

CREDIT RISK AND BANK

MARGINS IN STRUCTURED

FINANCIAL PRODUCTS:

EVIDENCE FROM THE GERMAN

SECONDARY MARKET FOR

DISCOUNT CERTIFICATES

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This study analyzes bank margins in the German secondary market for exchange-traded structured financial products, with particular emphasis on the influence of banks' credit risk. A structural model allowing for the incorporation of correlation effects between market and credit risk is applied to compare quoted and fair theoretical prices. For discount certificates, as the most popular type of structured financial products in Germany, an empirical study is conducted. Compared to earlier studies, total margins are found to be rather low, whereas the portion that draws back to credit risk appears to be a material part of the total margin.

INTRODUCTION

Beginning in the 1990s, U.S. banks, and later European banks, started offering retail customers structured financial products (SFPs), which quickly became very popular. The German market for SFPs has since become the largest derivatives market for private investors worldwide. SFPs combine elements of the spot market, such as bonds or stocks, with derivative components and are sold by the issuing bank as a single security. The private investor's benefit is a payoff profile, which he is usually unable to construct himself from the component elements due to market barriers and transaction costs. The issuer, however, is able to trade the single components on the respective markets and to net them in the trading book. The issuing bank is thus able to hedge the market risk of SFPs easily and in a cost-efficient manner.

Trading in the German secondary market takes place mainly at the European Warrant Exchange (EUWAX), which is a market maker-based segment of the Stuttgart Stock Exchange.¹ The market making insures sufficient liquidity of SFPs, as the market makers are obligated to continuously offer binding bid-ask quotes.² Hence, investors can buy and sell their certificates at any time and within seconds of placing their order. As the total outstanding number of certificates of each issue is usually small, the counterparty of an investor is typically the market maker, rather than another investor willing to buy or sell at the same time. At the EUWAX, the issuing bank usually acts as the market maker. As there is no realistic risk of arbitrage by other institutional or private investors, the bank can realize margins by selling the SFP for a price higher than the sum of its components. Several empirical studies analyze the price-setting behavior of issuers and find that banks quote prices that start above the fair value and converge to the fair value over time (e.g., Stoimenov & Wilkens, 2005). Hence, as the bank usually hedges its exposure from SFPs, it gains by diminishing overpricing in the course of product lifetimes.

¹ See Stuttgart Stock Exchange (2007) for details of the exchange rules.

² In the authors' later data sample, the average bid-offer spread equals 0.35%.

From the legal point of view, SFPs are senior unsecured bonds, which have the sole peculiarity of a redemption that depends on the values of other securities. As they are not guaranteed by a third authority in the event of the issuer's default, investors face a default risk similar to that of other bondholders. Consequently, even if the bank quotes a price equal to the SFP's default-free fair value, the "true" fair value is lower. This difference exactly represents the margin³ of the bank derived from its credit risk. Roughly speaking, the bank earns its own credit spread by incorporating its own risk in the payoff structure. This effect is, *ceteris paribus*, more pronounced for issuers with a higher default risk, which makes SFPs especially attractive for riskier banks.

The main contribution of this study is the theoretical and empirical analysis of the quotes and the respective "fair" theoretical values for discount certificates on the German secondary market in a framework that allows for integrating dependencies between market and credit risk. Its main focus is the influence of the issuers' default risk. This study is part of the empirical literature dealing with SFPs. The first empirical examinations were conducted for Market Index Certificates of Deposit and an S&P Index Note at the U.S. market by Chen and Kensinger (1990) and Chen and Sears (1990). They use the default-free theoretical value as a reference and compare it with prices quoted by the banks. The same holds for Wasserfallen and Schenk (1996) and Burth, Kraus, and Wohlwend (2001) in their analysis of capital-guaranteed products and reverse convertibles and discount certificates, respectively, in the Swiss market. Wilkens, Erner, and Röder (2003) are the first to consider credit risk when investigating the quotes of reverse convertibles and discount certificates in the German market. They use the Lehman Brothers bond indices to adjust the default-free discount factors for pricing the zero bond as part of the products considered. In contrast to this, they also value the embedded option as default-free. A similar approach is used by Grünbichler and Wohlwend (2005) for the Swiss market. Stoimenov and Wilkens (2005) analyze the pricing of various types of SFPs in Germany and incorporate the issuers' default risk by applying the model of Hull and White (1995) with an issuer-averaging and time-independent spread, assuming the independence of market risk from credit risk.

To the authors' knowledge, all empirical literature concerning the valuation of SFPs focuses on the banks' total margins, *i.e.*, the relative difference between the quoted price and the theoretically fair value. Further, these studies either totally neglect the possibility of the issuer defaulting or they neglect issuer-specific credit risk and dependencies between market risk and credit risk. In addressing these shortcomings, this study fills a gap, as it is the first

³Throughout this study, the term "margin" is used as a gross margin, *i.e.*, without any costs for structuring.

contribution that analyzes SFPs using a model that incorporates correlated market and credit risk.

The analysis is carried out for discount certificates as the most popular type of SFPs in Germany.⁴ The theoretical part presents a structural model to evaluate discount certificates that takes into account the risk of the issuer defaulting, similar to Klein (1996) with regard to vulnerable options.⁵ The default-free Black and Scholes (1973) and the Hull and White (1995) models, both of which can be regarded as special versions of the structural model, serve as benchmarks. The empirical part investigates quoted prices of discount certificates on DAX (Deutscher Aktienindex) stocks of five major issuers. For each issuer the total margin and the credit risk margin, i.e., the part of the total margin that draws back to credit risk, are analyzed. Compared to earlier studies, total margins are found to be rather low, whereas the credit risk margin appears to be a material part of the total margin. The analysis suggests that total margins have decreased over time, which may be caused by the rising competition among the issuers. The results imply that the credit risk margin, i.e., the issuers' credit risk, is an important source of bank profitability in the market segment analyzed here.

The study is organized as follows: in the second section, the model for the valuation of vulnerable discount certificates is presented and analyzed; the empirical study is the subject of the third section, whereas the fourth section concludes.

