Shapes & DOF: on the use of modal concepts in the context of parametric non-linear studies

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Abstract
Physical responses tend to lie within restricted subspaces even for parametric problems. For a given subspace, the choice of a basis defines Degree Of Freedom (DOF) and this choice may give interesting meaning to the associated amplitudes. Classical modal analysis builds subspaces combining modeshapes and static responses. Parametric loads for non-linear, damped, variable, ... structures are discussed to extend the theory and illustrated for test and simulation cases. Challenges in shape extraction and basis generation techniques are then detailed. Introducing the ability to manipulate models with variable junction properties, component material and geometry, load and operating conditions, ... opens new questions on the quantification and tracking of changes and objectives throughout design exploration. The definition of a reference linear system and the use of global and/or local modal DOF are shown to provide an interesting perspective.

1 Introduction

Modal analysis has been used extensively for many decades. The base notion is that the response is dominated by modal contributions corresponding to a linear combination of shapes constant in space and associated amplitudes depending on time, which are known as Degrees of Freedom (DOF) in mechanics and states in system modeling. Linear time invariant systems are no longer considered difficult, so the focus of this paper will be on parametric problems. Non-linearities, damping treatments, junctions, geometric variability, ... are such parameters that challenge current practice. In such studies the choice of relevant shapes and associated DOF plays a useful role in both speeding computations and providing ways to analyze tests.

As illustrated by figure 1, design groups use direct problem resolution: starting from geometry in the form of CAD models, well defined mechanical model equations are generated in the form of finite element models for most industrial applications. From these models, reduction techniques are used to extract shapes and generate system models that allow the prediction of frequency or time responses.

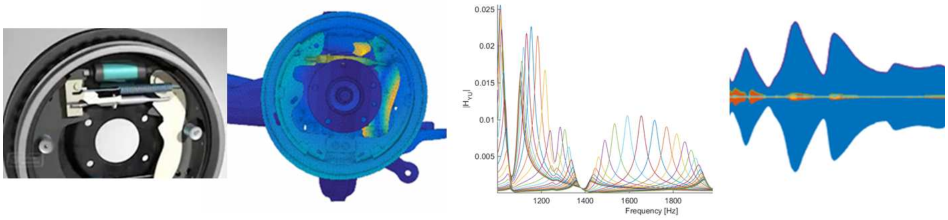


Figure 1: Direct problem: CAD, FEM modes/reduced models, transfers, time responses

Experimental groups go in the opposite direction, that is solve inverse problems. As shown in figure 2, time responses are measured, signal processing tools are then used to obtain experimental characterizations such as transfer functions, then multiple sensor reading are combined to identify system models containing shape information. Finally test and FEM models can be combined in modeshape expansion and updating processes.

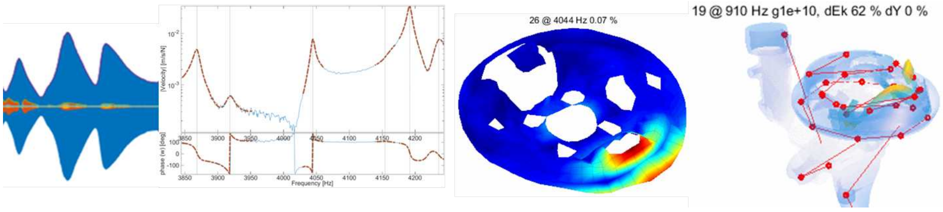


Figure 2: Inverse problem : a) time response, b) transfer function and identification, c) analysis of modeshapes, expansion

The meeting point between computational and experimental activities is a system with inputs and outputs as detailed in section 2. A key difficulty of system modeling is the definition of states/DOF and associated shapes. Bandwidth and load objectives are introduced to allow correct system representations. Within a given subspace the ability to choose DOF is discussed. Parametric problems and associated loads are then introduced. Finally current challenges in subspace techniques are detailed: alternative learning and classification strategies, interface reduction, sparsity preservation.

Considering cases where parametric models are available, section 3 addresses tools needed to analyze their evolution. A parallel with control theory shows the relevance of "coupling" studies that correspond to the adaptation of root locus computations. The definition of a reference linear system is used to track property changes in parametric computations and test. Finally, the issue of understanding component properties when integrated in a system is discussed and illustrated.

2 A system perspective of structural dynamics

2.1 Definition of system models

Considering vibration problems through the point of view of system dynamics is a useful perspective. In a system model, one considers

- inputs $\{u(t)\}$, which in vibration problems will typically be applied loads or enforced displacements. In experiments inputs are measured as electrical signals and thus limited to a series of scalar quantities, which may a notable approximation if the true loads are non-uniform fields.
- states $\{x(t)\}$, which are all the quantities needed to write a differential equation describing the evolution of the system in time. In a fairly general setting the evolution equation can be written as a non-linear state-space equation

$$\{\dot{x}\} = f(x(t), u(t), p, t) \tag{1}$$

For mechanical engineers, displacement Degrees of Freedom (DOF) $\{q\}$ are used and the state vectors combines displacement $\{q\}$ and velocity $\{\dot{q}\}$. Additional internal states are also introduced for non-linear material laws (plasticity, viscoelasticity, ...). Figure 3 illustrates a simple modified Oberst test which can be considered as a single DOF system characterizing the feature of interest which is the frequency and damping of the first bending mode, but will require many FEM DOF to allow the prediction of the strain energy field shown as color. Modal analysis or by extension model reduction, discussed in section 2.2, provides a clean strategy to relate these two definitions of DOF for the same system.

- outputs $\{y(t)\}$, which are measurable quantities in test (displacements, strains, loads, ...) and predictable quantities in models. Since states describe all the quantities describing the evolution of the system, outputs can be estimated at a given time through an observation equation

$$\{y\} = g(x(t), u(t), p, t) \tag{2}$$

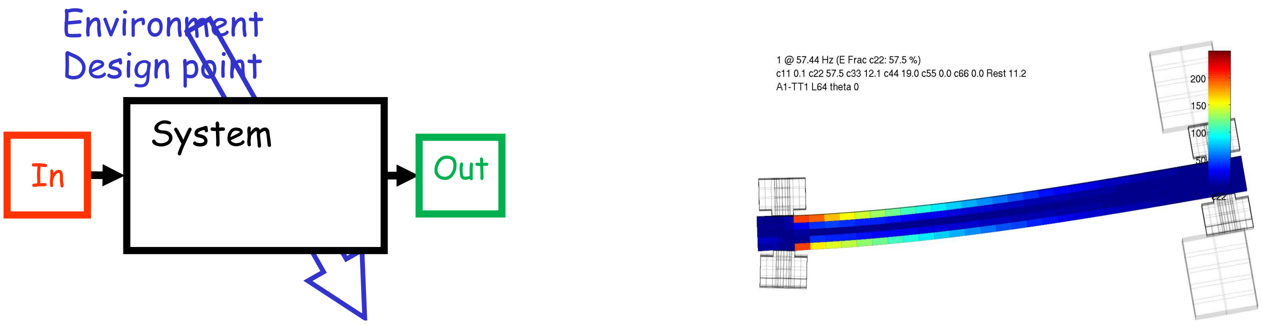


Figure 3: Left : parametric system representation. Right : sample case modified Oberst test

- environment variables or design points corresponding to parameters that are either constant or assumed to be independent of states. Section 2.3 will detail parameters of interest in vibration design problems.
- features are quantities that can be extracted from the response such as resonance frequencies, shapes, damping, mean levels, ...

