View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Publications at Bielefeld University

# Supplementary Data for the Paper entitled "An Analytical Study of Time of Flight Error Estimation in Two-Way Ranging Methods"

*Proof of Formula Derivation for Alternative Double-sided Two-Way Ranging Method

Cung Lian Sang, Michael Adams, Timm Hörmann, Marc Hesse, Mario Porrmann, Ulrich Rückert
*Cognitronics and Sensor Systems Group (CITEC)*
*Bielefeld University*, Bielefeld, Germany
{csang, madams, thoerman, mhesse, mporrman, rueckert}@techfak.uni-bielefeld.de

***Abstract*—The contents of this publication are intended for providing additional supplementary materials, i.e. proofs of formula derivations, presented in our paper entitled "An Analytical Study of Time of Flight Error Estimation in Two-Way Ranging Methods". The mentioned paper was submitted to "Ninth International Conference on Indoor Positioning and Indoor Navigation (IPIN) 2018", which will be held in Nantes, France on September 24-27, 2018.**

***Index Terms*—Two-Way Ranging, Alternative Double-sided Two-Way Ranging, TOF Error Model**

## I. Introduction

In this publication, we provide the additional material regarding formula derivation for an Alternative Double-sided Two-Way Ranging (AltDS-TWR) method presented in our paper [1], which was submitted to IPIN 2018 conference. The title of the mentioned paper is **"An Analytical Study of Time of Flight Error Estimation in Two-Way Ranging Methods"** [1].

This publication is organized as follows: In section II, the overview of AltDS-TWR method is presented. Then, a brief introduction of the system formulation is described in section III-A, see the details in [1]. It is followed by the corresponding formula derivations for AltDS-TWR method in section IV.

## II. Alternative Double-sided Two-Way Ranging Method

In double-sided TWR methods [2] (Fig. 1), the round trip time of a signal can be formulated as:

$$t_{roundA} = 2T_{tof} + t_{replyB} \tag{1a}$$

$$t_{roundB} = 2T_{tof} + t_{replyA} \tag{1b}$$

Where, $t_{roundA}$ and $t_{roundB}$ are the true round-trip time of device A and B respectively, and $t_{replyA}$ and $t_{replyB}$ are the true replied time of device A and B respectively.

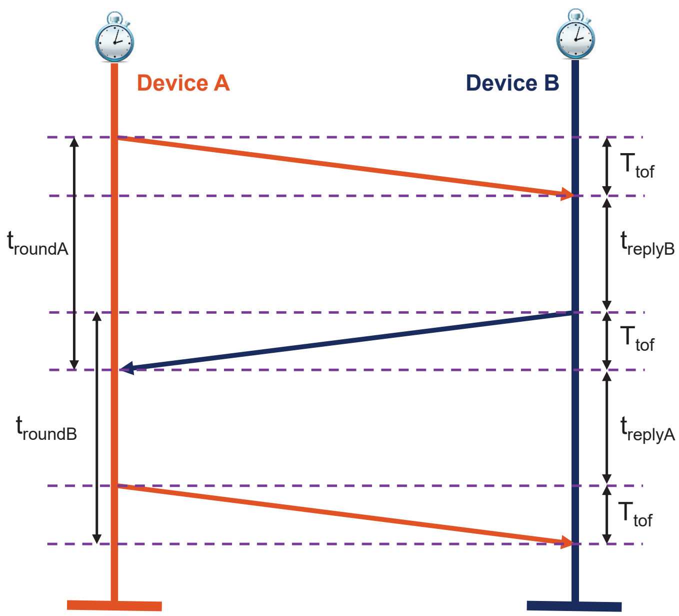


Fig. 1. Illustration of Double-sided Two-Way Ranging Method

The Alternative Double-sided Two-Way Ranging (AltDS-TWR) method [3] is achieved by multiplying (1a) and (1b) as follows. The illustration of the concept is depicted in Fig. 1.

$$t_{roundA} \cdot t_{roundB} = (2T_{tof} + t_{replyB}) \cdot (2T_{tof} + t_{replyA})$$

By simplifying the equation, $T_{tof}$ is obtained as follows [3]:

$$T_{tof} = \frac{t_{roundA} \cdot t_{roundB} - t_{replyA} \cdot t_{replyB}}{t_{roundA} + t_{replyA} + t_{roundB} + t_{replyB}} \tag{2}$$

The detailed derivation of the formula can be found in [3].

