International Journal of Applied Engineering Research ISSN 0973-4562 Volume 12, Number 24 (2017) pp. 15536-15543

Self-Centered Single Valued Neutrosophic Graphs

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Abstract

In this paper, we introduce the concepts of length, distance, eccentricity, radius, diameter, status, total status, median and central vertex of a single valued neutrosophic graph. We present the concept of self-centered single valued neutrosophic graph. We investigated some properties of self-centered single valued neutrosophic graphs.

Keywords: Length; distance; eccentricity; radius; diameter; central vertex; status; median; self-centered single valued neutrosophic graph.

INTRODUCTION

Fuzzy set [19] theory plays a vital role in complex phenomena which is not effortlessly described by classical set theory. Atanassov introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs(IFGs). Parvathi and Karunambigai[13] introduced the concept of IFG elaborately and analyzed its components. Authors of [9] introduced the concept of self-centered IFG. Smarandache[6]-[7] introduced the idea of neutrosophic sets by combining the non-standard analysis. Neutrosophic set is a mathematical tool for dealing real life problems having imprecise, indeterminacy and inconsistent data. Neutrosophic set theory, as a generalization of classical set theory, fuzzy set theory and intuitionistic fuzzy set theory, is applied in a variety of fields, including control theory, decision making problems, topology, medicines and in many more real life problems. Wang et al.[16] presented the notion of single-valued neutrosophic sets to apply neutrosophic sets in real life problems more conveniently. A single-valued neutrosophic set has three components: truth membership degree, indeterminacy membership degree and falsity membership degree. These three components of a single-valued neutrosophic set are not dependent and their values are contained in the standard unit interval [0, 1]. Single-valued neutrosophic sets are the generalization of intuitionistic fuzzy sets. Single-valued neutrosophic sets have been a new hot research topic and many researchers have addressed this issue. Akram et al.[1-4] has discussed several concepts related to single-valued neutrosophic graphs. Majumdar and Samanta [10] studied similarity and entropy of single-valued neutrosophic sets. Ye[18] proposed correlation coefficients of single-valued neutrosophic sets, and applied it to single-valued neutrosophic decision making problems.

In this paper, we introduce the concepts of length, distance, radius, eccentricity, diameter, status, total status, median and central vertex of a single valued neutrosophic graph. We present the concept of self-centered single valued neutrosophic graph. We also discuss some interesting properties besides giving some examples.

Definition 1.1 [17] Let X be a space of points. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. The functions $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or non standard subsets of $]0^-, 1^+[$. That is, $T_A(x): X \to]0^-, 1^+[$, $I_A(x): X \to]0^-, 1^+[$, $F_A(x): X \to]0^-, 1^+[$ and $0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

From philosophical point view, the neutrosophic set takes the value from real standard or non standard subsets of $]0^-, 1^+[$. In real life applications in scientific and engineering problems, it is difficult to use neutrosophic set with value from real standard or non standard subset of $]0^-, 1^+[$.

Definition 1.2 [3, 1] A single valued neutrosophic graph is a pair $G = (A, B)$, where $A: V \to [0,1]$ is single valued neutrosophic set in V and $B: V \times V \to [0,1]$ is single valued neutrosophic relation on V such that $T_B(xy) \leq \min\{T_A(x), T_A(y)\}$, $I_B(xy) \leq \min\{I_A(x), I_A(y)\}$, $F_B(xy) \leq \max\{F_A(x), F_A(y)\}$ for all $x, y \in V$. A is called single valued neutrosophic vertex set of G and B is called single valued neutrosophic edge set of G, respectively. We note that B is symmetric single valued neutrosophic relation on A. If B is not symmetric single valued neutrosophic relation on A, then $G = (A, B)$ is called a single valued neutrosophic directed graph.

Definition 1.3 A single valued neutrosophic graph $G = (A, B)$ is said to be complete if $T_B(v_i, v_j) = \min\left(T_A(v_i), T_A(v_j)\right)$, $I_B(v_i, v_j) = \min\left(I_A(v_i), I_A(v_j)\right)$ and $F_B(v_i, v_j) = \max\left(F_A(v_i), F_A(v_j)\right), \forall v_i, v_j \in V$.

SELF-CENTERED SINGLE VALUED NEUTROSOPHIC GRAPHS

Definition 2.1 Let $G = (A, B)$ be a single valued neutrosophic graph. Then the order of G is defined to be $O(G) = (O_T(G), O_I(G), O_F(G))$ where $O_T(G) = \sum_{u \in V} T_A(u)$, $O_I(G) = \sum_{u \in V} I_A(u)$, $O_F(G) = \sum_{u \in V} F_A(u)$.

Definition 2.2 The size of G is defined to be $S(G) = \left(S_T(G), S_I(G), S_F(G)\right)$ where $S_T(G) = \sum_{u,v \in V} T_B(u, v)$, $S_I(G) = \sum_{u,v \in V} I_B(u, v)$, $S_F(G) = \sum_{u,v \in V} F_B(u, u)$.

Definition 2.3 The neighbourhood of any vertex v is defined as $N(v) = (N_T(v), N_I(v), N_F(v))$ where
$N_T(v) = \{u \in V: T_B(u, v) = \min\{T_A(u), T_A(v)\}\}$,
$N_I(v) = \{u \in V: I_B(u, v) = \min\{I_A(u), I_A(v)\}\}$,
$N_F(v) = \{u \in V: F_B(u, v) = \max\{F_A(u), F_A(v)\}\}$
and $N[v] = N(v) \cup \{v\}$ is called closed neighbourhood of v.

