))$$

$\boldsymbol{R}$	Transformation matrix from Earth-fixed to inertial frame
$\boldsymbol{r}_{leo,e,0}$	LEO center of mass position in Earth-fixed frame
$\delta\boldsymbol{r}_{leo,e,ant}$	LEO antenna phase center offset in Earth-fixed frame

Kinematic positions $\boldsymbol{r}_{leo,e,0}$ are estimated for each **measurement epoch**:

- Measurement epochs **need not** to be identical with nominal epochs
- Positions are **independent** of models describing the LEO dynamics. Velocities cannot be provided

Kinematic Orbit Representation (2)



A kinematic orbit is an ephemeris at **discrete** measurement epochs

Kinematic positions are **fully independent** on the force models used for LEO orbit determination

(Švehla and Rothacher, 2004)

Kinematic Orbit Representation (3)

Measurement epochs
(in GPS time)

Positions (km)
(Earth-fixed)

```
*  2016  6  1  0  0 17.99999974
PL49  -2023.517746  -3061.130332   5742.844473      0.262691
*  2016  6  1  0  0 18.99999989
PL49  -2026.734429  -3066.703833   5738.746569      0.111976
*  2016  6  1  0  0 19.99999974
PL49  -2029.949393  -3072.273033   5734.641440      0.261262
*  2016  6  1  0  0 20.99999979
PL49  -2033.162630  -3077.837924   5730.529099      0.210548
*  2016  6  1  0  0 21.99999984
PL49  -2036.374136  -3083.398504   5726.409546      0.159835
*  2016  6  1  0  0 22.99999979
PL49  -2039.583909  -3088.954755   5722.282787      0.209119
*  2016  6  1  0  0 23.99999984
PL49  -2042.791949  -3094.506686   5718.148843      0.158440
*  2016  6  1  0  0 24.99999979
PL49  -2045.998248  -3100.054291   5714.007709      0.207778
*  2016  6  1  0  0 25.99999974
PL49  -2049.202791  -3105.597545   5709.859370      0.257064
*  2016  6  1  0  0 26.99999979
PL49  -2052.405584  -3111.136450   5705.703844      0.206352
```

Clock correction to nominal epoch (μs), e.g., to epoch 00:00:20

Excerpt of kinematic Swarm-C positions at begin of 1 June, 2016

The kinematic orbits may be downloaded at ftp://ftp.unibe.ch/aiub/LEO_ORBITS/

Dynamic Orbit Representation (1)

Satellite position $\boldsymbol{r}_{leo}(t_{leo})$ (in inertial frame) is given by:

$$\boldsymbol{r}_{leo}(t_{leo}) = \boldsymbol{r}_{leo,0}(t_{leo}; a, e, i, \Omega, \omega, u_0; Q_1, ..., Q_d) + \delta\boldsymbol{r}_{leo,ant}(t_{leo})$$

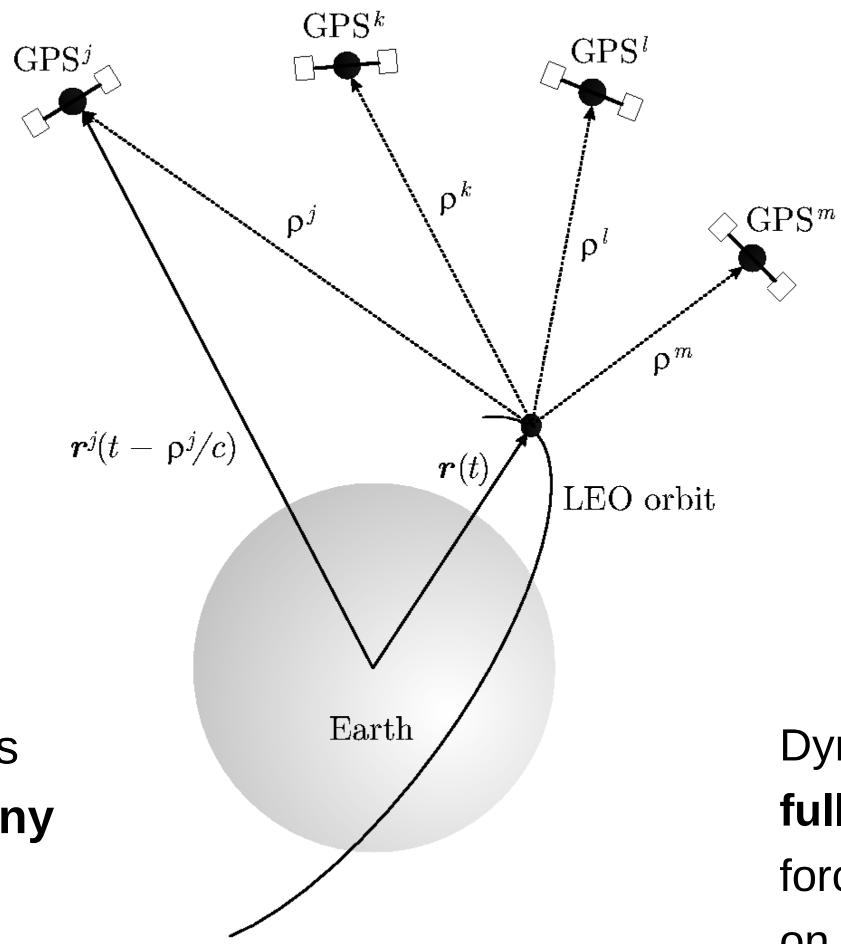
$\boldsymbol{r}_{leo,0}$	LEO center of mass position
$\delta\boldsymbol{r}_{leo,ant}$	LEO antenna phase center offset
$a, e, i, \Omega, \omega, u_0$	LEO initial osculating orbital elements
$Q_1, ..., Q_d$	LEO dynamical parameters

Satellite trajectory

$\boldsymbol{r}_{leo,0}$ is a particular solution of an **equation of motion**

- One set of **initial conditions** (orbital elements) is estimated per arc.
  Dynamical parameters of the force model on request

Dynamic Orbit Representation (3)



Dynamic orbit positions may be computed at **any epoch** within the arc

Dynamic positions are **fully dependent** on the force models used, e.g., on the gravity field model

Reduced-Dynamic Orbit Representation (1)

Equation of motion (in inertial frame) is given by:

$$\ddot{\boldsymbol{r}} = -GM\frac{\boldsymbol{r}}{r^3} + \boldsymbol{f}_1(t, \boldsymbol{r}, \dot{\boldsymbol{r}}, Q_1, ..., Q_d, P_1, ..., P_s)$$