The objective of a system model is to provide accurate prediction of the input/output relation and the associated features for a range of design points. Side objectives are to minimize the cost of obtaining and operating such a model and to have a proper understanding of its limitations.

2.2 Classical modal analysis/model reduction

Classical *modal analysis*, which is taken here to include Component Mode Synthesis (CMS), assumes Linear Time Invariant (LTI) systems so that the evolution equation can be written using constant matrices. Although the theory applies to all LTI problems, the paper will focus on mechanical problems of the form

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = [b]\{u(t)\} \tag{3}$$

In vibration problems, modeling typically considers two objectives: the accurate representation of the response to a restricted *set of loads* within a *bandwidth*. In an undamped LTI system, normal modes are harmonic solutions in the *absence of excitation* and thus verify

$$\left[-M\omega_j^2 + K\right]\{\phi_j\} = 0. \tag{4}$$

To achieve the bandwidth objective the system model must be able to represent all modeshapes within this band. Strategies to eliminate modes within the band can be based on modal participation factors or balanced realization arguments [1, 2], but in practice such truncation is nowadays rarely necessary for performance. The argument to use modes above the bandwidth, the common 1.5 times the bandwidth or any other factor, is actually quite weak and adding more modeshapes is generally sub-optimal in terms of model size.

To achieve the *loads objective*, it is also necessary to account for the additional associated information. This is achieved by computing static responses to loads

$$[q_{Residual}] = [K_{Flex}]^{-1} [b] \tag{5}$$

or by using the common alternative of static responses to enforced displacements on the areas of load application, which is known as Guyan condensation. The need for static terms is widely acknowledged: the contribution is called upper residuals in experimental modal analysis [3], attachment mode or residual flexibility for free mode component mode synthesis [4], D term in the observation equation of state-space models, *residual vector* in many FEM solvers, ... and yet many recognize to have difficulties with the notion.

Figure 4 illustrates a case with strong static contribution [5]. A piezo-actuator is glued on a honeycomb panel. At 10 Hz, the bending of the honeycomb skin is much larger than the overall panel bending. The response can be decomposed in modal (a), (b) and static (c) contributions. The first key here is to understand that static contribution comes from the piezo load, and thus is completely independent from the structural modes which do not depend of any load.

Once the necessity of static correction/residual flexibility understood, the challenge remains in the definition of reference loads $[b]$ for which a static correction is needed, discussed in section 2.3 and the understanding of how wrong results are when it is ignored.

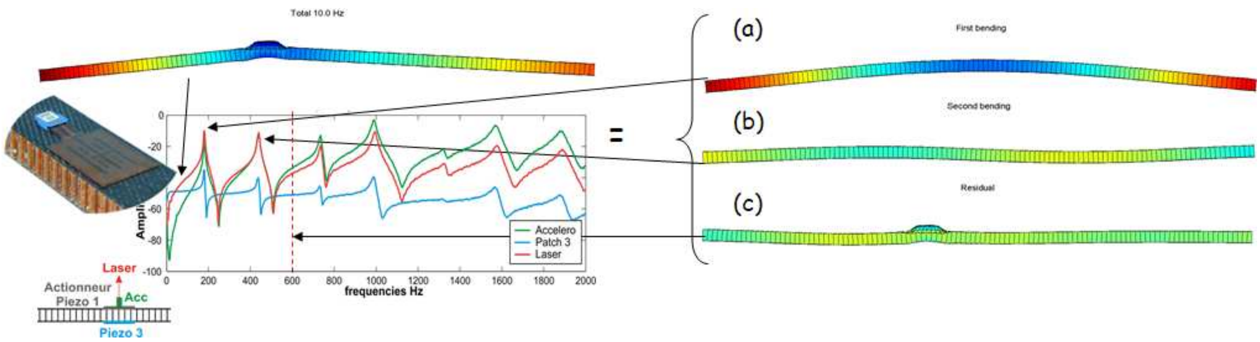


Figure 4: Sample case with strong static contribution [5]

While modes and static responses are results of interests, one of the key aspects of modal analysis is that they span the subspace where the response lies. For any basis T of that subspace, one defines generalized DOF $\{q_R\}$ and seeks an approximation of the form

$$[q_{Full}(t)] = [T] \{q_R(t)\} \tag{6}$$

leading in a Ritz-Galerkin approach to a reduced model still of the form (3) but using reduced matrices $M_R = T^T M T$, ...

The two basics bases [4, 6] are McNeal (free modes + static response to loads $[\phi_{1:NM} \;\; K^{-1}b]$) and Craig-Bampton (enforced static displacement /Guyan+ fixed interface modes), but any combination of mode and static response will give achieve the bandwidth and load objectives and thus lead to correct results. Conceptual limitations of most of the literature are

- the use of explicit formulas for bases rather than the implementation of numerical basis generation strategies to go from arbitrary sets of assumed vectors to bases [7, 8]. Thus many have rejected static corrections because it tends to generate poorly conditioned basis if iterative orthogonalization of loads b is not enforced. But the combination of free modes and Guyan vectors is also valid although rarely implemented in software packages.

- the use of modes computed on a single component rather than the trace of modes computed on a larger system (see section 2.4.3).

- the failure to recognize that the accuracy of a reduced model depends on the subspace spanned by T not on the base vectors and associated DOF used to generate that subspace. For example, a modal model in physical coordinates can be defined by taking the first few modes $[\phi_{1:NR}]$, retaining a subset of physical DOF R, and spanning the subspace with the basis defined as follows

$$[q_{Full}(t)]_N = [\phi_{1:NR}]_{N\times NR} [\phi_{1:NR}|_R]^{-1}_{NR\times NR} \{q_R(t)\}_{NR} \quad (7)$$

  Starting from physical DOF, basis generation techniques based in incomplete LU factorization will lead to generalized coordinates that still correspond to physical DOF, while eigenvalue/SVD based techniques will lead to modal coordinates.

