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# Active times for acoustic metamaterials

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A B S T R A C T

Initially proposed to achieve strong noise isolation levels beyond the mass-density law, acoustic metamaterials (AMMs) have now overturned the conventional views in all aspects of sound propagation and manipulation. In fact, within the last two decades, these artificial materials have enabled improved control over the propagation of sound waves by allowing one to engineer macroscopic effective properties well beyond what is naturally available. In this review, we first trace the development of passive AMMs from their initial realizations based on locally resonant structures to their more advanced versions, like space-coiling, holey and labyrinthine metamaterials, Willis materials, and subwavelength crystalline metamaterials, highlighting their basic working principles and applications. We then survey more recent research topics, including non-Hermitian, non-reciprocal, and topological acoustic metamaterials. Altogether, this paper provides a comprehensive overview of research on acoustic metamaterials, and highlights prominent future directions in the field, including topological and active metamaterials.

## 1. Introduction

Controlling wave propagation at will is an endeavor dating back to the dawn of our modern civilizations, when early scientists used conventional lenses made of properly shaped pieces of glass or plastic, to focus, bend or magnify images carried by light [1]. Despite being simple and commonplace, the approach to manipulate wave behavior with materials readily found in nature comes with several drastic restrictions. An important one is that, while in principle the laws of physics allow extreme wave functionalities like cloaking [2], (concealing an object from external radiation) or subwavelength focusing [3] (creating focal spots smaller than the diffraction limit), no natural material provides the appropriate values of constitutive parameters required to achieve these effects: natural substances only cover a small portion of the physically allowed material parameter space.

Within a time span of 20 years, the development of the field of metamaterials [4–16] has offered an unprecedented expansion of our ability to control wave propagation, be it of electromagnetic or mechanical nature. Metamaterials (MMs) are rationally designed composites composed of mesoscopic resonant inclusions placed in a host medium. These meta-atoms, being smaller than or comparable to the wavelength in the background medium, exhibit collectively an on-demand property, which can often (but not always) be described by effective media parameters. Such artificial or man-made composite structures have paved the way to realize useful novel wave manipulation functionalities such as cloaking and subwavelength resolution imaging, by going beyond the conventional wisdom of refraction laws.

Of our particular interest in this review are acoustic metamaterial (AMMs) [17–32] that, as their name suggests, are used to control airborne sound, the simplest but one of the most ubiquitous form of classical waves. In absence of sources, the equation governing the propagation of scalar pressure waves inside a homogenous linear fluid is expressed as

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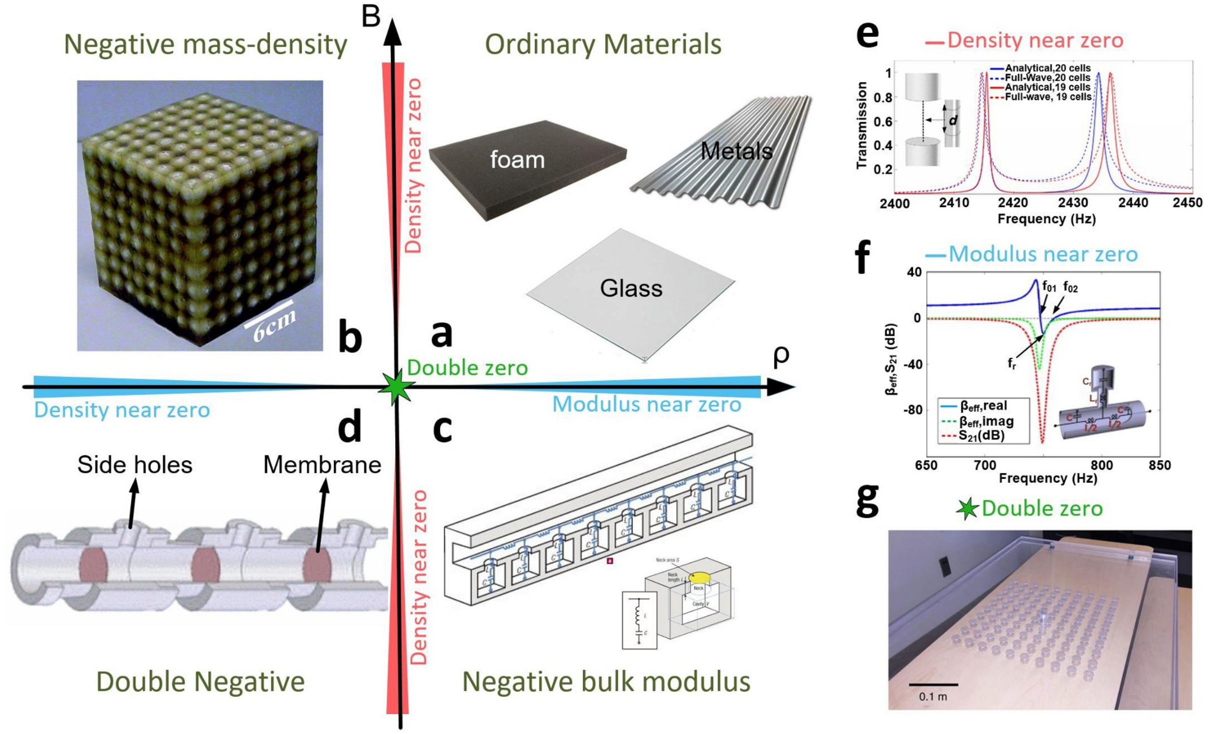


**Fig. 1.** Acoustic metamaterials with extreme constitutive parameters. (a) Ordinary materials found in nature are in the first quadrant of the figure, as they usually exhibit positive mass density ρ and bulk modulus *B*. (b) Realization of a single-negative acoustic metamaterial, exhibiting a negative effective mass density. The metamaterial is built from deeply subwavelength resonators (meta-atoms) made of metallic spheres, coated with a soft layer of silicon rubber [36]. (c) A 1D acoustic metamaterial exhibiting negative bulk modulus, composed of a waveguide, whose walls are loaded with Helmholtz resonators [46]. (d) Double negative AMMs: acoustic waveguide loaded with both resonant membranes (leading to a negative effective mass density) and Helmholtz resonators (responsible for negative bulk modulus) [61]. (e) Density near zero AMMs: The meta-atom is composed of two circular cylindrical waveguides connected through an ultra-narrow tube. The wave tunneling through the ultrathin channel results in a density near zero operation [91]. (f) Realization of a bulk modulus near zero AMM: The metamaterial is composed of an acoustic duct loaded with a Helmholtz resonator whose dispersive behavior is used to create a compressibility near zero response [93]. (g) A fabricated double zero AMM: The sample is composed of an array of symmetric blind holes whose band structure possesses a Dirac-like cone right at the center of Brillouin zone. The metamaterial exhibits zero refractive index at the Dirac frequency [96]. All figures are adapted with permission.

$$\nabla^2 P(r) - \frac{\rho}{B}\frac{\partial^2 P(r)}{\partial t^2} = 0 \tag{1}$$

where *P* is the acoustic pressure. In natural media, the parameters ρ and *B,* which stand for the mass density and bulk modulus, respectively, can only take positive values (Fig. 1a, first quadrant). AMMs, on the contrary, can be tailored in such a way that they exhibit arbitrary effective constitutive parameters, offering an extreme range of not only positive values (like very large or very small), but also of negative values. This is exactly what enables AMMs to support distinct physical phenomena that can be used to control sound propagation in unprecedented manners.

The review is organized as follows: the first part reviews the origins of the field, and conventional designs based on passive structures. By passive, we not only mean that the artificial medium cannot provide any energy to the wave, but also that its realization itself does not involve any form of external input or bias. In the second part, we move on to the recent topic of active acoustic metamaterials. Following our terminology, the word active here is to be understood in a general sense: it can either refer to systems in which the acoustic wave can receive energy from the material (acoustic gain), or to systems that do not provide energy to the wave but require an external input to function which allows either special properties, or their dynamic reconfiguration.