$P_1, ..., P_s$ Pseudo-stochastic parameters

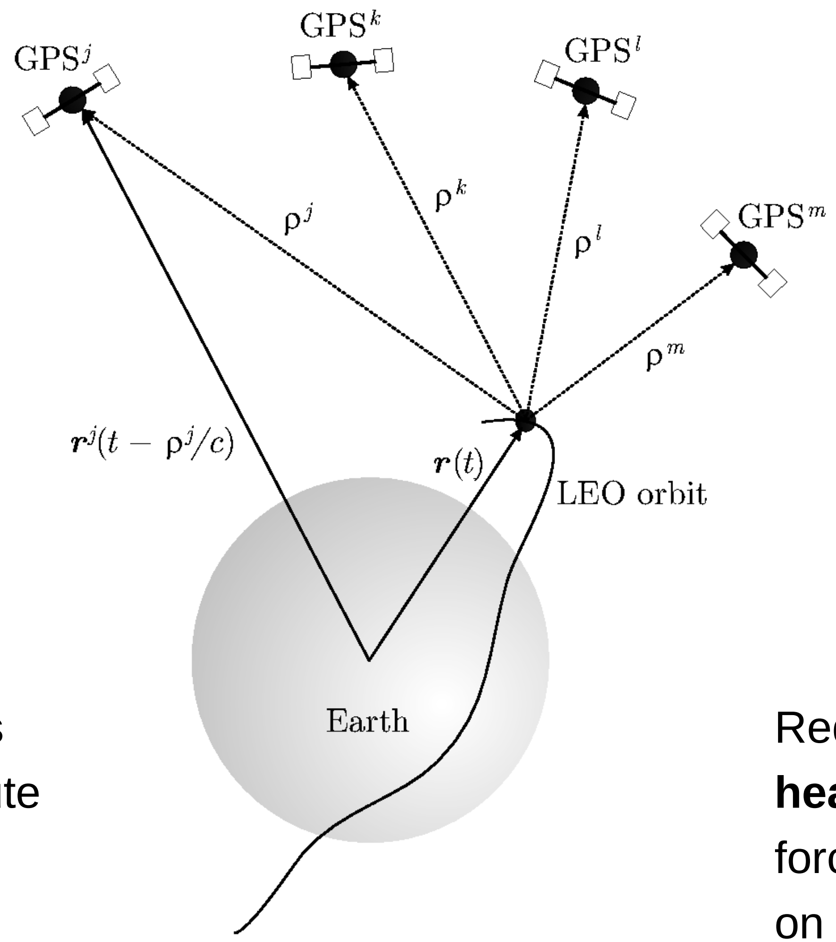
Pseudo-stochastic parameters are:

- additional empirical parameters characterized by a priori known **statistical properties**, e.g., by expectation values and a priori variances

- useful to **compensate** for deficiencies in dynamic models, e.g., deficiencies in models describing non-gravitational accelerations

- often set up as **piecewise constant accelerations** to ensure that satellite trajectories are continuous and differentiable at any epoch

Reduced-Dynamic Orbit Representation (2)



Reduced-dynamic orbits are well suited to compute LEO orbits of **highest quality**

(Jäggi et al., 2006; Jäggi, 2007)

Reduced-dynamic orbits **heavily depend** on the force models used, e.g., on the gravity field model

Reduced-dynamic Orbit Representation (3)

Position epochs
(in GPS time)

Positions (km) &
Velocities (dm/s)
(Earth-fixed)

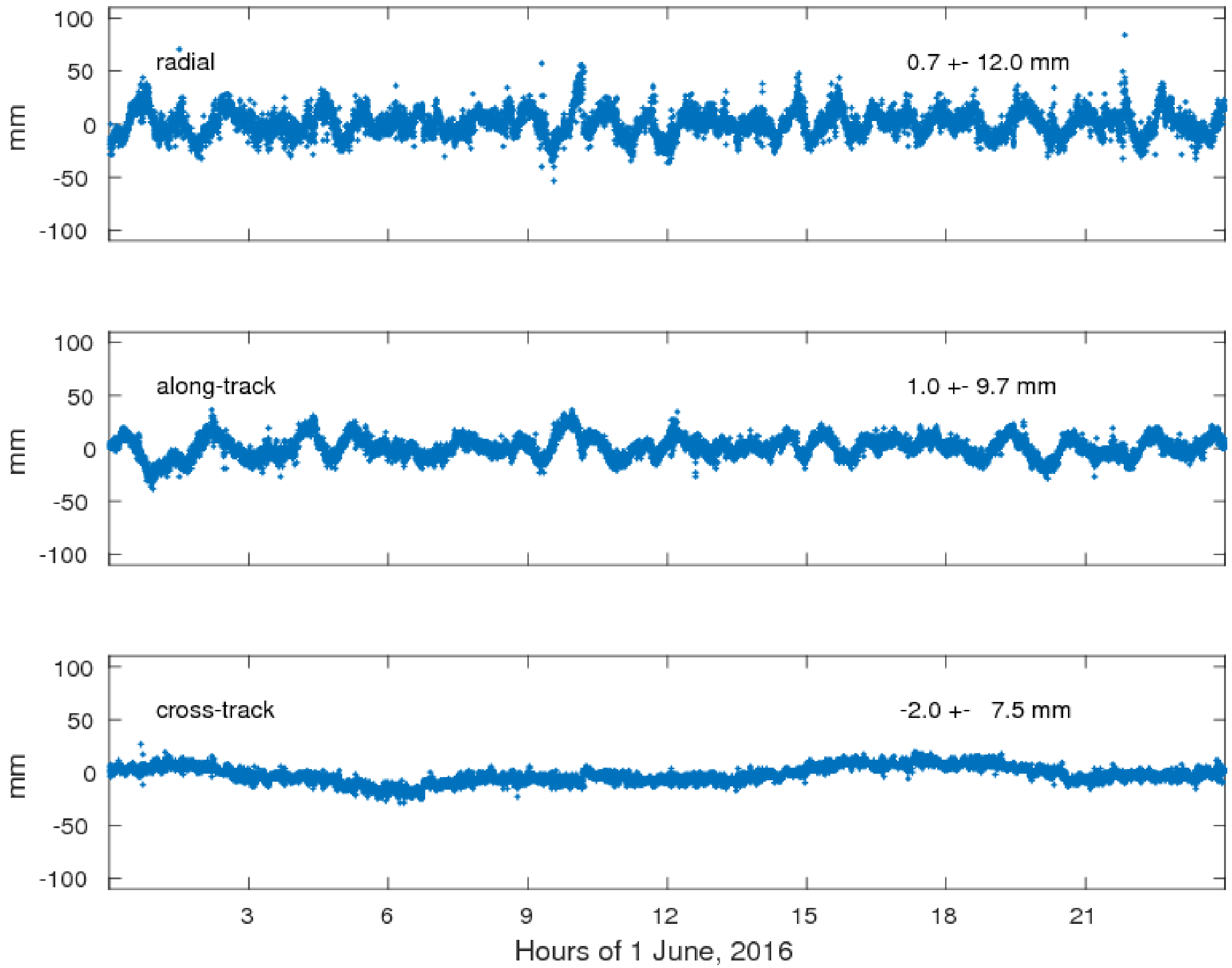
```
*  2016  6  1  0  0  0.00000000
PL49  -1965.328762  -2960.079621   5815.366063 999999.999999
VL49 -32476.530949 -56518.428574 -39633.949261 999999.999999
*  2016  6  1  0  0 10.00000000
PL49  -1997.722965  -3016.388318   5775.367094 999999.999999
VL49 -32311.097194 -56097.834133 -40363.154274 999999.999999
*  2016  6  1  0  0 20.00000000
PL49  -2029.949403  -3072.273033   5734.641439 999999.999999
VL49 -32141.000143 -55670.464832 -41087.301898 999999.999999
*  2016  6  1  0  0 30.00000000
PL49  -2062.003415  -3127.727011   5693.194205 999999.999999
VL49 -31966.250891 -55236.380456 -41806.300697 999999.999999
*  2016  6  1  0  0 40.00000000
PL49  -2093.880357  -3182.743574   5651.030585 999999.999999
VL49 -31786.861194 -54795.641569 -42520.059993 999999.999999
*  2016  6  1  0  0 50.00000000
PL49  -2125.575594  -3237.316095   5608.155863 999999.999999
VL49 -31602.843520 -54348.309592 -43228.489711 999999.999999
*  2016  6  1  0  1  0.00000000
PL49  -2157.084506  -3291.438018   5564.575411 999999.999999
VL49 -31414.211010 -53894.446726 -43931.500489 999999.999999
```