Definition 2.4 A path P in a single valued neutrosophic graph $G = (A, B)$ is a sequence of distinct vertices $v_1, v_2, \ldots, v_n$ such that either one of the following condition is satisfied
(i) $T_B(v_i, v_j) > 0, I_B(v_i, v_j) > 0$ and $F_B(v_i, v_j) = 0$ for some i and j. (ii) $T_B(v_i, v_j) = 0, I_B(v_i, v_j) = 0$ and $F_B(v_i, v_j) > 0$ for some i and j.

Definition 2.5 Let G be a single valued neutrosophic graph.
(i) [13] The length of a path $P: v_1, v_2, \ldots, v_{n+1} (n > 0)$ in G is n.
(ii) [13] The path $P: v_1, v_2, \ldots, v_{n+1}$ in G is called a cycle if $v_1 = v_{n+1}$ and $n \geq 3$. (iii) An single valued neutrosophic graph G is connected if any two vertices are joined by path.

Definition 2.6 The strength of a path $P: v_1, v_2, \ldots, v_n$, is defined as $S(P) = (S_T(P), S_I(P), S_F(P))$ where, $S_T(P) = \min(T_B(v_i, v_j))$, $S_I(P) = \min(I_B(v_i, v_j))$ and $S_F(P) = \max(F_B(v_i, v_j))$ for all i and j.

Note 2.1 In other words, the strength of a path is defined to be the weight of the weakest edge of the path. i.e the strength of a path $S(P)$.

Definition 2.7 A single valued neutrosophic graph $G = (A, B)$ is said to be a single valued neutrosophic bipartite if the vertex set V can be partitioned into two non empty sets V_1 and V_2 such that (i) $T_B(v_i, v_j) = 0, I_B(v_i, v_j) = 0$ and $F_B(v_i, v_j) = 0$, if $v_i, v_j \in V_1$ or $v_i, v_j \in V_2$, (ii) $T_B(v_i, v_j) > 0, I_B(v_i, v_j) > 0$ and $F_B(v_i, v_j) > 0$, if $v_i \in V_1$ or $v_j \in V_2$ for some i and j (or) $T_B(v_i, v_j) = 0, I_B(v_i, v_j) = 0$ and $F_B(v_i, v_j) > 0$, if $v_i \in V_1$ or $v_j \in V_2$ for some i and j (or) $T_B(v_i, v_j) > 0, I_B(v_i, v_j) > 0$ and $F_B(v_i, v_j) = 0$, if $v_i \in V_1$ or $v_j \in V_2$ for some i and j.

Definition 2.8 A single valued neutrosophic bipartite graph $G = (A, B)$ is said to be complete if $T_B(v_i, v_j) = \min(T_A(v_i), T_A(v_j))$, $I_B(v_i, v_j) = \min(I_A(v_i), I_A(v_j))$ and $F_B(v_i, v_j) = \max(F_A(v_i), F_A(v_j))$ for all $v_i \in V_1$ and $v_j \in V_2$. It is denoted by K_{v_1, v_2}.

Definition 2.9 Let single valued neutrosophic graph $H = (A', B')$ is said to be a single valued neutrosophic subgraph of a connected single valued neutrosophic graph $G = (A, B)$. If $T'_A(v_i) = T_A(v_i)$, $I'_A(v_i) = I_A(v_i)$, $F'_A(v_i) = F_A(v_i) \forall v_i \in V'$ and $T'_B(v_i, v_j) = T_B(v_i, v_j)$, $I'_B(v_i, v_j) = I_B(v_i, v_j)$, $F'_B(v_i, v_j) = F_B(v_i, v_j) \forall (v_i, v_j) \in E'$.

Definition 2.10 Let $G = (A, B)$ be a connected single valued neutrosophic graph.
(i) The T-length of a path $P: v_1, v_2, \ldots, v_n$ in G, $l_T(P)$ is defined as $l_T(P) = \sum_{i=1}^{n-1} \left(\frac{1}{T_B(v_i v_{i+1})}\right)$
(ii) The I-length of a path $P: v_1, v_2, \ldots, v_n$ in G, $l_I(P)$ is defined as $l_I(P) = \sum_{i=1}^{n-1} \left(\frac{1}{I_B(v_i v_{i+1})}\right)$
(iii) The F-length of a path $P: v_1, v_2, \ldots, v_n$ in G, $l_F(P)$ is defined as $l_F(P) = \sum_{i=1}^{n-1} \left(\frac{1}{F_B(v_i v_{i+1})}\right)$
The (T,I,F)-length of a path $P: v_1, v_2, \ldots, v_n$ in G, $l_{(T,I,F)}(P)$ is defined as $l_{(T,I,F)}(P) = (l_T(P), l_I(P), l_F(P))$.

Definition 2.11 Let $G = (A, B)$ be a connected single valued neutrosophic graph.
(i) The T-distance $\delta_T(v_i, v_j)$ is the minimum of the T-length of all the paths joining v_i and v_j in G, where $v_i, v_j \in V$. i.e $\delta_T(v_i, v_j) = \min\{l_T(P)$: P is a path between v_i and $v_j\}$,

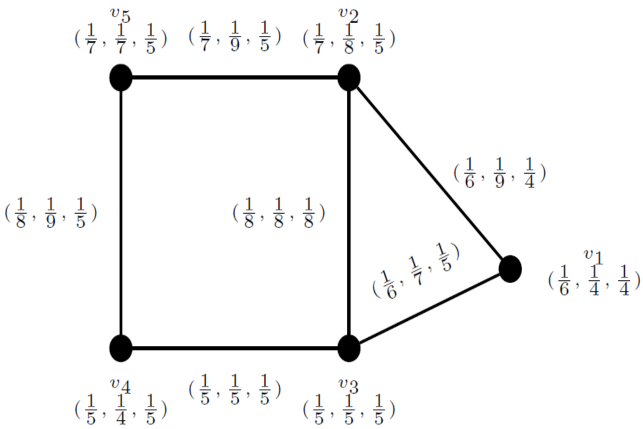
(ii) The I-distance $\delta_I(v_i,v_j)$ is the minimum of the I-length of all the paths joining v_i and v_j in G, where $v_i,v_j \in V$. i.e $\delta_I(v_i,v_j) = \min\{l_I(P)$: P is a path between v_i and $v_j\}$,
(iii) The F-distance $\delta_F(v_i,v_j)$ is the minimum of the F-length of all the paths joining v_i and v_j in G, where $v_i,v_j \in V$. i.e $\delta_F(v_i,v_j) = \min\{l_F(P)$: P is a path between v_i and $v_j\}$,
The distance $\delta_{(T,I,F)}(v_i,v_j)$ is defined as $\delta_{(T,I,F)}(v_i,v_j) = (\delta_T,\delta_I,\delta_F)$.

