Closed-Form Solution to Disambiguate Defocus Blur in Single-Perspective Images

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Abstract: Depth-from-defocus techniques suffer an ambiguity problem where depth planes on opposite sides of the focal plane have identical defocus. We solve the ambiguity by relying on the wavelength-dependent relationship between defocus and depth. We conduct a robustness analysis and validation on consumer lenses.   © 2019 The Author(s)

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1.   Mathematical Framework and Solution

Our proposed solution is developed in a simple lens framework. Blur is characterized by the radius of a circle of confusion, representing the image of a point light source, and is wavelength dependent. For a wavelength channel C, the blur radius is $r_C(d) = L|1 - x/f_C + x/d|$, where L is the simple lens aperture radius, x is the distance between the center of the lens and the sensor plane, f_C is the lens' focal length for channel C and d is the depth of the source point relative to the lens. For a given channel C and sensor position, the focal plane is at depth d_C^0, at which the blur radius is minimal: $r_C(d_C^0) \to 0$. This depth is given by $d_C^0 = x(x/f_C - 1)^{-1}$. Varying the depth d, under a realistic scenario where $x > f_C$, the blur radius is a convex function of d, with a minimum at the focal plane. This creates an ambiguity that was solved recently using coded-aperture lenses [1] or calibrated and memorized for modified lenses over shallow pre-defined depth ranges [2]. Here, we leverage the differential blur across spectral channels. We consider two channels A and B of different wavelength and set w.l.o.g. $f_B > f_A$. Noticing that the blur radius is wavelength-dependent, we study the measure $\Delta_{B,A}(d) \triangleq r_B(d) - r_A(d)$ given by

$$
\Delta_{B,A}(d) = \begin{cases} \alpha \triangleq L(x/f_A - x/f_B) & d \leq d_A^0 \\ 2L(1+x/d) - L(x/f_A + x/f_B) & d \in [d_A^0, d_B^0] \\ -\alpha = L(x/f_B - x/f_A) & d \geq d_B^0. \end{cases} \tag{1}
$$

For d smaller than d_A^0 or larger than d_B^0, $\Delta_{B,A}(d)$ is constant and only depends on the camera parameters (L and x) and the focal lengths of the two spectral channels A and B. In practice, f_A and f_B are close between RGB channels, making the depths d_A^0 and d_B^0 close, which in turn decreases α. Also, complex lenses are designed to correct color chromatic aberration and minimize the shift between color focal planes. Sec. 3 shows that the shift is nevertheless detectable even with complex lenses, and that with near-infrared (NIR) the corresponding α value becomes noticeably larger, allowing for a more robust solution (Fig. 1).

The Δ measure can be estimated directly from an input image. For a given image patch $I(d)$, captured at wavelength λ_C, we observe a blurred version $I_b(d, \lambda_C) = I(d) * PSF_{eq}(d, \lambda_C, u, v) = I(d) * H_{def}(d, \lambda_C, u, v) * H_0(u, v)$, where d is the depth of the object in pixel coordinates (u, v). H_{def} represents defocus blur in channel C, and H_0 all wavelength-invariant blur (e.g., motion blur). The variance of PSF_{eq} is $\sigma_{eq}^2(d, \lambda_C, u, v) = \sigma_{def}^2(d, \lambda_C, u, v) + \sigma_0^2(u, v)$. When the blur radius $r_C(d)$ is estimated, $\sigma_{eq}^2(d, \lambda_C, u, v)$ is approximated instead of the desired $\sigma_{def}^2(d, \lambda_C, u, v)$, modifying the estimated value of $r_C(d)$ by an offset shift [3]. As $\sigma_0^2(u, v)$ is invariant with respect to wavelength, the measure $\Delta_{B,A}(d)$ defined as $\sigma_{def}^2(d, \lambda_B, u, v) - \sigma_{def}^2(d, \lambda_A, u, v)$, can be estimated by the pixel-wise subtraction $\sigma_{eq}^2(d, \lambda_B, u, v) - \sigma_{eq}^2(d, \lambda_A, u, v)$ that cancels the undesirable offset. The sign of this estimated $\Delta_{B,A}(d)$ is sufficient to resolve the blur ambiguity, and obtain a bijective blur-depth mapping.