2.3 Parametric problems of interest beyond LTI modal analysis

While LTI modal analysis remains very useful the challenge has now moved to the handling of parametric problems. The common approach is to continue using classical CMS methods and to reformulate the problem as a linear reduced model with all other aspects of interest (coupling, non-linearity, and environment) implemented as external load or feedback. This approach is well mastered for point to point connections. Thus many multi-body simulation software support flexible component reduction using Craig-Bampton reduction with residual terms for viscous loads, distributed inertia (inertial relief modes [4]). But much more should be done.

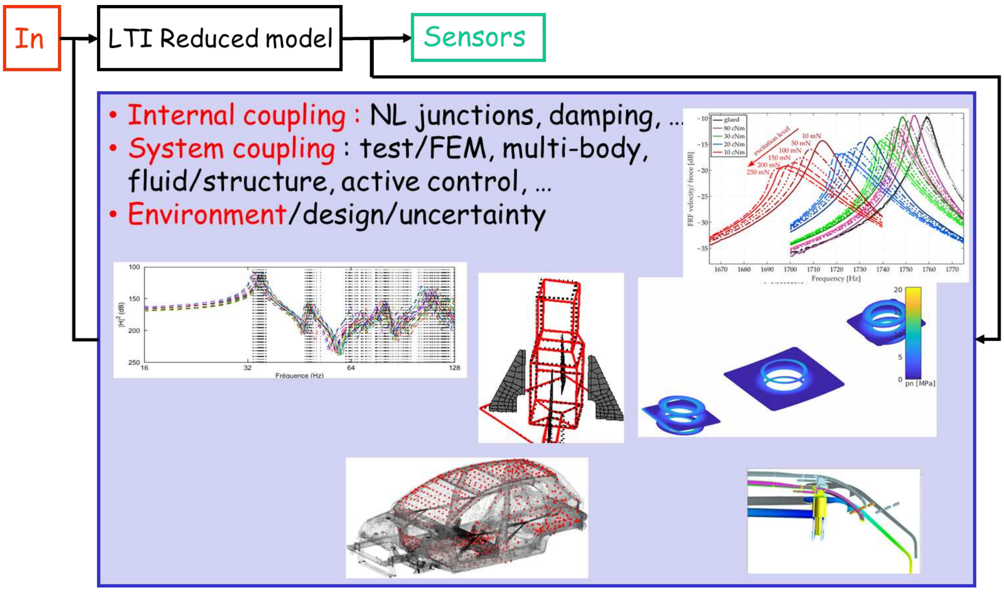


Figure 5: Formulation as a combination of LTI problem with additional loads representing parametric effects.

Selecting an underlying linear system, associated with constant matrices, M_0, C_0, K_0, one can always rewrite the equations of motion (3) by the defining a parametric load $\{f_p\}$ in the right hand side of the equation

$$[M_0]\{\ddot{q}\} + [C_0]\{\dot{q}\} + [K_0]\{q\} = [b]\{u(t)\} + \{f_p(M(p) - M_0, K(p) - K_0, q, \dot{q}, p)\} \quad (8)$$

Figure 12 illustrates a parametric contact problem. The contact between a main press formed plate and a small "cable guide" component is modeled at gauss points shown in (a). Due to manufacturing the contact does not occur on the whole surface and different colored dots correspond to different surfaces. In the frequency tracking, cross markers correspond to variable surface computations. They overlay nearly exactly with solid lines which correspond to variable distributed stiffness. The selected modes illustrate that fairly flat lines correspond to global bending of the plate while the frequency traversing the full frequency range from 700 to 1900 Hz corresponds to a local bending shape of the cable guide.

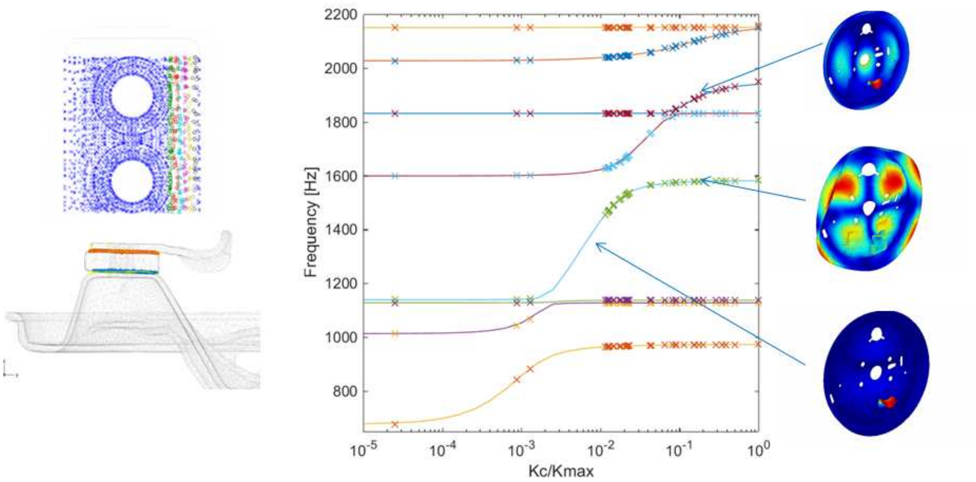


Figure 6: Influence of contact on the modal properties of a brake part [9]. Left: Zoom on cable guide, right: frequency evolution with contact surface/stiffness

The idea of parametric reduced models is to construct a basis independent of the amplitude of parameter loads just as traditional CMS is independent of the amplitude of external inputs. Here the parameter loads are forces applied on all contact points. As there are many points, computing a static correction (5) for all contact loads will lead to large reduced models. Alternatives [10] are to build a multi-model basis from modes computed using different contact conditions or the compute the static correction to contact loads found for the nominal modes (first order correction). These techniques are similar to interface reduction discussed in 2.4.1.

The representation of static correction corresponds to the use of additional out of band DOF. These have a strong impact on zeroes (by opposition to in band modes which generate poles) of transfers associated with parameter loads and thus on the prediction of closed loop configurations where these have a significant impact. A key result is that the added shapes are very different from the following modes so that seeking convergence by just adding more modes is rarely a good idea. A challenging question is the analysis of the minimum number of out of band DOF necessary to represent all configurations shown in figure 12 right.

A very partial list of problems that can be addressed by including parametric loads into classical CMS are

- Non-linear responses are parametric in the sense that local properties depend on inputs. Figure 5 thus illustrates the amplitude dependence of a junction problem [11] where the parametric loads are contact/friction forces distributed on junction surfaces also shown in the figure.
- Structures damped with viscoelastic materials have parametric volume loads associated with viscoelastic stresses which depend on temperature and frequency (figure 11 gives an illustration). Thickness variations, often used as design parameters, can also be represented as a parametric load.
- Structural dynamic modification techniques [12] seek to combine a linear test model with parametric models of possible modifications.
- Geometry variations due to design or variability in the manufacturing process are also of major interest [13, 14].