Clock corrections
are not provided

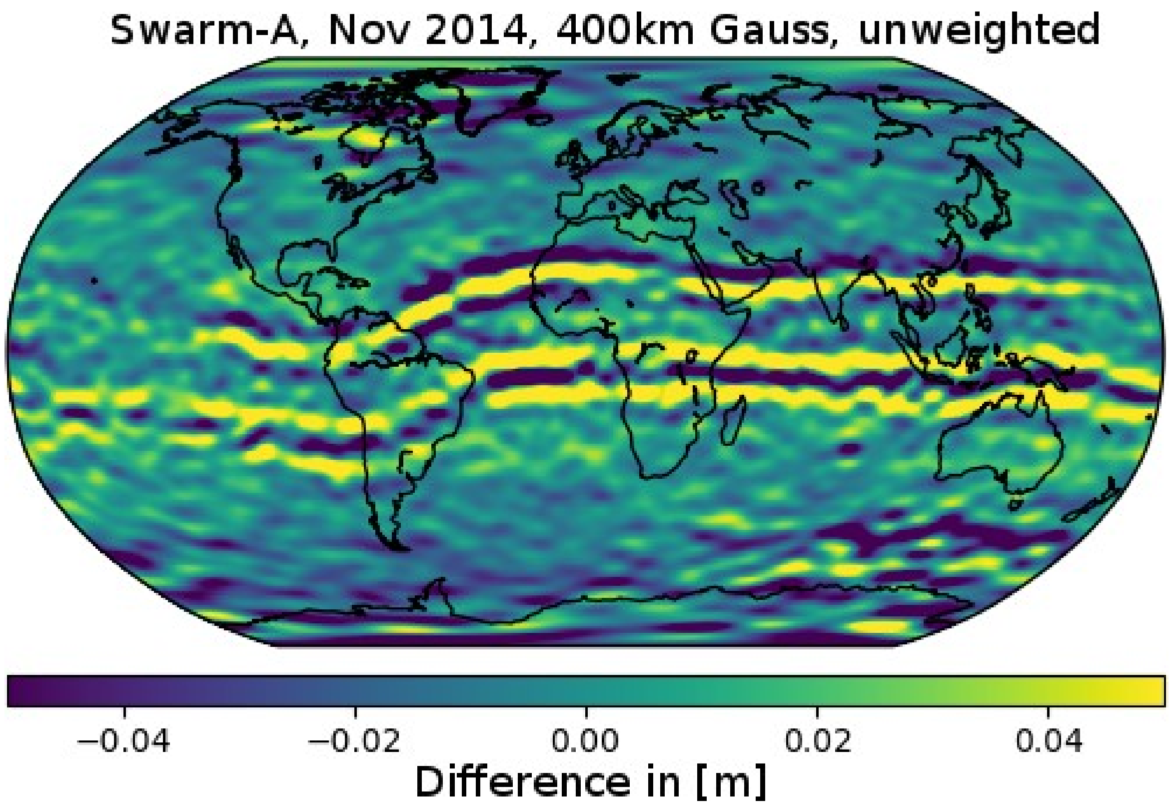
Excerpt of reduced-dynamic Swarm-C positions at begin of 1 June, 2016

Orbit Differences KIN-RD (Swarm-C)