VALUING VULNERABLE DISCOUNT CERTIFICATES

Replicating Portfolio

A discount certificate is based on an underlying security, usually a stock or an index. At maturity date T , it promises the investor either an amount equal to the price of the underlying security or a fixed cap, whichever is smaller.⁶ Thus, the promised payoff, which equals the payoff of a default-free issuer, is given by

$$DC_T^{df} = \min\{S_T; X\} = S_T - \max\{S_T - X; 0\} = X - \max\{X - S_T; 0\} \quad (1)$$

⁴ See Stoimenov and Wilkens (2005) for the development of this market segment in Germany.

⁵ Structural models allow for integrating correlation between market risk and credit risk. The first contribution to this approach in pricing vulnerable options was made by Johnson and Stulz (1987), who assume that the vulnerable option is the sole liability of the issuer. The model of Klein (1996) revokes this assumption. The Klein approach has been modified in several ways. Klein and Inglis (1999) extended the original Klein model by incorporating stochastic interest rates. Klein and Inglis (2001) combined the models of Johnson and Stulz (1987) and Klein (1996). Hui, Lo, and Lee (2003) introduced early default into a simplified version of Klein's model.

⁶ In practice, this amount is often multiplied by a cover ratio, because one certificate generally refers to a fraction or a multiple of the underlying. Furthermore, besides cash settlement, physical delivery of the underlying is also possible.

where S_t denotes the price of the underlying security at time t , and X is the cap, also called the strike price. In academic literature dealing with SFPs, these products are usually valued as the sum of the fair values of the components of a replicating portfolio. Obviously, the (default-free) payoff scheme (1) can be duplicated in two ways: a long position in the underlying security and a short position in a European call with strike price X and maturity T , or a long default-free zero bond with maturity T , face value X , and a short European put with strike price X and maturity T .⁷

If the risk of an issuer's default is taken into consideration, the amount DC_T^{df} is only paid if the issuer has not defaulted during the lifetime of the discount certificate. To keep things simple, it is assumed that default can only occur at maturity of the certificate. Following Hull and White (1995), it is assumed further that in the event of a default, the final payment is reduced to a constant fraction δ of the no-default value.⁸ Thus, the payoff of a defaultable discount certificate is given by

$$DC_T^d = \min\{S_T; X\}(1 + (\delta - 1)I_{\{Default\}}) \quad (2)$$

$$= (S_T - \max\{S_T - X; 0\})(1 + (\delta - 1)I_{\{Default\}}) \quad (3)$$

$$= (X - \max\{X - S_T; 0\})(1 + (\delta - 1)I_{\{Default\}}). \quad (4)$$

The component $X(1 + (\delta - 1)I_{\{Default\}})$ in (4) can be interpreted as a bank's zero bond ranking *pari passu* with the discount certificate. The second component, $-\max\{X - S_T; 0\}(1 + (\delta - 1)I_{\{Default\}})$, is "similar" to a vulnerable put. But it cannot be interpreted as a short put bought by the bank from the investor. The payoff of an investor's standard short put position does not depend on the bank's default because claims to a bank's counterpart are typically not reduced to any degree if the bank goes into default. Consequently, the second component of (4) does not equal the payoff of a short put, but of a vulnerable long put multiplied by -1 .⁹ Thus, in contrast to the default-free case, the replication has to be considered a "synthetic" one. The elements of the replication portfolio, seen as

⁷ To be precise, the first replication is only correct if the underlying security pays no dividends within the lifetime of the discount certificate. To keep things simple, this additional assumption in the theoretical analysis is made. It is straightforward to generalize the analysis to incorporate dividend payments or a continuous dividend yield.

⁸ This is analogous to the recovery of treasury value assumption, which is widely used in credit risk bond pricing literature. For an overview of different recovery assumptions, see, e.g., Schönbucher (2003, pp. 131–143).

⁹ However, (4) can be interpreted as a combination of a long zero bond and a short put in conjunction with a netting effect. If netting is allowed in the event of default, the bank's claim from the put's payoff can be netted by the investor's claim from the zero bond. The investor's nominal claim is then $X - \max\{X - S_T; 0\}$, which is reduced by the fraction $(1 - \delta)$ in case of default.

isolated securities, are not necessarily traded in the real world, but can nonetheless be used to value the discount certificate by

$$DC_0^d = ZB_0^d X - p_0^d \quad (5)$$

where ZB_0^d denotes the value of a bank's zero bond with notional amount 1 and p_0^d the value of a vulnerable long put. The same observations hold true for the other replication (see (3)) portfolio built by the stock (a vulnerable stock certificate S_0^d) and a vulnerable long call (c_0^d), so that

$$DC_0^d = S_0^d - c_0^d \quad (6)$$

is also fulfilled.¹⁰

On the basis of these general replicating portfolios, a structural model for vulnerable options is specified in the following, which allows us to value discount certificates subject to correlated market and credit risk. The structural model contains the default-free Black and Scholes (1973) model and the Hull and White (1995) model as special cases. Accordingly, the results from the structural model and from the models that have been applied so far in empirical literature (see the Introduction section) can be compared in a unified framework. The structural model consists of two parts: the first part models the behavior of the issuer's asset value. A default event occurs if the asset value falls below a default point, as in the classical Merton (1974) paper. The second part models the behavior of the underlying stock. Basically, the approach follows Klein (1996), who assumes correlated geometric Brownian motion for the issuer's asset value and the underlying stock.¹¹

The Issuer Model

In addition to the common assumptions of an arbitrage-free, complete, and frictionless market with flat and constant interest rates r , the following assumptions concerning the issuing bank are made:

II. Under the risk-neutral measure Q , the total asset value V_t of the issuer follows geometric Brownian motion with variance rate σ_V^2 :

$$dV_t = rV_t dt + \sigma_V V_t dz_V.$$

¹⁰ Note that the put-call parity holds.

¹¹ We have also applied and analyzed a model in which the asset value of the underlying firm, rather than the underlying stock, follows geometric Brownian motion. However, the results for the data sample are very similar.

- I2. A default event occurs if and only if the asset value at expiry date T of the discount certificate V_T is less than the default point D .
- I3. In the event of a default, the holder of the discount certificate receives a constant fraction δ of the no-default value.
- I4. There is a zero bond issued by the bank with maturity T that ranks *pari passu* with the discount certificate. In the event of a default, the holder of the zero bond receives a constant fraction δ of the no-default value.
- I5. The volume of the discount certificate is negligible in comparison to the issuer's other liabilities.