## 2. Passive acoustic metamaterials

### *2.1. Locally resonant AMMs*

#### *2.1.1. Negative mass density and bulk modulus*

In common sound blocking panels, sound attenuation generally follows the so-called mass-density law, stating that the transmission loss (*TL*) of sound through a wall is, as a rule of thumb, proportional to $TL \propto \rho d\omega$ [33], where *d* and ρ are the thickness and mass density of the wall, respectively, and $\omega$ is the angular frequency. Low frequencies, as a consequence, are easily transmitted through walls, especially when their thickness is smaller than or comparable to the associated acoustic wavelength. An alternative to the use of a wall in order to block sound, reported for the first time by Martinez-Sala et al. [34], is the possibility of employing sonic crystals, which are made of periodic arrangement of solid scatterers, for instance solid cylindrical rods. Sonic crystals can exhibit complete band gaps, which are forbidden frequency bands for sound. These band gaps, originating from destructive Bragg interferences as waves scatter off distinct crystal planes, require the distance between adjacent scatterers to be approximately half a wavelength (180° phase shift). This implies that blocking an A tone at 440 Hz would require a crystal of periodicity around 40 cm. This large crystal periodicity results in extremely bulky structures considering the fact that at least 3 crystal planes are needed for the interference to start (see [35] for an example). In 2000, however, an article [36] was published, reporting at least 60 dB sound isolation at 400 Hz for a synthetic structure as thin as 2 cm! How was this achieved? Well, the idea is the following: instead of relying on the phase accumulated by the scattered wave as it travels from one crystal plane to the next, this metamaterial exploited the out-of-phase response of subwavelength resonant scatterers (which occurred above their resonance frequency) to produce the desired destructive interference. The destructive interference provided by these types of scatterers was, however, independent of the spatial periodicity of the material, allowing one to scale down the medium to deep subwavelength sizes. In other words, Bragg interferences were replaced by Fano interferences (which are interferences occurring between a continuum of states and a sharply localized state [37]) at the level of the small resonant inclusions (called meta-atoms). The use of locally resonant subwavelength scatterers whose resonance frequency sets the frequency of operation is the hallmark of acoustic metamaterials when compared to sonic crystals, whose band gap frequencies scale with the lattice period. Fig. 1b shows a photograph of the fabricated metamaterial, built from atoms of a metallic sphere coated with a soft layer of silicon rubber, emulating a simple mass-spring resonator. These atoms were arranged in an $8 \times 8 \times 8$ matrix array with a lattice constant much smaller than the operation wavelength. Above the resonance, the out-of-phase motion of the mass was at the origin of the Fano interference effect, which could also be interpreted as a negative effective mass density, preventing wave propagation. Note that any negative effective mass must be dispersive, as a direct consequence of Kramers-Kronig relations which apply to any causal and passive response function [38]. Similar negative mass behavior was also observed in one-dimensional scenarios using membrane inclusions arranged along a pipe [39–45].

An alternative strategy for achieving high levels of sound isolation is to realize artificial structures exhibiting dispersive bulk moduli [46–50]. For this, the Helmholtz resonator is a perfect candidate as it involves compression-extension of air. In [46], Fang et al. first demonstrated anomalous bulk modulus response employing a waveguide loaded by a periodic set of Helmholtz resonators (Fig. 1c). Thanks to the strong resonant behavior of the local Helmholtz resonators, the bulk modulus of the composite structure could become negative, resulting in high sound isolation levels from subwavelength structures.

#### 2.1.2. Double negative and near zero acoustic metamaterials

In the context of locally resonant AMMs, it has been shown that the spatial symmetries of the relevant resonant modes can be used as an indicator of whether the mass density or the bulk modulus of the composite structure becomes dispersive. More specifically, it has been theoretically proven that modes with monopolar symmetry mostly generate a compressive-extensional response (leading to a dispersive bulk modulus), whereas those with dipolar symmetry contributes more to the displacement of the center of mass (creating a dispersive mass density) [51]. By overlapping the frequency spectra of two modes with different symmetries, it is however possible to realize artificial media whose effective mass-densities and bulk moduli are both dispersive, and can become negative within the same frequency range. These so-called double negative AMMs [52–61] can, for example, be realized using an acoustic waveguide loaded by coupled membrane masses, generating an effective mass-density response, and shunted by Helmholtz resonators, giving rise to anomalous bulk modulus behavior (Fig. 1d). Such composite structures, corresponding to a negative index of refraction, have inspired peculiar, but useful, applications comprising super-focusing [62–67], subwavelength imaging [67–73], reverse Doppler effect [74,75], cloaking [76–86], and leaky wave acoustic rainbow devices [87,88], some of which are explained in Section 2.3.

By now, we have fully explored all four possible portions of the material parameter space. We now have a closer look at the regions close to the axes (marked with blue and red colors in Fig. 1), corresponding to the so-called near zero AMMs. In these kinds of media, sound propagates with an extremely large phase velocity, leading to a quasi-static spatial distribution for time-harmonic fields. Such an exotic behavior holds promise for a variety of exciting applications such as tailoring radiation phase patterns [89], and cloaking [90]. In [91], Fleury and Alù proposed the realization of a density near zero AMM, demonstrating the acoustic equivalent of a phenomenon called supercoupling in electromagnetism, which allows perfect tunneling of sound through very narrow pipes. They considered two identical circular acoustic waveguides connected to each other through an ultra-narrow channel (Fig. 1e), and demonstrated that sound could be transmitted perfectly through the narrow channel provided that the fluid filling it has near-zero effective mass density. Unlike usual Fabry–Pérot resonant transmission, such a density-near-zero tunneling effect does not depend on the channel length or the presence of bends due to the quasi-static nature of propagation (this effect is similar to electrical cables at kHz frequencies, in which the electrical wavelength is so large that they can be bent at will). This phenomenon was recently demonstrated experimentally [92]. Proposals have also been reported to achieve a dual effect, with a compressibility near zero response, leveraging the frequency dispersive behavior of Helmholtz resonators (Fig. 1f) [93].

Single near-zero acoustic metamaterials hold a great potential for wavefront engineering since the transmitted wavefront simply follows the shape of the output surface [94]. However, they suffer from low-transmission due to the impedance mismatch with air. More recently, motivated by their analogues in electromagnetism [95], double zero AMMs, exhibiting simultaneous zero effective density and bulk modulus, have been demonstrated making use of a square lattice of $10 \times 10$ symmetric cylindrical blinds (Fig. 1f) [96]. The underlying idea of such structures is that if both constitutive parameters tend to zero while keeping their ratio (the impedance) constant, a near-zero-index metamaterial can remain matched to air, allowing high transmission levels.

