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Computationally Efficient Leakage Inductance Estimation of Multi-Winding Medium Frequency Transformer

M. Mogorovic and D. Dujic

POWER ELECTRONICS LABORATORY
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Computationally Efficient Leakage Inductance Estimation of Multi-Winding Medium Frequency Transformers

Marko Mogorovic, Drazen Dujic
Power Electronics Laboratory (PEL), École Polytechnique Fédérale de Lausanne (EPFL), Switzerland
Corresponding author: Marko Mogorovic, marko.mogorovic@epfl.ch
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Abstract

This paper describes the technical challenges of leakage inductance estimation of multi-winding medium frequency transformers for design optimization purposes, where besides the estimation accuracy, the computational cost represents an equally important figure of merit. While it is possible to very accurately model the leakage inductance of typical 2-winding transformer geometries, using light-weight analytical models, in case of multi-winding structures, all of the available methods resort to some type of computationally intensive numerical methods, e.g. solving of numeric integrals or finite elements method. These methods offer good accuracy, but their execution time is often a limiting factor when it comes to multi-variable optimization. To that end, this paper proposes a computationally efficient leakage inductance model for multi-winding transformers, developed to yield very fast execution.

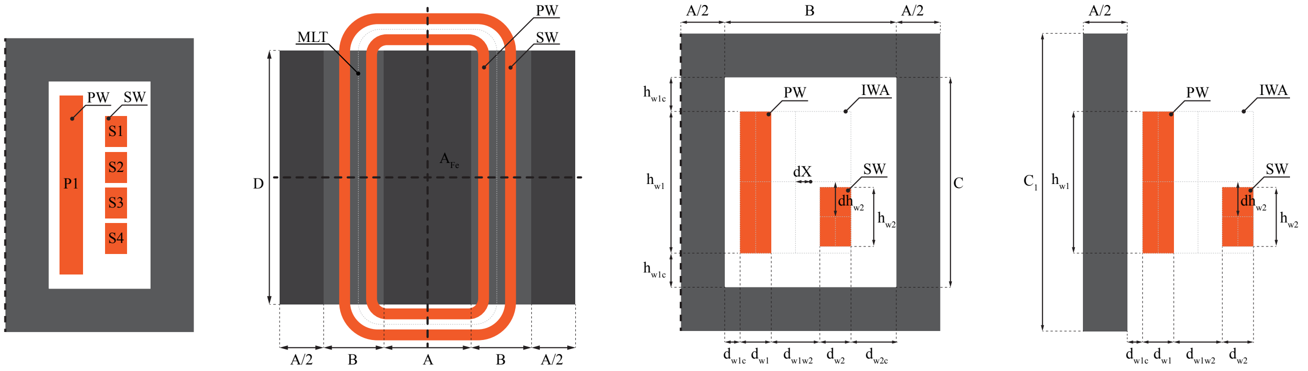
1. Introduction

The proliferation of high power semiconductor devices and advances in magnetic materials have given life to the so called solid state transformer (SST) [1] or power electronic transformer (PET) [2] concept. Driven by the new requirements of the emerging MVDC distribution grids and various potential improvements of the existing traction systems, these concepts have gained popularity in the recent years. Especially now, the advancements in power semiconductor industry and the expected proliferation of the wide-band gap high power semiconductor devices such as SiC and GaN, are further promoting the SST concept.

To that end, the research interest into various aspects of medium frequency transformer (MFT) design, as a key enabling SST technology, has dramatically increased [3]–[8]. Accurate design of the MFT electric parameters is paramount for proper operation of these converters. Especially, high precision of the MFT leakage inductance is usually required [9]. While there are tools for very precise leakage inductance modeling (e.g. FEM, Roth [10] etc.), they all resort to some type of computationally intensive techniques. When it comes to multi-physics design optimization, involving a multitude of design variables, the execution time of these models is often a limiting factor. As a result, one must usually resort to some type of sufficiently accurate analytical model with low computational cost.

