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The Evolution of College Algebra: Competencies and Themes of a Quantitative Reasoning Course at the University Of Kentucky

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THE EVOLUTION OF COLLEGE ALGEBRA:
COMPETENCIES AND THEMES OF A QUANTITATIVE REASONING COURSE
AT THE UNIVERSITY OF KENTUCKY

A Dissertation
Presented to
The Faculty of the Educational Leadership Doctoral Program
Western Kentucky University
Bowling Green, Kentucky


In Partial Fulfillment
Of the Requirements for the Degree
Doctor of Education


By
Scott Taylor

December 2017

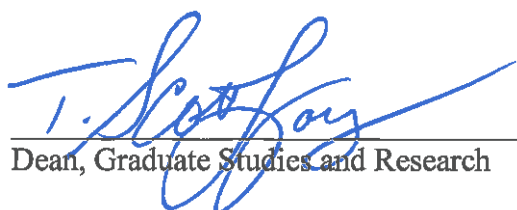
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COMPETENCIES AND THEMES OF A QUANTITATIVE REASONING COURSE
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For many institutions, especially community colleges, college algebra has been the *default* mathematics or quantitative reasoning requirement. However, the topics that have been taught in college algebra, teaching methods, and the goals of a quantitative reasoning requirement have changed and vary over time and among different institutions. Because of history, policy, and political influences, this study sought to explore commonalities and disparities of college algebra as it has evolved through the University of Kentucky. The three central research questions were *What have been the common topics or themes of the competencies and topics covered in CA over the years at UK?* (RQ1), *What internal forces have led to topic coverage or attribute changes in CA?* (RQ2), and *How has QR evolved at UK?* (RQ3).

Through a review of literature, common topics were discovered among Kentucky college algebra course descriptions. These commonalities were used as a foundation by which, through the qualitative lens of historical methods, the history of college algebra was measured and studied. The origins and motivations for these changes were explored using multiple sources of data.

CHAPTER I: STATEMENT OF THE PROBLEM

Introduction

Within a general education curriculum, most institutions require a mathematics or statistics course for the purpose of meeting a quantitative reasoning (QR) requirement. The purpose of a general education curriculum in Kentucky has traditionally grown from a liberal arts education philosophy that insisted all students have a broad, common knowledge base in order to graduate not only with intense knowledge of their major discipline, but also with breadth of knowledge from many areas (Eastern Kentucky University, n.d.; Kentucky State University, 2014a; Northern Kentucky University, n.d.a; Southern Association of Colleges and Schools [SACSCOC], 2012; University of Kentucky, 2016a). QR has historically been one of those areas. Any approved QR course, therefore, could serve myriad degree programs unless a particular major prescribes specific QR or mathematics coursework (Latzer, 2004). For example, a degree program in chemistry may mandate two semesters of calculus, for which College Algebra (CA) would typically be the prerequisite. If all three courses in that sequence met institutional QR requirements, no chemistry major had to worry about failing to meet the general education requirement of QR.

However, history majors may not have an explicit QR course outlined in their program. Therefore, in order to meet the QR requirement of the core curriculum, they may have chosen a course they wanted in order to meet this requirement, assuming the institution offered a variety that satisfied the QR requirement. In many instances, the QR course of choice appears to have been, by default, a mathematics or statistics course, especially at community colleges. Despite the range of potential courses—mathematics

or non-mathematics—that could satisfy QR requirements, CA has been the *default mathematics requirement* in the thinking of many institutional policy makers (Vandal, 2015).

This study investigates the content that has been covered in CA at the University of Kentucky (UK) as the course has evolved over the years, examining reasons for content change. This qualitative research focuses on historical events at the university, state, and the national levels that have played a role in the evolution of mathematics curriculum at UK. By using historical methods (document analysis), changes to the course competencies and course description are highlighted for the purposes of determining the reason the current incarnation of CA covers specific topics while excluding others. The discernments gleaned from this project will be useful in establishing (a) what CA is, (b) why it contains the specific material taught, and (c) historical context that will challenge why CA seems to be the default quantitative reasoning class of choice for many institutions, especially community colleges.

College Algebra

Every year over a million college students enroll in CA, a proverbial cash cow of the department and institution, yet close to half fail the course (Gordon, 2008). Further, as with most college classes, material covered in CA varies from institution to institution. While some topics may be common to many colleges, there are invariably differences in content and focus, as no national consensus or uniformity of curriculum exists among colleges and universities for any general education curriculum; in fact, the SACSCOC allows for variation (SACSCOC, 2012; Toombs, Amey, & Chen, 1991). While this in itself may not necessarily constitute a problem, any expectations of consistency would be

an issue. As CA typically serves as a prerequisite for other mathematics coursework such as calculus (Vandal, 2015), taking CA at one institution while taking calculus at another may represent a conundrum under the fallacy of consistency. This research reveals the deficit of uniformity in definition as to that which has constituted college-level algebra. In addition, within any individual institution, there will be a course description outlining the topics that an aforementioned institutional class covers, although depth of topic emphasis is at the discretion of the instructor. Many times instructors pick their books, so different sections of the same course may manifest themselves in radically different fashions. One instructor may mention a particular topic in passing, while another spends several weeks working with it. As such, there has been no consensus as to what CA should entail across the nation or even within a single college. CA textbooks may also play a role in the selection of topic coverage. Instructors, especially adjuncts, whose numbers are starting to increase with the reduction of full-time college instructors (Jolley, Cross, & Bryant, 2014), may follow a textbook's organizational structure more so than their own particular thoughts (or that of the institution) about what should be emphasized.

Within any given institution, common competencies or course descriptions would allow for continuity among different sections and instructors. Western Kentucky University (WKU) has regularly offered trigonometry; in fact, students could choose from 13 sections taught by eight different instructors in the fall 2016 semester, all of which shared the similar course description asserting the course would include "unit circle, trigonometric functions and graphs, trigonometric identities and equations, right triangle trigonometry, laws of sines and cosines, DeMoivre's Theorem, vectors and applications of trigonometry" (WKU, 2016, p. 256). While the course description

outlined specific topics to be covered, the length of time each instructor spent on each topic may depend upon instructor discretion. The books used by individual section also varied by instructor—per WKU’s online bookstore, different sections of the same course required different textbooks (WKU Store, 2017). Additionally, course descriptions have never precluded topics; they have simply stated what will allegedly assuredly be covered. Professors have enjoyed the academic freedom of electing the material they wish to supplement to their courses as it benefits their field (Post, 2008; Stone, 2006). As such, instructors have always enjoyed the liberty of appending relevant topics at their discretion. The assortment in textbook selection, depth of topic, and any section-specific material supplementation has resulted in discontinuity among various sections of the same class within the same institution.

While no formal legislation has mandated all colleges, universities, or instructors to conform to homogeneous placement guidelines, curricular content, textbooks, or depth of topic coverage (nor, under the ideas of academic freedom, should they), individual institutions or departments may forge their own internal policies, rules, or agreements. However, even in the scenario wherein a department has established the implementation of a practice in which all instructors work from the same text, have the same number of tests (even conceivably authored from commonly-adopted test templates), and operated on a shared grading scale, bias inherently would influence individual professor appraisal of student work. Perception as to the degree of an error’s significance would likely vary among instructors when trying to establish partial credit, even with the application of a common rubric. That which one teacher felt was a major error, another may have found trivial. On a single exam or assignment, elements need not be evenly distributed. One

mathematics instructor may have an exam with 20 questions, all worth five points apiece. Another may have a 20-question exam on which some problems are worth more than others. Likewise, weight of examinations, homework, and other assignments to the final overall course grade may not be parallel among a department. For example, per syllabi, one section of WKU's CA course listed exams as being worth 50% of the total course grade (Wilson, 2017), while another listed exams as being worth 60% of the total course grade (Wells, 2017).

More research is needed to determine what, if any, consistencies exist among sections within an institution, a geographical region, and nationally to establish a commonly-accepted notion of what has been taught in a given section of CA and the competencies or learning outcomes therein. Further, it should be noted I have not claimed inconsistencies themselves have represented problems in need of solution, with exception of expectations of consistency under a prerequisite model of mathematical hierarchy. Rather, the aim is to see to what degree there has or has not been an effort to establish commonly-accepted definitions.

Quantitative Reasoning

Quantitative Reasoning, *Quantitative Literacy*, *Mathematical Reasoning*, *Numeracy*, *Quantitative Thinking*, and *Mathematical Thinking* have been, depending upon the source, synonyms that can either be used quite interchangeably or differentiated through rigorous minutiae in definition. Despite that some educational and mathematical philosophers have meticulously worked to delineate among these terms, for the purposes of this piece the terms will be used interchangeably and, except in cases in which scholars have made deliberate and overt effort to identify differences between or among the terms,

when a referenced work uses one name, this piece shall assume synonymy with all others. To distinguish the minutiae among these terms goes beyond the scope of this research, and the overall intent of these topics within the framework of higher education will generally be to address a graduation requirement for a baccalaureate credential. Therefore, philosophical nuances of meaning will be irrelevant to the purpose of this work.

Many definitions for QR have been suggested. Kirsch and Jungeblut (1990) defined it as “the knowledge and skills needed to apply arithmetic operations, either alone or sequentially, that are embedded in printed materials, such as in balancing a checkbook, figuring out a tip, completing an order form, or determining the amount of interest from a loan advertisement” (p.4). Steen (1997) defined QR over five dimensions: “practical, for immediate use in the routine tasks of life; civic, to understand major public policy issues; professional, to provide skills necessary for employment; recreational, to appreciate games, sports, lotteries; and cultural, as part of the tapestry of civilization” (pp. 6-7). Boersma, Diefenderfer, Dingman, and Madison (2011) identified six core competencies for quantitative reasoning:

...a ‘habit of mind,’ competency, and comfort in working with numerical data.

Individuals with strong QL skills possess the ability to reason and solve quantitative problems from a wide array of authentic contexts and everyday life situations. They understand and can create sophisticated arguments supported by quantitative evidence and they can clearly communicate those arguments in a variety of formats (using words, tables, graphs, mathematical equations, etc., as appropriate) (p. 3).

The International Life Skills Survey (as cited in Steen, 2001) defined QR as “an aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem-solving skills that people need in order to engage effectively in quantitative situations arising in life and work. Dwyer, Gallagher, Levin, and Morley (2003) defined QR as including the following:

...reading and understanding information given in various formats, such as in graphs, tables, geometric figures, mathematical formulas or in text (e.g., in real-life problems); interpreting quantitative information and drawing appropriate inferences from it; solving problems, using arithmetical, algebraic, geometric, or statistical methods; estimating answers and checking answers for reasonableness; communicating quantitative information verbally, numerically, algebraically, or graphically; recognizing the limitations of mathematical or statistical methods (p. 13).

Hughes-Hallett (as cited in De Lange, 2003) insisted that QR required students “to stay in context. Mathematics is about general principles that can be applied in a range of contexts; quantitative literacy is about seeing every context through a quantitative lens” (p. 94). Rocconi, Lambert, McCormick, and Sarraf (2013) leaned on several other definitions (including Steen’s 1997 definition) to say QR were the skills necessary to be quantitatively literate, and quantitatively literate included “an everyday understanding of mathematics; in other words, the ability to use numerical, statistical, and graphical information in everyday life” (p. 1). The general theme of the definitions of QR has been application of mathematical thinking to contexts beyond academia—that those who are engaging in QR are not only learning some general form of mathematics, statistics, or

algebra, but the knowledge has authentic meaning to the student.

QR (or some mathematics coursework) requirements are typically encouraged or mandated by regional accrediting agencies and state advisory agencies (such as the SACSCOC and the Kentucky Council on Postsecondary Education [CPE], respectively) (CPE, 2011; SACSCOC, 2012). It is, however, up to the individual institution to decide what courses meet the QR requirement. The goals of QR have typically been established as encouraging students to think abstractly, demonstrate an understanding of critical thinking, or apply mathematics to real-world situations (Elrod, 2014; CPE, 2011). While most colleges and universities make explicit the reason for a QR requirement as a part of their general education curriculum, it has not necessarily been clear why the particular classes, including CA, were the courses offered to satisfy QR requirements. For example, trigonometry satisfied the QR requirement for the Kentucky Community and Technical College System (KCTCS) Associate of Arts (AA) degree, but a class called applied mathematics did not (Kentucky Community and Technical College System, 2016). Furthermore, why mathematics courses have typically served as the classes designated to meet the QR requirement has not been established. As one of the purposes of a QR requirement under the CPE definition was to apply mathematics to real-world situations, it has not necessarily been made explicit why applied mathematics has not satisfied the AA degree QR requirement. A possible factor in determining why applied mathematics, or any particular course, would be precluded from an accepted QR course might be rigor. If rigor were a factor, then while trigonometry may be a more collegiate-level course, to my knowledge, no evidence has been demonstrated that trigonometry—or any of the KCTCS AA QR-certified courses—has met CPE stipulations for QR status.

While newly-created classes may have to undergo a process to certify they meet the requirements of general education QR status (KCTCS, 2017), this study furthers the research into whether preexisting courses, which have been granted QR status, have been designed in a fashion which reflected the aims of a QR requirement. Furthermore, it has not been established why non-mathematics courses have seldom been awarded QR status. There have been exceptions; the University of Kentucky (UK) has allowed certain science and philosophy classes to meet their QR requirement (UK, 2016a). However, only mathematics and statistics courses have satisfied the KCTCS QR requirement for degree-seeking students (Kentucky Community and Technical College System, 2016).

Additionally, as there may have been a disconnect between course design and course application (i.e., the teaching of the course), this study ascertains to what extent the course has reflected the aims of a QR requirement.

College Algebra as a Quantitative Reasoning Course

As aforementioned, the evolution of all college courses, including CA, has been subject to independent historical paths particular to each college and to each department within the college. Hence, discrepancies have existed between the content of CA among higher education schools, as well as between CA and the QR requirement. This discrepancy grew from a general education QR requirement—which was set forth by forces external to the college—that has been met by courses potentially predating QR legislation that were not designed with QR-specific goals in mind. However, since there has been no consensus as to what CA means (e.g., what competencies it should include, what admissions or prerequisites should be, i.e., ACT score, depth of competency coverage, etc.), sometimes even among faculty within the same institution, it cannot be

guaranteed that CA has satisfied the purposes underpinning a QR requirement. In addition to research into why any given CA class covers the specific topics of its course description, research should be conducted to determine whether that course should be used to satisfy its purported QR requirement. Once a sense is gained as to why CA has manifested itself in its current form, the findings of the study can be used to evaluate if it is the best choice for meeting QR requirements of a general education core that serves a multitude of majors. Ultimately, this study will gain an idea of what CA actually is.

Historical Influences

Many national, statewide, and institutional historical influences have altered the landscape of higher education. At the national level the STEM race of the 1950s encouraged curriculum across America to re-emphasize mathematics and science. Due to Kennedy's appeal to put a man on the Moon by the end of the decade, not only were science and mathematics emphasized in curricula, but also specifically the mathematics and science necessary to put a man on the Moon. Thus, the prerequisite engineering and physics knowledge needed for astronomy and ballistics operations were purposely targeted, giving rise to an explicit subset of mathematics topic coverage (Wissehr, Concannon, & Barrow, 2011), namely algebra and calculus. However, algebra and calculus do not comprise all the branches of mathematics, yet mathematics curricula have been dominated by algebra for decades, arguably due to political motivations no longer germane to the general public and society. Logic, set theory, proof theory, number theory, computation theory, non-Euclidean geometry, topology, analysis, graph theory, and complex analysis are some subfields, to my knowledge, that have not been regularly covered at the precollege level, which has consequently cultivated a postsecondary

overemphasis on algebra and calculus.

Problematic, then, has been that the fields into which these other underrepresented areas feed have suffered precollege representation. For example, logic would benefit one who has interest in professionally working within philosophy or law (Geach, 1979). Further, if a goal of higher education includes fostering critical thinking skills, research has shown studying formal logic improves scores on critical thinking skills—an example being experimentation conducted at UK measuring analytic prowess before and after taking a course in logic (Melzer, 1949). Another example would be topology, which traditionally might be considered an upper-level baccalaureate mathematics course explicitly reserved for mathematics majors. According to Hilton (1971), the field has not been taken seriously by professionals and therefore disregarded as a “fun” subject of “rubber sheet geometry” (p. 437). However, topics covered in a high school topology class would “penetrate so many other disciplines that it must be learnt by any one wanting to become conversant with modern mathematics at large” (p. 438), and “are among those most immediately apprehended by our intelligence when coupled through our senses with the world of experience” (p. 436). Hilton also commented that a high school topology course would better prepare students for calculus and make future mathematics Ph.D. students better understand their field before enrolling in college.

Even within algebra and calculus, specific topics are considered rudimentary (although which topics might vary by school or institution), while other topics have been ignored. For example, the KCTCS CA courses cover polynomial graphs, but not partial fraction decomposition (KCTCS, 2016). Additional research would establish the historical influences of politics on mathematics curriculum today and determine if other

areas of mathematics have been needlessly ignored or overlooked in light of a now arbitrary overemphasis on CA.

At the Kentucky state level, higher education has been supported by the Kentucky Council on Public Higher Education from 1934 to 1977, the Kentucky Council on Higher Education from then up to 1997, and by CPE from 1997 to present (Ellis, 2011). Political forces caused postsecondary educational reform in Kentucky independent from, and co-correlated with, national politics. For example, CPE formed when House Bill 1 simultaneously separated the community college system from UK while combining Kentucky's technical colleges with the community colleges under the KCTCS (Commonwealth of Kentucky, 1997). This historical event, which forced technical and general education faculty departments to merge, brought about countless policy changes to curriculum and academic policies (Warren, 2008). Individual colleges invariably have had their own historical political influences (e.g., factions of faculty, long-term faculty retiring, and new faculty with innovative ideas) that have prompted curriculum changes independent from their department and institution.

This study investigates the content that has been covered in CA at UK as the course has evolved over the years, examining reasons for content change. This qualitative research focuses on historical events at the university, in Kentucky and at the national level that have played a role in the evolution of mathematics curriculum at UK. By using historical methods (document analysis), changes to the course competencies and course description are highlighted for the purposes of determining why the current incarnation of CA covers specific topics while excluding others. The discernments gleaned from this project will be useful in establishing (a) what CA is, (b) why it contains the specific

material taught, and (c) historical context that will challenge why CA seems to be the default quantitative reasoning class of choice for many institutions, especially community colleges.

UK and KCTCS. UK in Lexington, Kentucky was founded in 1865 via the Morrill Land Grant Act in 1862 and a state legislative act on February 22, 1865 (The Kentucky Encyclopedia, 2000). The campus has stretched over seven hundred acres, and had undergone three iterations before becoming the University of Kentucky in 1916 (The Kentucky Encyclopedia, 2000). It was a private, denominational institution called the Agricultural and Mechanical (A&M) College of Kentucky University from 1865 through 1878 before becoming the Agricultural and Mechanical College of Kentucky (The Kentucky Encyclopedia, 2000). It was called State University of Lexington from 1908 through 1916 (The Kentucky Encyclopedia, 2000). UK was ranked number 133 under the *US News & World Report's National Universities* category (2017). In 1960 the Northwest Center of the University of Kentucky opened in Henderson County, and the campus was renamed Henderson Community College (HCC) four years later (Henderson Community College, n.d.). From 1919 through 1997, the community college system in Kentucky fell under the jurisdiction of UK through both independent community colleges as well as extension centers (Commonwealth of Kentucky, 1997; KCTCS, 2008). The separation of the community colleges from UK and the creation of KCTCS was controversial, and many students, faculty, and staff were opposed to the legislative decision (Kentucky Community & Technical College System, 2008). However, to study the history of CA at the community colleges in Kentucky before 1997 would have been to study UK.

Purpose and Central Research Questions

This study brings together the issues described previously. There is no established, commonly-accepted definition of CA nor the competencies therein. QR requirements, while defined by regional accreditation and state authorities, are met through coursework as designated by individual institutions, but seldom have sufficient justification as to why those courses—which are primarily mathematics—were designated to meet QR requirements nor if they reflect QR purposes or definition. Specifically, CA may not be sufficient to satisfy the purpose of a QR requirement of a general education program. Finally, historical influences and past political agendas have impelled mathematics curricula at the postsecondary level to cultivate an inequitable emphasis on algebra and specific topics therein.

The purpose of this qualitative research project is to investigate the history of CA at a research facility, namely UK, as well as the oldest community college in Kentucky—HCC—to see how and why the course has changed over the years. Data sources include course catalogs and other records of the UK archives, government regulatory and memorandum documents, and scholarly works on historical influences in mathematics curriculum in higher education. To do so, document analysis will be used within an historical research framework, which will follow prescribed coding techniques later defined.

Once the evolutionary track has been established, the findings can be used as a springboard for further research into the validity of widespread CA coursework as an answer to quantitative reasoning, along with a better understanding as to what CA, as a class, means to a research one facility and, historically, why. The central research

question will be “what forces have influenced the growth of CA competencies at UK?”

Empirical Research Questions. Empirical research questions include the following:

1. What have been the common topics or themes of the competencies and topics covered in CA over the years at UK? (RQ1)
2. What internal forces have led to topic coverage or attribute changes in CA? (RQ2)
3. How has QR evolved at UK? (RQ3)

The answers to these questions will allow for research on some of the deficiencies aforementioned, which will add to the knowledge of the field. By understanding how CA and QR requirements have progressed in the current state of affairs, challenges to the status quo, growth, and productive change can be achieved through an understanding of how potentially antiquated ideals are no longer relevant in the current landscape of higher education.

Additionally, educational leaders—especially those within the KCTCS—should understand how history and other political motivations have shaped the current understanding of CA and QR when making policy and curricular decisions in the current climate, in which such issues as performance-based funding, accreditation, and external policy makers (i.e., Kentucky Governor Matt Bevin and newly-elected President Trump) are having an impact on the activities of higher education. For example, under performance-based funding, institutions would likely be expected to enable students to take their gateway mathematics coursework without remediation. Understanding what *should* be in CA, or in a QR-sanctioned class versus what historically *has been* in CA or in a QR-sanctioned class, would allow policy stakeholders to make informed decisions.

Chapter I summary. This study was motivated by the central research questions and the aforementioned issues. However, prior to the necessary steps in tracing the evolutionary pathway of CA at UK, a review of the pre-existing research within the field follows in the next chapter. The literature review establishes some background of CA, CA in Kentucky, the history of education reform, and national government and politics. Following the literature review is a chapter discussing the qualitative methodology, methods, data collection, researcher biases, and limitations/delimitations of the study. The fourth chapter provides results of the research, which will be organized by the three research question and divided among the different types of documents analyzed. The fifth and final chapter provides discussion of the findings and relevance to educational leadership, along with suggestions for further research.

CHAPTER II: REVIEW OF LITERATURE

This study investigates the content that has been covered in CA at UK as the course has evolved over the years, examining reasons for content change. This qualitative research focuses on historical events at the university, in Kentucky and at the national level that have played a role in the evolution of mathematics curriculum at UK. By using historical methods (document analysis), changes to the course competencies and course description are highlighted for the purposes of determining why the current incarnation of CA covers specific topics while excluding others. The discernments gleaned from this project will be useful in establishing (a) what CA is, (b) why it contains the specific material taught, and (c) historical context that will challenge why CA seems to be the default quantitative reasoning class of choice for many institutions, especially community colleges.

According to Randolph (2009), while the most common function of a literature review is to focus on research outcomes, “the scientific reasons for conducting a literature review are many” (p. 2). Cooper and Cooper (1998) suggested a literature review can be described through six characteristics: focus, goal, perspective, coverage, organization, and audience. This literature review (a) focuses on practices and applications; (b) seeks explication of an argument; (c) adopts a qualitative perspective of admitting authorial bias; (d) approaches the literature with purposive sampling (e.g., selecting literature I perceive as pivotal to the central research goals); (e) follows a conceptual organization wherein relevant constructs will be reviewed by topic; and (f) addresses academic audiences (Cooper & Cooper, 1998; Randolph, 2009). The overall goal of the literature review is to justify the material to be presented. Because the goal is to seek explication

based on historical practices, influences, and applications, I have adopted a coverage philosophy of Cooper and Cooper's notion toward purposive sampling; therefore, the literature is not limited to peer-reviewed scholarly research and dissertations, and much of the supportive literature is historical analyses and policy documents.

Specifically, literature on the purposes and the history of higher education mathematics curricula provided legitimacy for the study. For example, according to Tucker (2013), in the late 1800s most college students took algebra in their freshman and sophomore years, while "Well prepared students at better colleges took calculus in the sophomore year" (p. 2). However, as higher education progressed, in the second half of the 20th century the proliferation of computer science, physics, and engineering required emphasizing calculus-based mathematics curricula. Additionally, "The launching of Sputnik in 1957, in the larger context of the Cold War competition with the Soviet Union, made mathematicians, scientists, and engineers the country's Cold War heroes" (p. 9), awarding the mathematical constructs used to achieve this feat more prestige than pure and abstract mathematics. This tradition of following calculus-based curricula in the mathematics undergraduate degree programs (for which college algebra is a prerequisite) made college algebra the natural QR course of choice for the general education programs because it prepared students for calculus (Vandal, 2015).

Literature on the purposes and the history of quantitative reasoning also provided legitimacy for the study. The 2001 work by The National Council of Education and the Disciplines (NCED) established commonly-accepted definitions for quantitative reasoning as well as numerous purposes. According to Ewell (as cited in the work of The NCED, 2001), there has been a misunderstanding of the difference between mathematics

and quantitative reasoning.

While the aforementioned pieces are examples of supporting literature, part of the deficiency in the field has been a lack of research on why college algebra has been the choice course for satisfying the QR requirement. Furthermore, according to Ewell (as cited in the work of The NCED, 2001), college algebra has not addressed the real-world applications necessary to address differences between mathematics coursework and QR.

College Algebra

Nationally, CA has been offered at most public universities and has been a staple among community colleges, in which CA tends to be the commonly-accepted gateway course (Simmons, 2014). Despite perceptions that the course is universally understood, differences among universities exist. While these differences themselves may not necessarily constitute a problem, assumptions of congruence of content and uniformity can be problematic for student transfer. For example, a student who takes college algebra at one university who transfers to another may discover the transfer institution's calculus instructors assume certain knowledge was covered in college algebra. Specifically, the KCTCS course description of CA does not include sequences, and to my knowledge, sequences have generally not been taught in the KCTCS CA curriculum. However, Morehead State University's (Morehead) CA course description explicitly identifies sequences as a topic to be covered (Morehead, 2016a), and presumably a KCTCS student who transfers to Morehead may be expected to know sequences prior to enrolling in calculus. Additionally, assumptions of college readiness and prerequisite placement differences may cause considerable complications. Differing QR requirements may additionally be frustrating for students who took college algebra at a college and then

transferred to UK or Northern Kentucky University (NKU), where college algebra currently does not satisfy their QR requirement.

College Algebra in Kentucky

As this study focuses on CA at UK, most of this dissertation, and a substantive amount of the literature review, is written with heavy emphasis on events in and about Kentucky. Every public postsecondary institution in Kentucky offers college algebra (Eastern Kentucky University [EKU], 2016; KCTCS, 2016; Kentucky State University [KSU], 2016; Morehead, 2016a; Murray State University [MSU], 2016; NKU, 2016; UK, 2016b; University of Louisville [UL], 2002; WKU, 2016). At ECU, the course focused on “real and complex numbers, integer and rational exponents, polynomial and rational equations and inequalities, graphs of functions and relations, exponential and logarithmic functions,” and the “use of graphing calculators” (EKU, 2016, p. 330), which is the only mention of graphic calculators in the official course description of any public institution (although WKU’S description of the course stated that a graphing calculator was required).

At KSU, the course aimed to develop “the algebraic skills necessary for further studies in mathematics,” and covers “the algebra of functions; graphing techniques; quantitative and qualitative analysis of polynomial, rational, exponential and logarithmic functions, including limits at infinity and infinite limits; and appropriate applications,” (KSU, 2016, p. 381). Kentucky State University was the only public university in Kentucky that explicitly included limits in college algebra.

Morehead’s course included “field and order axioms; equations, inequalities; relations and functions; exponentials; roots; logarithms; [and] sequences,” (Morehead,

2016a, p. 269). Morehead was the only public institution which included sequences in its course description of college algebra.

At MSU, college algebra was designed to develop and extend “the student's basic algebra concepts and problem-solving skills in the context of functions, models, and applications,” (MSU, 2016, p. 516). The course covered “exponents and radicals; graphing; setting up and solving equations in linear, quadratic, and other forms; systems of equations; and operations on functions;” additionally, the course addressed “properties and applications of linear, quadratic, polynomial, rational, exponential, and logarithmic functions” (MSU, 2016, p. 516). MSU was the only public institution to address modeling explicitly, although many colleges mention applications, under which modeling might fall.

The UL course included “advanced topics in algebraic and rational expressions and factoring; polynomial, rational, exponential, and logarithmic functions; [and] applications,” (UL, 2002), which was the only public university that explicitly addressed rational expressions (although most, including UL, include rational equations, which can be taught independently of rational expressions).

At WKU, the course included “graphing and problem solving” that were “integrated throughout the study of polynomial, absolute value, rational, radical, exponential, and logarithmic functions” (WKU, 2016, p. 312), which was the only course description to include absolute value functions.

NKU had a class called “Algebra for College Students,” that reviewed “advanced topics from Algebra II essential for success in MAT 112 and MAT 119,” which are courses in applied calculus and calculus I, respectively (NKU, 2016, p. 329). This course,

which did not count toward the general education requirement for the institution, seemed to read more like a developmental course than a gateway course.

UK's college algebra aimed to develop "manipulative algebraic skills and mathematical reasoning required for further study in mathematics," and included "brief review of basic algebra, quadratic formula, systems of linear equations, [and] introduction to functions and graphing" (UK, 2016b). UK's CA did not meet their QR general education requirement from the 2010-2011 to the 2016-2017 academic years (UK, 2011).

