The Impact of Market Structure on Innovation Incentives

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Preface

First of all, I am particularly indebted to my supervisor Prof. Dr. Herbert Dawid, who offered me the possibility for my doctoral studies and attracted my attention to differential games. His guidance and expertise helped and encouraged me all the time. Working with him has always been educational. To support my research, he enabled me to spend a research semester at HEC Montréal and GERAD, which was formative for my doctoral project.
I would like to express my gratitude to my second advisor Prof. Georges Zaccour, Ph.D., who I got to know during my visit in Montréal. Discussions with him at conferences and during visits in Montréal improved my research every time.
Both professors have been on hand with help and advice for me during my doctoral studies.

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The BiGSEM program has been rewarding due to the course program and the discussions in the doctoral seminar. Thanks to financial support of BiGSEM, I was able to complete my dissertation.

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Last but not least I am very thankful for my family. Even though I was not always very communicative about my progress, you have accompanied and encouraged me throughout this time.

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Chapter 1

General Introduction

1.1 Motivation

Depending on the time horizon and the flexibility, firms have different economic decision variables: Structural (long-term) and operational (short-term) decision variables. Structural decisions are usually a long-run commitment of the firms over a certain time horizon. They are irreversible in the short-run and affect the market position of the firm. Contrary, operational decisions (e.g. price, quantity, investment) are flexible, firms can easily adjust operational decisions if a shift in the market structure occurs or competing firms change their variables. A lot of economic models use a multi-stage structure to represent timing and flexibility of decision variables. From least flexible (long-run) to most flexible (short-run), firms decide sequentially about their variables. Even though this form gives some insights about the strategic interaction, all decisions are one-time decisions. To analyze firm behavior over a certain time interval, a different approach is necessary.

A differential game extends the strategic behavior of the firms to a longer time horizon and firms can adjust their operational decisions at every instant of time.[1] A differential game is able to shape strategic interactions of the players as well as long-run impact of operational decisions. Generally, the initial setting for a differential is given, but what if structural decisions define endogenously the initial setting?

This dissertation examines the significance of structural decisions and market structure on

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[1] For the definition of a differential game see Dockner et al. (2000), chap. 2-3.

operational decisions. How do structural and operational decisions affect each other and what is the meaning for long-run profitability and long-run consumer surplus? The general research questions for this dissertation are:

- Do firms' structural decisions influence each other?
- What is the impact of market structure on operational decisions?
- What is the long-term consequence of market structure and operational decisions on profits and consumer surplus?

Structural changes (endogenous or exogenous) in the following three chapters are vertical integration, corporate social responsibility and market entry. Operational decisions refer to quantities, prices and R&D investment.

Organization of this work

This dissertation consists of three chapters covering the impact of structural market changes on operational decisions, especially innovation incentives. Additionally, they consider how structural changes and operational decisions influence profitability in the long-run.

Chapter 2 addresses the interaction of an endogenous structural decision (vertical integration) and operational decisions (quantity and process innovation). More precisely, two suppliers and two retailers and can merge vertically in a supply chain. Therefore, the market consists of one or two integrated supply chains or is not integrated at all. To the best of my knowledge, Laussel and Van Long (2012) is the only economic model which examines vertical integration in a dynamic framework. Although mergers are structural commitments and affect firm profits in the short- and long-run, most economic models use multi-stage models with sequential moves. To model long-run effects of vertical mergers in oligopolistic industries, here a differential game is used. The focus is on the efficiency and competitive effect of a vertical merger. For this reason, the model uses a non-cooperative differential game for three different vertical structures (no integration, partial integration, full integration). In the absence of integration, the industry consists of two competing downstream and two competing upstream firms. Partial integration refers to one whereas full integration refers to both supply chains. In all three scenarios R&D and production patterns in the corresponding Markov perfect equilibria with linear feedback strategies are considered. The model sheds light on investment incentives and profitability of a vertical merger. First, the investment incentive depends on the vertical structure, degree of competition and spillover. Second, vertical integration is always profitable for the integrating firms and can be profitable for the competing downstream firm if spillovers are high and competition low. In addition, the vertical structure also affects the intertemporal strategic effect, which does not occur in a static setting. Depending on spillover and intensity of competition, the intertemporal strategic effect is different for all three vertical structures.

