

# Valuation differences between credit default swap and corporate bond markets

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*This paper quantifies and explains the valuation differences between credit default swaps (CDSs) and corporate bonds from a sample of European investment-grade firms. Based on all information gained through the calibration of a stochastic intensity credit model to the time series of the issuer's CDS curve, we define a new corporate-bond-specific measure of the valuation difference. Our results show that, on average, risk premiums implied in corporate bonds exceed those in CDS markets by a much smaller extent than found in previous studies. Using panel data analysis, we detect a cross-sectional influence of bond liquidity measures and find a significant impact of the general level of credit risk on the time series variation of the valuation difference.*

## 1 INTRODUCTION

The rise of the credit default swap (CDS) markets during the last decade created, alongside corporate bond markets, an additional source of information about the default risk of individual firms. Under the efficiency assumption, credit risk premiums in both markets should be identical. However, in reality, this identity is perturbed by a wide range of different factors such as liquidity or differing supply and demand conditions.

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Numerous studies have recently investigated the interrelations between these two credit markets, analyzing either their comovements or the market price deviations between them. Most studies examining deviations in market prices either base their analyses on the difference between the credit spread and the CDS spread for a certain maturity – called the “basis” – or calibrate a credit model to a sample of corporate bonds to obtain model CDS spreads, which they compare with market CDS spreads. The first approach ignores contractual differences between the two credit instruments and therefore leads by construction to a distortion of the actual valuation difference. The drawback of the second approach results from the varying and often very small number of bonds that in most cases does not adequately reflect the full maturity spectrum. Calibrating a credit model to such data can lead to unreliable parameter estimates and can thus yield inadequate model-implied CDS spreads.

The aim of this paper is to define and calculate a new measure of the valuation difference between CDS and corporate bond markets and identify the factors driving its magnitude. We avoid the shortcomings of both abovementioned approaches by tackling the problem in question from the other end: we calibrate our credit model to the CDS curve rather than to corporate bonds, carefully integrate the differing contractual features in the valuation formulas and compare the resulting model prices for corporate bonds with their actual market prices.

In detail, our contribution to the literature is threefold: first, to the best of our knowledge, this paper is the first to source the pricing information solely from the CDS market rather than from corporate bonds as done in previous studies. We calibrate a stochastic intensity credit model to the time series of the ten-year CDS curve in order to contrast CDS market-implied corporate bond prices with their market counterparts. In doing so, we avoid the estimation problems resulting from the typically limited number of corporate bonds. In addition to estimational advantages, we consider this approach to be the more natural choice from an economic point of view, as, for market participants, the CDS curve serves as the primary seismograph of a firm’s healthiness and provides a more direct measure of its credit risk that is much less contaminated by nondefault components.

Second, our procedure allows us to calculate bond-specific rather than firm-specific valuation differences, as is done for the five-year maturity in most related papers. In doing so, we can investigate the impact of bond-specific factors on a bond’s valuation difference in more detail.

Third, we use panel data analysis in order to examine determinants of the valuation differences, rather than time series or cross-sectional ordinary least squares (OLS) as in related empirical studies. This allows us to capture the time series and the cross-sectional dimension of our data. In addition, panel data analysis also allows us to account for unobservable bond-specific heterogeneities and to obtain more efficient parameter estimates than through OLS.

Using a sample of highly liquid European investment-grade bonds, we find that, on average, risk premiums implied in the corporate bond market exceed those in the CDS market to a much smaller extent than found in previous studies during comparable time periods. Bond liquidity measures, such as the coupon rate and the amount outstanding of a bond, play an important role in the cross-sectional variation of the valuation difference, whereas the level of market-wide credit risk has a significant influence in the time series dimension.

The remainder of this paper is organized as follows. Section 2 provides an overview of the related literature. Section 3 presents the credit model we calibrate to the CDS panel data. Section 4 describes the data and the model estimation methodology and illustrates the results. Section 5 contrasts some statistics about the resulting valuation differences with previous studies. Section 6 introduces proxies for various fundamental and market factors potentially influencing the valuation difference and conducts a panel data analysis to determine their impact. Section 7 concludes.

## 2 RELATED LITERATURE

There are two main strands of research investigating the interrelations between CDS and corporate bond markets. The first one, pioneered by Blanco *et al* (2005), Zhu (2006) and Norden and Weber (2009), examines the comovements between the two credit markets. They find that CDS markets generally lead corporate bond markets in the price discovery process.

In this paper, we contribute to the second strand of studies that investigates deviations between the two markets. Hull *et al* (2004) calculate five-year corporate bond yields by regression and compare the resulting credit spreads, which they adjust for accrued interest, with five-year CDS spreads. Houweling and Vorst (2005) calibrate three different deterministic reduced-form models to corporate bond data using three different proxies for the risk-free benchmark. They contrast calculated model CDS spreads with market CDS spreads. Since using the swap curve rather than the government curve as the risk-free benchmark yields the smallest price differences, both studies conclude that markets started considering the swap curve as the new benchmark. In contrast to these studies, we consider the government curve to be the risk-free benchmark, especially in the face of the money market disruptions in 2008.

Cossin and Lu (2005) break down five-year credit spreads into liquidity and credit risk components and find that, on average, the difference between the five-year credit component and the five-year CDS spreads becomes almost zero. They show that, alongside the five-year CDS spread, the liquidity component plays an important role in explaining the time series variation of the credit spread, especially for firms with high credit quality. Jankowitsch *et al* (2008) consider the cheapest-to-deliver option in CDS contracts to be the primary cause for the valuation difference. They estimate

the expected minimum recovery rate implied in CDS and find that cheapest-to-deliver proxies rather than liquidity proxies explain a substantial part of their cross-sectional variation across firms. In contrast to these two studies, we do not focus on any particular effect but examine the influence of a wide range of different factors.

Our approach is closest to the studies conducted by Levin *et al* (2005) and Longstaff *et al* (2005). Levin *et al* (2005) find that, compared with general market factors, firm- and bond-specific factors play a dominant role in explaining the variation of the difference between the corporate bond's credit spread over the swap rate and the issuer's CDS spread for the same maturity. In contrast to them, we use a credit model which guarantees that our calculated measure of the valuation difference, unlike the basis, is not distorted by factors such as different payment frequencies of CDS and corporate bonds or by the fact that the corporate bond trades away from par.

Longstaff *et al* (2005) calculate a nondefault component in bond credit spreads by simultaneously calibrating a credit model to corporate bond yields and five-year CDS spreads. We differ from their approach in three aspects. First, we derive our hazard rate parameters solely from the information contained in the time series of the issuer's CDS curve without relying on any information contained in its corporate bonds. In doing so, we obtain hazard rate parameters that are completely independent of the issuer's corporate bonds whose valuation differences we calculate. Therefore, our resulting valuation difference for a certain corporate bond is not affected by other bonds of the same issuer being included in our data set. Second, we estimate a more realistic credit model by allowing for correlation between the hazard rate and short-rate processes. Third, we examine the impact of several additional factors on the valuation difference beyond liquidity.

### 3 CREDIT MODEL

A CDS is an over-the-counter contract between two parties, the protection buyer and the protection seller. Under the contract, the protection buyer can gain insurance against the default of a specified company by making fixed quarterly payments to the protection seller until the contract matures or a specified credit event is triggered, whichever occurs first. In the case of a credit event prior to maturity, the protection buyer has the right to deliver a corporate bond of the defaulted company to the protection seller and to receive the payment of the full face value of the bond.

#### 3.1 Credit default swap valuation

For the valuation of CDS and corporate bonds we choose the reduced-form approach in the sense of Duffie (1998), Lando (1998), Duffie and Singleton (1999) and others. Hence, we fix a probability space  $(\Omega, \mathcal{F}, \mathbb{Q})$  endowed with a filtration  $(\mathcal{F}_t)_{t \geq 0}$ .

satisfying the usual conditions, ie, it is complete and right-continuous. The filtration models the flow of information available to the market, ie,  $\mathcal{F}_t$  represents all events observable up until time  $t$ .  $\mathbb{Q}$  is an equivalent martingale measure, in the sense that all discounted assets are  $\mathbb{Q}$ -martingales, where the discounting is defined with respect to a money market account that accumulates at a default-free short-rate process  $r(t)$ . The physical probability measure, to which  $\mathbb{Q}$  is equivalent, is denoted by  $\mathbb{P}$ . A company's default time is characterized by the first jump time  $\xi$  of a Cox process. We denote the default indicator function by  $\mathbf{1}_{\{\xi \leq t\}}$  and the associated intensity process by  $h(t)$ . The value at time  $t$  of a promised payment of one unit of currency in  $s > t$  with a recovery of zero in the case of default is

$$\mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^s r(u) du \right) \mathbf{1}_{\{\xi > s\}} \right] = \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^s [r(u) + h(u)] du \right) \right], \quad (3.1)$$

where we set  $\mathbb{E}_t^{\mathbb{Q}}[\cdot] := \mathbb{E}^{\mathbb{Q}}[\cdot | \mathcal{F}_t]$ . The value at time  $t < \min(s, \xi)$  of a recovery payment  $\omega(\xi)$  due to a default during the time period  $[t, s]$  is given by

$$\begin{aligned} \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^{\xi} r(u) du \right) \mathbf{1}_{\{\xi \leq s\}} \omega(\xi) \right] \\ = \text{REC}(t) \int_t^s \mathbb{E}_t^{\mathbb{Q}} \left[ h(u) \exp \left( - \int_t^u [r(z) + h(z)] dz \right) \right] du, \end{aligned} \quad (3.2)$$

where  $\text{REC}(t)$  denotes the risk-neutral expectation in  $t$  of the recovery payment  $\omega(\xi)$ , which we assume to be independent of the processes  $r(t)$  and  $h(t)$ . Technical proofs of the identities (3.1) and (3.2) are given in Duffie (2001). These two building blocks can be used to express both sides of the CDS contract.

The premium leg of the contract at time  $t$  for an integer-valued contract maturity  $\tau$  is given by

$$\begin{aligned} \text{PL}(t, \tau) = \delta \text{CDS}(t, \tau) \sum_{n=1}^{4\tau} \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^{T_n} [r(u) + h(u)] du \right) \right] \\ + \delta \text{CDS}(t, \tau) \sum_{n=1}^{4\tau} \int_{T_{n-1}}^{T_n} \frac{u - T_{n-1}}{T_n - T_{n-1}} \\ \times \mathbb{E}_t^{\mathbb{Q}} \left[ h(u) \exp \left( - \int_t^u [r(z) + h(z)] dz \right) \right] du, \end{aligned} \quad (3.3)$$

where  $\text{CDS}(t, \tau)$  is the fixed annual  $\tau$ -year CDS premium, and  $\delta := \frac{1}{4} \frac{365}{360}$  is the day count fraction for quarterly premium payments, approximating the ACT/360 day count convention used in CDS markets. The first term of the premium leg is the present value of the quarterly-paid premiums, with the first payment being made in

$T_1$ . The second term is the present value of the accrued premiums to be paid in case of the occurrence of a credit event between two premium payment dates  $T_{n-1}$  and  $T_n$ .

The default leg of the contract can be expressed as

$$DL(t, \tau) = [1 - \text{REC}(t)] \int_t^{t+\tau} \mathbb{E}_t^{\mathbb{Q}} \left[ h(u) \exp \left( - \int_t^u [r(z) + h(z)] dz \right) \right] du. \quad (3.4)$$

Following the traditional “par spread” quotation instead of the recently introduced (economically equivalent) “upfront” quotation method for CDS, the two legs of the contract have to be equal at the inception date  $t$ . Consequently, the CDS premium at time  $t$  with contract maturity  $\tau$  can be expressed as

$$\begin{aligned} \text{CDS}(t, \tau) &= [1 - \text{REC}(t)] \int_t^{t+\tau} \mathbb{E}_t^{\mathbb{Q}} \left[ h(u) \exp \left( - \int_t^u [r(z) + h(z)] dz \right) \right] du \\ &\quad \times \left\{ \delta \sum_{n=1}^{4\tau} \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^{T_n} [r(u) + h(u)] du \right) \right] \right. \\ &\quad \left. + \delta \sum_{n=1}^{4\tau} \int_{T_{n-1}}^{T_n} \frac{u - T_{n-1}}{T_n - T_{n-1}} \right. \\ &\quad \left. \times \mathbb{E}_t^{\mathbb{Q}} \left[ h(u) \exp \left( - \int_t^u [r(z) + h(z)] dz \right) \right] du \right\}^{-1}. \quad (3.5) \end{aligned}$$

### 3.2 Corporate bond valuation

While in the case of the CDS contract we assumed the maturity  $\tau$  to be integer-valued, this restriction does not apply to corporate bonds. Hence, in order to value corporate bonds, we also account for non-integer-valued maturities  $\tau$ . Using the same reduced-form framework as above, the value of a corporate bond with term to maturity  $\tau$  and annual coupon rate  $C$ , issued by the same firm as the reference entity of the CDS contract, can be expressed as

$$\begin{aligned} \text{CB}(t, \tau) &= C \sum_{n=1}^{N(\tau)} \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^{\mathcal{T}_n} [r(u) + h(u)] du \right) \right] \\ &\quad + \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^{t+\tau} [r(u) + h(u)] du \right) \right] \\ &\quad + \text{REC}(t) \int_t^{t+\tau} \mathbb{E}_t^{\mathbb{Q}} \left[ h(u) \exp \left( - \int_t^u [r(z) + h(z)] dz \right) \right] du. \quad (3.6) \end{aligned}$$

The number of coupon payments is defined by  $N(\tau) := \lfloor \tau \rfloor + \mathbf{1}_{\mathbb{R}^+ \setminus \mathbb{N}}(\tau)$ , where  $\lfloor \tau \rfloor$  denotes the integral part of the nonnegative real number  $\tau$ . The coupon payment dates are  $\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3, \dots, \mathcal{T}_{N(\tau)} \equiv t + \tau$ . The first term of this expression is the present value

of the coupon rates and the second term is the present value of the face value of the bond repaid at maturity. The last term is the present value of the expected recovery payment in the case of a default of the company.

