Sampling at the rate of innovation for ultrasound imaging and localization

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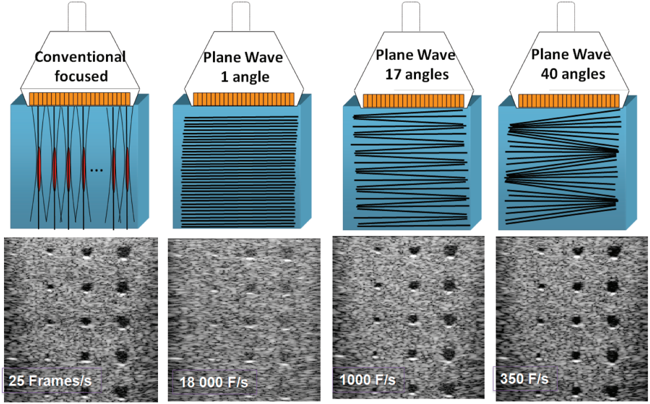
Outline

Motivation

How can we reduce data rate / increase frame rate?

To what extent can recent signal processing techniques (FRI, EDM) be applied to ultrasound imaging?

Ultrasound imaging



Comparing B-mode imaging transmission schemes. From “Ultrafast imaging in biomedical ultrasound” [Tanter and Fink, 2014].

SAMPLING ALONG CHANNELS

Rate of innovation

Rate of innovation: *Number of degrees of freedom per unit of time.*[1]

Classic examples with *finite* rate of innovation (FRI):

- Bandlimited at baseband $\Rightarrow$ RF signal.
- Bandlimited at carrier frequency $\Rightarrow$ IQ demodulation.
- Finite pulse stream.

[1] Coined by "Sampling signals with finite rate of innovation" [Vetterli et al., 2002].

Pulse stream sampling

Assume RF signal can be modeled as T-periodic *pulse stream*:

$$y(t) = \sum_{n\in\mathbb{Z}} \sum_{k=0}^{K-1} c_k \underbrace{h(t - t_k - nT)}_{\substack{\text{known pulse at} \\ \text{arbitrary positions}}}. \tag{1}$$

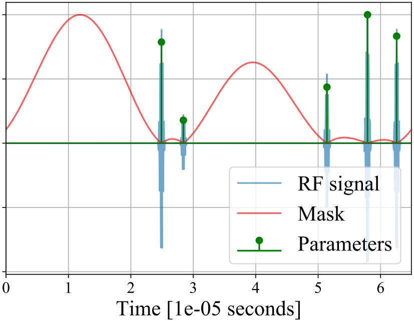
$2K$ parameters $\{c_k, t_k\}_{k=0}^{K-1}$ every period.

$$\text{FRI} = (2K/T).$$

Pulse stream sampling

Unlike BL case, FRI parameters not given directly by samples!

Reconstruction involves finding mask that will yield parameters.[2]



[2] Also known as "annihilating filter" in "Sampling signals with finite rate of innovation" [Vetterli et al., 2002].

PULSE STREAM MODEL FOR ULTRASOUND

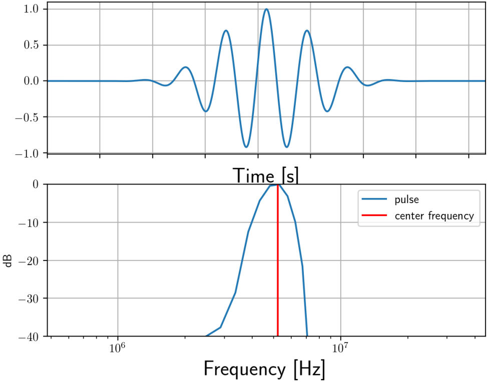
Propagation

Assumptions:

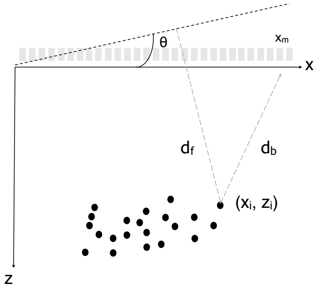
- Homogeneous medium $\Rightarrow$ constant speed of sound.
- No frequency attenuation while propagating through medium.
- Modeling transducer and reflective sources as point sources.
- Born approximation, no multiple scattering.
- Planar wavefront excitation.