2.4 Open challenges in shape extraction and basis generation techniques

Space/time separation (6) considering bandwidth and input objectives were shown to be the base ingredients of CMS. This leaves many choices open and this section seeks to illustrate areas still under development. Section 2.4.1 addresses alternative learning and vector classification techniques. Section 2.4.2 discusses issues with interface reduction, which occur in mechanical problems where applied loads correspond to fields. Finally, section 2.4.3 touches basis sparsity and associated memory/CPU issues.

2.4.1 Alternative learning and vector classification strategies

In traditional CMS methods, shapes are obtained by a combination of simple computations of modes and static responses. Even modes are actually computed through a series of static computations in Krylov iteration based strategies such as the Lanczos algorithm.

Another category of approaches uses learning techniques where a full order response is analyzed a posteriori. Such techniques can be used both in computations (for example to compute the high frequency impedance of a piezo [16]) and experiments [17]. Figure 7 illustrates a time varying resonance found in a squeal test. From the spectrogram of each sensor, a response at the frequency of the maximum amplitude can be extracted. The set of learning vectors is thus a matrix with spatial rows (different sensors) and temporal columns associated with the maximum response at different instants.

Given a set of learning vectors or the full possible space, a natural strategy is to classify directions by their relative contribution. Here the Singular Value Decomposition is used to extract shapes, shown in red, and associated time amplitudes (DOF) shown in blue. The usefulness of these amplitudes will be further discussed in section 3.2.

$$\begin{bmatrix} y(c,t) & \rightarrow & time \\ \downarrow & & \\ sensors & & \end{bmatrix} = \sum_i \begin{Bmatrix} U_i \\ \downarrow \\ sensors \end{Bmatrix} \{a_i(t) \quad \rightarrow \quad time\}$$

Figure 7: Shape and amplitude extraction in a time varying squeal event [17]

Classification of vectors in a large subspace, followed by truncation, is a key ingredient of most strategies. Modal analysis considers low frequency modes which for equal kinetic energy the shapes of interest are those that minimize strain energy [18]. Using strain and kinetic energy norms is not the only possibility and Ref [19] clearly points out that the literature is full of eigenvalue based subspace classification algorithms: SVD Singular Value Decomposition, KLD Karhunen-Loève Decomposition, PCA Principal Component Analysis, ...

In parametric problems, the body of literature on Proper Generalized Decomposition arguments that rather than limiting the decomposition to space and time, variable separation can made on space, time and parameter variations [20]. Finally, many other fields, including the very active image and text classification fields, also address dimensionality reduction [21]. It should be noted however that a lot of this literature deals with data reduction (representation of a parametric surface through variable separation) rather than model reduction (generation of small differential equations).

2.4.2 Interface reduction

The global application of subspace classification algorithms without reference to loads leads to modes. To continue addressing the objective of correct representation of inputs, local or interface problems must be considered. Interfaces corresponds to the areas where external or parametric loads are applied (surfaces in junctions, volumes for viscoelastic models, volumes for geometry changes, ...).

Taking the crankshaft shown in figure 5, there are about 5 000 DOF connected to the bearing interfaces (red dots) and the FEM model contains 2 million DOF to allow accurate prediction of stresses. If no special care is taken to build vectors that are zero on large parts (possible here by considering interfaces that contain all nodes of a perpendicular surface), the static part of the basis shown in green will use $2e6 \times 5e3$ doubles $\approx 74GB$. The reduced matrix also has a full block associated with the interface leading to a $5e3 \times 5e3$ matrix requiring 120 MB storage which must be passed from computer memory to processor at each time step of a time integration thus creating a computational bottleneck.

The Craig-Bampton approach however considers unit displacements at DOF (indicated by the red identity in basis T): one of the interface nodes moves while no other does. Unit displacements guarantee independence but any basis on the interfaces will achieve the same objective. Analytic decompositions (Fourier, orthogonal polynomials) and eigenvalue problems solved on a restricted mesh close to the interface provide much more bases of independent unit displacements that are more easily truncated. Such interface modes are used in automated procedures such as AMLS [22], but can also be quite useful in analyzing loads transmitted through surfaces in contact [23, 24]. Similar techniques can be used to deal with periodic structures [8, 25].

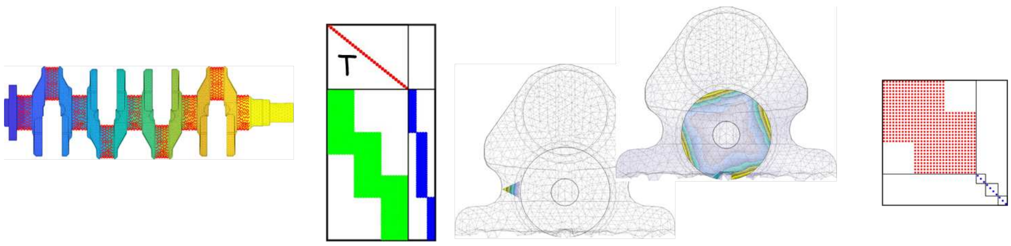


Figure 8: Redefining unit displacements : (a) sample crankshaft example, (b) topology of reduced basis, (c) unit nodal displacement/interface mode (d) topology of reduced matrix

The limited success of interface reduction methods outside automated procedures may be due to issues with the choice of coordinates (see (7) for a bypass) and the need to automate precision checks (see, for example, the work on residue iteration [26]).

2.4.3 Piece wise decompositions and sparsity

The discussion of interface modes motivated that computation and memory costs are critical objectives for computational model reduction methods. The variable separation techniques (6) need to be extended by acknowledging that piecewise decompositions will lead to much sparser matrices and thus lower memory and/or CPU costs. Figure 9a illustrates that a full model response $\{q\}$ can be represented as a combination of bases on components (here individual blade sectors shown with different colors) and associated generalized DOF $\{q_{Ri}\}$. If the DOF sets associated with each component are disjoint (this is always possible), the reduced matrix is sparse as shown in 9b. In the test case [8], reduction is essential both to allow computations and to store 1500 modes on a 60e6 DOF model which would require 670 GB.

