# ACCOUNTING APPROACH
# AND MULTISECTORAL MODELLING

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K. Velupillai contributed to this paper in many ways, but, first of all, by calling my attention to a beautiful (and I believe, little known) article by Hicks (1956).
Francesca di Brisco read and commented various versions of the paper: her help has been really valuable. As usual, I alone have to be blamed for any remaining errors.

"Imagination is more important than knowledge" (A. Einstein).
"What we need is imagination. We have to find a new view of the world" (R.P. Feynman).


## Abstract

There are two versions of the multisectoral model: the "formalistic" one, based on an axiomatics of production relations and the "empiristic" version based on an accounting/algebraic approach. For this reason, the multisectoral model provides a good case study for an issue that has dominated the development of economic analysis in this century: the opposition between formalistic and the empiricist philosophies. In this paper a reconstruction of this latter one is suggested.

## Zusammenfassung

Es gibt zwei Versionen von multisektoralen Modellen: eine "formalistische", die auf der Axiomatik von Produktionsbeziehungen basiert und eine "empirizistische" Version, der ein algebraischer Ansatz unter Verwendung von Buchhaltungsbeziehungen zugrundeliegt. Aus diesem Grund stellt das multisektorale Modell ein gutes Fallbeispiel für ein Thema dar, das die Entwicklung der Wirtschaftsanalyse in diesem Jahrhundert dominiert hat: Der Gegensatz von formalistischer und empirizistischer Philosphie. In dieser Arbeit wird eine Rekonstruktion des letzten Standpunkts nahegelegt.

# CONTENT

## INTRODUCTION

There are two really distinct versions of the multisectoral "model": the axiomatic-formalistic one (associated with the name of von Neumann and with the modern Activity Analysis), and the accounting-empiristic version. This is associated with the name of W. Leontief, but, for reasons that will become clear later, I will also associate Sraffa and Goodwin of the Linear Models. They are distinct approaches, though they tend to be assimilated. For this reason, the multisectoral model provides a good case study for an issue that has dominated the development of economic analysis in this century: the opposition between the empiricist and the formalistic lines. While empiricism comes to us from the last century, formalism is really a modern product: for this reason, formalists have always presented themselves as representatives of serious scientific work and of scientific progress. The fight is not probably over yet, and the issue has come up again in the recent debate on 'Microfoundations'. Therefore, it is hoped that the reader can find something to his taste, even if he is not interested in multisectoral modelling.

The axiomatic-formalistic version is assumed to be familiar: its approach is a typical General Equilibrium Approach; its structure is formal, axiomatic and rigorous; its 'informative content' fairly understood (though there may be disagreement on this issue). It is the general status of the empiricist version that is not clear: it does not even seem clear whether it forms a coherent proposal about how to model. To this approach, therefore, is this paper devoted, in an effort to provide some hints.

There is a preliminary difficulty when we discuss the empiricist version: namely, there does not seem to be any programmatic declaration to which one can refer to build a story (something of the sort of Koopmans' Three Essays). The story has to be re-constructed from bits and pieces, and obviously, there will always remain doubts, as to whether it reflects only the way the author likes to see it, or it corresponds to "truth". However, I still hope that what I offer can be accepted as a re-construction. This develops very abstractly till we come to section 5: the abstract character of the discussion is motivated by the intent of developing a unified framework against which individual models can be evaluated. It is hoped that the reader will remember the sort of models we are focusing on, and so will be able to follow the general thread.

In section 4, we refer to certain work by J. Hicks, for his work provides one of the themes of the whole paper. The distinctive feature of the version of multisectoral models we focus on is their accounting character, and this has to be interpreted: it is to do this that we borrow an idea from Hicks.

In September 1933, Hicks and von Neumann went for a walk in Buda, the right bank town of Budapest, on the Danube. (The meeting had been organized by N. Kaldor). Von Neumann explained his model of growth, but Hicks did not understand what the plot was about. My explanation is that Hicks was following a different track, and had a different (classical) tradition behind him. This became clear later.

In his contribution to Lindahl's Festschrift (1956) Hicks discusses the relation of methods of dynamic analysis and accounting structures. He argues that the very development of a dynamic analysis in Economics has been made possible by the application of accounting procedures and structures which had been elaborated in a parallel way for an entirely different purpose. Of these structures, two major one are distinguished: namely, i) stock-flow business accounting (which would have led to Keynes' analysis), and ii) the ex-post/ex-ante budget account typical of Public Finance (which would be at the basis of Lindahl's analysis). The above is connected with the balance-sheet, the latter with the transaction account. It is this latter framework that Hicks himself traces in the dynamical analysis he developed in Value and Capital. That of a transaction account is the typical viewpoint also of classical analysis: in their elaboration on national economies, from Quesnay to Ricardo, and possibly Marx himself, they were striking the balances of the commodity and labour flows of their economic systems. From this flow-account framework, the classics derived propositions like the existence of a surplus value, or theoretical descriptions like, the reproduction schemes or the Tableau. In all cases, social accounting equations were used to determine the conditions of reproducibility of the system. The balance-sheet was being developed at the same time when economists were trying their very first steps, at the time of the Industrial Revolution, and this was the historical reflection of the development of more and more capital intensive and indirect techniques of production. The balance-sheet came to Economics later, possibly with the Austrians, after the first (and possibly the second stage of) the Revolution had been accomplished.

The transactional viewpoint led the classics to develop a formal analysis where the problems of value and distribution were central: capital and growth were dealt with in the non formalized section of the theory. The strict distinction between the problems of distribution and those of development, the former being dealt with in an accounting framework, gave their models a static flavour. Using the notion of a capital account, the Austrians tried to develop a dynamical analysis, but did not find many followers (not even recently, after Hicks has revived their viewpoint). Growth re-appeared in a sort of classical setting, in the von Neumann model, but the accounting viewpoint had been lost and replaced by the General Equilibrium viewpoint. Hicks could not understand von Neumann.

The modern Empiricist approach to multisectoral models retains the concern, typical of classics, with flow balances as reflected in a transaction account for the whole economy, which is conceived as being made up of separate industries. Out of the two phenomena of a production and exchange system, they seem to privilege the 'visible' phenomenon of market transactions, as they are recorded by the Social Accounting Identities. The reason is that they take an observational viewpoint: their data must come from an empirical and observable system, taken as a whole.