### 3.3 Term structure and hazard rate model

In order to use a more realistic credit model than previous studies, one which allows for correlation between the short-rate and hazard rate processes, we parameterize our model following the idea of Duffee (1999). We specify the short-rate  $r(t)$  as a three-factor generalization of the short-rate process proposed by Cox, Ingersoll and Ross (CIR) (Cox *et al* 1985), since we obtain a rather poor fit in the case of the one- and two-factor specifications, especially at the short end of the spot rate curve. Thus,  $r(t)$  is the sum of three economy-wide latent state variables:

$$r(t) = X_1(t) + X_2(t) + X_3(t), \quad (3.7)$$

with the three state variables following square-root processes, ie,

$$dX_i(t) = \kappa_i(\theta_i - X_i(t)) dt + \sigma_i \sqrt{X_i(t)} dW_i^{\mathbb{P}}(t), \quad i = 1, 2, 3. \quad (3.8)$$

To capture credit risk, we define a company-specific distress variable  $Z(t)$  also following a square-root process:

$$dZ(t) = \kappa_z(\theta_z - Z(t)) dt + \sigma_z \sqrt{Z(t)} dW_z^{\mathbb{P}}(t), \quad (3.9)$$

where  $W_1^{\mathbb{P}}(t)$ ,  $W_2^{\mathbb{P}}(t)$ ,  $W_3^{\mathbb{P}}(t)$  and  $W_z^{\mathbb{P}}(t)$  are independent standard Brownian motions under the physical measure  $\mathbb{P}$ . We specify the market price of risk for the four variables as  $\Gamma_i(t) = \lambda_i \sqrt{X_i(t)}/\sigma_i$  for  $i = 1, 2, 3$  and  $\Gamma_z(t) = \lambda_z \sqrt{Z(t)}/\sigma_z$  and use the Girsanov theorem to obtain their dynamics under the equivalent martingale measure  $\mathbb{Q}$  as

$$\begin{aligned} dX_i(t) &= [\kappa_i \theta_i - (\kappa_i + \lambda_i) X_i(t)] dt + \sigma_i \sqrt{X_i(t)} dW_i^{\mathbb{Q}}(t), \quad i = 1, 2, 3, \\ dZ(t) &= [\kappa_z \theta_z - (\kappa_z + \lambda_z) Z(t)] dt + \sigma_z \sqrt{Z(t)} dW_z^{\mathbb{Q}}(t), \end{aligned}$$

where  $W_1^{\mathbb{Q}}(t)$ ,  $W_2^{\mathbb{Q}}(t)$ ,  $W_3^{\mathbb{Q}}(t)$  and  $W_z^{\mathbb{Q}}(t)$  are independent standard Brownian motions under  $\mathbb{Q}$ .

Using the specification of Duffee (1999), we define the hazard rate process  $h(t)$  as an affine-linear function of the four variables:

$$h(t) = \Lambda_0 + \Lambda_1[X_1(t) - \bar{X}_1] + \Lambda_2[X_2(t) - \bar{X}_2] + \Lambda_3[X_3(t) - \bar{X}_3] + Z(t), \quad (3.10)$$

where  $\Lambda_1$ ,  $\Lambda_2$  and  $\Lambda_3$  capture the correlation between the short-rate and the hazard rate process, and  $\bar{X}_1$ ,  $\bar{X}_2$  and  $\bar{X}_3$  denote the time series averages of  $X_1(t)$ ,  $X_2(t)$  and  $X_3(t)$ , respectively.

### 3.4 Computational feasibility

In order to ensure the computational feasibility of our credit model, we need closed-form expressions of the conditional expectations on the right-hand sides of the identities (3.1) and (3.2). Therefore, we follow the concept developed by Duffie *et al* (2000) and the implementation by Zhang (2008), and define the conditional discounted characteristic function of  $h(t + \tau)$ :

$$\Phi(t, t + \tau; \phi) := \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^{t+\tau} [r(u) + h(u)] du + i\phi h(t + \tau) \right) \right], \quad (3.11)$$

with the boundary condition  $\Phi(t + \tau, t + \tau; \phi) = \exp(i\phi h(t + \tau))$  and the imaginary unit  $i := \sqrt{-1}$ . The characteristic function has a closed-form solution of the exponentially affine form

$$\Phi(t, t + \tau; \phi) = \exp \left( \mathcal{A}(\tau; \phi) - \sum_{i=1}^3 \mathcal{B}_i(\tau; \phi) X_i(t) - \mathcal{B}_z(\tau; \phi) Z(t) \right), \quad (3.12)$$

where the functions  $\mathcal{A}$ ,  $(\mathcal{B}_i)_{i=1,2,3}$  and  $\mathcal{B}_z$  are given in Appendix A. By evaluating the characteristic function and its partial derivative with respect to  $\phi$  in  $\phi = 0$ , we can express both the conditional expectations on the right-hand sides of (3.1) and (3.2) in closed form.

Therefore, using the characteristic function, the premium leg of the CDS contract in (3.3) can be stated as

$$\begin{aligned} \text{PL}(t, \tau) &= \delta \text{CDS}(t, \tau) \sum_{n=1}^{4\tau} \Phi(t, T_n; \phi = 0) \\ &+ \delta \text{CDS}(t, \tau) \sum_{n=1}^{4\tau} \int_{T_{n-1}}^{T_n} \frac{u - T_{n-1}}{T_n - T_{n-1}} \frac{1}{i} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} du, \end{aligned} \quad (3.13)$$

where the details of the closed-form solutions of  $\Phi(t, t + \tau; \phi = 0)$  and

$$\frac{1}{i} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0}$$

are given in Appendix B. Accordingly, the default leg in (3.4) can be written as

$$\text{DL}(t, \tau) = [1 - \text{REC}(t)] \int_t^{t+\tau} \frac{1}{i} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} du. \quad (3.14)$$



Hence, the CDS premium in (3.5) can be expressed as

$$\begin{aligned} \text{CDS}(t, \tau) = & [1 - \text{REC}(t)] \int_t^{t+\tau} \frac{1}{i} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} du \\ & \times \left( \delta \sum_{n=1}^{4\tau} \Phi(t, T_n; \phi = 0) \right. \\ & \left. + \delta \sum_{n=1}^{4\tau} \int_{T_{n-1}}^{T_n} \frac{u - T_{n-1}}{T_n - T_{n-1}} \frac{1}{i} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} du \right)^{-1}. \end{aligned} \quad (3.15)$$

Finally, in this same valuation framework, the value of the corporate bond in (3.6) is given by

$$\begin{aligned} \text{CB}(t, \tau) = & C \sum_{n=1}^{N(\tau)} \Phi(t, T_n; \phi = 0) + \Phi(t, t + \tau; \phi = 0) \\ & + \text{REC}(t) \int_t^{t+\tau} \frac{1}{i} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} du. \end{aligned} \quad (3.16)$$

## 4 DATA AND ESTIMATION METHODOLOGY

### 4.1 Data description

#### 4.1.1 Credit default swap data set

We extract our credit default swap data from the Markit database. Markit composes its CDS quotes from pricing information contributed by a broad range of market makers on a daily basis. Markit quotes are widely used by financial institutions for mark-to-market and risk management purposes. We downloaded the CDS pricing information from January 2001 to January 2005 for euro-denominated contracts on senior unsecured debt for the 125 constituent companies of the iTraxx Europe index, which are the most liquid European investment grade companies. The restructuring clause of the contracts is complete restructuring (CR), under which any restructuring type qualifies as a credit event and senior unsecured bonds with maturities up to thirty years count as deliverable obligations. The CDS data set consists of daily CDS spreads for the one-, two-, three-, five-, seven- and ten-year maturities and the expected recovery rate for every company. As the CDS time series during the first two years are characterized by a large proportion of missing or stale spreads, we limit our analysis to the sample period from January 2003 to January 2005. Furthermore, we exclude all companies that have more than two consecutive stale spreads in any of the six available contract maturities, except for year-end holidays. We treat the remaining daily stale spreads as missing values and fill these gaps by linear interpolation.

### 4.1.2 Corporate bond data set

We download daily corporate bond quotes from Datastream for all available iTraxx Europe constituent companies for the sample period. We only consider euro-denominated senior unsecured plain vanilla bonds. The corporate bond data set contains clean prices for 970 bonds issued by 106 companies. We only consider bonds that touch the maturity range from one to ten years during our sample period and have an issued amount of at least €125 million. Furthermore, we exclude all bonds that have more than two consecutive daily stale prices, except for year-end holidays. Finally, we only consider companies with at least two remaining bonds.

Imposing these quality criteria on the daily time series and merging the corporate bond and CDS data sets leaves 137 bonds issued by 29 companies and their CDS curves, which form our final data sample. Since we use a stochastic credit model in our analysis, we reduce our data sample to a weekly frequency by choosing the observations on every Wednesday between January 1, 2003 and January 5, 2005, resulting in 106 data points.

Although these strict filtering criteria, which we apply to our data in order to guarantee a meaningful analysis, eliminate most companies and corporate bonds, the extent of our final data sample is still comparable with the data samples used in previous studies and ensures the empirical representativeness of our results. Table 1 on page 14 illustrates the maturity structure of our corporate bond data for every company and reveals that we have a sufficient number of bonds in all nine maturity bands.

### 4.1.3 Interest rate data set

Contrary to some of the previous studies considering the swap curve as the risk-free benchmark, we choose interest rates of euro-denominated German government bonds as our risk-free rates. This choice is supported not only by Duffie and Singleton (1997) and Liu *et al* (2006), who find both liquidity and default risk components in swap rates, but also by the recent money market disruptions in the year 2008. Therefore, we use the ten-year spot rate curve, calculated by the Deutsche Bundesbank from German government bond yields using the method proposed by Svensson (1994). The spot rate curve consists of the ten maturities of the interest rate curve from 1Y to 10Y, ie, 1Y, 2Y, 3Y, 4Y, 5Y, 6Y, 7Y, 8Y, 9Y and 10Y.

## 4.2 Estimation of the credit model

We calibrate our credit model in two steps: in the first step, we calibrate the short-rate process to our interest-rate data using the linear Kalman filter combined with a quasimaximum likelihood (QML) estimation. In the second step, we use the results obtained from the first step and carry out a QML estimation for each of the twenty-nine

companies separately in order to obtain their hazard rate parameters. Since we use panel data for both the interest rate and the CDS calibration, we are able to estimate the market price of risk parameters and the time series of all state and company-specific variables.

#### 4.2.1 Estimation of the interest rate parameters

The Kalman filter technique has been applied to the calibration of multifactor CIR models, by, among others, Duan and Simonato (1999), Geyer and Pichler (1999), de Jong (2000), Chen and Scott (2003) and Hördahl and Vestin (2005). The details of the calibration methodology for the three-factor CIR model are described in Appendix C. Table 2 on page 16 shows the estimates of the interest rate parameters with their standard errors in parentheses.