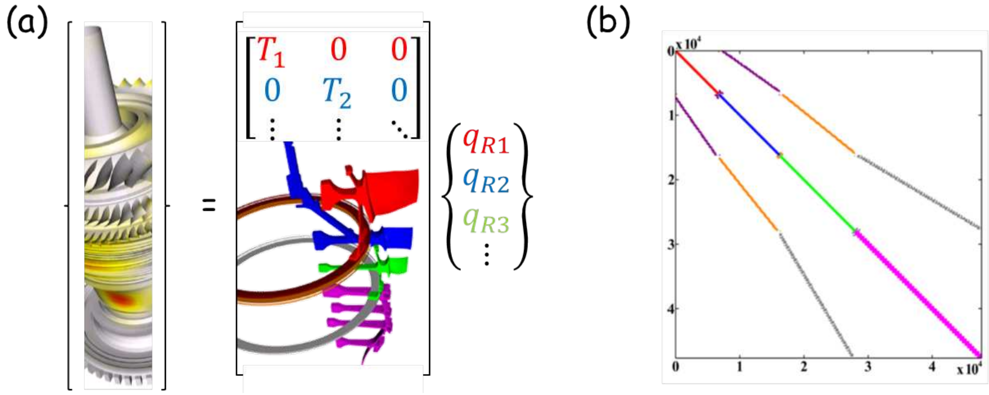


Figure 9: Reduction and sparsity: (a) reduced bases with disjoint support (b) topology of reduced matrix [8]

Obviously a model reduced this way should still achieve the general load/bandwidth accuracy objectives introduced in section 2.1. This requires fairly automated procedures to be applicable without expert knowledge. When performing a spatial decomposition, loads will be transmitted between components and if the components are reduced independently, as traditionally assumed [4], large interfaces are found and truncation, discussed in section 2.4.2, does not solve everything.

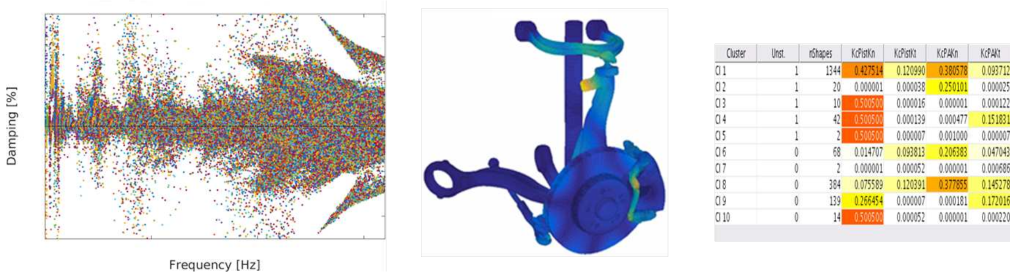
A simple procedure to obtain *exact interface reduction* in the nominal model consists in starting with a basis T valid for the full model and using the subspace generated by the trace of this basis on each component at component basis T_i. Obviously the basis is no longer exact if parameters vary, but parameter variations can be accounted for by enriching T. For more details see applications to multi-stage cyclic symmetry [8], brake squeal [27], train/track interaction [25].

It is finally interesting to note that sparsity issues can also be applied to the time domain (Harmonic Balance Method) or to PGD strategies.

3 Parametric problems :*coupling*, modal interaction, generalized DOF

While different orientations for reduced model representations were discussed in section 2.4, more interesting challenges are associated with the analysis of parametric problems.

Figure 10 illustrates a brake squeal instability study. Model complexity is due to the fact that lower squeal frequencies (1-4kHz) tend to have significant interactions with the suspension system. A state-of-the art industrial model thus features between 2 and 5 million DOF (geometric complexity), and between 300 and 600 modes in the frequency band of interest below 16 kHz (dynamic complexity). 8 to 15 components, which exhibit material and geometric variability, are coupled by interfaces whose properties vary with operating conditions requiring additional DOF (parametric complexity). Design exploration typically considers sampling over a subset of sensitive parameters to search for robust countermeasures or update parameters when test is available. A full complex eigenvalue extraction for a design point can require a few hours of computation time, challenging relevant design exploration strategies. Here computing 1000 design points would require over 80 days and generate 22 TB of data without post-treatment – clearly unacceptable in terms of design cycles timing and storage capabilities.



Cluster	Unst.	nShapes	KcPistKn	KcPistKt	KcPAKn	KcPAKt
Cl 1	1	1344	0.427514	0.120990	0.380578	0.093712
Cl 2	1	20	0.000001	0.000038	0.250101	0.000025
Cl 3	1	10	0.500500	0.000016	0.000001	0.000122
Cl 4	1	42	0.500500	0.000139	0.000477	0.151831
Cl 5	1	2	0.500500	0.000007	0.001000	0.000007
Cl 6	0	68	0.014707	0.093813	0.206383	0.047043
Cl 7	0	2	0.000001	0.000052	0.000001	0.000686
Cl 8	0	384	0.075589	0.120391	0.377855	0.145278
Cl 9	0	139	0.266454	0.000007	0.000181	0.172016
Cl 10	0	14	0.500500	0.000052	0.000001	0.000220

Figure 10: Left : poles for a 1000 design points, center : sample mode, right : tabular representation of sensitivity parameters. Test case courtesy of Daimler AG.

Model reduction is key to make the model practical in the sense that rather than doing a few hundred computations lasting a couple hours, one spends a couple hours reducing the model and computes a few thousands computations lasting tens of seconds. Another aspect of model reduction is that if you perform restitution on the fly (when you click on a point of (a) you launch the restitution of the shape in (b)), you do not deal with TB of data but just hundreds of MB. There are thus no theoretical limitations to making parametric tools accessible on workstation, rather than large cluster.

The figure however illustrates that the raw result of a parametric experiment is not very understandable. Questions of interest are : is one of those points closer to my experiment, is the experiment sensitive to small changes, is one of the designs better and robust (meaning with little sensitivity to things that are not controlled).

A key question in analyzing parametric problems is thus to have objective strategies to quantify what remains constant and what corresponds to small or large changes. Section 3.1 introduces the idea that many parametric problems can be analyzed as system *coupling* with variable stiffness studies providing very relevant information about damping, non-linear or variability effects. Tools to track shape changes are then addressed in section 3.2. Finally, section 3.3 addresses the relation between local and global modes.

3.1 *Coupling* : a measure of change in parametric dynamic problems

In control applications, it is common to compare open and closed loop performance. Taking the system perspective, the equivalent of a feedback loop is the part of the dynamics not included in the nominal LTI system. Change can thus be measured through the impact of parameter loads and the section seeks to illustrate how this applies to structural problems.

Following resonance frequencies is a measure of change that is very natural to modal analysts. Figure 11 illustrates the impact of random excitation level on steam exchanger structure with contact non-linearity and the impact of temperature and viscoelastic material thickness in a bracket junction. In both cases, the modal frequency increases with a parameter and in the range where the frequency sensitivity is highest, higher damping levels are achieved. As argumented in [10], frequency sensitivity is very much related to the level of energy in the design area (here the contact non-linearity or the viscoelastic joint). The notion is also used to estimate electromechanical coupling in active systems [28], hence the name *coupling* retained here.