Except for I4 and I5, no further assumption about the capital structure of the issuer and the structure of the liabilities is necessary. The introduction of a constant recovery rate renders it possible to focus on just one part of the liabilities side. In other models, where the asset value upon default is distributed to different forms of debt, the absolute priority of the debt has to be considered. Therefore, recovery would depend on other liabilities as well. To achieve realistic average recovery rates, deadweight costs would have to be introduced, which would reduce the asset value at default (e.g., Klein, 1996). The estimation of an empirical recovery rate would then be replaced by the estimation of deadweight costs. A drawback to the assumption of a constant recovery rate is the fact that it ignores the variability of recovery rates and the negative relationship between the probability of default and the level of recovery rates (e.g., Frye, 2000). Hence, assuming a constant (average) recovery rate will overestimate the value of discount certificates when the credit quality of the issuer deteriorates. However, the assumed constant recovery rate makes the model much more tractable. Furthermore, the constant recovery rate will ensure that the empirical results presented later are independent of the non-observable variables: asset value, asset volatility, and default point.

Under the assumptions above, today's value of a European claim H_T at T on the bank which is reduced by a fraction $(1 - \delta)$ in the event of default can be written as

$$H_0^d = e^{-rT} E_Q(H_T(1 + (\delta - 1)1_{\{\text{Default}\}})) \quad (7)$$

where expectation is taken with respect to the risk-neutral measure Q . In this framework, it can easily be shown, using Equation (7), that the value of a bank's zero bond with notional amount 1, i.e., $H_T = 1$, and maturity T is given by

$$ZB_0^d = e^{-rT} E_Q(1 + (\delta - 1)1_{\{V_T < D\}}) = e^{-rT}(1 + (\delta - 1)N(-b_2)) \quad (8)$$

where $N(\cdot)$ represents the cumulative standard normal distribution function. $N(-b_2)$ equals the risk-neutral T -year default probability, and b_2 is given by

$$b_2 = \frac{\ln(V_0/D) + (r - \sigma_V^2/2)T}{\sigma_V \sqrt{T}}. \quad (9)$$

Let s_T denote the continuously compounded spread of this zero bond:

$$s_T = -\frac{1}{T} \ln(1 + (\delta - 1)N(-b_2)). \quad (10)$$

The Underlying Model

The assumptions concerning the underlying are:

U1. Under the risk-neutral measure Q , the stock price S_t of the underlying firm follows geometric Brownian motion with variance rate σ_S^2 :

$$dS_t = rS_t dt + \sigma_S S_t dz_s.$$

U2. The issuer's asset value V and the stock price S are instantaneously correlated by $\rho_{V,S}$:

$$\text{Corr}(dz_V, dz_S) = \rho_{V,S}.$$

Following these assumptions, the value of a defaultable European put in the structural model with expiry date T and strike price X can be calculated in a straightforward manner by applying Equation (7) and setting $H_T = \max\{X - S_T, 0\}$:

$$p_0^{d,SM} = -S_0(N(-a_1, a_2, -\rho_{V,S}) + \delta N(-a_1, -a_2, \rho_{V,S})) + e^{-rT} X(N(-b_1, b_2, -\rho_{V,S}) + \delta N(-b_1, -b_2, \rho_{V,S})) \quad (11)$$

with

$$a_1 = \frac{\ln(S_0/X) + (r + \sigma_S^2/2)T}{\sigma_S \sqrt{T}} \quad (12)$$

$$b_1 = \frac{\ln(S_0/X) + (r - \sigma_S^2/2)T}{\sigma_S \sqrt{T}} = a_1 - \sigma_S \sqrt{T} \quad (13)$$

$$a_2 = \frac{\ln(V_0/D) + (r - \sigma_V^2/2 + \rho_{V,S} \sigma_S \sigma_V)T}{\sigma_V \sqrt{T}} \quad (14)$$

$$b_2 = \frac{\ln(V_0/D) + (r - \sigma_V^2/2)T}{\sigma_V \sqrt{T}} = a_2 - \rho_{V,S} \sigma_S \sqrt{T} \quad (15)$$

where $N(\cdot, \cdot, \rho)$ represents the bivariate standard normal cumulative distribution function with correlation ρ . The Appendix provides the analogue values of a vulnerable stock certificate and a vulnerable European call, as well as a sketch of the proof. According to the previous analysis (see (5), (8), (11)), the value of a defaultable discount certificate in the structural model is given by

$$\begin{aligned} DC_0^{d,SM} = & e^{-rT}(1 + (\delta - 1)N(-b_2))X \\ & + S_0(N(-a_1, a_2, -\rho_{V,S}) + \delta N(-a_1, -a_2, \rho_{V,S})) \\ & - e^{-rT}X(N(-b_1, b_2, -\rho_{V,S}) + \delta N(-b_1, -b_2, \rho_{V,S})). \end{aligned} \quad (16)$$

Equation (16) contains the default-free Black and Scholes (1973) formula and the Hull and White (1995) formula for discount certificates as special cases. If the issuer's credit risk tends to zero, e.g., for $V_0 > \exp(-rT)D$ and $\sigma_V \rightarrow 0$, which makes the issuer's default impossible, (16) converges to

$$DC_0^{df,BS} = e^{-rT}X + S_0N(-a_1) - e^{-rT}XN(-b_1) \quad (17)$$

where $N(\cdot)$ denotes the standard normal cumulative distribution function (17) equals the value of a default-free zero bond and a short put according to the standard Black and Scholes (1973) option pricing formulas. Otherwise, in the case of independence between V_t and S_t , i.e., $\rho_{V,S} = 0$, (16) reduces to¹²

$$DC_0^{d,HW} = e^{-s_rT}(e^{-rT}X + S_0N(-a_1) - e^{-rT}XN(-b_1)) = e^{-s_rT}DC_0^{df,BS} \quad (18)$$

which is the value of a default-free, but in other respects identical, certificate that is discounted with the spread of a zero bond ranking pari passu with the discount certificate. If independence is assumed, the value equals the value in the Hull and White (1995) model, which is not surprising, because the Hull and White result is distribution-free. For simplicity, (17) and (18) will be referred to as the "Black and Scholes value" and the "Hull and White value", respectively.