The parameter estimates are all in line with previous studies, ie, they all have the expected sign and a realistic magnitude. The rate of mean reversion parameter estimates  $\kappa_i$  indicate a mean half-life of 0.55, 1.20 and 3.31 years for the three respective state variables. The long-term mean parameter estimates  $\theta_i$  add up to about 1.6. The expected negative sign and the magnitude of the market price of risk parameter estimates  $\lambda_i$  lead to a negative speed of mean reversion ( $\kappa_i + \lambda_i$ ) under the measure  $\mathbb{Q}$  for the second and third state variables. Parallels to previous studies also include the fact that most estimates lack statistical significance. While the volatility parameters  $\sigma_i$  are statistically significant, the long-term mean parameter estimates especially are only fractions of their estimated standard errors. However, as we intend not to forecast future movements of the spot rate curve but instead to capture the interest rate process during our sample period, the lack of statistical significance is not an issue of concern. Table 3 on page 16 demonstrates that our interest rate model is capable of performing this task quite well.

The average mean absolute error (MAE) over all ten maturities is 0.77bps. Except for the MAE of the one-year maturity (3.96bps), where affine term structure models like the three-factor CIR usually show a poorer fit due to a higher volatility at the short end of the interest rate curve, all MAEs are well below 1bp. In relative terms, the mean absolute percentage error (MAPE) of the one-year maturity is 1.79%, whereas the MAPEs for all other maturities are less than or equal to 0.25%.

#### 4.2.2 Estimation of the hazard rate parameters

We use the estimates of the interest rate parameters and the associated time series of the three state variables as inputs and calibrate the hazard rate process of our credit model to the CDS data for each of the twenty-nine companies separately. We use the inversion methodology also used in Zhang (2008), under which we assume that the five-year CDS spreads are observed without error and that the one-, three- and

**TABLE 1** Corporate bond maturity structure. [Table continues on next page.]

Company	Maturity band										Σ
	1Y-2Y	2Y-3Y	3Y-4Y	4Y-5Y	5Y-6Y	6Y-7Y	7Y-8Y	8Y-9Y	9Y-10Y		
Abbey National	2	—	2	—	1	1	—	—	—	—	6
Allianz	—	—	—	2	1	—	—	—	1	—	4
Allied Domecq	—	1	1	—	—	1	—	—	—	—	3
Arcelor	—	—	—	—	1	—	1	—	—	—	2
HypoVereinsbank	—	—	1	—	—	—	1	—	—	—	2
British American Tobacco	1	—	1	—	—	1	1	—	1	—	5
Carrefour	1	—	1	1	—	1	1	1	—	—	6
Casino Guichard	—	1	1	1	1	—	2	—	1	—	7
Commerzbank	—	—	—	—	—	1	1	1	—	—	3
St. Gobain	—	1	—	—	1	1	1	—	1	—	5
DaimlerChrysler	1	2	3	2	—	—	—	1	—	—	9
Danone	1	—	1	—	1	—	—	—	—	—	3
Deutsche Bank	1	—	—	—	—	2	—	1	2	—	6
France Télécom	—	2	1	2	1	1	1	1	1	—	10
Gallagher	—	1	1	—	1	—	1	—	—	—	4

BAT is British American Tobacco.

TABLE 1 Continued.

Company	Maturity band									Σ
	1Y-2Y	2Y-3Y	3Y-4Y	4Y-5Y	5Y-6Y	6Y-7Y	7Y-8Y	8Y-9Y	9Y-10Y	
Iberdrola	—	1	—	—	—	1	1	—	1	4
ING	—	—	—	1	—	1	1	—	—	3
Investor	—	—	1	—	1	—	1	—	1	4
KPN	1	2	1	—	1	—	1	—	—	6
Lafarge	1	1	1	1	1	—	—	—	2	7
LVMH	—	1	1	—	1	—	2	—	—	5
Repsol	—	—	1	—	—	—	1	—	2	4
Rolls-Royce	—	1	—	1	—	—	1	—	—	3
RWE	—	—	—	1	1	—	—	—	2	4
Telefonica	—	1	1	1	—	1	—	—	1	5
Vattenfall	—	—	—	1	—	1	1	1	—	4
Vodafone	1	—	2	—	1	2	—	—	—	6
Volvo	—	1	1	1	—	—	1	—	—	4
Wolters Kluwer	—	1	1	—	—	—	—	—	1	3
All companies	10	17	23	15	14	15	20	6	17	137

This table shows, for each company, the number of available corporate bonds in the nine maturity bands at the beginning of the sample period on January 1, 2003, or at a later date in the case of a number of corporate bonds that were issued later during the sample period.

**TABLE 2** Interest rate parameter estimates of the three-factor Cox–Ingersoll–Ross model.

$i$	$\kappa_i$	$\theta_i$	$\sigma_i$	$\lambda_i$
1	1.2743 (0.0892)	0.0037 (0.1519)	0.1562 (0.0680)	−0.0007 (0.2133)
2	0.5748 (0.0719)	0.0078 (0.0951)	0.3447 (0.0117)	−0.7706 (0.0604)
3	0.2091 (0.0798)	0.0051 (0.0792)	0.1356 (0.0103)	−0.3543 (0.0382)

This table shows the estimates of the interest rate parameters of the three-factor CIR model using the Kalman filter combined with a QML method. The logarithm of the estimated likelihood value is 7878.93. In the three-factor CIR model the short rate  $r(t)$  is defined as the sum of three economy-wide latent state variables (see (3.7)) where each of them follows a square-root process, ie, (3.8), under the physical measure  $\mathbb{P}$ .  $\kappa_i$  is the speed of mean reversion,  $\theta_i$  is the long-term mean level,  $\sigma_i$  is the volatility parameter and  $\lambda_i$  is the market price of risk parameter of the according state variable  $X_i$ . The standard errors of the point estimates in the parentheses are calculated as proposed by Bollerslev and Wooldridge (1992).

**TABLE 3** Fit of the three-factor CIR model.

	Maturity in years										Mean
	1	2	3	4	5	6	7	8	9	10	
MAE (bps)	3.96	0.39	0.67	0.43	0.32	0.41	0.43	0.36	0.26	0.48	0.77
MAPE (%)	1.79	0.17	0.25	0.15	0.11	0.12	0.12	0.10	0.07	0.12	0.30

This table reports the fit of the three-factor CIR model. MAE stands for the mean absolute error and MAPE stands for the mean absolute percentage error.

ten-year CDS spreads are observed with errors. The technical details of the QML estimation method are presented in Appendix D. Table 4 on the facing page reports the hazard rate parameter estimates.

Overall, the hazard rate parameter estimates are in line with our expectations. The estimates of the mean reversion parameter  $\kappa_z$  are positive, except for seven companies where they are negative. All estimates for the market price of risk parameter  $\lambda_z$  have the expected negative sign. For seven companies, the estimates of the long-term level parameter  $\theta_z$  have a negative sign. The volatility parameter estimates  $\sigma_z$  range between 0.05 and 0.36. The estimates of the interest rate sensitivity parameters  $\Lambda_i$  do not confirm the expected negative correlation with the short rate as found in Duffee (1999). Table 5 on page 18 reports the errors of the credit model and shows that it fits the market data quite well.

The average MAE over all companies and all maturities (except for the five-year maturity which is assumed to be observed without error) is 2.22bps and the according average MAPE is 6.47%. As, in general, the weakest fit is found at the two ends of

**TABLE 4** Hazard rate parameter estimates.

Company	$\kappa_z$	$\theta_z$	$\sigma_z$	$\lambda_z$	$\Lambda_1$	$\Lambda_2$	$\Lambda_3$
Abbey National	0.0240	0.0050	0.1910	-0.4331	0.0034	-0.0167	0.1199
Allianz	-0.0041	0.0031	0.1610	-0.1928	0.0757	0.0399	0.5607
Allied Domecq	0.1123	0.0228	0.2281	-0.1968	-0.0741	-0.0890	-0.1854
Arcelor	0.0058	-0.0139	0.1071	-0.0573	0.0099	0.1190	1.0999
HypoVereinsbank	0.0021	0.0021	0.1935	-0.3259	0.1631	0.1765	0.3699
BAT	0.2034	0.0277	0.2388	-0.3104	0.2569	-0.3105	-0.5821
Carrefour	0.0801	0.0258	0.2800	-0.3168	0.1064	-0.1579	-0.1950
Casino Guichard	0.2118	0.0227	0.0503	-0.1999	-0.1501	-0.0332	-0.8568
Commerzbank	0.0731	-0.0020	0.1554	-0.2999	0.1699	0.1319	0.5501
St. Gobain	0.0141	0.0063	0.1108	-0.0788	-0.0933	-0.0448	0.8920
DaimlerChrysler	0.1809	0.0215	0.3611	-0.4571	0.3969	0.1578	-0.0598
Danone	-0.0689	0.0020	0.1222	-0.1606	0.0189	-0.0658	0.5875
Deutsche Bank	-0.0105	0.0059	0.1730	-0.2727	0.0819	0.0293	0.4311
France Télécom	0.0400	0.0334	0.1361	-0.0847	-0.2599	-0.0999	1.0999
Gallaher	0.0555	0.0595	0.2852	-0.2103	-0.1366	-0.1621	-0.2195
Iberdrola	0.0075	0.0195	0.1384	-0.1611	-0.0299	-0.1245	0.6904
ING	0.0036	-0.0049	0.1583	-0.2811	0.1204	0.0568	0.2316
Investor	0.0321	-0.0009	0.1050	-0.0969	0.0839	0.0971	0.8499
KPN	0.0796	0.0131	0.2001	-0.2722	0.0499	-0.2616	1.2846
Lafarge	0.1651	0.0110	0.2301	-0.2939	-0.0999	0.0674	-0.1311
LVMH	-0.0023	-0.0004	0.0925	-0.0725	-0.0924	-0.0723	1.2798
Repsol	0.0606	0.0069	0.0779	-0.0662	-0.0163	-0.0883	1.1999
Rolls-Royce	-0.0249	-0.0081	0.1012	-0.0292	0.0501	0.0109	1.1949
RWE	0.0452	0.0054	0.1321	-0.1578	0.0256	-0.0883	0.7899
Telefónica	0.0664	0.0341	0.2233	-0.2268	-0.0696	-0.1291	-0.1699
Vattenfall	-0.0281	0.0003	0.1177	-0.1285	0.0349	-0.0174	0.7381
Vodafone	0.0865	0.0075	0.1641	-0.3072	0.0384	-0.3197	1.0767
Volvo	-0.0219	-0.0106	0.1177	-0.0503	-0.0201	-0.1510	1.1999
Wolters Kluwer	0.0529	0.0360	0.2148	-0.0883	0.0499	0.0812	0.0601

This table shows the estimates of the hazard rate parameters of the credit model calibrated to the CDS data, where the short rate is defined as the sum of three economy-wide state variables  $X_1(t)$ ,  $X_2(t)$  and  $X_3(t)$ , and where the intensity of company  $j$  is defined as

$$h_j(t) = \Lambda_{0,j} + \Lambda_{1,j}[X_1(t) - \bar{X}_1] + \Lambda_{2,j}[X_2(t) - \bar{X}_2] + \Lambda_{3,j}[X_3(t) - \bar{X}_3] + Z_j(t),$$

with the name-specific distress variable  $Z_j(t)$  of company  $j$  following a square-root process:

$$dZ_j(t) = \kappa_{z,j}(\theta_{z,j} - Z_j(t)) dt + \sigma_{z,j} \sqrt{Z_j(t)} dW_{z,j}^{\mathbb{P}}(t)$$

under the physical measure  $\mathbb{P}$ . In order to reduce the dimensionality of the optimization problem, we set  $\Lambda_{0,j} = 0$  for all companies. The optimization procedure minimizes the sum of the MAEs of the one-, three- and ten-year contracts, in order to find the optimal hazard rate parameters.

TABLE 5 Fit of the credit model to the CDS data. [Table continues on next page.]