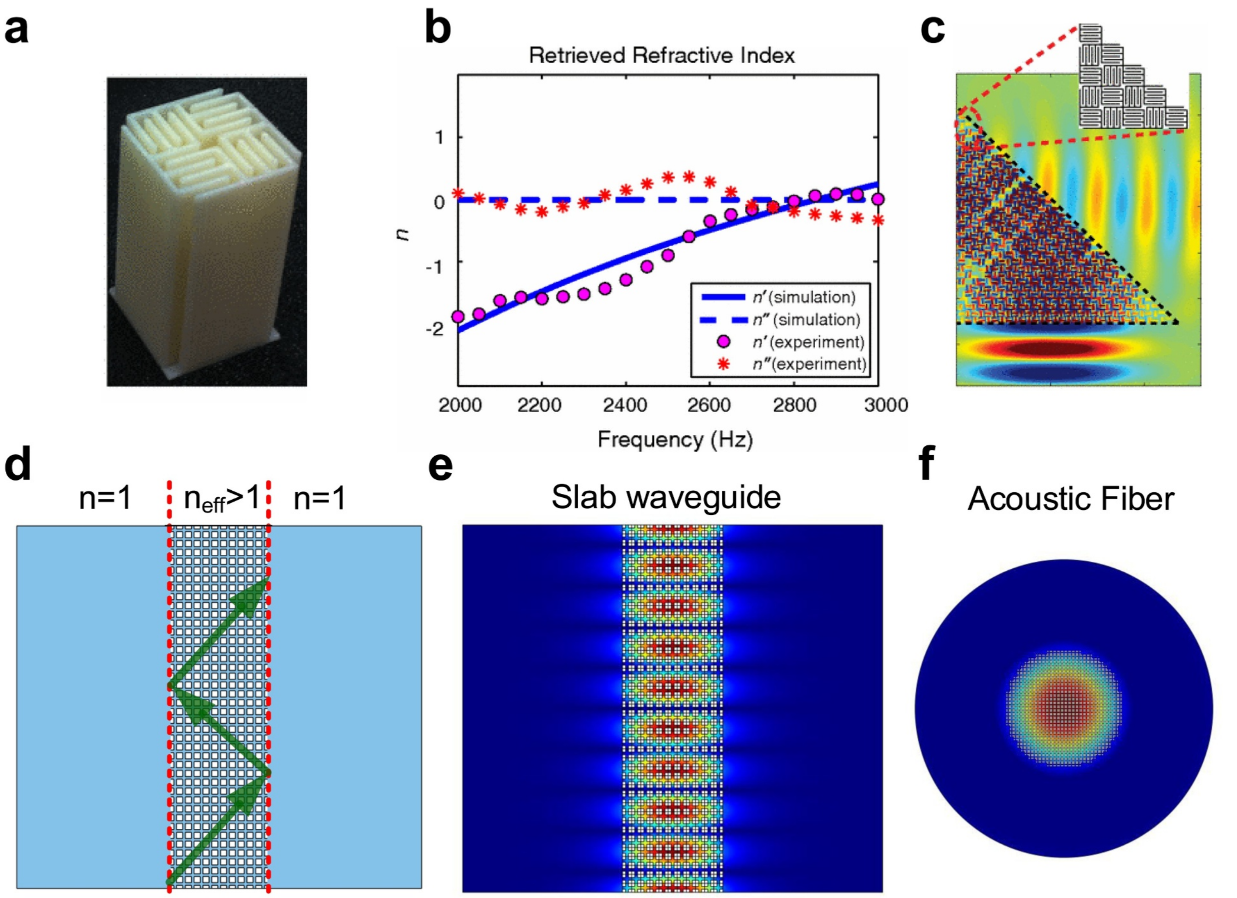


**Fig. 2.** Space coiling and holey AMMs. (a) Photograph of a fabricated labyrinthine AMM. (b) By coiling-up space, the mazelike composite structure in panel a provides a negative refractive index over a wide frequency range. Adapted with permission from [97]. (c) Negative refraction of sound waves impinging into a flat plate of a space coiling AMM, adapted with permission from [98]. (d) An open high-index acoustic waveguide, confining sound via total internal refraction, is built by sandwiching a finite piece of a holey AMM between two lower index media [107]. (e) Profile of the fundamental mode of the slab waveguide. (f) First guided acoustic mode in a cylindrical high-index acoustic fiber based on the same principles (transverse cross-section): sound is guided along the fiber by total internal reflection, like in optical fibers.

### *2.2. Non-resonant acoustic metamaterials*

Although locally-resonant AMMs have allowed realization of media with unusual effective properties, they are accompanied with challenges associated with the available bandwidth of operation due to their resonant nature. Extensive research is therefore ongoing to realize AMMs using units with much lower dispersion, providing larger bandwidth. In the following, we discuss two established and common techniques to realize AMMs without resonating inclusions, namely labyrinthine AMMs [97–103], and holey AMMs [104–108].

#### *2.2.1. Space coiling acoustic metamaterials*

Fig. 2a shows a photograph of a fabricated space coiling AMM [97]. By properly designing its subwavelength curled cross-sections, the effective properties of such a mazelike structure can be tailored to achieve unusual effective properties, including negative refractive indices, over a broader frequency range (Fig. 2b). These types of AMMs indeed owe their unusual properties to their subwavelength curled channels rather than highly-resonant inclusions. This provides them with two important advantageous features: (i) Unlike locally resonant AMMs, they provide a larger bandwidth for the on-demand effective parameters, (ii) They avoid high levels of sound attenuation associated with resonating peaks (although visco-thermal losses are still present in their channels, which may hinder their applicability especially in the higher audible and ultrasonic range). Just like their resonance-based counterparts, however, these types of AMMs are able to induce exotic wave-phenomena such as negative refraction of sound as seen in Fig. 2c [98]. Furthermore, the curled perforations of such structures have established themselves as a relevant way of slowing down sound waves so as to achieve acoustic media with large refractive indices, a property which is not readily achievable in airborne acoustics as the speed of sound in air is lower than that of any solid material.

An alternative route to achieve large refractive indices is the use of so-called holey AMMs, built from a rigid block perforated with subwavelength pores of arbitrary shape (for instance square in Fig. 2d). This type of non-resonant metamaterial also exhibits high refractive indices as the effective mass density is lower than homogenous air (the air filling it occupies less volume). Such high-index property has recently been leveraged to realize acoustic analogues of optical waveguide components, such as slab waveguides (Fig. 2e) and fibers (Fig. 2f) [107].

### 2.3. Applications of passive acoustic metamaterials

As explained previously, the target application of AMMs is to realize novel wave functionalities, most of which are not achievable with regular materials. One of the most popular is transformation-based cloaking. Consider again the acoustic wave equation given in Eq. (1). Suppose a coordinate transformation $r'(r)$, corresponding to the desired wave functionality (in terms of guiding the acoustic power flow), is applied to all of the spatial points $r$. In the context of transformation acoustics (TA) [109], it has been shown that the acoustic wave equation is invariant under coordinate transformations. As a consequence, acoustic power transport in a curved space can be effectively obtained by engineering effective parameters (which are generally inhomogeneous and anisotropic) in the real space, given by:

$$\rho'(r') = \frac{J\left[\rho(r)\right]J^t}{\det(J)}, \quad B'(r') = \det(J)B(r) \tag{2}$$

where $J$ denotes the Jacobian matrix of the coordinate transformation. Now, assume our aim is to manipulate impinging sound waves in a way that a spherical object with a radius of $a$, for example, becomes completely hidden, meaning that all incident acoustic power goes around the spherical region without entering it and being scattered. To do so, a proper coordinate transformation is to expand all points inside the sphere into a spherical shell with the inner radius of $a$ and an outer radius $b$ larger than $a$. The associated effective material parameters can then be found using the formula given in Eq. (2), and is available in the literature [17]. Despite being theoretically achievable, the required material properties for cloaking imply relatively large levels of anisotropy most of the time, which is difficult to implement. Alternative strategies have therefore been proposed to mitigate the anisotropy level required for concealing scatterers, among which the so-called ground-plane or carpet cloak [110–112] has attracted the most attention. Owing to their simpler quasi two-dimensional geometries, these types of cloaks minimize the anisotropy levels of the index profile, enabling successful experimental realization of acoustic cloaking (see Fig. 3a) [112].

Another important application of AMMs is to achieve imaging systems with resolutions below the so-called diffraction limit. It has been shown that, similar to the case of Pendry's lens in electromagnetism [3], double negative acoustic media are also able to retrieve information carried by evanescent waves, whose exponential decay is the major cause of a limited resolution in conventional far-field imaging. Fig. 3b illustrates an example of such ultra-high resolution sound focusing, accomplished at the common corner of two negative index ($n \approx -1$) acoustic metamaterial square pieces [113,114]. Alternatively, imaging below diffraction limit can be achieved using the so-called hyperlens [115–117]. Shown in Fig. 3c is a photograph of the first experimental realization of such lenses, used to construct the magnified image of a subwavelength source. By engineering the dispersion relation in such a way that

