PROCEEDINGS OF SCIENCE

Charmless Non-Leptonic B decays

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We review the theoretical and phenomenological status of two- and three-body charmless non-leptonic B decays, with an emphasis on factorization approaches.

Flavor Physics and CP Violation,
6-9 June 2016
Caltech, Pasadena CA, USA

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PoS(FPCP2016)007

1. Introduction and Motivation

1.1 Definitions

Non-leptonic B decays are exclusive decays of the form $B \to h_1 \cdots h_n$ with h_i any heavy or light hadrons. Charmless non-leptonic decays are non-leptonic decays with no charmed hadrons in the final state (and excluding $c\bar{c}$ states). We will mostly focus on charmless decays to two ($B \to M_1 M_2$) and three ($B \to M_1 M_2 M_3$) final mesons, but the generalities are common to all non-leptonic modes.

1.2 Non-leptonic B decays within the global arena of particle physics

There are a number of open issues in our understanding of the physics of the elementary particles and their interactions which our current theory –the Standard Model (SM)– does not seem to be able to answer. These questions are related to gauge symmetry, electroweak symmetry breaking, flavor and CP (including baryogenesis), astrophysics and cosmology (dark matter, dark energy and inflation), and gravity. Many extensions of the SM addressing some of these issues have been put forward, and many are perfectly plausible given our current theoretical knowledge and experimental record. While it is possible that purely theoretical work may narrow down in the future the number of viable models, it is clear that the fast track is to obtain hints from experiment. A more complete theory that addresses all or some of these issues, while at the same time sharing the many outstanding successes of the SM, will very likely as well modify the predictions for current and future laboratory experiments which study collisions and decays of known particles. Establishing such deviations with respect to SM expectations will not only provide *direct* evidence for the need of a non-trivial extension of the SM, but also specific hints of what this extension should look like. This is arguably the most important task in particle physics today.

Testing the SM requires first to measure its free parameters precisely and to understand how to make precision calculations. Most of the free parameters of the SM are related to flavor, such as the entries of the CKM matrix –which govern the physics of flavor in the quark sector (flavor transitions of hadrons). Non-leptonic B decays are an essential input in CKM fits, and necessary for the direct measurement of the CKM angles α, β and γ (see *e.g.* [1]), thus providing, in addition, valuable tests of the SM mechanism of CP violation. They also provide direct access to the study of $B_q - \bar{B}_q$ mixing ($\Delta B = 2$ transitions) through the interference of CP-conjugated decays into final CP eigenstates. From the huge number of different non-leptonic B decays accessible experimentally, some are mediated at tree level in the SM, while some arise only at the loop level; some are dominated by a single SM amplitude, while some are the result of interference of two amplitudes of similar size with different weak and strong phases. This results in very broad phenomenological applications from SM studies to New Physics (NP) searches, and including hadronic physics.

1.3 Non-leptonic B decays in the context of strong interactions

Any process involving hadrons is probing the physics of QCD bound states in some way. Therefore one is forced to either make full computations in a non-perturbative regime, or to isolate the contributions sensitive to infrared (IR) physics, parametrize them by a few "universal" quantities, and subsequently (a) calculate them, (b) extract them from experiment, or (c) build observables where these cancel out. So far, we can only calculate non-perturbatively a few simple objects such as decay constants (matrix elements like $\langle 0|\bar{d}_L\gamma^\mu b_L|\bar{B}\rangle$) or form factors (matrix elements such as $\langle\pi|\bar{d}_L\gamma^\mu b_L|\bar{B}\rangle$). These calculations are based on numerical simulations (in the framework of Lattice QCD, see *e.g.* [2]), or on operator product expansions and dispersion relations (within the framework of QCD sum rules, see *e.g.* [3]). Decay constants and form factors are enough for predictions of leptonic (*e.g.* $B^-\to\ell^-\bar{\nu}_\ell$ or $B_s\to\ell^+\ell^-$) and semileptonic (*e.g.* $B^-\to\pi^0\ell^-\bar{\nu}_\ell$) decays –at least to leading order in QED–, but not for non-leptonic decays such as $B\to\pi\pi$.

Isolating IR effects is a particular case of *scale separation* in quantum field theory, which –if the scales are widely separated– is achieved most conveniently in the framework of effective field theory (EFT). In the case of weak meson decays, a first step is to separate the scale of weak interactions ($\sim M_W$) from the scale of hadronic physics (m_b or lower). These leads to the *Weak Effective Theory* (see *e.g.* [4]) where weak transitions are mediated by dimension-six operators:

$$\mathscr{L}_W = \mathscr{L}_{QCD+QED}^{\text{no top}} - \frac{4G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(D)}\left[C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3\cdots 6,8} C_i Q_i\right] + \cdots \tag{1.1}$$