Numerical Example

To illustrate the effect of the correlation $\rho_{V,S}$ between credit and market risk (i.e., between the issuer's asset value and the underlying stock price) on the value of the vulnerable discount certificate, a numerical example is considered. The features of this discount certificate, together with the relevant market data and values, are summarized in Table I. The value of the defaultable zero bond

¹² Here $N(x, y, 0) = N(x)N(y)$, $N(-x) = 1 - N(x)$, and $a_2 = b_2$ for $\rho_{V,S} = 0$ are used.

with notional amount $X = 95$ is $ZB_0^d X = 89.95$ (default-free: $ZB_0^{df} X = 90.82$), which corresponds to an annualized spread of $s_T = 0.64\%$. The value of the embedded put is $p_0^{d,SM} = 9.51$, according to the structural model (default-free: $p_0^{df} = 9.79$). Thus, the total value of the certificate is $DC_0^{d,SM} = 89.95 - 9.51 = 80.44$ (default-free: $DC_0^{df,BS} = 90.82 - 9.79 = 81.03$), whereas the Hull and White value is $DC_0^{d,HW} = \exp(-0.0064 \cdot 1.5) 81.03 = 80.26$.

The relative difference between the default-free value $DC_0^{df,BS}$ and the defaultable value $DC_0^{d,model}$ is defined as credit risk margin CRM^{model} :

$$CRM^{model} = \frac{DC_0^{df,BS} - DC_0^{d,model}}{DC_0^{d,model}}, \text{ model} = SM, HW. \quad (19)$$

The credit risk margin is the relative value of the bank's credit risk in conjunction with discount certificates. It represents that part of the bank's total margin that is caused by its default risk and is, therefore, the major point of focus in this study. In the example, the credit risk margin in the structural and the Hull and White models equals $CRM^{SM} = 0.73\%$ and $CRM^{HW} = 0.96\%$,

TABLE I
Exemplary Discount Certificate, Exemplary Market Data, and Fair Values

<i>Discount certificate</i>			
Time-to-maturity	T		1.5
Cap	X		95
Continuously compounded interest rate	r		3%
<i>Issuer</i>			
Initial total asset value	V_0		10,000
Default point	D		9,500
Volatility of asset value	σ_V		3.75%
Recovery rate	δ		0.5
<i>Underlying</i>			
Initial stock price	S_0		100
Stock price volatility	σ_S		30%
Correlation	ρ		0.5
<i>Fair values</i>			
	<i>Black and Scholes</i>	<i>Structural model</i>	<i>Hull and White</i>
Zero bond	90.82	89.95	89.95
Put	9.79	9.51	9.69
Discount certificate	81.03	80.44	80.26
Credit risk margin (CRM)	–	0.73%	0.96%

Note. This table shows an exemplary discount certificate and exemplary input data for the valuation models. Based on these data, the zero bond, the put, and the discount certificate are valued according to three models: the default-free Black and Scholes (1973) model (see (17)), the structural model based on Klein (1996) (see (16)), and the Hull and White (1995) model (see (18)). The credit risk margin (CRM) is defined as the relative difference between the Black and Scholes value and the Hull and White value, and the value of the discount certificate according to the structural model, respectively (see (19)).

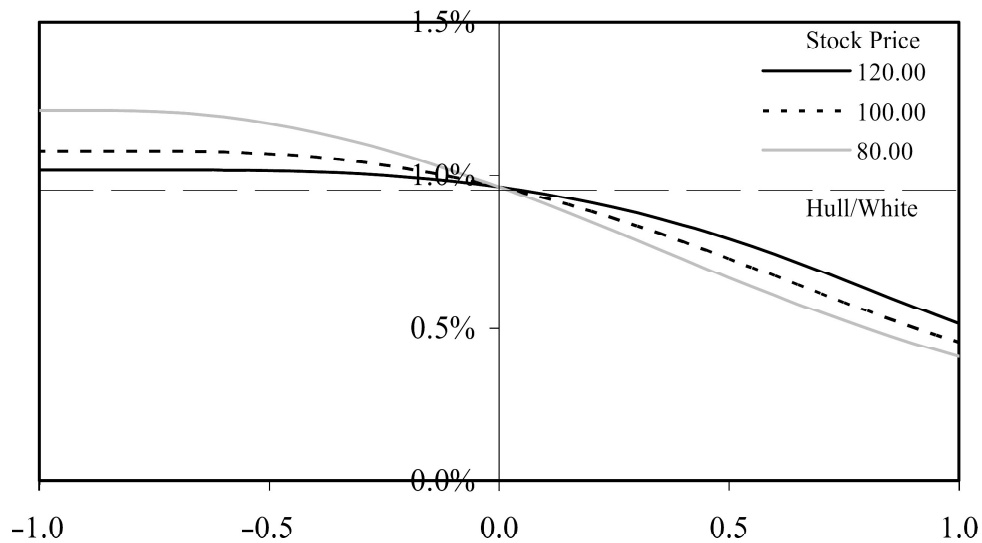


FIGURE 1
Credit Risk Margin of the Discount Certificate with Respect to Correlation for Different Stock Prices.

Note. This figure shows the credit risk margin of the exemplary discount certificate (see Table 1) with respect to correlation between the issuer's asset value and the stock price for different stock prices. The credit risk margin (*CRM*) is defined as the relative difference between the Black and Scholes value and the Hull and White value (slim dashed line), and the value of the discount certificate according to the structural model (other lines), respectively (see (19)).

respectively. As the credit risk margin is based on model values, it is, in the authors' later empirical analysis, a theoretical margin rather than a strictly "observable" margin. It depends on the respective model assumptions and on the calibration of the models to the market data.

Before presenting the empirical analysis, the effect of the correlation between the issuer's asset value and the stock price, i.e., the underlying of the discount certificate, is briefly analyzed. Figure 1 shows the credit risk margin for the structural model with respect to the correlation $\rho_{V,S}$ for different stock prices. The respective value for the Hull and White model is included as a benchmark (dashed line). Obviously, the credit risk margin is a decreasing function of correlation. This was to be expected, because the final payoff of the certificate is a non-decreasing function of the stock price; so a positive correlation reduces the impact of default risk. Furthermore, for positive correlations, the credit risk margin decreases with the moneyness (stock price/cap) of the embedded put option.

