# Numerical simulations of plasma fuelling in tokamaks using the GBS code

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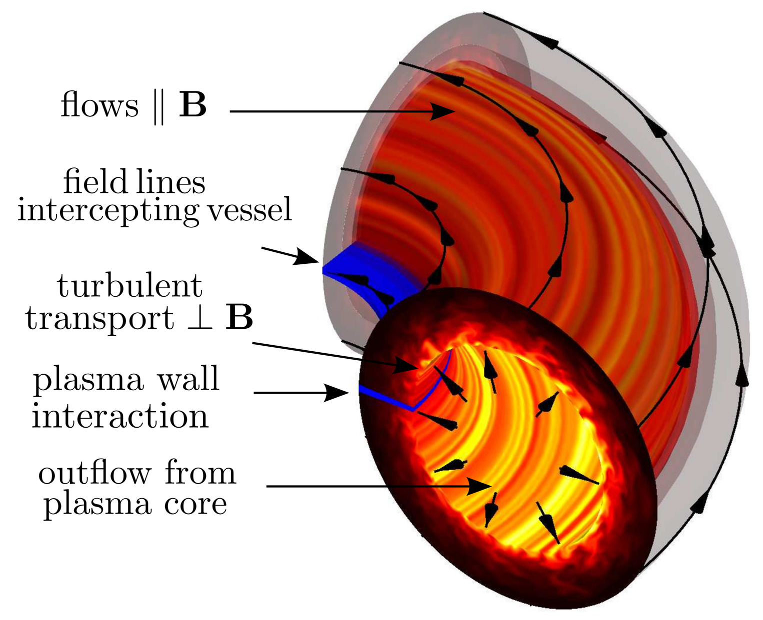
## Introduction




- In tokamaks Scrape-Off Layer (SOL), **magnetic field lines intersect the walls** of the fusion device
- **Heat and particles** flow along magnetic field lines and are **exhausted to the vessel**
- **Turbulence** amplitude and size **comparable** to **steady-state** values
- **Neutral** particles interact with the plasma
- SOL plays a key role on determining the **refuelling** of the plasma

The **Global Braginskii Solver (GBS) code:**
**a 3D, flux-driven, global turbulence code in limited geometry**
used to study **plasma turbulence in the SOL**

- GBS is a simulation code to evolve plasma turbulence in the edge of fusion devices. [Halpern *et al.*, JCP 2016], [Ricci *et al.*, PPCF 2012]
- GBS solves 3D **fluid equations for electrons and ions**, Poisson's and Ampere's equations, and a **kinetic equation for neutral atoms**.

## The Global Braginskii Solver (GBS) code

### Two fluid drift-reduced Braginskii equations, $k_\perp^2 \gg k_\parallel^2$, $d/dt \ll \omega_{ci}$

$$\frac{\partial n}{\partial t} = -\frac{\rho_*^{-1}}{B}[\phi, n] + \frac{2}{B}[C(p_e) - nC(\phi)] - \nabla\cdot(n v_{\parallel e}\mathbf{b}) + \mathcal{D}_n(n) + S_n + n_n\nu_{iz} - n\nu_{rec} \quad (1)$$

$$\frac{\partial \Omega}{\partial t} = -\frac{\rho_*^{-1}}{B}\nabla_\perp\cdot[\phi,\omega] - \nabla_\perp\cdot\left[\nabla_\parallel\left(v_{\parallel i}\omega\right)\right] + B^2\nabla\cdot(j_\parallel\mathbf{b}) + 2BC(p) + \frac{B}{3}C(G_i) + \mathcal{D}_\Omega(\Omega) - \frac{n_n}{n}\nu_{cx}\Omega \quad (2)$$

$$\frac{\partial U_{\parallel e}}{\partial t} = -\frac{\rho_*^{-1}}{B}[\phi, v_{\parallel e}] - v_{\parallel e}\nabla_\parallel v_{\parallel e} + \frac{m_i}{m_e}\left[\frac{\nu j_\parallel}{n} + \nabla_\parallel\phi - \frac{\nabla_\parallel p_e}{n} - 0.71\nabla_\parallel T_e - \frac{2}{3n}\nabla_\parallel G_e\right] + \mathcal{D}_{v_{\parallel e}}(v_{\parallel e}) + \frac{n_n}{n}(\nu_{en} + 2\nu_{iz})(v_{\parallel n} - v_{\parallel e}) \quad (3)$$

