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FUZZY P-SPACES GAMES AND METACOMPACTNESS

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Abstract. Fuzzy P-spaces are introduced and a characterization for the same in terms of a particular type of fuzzy topological game is obtained. Further some applications of fuzzy P-spaces in product α-metacompact spaces are also investigated.

1. Introduction

The concept of P-spaces was introduced by K. Morita and a characterisation for the same was given by Telgarsky in [10]. Just like the applications of P-spaces in general topology, fuzzy P-spaces help the study of covering properties in fuzzy topological spaces. In [8] and [9] the author introduced metacompactness for $[0, 1]$ and L-Fuzzy Topological Spaces respectively and in [8] it is shown that the product of two α-metacompact spaces need not be α-metacompact. But if we impose some conditions on one of these spaces, we can make the product α-metacompact. This is done in terms of fuzzy topological games and fuzzy P-spaces and this was the main motivation behind the study of fuzzy P-spaces. For this reason, we generalize the concept of P-spaces to fuzzy topological spaces (fts) and a characterization for the same in terms of some particular kind of fuzzy topological game is obtained. Some basic definitions and results regarding fuzzy topological games are given in [6] by the author.

2. Basic definitions and results

In this section we collect the basic definitions and results regarding metacompact spaces, fuzzy topological games and fuzzy P-spaces.

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Definition 2.1. [1] *Let* (X,T) *be a fts and* $\alpha \in [0,1)$. *A collection* $\mathbf{U}$ *of fuzzy sets is called an* α*-shading (resp. (*α^**-shading) of* X *if for each* $x \in X$, *there exists* $g \in \mathbf{U}$ *with* $g(x) > \alpha$ *(resp.* $g(x) \geq \alpha$*).*

Definition 2.2. [3] *A family* $\{a_s : s \in S\}$ *of fuzzy sets in a fts* (X,T) *is said to be point finite if for each* x *in* X, $a_s(x) = 0$ *for all but at most finitely many* s *in* S *(or equivalently as* $a_s(x) > 0$ *for at most finitely many* s *in* S*). Where* S *is an indexing set.*

Definition 2.3. [3] *Let* (X,T) *be a fts and* $\alpha \in [0,1)$. *Let* $\mathbf{U}$ *and* $\mathbf{V}$ *be any two* α*-shadings (resp.* α^**-shading) of* X. *Then* $\mathbf{U}$ *is a refinement of* $\mathbf{V}(\mathbf{U} < \mathbf{V})$ *if for each* $g \in \mathbf{U}$ *there is an* $h \in \mathbf{V}$ *such that* $g \leq h$. *Also a refinement* $\{b_t : t \in T\}$ *of* $\{a_s : s \in S\}$ *is said to be precise if* $T = S$ *and* $a_s \leq b_s$ *for each* $s \in S$. *Where* S *and* T *are indexing sets.*

Definition 2.4. [1] *A fuzzy topological space* (X,T) *is* α*-compact (resp. countably* α*-compact) if every* α*-shading of* X *by open fuzzy sets has a finite (resp. countable)* α*-sub shading.*

Definition 2.5. [8] *A fuzzy topological space* (X,T) *is said to be* α *-metacompact if each* α*-shading (resp. (*α^**- shading) of* X *by open fuzzy sets has a point finite* α*-shading refinement by open fuzzy sets.*

Definition 2.6. [2] *A fuzzy topological space* (X,T) *is said to be fuzzy regular if and only if for every fuzzy point* p *in* X, *and for every open fuzzy set* U *containing* p, *there exists an open fuzzy set* W *such that* $p \leq W \leq cl\, W \leq U$. *Where* $p \leq W$ *means that* $p(x) \leq W(x)$, x *being the support of the fuzzy point* p.

Definition 2.7. [12] *Let* $\{X_i\}_{i \in I}$ *be a family of fuzzy topological spaces. Let* $\mathbf{X} = \prod_{i \in I} X_i$ *be the usual Cartesian product and let* P_i *be the projection from* X *on to* X_i *for each* $i \in I$. *The set* X *with fuzzy topology having the family* $F = \{P_i^{-1}(B) : B \in T_i, i \in I\}$ *as a subbase is called the product fuzzy topological space.*

Definition 2.8. *Let* $X \times Y$ *be a fuzzy product space. A subset of the form* $R = R_1 \times R_2$ *where* R_1 *and* R_2 *are projections of* R *in to* X *and* Y *respectively is called a fuzzy rectangle in* $X \times Y$.