Commonalities. Regardless of the potentially commonly-held notion that all college algebra courses cover the same material, few topics were common to all descriptions. *Functions* was the unequivocal front-runner for most-often-appearing term. With exception of NKU, functions were explicitly identified in every course description; however, *function* is an exceptionally vague term. To cover linear functions, for example, would be radically different from covering exponential functions. In essence, *functions* would likely be more of a category than a competency. Thus, the second most-often-appearing terms, exponential and logarithmic functions, which were identified in six of the eight public universities, might be construed as the most representative topics of CA. It should be noted that exponential and logarithmic functions always followed each other, which would make sense as logarithmic functions are inverse functions of exponential functions (which could possibly imply that inverse functions were also covered at these institutions, although inverse functions were not mentioned by name in any description). Polynomial and rational functions were next, being cited in five of the course descriptions. No other competency was listed at more than three instances. While it is

possible that some topics—such as linear, quadratic, radical, or inverse functions—have been taught in all university CA courses, based on the course descriptions, this would not be certain without looking into course syllabi or exams at all the institutions. Further, while some topics may be covered beyond the course descriptions, the absence of quadratic functions, for example, may reveal emphases or institutional value has not been the same across Kentucky universities. However, it should be noted that absences within a description does not automatically preclude coverage; the inclusion within a curriculum may be inherently understood at that university. No one at ECU, for example, might teach CA without spending a lecture or two covering linear functions in detail; nonetheless, from an outsider's perspective there has been no guarantee this competency was addressed.

Disparities. Differences were more prevalent than commonalities based on the university course descriptions, i.e., WKU was the only institution that explicitly identified absolute value functions. Further, it would seem unlikely that absolute value functions would be covered without including some linear functions, although linear functions were not identified explicitly. ECU identified rational inequalities, rational exponents, complex numbers and graphing calculators—topics no other course description addressed. Complex numbers would likely be considered pre-college material at most universities. Rational exponents may also be considered pre-college material if what was meant was *real numbers with rational exponents*; however, if what was meant was *algebraic expressions with rational exponents in an equation*, then the difficulty level would arguably be much more collegiate, especially if ECU CA students are expected to solve and graph them. However, because ECU explicitly identified graphing

calculators, it may be possible that some instructors have taught the class entirely through numerical or technological methods. Teaching students to graph rational equations without a graphic calculator would likely imply many skills relying on algebraically determining vertical, horizontal, and oblique asymptotes, removable discontinuities, and understanding the effects of odd and even powers on linear factors as they pertain to defining *x-intercepts*. However, technological approaches could circumvent an effort to compel students to learn those algebraic skills. The controversy of technology in the classroom has been prevalent for decades; in fact, a study in the 1940s argued against teaching the slide rule until high school, for fear students would become too reliant on technology and not grasp mathematical concepts (Hartung, 1942). Although theoretically this approach might be present at any university, it would seem using a graphic calculator to some degree has been explicitly encouraged at ECU. Again, while this may not necessarily constitute a problem or deficiency at ECU, it certainly would constitute inconsistencies on curricular delivery among the universities.

Instrument variation and the myth of college readiness. Instrument variation—both in physical differences among instruments and utilization policies on instrument scores—as well as differences among the universities have led to an unintended consequence. ECU required students to earn a score a 22 on the mathematics portion of the ACT exam (math ACT score of 22), earn a score of 510 on the mathematics portion of the SAT (math SAT score of 510), or earn a “passing score on an algebra placement test” in order to enroll in CA (ECU, 2016). Murray, however, allowed students to have a math ACT score of 21 (MSU, 2016). Two students with identical ACT scores, for example, would be placed into different categories depending on which Kentucky

university they attended. While college readiness may be at the forefront of many policies and political agendas, numerous nontrivial challenges have prevented this objective from being an attainable goal. Particular examples of these barriers include a lack of uniformity of admissions standards among postsecondary institutions, a lack of uniformity of individual discipline readiness indicators—even with respect to the same assessment and placement instrument such as COMPASS, which was a computer-based assessment designed for placement testing for students who had not taken the ACT or who had not scored well on the ACT (MyCompassTest, 2014)—a lack of uniformity of content skills taught within the same discipline but different among colleges, a lack of uniformity of content skills taught within the same discipline and within the same college, and inconsistencies among instructors within a single school regarding depth of content, grading, and assessment of that grading. College readiness has implied different skill sets to different stakeholders in both the postsecondary and K-12 arenas. Some might hear the term and immediately assume being college ready means having content knowledge necessary to be successful in a college-level course. However, others might believe the word applies to assessment and admissions metrics. Their conclusion could be that college readiness implies content knowledge necessary to test into a credit-bearing college class. The ideal interpretation may be the conjunction of both placement and success in a college-level course, but such an interpretation assumes college readiness speaks specifically to content knowledge. Moreover, before a student can successfully pass a college-level course, the student must apply, be accepted, and pay for the first semester. Operating under this perception, the admissions counselor might assume college readiness relies more on knowledge about the college process rather than rote

knowledge of specific subject disciplines. A rudimentary understanding of what college is, what kinds of degree programs and majors exist, what processes are necessary to gain entrance into an institution (application, orientation, FAFSA, etc.), and institution-specific policies and practices may be challenging to students who are unaware of postsecondary culture, especially to first-generation students (first in their families to attend college). Once students navigate through the processes necessary to enroll in college-level courses, retention then becomes the next item for scrutiny. Even if students succeed well in their first semester, many discover that college is simply not for them. The most current data from the National Center for Educational Statistics (NCES) indicate less than 60% of students “who began seeking a bachelor's degree at a 4-year institution in fall 2007 completed that degree within 6 years” (NCES, 2015, p. 10). Theoretically, students who performed exceptionally well in high school might discover that college success relies heavily on a student-based accountability model as opposed to a teacher-based model. In this sense, students who had near-perfect GPAs were not college ready because of a general lack of understanding of the mentality and practices needed to be successful in a college setting. While many interpretations and definitions of college readiness have been researched, this article follows the notion of content knowledge necessary to gain access (and complete) a college-level course. Borrowing from Conley (2007), this work will use the definition that college readiness means “the level of preparation a student needs in order to enroll and succeed—without remediation—in a credit-bearing general education course at a postsecondary institution that offers a baccalaureate degree or transfer to a baccalaureate program” (p. 5). The central idea of this piece emphasizes the nonexistence of an overarching concept of

college readiness. While the claim would still be valid for many meanings of college readiness, including aforementioned definitions addressing college culture and mentality, the Conley definition likely encapsulates the most prevalent understanding of the term.

Another comment should be made about a subtle difference between attaining and measuring college readiness. Ideally, it would seem having graduated high school or earning a GED would denote a student has achieved college readiness. However, nearly 60% of community college students must take at least one developmental education course (Bailey, 2009), and this assumes every student who ought to take a developmental course actually enrolls in one; in fact, most KCTCS who tested into developmental courses typically did not immediately enroll in college if at all (Complete College America, 2007). Determining if a student meets college readiness indicators may be accomplished through high school GPA, standardized assessment and placement instruments such as ACT score or COMPASS, or individual institutional practices which might include multiple measures, portfolios, interviews, and so forth. While these constructs will be scrutinized later, the point being made here revolves around the delineation between a student's being college ready and a student's measurement of that degree of college readiness; the two sets are not isomorphic.

The first barrier to realizing universal college readiness lives at the forefront of every high school senior's mind when awaiting the dreaded acceptance letter from the university of choice. For example, the admissions standards for Berea College and HCC, both in Kentucky, have differed considerably. Any given public institution will have radically different admissions standards from a private school such as Berea. However, perhaps college readiness would imply the *normal* four-year institution, such as WKU,

Murray State, or the UK. Referring back to the established definition for this section would reveal virtually no demarcation regarding institutional type, whether it be open, selective, or highly selective admissions. Our definition simply spoke to a postsecondary institution offering a bachelor's degree or a degree leading to a bachelor's degree. Herein lies the situation: if college readiness means any college, then surely all high school graduates could get into some college somewhere. As this fatuous claim simply does not embody the spirit of the meaning of college readiness, regional colleges relative to a given high school might be the target of said college readiness (later this too will be refuted). As such, community colleges and regional colleges seem to be fair game for comparison; therefore, excluding the Research 1 and private colleges will allow the exploration to continue. However, admissions standards even among regional institutions prove no regularity. For example, to be admitted to MSU, students must have a minimum high school GPA of a 2.0 (MSU, n.d.a). WKU, which is 125 miles away, has required their students to have a high school GPA of a 2.5 or higher before they may be admitted (WKU, n.d.a). These two universities are not anomalies as there are no universal admissions standards for university type, even within a regional geographic area. However, the general admissions standards do not necessarily speak to content knowledge needed to enroll and succeed in a credit-bearing course. Not only do minimum admissions criteria fall more into the culture of college readiness definition more so than the academic definition (although clearly overlap exists), GPA may not necessarily be the most accurate measure of college readiness and is seldom used for individual course placement. Additionally, other admissions conditions, such as ACT or COMPASS scores, typically either allow students to bypass the GPA requirement or,

quite possibly, add to the list of preadmissions requirements.

Ignoring general school admissions requirements, the next issue can be found in individual discipline readiness indicators. College may use ACT, SAT, COMPASS, or other national standardized testing instruments, or they may use their own internal assessment for placing students into either credit-bearing courses that count toward graduation or remedial coursework. The inconsistency with institution-specific assessments would be straightforward to understand, but such common practices as ACT-based placement present less than obvious issues. For example, while WKU has had a general admissions requirement of an ACT composite score of 20 or higher, in order to enroll in their college-level English course, students must have earned a 16 or higher on the ACT English section (WKU, n.d.b). At MSU, the equivalent class prerequisite has been an 18 on the ACT English section (MSU, n.d.a). So, while two students might both have identical ACT scores, one would be considered college ready at one regional university and the other considered underprepared at another. No standardized ACT score exists among postsecondary facilities, even within the same state or geographic locale, and this has been the case not just for the ACT exam; no such agreement exists for COMPASS, SAT, or any other testing instrument.

While the ACT is the same general assessment, the test, which has been offered six times per year (ACT, 2016), has had slight question variation. While the overall content remained unchanged, the individual questions varied among tests. This slight question exchange has introduced a small, possibly nominal, threat to test validity. Institutions that utilize COMPASS introduce a new level of discrepancy. While ACT test questions change slightly among versions, the overall test has remained more or less

constant. COMPASS, owned by ACT, has had many versions and can be customizable to a certain degree by the institution. While the ACT has had a set number of questions and unable to be edited by any unique school, COMPASS testing has allowed for more user discretion with diagnostics versions, pre-algebra and algebra initial domains, and variation of question number (MyCompassTest, 2014). In addition, the COMPASS adapts to the user's answers. One student's mathematics COMPASS test may be five questions, while another's may be four times that number. As students correctly or incorrectly answer questions, the test changes in complexity and length (MyCompassTest, 2014). The issue has not been that the COMPASS is more or less valid than the ACT or SAT; it has been that these assessments are drastically different in structure with no agreement as to cut score from one institution to another. Yet, they all presume to establish the same result: measure the college readiness of a student.

One final comment should be made regarding college readiness among high schoolers about a regional university: even if some collaborative effort established all high school graduates within a regional college feeding system had sufficient knowledge to be prepared for their closest postsecondary school of choice, such a system assumes a one-to-one correlation between the student populations at both high school and college. Not all college students come from within a geographic location (although most typically do), and not all high school students stay within a given number of miles from home. As such, even achieving agreement of curriculum and skill among any fixed set of high schools and colleges would at best satisfy the needs of a majority of students. As has been demonstrated, no such pact exists or can exist regionally, national and international college readiness are concepts beyond unreachable. While a student can be *a college*

ready, few students can be *college ready*.

While I do not suggest all universities should be compelled to adopt a uniform policy or be subject to legislation, college leadership should acknowledge that no such level has ever existed where all high school graduates are ready for college, despite the long history of attempts to establish such standards. Throughout American history, both at the college and K12 levels, there have been attempts both formally and informally to adopt common practices, policies, and laws to assume universal standards and expectations of mathematics curriculum and performance. This history of such reforms illustrates that colleges in the US have never been united. There was never a golden age in mathematics higher education where all universities and instructors were in agreement about content, philosophy, and content definitions.

History of Educational Reform

It has been established that, currently, college algebra (CA) has appeared to be different among the public universities in Kentucky—or, at a minimum, the course descriptions have seemed to imply different emphases or values currently preside over the CA curricula across the state. However, there were commonalities among topics. Noticeably, CA in Kentucky and various orders of functions appeared to be parallel. However, why all Kentucky universities have come to incorporate functions (with exponential and logarithmic appearing most often), but not partial fraction decomposition, has yet to be explored. It would seem that other, possibly larger forces, have influenced higher education.

Mathematics and Early American Colleges

In colonial times, American colleges resembled British universities and primarily

served to provide training for ministers (Thelin, 2011; Tucker, 2013). As such, the church had considerable influence over curriculum (Nichols, Smith, & Ginsberg, 1934; Thelin, 2011; Tucker, 2013). Colonial colleges avoided mathematics until the late 18th century (Brubacher & Rudy, 2008). The exceptions were Yale and Harvard, which offered courses in consumer as well as higher-level courses in algebra and what would today be called calculus (Hornberger, 1945). When they did offer courses in mathematics, early colleges focused mostly on Euclidean geometry and arithmetic (Cajori, 1890). The curricula of these early colleges were modeled after colleges and universities with which the colonists were familiar, so topic coverage and course offerings resembled the classics of the European tradition, hence the reason early American mathematics resembled an amalgamation of customary ideas borrowed from Europe, although many mirrored the current curriculum of Cambridge (Cohen & Kisker, 2010). Harvard, a leader in all matters concerning mathematics curriculum, heralded such instructors as Isaac Greenwood who taught algebra, focusing on quadratic equations, cubic equations, and converging series (Nichols et al., 1934). However, Euclidean geometry and practical topics (such as elliptical functions and projections as they pertain to astronomy) became the norm, although topics in early American colleges were constantly changed from a combination of desire for American colleges to distinguish themselves from their European counterparts as well as new professorships being established via attrition (Cohen & Kisker, 2010).

Reform of curriculum has been, in many senses, an American tradition; colleges have responded to desires for growth and change of disciplines and topics represented (Wills, 1936). However, as there were no organizations such as the US Department of

Education, which did not appear until 1979 (Stallings, 2002), tradition and political influence of other institutions established how college curricula articulated. As such, any advanced college mathematics focused primarily on geometry during early America (Hofstadter & Smith, 1961; Millett, Hofstadter, & Hardy, 1954). However, outside higher education, the landscape of mathematics was mostly barren. Mathematical research in the US did not appear until the first half of the 19th century, and it was not until 1888 when there was a deliberate effort to establish a venue for publication and comradery among professional mathematicians via the New York Mathematical Society, which eventually led to the installation of the American Mathematical Society (AMS) (Archibald, 1938).

In short, early America was not respected by professional mathematicians in Europe because mathematics was taught at the universities, but there were no renowned U.S. mathematicians (Grabiner, 1977). Additionally, priorities of the early colonists were hewing out life in a new world, dealing with diseases such as smallpox, and basic survival, so colonists' needs of mathematics were little more than the limited arithmetic needed for basic survival (Cohen, 1983; Cremin, 1988; Dewey, 1985). Thus, the focus on geometry was more a byproduct of tradition over high academic standards. It was the Mathematical Association of America (MAA) which prompted revisions to the curriculum. Harvard, once again being the leader of mathematics curriculum trends, led the movement to establish the MAA. Both the MAA and the influences of compulsory high school attendance were putting pressures on colleges to improve standards in mathematics and to be consistent regarding offerings (Duren, 1967).

The Mathematical Association of America (MAA)

The MAA was founded in 1915 to address concerns in the K12 arena about the

status of the nation's mathematics preparation for college (Hedrick, 1916). In response to concerns about subpar K12 mathematics, the MAA established the National Committee on Mathematical Requirements in 1916. Part of the charge of the committee was to make recommendations for specific topics to be covered in mathematics and to provide power for a unified effort for reform movements (Boyer, 1972). As many high schools did not have algebra or geometry requirements, graduates were unprepared for intense algebra or calculus, many colleges had little mathematics required for graduation, and those who did offer baccalaureate degrees in mathematics had lackluster programs (Tracey, 1937). As such, there was concern not only with the rigor of the mathematics majors themselves (i.e., one who majored in undergraduate mathematics at one university may not have the same broad exposure as one at another), but for mathematics in college curriculum altogether. Many forces in the early 20th century would have foundational effects still seen today. For example, an influential force was public desire for college entrance requirements and the development of regional and national testing programs (Jones, 1972).

While the MAA was well established at the end of the First World War, it was not until 1920 when the organization became incorporated (Bennett, 1967). Shortly thereafter, the organization focused on the first two years of postsecondary mathematics curricula (MAA, 1928). A significant criticism of mathematics in the liberal arts education was that the content was disorganized and undergraduates did not see the relevance to practical fields such as technology, business, finance, or industry (Schaaf, 1937). While some factions in higher education were advocating for its removal, the MAA supported keeping mathematics, but encouraged curricular revision. There was,

however, much disagreement about what should be covered. At the 1921 meeting of the National Committee on Mathematical Requirements, some members advocated for college algebra, solid geometry, and analytic geometry; others advocated for trigonometry, analytic geometry, and calculus; some even advocated for calculus to be a freshman-level course (Boyer, 1972). In 1927, discussion about including material designed to make freshman and sophomore mathematics curricula more interesting to students led to the proposal of non-routine topics to be included in mathematics, including “historical, biographical, recreational, practical, philosophical, and aesthetic” (Boyer, 1972, p. 30) aspects of mathematics, including portions of class dedicated to student discussions.

Even if a consensus as to what should constitute a postsecondary curriculum had come of these MAA meetings, the nature of CA at the time was possibly less clear than today. Rietz (1910) commented that the topics that fall under college algebra typically exceed the time allotted for the course, and the chief danger in selection of material “is that it is likely to be a sort of scrap heap of disconnected or rather remotely connected topics, rather than an organized body of knowledge” (p. 51). It should be pointed out that Rietz insisted that determinants (recall that no Kentucky university mentioned matrices or determinants in their course descriptions today), limits, and infinite series are paramount topics that should be covered in unifying CA, but admitted there was disagreement among CA professors in including limits and series.

At the first summer meeting of the MAA, member Cairns suggested that rates of change and basic integral problems (typically considered topics in calculus, not algebra) be included in CA (Hedrick & Cairns, 1916). At the seventh summer meeting of the

MAA, member C.E. Comstock of the Bradley Polytechnic Institute called CA “a complex of somewhat unrelated topics, such as the solution of equations and the transformation of expressions containing the common functions of algebra” (MAA, 1922, p. 284). Smith (1939) suggested that mathematical induction should be used in tandem with combinations to prove the Binomial Theorem, which was covered in “most good texts” (p. 346), but not mentioned in any course description of CA in Kentucky today. Danieley (1948) nonchalantly spoke of quadratic equations, exponents, radicals and progressions (similar to series and sequences) when speaking of the pedagogy of teaching CA.

Nevertheless, distinct themes could be seen in college algebra textbooks. In Lehmer’s (1917) review of *College Algebra with Applications* by E. J. Wilczynski and H. E. Slaught, he commented on the author’s claim that the textbook “probably contains everything ever given under the title *College Algebra* in any American college” (p. 230). Further, Lehmer mentioned the organization of content, which begins with irrational and complex numbers, then moved to linear functions, then quadratic functions, then high-degree functions, then fractional functions (probably what we would call rational functions today), then irrational functions, and then power functions (probably what we would call exponential functions today), along with chapters over determinants (implying matrices), a chapter on permutations, and a chapter on probabilities. The last chapters of the book include limits, series, and convergence. Lehmer suggested that the book would likely not be adopted by many instructors, who might choose to omit a chapter, and the act of doing so would likely make the student think the instructor does not know that material very well. In short, the book contains too much material. In Wells’ 1918 review

of *A First Course in Higher Algebra* by H. A. Merrill and C. E. Smith, she commented on how well the authors established theorems and concepts early in the first chapter on integers to better prepare students for later chapters on limits, series, and convergence. Further, she mentioned Cauchy tests, Maclaurin's expansions, and first and second derivatives of algebraic functions, which would today be considered calculus, not algebra. She also mentioned finding undetermined coefficients, which might be seen in a matrix algebra or differential equations class today more so than a CA course. As with the Wilczynski and Slaughter text, the Merrill and Smith text included material that would today be considered calculus, not algebra. In Burgess' 1920 review of the second edition of *College Algebra* by H. L. Rietz and A. R. Crathorne, mathematical induction and proofs were mentioned for undetermined coefficients, which is a proof-based method for exploring a topic typically seen in a matrix algebra course. Weaver's 1928 review of the revised edition of *College Algebra* by W. B. Ford lamented the exclusion of "advanced" topics such as "partial fractions and limits and series" (p. 32), which Weaver indicated seemed to be a trend as of late. However, Weaver commented on both how the derivative was defined and used to maximum and minimum values and how "most readers will be pleased to find Sylvester's method of elimination" (p. 33). These reviews would suggest some common themes among CA textbooks, and therefore presumably CA courses.

First, most of the textbooks included content that would not likely be in any CA course today, such as limits, derivatives, and undetermined coefficients. Second and possibly more important, the textbooks showed a much purer form of algebra than is seen today. There was a distinct tendency for the reviewers to praise rigor and depth of proof of theorems rather than execution of method. While the overarching theme seemed to

suggest algebra was less rote a process and more understanding the underlying theory, agreement to specific topic coverage seemed to vary among authors and professors.

Despite that CA was still largely lacking continuity, the MAA continued to push for a unified postsecondary requirement of mathematics at a time when the organization had little influence on community colleges (Boyer, 1972). In response, the MAA created in 1939 the Committee on Collegiate Curricula, which was to “collect, review, and collate facts pertinent to mathematical instruction in the colleges” (Boyer, 1972, p. 43).

The MAA was founded during the First World War, and it probably changed the most during the Second World War. Between the 1941-1942 and 1942-1943 academic years, enrollment in mathematics courses at the university level increased an average of 30%, although some institutions reported increases as high as 300% (Price, 1943). This increase in enrollment occurred at a time when many faculty were joining the war effort, causing a shortage of mathematics professors. As such, in addition to increased workloads, fewer vacation days, and recalling retired professors, higher education curricula experienced changes. While non-essential courses were being eliminated, new content was introduced, including “spherical trigonometry and navigation, dynamics, aeronautics, meteorology, ballistics, [and] cryptanalysis,” (Stark, 1972, 56). In 1941, the MAA made recommendations to both secondary and post-secondary faculty as to what curriculum would be most beneficial to the armed forces, including a college course on war mathematics that focused on artillery and machine gun, army engineering, and aviation problems (Hart, 1941).

By many accounts, there have been two competing factions within the discipline of mathematics—the pure and the applied. Pure mathematicians work with axioms and

theorems. Kline (1963, not to be confused with German mathematician Felix Klein) once said that “Mathematicians never know whether what they are saying is true because, as pure mathematicians, they make no effort to ascertain whether their theorems are true assertions about the physical world” (p. 167). Prior to World War II, the MAA focused primarily on pure mathematics, and therefore the beginning of the war marked a shift in paradigm when many purists made the claim that those who argued for application ceased to be mathematicians (Stark, 1972). By the end of the war, however, the MAA had largely become an agency working in part for the application of mathematics (Hart, 1941; Rees, 1980; Rosenbaum, 1967; Stark, 1972).

Because of this shift in paradigm, CA moved from a theoretical, proof-based course to more of what one might expect to see today—little proof and more algorithmic processes. As early as 1934, this change in paradigm seemed to have started. Bell (1934) passionately lamented the new revolution where rigor was being replaced in college mathematics; proof was falling out of the textbooks, which meant a textbook simply gave formulae and theorems and math students were relying on faith, not their own logical faculties. Knaebel (1952) said that *College Algebra* by E. B. Miller and R. M. Thrall was an endeavor to meet the requirements of students who wished to either pursue mathematics or fields requiring mathematics, so the authors took the middle road between “a brief treatment of the various topics and one offering a proof for every statement” (p. 480). In Feinstein’s (1955) review of *College Algebra* by H. G. Apostle, the author reportedly “tried to present the conventional topics of algebra as logical principles of science, employing both deductive and inductive methods . . . with numerous applications from the fields of physics, mechanics, engineering, commerce,

etc.” (p. 173). While Feinstein suggested this text attempted to present algebra within the context of its basic theorems and axioms, the application of mathematics to other disciplines was explicitly mentioned, showing movement from one era to another. Additionally, a new feature in many of the reviews was the mention of pedagogy and teaching methods, which were mostly present from earlier reviews that focused primarily on topics covered and organization; however, many reviews pointed to the psychological and educational merits of explaining material (Grant, 1954; Russell, 1950; Scott, 1947; Strehler, 1947; Wagner, 1948; Wegner, 1948).

Postwar MAA reflected this paradigm shift as well. While the MAA alleged that their primary function was pedagogical in nature, most of their activities mirrored that of the AMS. Duren (1967) called the postwar era the *revival* of the MAA. It was 1953 when MAA President Edward McShane created the Committee on the Undergraduate Mathematical Program (CUP, later CUPM), and the organization gained significantly more influence in higher education (Zitarelli, 2015). CUPM sought to increase training in college mathematics instructors; unify undergraduate mathematics; and, possibly most famously, promote the creation of *Universal Mathematics*, a freshman course in mathematics for all students regardless of major (Evans, 1956). While the effort to create such a course never came to fruition, it was CUPM that ignited the role of the MAA as a notable force and leader in national mathematics curriculum work (Duren, 1967).

In the 1960s and 1970s, the MAA continued its focus on application of mathematics over pure mathematics research and began to integrate areas of computer science and engineering, both in the mathematics of computers and the use of computers in mathematics (Rosenberg, 1973; Tarwater, 1981). The space age created new demand

for mathematics majors, and the MAA experienced a time where better mathematics students were enrolling in colleges, but the number of quality mathematics teachers was the lowest in 50 years (Duren, 1967). Alan Tucker, son of MAA President Albert Tucker, referred to 1955-1974 as the “Golden Age of Mathematics Majors” (Tucker, 2013, p. 9; Zitarelli, 2015, p. 18). It was in this so-called *Golden Age* that the community college boom of the 1960s, partially fueled by the GI Bill and other political factors, led to the rise of two-year colleges (Vaughan, 1985). In 1967 the New York State Mathematics Association of Two-Year Colleges was formed to act as a resource and decision-making entity for their community colleges, which led to American Mathematical Association of Two-Year Colleges (AMATYC) ironically following a similar pattern as the AMS (Blair & Cheifetz, 1999).

In 1999, the MAA’s CUPM formed the subcommittee known as the Committee on Renewal and the First Two Years (CRAFTY), which focused specifically on renewing college algebra (Ganter & Haver, 2011). CRAFTY made recommendations on course goals, competencies, emphases in pedagogy, and assessment that were endorsed by CUPM in 2007 (Ganter & Haver, 2011). Course goals included students’ (a) involvement with meaningful mathematical experiences; (b) opportunity to analyze, synthesize, and work collaboratively; (c) development of reasoning skills; (d) strengthening of algebraic and quantitative abilities; (e) development of algebraic techniques necessary for solving problems and modeling; (f) improvement of abilities to communicate mathematical ideas clearly; (g) development of competence in problem-solving ability; (h) development of ability to use technology; and (i) encouragement and ability to take additional coursework in mathematics (Ganter & Haver, 2011). Competencies included problem solving (real-

world situations, modeling, and problem-solving techniques); functions and equations (rates of change; symbolic manipulation; graphing; numeric processes; linear, polynomial, logarithmic, exponential, and periodic functions; and systems of equations); and data analysis (collecting data and presenting them in various forms to apply prediction), although specific recommendations were not included (Ganter & Haver, 2011). As a result, institutions have been encouraged to restructure their CA courses to a modelling approach and expand CA to be a QR course applicable to both inside and outside the world of academia (Edwards, 2011).

MAA and QR. Many mathematicians and educators connected with the MAA have affirmed that the QR movement is relatively new in education. Bookman, Ganter, and Morgan (2008) claimed QR “is a relatively new and unexplored area in higher education” (p. 911) that has only been scrutinized since the 1980s. Bullock (1994) referred to quantitative literacy as a “popular buzzword” (p. 743). However, the concepts of quantitative literacy, mathematical reasoning, mathematical literacy, and my preferred usage of QR, have been argued and advocated throughout the history of the MAA, albeit possibly in a different language or framing. Rietz (1919) confessed great satisfaction in discovering his former calculus students acquired better quantitative thinking skills long after college. Allendoerfer (1947) discussed the purposes of the so-called freshman standard course, which he defined as a “year of algebra, trigonometry, analytic geometry, and occasionally calculus” (p. 574). He felt such a course was necessary to (a) understand numbers, (b) improve the mind for reasoning, and (c) attain understanding of mathematics and its contribution to culture (Allendoerfer, 1947). However, Allendoerfer insisted the following:

The ability to construct a sound mathematical argument is popularly supposed to increase our reasoning powers in other fields of endeavor. I insist this position is unsound, and furthermore that our freshman course does not even sponsor sound mathematical reasoning. In my opinion our standard textbooks train the students in a limited number of routine processes and rarely call upon them to carry out original logical thought processes (p. 574).

He further criticized textbooks by comparing them to cookbooks, claiming they were designed to help students find the answers by following prescribed steps, which worked to help them pass standardized exams but did little to cultivate reasoning (Allendoerfer, 1947).