The following chapter 3 is a joint work with Björn Brand[2] where two firms decide endogenously about their market label. Future investments can either be conservative and have some kind of advertising character or they can be socially responsible to attract new consumers. A social label or social commitment (structural decision) also requires socially responsible investment (operational decision). As a result, no, one or two socially responsible firms can compete on the market. The sales of each firm depend on their pricing strategy and their goodwill stock. In addition to R&D costs, socially responsible investment increases the production costs as well but it is more sustainable in the long-run. We analyze the conditions and incentives for socially responsible investment in the equilibrium. To determine the equilibrium outcome and the optimal investment strategies, a linear quadratic differential game is used to derive Markov perfect strategies. Depending on the relationship of additional production costs and sustainability of socially responsible investment, two equilibria emerge. Either both firms choose advertising or both invest in social projects. The equilibrium with two socially responsible firms is Pareto superior if the investments attract new consumers and do not move consumers from one firm to another. The consumers' preferred market structure in terms of social and non-social firms deviates from the firms' point of view due to competition effect of product quality.

Whereas in chapter 2 and chapter 3 the market can be endogenously asymmetric, the market is asymmetric by assumption in chapter 4. The structural market shift results from an

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[2] Institute of Organization and Economics of Institutions, University of Graz, Graz, Austria.

entry of an open source firm into a monopolistic market with one proprietary software incumbent. Both software firms sell differentiated bundles of hardware and operating system for $p_i > c_i$. The later the entry, the fewer users buy the open source bundle and contribute to the quality of the open source software. To analyze the connection between market entry and user contributions, a multi-mode differential game with a monopoly and a duopoly mode is used. The entry time is exponentially distributed and determines the switching from the monopoly mode into the duopoly mode. The Markovian strategies for prices and investment are derived for both modes. The anticipation of an entry and expected competition forces the proprietary software firm to decrease its monopoly price before the entry actually takes place. If users can contribute to the quality of the open source product, the proprietary software firm lowers its monopoly price further such that less users buy the open source product and contribute afterwards. Open source firms have higher investment and lower pricing incentives than proprietary firms and are able to maintain a higher quality level of their products.

Software used for this work

Mathematica 9 was used for all numerical calculations and the source code is available on request.

1.2 Methodology

To analyze the interaction of structural (long-term) decisions and operational (short-term) decisions, in each chapter from 2 to 4 a differential game is used. A very general definition by Dockner et al. (2000, p. 27) of a differential game is: *" A differential game is a dynamic game, played in continuous time. Two distinguishing features of a differential game are:*

(i) the modeller introduces a set of variables to characterize the state of the dynamical system at any instant of time during the play of the game, and

(ii) the evolution over time of the state variables is described by a set of differential equations.

Feature (i) makes the dynamic game a state space game and feature (ii) makes the game a differential game." The state of the dynamical system is characterized by a state vector $x(t)$. The operational decisions by each player are defined by a control vector $u_i(t)$.

In chapter 2 to 4 two players compete with each other and decide about their structural and operational variables. Within a differential game an operational variable is called control variable. Each player maximizes his payoff stream for a given time interval $[0, \infty]$. Both players select Markovian strategies $u(t) = \phi(x(t), t)$ for their optimal control path. If all opponents of player i use Markovian strategies, then player i faces a control problem[3]

$$\begin{aligned} \max_{u_i} J^i_{\phi^{-i}}(u^i(\bullet)) &= \int_0^\infty e^{-rt} F^i_{\phi^{-i}}(x(t), u^i(t), t)\, dt \\ \text{subject to}\, \dot{x}(t) &= f^i_{\phi^{-i}}(x(t), u^i(t), t), \\ x(0) &= x_0 \\ u^i(t) &\in \mathbb{R}_0^+, \end{aligned} \tag{1.2.1}$$

where J^i is the objective functional, F^i the instantaneous payoff (here: profit), f^i the dynamics and $r > 0$ the discount rate.