Company	Contract maturity											
	1Y		2Y		3Y		7Y		10Y		Mean	
	MAE	MAPE	MAE	MAPE	MAE	MAPE	MAE	MAPE	MAE	MAPE	MAE	MAPE
Abbey National	1.45	19.37	0.88	7.45	0.61	4.76	1.18	5.58	0.95	3.86	1.01	8.20
Allianz	2.48	20.84	2.20	9.29	1.55	5.12	1.15	3.08	1.06	2.49	1.69	8.16
Allied Domecq	2.59	9.86	2.20	6.00	1.48	3.67	1.16	1.98	2.60	3.96	2.01	5.09
Arcelor	3.19	11.55	3.74	8.64	1.70	2.89	2.62	3.53	2.64	2.67	2.78	5.86
HypoVereinsbank	4.14	19.23	3.33	8.85	2.65	6.10	2.58	4.20	1.75	3.17	2.89	8.31
BAT	5.86	17.36	4.65	9.12	3.42	5.62	2.18	2.68	3.66	3.54	3.95	7.66
Carrefour	1.58	12.92	1.37	7.62	1.13	5.20	1.23	3.56	1.92	4.54	1.45	6.77
Casino Guichard	5.63	20.15	4.63	10.84	3.00	5.37	1.90	2.44	3.37	3.30	3.71	8.42
Commerzbank	3.78	18.20	2.23	7.60	1.47	5.06	2.02	3.95	1.72	2.80	2.24	7.52
St.Gobain	1.64	6.88	1.62	5.03	1.21	3.45	1.69	3.37	1.92	3.59	1.62	4.46
DaimlerChrysler	4.43	8.55	4.70	6.24	2.69	3.57	2.99	3.00	3.33	2.63	3.63	4.80
Danone	1.65	16.48	1.40	10.28	1.23	7.41	1.36	5.46	1.83	6.32	1.49	9.19
Deutsche Bank	1.17	13.58	1.05	8.95	0.72	4.99	1.17	4.73	0.78	2.62	0.98	6.97
France Télécom	5.35	11.22	5.38	11.29	4.10	7.00	1.81	1.88	1.93	1.70	3.71	6.62
Gallagher	4.38	15.62	3.11	8.97	2.09	4.97	2.53	4.02	3.57	4.75	3.14	7.67



TABLE 5 Continued.

Company	Contract maturity													
	1Y		2Y		3Y		7Y		10Y		Mean			
	MAE	MAPE	MAE	MAPE	MAE	MAPE	MAE	MAPE	MAE	MAPE	MAE	MAPE		
Iberdrola	1.44	9.19	1.05	5.09	0.91	3.70	1.05	2.82	1.14	2.93	1.12	4.75		
ING	1.16	16.14	1.15	10.95	0.76	5.88	0.87	4.35	1.04	3.92	1.00	8.25		
Investor	2.93	11.10	2.53	5.62	1.50	3.38	1.94	2.72	3.29	4.09	2.44	5.38		
KPN	2.50	9.97	3.07	9.50	3.60	8.57	1.97	2.95	3.26	4.79	2.88	7.16		
Lafarge	2.19	6.33	2.15	4.94	2.47	4.17	2.08	3.12	3.02	3.42	2.38	4.40		
LVMH	2.90	9.91	2.48	6.41	1.71	4.14	1.59	2.53	1.86	2.79	2.11	5.16		
Repsol	5.09	16.23	3.56	7.29	4.32	7.45	1.88	2.73	2.05	2.61	3.38	7.26		
Rollis-Royce	3.42	9.36	3.04	7.77	1.59	2.81	1.74	2.14	1.96	2.27	2.35	4.87		
RWE	2.35	12.71	1.67	6.91	1.05	3.22	1.17	2.56	1.09	2.39	1.47	5.56		
Telefónica	2.13	10.06	1.70	6.47	1.47	5.49	1.15	2.25	3.02	5.20	1.89	5.89		
Vattenfall	2.17	14.09	1.24	5.59	0.96	3.62	1.39	3.41	1.84	3.62	1.52	6.07		
Vodafone	2.37	15.44	1.93	8.22	1.61	5.68	1.23	2.64	2.57	4.68	1.94	7.33		
Volvo	1.66	7.61	1.30	4.52	1.30	3.79	1.11	2.34	1.76	3.35	1.43	4.32		
Wolters Kluwer	2.74	11.02	2.08	5.73	1.88	4.43	1.77	3.05	2.32	3.35	2.16	5.52		
Mean	2.91	13.14	2.46	7.63	1.87	4.88	1.67	3.21	2.18	3.49	2.22	6.47		

This table reports the pricing errors between the credit model and the CDS data, where the five-year contract is assumed to be measured without error. MAE stands for the mean absolute error and MAPE stands for the mean absolute percentage error.

the curve. The average MAE over all companies for the one-year maturity is 2.91, for the two-year maturity is 2.46 and for the ten-year contracts is 2.18bps.

## 5 VALUATION DIFFERENCES

In order to establish our measure of the valuation difference, we first calculate the actual yield to maturity of every corporate bond in our sample, denoted by  $YTM^{CB}$ . Second, given the interest rate parameters and the hazard rate parameters of every firm, extracted from its CDS data, we use the formula in (3.16) to compute CDS market-implied prices for every corporate bond in our sample, and calculate their associated yields to maturity, which we denote by  $YTM^{CDS}$ . We define the valuation difference for corporate bond  $i$  at time  $t$  as

$$\Delta_{it} := YTM_{it}^{CB} - YTM_{it}^{CDS}. \quad (5.1)$$

Defining the valuation difference between corporate bonds and CDSs in this manner differs from the approaches taken in previous studies in at least one of the following three aspects. First, in contrast to many previous studies, which consider the valuation difference to be a maturity-specific measure, in most cases for the five-year maturity, our measure  $\Delta_{it}$  is corporate-bond-specific. This choice allows us to examine the influence of individual characteristics of different bonds, such as coupon rates or amounts outstanding, even if these bonds have the same maturity and are issued by the same firm.

Second, many previous studies found their analyses on the basis defined as the difference between the credit spread and the CDS spread of the same maturity. This choice, however, leads to a distortion of the true valuation difference due to the contractual differences in the two instruments, like the accrued premiums in CDS contracts or the fact that the bond trades away from par (see, for example, Adler and Song 2010). For this reason, we make use of our credit model to account for the specific characteristics of CDS and corporate bonds and to create a common basis for comparison by calculating the yields to maturity  $YTM_{it}^{CB}$  and  $YTM_{it}^{CDS}$ .

Third, the chosen credit model captures the information provided by the entire CDS curve of an issuer, which reflects its pure default risk and is considered by financial markets as the main detector of changes in its healthiness. Calibrating the model to the time series of the CDS curve rather than to the time series of a sample of the firm's corporate bonds, as done in previous studies, also yields more efficient model estimates. Instead of dealing with a varying and possibly very small number of corporate bonds entering the estimation, we can rely on a constant number of CDS spreads spanning the entire ten-year maturity structure throughout our investigated time period.

Table 6 on the next page shows some statistics of the resulting valuation differences aggregated at the individual firm level.

The average of the valuation difference across all corporate bonds is 8.41bps. Thus, on average, yields in the corporate bond market are higher than those implied by the CDS market. Despite this rather small average valuation difference across all bonds, we find a substantial variation across individual firms, ranging from  $-11.12$ bps for British American Tobacco (BAT) to 36.56bps for Volvo. Table 6 on the next page also reveals considerable variation in the time series dimension from  $-91.33$ bps for France Télécom to 110.70bps for Volvo. Although not shown, we also find substantial variation between the valuation differences of individual bonds issued by the same firm. These observations deliver strong evidence for the existence of not only firm-specific but also corporate-bond-specific determinants of the valuation difference.

Before investigating the impact of these determinants in the next section, we first turn our attention to the empirical results of previous studies in order to put our results into perspective. Table 7 provides an overview of the mean valuation differences found in previous studies for investment-grade firms along with the corresponding definition, the data source and the currency.

Most of these results are multiples of our results, ranging from 11.6bps found by Cossin and Lu (2005) to 65.0bps found by Longstaff *et al* (2005) (leaving out the studies by Levin *et al* (2005) and Jankowitsch *et al* (2008), as they base their analyses only on the swap curve as the risk-free benchmark). However, these large deviations between the results of previous studies and our results are probably not caused solely by different definitions of the valuation difference and model specifications. Since the average valuation differences of previous studies in Table 7 on page 24 seem to decrease with the use of increasingly recent data, they are probably also attributable to the improved data quality in the CDS markets since 2003, in the course of the publication of the 2003 ISDA Credit Derivative Definitions.

In order to examine what specific influence the definition of the valuation difference has on its magnitude, we compared our results from Table 6 on the next page with the results (not reported here) obtained using an alternative measure of the valuation difference that has a naive form in the sense that it ignores all contractual differences between the two credit instruments. We calculated this naive measure for corporate bond  $i$  at time  $t$  as

$$\Delta_{it}^{\text{NAIVE}} := \text{YTM}_{it}^{\text{CB}} - \text{YTM}_{it}^{\text{GOV}} - \text{CDS}_{it}. \quad (5.2)$$

$\text{YTM}_{it}^{\text{CB}}$  is, as in (5.1), the actual yield to maturity of corporate bond  $i$ .  $\text{YTM}_{it}^{\text{GOV}}$  denotes the risk-free par yield for the maturity of bond  $i$  calculated from our interest rate data set using linear interpolation.  $\text{CDS}_{it}$  is the bond issuer's CDS spread for the maturity of bond  $i$ , calculated using linear interpolation. Across all bonds, the

TABLE 6 Valuation differences  $\Delta_{it}$  at the individual firm level. [Table continues on next page.]

Company	N.Obs.	Mean	MAD	SD	Skew	Kurtosis	Min	1. Quartile	Median	3. Quartile	Max
Abbey National	467	9.89	11.29	9.13	-0.24	-0.13	-18.82	3.96	9.81	16.57	35.95
Allianz	424	9.30	11.96	10.72	-0.56	0.38	-26.26	3.11	9.63	17.09	30.81
Allied Domecq	314	14.21	14.44	10.59	0.90	0.75	-6.59	5.75	12.76	19.81	50.72
Arcelor	175	32.30	32.41	14.77	0.36	0.12	-9.88	19.95	33.01	41.33	71.48
HypoVereinsbank	212	10.17	17.23	16.32	-2.03	3.58	-47.92	8.74	15.48	19.25	28.32
BAT	345	-11.12	20.42	23.54	-0.25	-0.88	-63.33	-30.24	-7.07	4.75	33.79
Carrefour	495	-9.70	11.49	9.78	0.19	-0.48	-33.39	-16.89	-9.87	-2.81	19.05
Casino Guichard	611	-5.83	11.90	13.43	0.01	-0.45	-39.07	-14.40	-6.53	3.53	25.89
Commerzbank	318	5.09	19.60	22.88	-1.52	1.43	-77.01	2.98	12.23	19.27	33.07
St.Gobain	437	15.50	16.08	8.94	-0.28	0.99	-19.10	10.18	16.01	20.99	41.84
DaimlerChrysler	705	-7.67	14.60	16.12	-0.16	-0.60	-57.58	-19.49	-7.71	4.70	25.44
Danone	229	8.84	9.35	6.94	0.38	1.72	-17.96	4.62	8.02	12.88	35.61
Deutsche Bank	508	19.80	19.93	15.28	1.30	1.29	-10.19	9.96	15.09	24.46	73.20
France Télécom	945	6.14	17.25	23.57	-0.42	2.17	-91.33	-4.05	4.23	19.22	108.24
Gallaher	301	7.79	12.22	12.44	-0.17	-0.70	-21.19	-0.81	7.81	17.04	34.67

TABLE 6 Continued.

Company	N.Obs.	Mean	MAD	SD	Skew	Kurtosis	Min	1. Quartile	Median	3. Quartile	Max
Iberdrola	341	10.24	11.00	8.00	0.05	0.57	-16.76	5.09	10.00	14.59	35.50
ING	318	12.80	13.51	9.49	-0.13	-0.31	-16.02	6.38	12.66	20.01	36.65
Investor	389	34.80	34.80	13.22	0.72	0.44	1.36	25.20	33.10	42.45	76.93
KPN	448	7.67	14.78	19.81	1.13	2.47	-37.57	-3.20	5.13	16.41	83.43
Lafarge	491	7.33	9.81	9.01	-1.13	2.86	-42.79	2.40	8.74	13.41	26.21
LVMH	393	11.94	14.33	11.49	-0.45	-0.08	-18.94	5.73	12.59	20.24	37.46
Repsol	301	19.85	20.44	13.45	0.22	-0.24	-16.10	10.49	18.73	29.42	60.21
Rolls-Royce	255	8.73	15.64	18.39	-2.31	7.25	-82.97	3.10	11.71	19.56	33.42
RWE	342	3.39	10.03	11.00	-1.04	0.36	-28.41	-2.86	6.84	10.71	23.05
Telefónica	507	-3.88	11.03	13.19	-0.20	-0.58	-38.88	-12.64	-2.91	6.00	30.51
Vattenfall	424	19.00	19.03	9.59	0.53	-0.10	-4.19	11.77	17.87	24.65	47.36
Vodafone	561	2.10	9.58	11.82	0.14	-0.25	-28.87	-6.45	2.31	9.47	33.47
Volvo	413	36.56	36.60	28.59	1.06	0.06	-3.67	16.30	24.95	50.21	110.70
Wolters Kluwer	258	15.33	17.82	13.60	-0.51	0.52	-20.54	6.06	18.91	24.03	55.87
All bonds	11 927	8.41	16.01	19.28	0.19	3.02	-91.33	-1.55	9.08	18.61	110.70

This table shows valuation difference statistics in bps aggregated at the firm level. The valuation difference for corporate bond  $i$  at time  $t$  is defined as in (5.1) where  $YTM_{i,t}^{CF}$  is the yield to maturity of the corporate bond  $i$  at time  $t$  and  $YTM_{i,t}^{CDS}$  is the yield to maturity of the CDS market-implied model price of the same corporate bond. N.Obs is number of observations. SD is standard deviation. 1.Quartile and 3.Quartile are the first and third quartiles, respectively.