As a generalization of Topological Game $G(\mathbf{K}, X)$ introduced by Telgarsky [10], the author [6] introduced the Fuzzy Topological Game $G'(\mathbf{K}, X)$ in the following way.

Definition 2.9. [6] *Let* K *be a non empty family of fuzzy topological spaces, where all spaces are assumed to be* T_1 *(fuzzy singletons are fuzzy closed).* $\underline{I^x}$ *denote the family of all fuzzy closed subsets of* X. *Also* $X \in \mathbf{K}$ *implies* $\underline{I_x} \subseteq \mathbf{K}$. *Let* $X \in \mathbf{K}$. *Then the fuzzy topological game* $G'(\mathbf{K}, X)$ *is* *defined as follows. There are two players Player I and Player II. They alternatively choose consecutive terms of the sequence* $(E_1, F_1, E_2, F_2, \ldots)$ *of fuzzy subsets of* X*. When each player chooses his term he knows* $\mathbf{K}$*,* X *and their previous choices. A sequence* $(E_1, F_1, E_2, F_2, \cdots)$ *is a play for* $G'(\mathbf{K}, X)$ *if it satisfies the following conditions for each* $n \geq 1$*.*

1. E_n *is a choice of Player I*
2. F_n *is a choice of Player II*
3. $E_n \in \underline{I_x} \cap \mathbf{K}$
4. $F_n \in \underline{I_x}$
5. $E_n \vee \overline{F_n} < F_{n-1}$ *where* $F_0 = X$
6. $E_n \wedge F_n = 0$

Player I wins the play if $\inf_{n \geq 1} F_n = 0$*. Otherwise Player II wins the Game. A finite sequence* $(E_1, F_1, E_2, F_2, \ldots, E_m, F_m)$ *is admissible if it satisfies conditions (1) – (6) for each* $n \leq m$*.*

Definition 2.10. *Let* S' *be a crisp function defined as follows*

$$S' : \cup(\underline{I_x})^n \ \underrightarrow{into} \ \underline{I^x} \cap \mathbf{K} \quad n > 1 \tag{2.1}$$

Let $S_1 = \{X\}$*,* $S_2 = \{F \in \underline{I_x} : (S'(X), F)$ *is admissible for* $G'(\mathbf{K}, X)\}$*. Continuing like this inductively we get* $S_n = \{(F_1, F_2, F_3, \ldots, F_n) : (E_1, F_1, E_2, F_2, \ldots E_n, F_n)$ *is admissible for* $G(\mathbf{K}, X)$ *where* $F_0 = X$ *and* $E_i = S'(E_1, F_1, E_2, F_2, \ldots F_{i-1})$ *for each* $i < n\}$*. Then the restriction* S *of* S' *to* $\cup_{n>1} S_n$ *is called a fuzzy strategy for Player I in* $G'(\mathbf{K}, X)$*. If Player I wins every play* $(E_1, F_1, E_2, F_2, \ldots E_n, F_n, \ldots)$ *such that* $E_n = S(F_1, F_2, \ldots, F_{n-1})$*, then we say that* S *is a fuzzy winning strategy.*

Definition 2.11. $S : \underline{I^x} \ \underrightarrow{into} \ \underline{I^x} \cap \mathbf{K}$ *is called a fuzzy stationary strategy for Player I in* $G'(\mathbf{K}, X)$ *if* $S(F) < F$ *for each* $F \in I_x$*. We say that* S *is a fuzzy stationary winning strategy if he wins every play* $(S(X), F_1, S(F_1), F_2, \ldots)$

Definition 2.12. *A collection* $\{U_i : i = 1, 2, 3, \ldots\}$ *of fuzzy subsets of a set* X *is called an increasing family if* $U_i < U_{i+1}$ *for every* $i = 1, 2, 3, \ldots$*.*

As a generalization of P-spaces defined by K. Morita, Fuzzy P-spaces are defined as follows.