Here the CKM prefactors $\lambda_p^{(D)} \equiv V_{pb}V_{pD}^*$ ensure the Wilson coefficients C_i are independent of CKM elements in the SM once CKM unitarity is used, and we have only written down explicitly the dimension-six operators most relevant for charmless non-leptonic $b\to D\bar{q}q$ transitions, with $D=\{d,s\}$ and $q=\{u,d,s\}$. These include current-current operators $Q_{1,2}^p\sim(\bar{D}p)(\bar{p}b)$, QCD penguin operators $Q_{3\cdots 6}\sim\sum_q(\bar{D}b)(\bar{q}q)$ and the chormomagnetic operator $Q_8\sim\bar{D}\sigma^{\mu\nu}G_{\mu\nu}b$ (see *e.g.* [5]). Additional "evanescent" operators are needed for renormalization in dimensional regularization at higher orders in QCD (see *e.g.* [6]). Non-leptonic B-decay amplitudes are then given by (this notation is *not* standard):

$$A(\bar{B}\to f) = \lambda_u^{(D)}(T_f^u + P_f) + \lambda_c^{(D)}(T_f^c + P_f) \tag{1.2}$$

with

$$T_f^p = -\frac{4G_F}{\sqrt{2}}\sum_{i=1,2} C_i(\mu)\langle f|Q_i^p(\mu)|\bar{B}\rangle\ , \quad P_f = -\frac{4G_F}{\sqrt{2}}\sum_{i=3\cdots 6,8} C_i(\mu)\langle f|Q_i(\mu)|\bar{B}\rangle\ . \tag{1.3}$$

Note that in the case of charmless decays T_f^c is purely the result of a penguin contraction (and thus "T" does not necessarily mean "Tree").

For $\mu\sim m_b$, the matrix elements of the operators do not depend on any scale larger than m_b (all the dependence on the weak and, possibly, NP scales is contained in $C_i(\mu)$). At the same time the Wilson coefficients do not depend on any IR scale and are thus perturbatively calculable. The values of the Wilson coefficients $C_i(\mu)$ in the SM at the renormalization scale $\mu\sim m_b$ can be calculated via the usual matching-and-running procedure, and are known to next-to-next-to-leading logarithmic (NNLL) accuracy [7, 8, 6, 9, 10].

The challenge is to calculate the matrix elements $\langle M_1M_2\cdots|Q_i(\mu)|\bar{B}\rangle$ in QCD. This is a very complicated task, not completely understood so far, and which constitutes yet another strong motivation for the study of non-leptonic B decays: they teach us about QCD. One example: the soft-collinear effective theory (with a wide range of applications, from heavy-meson decays to collider physics and gravity, see *e.g.* [11]), was first developed to describe B-meson decays [12, 13, 14, 15].

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1.4 Soft-collinear factorization

The matrix elements $\langle M_1 M_2|Q_i(\mu)|\bar{B}\rangle$ at $\mu \simeq m_b$ depend on three different momentum scales: (1) a "hard" scale $p_h^2 \sim m_b^2$ associated to the energy of the process and the choice of renormalization scale; (2) a "soft" scale $p_s^2 \sim \Lambda_{\rm QCD}^2$ associated with the dynamics of light degrees of freedom within the B and light mesons; and (3) a "hard-collinear" scale $p_{hc}^2 \sim m_b\Lambda_{\rm QCD}$ associated with a momentum transfer that would give a soft light parton in the B-meson a large energy ($\sim m_b$) and a low virtuality ($\sim \Lambda_{\rm QCD}$), so as to become part of one of the final mesons. Such a large-energy-low-virtuality momentum is called a "collinear" momentum p_c (note that $p_c^2 \sim \Lambda_{\rm QCD}$). In the two-body final state there are two different collinear momenta: p_c and $p_{\bar{c}}$ in opposite directions, corresponding to partons in M_1 and M_2. In the heavy-quark limit these three scales are widely separated: $p_h^2 \gg p_{hc}^2 \gg p_{s,c,\bar{c}}^2$, calling for a scale separation within EFT. Scale separation leads often to useful factorization "theorems"; in this case integrating out hard scales at the leading power leads to "soft-collinear factorization" (see *e.g.* [16]) with decoupling of anti-collinear modes.

The first step is to integrate out from QCD the scale p_h^2. This leads to an EFT called SCET-1. The matching condition for a QCD operator Q_i in terms of SCET-1 operators O^I, O^{II} is given by:

$$Q_i = \int dt\, \widetilde{T}^I(t) O_i^I(t) + \int dt\, ds\, \widetilde{T}^{II}(t,s) O_i^{II}(t,s) \tag{1.4}$$

$$O_i^I(t) = \big[(\bar{\chi}W_{\bar{c}})(tn_-)\Gamma_i^1(W_{\bar{c}}^\dagger\chi)(0)\big]\big[(\bar{\xi}W_c)(0)\Gamma_i^2 h_v(0)\big] \tag{1.5}$$

$$O_i^{II}(t,s) = \big[(\bar{\chi}W_{\bar{c}})(tn_-)\Gamma_i^3(W_{\bar{c}}^\dagger\chi)(0)\big]\big[(\bar{\xi}W_c)(0)\Gamma_i^4(W_c^\dagger i\not{D}_{\perp c}W_c)(sn_+)\Gamma_i^5 h_v(0)\big] \tag{1.6}$$

where χ, ξ and h_v are collinear, anti-collinear and heavy quark fields, $W_{c,\bar{c}}$ are collinear and anti-collinear Wilson lines, $n_\pm$ are unit light-cone vectors in the collinear and anti-collinear directions, and Γ_i^j are Lorentz structures. The functions $\widetilde{T}^{I,II}$ are perturbative Wilson coefficients that depend only on the hard scale. In SCET-1 there are no leading power interactions between anti-collinear and soft or collinear modes, and the anti-collinear sector decouples. Thus the matrix elements of Q_i are proportional to a light-cone distribution amplitude (LCDA) of a light meson:

$$\langle M_2|(\bar{\chi}W_{\bar{c}})(tn_-)\Gamma_i(W_{\bar{c}}^\dagger\chi)(0)|0\rangle \sim \phi_{M_2}(t)\ , \tag{1.7}$$