TABLE 7 Overview of related studies.[Table continues on next page.]

Study	Market	Data	Definition of valuation difference	Mean diff.
Hull <i>et al</i> (2004)	USD	1998–2002, 5Y CDS spreads from GFI and corporate bonds from Bloomberg for 31 firms.	Difference between 5Y bond par yield, obtained by regression and adjusted for accrued interest, Treasury rate and 5Y CDS spread.	62.87
Blanco <i>et al</i> (2005)	USD, EUR	2001–2, 5Y CDS spreads from CreditTrade and corporate bonds from Bloomberg for 33 firms	Difference between 5Y yield, obtained by interpolation, 5Y gov. yield and 5Y CDS spread	40.8
Cossin and Lu (2005)	EUR	2002–3, 5Y CDS spreads from Bloomberg and corporate bond prices from Reuters for 39 firms	Difference between 5Y credit spread calculated from bond par yields and adjusted for accrued interest and 5Y CDS spread	11.6
Houweling and Vorst (2005)	USD, EUR	1999–2001, CDS of different maturities from creditex, CreditTrade and several banks and corporate bonds from Reuters for 225 firms	Difference between corporate bond market-implied model CDS spreads and market CDS spreads	23.5
Levin <i>et al</i> (2005)	USD	2001–5, CDS from Markit and corporate bonds from Merrill Lynch for 306 firms	Difference between credit spread over swap rate of a bond and the CDS spread with the same maturity	3.99*

TABLE 7 Continued.

Study	Market	Data	Definition of valuation difference	Mean diff.
Longstaff <i>et al</i> (2005)	USD	2001–2, 5Y CDS spreads and corporate bond prices from Citigroup for 68 firms	Nondefault component of the 5Y credit spread, obtained by fitting a credit model to corporate bonds and 5Y CDS spreads	65.0
Zhu (2006)	USD, EUR	1999–2002, 5Y CDS spreads from CreditTrade and corporate bonds from Bloomberg for 24 firms	Difference between 5Y credit spread, obtained by linear interpolation and 5Y CDS spread	52.26
Jankowitsch <i>et al</i> (2008)	EUR	2003–5, CDS of different maturities and corporate bonds from Reuters for 12 firms	Difference between corporate bond market-implied CDS spread, obtained by the Nelson–Siegel specification and using the swap curve as risk-free curve and market CDS spread	–19.57*
Norden and Weber (2009)	USD, EUR, GBP	2000–2002, 5Y CDS quotes from CreditTrade and a bank and corporate bonds from Bloomberg for 58 firms	Difference between 5Y yield obtained by regression, gov. bond yield and 5Y CDS spread	32.44

This table provides an overview of related studies. Mean diff. stands for the average valuation difference based on the chosen definition. Values are given for investment grade firms; however, Cossin and Lu (2005), Longstaff *et al* (2005) and Norden and Weber (2009) also include a negligible number of speculative grade issuers. \*With the exception of Levin *et al* (2005) and Jankowitsch *et al* (2008), who use solely the swap curve as the risk-free benchmark, all other values are based on the government curve.

mean absolute deviation (MAD) between our valuation difference  $\Delta_{it}$  and this naive valuation difference  $\Delta_{it}^{\text{NAIVE}}$  is 7.53bps, ranging from 1.37bps for Abbey National up to 31.40bps for BAT. These results provide evidence that measures of the valuation difference that are based on such simplifying assumptions can substantially misrepresent its magnitude as captured by our measure  $\Delta_{it}$ .

## 6 EMPIRICAL ANALYSIS

### 6.1 Factors driving the valuation difference

The literature identifies a wide range of factors that can cause prices in both markets to diverge. O’Kane and McAdie (2001) separate these factors into fundamental and market factors. Fundamental factors arise from the difference between the specific structures of CDS contracts and corporate bonds. Market factors result from liquidity and supply and demand conditions in both markets.

#### 6.1.1 Fundamental factors

Fundamental factors include contractual differences:

- (i) different payment frequencies;
- (ii) different day count conventions;
- (iii) accrued premiums in CDS contracts.

They can distort the valuation difference when it is calculated as the difference between the CDS premium and the credit spread. Since we account for these contractual differences in our pricing formulas, they do not bias our measure of the valuation difference.

An additional effect results from the possibility of the corporate bond trading away from par. Whenever a bond departs from its par value, an investor in that bond faces a different loss given default than a market participant selling protection on the issuer of that bond. A protection seller’s loss given default is always the difference between the par value and the recovery rate. On the other hand, an investor in a premium bond faces a higher loss given default – and an investor in a discount bond a lower one – than the protection seller, given the same recovery rate. Therefore, the credit spread of a premium bond is above, and that of a discount bond is below, the corresponding CDS spread. We eliminate this effect by creating a common base of comparison through our model-based approach, which compares the yields to maturity of market and model bond prices.

In the following, we consider two fundamental factors: the cheapest-to-deliver option and the counterparty credit risk in CDS contracts.



### The cheapest-to-deliver option

The cheapest-to-deliver option in a physically settled CDS contract is based on the right of the protection buyer to deliver the cheapest deliverable bond in the case of a credit event. Its value depends on the restructuring clause of the CDS contract, the type of credit event and the number of deliverable bonds. To capture the effect of the cheapest-to-deliver option, we calculate a proxy also used in Jankowitsch *et al* (2008), namely the maximum bond price difference, defined as the difference between the highest and the lowest market bond prices of an issuer at each date. The maximum bond price difference intends to capture the fact that different deliverable corporate bonds of the same issuer can trade at substantially different prices in the aftermath of a credit event. We assume this proxy to have a positive effect on the value of the cheapest-to-deliver option and a negative effect on the valuation difference.

### Counterparty default risk

The counterparty risk in CDS contracts is due to the exposure of the two parties to each other's default risk. Generally, as highlighted in Hjort (2003), the protection seller's risk of losing potential mark-to-market gains on their position is almost negligible compared with the protection buyer's risk of not receiving the payment of par upon the occurrence of a credit event. As, according to O'Kane and McAdie (2001), banks and insurers are the dominant players in the CDS market, we calculate a counterparty credit risk index, defined as the average five-year CDS spread of the six financial companies in our data sample (Abbey National, Allianz, HypoVereinsbank, Commerzbank, Deutsche Bank and ING), in order to proxy the development of the general level of counterparty risk in the market.

As a second proxy for the credit risk of financial institutions, we choose the one-year EUR swap spread. Similar to the role of the TED spread<sup>1</sup> in the US market, the one-year EUR swap spread, defined as the difference between the one-year EUR swap rate and the one-year EUR spot rate (estimated from German government bond yields as outlined at the end of Section 4.1), measures excess interbank lending costs in the eurozone.

Our conversations with market practitioners revealed that counterparty default risk was not a major concern among market participants during our sample period. Therefore, in the unlikely case that we detect that our counterparty default risk proxies have a significant impact on the valuation difference, we expect a positive sign for both of them.

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<sup>1</sup> The TED spread is the difference between the interest rates on interbank loans and on short-term US government debt ("T-bills"). TED is an acronym formed from T-Bill and ED, the ticker symbol for the Eurodollar futures contract.

### 6.1.2 Market factors

The market factor proxies we include in our analysis can be divided into six groups.

#### Liquidity

The first group of market factor proxies we take into account deals with liquidity. We consider both bond-specific and CDS-related liquidity measures. The bond-specific liquidity measures we include are the coupon rate, maturity, age and issued amount of a bond. Due to clientele effects among investors discussed in Longstaff *et al* (2005), we assume the coupon rate and the maturity of the bond to have a negative effect on bond liquidity and thus a positive effect on the valuation difference. We hypothesize, as in Houweling *et al* (2005), that the age of a bond also has a negative effect on bond liquidity due to buy-and-hold investors. The issued amount of a bond should be positively related to its liquidity, as large issues are assumed to trade more often. Therefore, we expect a negative effect of the issued amount on the valuation difference.

We now turn to the proxy capturing liquidity in the CDS market. Like Levin *et al* (2005), we use the number of financial institutions that provide CDS quotes for the five-year maturity from our Markit data set. In order to make an assumption about the effect of this CDS liquidity proxy on the valuation difference, we should point out that the above hypotheses about the effect of bond liquidity are based on the premise that illiquid assets always trade at a discount in markets with a positive net supply, such as the corporate bond market. By developing a new equilibrium asset pricing model that incorporates liquidity risk and short-selling, Bongaerts *et al* (2011) show that short-selling dramatically changes the effect of liquidity on asset prices. This is especially true for derivatives markets, such as the CDS market, where short-selling plays a crucial role. In such markets with zero net supply the effect of liquidity can be zero, negative or positive. Therefore, determining the effect of our CDS liquidity proxy is not as straightforward as in the case of the bond liquidity measures. Bongaerts *et al* (2011) implement their asset pricing model for the US CDS market and find that protection sellers earn a liquidity premium. Based on their empirical results, we assume that a higher number of contributing institutions reduces the liquidity premium earned by protection sellers, which leads to a higher valuation difference.

#### Credit risk

The second group of proxies attempts to measure the credit risk at three different levels. The first proxy, the iTraxx Europe CDS index for the five-year maturity, captures the market-wide level of credit risk. Since the index was launched only in the second half of 2004, we calculate the missing index values using those 121 of the 125 constituent companies for which we have reliable five-year CDS spreads. The second proxy, the five-year CDS spread, mimics the credit risk at the individual firm level. Finally,

the third proxy measures the credit risk at the individual corporate bond level and is defined as the CDS spread for the same maturity as the corporate bond, calculated by linear interpolation. We expect a negative effect of these three proxies on the valuation difference since, as found by Cossin and Lu (2005), a declining credit quality leads to a decrease of the liquidity component of credit spreads for AA- and A-rated firms whose bonds constitute the vast majority of our data sample.

### Firm-specific factors

The third group of market factor proxies aims to capture firm-specific influences through the stock market. These are the equity return and its one-month historical volatility. The equity return aims to capture an effect detected by Norden and Weber (2009), ie, that CDS spreads are more sensitive to changes in equity returns than corporate bond spreads. Therefore, we expect that a higher equity return will decrease prices in the CDS market more strongly than in the bond market. As a result, we expect a positive relationship between equity returns and valuation differences. We include the equity volatility, similar to Levin *et al* (2005), in order to capture periods of firm-specific stress. A higher volatility should increase CDS premiums more than credit spreads in the bond market and thus have a negative effect on the valuation difference.

### Supply in the CDS market

The fourth group, consisting of only one proxy, deals with an effect not yet explored in previous studies, namely the impact of synthetic CDO transactions. As pointed out by O’Kane and McAdie (2001) and Hjort (2003), these transactions create a massive additional supply of protection, tightening the CDS spreads of the names in the underlying portfolio. We download the monthly time series of the global synthetic CDO issuance volumes from the CreditFlux database. In order to convert the data into a weekly frequency, we assume that the monthly issuance volumes are uniformly distributed over time. We expect a positive effect of the issuance volume on the valuation difference.

### Macroeconomic uncertainty

The fifth group of market factor proxies deals with macroeconomic uncertainty. Similarly to Levin *et al* (2005), in order to capture uncertainty concerning central bank decisions, we calculate the one-month historical volatility of the three-month EURIBOR. In order to measure inflation risk, we use the one-month historical volatility of the ten-year spot rate. We expect these volatilities to have a negative effect on the valuation difference for the same reasons as in the case of the equity volatility.

### Lagged valuation differences

The sixth group of market factor proxies aims to explore the influence of basis trades in a way similar to Zhu (2006). We assume market participants will set up such trades so as to profit from arbitrage opportunities due to nonzero valuation differences, which should lead to a reduction of the extent of the valuation difference during subsequent periods. We therefore include two lagged valuation differences with a lag of one week,  $\Delta_{i,t-1}$ , and two weeks,  $\Delta_{i,t-2}$ .