EMPIRICAL ANALYSIS

Preliminaries

The aim of this empirical analysis is to measure the share of the margin incorporated in the quotes of discount certificates, which is due to credit risk (the credit risk margin). To analyze the relative importance of the credit risk margin, it is

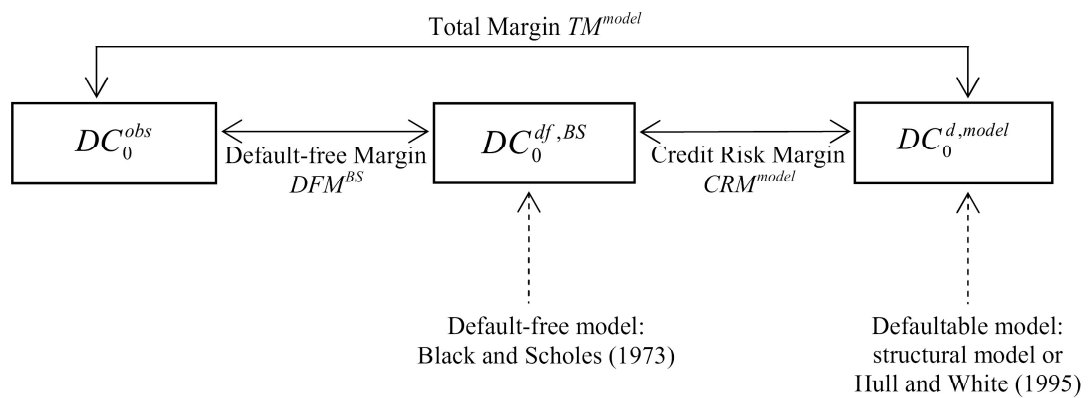


FIGURE 2
Decomposition of the Total Margin.

compared with the total margin TM . The total margin is defined as the relative difference between the observed quote DC_0^{obs} and the theoretical fair value according to the structural model and the Hull and White model, respectively:

$$TM^{model} = \frac{DC_0^{obs} - DC_0^{d,model}}{DC_0^{d,model}}, \text{ model} = SM, HW. \quad (20)$$

The default-free margin DFM^{BS} , which is equivalent to the total margin for the Black and Scholes model, is defined analogously¹³:

$$DFM^{BS} = \frac{DC_0^{obs} - DC_0^{df,BS}}{DC_0^{df,BS}}. \quad (21)$$

The total margins TM^{HW} and TM^{SM} can be “decomposed”¹⁴ into the respective credit risk margins CRM^{HW} and CRM^{SM} , which reflect the bank’s gain related to its credit risk, and the default-free margin DFM^{BS} , which reflects the bank’s gain if it were default-free. To facilitate clarity, Figure 2 summarizes the margin definitions.

Data and Empirical Methods

The data sample consists of discount certificates on DAX stocks issued by European banks. The analysis is based on data from February 27, 2004. Selected for analysis are those exchange-listed issuers whose number of outstanding discount certificates exceeds 50: Commerzbank (CBK), Deutsche Bank (DBK),

¹³ Similar to the credit risk margin and in line with all earlier empirical studies, both the total margin and the default-free margin should be considered as theoretical margins as they are based on model values and, hence, depend on the respective model assumptions.

¹⁴ Note that this is not a perfect decomposition, as the Black and Scholes total margin and the credit risk margin do not add up to the total margin because relative differences are considered. However, the differences are negligible.

BNP Paribas (BNP), Société Générale (SGE), and UBS. To have a homogenous sample, only certificates with a remaining time to maturity of between 0.5 and 2.0 years are considered. It appears to be impossible to estimate the issuers' credit spreads properly for shorter maturities. Certificates with longer maturities are relatively rare and options with these maturities are rather illiquid. Further, those certificates for which the corresponding implied EUREX option volatility could not be properly estimated are excluded. From the five issuers, 1,722 discount certificates meet these criteria (see Table II for details).

Quotes for discount certificates were obtained from the EUWAX in Stuttgart, which is the most commonly used exchange for SFPs in Germany. According to the market making of the issuers, these quotes are updated on a high-frequency basis. Mid-quotes were calculated from the bid-ask quotes.

For the default-free spot rate structure, data provided by the Deutsche Bundesbank and the Schweizerische Nationalbank were applied.¹⁵ Both central banks use an extension of Nelson and Siegel (1987) to estimate governmental spot rate curves, which are assumed to coincide with default-free interest rates.

In contrast to earlier studies, this study concentrates on bank-specific credit risk and does not rely on issuer-averaging (rating-specific) or maturity-averaging spread curves offered by investment banks or exchanges. To estimate bank-specific zero spread curves, quotes of senior unsecured EUR-denominated straight bonds issued by the five banks with maturities between 0.5 and 2.5 years are used.¹⁶ It is well known that bond liquidity usually declines with decreasing remaining time to maturity and decreasing amount issued. Furthermore, available matrix quotes offered in financial information systems are often not to be trusted. Those bonds are identified for which it can be ascertained that they have been traded at the main exchanges in Germany, France, or Switzerland on the analysis date or for which are found binding bid-ask quotes offered by a market maker of the bonds or by main brokers with reasonable amounts on each side. These data are used to directly estimate a linear zero spread curve over the German governmental spot rate curve for each issuer. Considering the short maturity period of 2 years, the linear shape makes sense. The curve is fitted to minimize the sum of squared differences between the observed quotes and the calculated prices applying nonlinear optimization procedures.

¹⁵ Throughout the analysis, the EUR-spreads over the German governmental spot rate curve are estimated. Only a small number of EUR-denominated bonds issued by UBS are found that fulfill certain criteria defined below. To solve this problem, the CHF spot rate curve provided by Schweizerische Nationalbank and the spot exchange rate (EUR-CHF) are used to estimate forward exchange rates. With these exchange rates, EUR-spreads from CHF-denominated bonds can be calculated.

¹⁶ As there are several bonds in the range between 2 and 2.5 years, it was decided to include them in the authors' sample, even though discount certificates with these maturities were not taken into consideration.