For the 2-winding transformer case, it is possible to quite accurately analytically estimate the frequency dependent leakage inductance via Dowell's winding geometric equivalence [11] and some simple corrections [9] that extend its validity to the realistic window area utilization of the MV transformer structures. This type of equivalence cannot in general be applied to any of, frequently encountered, multi-winding transformers with non-symmetric winding structure [12], [13], such as shown in Fig. 1a.

This paper proposes a computationally efficient model that allows for a very fast DC leakage inductance estimation of multi-winding transformers. It is based on multi-variable polynomial fitting of the results of the FEM analysis generated on the two representative generalized winding geometry primitives, as given in Figs. 1c and 1d. Thus the proposed model keeps the inherent precision of the FEM as the most precise modeling approach, whereas its execution boils down to several low order matrix multiplications, as one of the most primitive processor functions, resulting in very fast execution.



(a) (b) (c) (d)

Fig. 1: (a) Considered multi-winding transformer with non-symmetric winding structure; (b) MFT symmetry showing two generalized winding cross-sections with different electromagnetic properties, inside (c) and outside (d) of the core window area;

Tab. 1: Definition of normalized variables based on generalized winding geometry detail, as shown in Fig. 1c

Variable	/	/	x_1	x_2	x_3	x_4	x_5	x_6
Definition	$\frac{d_{w1}}{h_{w1}}$	$\frac{d_{w2}}{h_{w1}}$	$\frac{d_{w1w2}}{h_{w1}}$	$\frac{h_{w2}}{h_{w1}}$	$\frac{2dh_{w2}}{h_{w1}-h_{w2}}$	$\frac{B}{d_{w1}+d_{w1w2}+d_{w2}}$	$\frac{C}{h_{w1}}$	$\frac{2dX}{B-d_{w1}-d_{w1w2}-d_{w2}}$
Range	0.1	0.1	$0-0.7$	$0.2-1$	$0-1$	$1-2$	$1-2$	$-1-1$

Note that h_{w1} is taken as a reference when defining the geometry ratios. Winding widths (d_{w1} and d_{w2}) are fixed as their effect on the leakage is quite linear and can be taken into account through a correction of d_{w1w2}.

Tab. 2: Definition of normalized variables based on generalized winding geometry detail, as shown in Fig. 1d

Variable	/	/	x_1	x_2	x_3	x_4	x_5
Definition	$\frac{d_{w1}}{h_{w1}}$	$\frac{d_{w2}}{h_{w1}}$	$\frac{d_{w1w2}}{h_{w1}}$	$\frac{h_{w2}}{h_{w1}}$	$\frac{2dh_{w2}}{h_{w1}-h_{w2}}$	$\frac{d_{w1c}}{d_{w1}+d_{w1w2}+d_{w2}}$	$\frac{C}{h_{w1}}$
Range	0.1	0.1	$0-0.7$	$0.2-1$	$0-1$	$0-1$	$1-2$

Note that compared to the geometry detail inside the core window area (Fig. 1c), the leakage inductance is in this case fully defined by five normalized variables. Instead of x_4 and x_6 from Tab. 1, it is enough to define x_4 as given in Tab. 2.

2. Proposed Modeling Method

First phase of modeling is the magnetostatic FEM analysis, allowing to identify the effect of various geometric parameters on the transformer DC leakage inductance. As shown in [14], the total (3D) leakage inductance of the transformer can be very well estimated using two 2D models, representing the magnetic field energy inside and outside of the core widow area, as depicted in Fig. 1. Therefore, the estimation of the total (3D) leakage, boils down to the ability to accurately estimate the magnetic energy in these two 2D geometries.