It was not until 1989 when the MAA formed the Subcommittee on Quantitative Literacy Requirements, which published in 1994 specific recommendations to help reshape the notion of QR: namely that colleges and universities should (a) treat quantitative literacy as legitimate and necessary for graduates, (b) expect every graduate to apply mathematical methods to real-world problems, (c) develop QR programs, and (d) manage their QR programs through measurement instruments and assessments (MAA, n.d.a). In 2001, the MAA published *Mathematics and Democracy: The case for quantitative literacy*, an in-depth anthology of the history, need, and future of QR (MAA, n.d.b; Steen, 2001). It was followed by anthological publications in 2003, 2004, 2006, and 2008 (MAA, n.d.b). Through these publications, it can be seen that the MAA has, in recent years, shifted from the stance that CA should be taught for the purposes of instilling into students a strong sense of QR to acknowledging that the two are not necessarily synonymous (Best, 2008; Cohen, 2003; De Lange, 2003; Lutsky, 2008;

Rosen, Weil, & Von Zastrow, 2003; Schield, 2008; Taylor, 2008).

Quantitative Reasoning Requirement

While quantitative reasoning had been discussed implicitly by mathematicians and educators for years, it was allegedly the 1940s when quantitative reasoning became a focus in mathematics and educational curriculum to encourage students to be good citizens and fight propaganda (Dwyer et al., 2003; Presseisen, 1987). Bloom's Taxonomy asserted that judgments in terms of external criteria—the highest echelon of his taxonomy—in order to be satisfied must include “the techniques, rules, or standards by which such works are generally judged; or the comparison of the work with other works in the field” (Engelhart, Hill, Furst, & Krathwohl, 1956, p. 190), which would imply quantitative reasoning (as meant by application of information outside academia) would meet higher-level educational objectives. Nevertheless, modern emphasis on QR *requirements* seems to have developed in the late 1990s and early 2000s. The CUPM-created Quantitative Literacy Subcommittee gave a description of recommendations and guidelines for QR programs in 1996 (Sons, 1996). The foundational and most cited work was likely the NCED work *Mathematics and Democracy: The Case for Quantitative Literacy*, edited by Steen (2001). This publication gained much attention and led to the rise of formalized QR requirements in governing agencies. The National Numeracy Network (NNN) was formed in 2000 as the outreach component of the NCED, focusing on QR as its primary concern (The National Numeracy Network, n.d.).

Current administrative policies. Kentucky public postsecondary education institutions have been members of the SACSCOC, which has set forth the principles of accreditation for its members. Part of this accreditation has been the general education

component for associate and baccalaureate degrees. Contrary to popular belief, however, SACSCOC has not obliged member colleges to require a course in mathematics for graduation. Policy 2.7.3 of the general education requirements stated that core requirements were “to be drawn from and include at least one course from each of the following areas: humanities/fine arts, social/behavioral sciences, and natural science/mathematics” (SACSCOC, 2012, p. 19). Within Kentucky, all public postsecondary institutions have also met the standards set forth by the Kentucky Council on Postsecondary Education (CPE), whose definition of QR follows that of the Liberal Education and America’s Promise (LEAP). LEAP was a national public advocacy initiative launched in 2005 by the Association of American Colleges and Universities (AACU) (Association of American Colleges & Universities, n.d.). In order to satisfy CPE policy regarding qualifying as a QR requirement, a course must meet all five of the following student learning outcomes, as defined by LEAP:

1. Interpret information presented in mathematical and/or statistical forms.
2. Illustrate and communicate mathematical and/or statistical information symbolically, visually, and/or numerically.
3. Determine when computations are needed and when to execute the appropriate computations.
4. Apply an appropriate model to the problem to be solved.
5. Make inferences, evaluate assumptions, and assess limitations in estimation modeling and/or statistical analysis (CPE, 2011, p. 10)

Further, CPE requires that, in order to meet state general education requirements, degrees must include three to six hours of QR (CPE, 2011). Institutions may have more specific

requirements; i.e., in order to earn an associate in science from the KCTCS, students must have earned a minimum of six hours of QR, but must have had an additional six hours of either QR or approved natural science (KCTCS, 2016). At UK, students must have met three hours of QR and another three hours of statistical inferential reasoning (for which no mathematics can satisfy). Approved QR courses included classes from the disciplines of computer science, earth and environmental sciences, forestry, mathematics, and philosophy (UK, 2016a).

Institutional missions & philosophies of QR. Kentucky universities have had different attitudes toward justifying their QR or other general education requirements. Some universities have taken much effort in explaining their general education and QR programs, including tying them to student learning outcomes, while others have simply presented their requirements without much rationale.

EKU's general education site defended their general education program with an analogy. They likened the knowledge of a single discipline to that of a hammer—a powerful tool for dealing with problems, but other problems less nail-like in nature would render a hammer useless, so having many different tools available would enable students' problem-solving skills more diverse (EKU, n.d.). Further, it should be noted that EKU does not have a QR requirement as much as a mathematics requirement with specific mathematics and statistics courses identified as satisfying such a requirement (EKU, 2015).

KSU's page dedicated to explaining their general education program purports that "Liberal studies education provides the tools by which people come to understand the world, one another, and themselves. In short, liberal studies develop independent and

critical thinking” (KSU, 2014a). Further, KSU has required students to take either CA or contemporary college mathematics, depending on their major (KSU, 2014b).

Morehead’s site claimed their general education has provided “a foundation of knowledge and skills vital for all students” and “the attributes needed to participate intelligently and responsibly in the discourses that shape the communities in which they live” (Morehead, 2016b). However, Morehead did not provide an explicit justification for QR, such as mention of *critical thinking, numeracy, or problem-solving skills* in their overview. Additionally, Morehead has required their students to take one of the following mathematics courses to meet their requirement: Problem Solving, Mathematics for Technical Students, CA, Pre-Calculus, or Calculus I (Morehead, 2016c).

MSU’S University Studies component aimed to provide “students with a broad-based, liberal arts and sciences education as a foundation for their academic specialty” (MSU, n.d.b), and partitioned their general education courses into five themes, one of which included QR. Only mathematics and statistics courses can satisfy their requirement (MSU, n.d.c).

NKU’s general education program has been predicated on a foundation of knowledge, designed around a set of student learning outcomes (NKU, n.d.a). Some of the student learning outcome categories have included critical thinking and science and technology outcomes (NKU, n.d.b). Within their foundation of knowledge, the general education program was partitioned within five categories, including scientific and QR (NKU, n.d.c). In order to satisfy the QR requirement, students at NKU have had to take three hours of mathematics coursework, although both the disciplines of statistics and philosophy were represented among the course choices (NKU, 2014).

According to their site, the purpose of the general education core at UK has been “designed to broaden the students’ understanding of themselves, of the world we live in, of their role in our global society, and of the ideals and aspirations that have motivated human thought and action throughout the ages” as well as “provide the bases for critical thinking and problem solving, and to develop life-long learning habits” (UK, 2016a). While QR was not explicitly addressed, the *critical thinking* and *problem solving* components of their core mission statement would correlate to the QR component.

The UL general education program purported to foster “active learning by asking students to think critically, to communicate effectively, and to understand and appreciate cultural diversity” (UL, n.d.). Their requirements explicitly identified mathematics as an area under their general education program, and only mathematics courses could satisfy this requirement (UL, 2017).

Students who entered WKU as of 2014 or later must have met the university’s Colonnade Requirements Framework, which included three hours of QR for their baccalaureate degree (it should be noted that WKU’s associate degree required three hours of QR or science), which would have fallen under their *foundations* subcategory (WKU, n.d.b). WKU has allowed computer science and philosophy coursework to satisfy their QR requirement in addition to mathematics (WKU, 2017).

While some universities solely relied on a mathematics coursework to fulfil their QR requirement (or, in some cases, there was not QR requirement, but rather, only a mathematics requirement), others included disciplines such as computer science, statistics, or philosophy. UK had the most diverse QR course list from which to choose. The wide variety of choices has likely been predominantly mathematics. Thus far, all

conversation on CA and QR has focused on policies and forces internal to the field of mathematics or the university. However, other influences have impacted the topic selection of mathematics coursework and the notion of QR.

Government, Politics, and War

Previously addressed was the influence the Second World War had on postsecondary mathematics through the MAA. However, government, politics, and war have had impacted postsecondary mathematics through other mediums as well. WWII, aside from encouraging the curricula to incorporate mathematics for wartime, also led to the development of many technologies, new fields of study, and opportunities for mathematics to grow, thus causing changes in the curricula. For example, prior to WWII, mathematicians worked in their own silos and focusing on their own interests, but the world war forced mathematicians to think outside their specialties, which gave birth to new branches such as cryptanalysis (Hilton, 1984; Rees, 1980). Following WWII, however, were influences that allowed opportunities for veterans and their families to attend college.

WWII/GI Bill. While the influence of WWII was previously addressed in relation to MAA, other considerations should be addressed as well. Because of the needs of the military, many new professional organizations in mathematics have formed, and many universities' mathematics departments started offering courses such as operations research, linear programming, and other similar applied mathematics classes (Rees, 1980). Further, because of the financial strains caused by WWII, many social programs were introduced as a way to help boost the economy. The pay-as-you-earn tax (PAYE) system of withholding tax per paycheck to be used as advance payments of income tax

due was a direct result of WWII (Davies & Stammers, 1975). Probably the most famous example of economic programming was the Servicemen's Readjustment Act of 1944 (G.I. Bill), which, according to some, has democratized American higher education and created the middle class (Batten, 2011; Murray, 2008). The number of veterans who used the G.I. Bill to attend college has widely surpassed all predictions (Olson, 1973).

The most notable impact of WWII on higher education, as well as the G.I. Bill and other such programs, has therefore been a new demographic in college classrooms. While prewar students were arguably a set of individuals who were interested in pursuing academics because they were (a) interested in doing so and (b) equipped with the financial resources to attend specific institutions, postwar students who were veterans or dependents of veterans had a higher level of economic access. (US Department of Veterans Affairs, n.d.) As such, higher education experienced an era of student diversity where socioeconomic status was less of an entrance barrier (US Department of Veterans Affairs, n.d.). Further, the U.S. general population had become captivated with the Cold War—specifically the Space Race—and one cannot explore space without focusing on science, technology, engineering, and mathematics (STEM).

The Space Race—an essential STEM race. Within the context of higher education, the term STEM has referred to the teaching of and departments containing science, technology, engineering, and mathematics and has origins typically attributed to the launching of the Sputnik in 1957 (Gonzalez & Kuenzi, 2012). Sputnik arguably created a crisis in America when many felt the Russians were spying on the country, leading to a call to fund the sciences (Altbach, Berdahl, & Gumport, 2005; Axelrod, 2009; Hansen, 2005). For a decade afterward, much emphasis was put on sciences and

mathematics at the university level, as well as pressure on Washington to fund such endeavors (Altbach et. Al., 2005). The STEM movement was thus born from and sustained through the Space Race and the National Defense Act of 1958, which established legitimacy of federal funding in postsecondary education and focused on education in science and mathematics (Grubbs, 2014; United States Senate, n.d.). Further, the National Science Foundation (NSF) was established in 1950 (NSF, n.d.), and money from the NSF prompted curricular rewrites in mathematics (Hoff, 1999).

In 1952, the NSF founded the Graduate Research Fellowship Program (GRFP), a fellowship program directly supporting over 50,000 graduate students in STEM degrees since its inception (GRFP, n.d.). While STEM has been a major focus of many current agendas in politics and higher education, the climax of the STEM focus on ballistics technology can be attributed to the International Geophysical Year (IGY) scientific project, which narrowed specifically on projectiles and rocket technology (Osman, 1983). While many attribute the Space Race as having started in the mid-1950s when the U.S. and the Soviet Union implied the launching of a satellite would correspond with the IGY (Alexander, 1964; Benford & Wilkes, 1985; D'Antonio, 2014; Neal, Lewis, & Winter, 1995; Ordway & Sharpe, 1982; Osman, 1983; Von Braun, Ordway, & Dooling, 1985), attempts to launch rockets date back as far as 1915, with mathematicians speculating about viable planetary ejection as early as 1883 (Von Braun et al., 1985).

Attempts and tests to break the atmosphere had been many, and the origin of space flight has multiple origins (Lewis, 1969). However, Kennedy's promise in 1961 to have a man on the Moon by 1970 likely expedited the growth of space technology and caused the lunar landing to occur years before it otherwise would have (Hansen, 2005).

Organizations such as NASA were forced to settle debates, such as the pros and cons of which theoretical method might be best to put a human safely on the Moon, and to make decisions on how to proceed with the selected method (Ordway & Sharpe, 1982). Further, more than just orbital mechanics were needed to make the plan succeed—arguments of materials engineering, rocket fuel, oxygen delivery systems, waste management, and sustenance planning were necessary engineering problems to solve (Alexander, 1964; Ordway & Sharpe, 1982; Osman, 1983). As such, an age of applied research began by political pressure compelling physicists and mathematicians to develop the field to meet Kennedy’s challenge. As previously mentioned, this became what Tucker (2013) referred to as the “Golden Age of Mathematics” (Tucker, 2013, p. 9; Zitarelli, 2015, p. 18).

National education reform. Because of the increase in students majoring in the biological and physical sciences, those majoring in liberal arts and social science decreased in the 1960s and early 1970s (Hassenger, 1978). Students were not only changing their majors, but their attitudes were also considerably different; they were much more vocal and opinionated: protests, sometimes violent, became the norm on many college campuses (Spalding, 1973). In addition, the college curriculum was under proverbial fire. By the accounts of educators, students, the general public, and government agencies, the general education programs of colleges were criticized as lacking quality, consistency, and breadth (Altbach et al., 2005; Lucas, 2006; Murphy, 1989; Spalding, 1973). Tucker (1974) commented on the same poor quality of mathematics teachers over the same period. Because of myriad sources of criticisms of curriculum, shortfalls in enrollment, and presidential turnover, many colleges were pressured to respond, making broadband changes in their curriculum (Finkelstein, Farrar,

& Pfnister, 1984).

From 1976-1993, colleges added more coursework than they dropped, and the curriculum accreted (Cohen & Kisker, 2010). With regard to mathematics, Robitaille and Dirks (1982) described four forces that have influenced the curriculum changes in mathematics: sociological (factors beyond the control of the school itself), psychological (beliefs educators hold about mathematics students and their learning, pedagogical (methods and materials used in educating mathematics students), and technological (using media and computers in teaching). Ralston (1981) reinforced the influences mentioned by Robitaille and Dirks (1982) and the accretion of courses mentioned by Cohen and Kisker (2010) when he commented on the supplement of computer science courses in colleges—both within mathematics departments and independent of them—as well as the need to offer discrete mathematics courses to complement a computer-rich curriculum. Ralston emphasized a need to add, but not replace, discrete mathematics course requirements to majors that required calculus. Because of majors such as computer science that required mathematics coursework, along with an overall increase in college enrollment, there has been a surge in the number of students enrolling in mathematics coursework, increasing both the number of mathematics classes and types of courses offered (Tucker, 2013).

Effects of Economics and Funding. Selingo (2013) lamented the *lost decade* of 1999-2009, in which American postsecondary education lost sight and track of its purpose, whose end was marked by the economic crises of 2008, when colleges suddenly found themselves in a situation at which enrollment plummeted, market demand no longer catered to the egregious tuition bills, and administrative bloat and overall

operational costs of college campus soared. Since 2007, the number of colleges operating in the red has increased by more than 33%, although this rating system only takes in account colleges with “strong balance sheets to begin with” (Selingo, 2013, p. 60). In 2008 financial markets were immobilized, banks stopped lending money to each other, and Congress was asked to intervene (Spiegel, 2011). Even the wealthier institutions typically immune from the effects of such widespread economic downturns suffered damages (Geiger, 2015). Arguably, this economic crisis has expedited efforts to tie funding to academic performance (Douglas-Gabriel, 2016). Performance-based funding has been a concept in higher education for decades, but has recently experienced a resurgence (McLendon & Hearn, 2013). In most states that adopt performance-based funding, pressure has been put on colleges to seek ways of enrolling underprepared students in gateway courses, including college-level mathematics coursework (Mangan, 2015; McLendon & Hearn, 2013). Some states have received funding based on the number of students who complete their mathematics coursework, and some by the number of STEM majors (National Conference of State Legislatures, 2015). Whether prompted by economic downturns or performance-based forming, Kentucky has not been an exception to the co-requisite movement. In 2015 CPE published its *Guiding Principles for Developmental Education and Postsecondary Intervention Programming*, which stated:

Default placement for students not meeting mathematics benchmarks should be in credit-bearing quantitative reasoning courses linked to the degree pathway of the student. Quantitative reasoning pathways should include a foundational pathway for occupational programs; statistical pathways for most health care, behavioral

and social sciences, and business management programs; broad-based general education pathways for most liberal arts programs; and algebraic pathways for science, technology, engineering, and mathematics (STEM) pathways. The enhanced credit-bearing course or linked course should not carry more than two additional credit hours (p. 2).

The implication of such policies has been that colleges have been enrolling students previously considered underprepared, into such courses as CA.

Chapter II summary. Higher education mathematics has been mostly influenced by a combination of societal forces outside mathematics departments, as well as tradition. Early American colleges taught arithmetic because that was the rudimentary mathematics necessary for the colonists to survive, but they also included geometry as a throw to the classics of academia. Later, undoubtedly the MAA had the most influence on postsecondary mathematics, as it was the first professional organization dedicated to mathematics education. WWII, the Cold War/Space Race, and corresponding political pressures introduced a modern take on mathematics where the focus drifted from classic proof and theory to applied mathematics and material useful to evolving fields such as computer engineering. Further, reform and other political forces have caused drastic changes in content and a heightened emphasis on pedagogy and teaching methods.

However, absent from these scholarly reports and resources is an important aspect in the field: specificity. The literature has been overarching and general, but little has been researched as to the specific topics included in the courses, why certain topics are specific to college algebra, and why college algebra has still been the default gateway course for most college majors, despite the absence of necessity for those specific

competencies. Additionally, while the literature has been mostly global, even less research has been conducted at the state level, let alone at specific institutions.

The conclusion I made based on the current body of literature explored above left me to realize there have been deficiencies in the field. This recognition has prompted me to conduct my own research, which will add to the field and grow the knowledge base of higher education. However, before I can conduct this research, the methodology and research designs must be explained; the Chapter III has accomplished this task and explores the research design, role of the researcher (including trustworthiness and authorial biases), sources of data, overview of instrumentation, procedures and data collection, as well as the analysis plan.

CHAPTER III: METHODOLOGY

This study investigated the content that has been covered in CA at UK as the course has evolved over the years, examining reasons for content change. This qualitative research focused on historical events at the university, in Kentucky and at the national level that have played a role in the evolution of mathematics curriculum at UK. By using historical methods (document analysis), changes to the course competencies and course description were highlighted for the purposes of determining why the current incarnation of CA covers specific topics while excluding others. The discernments gleaned from this project will be useful in establishing (a) what CA is, (b) why it contains the specific material taught, and (c) historical context that will challenge why CA seems to be the default quantitative reasoning class of choice for many institutions, especially community colleges.

To understand better the current nature of postsecondary mathematics curricula, this study followed an historical qualitative methodology, leaning predominantly on document analysis as its principal method. Through analyzing documents such as course catalogs, syllabi, textbooks, and other primary documents, this study sought to compare and contrast changes in content and theme of the nature of mathematics education in higher education. This chapter describes the research design of the study and the sources of data. Additionally, an overview of instrumentation is provided, along with a discussion of procedures, data collection, and analysis plan.

Research Design

Creswell (2013) suggested that researchers must first state their philosophical position in an inquiry. As such, my epistemological position will follow the postpositive

interpretive framework in which reality can be approximated “through research and statistics” (p. 36). Ontologically, I would concur with the notion that reality exists beyond human experience and interpretation, and the researcher may not have access to “understand or get to it because of absolutes” (p. 36); additionally, reality may further elude the researcher due to the complexities in ascertaining the historical reasons for the culmination of any one event. Because this qualitative investigation followed the historical methods approach, an apparent contradiction would seem evident with Creswell’s ideas of the axiological position for postpositivism. While Creswell claimed a postpositive researcher and the study should be distinct and differentiated, an historical researcher would acknowledge that “the facts in history are not necessarily value free and possess an objective reality which is the same for all historians” (McDowell, 2002, p. 11). This apparent rivalry between the desire to be objective and acknowledgement of personal bias would achieve reconciliation through two differentials: (a) historical research is a qualitative methodology, whereas most postpositivists have engaged in quantitative research; and (b) historical researchers would understand that “ironically, however, there is perhaps no scholarly discipline in the humanities or social sciences in which the goal of pure objectivity has been more ardently sought, more obsessively worried over” (Howell & Prevenier, 2001, p. 146).

Moreover, as this investigative inquiry constituted an historical look into higher education, it followed the rules for such examination. First, any college or university has never been dissimilar to a “living, breathing organism that consumes resources, grows, has dreams, makes friends and enemies, makes mistakes and, on very rare occasions, achieves greatness” (Gasman, 2010, p. 13). A corollary to this personification of the

higher education institution would emerge when analyzing any of their publications, regardless of the inherent impartiality one might expect from such a document. That is, even in a catalog of course descriptions, there will undoubtedly appear some statement of mission or purpose which will speak to the superiority of this particular college over its rival institutions. In short, all documents and publications from an institution will have some marketing overtones or bias in promoting the school.

Role of the Researcher

Following the notion of Denzin and Lincoln (2010) that the researcher is an instrument of the data collection, my role was to read, compare, analyze, interpret, and report the findings from the document. As not only a community college mathematics faculty member, but also a person who has taught CA at HCC, I have had a personal connection with both the content of the field as well as the institutions in which the data have been drawn. As coding was completed manually, and the narratives were written through the themes and observations I perceived, in addition to being the instrument of data collection, I am also the mechanism of data analysis.

Trustworthiness. Merriam (2009) asserted the aim of qualitative studies tends to establish understanding more so than faithfully following the strict guidelines and procedures of a quantitative study. While both qualitative and quantitative researchers would be concerned with validity and reliability, their methods of protecting such integrity have been different. Merriam described several tactics for establishing credibility, including triangulation, which would include seeking multiple sources of data. To this end, I have examined several different documents, which are outlined under the *Sources of Data* section. Another strategy for ensuring credibility was reflexivity,

which is described over the next three subsections. It should be known that two guiding principles have prompted my pursuit of this research: the objection to the existence of a uniform understanding of what material constitutes CA (the denial of the one-to-one function, as I will call it), and the notion that CA should be the *default* QR requirement for most college students (e.g., those who are not in a STEM or STEM-H field).

Denial of the one-to-one function. The one-to-one function to which I refer sprang from anecdotal conversations with and perceptions I have of colleagues in my field. The one-to-one function would assume that there has been a body of material that, when listed, would fall under that category of *CA material*. That is, were I to list these topics, a mathematician would agree these all fall under the umbrella of CA. Further, the converse would also be true: that the category of CA material would also generate an isomorphic list. That is, were a mathematician to list all the material that falls under the umbrella of CA, the same list would be reconstructed. This one-to-one function, I have concluded, does not exist, except perhaps in the minds of individual mathematics faculty who operate in silos. Based on the analysis in Chapter II, there has been demonstrated variation in course descriptions among the public universities in Kentucky. Further, as previously stated, issues such as academic freedom, variation in textbook utilization (even within the same institution), and individual instructor emphasis will further variation of CA delivery from section to section. While I am not claiming these differences themselves are inherently good or bad, I am making the assertion that these differences exist and have always existed. Not only have they existed in content covered, but they also exist in depth and emphasis of content coverage, pedagogy, evaluation of content covered, and overall CA course evaluation.

Because I deny the one-to-one function exists, I have specifically and deliberately utilized a lens to uncover these disparities. The philosophical view undertaken was that, to prove the absence of a construct, it must be therefore necessary to prove the existence of the counterpart of such a construct. In proving the absence of the one-to-one function, I have endeavored to demonstrate material has been different over time due to political or internal reasons, but not due to discipline-specific agreement about what CA should cover and what material automatically falls under the CA umbrella. Because of this lens, the content analysis and themes explored within the documents ascribed to my desires and objectives as a researcher.

Rejecting CA as the default QR. The tautological and circular argument presented, again anecdotally, would be that CA should be the default QR requirement because it has been universally accepted as the default (a similar argument has been made for transfer: CA should be the default QR because it has transferred everywhere, but it has transferred everywhere because it has been the default QR). While some may claim that CA should be the default because other courses (i.e., liberal arts mathematics, technical mathematics, statistics, or business mathematics) lack the rigor or level of respectability as CA. A similar argument has been made by some of my colleagues regarding non-mathematics courses with QR status; a graduate with a college degree should have at least one course in mathematics, and that course should be CA. My contention with these standards has been that they have been seemingly arbitrary. If the purpose of QR is to teach students to think quantitatively, it seems egotistic to assume only coursework in mathematics can accomplish this goal. The notion that all students should have a course in mathematics in order to obtain a degree as a self-evident

argument can be made by any discipline, and the number of ever-growing disciplines would inflate a baccalaureate general education core beyond its standard two-year time frame.

The fundamental reason I reject CA as the default QR requirement, however, comes from the content of CA. The role CA plays in mathematics has historically been to serve as a prerequisite for calculus (Vandal, 2015). However, most students do not take calculus. Further, other courses both in and outside mathematics could serve to teach students to think mathematically within their discipline. While the purpose of a liberal arts degree may be to expose students to myriad disciplines and manners of thinking, that the only course under the QR or mathematics banners that can accomplish this has been CA appeared to be false and ironic in narrow and linear thinking. Coupling this restrictive notion with the absence of the one-to-one function has led me to believe momentum and reluctance to change have been the genuine reasons many math faculty have not been as open to allowing other coursework to serve as a QR (Kentucky Council on Postsecondary Education, 2011).

Other values. As an instructor of CA and other mathematics coursework at HCC, issues of rigor versus sensibility have been wrestling in my mind. For example, when looking at textbooks, course descriptions, and competencies, I made observations regarding levels of difficulty. That is, while I have maintained that there has been no uniform agreement about what material *is unequivocally CA* (nor, when one hears the term CA, what material comes to mind), I would disagree that the converse has also been true: that there has been material *unequivocally not CA*, at least in Kentucky. I would maintain that integration by parts, for example, has never been considered material found

in any CA course. Therefore, proofs based in a calculus line of thinking, which have appeared in some textbooks, I considered to be difficult given the level of CA as a course. Other such similar comments were made, such as giving a definition and then following with an example of a special case or a more advanced situation than the definition (as opposed to giving a definition and then an example that mirrored the definition, then slowly working up to the more advanced example). These observations were based on values of pedagogy from my personal experiences with the content.

Consistency, according to Merriam (2009), is analogous to reliability; however, the goal of the qualitative researcher is to establish that the results are to “convince the outsider to concur that, given the data collected, the result make sense—they are consistent” (p. 221). A tactic suggested was the use of an *audit trail* showing how the results were determined from the data. I have tried to do this by (a) being thorough with presenting screenshots from the documents analyzed as figures; (b) describing in detail the historical contexts surrounding the documents, including an occasional chart or table both in-text and in various appendices; and (c) when documents were unpublished, making them available in an appendix.

While quantitative researchers seek generalizing their findings from a random sample to a greater population, transferability in qualitative research cannot accomplish this feat (Merriam, 2009); however, qualitative researchers can “find a general” and therefore “extract a universal from a specific” (p. 226). A strategy for enhancing such transferability was *rich, thick descriptions* that normally would apply to contextualizing the settings of a study and or its participants (Merriam, 2009). However, in this study, I have attempted to contextualize eras of the mathematics department of UK by including

annual reports, including pictures and small biographical information regarding some of the key historical figures, and by including newspaper clippings in addition to formal documents to enhance the “presentation and setting of the study” (Merriam, 2009, p. 227).

Finally, I would add some introspective about the conclusions and inferences in Chapters IV and V: my perspective as an instructor of mathematics has likely influenced my statements. For example, when a textbook changed or a change in departmental leadership occurred, I made assumptions that impacts to the curricula occurred. My experiences as a faculty member within the field has given me anecdotal insight to how content within a course has changed when staff, textbook, funding, or course descriptions change. Further, when a textbook goes into a new edition, I have seen curriculum adapt. While these have been my experiences, I can only assume these would be mirrored by mathematics faculty in the genesis era through the dark ages.

Sources of Data

This piece focused mostly on official university and government documents, specifically course descriptions from college catalogs and course syllabi, as well as sequential editions of specific textbooks that have been used by UK or HCC. Other documents, such as annual reports, self studies, and memorandums, will be analyzed when available. While the amount of authorial bias would be considerably less prevalent than in a deeper investigation into the history of the institution or any subset academic units, course descriptions have typically been straightforward. Nevertheless, course descriptions have not been without bias. A course description will have outlined the content of any given class, but the specific topics covered and, more subtly, the topics not

covered have had historical considerations that may not be easily scientifically derived. That is, both political and personal experience may have shaped the content of the class. For example, a department chair or influential professor might have persuaded the academic decision makers to include or exclude certain topics for personal, political, or professional reasons unrelated to some pedagogical or discipline-based contention. As such, when evaluating documents from institutions, the criticism of sources must transcend what Howell and Prevenier (2001) referred to as “external characteristics” and target on “internal criteria” (p. 60), specifically considering the authors’ intentions. Unlike most historical artifacts, however, academic literature would be atypical, in that attempts to measure validity and reliability have been more apparent; i.e., one does not necessarily question the accuracy of, for example, the degree requirements in the catalog. If the college literature indicated CA was required for a business major, it would generally be accepted this was a true statement; strong accuracy of information would be expected in formal publications. Nonetheless, why CA has been required for a business major must be considered and has presented a subtler problem. Additionally, any syllabus analyzed has had the added personal bias supplemented to the document, regardless of the degree to which it is official. As syllabi have been authored by individual instructors regardless of their officiality, they may lack peer-review or other scrutiny.

College catalogs. College catalogs allowed the researcher to access the most official understanding of the courses that were analyzed. Catalogs were taken from the UK website for the university archives and from the UK website of the registrar. They included the academic year 1865-1866 up through the academic year 2016-2017. Sections specifically examined were the course descriptions for CA. Catalogs were

primarily used in addressing RQ1.

