Forced response analysis of hydroelectric systems

**S Alligné[1], PCO Silva[2], A Béguin[1], B Kawkabani[2], P Allenbach[2],
C Nicolet[1], F Avellan[3]**

[1] Power Vision Engineering Sàrl , Ecublens, Switzerland
[2] Group of Electrical Machines, EPFL, Ecublens, Switzerland
[3] Laboratory for Hydraulic Machines, EPFL, Lausanne, Switzerland

E-mail: sebastien.alligne@powervision-eng.ch

Abstract. At off-design operating points, Francis turbines develop cavitation vortex rope in the draft tube which may interact with the hydraulic system. Risk resonance assessment by means of eigenmodes computation of the system is usually performed. However, the system response to the excitation source induced by the cavitation vortex rope is not predicted in terms of amplitudes and phase. Only eigenmodes shapes with related frequencies and dampings can be predicted. Besides this modal analysis, the risk resonance assessment can be completed by a forced response analysis. This method allows identifying the contribution of each eigenmode into the system response which depends on the system boundary conditions and the excitation source location. In this paper, a forced response analysis of a Francis turbine hydroelectric power plant including hydraulic system, rotating train, electrical system and control devices is performed. First, the general methodology of the forced response analysis is presented and validated with time domain simulations. Then, analysis of electrical, hydraulic and hydroelectric systems are performed and compared to analyse the influence of control structures on pressure fluctuations induced by cavitation vortex rope.

1. Introduction

In the recent years due to tremendous development and integration of renewable energy resources (NRE), water turbines and pump-turbines are key technical components to achieve both primary and secondary power grid control. The fast change of power generation by NRE is impacting the required operating range of hydro units going from overload down to part load. At this off-design operating points, hydraulic machines experience flow instabilities being excitation source for the complete hydroelectric system. For instance, Francis turbines develop cavitation vortex rope in the draft tube which may interact with the hydroelectric system and resonance or instability phenomena can be experienced. Usually, modal analysis of the system is performed to identify the eigenmodes of the system. This can be completed by a forced response analysis yielding the contribution of each

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[1] To whom any correspondence should be addressed.

eigenmode into the system response depending on the system boundary conditions and the excitation source location.

Stability analysis of hydroelectric systems including hydraulic system, rotating train, electrical system and control devices based on eigenmodes computation is usually performed [1]. However, the forced response in the frequency domain was limited to the analysis of hydraulic systems [2-7], electrical systems [8] or hydroelectric systems with simplified transfer function of the hydraulic part [9,10]. This paper goes further and presents the application of this method to hydroelectric systems including detailed hydraulic models, even if similar study was performed with time domain simulations [11,12].

In the first section, the general methodology of the forced response analysis is presented and validated with time domain simulations. Then, analysis of electrical, hydraulic and hydroelectric systems are performed and compared to analyse the influence of control structures on pressure fluctuations induced by cavitation vortex rope developed in the turbine draft tube.

2. Forced Response Technique

2.1. Transfer Matrix

For SIMSEN, system dynamics is described according to the following set of first order differential equations:

$$[A] \cdot \frac{d\vec{X}}{dt} = [B] \cdot \vec{X} + [C] \cdot \vec{U} + \vec{V}(\vec{X}, \vec{U}) \tag{1}$$

With:

- $\vec{X}$ the state vector with n components;
- $\vec{U}$ the input vector with p components;
- $[A]$ and $[B]$ the nonlinear state global matrices of dimensions $[n \times n]$;
- $[C]$ the input matrix of dimension $[n \times p]$;
- $\vec{V}(\vec{X}, \vec{U})$ the boundary conditions vector with n components.

System's outputs $\vec{Y}$ are formulated through the following set of equations:

$$\vec{Y} = [D].\vec{X} + [E]\vec{U} \tag{2}$$

With:

- $\vec{Y}$ the output vector with q components;
- $[D]$ the output matrix of dimensions $[q \times n]$;
- $[E]$ the feedthrough matrix of dimensions $[q \times p]$.

Frequency response technique is based on linearized set of equations [13]. Indeed, sinusoidal input to a linear system generates sinusoidal response at the same frequency. Hence, the set of equations is first linearized around the solution $\{\vec{X}_0, \vec{U}_0\}$ by considering small perturbations:

$$\begin{cases} \vec{X}_0 + \delta\vec{X} \\ \vec{U}_0 + \delta\vec{U} \end{cases} \tag{3}$$

The resulting linearized set of equations is written as follows:

$$\begin{cases} [A_l] \cdot \dfrac{d\delta \vec{X}}{dt} = [B_l] \cdot \delta \vec{X} + [C_l] \cdot \delta \vec{U} \\ \delta \vec{Y} = [D_l] \cdot \delta \vec{X} + [E_l] \cdot \delta \vec{U} \end{cases} \tag{4}$$