By using (2), the estimated TOF for AltDS-TWR method can be considered as:

$$\hat{T}_{tof} = \frac{\hat{t}_{roundA} \cdot \hat{t}_{roundB} - \hat{t}_{replyA} \cdot \hat{t}_{replyB}}{\hat{t}_{roundA} + \hat{t}_{replyA} + \hat{t}_{roundB} + \hat{t}_{replyB}}$$

Therefore, the clock drift error between the estimated and true value of TOF for AltDS-TWR can be resolved as:

$$\hat{T}_{tof} - T_{tof} = \frac{\hat{t}_{roundA} \cdot \hat{t}_{roundB} - \hat{t}_{replyA} \cdot \hat{t}_{replyB}}{\hat{t}_{roundA} + \hat{t}_{replyA} + \hat{t}_{roundB} + \hat{t}_{replyB}} - \frac{t_{roundA} \cdot t_{roundB} - t_{replyA} \cdot t_{replyB}}{t_{roundA} + t_{replyA} + t_{roundB} + t_{replyB}} \quad (3)$$

## III. Formulation of the System

The detailed formulation is provided in our paper [1]. Here, we present the minimum requirements for deriving the corresponding formulas of AltDS-TWR method presented in [1].

### A. TOF Error Estimation Model for TWR Methods

The analytical formulas for a TOF error estimation model (TEEM) are provided as follows in reference to Fig 1, see the details on our submitted paper to IPIN 2018 [1]:

$$\hat{t}_{roundA} = (1 + e_A + \xi_{ABA}) t_{roundA} \quad (4a)$$

$$\hat{t}_{replyA} = (1 + e_A) t_{replyA} \quad (4b)$$

$$\hat{t}_{roundB} = (1 + e_B + \xi_{BAB}) t_{roundB} \quad (4c)$$

$$\hat{t}_{replyB} = (1 + e_B) t_{replyB} \quad (4d)$$

Where, $\xi_{ABA}$ and $\xi_{BAB}$ are additional delays in a single round trip time of a signal measured at device A and B respectively. $\hat{t}_{roundA}$ and $\hat{t}_{roundB}$ are the estimated round-trip time of device A and B respectively, $t_{roundA}$ and $t_{roundB}$ are the true round-trip time of device A and B respectively, $\hat{t}_{replyA}$ and $\hat{t}_{replyB}$ are the estimated replied time of device A and B respectively, $t_{replyA}$ and $t_{replyB}$ are the true replied time of device A and B respectively, and $e_A$ and $e_B$ are the clock drift error introduced by the device A and B respectively.

### B. Classification of Error Model in Three Types

Three types of assumptions are defined, see the details on our submitted paper to IPIN 2018 [1], as follows:

- **Type I Assumption**: This is an ideal case. Assume $e_A = e_B = e$, $t_{replyA} = t_{replyB} = t_{reply}$ and $T_{tof} << t_{reply}$ or $T_{tof}$ is negligible compared to $t_{reply}$. In this assumption, not only there is no clock drift errors between the two evaluated devices, but also the reply time are assumed to be absolutely the same.
- **Type II Assumption**: This is a special case. Assume $t_{replyA} = t_{replyB} = t_{reply}$ and $T_{tof} << t_{reply}$ or $T_{tof}$ is negligible compared to $t_{reply}$. In this assumption, the clock drift error does exist in the evaluated two devices. However, the reply time are assumed to be absolutely the same between them.
- **Type III Assumption**: This is a typical case. Assume $t_{replyA} \neq t_{replyB}$ and $T_{tof} << t_{reply}$ or $T_{tof}$ is negligible. In this assumption, not only the clock drift error does exist in the evaluated two devices but also the reply time between them is different.

## IV. Formula Derivation for AltDS-TWR Method

As it can be seen, if (4a) to (4d) from section III-A are directly substituted into (3), the equation will lead to a very complicated and hard to solve one. Therefore, we derive three different formula for AltDS-TWR method in accordance with three types of assumption characterized in section III-B. The proof of formula derivation for each of the assumptions are present in the Appendix A (Type I assumption), Appendix B (Type II assumption), and Appendix C (Type III assumption) respectively.