Course syllabi. Syllabi were collected through the UK website and through the UK Department of Mathematics. While official documents, course syllabi allowed the researcher to achieve a more intimate understanding of individual course instructor values, philosophy, and emphases. According to the current version of UK’s bulletin, the syllabus “is the first indicator of the instructor’s expectations” (UK, 2017, p. 83). This document should contain detailed descriptions of assignments and course content and should be thought of as a contract between the instructor and student (UK, 2017). Not only should it contain course-specific information and policies, but it must also provide students with resources for out-of-class assistance, including faculty office hours.

Further, a deeper sense of priority was gleaned from the syllabi. In HCC mathematics professor Maura Corley’s CA syllabus from fall 2006, she addressed attendance in the figure below.

ATTENDANCE:
Students are expected to be at each class meeting. Attendance will be taken every day.

All students start with 30 bonus points. For each day of missed class the student will lose five bonus points until the number of bonus points reaches zero. The loss of bonus points occurs **regardless of the reason for the absence.** These are extra credit points given for attendance. Since homework will not be collected for a grade, students may use these points to supplement their exam scores. If a student misses an exam this does not affect his or her bonus credit. Make-up exams will not be given.

If one of the exams is missed the comprehensive final will replace the score from that exam. If more than one exam is missed the comprehensive final increases in value 100 points for each additional exam missed. Any student who has not missed an exam may take the comprehensive final if he/she chooses to do so. In that event, the lowest score of all exams including the final will be dropped.

Figure 1. Excerpt from Maura Corley’s CA fall syllabus (2006) at HCC.

From this document, information was gathered that would not appear in a policy or catalog, so a personal sense of Ms. Corley’s values and attitudes can be seen in the policy

itself, as well as her use of bold and underlining. Syllabi were primarily used in addressing RQ1.

Mathematics textbooks. The textbooks analyzed included *Algebra & Trigonometry* by Aufman, Barker, and Nation (ABN), all editions from the first to the eighth, as well as *Fundamentals of College Algebra* by Miller, Lial, and Scheider (MLS), all editions from the first to the fourth. These textbooks have been used by both HCC and KCTCS, although all editions of the books have not necessarily been used. Despite this, the adoption of such textbooks would suggest to the researcher that the topic coverage, pedagogy, and depth of coverage were attractive to the majority of decision makers to the extent that the overall content and presentation of the crux of the book aligned with the values of the faculty.

In deciding which material from the textbooks would be analyzed versus which would be overlooked, I decided to look at the aforementioned results from course descriptions from across the state. By examining the course descriptions for CA from all public postsecondary colleges, it was observed that *functions* was the unequivocal front-runner for most-often-appearing term. With the exception of NKU, functions were explicitly identified in every course description; however, *function* is an exceptionally vague term. To cover linear functions would be radically different from covering exponential functions. In essence, *functions* would likely be more of a category than a competency. However, *functions* is still a term that would need to be defined and there are concepts within *functions* that would need to be explored, so the concept was included. The second most-often-appearing terms, exponential and logarithmic functions, which were identified in six of the eight public universities, might be construed as the

most representative and definitive topics of CA. It should further be noted that exponential and logarithmic functions always followed each other, which would make sense, as logarithmic functions are inverse functions of exponential functions. Therefore, the topics of logarithmic and exponential functions were analyzed from the textbooks.

Polynomial and rational functions were the next most-often-appearing concepts, being cited in five of the course descriptions. No other competency was listed at more than three instances. Therefore, polynomial and rational functions were also analyzed. It should be noted that absences within a description does not automatically preclude coverage; the inclusion within a curriculum may be inherently understood at that university. Nevertheless, based on the current mentioning of these terms in most descriptions, by examining the concept of functions themselves, as well as logarithmic, exponential, polynomial, and rational functions, an idea of how CA has changed over the years can be gleaned from how these four topics have been presented.

In the analysis, the topic analyzed were considered in as broad a definition as the book defines. When looking at the concept of *functions*, only the basic notion of what a function is could be considered. However, applications of functions, finding the domain or range of a function, odd and even functions, one-to-one functions, and other concepts tangent to the notion of a function have been included within the section, so they were included in the analysis. However, if the book has a section dedicated to functions and a separate section dedicated to odd and even functions, then only the former section was considered. However, if concepts were included within a section of an edition of the book that, in later editions, were removed or expanded into their own section, this was noted. Textbooks were primarily used in addressing RQ1.

Other documents. When available, other documents were also analyzed. The documents included annual reports made to the division chair by the department head, memorandums sent out by the department head, minutes from the boards of trustees, official committee reports, and self-study final reports. While these documents were not always available, detailed, or containing relevant information, they helped (a) fill knowledge gaps; (b) give insight to contextualize changes observed in catalogs, syllabi, or other documents; and (c) lend to reinforcing validation/trustworthiness strategies of “corroborating evidence through triangulation” (Creswell, 2013, p. 260). These documents were the primary source for answering RQ2 and RQ3; they were also used in assisting with answering, and contextualizing, RQ1.

Overview of Instrumentation

The primary approach for this research was the method of document analysis, specifically of course catalog descriptions, syllabi, and official documents for UK curriculum-related committee minutes and reports. While any course description or obligatory competency listing in a course has prescribed a topic for coverage, its profundity and prevalence in the content has almost always been contingent upon the individual instructor of the course. While one professor may have spent several class periods exploring every facet of an issue, another may only have spent a few minutes and moved on. This element of personal subjectivity therefore required course syllabi to be used to interpret scope of individual course coverage.

From these documents, however, neither has more *truth* to it than the other; meaning and interpretation fall to the reader (Hodder, as cited in Denzin & Lincoln, 2010). Therefore, thematic analysis of documents was used to interpret subtle patterns

and emphases and to decipher meaning among the documents.

Procedures/Data Collection

Documents were collected via UK's online bulletin database for course descriptions from the college's catalogs; these have been available to the public. Syllabi and tests were collected through communication with UK's mathematics department and through the department's website, as well as through local records within my department at HCC (as HCC and UK used to fall under the same administrative umbrella). Annual reports, newspaper clippings, meeting minutes, and memorandums were collected through the UK archives of the special collections library (which I have henceforth called the *physical archives* for brevity).

Analysis Plan

Coding for this project varied among document types. Meeting minutes and annual reports tended to be more formal and factual, so ascertaining beliefs and worldviews of the individuals authoring them required a form of value coding (Saldaña, 2013) where I noted what was said and emphasized more so than trying to develop themes within the document (unlike textbook coding). Course descriptions from catalogs required the usage of a form of evaluation coding (Saldaña, 2013) by examining common terms or concepts from the paragraph, such as *solving polynomial equations*, or *graphing logarithmic functions* and judging their occurrence, changes, or absence over time. Terms and concepts would often disappear, only to reappear later. Sometimes the course description topics would be *closely related*, by which I mean following similar themes of subcategories of math; however, these topics would later change completely to a different set of closely-related topics, thus being a distinct subcategory from the first. These

subcategories were not formally defined under an official mathematical taxonomy but, rather, were interpreted from my perspective as a mathematics faculty member.

Saldaña (2013) suggested codes lead to themes, thus a theme could be derived from those data falling under such categories as *symbolic manipulation*, *visual representation of concepts*, and/or *critical thinking/problem-solving skills*, which may or may not potentially include application of mathematics to real-world scenarios or other instructional or pedagogical end goals. Similar codes were used for categorizing question types on examinations.

As this was both a first-time and small-scale study, all coding was completed manually as opposed to some CAQDAS software to “touch the data” (Saldaña, 2013, p. 22) and to avoid the hypothetical overwhelming nature of software. Further, because the coding was completed over official documents serving formal purposes and not personal interviews or letters, coding followed the applicable logic as I saw fit. While codes typically have been categorized as one or two words describing an observation (Saldaña, 2013), these were sometimes insufficient in describing complex mathematical constructs. Precoding reflected in the document analysis protocols as well.

Keeping in mind that historians are “not reporters or detectives” in the professional sense but, rather, they are “interpreters of the past” (Howell & Prevenier, 2001, p. 60), it must be emphasized that, methodologically, this inquiry was more inquisitive than determinant in aim. While questions have been asked, it should be understood that, unlike mainstream postpositive projects, no definitive conclusions could be determined nor could any propounded theory be validated. The end objective of this research is to provide a baseline for further questions to be asked, so analysis of the data

collected was targeted at understanding how and why CA at UK evolved into its current form based on historical clues. Documents, memorandums, syllabi, and meeting minutes served to help understand how the course content, delivery, emphasis, and values thereto appertaining progressed over the years. Larger-scale research into personal, political, and educational agendas helped to shed light on what factors played a role in changes.

Delimitations and Limitations of this Study

The research in this study relied heavily on document analysis. As such, only documents that were available were analyzed. Certain documents, such as course syllabi, were limited in obtainability. Several of the catalogs were missing in the UK online archives. Some of the catalogs were combined with other catalogs of consecutive years. For example, the 1914-1915 and 1915-1916 catalogs were combined into a single listing in the archives, so it was not possible to determine if any given page from the catalog was from the academic year 1914-1915 or 1915-1916 (see Appendix C). Further, because of these observed mistakes, an element of credibility was threatened as I cannot claim with perfect certainty that other pages were labelled correctly. The 1918-1919 catalog was in the archives, but the 1919-1920 catalog was missing; however, I cannot affirm that half of each catalog was missing, but an archivist mistakenly blended the two into one. While unlikely, there could be trustworthiness issues when working with archival material. Additionally, while documents were valuable to this study (I could scan, save, and reread them as many times as I wanted), by only relying on documents and not interviewing instructors, triangulation of this study was limited.

Textbooks were another limitation. The two sets of textbooks were analyzed because they have both been used in the past at UK; however, the current textbook was

not analyzed because of availability (and because not all editions have been used). Additionally, a larger study using more than just two of the textbooks might have been more revealing if textbooks used over the past several decades were analyzed; however, because syllabi have not been archived, it is not known which textbooks have been used in the past outside limited library records.

Another limitation was the choice of topic within textbook analysis. This limitation is twofold. The first aspect of this limitation was the decision to look at introductory topics concerning functions, polynomial functions, rational functions, logarithmic function, and exponential functions. This was based on the common topics of course descriptions from public universities around Kentucky. While these were the current most-commonly-appearing topics, it could be that different terms would have been more commonly appearing in the past. However, this decision was, to a degree, arbitrary. Another decision could have been to look at topic coverage from the current UK CA course description, although that would have offered a different limitation. Unless all possible topics that have ever been covered in any CA course were analyzed over all textbooks ever used to teach CA, there will always be a limitation of scope of such topic analysis.

The second aspect of this limitation is the absence of research on how the other course descriptions from public universities around Kentucky have evolved. While the common topics of functions, polynomial functions, rational functions, exponential functions, and logarithmic functions were most prevalent, their evolutionary paths would have been different from UK's CA course description history. While it could be logical or explainable that different public universities in Kentucky eventually came to cover

many of the same topics in their CA courses through some common state or national recommendation or legislation, it could also be coincidental.

This phenomenon led to another limitation: only UK's history was studied. The reasons UK was the focus of this study include (a) it used to be the administrative body of the community colleges in Kentucky, and (b) it is considered to be the only research university in the state by many legislative and political metrics. Nevertheless, by studying only a single institution, the study was limited by the perspectives and practices of one university.

The final limitation of the study was my choice to include HCC syllabi, but no other documents from HCC. The decision to include these documents was based on (a) the lack of syllabi available to analyze; and (b) the line of thinking that HCC, being a former satellite site of UK, was a pseudo-extension of UK, which meant that the syllabi of the faculty who used to be UK employees would be somewhat reflective of the values of the UK mathematics department.

All syllabi analyzed were from HCC faculty who were, at one point, UK employees. Nevertheless, this decision, while attempting to triangulate the study and add to trustworthiness, approximated the goal of attempting to study UK; UK and a small part of HCC were essentially studied.

Chapter III summary. This description of how the study was conducted has provided not only descriptive and technical information for how the findings were cultivated, but has also given insight into how I have endeavored to uncover the information evaluated. Therefore, the following chapter serves to recapitulate the findings from the data I have collected. The summaries are organized by research question, so all

documents pertaining to RQ1 are discussed, followed by documents pertaining to RQ2. Finally, the chapter closes with the results from the documents pertaining to RQ3.

CHAPTER IV: FINDINGS

This study investigated the content that has been covered in CA at UK as the course has evolved over the years, examining reasons for content change. This qualitative research focused on historical events at the university, in Kentucky, and at the national level that have played a role in the evolution of mathematics curriculum at UK. By using historical methods (document analysis), changes to the course competencies and course description were highlighted for the purposes of determining why the current incarnation of CA covers specific topics while excluding others. The discernments gleaned from this project established (a) what CA is, (b) why it contains the specific material taught, and (c) historical context that will challenge why CA has been the default quantitative reasoning class of choice for many institutions, especially community colleges.

Empirical research questions include the following:

1. What have been the common topics or themes of the competencies and topics covered in CA over the years at UK? (RQ1)
2. What internal forces have led to topic coverage or attribute changes in CA? (RQ2)
3. How has QR evolved at UK? (RQ3)

The answers to these questions allowed for research on some of the deficiencies aforementioned, adding to the knowledge of the field. By understanding how CA and QR requirements have progressed over time, challenges to the status quo, growth, and productive change can be achieved through an understanding of how potentially antiquated ideals are no longer relevant in the current landscape of higher education. Through document analysis, thematic comparisons were used to answer RQ1 through

RQ3 among textbooks, course descriptions taken from the UK online archive database, course syllabi, and other documents as available in the UK online archives.

The documents in the following sections were organized primarily by the research question they served to address. For example, textbooks analyzed and course descriptions from catalogs lent themselves primarily to RQ1, so they follow immediately. For clarity, each subsection related to RQ1 was prefixed with a *common topics* heading. Documents from both the UK physical and online archives (as well as from the mathematics department website) primarily answered RQ2; for clarity, each subsection related to RQ2 were prefixed with an *internal forces* heading. Documents related from the self-study effort from the UKCore website answered RQ3; for clarity, each subsection related to RQ3 were prefixed with a *QR evolution* heading.

Common Topics—Textbooks Once Used in CA

To analyze functions in the ABN textbooks, sections utilized for data analysis were those in which the term *functions* was defined. To analyze logarithmic functions, the sections that defined logarithmic functions were used. It should be noted that the ABN textbooks included a section about logarithmic expressions prior to the section where logarithmic functions were defined; however, these sections were not included in the data collected. To analyze exponential functions, the sections in which exponentials were defined were utilized. To analyze polynomial functions, as multiple sections were dedicated to operations on polynomial, the decision was made to use only the section regarding graphing polynomials. Similarly, to analyze rational expressions, the decision was made to use only the section regarding graphing rational functions.

Common topics—functions. *Functions* was the topic identified as the most

commonly-appearing term in Kentucky course descriptions of CA, as identified in Chapter II. Because functions of some sort were identified in seven of the eight public universities in Kentucky, an argument could be made that CA in Kentucky is a study of functions. However, *function* is an exceptionally vague term. To cover linear functions, for example, would be radically different from covering exponential functions. Therefore, *functions* should be considered more of a category of topics than a singular competency. The two sets of books that were analyzed both contained a section on functions. Functions were analyzed by textbook through comparing changes in content and delivery presentation. Two subheadings were used: (a) definitions of functions and Example 1 and (b) other topics related to functions.

ABN functions. The textbook used by some UK and HCC professors in the mid-1990s to the present is *College Algebra and Trigonometry* by Aufmann, Barker, and Nation (although Barker was dropped starting with the eighth edition). For simplicity, ABN will be used to refer to this set of texts, albeit slightly inaccurate when discussing the eighth edition. The textbook's eighth edition is still in use currently at HCC. ABN functions, Example 1, and other topics relating to functions were analyzed with two rounds of coding. The first round was an informal scrutinization of observations of organization, presentation, definition, and technique. The second round included comparisons of the aforementioned traits from the first to the eighth editions.

Definition of function and Example 1. From the first to the second edition of ABN, the first six paragraphs were essentially the same. The definition of *function* was likewise the same, as shown in Figure 2.

Definition of a Function

A function f from a set D to a set R is a correspondence, or rule, that pairs each element of D with exactly one element of R . The set D is called the **domain** of f , and the set R is called the **range** of f .

Figure 2. Definition of a function from ABN edition 2, page 130.

While this definition did not explain the connection between an equation and a rule, it did, however, appear that the definition lent itself to equations as functions that map elements of one set onto another set. Both the first and second editions followed this definition with paragraphs explaining how the correspondence in a table and an equation are not functions, as well as describing how to evaluate a function given a value. The second edition added a graph representing the motion of a pendulum swinging to illustrate how functions could be represented visually. The second edition also added an *alternate* definition of functions that introduced the idea of sets. Both editions gave Example 1 as *Evaluating Functions* with equations (see Figure 3).

EXAMPLE 1 Evaluate Functions

For the function f defined by $f(x) = x^2 - 11$, evaluate each of the following:

- a. $f(7)$ b. $f(-5)$ c. $f(3h)$ d. $f(w + 3)$

Solution

- a. $f(7) = 7^2 - 11 = 49 - 11 = 38$
b. $f(-5) = (-5)^2 - 11 = 25 - 11 = 14$

Figure 3. Evaluating functions as Example 1 from ABN edition 2, page 130.

The third edition of ABN split the first paragraph into two separate paragraphs and added the notion of relations to the end of the first paragraph, defining them as a set of ordered pairs. It also added a subheading above the opening paragraph labelled *Relations*. This edition also modified its opening table that showed the correspondence between scores

and grades by including brackets and parentheses around score classes. The first equation given, $d = 16t^2$, appeared within the paragraphs of the first two editions; edition three moved this equation between paragraphs, centering it in the textbook. Further, the pendulum motion graph was moved from the margin in the second edition to the main body. Following the pendulum motion graph, another subheading reading *Functions* was added, and a short sentence introduced functions, followed by a heavily revised definition, as given in Figure 4.

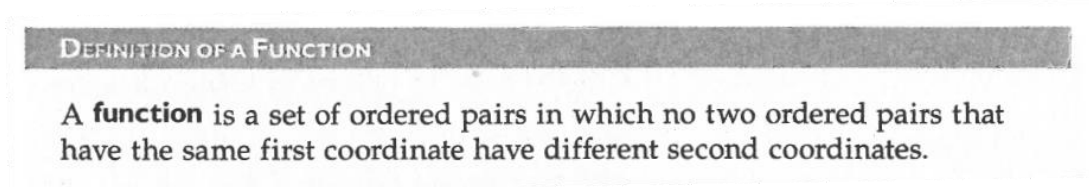


Figure 4. Revised definition of a function from ABN edition 3, page 144.

This revised definition of function, which resembled the alternate definition from the second edition, removed the symbolic use of f , D , and R while eliminating the idea of range and domain from the definition, yet adding the idea of ordered pairs. This edition also added explanation of sets as functions following the initial definition. Domain and range were then explained within paragraphs after the set explanation, as opposed to mentioning them as part of the definition. Following this explanation of domain and range, independent and dependent variables were addressed, along with notation when using independent and dependent variables. Prior to Example 1, another heading was then added called *Functional Notation*, which was still evaluating functions. Example 1, which previously had four parts, now included a fifth part of mild algebra of functions, as shown in Figure 5. In Example 1, steps were added to show how to solve the problems, as opposed to the previous editions, which simply gave the answers to the problems.

EXAMPLE 1 Evaluate Functions

Let $f(x) = x^2 - 1$, and evaluate.

- a. $f(-5)$ b. $f(3b)$ c. $3f(b)$ d. $f(a + 3)$ e. $f(a) + f(3)$

Solution

-
- | | |
|---|--|
| a. $f(-5) = (-5)^2 - 1 = 25 - 1 = 24$ | • Substitute -5 for x, and simplify. |
| b. $f(3b) = (3b)^2 - 1 = 9b^2 - 1$ | • Substitute $3b$ for x, and simplify. |
| c. $3f(b) = 3(b^2 - 1) = 3b^2 - 3$ | • Substitute b for x, and simplify. |
| d. $f(a + 3) = (a + 3)^2 - 1$
$= a^2 + 6a + 8$ | • Substitute $a + 3$ for x.
• Simplify. |
| e. $f(a) + f(3) = (a^2 - 1) + (3^2 - 1)$
$= a^2 + 7$ | • Substitute a for x; substitute 3 for x.
• Simplify. |

Figure 5. Bold bullets were added in the third edition of ABN (page 147) to explain how to evaluate functions in Example 1.

A theme starting in this edition was the notion of reminding readers of aforementioned concepts and repetition of ideas. For example, on page 146, when explaining domain and range, the idea that the first coordinate of an ordered pair cannot be repeated was reinforced. A final addition was made in the margin of the textbook: a *Point of Interest* section was included to give some background about Euler's coining of the term *functions*, adding some content beyond the math, presumably to make material more interesting to the typical student.

The fourth, fifth, and sixth editions of ABN had no substantive changes in the definition of function nor Example 1, only minor changes in typography and color were observed. However, the sixth edition introduced a *Prepare for this Section* piece prior to the *Relations* heading that included review topics from previous chapters necessary for a student to understand and work the material over functions. This addition continued the

growing theme of repetition and reinforcement of previously-presented material.

The seventh edition contained the biggest revisions since the third. Example 1 was changed to *identifying functions* rather than *evaluate functions*, and appeared earlier in the text, coming now before the paragraph explaining function notation. The paragraph explaining functions appeared before Example 2, which was changed to *evaluating functions*. As the evaluative instrument, I would comment that this reorganization therefore placed more emphasis on defining and explaining functions as *Identifying Functions* logically flowed immediately following defining and explaining rather than using functions (e.g., evaluating functions). That is, the prior editions explained, used, and explained further while the seventh and later edition explained, explained further, and then used (see Figure 6).

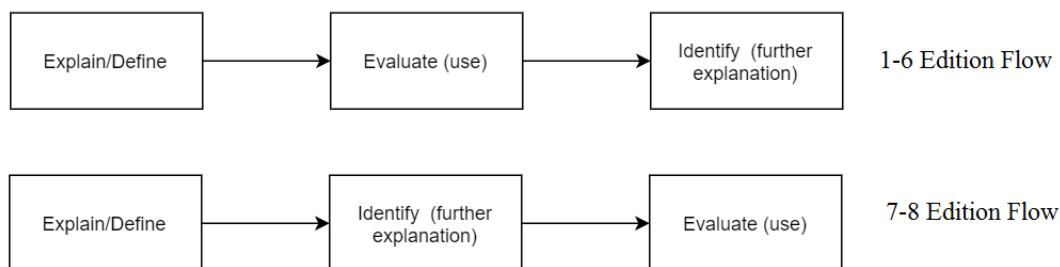


Figure 6. Flow of material from definitions to Example 1 to Example 2.

There were no substantive changes from the seventh to the eighth editions regarding defining functions and Example 1. Only minor changes in colors schemes were observed. One overt, if insignificant, change was noted: a typo in the opening paragraph in the form of a gratuitous *the* before the word *sets* was made; this paragraph has otherwise been unchanged since the first edition.

Other topics related to functions. The first edition followed Example 1 with a brief description of independent and dependent variables, succeeded by an explanation of

how “some equations do not define functions” (Aufmann, Barker, & Nation, 1990, p. 149), while the second edition reworded this sentence as “not all equations, however, define functions” (ABN, 1993, p. 132). Both the first and second edition then gave a brief algorithm as to how to determine if an equation defined a function, and then Example 2 was *Identify Functions*. It should be noted that the algorithm and example only identified functions that were equations; sets and graphs were not mentioned. The first edition then explained domain of a function briefly, gave Example 3 as *Find the Domain of a Function*, and then moved into Odd and Even Functions. Because all later editions of ABN classified Odd and Even Functions under a different section of the chapter, for the purposes of this analysis, the first edition’s section of functions ends after *Find the Domain of a Function*. Therefore, most notable are the concepts absent from the first edition, which would later include functions as sets of ordered pairs, graphing functions, vertical line test, sets as functions, 1-1 functions, horizontal line test, increasing/decreasing/constant functions, piecewise functions, and greatest/lower integer functions. The second edition added these concepts in the aforementioned order. Further, these concepts emphasized graphing functions, almost entirely absent in the first edition. For example, the piecewise function, Example 6, was *Graphing a Piecewise-Defined Function*.

The third edition changed piecewise functions to Example 2, but changed the *Graphing a Piecewise Function to Evaluate a Piecewise-Defined Function*. Example 3 remained *Identifying Functions*, but a set as a function was added as one of the sub-problems, and a picture of a set as a function was included to further explain this concept. The definition of domain was also revised in the third edition to be broader: previous

editions identified the domain as all real numbers where division by zero and complex roots were avoided; the third edition stated the domain includes the set of all real numbers where the answer makes sense and is real. This definition would work for more functions than those found only in this section or textbook (e.g., logarithmic, trigonometric, or non-elementary functions). Also notable in the third Edition was the introduction of using technology to graph and work with functions. Keeping with the theme of repetition and reinforcement starting in this edition, *Topics for Discussion* was added prior to the homework that included review questions over the section.

The fourth, fifth, and sixth editions had few substantive additions. The fourth edition added more sentences and paragraphs that continued the theme of repetition and reinforcement. The fifth edition added more examples and text on integrating technology. The sixth edition had small changes in wording but was mostly indistinguishable from the fifth edition.

The seventh edition added the concept of finding the zero of a function. This concept was explained using a single quadratic in Example 7, and following this example was a sentence connecting the concept of a zero of a function with an x-intercept. While no other type of function was addressed, zeros of a function would later be covered in the chapter on polynomial functions. *Topics for Discussion* was also removed in this edition.

The eighth edition explored the idea of zeros of a function in far greater detail than the seventh edition, adding a figure dedicated to defining and explaining zeros of a function that spanned over half the page. A similarly-sized figure was added to explain 1-1 functions. Otherwise, no substantive changes were made from the seventh to the eighth edition.

MLS functions. The textbook used by some UK (and HCC) professors in the 1980s and early 1990s was *Fundamentals of College Algebra* by Miller and Lial (and Schneider, starting with the third edition). For simplicity, MLS will be used to refer to this set of texts, albeit slightly inaccurate when discussing the first and second editions. The textbook ended publication after the fourth edition, and UK/HCC professors switched to the ABN textbook series. MLS functions, Example 1, and other topics relating to functions were analyzed with two rounds of coding. The first round was an informal scrutinization of observations of organization, presentation, definition, and technique. The second round included comparisons of the aforementioned traits from the first to the fourth editions.

Definition of function and Example 1. The first edition of the MLS text initiated the section on functions by describing the set of all students studying the textbook on a Monday evening at a pizza parlor, and then setting up a visual correspondence between the names of the students and their approximate integer weights. The correspondence was defined as a function, and a formal definition was given, leaning on the correspondence of an element to exactly one of another element. As with the early ABN definitions, this definition did not explain the connection between an equation and a rule. Further in the chapter, an alternative definition of a function was given that related functions to relations, and then examples were given to support that definition. The second edition of the MLS text did not change much, except the analogy of student weight was changed to student test score. For both the first and second edition, Example 1 was a multipart question asking students to identify if a given figure represented a function (see Figure 7).

Example 1 Decide whether or not the following diagrams represent functions.

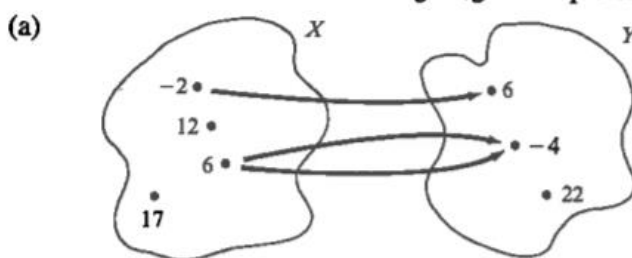


Figure 7. Example 1, part a, from the first edition of the MLS text, page 152.

Following the alternative definition of a function, a connection was made to graphing, but no examples were given for students to work about this alternate definition, nor its connection to graphing.

As with the ABN text, the third edition of the MLS text had substantial edits made to the functions section. The introductory paragraph with the pizza parlor was omitted entirely, and a paragraph about relations was added, referring back to a previous chapter in the book. The definition of function was revised to combine components of the prior two editions' definitions: "a function is a relation that assigns to each element of a set X exactly one element of a set Y" (MLS, 1990, p. 134). This definition was followed by an expanded Example 1, which included the previously-used pictures of sets, but also included algebraic examples as well as sets in roster notation.

The fourth edition had a revised introductory paragraph, discussing relations and adding a business analogy. The definition of function was unchanged from the third edition, but Example 1 was edited from six to four subparts; the pictures of sets were removed, the word problem was removed, and more algebraic problems were added.

While the ABN series had, to date, eight editions, the MLS text ended at the fourth edition. From the first to the fourth edition, the finding I would report would be the heavy revision of the definition of a function and the move toward a more symbolic

approach to functions as opposed to a correspondence argument. The move away from the pictures and inclusion of more symbolic algebra seemed to be the overarching themes.

Other topics related to functions. The first and second editions of MLS emphasized domain and range heavily before moving on to evaluating functions. Further, when addressing evaluation of functions, the texts provided no support or explanation on how to evaluate functions, nor was function notation addressed in detail. I also found it notable that the difference quotient was included in this section of the first two editions, as opposed to being in a section about the algebra of functions. Further, the early editions seemed to include many topics that would later be moved into their own sections, such as odd and even functions (added in the second edition); increasing, decreasing, and constant functions; translations of graphs of functions, and the greatest integer function. In the third edition, these concepts were moved into other sections and the authors revised much of the (what I would call) issues, including the absence of function notation prior to evaluating functions, adding vertical line test to the graphing functions explanation, and de-emphasizing domain and range, which were moved to Example 3.

Starting with the third edition, concepts of maximum and minimum were added with domain and range, although they were defined as global maximum and minimum values (as opposed to differentiating local versus global) and tied with the concept of a restricted versus unbounded domain and range. A graphical argument was used with interval notation.