where it is assumed that M_2 has anti-collinear momentum. The matrix elements of heavy-collinear currents between B and M_1 still depend on the hard-collinear scale. Hard-collinear modes are integrated out at a second step, leading to an EFT called SCET-2, containing only soft and (anti)collinear modes. Hard-collinear factorization works for O^{II}, leading to:

$$\langle M_1|(\bar{\xi}W_c)(0)\Gamma_i(W_c^\dagger i\not{D}_{\perp c}W_c)(sn_+)\Gamma_i' h_v(0)|\bar{B}\rangle \sim \int d\omega\, du\, J_i(s,w,u)\phi_B(\omega)\phi_{M_1}(u) \tag{1.8}$$

where $J_i(s,\omega,u)$ is a hard-collinear matching coefficient, which is perturbative provided the hard-collinear scale $\sqrt{m_b\Lambda_{\rm QCD}}$ is perturbative. Hard-collinear factorization fails for O^I, so that the form factor $\langle M_1|(\bar{\xi}W_c)(0)\Gamma h_v(0)|\bar{B}\rangle$, which depends on soft and hard-collinear momenta, cannot be factorized. This is a long-standing problem [17, 18, 19]. In practice, this is part of the full QCD $B\to M_1$ form factor F^{BM_1}, which appears in factorization formulas.

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2. Two-body decays

2.1 Factorization formula for two-body decays at the leading power

The arguments laid down in Section 1.4 lead to a factorization formula for charmless two-body B decays at the leading power in $\Lambda_{\rm QCD}/m_b$, first put forward in [20, 21]. It should be remarked that after 20 years of intense research these papers are not outdated in any way and remain state-of-the-art: much has been understood conceptually since then but the formulation has not changed a bit. In essence, the matrix element of an operator Q_i is given by:

$$\langle M_1 M_2|Q_i|\bar{B}\rangle = F^{BM_1}\int du\, T_i^I(u)\phi_{M_2}(u) + \int d\omega\, du\, dv\, T_i^{II}(\omega,u,v)\phi_B(\omega)\phi_{M_1}(u)\phi_{M_2}(v), \qquad (2.1)$$

where F^{BM} is a form factor in QCD, ϕ_M are LCDAs of light and heavy mesons, and the (perturbative) "hard-scattering kernels" $T_i^{I,II}$ are related to the SCET matching coefficients $\widetilde{T}^{I,II}$ and J_i in Section 1.4. The notation is such that M_1 picks the B-meson spectator quark; if M_2 can also pick the spectator, there is an additional corresponding term proportional to F^{BM_2}. $T^I(u) = 1+\mathcal{O}(\alpha_s)$ arises from vertex corrections already at the leading order, while $T^{II}(\omega,u,v) = \mathcal{O}(\alpha_s)$ starts at next-to-leading order and involves spectator scattering, and it is power suppressed if M_1 is heavy (not in charmless decays).

The factoriztion formula (2.1) is valid only up to $\mathcal{O}(\Lambda/m_b)$ corrections but (presumably) to all orders in α_s. Formally, this has been proven explicitly up to NNLO. Assuming that the SCET contains all the relevant IR degrees of freedom (which is the standard assumption), leads to an all-order proof. But this wouldn't be the first time IR modes are missed.

2.2 Status of perturbative calculations

The original papers on QCDF (*e.g.* [22]) contain already all next-to-leading order (NLO) corrections (*i.e.* $\mathcal{O}(\alpha_s)$) to the hard-scatering kernels T^I, T^{II} for both tree and penguin topologies. The calculation of next-to-next-to-leading order corrections (*i.e.* $\mathcal{O}(\alpha_s^2)$) is a much more demanding task, which has been almost completed during the last decade. These include: two-loop vertex corrections [23, 24, 25] and two-loop penguin and one-loop chromomagnetic operator contributions [26, 27] to T^I, as well as one-loop vertex corrections [28, 29, 30] and one-loop penguin contributions [31, 32] to T^{II}. This is summarized in Table 1. NNLO penguin contributions to T^I from $Q_{1,2}^c$ are particularly difficult as they require the evaluation of a large number of two-loop Feynmann integrals with three scales (m_b,m_c, um_b), with a non-trivial threshold at $\bar{u}m_b^2 \sim 4m_c^2$ [33]. Missing NNLO pieces include two-loop vertex and penguin corrections from the penguin operators $Q_{3\cdots 6}$, which are nevertheless numerically subleading for tree decays.

At leading power, strong phases appear first at NLO. Therefore the first correction to CP asymmetries comes from NNLO corrections. This is the main motivation for the NNLO calculation. CP asymmetries will be discussed below.