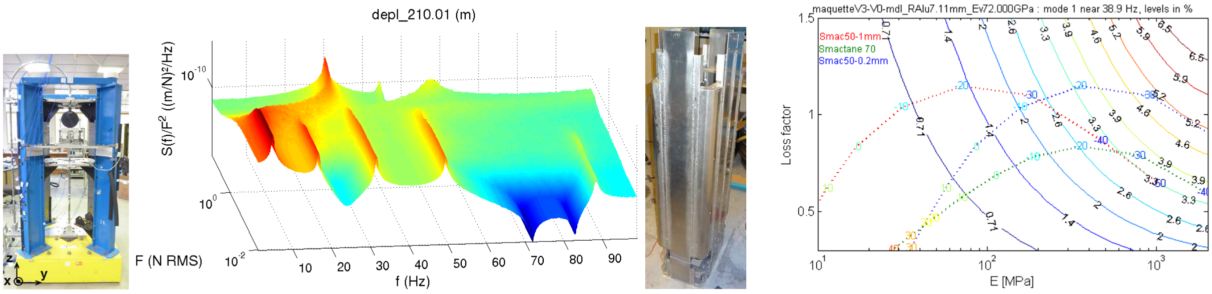


Figure 11: Left : effect of excitation level for a structure with impact non-linearity [29]. Right : effect of temperature in a structure with viscoelastic damping [10]

As motivated in section 2.3, it always possible to represent non-linear parametric problems as an underlying LTI problem (at design point p_0) with parametric loads. It is further possible to use modal states of this LTI problem to write system equations under the form

$$[I + \Delta M(p)]\{\ddot{q}_R\} + [\Delta C(p)]\{\dot{q}_R\} + \left(\left[\backslash s^2 + \omega_j^2(p_0)_{\backslash}\right] + \Delta K(p)\right)\{q_R(s)\} = \{f_p(q_R, \dot{q}_R, p)\} \quad (9)$$

In this formulation, one distinguishes the nominal LTI model, and perturbations on mass $\Delta M(p)$, damping $\Delta M(p)$ and stiffness $\Delta K(p)$. These are however not unique since their effect can also be represented by the parametric force f_p. In the contact problem of figure 11 left, the linear system can be considered to correspond to the tube without contact, the tube in full contact or a tube with an intermediate contact stiffness that depends on amplitude. From the point of view of resonances visible in the spectra, the variable contact stiffness/damping provides a good representation of the frequency transition or an envelope for the non-linear response.

A key consequence of illustrations shown here is that in many parametric problems, initial design can simply be performed by doing multiple linear elastic computations with variable stiffness, which is much more accessible than full-fledged non-linear or damping computations.

3.2 Observing *Degree of Freedom* associated with shapes

While observing frequency shifts is very useful, properly understanding modal crossing can be very necessary to understand responses. Section 2.3 illustrated how this can be done in experiments but more efficient tools can be inspired by analysis.

Computing modal amplitudes is much more efficient than using the MAC (shape correlation) for tracking. Using the orthogonality condition for modes of the reference system $\{\phi_j(p_0)\}^T [M(p_0)] \{\phi_k(p_0)\} = \delta_{jk}$ leads to the estimation of modal amplitudes from non-reduced response $\{q\}$ as

$$\{q_{jR}(t)\} = [\phi(p_0)]^{-1}\Big|_j \{q(t)\} = \left[\{\phi_j(p_0)\}^T [M(p_0)]\right]\{q(t)\} \quad (10)$$

which corresponds to the perfect modal filter [30].

On the vibro-impact case of figure 11 left, the estimation from operational deflection shapes shown in figure 12, clearly illustrates that the shape of the dominant resonance transitions from the low frequency bending with no effect of the support plate to an high frequency shape combining the first two modes of the free-free tube. During the transition some torsion effects become visible. The shape transition corresponds directly with the resonance transition visible in 11 left (5 Hz for low amplitude to 30 Hz at high amplitude with additional damping visible during the transition).

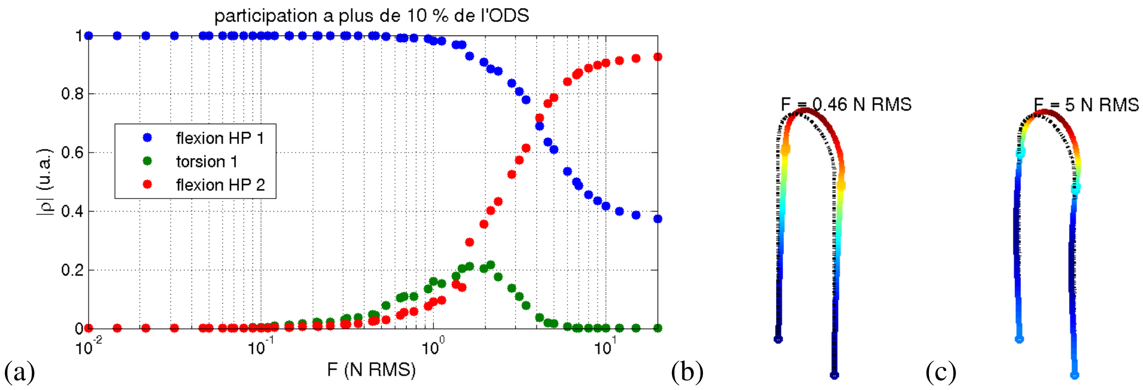


Figure 12: a) Evolution of modal participations with excitation amplitude, b) Low amplitude shape, c) high amplitude shape.

Other analysis applications of modal amplitude tracking are tracking unstable modes in time simulation of brake squeal [27], study of blade mistuning [8], tracking of waves propagating in the soil [25].

Experimentally, modal amplitude estimation was illustrated for time varying squeal (figure 7 and [17]), is applied to auto-resonance techniques in [31]. Expansion [32] in subspaces of modes or static responses are related linear observation schemes, which use the current measured shapes to estimate the generalized amplitudes. These techniques can however be usefully extended to estimate more DOF than sensors in the frequency domain (Minimum Dynamic Residual Expansion or Error in Consitutive Law) or in the time domain (Kalman Filtering possibly extended with the estimation of parameters).