The two 2D generalized geometry primitives are defined and parametrised, as described in Figs. 1c and 1d, and normalized, as shown in Tabs. 1 and 2. As can be seen, they are fully defined with eight and seven normalized parameters, respectively. Due to the largely linear effect on the leakage inductance, relative winding widths can be considered as constant. Based on the preservation of the total magnetic energy (S_W), a winding geometry equivalence with infinitely thin windings can be derived, as given in Fig. 2. The effect of the winding (d_{w1} and d_{w2}) and inter-winding (d_{w1w2}) widths on the leakage inductance can be merged into an equivalent inter-winding distance (1).

$$d_{w1w2.eq} = d_{w1w2} + \frac{1}{3}(d_{w1} + d_{w2}) \tag{1}$$

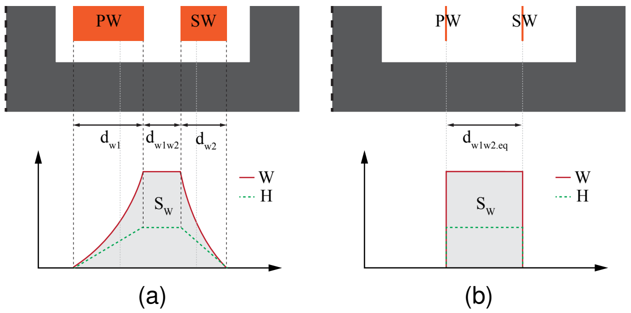
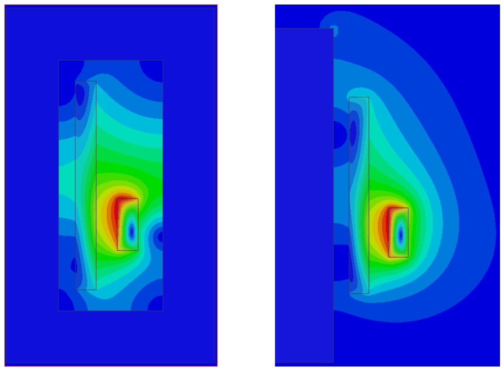


Fig. 2: Winding geometry equivalence in respect to total magnetic energy (S_W) preservation: (a) Standard 2-winding example (b) Theoretical equivalence with infinitely thin windings



(a) (b)

Fig. 3: An example of the 2D magnetostatic FEM simulation of the magnetic field inside (a) and outside (b) of the core window area;

Using this equivalence, any winding width configuration can be transformed into any other, while preserving the total magnetic energy and consequently the resulting leakage inductance.

A 2D magnetostatic FEM parametric sweep has been performed, generating the set of leakage inductance estimations (50’000 simulations) within the geometry range of interest according to Tabs. 1 and 2, covering all of the practically relevant geometry ratios, as displayed in Figs. 3a and 3b.

Based on these results, and taking into account the minimum vector of influential variables (x), as summarized in Tabs. 1 and 2, a multivariate polynomial fitting is done variable by variable, as shown in Fig. 4, allowing to always choose the minimum adequate polynomial order while inherently ensuring convergence. The final result of this fitting is a multi-dimensional array of polynomial parameters. For the sake of illustration, an example of the described model, for a two variable case is shown in (2).

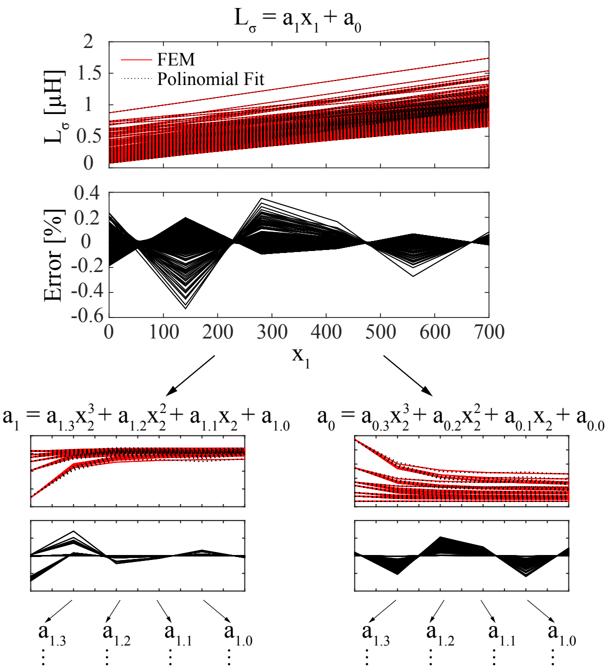


Fig. 4: Multi-variable polynomial fitting of the MFT leakage inductance to the results of the 2D FEM sweep (50’000 simulations)