2.3 Tree decays

Tree decays are those receiving vertex contributions from current-current operators. We take

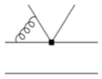
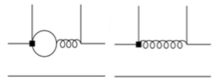
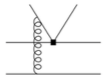
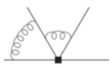
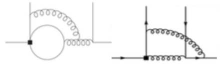
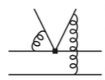
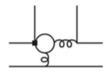
	T^I, tree	T^I, penguin	T^{II}, tree	T^{II}, penguin
LO: $\mathcal{O}(1)$				
NLO: $\mathcal{O}(\alpha_s)$ BBNS '99-'04				
NNLO: $\mathcal{O}(\alpha_s^2)$	 Bell '07,'09 Beneke, Huber, Li '09	 Kim, Yoon '11, Bell Beneke, Huber, Li '15	 Beneke, Jager '05 Kivel '06, Pilipp '07	 Beneke, Jager '06 Jain, Rothstein, Stewart '07

Table 1: Summary of the status of perturbative calculations of charmless two-body B decays in QCDF.

as an example $B \to \pi\pi$. In this case the amplitudes are given by:

$$\sqrt{2}A(B^- \to \pi^-\pi^0) \;=\; \lambda_u^{(d)}\,\big[a_1(\pi\pi) + a_2(\pi\pi)\big]\,A_{\pi\pi} \tag{2.2}$$

$$-A(\bar{B}^0 \to \pi^0\pi^0) \;=\; \lambda_u^{(d)}\,\big[a_2(\pi\pi) - \hat{\alpha}_4^u(\pi\pi)\big]\,A_{\pi\pi} - \lambda_c^{(d)}\,\hat{\alpha}_4^c(\pi\pi)\,A_{\pi\pi} \tag{2.3}$$

and $A(\bar{B}^0 \to \pi^+\pi^-) = A(\bar{B}^0 \to \pi^0\pi^0) + \sqrt{2}A(B^- \to \pi^-\pi^0)$. Here we have ignored contributions from electroweak penguins and annihilation topologies (although $\hat{\alpha}_4^p = \alpha_4^p + \beta_3^p$ contains an annihilation contribution β_3^p). a_1 and a_2 are color-allowed and color-suppressed tree amplitudes. α_4^p contains penguin contractions of current-current operators, and will be considered later. Since $a_{1,2} \gg \alpha_4$, and $\lambda_u^{(d)} \sim \lambda_c^{(d)}$, tree decays are dominated by the tree amplitudes $a_{1,2}$. At NNLO [25]:

$$\begin{aligned} a_1(\pi\pi) &= 1.009 + [0.023 + 0.010\,i]_{\text{NLO}} + [0.026 + 0.028\,i]_{\text{NNLO}} \\ &\quad - \left[\frac{r_{\text{sp}}}{0.485}\right]\Big\{0.015 + [0.037 + 0.029\,i]_{\text{NLOsp}} + [0.009]_{\text{tw3}}\Big\} = 1.00 + 0.01\,i\,, \end{aligned} \tag{2.4}$$

$$\begin{aligned} a_2(\pi\pi) &= 0.220 - [0.179 + 0.077\,i]_{\text{NLO}} - [0.031 + 0.050\,i]_{\text{NNLO}} \\ &\quad + \left[\frac{r_{\text{sp}}}{0.485}\right]\Big\{0.123 + [0.053 + 0.054\,i]_{\text{NLOsp}} + [0.072]_{\text{tw3}}\Big\} = 0.26 - 0.07\,i\,, \end{aligned} \tag{2.5}$$

where $r_{\text{sp}} = 9 f_\pi \hat{f}_B/(m_b f_+^{B\pi}(0)\lambda_B)$ is a normalization related to the hard-spectator contributions (*i.e.* T^{II}), most notably proportional to the inverse moment λ_B^{-1} of the B-meson LCDA. The perturbative expansion is seen to be well behaved, taking into account that the NLO contribution to a_2 lifts color suppression, while the opposite is true for a_1. These two amplitudes must be scale-independent, and indeed the μ-dependence estabilizes at NNLO for the real parts (no so much for the imaginary parts, as the LO contribution is real). Radiative corrections are relatively large, but significant cancellations occur between the form factor and spectator terms. The color suppressed amplitude is dominated by the spectator scattering contribution, and therefore the amplitude $\bar{B} \to \pi^0\pi^0$ has a strong dependence on λ_B (one finds $a_2 \sim 0.26 \to 0.51$ when $\lambda_B \to \lambda_B/2$).

For $\lambda_B(1\text{ GeV}) = 0.35 \pm 0.15$, all branching fractions for tree decays ($B \to \pi\pi$, $B \to \pi\rho$, $B \to \rho\rho$) agree well with experimental measurements within uncertainties, except for very slight tensions in $\bar{B} \to \pi^+\pi^-$, $\bar{B} \to \pi^-\rho^+$ and $\bar{B} \to \pi^0\rho^0$, and a persistent significant tension in $\bar{B} \to \pi^0\pi^0$: $10^6 BR(\bar{B} \to \pi^0\pi^0)_{\rm th} = 0.33^{+0.11+0.42}_{-0.08-0.17}$ vs. $10^6 BR(\bar{B} \to \pi^0\pi^0)_{\rm exp} = 1.55 \pm 0.19$. It turns out that a lower value for $\lambda_B(1\text{ GeV}) \sim 0.20$ improves the agreement of all these modes, and enhances significantly $10^6 BR(\bar{B} \to \pi^0\pi^0) = 0.63^{+0.12+0.64}_{-0.10-0.42}$, bringing it closer to the experimental average, but still far away. Notably, a new Belle analysis [34] reports $10^6 BR(\bar{B} \to \pi^0\pi^0)_{\rm Belle} = 0.90 \pm 0.16$, and would agree within uncertainties with the theory prediction, assuming such a low value for λ_B.