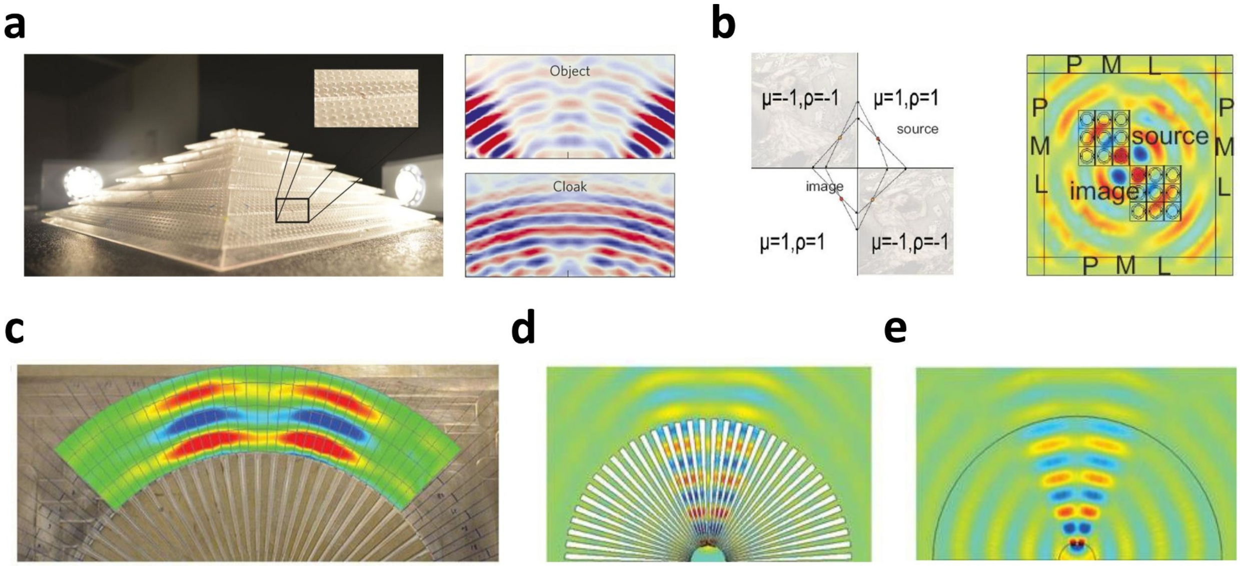


**Fig. 3.** Examples of acoustic metamaterial devices. (a) Acoustic carpet cloak concealing scatterers on top of a reflective flat plate [112]. (b) Acoustic super-focusing using metamaterials. Similar to the case of Pendry's lens in electromagnetism, double-negative acoustic media are capable of imaging with resolution below the diffraction limit [113]. c, Experimental realization of an acoustic hyperbolic lens using a metamaterial consisting of fins in air embedded on a brass substrate. The composite structure magnifies the image of a sub-diffraction-limited source [115]. (e,d) Numerical simulations demonstrate the proposed hyperbolic imaging technique. All figures are adapted with permission.

the associated equifrequency contours becomes elliptical with a large eccentricity, the proposed structure allowed one to access large $k$ components, enabling super-resolution without the stringent requirement of negative parameter values, required for hyperbolic dispersion (Fig. 3d and e).

AMMs have also been utilized for analog computing purposes, like their counterparts in electromagnetism [118–120]. The underlying principle of such computational scheme is that, by properly tailoring the constitutive parameters of AMMs, they can in principle realize arbitrary Green's functions associated with an operator of desire such as differentiation, integration, convolution, etc. Such “on-the-fly” wave-based computational scheme comes with several important advantageous features, compared to their digital alternatives, including ultrafast, real-time and high throughput operation [121–124].

As already discussed, an important issue hindering the applicability of AMMs, especially those working based on high quality factor resonances, in practical applications is visco-thermal losses [19,125,126]. Although loss and absorption are often considered as a limiting factor in the context of passive acoustic metamaterials, they can be turned into as an asset in some specialized applications such as acoustic absorbtion, shielding and noise blocking [127–132].

Another important issue mitigating the applicability of passive acoustic metamaterials is their bulky configuration. The quest to accomplish wave manipulation tasks through the smallest possible structures has inspired the concept of metasurfaces (MSs), which are essentially the 2D versions of metamaterials. MSs are planar array of subwavelength elements arranged in a periodic, aperiodic, or even random manner, offering exceptional control of wave-fronts over thicknesses that are much smaller than the wavelength [133–139]. Just like their three-dimensional counterparts, MSs have enabled realization of wave functionalities of interest by imparting controllable amplitude and phase modulation to impinging fields using their subwavelength planar inclusions. However, MSs have two advantages compared to bulky AMMs, which are due to their two-dimensional geometry and ultrathin profile: (i) they are easier to fabricate and (ii) they are less sensitive to losses. In acoustics, these so-called 2D metamaterials are usually built from resonant membranes, labyrinthine paths, or ultrathin Helmholtz resonators units, and are capable of supporting exotic effects such as anomalous refraction of sound [139–142].

### 2.4. Spatially non-local metamaterials

By now, we have shown how frequency dispersion, arising from the fact that the response of a material always comes with a delay with respect the excitation (inertia), can be leveraged to induce anomalous effective mass-densities and bulk moduli. However, frequency dispersion is not the only dispersive effect on which one can play to tailor the acoustic response of a medium. More specifically, if the response (motion) of an acoustic medium at a given point depends not only on excitation (force) at that point, but also on the excitation at other points, then another dispersive effect, namely spatial dispersion, takes place. Such type of dispersion, also known as spatial non-locality, gives rise to the dependency of effective material parameters on the wave-vector, which can be exploited as another degree of freedom to achieve AMMs with unusual effective properties. Depending on whether or not the composite structure can be homogenized into a uniform medium, these kinds of AMMs can be divided into two classes, namely Willis media, and metamaterial crystals, both of which are discussed in details in the following.

#### 2.4.1. Willis metamaterials

In electromagnetism, the so-called bi-anisotropic media are known as materials in which electric and magnetic responses are coupled. The giant magneto-electric coupling of such media can be used to realize circular polarizers [143], and breaking the reciprocity of wave propagation (the latter only applies to a particular form of bianisotropy involving a non-reciprocal material as specified in [144]). The acoustic equivalent of bi-anisotropy, theoretically predicted by Willis several decades ago [145], involves coupling between momentum and strain, a very weak phenomenon in conventional material. The emergence of AMMs, however, has allowed the realization of artificial acoustic media with large Willis coupling, i.e. “acoustic bi-anisotropy. In [146], Koo et al. first demonstrated acoustic bi-anisotropy in a meta-atom consisting of atoms of rectangular aluminum prisms coated with polyethylene membranes (Fig. 4a). The proposed AMM was spatially dispersive in the sense that it owed its property not only to a locally resonant effect, but also to a non-local one induced by the spatial asymmetry of its atoms. Using first principle calculations, it was shown that, in the framework of effective medium theory, such medium can be homogenized to a Willis model.

Although the above-mentioned study reported the first experimental observation of Willis coupling, it did not suggest an approach to predict and/or measure the effective properties that such kinds of AMMS exhibit. Later on, Muhlestein et al. [147] presented an interesting experimental procedure by which the material effective properties of such spatially dispersive composites could be extracted by simple scattering measurements (Fig. 4b). These findings altogether promise encouraging applications such as pressure-velocity impedance convertors, reflection-transmission decoupled wavefront shaping [146–149], and enhancing refraction efficiency of sound at higher angles [150].

At the end, it is instructive to point out that Willis coupling is equivalent to keeping spatially nonlocal terms up to first order in the Taylor expansion of the response function of the medium. As known in electromagnetics, second order spatial dispersion is a necessary condition for negative refraction to occur [151]. Hence, also in acoustics, any medium supporting negative refraction implies at least second-order spatial dispersion and should, in principle, always be homogenized employing a non-local model including some form of coupling between momentum and strain. This has been recognized only recently in the literature [152].

#### 2.4.2. Crystalline metamaterials: subwavelength sonic crystals

As mentioned above, Willis coupling is equivalent to second order spatial dispersion. However, there exist metamaterials with non-locality well beyond second-order. For these media, it may not be reasonable to find or define homogenous parameters, as it