$$L'_{\sigma.fit} = \begin{bmatrix} x_1 \\ 1 \end{bmatrix}^T \begin{bmatrix} a_{1.3} & a_{1.2} & a_{1.1} & a_{1.0} \\ a_{0.3} & a_{0.2} & a_{0.1} & a_{0.0} \end{bmatrix} \begin{bmatrix} x_2^3 \\ x_2^2 \\ x_2 \\ 1 \end{bmatrix} \qquad (2)$$

As can be seen, the evaluation of the model boils down to simple low order matrix multiplication. While this operation executes very fast on the processor, depending on how many variables (N) are involved, and what are the orders of the polynomial fittings (n_i), there is $\prod_{i=1}^{N} n_i$ polynomials to be solved. Although this model is still executing very fast for the case of 6-variables, described by Figs. 1c and 1d and Tabs. 1 and 2 (three orders of magnitude faster compared to FEM), it is possible to further optimize the model. Rearranging the order in which the polynomial is executed based on the algorithm allows for a part of the multi-variable polynomial, involving variables that have been defined, to be pre-calculated and only execute the part of the model dealing with optimization variables within the optimization loop. Doing this, in case that not all of the variables (x) are being actively changed in the most nested optimization loop, it is

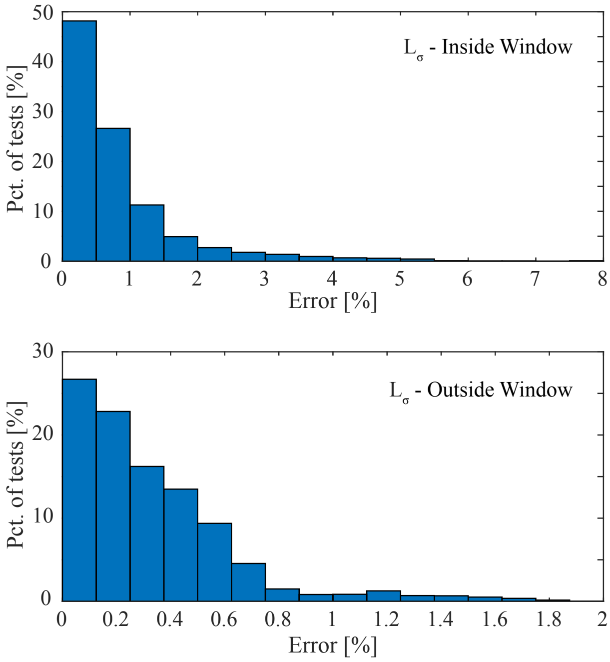


Fig. 5: Histograms of the relative leakage inductance estimation error, referred to the FEM results (50’000 test samples)

possible to even further improve the overall execution time.

The resulting multi-variable polynomial models of the two geometric primitives from Figs. 1c and 1d have been compared to the results of the FEM sweep, as displayed in Fig. 5. As can bee seen, the errors are very low (below 4% for any practical design), confirming the good precision of the model.

3. Application of the Model

Calculation of the total 3D leakage inductance boils down to a few simple steps. First, the dimensions of the two geometric profiles of the MFT, as seen in Figs. 1c and 1d, need to be identified and transformed using (1) according to Tabs. 1 and 2. This leads to

$$d_{w1w2.eq01} = d_{w1w2} + \frac{1}{3}(d_{w1} + d_{w2} - \frac{2}{3}0.1h_w) \quad (3)$$

where $d_{w1w2.eq01}$ is the equivalent inter-winding dielectric distance of the generalized geometry primitive with selected constant normalized winding widths ($0.1h_w$).