2.4 Penguin decays

Penguin decays are those for which vertex contributions from current-current operators are either absent, or CKM suppressed with respect to penguin contractions (of $\bar{c}b\bar{s}c$ operators) . Taking as an example $\bar{B} \to K\pi$, we have (ignoring electroweak penguins and annihilation topologies):

$$A(B^- \to \pi^-\bar{K}^0) = \lambda_u^{(s)}\ \hat{\alpha}_4^u(\pi\bar{K})\, A_{\pi\bar{K}} \qquad\qquad + \lambda_c^{(s)}\ \hat{\alpha}_4^c(\pi\bar{K})\, A_{\pi\bar{K}} \quad (2.6)$$

$$\sqrt{2}A(B^- \to \pi^0K^-) = \lambda_u^{(s)}\ \Big\{\big[a_1(\pi\bar{K}) + \hat{\alpha}_4^u(\pi\bar{K})\big]\, A_{\pi\bar{K}} + a_2(\bar{K}\pi)\, A_{\bar{K}\pi}\Big\} + \lambda_c^{(s)}\ \hat{\alpha}_4^c(\pi\bar{K})\, A_{\pi\bar{K}} \quad (2.7)$$

$$A(\bar{B}^0 \to \pi^+K^-) = \lambda_u^{(s)}\ \big[a_1(\pi\bar{K}) + \hat{\alpha}_4^u(\pi\bar{K})\big]\, A_{\pi\bar{K}} \qquad\qquad + \lambda_c^{(s)}\ \hat{\alpha}_4^c(\pi\bar{K})\, A_{\pi\bar{K}} \quad (2.8)$$

and $\sqrt{2}A(\bar{B}^0 \to \pi^0\bar{K}^0) = -A(B^- \to \pi^-\bar{K}^0) + \sqrt{2}A(B^- \to \pi^0K^-) - A(\bar{B}^0 \to \pi^+K^-)$. Note that $\lambda_u^{(s)}/\lambda_s^{(s)} \sim \lambda^2 \sim 0.04$ (with λ the Cabibbo parameter), so tree amplitudes are (at best) CKM suppressed. The full penguin amplitude $\hat{\alpha}_4^p(\pi\bar{K}) = a_4^p(\pi\bar{K}) + r_\chi^K a_6(\pi\bar{K}) + \beta_3^p(\pi\bar{K})$ contains a scalar penguin amplitude a_6^p and an annihilation amplitude β_3^p. Both contributions are formally power corrections and will be discussed separately below. The contributions from $Q_{1,2}$ to the leading penguin amplitudes a_4^p have been recently calculated at NNLO [27]:

$$\begin{aligned} a_4^u(\pi\bar{K})/10^{-2} &= -2.87 - [0.09 + 0.09i]_{\rm V_1} + [0.49 - 1.32i]_{\rm P_1} - [0.32 + 0.71i]_{\rm P_2} \\ &+ \left[\frac{r_{\rm sp}}{0.434}\right]\Big\{[0.13]_{\rm LO} + [0.14 + 0.12i]_{\rm HV} - [0.01 - 0.05i]_{\rm HP} + [0.07]_{\rm tw3}\Big\} = -2.46 - 1.94\,i\,, \end{aligned} \quad (2.9)$$

$$\begin{aligned} a_4^c(\pi\bar{K})/10^{-2} &= -2.87 - [0.09 + 0.09i]_{\rm V_1} + [0.05 - 0.62i]_{\rm P_1} - [0.77 + 0.50i]_{\rm P_2} \\ &+ \left[\frac{r_{\rm sp}}{0.434}\right]\Big\{[0.13]_{\rm LO} + [0.14 + 0.12i]_{\rm HV} + [0.01 + 0.03i]_{\rm HP} + [0.07]_{\rm tw3}\Big\} = -3.34 - 1.05\,i\,. \end{aligned} \quad (2.10)$$

Spectator scatering (proportional to $r_{\rm sp}$) is numerically small. The NNLO contribution is labeled '$\rm P_2$', and it is found to be rather large and of the same order of the NLO penguin contributions. It should be noted that in the case of a_4^c there is a strong cancellation at NLO (in the term labeled '$\rm P_1$') between the two Q_1 contributions with different color topologies. Thus the fact that the NNLO correction is much larger than the NLO seems accidental. Stabilization of the μ dependence of the real parts suggests the perturbative expansion is well behaved [27]. Again, the scale dependence of the imaginary part is not significantly reduced at NNLO since the LO contribution is real.

Full NNLO phenomenology for penguin decays would require the missing two-loop matrix elements of penguin operators $Q_{3\cdots6}$.

2.5 CP asymmetries

Direct CP asymmetries require the interference of two amplitudes with different weak and strong phases. Therefore they are governed by the penguin amplitude α_4^c and the imaginary parts in tree and penguin amplitudes (strong phases). Since the leading-power leading-order amplitudes are real, strong phases are either $\mathcal{O}(\alpha_s)$ or $\mathcal{O}(\Lambda/m_B)$. Since $\alpha_s/\pi \sim \Lambda/m_b$, it is plausible that power corrections are $\mathcal{O}(1)$ effects in direct CP asymmetries. In addition, perturbative corrections to CP asymmetries require tree and penguin amplitudes to NNLO, which is one of the main motivations behind the calculations in Refs. [25, 27].