Differences at epochs of kin. positions

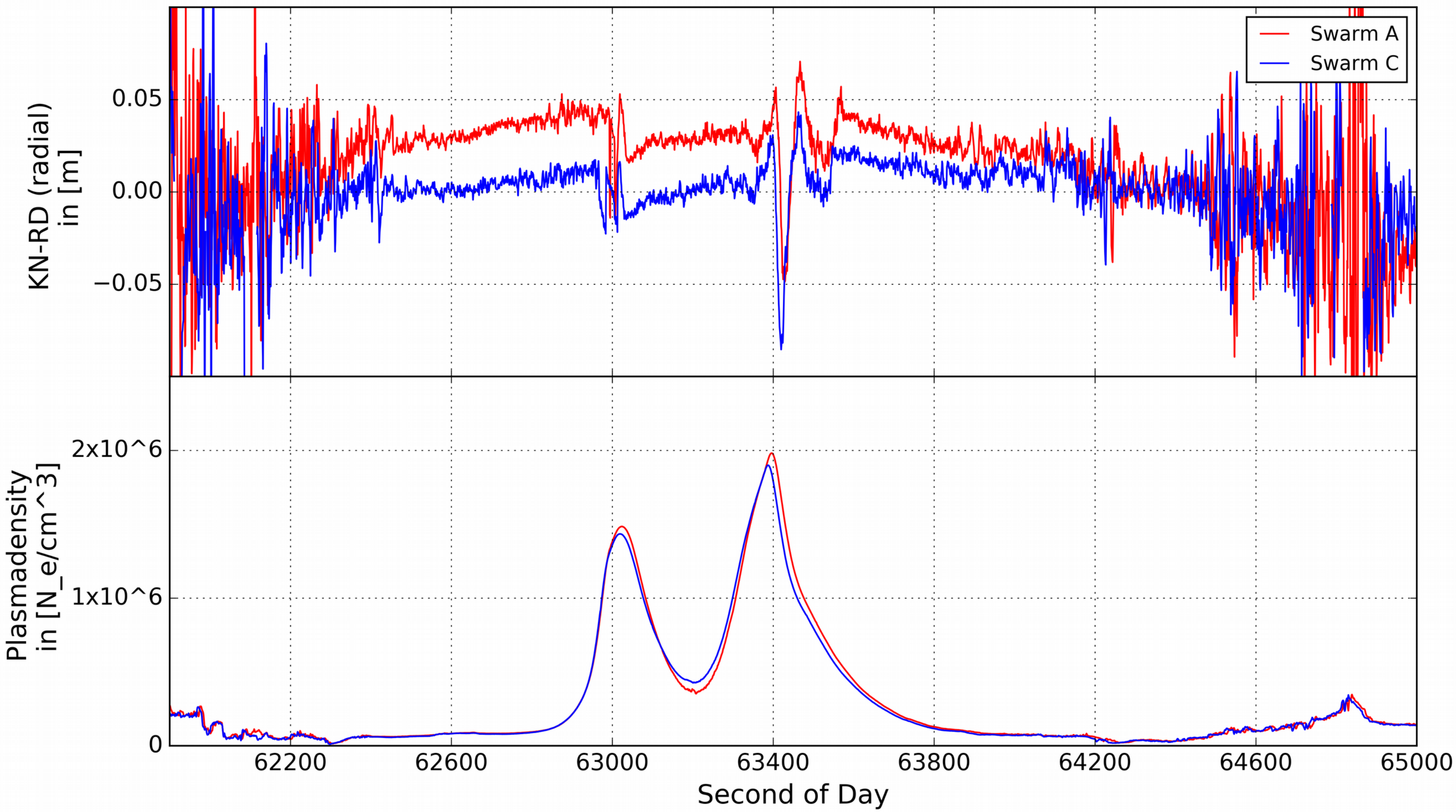


Swarm A Gravity Field

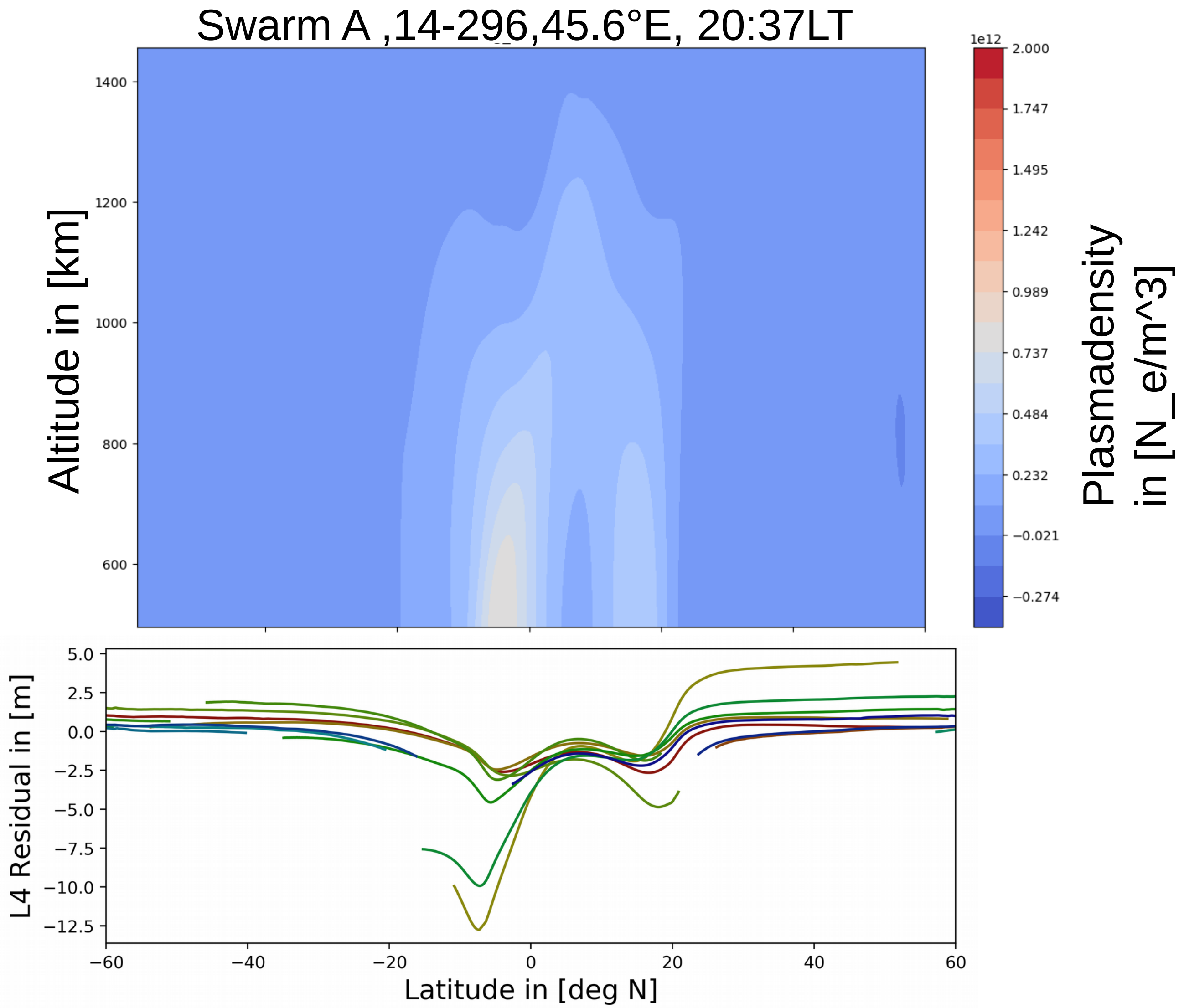


Geoid hight differences, static GRACE gravity field AIUB-GRACE03S - Swarm A gravity field, November 2014