Common topics—polynomial functions. Polynomial functions were listed in the CA course descriptions in five out of the eight public universities in Kentucky. While

technically linear and quadratic functions are polynomial functions, I would argue when *polynomials* were identified as a singular topic, it has been understood that, in addition to linear and quadratic functions, cubic functions and functions of a higher degree were implied. This was the case in the two textbooks that were analyzed. Two subheadings were used that included (a) definitions of polynomial functions and Example 1 and (b) other topics related to polynomial functions.

ABN polynomial functions. The ABN textbook never gave an explicit definition of polynomial until the eighth edition. However, unlike its approach with functions, the general idea of a polynomial remained mostly unchanged among the editions. While an entire chapter was dedicated to polynomial functions in the ABN texts, the first section that addressed graphing polynomial functions was analyzed. The ABN texts typically started the chapter with the first section as synthetic and long division, then covered graphing polynomials in section two, addressed zeroes of polynomial functions in section three, and then focused on Fundamental Theorem of Algebra and complex zeroes in section four. Therefore, for the purposes of this work, section two was analyzed. ABN graphing of polynomial functions, Example 1, and other topics relating to polynomial functions were analyzed with two rounds of coding. The first round was an informal scrutinization of observations of organization, presentation, definition, and technique. The second round included comparisons of the aforementioned traits from the first to the eighth editions.

ABN polynomial definitions and Example 1. The first edition of ABN introduced the section by revisiting constant, linear, and quadratic functions as special cases of polynomial functions. After covering how these functions' graphs behaved, the first

edition then made comment about how the graphs of polynomial functions are smooth and continuous, although rigorous definitions of *smooth* and *continuous* were not given. This claim of smooth and continuous remained unchanged among all editions. Following this claim, the first edition then moved into a conversation about far-left and far-right behavior of a polynomial function (FL/FR behavior), using language that “a polynomial $P(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$ ” (ABN, 1990, p. 219). However, no further explanation of this designation was given. It was not until the 6th edition that the subscripted components of this definition were further explained in the margin, and it was not until the eighth edition that this marginal explanation reiterated that a polynomial function follows the general form $P(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$.

Following the pseudo-definition, the first edition then covered a *leading term test*, which served as a segue into FL/FR behavior, for which a chart was provided to clarify the concept further. Example 1 required students to identify the FL/FR behavior over four parts, all of which were in standard form. Example 1 remained unchanged among all editions, so at no point in this section were students asked to determine the FL/FR behavior of a polynomial in factored form. Notable was that in all editions the answers to Example 1 included both algebraic and graphic explanations.

Other topics relating to polynomials. The first edition, following Example 1, focused on the Remainder Theorem and used synthetic division to find values for $P(x)$. This remained unchanged in the second edition, but was replaced with a word problem in the third and subsequent editions. The Remainder Theorem was moved to section one after the second edition.

Following Example 2, turning points were addressed, and the relationship among

zeroes, x-intercepts, linear factors, and roots was established (this was never addressed in the MLS text). Following this argument, synthetic division was used to find zeroes/x-intercepts and then to graph polynomial functions. Zero Locator Theorem (what most would probably call the Intermediate Value Theorem) concluded the chapter in the first and second editions.

The third edition included some significant changes in color and organization. Many charts were added to explain concepts, several graphing calculator illustrations were included, and a *Topics for Discussion* section was added at the end of the chapter. Most notable in the third edition was the addition of global and local maximum and minimum points, along with a graphic to illustrate them. This addition remained in subsequent editions.

While most of the fourth edition changes were cosmetic, the most prominent addition was the impact of higher multiplicities on linear factors and the influence odd and even powers on a linear factor have on the corresponding x-intercept (another concept never addressed in the MLS text). After the fourth edition, the only changes not already addressed were color, organization, or font-based modifications for emphasis or clarity. As the editions were published, there were noticeably more technology examples given.

MLS polynomial functions. The MLS textbook had vastly different interpretations of polynomial functions among the four editions. While the ABN book seemed to settle on an approach, the MLS textbook had four different approaches, changing dramatically from each edition to the next. MLS graphing of polynomial functions, Example 1, and other topics relating to polynomial functions were analyzed

with two rounds of coding. The first round was an informal scrutinization of observations of organization, presentation, definition, and technique. The second round included comparisons of the aforementioned traits from the first to the fourth editions.

MLS Polynomial Definitions and Example 1. The definition of polynomial functions stayed mostly consistent among all the MLS editions (see Figure 8).

As mentioned earlier, any function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0,$$

is called a polynomial function of degree n , for real numbers $a_n \neq 0$, a_{n-1}, \dots, a_1, a_0 , and any nonnegative integer n . Because the simplest polynomial func-

Figure 8. Definition of a polynomial function from the first edition of MLS, page 233.

While this definition was consistent, it offered no explanation to the reader how to read the symbolically-thick language, nor was an example provided to show how to interpret the terms, the coefficients versus the constant, and so forth. Following the definition, the first edition immediately made a graphing interpretation of a polynomial function. Readers were instructed to plot several points to get an idea of the shape of the graph. However, I should note that without a conversation about FL/FR, this method of explanation can be inaccurate and relies heavily upon assumptions the student must make about polynomial graph behavior.

The first edition then graphed polynomials in the form $y = x^n$. Stretching and compression arguments were made for $y = ax^n$ where $a > 0$ in cases where the leading coefficient was greater than one or a proper fraction. Example 1 required students to graph a polynomial following this form.

The second edition had a significant reorganization of the polynomial chapter. The first section defined polynomials, but then dedicated the remainder of the section to linear functions as special cases of polynomial functions. The chapter then focused on

quadratic functions as degree two polynomials, followed by conic sections. Polynomials as understood by the first edition were moved to section six of the polynomial chapter. Within that section, however, the material remained largely unchanged with exception of Example 1, which was to graph a polynomial in the form $y = x^n$, with the former Example 1 moved to Example 2 and so forth.

The third edition of MLS, as with the topic of *functions*, had major revisions. Linear functions, quadratic functions, and conic sections were removed from the chapter. The first section covered synthetic division and interpretations of the remainder; the second section covered complex roots of polynomial functions; the third section gave roots of polynomial functions its own segment. Section four then became graphing polynomials, although the arguments of plotting points were dropped.

In this edition, the Rational Zero Theorem was used to determine the possible rational zeroes of a polynomial, although the connection between zeroes and x-intercepts of a polynomial was not made explicit. The Intermediate Value Theorem was used to determine the existence of zeroes between arbitrary points. The Upper and Lower Bound Theorem was used to find boundary of zeroes and determine over which intervals all x-intercepts would lie, although the connection between zeroes and x-intercepts was still never described. Descartes' Rule of Signs was used to determine the number of positive and negative real zeroes. These theorems were used together to sketch graphs of polynomial functions, although higher-degree polynomials (meaning polynomials greater than a cubic) were not factored into linear factors. There was also no mention of how to address zeroes with a multiplicity greater than one, and most of the graphing did not use synthetic division with the Rational Zero Theorem to find rational roots (although this

was Example 1, which required the student to factor a polynomial in standard form into factored form); much of the graphing required students to approximate zeroes/x-intercepts.

The fourth edition had major revisions from the third. Section 1 was changed to quadratics, and section 2 covered both synthetic division and complex zeroes. Section three then became finding zeroes of polynomial functions. Section four was graphing polynomial functions.

The most notable change in the fourth edition was the inclusion of FL/FR behaviors in the graphing polynomials section. This concept started the section, and a chart was given to show the four FL/FR behaviors based on polynomial degree and sign of the leading coefficient. The polynomial given was in standard form. However, Example 1 was to graph a polynomial function in factored form, but there was no explanation given to the relationship between standard form and factored form.

Other topics relating to polynomials. The first and second editions of the book neglected to include many topics that were included in later editions, such as Rational Zero Theorem, Fundamental Theorem of Algebra, Conjugate Pair Theorem, and FL/FR behavior. What was most notable was the absence of zeroes of a polynomial function prior to graphing polynomial functions. Further, finding the y-intercept was almost entirely ignored, albeit sometimes included in charts of value. Additionally, while the critical value method was used to discuss positive and negative regions of curves, powers on linear factors were never explicitly addressed, and therefore even versus odd powers on linear factors could not be linked with behavior about an x-intercept (e.g., crossing versus bouncing off the x-axis). While the earlier editions gave examples that lent

themselves to rough sketches, later editions became much more accurate by providing more theorems and methods for determining where zeroes lay, how many exist, and if they were real, rational, positive, negative, and so forth.

Common topics—rational functions. Rational functions were identified in CA course descriptions in five out of eight of the public universities in Kentucky. Further, whenever a course description identified polynomial functions, rational functions were also mentioned. Because of the frequency of appearance, rational functions will be analyzed over all available editions of two textbooks. Two subheadings of (a) definitions of rational functions and Example 1 and (b) other topics related to rational functions.

ABN rational functions. Rational functions appeared in the same chapter as polynomial functions; this was the last section, following complex zeroes of polynomial functions. As such, graphing rational functions leaned on knowledge of polynomials, although many of the concepts (Descartes' Rule of Signs, Rational Zero Theorem, synthetic division, and so forth) never made appearances, despite opportunities. ABN graphing of rational functions, Example 1, and other topics relating to rational functions were analyzed with two rounds of coding. The first round was an informal scrutinization of observations of organization, presentation, definition, and technique. The second round included comparisons of the aforementioned traits from the first to the eighth editions.

ABN rational definitions and Example 1. Unlike previous topics, the first sentence in the rational functions section was a definition of rational functions (see Figure 9).

If $P(x)$ and $Q(x)$ are polynomials, then the function F given by

$$F(x) = \frac{P(x)}{Q(x)}$$

is called a **rational function**. The domain of F is the set of all real numbers except for those for which $Q(x) = 0$. For example, the domain of

Figure 9. Definition of a rational function from the second edition of the ABN textbook, page 228.

This definition remained constant among all editions. The definition was succinctly put as a quotient of two polynomials, which made rational functions' placement in the same chapter as polynomial functions a convenient organizational decision. Following this definition, a conversation about domains addressed the issue with values that would make the denominator polynomial zero. Again, this remained constant among all editions of the ABN textbook. Two examples of rational functions followed the domain conversation. Both centered about the idea of restrictions of the domain due to zeroes in the denominator and the asymptotic behavior of the rational about said zero (although the early editions did not use the term asymptote until further in the chapter).

The first edition then followed with another example, giving a five-part explanation of the graph of a rational, using the notions of increasing and decreasing without bound about a value. It was after this discussion that the first edition addressed vertical asymptotes (VA). However, the first edition did not address the four behaviors of a VA and the relationship between them and powers on the linear factors from which the zeroes are defined. The four behaviors were not added until the sixth edition. Following the definition of VA, the first edition then defined horizontal asymptotes (HA). Both definitions of VA and HA leaned on the idea of limits (although such language was not used), giving a calculus line of thinking when concerning asymptotic behavior. A

connection was then made between VA and the zeroes of the denominator. Example 1 then required students to determine the VA of two rational functions—the first had no VA and the second required students to factor a quadratic. Example 1 remained the same among all eight editions.

Other topics relating to rational functions. Following Example 1, a three-part description of HA related the degrees of the numerator and denominator, followed by a proof of the three cases with HA. However, the first two editions did not explicitly address cross tests for HA (nor oblique asymptotes). A sign argument over intervals was made for behavior about asymptotes, and then general procedures for graphing rational functions were stated. The next two examples required students to graph rational functions based on the general procedures. These components remained unchanged among all eight editions. After Examples 3 and 4, oblique asymptotes and removable discontinuities were covered. The third and later editions then provided one or more word problems. Otherwise, the overall content, sans minor changes in color and typography, remained steady among all the editions.

MLS rational functions. Unlike the MLS section on graphing polynomials, the MLS section of rational functions did not have significant changes among the four editions. While some organizational changes were made, the presentation was overall similar. MLS graphing of rational functions, Example 1, and other topics relating to polynomial functions were analyzed with two rounds of coding. The first round was an informal scrutinization of observations of organization, presentation, definition, and technique. The second round included comparisons of the aforementioned traits from the first to the fourth editions.

MLS Rational definitions and Example 1. The definition among all four chapters remained similar (see Figure 10).

A function of the form

$$f(x) = \frac{p(x)}{q(x)},$$

where $p(x)$ and $q(x)$ are polynomial functions, is called a **rational function**. Since any values of x such that $q(x) = 0$ are excluded from the domain, a rational function usually has a graph which has one or more breaks in it.

Figure 10. Definition of a rational function from the first edition of MLS, page 261.

The definition from the first edition excludes any values for which the denominator is zero. The third and later editions would slightly modify this to say the denominator cannot be zero. Nevertheless, the idea of a rational function as a quotient of two polynomial functions remained the same throughout the editions of the MLS texts. All editions start with the problem of graphing $f(x) = \frac{1}{x}$, which was identified as the simplest form with variable in the denominator. From this problem, a chart argument is made to explore the VA of $x = 0$. After exploring the chart, the editions then gave a definition that resembled a calculus line of thinking: a VA means as $|f(x)| \rightarrow \infty$ as $x \rightarrow a$, then $x = a$ is the VA.

The texts then explored the idea of HA, earlier editions leaning again on a chart argument. HA were defined as follows: if $y \rightarrow a$ as $|x| \rightarrow \infty$, then $y = a$ is the HA. Again, this has a calculus line of thinking. None of the editions addressed how to identify the crossing point of a rational function's graph which intersects a HA, although in the fourth edition, such a graph was shown. Further, the first example remained the same among all editions; graph $y = \frac{-2}{x}$. All editions made the same argument when addressing this graph, which was a special case of the function $f(x) = \frac{1}{x}$.

Other topics relating to rational functions. None of the editions addressed the four behaviors associated with VA. All of the editions instructed students to plot points to determine behavior about a VA. However, since powers on linear factors were not addressed in the section on graphing polynomials, by not addressing powers on linear factors in this section, there would be no other way to discover behavior about a VA. While the first edition did address HA, the first examples given were in the form where the degree of the numerator was larger than the degree of the denominator. HA where the degrees of the numerator and denominator were the same were addressed toward the end of the section. Oblique asymptotes were also addressed in the first edition, but, much as was the case with HA, cases where the rational functions graph crossed the oblique asymptote were not addressed. Removable discontinuities were never addressed in the first edition.

The second edition was largely unchanged, except VA and HA were defined in the same box for clearer organization. Unlike the first edition, rational functions that were not in factored form were graphed. HA were expanded greatly, and a division by a common factor of all terms within a rational function in standard form was used to find the equation of a HA. This method was very reminiscent of a calculus technique to find the limit of a rational function. A word problem was also added to the section.

The third edition was largely unchanged, with exception of including removable discontinuities in the section. This replaced the word problem. Fourth edition changes were cosmetic only in nature; additional graphics were added to clarify HA and VA, and the recipe for graphing rational functions was modified.

Common topics—exponential functions. Aside from functions, exponential

functions were identified in more CA course descriptions at the Kentucky public universities than any other topic. Out of the eight universities, exponential functions appeared in six of the CA course descriptions. Because of this frequency, exponential functions were analyzed across all available editions of two textbooks. Two subheadings were used: (a) definitions of exponential functions and Example 1 and (b) other topics related to exponential functions.

ABN exponential functions. The ABN coverage on exponential functions underwent changes, and some of the changes were in approach. While the definition remained largely unchanged, the ABN works vacillated regarding review. For example, the early editions of the book covered inverse functions in the same chapter as functions, while the fourth and later editions moved inverse functions to the chapter covering exponential functions and logarithmic functions. Unlike with polynomial functions, defining, evaluating, and graphing exponential functions always took place in the same section; however, solving exponential equations was organized in the same section as solving logarithmic equations. Therefore, for the purposes of this piece, the section that introduced, defined, and graphed exponential functions was analyzed. ABN graphing of exponential functions, Example 1, and other topics relating to exponential functions were analyzed with two rounds of coding. The first round was an informal scrutinization of observations of organization, presentation, definition, and technique. The second round included comparisons of the aforementioned traits from the first to the eighth editions.

ABN exponential definitions and Example 1. The definition of exponential functions remained essentially unchanged among all editions of the ABN textbooks (see Figure 11).

The exponential function f with base b is defined by

$$f(x) = b^x$$

where b is a positive constant other than 1 and x is any real number.

Figure 11. Definition of an exponential function from the first edition of the ABN textbook, page 261.

However, while the first two editions began with a review of properties of exponents, followed with a definition of exponential functions, then covered graphing exponential functions, the third and later editions spent time explaining, for example, why the base must be positive and cannot be one. Example 1 changed several times among the editions. While the first edition focused on graphing exponentials (because a review of exponential properties preceded the definition), a change was made in the third edition to include graphing exponential functions using translations. However, in the third edition, more explanation was given to graphing exponential functions prior to Example 1. The decision to return to more review was made in the fifth edition when Example 1 became evaluating exponential functions, thereby combining elements of exponential properties review with the definition of an exponential function.

Other topics relating to exponentials. Starting with the first edition, there was more graphing technology integrated with this section than any other analyzed. While some editions focused more on the graphing technology than others, every edition included some calculator technology.

Further, there were properties of exponential graphs given in every edition. The first three editions defined seven properties, which were reduced to six properties in the fourth edition via collapsing properties together, and then four in the fifth and subsequent

editions again through collapsing properties together. Remarkably, a property of exponentials that was never explicitly given was *opposite powers of x yield reciprocal powers of y* , which is a property I always cover when teaching. Despite the absence of this property, it is illustrated in the graphs of later editions. All editions define e and address $y = e^x$, although the first three editions afforded more attention than did later editions. Beginning with the fourth edition, there was more emphasis on applications.

MLS exponential functions. The MLS textbook had some minor changes among the four editions when covering exponential functions, although the overall message remained largely unchanged. Unlike the ABN textbook, specific topic coverage remained mostly consistent; the only changes were organizational and supplementary.

MLS exponential definitions and Example 1. The first two editions opened with a reference to the first chapter wherein the student is asked to recall how to work with the expression a^m when m is rational. While the ABN textbook dedicated formal review of laws of exponents, the MLS then moved on to a conversation about considering the expression $2^{\sqrt{3}}$, leaning on the notion that $\sqrt{3} \approx 1.7 \approx 1.73 \approx 1.732$ and making connections to rational exponential notation with these rational values. Figure 12 is given to illustrate the graph of an exponential function with the domains of whole number, rational numbers, and real numbers, respectively.

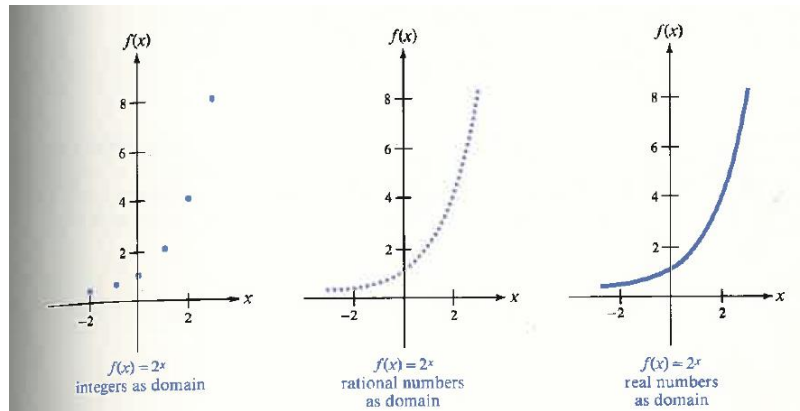


Figure 12. The graph of the same exponential function with different domains in the first edition of MLS, page 277.

Avoiding proofs, the first edition then made the assumption that the laws of exponents apply equally to real exponents of a function as they would rational numbers, and a four-part theorem was introduced (see Figure 13).

For $a > 0$, $a \neq 1$, and any real number x

- (a) a^x is a unique real number.
- (b) $a^b = a^c$ if and only if $b = c$.
- (c) If $a > 1$ and $m < n$, then $a^m < a^n$.
- (d) If $0 < a < 1$ and $m < n$, then $a^m > a^n$.

Figure 13. A four-part theorem regarding properties of exponential expressions from the first edition of the MLS textbook, page 278.

To keep exponential functions defined as functions of real numbers, the MLS text defended that a must be positive, and defined exponential functions (see Figure 14).

The function

$$f(x) = a^x, \quad a > 0 \text{ and } a \neq 1,$$

is the **exponential function** with base a .

Figure 14. The MLS definition of an exponential function from the first edition of the MLS textbook, page 278.

This definition remained unchanged among all four editions. Example 1 of the MLS first and second editions were evaluation of an exponential function, which followed the definition.

The third edition only changed by adding an introduction prior to the section that included a paragraph giving a real-world example of an exponential function (doubling a penny a day salary versus \$1000 per month), and organizational changes were made to concepts. There was also an informal definition of exponential functions given in words prior to the definition box. Further, the third edition changed the first example from evaluation of an exponential expression to solving an exponential equation (for an exponent). The only substantive change to definition and the first example in the fourth edition was to move the *properties of an exponential* box prior to the first example.

Other topics relating to exponential functions. After Example 1 in the first edition of the MLS textbook, explanation regarding the one-to-one nature of exponentials was given. The authors then addressed the notion that exponential functions are asymptotic to the x -axis (albeit translations were not mentioned, making this slightly inaccurate for all cases). Example 2 required the student to graph $f(x) = 2^{-x^2}$ (which I find a bit daunting for only the second example, especially since something as simple as $f(x) = 2^x$ has yet to be graphed), succeeded immediately by Example three, which was to graph an exponential with a fractional base. Example 4 asked students to solve an exponential for the base. This was immediately followed by a definition of Euler's number using compound interest. The fifth and sixth examples of the first edition covered exponential growth and decay.

In the second edition, Example 2 was changed to a two-part problem asking

students to graph $f(x) = 2^x$ and $f(x) = \left(\frac{1}{2}\right)^x$. Figures were then added to show the shapes of various exponentials depending on their bases. Compound interest was further explored, and the compound interest formula was given a definition box. Euler's number was expanded upon, and a box showing the number to nine places was given.

The third edition had the greatest number of substantive changes. Example 2 was changed to solving an exponential equation for the base (Example 1 in this edition was solving for the exponent), along with a caution about extraneous solutions, which never appeared in the previous editions. Example 3 was changed to evaluation of exponential functions, Example 4 was graphing $f(x) = 2^x$ and $f(x) = \left(\frac{1}{2}\right)^x$ (Example 2 in the previous edition), and Example 5 introduced translations of exponential functions, which was never addressed in previous editions. The fourth edition had no major changes from the third edition.

Common topics—logarithmic functions. Along with exponential functions, logarithms were identified more than any other topic (sans *functions*) in the Kentucky public universities' CA course description. Whenever exponential functions were named, the course description also included logarithms, so six of the eight public universities in Kentucky identified logarithms as a part of CA. Because of this frequency of appearance, logarithms were analyzed in all available editions of two textbooks. Two subheadings were used: (a) logarithmic definitions and Example 1 and (b) other topics related to the logarithm.

ABN logarithmic functions. Much like the ABN approach with exponentials, the ABN approach to logarithms also changed significantly, mostly between the third and fourth editions. While the first three partitioned logarithmic properties separately from

graphing logarithmic functions, a substantial reorganization occurred in the fourth edition where some properties were deemed more basic properties and coupled with graphing, while other properties were moved to a later section and more application problems were added. Because the change in approach impacted how graphing logarithmic functions was organized, for the purposes of this analysis, the first section which introduced and defined logarithms will be analyzed. ABN definition of the logarithm, Example 1, and other topics relating to logarithmic properties and graphing were analyzed with two rounds of coding. The first round was an informal scrutinization of observations of organization, presentation, definition, and technique. The second round included comparisons of the aforementioned traits from the first to the eighth editions.

ABN logarithmic definitions and Example 1. The first edition introduced the concept of logarithms by reviewing previous knowledge on exponential functions and inverse functions, making a logarithm-is-an-inverse-of-an-exponential argument. The edition then showed the exponential form of a logarithm, while presenting the quandary that solving for the dependent variable was not possible using previously-established method, therefore introducing new notation for a logarithm. The logarithmic form was then defined (see Figure 15).

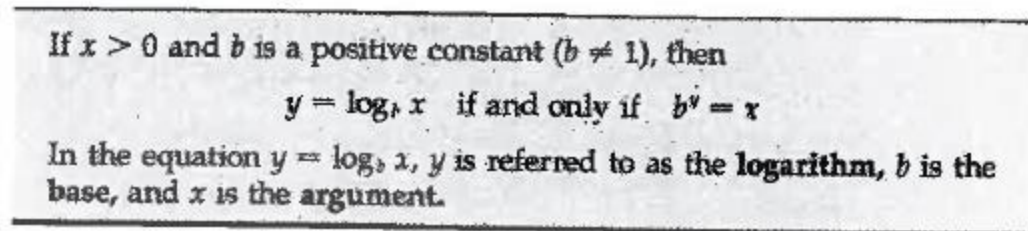


Figure 15. Definition of a logarithm from the first edition of the ABN text, page 268.

The nomenclature was then explained, and the congruency was explained between the exponential form and logarithmic form of a logarithmic equation. Example 1 therefore

required students to convert from logarithmic form to exponential form, and Example 2 required students to convert from exponential form to logarithmic form. Starting with the fourth edition, the relationship between exponentials and logarithms was expanded, but otherwise throughout the other editions, only minor changes were made to the definition and explanation of a logarithm and the first two examples.

Other topics relating to the logarithm. Other topics covered included equality of exponents theorem, which was moved to the following section after the third edition, eight properties of logarithms (see Figure 16), common and natural logarithmic definition, change of base, and antilogarithms.

Properties of Logarithms

In the following properties, b , M , and N are positive real numbers ($b \neq 1$), and p is any real number.

$$\log_b b = 1$$

$$\log_b 1 = 0$$

$$\log_b (b^p) = p$$

$$\log_b (MN) = \log_b M + \log_b N$$

$$\log_b \left(\frac{M}{N} \right) = \log_b M - \log_b N$$

$$\log_b (M^p) = p \log_b M$$

$$\log_b M = \log_b N \text{ implies } M = N$$

$$M = N \text{ implies } \log_b M = \log_b N$$

$$b^{\log_b p} = p \text{ (for } p > 0)$$

- An inverse property

- Product property

- Quotient property

- Power property

- One-to-one property

- Logarithm of each side property

- An inverse property

Figure 16. Properties of logarithms from the first edition of the ABN textbook, page 270.

Of the eight properties identified, I would note that *reciprocal values of x yield opposite values of y* was not among them. This would be consistent with the analogous property of

exponents—*opposite values of x yield reciprocal values of y* —which was omitted in the section on exponential functions.

In the first three editions, graphing logarithmic functions was not included in the introductory section on logarithms. However, in the fourth edition, the change of base rule and five of the eight properties were moved to the following section. Graphing logarithms were added, and a chart of values was included to show the relationship between exponential and logarithmic values. Translations of logarithmic graphs, domain of logarithmic functions, and applications of logarithms were also added. Starting with the fifth edition, a function composition argument was added to illustrate the inverse relationship between logarithms and exponents.

MLS logarithmic functions. The MLS textbook had more changes in its presentation of logarithms than it did of exponential functions. While there was usually one noticeable change from second to third edition, and other changes were minor or organizational, nearly every edition had an extreme change in definition, organization, or topic addition or deletion.

MLS logarithmic definitions and Example 1. In all four editions of the MLS textbook, the introductory conversation mentioned that, in the previous section on exponential functions, it was discussed that exponential functions were 1-1, which implies there must exist inverse function, and this section would therefore look at these inverse functions. The first edition defined the inverse function of an exponential $y = a^x$ as $y = \log_x x$. A box showing this definition of both exponential and logarithmic forms of a logarithm was then displayed, followed by proper language and vernacular (see Figure 17).

$$y = \log_a x \quad \text{if and only if} \quad x = a^y.$$

Figure 17. The definition of a logarithm from the first edition of the MLS textbook, page 287.

Example 1 was then a chart showing equivalent expressions of the two forms. Example 1 remained the same among all four versions. Following Example 1, a definition of logarithmic functions was then given. The definition of logarithmic function changed only slightly among all four versions (unlike the definition of a logarithm, which had several changes).

The second edition started with the same conversation about 1-1 functions, but then the notion of switching x and y is addressed, and the exponential form of a logarithm is given before the logarithmic form of a logarithm. A box then illustrated both forms, followed by the same language about vernacular. More explanation was then given about how the two form are related, including arrows connecting the same features among both forms. Example 1 and the definition were then presented.

In the third edition, the introduction remained unchanged, but the box that defined logarithms was removed (although the text remained), a new box with a new definition was given (see Figure 18).

A logarithm is an exponent: $\log_a x$ is the exponent on the base a that will yield the number x .

Figure 18. The definition of a logarithm from the third edition of the MLS textbook, page 261.

Example 1 then followed, still being the chart as given in the prior editions.

In the fourth edition, the 1-1 argument and definitions remained, but the definition of a logarithm changed again (see Figure 19).

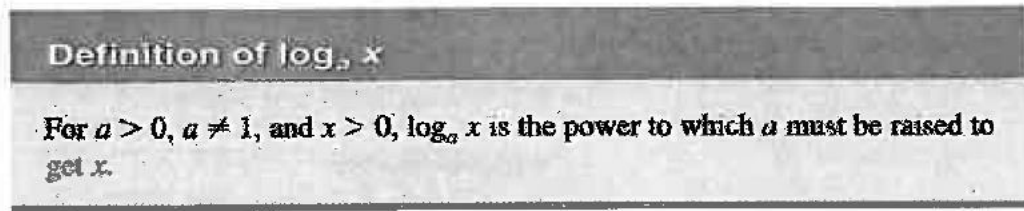


Figure 19. The definition of a logarithm from the fourth edition of the MLS textbook, page 265.

A *fill in the box* argument was made to explain the concept of logarithms' usefulness in determining unknown exponents. This was then used to explain the relationship between the two forms of a logarithm.

Notable is that the definition of a logarithm changed with every edition, although the change from the first to second editions was somewhat minor. The third edition definition was the only definition where a logarithm was explicitly defined as an exponent.

