CALCULATION OF A PRIORI POWER DISTRIBUTIONS FOR THE UNITED NATIONS

by Gerhard Schwödiauer

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I.) Introduction

As Luce and Raiffa [3] have pointed out, one of the most interesting and fruitful applications of n-person game theory to a sociopolitical problem is the estimation of the a priori power distribution as determined by the constitution of a voting committee.
In this paper we are trying to make such an application: Our problem here is the calculation of relative voting strengths of the members of the United Nations, the voting issue being world politics security questions (see articles 24 and 39 ff. of the Charta of the United Nations).

As we want to get an a priori power index showing only the members' possibilities based on the U.N.-Charta and ignoring "real" power relations due to certain coalition formation biases, the Shapley value [1] for the game representing United Nations (as concerned with security problems) should be adequate. The power index for the i^{th} nation, denoted by ϕ_i, is then given by the formula

$$(1) \qquad \phi_i(v) = \sum \frac{(|S| - 1)!\,(n - |S|)!}{n!} ,$$

where v is the relevant game and the summation is extended over all winning coalitions S in which the i^{th} nation is pivotal. $|S|$ means the number of nations in S , n is the number of members of the United Nations.

If v is a simple game, i.e. a characteristic function that attaches 1 to each winning coalition ("majority") and 0 to each losing one ("minority"), the Shapley value is of course automatically normalized between 0 (representing total absence of power) and 1 (representing absolute power) and gives the probability for each single nation to be a pivotal player, i.e. to be essential in transforming a losing coalition into a winning one, when all the possibilities of forming such a winning coalition are equally probable (this is the crucial - and very strong - assumption of aprioriness, which underlies the application of this concept to evaluating power distributions and could perhaps be modified).

In this paper we shall pose three questions and shall try to answer them in a quantitative manner:

1.) How does the power distribution in the United Nations look like nowadays?
2.) Did the 1963 amendment of the Charta (effective since Aug. 31, 1965) involve a change in power distribution or was it only a ficticious modification?
3.) What effect on the power distribution would the abolition of the veto of the "Big Five" in the Security Council have?

II.) The Model

As any voting committee the United Nations can be described as a simple game $G_{U.N.} = (N, \mathcal{W})$, i.e. an ordered pair, where N denotes the set of players (= nations) and $\mathcal{W}$ stands for the set of winning coalitions (see references [2] and [4]). The set of winning coalitions is implicitly given by the characteristic function v of the game in the above mentioned way.

In our model United Nations dealing with security problems consist of two bodies: U.N. Security Council, represented by the game $G_{S.C.} = (M, \mathcal{W}_1)$ and the characteristic function u , respectively, and U.N. General Assembly, represented by the game $G_{G.A.} = (N, \mathcal{W}_2)$ with characteristic function w.

The set of members N is in general made up of four groups of players: The Soviet Union, which disposes of a veto in

the Security Council and of three votes in the General Assembly (together with the Ukraine and White-Russia), is denoted by capital letter A; the other (privileged) permanent members of the Security Council, denoted by b, c, d and e; the non-permanent members (with no veto) of the Security Council, denoted by Roman numbers I, II, III, ...; and the common members of the United Nations, denoted by 1, 2, ...
Within these groups there is full symmetry.
Today the prominent feature of the Security Council is the vetoes of the "Big Five". Since the amendment of 1963 there must be nine affirmative votes always including those of the permanent members to pass a substantive resolution in the Security Council, where the set of all members is given by

$$M = \{A, b, c, d, e, I, II, \dots, X\} .$$

This voting system can be described by the characteristic function u, such that

$$
\begin{aligned}
& u(i) = 0, \qquad \text{for all } i \in M, \\
& \vdots \\
& u(A, b, c, d, e, I, II, III) = 0, \\
& \vdots \\
(2) \qquad & u(A, b, c, d, I, II, \dots, X) = 0, \\
& \vdots \\
& u(A, b, c, d, e, I, II, III, IV) = 1, \\
& \vdots \\
& u(M) = 1.
\end{aligned}
$$

For the same voting system a more elegant <u>weighted majority representation</u> (see [4]), which enables us to apply certain efficient computational techniques in calculating the Shapley value (see references [6] and [7]), can be given:

(3)
$$G_{S.C.} = [39; 7, 7, 7, 7, 7, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1],$$

where 39 stands for the quota, and 7, 1, respectively, are the weights of the permanent and non-permanent members.