3.3 Component within a system, local and global modes

The next issue of interest is that one typically redesigns a component with the objective of improving the system performance. However, the component modes of interest are usually not the fixed interface modes found in a Craig-Bampton model. The *Component Mode Tuning* (CMT) method [14, 27] was thus introduced to provide a reduced model with DOF that explicitly correspond to the amplitude of free-free component modes. In this algorithm, components are assumed disjoint (this is always possible if it is accepted to use an element layer as interface) and for each component one generates a mass and stiffness orthogonal basis spanning the subspace combining NC component free modes and NG trace on the component of system modes

$$[T_c] = \left[\left[\phi_{\text{local}} \right]_{1:NC} \ \left[\phi_{\text{global}} \big|_c \right]_{1:NG} \right]_{\perp} \tag{11}$$

Any appropriate numerical orthogonalization procedure can then be used to generate a well-conditioned basis T_c.

In this basis, the first generalized DOF of each component correspond to its retained free modes and the last to corrections needed to recover the nominal system modes exactly. Figure 13 illustrates that in the resulting basis, the mass is an identity matrix, the stiffness is diagonal with squared component mode frequencies ω_{jc}^2 for the first elements, and component coupling leads to block diagonal terms respecting the topology of component connections. The *tuning* in the name CMT reflects the fact the specific form of matrices makes it simple to perform sensitivity studies on the frequency of component modes. Similarly it is quite simple to sum modal energies to display component energies as illustrated in figure 13 right.

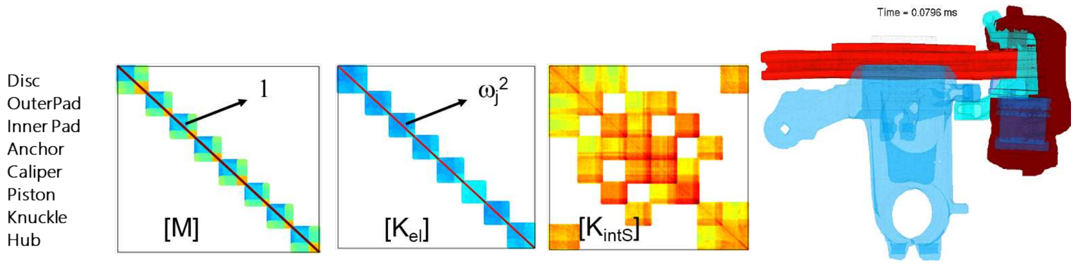


Figure 13: Left : reduced matrix topology in a CMT reduced basis. Right : sample post-processing of component energies

The idea of tracking modal amplitude is quite useful in this context. The CMT basis decomposition provides a sound basis to define local modes : significant fraction of energy associated with a given component. For small components, such as the cable guide of figure 6, free component modes may not be relevant and variants of the base CMT procedure were introduced in [14].

The notions of local and global modes shown here are very much related with the introduction of stochastic models in Ref [33, 34]. These discussions paved the way for the work in [35] which demonstrates how a large number of randomly distributed local modes can lead to significant damping : actually the major source of damping in aircraft modes. The fact that such modes come from so called *non structural* mass opens a very interesting design perspective.

4 Conclusion

The presentation sought to illustrate notions related to shapes and modes. Physical responses tend to lie within restricted subspaces even for parametric systems. DOF are amplitudes associated with base vectors generating the subspace of interest. Since a given subspace has an infinite number of bases, DOF are arbitrary in the sense that they can be changed without affecting the response. But some choices make more sense. Modal amplitude DOF can be defined at the system and component levels and tracking these amplitudes is of interest both in tests and analysis. Amplitude extraction seems particularly promising for non-linear, damped, and time varying problems which commonly share the fact that the varying parameter induces coupling. On the computation side, DOF selections leading to sparse bases can lead to gains in CPU and memory requirements which become interesting when the one or more order of magnitude is gained.

Problems of interest go beyond the traditional theory of linear time invariant systems. Extensions of traditional CMS, considers parametric loads, interface reduction, constant basis reduction for parametric models, and sparsity enhancing basis generation. With such models parametric studies with thousands of design points become routine and lead to new challenges in defining procedures to base decisions on large data sets. Finally, the possibility of state (expansion) and parameter (updating) estimation is very much linked to the ability to build a reduced parametric model but the relation between the two needs to be clarified.

This presentation will conclude with a few general comments, partially inspired by challenges in A. Pfeiffer's ISMA/USD 2016 keynote.

- The main cost of a model is not computational time but engineering time spent : preparing the right computation, navigating between results of a parametric experiment, designing and analyzing tests, progressing in a design cycle. This implies that each design group should have a digital work-flow allowing parametric studies.
- When everything goes well, the new routine practice is to deal with tens of configurations each containing thousands of computations. The need for companies to manage learning costs typically implies that software vendors pre-package GUI operated processes. But new processes devised for parametric studies typically imply the writing of custom steps (computation of criteria, automated generation

of images, ...). A trade-off thus exists between process standardization, implemented as a GUI, and adaptability implemented in the form of a script. On the academic side, the need to focus on teaching GUI or script operation will certainly continue to be an object of debate.

- Outside user process, a few software issues with parametric studies are: the need to perform reduced order computations on the fly using interaction with initial data; the computational architecture between off-line computations (cluster, background) and workstation near real time interaction; scalability of licensing models devised for a few computations rather than thousands.
- It is unlikely that any software provider will ever cover more than 50-70% needs. This is even not desirable since it would mean other groups would stop developing new intelligent strategies. Efficient inter-operability between software packages is thus key. Open source efforts are very good at broad problems with a very large number of users, but often lack vision and focus to answer very technical needs of niche markets. And the noise and vibration community is a collection of such niches. So the inter-operability impulse should come from major user companies forcing/giving incentives to the software providers to develop open API that allow access not only at pre-processing and post-processing but also at solver levels.
- Non-linearity, damping, junction surfaces, variability, local modes associated with non-structural mass, ... correspond to both challenges (they are difficult to account for in test and analysis) and opportunities (since they have a strong impact on the dynamics, they can be used to improve performance).
- Academics will not convince industry that they are dealing with real problems when using mass-spring-beam models. Distinguishing geometric complexity (a FEM with the true geometry and thus many DOF rather than analytic equations), system complexity (a CMS model accounting for both bandwidth and loads), and parametric complexity (multiple designs, boundary conditions, operating conditions, ...) is an often ignored path to demonstrating scalability.

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References

[1] A. Girard, N. Roy, *Structural Dynamics in Industry*, ISTE, London, UK (2008).

[2] D. C. Kammer, M. J. Triller, *Selection of Component Modes for Craig-Bampton Substructure Representations*, Journal of Vibration and Acoustics, Vol. 118, No. 2, (1996), pp. 264.

[3] W. Heylen, P. Sas, *Modal Analysis Theory and Testing*, Katholieke Universteit Leuven, Departement Werktuigkunde (2006).

[4] R. J. Craig, *A Review of Time-Domain and Frequency Domain Component Mode Synthesis Methods*, Int. J. Anal. and Exp. Modal Analysis, Vol. 2, No. 2, (1987), pp. 59–72.