TABLE II
Data Sample: Discount Certificates on DAX Stocks

<i>Rating</i>	<i>BNP</i>	<i>CBK</i>	<i>DBK</i>	<i>SGE</i>	<i>UBS</i>	
<i>Moody's</i>	<i>AA2/P-1</i>	<i>A2/P-1</i>	<i>AA3/P-1</i>	<i>AA3/P-1</i>	<i>AA2/P-1</i>	
<i>S&P</i>	<i>AA-/A-1+</i>	<i>A-/A-2</i>	<i>AA-/A-1+</i>	<i>AA-/A-1+</i>	<i>AA+/A-1+</i>	
Underlying						Sum
ADS	8	26	9			43
ALT	7		5	2		14
ALV	7	27	17	7	51	109
BAS	7	16	8	3	32	66
BAY	6	34	13	3	25	81
BMW	8	13	8	1	10	40
CBK	4	12	5	3		24
CON	4	17	6			27
DB1			7	3		10
DBK	10	18	18	4	46	96
DCX	6	20	16	3	44	89
DPW	9	17	7			33
DTE	7	26	14	4	50	101
EOA	8	17	11	2	30	68
FME	8	1	7	1		17
HEN3	8		6			14
HVM	5	26	13	4		48
IFX	11	17	13	3	16	60
LHA	8	13	12	2	28	63
LIN	8		6			14
MAN	8		9			17
MEO	7		9			16
MUV2	15	36	23	8	48	130
RWE	9	22	15	1	22	69
SAP	12	29	17	3	72	133
SCH	8	11	10	1	24	54
SIE	11	34	21	4	61	131
TKA	6	23	7		10	46
TUI	6	22	10	4		42
VOW	10	10	19	3	25	67
Sum	231	487	341	69	594	1,722
Maturity						
Avg.	1.09	1.17	0.97	1.11	1.08	
Min.	0.80	0.58	0.57	0.56	0.59	
Max.	1.30	1.82	1.33	1.61	1.82	
Std.	0.25	0.39	0.27	0.23	0.37	

Note. This table shows the number of discount certificates in the data sample for each issuer (columns, abbreviated by ticker symbols, except for UBS) and underlying (rows, abbreviated by ticker symbols). The reference date is February 27, 2004. The last four rows show the average, minimum, maximum, and standard deviation of the maturities of the discount certificates for each issuer.

Stock prices are taken from the electronic trading system Xetra, and from floor trading on the Frankfurt Stock Exchange.¹⁷ Expected dividends are extracted from the financial data provider OnVista (www.onvista.de). Dividend payment dates are assumed to coincide with the day after the expected meeting dates of shareholders, which is common practice.

To estimate stock price volatility, daily settlement prices of put options on DAX stocks traded on the EUREX are applied. For each option, that stock price is used which is available from the Frankfurt Stock Exchange at the time the settlement price of the option is obtained. With these prices, the implied volatility surface is calculated. As stock options on the EUREX are American-style, a finite difference method is applied, which allows for the treatment of discrete dividends. The calculated market volatility is then used as the basis for an interpolation scheme to estimate the volatilities of the underlyings of the discount certificates. The procedure includes linear interpolation in the two-dimensional space formed by time to maturity and strike price. Basically, the implied volatility for the underlying of a single certificate is interpolated using those four EUREX options with maturity and strike price as close as possible to the respective values of the certificate.

Equity correlations between the banks and the underlying firms are estimated from historical continuously compounded daily stock returns of the relevant companies over a 250-day period. These correlations are used as proxies for the correlations between the asset returns of the banks and the returns of the underlyings.¹⁸ Finally, throughout the examination, a recovery rate of 50% is applied.

Calibration of the Model

This section describes the calibration of the models to the market data carried out to value a discount certificate with maturity T and cap X . Generally, the interest rate r is identified with the German governmental T -year spot rate. The calibration of the default-free Black and Scholes model is rather simple given the processing of the data as described before: the stock price of the underlying is observed, and the stock price volatility σ_S is extracted from the options

¹⁷ Some of these data were kindly made available to us by The Karlsruher Kapitalmarktdatenbank.

¹⁸ This proxy is also applied by the widely used credit risk model CreditMetrics; see Gupton, Finger, and Bhatia (1997). From a theoretical point of view, this assumption is not critical, because it is easy to show that the differences between the correlations are negligible in the authors' setting. On the contrary, there is empirical evidence that asset correlations are significantly smaller than the observed equity correlations, as shown, e.g., in the empirical study of Rösch (2003) for the German market. However, these studies generally focus on small and medium-sized enterprises, which can be assumed to also have much lower asset correlations than larger companies. The blue chips considered here should be much better diversified and hence more highly correlated. See Hahnenstein (2004) for empirical evidence. Equity correlations still seem to be the best available proxy for the unobservable asset correlations. For a detailed discussion, see Düllmann and Scheule (2003).

market, using the already-explained procedure for strike price X and expiry date T . Throughout the calibration, the stock price is adjusted by the net present value of dividends that are expected to be paid before T (e.g., Hull, 2006, p. 302–303). The Hull and White value is obtained by discounting the default-free value by the bank-specific T -year credit spread.

Within the structural model, the parameters concerning the issuer have to be calibrated. As a benchmark, the model should adequately explain the observed market values of standard securities issued by the bank. As discount certificates partly equal the straight zero bonds of the bank (see the Replicating Portfolio section), the model is calibrated in such a way that it explains the market prices of (synthetic) straight zero bonds. More precisely, the T -year credit spread s_T of the issuer must be matched. Through this matching, the market's evaluation of the issuer's credit risk can be transformed into a "spread" for the discount certificate that is consistent with the market. As there are discount certificates with different maturities in the authors' data sample, the model is recalibrated for each certificate.

The parameters determining the spread s_T (see (10) and (9)) are the pseudo-leverage of the bank V_0/D , the volatility of the asset value σ_V , and the recovery rate δ . In practice, however, V_0/D and σ_V are not directly observable. If δ is fixed, there remains 1 degree of freedom. This might represent a source of uncertainty or model risk. However, the values of the discount certificate turn out to be independent of the concrete values of the issuer's parameters V_0/D and σ_V , as long as the parameter combination perfectly explains the observed issuer's spread s_T . To prove this, note that the spread depends on b_2 . Consequently, if the unobservable variables are chosen to explain the spread, b_2 is independent of the concrete values. This consideration applies to a_2 , as well. a_1 and b_1 are independent of the issuer's variables, anyway. As the value of the discount certificate does not depend on V_0/D and σ_V , but on a_1 , a_2 , b_1 , and b_2 (see (16) and (12)–(15)), the spread directly transfers into the value of the discount certificate.