Pulse shape (pulse-echo wavelet in literature)

$$h(t) = excitation * transducer_{(elec \rightarrow acoust)} * transducer_{(acoust \rightarrow elec)}.$$



Plane wave insonification for higher frame rate



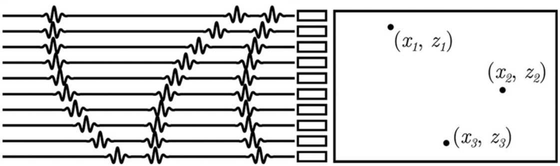
Time-of-flight (TOF) for element at x and point at $\mathbf{r}_k = [x_k, z_k]^T$:

$$\begin{aligned}\tau(\mathbf{r}_k, x, \theta) &= \tau_{tx}(\mathbf{r}_k, \theta) + \tau_{rx}(\mathbf{r}_k, x) \\ &= (x_k \sin\theta + z_k \cos\theta)/c + \|\mathbf{r}_k - [x, 0]^T\|_2/c. \qquad (2)\end{aligned}$$

Measurements / pulse parameters

For K reflectors $\{\mathbf{r}_k = [x_k, z_k]^T\}_{k=0}^{K-1}$ with reflectivity strengths $\{a_k\}_{k=0}^{K-1}$, we measure:

$$y(x,t) = \sum_{k=0}^{K-1} \underbrace{\frac{a_k}{2\pi \|\mathbf{r}_k - [x,0]^T\|_2}}_{c_k} h\Big(t - \underbrace{\tau(\mathbf{r}_k, x, \theta)}_{t_k}\Big). \qquad (3)$$



"Time domain compressive beam forming of ultrasound signals" [David et. al, 2015].

Typical values in medical ultrasound

- $K \gg 1'000$ pulses.
- Finite duration (period) of $T \approx 65\,\mu s$.[3]
- RF sampling rate of $f_s = 20.8\,\mathrm{MHz}$.
- IQ sampling rate of $f_s = 3.47\,\mathrm{MHz}$.
- $M = 32 - 300$ channels.

[3] For $c = 1540\,\mathrm{m\,s^{-1}}$ and imaging depth of 5 cm.

Consequences for pulse stream recovery

Sampling rate for pulse stream given by $f_s \geq (2K/T)$.

Maximum K for given f_s and $T (= 65\,\mu s)$:

- RF: 675 pulses.
- IQ: 112 pulses.

Upper bound on model order for sampling/data rate reduction!

- Sparse strong reflectors are most suitable, e.g. tracking microbubbles.
- Need to "oversample" for robustness to noise.

PULSE STREAM UNDER NOISE

Types of noise

1. Additive noise.
2. Pulse shape estimate error.
3. Model order error.

Increasing sampling rate to achieve robustness against noise.

Denoising approaches

1. Cadzow iterative denoising and total least-squares (standard).[4]
   - **Separately** denoise and obtain parameters.
   - With more measurements, exploit low rank property.
   - Fast(er) but sensitive to pulse estimate errors.
2. Pan's generalized FRI framework (general).[5]
   - **Simultaneously** denoise and obtain parameters.
   - Cast as inverse problem (non-convex).
   - Can be less sensitive to modeling errors but expensive.

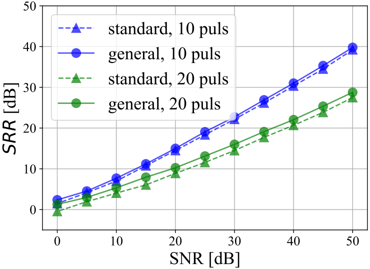
[4] "Sparse Sampling of Signal Innovations" [Blu et al., 2008].

[5] From "Towards Generalized FRI sampling with an application to source resolution in radioastronomy" [Pan et al., 2017].

Comparing denoising approaches

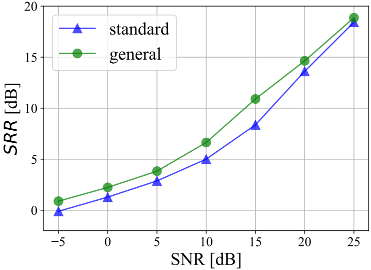
Additive noise to samples of:

- 10 pulses oversampled 10×.
- 20 pulses oversampled 5×.