Definition 2.13. *A fts* X *is said to be a* P_α*-space if for every increasing family* $\mathbf{U} = \{U(a_1, a_2, \ldots, a_i)/a_1, a_2, \ldots, a_i \in A,\ i = 1, 2, 3, \ldots\}$ *of open fuzzy sets in* X*, there exists a precise refinement*

$$\mathbf{F} = \{F(a_1, a_2, \ldots, a_i)/a_1, a_2, \ldots, a_i \in A,\ i = 1, 2, 3 \ldots\}$$

by closed fuzzy sets satisfying the condition that if $\mathbf{U}$ *is an* α*-shading of* X*, then* $\mathbf{F}$ *is also an* α*-shading of* X *where* $\alpha \in [0, 1)$*.*

Theorem 2.14. *A fts X is a P_α-space if and only if there exists a crisp function*

$$p : \cap \mathbf{G}^n \to \mathbf{F}$$

such that

1. *If $(G_1, G_2, G_3, \ldots, G_n) \in \mathbf{G}^n$, $n \in N$ then $p(G_1, G_2, G_3, \ldots) < \sup\{G_k : 1 \leq k \leq n\}$*
2. *If $\{G_1, G_2, G_3, \ldots\}$ is an α-shading of X, then so is $\{p(G_1), p(G_1, G_2), p(G_1, G_2, G_3), \ldots\}$. Where $\mathbf{G}$ and $\mathbf{F}$ represent the family of all open and closed fuzzy subsets of X respectively.*

Proof. Let X be a P_α-space. Let $(G_1, G_2, G_3, \ldots G_n) \in \mathbf{G}^n$ and take $a_i = G_i$ in the definition of P_α-spaces and define

$$U(a_1, a_2, \ldots a_n) = U(G_1, G_2, G_3, \ldots G_n) = \sup\{G_i : 1 \leq i \leq n\}.$$

Then clearly $U(G_1, G_2, G_3, \ldots G_n) < U(G_1, G_2, G_3, \ldots G_{n+1})$. Then from the definition of P_α-spaces the remaining follows.

Conversely let $\mathbf{U} = \{U(a_1, a_2, \ldots a_i)\ a_i \in A,\ i = 1, 2, 3 \ldots\}$ be an increasing family of open fuzzy sets in X. Now corresponding to each $U(a_1, a_2, a_3, \ldots a_i)$ in $\mathbf{U}$, we define

$$\begin{aligned} F(a_1, a_2, \ldots a_i) &= p(U(a_1), U(a_1, a_2,)U(a_1, a_2, a_3), \ldots U(a_1, a_2, a_3, \ldots a_n)) \\ &< \sup\{U(a_1, a_2, \ldots a_i) : 1 \leq i \leq n\} \\ &= U(a_1, a_2, \ldots a_n) \text{ since } \mathbf{U} \text{ is increasing.} \end{aligned}$$

Now if $\mathbf{U}$ is an α-shading of X, for every $x \in X$, there exists a $U(a_1,\ a_2,\ a_3, \ldots a_k)$ such that $U(a_1, a_2, a_3, \ldots a_k)(x) > \alpha$. Now clearly by definition, we have $F(a_1, a_2, a_3, \ldots a_k)(x) > \alpha$ and hence $\{F(a_1, a_2, a_3, \ldots a_i) : a_i \in A, i = 1, 2, 3, \ldots\}$ is an α-shading of X. Hence X is a P_α-space. ◻

From the definition of P_α-Spaces and Theorem 2.14, next theorem follows clearly.

Theorem 2.15. *A fuzzy topological space X is a P_α-space if and only if there is a crisp function p defined from the family of all increasing finite sequences of open fuzzy sets $\mathbf{G}$ to the collection of all closed fuzzy sets $\mathbf{F}$ with $p(G_1, G_2, G_3, \ldots G_n) < G_n$ where $(G_1, G_2, G_3, \ldots G_n) \in \mathbf{G}_n$ and if $G_n < G_{n+1}$ for each $n \in N$ and if $\{G_1, G_2, G_3 \ldots G_n\}$ is an α-shading then so is $\{p(G_1), p(G_1, G_2), p(G_1, G_2, G_3), \ldots\}$.*

Theorem 2.16. *A fts X is a P_α-space if and only if there exists a crisp function $p : \cup(\mathbf{F})^n \to F$ such that*

i) *For each $(F_0, F_1, \ldots F_n) \in (\mathbf{F})^n$, $n \geq 0$*

$$p(F_0, F_1, \ldots F_n) \wedge \inf_{i \leq n} F_i = 0.$$

ii) *For each* $(F_0, F_1, \ldots) \in (\mathbf{F})^\infty$ *with* $\inf_{n\geq 1} F_n = 0$*, the collection* $\{p(F_0, F_1, \ldots F_n) : n \geq 0\}$ *is an* α*-shading of* X*.*