Accordingly, reasonable assumptions about the issuer's default point and the asset value can be made. For the default point, the notional amount of outstanding debt is chosen, and the asset value is approximated by the market value of equity plus the book value of debt. Using the zero bond spread (10) implied by the model, together with (9), the (implied) asset volatility σ_V can then be calculated by solving the resulting quadratic equation, which yields

$$\sigma_V = -\frac{N^{-1}(u)}{\sqrt{T}} + \sqrt{\frac{N^{-1}(u)^2}{T} + 2r + \frac{2\ln(V_0/D)}{T}} \quad (22)$$

with

$$u = \frac{e^{-s_T T} - \delta}{1 - \delta}. \quad (23)$$

After the calibration of the issuer, the underlying can be treated the same way as in the Black and Scholes model. The remaining parameter is the correlation $\rho_{V,S}$ between the asset value of the issuer and the stock, which is identified with the historical equity correlation as discussed above.

As already shown, the model outputs are invariant with respect to the choice of the pseudo-leverage V_0/D , as long as the observed spread is explained in the model. Furthermore, several robustness checks have shown that the later empirical results are very insensitive with respect to the recovery rate δ , which allows us to apply the rather rough estimate of 50%.

Empirical Results

Figure 3 shows the credit risk margin (19) as a function of the time-to-maturity. Each dot refers to a single discount certificate in the data sample when the structural model is applied. Somehow surprising is the large range of values that occur for all maturities, even for one issuer, indicating that the other model parameters, particularly correlation, can have a strong impact on the empirical credit risk margin. This result confirms the demand for an advanced model, because the Hull and White approach yields the same credit risk margin for all certificates with identical time-to-maturity (see Figure 3).

Table III shows average empirical margins for the authors' sample, grouped by issuer. Given that the correlation between issuer and underlying is positive in all cases, the Hull and White total margin is always larger than the

TABLE III
Average Margins by Issuer and Model

	BNP (%)	CBK (%)	DBK (%)	SGE (%)	UBS (%)
<i>Total margin (TM)</i>					
Hull and White	1.39	1.01	0.78	2.39	0.87
Structural model	1.29	0.91	0.67	2.27	0.84
<i>Default-free margin (DFM)</i>					
Black and Scholes	0.97	0.65	0.39	1.97	0.60
<i>Credit risk margin (CRM)</i>					
Hull and White	0.42	0.36	0.40	0.40	0.27
Structural model	0.32	0.25	0.29	0.29	0.24
<i>Fraction CRM/TM</i>					
Hull and White	30	35	51	17	31
Structural model	25	28	42	13	28

Note. This table reports average margins in the quotes of discount certificates (see Table 2) calculated for each issuer (columns) and model (rows). The total margin (*TM*) is the relative difference between the quoted price and the Hull and White value (see (18)) and the value of the discount certificate according to the structural model (see (16)), respectively. The default-free margin (*DFM*) is the relative difference between the quoted price and the Black and Scholes value (see (17)). The credit risk margin (*CRM*) is defined as the relative difference between the Black and Scholes value and the Hull and White value, and the value of the discount certificate according to the structural model, respectively (see (19)).

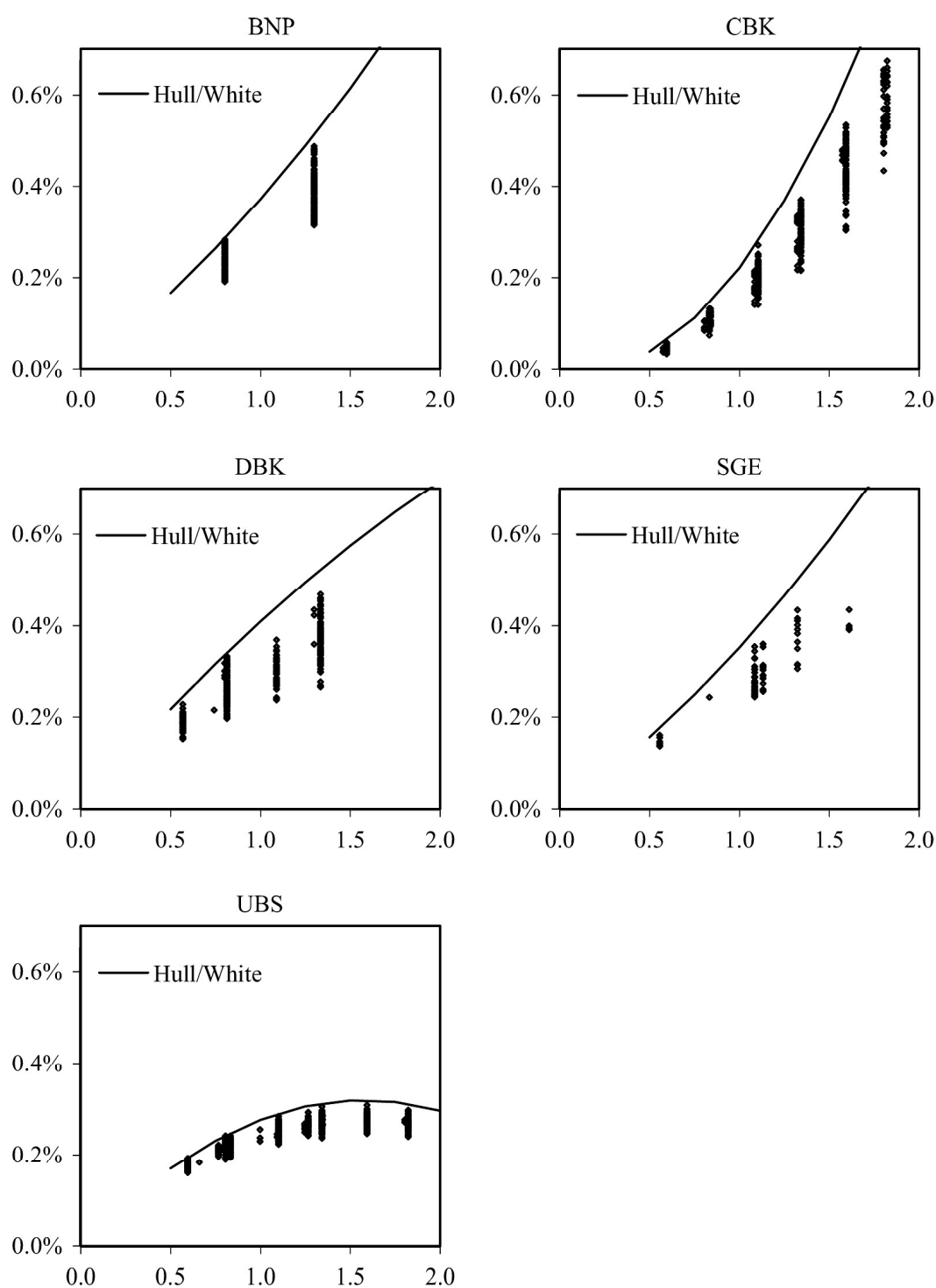


FIGURE 3

Credit Risk Margin for Each Issuer with Respect to the Time-to-Maturity.

