



## Using spatial and spatial-extreme statistics to characterize snow avalanche cycles

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### Abstract

In December 2008, an intense avalanche cycle occurred in the eastern part of the southern French Alps. Using this case study, this paper illustrates how spatial statistics can be used to analyse such abnormal temporal clusters of snow avalanches. Spatial regression methods are used to quantify aggregation and gradients and highlight the three day snowfall as the main explanatory factor. A max-stable model is developed to evaluate the snowfall return period, so as to compare the studied cycle with previous ones and with empirical return periods for avalanche counts.

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Selection and peer-review under responsibility of Spatial Statistics 2011

*Keywords:* Snow avalanche cycle, snowfall, spatial regression, max-stable process, return period

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### 1. Introduction

An avalanche cycle is an abnormal temporal cluster of snow avalanche events occurring during a few days in a given mountain range (e.g. Birkeland et al. 2001 [1]). The cause is generally a severe storm bringing high snowfalls, but strong temperature variations causing snowmelt and/or fluctuations of the freezing level can also be involved. Understanding the links between avalanche cycles and climatic factors is important for hazard forecasting and risk mitigation, especially in the current context of climate change that increases variability at high altitudes.

While it now is relatively straightforward to precisely evaluate the magnitude/frequency relationship on given paths (e.g. Eckert et al. 2010 [2]), quantifying the magnitude of an avalanche cycle that affects a

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whole mountain range over several days is more difficult. Different data sources must be used and confronted, and the spatial and temporal scales that provide the best description of the phenomenon must be found to make sound historical comparisons.

In December 2008, an intense avalanche cycle occurred in the southern French Alps. Southerly atmospheric fluxes that progressively evolved into an easterly return caused important snowfalls during three days. Cold temperatures and drifting snow had important aggravating effects, so avalanche activity was high during 5 days, with a total of 209 avalanches recorded in the French “EPA” avalanche database (Fig 1). Some of the avalanches had very long runouts that exceeded historical limits recorded in the avalanche atlas (Gaucher et al. 2010 [3]). Two massifs of the southern French Alps were strongly affected: Queyras and Mercantour, where important roads were closed over several consecutive days. Villages such as Ristolas (Queyras massif) and ski resorts such as Isola 2000 (Mercantour massif) were thus isolated. A few buildings were partially destroyed, for instance in Saint Etienne de Tinée (Mercantour massif), and ski lifts as well as forests were damaged.

This cycle has been deeply analysed in terms of snow avalanche climatology in Eckert et al. 2010 [4]. The aim of this paper is to emphasize the methodological approach, mainly the use of relatively simple spatial statistics tools to highlight the 3-day snowfall as the main explanatory factor of the cycle, and the use of a max-stable model to characterise its return period spatially. An important constraint is that different data structures had to be considered: avalanche reports, point measurements of continuous weather fields, and gridded results from numerical modelling of meteorological and snow cover variables. For the latter, the scale is the massif which is used for data assimilation and avalanche forecasting in an operational context (Durand et al. 1999 [5]).

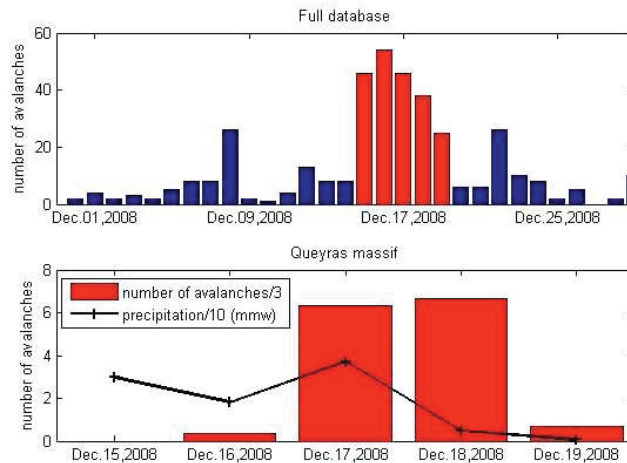


Fig. 1. Number of avalanches per day recorded in the French EPA database derived from Eckert et al. 2010 [4]. In red, the 5 days defined as the avalanche cycle. Top corresponds to the full database and bottom to one of the two most affected massifs, Queyras.