To determine the Markovian strategies, the dynamic programming approach and, therefore, the Hamilton-Jacobi-Bellmann (HJB) equation is used. The necessary value function

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[3] For more details about differential games see Dockner et al. (2000), chap. 2-4.

$V^i : X \times [0,T] \mapsto \mathbb{R}$ has to satisfy the following equation[4]

$$rV^i(x,t) - V^i_t(x,t) = \max_{u^i}\{F^i_{\phi^{-i}}(x,u^i,t) + V^i_x(x,t)f^i_{\phi^{-i}}(x,u,t) \mid u^i \in \mathbb{R}^+_0\}, \tag{1.2.2}$$

as well as the terminal condition

$$\lim_{t\to\infty} e^{-rt}V_i(x(t),t) \leq 0 \quad i = 1,2. \tag{1.2.3}$$

The differential game in chapter 4 is more complex because the system switches from a monopoly mode into a duopoly. This game is a multi-mode differential games (see Dockner et al. (2000), chap. 8).

The respective transversality conditions for chapter 2 to 4 are

$$\lim_{t\to\infty} e^{-rt}\, V_i(K_i(t),K_j(t),t) \leq 0, \quad i,j = 1,2; i \neq j \quad \text{and} \quad K_i, K_j \in \mathbb{R}^+_0 \tag{1.2.4}$$

$$\lim_{t\to\infty} e^{-rt}\, V_i(G_i(t),G_j(t),t) \leq 0, \quad i,j = 1,2; i \neq j \quad \text{and} \quad G_i, G_j \in \mathbb{R}^+_0 \tag{1.2.5}$$

$$\lim_{t\to\infty} e^{-rt}\, V_i(A(t),N(t),K_1(t),K_2(t),t) \leq 0, \quad i,j = 1,2; i \neq j \quad \text{and} \quad A, N, K_i, K_j \in \mathbb{R}^+_0. \tag{1.2.6}$$

where each value function is bounded from below.

Autonomous linear-quadratic games

All differential games in chapter 2 to 4 are autonomous and linear-quadratic which simplifies the HJB equation (1.2.2) with respect to multiple aspects.

- If the time horizon is $T = \infty$ and the payoff function, the dynamics and the controls do not explicitly depend on the time variable, the game is called autonomous. As a result, it is reasonable to look at stationary Markovian strategies $u_i(t) = \varphi_i(x(t))$. The equilibrium strategies and the value function are time independent as well $V_i(x(t))$.
- Due to time independence, V^i_t in equation (1.2.2) is equal to zero.
- If additionally the game is linear quadratic, the payoff function of each player is quadratic in state and control variable. In addition, the dynamics are linear in the

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[4] For a precise definition see Dockner et al. (2000, p. 41).

state and the control variable.

As a result, player i's equilibrium strategy is linear in the state, i.e. for one state $u_i = \varphi_i(x(t)) = a_i x + b_i$ and the value function is quadratic, $V_i(x(t)) = A_i x^2 + B_i x + C_i$.

Due to these properties the differential games are tractable.[5]

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[5] See Jørgensen and Zaccour (2007) for more details about analytically tractable games.