Note. This figure shows the credit risk margins for each issuer with respect to the time-to-maturity for the data sample (see Table II). The credit risk margin is defined as the relative difference between the Black and Scholes value (see (17)) and the Hull and White value (line) (see (18)) and the value of the discount certificates according to the structural model (dots) (see (16)), respectively (see (19)).

“true” margin calculated by the structural model. Naturally, the default-free margin is the smallest.

The absolute values of the empirical total margins among the different issuers are a point of interest on their own. On average, Deutsche Bank incorporates the lowest total margins (0.67%, structural model) in the sample, i.e., quoted prices are on average most customer-friendly. Deutsche Bank is followed by UBS (0.84%), Commerzbank (0.91%), and BNP Paribas (1.29%), whereas the highest average margin is found for Société Générale (2.27%). This ranking is consistent with the findings of Wilkens, Erner, and Röder. (2003). However, based on a data set from late 2001, they apply different theoretical and empirical methods and report much higher margins.¹⁹ The low total margins found in the authors’ analysis may also reflect the rising competition in this segment of the German retail market since 2001.

The low total margins underline the importance of credit risk, because the credit risk margin becomes a growing share of the total margin. For Deutsche Bank for instance, it reaches an average proportion of more than 40%. Credit risk margin should likewise not be neglected for the other issuers. Interestingly, with the exception of UBS, the Hull and White model overestimates the credit risk margin by up to 50%. In other words, assuming an average correlation of zero leads to strongly biased results. For the non-Euro-land-based UBS, however, correlations are significantly lower, so the differences are smaller as well.

CONCLUSIONS

In this study, three models for valuing discount certificates are compared: the standard default-free model of Black and Scholes (1973), the Hull and White (1995) model assuming independence between market and credit risk, and a structural model allowing for the incorporation of correlation effects. It is illustrated that the Hull and White approach typically overestimates the credit risk margin.

The empirical investigation pointed out that the value of credit risk for discount certificates is an important part of the banks’ margins, up to 42% of the total margin in the authors’ sample. Therefore, the importance of the credit risk margin is not solely an academic phenomenon. Given the fact that total margins seem to have decreased compared to earlier studies, credit risk margins become

¹⁹Wilkins and Röder (2003) analyze the impact of stochastic volatilities on the valuation of default-free discount certificates. They find in an example-driven study that discount certificates may be undervalued when constant volatility is assumed. This could imply that bank margins reported in earlier studies might be clearly too high due to model misspecification. However, for judging the final effect of stochastic volatilities on the estimation of bank margins, both the standard Black and Scholes model and the extended model with stochastic volatilities should be calibrated to real data from the option market.

more important. These margins will gain even more importance if the unusually small financial bond spreads in Europe in 2004 rise to a historical average level in the future and if markets for SFPs become even more competitive.

Private investors participating in derivative markets will certainly become more familiar with standard pricing models for default-free securities. In contrast, they will generally still ignore the issuers' credit risk or at least the correlation effects. Hence, the proper credit risk margin can be seen as a partly hidden margin. Thus banks could, *ceteris paribus*, increase their margin income by choosing underlyings for their discount certificates with low correlations.

APPENDIX FORMULAS

The Issuer

Under the risk-neutral measure, the asset value of the issuer V_t follows geometric Brownian motion with drift r and variance rate σ_V^2 . The issuer defaults if and only if this value falls below the default point D at maturity T . In this case all creditors, including the holders of the discount certificate with maturity T and cap X , receive only a fraction δ of their claim.

Vulnerable zero bond with face value 1

$$ZB_0^d = e^{-rT}(1 + (\delta - 1)N(-b_2))$$

with b_2 defined below.

The Underlying

Under the risk-neutral measure, the underlying stock S_t follows geometric Brownian motion with drift r and variance rate σ_S^2 . The processes V_t and S_t are correlated with $\rho_{V,S}$. The respective values can be calculated as follows.

Vulnerable stock certificate

$$S_0^{d,SM} = S_0(\delta + (1 - \delta)N(a_2))$$

Vulnerable call

$$c_0^{d,SM} = S_0(N(a_1, a_2, \rho_{V,S}) + \delta N(a_1, -a_2, \rho_{V,S})) \\ - e^{-rT} X(N(b_1, b_2, \rho_{V,S}) + \delta N(b_1, -b_2, -\rho_{V,S}))$$

Vulnerable put

$$p_0^{d,SM} = -S_0(N(-a_1, a_2, -\rho_{V,S}) + \delta N(-a_1, -a_2, \rho_{V,S})) \\ + e^{-rT} X(N(-b_1, b_2, -\rho_{V,S}) + \delta N(-b_1, -b_2, \rho_{V,S}))$$

with

$$a_1 = \frac{\ln(S_0/X) + (r + \sigma_S^2/2)T}{\sigma_S \sqrt{T}}$$

$$a_2 = \frac{\ln(V_0/D) + (r - \sigma_V^2/2 + \rho_{V,S} \sigma_S \sigma_V)T}{\sigma_V \sqrt{T}}$$

$$b_1 = \frac{\ln(S_0/X) + (r - \sigma_S^2/2)T}{\sigma_S \sqrt{T}} = a_1 - \sigma_S \sqrt{T}$$

$$b_2 = \frac{\ln(V_0/D) + (r - \sigma_V^2/2)T}{\sigma_V \sqrt{T}} = a_2 - \rho_{V,S} \sigma_S \sqrt{T}.$$

Proof: The value of a vulnerable call is given by Ammann (2001, p. 83). The value of a vulnerable stock certificate can easily be obtained by $S_0^{d,SM} = \lim_{X \rightarrow 0} c_0^{d,SM}$. As the put-call parity holds, the value of a vulnerable put is $p_0^{d,SM} = ZB_0^d X - S_0^{d,SM} + c_0^{d,SM}$.

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