This focus on the transaction aspect of a specialized system and on its 'cash-flow' account is due to the distinctive feature that this account has in comparison with the balance-sheet (see Hicks (1965)). Transactions are 'visible' transactions, their account is made up of certain entries, both quantities and prices (the quantities actually produced and exchanged and the current prices at the time of the exchange). In contrast to this, the balance-sheet includes uncertain values, that is imputed values of unfinished goods and of held stocks. These are really ascertained at the level of firm, for they belong to the black-box of the transformation laws and to the sphere of microeconomic decisions. A balance-sheet can become an instrument of analysis only if combined with the knowledge of what goes on in these black-boxes.

The empiricist viewpoint does not allow the introduction of the hypothesis that technology is known, for it is not independently observable: only indirect and experimental information can be obtained about it. Thus, the choice of the transactional framework combines with and is motivated by the strict empiricist (observational) viewpoint.

For this reason, empiricists cannot develop, I believe consciously, an analytical representation of production, as Georgescu-Roegen (1971) points out for Input-Output analysis. Nor can they develop dynamics along Hicksian lines. However, I consider this a choice more than an undesired short-coming. They try to compensate for this weakness with the notion of the (productive) core of the system.

As we said, basic to the empiricist viewpoint, is the idea that data can only come from observations taken on an empirical system. This requires a notion of system and of state; moreover, it requires a solution to the problem: how is a system identified. The empiricist has his solution: a system is identified by the number of produced commodities, and this is ascertained directly by putting together the Social Accounting Identities, which reflect certain information about the metabolic flows within a producing system as made up of specialized parts. From this intuition, they derive a number of empirical terms, like commodity and industry, that identify their theoretical context in opposition to the formalistic approach.

This vision of the system as made of already connected parts is supported by the 'intuitive truth' that the system is a goal-oriented or norm-governed entity. As a consequence, in contrast with the formalistic use of "proof", no empiricist statement can violate this fundamental axiom.

This is why the class of models defined as empiricist share, as distinctive features, the algebraic approach, and the same set of empirical terms and methods of proof. The term Empiricism is borrowed, with a certain amount of freedom, from Lakatos, who identifies it with the (mathematical) philosophy according to which "truth value is injected at the bottom" that is at the level of the basic statements (empirical terms)"[1].

However, to mark the distance of the approach we will focus on from Logical Empiricism, not only the basic statements but also the primitive terms at the top are made up of 'perfectly known terms'.

What follows is a sort of (rational) re-construction of the approach just outlined. It will be clear that the set of models characterized by the Accounting Approach and the set of 'Empiricist models', in the multisectoral literature at least, are such that the former strictly includes the latter.
(To help the reader through, footnotes are kept at a minimum).

The paper is organized as follows: the construction of a trial model called 'snap-shot model' occupies sections from 1 to 3 included. The snap-shot formalizes the intuitive idea of data collected from an observed state of the system. The process by which data are given a format is the centre of the discussion, together with various properties of the model description so derived. In section 4, Hicks' approach is used as a reference for comparison, to argue that Hicks' analysis is properly based on the snap-shot: the empiricist is not. Section 5 clarifies that the snap-shot hides an internal contradiction which would force into dynamical analysis. The Empiricist approach avoids this issue by concentrating on the 'core' of the snap-shot. All remaining sections are devoted to illustrate the procedure of isolating the core.

§ 1.

The first claim of the empiricist is that Basic Data are taken from observations of the system moving over time (this is called "observational viewpoint"). The collection of such data gives information both of a qualitative and a quantitative nature on the state of the system and on its internal properties. The first important question that this claim raises is: how is the system identified and what representation can we give.

Taking the claim seriously, it implies a sort of snap-shot theory: in other words, the system is photographed at a given point of time. We will develop this snap-shot model as a starting point for our discussion. This will be useful to see the difference between the two types of empiricism we referred to. The hard version is usually identified with the snap-shot model: however, this version of empiricism is not really very popular. The value of the snap-shot model is in that it provides a framework for evaluating the empiricist approach in general, and to understand certain basic characteristics: namely, why the format of their model is square (n x n), where n is the number of "commodities".

The Snap-shot

We start by imagining that we are observing an isolated system moving over time, and we are taking a camera picture. We assume that the system is an exchange system made up of k parties.

The snap-shot gives us a triple of data, $D = (Z,p,q)$. The vector q is the vector of actual sales of goods, vector p the actual price vector and Z is the transaction matrix recording all transactions carried out at the time the snap-shot is being taken. All quantities are, by hypothesis, actual quantities and must be thought of as carrying implicitly a time sub-script: it is only because we are taking a single snap-shot that we need not to write t explicitly.

Recalling that Z is a transaction matrix and therefore is expressed in value terms, the following definitional relations hold along the rows and the columns

(1) $Z\,1 = r$

(2) $1'Z = s$

where 1 stands for the unit sum vector (and the sign ' denotes transpose). Relations (1) and (2) define, respectively, receipts from sales of the j-th party $r_j$ and his corresponding outlays, $s_j$.

So far, there is no need for each party's receipts to be equal to its outlays, and in general $r_j \neq s_j$.

We have not yet discussed the size of our data, Z, p,q: Z is a generic rectangular matrix, and p and q conformable vectors. We can now be more specific, and introduce the assumption that our system exchanges n commodities, a commodity being defined as a good with a positive price: if we put 'parties' along the columns, then commodities will be written along the rows, and the size of Z will be (n x k). (Likewise, p is a row vector (1 x n) of all positive entries). If we think of q as the quantity sold (measured in phsyical units), then (1) becomes

$$(3) \qquad Z\,1 = r = pq$$

There are n such relations, representing in each market the balance between actual quantity supplied and actual quantity demanded. I will call these Social Accounting Identities (SAI), as they hold for the ex-post demands and supplies. It should be remarked that the SAI refer not to "goods" generally speaking, as abstractly conceived, but to goods with positive prices. Any other relation referring to goods with a zero price would add nothing to the above set of relations, for it would only add rows identically zero. (On the other hand, the same would happen if the good were not supplied, $q_j = 0$). As a consequence, taking the point of view implicit in the SAI, we are restricting our attention to the subset of all conceivable goods that are actually <u>exchanged</u> at a positive price (p and q are thus both positive vectors!).