The Security Council voting system corresponding to the Charta before the 1963 amendment, which may be described by a characteristic function u', can also be easily represented by a weighted majority game:

(4) $$G'_{S.C.} = [27; 5, 5, 5, 5, 5, 1, 1, 1, 1, 1, 1],$$

the set of members is now given by

$$M' = \{A, b, c, d, e, I, II, III, IV, V, VI\} .$$

If we assume the vetoes of the "Big Five" to be abolished, the set of members being now

$$M'' = \{A, I, II, \ldots, XIV\},$$

the Security Council voting scheme $G''_{S.C.}$, described by a characteristic function u'', will reduce to a majority game with 15 players and a quota of 9.

In the General Assembly the situation is much simpler: A majority can always be reached by two thirds of the number of members n (currently n = 122), the Soviet Union disposing of three votes, however. The present General Assembly voting system, where the set of members is given by

$$N = \{A, b, \ldots, e, I, \ldots, X, 1, \ldots, 107\},$$

can be described by the characteristic function w, such that

$$
\begin{aligned}
&w(i) = 0, \qquad \text{for all } i \in N,\\
&\vdots\\
&w(A, 1, 2, \ldots, 80) = 1,\\
&\vdots\\
(5)\quad &w(b, 1, 2, \ldots, 82) = 1,\\
&\vdots\\
&w(I, 1, 2, \ldots, 82) = 1,\\
&\vdots\\
&w(1, 2, \ldots, 83) = 1,\\
&\vdots\\
&w(N) = 1.
\end{aligned}
$$

For this voting scheme again a weighted majority representation is possible:

$$(6) \qquad G_{G.A.} = [83; 3, 1, 1, \ldots, 1],$$

where 83 is the quota, and 3, 1 are the weights of the Soviet Union and the other members of the United Nations, respectively. For the games $G'_{G.A.}$ (Charta before 1963) and $G''_{G.A.}$ (without vetoes) analogous weighted majority representations can be given.

It is not very difficult to calculate a priori power indices for the above varieties of voting systems. Even for very large weighted majority games (but only for those!) we have both approximative and exact methods of evaluation which will deliver results within a reasonable span of time. (See references [6] and [7].) But as in this paper our aim is to evaluate a <u>complex voting system</u> (called United Nations), we have to remember the subtle interrelationship between its two <u>subsystems</u> (called Security Council and General Assembly): Article 12 of the Charta, confirmed and strengthened by the so-called "Uniting for Peace Resolution" (1950), tells us that as long as an agreement on peace-keeping measures is attained in the Security Council the General Assembly is not concerned with such problems. But if (and only if) U.N. Security Council is blocked - what is now possible by at least one permanent or seven non-permanent members - the General Assembly is entitled to pass the respective resolution and/or to decide on certain peace-keeping actions by a two-thirds majority. This complex voting system $G_{U.N.}$ can be formally described by the characteristic function v in the following way:

$$\begin{aligned}
&v(i) = 0, \qquad \text{for all } i \in N,\\
&\vdots\\
&v(A, b, c, d, e, I, II, III) = 0,\\
&\vdots\\
&v(A, b, c, d, I, II, \ldots, X) = 0,\\
&\vdots\\
&v(A, b, c, d, e, I, II, III, IV) = 1,\\
&\vdots\\
(7) \qquad &v(1, 2, \ldots, 107) = 0,\\
&\vdots\\
&v(A, 1, 2, \ldots, 80) = 1,\\
&v(b, 1, 2, \ldots, 82) = 1,\\
&\vdots\\
&v(I, II, \ldots, VII, 1, 2, \ldots, 76) = 1,\\
&\vdots\\
&v(N) = 1.
\end{aligned}$$