The input vector $\delta \vec{U}$ contains both set points of regulators $\delta \vec{U}_{reg}$ and excitation sources $\delta \vec{U}_{sources}$:

$$\delta \vec{U} = \begin{bmatrix} \delta \vec{U}_{reg} \\ \delta \vec{U}_{sources} \end{bmatrix} \tag{5}$$

Applying the Laplace transformation to the set of differential equations, it yields:

$$[A_l] \cdot s \cdot \delta \vec{X}(s) = [B_l] \cdot \delta \vec{X}(s) + [C_l] \cdot \delta \vec{U}(s)$$

$$\delta \vec{X}(s) = \left(sI - [A_l]^{-1} \cdot [B_l]\right)^{-1} \cdot [A_l]^{-1} \cdot [C_l] \cdot \delta \vec{U}(s) \tag{6}$$

Using equation (2), the relation between the system's output and the input is made:

$$\delta \vec{Y}(s) = \left[[D_l] \left(sI - [A_l]^{-1} \cdot [B_l]\right)^{-1} \cdot [A_l]^{-1} \cdot [C_l] + [E_l] \right] \delta \vec{U}(s)$$

$$\delta \vec{Y}(s) = [M(s)] \cdot \delta \vec{U}(s) \tag{7}$$

With $[M(s)]$ the matrix transfer functions of dimension $[n \times l]$ which relates the system output $\delta \vec{Y}$ to the input vector $\delta \vec{U}$. Considering a system with 2 input excitation sources, the third system's output δY_3 can be represented through the block diagram given in Figure 1a. The frequency response of a system whose matrix transfer function is $[M(s)]$ is given by:

$$[M(j\omega)] = [M(s)]\big|_{s \to j\omega} \tag{8}$$

Since input excitation, forced response and the system itself are sinusoids, an alternate way is to use phasors, see Figure 1b.
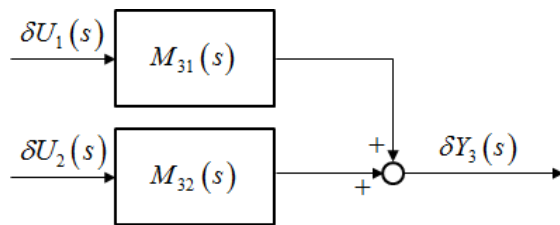

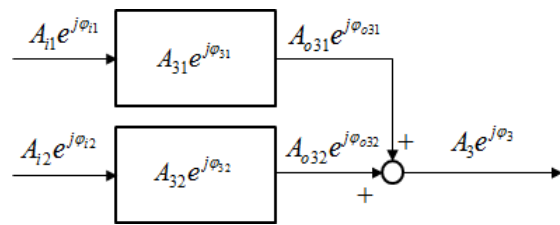**Figure 1a.** Block diagram of the frequency response.

Figure 1b. Block diagram expressed with phasors for sinusoids inputs.

2.2. Validation of the forced response

A forced response analysis tool has been integrated to the SIMSEN software based on the above mentioned methodology. This tool uses the system matrices provided by SIMSEN and the user

specifies excitation sources into the system in a frequency range to analyze the response in the frequency domain. Since time integration of differential equations is not performed, the forced response technique is less time consuming than performing time domain simulations including excitation sources. This new tool is validated by comparing time domain simulation with frequency response.

2.2.1. Hydraulic system

The first validation is performed for a hydraulic system featuring a Francis turbine generating unit at constant speed (dynamics of rotating inertia are neglected) connected to a penstock of 1'100 m length (named "Pipe1"), see Figure 2. The draft tube (named "Pipe2") of the turbine is modeled with a downstream pipe of 36m length in which momentum excitation source is applied to represent pressure fluctuations induced by a cavitation vortex rope. Standard pipe model of constant cross section with a wave speed of 300m/s is used for the draft tube part. However advanced models of draft tube could be used for this study taking into account divergent geometry and convective terms [14]. With a gross head of 315m, the turbine nominal power is 200MW with a rotational speed of 375 rpm.
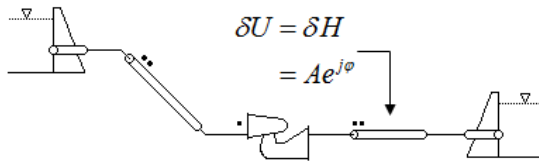

Figure 2. Forced response of a hydraulic system with momentum excitation source δH in the turbine draft tube.