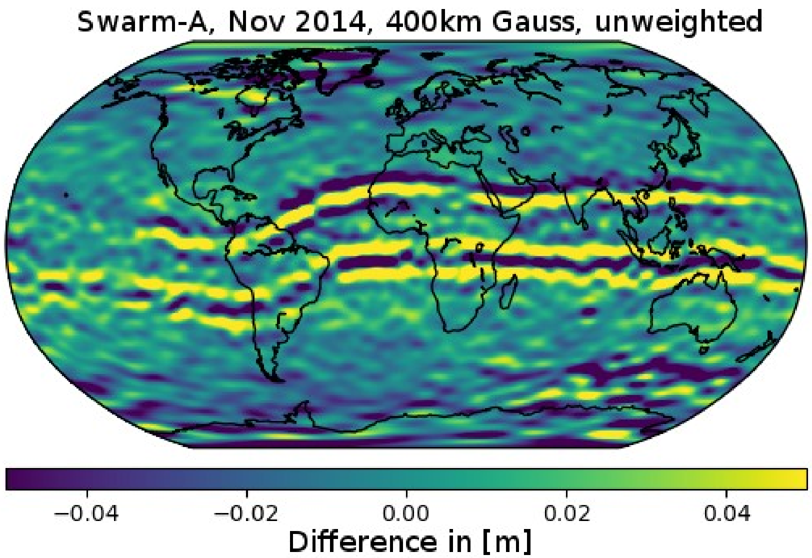
Orbit Differences KIN-RD and Plasmadensity

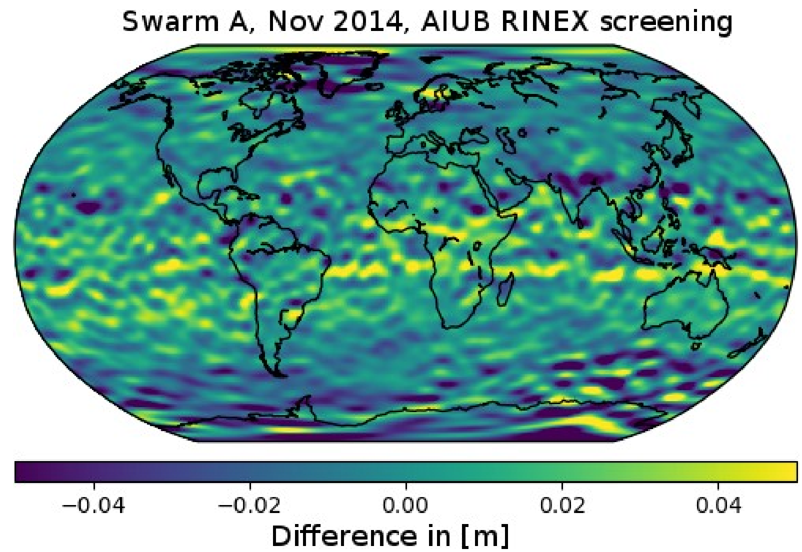


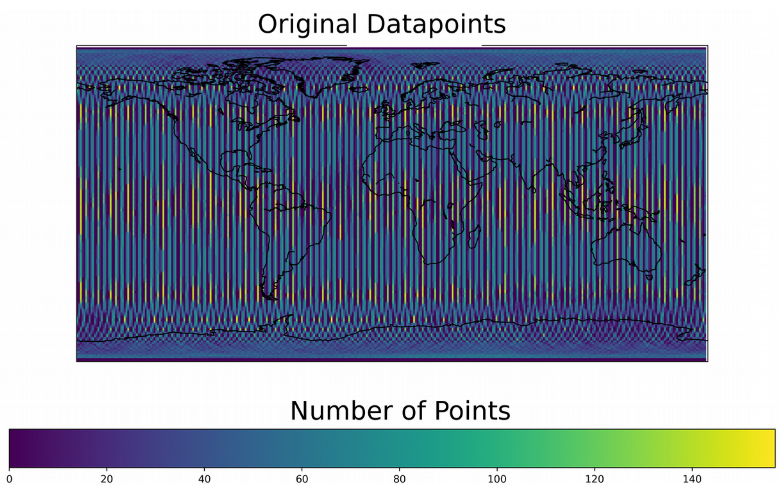
Orbit Differences KIN-RD and Plasmadensity