## 6.2 Results

Previous studies investigated valuation differences separately by time series or by cross-sectional OLS. In this study we use panel data analysis for the following reasons. First, by capturing both the time series and the cross-sectional dimension of our data with one regression model, this methodology allows us to investigate questions that simple OLS cannot answer. Second, panel data analysis yields more efficient parameter estimates than OLS. Finally, and most importantly, panel data analysis seems to be the natural choice to analyze corporate-bond-specific valuation differences, especially for our study, as we are able to account for unobservable bond-specific heterogeneities such as the average bid–ask spread of the bond across time and market makers or the average cost of shorting the bond via the repo market.

In the following we estimate seven different panel data models and discuss their results. In the first five regression models we choose a random-effects specification for the 137 unobservable bond-specific effects and for the last two models, which involve lagged valuation differences, a fixed-effects specification. This choice is supported by the Hausman test (where applicable) and enables us, in the case of the random-effects specification, to investigate the influence of time-invariant bond-specific factors, eg, the coupon rate and the issued amount.

Thus, to examine the impact of our factors on the valuation difference  $\Delta_{it}$  of bond  $i$  in  $t$ , we estimate the following panel data model:

$$\Delta_{it} = \alpha + \beta^T \mathbf{x}_{it} + \alpha_i + \varepsilon_{it}, \quad (6.1)$$

where the vector  $\mathbf{x}_{it}$  represents our factors,  $\alpha_i$  is the bond-specific effect, which is specified as random in the first five models and fixed in the last two,  $\varepsilon_{it}$  is the unsystematic residual and  $\alpha$  and the vector  $\beta$  are the regression coefficients. In order to account for serial correlation and intrafirm cross-sectional dependence of residuals, we consider firm-clustered standard errors as derived by Froot (1989) and computed based on Rogers (1993). Table 8 on the facing page reports the results of the seven panel data regressions.

In column (1) of Table 8, we examine the most widely investigated effect on valuation differences, namely liquidity. Similarly to the cross-sectional regression in

**TABLE 8** Panel data analysis of the valuation difference  $\Delta_{it}$ . [Table continues on next two pages.]

EV	(1)	(2)	(3)	(4)	(5)	(6)	(7)
<i>CDT option</i>							
Max. bond price difference (-) (in EUR)				0.082 (0.862)		-0.048 (0.588)	
<i>Counterparty risk</i>							
Counterparty credit risk index (+) (in bps)				-0.200 (0.002)			
1Y Swap spread (+) (in bps)				-0.154 (0.247)			
<i>Liquidity</i>							
Bond coupon rate (+) (in %)	3.938 (0.015)	4.565 (0.006)	6.413 (0.000)				
Bond maturity (+) (in years)	-1.706 (0.297)						
Bond age (+) (in years)							
Bond amount outstanding (-) (in EUR bn)	-5.731 (0.041)	-6.914 (0.003)	-5.999 (0.023)				
Depth of 5Y CDS quote (-) (in number of banks)	0.109 (0.485)						
<i>Supply in CDS market</i>							
Synthetic CDO issuance (+) (in USD bn)					-0.133 (0.000)	-0.025 (0.000)	

Longstaff *et al* (2005), we include all our liquidity-related factors except for the age of the bond, as it is highly negatively correlated with its maturity in the time series dimension. Both the coupon rate and the amount outstanding of the bond are statistically significant and have the predicted signs. In contrast to Longstaff *et al* (2005) and Levin *et al* (2005), we find no maturity effect in the valuation difference. An additional univariate between regression and a separate univariate within regression

TABLE 8 Continued.

EV	(1)	(2)	(3)	(4)	(5)	(6)	(7)
<i>Credit risk</i>							
iTraxx Europe 5Y CDS index (–) (in bps)		–0.180 (0.000)				–0.037 (0.002)	
5Y CDS (–) (in bps)			–0.195 (0.000)				–0.048 (0.000)
CDS spread acc. to bond maturity (–) (in bps)					–0.230 (0.000)		
<i>Firm-specific factors</i>							
Equity return (+) (in %)			0.184 (0.000)			0.278 (0.000)	0.251 (0.000)
Equity volatility (–) (in %)				–0.791 (0.327)		0.087 (0.701)	
<i>Macroeconomic uncertainty</i>							
3 month EURIBOR volatility (–) (in %)					0.981 (0.285)		
10Y Spot rate volatility (–) (in %)					–0.863 (0.255)		0.282 (0.311)

of the valuation difference on the bond maturity reveal the lack of any explanatory power in both the cross-sectional and the time series dimensions.

The CDS liquidity proxy, ie, the number of institutions providing five-year CDS quotes, is not statistically significant, which is in stark contrast to Levin *et al* (2005), who find a positive effect in the cross-sectional dimension and a negative effect in the time series dimension between 2001 and 2005. This finding supports our view that the CDS market went through a fundamental maturity process which culminated in the publication of the 2003 ISDA Credit Derivative Definitions. We thus suspect that the effects found by Levin *et al* (2005) are mainly caused by this maturity process in the 2001–3 period, which flattened in the aftermath during our sample period from 2003 to 2005.

Finally, a univariate regression of the valuation difference on the the age of the bond, not shown in Table 8, confirms the expected positive relationship, which is,

TABLE 8 Continued.

EV	(1)	(2)	(3)	(4)	(5)	(6)	(7)
<i>Lagged valuation difference</i>							
$\Delta_{i,t-1}$ (in bps)						0.807 (0.000)	0.520 (0.000)
$\Delta_{i,t-2}$ (in bps)							0.300 (0.000)
Intercept	-1.494 (0.865)	-0.890 (0.909)	-10.052 (0.237)	15.910 (0.002)	23.687 (0.000)	3.577 (0.000)	4.301 (0.000)
Number of observations	11 927	11 927	11 036	11 036	11 927	10 909	10 782
$R^2$ within	0.0099	0.0991	0.2218	0.1071	0.2696	0.6936	0.7291
$R^2$ between	0.0389	0.1301	0.2416	0.000	0.0677		
$R^2$ overall	0.0384	0.1201	0.2179	0.0321	0.1390		
Model specification	RE	RE	RE	RE	RE	FE	FE

This table reports the results of seven panel data regressions of the valuation difference  $\Delta_{i,t}$  (in bps). Models (1)–(5) are random-effects (RE) regressions and models (6) and (7) are fixed-effects (FE) regressions. Explanatory variables are presented with their expected signs in parentheses. The coefficient estimates are reported with firm-clustered  $p$ -values in parentheses. The inclusion of equity-related proxies in models (3), (4), (6) and (7) reduces the number of observations since Abbey National and Vattenfall are not listed companies. Coefficients of determination are defined according to STATA. EV stands for the explanatory variable.

however, not statistically significant. This lack of significance is consistent with the results of Longstaff *et al* (2005).

In column (2) of Table 8 on page 31, we combine the two significant liquidity proxies from regression (1) with the iTraxx Europe index, which reflects the market-wide level of credit risk. The coupon rate and the issued amount of the bond remain statistically significant. The iTraxx Europe index is negatively correlated with the valuation difference. Thus, with a higher level of market-wide credit risk, the two credit markets tend to converge. This relationship remains unchanged whether we use the five-year CDS spread or the bond-maturity-equivalent CDS spread instead of the iTraxx index.

These findings are in line with the results of Huang and Huang (2002) and Cossin and Lu (2005), who find that a decline in the firm's credit quality diminishes the role of the liquidity component in the credit spread of its bonds. Hence, in our case this decline of the non-default-related components in a bond's credit spread results in a lower valuation difference. Our findings show that the relationship detected by Huang and Huang (2002) and Cossin and Lu (2005) generalizes to the case where the market-wide level of credit risk is considered instead of the firm-specific credit quality.

Column (3) presents our benchmark model, which is the most parsimonious and yields maximal  $R^2$  statistics among our five random-effects regressions. All four included variables are statistically significant. The two liquidity proxies and the credit market proxy have the predicted signs. The fourth included proxy, the equity return, has the expected positive effect on the valuation difference. This finding supports our hypothesis and the results of Norden and Weber (2009) that CDS markets are more sensitive to equity returns than bond markets. Hence, this difference in the responsiveness of the two credit markets to movements in the stock market causes deviations between them.

In column (4), we examine the influence of the equity volatility along with the impact of fundamental factors. Thus, in addition to the equity volatility, we include the counterparty risk index and the one-year EUR swap spread to account for counterparty credit risk and the maximum bond price difference to proxy the influence of the cheapest-to-deliver option. We find that the conclusion drawn from regression (3), that the two credit markets exhibit different sensitivities to equity returns, does not apply to its volatility. The volatility has the expected negative sign; however, it lacks statistical significance. We cannot therefore confirm the findings of Cossin and Lu (2005) and Levin *et al* (2005), who report a negative effect of the equity volatility.

Among the two counterparty proxies, only the counterparty risk index is statistically significant. However, contrary to our expectation, it has a negative sign. The fact that the counterparty risk index has the same sign as the iTraxx Europe index in column (2) of Table 8 on page 31 leads us to the conjecture that there was no change in the credit quality of these financial institutions relative to the market-wide credit quality sufficient to induce a significant change in CDS spreads that can be related to counterparty credit risk. All in all, these results seem to confirm our initial hypothesis based on conversations with market practitioners that counterparty default risk was not a major concern among CDS market participants during our sample period.

Finally, the cheapest-to-deliver option proxy, ie, the maximum bond price difference, is not statistically significant. Thus, we cannot confirm the results of Jankowitsch *et al* (2008), who also use this proxy and find the cheapest-to-deliver option to be the predominant cause for the valuation difference.

Column (5) investigates the influence of macroeconomic variables along with the synthetic CDO issuance volumes. Neither of the two macroeconomic variables is statistically significant. This is different from the results of Levin *et al* (2005), who detect a positive and statistically significant relationship between the ten-year Treasury yield volatility and the basis, and conclude that bond markets exhibit a higher sensitivity to macroeconomic uncertainty than CDS markets. We cannot confirm their results since our findings reveal a rather similar responsiveness of the two credit markets to both macroeconomic proxies, ie, to the volatilities of the three-month EURIBOR rate and the ten-year spot rate. Given the findings of Norden and Weber (2009) that CDS



markets generally lead corporate bond markets in the price discovery process, we find it improbable that the credit market with the higher diagnosed degree of inertia is more susceptible to changes in macroeconomic conditions.

Finally, the synthetic CDO issuance volume is statistically significant but does not have the expected positive sign. In a univariate regression, however, it is not significant. These contradicting results could indicate that the excess supply in credit protection generated by the issuance of synthetic CDOs could not drive a substantial wedge between the two credit markets during our observation period.

In the fixed-effects regressions of columns (6) and (7), we include lagged valuation differences in order to explore whether the results we have found so far are robust to the impact of potential basis trades set up by market participants. We use the fixed-effects estimator instead of the estimator developed by Arellano and Bond (1991) for computational reasons, since for long panels the number of instrumental variables becomes too high, and, as pointed out by Roodman (2006), the dynamic panel bias becomes insignificant in such cases. In column (6), we include  $\Delta_{i,t-1}$ , ie, the valuation difference with a lag of one week, and in column (7) we combine the lagged values  $\Delta_{i,t-1}$  and  $\Delta_{i,t-2}$ , ie, the valuation differences with lags of one and two weeks. In both models, the lagged valuation differences are statistically significant. However, the results for the other explanatory variables included are qualitatively similar to the random-effects regressions in columns (1)–(5), confirming our results found so far.

### 6.3 Complementary and robustness tests

First, we test whether movements of the ten-year spot rate curve have an effect on the valuation difference. We examine three factors:

- (1) the level of the curve, proxied by the ten-year spot rate;
- (2) the slope of the curve, defined as the difference between the ten-year and the one-year spot rates;
- (3) the curvature at the five-year maturity.

We find no explanatory power for any of these three factors and conjecture that both credit markets exhibit comparable sensitivities to interest rate changes. This finding is consistent with the results from regression (5) in Table 8 on page 31 that the two credit markets react in a very similar way to changes in macroeconomic conditions.

Second, in order to assess the influence of our specific credit model choice on the resulting valuation differences, we carry out a random-effects regression of the valuation difference on the interest rate model error and the CDS model error. The interest rate model error is defined as the difference between the market and the model spot rate for the same maturity as the corporate bond. Likewise, the CDS

model error is the difference between the market and the model CDS spread for the same maturity as the bond. Both proxies are calculated by linear interpolation from the available maturities (ten and six, respectively). Neither of the two model errors has any explanatory power, which indicates that the calculated valuation differences are not systematically distorted by the quality of the model fit.