$$\frac{\partial v_{\parallel i}}{\partial t} = -\frac{\rho_*^{-1}}{B}[\phi, v_{\parallel i}] - v_{\parallel i}\nabla_\parallel v_{\parallel i} - \frac{\nabla_\parallel p}{n} - \frac{2}{3n}\nabla_\parallel G_i + \mathcal{D}_{v_{\parallel i}}(v_{\parallel i}) + \frac{n_n}{n}(\nu_{iz} + \nu_{cx})(v_{\parallel n} - v_{\parallel i}) \quad (4)$$

$$\frac{\partial T_e}{\partial t} = -\frac{\rho_*^{-1}}{B}[\phi, T_e] - v_{\parallel e}\nabla_\parallel T_e + \frac{4T_e}{3B}\left[\frac{C(p_e)}{n} + \frac{5}{2}C(T_e) - C(\phi)\right] + \frac{2T_e}{3n}\left[0.71\nabla\cdot(j_\parallel\mathbf{b}) - n\nabla\cdot(v_{\parallel e}\mathbf{b})\right] + \mathcal{D}_{T_e}(T_e) + \mathcal{D}^\parallel_{T_e}(T_e) + S_{T_e} + \frac{n_n}{n}\nu_{iz}\left[-\frac{2}{3}E_{iz} - T_e + \frac{m_e}{m_i}v_{\parallel e}\left(v_{\parallel e} - \frac{4}{3}v_{\parallel n}\right)\right] - \frac{n_n}{n}\nu_{en}\frac{m_e}{m_i}\frac{2}{3}v_{\parallel e}(v_{\parallel n} - v_{\parallel e}) \quad (5)$$

$$\frac{\partial T_i}{\partial t} = -\frac{\rho_*^{-1}}{B}[\phi, T_i] - v_{\parallel i}\nabla_\parallel T_i + \frac{4T_i}{3B}\left[\frac{C(p_e)}{n} - \frac{5}{2}\tau C(T_i) - C(\phi)\right] + \frac{2T_i}{3n}\left[\nabla\cdot(j_\parallel\mathbf{b}) - n\nabla\cdot(v_{\parallel i}\mathbf{b})\right] + \mathcal{D}_{T_i}(T_i) + \mathcal{D}^\parallel_{T_i}(T_i) + S_{T_i} + \frac{n_n}{n}(\nu_{iz} + \nu_{cx})\left[\tau^{-1}T_n - T_i + \frac{1}{3\tau}(v_{\parallel n} - v_{\parallel i})^2\right] \quad (6)$$

$$\rho_\star = \rho_s/R_0, \qquad \mathbf{b} = \frac{\mathbf{B}}{B}, \qquad [A,B] = b\cdot(\nabla A\times\nabla B), \qquad C(A) = \frac{B}{2}\left[\nabla\times\frac{b}{B}\right]\cdot\nabla A, \qquad \nabla_\parallel f = \mathbf{b}_0\cdot\nabla f + \frac{\beta_{e0}}{2}\frac{\rho_*^{-1}}{B}[\psi, f]$$

$$p = n(T_e + \tau T_i), \qquad U_{\parallel e} = v_{\parallel e} + \frac{\beta_{e0}}{2}\frac{m_i}{m_e}\psi, \qquad \Omega = \nabla\cdot\omega = \nabla\cdot(n\nabla_\perp\phi + \tau\nabla_\perp p_i)$$

- Equations implemented in GBS, a **flux-driven** plasma turbulence code with limited geometry to study SOL heat and particle transport
- System completed with **first-principles boundary conditions** applicable at the magnetic pre-sheath entrance where the magnetic field lines intersect the limiter [Loizu *et al.*, PoP 2012]
- Parallelized using domain decomposition, **excellent parallel scalability** up to $\sim$ 10000 cores
- Gradients and curvature discretized using **finite differences**, Poisson Brackets using Arakawa scheme, integration in time using **Runge Kutta method**
- Code **fully verified** using method of manufactured solutions [Riva *et al.*, PoP 2014]
- Note: $L_\perp \to \rho_s$, $L_\parallel \to R_0$, $t \to R_0/c_s$, $\nu = ne^2R_0/(m_i\sigma_\parallel c_s)$ normalization