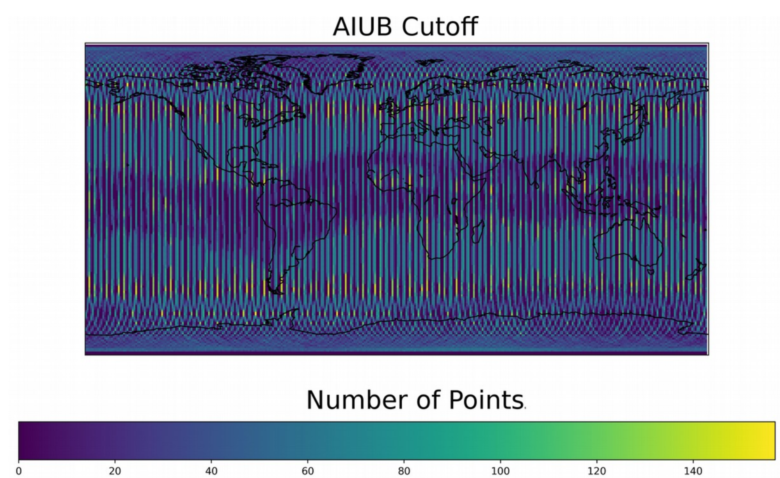


Previous Studies: AIUB-Cutoff





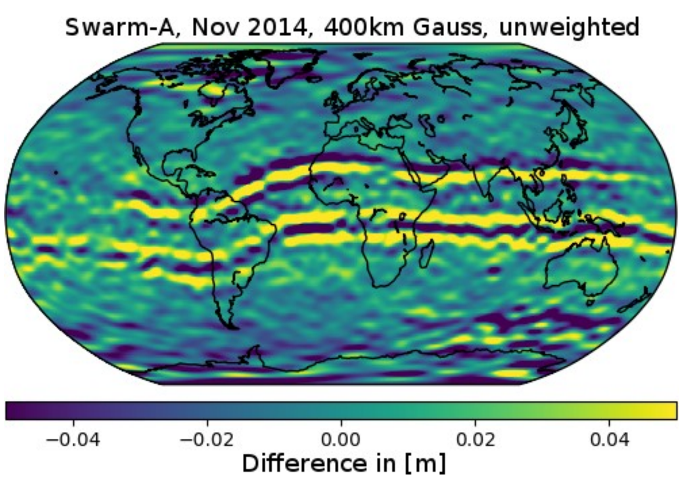


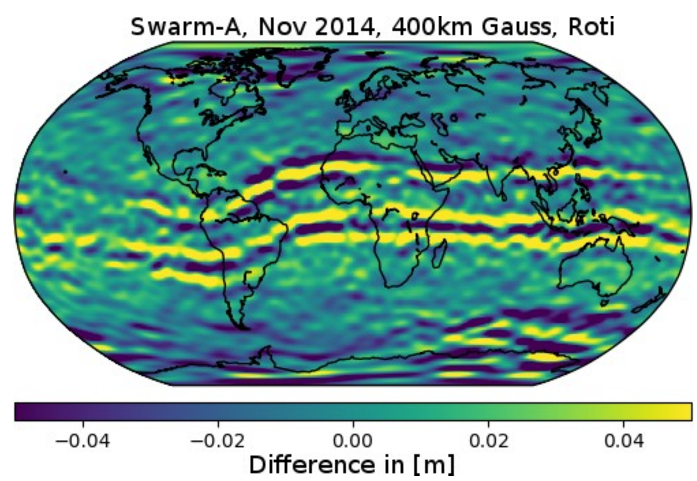


Previous Studies: Graz-ROTI

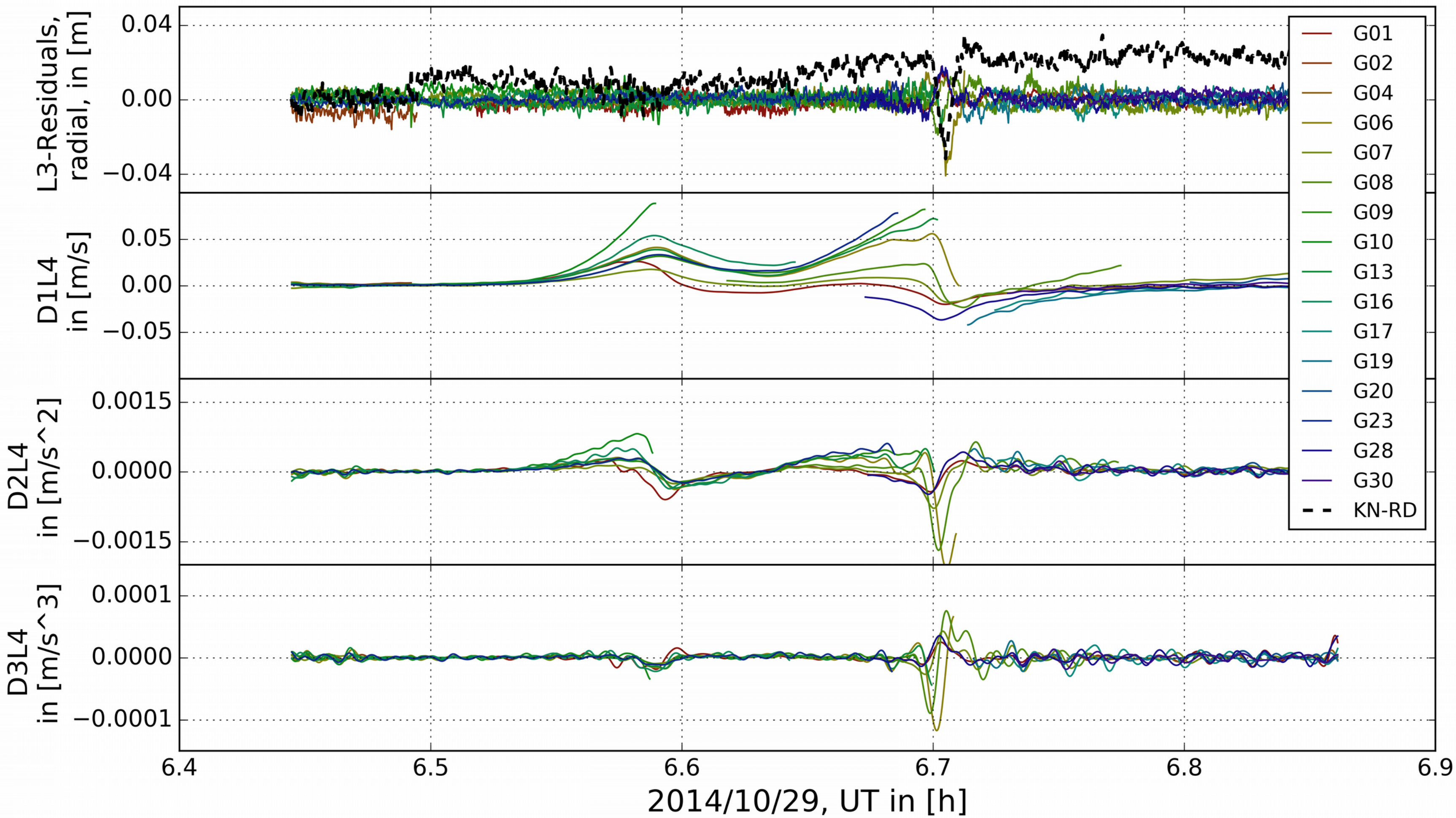
$$ROTI = \sqrt{\frac{\langle \Delta TEC^2 \rangle - \langle \Delta TEC \rangle^2}{dt^2}}$$

- Applied to ΔTEC, moving window, 31s
- $\sigma = 10 \cdot ROTI \cdot \sigma_0$ if $ROTI > 0.1$
- Developed for GOCE