The Charta before 1963 taken as a basis, the characteristic function v' that corresponds to $G'_{U.N.}$ looks as follows:

$$\begin{aligned}
&v'(i) = 0, \qquad \text{for all } i \in N',\\
&\vdots\\
(8) \qquad &v'(A, b, c, d, e, I) = 0,\\
&\vdots
\end{aligned}$$

$$v'(A, b, c, d, I, II, \ldots, VI) = 0,$$
$\cdot$
$\cdot$
$$v'(A, b, c, d, e, I, II) = 1,$$
$\cdot$
$\cdot$
$$v'(1, 2, \ldots, 111) = 0,$$
$\cdot$
$\cdot$
$$v'(A, 1, 2, \ldots, 80) = 1,$$
$$v'(b, 1, 2, \ldots, 82) = 1,$$
$\cdot$
$\cdot$
$$v'(I, II, \ldots, V, 1, 2, \ldots, 78) = 1,$$
$\cdot$
$\cdot$
$$v'(N') = 1.$$

If no member of the Security Council had a veto, we should get the characteristic function v'' describing the game $G''_{U.N.}$:

(9)

$$v''(i) = 0, \qquad \text{for all} \quad i \in N''$$
$\cdot$
$\cdot$
$$v''(A, I, \ldots, VII) = 0,$$
$$v''(I, II, \ldots, VIII) = 0,$$
$\cdot$
$\cdot$
$$v''(A, I, \ldots, VIII) = 1,$$
$$v''(I, II, \ldots, IX) = 1,$$
$\cdot$
$\cdot$
$$v''(1, 2, \ldots, 107) = 0,$$
$\cdot$
$\cdot$
$$v''(A, I, \ldots, VI, 1, 2, \ldots, 74) = 1,$$
$$v''(I, II, \ldots, VII, 1, 2, \ldots, 76) = 1,$$
$\cdot$
$\cdot$
$$v''(N'') = 1.$$

Obviously the voting schemes $G_{U.N.}$, $G'_{U.N.}$, and $G''_{U.N.}$ cannot be regarded as <u>compositions</u> (products or sums) of $G_{S.C.}$ and $G_{G.A.}$, $G'_{S.C.}$ and $G'_{G.A.}$, $G''_{S.C.}$ and $G''_{G.A.}$, respectively, because $M \subset N$, $M' \subset N'$, and $M'' \subset N''$. (See reference [4].)

But the most prominent and most unwelcome feature of the above functions v, v', and v'' is the impossibility of finding any weighted majority representation. This unpleasant property suggests the following question: Is it possible to construct the simple game v as the superposition of the characteristic functions u and w, such that for each $S \subset N = M \cup N$

$$v(S) = u(S) + w(S),$$

where $u(S) = u(M \cap S)$ and $w(S) = w(N \cap S)$? In this case the important property of the Shapley value that

$$\phi_i(v) = \phi_i(u + w) = \phi_i(u) + \phi_i(w)$$

would tremendously facilitate our computational problems. We should only have to add $\phi_i(u)$ and $\phi_i(w)$, which, as already mentioned, could be received very easily, in order to get our a priori power index $\phi_i(v)$ for the United Nations as a whole (see references [2], [3], and [1]).

A short examination will convince us, however, that the superposed functions u + w are not reflecting the structure of interaction between Security Council and General Assembly described above, but are treating $G_{S.C.}$ and $G_{G.A.}$ as if they were played as two separate games.