### The Poisson and Ampere equations

- **Generalized Poisson equation**, $\nabla\cdot(n\nabla_\perp\phi) = \Omega - \tau\nabla_\perp^2 p_i$
- **Ampere's equation** from Ohm's law, $\left(\nabla_\perp^2 - \frac{\beta_{e0}}{2}\frac{m_i}{m_e}n\right)v_{\parallel e} = \nabla_\perp^2 U_{\parallel e} - \frac{\beta_{e0}}{2}\frac{m_i}{m_e}nv_{\parallel i}$
- Stencil based **parallel multigrid** implemented in GBS
- Elliptic equations separable in parallel direction allow for **independent 2D solutions** for x-y plane

### The kinetic neutral atoms equation

$$\frac{\partial f_n}{\partial t} + \vec{v}\cdot\frac{\partial f_n}{\partial \vec{x}} = -\nu_{iz}f_n - \nu_{cx}n_n\left(\frac{f_n}{n_n} - \frac{f_i}{n_i}\right) + \nu_{rec}f_i \quad (7)$$

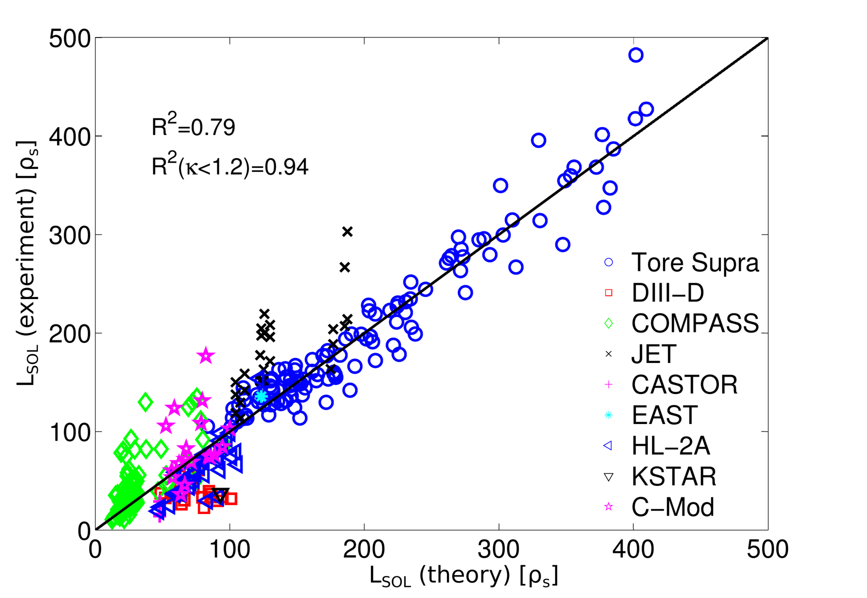
- **Method of characteristics** to obtain the formal solution of $f_n$ [Wersal *et al.*, NF 2015]
- **Two assumptions**, $\tau_{\text{neutral losses}} < \tau_{\text{turbulence}}$ and $\lambda_{\text{mfp, neutrals}} \ll L_{\parallel,\text{plasma}}$, leading to a 2D steady state system for each x-y plane
- **Linear integral equation** for neutral density obtained by integrating $f_n$ over $\vec{v}$
- **Spatial discretization** leading to a linear system of equations

$$\begin{bmatrix} n_n \\ \Gamma_{out}\end{bmatrix} = \begin{bmatrix} K_{p\to p} & K_{b\to p} \\ K_{p\to b} & K_{b\to b}\end{bmatrix}\cdot\begin{bmatrix} n_n \\ \Gamma_{out}\end{bmatrix} + \begin{bmatrix} n_{n,rec} \\ \Gamma_{out,rec} + \Gamma_{out,i}\end{bmatrix} \quad (8)$$

- This system is solved for neutral density, $n_n$, and neutral particle flux at the boundaries, $\Gamma_{out}$, with the threaded LAPACK or MUMPS (serial or parallel) solvers.

## Past achievements of GBS

- Characterization of **non-linear turbulent regimes** in the SOL
- **SOL width scaling** as a function of dimensionless / engineering plasma parameters
- Origin and nature of **intrinsic toroidal plasma rotation** in the SOL
- Mechanisms regulating the SOL **equilibrium electrostatic potential**




## Open questions on plasma fueling in tokamaks

- Where is the plasma created and how is it transported?
- How do neutral flows influence the plasma density profile?
- How is fueling affected by the $n$, $T$ profiles and the poloidal location of the gas puffs?