As the description is restricted to account for commodities, we may be more specific about their origins: the characterictic of our system is that all commodities are produced goods. Our snap-shot represents a production and exchange system, where then q (the total amount of sales) stands for a proxy of levels of production, as it represents total outputs plus decumulation of stocks. However, to be faithful to the assumption that our transactions are 'observed' transactions, the pattern of matrix Z will show all zeroes along the main diagonal (for these entries stand usually for accumulation of stocks at the producers). With this convention, thus, the vector q will stand for actual (current) production.

As a consequence, we change the interpretation of the accounting definitions (2), for now outlays are productive outlays (purchases of goods and services for productive purposes).

We now have to deal with the number of parties, which so far has been assumed to be a generic k. Again, conceptually, if we start with a world where produced commodities are flowing among the parties, the easiest assumption is that each commodity is produced by a separate 'industry' (that is precisely what is done in National Accounting). Thus, to n commodities and markets as represented by the set of Social Accounting Identities, there must correspond (at least) n industries. The size of the transaction matrix is now fully specified: it is a square matrix of an order equal to the number of commodities, while vector q is now interpreted as a (n x 1) vector of industrial production levels (in the ex-post sense). The order of the transaction matrix determines logically the number of parameters needed to represent the system state. This set, synthetically collected in Z, is thought of as a sort of core for our system, and will play a role soon.

The Social Accounting Identities of which we have made so much, are a sort of giant (consolidated) revenue account for the whole system, as made up of specialized industries, themselves giant firms. The dual definitions (2) reflect, instead, the production (or running) cost side of the same system. However, it is commodities that starting from SAI we focus on: they make the principal ingredient of our representation of the system. We could have started from the production side, instead: let us do this to see where it would have led us. Hopefully it is going to make clearer the nature of commodity description that our snap-shot gives. We will keep the assumption that Z is a rectangular matrix (n x k), and p and q are conformable vectors. Moreover, we keep the assumption of an isolated system of production and exchange, but, just for the sake of the exercise, we will dismiss the assumption that there is an industry behind each produced good.

Thus, we have that each column of Z is a sort of aggregate process (or method of production), whose cost side is described by (4), namely

(4) $\quad 1\, Z^j = s_j$, for all $j = 1,2,\ldots,k$

Relation (4) records the cost incurred in the production of the output. We know that, typically, in our snap-shot economy, there is no need for costs to be equal to revenue: this ensures the generality of the situation described by the snap-shot. However, assume we are striking the balance at the end of the year: output q has been sold at the average price p, ruling at the moment of the transactions, so that the accountant of the giant firm can obtain profits as a firm figure, say $\pi_j$. Adding this figure to the left side, we obtain

(5) $\quad 1\, Z^j + \pi_j = r_j \qquad j = 1,2,\ldots,k$

where $r_j$ is revenue from sales of produced output. The left side is thus interpreted as the (ex-post) cost of production allowance being made for (gross) profits, as they come out from the transaction account of the firm. There are k relations like (5), one for each 'process' that has been operated and has therefore led to saleable (and sold) output. On the other hand, we have n produced 'goods'. It is clear then that, if there are more produced goods than 'processes' actually used, some of them have produced more than one good: no industry model can be obtained in this case. It is only if n is not <u>larger</u> than k, that we can try to recover the commodity/industry model, because only in this case does the assumption on which it is based make sense. In any case, it is the number of processes used that is important: the number of goods should be at least k and is determined by introducing an extra assumption, totally dual to the one introduced before.

Starting with the production side of the transaction matrix of our system, we tend to obtain a description that is based on the notion of 'process' or of 'firm' more than one based on 'commodity'. 'Process' is a technological notion and yields a process description; the description derived from the revenue side is an (economic) description (based on the notions of commodities and symmetrically of industries). This points out that, even if we start from an "observational viewpoint" as empiricists claim they are doing, there is a choice among alternative descriptions. However, these are usually not perceived as logically distinct, because of the assumption that commodities imply specialized industries. It is this assumption that introduces a logical symmetry between 'rows' and columns of the transaction matrix Z. This is often taken to imply more information content than the matrix Z really has.

## § 2. Alternative, equivalent representations

From the empiricist viewpoint, therefore, the number of commodities is ascertained from the number of ("meaningful") Social Accounting Identities and determines the space where the state of the system (as represented by the equalities of actual quantities produced with their demand) is being represented. The 'industry' assumption introduces a technological flavour to the reading of the transaction matrix columnwise. For this reason, which unfortunately will be the cause of much misunderstanding, the transaction matrix Z will be said to represent the core of our representation, and the core is automatically represented by a square matrix of order and rank equal to the number of commodities.[2)]

The commodity space is taken to be $R^n$, with the usual set of orthonormal coordinates. In $R^n$, the representation of matrix Z yields n points, one for each industry, bound by a composition rule (or law) represented by the SAI. However, these points have not yet been obtained.

In fact, our snap-shot data (Z,p,q) has the (not so peculiar) property that transactions are expressed in value terms: i.e. they are volumes of commodities exchanged at the price p, though we know, on the basis of our interpretation of what the 'system' represents, that in each row there is a different commodity, not just flows of 'money'. Being expressed in the same unit, entries in Z are homogeneous, both along columns and along rows, and physically distinguishable commodities do not appear. The industry as the dual unit for commodity is on the forefront with its network of buying-selling relations with all other industries and we have to think that there is an axis for each such industry in the coordinate system where Z is represented. However, due to the above assumption, by construction there are as many industries as produced commodities: this is the clue for the smooth change of coordinates that we need to get commodity coordinates. Starting from the Social Accounting Identity, this is very easy. In fact, we have found out that it only refers to 'observed' transactions of produced commodities, which in our industry framework means that both p and q are positive.

The change of coordinates is thus easily performed by using actual prices and quantities in diagonal form, Dp and Dq. From the SAI, i.e. from

$$Z\,1 = Dp\,Dq\,1$$

we obtain

$$Dp(Dp^{-1})Z(Dq^{-1})Dq\,1 = Dp\,Dq\,1 \qquad (6)$$

so that, putting $(Dp^{-1})Z(Dq^{-1}) = A$, we can rewrite (6) as

$$Dp\,A\,Dq\,1 = Dp\,Dq\,1 \qquad (7)$$

Now, A is the new matrix representing system's coordinates in commodity space: in fact, each entry $a_{ij}$ is the amount of commodity i bought by industry j per unit of its output, $q_j$. The entry is still an ex-post quantity but it is a quantity in natural units of commodity i (tons, yards, gallons, etc.), so that, now, entries on different rows are not homogeneous.