Other topics relating to logarithm. The first edition's Example 2 required students to graph a logarithmic function with a translation (which I found daunting because a graph without translations had not yet occurred). Example 3 in the first edition was then a graph of a logarithm with its argument under an absolute value. Example 4 was then solving a logarithmic equation for its base. Following were five properties of logarithms, as shown in Figure 20.

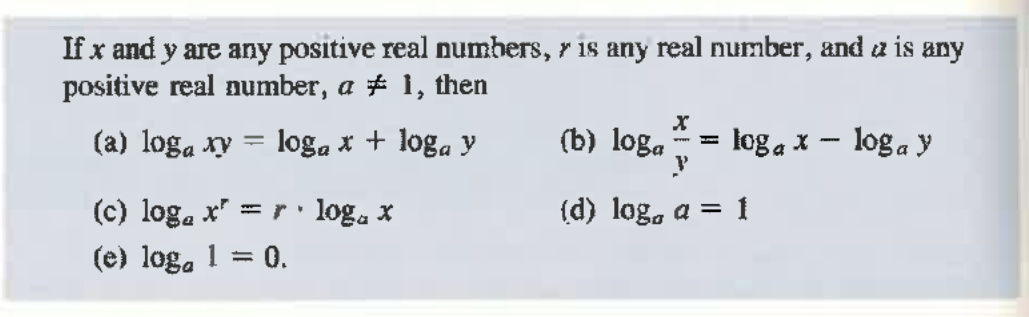


Figure 20. Five properties of logarithms as given in the first edition of ABN, page 290.

These five properties appeared in all four editions of the MLS textbook. Examples 6 and 7 were then expansion and condensing of logarithms, respectively, and a theorem and example of exponentials within the argument of a logarithm ended the chapter.

The second edition inserted a new Example 2 to a graph of a logarithmic function without translation, leaving a new Example 3 to be a graph with a translation. The only other substantive change in the second edition was Example 5 (the former Example 4), which became a two-part example in which students were asked to solve a logarithmic equation for both the base and the argument.

In the third edition, Example 2 became the two-part example in which students were asked to solve a logarithmic equation for both the base and the argument. The newly-formed definition of a logarithmic function then followed Example 2, which was a significant reorganization given that the definition of a logarithmic function was moved after two examples. Example 3 was changed to a two-part graphing problem with no translations, and the figure that showed the inverse relationship between logarithms and exponentials was moved to follow Example 3. Example 4 then became graphing with a translation, succeeded by the graphing of a logarithm with an absolute value in its argument (which had been deleted in the second edition), and the property of exponents in the argument was moved from the end of the section to become Example 6. The five

properties of logarithms were numbered (they had previously been denoted with letters) and inserted after Example 6, leaving Examples 7 and 8 to be expansion and condensing, respectively. Example 9 was evaluation of logarithms given assumptions, and a history of logarithms and John Napier was added to end the section.

In the fourth edition, Example 3 became a graph using a single base (as opposed to the two-part with different bases from prior editions), and a box was added to illustrate features of a logarithmic graph, adding concepts about (a) the point $(1,0)$ being on the graph, (b) increasing versus decreasing logarithmic functions, (c) being asymptotic to the y -axis (which is technically incorrect if there is a horizontal translation), and (d) domain and range. Two new concepts were also introduced in the fourth edition: the natural logarithm and the Change of Base property. Examples were added to reflect these new changes. Concluding the section was a new word problem that incorporated diversity of ecology.

Common Topics—Relating RQ1 with Textbooks

Reporting findings over these two books among their many editions revealed to me a couple of noticeable themes. The most blatant would be that later editions included more detail, more material, and more examples. Earlier editions tended to give a definition and then move into content. Examples may not necessarily reflect explanation or definition and would often be a special or more advanced case than the immediate preceding content. Later editions tended to give a definition, explain, and then give an example immediately relevant to the explanation.

The second theme would be the increasing inclusion of more real-world application problems. While the early editions of both textbooks would include at least

one application, the later editions included several.

Another theme would be increased supplemental background material. Later editions would include information about the mathematicians who helped develop the content covered, historical or cultural references of the material, or other information not pertinent to the mathematics itself that only served to make the content more interesting or enriched.

Common Topics—Course Descriptions from Catalogs

The second type of document analyzed for RQ1 was course descriptions taken from catalogs. Course descriptions from the UK catalogs were gathered from the online special collections database. The course descriptions and catalogs will primarily be used to answer RQ1, although some other information may be embedded within the catalogs, especially earlier catalogs that did not resemble the modern format of such publications. The oldest catalog in the UK online archives was from 1865. Unlike modern catalogs, which have typically included a course description section outlining topics to be covered in a particular class, catalogs of UK from this era resembled more of a schedule of classes. Rather than describe what classes a student would take with a description of those classes, early catalogs listed what book would be used within a discipline based on student rank (see Figure 21).

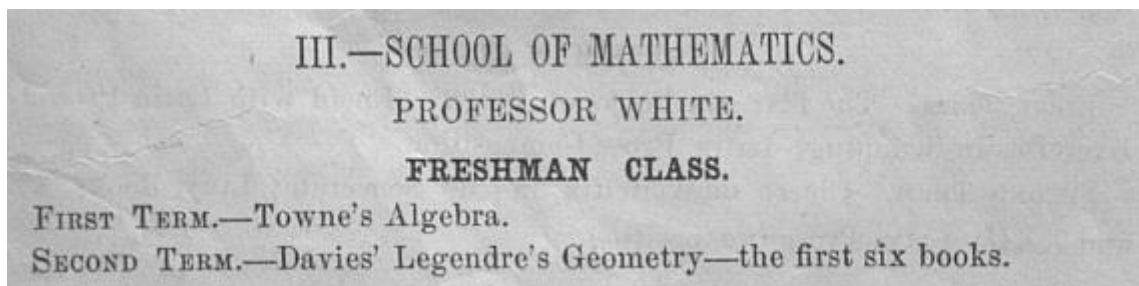


Figure 21. Screenshot of the UK catalog from 1865, page 24.

From academic years 1865-1866 through 1876-1877, the catalog simply listed “Towne’s Algebra” under the first term for the school of mathematics (UK Catalogs 1865-1876), although the catalogs were missing for the academic years 1874-1875, and 1877-1878. Because specific topics were not identified from Towne’s work, it is unknown what specifically was covered in the course.

However, it is conceivable all chapters over the 282 pages of content were addressed. Some of the topics in Towne’s textbook included basic algebraic expressions, factoring, rational expressions, linear equations, systems of linear equations, logarithmic functions, quadratic functions, radical expressions, and polynomial functions (Towne, 1865). It should be noted that absent were exponential functions, which always appeared with logarithmic functions in the Kentucky public university course descriptions. While the 1877-1878 catalog was not in the online archives, in the 1878-1879 edition, Towne’s textbook was replaced with Peck’s *Manual of Algebra* through chapter eleven. Peck’s textbook, as Towne’s, included basic algebraic expressions, factoring, rational expressions, linear equations, systems of linear equations, logarithmic functions, quadratic functions, radical expressions, and polynomial functions, but also covered sequences and series (Peck, 1875).

In 1882, the catalog switched to Wentworth’s *Complete Algebra (Elements of Algebra)*. The Wentworth textbook, in addition to the aforementioned topics, also included material over loci of equations, inequalities, and limits (Wentworth, 1881). Wentworth’s textbook was the first of the textbooks to cover graphing in detail. In 1883, specific chapters of Wentworth’s work were identified, indicating that topics in the first semester mathematics course were simultaneous quadratic equations, simple

indeterminate equations, inequalities, exponents (including basic exponents, radical expression, radical equations, and reciprocal equations), variation, series, interest, indeterminate coefficients, number theory, and polynomial equations. It should be noted that the chapter on logarithms was specifically skipped per the catalog, possibly implying logarithms were not typically covered, a juxtaposition against the prevalence of logarithms in modern course descriptions. Selected chapters of Wentworth's textbook continued to be the closest approximation of a course description through the 1891-1892 academic year. However, in the 1892-1893 academic year, more information was added to the catalog:

A thorough knowledge of Arithmetic and Algebra through equations of the second degree is required for this class. The first five months of the session is occupied in studying the Algebra, beginning with chapter XVI. The remainder of the session is devoted to the study of the first five books of Geometry (UK, 1891).

This paragraph was the first instance of prerequisite information appearing in the catalog. The following year, aside from minor revisions to the prerequisite information (specifying quadratics over *equations of the second degree*), paragraphs—as opposed to short sentences—were used to describe not only textbook information, but also to give more information about how the freshmen semesters would unfold, as shown in Figure 22..

A thorough knowledge of Arithmetic and of Algebra through quadratic equations, as presented in Wentworth's Higher Algebra is required for admission to the Freshman class.

FRESHMAN YEAR.

TEXT BOOKS—Wentworth's Higher Algebra, Wentworth's Plane and Solid Geometry, Wentworth's Trigonometry.

During the first term Algebra and Geometry are studied simultaneously; the work in Geometry being Books I to V inclusive, that in Algebra chapters XXII to XXXIV inclusive.

The second term is devoted to Plane Trigonometry and to the completion of the Higher Algebra.

Figure 22. Screenshot of the UK catalog from 1892, page 74.

By explaining the prerequisite information, identifying the textbook to be used (which implicitly gives some notion to what topics will be covered), and forecasting the procession of curriculum, the 1892-1893 catalog was the closest iteration of early bulletins to give what would likely be considered a modern course description, albeit the focus was still on the semester as opposed to individual course specificity.

For the next three years, the catalogs continued the practice of describing the details of the first few semesters of mathematics, but the 1896-1897 catalog omitted information previously given, and the 1898-1899 catalog had almost no information at all, giving only prerequisite information and book title. This custom remained consistent until 1908 (the 1907-1908 catalog was missing from the online archives), when a comprehensive revision was made in the catalog. In the 1908-1909 catalog, the student classification system was dropped in determining what course was required, and instead

course names were listed. Further, full paragraphs were given following the names of the course, fully resembling modern day course descriptions. Most notably, the first appearance of the course title *college algebra* was used. In essence, college algebra as a singular, differentiable concept, came into existence during this time (see Figure 23).

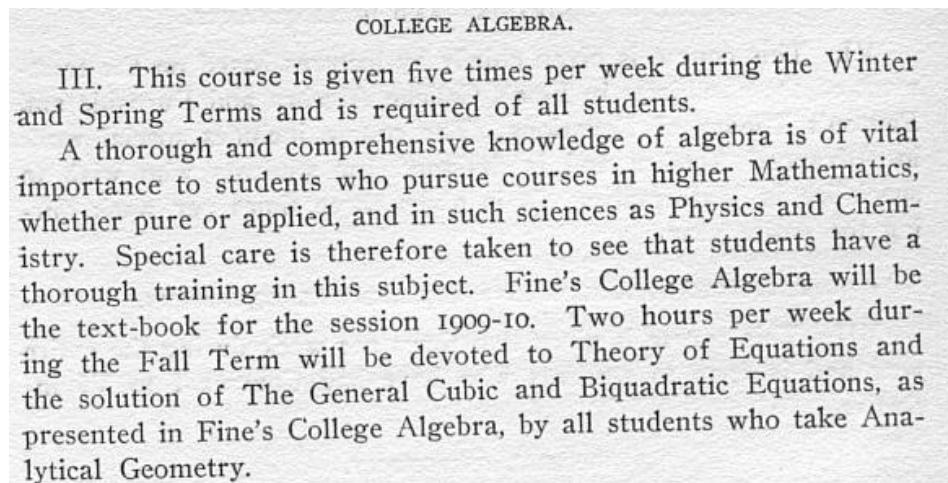


Figure 23. The first mention of college algebra in the 1908-1909 catalog, page 105.

The specific topics of general cubic and biquadratic equations (quartic equations that are quadratic in form—i.e., three terms with no odd variable powers), are specific forms of general polynomial equations. The topic theory of equations could be interpreted as polynomial or otherwise; equations need not be functions, so without looking at the textbook, there is no way to understand what constituted theory of equations.

The 1909-1910 through 1911-1912 catalogs were not available in the online catalog system, and the specific topics were omitted in the 1912-1913 catalog. Specific topics returned in the 1913-1914 catalog, but neither mentions of theory of equations, cubic functions, nor biquadratic functions appeared (See Figure 24).

3. College Algebra. Elementary algebra is first reviewed in a way to give greater clearness as to assumptions, the number concept and the functional idea, and to introduce graphs and determinants. Following this, topics are taken up that will, together with those included in courses 1 and 3, furnish the student with a fairly complete view of the subject.

Figure 24. The return of specific topics in CA from the 1913-1914 catalog, page 90.

Starting with this catalog, functions, graphs, and determinants are identified. However, as aforementioned in chapter two, *functions* would be more a general category than a specific topic, and I would claim *graphs* would also be more of a category than a topic as well. For example, a graph could be a linear function or even a set of disjoint points in a plane, and the skill set necessary to plot a single point would not be nearly as rigorous as the skill set necessary to graph a rational function with asymptotes. As such, the 1913-1914 catalog, while providing some information, was not as specific as the 1908-1909 catalog.

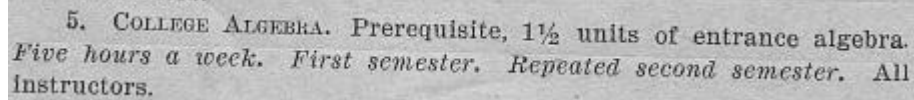
This course description remained unchanged until the 1918-1919 catalog (the 1917-1918 catalog was unavailable in the online archives), when the mention of the word function was removed and added were review of elementary algebra, the number concept, and the fundamental idea (see Figure 25).

5. College Algebra. Elementary algebra is first reviewed in a way to give greater clearness as to assumptions, the number concept and the fundamental idea, and to introduce graphs and determinants. Following this, topics are taken up that will, together with those included in Course 4, furnish the student with a fairly complete view of the subject. Prerequisite $1\frac{1}{2}$ units of entrance Algebra. Five hours a week. First semester. Repeated second semester. All instructors.

Figure 25. The removal of functions in the 1918-1919 catalog, page 166.

The 1919-1920 and 1920-1921 catalogs were unavailable, and the 1921-1922 catalog removed all mention of specific topics; the course description was severely

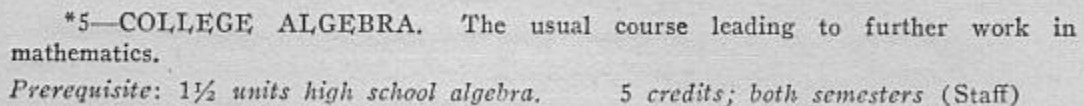
reduced in specificity (see Figure 26).



5. COLLEGE ALGEBRA. Prerequisite, 1½ units of entrance algebra. Five hours a week. First semester. Repeated second semester. All instructors.

Figure 26. The 1921-1922 catalog returned to a limited information format, page 96.

This course description remained unchanged until the 1931-1932 catalog (the 1930-1931 catalog was unavailable), when a sentence was added reading “the usual course leading to further work in mathematics” (UK, 1931), as shown in Figure 27.

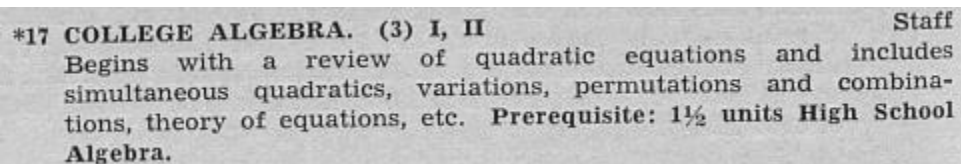


*5—COLLEGE ALGEBRA. The usual course leading to further work in mathematics. Prerequisite: 1½ units high school algebra. 5 credits; both semesters (Staff)

Figure 27. The 1931-1932, catalog returned to a limited information format page 126.

The language *the usual course* would seem to imply mathematicians has a general understanding of the content that would be in a college algebra course; however, through examining the changes in catalog so far, such a common understanding has not been demonstrated.

This language remained constant through all catalogs until the 1940-1941 edition, which saw the return of specific topics, including quadratics, variations, permutations, combinations, and theory of equations (see Figure 28).



*17 COLLEGE ALGEBRA. (3) I, II Staff
Begins with a review of quadratic equations and includes simultaneous quadratics, variations, permutations and combinations, theory of equations, etc. Prerequisite: 1½ units High School Algebra.

Figure 28. The 1940-1941 catalog, page 182, included specific topics.

The topics of variations, permutations, and combinations, however, were not mentioned in any previous version. Therefore, the decade-long description of the *standard course*

connects dissimilar topics.

This version of the course continued essentially the same until the 1943-1944 catalog, which introduced the sentence “A standard course.” prior to the unchanging description. From the 1943-1944 academic year until the 1949-1950 academic year, the course description remained unchanged. However, in the 1950-1951 catalog, the description returned to the practice of not mentioning specific topics, simply reading “a standard course” and giving prerequisite information (see Figure 29).

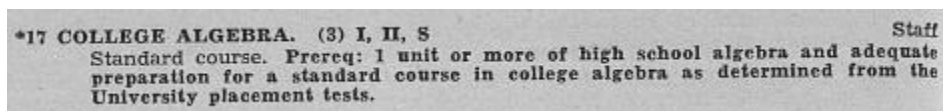


Figure 29. The 1950-1951 catalog, page 190, excluded specific topics.

From the 1950-1951 academic year to the 1976-1977 academic year (the 1975-1976 catalog was unavailable), the course description of CA was simply *Standard course*. It should be noted that, from 1967-1977, CA was non-credit bearing, making calculus I the first credit-bearing course. Therefore, for 27 years, using the course description alone, it was unknown what topics CA included at UK.

In the 1976-1977 academic year, substantive changes occurred to CA. Not only did it become credit-bearing again, but it was also renumbered from MA 111 to MA 109, and a full course description returned, including specific topics. This version of CA and the overall course descriptions resembled the contemporary look of a college catalog. There were no nontrivial changes in the course description from the academic year 1976-1977 to the year 2008-2009. In the academic year 2008-2009, the topic of conic sections was removed, and that was the only substantive change. Since 2008-2009, there have been no substantive changes in the course description (see Figure 30).

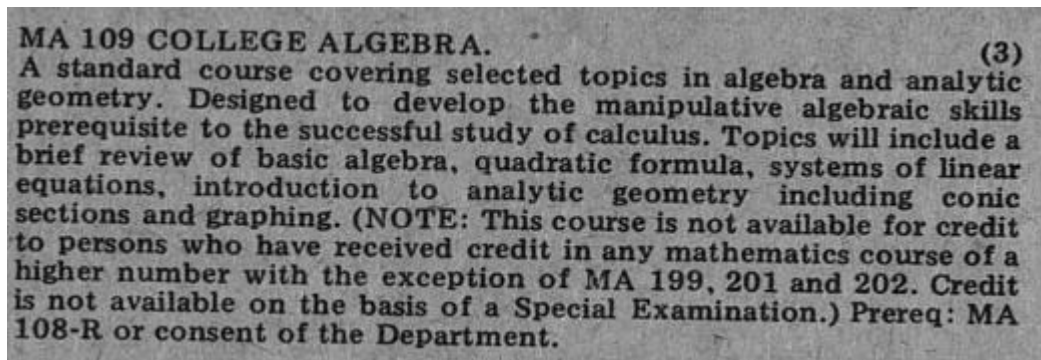


Figure 30. The wording from the 1976-1977 catalog, page 181, was mostly unchanged until 2008.

Aside from removing conics in 2008, the course description of CA at UK has therefore been unchanged since 1976. I suggest there have therefore been six eras of relevant history, for which I will introduce nomenclature in the interests of discussion: the genesis era spanning 1865 through 1907 where books were used to identify the first-semester course (which may not align well with the concept of CA) with fluctuations in level of specificity of coverage in the first-semester course, the inception era from 1908 to 1920 when CA was identified by textbook and specific topics were usually addressed (many catalogs were missing in this era), the prewar era from 1922-1939 when course descriptions did not include much in the way of identifying course aims, the war era from 1940-1950 when specific ideas were addressed and the course remained largely unchanged, the dark ages from 1950-1975 where *a standard course* was the extent of the description, and 1976 to the present, which will be called the modern era with the current topics identified.

To help to visualize the topics covered in CA by era, I have provided Table 1, which shows select eras, e.g., eras in which substantive information was provided in course descriptions. By comparing course descriptions over time, changes and similarities can be seen.

Table 1

Topics covered in CA identified by era and starting year

Era	Beginning Year	Topics Identified	Notes
inception era	1908	Theory of Equations The General Cubic Biquadratic Equations	Textbook identified
	1913	Elementary Algebra The Number Concept The Functional Idea Introduction to graphs Introduction to determinants	
war era	1940	Review of Quadratic Equations Simultaneous Quadratics Variations Permutations Combinations Theory of Equations	
modern era	1976	Brief Review of Basic Algebra Quadratic Formula Systems of Linear Equations Introduction to Analytic Geometry including Conics and Graphing	
	2008	Brief Review of Basic Algebra Quadratic Formula Systems of Linear Equations Introduction to functions and graphing	Conics removed

Common topics—Relating RQ1 with Course Descriptions

Using course descriptions in catalogs along with early textbooks, RQ1 was addressed through analyzing common themes. In the genesis era, individual books, such

as Towne's Algebra or Peck's Manual of Algebra were the only qualifying data given for topic coverage. While topics or chapters from Towne's Algebra were never explicitly identified, the content spanned eight chapters and included what would today be considered basic algebra, linear functions, quadratic functions, logarithmic functions, polynomial functions, systems of linear equations, and series and sequences, although it was not possible to determine what, if any, topics were omitted. Further, because, in the genesis era, the freshman level course was not identified as CA, including material in this era as an integral part of the progression of the course would be fallacious. The inception era had two sets of topics identified under CA course descriptions. The first included (a) theory of equations, (b) the general cubic, and (c) biquadratic equations. The second included (a) elementary algebra, (b) the number concept, (c) the functional idea, (d) introduction to graphs, and (e) introduction to determinants. Based on the changes from 1908 to 1913, the theme seemed to be to reach higher-level polynomial functions. By specifically identifying cubic and biquadratic, the underlying assumption I would make is that linear and quadratic functions must either first be covered or assumed to be known. The latter would be implied in the war era course description that included review of quadratic equations in many of the descriptions. In the war era, variations, combinations, and permutations were added to the polynomial-heavy topics. However, these topics were removed from the descriptions by the modern era, and analytic geometry returned.

The finding that I can report, based on course descriptions, is that CA at UK always included elements of polynomial functions with heavy focus on quadratics. Quadratic functions, equations, or forms appeared in almost every course description where topics were identified. Further, it would seem that analytic geometry has been a

longstanding tradition at UK, having appeared not only in modern course descriptions, but also in some inception era descriptions. What topics seemed to have been abandoned are those relating to matrix algebra, series, sequences, combinations, and permutations.

While polynomials, quadratics, and graphing have been topics that have endured, it should also be noted that logarithmic and exponential functions have never explicitly been identified in the course descriptions at UK, while most Kentucky postsecondary institutions have included them in current publications. While this has not precluded topic coverage of logarithms nor exponentials, emphasis at the publication level has never been a priority.

Summary of RQ1—transition to RQ2. RQ1 was analyzed with textbooks and course descriptions taken from textbooks. Documents taken from the UK online archives, the physical archives at the UK Special Collections Library, and from the UK Mathematics Department website were used to answer RQ2: What internal forces have led to topic coverage or attribute changes in CA?

Internal Forces—Documents from the UK Archives and the Math Website

Both the online special collections database as well as the physical UK archives contained numerous miscellaneous documents from which internal political or historical influences have undoubtedly had an impact on CA or the mathematics department. Additionally, online examinations have been stored on the UK mathematics department website since 2011. From these documents, RQ2 will be addressed.

The unequivocally most powerful internal force in the genesis and inception eras was the White family. Professor James G. White, the first mathematics faculty member in the genesis era, led the mathematics and astronomy department from 1865 through his

death in 1913 (Cone, 2015; Lexington Herald, 1934). Most of the catalogs of the genesis era identified White as the sole professor within the mathematics and astronomy department. However, the 1908 edition of *The Kentuckian* showed the images of three mathematics faculty, one of whom was Martha White, although the last name could be coincidental (see Figure 31).

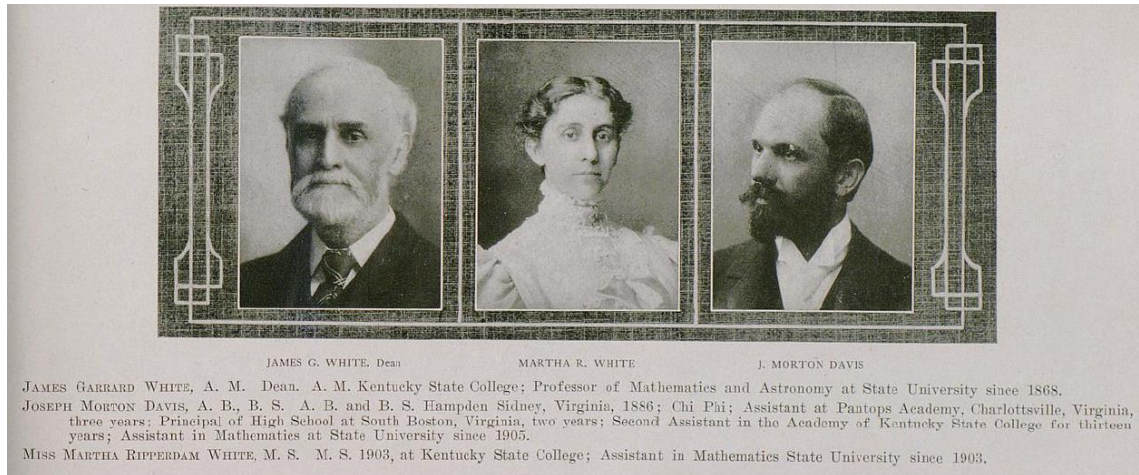


Figure 31. Another professor White in the 1908 edition of *The Kentuckian*, page 29.

In the 1909 edition of *The Kentuckian*, J.G. White was described as an able and thoughtful man (see Figure 32).

The Department of Mathematics is under the very able direction of Professor James G. White, who not only guides his students to a thoughtful and practical choice of work, but aids in contributing to the welfare of the students at the University. Associated with him is Professor J. M. Davis, who requires duty to be done at all times; yet his support in giving the students a square deal is greatly appreciated by them. Although the department suffered the loss, last December, of Miss Martha White, her influence is still manifest. Mr. J. L. Rees, a recent graduate of Kentucky State, and a man of marked mathematical ability, is her successor.

Figure 32. The 1909 edition of *The Kentuckian*, page 60, described J.G. White.

It was also revealed in this edition that aforementioned M. White had died. Regardless of potential nepotism, I would claim that J.G. White was the single most

influential person in the early years of the history of CA at UK because it was under his leadership the course came into named existence in the 1908-1909 college catalogs. As White was the dean of the department, his vision of the college likely shaped the topics identified early in the course's history.

In the 1910 edition of *The Kentuckian*, language regarding the mathematics department seemed to imply there were more mathematics majors than any other, and specific mentions of the BA and the BS in mathematics were made (see Figure 33).

IF that which is most sought is most loved, then without doubt the Department of Mathematics is the most (not to say the best) loved department of the University. For does not every student pursuing any of the courses leading to a degree offered by any of the colleges of the University, come to this department for from one to four years of Mathematics, to be served to them not as a dessert two or three times a week but as the staff of life, daily?

Not only has Mathematics many lovers, but there are some who love it best. To meet their desires there has been developed in the last two years two courses of study with mathematics as a major; one, in which language is given a prominent place, leading to the degree of Bachelor of Arts; the other, in which science is given emphasis, leading to the degree of Bachelor of Science. In both courses there is given in addition to the regular courses in Plane and Spherical Trigonometry, College Algebra, Plane and Solid Analytical Geometry, and Differential and Integral Calculus,, the following courses in Higher Mathematics: A course of lectures on Theory of Equations, Determinants, and Vector Analysis given in the Junior year. An extension of the course in Calculus and a course in Differential Equations given in the Senior year. Besides the above courses there will be offered other courses varying from year to year and so graduated that the work of this department will be kept fully abreast with the growing demands of the expanding University.

The Mathematical Society which meets bi-weekly and discusses subjects of interest to mathematicians is another enterprise of the year, which, it is hoped, will awaken and stimulate an interest in Mathematics.

Figure 33. The 1910 *The Kentuckian*, page 122, described CA and baccalaureate information.

This edition of the publication also addressed basic courses required in the degrees, for which CA was required in both.

Departmental annual reports in the early 1950s included insight into the enrollments of courses such as CA, the activities of the math faculty, and the political and workload struggles experienced by the department. For example, in 1954, 371 students took CA (at the time, there was a three-hour CA and a five-hour CA; the former was the more popular choice), which represented 21 percent of the entire math student population (Brown, 1954). The increasing number of students in mathematics coursework prompted the department to seek funding for a *large class* study the following year. The Ford Foundation Grant allowed \$12,000 to be invested in a project that eliminated several smaller sections (which had previously an average of 35 students per section [Brown, 1954]) and replace them with fewer large sections of 100 to 150 students per section (Courier-Journal, 1965; Eaves, 1956; Lexington Leader, 1956). In department chair reports from 1960 and 1963, while exact numbers of CA were not given, both documents noted growth; specifically, in the 1960 departmental report, the number of students majoring in math had increased from six to 26 from 1955 to 1960 (Eaves, 1960a, 1963). With more students, invariably problems arose. Eaves identified an increase in students repeating CA for the third time (Eaves, 1960a), which brought about a policy (still being followed at HCC) in which third-time enrollments required departmental permission prior to registration approval (Eaves, 1960b). Throughout the 1960s, the mathematics department continued to grow. From 1963 to 1967, the department had grown from 11 faculty members to 25 (Tevis, 1985).