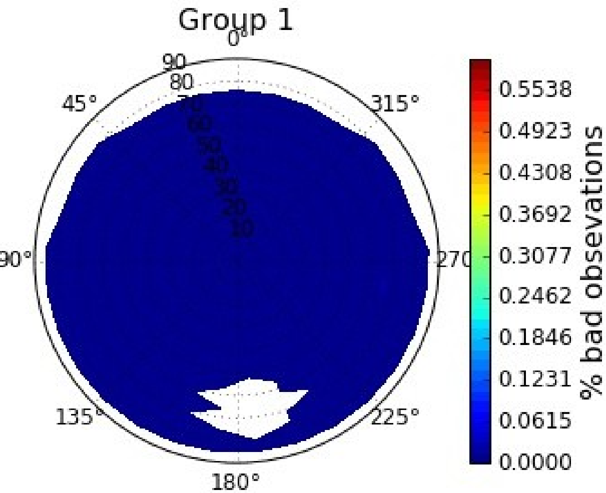


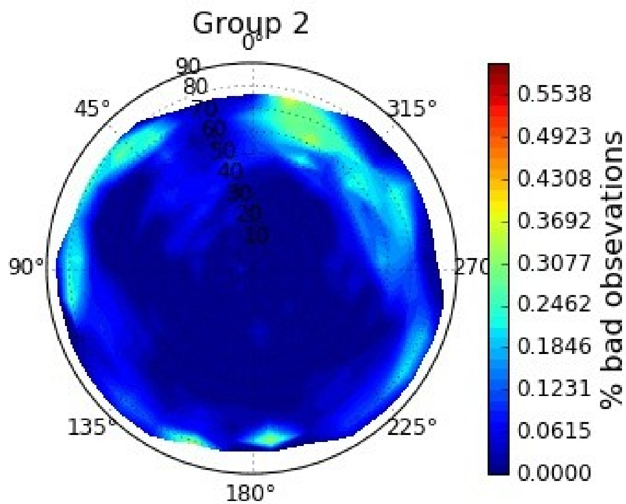


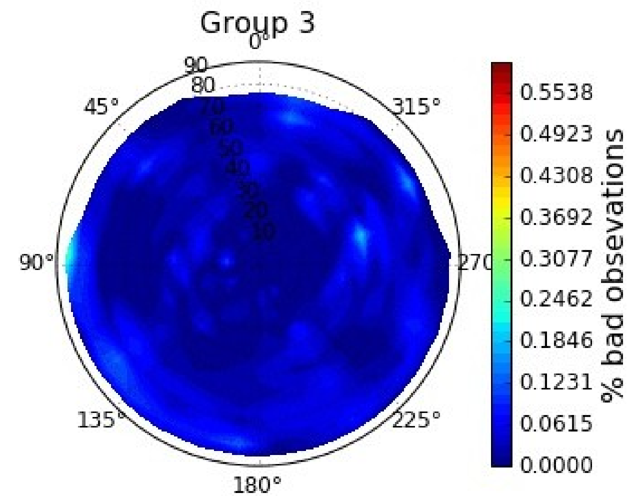
Phase Residuals and Time derivatives *L4*

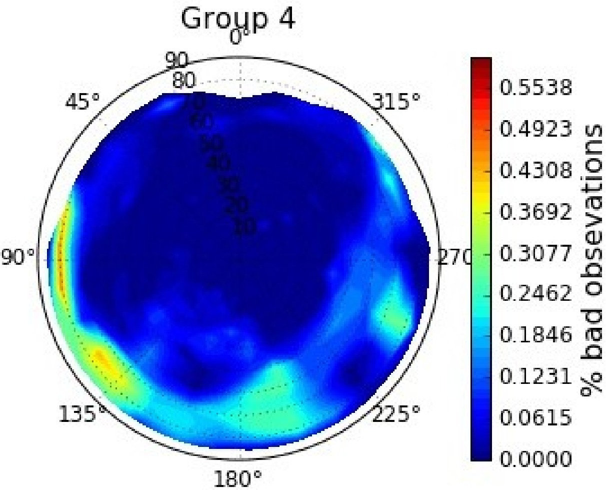


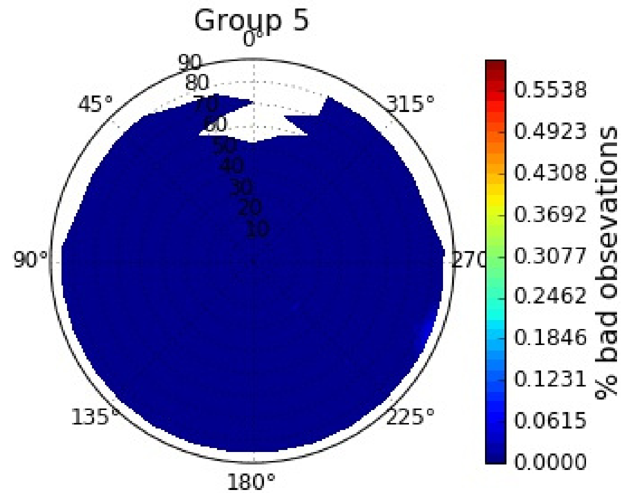
Elevations and azimut of affected GPS satellites