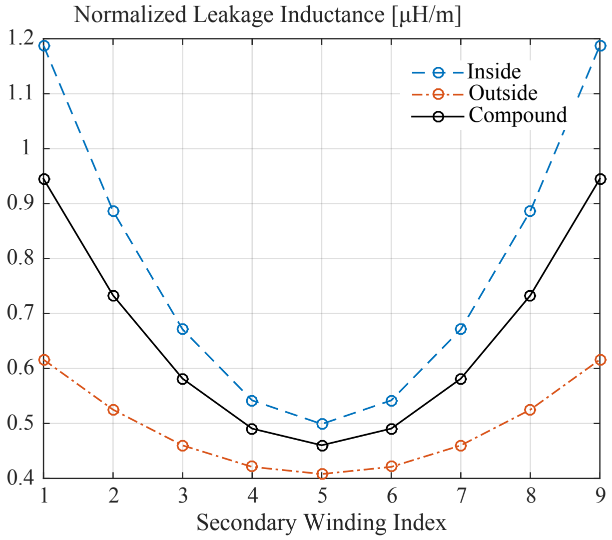


Fig. 6: An example of the per-length primary to i-th secondary winding leakage inductance calculation of a multi-winding transformer with a geometry profile, as shown in Fig. 1a, in case of nine secondary windings, according to the prototype, as given in Fig. 7.

Taking into account (3), the two multi-variable polynomial models (f_1 and f_2), describing the magnetic energy inside and outside of the core window area are evaluated leading to per-length leakage inductances

$$L'_{\sigma.in} = f_1(\boldsymbol{x}_1) \quad (4)$$

$$L'_{\sigma.out} = f_2(\boldsymbol{x}_2) \quad (5)$$

where $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are the normalized variable vectors from Tabs. 1 and 2, respectively. Weighted sum of these two values in respect to the portions of the MLT inside and outside of the core window area (see Fig. 1b) leads to the equivalent per-length leakage inductance

$$L'_{\sigma.eq} = \frac{1}{A+D}(DL'_{\sigma.in} + AL'_{\sigma.out}) \quad (6)$$

An example of these values for a nine-secondary multi-winding transformer prototype, as displayed in Fig. 7, with a geometry profile, as shown in Fig. 1a, is given in Fig. 6.

Finally, the total leakage inductance referred to the primary winding of the transformer can be calculated, as shown in (7).

$$L_{\sigma.total} = N_1^2(MLT)L'_{\sigma.eq} \quad (7)$$

Depending on the design task, beside the direct estimation, this fast executing model also facilitates

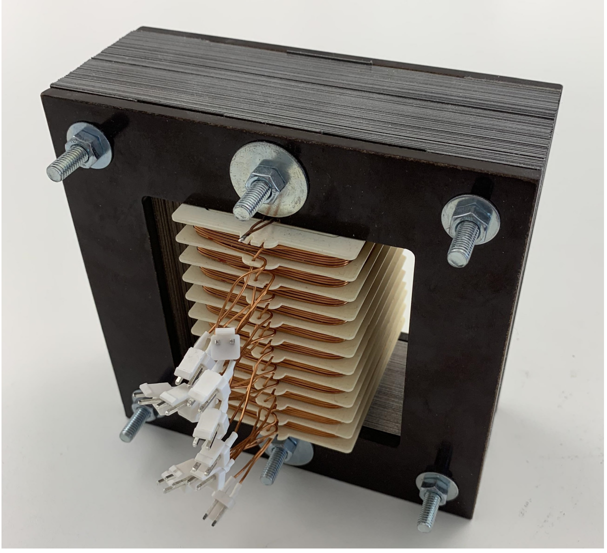


Fig. 7: Multi-winding transformer test setup for experimental verification [15]

simple and numerically efficient inverse calculation of any single or arbitrary combination of geometric dimensions $\boldsymbol{x}$ (e.g. d_{w1}, d_{w1}, d_{w1w2}) in order to match the reference leakage inductance.