In 1967, CA became non-credit bearing; calculus became the first gateway course. It was also during this academic year that the Board of Trustees' minutes reflected the creation of the school of mathematical sciences with the appointment of Dr.

W. C. Royster as its chair (Office of the President, 1967). Further, in the next academic year, an NSF grant close to one million dollars was awarded to UK to strengthen the mathematics program (Lexington Herald, 1968; Office of the President, 1968).

The creation of the school of mathematical sciences, the awarding of a million dollar NSF grant, and the demotion of CA to non-credit bearing all corresponded with Royster's leadership, which seemed to have a theme of continuing to increase the size of the mathematics department, both in faculty and students. Further, language suggesting the department's curricula were being improved, coupled with compelling all students to take calculus as their first credit-bearing course, would suggest that Royster had a vision in place for the rigor associated with the department. As CA had, up until 1967, been a credit-bearing integral part of both baccalaureate programs in mathematics, this transition period illustrated a time in which the department sought more mathematics as a part of their students' general education curricula.

The decision to make calculus the first credit-bearing course was reversed in the April faculty senate meeting 1976 (Department of Mathematics, 1976), and it was in this year the last major revision of CA to date took place. Royster was still a member of the mathematics faculty during this time, but reasons for the changes were not available in the online nor physical archives.

Internal forces—examinations. Beginning in the Spring 2011 semester, the UK mathematics department began to upload CA examinations on their website. Looking at these tests over the past twelve semesters, I analyzed changes. I broke the types of questions into four categories: algebra (symbolic manipulation), critical thinking (word problems), arithmetic (no variables used), and analytic geometry (graphs). Among those,

answer choices were divided between multiple choice (MC) or short-answer (SA). I then noted the number of each in a table (see Appendix E). Most of the tests were 20 questions, although a couple were 18 in length during the 2011-2012 academic year. While the balance between MC and SA questions favored MC for several semesters, as of Fall 2014, all tests have become completely MC. Analytic Geometry has become more prevalent in the examinations, now accounting for 20% of the questions. No critical thinking questions have been asked in the past three semesters. There are more arithmetic questions being asked than there were in 2011.

While examinations were analyzed with the aim of exploring RQ2, it should be noted that they could have reinforced an element of RQ1: What have been the common topics or themes of the competencies and topics covered in CA over the years at UK? Specifically, examinations illustrated that the topics in the course descriptions were on the exams (and some exam content were not in the course descriptions). While the number available examinations were limited to the past few years, they did give insight to content in the course, which presumably influenced how instructors taught the course, considering the examinations were uniform across the department.

Internal forces—syllabi. Course syllabi have not historically been preserved at UK. They were not in the online nor physical archives, and the UK mathematics faculty liaison assigned to answer my questions was unable to produce very many syllabus outside the current version used by all CA instructors. In all, 23 syllabi were collected from both UK and HCC (far more were sent to me, but as a vast majority of them were mine, I excluded them). One HCC syllabus was from 2007; it belonged to an adjunct instructor and was from an evening class. The remaining HCC syllabi were from 2011 to

the present with several gaps. All the UK syllabi were from 2010-2012. A fall 2016 syllabus was located on the department chair's webpage, and the fall 2017 syllabus was on the department website.

The UK syllabi were essentially carbon copies. The only differences among all of them were instructor names and office times. No other substantive changes were observed as UK has a *group syllabus* format in which all sections of CA follow the same evaluation and course design. This notion would be reinforced by uniform CA examinations. As such, CA at UK has included tests, homework, written assignments, and the instructor score (attendance, pop quizzes, etc.). The weight of these assignments was also homogeneous. It appeared that the instructor score was the opportunity for individual instructors to have their own flexibility in course management.

All the 2010-2012 UK syllabi indicated homework (18% of overall grade) was 90 points, three exams were worth 90 points each (18% of overall grade), the final exam was worth 90 points, the written assignments were worth a total of 20 points (four percent of the overall grade), and the instructor score was worth 30 points (six percent of the overall grade). Interestingly, the current syllabus for fall 2016 did not include the written assignment, and the points were changed to an overall 550 points. The fall 2017 syllabus, scaled to include 500 overall points, included a *written project* that was 20 points and therefore worth four percent of the overall grade. Among all the syllabi were links to online homework resources and software; UK has been embracing technology in CA since at least 2010.

HCC syllabi also followed the notion of a group syllabus. As with UK, all the syllabi (again, excluding my own) at HCC were essentially carbon copies with only

minor edits to them (i.e., faculty and staff room changes or updated website URL changes). All syllabi indicated there were five exams, each worth 100 points for a total of 500 points overall. The final exam replaced the lowest or a missing exam grade. Homework was not a grade in any syllabus. All syllabi indicated that 30 bonus points were awarded at the end of class to any student who had never missed a day. Missing a day for any reason resulted in a deduction of five points. The final exam was the only makeup opportunity for a missed exam. None of the syllabi indicated any sort of online assignments or software. From personal experience, I would comment that, while UK had ample online assignments and resources, HCC has always been vehemently opposed to the integration of technology in the classroom with very few exceptions (my being one of them).

Overall, the surprising finding to report on syllabi was the overall absence of diversity. While a community college might argue that uniform execution of a course would be the priority in course design to ensure as smooth a transfer experience as possible, it did surprise me that the only research institution in Kentucky had as homogeneous a syllabus and course design as a community college. Additionally, I found it contrary that there have been near-zero changes in the syllabus and course design since 2007 at HCC and 2010 at UK. Technology, people, pedagogical theory, and culture all change. I would have thought some major adjustments would have been made to CA based on trends in higher education. Another finding I would report was the difference between the grading between a community college and a research university. While UK included attendance, homework, and written assignments to assist with grading outside summative and high-stakes assessment, the only grade at HCC was the exam, coupled

with an attendance-based bonus score. Additionally, UK allowed a modest opportunity for individual instructor flexibility; HCC did not. It has been my goal to present findings bereft of value statements, but this would seem to indicate the standards of the community college have been higher than that of UK, which might seem counterintuitive.

Internal Forces—Relating RQ2 with archival and website documents

While archival information for catalogs allowed for analysis to span from the inception to modern eras, annual reports both online and in the physical archives were limited mostly to the 1950s through the 1970s. Prior years were not retrievable in the online archives and the UK archivist was unable to locate anything prior to 1954. While I am sure the documents are still maintained and housed at UK, they were not available to me during this research. Syllabi, exams, and other instructor-specific documents have not been historically preserved at UK. From these documents, the idea previously propounded that not much has changed since 1976 extends more than just to course descriptions. While grants, growth of department, enrollment, and technology all prompted changes in the 1950s and 1960s, the modern era seemed to have little in the way of changing content.

Summary of RQ2—transition to RQ3. Examinations, syllabi, and other documents from the UK website were used to answer RQ2: What internal forces have led to topic coverage or attribute changes in CA? However, these documents were insufficient for addressing RQ3: How has QR evolved at UK? Because QR was a relatively new notion at UK, institutional documents relating to the UK self-study effort were used.

QR Evolution—Documents from the Self-Study

When researching the history of QR through the online and physical archives, it became quite clear the notion of QR has not been a longstanding concept at UK. In fact, the term *quantitative reasoning* did not appear in any of the online catalogs prior to 2011 to refer to an academic requirement, although synonymous terms were used among disciplines such as nursing within course descriptions. Prior to the QR requirement, catalogs had an *inference requirement* for several years, and a *mathematics-philosophy* requirement since at least 1972. Catalogs prior to 1972 indicated a general studies component for degrees, but these were not made explicit under graduation requirements, nor did entries exist in the indices. It should be noted that, since at least 1972, one did not have to take CA, nor even a mathematics or statistics course, to earn a baccalaureate degree if one opted to take specific philosophy courses. While an argument could be made that the differences between a QR requirement and an inference requirement are trivial, to narrow the scope of the research, QR was specifically researched in RQ3. Because of this absence of terms, the online and physical archives were insufficient in addressing RQ3. Therefore, other institutional documents were used to answer RQ3.

In the 2010-2011 academic year, the catalog included language that the inference requirement for graduation was being revised (UK, 2010). In the 2011-2012 academic year, however, the inference requirement for graduation was replaced with the QR requirement under the new UK Core; it was this year in which CA no longer satisfied the QR (inference) requirement (UK, 2011). This decision was reversed effective fall of the 2017-2018 academic year. In the November 2016 meeting of the UK Mathematics Department, the faculty discussed their desire to convert CA back to a general education

course, re-endowing it with QR status (UK Department of Mathematics, 2016a). The process of acquiring QR status included an application with a rubric complete with examples and a syllabus-based course review (UK Department of Mathematics, 2016b)

The formal origin of this change can be traced to the recommendations of a 2004 self-study report of the University Studies Program (USP) that preceded the UK Core. The self study reported there were myriad differences of opinion about the value of the USP, the role of general education, the role of assessment and evaluation of the USP, and the goals of the USP (University Self Study Committee, 2004). Further, the USP report urged that, at a minimum, the USP goals needed to be revisited and compared with a student learning and development framework. These goals were partitioned into three groups relating to (a) interdisciplinary and liberal arts knowledge, (b) common skills across disciplines, and (c) broader outcomes over different forms of reasoning or social responsibility/citizenship (University Self Study Committee, 2004). Additionally, while the USP self study was taking place, the report identified that “other groups on campus were recognizing the need to evaluate general education and were making plans for innovative methods” (University Self Study Committee, 2004, p. 36).

Another body (possibly referenced as one of the *other groups* above) created during this time was the External Review Committee (ERC) which also released a report on the USP. In 2006, the ERC report examined other universities’ general education programs to analyze the requirements of 19 other institutions around the US. The ERC report’s first recommendation was that the USP be restructured based on the following four curriculum objectives: (a) provide students with eight specifically-identified essential skills and three specifically-identified basic skills, (b) enable students to think

from perspectives across different disciplines, (c) require students to inquire, analyze, and reflect, and (d) include a citizenship/cultural component (ERC, 2006). The ERC deliberately avoided making specific recommendations about requirements, programs, or curricula, but the body did urge a larger conversation take place at UK based on data-driven and collaborative research into how the USP could be renovated.

In 2005, members of the UK Senate Council and its chair worked with both Miami University and a team from Indiana University-Purdue University—Indianapolis (IUPUI) to study their general studies programs. After researching their program, the Senate Council and the UK Provost’s office formed the General Education Reform and Assessment (GERA) Committee to encourage campus conversation about USP reform and the necessary assessment associated with such reform (GERA, 2006). GERA Committee took much time during its formative months to scrutinize the findings and recommendations of the ERC final report. Through forums and other feedback, the GERA Committee collected and published comments and concerns about the USP. The final report addressed issues of individual departments having to provide general education requirement services for the entire university (GERA, 2006). For example, providing mathematics skills for the USP lay entirely on the mathematics and philosophy departments. Ultimately, the GERA final report indicated that reform must occur only if the faculty were able to “teach differently and with the prospect of freeing up more of their time for research and graduate teaching” (GERA, 2006, p. 9).

These three reports were then summarized in a self-proclaimed whitepaper by the UK provost.

I share the ERC’s conclusion that the starting point for a reconceptualization of

general education is the articulation of a new curricular framework within which our current or some new set of courses would be embedded. Indeed, by way of foreshadowing, I believe our current set of disciplinary courses, relying as they do on the discrete subject matter of pre-major courses, are ill-suited for the curricular framework of an ambitious program of liberal education (Subbaswamy, 2006, p. 2).

Subbaswamy echoed the notion put forth by the ERC that the UK USP should avoid thinking in terms of specific courses faculty want students to take, addressing math by name, and to think about the knowledge the faculty want to be transmitted. Subbaswamy was forthright in admitting the whitepaper was influenced heavily by ideas and values of LEAP and quoted several LEAP standards and research.

The provost's report, along with the efforts of the ERC and GERA, were clearly grounds for action; a report from the University Committee on Academic Planning and Priorities Undergraduate Education Domain Subcommittee (UCAPPUEDS) stated that they were "keenly aware of the other groups on campus working in tangential areas, e.g. the USP Reform Steering Committee..." (UCAPPUEDS, 2007). The following year, the UK Senate adopted the recommendations of this steering committee (UKCore, n.d.-a), which were to go into effect during the 2011-2012 academic year (UKCore, n.d-b). It was from this decision that *quantitative reasoning* first became a concept as a requirement at UK.

The year in which the UK core became live excluded CA from the list of approved QR courses. However, in November of 2016, the mathematics department agenda included an item to add CA to the UK core (UK Department of Mathematics,

2016). Included on their website was a document demonstrating how CA could meet all the requirements of QR. In the current year's catalog, CA reappeared on the list of courses with QR status (UK Core, 2017).

QR Evolution—Relating RQ3 with Self-Study Documents

Based on the website of the UKCore, the documents provided on that website, and documents collected from related committee websites, QR as a requirement at UK was heavily influenced by research from other institutions' general education formats, as well as from the office of the provost, whose perspective was heavily influenced in turn by LEAP. The overarching idea of QR grew from the notion that student learning outcomes, not courses out of specific disciplines, should dictate what classes would satisfy general education coursework, which explained how CA was able to lose QR status and also how several other non-mathematics courses were able to attain the QR attribute.

Chapter IV summary. Findings from this chapter have revealed much data were available from the myriad sources in the physical archives, online archives, various departmental websites, as well as in library records. These data were, by themselves, demonstrative of many personal and political factors that have shaped CA throughout the years. However, wider spread implications and conclusions can be made to how these findings can influence the broader landscape of higher education. Chapter five has served to explore these conclusions.

CHAPTER V: CONCLUSIONS

This study researched, investigated, and analyzed the content that has been covered in CA at UK as the course has evolved over the years, examining reasons for content change. Additionally, themes in pedagogy and internal UK politics were also developed. This qualitative inquiry focused on historical events through document analyses. Changes to course descriptions, themes developed in textbooks, observations made regarding examinations, comparisons of syllabi, and interpretation of official documents were highlighted for the purposes of determining how the current incarnation of CA has evolved. The discernments gleaned from this project were useful in establishing (a) what CA is, (b) why it contains the specific material taught, and (c) historical context that challenge why CA has been the default quantitative reasoning class of choice for many institutions, especially community colleges.

Empirical research questions include the following:

1. What have been the common topics or themes of the competencies and topics covered in CA over the years at UK? (RQ1)
2. What internal forces have led to topic coverage or attribute changes in CA? (RQ2)
3. How has QR evolved at UK? (RQ3)

Summaries on RQ1

RQ1 was *what have been the common topics or themes of the competencies and topics covered in CA over the years at UK?* The most prevalent conclusion that I have made was that CA has evolved through the years based on internal values and beliefs of the institution. Evidence from chapter two research regarding the disparity of topic

coverage revealed that institutions in Kentucky have different ideas about what material should be included, although there were commonalities and themes as well. However, catalogs from the late 1800s found in the online UK archives revealed that textbooks were the guiding principle for topic coverage. Additionally, the content of these early CA courses was radically different from the content of any course currently being taught in Kentucky.

After 1907, the practice of using books to identify the material of CA was dropped, and descriptive text resembling modern day course descriptions appeared in the catalogs. However, their usage was inconsistent. A common practice in the early to middle part of the twentieth century was to use *a standard course* as the description, giving me the impression the faculty had great leeway as to what was covered. By the late 1970s, this practice fell out of use, and the modern-day course description has been nearly unchanged since. However, the topics identified in this description were vastly different than the topics identified in the course descriptions prior to the *standard course* years. Because the course description has remained largely the same for over forty years, the idea that there has been agreement about the content and material found within CA would lend credence to (for example) a seasoned mathematics faculty who made this observation. This notion would be further supported by the analysis of syllabi among both UK and HCC faculty, as well as the uniformity of examinations observed on UK's mathematics department website. A common syllabus was used by all faculty at both institutions for CA, although these documents have not historically been preserved, and no syllabus older than ten years was found.

However, much evidence was discovered that has suggested there has been less

agreement than a cursory glance at these documents would reveal. In addition to the differences discovered among other public universities in the state, the textbooks that have been used to teach CA showed differences. There were organizational, pedagogical, and content changes with new editions. Because the authors of the textbook made changes, I assume the faculty adjusted their courses to match the changes. Even if seasoned faculty refused to change the manner in which they taught their courses, I find it highly likely newer faculty (who likely never saw older editions) would be teaching their classes differently from their colleagues who had been using the prior editions. Further, that textbooks *could* have organization and topic differences from older to newer editions suggested to me that there have been changes in material of the curriculum at a more national level.

Examinations were another source of evidence that suggested there has been no universal agreement as to what CA should be. In the examinations from 2011 to spring of 2014, there were multiple choice questions as well as short answer questions. Starting in the fall of 2014, all examinations had become completely multiple choice. While the reason for this may be, in part, scarce resources on the part of the department (grading short answer questions takes more time and scrutiny than multiple choice), there were implications from this change. If CA has been a course where students were being taught how to think quantitatively, reason, and draw logical conclusions, then a short answer examination would be appropriate for assessing that form of thinking. However, if CA has been a course where specific skills have been sought—that the emphasis was on a student’s ability to solve particular types of questions accurately—then either multiple choice or short answer would be appropriate for assessment. That is, if accuracy of

student computation were the most important goal, then multiple choice examinations were suitable for course objectives. However, if CA, which did not satisfy QR requirements at UK during the time this research was conducted, was supposed to be a course where final answer was not as important as the manner of thinking, then a multiple-choice examination was ill-suited to measure student performance. This claim was further corroborated by the decline and eventual absence of *critical thinking* questions from the exams. While the overall test material did not change much over the past six years, the delivery and composition of question type did transform.

Examinations, therefore, changed over six years. However, a change in test questions themselves does not constitute an absence of agreement in CA, but they do when considering the syllabi that were analyzed. In all the HCC syllabi reviewed, a *written work* policy was included that explicitly outlined how written work was necessary to receive any credit for a given problem (see Appendix F). Therefore, while the tests and syllabi I found have not shown explicit differences in CA material, they have demonstrated variance in consensus as to how CA should be delivered and philosophical differences on CA objective and purpose.

Syllabi have also revealed that assessment of the course has changed at UK, and the UK syllabi were considerably different than the HCC syllabi. These changes and differences exposed incongruence between the grading and emphases between a community college and a research university. Surprisingly, however, I determined that the rigor and difficulty level was more strenuous at the community college than it was at UK. HCC's tests, by prohibiting multiple choice, relied entirely on student work (one can guess on a multiple-choice exam). In addition, a student, when given answers, could back

track or test them against a function or equation, giving the student an advantage. Finally, the UK syllabi indicated homework, attendance, and other grades were considered in student assessment. The grade composition of the HCC syllabi allowed only for examinations to be considered.

The overarching conclusion, therefore, is that CA has changed in content at UK. Competencies and topics covered have been different, and this was observed over long periods of time. While the course has consistently looked at functions, quadratics, and analytic geometry, early concepts such as sequences and matrices have disappeared from the course. In the past four decades, the course description has been relatively constant per the course description, but resources such as technology and funding have altered the fashion in which the course has been taught and assessed.

Summaries on RQ2

RQ2 was *what internal forces have led to topic coverage or attribute changes in CA?* Internal forces have had considerable impact on topic coverage and attribute changes. In the earliest available documents, individual instructors seemed to have great influence on what material CA would cover, considering the course descriptions were just a list of what chapters would be covered in a textbook. Professor James G. White was the first and only mathematics faculty member for most of the genesis era (1865 through 1908). As the department grew, an effort to make the topic coverage uniform was evident with the appearance of course descriptions, but even in the absence of course description, there were internal forces that changed course attributes, especially in the war era (1940-1975).

Examinations did not date back more than a few years, but they were invaluable

in giving me insight to the content of the class. Because examinations were uniform across the university, I would think that instructors taught material to prepare their students for the exams. This would imply not only examinations, as they changed, showed how course attributes changed, but these tests likely reinforced notions from RQ1 regarding the topics of the course. If examinations were available across all six eras, they could be the most useful documents for answering RQ1. Examinations also raised a subtle question regarding the purpose of CA. If the goal of CA has been to instill in students a set of skills, then multiple choice was an appropriate delivery method for the examinations. However, if CA should compel students to learn to reason quantitatively, then the simple right or wrong answer of a multiple-choice exam would not appropriately measure this outcome. For example, if a student were to work a 15-step problem, but erroneously drop a negative in the seventh step, the student would almost certainly arrive at an incorrect answer; however, that student might have reasoned through the problem exceptionally well and quantitatively. Further, examinations likely led to grading consistency. At both HCC and UK, examinations have been uniform. At UK, the same examinations were used among all sections of CA, and the multiple-choice format forces instructors to weigh problems equally. At HCC, templates have been used to ensure all examinations had the same number of questions and addressed the same competencies, although differences in grading might introduce some inconsistencies.

Annual reports and departmental minutes revealed that funding opportunities and internal politics impacted CA. Grants in the 1950s and 1960s prompted many changes and created new programs in the UK mathematics department. The department grew on both the student and faculty side, class sizes were increased, the department was

separated from astronomy and defined as its own business unit, and facilities, including computer equipment and library services, were added. Several annual reports included language about how the department's curricula were being improved, including a change in 1967 where CA was no longer credit bearing. This was the most substantive change in course attribute and occurred close to two events: a near-million dollar NSF grant awarded to the department and the appointment of Dr. Royster as the chair. The decision to make calculus the first credit-bearing course was reversed in the April faculty senate meeting of 1976, starting the modern era, where the course description has remained mostly unchanged (conic sections were dropped in 2008).

Despite that the CA course description has hardly changed since 1976, the second most significant attribute change to CA occurred in 2011: the course lost its QR status. While the course remained credit bearing, simply completing CA did not satisfy the university QR requirement following several years of self study and research. Despite that CA is still the default QR of choice at the community colleges in Kentucky, the university that used to administrate these institutions had ended this practice, only to reverse it effective fall 2017.

Summaries on RQ3

RQ3 was *how has QR evolved at UK?* The term *quantitative reasoning* is relatively new at UK. Other terms or requirements have been used to describe a general mathematics or reasoning requirement at the university, but since at least 1972, UK has required a mathematics/philosophy course in order to graduate with a bachelor's degree. While this requirement would later include statistics (and I have deliberately avoided engaging in the *is statistics mathematics?* conversation), those two fields have dominated

the requirement until the UK Core replaced the USP in 2011. This was an unexpected finding, and it represented a significant issue in one of my underlying assumptions—that CA was the default QR course at UK for a clear majority of the college’s history.

Without doing further research, I cannot say with certainty how many students took CA versus symbolic logic. I can speak locally at HCC by running an internal report on how many sections of symbolic logic have been offered at HCC over the past fifteen years (zero), so I can say with certainty that CA has been the default QR at HCC, but further research would have to be conducted to make the same assertion about UK.

The origin of the term QR was traced to a 2004 self-study report of the USP that spawned several committees. After conducting much external research and encouraging campus communication and discussion among faculty, staff, students, and administration, UK decided that the idea of a general education should be knowledge learned, not classes required. Because of this seven-year dialogue and research, QR (specifically called *quantitative foundations*) was not only defined, but other courses outside mathematics and philosophy attained QR status.

I would conclude, based on statements from the documents, that the faculty’s desire to research and teach graduate classes played a small role in motivating the UK mathematics and philosophy departments to encourage (or at least not fight) other departmental courses from attaining QR status. While I was unable to ascertain whether there was considerable encouragement or argument regarding QR to apply to other disciplines, the conversations were not limited to the philosophical questions of student knowledge versus required courses, but were personal in nature to faculty time and resources.

Significance to Educational Leadership

This study had several aims. It examined how CA has evolved at UK in terms of content, reasons for content changes, and the development of QR requirements. These examinations were completed to define CA—to discover what material has constituted CA and if CA constituted covering that material, which I defined as the *one-to-one function*. Further, these examinations served to explore why CA has been the default QR requirement at UK and the community colleges (although I now know that there could be several students who have taken symbolic logic instead of CA at UK). This study therefore bears significance to the educational leader, especially at the community college level in Kentucky, when looking at making curricular decisions.

Performance-based funding. In the past couple of years, on the forefront of many educational leaders' minds has been Senate Bill 153 (SB 153), the legislation for performance-based funding (Kentucky Legislature, 2017). SB 153 described a proposal for distributing state-allocated money for both public universities and community colleges through a model where 35 percent of funding would be contingent upon performance outcomes, 35 percent upon credit hours earned, and 30 percent to operational costs (Spalding, 2017). Specifically, SB 153 would allow state funding to be “based on rational criteria, including student success, course completion, and operational support components” (SB 153, 2017, p. 5).

Because student success and course completion were explicitly identified, recognizing the challenges of requiring all community college students to take CA would undoubtedly have an impact on funding. While CA may be appropriate for STEM majors, a student majoring in the liberal arts, for example, may find CA far more

challenging than a course such as liberal arts mathematics. An educational leader might choose to research completion rates of such a course versus CA. Additionally, other non-mathematics coursework that could satisfy LEAP definitions of QR might also be substituted to increase student success and completion rates.

Pathways and meta-majors. The Kentucky CPE’s Guiding Principles (2015), while focusing mostly on co-requisite models and courses, addressed the idea that “specific connections between the Individual Learning Plan (ILP) for secondary students and the student meta-majors or career pathways should be used, where available, by academic advisors and career counselors” (p. 3). Pathways and meta-majors have been a continuous topic of discussion and is currently being pursued by CPE; the college admissions regulation (13 KAR 2:020) is being reviewed, and the addition of QR pathways relevant to student credential is a CPE suggestion for consideration (Cain, 2017).

Regardless of one’s personal views regarding co-requisite remediation, pathways, and the influence of political bodies such as CPE in higher education, leaders must be cognizant of these political factors and agendas. Because CPE is actively seeking to incorporate QR pathways into state legislation, educational leaders should be aware of this to prepare for a potential new law. By planning and anticipating these changes, lessening their overall operational and curricular impact through proactive implementation makes compliance management and expense manageable. By understanding the history of CA and knowing the story of QR development at UK, leaders can make more logical and compelling knowledge-driven arguments to faculty as to how accomplishing such goals as QR pathway development can be done while

maintaining academic integrity.

Liberal arts philosophy and academic integrity. The idea of liberal arts dates to the Roman Republic (O'Banion, 2016; Wintrol, 2014), and the underpinning idea of a general education rests heavily in liberal arts philosophy (O'Banion, 2016; Vander Schee, 2011). While the purpose and value of a general/liberal arts education may be greatly debated (Lytle, 2013; McGrath, 1944; O'Banion, 2016; Vander Schee, 2011; Wintrol, 2014), the overarching idea has been to expose students to myriad disciplines and skills for some sense of academic versatility (some authors and researches have taken great measures to differentiate among liberal arts, liberal education, and general education; I have not). The common theme of these requirements has been to instill in students certain skills pertaining to critical thinking, cultural awareness, and stellar citizenship (Dwyer et al., 2003; Lytle, 2013; McGrath, 1944; O'Banion, 2016; Presseisen, 1987; University Self Study Committee, 2004; Vander Schee, 2011; Wintrol, 2014). Because the aims of liberal arts and general education have been to broaden students' perspectives and thinking, and because the literature has indicated this should be accomplished through different perspectives and thinking across the disciplines, to compel students to take any specific course (and therefore a single approach to quantitative thinking) would ironically contradict the latent idea of liberal arts and the traditional meaning of general education.

If an educational leader fully supports the ideas and philosophies of liberal arts, then students should have more than a single choice to satisfy any general education requirement to protect academic integrity. If any general education has a set of specifically-prescribed courses, then breadth of knowledge being taught follows a specific set of skills, not a broad set of knowledge. While I am not making any claims of

value to which set should be present in higher education, curriculum decision makers should be cognizant of the subtle differentiation.

This study would serve the educational leader, especially at the community college, in making informed decisions about general education, quantitative reasoning status and attribute, and policies about CA—whether approving curriculum changes or concerning its role as a QR requirement. Contemporary educational leadership depends on decisions with regards to forming a more flexible general education curriculum, considering the future of higher education with the variables comprising performance-based funding, establishing QR pathways, and developing/implementing meta-majors.

Suggestions for Further Research

Because this research relied heavily on document analysis, and many of the documents were unavailable through online or physical archives, a member of the UK community might have better access or resources to locate documents that I could not. Along those lines, such a UK community member may also be able to discover what textbooks were used in the past and could complete research on those textbooks in addition to or instead of the MLS and ABN textbooks. The documents that I sought were those that were logical to me to analyze; however, future research may seek different documents to complete this study from a different perspective.

Topics within the textbooks, and the lenses that I used to scrutinize those topics, were only a small perspective. Future research could investigate different topics and analyze them using different coding to develop different themes. While I looked at definitions, Example 1, and organization, a stronger emphasis on pedagogy, semiotics, or other metrics could present research opportunities.

Because UK was the focus of research, many other institutions could be examined. This study could be replicated at other postsecondary institutions in Kentucky. Further, this study could be completed at other research institutions in other states or countries. This study could be repeated at community colleges or could be modified and completed at private colleges. Further, a quantitative instrument could be developed or located, and a similar study focusing on elements such as course descriptions or elements within a syllabus or policy could be conducted on several institutions. While CA and QR were the primary subjects scrutinized, calculus, trigonometry, or other courses within mathematics could also be considered. Likewise, other disciplines or requirements outside of QR (such as written communications or social and behavioral science) could also be investigated.

Finally, a study could be completed to measure what mathematics faculty perceive as the fundamental content or crucial topics in CA. A survey, interview, or case study into faculty perspectives would be revealing in determining what they feel CA should include. It would also be evident if disparities of perception existed in establishing the essential topics of the course. Such research would be definitive in disproving the absence of the *one-to-one function*.

Conclusions

CA has been a staple not only at UK, but around the nation. It has been a default QR, although many other courses allow for quantitative reasoning. While students entering the STEM field certainly need the material to be successful in their academic and professional careers, to compel blindly all students to take the course does not serve student (and often institutional) best interest. Further, there has been much evidence to

suggest there has not been a consistent delivery and agreement as to what material constitutes CA, nor what material comes to the minds of mathematicians if they were to be asked to answer *what is college algebra?* While this staple course has endured for several decades, its popularity may decline as educational leaders start to question whether CA is the best fit for most of their students. In the vacuum created by such a decline of CA offerings, other QR-worthy courses may further diversify the curricula of higher education.