As it is neither possible to find any weighted majority representations for the functions v, v', and v'' nor even to construct them as superpositions of finite numbers of weighted majority characteristic functions, we have to resort to basic combinatorics when computing the Shapley value. Though this method is not very pleasant, it is nevertheless feasible owing to the "partial symmetry" in the set of players.

III.) The Results

Our first question had asked for the present power distribution in the United Nations. Now we are in a position to give the promised quantitative answer. The Shapley value $\phi(v)$ for the game $G_{U.N.}$ delivers the following a priori power index:

$$\phi_A(v) = 0{,}044409 = 4{,}4409\ \%\ ,$$

(10)
$$\phi_{b,c,d,e}(v) = 0{,}019885 = 1{,}9885\ \%\ ,$$

$$\phi_{I,\dots,X}(v) = 0{,}016439 = 1{,}6439\ \%\ ,$$

$$\phi_{1,\dots,107}(v) = 0{,}006651 = 0{,}6651\ \%\ .$$

This index means that the probability for the Soviet Union to be a pivotal player is 0,044409, we can also say that the "power-share" of the Soviet Union amounts to 4,4409 %. In this sense the power-shares of the other permanent members and of the non-permanent members of the Security Council are 1,9885 % and 1,6439 %, respectively. A common member of the United Nations has therefore only 0,6651 % of the "whole power" in his hand.

But these figures alone do not say too much. The strong point of this analysis lies in its comparative use. It offers a good method for measuring exactly the effect of a revision of a certain voting system. Let us then look at the effect of the 1963 amendment of the Charta. The power index $\phi(v')$ for the game $G'_{U.N.}$ shows the following picture:

$$\phi_A(v') = 0{,}045036 = 4{,}5036\ \%,$$

(11)
$$\phi_{b,c,d,e}(v') = 0{,}020512 = 2{,}0512\ \%,$$

$$\phi_{I,\dots,VI}(v') = 0{,}014691 = 1{,}4691\ \%,$$

$$\phi_{1,\dots,111}(v') = 0{,}007070 = 0{,}7070\ \%.$$

As expected the 1963 amendment has reduced the power of the "Big Five": the power of the Soviet Union by 1,39% and the power of the other permanent members by 3,o5%, whereas it increased the power of the non-permanent members of the Security Council by 11,9 % and lowered that of the common members of the United Nations by 5,92%. On the whole the importance of the Security Council increased by 9,32%. But this comes as no great surprise to us - we had to suspect that the enlargement of an oligarchic group like U.N. Security Council, which moreover makes the process of majority formation within this group more flexible, should have such an effect.

More amazing is the effect of an abolition of the vetoes in the Security Council. Let us take a look at the Shapley value $\phi(v'')$ of the corresponding game $G''_{U.N.}$:

$$\phi_A(v'') = 0{,}061129 = 6{,}1129\ \%,$$

$$\phi_{I,\dots,XIV}(v'') = 0{,}056614 = 5{,}6614\ \%, \tag{12}$$

$$\phi_{1,\dots,107}(v'') = 0{,}001363 = 0{,}1363\ \%.$$

It is certainly not very surprising that abolishing the vetoes of the permanent members of the Security Council would raise the power of the non-permanent ones, but this increase is tremendous, it amounts to 244,4 % (the present Charta taken as a basis of reference). But the - at least at the first glance - unexpected effect of such an modification of the U.N. Charta would be that even the power of the Soviet Union and the other permanent members would be increased by 37,65 % and 184,71 %, respectively, whereas the power of the common members would be reduced by 79,57 %.

We can say therefore that the common members of the United Nations should have an interest in the vetoes of the "Big Five" - at least as long as there exists a body like Security Council!

But also this astonishing result can be given a very natural and plausible interpretation: By abolishing the vetoes the number of possible blocking coalitions is reduced drastically - this will raise the power of even those, who are deprived of their privileges. It seems that equality within an oligarchic group maximizes the oligarchs' power.

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