This requires:

- Quantitative assessment of **plasma and neutral flows**
- **Mass-conserving model** (total ions + neutrals kept constant within the simulation)

These will also allow to address:

- Influence of neutrals in the formation of **different SOL regions**
- High density effects (formation of the **density shoulder**, **Greenwald density limit**...)

## Moving towards a mass-conserving model

GBS was modified to ensure **mass conservation** (ions + neutrals):

- 2 changes were implemented to make the **continuity equation exactly satisfied**
  - **Radially variable inverse aspect ratio** $\epsilon = \frac{r}{R_0}$ to take into account **curvilinear geometry**
  - **Parallel gradient terms** included in Poisson brackets and curvature operators

$$[\phi, A] = P_{yx}[\phi, A]_{yx} + \mathbf{P}_{\mathbf{x}\parallel}[\phi,\mathbf{A}]_{\mathbf{x}\parallel} + \mathbf{P}_{\parallel\mathbf{y}}[\phi,\mathbf{A}]_{\parallel\mathbf{y}}, \quad C(A) = C^x\frac{dA}{dx} + C^y\frac{dA}{dy} + \mathbf{C}^\parallel\nabla_\parallel\mathbf{A}$$

$$[\phi, A]_{uv} = \frac{d\phi}{du}\frac{dA}{dv} - \frac{d\phi}{dv}\frac{dA}{du}, \quad P_{yx} = \frac{a_0}{Jb^\varphi}, \quad \mathbf{P}_{\mathbf{x}\parallel} = \frac{\mathbf{b}_{\theta^*}}{\mathbf{J}\mathbf{b}^\varphi}, \quad \mathbf{P}_{\parallel\mathbf{y}} = \frac{\mathbf{a_0 b_r}}{\mathbf{J}\mathbf{b}^\varphi}$$

$$C^x = -\frac{2B}{J}\frac{dc_\varphi}{d\theta^*}, \quad C^y = \frac{a_0B}{2J}\left[\frac{dc_\varphi}{dr} + \frac{1}{q}\left(\frac{dc_{\theta^*}}{dr} - \frac{dc_r}{d\theta^*}\right)\right], \quad \mathbf{C}^\parallel = \frac{\mathbf{B}}{\mathbf{2Jb}^\varphi}\left(\frac{\mathbf{dc_r}}{\mathbf{d}\theta^*} - \frac{\mathbf{dc}_{\theta^*}}{\mathbf{dr}}\right)$$

Straight-field-line right-handed coordinates set: $(y, x, z) = (a_0\theta^*, r, R_0\varphi)$

$\theta^*$ defined by $b^\varphi = qb^{\theta^*}$ (with $q$ the safety factor) $\quad c_i = \frac{b_i}{B} \quad J = rR_0\frac{\left(1-\epsilon^2\right)^{3/2}}{(1-\epsilon\cos(\theta^*))^2}$

- **Neutrals generated by boundary recycling** were made to match the **plasma outflow**
  - **Boundary conditions** were changed by adding the diamagnetic and $E\times B$ contributions

$$(n\vec{v})^{\theta^*} = nv_{\parallel e}b^{\theta^*} + (\mathbf{n}\vec{\mathbf{v}}_{de})^{\theta^*} + (\mathbf{n}\vec{\mathbf{v}}_{E\times B})^{\theta^*}, \qquad (n\vec{v})^r = (\mathbf{n}\vec{\mathbf{v}}_{de})^r + (\mathbf{n}\vec{\mathbf{v}}_{E\times B})^r$$

$$\vec{v}_{de} = \frac{1}{B^2}\vec{\nabla}p_e\times\vec{B}, \qquad (n\vec{v}_{de})^{\theta^*} = -\frac{1}{JB^2}\left[\frac{\partial p_i}{\partial x}B_\phi - \frac{\partial p_i}{\partial z}R_0B_r\right], \qquad (n\vec{v}_{de})^r = -\frac{1}{JB^2}\left[-\frac{\partial p_i}{\partial y}a_0B_\phi + \frac{\partial p_i}{\partial z}R_0B_{\theta^*}\right]$$