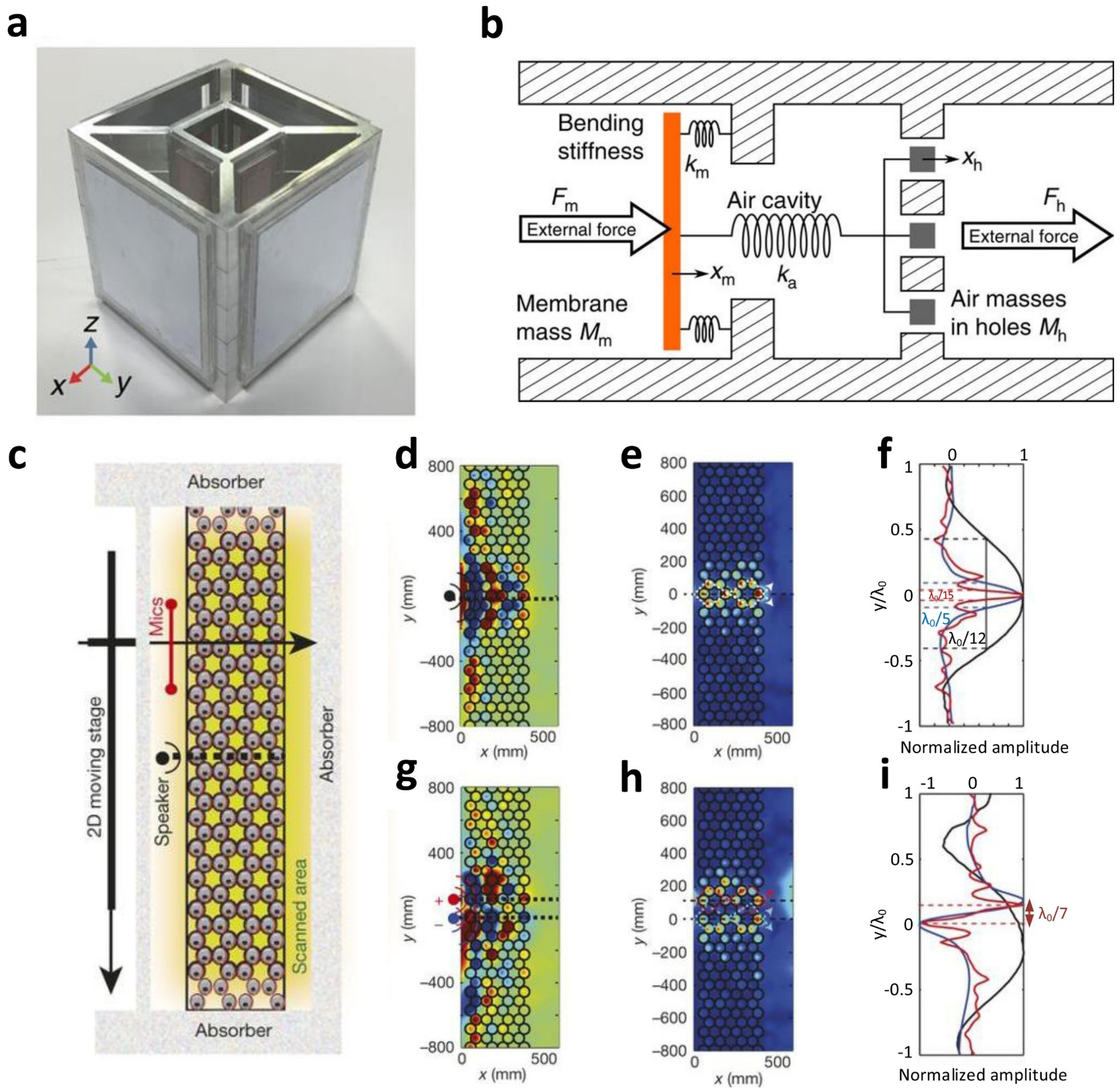


**Fig. 4.** Spatially non-local AMMs. (a) An acoustic Willis metamaterial composed of meta-atoms consisting of rectangular aluminum prisms asymmetrically coated with polyethylene membranes. The metamaterial exhibits second-order spatial dispersion which is equivalent to acoustic bi-anisotropy [146]. (b) An experimental appatus proposed in [147], by which the material effective properties of Willis-type composites can be readily extracted. (c) Concept of crystalline metamaterials. Through multiple scattering and Fano interferences, the subwavelength-scaled Helmholtz resonators of a periodic array can create spatial dispersion well beyond second order. Such a much-sought property is leveraged for super-focusing of a one (d–f) and two (g–i) acoustic sources [154]. Figures are all adapted with permission.

would result in a complicated set of constitutive relations/boundary conditions that defeats the initial purpose of defining effective parameters (simplicity). In the worst cases, homogenization may not be possible at all in the sense that the structure cannot be considered as a continuous medium. However, when these systems are periodic, they can still be treated in the same way as sonic crystals, by solving the full complexity of their Bloch dispersion bands and eigenmodes in the first Brillouin zone [153]. An important example of such a crystalline approach is the metamaterial proposed by Kaina et al. in [154], and represented in Fig. 4c–i. In contrast to other double negative AMMs, the proposed medium is only composed of subwavelength resonant units (a soda can, which is by itself a spatially-local Helmholtz resonator), relaxing the requirement of multiple overlapping resonances. By engineering the subwavelength spatial arrangement of these local subwavelength resonators, it is possible to induce strong non-local effects such as negative refractive index and super-focusing, as seen in the figure. These locally-resonant metamaterial crystals are, remarkably, the subwavelength analogs of non-resonant phononic crystals: by engineering multiple scattering and Fano interferences (rather than Bragg interferences), they allow spatially non-local field manipulation at the deep subwavelength scale [155].

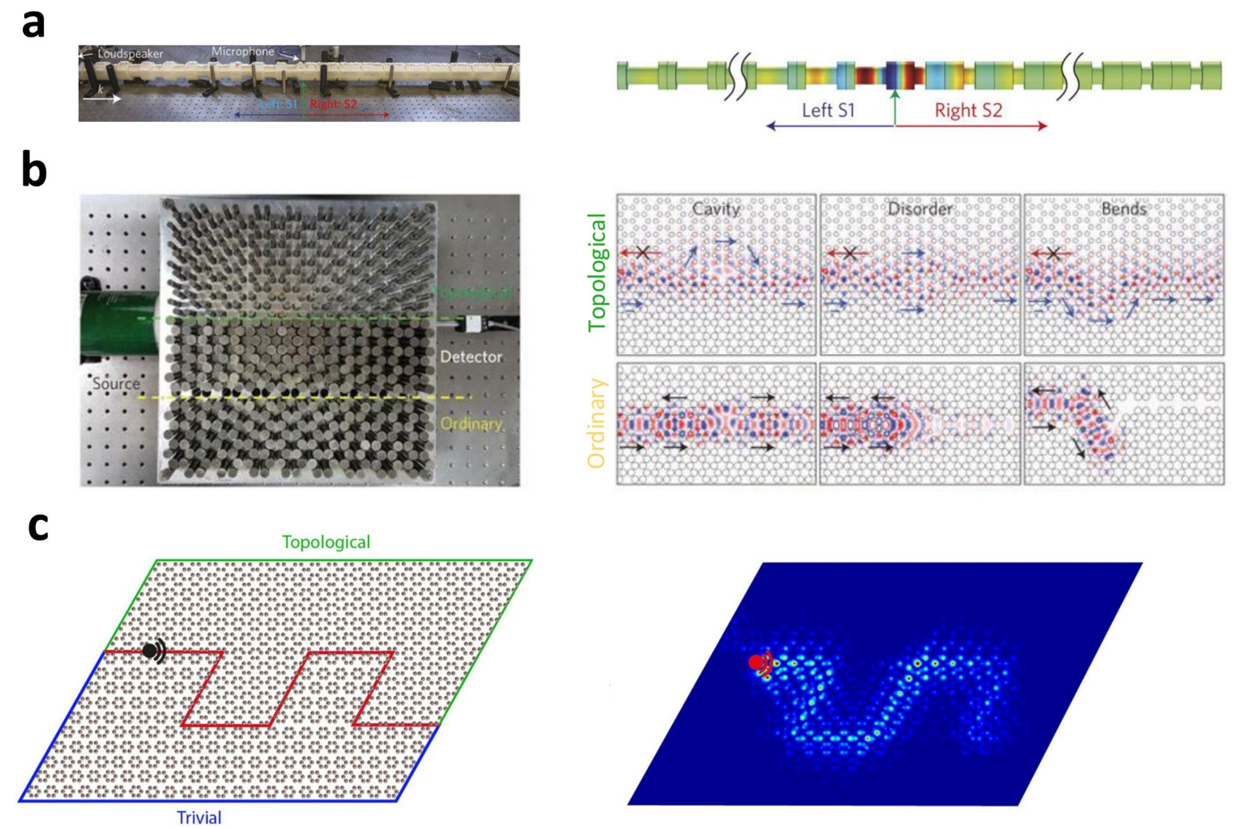


**Fig. 5.** Acoustic topological insulators with preserved time reversal symmetry. (a) Acoustic edge mode formed at the interface between two one-dimensional acoustic topological insulators with different topological invariants [161]. (b) A two dimensional acoustic topological insulator built from two graphene-like lattices with the same lattice constant but different atom radii. By mimiking the quantum spin hall effect, the structure supports spin-polarized protected edge modes [156]. (c) Acoustic edge mode formed at the domain wall between two locally-resonant acoustic metamaterials with distinct topological properties. The existence of the edge mode is a consequence of the different topological properties of the surrounding band gaps and is garanteed regardless of how the interface is drawn [180]. Figures are all adapted with permission.