4. Experimental Verification

The presented modeling is verified with a full 3D FEM model and measurement on a muti-winding transformer prototype [15], as displayed in Fig. 7, with geometry profile, as shown in Fig. 1a, with nine secondary windings. All of the geometric dimensions, construction details and leakage inductance measurement between each secondary and primary winding can be found in [15].

Plots of the measured and estimated total leakage inductance between the primary and each of the secondary windings and relative estimation errors are displayed in Fig. 8. Estimation is done both with proposed statistical data-driven model (SDDM) and full 3D FEM for comparison purposes. A slight under estimation of the total leakage can be observed for both models. This is most likely due to a small additional leakage of the extended termination of the windings and connection cables. As expected the proposed SDDM performs within the 5% error compared to FEM, achieving good accuracy with less than 9% error compared to the measurement even in case of such extreme geometry ratios.

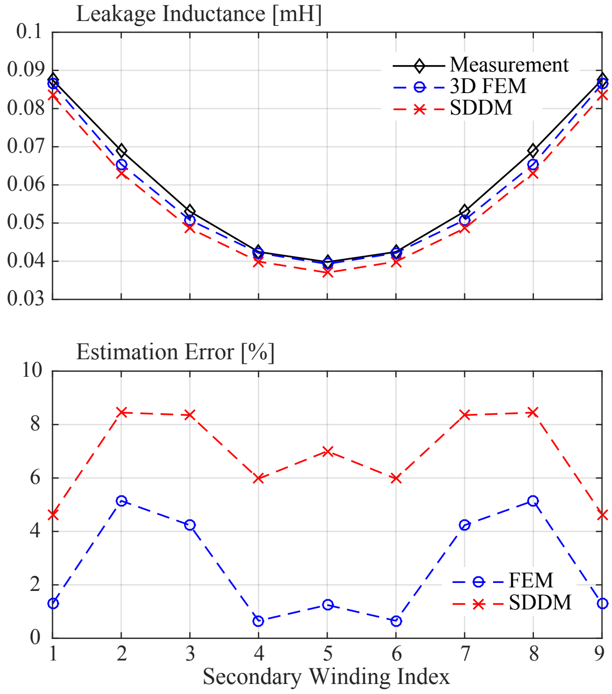


Fig. 8: Top: plots of the measured and estimated (both with the 3D FEM and the proposed SDDM) total primary to i-th secondary winding leakage inductance of the multi-winding transformer prototype from Fig. 7 referred to each secondary winding. Bottom: Plots of the relative estimation errors of the 3D FEM and the proposed SDDM referred to the measurements.

5. Conclusion

Appropriate MFT modeling for design optimization is not straightforward. Besides the accuracy requirements, facilitating any optimization scheme implies hard constraints in terms of computational cost, thus in general excluding the possibility of direct application of very sophisticated numeric tools such as FEM. This paper proposes a computationally efficient, yet sufficiently accurate, statistical data-driven DC leakage inductance model for multi-winding transformers based on FEM simulation of simple geometry details and muti-variable polinomial fitting.

The proposed modeling framework has been described in detail, showing how it is possible to transform, generalize and normalize a numerically difficult problem up to the point where a sufficiently small set of significant influences (variables) can be very efficiently captured via a neural-network inspired multi-variable polynomial model. Although already very fast (three orders of magnitude compared to simple 2D FEM models), this model can also be reorganized for most optimal execution depending on the specific design optimization algorithm and potentially achieve an even more drastic speed improvement. Moreover, a good estimation accuracy is achieved, with errors less than 5% relative to the 3D FEM and less than 9% relative to the measurement.