The introduction of A therefore reflects only a coordinate change, and parallel to it a change in the measure units, and does not entail anything else. The system state was represented before by value flows, now by physical flows of commodities coordinated in a network. This is confirmed by the fact that we derived A by mere manipulation of Z, without introducing any further assumptions: ex-post quantities remain ex-post, and we do not know what generated those decisions of buying and selling, nor the mechanism that led to the observed price p. It is enough that we know the snap-shot data as qualified by the SAI.

The SAI sets a composition rule on the total amounts of commodities being exchanged. The change in coordinates must preserve this property: (ex-post) demand and supply must be equal also in physical terms, as they are by construction in value terms. We can check whether, with the new representation of A, this still holds true, otherwise we have to reject it.

This is easily done by exploiting again the fact that Dp is a positive (diagonal) matrix. From (7), multiplying to the left by $Dp^{-1}$, and remembering the definition of A, we obtain

(8) $A\,q = q$

or

(8') $(I-A)q = 0$

which says that equality of demand and supply holds for each commodity. The composition law on which the SAI has been constructed is satisfied also by the new representation of the system. (By the way, it also says that in the matrix (I-A) at least one row or column is linearly dependent on all others, something which will turn up in later discussion). Again, the relation (8') too is true in virtue of its nature of accounting identity.

Let us now turn to the cost or production side of our industries accounts. (This means we start from relations (2)). We have seen that in general row-sums and corresponding column-sums need not to be equal in the snap-shot. This means that, by applying the above transformation (using actual prices and quantities!) from the snap-shot transaction matrix Z to A, we will have, in general,

(9) $1\,Dp\,A\,Dq = s \neq r$

Substituting $r = pq$, we obtain

(10) $1\,Dp\,A\,Dq \neq 1\,Dp\,Dq$

and dividing both sides by $Dq^{-1}$, we have the fundamental inequality characterizing our snap-shot model:

(11) $p\,A \neq p$

holding at the actual prices. Unless we are prepared to assume that the left side of (11) (the cost incurred in the production of a unit of the output) and its (observed) market price are different. For some commodities it will be higher, for some others lower or equal. That snap-shot data give us equality for all is but a happy coincidence.

## § 3. From the transaction matrix to the transaction account

Our snap-shot is not normalized according to any accounting procedure. It gives a set of 'brute' data, which are only related by definitions like (1) and (2).An accounting structure requires more than this. What sort of accounting structure may we build with our data? The snap-shot records only "visible" transactions which have taken place on markets, and does not embody more information (say about what happens on production lines for they lie in the "industries" black boxes, nor about stocks). If we are coherent with the "observational assumption", we cannot have a balance sheet. The only account on the basis of the available information as embodied in (Z,p,q), is a transaction (or income) account.

Its typical procedure requires that transaction data be related in such a way that total revenue is equal to total "outlays". This procedure comes from the everyday practice of double-entry bookkeeping of merchant and productive firms. It implies that in striking the balance of the running or transaction account for each industry, data are to be so manipulated that value of sales for each industry is equal to its outlays. As explained in the previous paragraph, the only way to make them balance for our snap-shot is by adding to the outlays the difference, $r_j - s_j = \pi_j$, as realized profits (or losses).

If we now border the transaction matrix Z with the row of 'profits' (a loss being a negative profit), we obtain a new matrix, $\hat{Z}$:

$$\hat{Z} = \frac{Z}{\pi}$$

where vector $\pi$ stands for profits. Obviously, the new accounting matrix $\hat{Z}$ has now n+1 rows, as we added to the 'core' the accounting residual. If we use $\hat{Z}$, then it is true by construction that

$$(12) \qquad 1\,\hat{Z} = 1\,Dp\,Dq = r$$

in other words, that the revenue side and production side of the account balance off. If we, now, extend the vector $\hat{p} = (p_1, p_2, \ldots, p_n, 1)$, then we may transform $\hat{Z}$ first into $\hat{A} = \frac{A}{\hat{\pi}}$, where $\hat{\pi}_j = v_j/q_j$, or realized profit per unit of output, and finally we may consider the analogy to (11)

(12') $\hat{p}\,\hat{A} = p$

which is now obviously satisfied by construction.

This reminds the reader that snap-shot transaction data as represented by (Z,p,q) are not yet the transaction account of our system: this is only obtained if we border Z (observed transactions) by the accounting (residual) profits or losses and operate with the new $\hat{Z}$. Before leaving this discussion let me remark that both relation (8'), which binds our snap-shot data, and (12) are found to be satisfied for different reasons: the former because it only rewrites the SAI, the latter because, like any good accountant, we took account of profits. No assumption as to Z (nor, for that matter, on A) has been introduced: so far, we have been very general, so that, if Z reflects acts and decisions of production and of exchange as it surely does, this does not appear. Nor could it appear, given the purely accounting manipulations that we have made.

Let us call the problem of finding the row-vector $\pi$ (and therefore matrix $\hat{Z}$) an 'accounting problem'. It is obviously a specially easy problem, involving only plus and minus. However, its characteristic is in that it takes as parameters the entries of the transaction matrix: this definition can be used in a more general context. At any rate, making up the transaction account for our system requires solving an accounting problem.

§ 4. The transaction account as a starting point

The account made according to the procedure described in the previous paragraph, the transaction account of our system, is distinct from the snap-shot data (Z,p,q) and from the only relation binding them, the (matrix) Social Accounting Identity. The account, now in terms of the bordered transaction matrix $\hat{Z}$, incorporates the SAI but has more than that, for it is constructed in such a way as to comply with a balancing rule.

The transaction account shows whether in the actual (observed) situation there are realized profits and losses and, moreover, how different they are among the various industries: this could be the starting point of two sorts of analysis which, logically, focus on the last row of the account matrix, the row where profits are recorded. These two lines are classical dynamics and sequential analysis, as the analysis developed by Lindahl and Hicks in the thirties.

The Hicks-Lindahl method[3] starts by comparing the outcomes of the past production activity (realized profits) with the expected profits: it is the comparison of the (ex-post) matrix account $\hat{Z}$ with an ex-ante account. However, to do this we need to introduce time segments and distinguish past, present and future, as Hicks does, in his Temporary Equilibrium Analysis. If we introduce Time, then reading the last column of the transaction account is one of the roads leading to (one of the methods, Hicks would say, of) dynamic analysis.