Chapter 2

Vertical Integration

2.1 Introduction

If firms want to maximize their profits, they have different decision variables: Short-run and very flexible variables like price, output or R&D investment and long-run or strategic decisions like cooperations or mergers. Nevertheless, both kinds of decisions affect each other and this interaction is the main topic of this paper. The strategic decision is a vertical integration, more specific a vertical merger.[1] In the literature, integrations are classified as horizontal and vertical. Horizontal integrations occur between firms on the same market, whereas a vertical integration includes two firms of a supply chain. The aim of this paper is to analyze the impact of different vertical structures on quantity decisions and investment in process innovation. Therefore, a infinite-horizon differential with capital accumulation is used for three different vertical structures (no integration (NI), partial integration (PI), full integration (FI)). Initially, two downstream firms compete for quantities in a differentiated market and they have to buy their homogeneous inputs from a upstream market. The upstream markets consists of two suppliers, who decide on their quantities and investment in process innovation. The inputs are homogeneous, and the R&D incentives are affected by knowledge spillovers. A vertical integration of an upstream and downstream firm, labeled as partial integration, increases the efficiency of the integrating firm and reduces the competition on

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[1] Vertical integration is used in terms of a vertical merger. For a definition of vertical integration and specification see Perry (1988).

the input market.[2] An integration of the remaining supply chain reduces the model to a Cournot duopoly with investment spillover, which is the full integration case. In contrast to other vertical integration models, which are usually two stage games, the infinite-horizon differential game framework contains long-run effects and is more adequate to model strategic decision like an integration.

This paper focuses on vertical integration, but even vertical integration can be split in different aspects, which are already examined by the existing literature.

One class of papers covers competitive and foreclosure effects. Salop and Scheffman (1987) identify vertical integration as a strategy for *"raising rivals' costs"* because downstream competition is reduced and, therefore, can be anticompetitive. More general,Salinger (1988) shows that vertical integration between retailers and suppliers, who compete on oligopolistic markets, can lead to higher or lower final good prices because of two opposing effects of vertical integration[3]. First, an integration lowers the competition of suppliers which increases the wholesale price and of course the price for the final good. Second, the merged firm produces more of the final good which lowers the price for consumers. The dominating effect depends on the parameters. The anticompetitive effect in a asymmetric market with a dominant firm, which can be partially integrated, and fringe firms, who want to enter the market, is investigated by Riordan (1998). A game-theoretic model by Ordover et al. (1990) uses bidding stages to model integration incentives. The downstream firms can make offers and counter offers to the upstream firms to determine the vertical structure (separation, partial- and full integration). In contrast to the previous two papers partial integration and increased input costs can be avoided endogenously by a counter offer of the remaining downstream firm.

In addition to the downstream market, vertical integration impacts also the upstream market. Thus, Chen (2001) extends his analysis on competitive effects on both markets[4] and the results show that vertical integration affects firm's behavior on both markets if multiple suppliers are available: competitor's choice of a supplier, independent upstream firm's and

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[2] Due to the fact that both markets consist of the same number of firms, a possible "foreclosure effect" of a vertical integration is excluded.

[3] The paper is referred to market foreclosure in the sense that an integrated firm can not participate to the intermediate good market.

[4] A general survey of competition effects of vertical integration can be found in Riordan (2008).

integrated downstream firm's pricing.

Abiru et al. (1998) find that the equilibrium structure (no integration, partial integration, full integration) depends on the initial number of suppliers and downstream firms. If the number is unequal, all three structures are possible.[5]Nevertheless, the mergers are still exogenous. Vertical mergers are endogenous if offers must be accepted by each of the potentially integrated firms (Atallah (2007)). Moreover, the offers are limited because they must be lower than the increase in profits as a result of the integration. In exogenous models the amount of the selling price of integrated firm is not taken into consideration. This context raises the question if vertical integration is always preferable (Lambertini and Rossini (2008))[6] or if vertical separation and integration can coexist (Jansen (2003))[7]?

To the best of my knowledge just a few papers deal with innovations in vertically related markets, especially process R&D in conjunction with vertical integrated markets. Banerjee and Lin (2003) pick up the raised input prices and add additional R&D investment. If upstream suppliers are connected via a common supplier, investment in downstream process R&D raises demand which leads to higher input prices. Accordingly, additional R&D investment can enforce the *"raising rivals' costs"* effect. Brocas (2003) examines the relation of vertical market structure and cost-reducing investment and the effect of vertical integration on innovation incentives. In contrast to this paper, she looks on licenses and switching cost whereas the following model fixes supply chains and compares the effect of vertical integration decisions on production and R&D investment.