Eigenmodes of this system are given by the eigenvalues and the eigenvectors of the linearized state global matrix defined by $\left[A_l \right]^{-1} \left[B_l \right]$. Francis turbine is modeled as a quasi-steady head $H_t \left(Q, \omega, y \right)$ defined from turbine characteristics work which is nonlinear function of the flow rate, the rotational speed and the guide vane opening. Linearization of this function is performed to obtain the linearized state global matrix [15]. Damping and oscillation frequency of the eigenmodes are given respectively by the real part and the imaginary part of the complex eigenvalues $s = \alpha + j\omega$ given in Table 1. The method also provides purely real eigenvalues corresponding to system time constants, which are not reported in this paper. With a spatial discretization of 20 pressure nodes in the penstock and 3 pressure nodes in the draft tube, 23 hydraulic eigenmodes, named Hi, are found from 0.37 Hz to 7.7 Hz.

Table 1. Eigenvalues of the hydraulic system.

Name	$s = \alpha + j\omega$	f (Hz)
$H1$	$-1.61 + j2.35$	0.37
$H2$	$-1.13 + j5.42$	0.86
$H3$	$-0.75 + j8.48$	1.35
$H4$	$-0.26 + j11.35$	1.81
$H5$	$-0.06 + j13.59$	2.16
$H6$	$-0.57 + j16.19$	2.58
$H...$	...	...
$H23$	$-0.10 + j48.36$	7.70

Combined with the computation of the eigenmodes, forced response is performed with the momentum excitation source in the draft tube which frequency is defined between 0 Hz and 5 Hz. The amplitude is set to 3 m corresponding to 1% of the gross head and the phase is set to $\pi/4$. The resulting system response is shown in Figure 3 at two pressure nodes in the penstock (Hc7 and Hc14 respectively corresponding to 32.5% and 70% of the length of the penstock) and at one pressure node (Hc2 located in the middle of the draft tube) in the draft tube.
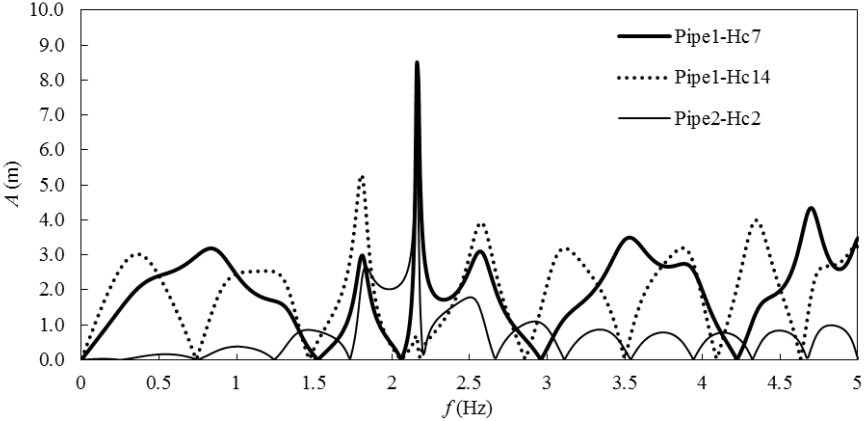
Figure 3. Forced response of the hydraulic system.

Figure 3 represents for each frequency of the momentum excitation source between 0 Hz and 5 Hz, the amplitude of the system response. Results are in agreement with the eigenvalues since resonances are observed at the eigenfrequencies and amplitudes are in accordance with the damping of the eigenvalues. For the lowest damping, amplitude response will be the highest, see eigenmode H5 at 2.16 Hz. In Figure 4, the comparison of pressure fluctuations in the piping system between forced response analysis and time domain simulation is made with an excitation source set to the first eigenfrequency H1 of the system. To compare with the forced response, spectra of pressure fluctuations in the time domain simulation are performed and both amplitude and phase at the frequency H1 are depicted and compared. A very good matching is obtained between forced response and time domain simulation.
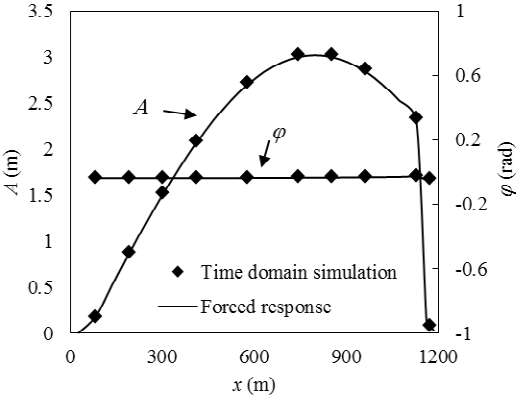

Figure 4. Comparison of pressure fluctuations in the piping system between forced response and time domain simulation with excitation source at the first eigenfrequency H1.