## 2. Spatial analysis of the December 2008 avalanche cycle

To characterise spatial patterns at the massif scale, the classical Moran index  $I$  has been used:

$$I_k = \frac{N \sum_{i=1}^N \sum_{j=1}^N \left( \omega_{ij}^{(k)} \left( y_i - \bar{y} \right) \left( y_j - \bar{y} \right) \right)}{\sum_{i=1}^N \sum_{j=1}^N \left( \omega_{ij}^{(k)} \right) \times \sum_{i=1}^N \left( y_i - \bar{y} \right)^2} \quad (1)$$

where  $N$  is the number of massifs,  $y$  is the variable of interest,  $\bar{y}$  is the mean of  $y$  and  $\omega_{ij}$  is a matrix of spatial weights (inverse distances). The notation  $I_k$  and  $\omega_{ij}^{(k)}$  denotes that the index can be computed for different distance classes (lags), so as to investigate how spatial autocorrelation varies with distance. The expected value of Moran's  $I$  under the null hypothesis of no spatial autocorrelation is  $E(I_k) = -1/(N-1)$ . Lower values indicate a negative spatial autocorrelation, i.e. anti-clustering. Conversely, higher values indicate presence of clusters in the observed spatial patterns. Significance of the observed aggregation/repulsion patterns were tested against the null hypothesis of random spatial sampling. Computations were made using a very simple weight matrix based on a minimal number of nearest neighbours at a given lag. Results clearly show the strong spatial heterogeneity of avalanche activity during the cycle across the French Alpine massifs, and the very similar patterns for the 3-day snowfall evaluated at the massif scale by the meteorological model after data assimilation: a strongly significant Moran's  $I$  at small distances and a significant repulsion at lag 5 (Fig 2).

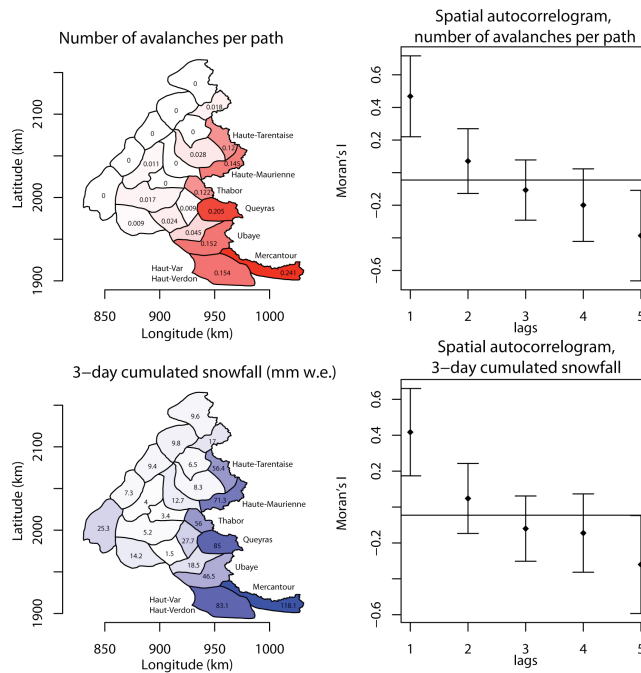


Fig. 2. Spatial patterns during the cycle at the massif scale derived from Eckert et al. 2010 [4]: (a) number of avalanches per path; (b) associated spatial autocorrelogram; (c) three-day snowfall at the massif scale; (d) associated spatial autocorrelogram