## References


[1] C. Lian Sang, M. Adams, T. Hörmann, M. Hesse, M. Porrmann and U. Rückert, "An analytical study of time of flight error estimation in Two-Way Ranging method," unpublished (submitted to IPIN 2018).

[2] "IEEE Standard for Local and metropolitan area networks–Part 15.4: Low-Rate Wireless Personal Area Networks (LR-WPANs)," in IEEE Std 802.15.4-2011 (Revision of IEEE Std 802.15.4-2006) , vol., no., pp.1-314, Sept. 5 2011.

[3] D. Neirynck, E. Luk and M. McLaughlin, "An alternative double-sided two-way ranging method," 2016 13th Workshop on Positioning, Navigation and Communications (WPNC), Bremen, 2016, pp. 1-4.


## Appendix A
## Proof of Type I Assumption for AltDS-TWR

According to Type I assumption described in our paper [1], the equation (1a) and (1b) becomes:

$$t_{roundA} \approx t_{replyB} \quad (5a)$$

$$t_{roundB} \approx t_{replyA} \quad (5b)$$

and the proposed model from (4a) to (4d) becomes:

$$\hat{t}_{roundA} \approx (1 + e + \xi) t_{roundA} \approx C_A t_{roundA}$$

$$\hat{t}_{replyA} \approx (1 + e) t_{replyA} \approx C_B t_{replyA}$$

$$\hat{t}_{roundB} \approx (1 + e + \xi) t_{roundB} \approx C_A t_{roundB}$$

$$\hat{t}_{replyB} \approx (1 + e) t_{replyB} \approx C_B t_{replyB}$$

Where, $C_A = 1 + e + \xi$ and $C_B = 1 + e$.

Therefore, the TOF error between the estimated and true value introduced by AltDS-TWR method in (3) becomes:

$$\hat{T}_{tof} - T_{tof} \approx \frac{C_A^2 t_{roundA} \cdot t_{roundB} - C_B^2 t_{replyA} \cdot t_{replyB}}{(C_A(t_{roundA} + t_{roundB}) + C_B(t_{replyA} + t_{replyB}))} - \frac{t_{roundA} \cdot t_{roundB} - t_{replyA} \cdot t_{replyB}}{t_{roundA} + t_{replyA} + t_{roundB} + t_{replyB}}$$

Substituting (5a) and (5b) in the equation and assuming $t_{replyA} = t_{replyB} = t_{reply}$ yields:

$$\hat{T}_{tof} - T_{tof} \approx \frac{(C_A + C_B)(C_A - C_B) t_{reply}^2}{2(C_A + C_B) t_{reply}}$$

$$\hat{T}_{tof} - T_{tof} \approx \frac{1}{2}(C_A - C_B) t_{reply}$$

Substituting $C_A$ and $C_B$ in the equation becomes:

$$\hat{T}_{tof} - T_{tof} \approx \frac{1}{2} \xi t_{reply} \quad (9)$$

## Appendix B
## Proof of Type II Assumption for AltDS-TWR

According to Type II assumption defined in our paper [1], the equation (1a) and (1b) will now become:

$$t_{roundA} \approx t_{replyB} \tag{10a}$$

$$t_{roundB} \approx t_{replyA} \tag{10b}$$

and the proposed model from (4a) to (4d) will become:

$$\hat{t}_{roundA} \approx (1+e_A+\xi_{ABA})t_{roundA} \approx C_A + \xi_{ABA}t_{reply}$$

$$\hat{t}_{replyA} \approx (1+e_A)t_{replyA} \approx C_A$$

$$\hat{t}_{roundB} \approx (1+e_B+\xi_{BAB})t_{roundB} \approx C_B + \xi_{BAB}t_{reply}$$

$$\hat{t}_{replyB} \approx (1+e_B)t_{reply} \approx C_B$$

Where, $C_A = (1+e_A)t_{reply}$ and $C_B = (1+e_B)t_{reply}$.