2.2.2. Electrical system

The second validation is performed for an electrical system featuring a synchronous machine connected to a grid, see Figure 5. This electrical machine is set to be connected to the hydraulic system presented above with a terminal voltage of 17'500 V. An excitation torque on the mechanical mass is applied to simulate mechanical power fluctuations on the shaft induced by a cavitation vortex rope development in the turbine draft tube.
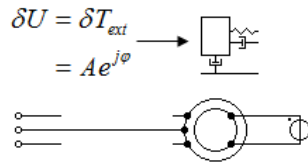

Figure 5. Forced response of an electrical system with torque excitation source δT_{ext}.

Table 2 presents the eigenmodes of the electrical system computed from the linearized state global matrix defined by $\left[A_l \right]^{-1} \left[B_l \right]$. The eigenmode E1 represents the oscillations of the machine against the power grid and is called "local mode". The second eigenmode E2 corresponds to the system frequency since stator currents state variables of the machine are expressed in a rotating reference frame attached to the rotor.

Table 2. Eigenvalues of the electrical system.

Name	$s = \alpha + j\omega$	f (Hz)
E1	$-3.70 + j11.30$	1.80
E2	$-3.48 + j313.99$	49.97

Forced response of this electrical system is performed with a torque excitation source which frequency is defined between 0 Hz and 5 Hz. The amplitude is set to 50'000 Nm corresponding to1 % of the nominal torque and the phase is set to $\pi/4$. The resulting system response is shown in Figure 6 for the stator currents (ia, ib, ic) and the excitation current of the rotor (if).
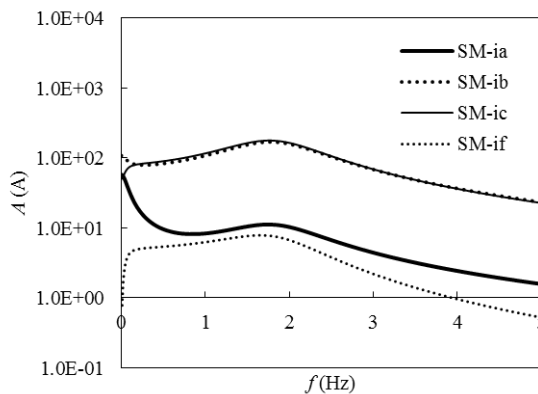

Figure 6. Forced response of the electrical system.

Figure 6 represents for each frequency of the torque excitation source between 0 Hz and 5 Hz, the amplitude of the system response. The local mode of the synchronous machine is found at 1.8 Hz in accordance with the eigenfrequencies of Table 2. Since the system matrices for electrical systems are

expressed in a rotating frame attached to the rotor, the frequency response must be interpreted in this rotating frame whereas time domain simulation is in a fixed frame. Hence, for validation and comparison, time domain simulation results must be expressed in the rotating frame. This transformation is made in three steps [16]. The first one is to define from the three phase currents a-b-c, a spatial phasor $\underline{i}^s$ in a fixed frame (superscript s) with coordinates $\left\{ i_\alpha^s ; i_\beta^s \right\}$:

$$\underline{i}^s = i_\alpha^s + j i_\beta^s \qquad \begin{cases} i_\alpha^s = \dfrac{1}{3}\left(2i_a - i_b - i_c\right) \\ i_\beta^s = \dfrac{1}{\sqrt{3}}\left(i_b - i_c\right) \end{cases} \tag{9}$$

Then, considering a coordinates system (k) shifted with an angle θ_k from coordinates system (s) and moving with angular frequency ω_0 corresponding to the rotor frequency of the machine, the spatial phasor in the new coordinates system (k) is expressed as:

$$\theta_k = \omega_0 t \qquad \begin{cases} i_\alpha^k = i_\alpha^s \cos\theta_k + i_\beta^s \sin\theta_k \\ i_\beta^k = i_\beta^s \cos\theta_k - i_\alpha^s \sin\theta_k \end{cases} \tag{10}$$

Finally, from this spatial phasor in the rotating coordinates system (k), instantaneous three phase currents a-b-c can be reconstituted:

$$\begin{cases} i_a^k = i_\alpha^k \\ i_b^k = -\dfrac{1}{2}i_\alpha^k + \dfrac{\sqrt{3}}{2}i_\beta^k \\ i_c^k = -\dfrac{1}{2}i_\alpha^k - \dfrac{\sqrt{3}}{2}i_\beta^k \end{cases} \tag{11}$$