Direct CP asymmetries at NNLO for penguin decays have been discussed in [27]. In Table 2 we reproduce some of the results for $B \to K\pi$ direct CP asymmetries. In this case NNLO corrections are small because a_4^p is only a part of the penguin amplitude $\hat{\alpha}_4^p$, and a_6^p is numerically large, thereby diluting the effect. The 'NNLO' column does not include the annihilation contribution β_3^p nor the twist-3 spectator scattering contributions. These are power suppressed but not calculable, and induce a significant error in the predictions. Using a similar model for power suppressed non-factorizable contributions as in [22] these are included in the column labeled 'NNLO+LD', with the annihilation contribution β_3^p giving the dominant effect. In this case the agreement with data is improved, although uncertainties are inflated considerably. The prediction and experimental number for the quantity $\delta(\pi K) \equiv A_{\rm CP}(\pi^0 K^-) - A_{\rm CP}(\pi^+ K^-)$ is also given, which remains a "puzzle" (see *e.g.* [35]). In the case of PV and VV final states such as ρK, πK^*, ρK^*, the NNLO contribution to CP asymmetries can be important, depending on the role of the scalar penguin amplitude a_6^c. In any case it is a general feature that the long-distance annihilation contribution is very important numerically. But experimental results for these PV and VV modes are still quite uncertain.

f	NLO	NNLO	NNLO + LD	Exp
$\pi^-\bar{K}^0$	$0.71^{+0.13+0.21}_{-0.14-0.19}$	$0.77^{+0.14+0.23}_{-0.15-0.22}$	$0.10^{+0.02+1.24}_{-0.02-0.27}$	-1.7 ± 1.6
π^0K^-	$9.42^{+1.77+1.87}_{-1.76-1.88}$	$10.18^{+1.91+2.03}_{-1.90-2.62}$	$-1.17^{+0.22+20.00}_{-0.22-6.62}$	4.0 ± 2.1
π^+K^-	$7.25^{+1.36+2.13}_{-1.36-2.58}$	$8.08^{+1.52+2.52}_{-1.51-2.65}$	$-3.23^{+0.61+19.17}_{-0.61-3.36}$	-8.2 ± 0.6
$\pi^0\bar{K}^0$	$-4.27^{+0.83+1.48}_{-0.77-2.23}$	$-4.33^{+0.84+3.29}_{-0.78-2.32}$	$-1.41^{+0.27+5.54}_{-0.25-6.10}$	1 ± 10
$\delta(\pi\bar{K})$	$2.17^{+0.40+1.39}_{-0.40-0.74}$	$2.10^{+0.39+1.40}_{-0.39-2.86}$	$2.07^{+0.39+2.76}_{-0.39-4.55}$	12.2 ± 2.2

Table 2: Direct CP asymmetries (in percent) for πK final states (from Ref. [27]).

Direct CP asymmetries will most certainly lead to a clear picture of successes and failures of leading-power factorization. So far the situation is rather confusing, with an "ununderstood pattern of agreements and disagreements" (quoting [16]). More precise data will also contribute to clarify the situation, with good prospects from LHCb and Belle-II.

2.6 Power corrections

Power corrections are the main source or uncertainty in the prediction of non-leptonic two-body B-decay amplitudes. Some power corrections are calculable and numerically important. This is the case of the scalar penguin amplitude $a_6^p(M_1M_2)$, which contributes to the full penguin amplitude $\hat{\alpha}_4^p(M_1M_2) = a_4^p(M_1M_2) \pm r_\chi^{M_2} a_6(M_1M_2) + \beta_3^p(M_1M_2)$. Here the plus (minus) sign applies

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when M_1 is a pseudoscalar (vector) meson, and $r_\chi^{M_2}$ is a "kinematic" factor that contains a power suppression and a chiral enhancement, *e.g.* $r_\chi^K = 2m_K^2/[m_b(m_q+m_s)]$. Numerically $r_\chi \simeq 1$, so although the scalar penguin amplitude is power suppressed, it is numerally leading. This is not a problem since this amplitude factorizes and it is therefore, calculable.

Other power corrections come from annihilation (*e.g.* β_3^p) and spectator scattering contributions (*e.g.* the terms labeled 'tw3' above), and do not always factorize. As discussed above, annihilation contributions are relevant for CP asymmetries. Modelling these power-suppressed contributions leads to large uncertainties in the QCDF predictions.

One possibility is to parametrize the weak annihilation (WA) contributions and determine whether some pattern for WA can accomodate the data. A global fit to most the available data on $B_{u,d,s} \to PP, VP, VV$ modes [36] finds that the SM can reproduce the experimental results (with a few exceptions) using one universal WA parameter for each decay system, and with no anomalously large values for these parameters (that is, consistent with the most popular model *e.g.* [22]). The exceptions are $\delta(\pi K)$ (thus not resolving the "ΔA_{CP} puzzle") and, less significantly, the branching ratio of $B^0 \to K^{*0}\phi$, with a pull around $\sim 2\sigma$. Removing the "universality" assumption for WA will however ease all tensions (including ΔA_{CP}), at the obvious cost of more freedom and little predictivity. A similar analysis can be found in [37].

Another possibility is to look for theoretical quantities where non-factorizable contributions cancel, either completely or approximately. An example is given in Ref. [38], where it is shown how this cancellation takes place in the quantity $\Delta_f \equiv T_f^u - T_f^c$ [in the notation of Eq. (1.2)] for certain penguin-mediated decays (for a list of such modes see [39]). Using the QCDF prediction for this quantity one can predict certain relationships between observables which can help to test branching ratios and direct CP asymmetries [38, 40] or to extract mixing angles from data [41, 39].

Now that perturbative calculations have reached the NNLO level, progress in the theoretical study of non-leptonic two-body B decay amplitudes requires addressing power corrections systematically. This is strongly motivated given the experimental prospects for measurements of branching fractions and CP asymmetries in two-body charmless B decays.