Definition 2.12 Let $G = (A,B)$ be a connected single valued neutrosophic graph.
(i) For each $v_i \in V$, the T-eccentricity of v_i, denoted by $e_T(v_i)$ and is defined as $e_T(v_i) = \max\{\delta_T(v_i,v_j): v_i \in V, v_i \neq v_j\}$.
(ii) For each $v_i \in V$, the I-eccentricity of v_i, denoted by $e_I(v_i)$ and is defined as $e_I(v_i) = \max\{\delta_I(v_i,v_j): v_i \in V, v_i \neq v_j\}$.
(iii) For each $v_i \in V$, the F-eccentricity of v_i, denoted by $e_F(v_i)$ and is defined as $e_F(v_i) = \min\{\delta_F(v_i,v_j): v_i \in V, v_i \neq v_j\}$.
For each $v_i \in V$, the eccentricity of v_i denoted by $e(v_i)$ and is defined as $e(v_i) = (e_T(v_i), e_I(v_i), e_F(v_i))$.

Definition 2.13 Let $G = (A,B)$ be a connected single valued neutrosophic graph.
(i) The T-radius of G is denoted by $r_T(G)$ and is defined as $r_T(G) = \min\{e_T(v_i): v_i \in V\}$.
(ii) The I-radius of G is denoted by $r_I(G)$ and is defined as $r_I(G) = \min\{e_I(v_i): v_i \in V\}$.
(iii) The F-radius of G is denoted by $r_F(G)$ and is defined as $r_F(G) = \min\{e_F(v_i): v_i \in V\}$.
The radius of G is denoted by $r(G)$ and is defined as $r(G) = (r_T(G), r_I(G), r_F(G))$.

Definition 2.14 Let $G = (A,B)$ be a connected single valued neutrosophic graph.
(i) The T-diameter of G is denoted by $dia_T(G)$ and is defined as $dia_T(G) = \max\{e_T(v_i): v_i \in V\}$.
(ii) The I-diameter of G is denoted by $dia_I(G)$ and is defined as $dia_I(G) = \max\{e_I(v_i): v_i \in V\}$.
(iii) The F-diameter of G is denoted by $dia_F(G)$ and is defined as $dia_F(G) = \max\{e_F(v_i): v_i \in V\}$.
The diameter of G is denoted by $dia(G)$ and is defined as $dia(G) = (dia_T(G), dia_I(G), dia_F(G))$.

Example 2.1 Consider a single valued neutrosophic graph, $G = (A,B)$ such that $V = \{v_1,v_2,v_3,v_4,v_5\}$ $E = \{(v_1,v_2), (v_2,v_3), (v_3,v_1), (v_3,v_4), (v_4,v_5), (v_5,v_2)\}$.



Then the eccentricity of v_i are $e(v_1) = (13,18,4)$, $e(v_2) = (13,13,4)$, $e(v_3) = (13,14,5)$, $e(v_4) = (13,13,5)$, $e(v_5) = (13,18,5)$. Radius of G is $r(G) = (13,13,4)$ and Diameter of G is $d(G) = (13,18,5)$.

Definition 2.15 A vertex $v_i \in V$ is called a
(i) T-central vertex of a connected single valued neutrosophic graph G, if $r_T(G) = e_T(v_i)$.
(ii) I-central vertex of a connected single valued neutrosophic graph G, if $r_I(G) = e_I(v_i)$.
(iii) F-central vertex of a connected single valued neutrosophic graph G, if $r_F(G) = e_F(v_i)$.
(iv) Central vertex of a connected single valued neutrosophic graph G, if $r_T(G) = e_T(v_i)$, $r_I(G) = e_I(v_i)$ and $r_F(G) = e_F(v_i)$ and the set of all central vertices of a single valued neutrosophic graph is denoted by $C(G)$.

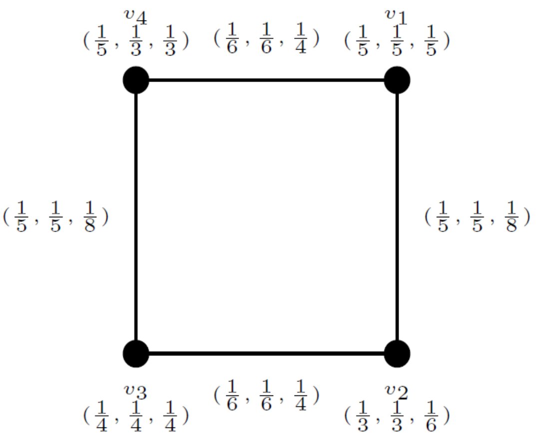
Definition 2.16 $< C(G) >= H: (A', B')$ is a single valued neutrosophic subgraph of $G = (A,B)$ induced by the central vertices of G is called the center of G.