In Figure 7a the amplitude spectrum of the stator current on phase a is plotted. A modulation of the 50 Hz network frequency at 1.8 Hz (E1) is observed with peak amplitudes at $50\ \mathrm{Hz} \pm 1.8\ \mathrm{Hz}$. Transformation in the rotating frame yields the amplitude spectrum given in Figure 7b corresponding to harmonic oscillations at the local mode frequency 1.8 Hz (E1). Both amplitude and phase at the frequency E1 of the transformed current are depicted and compared to the forced response. A very good matching is obtained between forced response and time domain simulation in rotating frame.
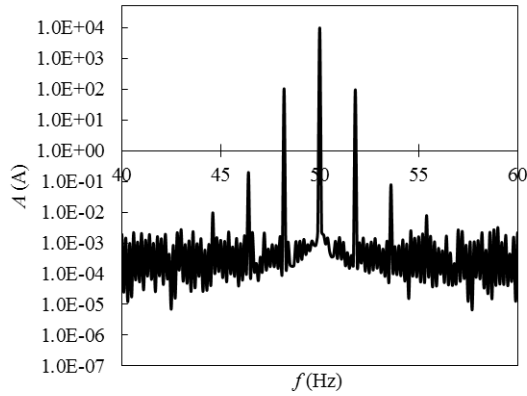
Figure 7a. Amplitude spectrum of stator current on phase a in the fixed frame (s) obtained by time domain simulation.
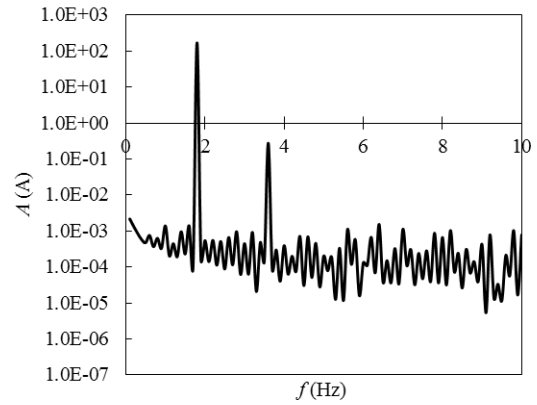

Figure 7b. Amplitude spectrum of stator current on phase a in the rotating frame (k) obtained by time domain simulation.

3. Case study

The case study is a complete hydroelectric power plant with controllers such as speed regulator and generator excitation system with ABB Unitrol voltage regulator including power system stabilizer of type IEEE PSS2B, see Figure 8. Both hydro-mechanical and electro-mechanical parts with their own controllers will be analyzed separately with the forced response technique. Then, the analysis of the complete system will be performed. Hydraulic and electrical systems studied for validation of the forced response in subsection 2.2 are constituting this case study.

4. Forced Response Analysis

4.1. Hydro-mechanical part

The hydro-mechanical part includes the Francis turbine, the hydraulic layout, the speed governor and the two coupled rotating inertias. Eigenvalues of this system are computed and summarized in Table 3. By comparing with Table 1, two additional eigenmodes are found: one related to the hydraulic regulation (HR1) and the other one is the torsional mode between the two rotating inertias which frequency and damping depend on the parameters of the mechanical system with coupling (M1). Moreover, it can be noticed that taking into account both regulation and rotating inertias, increases the frequency of the two first hydraulic eigenmodes (H1-2) respectively by 67% and 13%.
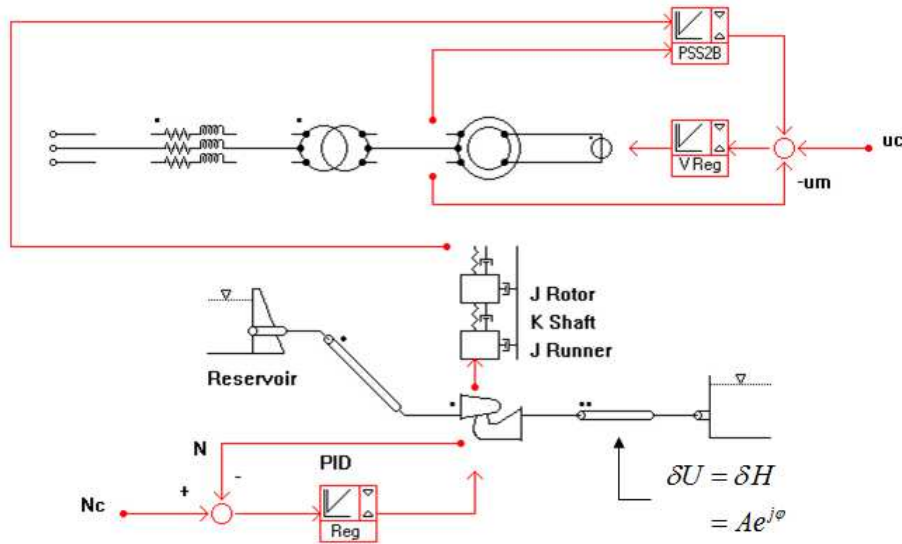
Figure 8. Hydroelectric case study for forced response analysis with momentum excitation source δH in the turbine draft tube.

Table 3. Eigenvalues of the hydro-mechanical part.