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APPENDIX C: CATALOG NOTES

Catalog	Notes
1865-1866	First catalog available in online archives
1874-1875	Missing
1876-1877	Last mention of Towne's Algebra
1877-1878	Missing
1878-1879	First mention of Peck's Manual of Algebra through XI
1879-1880	Missing
1881-1882	Peck's Manual of Algebra through XI
1882-1883	Switched to Wentworth's Algebra
1883-1884	First mention of specific chapters
1891-1892	First attempt at course descriptions
1892-1893	Sentences used to describe semester
1896-1897	Less descriptive than prior editions
1898-1899	Back to almost no description
1907-1908	Missing
1908-1909	Introduced course names, CA, and course descriptions with specific topics
1909-1910	Missing
1910-1911	Missing
1911-1912	Missing
1912-1913	Specific topics removed.
1914-1915	This and the next year's were merged in archives. May be reversed.

1915-1916 This and the prior year's were merged online archives. May be reversed.

1917-1918 Missing

1919-1920 Missing

1920-1921 Missing

1921-1922 Course descriptions removed

1926-1927 Missing

1928-1929 Missing

1930-1931 Missing

1931-1932 Started "The usual course"

1940-1941 Descriptions of topics returned

1943-1944 Description changed slightly. Included "A standard course" and topics

1950-1951 Description disappeared; just "A standard course"

1961-1962 Missing

1966-1967 Introduction of "CA and Trig"

1967-1968 "CA & Trig" and CA became non-credit. Calculus was 1st credit-bearing

1972-1973 Introduction of Pre-calculus

1975-1976 Missing CA-related pages in catalog

CA returned to credit-bearing; descriptions listed separate from dept.

1976-1977

Contemporary organization of descriptions

1980-1981 This and the next year's were merged in archives. May be reversed.

1981-1982 This and the prior year's were merged in archives. May be reversed.

2008-2009 Conics removed. First change since 1977. Functions added.

APPENDIX D: FIRST-ROUND CODING ON TEXTBOOKS

Functions

1st Ed. ABN:

- Assumed only equations (mentioned sets and graphs, but none used in examples)
- Lots of “not functions” used in explanation
- Used the word *correspondence* many times without explaining/defining
- In the *Identify Functions* example, only equations are given; students not shown how to identify graphs or sets as functions despite defining functions w/ graphs & sets
- No graphing of functions at all

2nd Ed. ABN:

- Added “alternate definition” using sets
- Added sets to examples/explanations
- Added graphing functions to examples/explanations
- Added horizontal line test to examples/explanations
- Added vertical line test to examples/explanations
- Added increasing/decreasing/constant functions to examples/explanations
- Added piecewise functions to examples/explanations
- Added greatest integer function to examples/explanations

3rd Ed. ABN:

- Relations heading added 1st

- Added relation prior to talking about functions
- Defined correspondence using a table/equation/graph
- Defined functions using “set of ordered pairs”
- Still using a lot of “not a function” wording
- Uses a set in an example on page 146
- Uses a picture of a set on page 148
- Defined domain to be more broad on page 149
- Differentiated graph of a set page 150
- Many repeated concepts; i.e., page 146. Domain/range and 1-1 concepts repeat/reinforce concept of function having no repeating x values
- Repeated concept on page 148 “Recall that a function is...”
- “Point of Interest” added about Euler on page 145 in margin
- Added graphing functions to examples/explanations
- Added integrating technology to examples/explanations
- Added mild algebra of functions (arithmetic of functions) to Example 1 (e) on page 147
- Added “topics for discussion” on page 191, a section summary/review of new concepts

4th Ed ABN:

- “Point of Interest” renamed to “Math Matters” on page 148

5th Ed. ABN:

- Added more integrating technology to examples/explanations

- Very little substantive changes noted

6th Ed. ABN:

- Added “prepare for this section” on page 177, a review of concepts prior to starting the section on functions

7th Ed. ABN:

- In 6th Ed., font face changed to a sans serif when bold (to emphasize a vocabulary word), but in this edition, bold words remained the same font face (Times New Roman?)
- Example 1 switched with the paragraph on function notation
- Example 1 changed to identify function instead of evaluate function (which makes more sense to me as being the first example; mention in Chapter 5)
- Piecewise example changed from a word problem to an evaluate/algebra problem (which makes more sense to me as an easier/more effective first problem for students who have never seen a piecewise function before; mention in Chapter 5)
- Zero of a function added to examples/explanations
- “Topics for discussion” removed at the end of the section that summarized new concepts

8th Ed. ABN:

- Odd typo in the first sentence on page 164; this sentence has otherwise remained the same since its first appearance in the first addition.

- Domain moved ahead of piecewise functions
- Graph added to piecewise functions, so not just algebra anymore
- A figure/box was added prior to zero of a function that explained the concept more in-depth; box takes up over half a page
- Colors/graphics changed throughout section

1st Ed. MLS:

- Functions as a correspondence between two sets
- Domain briefly addressed
- Three-part description of functions
- EX1 picture of sets
- Naming functions after EX1
- Value/image used to address range
- Domain further defined as “largest possible set of \mathbb{R} where formula is meaningful”
- EX2 is domain/range
- Independent/dependent variable before EX3
- EX3 evaluate (no explanation of how to evaluate)
- EX4 is difference quotient
- Alternate definition of function a set of ordered pairs
- Graphing and vertical line test
- Increasing/decreasing functions
- EX5 increasing/decreasing functions; part b was constant function
- EX6 was greatest integer function

- Translations

2nd Ed. MLS:

- Odd/even functions added prior to increasing/decreasing functions
- Weight changed to test scores in opening paragraph

3rd Ed. MLS:

- Relations added prior to definition of function
- Pizza parlor intro deleted
- Definition of function revised to relation
- EX1 heavily revised with six parts, including sets and word problems
- Next text regards graphs and vertical line test
- EX2 graphs points prior to sections
- Following is function notation
- EX3 is domains
- Maximum and minimum added
- Odd/even, increasing/decreasing, translations, and greatest integer function are all deleted
- EX6 is a word problem

4th Ed. MLS:

- Relation paragraph heavily revised; Replaced with business analogy

Polynomial Functions

1st Ed. ABN

- 4.1 long and synthetic division; included Factor Theorem
- 4.2 graphs
- Starts w/ review using table of constant, linear, and quadratics as special cases of polynomials
- Graphs are first described as smooth and continuous with a figure to illustrate
- Polynomial function is designated as $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, although no attention is given to explaining this
- Then “leading term test” and moves into FL/FR
- Charts used for FL/FR
- EX1 is FL/FR w/ four parts all in standard form
- Answers to EX1 include both algebraic and graphic explanations
- EX2 is Remainder Theorem
- Turning points given after EX2
- Relationship established among zeroes, x-intercepts, linear factors, and roots
- EX3 is intercepts and graphing polynomials, using synthetic division
- Then “Zero Location Theorem” (Intermediate Value Theorem)

2nd Ed. ABN

- Content largely the same; many graphics moved into the margins
- All examples are in standard form

3rd Ed. ABN

- Section renamed from “graphing polynomial functions” to “introduction to polynomial functions”
- Table explaining constant, linear, and quadratic functions moved immediately below paragraph explaining them (prior editions it was at the bottom of the page)
- Chart explaining FL/FR had color added
- EX1 largely the same
- FL/FR behavior graphing calculator exercise added
- Local and absolute minimum and maximum added with figure to explain
- EX2 changed to word problem
- Numerous graphing utility examples added
- “Topics for discussion” added at the end of the section

4th Ed. ABN

- Section renamed to “polynomial functions”
- Relative minimum and maximum expanded; includes intervals
- FL/FR graphing utility explanation removed
- Relationship among zeroes, x-intercepts, linear factors, and roots moved into colored box for emphasis
- Powers on linear factors added and x-intercept behavior established

5th Ed. ABN

- Section renamed to “polynomial functions of higher degree”
- Graphing technology put back in for maximum and minimum

- EX3 changed to factoring a cubic into linear factors to find x-intercepts
- General graphing procedures added to the end of the section

6th Ed. ABN

- “Prepare for this section” added
- After polynomials are designated as $P(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$, a “take note” is added in the margin to explain the subscripted pieces
- Large table added with graphics to explain FL/FR
- Turing points moved under maximum and minimum heading
- Zero Location Theorem renamed Intermediate Value Theorem
- Graphing technology added to explain powers on linear factors

7th Ed. ABN

- Mostly font/color changes
- Additional word problem with technology added at the end of the section

8th Ed. ABN

- In the margin, the definition of the general form of a polynomial is made more explicit and subscripted components explained

1st Ed. MLS:

- Definition given in thick mathematical notation with no clarifying language
- Graphing interpretation immediate
- Plot several points to get shape

- No FL/FR behavior argument given
- First forms graphed are of the form $y = x^n$
- Stretching/compression argument made for $y = ax^n$ where $a > 0$
- Example 1 is graphing the form $y = ax^n$ where $a > 0$
- Translation arguments made for $y = x^n + k$
- Reflection argument made for $y = ax^n$ where $a < 0$
- Odd-powered polynomials have one real zero; pseudo FL/FR argument, although even-powered polynomials are not addressed
- Factoring argument given for general polynomial form
- Critical value method argument used between linear factors
- Odd and even powers on linear factors not addressed in depth

2nd Ed. MLS

- Moved to section 6 and linear, quadratic, and conics covered first
- Much the same, except EX1 changed to graph the form $y = x^n$, rest examples stay the same but pushed back

3rd Ed. MLS

- Major revisions in 3rd edition.
- Linear and quadratic moved out of the chapter
- Chapter starts with synthetic division section (section 1)
- Then complex root section (section 2)
- Section for polynomial roots follows (section 3)
- Graphing polynomials is its own section (section 4)

- Rational Zero Theorem used to find zeros of a polynomial, although connection between zeroes and x-intercepts is not made explicit
- Intermediate Value Theorem used to determine the existence of zeroes
- Upper and Lower Bound Theorem introduced to find boundary of zeroes
- DesCartes Rule of Signs introduced to determine the number of positive and negative real zeroes
- Higher-degree polynomials not factored into linear factors
- Still no mention of powers on linear factors
- Much approximation of zero graphing

4th Ed. MLS

- Section 1 changed to quadratics
- Section on synthetic division and complex zeroes collapsed into section 2
- Section 3 is zeroes
- Section 4 is graphing; starts with FL/FR behavior
- EX1 changed to graphing a polynomial in factored form
- No connection made between standard form and factored form

Rational functions

1st Ed. ABN

- First sentence in the section on rational functions is the definition
- Given as a quotient of two polynomial functions
- Following definition is a claim about domains (domain of F is all reals except those for which Q is zero)

- Example of definition is given immediately following the definition
- First part of explanation is domain
- Another example of definition is given
- Graph of second example is given with 5-part explanation
- Increasing function without bound language used, notation used to show f increasing to infinity as x approaches a value.
- Following notation is discussion of asymptotes
- VA defined; four behaviors not identified
- HA defined
- Graphics used to show VA and HA
- Following graphics is a theorem on VA and zeroes of the denominator
- EX 1 is finding VA of a rational function
- EX 1 has two parts – one with no VA and another requiring factoring
- Following EX1 is 3-part theorem on HA
- EX2 is HA with three parts
- No mention of cross test for HA
- Signed argument made for behavior about asymptotes
- Following EX2 is a proof of HA, using calculus line of thinking
- Following talk of HA is general procedures for graphing rationals
- EX3, EX4 graphing rational
- Following are oblique asymptotes
- Following oblique asymptotes are removable discontinuities

- Color schemes changed
- Minor organizational changes

3rd Ed. ABN

- Charts added to explain increasing w/o bound
- General procedures moved inside of colored box
- Cross test for HA addressed in colored box
- Word problem added at end of section
- “Topics for discussion” added at the end of the section

4th Ed. ABN

- Color schemes changed
- Minor organizational changes

5th Ed. ABN

- Cross test for HA made more explicit in the graph
- Color scheme adjustment; minor organizational changes

6th Ed. ABN

- *Prepare for this Section* added
- Four behaviors of VA added after EX1
- Relationship between four behaviors of VA and powers on linear factors added

7th Ed. ABN

- More word problems added
- Color and typographical changes made

8th Ed. ABN

- Color and typographical changes

1st Ed. MLS

- Definition as a fraction of polynomials
- Gives example of $f(x) = \frac{1}{x}$ as the simplest form with variable in the denominator
- Chart argument made to see behavior about VA
- VA defined: as $|f(x)| \rightarrow \infty$ as $x \rightarrow a$, then $x = a$ is VA
- HA defined: if $y \rightarrow a$ as $|x| \rightarrow \infty$, then $y = a$ is HA; cross test not mentioned
- EX1 graph $y = \frac{-2}{x}$
- Reflection argument made for EX1
- Four behaviors of VA not discussed (since powers of linear factors never taught, this makes sense).
- Graphs completed by charts of value and plugging in points to determine four behaviors of VA
- HA of leading coef/leading coef discussed later
- All graphs involving both VA and HA were in factored form
- SA addressed later, cross test not mentioned
- End of chapter, a recipe for graphing rationals is given
- Removable discontinuities never addressed

2nd Ed. MLS

- Asymptote definitions combined in one box
- EX 1 stayed the same
- Rationals not in factored form were graphed
- To find HA in non-factored form, division by all terms was used and a limit argument was used
- Word problem added

3rd Ed. MLS

- Definition slightly revised to say denominator not zero, as opposed to values are not included that make denominator zero
- EX1 same
- Word problem deleted; replaced with removable discontinuity

4th Ed. MLS

- Additional graphic added to illustrate HA and VA
- EX1 same
- Recipe at end of chapter modified

Exponential Functions

1st Ed. ABN

- Begin w/ review of laws of exponents
- Following review is exp. on calculator
- Definition exponential function
- EX1 two-parts
- Part (a) $b > 1$

- Part (b) $0 < b < 1$
- Following EX1 is conversation about plane scales
- Then seven properties: domain, range, $(0,1)$, asymptotic to x-axis, $1-1$, increasing $b > 1$, decreasing where $0 < b < 1$
- Graphs given to illustrate some of these properties
- Following are translations/reflections
- Definition natural log
- Calculator natural log
- Graphing natural log
- Graph average value function

2nd Ed. ABN

- Introduction about perfect numbers prior to review of exp
- Calculator review removed
- Seven properties put into colored box

3rd Ed. ABN

- Perfect numbers intro changed with optical illusion/ St. Louis Arch
- Prior to definition, graphic and paragraph about cd-rom sales added
- Review of exponents removed
- Following definition, review of exponents worked into definition explanation
- Table added to explain exponential functions
- Graphing calculator added back after table

- EX1 changed to graphing exponentials using translations
- Graphing utility explanation added after EX1
- Zeroes of an exponential function added at end of section
- Topics for discussion added at end of section

4th Ed. ABN

- Exponentials moved to second section of the chapter. Section 1 now inverse functions
- Cd-rom intro replaced with number of transistors on a chip
- Definition of exponential now in a colored box
- Additional tables added to explain exponential graphs
- Seven properties reduced to six (last two collapsed into one property)
- Word problem added after zeroes of a function

5th Ed. ABN

- EX1 changed to evaluate an exponential function
- Six properties reduced to 4. Several collapsed. Increasing/decreasing broken back into two
- EX2 changed to graphing an exponential where $0 < b < 1$
- EX3 now a translation
- EX4 now a reflection/stretching
- Average value removed
- Zeroes of a function removed

6th Ed. ABN

- *Prepare for this Section* added
- EX3 now just a translation; no stretching/compressions
- EX4 changed to stretching/compressions
- More information regarding history/famous mathematicians in margins

7th Ed. ABN

- Transistors on a chip changed to airport parking

8th Ed. ABN

- Color/typographical changes only

1st Ed. MLS

- Begins with “we know a^m ” when m is rational
- Conversation about $2^{\sqrt{3}}$ and how $\sqrt{3} \approx 1.7, \approx 1.73, \approx 1.732$
- Three graphs of different domains
- Assumption made that laws of exponents apply to reals as they do rats
- Four-part theorem
- Conversation about $a > 0$
- Exponential function defined
- EX1 is evaluate an exponential
- 1-1 functions
- x-axis is asymptote
- EX2 is graph $f(x) = 2^{-x^2}$
- EX3 is fractional base
- EX4 is solving an exponential equation for the base

- Then defining Euler's number using compound interest
- EX5 is exponential growth/decay
- EX6 is radioactive decay word problem

2nd Ed. MLS

- EX2 changed to graph $f(x) = 2^x$ and graph $f(x) = \left(\frac{1}{2}\right)^x$
- Figures added to show different bases
- EX3 graph $f(x) = 2^{-x^2}$
- EX4 fractional base
- EX5 solving exponential equation for base
- Compound interest formula given in box
- EX6 compound interest word problem and Euler's number
- Euler's number given to nine places
- EX7 population growth problem

3rd Ed. MLS

- Introduction added to section
- Opening conversation about doubling pennies
- Definition of exponential given in words
- Repeat concept from chapter 1 about a^m for rational values
- Conversation about $2^{\sqrt{3}}$ and how $\sqrt{3} \approx 1.7, \approx 1.73, \approx 1.732$
- Three graphs of different domains given
- Four-part theorem given
- EX1 is solving exponential equation for an exponent

- EX2 is solving an exponential equation for the base
- “Caution” added for extraneous solutions
- Definition of exponential given in box
- EX3 is evaluate
- EX4 is graph $f(x) = 2^x$ and graph $f(x) = \left(\frac{1}{2}\right)^x$
- Box of properties of the graph of an exponential, including (0,1) is a point; if $a > 1$, $f(x)$ increases, and if $0 < a < 1$, $f(x)$ is decreasing; x-axis is horizontal asymptote; and domain and range
- EX5 is translations
- EX6 is graph $f(x) = 2^{-x^2}$
- Compound interest box
- EX7 is compound interest problem and Euler’s number
- Euler’s number to ten places
- EX8 is population growth

4th Ed. MLS

- Properties box moved prior to EX1
- Euler’s number changed back to nine digits

Logarithmic Functions

1st Ed. ABN

- Reviews exponential functions and inverse functions
- Shows exponential form of logarithm (as inverse of exponential)
- Defined logarithm and logarithmic form

- Explained notation and nomenclature
- Explained relationship between exponential form and logarithmic form of logarithm
- EX1 change from logarithmic to exponential form
- EX2 change from exponential to logarithmic form
- Equality of exponents theorem
- EX3 evaluate logarithms 4-parts
- 8 properties of logarithms given
- Following are proofs of the 8 properties
- EX4 using properties expansions
- EX5 using properties given values
- EX6 condensing
- Common and natural log defined
- Calculator explanation of common and natural log
- Change of base
- Antilogarithms

2nd Ed. ABN

- Common and natural logarithms moved to colored boxes

3rd Ed. ABN

- EX3 changed to one part
- Using properties given values deleted
- EX5 changed to condensing

- *Topics for Discussion* added at end of section

4th Ed. ABN

- Combined graphing and definition into same section
- Explanation of inverse logarithms and exponential functions expanded
- Following EX2, three properties of logarithms given instead of eight
- EX3 applying basic properties of logs
- EX4 evaluating logs using the properties (not given values)
- Graphing logarithmic functions followed EX4
- Tables used to explain relationship between exponential and logarithmic values
- EX5 graphing a logarithm
- Properties of the graph of logarithms followed EX5
- Domain of logarithms
- EX6 domain of logarithms
- EX7 translations
- Common and natural logarithms
- Applications of logarithms
- *Topics for Discussion* added at end of chapter
- Other properties, change of base moved to next section, along with more applications

5th Ed. ABN

- Composition of exponentials and logarithms argument added

- Four properties instead of three given
- EX3 still applying basic properties, but EX4 deleted
- Application problem added at end of section

6th Ed. ABN

- *Prepare for this Section* added
- Graphics added to the answers of EX1 and EX2

7th Ed. ABN

- Color and typographical changes only

8th Ed. ABN

- More examples added to EX3 (apply basic logarithmic properties)

1st Ed. MLS

- Conversation previous section 1-1 implies there exists inverse function
- Look at these inverse functions
- Inverse of $y = a^x$ is $y = \log_x x$
- Box showing definition of log (both forms)
- Language and vernacular
- EX1 is chart of two forms (no work to be done)
- Definition of logarithmic function
- Figure showing log and exponential as inverses
- EX2 is graph a log with translation
- EX3 is graph a log with abs value
- EX4 is solving a logarithmic equation for base

- Then five properties of logs
- EX5 is expansion of logs
- EX6 is condense logs
- EX7 is log properties
- Then theorem on exponents in log expressions
- EX8 is exponents in log expressions

2nd Ed. MLS

- Conversation about 1-1
- X and y are switched before logarithmic notation given
- Conversation about solving for y given
- Then box with both forms, followed by vernacular
- More explanation about log vs exp form of a logarithm
- EX1 still chart
- Logarithmic definition follows
- EX2 changed to graph without translations
- EX3 graph with translations
- EX4 graph abs val
- EX5 solve for base but now also solve for argument

3rd Ed. MLS

- Intro same
- Box removed for definition of logarithm, vernacular same
- Following two forms conversation, a definition is given in a box different

than before

- EX1 still chart
- EX2 changed to two-part solve of base and argument
- Definition of log function then given
- EX3 changed to two-part graph no translation
- Figure showing inverse relationship between logs and exps moved here
- EX4 now graph translation
- EX5 graph abs val
- Exponent theorem moved here
- EX6 now the exp theorem problem
- 5 properties numbered instead of lettered are next
- EX7 is expand
- EX8 is condense
- EX9 is evaluate using properties
- History of Napier added

4th Ed. MLS

- Same 1-1 argument
- Definition given
- *Fill in the box* argument made
- Relationship between two forms explained
- EX1 still chart
- EX2 solve base/argument
- Definition given

- EX3 graph single base
- Box added to show four features of graph of log
- EX4 graph translation
- Natural log introduced
- EX5 is PH
- Exp properties of log
- EX6 is exp properties
- 5 properties of logs given
- EX7 is expansion
- EX8 is condense
- EX9 is evaluate given assumption
- Change of Base introduced
- EX10 is change of base
- EX11 word problem on diversity of ecology

APPENDIX E: EXAMINATIONS

Exam	Category	Sub-Total	MC	SA	Total
Exam 1 Spring 2011	Algebra	15	13	5	18
	Critical Thinking	1			
	Arithmetic	2			
	Analytical Geometry	2			
Exam 1 Fall 2011	Algebra	16	13	5	18
	Arithmetic	1			
	Analytical Geometry	1			
	Algebra	16			
Exam 1 Spring 2012	Critical Thinking	0	13	5	18
	Arithmetic	1			
	Analytical Geometry	1			
	Algebra	17			
Exam 1 Fall 2012	Critical Thinking	0	14	6	20
	Arithmetic	1			
	Analytical Geometry	2			
	Algebra	16			
Exam 1 Spring 2013	Critical Thinking	1	14	6	20
	Arithmetic	1			
	Analytical Geometry	2			
	Algebra	17			
Exam 1 Fall 2013	Critical Thinking	0	14	6	20
	Arithmetic	1			
	Analytical Geometry	2			
	Algebra	18			
Exam 1 Spring 2014	Critical Thinking	1	15	5	20
	Arithmetic	1			
	Analytical Geometry	0			
	Algebra	15			
Exam 1 Fall 2014	Critical Thinking	2	20	0	20
	Arithmetic	2			
	Analytical Geometry	1			
	Algebra	15			

Exam 1 Spring 2015	Algebra	16	20	0	20
	Critical Thinking	1			
	Arithmetic	2			
	Analytical Geometry	1			
Exam 1 Fall 2015	Algebra	17	20	0	20
	Critical Thinking	0			
	Arithmetic	1			
	Analytical Geometry	2			
Exam 1 Spring 2016	Algebra	11	20	0	20
	Critical Thinking	0			
	Arithmetic	6			
	Analytical Geometry	3			
Exam 1 Fall 2016	Algebra	11	20	0	20
	Critical Thinking	0			
	Arithmetic	5			
	Analytical Geometry	4			

APPENDIX F: SAMPLE HCC SYLLABUS

**SYLLABUS
MT 150 COLLEGE ALGEBRA
FALL 2006**

INSTRUCTOR: Maura Corley
OFFICE: AS 214

PHONE: OFFICE: 831-9683
CELLPHONE: 270-704-0862
E-MAIL: Maura.Corley@kctcs.edu

OFFICE HOURS: Office hours will be posted.

REQUIRED TEXT: College Algebra Aufman, Barker, Nation, 5th ed.

REQUIRED SUPPLIES: Math notebook, scientific calculator.

COURSE DESCRIPTION:

Selected topics in algebra and analytic geometry. Develops manipulative skills and concepts required for further study in mathematics. Includes linear, quadratic, polynomial, rational, exponential, logarithmic and piecewise functions; systems of equations and inequalities; and introduction to analytic geometry. Students may not receive credit for both MT 150 and MA 109 or for both MT 150 and MA 110. Credit not available on the basis of special exam. Lecture: 3 credits (45 contact hours). Prerequisites: One of the following: 1. Math ACTE score of 20 or above. 2. Math ACTE score of 18 or 19 with concurrent MT 100 workshop. 3. MT 120 or MT 122 or MT 125. 4. KCTCS placement exam recommendation.

GRADING POLICY:

90 - 100% = A
80 - 89% = B
70 - 79% = C
60 - 69% = D
Below 60% = E

There will be five exams, worth 100 points each. The final exam will be comprehensive (covering Chapters P through 4 and part of 9) and will be worth 100 points. The final is optional for those students who have not missed any exams. The final is mandatory for any student who missed one or more of the exams. Students will also have the opportunity to earn 30 bonus points for attendance (see attendance policy). There are 500 possible points from exams. No exam score may be dropped unless the final exam is taken.

ATTENDANCE:

Students are expected to be at each class meeting. Attendance will be taken every day.

All students start with 30 bonus points. For each day of missed class the student will lose

five bonus points until the number of bonus points reaches zero. The loss of bonus points occurs **regardless of the reason for the absence.** These are extra credit points given for attendance. Since homework will not be collected for a grade, students may use these points to supplement their exam scores. If a student misses an exam this does not affect his or her bonus credit. Make-up exams will not be given.

If one of the exams is missed the comprehensive final will replace the score from that exam. If more than one exam is missed the comprehensive final increases in value 100 points for each additional exam missed. Any student who has not missed an exam may take the comprehensive final if he/she chooses to do so. In that event, the lowest score of all exams including the final will be dropped.

WRITTEN WORK:

On exams mere answers without supporting steps will receive no points.

MAKE-UP WORK: See section on **ATTENDANCE POLICY**

ACADEMIC HONESTY POLICY:

The KCTCS faculty and students are bound by principles of truth and honesty that are recognized as fundamental for a community of teachers and scholars. The college expects students and faculty to honor, and faculty to enforce, these academic principles. The college affirms that it will not tolerate academic dishonesty including, but not limited to, violation of academic rights of students and student offenses. (*Rules of the Community College Senate, Section VII and Code of Student Conduct, Article II*)

Information about the academic rights of students and academic offenses and students' right to appeal can be found in the Kentucky Community and Technical College System *Code of Student Conduct, Article II - Academic Policies and Procedures*. The *Code of Student Conduct* is available at the following web site:
<http://www.kctcs.edu/student/studentcodeofconduct.pdf>.

REPEATING:

Any student repeating this class and desiring to replace the old grade with the new grade (if the new grade is higher) must complete an "Option to Repeat" form within the first two weeks of classes.

WITHDRAWAL:

Up until midterm, the student may withdraw and receive a "W". After midterm, the instructor shall consider each case individually. In general, a student must discuss the possible desire for a "W" with the instructor **before** midterm in order to obtain a "W" after midterm.

FINANCIAL AID REPAYMENT:

Students receiving some forms of federal financial aid, who do not officially withdraw by the scheduled deadline, may also face financial penalties. Students may be required to repay a portion of their financial aid or may not be able to receive future financial aid.

ADA NOTICE:

If you need an accommodation because of a documented disability, you are required to register

with Disability Services **each semester**. Student(s) should contact the Disability Services Coordinator, Larry Tutt, (Administration Building, Room 218) or at (270) 831-9783 or 1-800-696-9958 (in Western Ky.), ext 19783.

**TENTATIVE SCHEDULE
MT 150 – 02 MWF 7:40 – 8:30
FALL 2006**

AUGUST

21	P.1	22	23	P.2	24	25	P.3
28	P.4	29	30	P.5	31		

SEPTEMBER

Monday	Tuesday	Wednesday	Thursday	Friday
				1 P.6
4 Holiday	5	6 Review	7	8 Test 1
11 1.1	12	13 1.2	14	15 1.3
18 1.4	19	20 1.5	21	22 Review
25 Test 2	26	27 2.1	28	29 2.2

OCTOBER

Monday	Tuesday	Wednesday	Thursday	Friday
2 2.3	3	4 2.4	5	6 2.5
9 ----- --	10 ----FALL----	11 ---BREAK-- -	12 ----- -	13 -----
16 2.6	17	18 Review	19	20 Test 3
23 Prof Dev	24	25 3.1	26	27 3.2
30 3.3	31			

NOVEMBER

Monday	Tuesday	Wednesday	Thursday	Friday
		1 3.4	2	3 3.5
6 Review	7	8 Test 4	9	10 4.1

13	4.2	14	15	4.3	16	17	4.4
20	4.5	21	22 – Thanksgiving-		23 ----Break----- -	24 ----- -	
27	4.5	28	29	9.1	30		

<i>DECEMBER</i>				
Monday	Tuesday	Wednesday	Thursday	Friday
				1
4	Review	5	6	Test 5
7		8	Review	
11	Final Exam	12	13	14
				15

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