Definition 2.17 A connected single valued neutrosophic graph G is a
(i) T- self-centered single valued neutrosophic graph, if every vertex of G is a T- central vertex. (i.e) $r_T(G) = e_T(v_i), \forall v_i \in V$.
(ii) I- self-centered single valued neutrosophic graph, if every vertex of G is a I- central vertex. (i.e) $r_I(G) = e_I(v_i), \forall v_i \in V$.
(iii) F- self-centered single valued neutrosophic graph, if every vertex of G is a F- central vertex. (i.e) $r_F(G) = e_F(v_i), \forall v_i \in V$.
(iv) Single valued neutrosophic self-centered graph, if every vertex of G is a central vertex. (i.e) $r_T(G) = e_T(v_i)$, $r_I(G) = e_I(v_i)$ and $r_F(G) = e_F(v_i), \forall v_i \in V$.

Example 2.2 Consider a single valued neutrosophic graph, $G=(A,B)$ such that $V=\{v_1,v_2,v_3,v_4\}$ $E=\{(v_1,v_2),(v_2,v_3),$ $(v_3,v_4),(v_4,v_1),(v_1,v_3)\}$.

Path	v_1-v_2	v_1-v_3	v_1-v_4
Distance $\delta_{(T,I,F)}(v_i,v_j)$	(5,5,8)	(11,11,12)	(6,6,4)
Path	v_2-v_3	v_2-v_4	v_3-v_4
Distance $\delta_{(T,I,F)}(v_i,v_j)$	(6,6,4)	(11,11,12)	(5,5,8)

Path	v_1-v_2	v_1-v_3	v_1-v_4	v_1-v_5	v_2-v_3
Distance $\delta_{(T,I,F)}(v_i,v_j)$	(6,9,4)	(6,7,5)	(11,12,10)	(13,18,9)	(8,8,8)
Path	v_2-v_4	v_2-v_5	v_3-v_4	v_3-v_5	v_4-v_5
Distance $\delta_{(T,I,F)}(v_i,v_j)$	(13,13,10)	(7,9,5)	(5,5,5)	(13,14,10)	(8,9,5)



Then the eccentricity of v_i are $e(v_1)=(11,11,4)$, $e(v_2)=(11,11,4)$, $e(v_3)=(11,11,4)$, $e(v_4)=(11,11,4)$. Radius of G is $r(G)=(11,11,4)$ and Diameter of G is $d(G)=(11,11,4)$. Here $r(G)=e(v_i),\forall v_i\in V$. Hence G is a self-centered single valued neutrosophic graph.

Definition 2.18 Let $G=(A,B)$ be a connected single valued neutrosophic graph.
(i) The T-status of a node u of G is denoted by $s_T(u)$ and is defined as $s_T(u)=\sum_{v\in V}\delta_T(u,v)$,
(ii) The I-status of a node u of G is denoted by $s_I(u)$ and is defined as $s_I(u)=\sum_{v\in V}\delta_I(u,v)$,
(iii) The F-status of a node u of G is denoted by $s_F(u)$ and is defined as $s_F(u)=\sum_{v\in V}\delta_F(u,v)$,
(iv) The status of a node u of G is defined as $s(u)=(s_T(u),s_I(u),s_F(u))$.

Definition 2.19 Let $G=(A,B)$ be a connected single valued neutrosophic graph.
(i) The minimum T-status of G is defined as $m[s_T(G)]=min\{s_T(u): u\in V\}$,
(ii) The minimum I-status of G is defined as $m[s_I(G)]=min\{s_I(u): u\in V\}$,
(iii) The minimum F-status of G is defined as $m[s_F(G)]=min\{s_F(u): u\in V\}$.
(iv) The minimum status of G is denoted by $m[s(G)]$ and is defined as $m[s(G)]=(m[s_T(G)],m[s_I(G)],m[s_F(G)])$.

Definition 2.20 Let $G=(A,B)$ be a connected single valued neutrosophic graph.
(i) The maximum T-status of G is defined as $M[s_T(G)]=max\{s_T(u): u\in V\}$,
(ii) The maximum I-status of G is defined as $M[s_I(G)]=max\{s_I(u): u\in V\}$,
(iii) The maximum F-status of G is defined as $M[s_F(G)]=max\{s_F(u): u\in V\}$.
(iv) The maximum status of G is denoted by $M[s(G)]$ and is defined as $M[s(G)]=(M[s_T(G)],M[s_I(G)],M[s_F(G)])$.

Definition 2.21 Let $G=(A,B)$ be a connected single valued neutrosophic graph.
The total T-status of a node u of G is denoted by $ts_T(u)$ and is defined as $ts_T(u)=\sum_{u\in V}s_T(u)$,
The total I-status of a node u of G is denoted by $ts_I(u)$ and is defined as $ts_I(u)=\sum_{v\in V}s_I(u)$,
The total F-status of a node u of G is denoted by $ts_F(u)$ and is defined as $ts_F(u)=\sum_{v\in V}s_F(u)$.
The total status of G is denoted by $t[s(G)]$ and is defined as $t[s(G)]=(ts_T(u),ts_I(u),ts_F(u))$.

Definition 2.22 Let $G=(A,B)$ be a connected single valued neutrosophic graph. The median is defined as
$M(G)=(M_T(G),M_I(G),M_F(G))$, where $M_T(G)=\{v_i\in V: min\{s_T(v_i)\}\}$, $M_I(G)=\{v_i\in V: min\{s_I(v_i)\}\}$, $M_F(G)=\{v_i\in V: min\{s_F(v_i)\}\}$.

Example 2.3 Consider a single valued neutrosophic -graph, $G=(A,B)$ such that $V=\{v_1,v_2,v_3,v_4\}$, $E=\{(v_1,v_2),(v_2,v_3),$ $(v_3,v_1),(v_3,v_4),(v_1,v_4)\}$.