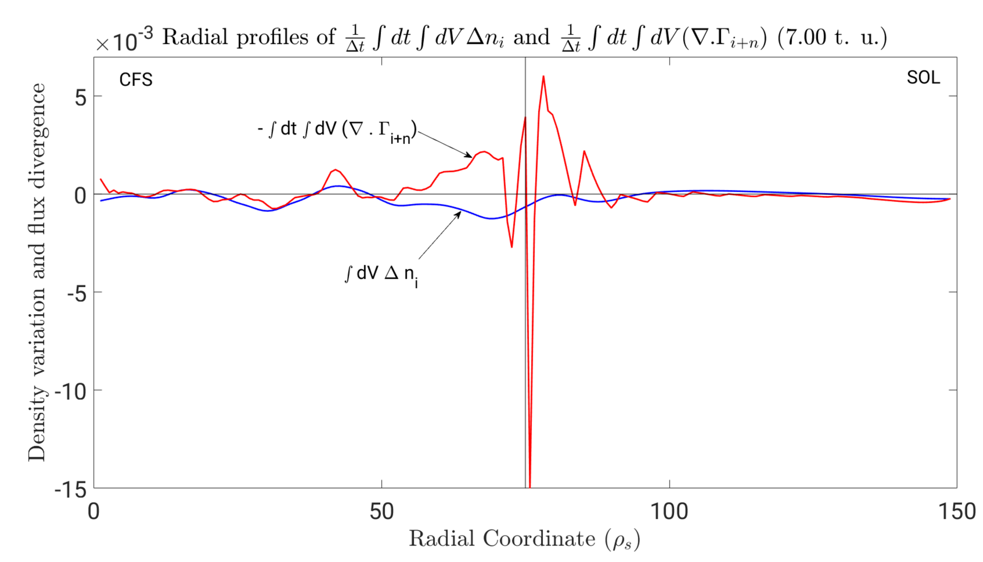
$$v_{\vec{E\times B}} = -\frac{n}{B^2}\vec{\nabla}\phi\times\vec{B}, \qquad (n\vec{v}_{E\times B})^{\theta^*} = -\frac{n}{JB^2}\left[\frac{\partial\phi}{\partial x}B_\phi - \frac{\partial\phi}{\partial z}R_0B_r\right], \qquad (n\vec{v}_{E\times B})^r = -\frac{n}{JB^2}\left[-\frac{\partial\phi}{\partial y}a_0B_\phi + \frac{\partial\phi}{\partial z}R_0B_{\theta^*}\right]$$

**Mass conservation** is evaluated by checking the balance of the number of particles:

- Continuity equation is integrated over volume and time
- **Neutral density is conserved** within the model, so $(n_n\nu_{iz}) = -\vec{\nabla}\cdot\vec{\Gamma}_{neutral}$
- Density balance given by $\int dt\int dV\,\frac{dn}{dt} = -\int dt\int dV\,(\vec{\nabla}\cdot\vec{\Gamma}_{ion} + \vec{\nabla}\cdot\vec{\Gamma}_{neutral})$