To explain these patterns, different regressions were performed using the generic linear model:

$$y_i = \beta_0 + \sum_{k=1}^P x_{ik} \beta_k + \varepsilon_i \tag{2}$$

where  $y_i$  is the number of avalanches per path and massif or the 3-day snowfall per massif,  $x_{ik}$  the considered covariates,  $(\beta_0, \beta_k)$  the parameters to be estimated and  $\varepsilon_i \sim N(0, \sigma_y^2)$  the local residuals. Spatial gradients were obtained by fitting the model with  $x = \{lat, long\}$ , where  $(lat, long)$  denotes the

latitude and longitude of the centre of each massif. For the two variables, both gradients are significant at the 95% confidence level. The direction of maximal gradient is between  $-14^\circ$  and  $-20^\circ$ , i.e. in the South-East direction. Spatial regression is therefore good ( $R^2=0.76$  for the number of events per path and  $R^2=0.68$  for the 3 day snowfall). Furthermore, corrected residuals (Cliff and Ord, 1981 [6]) are no longer spatially correlated according to Moran's  $I$  test against random spatial sampling. These similarities in spatial patterns explain why the 3 day snowfall, as a single covariate, explains extremely well ( $R^2=0.89$ ) the avalanche activity during the cycle at the massif scale and can be considered as the predominant climatic driver of this exceptional event.

### 3. Evaluating high return period snowfalls using max stable processes

A max-stable process  $Z(x)$  is the limit process of maxima of i.i.d. random fields  $Y_i(x)$  such as:

$$Z(x) = \lim_{n \rightarrow \infty} \frac{\max_{i=1}^n Y_i(x) - b_n(x)}{a_n(x)} \quad (3)$$

where  $a_n$  and  $b_n$  are suitable sequences of functions (De Haan, 1984 [7]). All marginal distributions are GEV, with location, scale and shape parameters depending on the spatial coordinates, with the limit case of a Gumbel distribution if  $\xi(x) = 0$ :

$$P(Z(x) \leq z) = \exp \left( - \left( 1 + \xi(x) \left( \frac{z - \mu(x)}{\sigma(x)} \right) \right)^{-1/\xi(x)} \right) \quad (4)$$

The extremal coefficient  $\theta(h)$  measures dependence between two locations separated by the distance  $h$  (Schlather and Tawn, 2003 [8]). Suitable transformation to Frechet unit margins  $Z^*(x)$  leads to:

$$P(Z^*(x) \leq z, Z^*(x+h) \leq z) = P(Z^*(x) \leq z)^{\theta(h)} = \exp(-\theta(h)/z) \quad (5)$$

A max stable model is an interesting way to combine extreme value theory and geostatistics, i.e. to quantify spatial dependence between tails of distributions. Several formulations are now available (Brown and Resnick, 1977 [9], Smith, 1990 [10], Schlather, 2002 [11]), leading to different expressions of the extremal coefficient  $\theta(h)$ . The pairwise likelihood is then accessible, which can be used to fit a set of block maxima in space by maximising the composite likelihood of all pairs (Padoan et al., 2010 [12]).

This framework has been applied to all the long snowfall series available in the French Alps for different durations. Data have been standardized to retire the orographic effect. Different formulations of the extremal coefficient have been tested, with the constraint of a simple smooth evolution of the GEV parameters, to provide easy access to any quantile such as the one corresponding to the 100-year return period in space. Competing models have been evaluated using an adapted penalized likelihood criterion (Takeuchi, 1976 [13]). Retained model for both one and 3 day snowfall is a modified Brown-Resnick one inspired from Blanchet and Davison, 2011 [14], so as to take anisotropy into account. Anisotropy is much more visible for one day than for 3 day snowfall (Fig 3), but necessary to obtain the best fit in both cases. By comparison to the tested Schlather and Smith formulations, the Brown-Resnick one has the advantage of allowing asymptotic independence at large distances, and to be relatively flexible at short distances.

In terms of smooth evolution of the GEV parameters, the selected model for the 3-day snowfall is:

$$\begin{cases} \mu(x) = -317.34 + 0.104lat + 0.131long \\ \sigma(x) = -64.14 + 0.076long \\ \xi(x) = 0.055 \end{cases} \quad (6).$$

This pleads for higher values in mean in the North West, and higher variability in the South East, which is compatible with the double Atlantic-Mediterranean influence on the climate of the French Alps. No influence is though inferred for the shape parameter  $\xi(x)$ , whose constant positive value indicates slightly higher quantiles than predicted by a Gumbel model.