Therefore, the TOF error between the estimated and true value introduced by AltDS-TWR method in (3) becomes:

$$\hat{T}_{tof} - T_{tof} \approx \frac{(C_A + \xi_{ABA}t_{reply}) \cdot (C_B + \xi_{BAB}t_{reply}) - C_A C_B}{2C_A + 2C_B + \xi_{BAB}t_{reply} + \xi_{ABA}t_{reply}} - \frac{t_{reply}^2 - t_{reply}^2}{4t_{reply}}$$

$$\hat{T}_{tof} - T_{tof} \approx \frac{C_A\xi_{BAB}t_{reply} + C_B\xi_{ABA}t_{reply} + \xi_{BAB}\xi_{ABA}t_{reply}^2}{2(C_A+C_B) + (\xi_{BAB}+\xi_{ABA})t_{reply}}$$

Substituting $C_A$ and $C_B$ in the equation yields:

$$\hat{T}_{tof} - T_{tof} \approx \frac{[e_A\xi_{BAB} + e_B\xi_{ABA} + (\xi_{BAB}+\xi_{ABA}+\xi_{BAB}\xi_{ABA})]t_{reply}^2}{(2e_A + 2e_B + 4 + \xi_{BAB} + \xi_{ABA})t_{reply}}$$

$$\hat{T}_{tof} - T_{tof} \approx \frac{K_A}{K_B} t_{reply} \tag{15}$$

Where, $K_A = \xi_{BAB}(1+e_A) + \xi_{ABA}(1+e_B) + \xi_{BAB}\xi_{ABA}$ and $K_B = 4 + 2(e_A+e_B) + \xi_{BAB} + \xi_{ABA}$ respectively.

## Appendix C
## Proof of Type III Assumption for AltDS-TWR

According to Type III assumption defined in our paper [1], the equation (1a) and (1b) will now become:

$$t_{roundA} \approx t_{replyB} \tag{16a}$$

$$t_{roundB} \approx t_{replyA} \tag{16b}$$

and the proposed model from (4a) to (4d) will become:

$$\hat{t}_{roundA} \approx (1+e_A+\xi_{ABA})t_{roundA} \approx (C_A + \xi_{ABA})t_{replyB}$$

$$\hat{t}_{replyA} \approx (1+e_A)t_{replyA} \approx C_A t_{replyA}$$

$$\hat{t}_{roundB} \approx (1+e_B+\xi_{BAB})t_{roundB} \approx (C_B + \xi_{BAB})t_{replyA}$$

$$\hat{t}_{replyB} \approx (1+e_B)t_{reply} \approx C_B t_{replyB}$$

Where, $C_A = (1+e_A)$ and $C_B = (1+e_B)$.

Therefore, the TOF error between the estimated and true value introduced by AltDS-TWR method in (3) becomes:

$$\hat{T}_{tof} - T_{tof} \approx \frac{(C_A+\xi_{ABA})t_{replyB} \cdot (C_B+\xi_{BAB})t_{replyA} - C_A t_{replyA} \cdot C_B t_{replyB}}{(C_A+\xi_{ABA})t_{replyB} + (C_B+\xi_{BAB})t_{replyA} + C_A t_{replyA} + C_B t_{replyB}} - \frac{t_{replyB} \cdot t_{replyA} - t_{replyA} \cdot t_{replyB}}{2(t_{replyA+t_{replyB}})}$$

$$\hat{T}_{tof} - T_{tof} \approx \frac{[(C_A+\xi_{ABA}) \cdot (C_B+\xi_{BAB}) - C_A C_B] t_{replyA} \cdot t_{replyB}}{(C_A+\xi_{ABA})t_{replyB} + (C_B+\xi_{BAB})t_{replyA} + C_A t_{replyA} + C_B t_{replyB}}$$

Substituting $C_A = (1+e_A)$ and $C_B = (1+e_B)$ in the equation and simplifying it yields:

$$\hat{T}_{tof} - T_{tof} \approx \frac{(\xi_{BAB} + e_A\xi_{BAB} + \xi_{ABA} + e_B\xi_{ABA} + \xi_{BAB}\xi_{ABA})t_{replyA}t_{replyB}}{(2+e_A+e_B+\xi_{BAB})t_{replyA} + (2+e_A+e_B+\xi_{ABA})t_{replyB}}$$

$$\hat{T}_{tof} - T_{tof} \approx \frac{C_1 t_{replyA} t_{replyB}}{C_2 t_{replyA} + C_3 t_{replyB}} \tag{23}$$

Where $C_1 = \xi_{BAB}(1+e_A) + \xi_{ABA}(1+e_B) + \xi_{BAB}\xi_{ABA}$, $C_2 = 2 + e_A + e_B + \xi_{BAB}$ and $C_3 = 2 + e_A + e_B + \xi_{ABA}$ respectively.