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 12, Number 24 (2017) pp. 15536-15543
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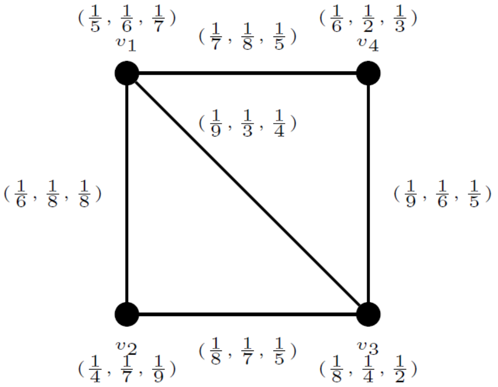
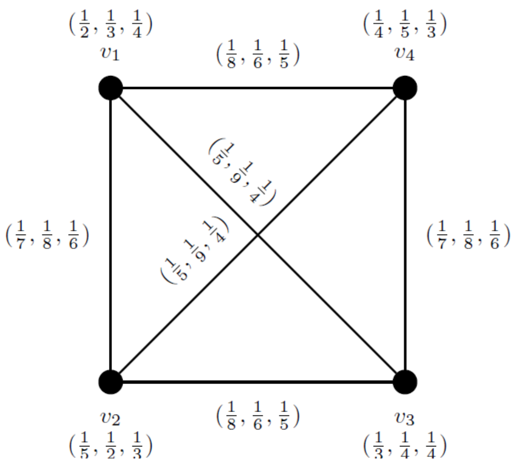


Here, status of the nodes are $s(v_1) = (22,19,17), s(v_2) = (27,28,23), s(v_3) = (26,16,14)$, $s(v_4) = (29,27,20)$. The minimum status of G is $m[s(G)] = (22,16,14)$. The maximum status of G is $M[s(G)] = (29,28,23)$. The total status of G is $t[s(G)] = (104,90,74)$.
The median is $M(G) = (\{v_1\},\{v_3\},\{v_3\})$.

Definition 2.23 A connected single valued neutrosophic graph $G = (A, B)$ is a self-median if all the nodes have the same status. In other words, a connected single valued neutrosophic graph $G = (A, B)$ is self-median if and only if $m[s(G)] = M[s(G)]$.

Example 2.4 Consider a single valued neutrosophic graph, $G = (A, B)$ such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1, v_2), (v_2, v_3),$
$(v_3, v_4), (v_4, v_1), (v_1, v_3), (v_2, v_4)\}$.
Here, status of the nodes are $s(v_1) = (20,23,15), s(v_2) = (20,23,15), s(v_3) = (20,23,15)$, $s(v_4) = (20,23,15)$. The minimum status of G is $m[s(G)] = (20,23,15)$. The maximum status of G is $M[s(G)] = (20,23,15)$. The total status of G is $t[s(G)] = (80,92,60)$.



The median is $M(G) = \{\{v_1, v_2, v_3, v_4\}, \{v_1, v_2, v_3, v_4\},$
$\{v_1, v_2, v_3, v_4\}\}$. Hence $G = (A, B)$ is called the self-median graph.

Theorem 2.1 If $G = (A, B)$ is a bipartite single valued neutrosophic graph then it has no strong single valued neutrosophic cycle of odd length.

Proof. Let G be a bipartite single valued neutrosophic graph with bipartition V_1 and V_2. Suppose that it contains a strong cycle of odd length, say $v_1, v_2, \dots, v_n, v_1$ for some odd n(vertices). Without loss of generality, let $v_1 \in V_1$. Since (v_i, v_{i+1}) is strong single valued neutrosophic for $i = 1,2,\dots,n-1$ and the nodes are alternatively in V_1 and V_2, we have v_n and $v_1 \in V_1$. But this implies that (v_n, v_1) is an edge in V_1 which contradicts the assumption that G is a bipartite single valued neutrosophic graph. Hence bipartite single valued neutrosophic graph has no single valued neutrosophic strong cycle of odd length.

Theorem 2.2 Every complete single valued neutrosophic graph G is a self-centered single valued neutrosophic graph and $r(G) = (\frac{1}{T_A(x)}, \frac{1}{I_A(x)}, \frac{1}{F_A(x)})$ where $T_A(x)$ and $I_A(x)$ are the least value and $F_A(x)$ is greatest value.

Proof. Let G be a complete single valued neutrosophic graph G. To prove that G is a self-centered single valued neutrosophic graph. That is we have to show that every vertex is a central vertex. First we claim that G is a T- self-centered single valued neutrosophic graph and $r_T(G) = \frac{1}{T_A(v_i)}$, where $T_A(v_i)$ is the least. Now fix a vertex $v_i \in V$ such that $T_A(v_i)$ is least vertex membership value of G.

(1) Consider all the $v_i - v_j$ paths P of length n in G, $\forall v_i \in V$.

Case (i) : If $n = 1$, then $T_B(v_i, v_j) = \min(T_A(v_i, v_j)) = T_A(v_i)$. Therefore, the T-length of $P = l_T(P) = \frac{1}{T_A(v_i)}$.

Case (ii) : If $n > 1$, then one of the edges of P possesses the T-strength $T_A(v_i)$ and hence, T-length of a $v_i - v_j$ path will exceed $\frac{1}{T_A(v_i)}$. That is T-length of P $= l_T(P) > \frac{1}{T_A(v_i)}$. Hence

$$\delta_T(v_i, v_j) = \min(l_T(P)) = \frac{1}{T_A(v_i)}, \forall v_j \in V. \qquad (1)$$

(2) Let $v_k \neq v_i$ in V. Consider all $v_k - v_j$ paths Q of length n in G, $\forall v_j \in V$.

Case (i) : If $n = 1$, then $T_B(v_k, v_j) = \min(T_A(v_k, v_j)) \geq T_A(v_i)$, since $T_A(v_i)$ is the least. Hence T-length of $Q = l_T(Q) = \frac{1}{T_B(v_k,v_j)} \leq \frac{1}{T_A(v_i)}$.