If data are taken from a snap-shot of a system moving in time, time should not prevent the empiricist from accepting and developing this same approach in the multisectoral framework. It should not be Time, therefore, responsible for the fact that this has not been done, but something deeper, that one can see remembering that in the empiricist framework we are not dealing with firms nor individuals, but with wholes, 'systems' and/or 'industries'.

In fact, we have already noted that reading the transaction matrix Z along the rows or the columns is not exactly the same thing: conceptually, in the former case, the basic notion is that of commodity (while we derive industry by assumption); in the latter, the basic notion is instead that, somewhat more abstract, of 'process'. To make it more 'concrete', at least from the point of view of the empiricist, we need to think, as Hicks implicitly does, of a process taking place within a firm. To develop this sort of dynamical analysis, we have to take the viewpoint of the business man, and of his accountant supplying expected and realized profit figures. From their point of view, technology is a certain (or fairly certain) datum on which they can make calculations and take decisions (form plans on the basis of expectations). To arrive at Hicksian theory, we would, therefore, need to supplement our observations on transactions with a theory of individuals' behavior, and a number of assumptions as to their knowledge of data (particularly, technical data).

The empiricist would obviously be skeptical about the possibility of deriving firm technical information from the accounting identities. Transactions are certain because they are market data; the technological transformation laws are there, and can be ascertained only indirectly. Technology is buried in black boxes, for the national accountant, and he has to either give up all hopes of knowing it, or just make more or less informed guesses (better so, if they are supplemented by 'experimental outcomes'). For this reason, maximizing behavior and technological data are never really formalized in empiricist models, and this gives them their deterministic flavour. Individuals' behavior is behind the scene, just like in the classical macroeconomics without micro-foundations.

§ 5. Empiricist use of the transaction account

The Hicks and Lindahl example illustrates the distinction between the Accounting Method (or approach), generally, and the use of it that is being made in the empiricist version of the multisectoral model. Both start with a transaction matrix, but Hicks-Lindahl theory requires really a fully developed business accounting structure (taken in both the ex-ante and the ex-post meaning), while empiricism, even when the (ex-post) structure appears to be used, uses it with qualifications. While the Accounting Approach can be conjugated with various theoretical structures (including those supplied by the microeconomic viewpoint), empiricists go accounting because they cannot accept but that method: for them, the revenue side is the 'primal', while the production side plays the role of a dual. The counterpart of this view is that it is hard to see how a (formal) dynamical theory could be developed.

This is not fortuitous: it is static theory that usually starts from the SAI. As a theory, opposed to observations, it normally tries to interpret the (ex-post) equalities between demand and supply as intersections of demand and supply functions. (This means that the observed point is interpreted as an equilibrium). In order to make this operational, we need to specify an 'interpreting structure' expressed in functional form (say, the curves going through the observed point), and a high degree of arbitrariness is involved in this specification. Conceptually, the interpreting structure should be conceived as lying in a different space from that of the observations, and tangent, so to say, to the latter at the observed point. For easy reference, I will call the (functional form of) the interpreting structure, the 'generating structure' for the observations on the system state condensed by the (Z,p,q). This is an ad hoc model constructed so as to fit the observations. (The Hicks-Lindahl procedure would require one such structure, to account for dynamical change).

With the aid of the above definition, empiricism in the multisectoral literature may be distinguished according as to: i) whether a generating structure is specified, and ii) whether the SAI are restricted to reflect certain states (and not just <u>any state</u>) in the system evolution as it is seen or perceived. To simplify the exposition, a table is provided to which one could refer for synthesis.

| | SRAFFA | GOODWIN(1951) | LEONTIEF | PASINETTI |
|---|---|---|---|---|
| SNAP-SHOT | self-reproducing state | Say's law of 2nd type | anyone | Say's law of 2nd type |
| MATRIX | Z | $\begin{pmatrix}\hat{Z}\\ \pi\end{pmatrix}=\begin{pmatrix}Z\\ 0\end{pmatrix}$ | $\begin{pmatrix}\hat{Z}\\ \pi\end{pmatrix}$ | $\begin{pmatrix}\hat{Z}\\ \pi\end{pmatrix}=\begin{pmatrix}Z\\ 0\end{pmatrix}$ |
| THEORETICAL MATRIX | Z | Z | Z | Z |
| STRUCTURE | non-linear | non-linear | linearized | linear |
| DEVELOPMENT | partial accounting (v is unknown because the self-reproducing state parameters are unknown) | partial dynamics (gradient dynamics) | stationary equilibrium | 'solution' |
| THEORETICAL MEANING | thought experiment | thought experiment | consistency analysis | justification |
| DATA | snap-shot | (Z,p,q) | AQ = Z | $A = P^{-1} Z Q^{-1}$ |

Row 3 ('theoretical matrix') shows that all versions share as a basic ingredient the transaction matrix (expressed in physical terms). This is obtained either by simply discarding the row of profits and losses, or by introducing assumptions (about the observed state) to the effect that profits and losses are all zero for the snap-shot. The purpose is to isolate for theoretical purposes the core from the rest of the accounting matrix. We are now in a position to appreciate why empiricism cannot possibly develop dynamical analysis either of the Hicksian or even of the classical type: it is because they restrict themselves to considering the operating on the core. While Hicks's and the classics' analysis would start with accounting for real, random, snap-shot data, the modern empiricist solves accounting problems for the core data only.

In our construction, the core represents a pure production system: as this is closed by assumption, all commodities are to be produced and exchanged between the parties of the system itself. This means that there will be as many industries as there are services (including services of labour), which may look like a bit artificial but it is just what modern National Accounting does. Anyway, this apparent artificiality is justified by the fact that we want to deal with various alternative interpretations ('formal models') from the vantage point of a unified framework, and all the models we will consider are of the closed type.

The focus on the core typical of the empiricist approach is justified by the belief that it reflects certain basic properties of the observed system. We will see later that the structure of the core is supposed to convey indirect information about the 'generating structure', and this information is gathered by a procedure of 'pattern recognition'.

## § 6. The 'hard empiricist/pure accounting' approach

There are, I believe, two versions of it: namely, the one due to Sraffa, which is well known, and a version provided by Goodwin (1951). A comparison is interesting for it shows two directions of development of empiricism.

6.1 Sraffa's model is usually presented as a snap-shot model (see for instance, Roncaglia (1981)): therefore, it is useful to take up this issue, in order to illustrate the difference between the qualitative and the quantitative content of the snap-shot, which is a major point where the various empiricist versions disagree.