Even beyond this specific model, this type of modeling framework allows the inclusion of complex effects, which cannot be analytically approximated within reasonable accuracy limits or numerically solved within reasonable time, within the design optimization loop.

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References

[1] J. E. Huber and J. W. Kolar, “Solid-State Transformers: On the Origins and Evolution of Key Concepts,” *IEEE Ind. Electron. Mag.*, vol. 10, no. 3, pp. 19–28, Sep. 2016.

[2] M. Claessens, D. Dujic, F. Canales, J. K. Steinke, P. Stefanutti, and C. Vetterli, “Traction Transformation: A Power-Electronic Traction Transformer (PETT),” *ABB Review, No: 1/12*, pp. 11–17, 2012.

[3] I. Villar, “Multiphysical Characterization of Medium-Frequency Power Electronic Transformers,” PhD thesis, EPFL Lausanne, Switzerland, 2010.

[4] G. Ortiz, “High-Power DC-DC Converter Technologies for Smart Grid and Traction Applications,” PhD thesis, ETH Zurich, Switzerland, 2014.

[5] M. Bahmani, “Design and Optimization Considerations of Medium-Frequency Power Transformers in High-Power DC-DC Applications,” PhD thesis, Chalmers University of Technology Gothenburg, Sweden, 2016.

[6] M. Mogorovic and D. Dujic, “100 kW, 10 kHz Medium-Frequency Transformer Design Optimization and Experimental Verification,” *IEEE Transactions on Power Electronics*, vol. 34, no. 2, pp. 1696–1708, Feb. 2019.

[7] M. Mogorovic and D. Dujic, “Sensitivity Analysis of Medium Frequency Transformer Designs for Solid State Transformers,” *IEEE Transactions on Power Electronics*, pp. 1–1, 2018.

[8] T. P. H. Guillod, “Modeling and Design of Medium-Frequency Transformers for Future Medium-Voltage Power Electronics Interfaces,” PhD thesis, ETH Zurich, Switzerland, 2018.

[9] M. Mogorovic and D. Dujic, “Medium Frequency Transformer Leakage Inductance Modeling and Experimental Verification,” in *IEEE Energy Conversion Congress and Exposition*, Cincinnatti, OH, USA, 2017., pp. 419–424.

[10] P. Hammond, “Roth’s method for the solution of boundary-value problems in electrical engineering,” *Proceedings of the Institution of Electrical Engineers*, vol. 114, no. 12, pp. 1969–1976, Dec. 1967.

[11] P. L. Dowell, “Effects of Eddy Currents in Transformer Windings,” *Proc. of the Institution of Electrical Engineers*, vol. 113, no. 8, pp. 1387–1394, Aug. 1966.

[12] S. Isler, T. Chaudhuri, D. Aguglia, and A. Bonnin, “Development of a 100 kW, 12.5 kV, 22 kHz and 30 kV Insulated Medium Frequency Transformer for Compact and Reliable Medium Voltage Power Conversion,” in *Proceedings of the 19th European Conference on Power Electronics and Applications (EPE 2017 - ECCE Europe), Warsaw, Poland*, 2017.

[13] M. Luo, D. Dujic, and J. Allmeling, “Leakage Flux Modeling of Medium-Voltage Phase-Shift Transformers for System-Level Simulations,” *IEEE Transactions on Power Electronics*, vol. 34, no. 3, pp. 2635–2654, Mar. 2019.

[14] A. Fouineau and et al., “Semi-Analytical Methods for Calculation of Leakage Inductance and Frequency-Dependent Resistance of Windings in Transformers,” in *IEEE Transactions on magnetics, 2018.*, pp. 1–10.

[15] M. Luo, D. Dujic, and J. Allmeling, “Leakage Flux Modeling of Multiwinding Transformers for System-Level Simulations,” *IEEE Transactions on Power Electronics*, vol. 33, no. 3, pp. 2471–2483, Mar. 2018.