Case (ii) : If $n = 2$, then $l_T(Q) = \frac{1}{T_B(v_k,v_{k+1})} + \frac{1}{T_B(v_{k+1},v_j)} \leq \frac{2}{T_A(v_i)}$, since $T_A(v_i)$ is the least.

Case (iii) : If $n > 2$, then $l_T(Q) \leq \frac{n}{T_A(v_i)}$, since $T_A(v_i)$ is the least. Hence

$\delta_T(v_k, v_j) = \min(l_T(Q)) \leq \frac{1}{T_A(v_i)}, \forall v_k, v_j \in V.$ (2)

From Equations (1) and (2), we have

$e_T(v_i) = \max(\delta_T(v_i, v_j)) = \frac{1}{T_A(v_i)}, \forall v_i \in V.$ (3)

Hence G is a T-self-centered single valued neutrosophic graph.

Now, $r_T(G) = \min(e_T(v_i))$

$= \frac{1}{T_A(v_i)},$ *since by equation* (3)

$r_T(G) = \frac{1}{T_A(v_i)},$ *where* $T_A(v_i)$ *is least.*

Next, we claim that G is a I- self-centered single valued neutrosophic graph and $r_I(G) = \frac{1}{I_A(v_i)}$, where $I_A(v_i)$ is the least. Now fix a vertex $v_i \in V$ such that $I_A(v_i)$ is least vertex membership value of G.

(1) Consider all the $v_i - v_j$ paths P of length n in G, $\forall v_i \in V$.

Case (i) : If $n = 1$, then $I_B(v_i, v_j) = \min(I_A(v_i, v_j)) = I_A(v_i)$. Therefore, the I-length of $P = l_I(P) = \frac{1}{I_A(v_i)}$.

Case (ii) : If $n > 1$, then one of the edges of P possesses the I-strength $I_A(v_i)$ and hence, I-length of a $v_i - v_j$ path will exceed $\frac{1}{I_A(v_i)}$. That is I-length of P = $l_I(P) > \frac{1}{I_A(v_i)}$. Hence

$\delta_I(v_i, v_j) = \min(l_I(P)) = \frac{1}{I_A(v_i)}, \forall v_j \in V.$ (4)

(2) Let $v_k \neq v_i$ *in* V. Consider all $v_k - v_j$ paths Q of length n in G, $\forall v_j \in V$.

Case (i) : If $n = 1$, then $I_B(v_k, v_j) = \min(I_A(v_k, v_j)) \geq I_A(v_i)$, since $I_A(v_i)$ is the least. Hence I-length of $Q = l_I(Q) = \frac{1}{I_B(v_k,v_j)} \leq \frac{1}{I_A(v_i)}$.

Case (ii): If $n = 2$, then $l_I(Q) = \frac{1}{I_B(v_k,v_{k+1})} + \frac{1}{I_B(v_{k+1},v_j)} \leq \frac{2}{I_A(v_i)}$, since $I_A(v_i)$ is the least.

Case (iii): If $n > 2$, then $l_I(Q) \leq \frac{n}{I_A(v_i)}$, since $I_A(v_i)$ is the least. Hence

$\delta_I(v_k, v_j) = \min(l_I(Q)) \leq \frac{1}{I_A(v_i)}, \forall v_k, v_j \in V.$ (5)

From Equations (4) and (5), we have

$e_I(v_i) = \max(\delta_I(v_i, v_j)) = \frac{1}{I_A(v_i)}, \forall v_i \in V.$ (6)

Hence G is a I-self-centered single valued neutrosophic graph.

Now, $r_I(G) = \min(e_I(v_i))$

$= \frac{1}{I_A(v_i)},$ *since by equation* (6)

$r_I(G) = \frac{1}{I_A(v_i)},$ *where* $I_A(v_i)$ *is least.*

Next, we claim that G is a F - self-centered single valued neutrosophic graph and $r_F(G) = \frac{1}{F_A(v_i)}$, where $F_A(v_i)$ is the greatest. Now fix a vertex $v_i \in V$ such that $F_A(v_i)$ is greatest vertex membership value of G.

(1) Consider all the $v_i - v_j$ paths P of length n in G, $\forall v_i \in V$.

Case (i): If $n = 1$, then $F_B(v_i, v_j) = \max(F_A(v_i, v_j)) = F_A(v_i)$. Therefore, the F - length of $P = l_F(P) = \frac{1}{F_A(v_i)}$.

Case (ii): If $n > 1$, then one of the edges of P possesses the F-strength $I_A(v_i)$ and hence, F-length of a $v_i - v_j$ path will exceed $\frac{1}{I_A(v_i)}$. That is F-length of P= $l_F(P) > \frac{1}{F_A(v_i)}$. Hence

$\delta_F(v_i, v_j) = \min(l_F(P)) = \frac{1}{F_A(v_i)}, \forall v_j \in V.$ (7)

(2) Let $v_k \neq v_i inV$. Consider all $v_k - v_j$ paths Q of length n in G, $\forall v_j \in V$.

Case (i): If $n = 1$, then $F_B(v_k, v_j) = \max(F_A(v_k, v_j)) \leq F_A(v_i)$, since $F_A(v_i)$ is the greatest. Hence F-length of $Q = l_F(Q) = \frac{1}{F_B(v_k,v_j)} \geq \frac{1}{F_A(v_i)}$.

Case (ii): If $n = 2$, then $l_F(Q) = \frac{1}{F_B(v_k,v_{k+1})} + \frac{1}{F_B(v_{k+1},v_j)} \geq \frac{2}{F_A(v_i)}$, since $F_A(v_i)$ is the greatest.