The following model is closely related to Buehler and Schmutzler (2008). Similar to Ordover et al. (1990), they use a model with two upstream and two downstream firms. In addition to the possible vertical structures (separation, partial- and full integration), firms can also invest in process R&D. Their focus is on the relation between vertical market structure and

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[5] If the number of firms is equal, full integration emerges as equilibrium structure

[6] Lambertini and Rossini (2008) use a supply chain model with one upstream and downstream firm and ask *"if vertical desintegration is preferable to integration when there is process R&D ?"*

[7] Jansen (2003) is similar to Abiru et al. (1998), but he discusses forward integration with more downstream than upstream firms. The bigger amount of downstream firms results in bargaining power of the upstream firm. If they don't integrate they can offer *"make take-it-or-leave-it offers"* to the downstream firms.

cost-reducing investment and they point out the *"intimidation effect"*: Vertical integration increases own investment and decreases competitor's investment. Vertical integration is endogenously and the downstream firm has to pay a fixed amount $F > 0$ for a supplier.

Similar to Buehler and Schmutzler (2008), the focal point is on investment incentives and profitability of vertical integration, but the setup deviates in some aspects. First, a differential game is used instead of a static four stage game. In all three vertical structures the innovating firms use feedback strategies and update their output and investment decisions at every instant of time. As a result, an intertemporal effect exists, which is different in all three vertical structures and does not occur in static games. Second, the final product is differentiated and spillover occur between R&D sections. Third, no price $F > 0$ has to be paid to integrate. An integration is profitable if the revenue of the integrating firm is higher than the summarized revenue of the separated firms. On the other hand, the model is limited to one configuration of market size and costs whereas Buehler and Schmutzler (2008) show that in equilibrium all structures (separation, partial- and full integration) are possible depending on market size and cost parameter.

The market structure of the supplier and the retailer market depends on the integration decision. Initially, two differentiated downstream firms have to purchase their homogeneous inputs from two innovative upstream firms, who can invest in process R&D to lower their production costs. A vertical integration removes one upstream firm from the upstream market and the R&D section is transferred to the downstream market. A knowledge flow always exists between R&D sections due to knowledge spillover. The analysis of the differential game for the three vertical structures (No Integration, Partial Integration, Full Integration) shows an interaction of market structure and parameters. First, the investment incentive depends on the vertical structure, degree of competition and spillover. If both markets are separated, competition and spillover lower investment incentives. Vertical integration results in a rise of investment for the integrated firm and a decline for the independent supply chain. With low competition, spillovers encourage investment, whereas with high competition spillovers discourage investment. The same effects occur with full integration, but the effects are stronger, because the independent supply chain weakens both effects. Second, vertical integration is always profitable for the integrating firms, because it eliminates double marginalization and increases the efficiency. It can be profitable for the remaining upstream firm if spillovers are high and competition low, but the remaining downstream firm is always punished with a weaker market power. A vertical integration of the remaining up- and downstream firms is always profitable for these firms and even profitable for the already integrated firm if spillovers are high and competition low.

The work is organized as follows. Section 2.2 presents the model. The Cournot feedback equilibria are presented in section 2.3 and compared in section 2.4. Section 2.5 concludes.

2.2 Framework

Initially, the structure is as follows: An innovative industry with two supplier, called upstream firms, produce a homogeneous good which is used by two retailers, called downstream firms. The downstream firms transform the input one-to-one into the differentiated final good. In addition, the upstream firms are able to invest in process innovation but have to take knowledge spillovers on the upstream market into account. Due to the innovation potential the upstream firms are possible targets for vertical mergers.[8] Firms compete in outputs on both markets.