[5] E. Balmes, C. Florens, *Validation of a vibration and electric model of honeycomb panels equipped with piezoelectric patch actuators*, Revue des composites et des matériaux avancés, Vol. 19, No. 3, (2009), pp. 319–338.

[6] D. de Klerk, D. Rixen, S. N. Voormeeren, *General Framework for Dynamic Substructuring : History, Review and Classification of Techniques*, AIAA Journal, Vol. 46, No. 5, (2008), pp. 1169–1181.

[7] O. Boiteau, *Modal Solvers and resolution of the generalized problem (GEP)*, Code_Aster, Version 5.0, R5.01.01-C, (2001), pp. 1–78.

[8] A. Sternchüss, *Multi-level parametric reduced models of rotating bladed disk assemblies*, Ph.D. thesis, Ecole Centrale Paris (2009).

[9] G. Martin, E. Balmes, T. Chancelier, *Review of model updating processes used for brake components*, *Eurobrake* (2015).

[10] C. Hammami, E. Balmes, M. Guskov, *Numerical design and test on an assembled structure of a bolted joint with viscoelastic damping*, Mechanical Systems and Signal Processing, Vol. 70–71, (2015), pp. 714–724.

[11] G. Chevallier, P. Butaud, *Design of an Assembly for Nonlinear Vibration Reduction*, *IMAC*, Springer International Publishing, Cham (2017), 331–336.

[12] M. Corus, *Amélioration de méthodes de modification structurale par utilisation de techniques d'expansion et de réduction de modèle.*, Ph.D. thesis, Ecole Centrale Paris (2003).

[13] T. França de Paula, G. Rejdych, T. Chancelier, G. Vermot des Roches, B. E, *On the influence of geometry updating on modal correlation of brake components.*, *Vibrations, Chocs & Bruit* (2012).

[14] G. Martin, *Méthodes de corrélation calcul/essai pour l'analyse du crissement*, Ph.D. thesis, CIFRE SDTools, Arts et Metiers ParisTech, Paris (2017).

[15] O. Vo Van, *Introduction of variability into pantograph-catenary dynamic simulations*, Ph.D. thesis, Ecole nationale supérieure d'arts et métiers - ENSAM (2016).

[16] E. Balmes, M. Guskov, M. Rebillat, N. Mechbal, *Effects of temperature on the impedance of piezoelectric actuators used for SHM*, *14th Symposium on Vibration, Shock and Noise (VISHNO)* (2014), 1–6.

[17] G. Martin, E. Balmes, G. Vermot Des Roches, T. Chancelier, *Squeal measurement with 3D Scanning Laser Doppler Vibrometer: Handling of the time varying system behavior and analysis improvement using FEM expansion*, *ISMA*, KUL (2018).

[18] E. Balmes, *Optimal Ritz vectors for component mode synthesis using the singular value decomposition*, AIAA Journal, Vol. 34, No. 6, (1996), pp. 1256–1260.

[19] G. Kerschen, J.-c. Golinval, A. F. Vakakis, L. A. Bergman, *The method of proper orthogonal decomposition for dynamical characterization and order reduction of mechanical systems: An overview*, Nonlinear dynamics, Vol. 41, No. 1, (2005), pp. 147–169.

[20] F. Chinesta, A. Huerta, G. Rozza, K. Willcox, *Model Reduction Methods*, E. Stein, R. de Borst, T. J. R. Hughes, Editors, *Encyclopedia of Computational Mechanics Second Edition*, John Wiley & Sons, Ltd, Chichester, UK (2017), 1–36.

[21] S. T. Roweis, *Nonlinear Dimensionality Reduction by Locally Linear Embedding*, Science, Vol. 290, No. 5500, (2000), pp. 2323–2326.

[22] J. Bennighof, M. Kaplan, M. Muller, M. Kim, *Meeting the NVH Computational Challenge: Automated Multi-Level Substructuring*, International Modal Analysis Conference, (2000), pp. 909–915.

[23] G. Vermot Des Roches, E. Balmes, H. Ben Dhia, R. Lemaire, *Compatibility measure and penalized contact resolution for incompatible interfaces*, European Journal Of Computational Mechanics, Vol. 19, (2010), pp. 317–329.

[24] E. Balmes, G. V. Vermot Des Roches, *From physical to macro-models of local assemblies for predictions of global dynamics.*, *Colloque Assemblages Mécaniques*, St Ouen (2015).

[25] E. Arlaud, S. Costa D'Aguiar, E. Balmes, *Receptance of railway tracks at low frequency: Numerical and experimental approaches*, Transportation Geotechnics, Vol. 9, (2016), pp. 1–16.

[26] A. Bobillot, *Méthodes de réduction pour le recalage : Application au cas d'Ariane 5*, Ph.D. thesis, Châtenay-Malabry, Ecole centrale Paris (2002).

[27] G. Vermot Des Roches, *Frequency and time simulation of squeal instabilities. Application to the design of industrial automotive brakes*, Ph.D. thesis, Ecole Centrale Paris, CIFRE SDTools (2011).

[28] N. W. Hagood, W. H. Chung, A. V. Flotow, *Modelling of Piezoelectric Actuator Dynamics for Active Structural Control*, Journal of Intelligent Material Systems and Structures, Vol. 1, No. 3, (1990), pp. 327–354.

[29] T. Thénint, E. Balmes, M. Corus, *Definition of a linear equivalent model for a non-linear system with impacts*, *ISMA* (2012).

[30] J. Bianchi, E. Balmes, G. Vermot des Roches, A. Bobillot, *Using modal damping for full model transient analysis. Application to pantograph/catenary vibration*, *ISMA*, Leuven (2010).

[31] S. Davis, I. Bucher, *Automatic vibration mode selection and excitation; combining modal filtering with autoresonance*, Mechanical Systems and Signal Processing, Vol. 101, (2018), pp. 140–155.

[32] E. Balmes, *Review and Evaluation of Shape Expansion Methods*, IMAC, (2000), pp. 555–561.

[33] A. Arnoux, A. Batou, C. Soize, L. Gagliardini, *Stochastic reduced order computational model of structures having numerous local elastic modes in low frequency dynamics*, Journal of Sound and Vibration, Vol. 332, No. 16, (2013), pp. 3667–3680.

[34] C. Soize, *A model and numerical method in the medium frequency range for vibroacoustics predictions using the theory of structural fuzzy.*, J. Acous. Soc. America, Vol. 94, No. 2, (1994), pp. 849–865.

[35] J. Loukota, J.-C. Passieux, A. Lucchetti, G. Michon, *Modeling of Nonstructural Components in the Dynamic Behavior of Aeronautical Structures*, Journal of Aircraft, (2016), pp. 1–9.