PROOF. Let $(F_1, \ldots F_n) \in (\mathbf{F})^n$. Then F_1^c, $F_1^c \wedge F_2^c$, $F_1^c \wedge F_2^c \wedge F_3^c, \ldots$ is an increasing family of open sets. Take $U(a_1) = F_1^c$, $U(a_1, a_2) = F_1^c \wedge F_2^c \ldots U(a_1, a_2 \ldots, a_n) = F_1^c \wedge F_2^c \wedge \ldots \wedge F_n^c$. Now since X is a P_α-space, there exists a collection $\{F(a_1), F(a_1, a_2)\ldots\}$ such that $F(a_1, a_2, \ldots a_i) < U(a_1, a_2, \ldots a_i)$ for each $i = 1, 2, 3, \ldots$

Now define

$$p(F_1, \ldots F_n) = \begin{cases} 0, & \text{if } \inf_{i\leq n} F_i \neq 0, \\ F(a_1, a_2, \ldots a_n), & \text{otherwise.} \end{cases}$$

Clearly p has properties (i) and (ii).

Conversely let $(G_1, G_2, \ldots G_n) \in \mathbf{G}^n$. Then $F_1 = G_1^c$, $F_2 = G_2^c, \ldots F_n = G_n^c$ are all closed and hence there exists a function $p' : (\mathbf{F})^n \to \mathbf{F}$ such that

$$p'(F_1, \ldots F_n) \wedge \inf_{i\leq n} F_i = 0.$$

Take $p(G_1, G_2, \ldots G_n) = p'(F_1, \ldots F_n)$ in Theorem 2.14, then

$$p(G_1, G_2, \ldots G_n) \wedge \inf_{i\leq n} F_i = 0.$$

Therefore

$$\begin{aligned} p(G_1, G_2, \ldots G_n) &< (\inf_{i\leq n} F_i^c) \\ &= \sup_{i\leq n} F_i^c \\ &= \sup_{i\leq n} G_i \end{aligned}$$

and hence p satisfies (i) and (ii) of Theorem 2.14 and hence X is a P_α-space. ∎

THEOREM 2.17. *If a fuzzy topological space* X *has a* σ*- closure preserving fuzzy closed* α*-shading by countably* α*-compact sets, then* X *is a* P_α*-space.*

PROOF. Let $\mathbf{F} = \cup\{\mathbf{F}_n : n \in N\}$ be an α-shading of X such that each $\mathbf{F}_n$ is closure preserving and every $F_n(\mathbf{F}_n$ is countably α-compact. Let $\{U(a_1, a_2, \ldots a_n) : a_i \in A, i = 1, 2, 3, \ldots\}$ be an increasing sequence of open fuzzy sets. Now corresponding to each $U(a_1, a_2, \ldots a_n)$ we define

$$F(a_1, a_2, \ldots a_n) = \sup\{F : F < U(a_1, a_2, \ldots a_n), F \in \cup_{i=1}^n \mathbf{F}_i\}$$

Since $\cup_{i=1}^n \mathbf{F}_i$ is closure preserving it follows that $F(a_1, a_2, \ldots a_n)$ is fuzzy closed and $F(a_1, a_2, \ldots a_n) < U(a_1, a_2, \ldots a_n)$ for each $n \geq 1$.

Again let $\{U(a_1, a_2, \ldots a_i) : i = 1, 2, 3 \ldots\}$ be an α-shading of X. Let $x \in X$. Now since $\mathbf{F}$ is an α-shading of X, there exists an $F_0 \in \mathbf{F}$ such that $F_0(x) > \alpha$. Let $F_0 \in \mathbf{F}_k$ for some k. Since F_0 is countably α-compact, and

$U(a_1, a_2, \ldots)$'s are increasing we can find out some $j \in N$ such that $j \geq k$ and $F_0 < U(a_1, a_2, \ldots, a_j)$.