The snap-shot interpretation of Sraffa's model maintains that production price analysis is restricted to find out certain properties that can be associated with an observed system, where production has already taken place and output (in level and proportions) is given and fixed. I will take the example of the "extremely simple society which produces just enough to maintain itself", at the beginning of Production of Commodities. The snap-shot of this system tells us which are the commodities and which are the methods of production, and that quantities used up and quantities produced satisfy the relation

(13) $$Z^{*}1 = q$$

where the star on Z indicates that the transaction matrix is already expressed in physical units of the commodities (just like in Sraffa's example). Matrix relation (13) supplies the set of parameters to determine prices (and therefore is auxiliary) from the equation

(14) $$vZ^{*} = vDq$$

which can be transformed into the more familiar from

(15) $$vA = v$$

by setting $A = (Z^{*}Dq^{-1})$. If $(Z^{*},q)$ are given, (14) is an accounting problem.

However, while (15) is clearly a (matrix) equation, the status of (13) has not yet been made clear, we just called it 'relation' defining the self-replacing state, and does look pretty much like an identity (derived by manipulating the social accounting identities). It is an (ex-post) identity only if the observed system is already in self-replacing state, but in Sraffa this is not assumed to be necessarily the case.[4)] In fact, we are warned in an important, and normally, overlooked footnote (1960, p.5) that, even if the system is not in a self-replacing state, it normally can be brought notionally into such a state by reproportioning: only after reproportioning are we allowed to use an amended balance flow relation like (13), for the price system (15). This states clearly that the snap-shot interpretation is not entirely correct, actually it is correct only by fluke.

However, this raises a great problem.Starting from an observed transaction matrix expressed in physical units, the operation of reproportioning to obtain a self-replacing 'virtual' state is itself solving an algebraic equation. Only after we have solved it, can quantities be taken as given, on one side, and parameters numerically determined for our price equation. Thus, there would be two equation systems, not just one.

There is here the typical theoretical trick of assuming that the 'quantity' equation has already been solved, so that parameters are at hand. The feasibility of the whole accounting exercise is thus based upon the qualification: let us assume that we have already solved the quantity equation.[5)]

The accounting nature of Sraffa's price problem comes from this accountant hypothesis, that parameters have been made certain so that the exercise can be performed. In this way, the hard empiricist view practices thought experiment, a thought experiment with an accountant viewpoint.

The snap-shot is therefore only used to identify the qualitative features of the system, nor could it do more: no self-replacing solution to the "quantity equation" could be interpreted as an alternative state of the system, for no technology (no assumption about returns) is specified. If we identify explanation with the 'generating structure', then there is no explanation of why certain data could come about. The quantity relation is central in Sraffa's version of empiricism: it is a pure accounting procedure for a purely theoretical state qualitatively identified but otherwise undetermined.

6.2 Sraffa offers a pure accounting problem, which is normally solvable, coupled with a consistency equation (the quantity equation) which is not solvable without further assumptions. This directs our attention to the 'necessity' of specifying a 'generating structure' to make the functional form of the equation precise. However, the meaning of Sraffa's Gedankenexperiment is fairly clear: solving an accounting problem related with a set of parameters coming from the 'isolated' core of the snap-shot. How this procedure is done is not clear and this is the problem.

There is an alternative to this approach and it is partial dynamics as proposed by Goodwin (1951). Partial dynamics is virtual dynamics experimentally obtained; on the other hand, it is "core" dynamics for it reflects only the properties of the core as could be seen 'in vitro'. This is the right perspective to evaluate it in comparison with classical dynamics and Hicks-Lindahl dynamical approach, which rely either on leaks of the closed system or on behavioral assumptions. It is the only dynamics that can be obtained from the closed system conceived as a mechanical body.

The basic ingredient is again the snap-shot and its transaction account. However, to isolate the core and its own properties from other phenomena (associated, say, with discrepancies between rates of profits), it is assumed that both profits and losses in the observed state are zero. Thus, as we have anticipated, $\hat{Z}$ and Z formally are the same matrix. This is an obvious restriction on the snap-shot, and qualifies the whole following analysis as a pure intellectual experiment, just in the same spirit of Sraffa.

The static part of the pure core model is easily dealt with: it should suffice to recall that we have dealt with it in paragraphs 2-3. We may rewrite the balance relation and the price/cost relations in terms of matrix $A = Dp^{-1} Z \, Dq^{-1}$, obtained from the observed transaction matrix, and there will appear equality of demand and supply of all goods and prices cover costs of production without profits. In fact, matrix A is really introduced for dynamical purposes. By simple manipulations, it can be shown that from A we may obtain

(18) $$Q = Dp \, A \, Dp^{-1}$$

and

(19) $$G = Dq^{-1} A \, Dq$$

so that, if A is a linear operator, so are both Q and G, and they are similar to one another. The two matrices, G and Q, supply two linear operators for two different experiments.[6)]

If prices do not change, matrix Q is a matrix of constants if A can be assumed not to change when quantities do not change, that is under constant returns to scale assumption. This assumption can thus be used to derive a partial dynamics of output, in the experimental ideal conditions represented by keeping prices fixed. This does not imply that, in the real evolution of output, both prices and quantities do not adjust, for the assumption (implying a restriction on the 'generating structure' behind A) is only instrumental to isolate lab's dynamical behaviors, just like the vacuum assumption in certain physical statements. A similar reasoning applies to the use of matrix G, with the roles for quantities and prices interchanged, and need not be repeated.

The experimental nature of the above exercises is important: we have two virtual core dynamics which are not put together, for the above procedure cannot provide a dual simultaneous dynamics of both quantities and prices. It is a different way of producing dynamics from 'objective properties' of the network as representing a body endowed with a homeostatic property.

## § 7. Soft empiricism: specifying the 'generating structure'

The difference of modern empiricist accounting approach and accounting approach as used by the old empiricists (the classics, in a sort of loose sense) is in that quantitative data may be drawn from observations but, conceptionally, the theoretical model is an auxiliary system: it is to this system that state (quantity) equations and the 'dual' accounting problem are referred. Moreover, given the conceptual hierarchy between the problems associated with the two sets of equations, the (empiricist version of the) accounting approach rejects simultaneous determination to rely, instead, on a two-step procedure.

How to solve the state equation is thus the real issue, for once we have devised a way to solve it, the solution of the (dual) accounting problem is but a matter of routine (linear) equation solving. The problem is how to determine the parameters of the accounting equation, and this raises the question of the generating structure of the observation, because it is only after we have made this precise, that we have a state equation in functional form.