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3. Three-body decays

3.1 Kinematics

While the kinematics of two-body decays is fixed, three-body decay amplitudes depend on two kinematic variables. We consider a decay $\bar{B}(p_B) \to M_1(p_1)M_2(p_2)M_3(p_3)$. It is customary to take these variables as two invariant masses of two pairs of final state particles (*e.g.* s_{12} and s_{13} with $s_{ij} \equiv 2(p_i \cdot p_j)/m_B^2$). All physical kinematic configurations thus define a two-dimensional region in the s_{12}-s_{13} plane, which in the limit where all final particles are massless is a triangle defined by $s_{12} > 0$, $s_{13} > 0$, $s_{12} + s_{13} < 1$. (We will assume massless decay products in the following for simplicity, all the conclusions remaining valid.) The density plot of the differential decay rate $d\Gamma/ds_{12}ds_{13}$ in that region is called a Dalitz plot.

The Dalitz plot can be divided in different regions with "characteristic" kinematics . The central region corresponds to the case where all three final particles fly apart with large energy ($E \sim m_B/3$) at $\sim 120^o$ angles. The corners correspond to the case in which one final particle is approximately at rest (*i.e.* soft), and the other two fly back-to-back with large energy ($E \sim m_B/2$). The central part of the edges correspond to the case in which two particles move collinearly with large energy and the other particle recoils back.

3.2 Three-body decays in QCDF

The two kinematic invariants on which the decay amplitudes depend, introduce two extra scales in the problem. Different forms of factorization theorems may be conjectured depending on the scaling of these momentum scales with m_b [42].

In the central region, where all invariant masses are of order m_B ($s_{12} \sim s_{13} \sim 1/3$), the following formula is proposed [43]:

$$\langle \pi\pi\pi|Q_i|\bar{B}\rangle_{s_{ij}\sim 1/3} = F^{B\to\pi}\, T_i^I \otimes \Phi_\pi \otimes \Phi_\pi + T_i^{II} \otimes \Phi_B \otimes \Phi_\pi \otimes \Phi_\pi \otimes \Phi_\pi\ , \tag{3.1}$$

where the convolutions of hard-scattering kernels and distribution amplitudes are written schematically. The hard kernels $T_i^{I,II}$ can be computed perturbatively in QCD. To the lowest order (at order α_s), only T_i^I is considered, and arises from diagrams with an insertion of the operator Q_i and all possible insertions of a hard gluon splitting into a quark-antiquark pair with large invariant mass. The convolutions of the resulting hard kernels T_i^I with the pion light-cone distributions can be computed without encountering end-point singularities, thus providing a check of the factorization formula. This is a non-trivial check since the kernels $T_i^I(u,v)$ already depend on the momentum fraction of the quarks at the leading order, so the convolutions are non-trivial.

At the edges of the Dalitz plot, one invariant mass becomes small, and low energy interactions between the corresponding pair of final state particles leads eventually to the formation of resonances. This is the case for *e.g.* $B \to \pi^+\pi^-\pi^+$ in the region where $m_{\pi^+\pi^-} \sim m_\rho$, and appears as a band in the Dalitz plot. The decay thus looks very much like a two-body decay, and one expects a similar factorization formula, except for the fact the one particle is, instead, two [43]:

$$\langle \pi^a\pi^b\pi^c|Q_i|B\rangle_{s_{ab}\ll 1} \;=\; F^{B\to\pi^c}\ T_c^I \otimes \Phi_{\pi^a\pi^b} + F^{B\to\pi^a\pi^b}\ T_{ab}^I \otimes \Phi_{\pi^c} + T^{II} \otimes \Phi_B \otimes \Phi_{\pi^c} \otimes \Phi_{\pi^a\pi^b}\ . \tag{3.2}$$

Here $\Phi_{\pi\pi}$ denotes a two-pion distribution amplitude (2πLCDA), and $F^{B\to\pi\pi}$ denotes a $B\to\pi\pi$ form factor. Conceptually, this factorization formula is at the same level of theoretical rigor as the factorization formula for two-body decays to unstable particles (*e.g.* $B\to\rho\pi$), but requires more complicated hadronic input (discussed below). This is the cost of generalizing quasi-two-body decays beyond the narrow-width approximation.

The three-body amplitude at the central region is both power- and α_s-suppressed with respect to the amplitude at the edge. The interpolation between one region and the other can be understood by noting that some parts of the central region amplitude arise from factorization of 2πLCDAs or $B\to\pi\pi$ form factors at large dipion masses, and one can check analytically the correspondence of such parts of the amplitudes. Numerically, it is found that the part of the amplitude at the central region corresponding to the large dipion limit of the 2πLCDA part of the amplitude at the edge agrees well with the latter only for $m_B \gtrsim 20$ GeV, but not for realistic values, suggesting that power corrections to (3.1) are too large in reality, precluding a description of the central region in terms of single pion states.