### 2.5. *Topological acoustic metamaterials with preserved time-reversal symmetry*

Another category of AMMs that has attracted a lot of attention in recent years includes artificial acoustic structures possessing non-trivial topological orders, which are known as acoustic topological insulators [156–181]. Topological insulators (TIs) are a new class of materials that, despite being insulator in the bulk, support conductive states across their edges, regardless of the shape of the edges. As opposed to polarization states of trivial media, the existence of the conductive states forming at the boundaries of a TI can be protected against structural imperfections by the topological order of the bulk insulator, an immunity that usually depends on the lattice symmetries. Such protection has paved the way for realizing striking wave functionalities such as back-scattering immune sound guiding, which is achievable neither with natural materials, nor with artificial but topologically-trivial acoustic media.

The simplest way of constructing an acoustic TI is to form a special array, known as SSH chain in the literature, making use of two one-dimensional sonic crystals with detuned nearest neighbor distances (Fig. 5a) [161]. A topological phase transition is known to occur at the interface between the two crystals, creating a topological edge mode. Although they have been exploited to induce interesting physical phenomena such as topological Fano interferences [179], these kinds of edge modes cannot be used for wave-guiding as they are confined to zero dimension. Instead, Fig. 5b illustrates a realization of a 2D acoustic TI, built from two graphene-like lattice with the same lattice constant but different atom radii [156]. By mimicking the quantum spin Hall effect [181] in condensed matter, such a configuration supports protected spin-polarized edge modes at the phase transition interface, which can flow with excellent transmission along arbitrary paths involving different types of disorder (Fig. 5b). These so-called helical topological edge modes have also been predicted in crystalline metamaterials, in which case they allow sound guiding at deep sub-wavelength scale [180], based on spatial dispersion (see Section 2.4.2). Fig. 5c depicts the edge mode emerging at a domain wall between two crystalline metamaterials with distinct topologies, constructed using two different arrangements of subwavelength Herlmholtz resonators. Regardless of the shape of the interface, the presense of the edge mode is guaranteed by the different topological properties of the surrounding bulk insulators, as confirmed in the right panel of the figure.

The TIs introduced above all support (*d-1*) dimensional gapless edge states, with *d* being the dimension of space. Very recently, the possibility of having (*d-n*) dimensional topological edge modes has been proposed, notably in electromagnetism, defining *d* dimensional TIs which are of *n*th order [182]. Hand in hand with their electromagnetic analogues, these so-called higher order TIs have also been realized in acoustics [183] and mechanics [184].

## 3. Active acoustic metamaterials

In the previous section, we specifically discussed passive, linear time-invariant (LTI) AMMs. While these properties result in a simpler and more straightforward realization, they also impose certain restrictions on the constitutive effective parameters. For instance, the dispersion of a passive, LTI acoustic medium is bounded by Kramers-Kronig relations, limiting the spectral range of the target effective properties. Furthermore, the efficiency of passive AMMs is most likely hindered by losses, as there is no compensation mechanism for the undesirable inherent visco-thermal dissipations. For these reasons, there has been recently an intense surge of interest for active AMMs, which could broadly enhance the real-world applicability of AMMs by overcoming the aforementioned challenges. In a very broad sense, one can divide all of the proposed active AMMs thus far into two categories according to whether the external bias provides energy to the wave or not: (i) Non-Hermitian AMMs involving acoustic gain, and (ii) Externally biased AMMs, enabling either a special property of interes or reconfigurability, but without exchanging power with the wave. These two types of active AMMs are discussed in details in the following.

### *3.1. Non-Hermitian acoustic metamaterials*

#### *3.1.1. Acoustic gain*

Non-Hermitian metamaterials possess effective parameters with significantly non-zero imaginary parts, meaning that a non-negligible amount of either material loss or gain is present, and strongly alters wave propagation. Note that we discuss here only non-trivial instances of such media, for which the amount of loss (or gain) is large enough so that the metamaterial cannot be considered as a small perturbation of a lossless system. These kinds of metamaterials offer more degrees of freedom in the design of new functionalities, by exploiting the interplay between loss, gain, and the coupling between individual resonances. Often, they rely strongly on our ability to achieve acoustic wave amplification, or acoustic gain. The strong response of piezoelectric materials to external voltage bias has been proposed to achieve large acoustic gain [185]. More specifically, it has been shown that when the electron drift velocity driven by the external voltage is higher than the speed of sound, Cherenkov radiation of phonons in a piezoelectric material results in acoustic gain. Such an amplification effect can be leveraged to realize *PT*-symmetric AMMs as we will see below. Other efficient mechanisms to obtain acoustic gain are the use of electroacoustic circuits [186], coupling between sound and hydrodynamic instabilities [187], or thermo-acoustic effects.

We further note that the strong response of a piezoelectric material to external electrical signals has also established a fertile ground for controlling effective parameters of AMMs in a dynamic manner [188,189]. Shown in Fig. 6a is an example of a reconfigurable AMM, whose effective properties are controlled via an external voltage, applied to piezoelectric materials.

#### *3.1.2. Parity-time symmetric acoustic metamaterials*

Of particular recent interest is a group of non-Hermitian AMMs respecting parity (*P*) and time (*T*) symmetries at the same time. In spite of their non-Hermitian Hamiltonian, such systems are found to support real spectra below a specific threshold, corresponding to an exceptional point. Within this regime the losses are perfectly compensated by the gain, constituting a perfect platform for creating loss-immune acoustic metamaterials. Above that threshold, the system moves to the *PT*-symmetry broken phase with complex conjugate eigenvalues [190–201], implying a superposition of exponentially increasing and decaying eigenstates, i.e. instability. These extraordinary spectral features, stemming from the special symmetry of the distribution of gain and loss in such systems, lead to exceptional scattering behavior such as anisotropic transmission resonances (ATR), also called unidirectional cloaking [186], and systems that behave simultaneously as coherent perfect absorbers and lasers [202,203].

In [187], Aurégan and Pagneux reported an experimental demonstration of a *PT* symmetric acoustic system, consisting of a pair of diaphragms submitted to air flow (see Fig. 6b). The acoustic gain was generated by coupling the acoustic wave with an hydrodynamic instability occurring in one of the diaphragm, while the other diaphragm was designed to induce absorption. By tunning the flow rate and the geometry of each diaphragm, a *PT* symmeteric system was then created and driven to the exceptional point of its scattering matrix, allowing one to achieve unidirectional invisibility. Another important example of such mechanical *PT* symmetric system is the work reported by Christensen et al. [204], where acoustoelectric effects in piezoelectric semiconductors were exploited to synthesize the required gain and loss elements (as described in the previous section). By tuning the gate voltage applied to the piezoelectric materials, the *PT* symmetric system could be brought to its exceptional points so as to achieve unidirectional suppressed reflectance (Fig. 6c).

In [186], Fleury et al. proposed to achieve the exceptional scattering properties of acoustic *PT* symmetric systems making use of loudspeakers loaded with non-Foster electrical circuits, by which the amounts of acoustic gain (or loss) were controlled (Fig. 6d), allowing for more design flexibility and an application of this concept to furtive sensing. By properly tuning the electroacoustic circuit, the existence of spontaneous symmetry-broken pairs of eigenstates and that of exceptional points with unidirectional invisibility was then observed. An alternative route to control *PT* phases that has recently been proposed is to consider the distance between gain and loss components as an additional degree of freedom. In [205], Shi et al. suggested the possibility of accessing the points with unidirectional invisibility solely by tuning the distance *L* between the involving gain and loss elements. The relevance of this approach for accessing the exceptional points has been experimentally verified (Fig. 6e).