Third, in order to further test the robustness of our results with respect to our specific credit model choice, we calibrate two additional credit models to our interest rate and CDS data. Instead of using a three-factor generalization of the Cox *et al* (1985) short-rate model, we specify the short-rate process in (3.7) using only one and two factors, respectively, everything else being equal. Estimating the seven models from Table 8 on page 31 for the resulting valuation differences in the two-factor case provides qualitatively similar results.

Remarkably, for the one-factor short-rate model we obtain very different regression results. In contrast to the two other credit models, in the one-factor case both liquidity measures, the coupon rate and the amount outstanding, lack statistical significance, whereas both macroeconomic proxies, the volatilities of the three-month EURIBOR rate and of the ten-year spot rate, become statistically significant. We suspect that these distortions of the regression results are caused by the poor fit of the one-factor interest rate model. The average MAE over all ten maturities is 4.91bps in the one-factor model and 0.77bps in the three-factor case. A univariate panel regression of the valuation difference on the interest rate model error supports this presumption since the effect is statistically significant and yields a value of 0.25 for  $R^2$  within. This finding highlights the importance of the use of advanced credit models in empirical investigations.

## 7 CONCLUSION

In this paper, we quantify and explain valuation differences between the credit default swaps and corporate bonds of a sample of European investment-grade companies. For our analysis, we first establish a corporate-bond-specific measure of the valuation difference by taking advantage of all information provided by the firm's entire CDS curve. We gain this information by calibrating a stochastic intensity credit model to the time series of the ten-year CDS curve of a firm and use it to calculate prices for its corporate bonds. The measure of the valuation difference is based on the difference between the yields to maturity of the actual market price and the CDS-implied model price of a corporate bond.

We find that, on average, risk premiums in the corporate bond market exceed risk premiums implied by the CDS market by 8.4bps, which is much less than found in previous studies. In spite of the low mean differences between the prices in these two

credit markets, we find substantial variation of the valuation difference both across individual corporate bonds and through time.

Using panel data analysis, we gain the following main insights. First, we find that liquidity plays a major role in the valuation difference. We detect a significant impact from the coupon rate and the amount outstanding of a bond, whereas the maturity and the age of a bond and the CDS liquidity measure are not statistically significant. Second, our results show that a common factor has a significant negative impact on the time series variation of the valuation difference in the form of the iTraxx Europe CDS index. Hence, with an increasing level of the market-wide default risk, default risk itself becomes a more dominant part of a bond's credit spread, diminishing the role of nondefault components. Third, we detect a positive significant effect of the equity return on the valuation difference, indicating that CDS markets respond more strongly to movements in the stock market than corporate bonds. This finding, however, does not extend to the volatility of the equity return. Fourth, we find no impact from fundamental factors such as the cheapest-to-deliver option and the counterparty credit risk on the valuation difference. Fifth, we come to the conclusion that both credit markets respond in a very similar way to changes in macroeconomic conditions as proxied by the volatilities of the three-month EURIBOR rate and the ten-year spot rate, since these two proxies do not influence the valuation difference. Finally, we find no evidence for an impact of synthetic CDO transactions on the valuation difference.

## APPENDIX A. DETAILS OF THE CHARACTERISTIC FUNCTION

The conditional discounted characteristic function of  $h(t + \tau)$  is defined as

$$\Phi(t, t + \tau; \phi) := \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^{t+\tau} [r(u) + h(u)] du + i\phi h(t + \tau) \right) \right], \quad (\text{A.1})$$

with the boundary condition  $\Phi(t + \tau, t + \tau; \phi) = \exp(i\phi h(t + \tau))$  and  $i := \sqrt{-1}$  the imaginary unit. Its closed-form solution has the exponentially affine form

$$\Phi(t, t + \tau; \phi) = \exp \left( \mathcal{A}(\tau; \phi) - \sum_{i=1}^3 \mathcal{B}_i(\tau; \phi) X_i(t) - \mathcal{B}_z(\tau; \phi) Z(t) \right), \quad (\text{A.2})$$

where

$$\mathcal{A}(\tau; \phi) = \sum_{i=1}^3 \mathcal{A}_i(\tau; \phi) + \mathcal{A}_z(\tau; \phi) + (i\phi - \tau) \left( \Lambda_0 - \sum_{i=1}^3 \Lambda_i \bar{X}_i \right), \quad (\text{A.3})$$

with

$$\mathcal{A}_i(\tau; \phi) = \frac{2\kappa_i \theta_i}{\sigma_i^2} \log \left( \frac{\gamma_i \exp((\kappa_i + \lambda_i)\tau/2)}{\gamma_i \cosh(\gamma_i \tau/2) + [(\kappa_i + \lambda_i) - i\phi \Lambda_i \sigma_i^2] \sinh(\gamma_i \tau/2)} \right), \quad (\text{A.4})$$

$$\mathcal{A}_z(\tau; \phi) = \frac{2\kappa_z \theta_z}{\sigma_z^2} \log \left( \frac{\gamma_z \exp((\kappa_z + \lambda_z)\tau/2)}{\gamma_z \cosh(\gamma_z \tau/2) + [(\kappa_z + \lambda_z) - i\phi \sigma_z^2] \sinh(\gamma_z \tau/2)} \right), \quad (\text{A.5})$$

and

$$\begin{aligned} \mathcal{B}_i(\tau; \phi) &= \frac{-i\phi \Lambda_i [\gamma_i \cosh(\gamma_i \tau/2) - (\kappa_i + \lambda_i) \sinh(\gamma_i \tau/2)] + 2(1 + \Lambda_i) \sinh(\gamma_i \tau/2)}{\gamma_i \cosh(\gamma_i \tau/2) + [(\kappa_i + \lambda_i) - i\phi \Lambda_i \sigma_i^2] \sinh(\gamma_i \tau/2)}, \end{aligned} \quad (\text{A.6})$$

$$\begin{aligned} \mathcal{B}_z(\tau; \phi) &= \frac{-i\phi [\gamma_z \cosh(\gamma_z \tau/2) - (\kappa_z + \lambda_z) \sinh(\gamma_z \tau/2)] + 2 \sinh(\gamma_z \tau/2)}{\gamma_z \cosh(\gamma_z \tau/2) + [(\kappa_z + \lambda_z) - i\phi \sigma_z^2] \sinh(\gamma_z \tau/2)}, \end{aligned} \quad (\text{A.7})$$

with

$$\gamma_i := \sqrt{(\kappa_i + \lambda_i)^2 + 2\sigma_i^2(1 + \Lambda_i)}, \quad \text{for } i = 1, 2, 3,$$

and

$$\gamma_z := \sqrt{(\kappa_z + \lambda_z)^2 + 2\sigma_z^2}.$$

## APPENDIX B. DETAILS OF THE CREDIT DEFAULT SWAP VALUATION FORMULA

The credit default swap valuation formula is given by

$$\begin{aligned} \text{CDS}(t, \tau) &= [1 - \text{REC}(t)] \int_t^{t+\tau} \frac{1}{i} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} du \\ &\quad \times \left( \delta \sum_{n=1}^{4\tau} \Phi(t, T_n; \phi = 0) \right. \\ &\quad \left. + \delta \sum_{n=1}^{4\tau} \int_{T_{n-1}}^{T_n} \frac{u - T_{n-1}}{T_n - T_{n-1}} \frac{1}{i} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} du \right)^{-1}. \end{aligned} \quad (\text{B.1})$$

First, the default-risk-adjusted discount factor in  $t$  for the maturity  $\tau$  is given by

$$\Phi(t, t + \tau; \phi = 0) = \exp \left( \mathcal{A}(\tau; 0) - \sum_{i=1}^3 \mathcal{B}_i(\tau; 0) X_i(t) - \mathcal{B}_z(\tau; 0) Z(t) \right). \quad (\text{B.2})$$

Since setting  $\phi = 0$  in the characteristic function causes all terms involving the complex factor  $i := \sqrt{-1}$  to vanish, formulas (A.3)–(A.7) reduce to

$$\mathcal{A}(\tau; 0) = \sum_{i=1}^3 \mathcal{A}_i(\tau; 0) + \mathcal{A}_z(\tau; 0) - \tau \left( \Lambda_0 - \sum_{i=1}^3 \Lambda_i \bar{X}_i \right), \quad (\text{B.3})$$

where

$$\mathcal{A}_i(\tau; 0) = \frac{2\kappa_i \theta_i}{\sigma_i^2} \log \left( \frac{\gamma_i \exp((\kappa_i + \lambda_i)\tau/2)}{\gamma_i \cosh(\gamma_i \tau/2) + (\kappa_i + \lambda_i) \sinh(\gamma_i \tau/2)} \right), \quad (\text{B.4})$$

$$\mathcal{A}_z(\tau; 0) = \frac{2\kappa_z \theta_z}{\sigma_z^2} \log \left( \frac{\gamma_z \exp((\kappa_z + \lambda_z)\tau/2)}{\gamma_z \cosh(\gamma_z \tau/2) + (\kappa_z + \lambda_z) \sinh(\gamma_z \tau/2)} \right), \quad (\text{B.5})$$

and

$$\mathcal{B}_i(\tau; 0) = \frac{2(1 + \Lambda_i) \sinh(\gamma_i \tau/2)}{\gamma_i \cosh(\gamma_i \tau/2) + (\kappa_i + \lambda_i) \sinh(\gamma_i \tau/2)}, \quad (\text{B.6})$$

$$\mathcal{B}_z(\tau; 0) = \frac{2 \sinh(\gamma_z \tau/2)}{\gamma_z \cosh(\gamma_z \tau/2) + (\kappa_z + \lambda_z) \sinh(\gamma_z \tau/2)}, \quad (\text{B.7})$$

with

$$\gamma_i := \sqrt{(\kappa_i + \lambda_i)^2 + 2\sigma_i^2(1 + \Lambda_i)}, \quad \text{for } i = 1, 2, 3,$$

and

$$\gamma_z := \sqrt{(\kappa_z + \lambda_z)^2 + 2\sigma_z^2}.$$

Second, the discounted density function of the risk-neutral default probability for  $t + \tau$  is given by

$$\begin{aligned} & \left. \frac{1}{i} \frac{\partial \Phi(t, t + \tau; \phi)}{\partial \phi} \right|_{\phi=0} \\ &= \Phi(t, t + \tau; \phi = 0) \\ & \times \left[ \left. \frac{1}{i} \frac{\partial \mathcal{A}(\tau; \phi)}{\partial \phi} \right|_{\phi=0} - \sum_{i=1}^3 \left. \frac{1}{i} \frac{\partial \mathcal{B}_i(\tau; \phi)}{\partial \phi} \right|_{\phi=0} X_i(t) - \left. \frac{1}{i} \frac{\partial \mathcal{B}_z(\tau; \phi)}{\partial \phi} \right|_{\phi=0} Z(t) \right], \end{aligned} \quad (\text{B.8})$$

where

$$\left. \frac{1}{i} \frac{\partial \mathcal{A}(\tau; \phi)}{\partial \phi} \right|_{\phi=0} = \sum_{i=1}^3 \left. \frac{1}{i} \frac{\partial \mathcal{A}_i(\tau; \phi)}{\partial \phi} \right|_{\phi=0} + \left. \frac{1}{i} \frac{\partial \mathcal{A}_z(\tau; \phi)}{\partial \phi} \right|_{\phi=0} + \left( \Lambda_0 - \sum_{i=1}^3 \Lambda_i \bar{X}_i \right), \quad (\text{B.9})$$

with

$$\frac{1}{i} \frac{\partial \mathcal{A}_i}{\partial \phi} \Big|_{\phi=0} = \frac{2\kappa_i \theta_i \Lambda_i \sinh(\gamma_i \tau/2)}{\gamma_i \cosh(\gamma_i \tau/2) + (\kappa_i + \lambda_i) \sinh(\gamma_i \tau/2)}, \quad (\text{B.10})$$

$$\frac{1}{i} \frac{\partial \mathcal{A}_z}{\partial \phi} \Big|_{\phi=0} = \frac{2\kappa_z \theta_z \sinh(\gamma_z \tau/2)}{\gamma_z \cosh(\gamma_z \tau/2) + (\kappa_z + \lambda_z) \sinh(\gamma_z \tau/2)}, \quad (\text{B.11})$$

and where

$$\begin{aligned} \frac{1}{i} \frac{\partial \mathcal{B}_i}{\partial \phi} \Big|_{\phi=0} &= \frac{1}{[\gamma_i \cosh(\gamma_i \tau/2) + (\kappa_i + \lambda_i) \sinh(\gamma_i \tau/2)]^2} \\ &\times \left( -\Lambda_i \left[ \gamma_i^2 \cosh^2 \left( \frac{\gamma_i \tau}{2} \right) - (\kappa_i + \lambda_i)^2 \sinh^2 \left( \frac{\gamma_i \tau}{2} \right) \right] \right. \\ &\quad \left. + 2\Lambda_i (1 + \Lambda_i) \sigma_i^2 \sinh^2 \left( \frac{\gamma_i \tau}{2} \right) \right), \end{aligned} \quad (\text{B.12})$$

$$\begin{aligned} \frac{1}{i} \frac{\partial \mathcal{B}_z}{\partial \phi} \Big|_{\phi=0} &= \left( -\left[ \gamma_z^2 \cosh^2 \left( \frac{\gamma_z \tau}{2} \right) - (\kappa_z + \lambda_z)^2 \sinh^2 \left( \frac{\gamma_z \tau}{2} \right) \right] \right. \\ &\quad \left. + 2\sigma_z^2 \sinh^2 \left( \frac{\gamma_z \tau}{2} \right) \right) \\ &\times \left( \left[ \gamma_z \cosh \left( \frac{\gamma_z \tau}{2} \right) + (\kappa_z + \lambda_z) \sinh \left( \frac{\gamma_z \tau}{2} \right) \right]^2 \right)^{-1}. \end{aligned} \quad (\text{B.13})$$