Now

$$F(a_1, a_2, \ldots, a_j)(x) = \sup_{F < U(a_1, \ldots a_j)} \{F(x) : F \in \cup_{i=1}^{j} \mathbf{F}_i\} \geq F_0(x) > \alpha.$$

Thus $\{F(a_1, a_2, \ldots, a_j) : a_i \in A, i = 1, 2, 3 \ldots\}$ is also an α-shading of X. This completes the proof. □

3. A characterisation of P_α-spaces using the game $G_\alpha(X)$

In this section we describe a game associated with P_α-spaces. Here $G_\alpha(X)$ denote the following infinite positional fuzzy topological game. Let $\mathbf{G}$ and $\mathbf{F}$ denote the collection of all open (resp. closed) fuzzy subsets of an fts X. There are two players Player I and Player II. Players alternatively choose fuzzy subsets of X so that each player knows X and first k elements when he is choosing the $(k+1)$th element.

We say that a sequence $(G_1, F_1, \ldots G_n, F_n)$ is a play for $G_\alpha(X)$ if for each $n \geq 1$, we have

i. $G_n \in \mathbf{G}$ is a choice of Player I.

ii. $F_n \in \mathbf{F}$ and $F_n < \sup\{G_k : 1 \leq k \leq n\}$ is a choice of Player II.

Player I wins the play $(G_1, F_1, G_2, F_2 \ldots)$ if $\{G_n : n \in N\}$ is an α-shading of X and $\{F_n : n \in N\}$ is not. And Player II wins if $\{F_n : n \in N\}$ or both $\{G_n : n \in N\}$ and $\{F_n : n \in N\}$ are α-shadings of X.

A strategy for Player I is a crisp function $s : \{0\} \cup_{n=1}^{\infty} \mathbf{F}^n \to \mathbf{G}$ and that of Player II is t: $\mathbf{G}^n \to \mathbf{F}$ such that $t(G_1, G_2, \ldots G_n) < \sup\{G_i : 1 \leq i \leq n\}$ for each $(G_1, G_2, \ldots G_n) \in \mathbf{G}^n$ and $n \geq 1$.

Now clearly for each pair of strategies (s, t) there exists a unique Play $(G_1, F_1, G_2, F_2, \ldots)$ of $G_\alpha(X)$ defined as follows.

Take $G_1 = s(0)$, $F_1 = t(G_1)$, $G_2 = s(F_1)$, $F_2 = t(G_1, G_2)$ and so on.

A strategy s (resp. t) is winning for Player I (resp. Player II) if he wins every play of $G_\alpha(X)$ using it.

From Theorem 2.17 and definition of $G_\alpha(X)$, we get the following game theoretic characterization of P_α-spaces.

Theorem 3.1. *A fuzzy topological space X is a P_α-space if and only if Player II has a winning strategy in $G_\alpha(X)$.*

4. Applications in metacompact spaces

Theorem 4.1. *Let X be a fuzzy regular α-metacompact P_α-space and Player I has a winning strategy in $G'(\mathbf{DC}, X)$, then $X \times Y$ is α-metacompact for every α-metacompact space Y. Where $\mathbf{DC}$ denote the class of all fts which have a discrete fuzzy closed α-shading by members of $\mathbf{C}$. Where C is the collection of all α-compact spaces.*

PROOF. We use the following notations. If $a = (a_1, a_2 \ldots, a_n)$ then $a \oplus \zeta = (a_1, a_2 \ldots a_n, \zeta)$, $a/k = (a_1, a_2, \ldots a_k)$ and $a- = a/n-1$. Also $'$ and $''$ represents the projections on X and Y respectively.

Given that Player I has a fuzzy winning strategy in $G'(\mathbf{DC}, X)$. Therefore by Theorem 2.4 of [8] it follows that Player I has a stationary winning strategy and let this be s. Let p be a function defined as in 2.16. We will prove that every α-shading $\mathbf{G}$ of $X \times Y$ by open fuzzy sets has a point finite α-shading refinement by open fuzzy rectangles.

Let $\mathbf{U}_0 = \{0\}$, $\mathbf{A}_0 = \{0\}$ and $R(0) = H(0) = X \times Y$. For each $n \geq 1$, we shall construct a collection $\mathbf{U}_n$ of open fuzzy rectangles and a collection $\{\{R(a), H(a)\} : a = (a_1, a_2, \ldots a_n) \in A_n\}$ of pairs consisting of fuzzy closed $\times$ open rectangle $R(a)$ and open rectangle $H(a)$ satisfying the following conditions.