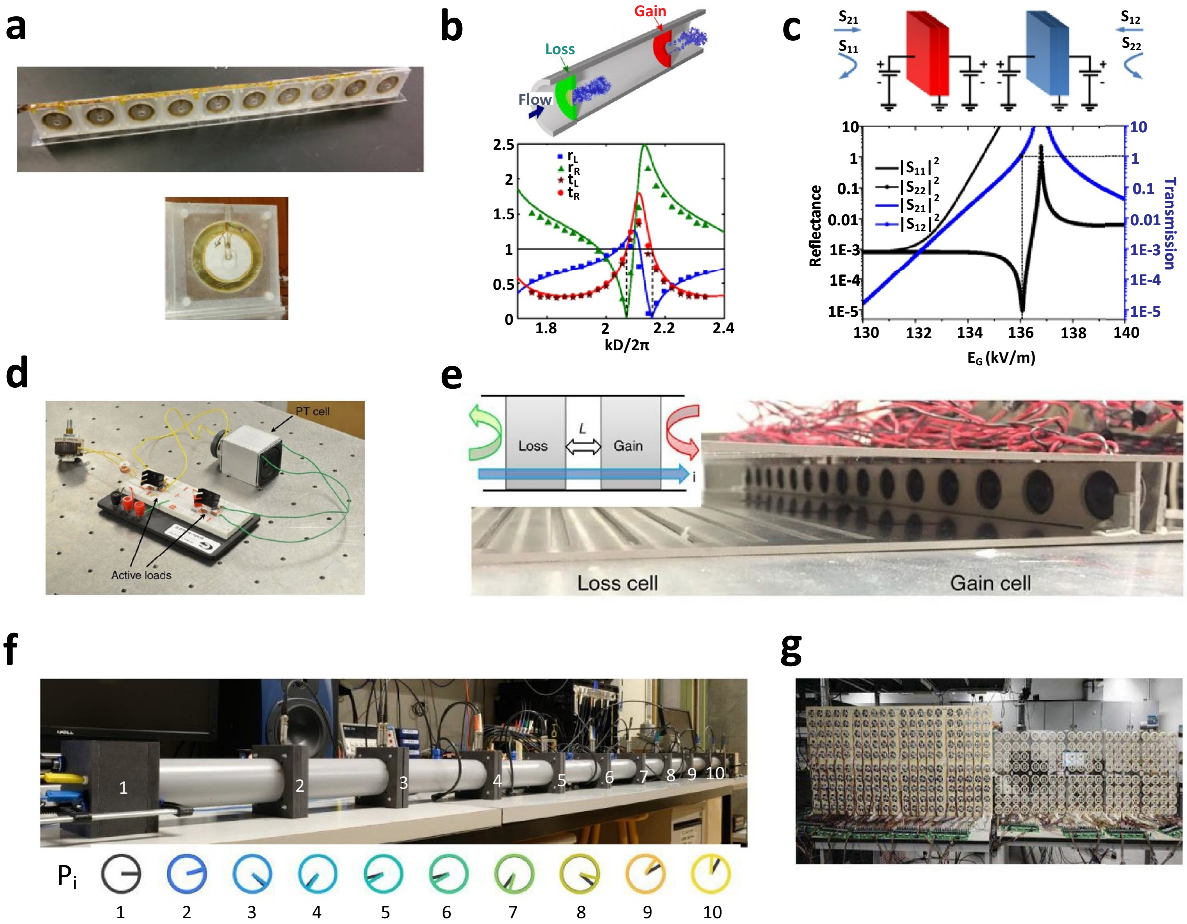


**Fig. 6.** Non-Hermitian acoustic metamaterials. (a) Photograph of a 1D active acoustic metamaterial whose effective properties are controlled via an external voltage applied to piezoelectric membranes [189]. (b) An acoustic *PT* symmetric system consiting of two units of diphragms (top). By tunning the flow rate and the geometry of each diaphragm, the *PT* symmetric system is tailored to its exceptional point, allowing one to achieve unidirectional invisibity (bottom) [187]. c, An acoustic *PT* symmeteric system achieved through electrically biased pizoelectric semiconductors, enabling unidirectional suppressed reflectance [204]. (d) An acoustic *PT* symmetric meta-atom, consisting of a pair of loudspeakers loaded with non-Foster electrical circuits, used to drive the scattering matrix near its exceptional point, which corresponds to an anisotropic transmission resonance, aka unidirectional cloaking [186]. (e) Optimizations of the distance between gain and loss elements in a *PT* symmetric system allows one to have a wider control for accessing the exceptional points by introducing an additional degree of freedom [205]. (f) Non-Hermitian acoustic metamaterial supporting constant amplitude pressure waves despite the presence of Hermitian disorder. Gain, loss and Hermitian disorder are synthezised using a FPGA-based feedback control loop. The bottom plots show the measured pressure phasors at 10 sites along the device [208]. (g) An active acoustic metasurface composed of circular elastic membranes units loaded with small magnet discs, used to control the reverberating sound fields in a room [213]. All figures are adapted with permission.

### 3.1.3. *Constant amplitude acoustic waves*

In disordered materials, signals cannot propagate efficiently due to multiple scattering, which is a severe limitation in wave engineering. Theoretical proposals, however, have predicted that engineering gain and loss distribution in a non-Hermitian system allows one to create a new family of waves whose intensity remains constant in space despite the presence of a strongly scattering medium [206,207]. These waves are indeed the generalization of plane waves to disordered non-Hermitian media. The practical implementation of these theoretical concepts, however, is quite challenging as it requires constructing continuous gain–loss profiles. In their proposal [208], Rivet et al. extended the notion of constant-intensity scattering states to a system with a discrete gain-loss distribution. More specifically, the work demonstrated that the required gain-loss distribution to achieve reflection-less states in non-Hermitian systems can be discretized while maintaining a discrete version of the constant intensity wave property. The successful operation of this new design principle was then established in a disordered acoustic waveguide loaded with set of non-Hermitian electroacoustic resonators, controlled with a specially designed feedback control scheme [209] (Fig. 6f). Each loudspeaker is connected to a FPGA controller, assigning a target acoustic impedance to it. By properly regulating the impedances of the resonators to add the proper discretized gain and loss distribution, an initially opaque disordered material was turned into a completely transparent one in which the acoustic pressure was uniform. This demonstrated the relevance of adding gain and loss to a system for overcoming both its opacity and sensitivity to losses.

### 3.2. Externally biased active AMMs

#### 3.2.1. Reconfigurable metasurfaces

Similar to the case of AMMs, the performance of a passive MS is hindered by its limited bandwidth and high absorption losses. Intense research is therefore ongoing to develop active acoustic MSs, suggesting exciting practical applications, for instance in noise control and mitigation, with the simplest possible realization. An important example includes the relevance of active MSs for binary-type phase modulators (SSM, for Spatial Sound Modulator), being capable of controlling reverberating sound in a room. Inspired by previous works performed in the microwave range [210–212], Ma. et.al proposed in [213] an active acoustic MS composed of circular elastic membranes units loaded with small magnet discs (Fig. 6g). By applying a dc voltage to the magnetic discs, the resonance frequency of the inner part of the membrane could be actively shifted between two frequencies. The obtained reflection phase shift in the corresponding intermediate frequency region was then leveraged to achieve the SSM. Such an effect was then used to control the reverberating sound fields in a room, inducing focalization of sound or silent zones. Other exciting applications of such active MSs includes digitizing 3D metamaterials bricks into binary units, dramatically simplifying design of AMMs and paves the way towards their commercial realization [214]. Altogether, not only do these two-dimensional versions of active AMMs overcome the shortcomings of bulk metamaterials with their light-weight and conformal designs, but also they allow the unique feature of dynamic reconfigurability.

#### 3.2.2. Non-reciprocal meta-atoms

Reciprocity is a fundamental, but not always desirable, property of wave propagation: when putting a source at point A, the received field at point B has the same magnitude and phase as it would have at point A if the source was placed at point B. In other words, if I can hear my neighbors through a common wall, they can also hear me. Unlike for electromagnetic waves, where conventional ferromagnetic materials have been used to induce non-reciprocal effects in the microwave range, acoustic non-reciprocity is not readily achievable as airborne sound waves do not interact with magnetic fields. Hence, approaches to break the conventional wisdom of sound reciprocity have mostly relied on non-linear effects that, unfortunately, are accompanied with unwanted features such as bulky structures, high power consumption, large signal distortion and sequential-only port operation [215–219].