### APPENDIX C. INTEREST RATE PARAMETER ESTIMATION WITH THE KALMAN FILTER

In order to estimate the interest rate parameters with the linear Kalman filter, we first transform the three-factor CIR model into the state space formulation, consisting of the transition and measurement equation. The transition equation describes the evolution of the three state variables over our 106 observation points:

$$\mathbf{X}(t+1) = \mathbf{c} + \mathbf{F} \cdot \mathbf{X}(t) + \boldsymbol{\eta}(t+1), \quad (\text{C.1})$$

where  $\mathbf{X}(t)$  is a  $3 \times 1$  vector with  $i$ th element  $X_i(t)$ . Since we replace the exact transition density of the state variables by a normal density,  $\mathbf{c}$  is a  $3 \times 1$  vector with  $i$ th element  $\theta_i (1 - e^{-\kappa_i \Delta t})$ ,  $\mathbf{F}$  is a  $3 \times 3$  diagonal transition matrix with  $i$ th element  $e^{-\kappa_i \Delta t}$ , and  $\boldsymbol{\eta}(t+1)$  is a  $3 \times 1$  disturbance vector with zero mean and a  $3 \times 3$  diagonal covariance matrix with  $i$ th element

$$\xi_i(t+1) = \frac{\theta_i \sigma_i^2}{2\kappa_i} (1 - e^{-\kappa_i \Delta t})^2 + \frac{\sigma_i^2}{\kappa_i} (e^{-\kappa_i \Delta t} - e^{-2\kappa_i \Delta t}) X_i(t). \quad (\text{C.2})$$

The measurement equation establishes the relationship between the ten spot rates of the spot rate curve and the three state variables as

$$\mathbf{R}(t) = \mathbf{d} + \mathbf{H} \cdot \mathbf{X}(t) + \boldsymbol{\varepsilon}(t), \quad (\text{C.3})$$

where  $\mathbf{R}(t)$  is a  $10 \times 1$  vector, containing the spot rates  $R_{\tau_1}(t), \dots, R_{\tau_{10}}(t)$ , with  $\tau_j = j$  years,  $\mathbf{d}$  is a  $10 \times 1$  vector with  $j$ th element

$$\sum_{i=1}^3 \frac{-A_i(\tau_j)}{\tau_j}.$$

$\mathbf{H}$  is a  $10 \times 3$  matrix of factor loadings with  $(j, i)$  element  $B_i(\tau_j)/\tau_j$ , where

$$A_i(\tau_j) = \frac{2\kappa_i\theta_i}{\sigma_i^2} \log \left( \frac{\tilde{\gamma}_i \exp((\kappa_i + \lambda_i)\tau_j/2)}{\tilde{\gamma}_i \cosh(\tilde{\gamma}_i\tau_j/2) + (\kappa_i + \lambda_i) \sinh(\tilde{\gamma}_i\tau_j/2)} \right) \\ (\equiv \mathcal{A}_i(\tau_j; \phi = 0, \Lambda_i = 0)), \quad (\text{C.4})$$

$$B_i(\tau_j) = \frac{2 \sinh(\tilde{\gamma}_i\tau_j/2)}{\tilde{\gamma}_i \cosh(\tilde{\gamma}_i\tau_j/2) + (\kappa_i + \lambda_i) \sinh(\tilde{\gamma}_i\tau_j/2)} \\ (\equiv \mathcal{B}_i(\tau_j; \phi = 0, \Lambda_i = 0)), \quad (\text{C.5})$$

and

$$\tilde{\gamma}_i = \sqrt{(\kappa_i + \lambda_i)^2 + 2\sigma_i^2},$$

are given by Cox *et al* (1985). Finally,  $\boldsymbol{\varepsilon}(t + 1)$  is a normally distributed  $10 \times 1$  disturbance vector with zero mean and a  $10 \times 10$  diagonal covariance matrix  $\mathbf{RHO}$  with  $j$ th element  $\rho_j^2$ . The implementation of the Kalman filter yields the logarithm of the quasilielihood function for the  $m = 10$  spot rates over the  $T = 106$  observation points:

$$\mathcal{L}(\boldsymbol{\Psi}) = -\frac{1}{2}mT \log(2\pi) - \frac{1}{2} \sum_{t=1}^T \log |\boldsymbol{\Omega}_t| - \frac{1}{2} \sum_{t=1}^T \boldsymbol{\xi}_t^T \boldsymbol{\Omega}_t^{-1} \boldsymbol{\xi}_t, \quad (\text{C.6})$$

with parameter vector

$$\boldsymbol{\Psi} = (\kappa_1, \kappa_2, \kappa_3, \theta_1, \theta_2, \theta_3, \sigma_1, \sigma_2, \sigma_3, \lambda_1, \lambda_2, \lambda_3, \rho_1, \dots, \rho_{10}),$$

forecast error  $\boldsymbol{\xi}_t$  and mean squared error  $\boldsymbol{\Omega}_t$  of the measurement equation. We maximize  $\mathcal{L}(\boldsymbol{\Psi})$  using the Nelder–Mead simplex search method.

## APPENDIX D. HAZARD RATE PARAMETER ESTIMATION WITH THE INVERSION METHOD

In order to estimate the hazard rate parameters of the twenty-nine sample companies, we take the interest rate parameter estimates and the associated time series of the three state variables as inputs and make use of the inversion method, where we assume that five-year CDS spreads are observed without error and that the one-, three- and ten-year CDS spreads are observed with errors. We assume the error vector for these three CDS spreads  $\mathbf{v}_t = (v_t^1, v_t^3, v_t^{10})^T$  to have zero mean and to be serially uncorrelated but jointly normally distributed with the time-invariant covariance matrix  $\boldsymbol{\Omega}_v$ . In order to carry out the QML estimation, we substitute the exact transition density of  $Z(t)$  with a normal density. Thus, the logarithm of the quasilielihood function is

$$\begin{aligned} \mathcal{L}_{\text{CDS}} = & -\frac{T-1}{2} \log(2\pi) - \frac{1}{2} \sum_{t=2}^T \log Q_z(t) \\ & - \frac{1}{2} \sum_{t=2}^T \frac{[Z(t) - \mu_z(t)]^2}{Q_z(t)} - \sum_{t=2}^T \log(|J_t|) \\ & - \frac{3(T-1)}{2} \log(2\pi) - \frac{T-1}{2} \log(|\boldsymbol{\Omega}_v|) - \frac{1}{2} \sum_{t=2}^T \mathbf{v}_t^T \boldsymbol{\Omega}_v^{-1} \mathbf{v}_t, \end{aligned} \quad (\text{D.1})$$

with the conditional expectation and variance of the transition density of  $Z(t)$ :

$$\mu_z(t) := \mathbb{E}^{\mathbb{P}}[Z(t) \mid Z(t-1)] = \theta_z(1 - e^{-\kappa_z \Delta t}) + e^{-\kappa_z \Delta t} Z(t-1), \quad (\text{D.2})$$

$$\begin{aligned} Q_z(t) &:= \text{Var}^{\mathbb{P}}[Z(t) \mid Z(t-1)] \\ &= \frac{\theta_z \sigma_z^2}{2\kappa_z} (1 - e^{-\kappa_z \Delta t})^2 + \frac{\sigma_z^2}{\kappa_z} (e^{-\kappa_z \Delta t} - e^{-2\kappa_z \Delta t}) Z(t-1), \end{aligned} \quad (\text{D.3})$$

and the time-dependent Jacobian term  $J_t$  of the variable transformation.  $J_t$  is the partial derivative of the CDS pricing function in (3.15) with respect to  $Z(t)$ :

$$\begin{aligned} J_t &= \frac{\partial \text{CDS}(t, \tau = 5)}{\partial Z(t)} \\ &= \mathcal{N} \left( \left[ \delta \sum_{n=1}^{20} \Phi(t, T_n; \phi = 0) \right. \right. \\ &\quad \left. \left. + \delta \sum_{n=1}^{20} \int_{T_{n-1}}^{T_n} \frac{u - T_{n-1}}{T_n - T_{n-1}} \frac{1}{i} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} du \right]^2 \right)^{-1}, \end{aligned} \quad (\text{D.4})$$



with the numerator

$$\begin{aligned}
\mathcal{N} = & \left[ [1 - \text{REC}(t)] \int_t^{t+5} \frac{\partial}{\partial Z(t)} \left[ \frac{1}{i} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} \right] du \right] \\
& \times \left[ \delta \sum_{n=1}^{20} \Phi(t, T_n; \phi = 0) \right. \\
& \quad \left. + \delta \sum_{n=1}^{20} \int_{T_{n-1}}^{T_n} \frac{u - T_{n-1}}{T_n - T_{n-1}} \frac{1}{i} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} du \right] \\
& - \left[ [1 - \text{REC}(t)] \int_t^{t+5} \frac{1}{i} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} du \right] \\
& \times \left[ \delta \sum_{n=1}^{20} \frac{\partial \Phi(t, T_n; \phi = 0)}{\partial Z(t)} \right. \\
& \quad \left. + \delta \sum_{n=1}^{20} \int_{T_{n-1}}^{T_n} \frac{u - T_{n-1}}{T_n - T_{n-1}} \frac{\partial}{\partial Z(t)} \left[ \frac{1}{i} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} \right] du \right],
\end{aligned}$$

where

$$\frac{\partial \Phi(t, T_n; \phi = 0)}{\partial Z(t)} = -\Phi(t, T_n; \phi = 0) \mathcal{B}_z(T_n - t; 0), \quad (\text{D.5})$$

and

$$\begin{aligned}
& \frac{\partial}{\partial Z(t)} \left[ \frac{1}{i} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} \right] \\
& = -\Phi(t, u; \phi = 0) \mathcal{B}_z(u - t; 0) \\
& \quad \times \left[ \frac{1}{i} \frac{\partial \mathcal{A}(u - t; \phi)}{\partial \phi} \Big|_{\phi=0} - \sum_{i=1}^3 \frac{1}{i} \frac{\partial \mathcal{B}_i(u - t; \phi)}{\partial \phi} \Big|_{\phi=0} X_i(t) \right. \\
& \quad \quad \left. - \frac{1}{i} \frac{\partial \mathcal{B}_z(u - t; \phi)}{\partial \phi} \Big|_{\phi=0} Z(t) \right] \\
& \quad - \Phi(t, u; \phi = 0) \frac{1}{i} \frac{\partial \mathcal{B}_z(u - t; \phi)}{\partial \phi} \Big|_{\phi=0}. \quad (\text{D.6})
\end{aligned}$$

Since maximizing the quaslikelihood function in (D.1) twenty-nine times using standard optimization routines turns out to be an unfeasible task, we make use of an iterated simulated QML procedure similar to that used in Saita (2006) and Anderson (2008).

We minimize the sum of the MAEs of one-, three- and ten-year maturities with respect to the parameter vector  $\Psi_{\text{CDS}} = (\kappa_z, \theta_z, \sigma_z, \lambda_z, \Lambda_1, \Lambda_2, \Lambda_3)$  using a multilevel grid search procedure. In order to reduce the dimensionality of our problem, we eliminate the parameter  $\Lambda_0$  by setting it to zero. We terminate the grid search when the sum of the three MAEs cannot be reduced by more than  $\varepsilon = 10^{-4}$  bps, which leads us to our optimal hazard rate parameters.

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