For each $n \geq 1$

(i) $\mathbf{U}_n$ is a point finite collection in $X \times Y$.
(ii) For every $U \times V(\mathbf{U}_n$, there is a $G \in G$ such that $U \times V < G$.
(iii) $\{H(a) : a \in A_n\}$ is point finite in $X \times Y$.
(iv) $\sup\{U : U(\mathbf{U}_n\} < \sup\{H(a) : a \in A_{n-1}\}$.
(v) $a_- \in A_{n-1}$.
(vi) $R(a) < R(a-)$ and $R(a) < H(a) < H(a-)$.
(vii) $S(R(a-)') \wedge R(a)' = 0$.
(viii) $R(a) \backslash \sup\{U : U \in \mathbf{U}_{n+1}\} < \sup\{R(a^+ \zeta)\ ;\ \{a^+ \zeta) \in A_{n+1}\}$.
(ix) $p(R(a/1)', \ldots, R(a/n-1),\ R(a)') \wedge H(a)'' = 0$.

Assume that for each $i \leq n$, the collections $\mathbf{U}_i$ and $\{R(a), H(a);\ a \in A_i\}$ have been constructed.

Now for any $a \in A_n$, let $\{C_\gamma : \gamma \in \Gamma(a)\}$ be a discrete collection of α-compact sets whose supremum is $S(R(a)')$. From the fact that X is fuzzy regular α-metacompact it follows that there exists point finite collections $\{W_\gamma : \gamma \in \Gamma(a)\}$ and $\{O_\gamma : \Gamma(a)\}$ of open fuzzy sets such that $C_\gamma < W_\gamma < clW_\gamma < O_\gamma < H(a)|\sup\{C_\beta : \beta \in \Gamma(a), \beta \neq \gamma\}$ for each $\gamma \in \Gamma(a)$. Now Y is α-metacompact and $R''(a)$ is open in Y. Now $R''(a)$ is α-metacompact (Since α-metacompact is hereditary with respect to open subsets) and hence for each $\gamma \in \Gamma(a)$, there exists a collection $\mathbf{U}_\gamma = U_{\delta,j} \times V_\delta : j$ $j = 1, 2, 3 \ldots m_\delta$ and $\delta \in \Delta(\gamma)\}$ such that

i) $C_\gamma < U_\delta = \sup_{i \leq \delta} U_{\delta,j} < W_\delta$ for each $\delta \in \Delta(\gamma)$.
ii) Each $U_{\delta,j} \times V_\delta$ is contained in some $G \in \mathbf{G}$.
iii) $\{V_\delta : \delta \in \Delta(\gamma)\}$ is point finite α-shading of $R(a)''$.

Set $\mathbf{U}_{n+1} = \cup\{\mathbf{U}_\gamma : \Gamma(a)$ and $a \in A_n\}$ and

$$A_{n+1} = \{a \oplus \delta : \delta \in \Delta(\gamma), \gamma \in \Gamma(a), a \in A_n\} \cup \{a + \theta : a \in A_n\}$$

Take any $a + \zeta \in A_{n+1}$. Then observe that $a/i \in A_i$ for all $i \in n$.

If $\zeta = \delta$ for some $\delta \in \Delta(\gamma)$ and, $\gamma \in \Gamma(a)$ put $R(a \oplus \delta) = [(clW_\gamma \backslash V_\delta) \wedge R(a)'] \times V_\delta$ and $H(a \oplus \delta) = O_\gamma \backslash p(R(a/1)', \dots R(a/n)', R(a+\theta)') \times V_\delta$.

If $\zeta = \theta$, put $R(a+\theta) = R(a)`\backslash \sup_{\gamma \in \Gamma(a)} W \times R(a)''$ and

$$H(a+\theta) = H(a)' \backslash p(R(a/1)', \dots R(a/n)', R(a+\theta)')\theta H(a)''$$

Then clearly U_{n+1} and $\{R(a), H(a) : a \in A_{n+1}\}$ satisfies conditions (i) – (ix).

Now take $\mathbf{U} = \cup_{n \geq 1} \mathbf{U}_n$. Now it can be shown that U is an α-shading of X and we will prove that $\mathbf{U}$ is also point finite. Also by (ii) $\mathbf{U}$ is a collection of open fuzzy rectangles in $X \times Y$ and any $U \times V \in \mathbf{U}$ is contained in some $G \in \mathbf{G}$.