Name	$s = \alpha + j\omega$	f (Hz)
$HR1$	$-0.48 + j0.94$	0.15
$H1$	$-1.13 + j3.92$	0.62
$H2$	$-1.64 + j6.12$	0.97
$H3$	$-0.93 + j8.58$	1.36
$H...$	...	...
$H23$	$-0.09 + j48.37$	7.70
$M1$	$-2.87 + j107.06$	17.04

Forced response is performed with the momentum excitation source in the draft tube which frequency is defined between 0 Hz and 5 Hz. The amplitude is set to 3m corresponding to 1% of the gross head and the phase is set to $\pi/4$. The resulting system response is shown in Figure 9 at two pressure nodes in the penstock (Hc7 and Hc14) and at one pressure node (Hc2) in the draft tube. The eigenfrequency of the speed regulator (HR1) at 0.15 Hz is found with the forced response and modifications of the two first hydraulic eigenfrequencies (H1-2) as well.
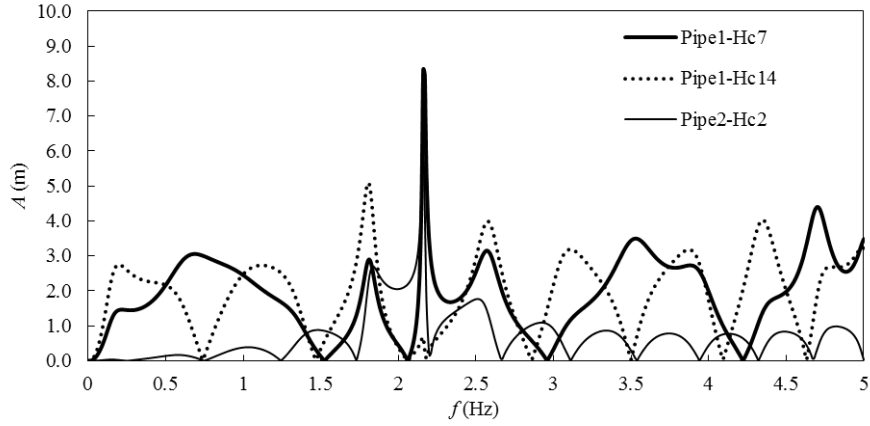
Figure 9. Forced response of the hydro-mechanical part

4.2. Electro-mechanical part

The electro-mechanical part includes the transmission line, the transformer and the synchronous machine with the two rotating inertias and the voltage regulator with power system stabilizer. Eigenvalues of this system are computed and summarized in Table 4 and influence of PSS is assessed. Only eigenvalues of interest are mentioned in Table 4. All remaining eigenvalues are either real or have a strong damping and consequently don't significantly affect the dynamical behavior of the system. The torsional mechanical mode (M1) is found with a damping much lower than the one found with the hydro-mechanical part. This additional damping is due to the linearization of the hydraulic torque turbine characteristics. Moreover, two regulated electrical eigenmodes (ER1-2) are found: one related to the voltage regulator (ER2) with high damping and one related to the power system stabilizer (ER1). Regarding the local mode (E1), damping is higher with the PSS and eigenfrequency is increased by 27%.

Table 4. Eigenvalues of the electro-mechanical part.

Name	With PSS		Without PSS	
	$s = \alpha + j\omega$	f (Hz)	$s = \alpha + j\omega$	f (Hz)
$ER1$	$-0.28 + j2.46$	0.39		
$ER2$	$-18.09 + j3.65$	0.58	$-19.85 + j6.70$	1.07
$E1$	$-2.22 + j10.28$	1.64	$-0.80 + j8.09$	1.29
$M1$	$-0.05 + j102.49$	16.3	$-0.05 + j102.49$	16.3

Forced response of this electro-mechanical part is performed with a torque excitation source which frequency is defined between 0 Hz and 5 Hz. The amplitude is set to 50'000 Nm corresponding to 1% of the nominal torque and the phase is set to $\pi/4$. The system responses with and without PSS are shown respectively in Figure 10a and 10b. The low eigenfrequency of the PSS (ER1) and the local mode (E1) are found with the forced response. Influence of the PSS on the damping and the frequency of the local mode (E1) is shown and is in accordance with the eigenvalues results.
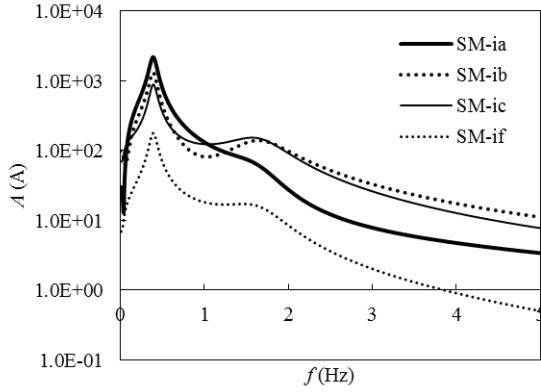
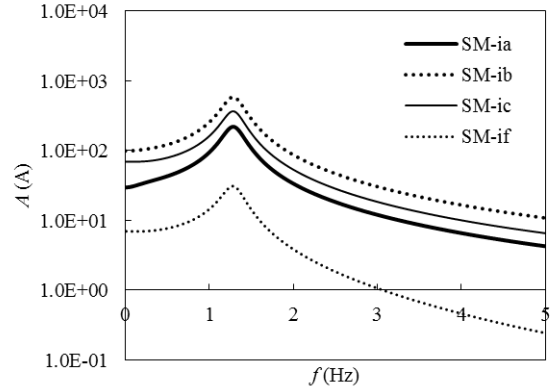**Figure 10a.** Forced response of the electro-mechanical part with PSS.