Pulse shape estimate error for denoising:

- 10 pulses oversampled 10×.



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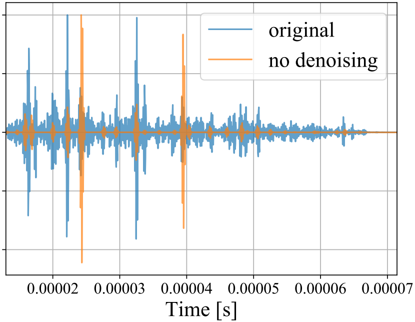
Model order error

Recovering 20 strong reflectors among $38'519$ reflectors with amplitudes 35 dB lower, emulating speckle.



Critical sampling of 30 pulses, no denoising.

Model order error

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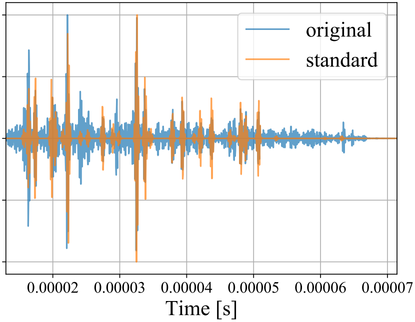

3× oversampling for 30 pulses; standard with 4 iterations of Cadzow.

Model order error

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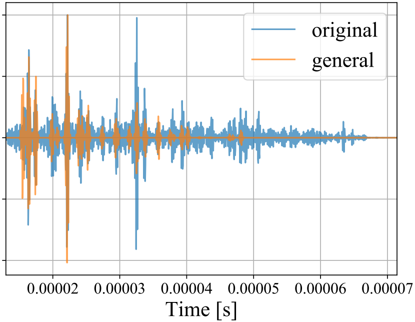
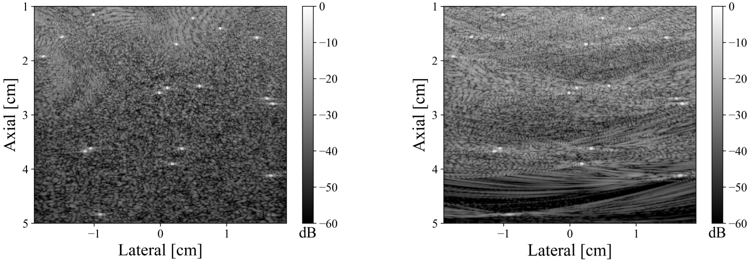


3× oversampling for 30 pulses; 1 random initialization.

Multichannel recovery for beamforming / imaging

Delay-and-sum beamforming with no apodization of:

- True RF signals (left).
- Resynthesized RF signals from estimated parameters with standard denoising (right).

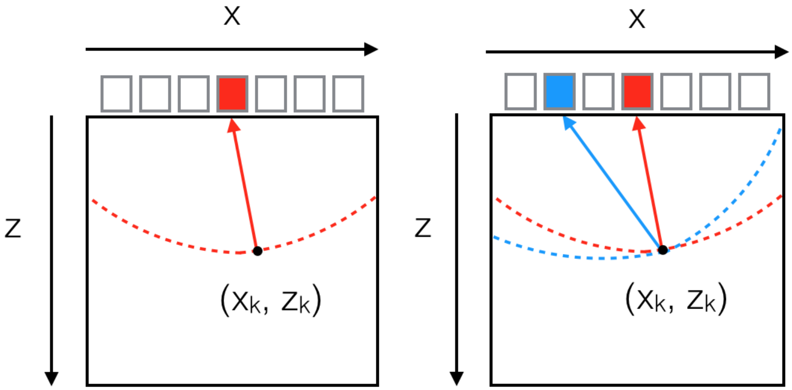


6.7/1.1 sampling rate reduction; 20/3.4 compression from RF/IQ.

SAMPLING ACROSS CHANNELS

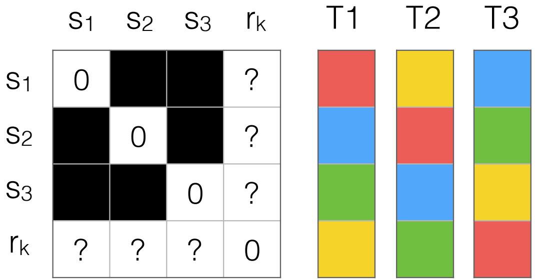
Localization with time-of-flights (TOFs)

Reflector position from TOFs of two elements.