This problem was first encountered in a model where no such structure is offered (and the reader is kindly invited to work out by himself what sort of structure(s) are compatible). In Sraffa, there is a sort of hidden assumption which entails a system selection procedure, to be implemented before formalizing the quantity equation.

The same procedure is (implicitly) implemented when we introduce in the "core" the assumption of constant returns to scale (see Leontief (1951)).[7] Let us assume that from the accounting matrix $\hat{Z}$, we have obtained the matrix $\hat{A}$, whence, by notionally setting the last row (profits or losses per unit of produced output), we isolated the core matrix A. This latter is a pseudo - or ex-post technological matrix. By pattern recognition, we come to the observation that matrix A is fairly constant whatever total output is, and from this we derive the 'experimental law' according to which returns to scale are approximately constant. (Note that the law has been derived from situation(s) where instead there might be reason to expect a dynamical change). However, pattern recognition is the justification provided by Leontief for the assumption of constant returns to scale (in the form of fixed coefficients) qualifying the core matrix A.

Whenever an assumption is introduced about the 'generating structure' and/or the observed state, a proof of consistency has to be supplied. This is what Leontief encounters after specifying A as a linear operator: the assumption makes the core self-sufficient and independent from the observed state. The core now coincides with the closed Leontief model, to which we attach the problem of finding a (stationary equilibrium) solution. This takes the form of two sets of equations in x (gross output levels) and in v (accounting prices). That is,

(20) $(I - A)x = 0$

(21) $v(I - A) = 0$

where 0 stands for the zero (row or column) vector. It must be stressed that (20) and (21) form a set of true equations in the true unknowns: their 'ability' to provide economically meaningful solutions must be shown explicitly, and the assumption that matrix A is empirically obtained is obviously not sufficient to ensure this result. Moreover, and this carries a certain weight in our reconstruction of empiricism, there is no assurance that, even if economically meaningful, the solutions will be such that commodities in 'the observed world' so to say, will remain commodities in the auxiliary (theoretical) system, and will still be produced. Actually we can expect that this in principle will happen, given the autonomy of the latter system. This problem arises from the fact that the assumption on constant returns to scale has introduced an ambiguity into the model: we now use, as primitive terms, commodities and industries and, at the same time, goods and processes (activities). The model has undergone a metamorphosis: it now looks like a close relative of Activity Analysis, and this has been the source of misunderstanding.[8)]

§ 8. Data and solving

The constant returns to scale assumption has a great defect: it tends to 'freeze' the technology and make it a very rigid, almost mechanical structure. This was not so in the observed system, it was true only in an experimental sense: output responses are on average proportional to inputs.

This attitude shows up in Leontief's proof that the closed system does have a solution. If we follow him, something can be said about the meaning of data and of proof as it comes out from the empiricist viewpoint.

This looks like a problem requiring an existence proof. As (20) and (21) are simple linear equations, the proof can follow two procedures: (i) a constructive approach, by which it is shown how the required solution (x,v) can be constructed (computationally obtained): this would be the method of Sraffa; (ii) otherwise, one could discuss whether there is any reason to expect that the mathematical condition for the existence of non trivial solutions (namely, the condition det (I - A) = 0 will be satisfied. With this latter procedure, we then have also to make sure that the mathematical solutions are also economically meaningful, so that it requires an extra step. This is the procedure adopted by Leontief.

It is obvious that even the first stage of the procedure is difficult, for the empiricity of matrix A does not imply that the det (I - A) will be zero. It may or may not be zero. To ensure that "in general" the condition is satisfied the last column which represents consumption coefficients is allowed to adjust so as to make it satisfied. However, it is not the technical aspect that interests us here, but the justification: the manipulation of the coefficients is justified because: "No economic system could possibly exist in which all the technical and consumption coefficients were independent of one another". So, technological and consumption coefficients are not really data in the mathematical sense, but reflect properties of empirical systems. To contrast it with formal proof, I have called this procedure 'implicit solving' and is typical of the empiricist viewpoint.

The overall unorthodox nature of this procedure is in that, while the process of proving goes on, assumptions are freely thrown in, so as to make it impossible to separate the set of data (in the mathematical sense!) from the features the model is expected to show: the deductive process is goal-oriented, in the sense that the formal model is constrained to show what the theorist justifies on the basis of common sense and intuition.[9)]

§ 9. Some conclusions: Comparing with an equilibrium approach

I maintain that with the qualifications he introduces during the process of proving, Leontief's proof is correct. But, what has he proved, after all? Technological data represent the prevailing conditions of production: no choice of technique is allowed in that model, so that even in the (theoretically determined) stationary equilibrium, the same technique must be used, and the (equilibrium) values of both quantity levels and prices are simply an alternative position in which the production system could be set. Equilibrium has a purely notional value, it really represents consistency and should be better named accordingly: say, economically "feasible" solution. We may contrast this result with the sketch of a proper equilibrium approach, along formalistic lines.

It is assumed that we know the blue-print book: our information is not just about the prevailing technique but about all available techniques, to produce a given bundle of goods, which are assumed to be n. Thus we have different primitive objects (processes and goods!) and options. Sets of n processes are assembled to form techniques (the assumption of no joint production sets this threshold number as a feasibility requirement). Finally, to make a closed model each technique is bordered with the corresponding vector of labour inputs and the (unique) vector of consumption goods per worker: this yields a set of feasible 'closed Leontief models' to choose from, and the problem is determinate only if we introduce a criterion: this is accomplished by stating that the only technique(s) entitled to represent equilibrium techniques are those that maximize the rate of capital accumulation above the consumption requirements per worker. To such techniques a set of competitive prices can be associated.

We see that in this way, by the method of activity analysis, one does obtain proper equilibrium solutions and Leontief's core has to be conceived as an equilibrium solution: but, and this is the point, they need not coincide.

To see this, perform the following mental exercise: assume that the maximum rate of growth allowed by the blue-print techniques is less than zero. In von Neumann's framework this makes good economic sense, though he would probably consent that the system would prove to be empirically irrelevant. The check on its empiricity comes after the system has been treated in an abstract way: it is a check on the relevance of the results and not of their logical stringency. Leontief's answer would be that this result makes no sense, for technology and consumption needs would adjust so as to make a stationary state feasible. No observable economic system is so bad as not to allow that rate. On the other hand, assume that the maximum rate is positive. Then, in Leontief, consumption would rise; in von Neumann, instead, the system (constrained in a stationary equlibrium) would only show a zero price vector, for there would be excess supply of all commodities.