Proceeding in a similar manner as in the proof of Theorem 2.4 in [7], we get if $\{a_n\}$ is a sequence such that $a_n \in A_n$ and $(a_n)- = a_{n-1}$ for each $n \geq 1$ where $a_0 = 0$, then

$$\inf_{n \geq 1} H(a_n)' = 0. \tag{4.2}$$

Again we claim that $\inf_{n \geq 1}(\sup \mathbf{H}_n) = 0$. Where $\mathbf{H}_n = \{H(a) : a \in A_n\}$. For if possible let there be an z_0 such that $\inf_{n \geq 1}(\sup \mathbf{H}_n)(z_0) > \eta =$ for some $\eta > 0$. Take $A_n(z_0) = \{a \in A_n : H(a)(z_0) \geq \eta\}$. By (iii) we get $A_n(z_0)$ is finite and by (v) and (vi) $a \in A_n(z_0) \Rightarrow a- \in A_{n-1}(z_0)$. Then by Konings Lemma, there exists $(\beta_1, \beta_2, \beta_3, \dots)$ such that $a_n \in (\beta_1, \beta_2, \dots \beta_n) \in A_n(z0)$ for each $n \geq 1$. Then $H(a_n)(z0) \geq \eta$ for each $n \geq 1$. Hence $\inf_{n \geq 1} H(a_n)(z_0) \geq \eta$. This is a contradiction to our claim.

Let $Z \in X \times Y$ then by claim above we can find an $m \geq 1$ such that $\sup H_m(z) = 0$. Now from (v) and (vi) it follows that $\sup \mathbf{H}_{n+1} < \sup \mathbf{H}_n$ for each $n \geq 1$. Since $\sup \mathbf{H}_n(z) = 0$ for each $n \geq m$, from (iv) we get that $\sup \mathbf{U}_n(z) = 0$ whenever $n > m$. Hence it follows from (i) that $\mathbf{U}$ is point finite in $X \times Y$. This completes the proof. □

THEOREM 4.2. [5] *If a fts X has a σ-closure preserving α-shading by fuzzy closed α-compact sets, then Player I has fuzzy winning strategy in $G'(\mathbf{DK}, X)$.*

From Theorems 2.17, 4.1, and 4.2 next corollary follows easily.

COROLLARY 4.3. *If X is a fuzzy regular α-metacompact space with a α-closure preserving α-shading by α-compact sets, then $X \times Y$ is α-metacompact for every α-metacompact space Y.*

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References

[1] T.E. Gantner, R.C. Steinlage and R.H. Warren, *Compactnesss in Fuzzy Topological Spaces*, J. Math. Anal. Appl. **62** (1978), 547–562.

[2] M.H. Ghanim, E.E. Kerre and A.S. Mashour, *Separation Axioms, Subspaces and Sums in Fuzzy Toplology*, J. Math. Anal. Appl. **102** (1984), 189–202.

[3] S.R. Malghan and S.S. Benchalli, *On Fuzzy Topological Spaces*, Glasnik Mathematički **16(36)** (1981), 313–325.

[4] J. Nagata, Modern General Topology, North Holland Pub., 1974.

[5] S.J. John, *Countable α-compactness, α-metacompactness and the fuzzy topological game*, $G'(DK, X)$ [Communicated]

[6] S.J. John, *Fuzzy Topological Games I*, Far East J. Math. Sci., Special Vol. (1999) Part III (Geometry and Topology), 361–371.

[7] S.J. John, *Fuzzy Topological Games II*, [Communicated].

[8] S.J. John, *Metacompactness in Fuzzy Topological Spaces through α-shadings*, [Communicated].

[9] S.J. John, *Metacompactness in the L-Fuzzy Context*, The Journal of Fuzzy Mathematics **8** (2000) 661–668.

[10] R.A. Telgarsky, *Characterisation of P-Spaces*, Proc. Japan Acad. **51** (1975), 802–807.

[11] R. Telgarsky, *Spaces defined by Topological Games*, Fund. Math. **88** (1975), 193–223.

[12] C.K. Wong, *Fuzzy Topology: Product and Quotient Theorems*, J. Math. Annal. Appl. **45** (1973), 512–521.

[13] Y. Yajima, *Topological Games and Applications*, in: Topics in General Topology, eds: K. Morita and J. Nagata, Elsevier Science Pub. 1989, 524–562.

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