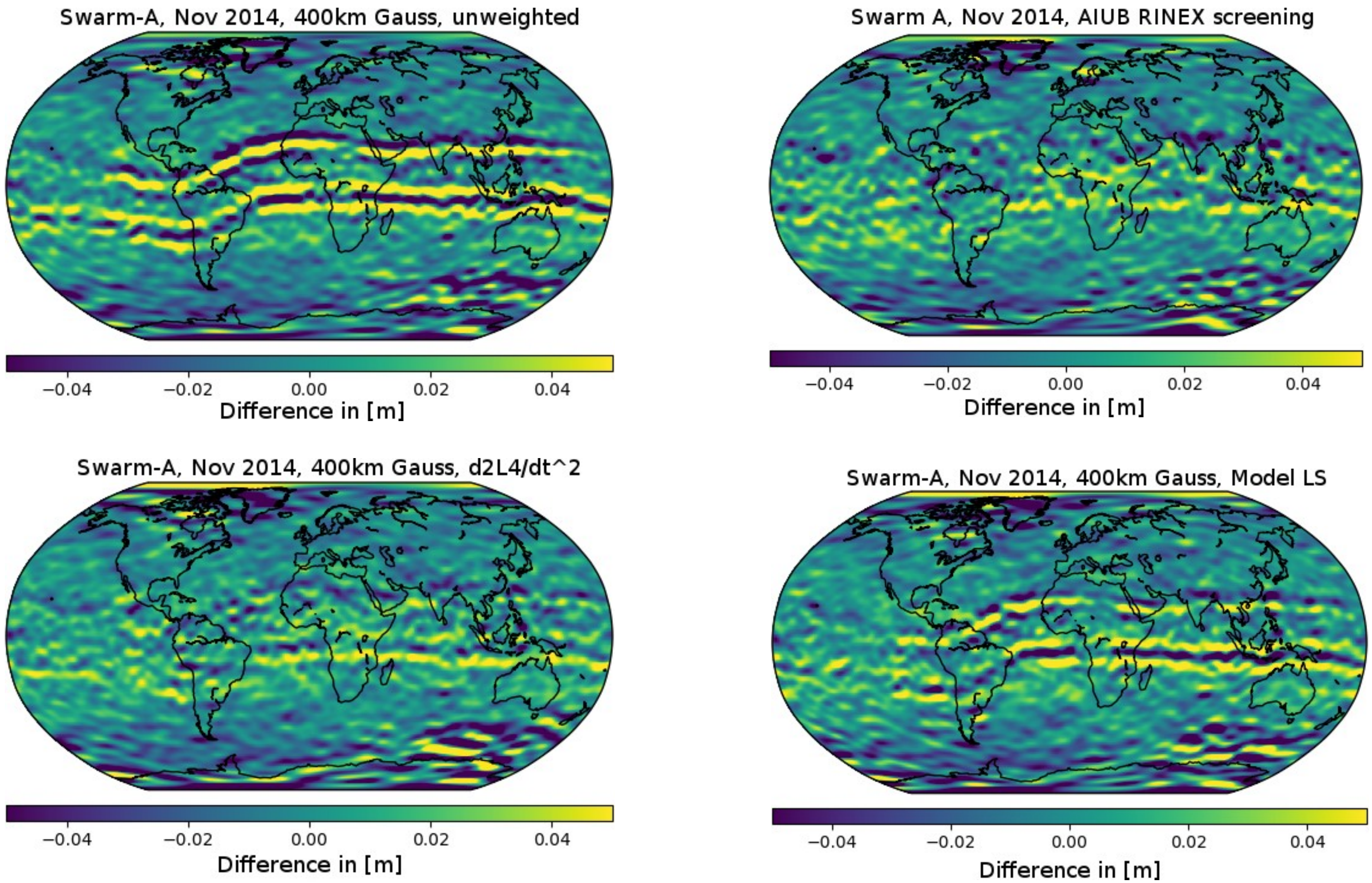




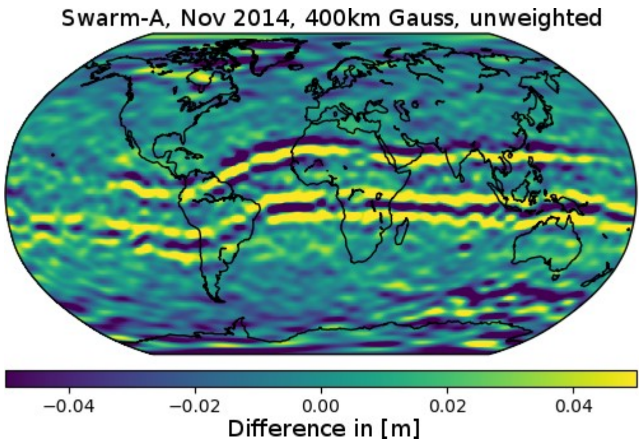


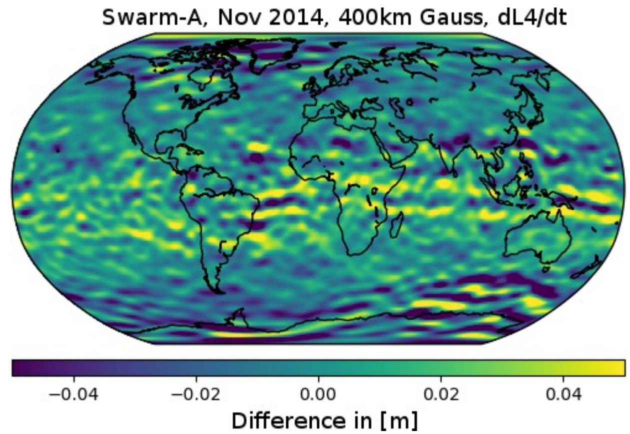
Group	MLAT(UP)
1	-35 → -25
2	-25 → -10
3	-10 → 10
4	10 → 25
5	25 → 35

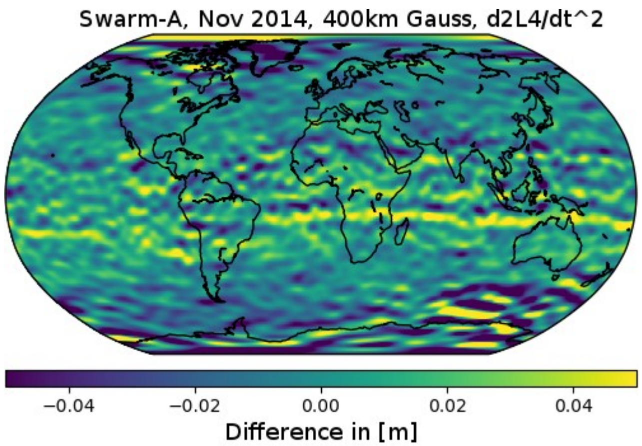
Elevations and azimut of affected GPS satellites

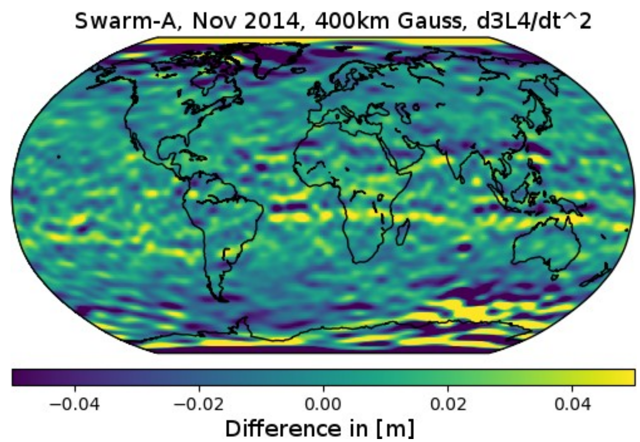


Elevations and azimut of affected GPS satellites









Evaluation of mehtods

SLR-validation of reduced dynamic orbits:

Method	Mean[*mm*]	Std[*mm*]	RMS[*mm*]
Original	2.6	16.6	16.8
ROTI	2.7	17.3	17.5
AIUB Cutoff	7.4	32.1	32.9
Model	3.4	14.0	14.4
D1L4	0.3	22.1	22.1
D2L4	2.8	16.0	16.1
D3L4	4.1	13.4	14.4

Thank you for your attention