We may now state a sort of temptative conclusion. Our previous exercise shows the main point of this paper: that is, that the empiricist and the Activity Analysis approach do represent two different mental experiments. They are based on two entirely different notions of systems, and accordingly, address the problem of identifying their generic properties in a totally different way. The Activity Analysis formalistic viewpoint treats the system as an abstract entity mechanistically made up of a number of qualitatively different and moreover independent building blocks. Only in equilibrium, the "meaning" or "empirical content" of both, the systems and its building blocks, can be ascertained, for only there coherence among the various individual parts emerges. The possibility of an equilibrium is thus the internal pre-requisite for the interpretation of the formalistic model. This motivation leads to focus <u>only</u> on limit behaviors or states (as equilibria are to be understood). Within this approach, therefore, the generic property of any meaningful system is that it allows for such limit behaviors: all other behaviors, including actual ones, appear as (transient) deviations.

The empiricists appeal to properties of systems somehow already empirically given. Obedience to intuitively perceived goals or norms constitutes the generic property identifying the class of systems they work with. While equilibrium is associated with some notion of optimality, goal or norms reflect minimal properties of feasibility (say, self-perpetuation). The empiricists formal model carve out of the over-all description of a given system that sub-set (the 'core') which is thought to be endowed with the system basic mechanism or driving force.

## FOOTNOTES

1. Lakatos (1978) pp.4-6

2. Leontief (1966) p.15

3. Hicks (1939), Hicks (1956), Hicks (1965) chpt.VIII, in particular footnote p.85

4. Sraffa (1960) p.5 footnote: "This formulation presupposes the system being in a self-replacing state; but every system of the type under consideration is capable of being brought to such a state by merely changing the proportions in which the individual equation enter it (...) Systems which are incapable of doing so under any proportions and show a deficit in the production of some commodity over their consumption even if none has a surplus do not represent viable economic systems and are not considered".

5. This experimental technique is discussed at length in my forthcoming 'La matematica di Sraffa', where I argue that the experimentalist attitude is typical of modern empiricism (as opposed to the 'classics') and I refer to a discussion in Lakatos (1978).

6. See Goodwin (1951), for a derivation of a price- and a quantity dynamics under the above experimental conditions. For a discussion about how crucial the assumptions are, see Tani (1977). The derivation of matrices Q and G from the transaction matrix is a matter of routine multiplication.

7. Leontief (1941), p.37: "The very nature of our study necessitates the introduction of quite definite assumptions concerning the shape of our production functions: and at the same time, it limits considerably the freedom of theoretical choice, because the numerical values of all parameters must be ascertainable on the basis of the available statistical information".

8. Because of this confusion, I think, Pasinetti (1977) is led to identify properties of the observed state with properties of the matrix A. (For this, see the discussion in Costa (1980)).

9. Let me note that this type of proofs (I mean the correct ones) is not typical of economic reasoning only: for good examples in other areas, see Lakatos (1978), "What does a mathematical proof prove?". Judging on the basis of Lakatos' reconstruction, this sort of proofs has a long and even glorious tradition not only in Economics, where they start with the Austrians, but in Mathematics. It should not be forgotten that the present understanding of the meaning and the function of a formal proof, in Economics at least, is fairly recent, and comes from the Vienna group around K. Menger. Actually, this group started by examining the pre-formal proofs (according to Lakatos' terminology) and the implicit solving technique of Austrians and of the Walras-Cassel model. (See Menger (1974), to find a good statement of why Leontief's proof as outlined in the text would not be correct). For a treatment of implicit solving as it comes up in Sraffa, see my (1980).

## BIBLIOGRAPHY


BRODY, A. (1970). Proportions, Prices and Planning. North Holland, New York.

COSTA, G. (1980). La legge di Say e la teoria della domanda effettiva. Guiffrè editore, Milano.

GEORGESCU- ROEGEN, N. (1971). The Entropy law and the economic process. Harvard University Press, Harvard.

GOODWIN, R.M. (1951). Static and Dynamic linear general equilibrium models. In: Input Output Relations. 1953, ed. Netherlands Economic Institute; (re-printed in R.M. Goodwin: Essays in Linear Economic Structures. MacMillan, (London: 1983).

HICKS, J. (1939). Value and Capital. Oxford at the Clarendon Press.

HICKS, J. (1956). Methods of Dynamic Analysis. Paper contributed to Festschrift in honour of E. Lindahl, published as: 25 Economic Essays. Ekonomisk Tidsckrift, Lund.

HICKS, J. (1965). Capital and Growth. Oxford University Press, London.

HICKS, J. (1973). Capital and Time. Clarendon Press, Oxford.

HICKS, J. (1983). The Social Accounting in Classical Models. In: J. Hicks, Classics and Moderns. Basil Blackwell, Oxford.

LAKATOS, I. (1978). What does a mathematical proof prove? In: I. Lakatos, Mathematics, Science and Epistemology. Philosophical Papers, vol. 2, Cambridge University Press, Cambridge.

LEONTIEF, W. (1941). The structure of the American Economy, 1919-1939. Oxford University Press, New York.

LEONTIEF, W. (1966). Input Output Economics. Oxford University Press, New York.

LEONTIEF, W. (1937). Implicit theorizing: a methodological criticism of the neo-Cambridge-school. Quarterly Journal of Econmics, vol. 51, n.2B.

MENGER, K. (1936). Remarks on the Law of Diminishing Returns. A study in Metaeconomics, re-printed in K. Menger, Selected Papers in Logic and Foundations, Didactics, Economics. The Vienna Circle Collection, D. Reidel Publishing Co. (Dordrecht: 1974).

PASINETTI, L. (1977). Lectures on the theory of production. MacMillan, (London-New York).

PASINETTI, L. (1981). Structural change and economic growth. Cambridge University Press, Cambridge.

PUNZO, L. (1980). Does the Standard Commodity exist?. Quaderni dell'Istituto di Economia, Siena.

SRAFFA, P. (1960). Production of Commodities by means of Commodities, Cambridge University Press, Cambridge.

TANI, P. (1977). Alcune questioni di teoria della produzione. Università degli studi di Firenze.

ZAGHINI, E. (1967). Sur la condition de résolubilitè du modéle fermè de Leontief. In: Economie Appliquée