Figure 10b. Forced response of the electro-mechanical part without PSS.

4.3. Hydroelectric system

Eigenvalues of the hydroelectric system are computed and summarized in Table 5. The local mode (E1) is modified by the PSS as mentioned previously by the analysis of the electro-mechanical part. Frequencies and dampings of the first hydraulic and regulated eigenmodes are slightly modified by the PSS. Finally, the torsional mode (M1) features damping and frequency which was obtained with the hydro-mechanical part, see Table 3. It can be also noted that the hydraulic eigenvalues H1 and H2 obtained with the hydroelectric system without PSS are close to those obtained for the first hydraulic model with constant rotational speed, see Table 1. Moreover, the frequency of the first hydraulic eigenmode H1 is affected by the presence of the PSS.

Table 5. Eigenvalues of the hydroelectric system.

Name	With PSS $s = \alpha + j\omega$	f (Hz)	Without PSS $s = \alpha + j\omega$	f (Hz)
$HR1$	$-1.21 + j0.91$	0.15	$-1.76 + j0.51$	0.08
$ER1$	$-0.19 + j2.27$	0.36		
$ER2$	$-17.81 + j3.43$	0.55	$-19.83 + j6.69$	1.06
$H1$	$-1.96 + j3.76$	0.60	$-1.28 + j2.49$	0.40
$H2$	$-1.32 + j5.22$	0.83	$-0.73 + j5.40$	0.86
$H3$	$-0.72 + j8.31$	1.32	$-1.14 + j9.49$	1.51
$E1$	$-1.77 + j10.79$	1.71	$-0.35 + j7.95$	1.26
$H23$	$-0.09 + j48.37$	7.70	$-0.09 + j48.37$	7.70
$M1$	$-2.87 + j107.07$	17.04	$-2.87 + j107.07$	17.04

The forced response of the synchronous machine when electrical or hydroelectric systems are considered are compared in Figure 11. The shape of the spectrum derived with the electrical system is reproduced with the hydroelectric system. Indeed, amplifications at the local mode E1 and at the PSS eigenmode ER1 are found. Around this mean shape, anti-resonances and hydraulic resonances are visible. Amplitudes of the different hydraulic eigenmodes are modulated by the spectrum of the electrical system.
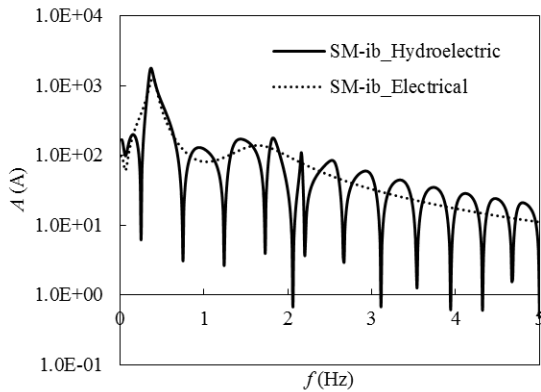

Figure 11a. Comparison of the forced response of the synchronous machine obtained with electrical and hydroelectric systems including PSS.
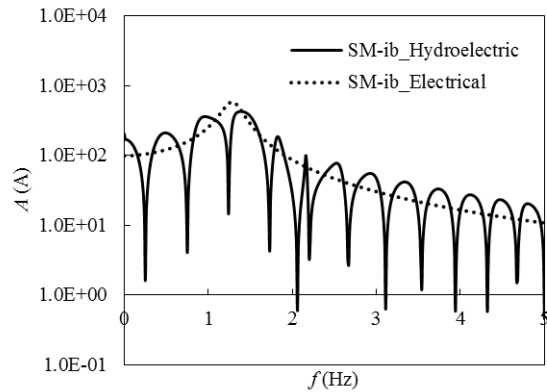
Figure 11b. Comparison of the forced response of the synchronous machine obtained with electrical and hydroelectric systems without PSS.