Very recently, however, it was shown [220,221] that a time-independent bias velocity applied to a stationary fluid can strongly break the time-reversal symmetry and induce acoustic non-reciprocity. Such an appealing effect was then exploited to construct an

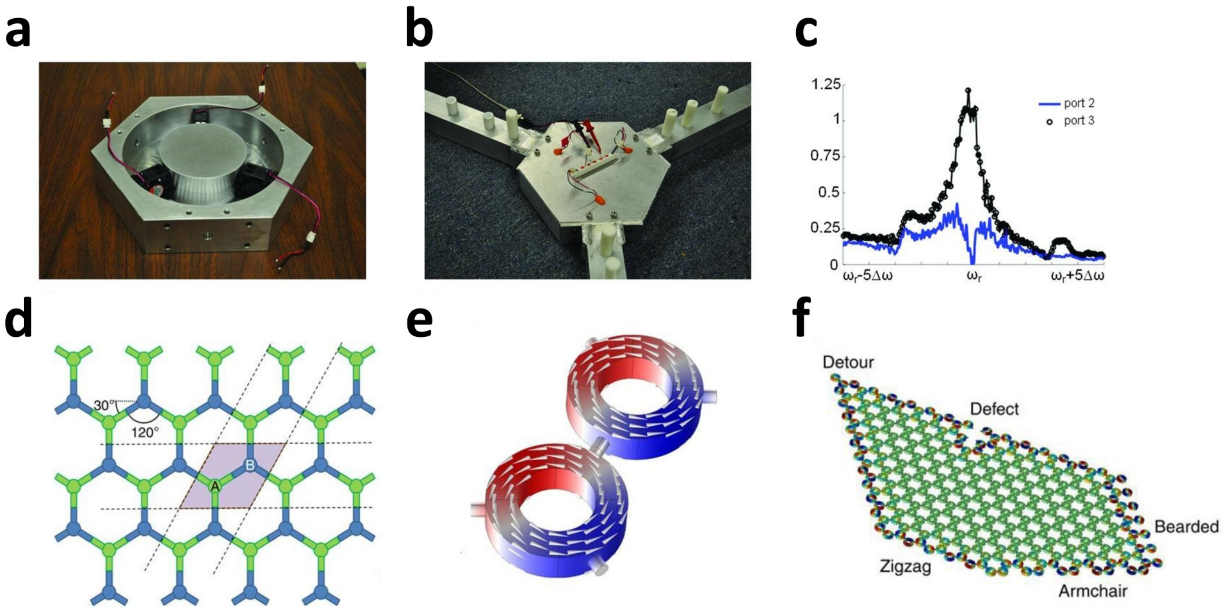


**Fig. 7.** Acoustic metadevices with broken time-reversal symmetry. (a) Imparting a constant rotating bias velocity flow to the air filling a ring cavity using fans allows one to break time-reversal symmetry and split the energies of degenerate counterpropagating modes, analogous to Zeeman splitting for electronic levels under magnetic bias [220]. (b) An acoustic circulator can be built by connecting three ports to the ring cavity represented in panel a. The measured scattering parameters in panel **c** demonstrates the behavior of the circulator as a largely non-reciprocal device. (d)–(f) A honeycomb lattice of circulators behaves like a graphene layer under a constant magnetic field. The Zeeman meta-atoms are connected via small pipes and biased by the same flow (panel **e**). When the bias is turned on, a topological band gap opens, associated with chiral edge states (panel f), which are unidirectional and protected against backattering, even in the presence of lattice defects or any type [224].

acoustic meta-atom steering sound to certain directions in a non-reciprocal manner. The meta-atom was composed of a ring resonator cavity filled with a fluid (air) on which a rotational bias velocity was applied (Fig. 7a). By analogy to the Zeeman effect in quantum mechanics, the applied rotational velocity resulted in energy splitting between degenerate counter propagating modes of the ring cavity, and for a proper choice of velocity induced giant non-reciprocity by modal interference. Such non-reciprocal behavior was then exploited to realize a compact acoustic circulator (shown in Fig. 7b), in which the power is only allowed to flow among the ports in a circular way, achieving a one-way acoustic roundabout (Fig. 7c).

### *3.2.3. Topological metamaterials with external drive*

Not only has the scheme explained above for breaking time reversal symmetry enabled the realization of non-reciprocal acoustic devices, but it also constituted a novel platform for translating interesting concepts in quantum physics, requiring broken time reversal symmetry, to acoustics. An important example is the concept of quantum Hall-effect that, after the notion of topological insulators, led to the invention of a new class of topological insulators known as Chern insulators [222]. Although these kinds of TIs had been demonstrated for microwave using magnetically biased photonic crystals [223], their extension to acoustics remained unexplored until very recently due to the absence of efficient ways to break time reversal symmetry of sound propagation. The Zeeman acoustic meta-atoms described in the previous section, however, opened the door for transposing these types of topological phases to acoustics. In [224], Khanikaev et al. proposed to realize an acoustic Chern insulator by constructing a hexagonal lattice (Fig. 7c) of the Zeeman meta-atoms shown in Fig. 7d (another work independently and simultaneously proposed a related idea [170]). When the bias velocity was turned on, one could open complete bandgaps surrounded by bands characterized by a non-trivial topological invariant. The associated edge modes have one-way character in the sense that, at one specific interface, they carry acoustic energy only along a given direction, even in the presence of defects along their way. Such protection against back-refection holds for any kind of disorder, as seen in Fig. 7e. Finally, we note an alternative route to achieve edge states with broken time reversal symmetry: the use of acoustic time-Floquet topological insulators, i.e. artificial acoustic media that are modulated in time. Fleury et al. [225] proposed an acoustic lattice of modulated resonators, characterized by non-trivial topological invariant due to the interaction between different Floquet harmonics [225,226].

## 4. Outlook and conclusion

Driven by a simple scientific curiosity, acoustic metamaterials have now turned into a well-established field with promising practical applications. We now know well how to design AMMs with macroscopic behavior which used to seem outlandish a decade ago. Despite this tremendous progress, there are promising directions that have been left largely uncharted. More specifically, the common reliance of AMMs on resonances poses critical issues, in terms of bandwidth and losses, on their applicability. Broadening the bandwidth and reducing the losses of AMMs is therefore a bottleneck for establishing their role in a large variety of technology-oriented applications. Fortunately, the nascent field of active acoustic metamaterials, discussed in the previous section, offers a unique solution to this challenge. As discussed, coupling electronic circuits to electromechanical transducers such as loudspeakers can be considered as a convenient way to supply gain to an acoustic system, and avoid bandwidth limitations imposed by passivity. Apart from its relevance for probing the physics of non-Hermitian systems and *PT* symmetric structures, such an elegant approach paves the way to fully compensate the intrinsic dissipations of AMMs and achieve lossless properties. Furthermore, the fact that acoustic dynamics occur on a much slower timescale than electrodynamics can be viewed as an advantage for achieving ultra-broadband AMMs using electro-acoustic systems in which the electrical side can respond almost instantaneously to changes occurring on the acoustic side. Such types of artificial structures with broadband characteristics may become even more interesting when combined with artificial intelligent systems, or learning machines. It is therefore exciting to imagine that a new generation of "auto-reconfigurable" AMMs may soon emerge, featuring much greater performance than the current state of the art, and enabling exotic wave functionalities such as broadband cloaking, tunable focusing, dynamic signal processing and programmable analog computing.

The research reviewed here mostly dealt with stationary media, resulting in constitutive effective parameters which are time independent. While time and space play the same role in sound propagation, the role of time modulation in AMMs has not been extensively studied. Just like the way that spatial engineering of material parameters overturned our conventional views on sound propagation, temporal modulation of constitutive parameters can also open up new research horizons in modern acoustics. Interestingly, in electromagnetism, it has been shown that removing the time invariance constraint in designing metamaterials can lead to unusual wave behavior such as instantaneous wave generation [227], and molding radiation patterns. Time-dependent media can also provide a full control over time-reversed waves, which are of great interest in acoustics imaging and medical therapy. We note, however, that these so-called four-dimensional metamaterials are undeniably more complex than their three-dimensional counterparts, as they involve active components with time-varying parameters, however they may be more easy to implement in acoustics than in electromagnetics, due to the slower time scales involved. We therefore envision a very bright future for acoustic metamaterials if we allow them to fully “go active” and merge wave engineering with other sciences including electronics, control systems or artificial intelligence.

Finally, we note that while this review focused on sonic metamaterials, which are generally built from solid materials and manipulate sound carried in the background fluid, the extension of many concepts presented here to elastic waves carried either by thins solid plates (such as Lamb waves), or thicker elastic solids (such as Rayleigh waves) is also possible [228–234].

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