Case (iii): If $n > 2$, then $l_F(Q) \geq \frac{n}{F_A(v_i)}$, since $F_A(v_i)$ is the greatest. Hence

$\delta_F(v_k, v_j) = \min(l_F(Q)) \geq \frac{1}{F_A(v_i)}, \forall v_k, v_j \in V.$ (8)

From Equations (7) and (8), we have

$e_F(v_i) = \min(\delta_F(v_i, v_j)) = \frac{1}{F_A(v_i)}, \forall v_i \in V.$ (9)

Hence G is a F-self-centered single valued neutrosophic graph.

Now, $r_F(G) = \min(e_F(v_i))$

$= \frac{1}{F_A(v_i)},$ *since by equation* (9)

$r_F(G) = \frac{1}{F_A(v_i)},$ *where* $F_A(v_i)$ *is greatest.*

From equations (3),(6), and (9), every vertex of G is a central vertex. Hence G is a self-centered single valued neutrosophic graph.

Theorem 2.3 A single valued neutrosophic graph $G = (A, B)$ is a self-centered single valued neutrosophic graph iff $\delta_T(v_i, v_j) \leq r_T(G)$, $\delta_I(v_i, v_j) \leq r_I(G)$ and $\delta_F(v_i, v_j) \geq r_F(G) \forall v_i, v_j \in V$.

Proof. ⇒ We assume that G is self-centered single valued neutrosophic graph G. That is $e_T(v_i) = e_T(v_j), e_I(v_i) = e_I(v_j),$ $e_F(v_i) = e_F(v_j), \forall v_i, v_j \in V,$ $r_T(G) = e_T(v_i), r_I(G) = e_I(v_i)$ and $r_F(G) = e_F(v_i), \forall v_i \in V$. Now we wish to show that $\delta_T(v_i,v_j) \le r_T(G), \delta_I(v_i,v_j) \le r_T(G)$ and $\delta_F(v_i,v_j) \ge r_F(G), \forall v_i, v_j \in V$. By the definition of eccentricity, we obtain, $\delta_T(v_i,v_j) \le e_T(v_i), \delta_I(v_i,v_j) \le e_I(v_i)$ and $\delta_F(v_i,v_j) \ge e_F(v_i), \forall v_i, v_j \in V$. When $e_T(v_i) = e_T(v_j)$, $e_I(v_i) = e_I(v_j)$, $e_F(v_i) = e_F(v_j), \forall v_i, v_j \in V$. Since G is self-centered single valued neutrosophic graph, the above inequality becomes $\delta_T(v_i,v_j) \le r_T(G), \delta_I(v_i,v_j) \le r_T(G)$ and $\delta_F(v_i,v_j) \ge r_F(G)$.

$\Leftarrow$ Assume that $\delta_T(v_i,v_j) \le r_T(G), \delta_I(v_i,v_j) \le r_I(G)$ and $\delta_F(v_i,v_j) \ge r_F(G), \forall v_i, v_j \in V$. Then we have to prove that G is self-centered single valued neutrosophic graph. Suppose that G is not self-centered single valued neutrosophic graph. Then $r_T(G) = e_T(v_i), r_I(G) = e_I(v_i)$ and $r_F(G) = e_F(v_i)$, for some $v_i \in V$. Let us assume that $e_T(v_i)$, $e_I(v_i)$ and $e_F(v_i)$ is the least value among all other eccentricity. That is

$$r_T(G) = e_T(v_i), r_I(G) = e_I(v_i) \; and \; r_F(G) = e_F(v_i) \tag{10}$$

Where $e_T(v_i) < e_T(v_j), e_I(v_i) < e_I(v_j), e_F(v_i) < e_F(v_j)$, for some $v_i, v_j \in V$ and

$$\delta_T(v_i,v_j) = e_T(v_j) > e_T(v_i), \delta_I(v_i,v_j) = e_I(v_j) > e_I(v_i) \tag{11}$$

$$and \quad \delta_F(v_i,v_j) = e_F(v_j) > e_F(v_i), \quad for \; some \quad v_i, v_j \in \; V.$$

Hence from equations (10) and (11), we have $\delta_T(v_i,v_j) > r_T(G), \delta_I(v_i,v_j) > r_I(G)$ and $\delta_F(v_i,v_j) < r_F(G)$, for some $v_i, v_j \in V$, which is a contradiction to the fact that $\delta_T(v_i,v_j) \le r_T(G), \delta_I(v_i,v_j) \le r_T(G)$ and $\delta_F(v_i,v_j) \ge r_F(G), \forall v_i, v_j \in V$. Hence G is a self-centered single valued neutrosophic graph.

Theorem 2.4 Let $G = (A, B)$ be a single valued neutrosophic graph. If the graph G is complete bipartite single valued neutrosophic graph then the complement of G is self-centered single valued neutrosophic graph.

Proof. A bipartite single valued neutrosophic graph G is said to be complete, if $T_B(v_i,v_j) = \min\left(T_A(v_i), T_A(v_j)\right)$, $I_B(v_i,v_j) = \min\left(I_A(v_i), I_A(v_j)\right)$, $F_B(v_i,v_j) = \max\left(F_A(v_i), F_A(v_j)\right)$,

$\forall v_i \in V_1, v_j \in V_2$

and

$$T_B(v_i,v_j) = 0, \tag{12}$$

$I_B(v_i,v_j) = 0,$

$F_B(v_i,v_j) = 0, \forall v_i, v_j \in V_1 \quad (or) \quad v_i, v_j \in V_2$

Now,

$$\overline{T}_B(v_i,v_j) = \min(T_A(v_i), T_A(v_j)) - T_B(v_i,v_j) \tag{13}$$

$\overline{I}_B(v_i,v_j) = \min(I_A(v_i), I_A(v_j)) - I_B(v_i,v_j)$

$\overline{F}_B(v_i,v_j) = \max(F_A(v_i), F_A(v_j)) - F_B(v_i,v_j).$

By using equation (12)

$$\overline{T}_B(v_i,v_j) = \min(T_A(v_i), T_A(v_j)) \tag{14}$$

$$\overline{I}_B(v_i,v_j) = \min(I_A(v_i), I_A(v_j)) \tag{15}$$

$\overline{F}_B(v_i,v_j) = \max\left(F_A(v_i), F_A(v_j)\right),$

$$\forall v_i, v_j \in V_1 \quad (or) \quad v_i, v_j \in V_2 \tag{16}$$

From equations (12), (14), the complement of G has two components and each is complete single valued neutrosophic graph, which are self-centered single valued neutrosophic by Theorem 2.2.Hence the proof.