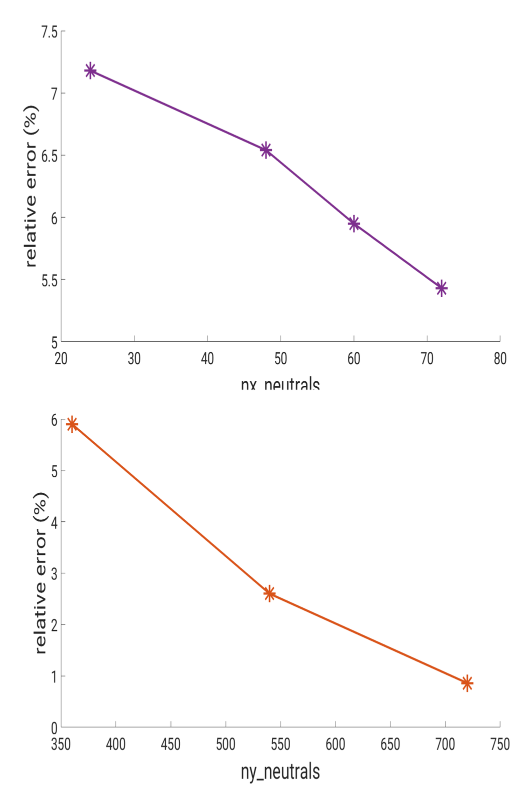
## 1D radial model

**Radial balance of particles by integrating over $\theta^*$ and $\phi$**



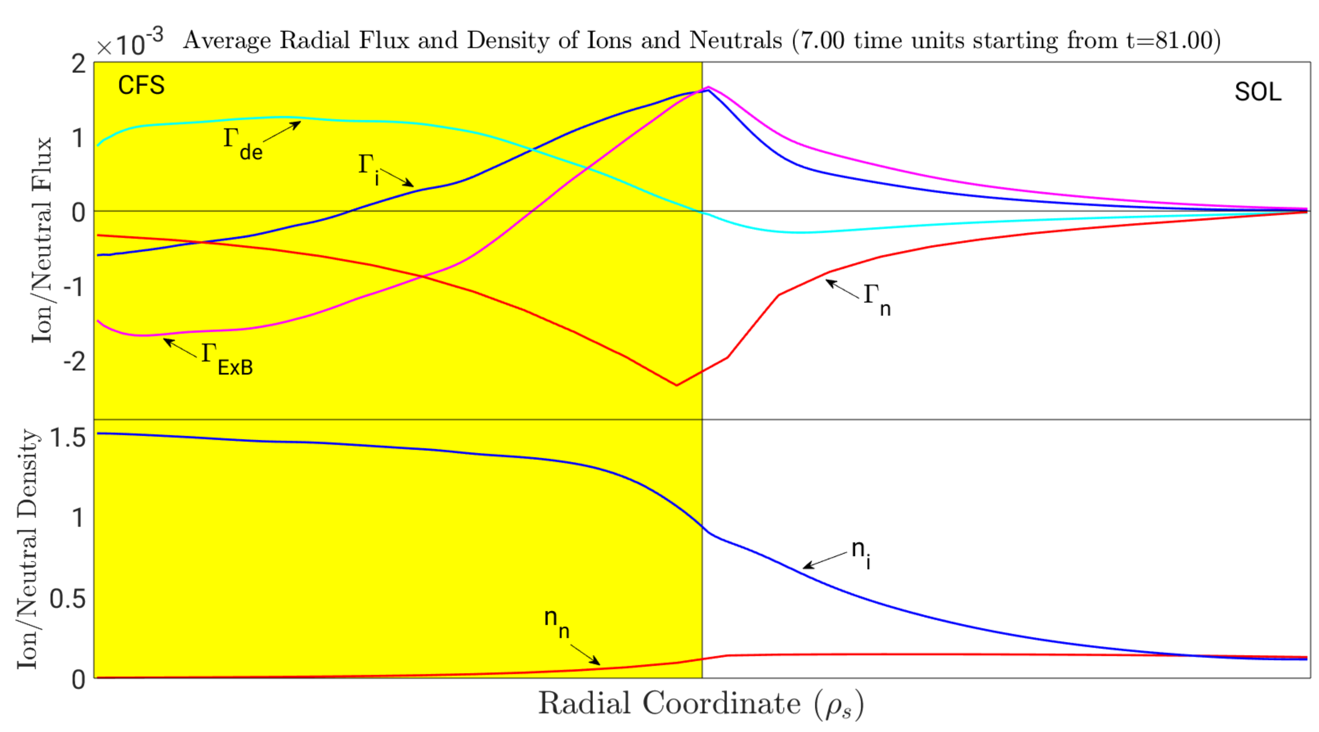

**GBS simulation parameters**:

- Circular magnetic equilibrium
- $R_0 = 500\rho_s$
- $L_x = 150\rho_s$, $L_y = 800\rho_s$
- Limiter at $x = 75 - 150\rho_s$
- 7 GBS time units
- Time step of $\Delta t = 3.75\times10^{-5}$




- **Density variation is slightly negative** almost everywhere as a result of **neutral and ion outflow** from the core
- Density variation profile is roughly flat (uniform variation rate)
- Reasonable matching at CFS region and SOL far from LCFS
- Curves strongly mismatch near LCFS due to very large gradients - much greater resolution required.
- Neutrals are conserved during calculation up to an error that **converges with grid resolution** (*nx_neutrals* and *ny_neutrals*)

**Quantitative assessment of ion and neutral fluxes**




- Ion and neutral fluxes profiles are similar but not symmetric since system is **not in a steady state**
- Both **ions and neutrals outflow to the core**
- Ion flux in the SOL is dominated by the **E × B flux** (outward pointing)
- Ion flux in CFS region determined by **competition between E × B and diamagnetic contributions**

EUROfusion

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