2.   Error Bounds Analysis for the Measure Δ

Channels capture radiation across a wavelength range determined by the sensor's color filter array. Looking at the intensity present in a channel C, the captured radiation may have any wavelength $\lambda_C \pm \delta_C$ where λ_C is the central wavelength and δ_C is a wavelength shift bound to the filter limits. In the extreme case, all radiation captured in the channel C has wavelength $\lambda_C + \delta_C^{max}$ instead of λ_C, where δ_C^{max} is the maximum deviation. δ_C^{max} corresponds to a shift γ_C^{max} in focal length. We also denote by e_C the algorithmic blur estimation error, and obtain the predicted blur
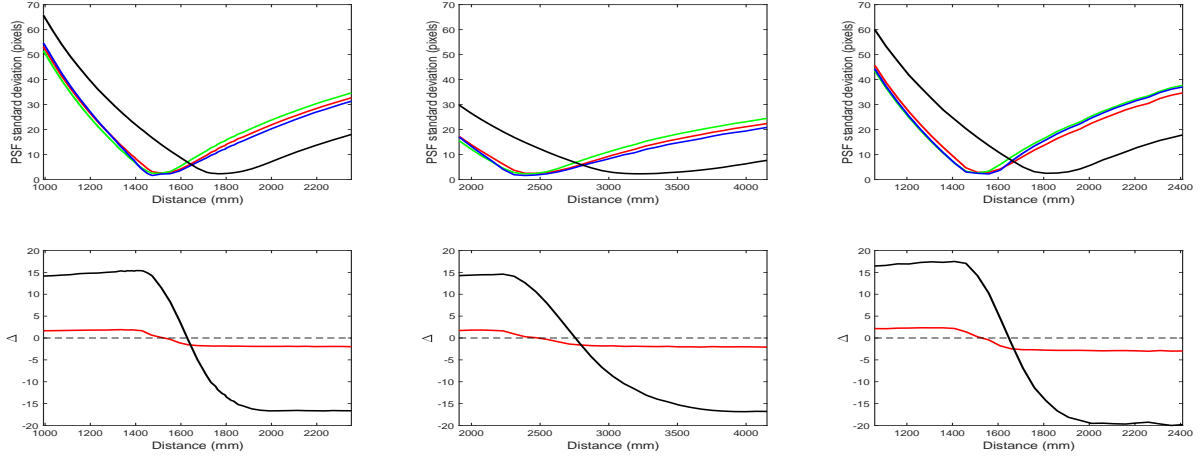
Fig. 1. Top: blur magnitude vs depth for the RGB (color) and NIR (black) channels, on 3 image sets. Blur is estimated as the standard deviation of a Gaussian fitted to the edge spread function, computed on a 5° ISO 12233 edge. Bottom: corresponding Δ plots for $\Delta_{R,G}$ (red) and $\Delta_{NIR,G}$ (black).

radius $r'_C(d) = L\left|1 - \frac{x}{f_C + \gamma_C^{max}} + \frac{x}{d}\right| + e_C$. The resulting difference $\Delta'_{B,A}(d) \triangleq r'_B(d) - r'_A(d)$ becomes

$$\Delta'_{B,A}(d) = \begin{cases} L\left(\frac{x}{f_A + \gamma_A^{max}} - \frac{x}{f_B + \gamma_B^{max}}\right) + E_{B,A} & d \leq d_A^{0'} \\ 2L\left(1 + \frac{x}{d}\right) - L\left(\frac{x}{f_A + \gamma_A^{max}} + \frac{x}{f_B + \gamma_B^{max}}\right) + E_{B,A} & d \in [d_A^{0'}, d_B^{0'}] \\ L\left(\frac{x}{f_B + \gamma_B^{max}} - \frac{x}{f_A + \gamma_A^{max}}\right) + E_{B,A} & d \geq d_B^{0'}, \end{cases} \qquad (2)$$

where $E_{B,A} \triangleq er_B - er_A$, and the estimated focal plane depth of channel C is $d_C^{0'}$ for $C \in \{A; B\}$, given by $d_C^{0'} = x(\frac{x}{f_C + \gamma_C^{max}} - 1)^{-1}$. The depth point of interest (d'_n) for our method is the point of intersection where the two channels show equal blur radii, yielding $\Delta'_{B,A}(d'_n) = 0$. Depth d'_n, assuming $E_{B,A}$ is not substantially large, is given by

$$d'_n = \frac{2Lx}{L\left(\frac{x}{f_A + \gamma_A^{max}} + \frac{x}{f_B + \gamma_B^{max}}\right) - E_{B,A} - 2L}. \qquad (3)$$

A mistake in region mapping (defined by the sign of Δ) can cause the loss of the bijective mapping only when $d'_n < d_A^0$ or $d'_n > d_B^0$. Shifts resulting in $d'_n \in [d_A^0, d_B^0]$ preserve a correct one-to-one mapping. This is because both A and B channels have an bijective mapping and either of them can be used in the range $[d_A^0, d_B^0]$. Therefore, discrepancies between d_n and d'_n can be tolerated and do not affect the disambiguation process as long as $d'_n \in [d_A^0, d_B^0]$.

3. Experimental Results

We show that the relationships derived between defocus blur, depth and wavelength extend to a typical lens, and demonstrate the generalization of our proposed measure Δ. PSF magnitude plots are obtained experimentally with a Canon EF 50mm f/2.5 lens for the first two plots (with different focal planes) and with a Canon EF 50mm f/1.8 II lens for the third plot (Fig. 1). For our proposed method to generalize, the main property of our Δ measure must still hold. Essentially, this means that its value must change signs only once with increasing depth, and exactly in the depth range delimited by the two focal planes corresponding to the chosen channels. It is advantageous to compute Δ between a color channel and the NIR channel due to the larger differential blur between them. This is due to the larger wavelength separation, and due to the uncorrected chromatic aberration in NIR (done only in superachromatic lenses [4, p. 105]). However, the property of Δ is valid even between spectrally adjacent channels, namely green and red. The Δ plots are shown in the bottom row of Fig. 1, where $\Delta_{R,G}$ and $\Delta_{NIR,G}$ are plotted in red and black, respectively. They each change signs correctly between their corresponding focal planes.

References

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