Theorem 2.5 Every self-median SVN-graph is a self-centered SVN-graph.

Proof. Let $G = (A, B)$ be a connected self-median SVN-graph with $V = \{v_1, v_2, v_3, \ldots, v_n\}$.

By definition,

$s_T(v_1) = s_T(v_2) = s_T(v_3) = \ldots = s_T(v_n),$

$s_I(v_1) = s_I(v_2) = s_I(v_3) = \ldots = s_I(v_n),$

$s_F(v_1) = s_F(v_2) = s_F(v_3) = \ldots = s_F(v_n).$

$\sum_{\substack{v_i \in V \\ i \ne 1}} \delta_T(v_1, v_i) = \sum_{\substack{v_i \in V \\ i \ne 2}} \delta_T(v_2, v_i) = \sum_{\substack{v_i \in V \\ i \ne 3}} \delta_T(v_3, v_i) = \ldots = \sum_{\substack{v_i \in V \\ i \ne n}} \delta_T(v_n, v_i),$

$\sum_{\substack{v_i \in V \\ i \ne 1}} \delta_I(v_1, v_i) = \sum_{\substack{v_i \in V \\ i \ne 2}} \delta_I(v_2, v_i) = \sum_{\substack{v_i \in V \\ i \ne 3}} \delta_I(v_3, v_i) = \ldots = \sum_{\substack{v_i \in V \\ i \ne n}} \delta_I(v_n, v_i),$

$\sum_{\substack{v_i \in V \\ i \ne 1}} \delta_F(v_1, v_i) = \sum_{\substack{v_i \in V \\ i \ne 2}} \delta_F(v_2, v_i) = \sum_{\substack{v_i \in V \\ i \ne 3}} \delta_F(v_3, v_i) = \ldots = \sum_{\substack{v_i \in V \\ i \ne n}} \delta_F(v_n, v_i).$

$\sum_{\substack{v_i \in V \\ i \ne 1}} \frac{1}{T_B(v_1,v_i)} = \sum_{\substack{v_i \in V \\ i \ne 2}} \frac{1}{T_B(v_2,v_i)} = \sum_{\substack{v_i \in V \\ i \ne 3}} \frac{1}{T_B(v_3,v_i)} = \ldots = \sum_{\substack{v_i \in V \\ i \ne n}} \frac{1}{T_B(v_n,v_i)},$

$\sum_{\substack{v_i \in V \\ i \ne 1}} \frac{1}{I_B(v_1,v_i)} = \sum_{\substack{v_i \in V \\ i \ne 2}} \frac{1}{I_B(v_2,v_i)} = \sum_{\substack{v_i \in V \\ i \ne 3}} \frac{1}{I_B(v_3,v_i)} = \ldots = \sum_{\substack{v_i \in V \\ i \ne n}} \frac{1}{I_B(v_n,v_i)},$

$\sum_{\substack{v_i \in V \\ i \ne 1}} \frac{1}{F_B(v_1,v_i)} = \sum_{\substack{v_i \in V \\ i \ne 2}} \frac{1}{F_B(v_2,v_i)} = \sum_{\substack{v_i \in V \\ i \ne 3}} \frac{1}{F_B(v_3,v_i)} = \ldots = \sum_{\substack{v_i \in V \\ i \ne n}} \frac{1}{F_B(v_n,v_i)}.$

$max\{\frac{1}{T_B(v_1,v_i)}\} = max\{\frac{1}{T_B(v_2,v_i)}\} = max\{\frac{1}{T_B(v_3,v_i)}\} = \ldots = max\{\frac{1}{T_B(v_n,v_i)}\},$

$max\{\frac{1}{I_B(v_1,v_i)}\} = max\{\frac{1}{I_B(v_2,v_i)}\} = max\{\frac{1}{I_B(v_3,v_i)}\} = \ldots = max\{\frac{1}{I_B(v_n,v_i)}\},$

$min\{\frac{1}{F_B(v_1,v_i)}\} = min\{\frac{1}{F_B(v_2,v_i)}\} = min\{\frac{1}{F_B(v_3,v_i)}\} = \ldots =$
$min\{\frac{1}{F_B(v_n,v_i)}\}$.
$max\{\delta_T(v_1, v_i)\} = max\{\delta_T(v_2, v_i)\} = max\{\delta_T(v_3, v_i)\} =$
$\ldots = max\{\delta_T(v_n, v_i)\}$,
$max\{\delta_I(v_1, v_i)\} = max\{\delta_I(v_2, v_i)\} = max\{\delta_I(v_3, v_i)\} =$
$\ldots = max\{\delta_I(v_n, v_i)\}$,
$min\{\delta_F(v_1, v_i)\} = min\{\delta_F(v_2, v_i)\} = min\{\delta_F(v_3, v_i)\} =$
$\ldots = min\{\delta_F(v_n, v_i)\}$.
$e(v_1) = e(v_2) = e(v_3) = \ldots = e(v_n)$.
Therefore G is self-centered.

CONCLUSION

In this paper, the concepts of length, distance, eccentricity, radius, diameter, status, total status, median and central vertex of a single valued neutrosophic graph have been investigated. We have presented the concept of self-centered single valued neutrosophic graph. Also some interesting properties of self-centered single valued neutrosophic graphs followed by some examples.

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