TOF matching with Euclidean Distance Matrices (EDMs)

- How to know which TOFs to pair?
- Modification of echo-sorting algorithm that exploits rank of EDMs.[6]



[6] "Euclidean Distance Matrices" [Dokmanic et al., 2015].

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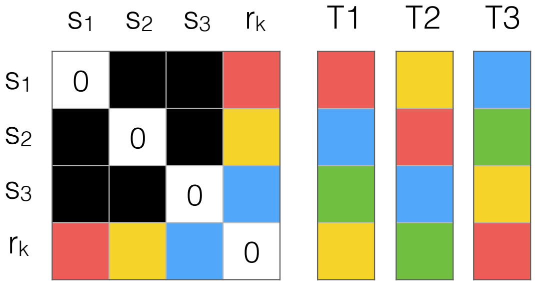

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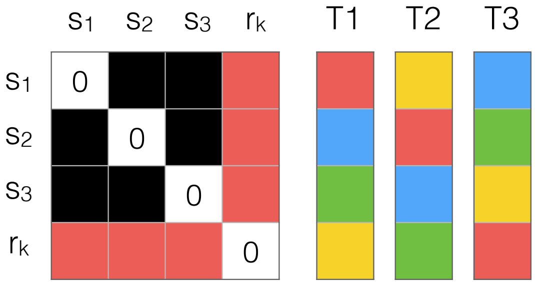


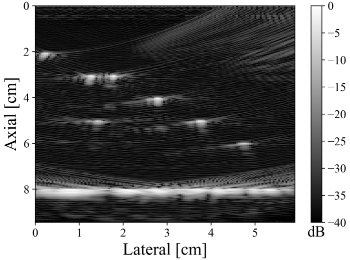
K TOFs, M channels $\Rightarrow K^M$ combinations.

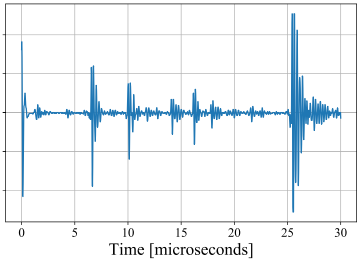
[6] "Euclidean Distance Matrices" [Dokmanic et al., 2015].

LOCALIZATION AT THE RATE OF INNOVATION

Non-destructive evaluation motivation

Drilled holes in aluminum block.

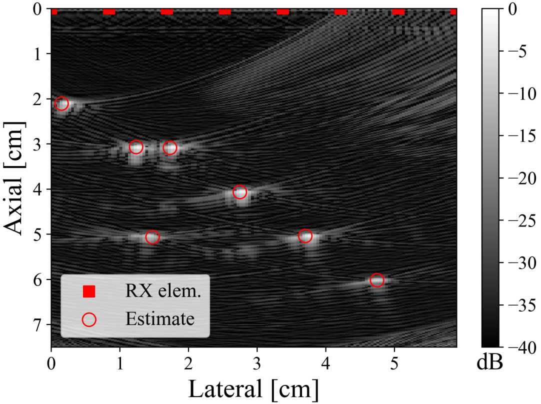




Proposed approach

1. Plane-wave imaging (increase frame rate).
2. Pulse stream recovery + denoising to estimate TOFs (reduction of sampling rate if K sufficiently small).
   - NDE: $f_s = 50\,\text{MHz} \rightarrow 7.14\,\text{MHz}$.
3. EDM-based test for TOF matching (reduce number of channels).
   - NDE: $M = 64 \rightarrow 8$.

Non-destructive evaluation example (cont.)



Concluding remarks

- Exploiting low rank (according to FRI) for:
  - Pulse stream recovery.
  - TOF matching.
- Finding the right application is key $\Rightarrow$ few strong reflectors for rate reduction.
  - For medical ultrasound, high model order is unsuitable.
  - Localization for NDE is promising. More data to validate.
  - Sufficient denoising is crucial.
- Extending to other transmit scheme so that $N_{tx} = N_{rx}$.