3.3 Generalized distribution amplitudes

The relevant 2πLCDA in Eq. (3.2) is given by the matrix element [43, 44]

$$\Phi^q_{\pi\pi}(z,\zeta,k_{12}^2) = \int \frac{dx^-}{2\pi} e^{iz(k_{12}^+x^-)} \langle \pi^+(k_1)\pi^-(k_2)|\bar{q}(x^-n_-)\not{n}_+ q(0)|0\rangle \,, \tag{3.3}$$

where $k_{12}^\mu = k_1^\mu + k_2^\mu \simeq (k_{12}^+/2)n_+^\mu$, $\zeta = k_{12}^+/k_1^+$, and we have suppressed a Wilson line that makes the non-local quark current gauge invariant. At the leading order the kernel T_c^I in Eq. (3.2) does not depend on z, and only the normalization for $\Phi_{\pi\pi}$ is needed:

$$\int dz\,\Phi^q_{\pi\pi}(z,\zeta,s) = (2\zeta-1)F_\pi(s) \tag{3.4}$$

where $F_\pi(s)$ is the pion time-like form factor. The absolute value of the pion form factor is well known experimentally in a wide range of energies (see Fig. 5 in [43]). Higher moments of the 2πLCDA are needed at higher orders, but these are much less known.

3.4 $B\to\pi\pi$ form factors

$B\to\pi\pi$ form factors are in principle accessible from measurements of $B\to\pi\pi\ell\nu$ observables [45]. The Lorentz structure of the leading-order $B^-\to\pi^-\pi^+\pi^-$ amplitude at low $m_{\pi^+\pi^-}$ is such that the relevant $B\to\pi\pi$ form factor is

$$F_t(\zeta,k_{12}^2) \equiv -\frac{1}{\sqrt{q^2}} \langle \pi^+(k_1)\pi^-(k_2)|\bar{u}\not{q}\gamma_5 b|B^-(p)\rangle \tag{3.5}$$

where $q = p - k_{12}$ (in our case $q^2 = m_\pi^2$). At low dipion masses, this form factor may be studied by means of light-cone sum rules. One may consider light-cone sum rules with two-pion distribution amplitudes [46] or with B-meson distribution amplitudes [47]. In the first case one arrives to a closed expression for F_t in terms of moments of the 2πLCDAs:

$$F_t(k_{12}^2,\zeta) = \frac{m_b^2\sqrt{m+\pi^2}}{\sqrt{2}f_B m_B^2}\int_{u_0}^1 \frac{du}{u^2}(m_b^2 - q^2 + u^2k_{12}^2)\,\Phi^q_{\pi\pi}(u,\zeta,k_{12}^2)\, e^{\frac{m_b^2}{M^2}-\frac{m_b^2-\bar{u}q^2+u\bar{u}k_{12}^2}{uM^2}} \,. \tag{3.6}$$

The disadvantage of this method is that moments of 2πLCDAs are not well-known.

In the second case, one obtains a sum-rule that depends on a weighted integral of the form factor [47]:

$$\int_{4m_\pi^2}^{s_0^{2\pi}} ds\, e^{-s/M^2}\, \frac{s\,\sqrt{q^2}\,[\beta_\pi(s)]^2}{4\sqrt{6}\pi^2\sqrt{\lambda}}\, F_\pi^\star(s)\, F_t^{(1)}(s,q^2) \tag{3.7}$$

$$= -f_B m_B^2 m_b \left\{ \int_0^{\sigma_0^{2\pi}} d\sigma\, e^{-s(\sigma,q^2)/M^2} \left[\frac{\sigma}{\bar{\sigma}}\phi_-^B(\sigma m_B) - \frac{1}{\bar{\sigma}m_B}\bar{\Phi}_\pm^B(\sigma m_B)\right] + \Delta A_0^{BV}(q^2,\sigma_0^{2\pi},M^2)\right\}$$

and depends on the B-meson LCDAs and the pion form factor. ΔA_0 denotes 3-particle contributions. This sum rule allows to test models for the $B\to\pi\pi$ form factor, and in the limit where the pion form factor is dominated by an infinitely narrow ρ meson, this sum rule reduces analytically to the know sum-rule for the $B\to\rho$ form factor $A_0^{B\rho}$ [48].

A factorization formula for $B\to\pi\pi$ form factors at large dipion masses has also been proven recently at NLO [49].

4. Conclusions and future prospects

QCD Factorization is by now very well established as a QCD-based approach to charmless non-leptonic two-body decays. Perturbative calculations of hard-scattering kernels have reached the NNLO precision, proving factorization to two loops and confirming a good behaviour of the perturbative expansion.

The pattern of branching fractions is understood qualitatively, although some tensions are observed, mostly in modes dominated by the color-suppressed tree amplitude. These tensions could be related to the spectator scattering contribution, which is proportional to λ_B, the inverse moment of the B-meson LCDA, and which is currently not very well known. Values of $\lambda_B \sim 200$ MeV are favored, much lower than sum-rule estimates. A direct experimental determination of λ_B must await to a precise measurement of $B \to \gamma \ell \nu$ at Belle-II.

On the other hand, the recent calculation of penguin amplitudes at NNLO provides the first perturbative corrections to CP asymmetries. However, in this case power corrections could be $\mathscr{O}(1)$ effects, explaining why the global picture in the comparison of theory and experiment is far from clear. In addition, the "$\Delta A_{\rm CP}$ puzzle" remains. One should add that experimental measurements of CP asymmetries to PV and VV final states are still not very precise.

Power corrections is now most probably the most pressing issue in order to make progress in the theoretical understanding of charmless two-body decays, but the prospects are rather modest.

Three-body decays are still mostly unexplored from the theoretical point of view, although detailed and exciting experimental analyses of branching fractions and CP violation are piling up. We also expect many results from Belle-II. Recent studies pursuing factorization methods for three-body decays look promising.

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