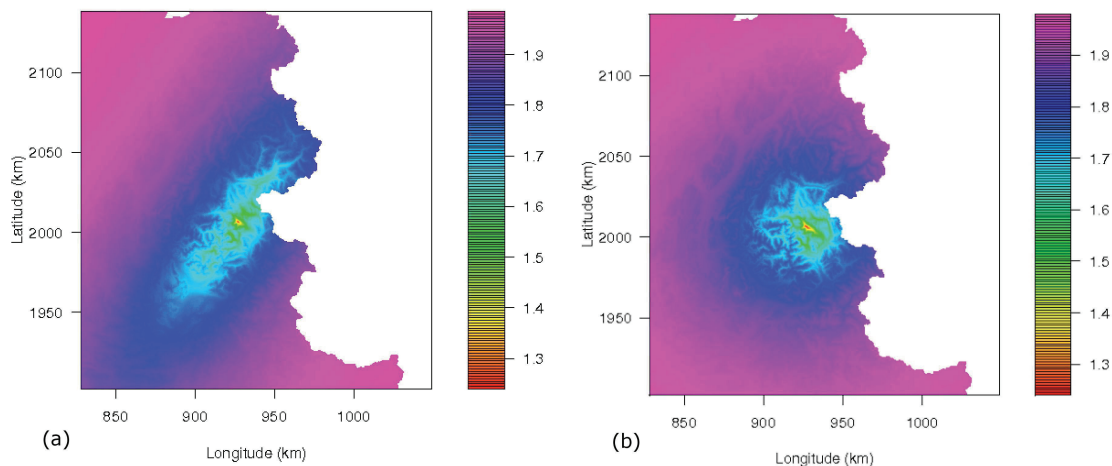


Fig. 3. Snowfall extremal coefficient  $\theta$ : (a) daily maximum; (b) 3-day snowfall. Reference station: Le Monetier les Bains.

#### 4. Return period of the December 2008 avalanche cycle

The max-stable model was used to evaluate the return period of the 3-day snowfall for all measurements made during the December 2008 cycle. Kriging was then used to obtain a continuous function of space. Highest 3-day snowfall return periods were close to 10 years in the extreme east of the Queyras massif, and in the Mercantour massif, but with very strong intra massif gradients (Fig 4).

For comparison, empirical return periods were computed at the massif scale for registered avalanche counts (Fig 5). Spatial patterns are similar, but largely enhanced, with highest values of 54 years in Queyras and Mercantour. This result is obtained because the cycle corresponds to the historical maximum since the early beginning of the EPA report in these two massifs. Discrepancies between 3-day snowfall and avalanche return periods can be attributed to aggravating factors, mainly snowdrift and cold temperatures, to the strong intra-massif snowfall gradients (most of the avalanche paths are close to the Italian border, where the snowfall has been the most intense), but also to the high imprecision related to the empirical estimation used for avalanche counts. This method has been chosen because of the discrete nature of avalanche counts that makes difficult direct use of extreme value theory. It highlights the need for, in the future, developing extreme values statistical models adapted to discrete observations.

#### Acknowledgements

This work was partially done within the ECANA project (French Ministry of the Environment, Risk Division-DGPR). It has also benefited from the European INTERREG ALCOTRA support through the DYNAVAL program.

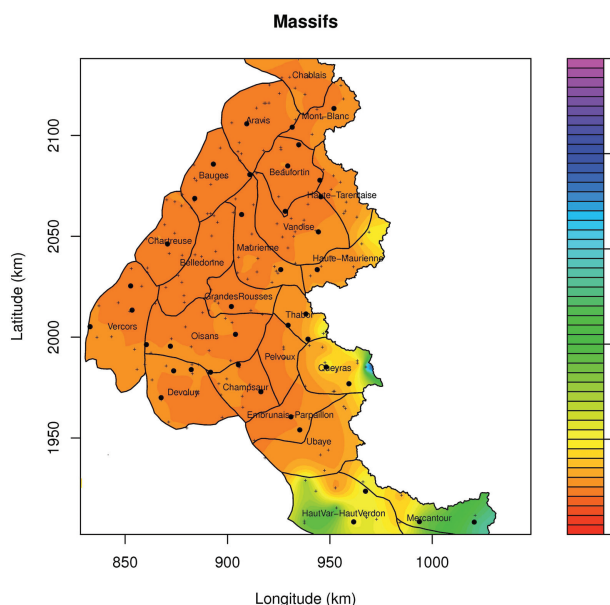


Fig. 4. December 2008 cycle's return period (years): three-day snowfall derived from Eckert et al. 2010 [4]. Dots: long series used for model calibration. Crosses: other available data for the December 2008 cycle