It is known that the cavitation vortex rope in the draft tube induces pressure fluctuations between 0.2 and 0.4 times the rotational frequency of the turbine runner corresponding respectively to 1.25 Hz and 2.5 Hz. Hence resonance between the system and the vortex rope is possible at the third hydraulic eigenmode frequency H3. Assuming a match of these frequencies, response of the synchronous machine is given in Table 6 and the influence of the PSS is shown. The PSS has a stabilizing effect by reducing the stator currents fluctuations since amplitude of the local mode E1 is reduced by the PSS and the third hydraulic eigenfrequency is closed. However, if resonance would occur with a hydraulic eigenmode which frequency is close to the PSS eigenmode ER1, the PSS would have a destabilizing effect by amplifying the response.

Table 6. Frequency response of the synchronous machine to the momentum excitation in the draft tube at frequency of the third hydraulic eigenmode H3.

Variable	With PSS		Without PSS	
	A (A)	φ (rad)	A (A)	φ (rad)
ia	88.704	1.97	134.645	1.38
ib	104.318	0.40	354.321	-1.84
ic	136.806	-2.03	220.311	1.26
if	16.934	1.92	17.707	-2.39
id	11.559	-0.03	8.497	-0.90
iq	3.274	1.05	12.850	-1.02

Despite of the stabilizing effect on the electrical part of the system, the damping of the third hydraulic eigenmode H3 is the lowest, see Table 5. Figure 12 shows the response of the hydraulic part of the system and compares the amplitudes of pressure fluctuations in the piping system if PSS is considered

or not. A good agreement is found with time domain simulation results. It can be seen that pressure fluctuations are higher if PSS is used. However, this behavior is specific to this eigenmode since the two first hydraulic eigenmodes feature a higher damping with the PSS, see Table 5.
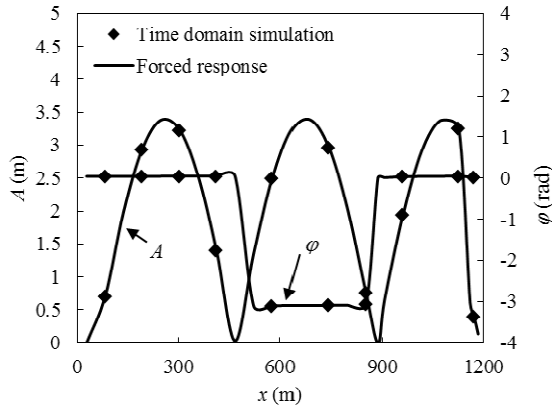

Figure 12a. Forced response in the piping system induced by excitation source set to the third hydraulic eigenmode frequency H3 (Hydroelectric system with PSS).
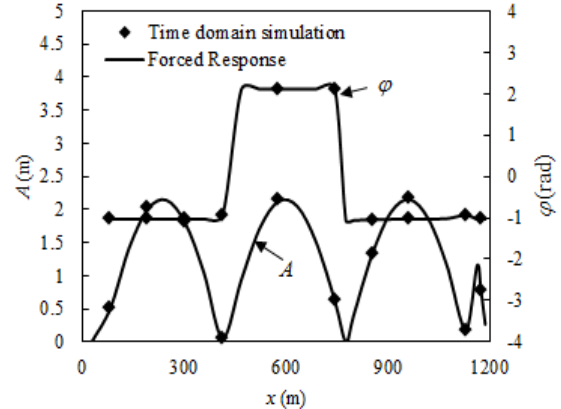
Figure 12b. Forced response in the piping system induced by excitation source set to the third hydraulic eigenmode frequency H3 (Hydroelectric system without PSS).

5. Conclusion

A forced response analysis tool of hydroelectric systems has been developed in SIMSEN. This method allows identifying the contribution of each eigenmode into the system response which depends on the system boundary conditions and the excitation source location. Combining forced response analysis with eigenmodes computation, SIMSEN is a powerful tool for stability analysis of hydroelectric systems including hydraulic system, rotating train, electrical system and control devices. In this paper, system response to hydraulic excitation induced by cavitation vortex rope in the turbine draft tube has been investigated. The study could be extended to any other excitation source which could be mechanical or electrical. This tool has been validated with time domain simulations from simplest test case to complex hydroelectric system including control structures. It is shown that this tool is complementary to modal analysis for stability assessment.

Moreover, the different test cases investigated in this paper have demonstrated that the hydraulic eigenmodes can be significantly affected by the operating conditions of the turbine (rotational speed constant or not), and by the control devices, including not only the turbine speed governor but also the generator voltage regulator and the power system stabilizer. It means for examples that the hydraulic system is expected to feature different hydraulic eigenmodes during the synchronisation phase, interconnected operation or isolated operation. Similar results have been found for the generator local mode which is affected not only by the voltage regulator and the power system stabilizer, but also by the hydraulic system.

6. Acknowledgments

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