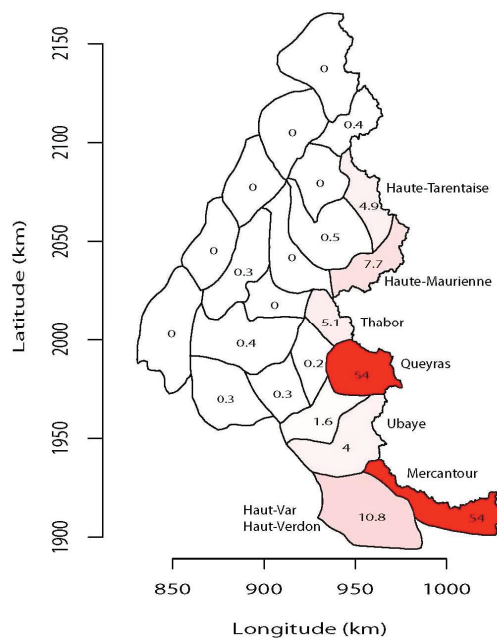


Fig. 5. December 2008 cycle's return period (years) derived from Eckert et al. 2010 [4]: number of events per massif.

## References

- [1] Birkeland KW, Mock CJ, Shinker JJ. Avalanche extremes and atmospheric circulation patterns. *Ann Glaciol* 2001;**32**:135-140.
- [2] Eckert N, Naaim M, Parent E. Long-term avalanche hazard assessment with a Bayesian depth-averaged propagation model. *J Glaciol* 2010;**56**(198):563-586.
- [3] Gaucher R, Escande S, Bonnefoy M, Pasquier X, Eckert N. A look back on the avalanche cycle in Queyras in December 2008. *Proceedings of the International Snow Science Workshop, Lake Tahoe, USA, 18-22 October 2010*:6p.
- [4] Eckert N, Coleou C, Castebrunet H, Giraud G, Deschates M, Gaume J. Cross-comparison of meteorological and avalanche data for characterising avalanche cycles: the example of December 2008 in the eastern part of the French Alps. *Cold Reg Sci Technol* 2010;**64**:119-136.
- [5] Durand Y, Giraud G, Brun E, Merindol L, Martin E. A computer based system simulating snowpack structures as a tool for regional avalanche forecasting. *J Glaciol* 1999;**151**:469-484.
- [6] Cliff AD, Ord JK. *Spatial processes: models and applications*. Taylor & Francis 1981.
- [7] De Haan L. A spectral representation for max-stable processes. *Ann Probab* 1984;**12**:1194–1204.
- [8] Schlather M, Tawn JA. A dependence measure for multivariate and spatial extreme values: Properties and inference. *Biometrika* 2003;**90**:139-156.
- [9] Brown BM, Resnick SI. Extreme values of independent stochastic processes. *J Appl Probab* 1977;**14**:732–739.
- [10] Smith RL. Models Max-stable processes and spatial extremes. *Unpublished manuscript* 1990:32p.
- [11] Schlather M, Models for stationary max-stable random fields. *Extremes* 2002;**5**:33-44.
- [12] Padoan SA, Ribatet M, Sisson SA. Likelihood-based inference for max-stable processes. *J Am Stat Assoc* 2010;**105**:263-277.
- [13] Takeuchi K. Distribution of informational statistics and a criterion of model fitting. *Suri-Kagaku* 1976;**153**:12-18.
- [14] Blanchet J, Davison, AC. Spatial modelling of extreme snow depth. Under review 2011.