Due to the structure, two vertical mergers are possible resulting in three different vertical structures: No Integration, Partial Integration, Full Integration. No Integration is the initial setting. If an upstream firm integrates with downstream firm, the upstream firm is removed from the upstream market but still linked with the remaining supplier because of the knowledge spillover. The merged firm controls now final output and R&D investment.[9] The second retailer is not foreclosed because the input can be bought from the remaining upstream firm and both act non-cooperative in a supply chain. In consequence of the integration, the remaining upstream firm has not to compete with another upstream firm for the demand. As a result, the independent supplier has higher market power and the independent retailer has to pay higher wholesale prices. A second integration eliminates the last independent supply chain and both downstream firms produce the input themselves. The model is now simplified

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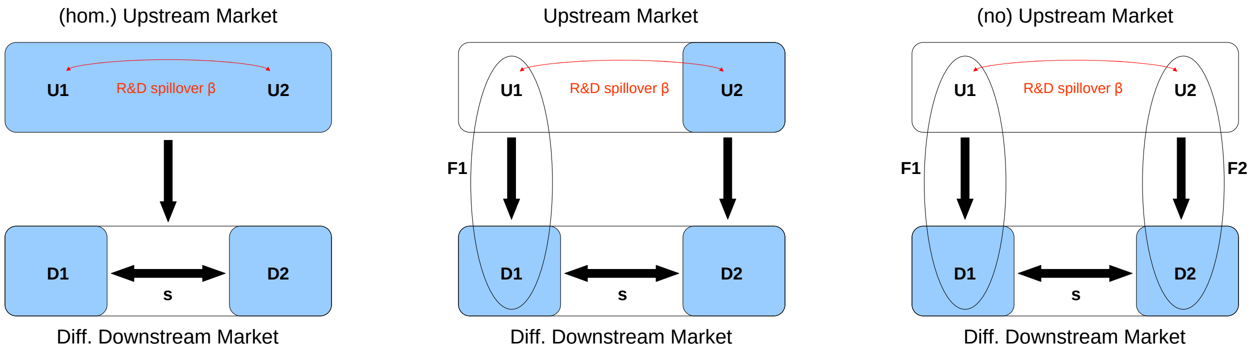
[8] This idea should reflect mergers between innovative start-up companies and incumbent companies.

[9] No distinction between forward and backward integration. For further details see Perry (1988).

to a Cournot duopoly with differentiated products and process innovation with spillover. Figure 2.1 shows the three vertical structures. To show the strategic long-run effect of a vertical integrations, a differential game with knowledge accumulation, to model process innovation, is used. As a result, the three structures are represented by an infinite-horizon differential game with four, three or two players. The three vertical structures (No Integration, Partial Integration, Full Integration) are compared to see the impact of an integration on R&D investment and profitability. Finally, the comparison concludes with the Nash-equilibrium for the Bimatrix game.



(a) No Integration (b) Partial Integration (c) Full Integration

Figure 2.1: Possible vertical structures

		Supply chain 2	
		separate	integrate
Supply chain 1	separate	No Integration	Partial Integration
	integrate	Partial Integration	Full Integration

Table 2.1: Two player Bimatrix Game

2.2.1 Consumers

Similar to Singh and Vives (1984), the utility function of the representative consumer is as follows

$$U(q_1, q_2) = A(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2) - sq_1q_2, \qquad A > 0,$$

where q_i is the output of firm i and $0 \leq s \leq 1$. The degree of product differentiation is represented by s. If s is equal to zero, the two goods are independent and homogeneous if s is equal to one. Otherwise, they are substitutes depending on the degree of differentiation. The utility function leads to the inverse demand

$$\begin{aligned} p_1 &= A - q_1 - sq_2 \\ p_2 &= A - sq_1 - q_2. \end{aligned} \tag{2.2.1}$$

2.2.2 Upstream Market

The structure of the upstream market depends on the the integration decision and can be seen in Figure 2.1. Three cases are possible:

1) No Integration

   The firms decide not to integrate and both markets are strictly separated. Two firms compete a la Cournot on a differentiated downstream market whereas two upstream firms compete in output and R&D investment in a homogeneous upstream market. The output decisions of the downstream firms transfer one-to-one to the demand function of the upstream market. For this reason , the demand for the upstream market can be written as[10]

   $$Q^u = D(w) = Q^d(w) = q_1^d(w) + q_2^d(w).$$

2) Partial integration

   Without loss of generality, downstream firm 1 decides to integrate with upstream firm 1. The upstream market gets the structure of a monopoly because upstream firm 1 is integrated, supplies downstream firm 1 and upstream firm 2 is left to supply downstream firm 2. Therefore, no foreclosure effect occurs.

3) Full integration

   Both downstream firms decide to integrate with one upstream firm. On the upstream market is no competition, because each upstream firm supplies his downstream division.

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[10] Assuming it is irrelevant from which supplier each firm buys its input. The summarized demand of the downstream market is produced by the upstream market and the market clearing condition holds.

The production of the input good on the upstream market is costly and depends on the following cost function

$$C_i(q_i, K_i, K_j) = (c_i - K_i - \beta K_j)q_i \tag{2.2.2}$$

$$\text{with} \quad c_i < A, \quad c_i - K_i - \beta K_j \geq 0, \quad 0 \leq \beta \leq 1.$$

Let K_i be the accumulated knowledge stock of R&D of firm i. The total production costs of firm i depend on the produced quantity and on both R&D stocks. β is the knowledge spillover between the upstream divisions.[11] The spillover β is exogenously given and the right-hand side of (2.2.2) is strictly positive.

Denote by I_i the R&D investment of firm i. The resulting investment cost R_i are quadratic

$$R_i(I_i) = \frac{\gamma}{2}(I_i)^2 \quad \gamma > 0. \tag{2.2.3}$$

The capital stock of firm i evolves over time according to the following standard accumulation process

$$\dot{K}_i = I_i - \delta K_i, \qquad K_i(0) = K_0, \tag{2.2.4}$$

where $0 \leq \delta < 1$ is a constant depreciation rate.

If the upstream firms are not integrated, the demand for the input good depends on the wholesale price $w = D(Q^u)^{-1}$ which depends on the demand of the downstream market. As a consequence, the upstream firms solve the following maximization problem

$$\begin{aligned} &\max_{q_i, I_i} \left\{ \Pi_i^u = \int_0^\infty e^{-rt}[w(q_i + q_j)q_i - C_i(q_i, K_i, K_j) - R_i(I_i)]\, dt \right\} \\ &s.t. : \dot{K}_i = I_i - \delta K_i, \quad K_i(0) = K_0. \end{aligned} \tag{2.2.5}$$

with the discount rate $0 < r \leq 1$.

2.2.3 Downstream Market

If the downstream firms are not integrated, they have to purchase their input good from the upstream market. Therefore, they pay w for each unit of input.[12] The downstream firms are price taker.

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[11] The knowledge spillover always exists, regardless whether the upstream firm is integrated or not.

[12] Assume that the transformation costs are equal to zero and the transformation is one-to-one.

If a downstream firm integrates with an upstream supplier, the integrated firm is able to produce the input good itself and can transfer the input one-to-one to the downstream division without additional cost.[13] The R&D section has always (regardless whether the upstream division is integrated or not) the possibility to invest in R&D to reduce the production costs. An integration decision gives the integrated firm control about the output of the whole supply chain and R&D investment. The maximization problem for the independent downstream firm is as follows

$$\max_{q_i} \pi_i^d = (p_i - w)q_i. \tag{2.2.6}$$

The downstream firm faces a simple maximization problem, because the output decision does not depend directly on any state dynamics. The output decision depends only on the wholesale price w and the downstream firm is a price taker.[14] The wholesale price varies over time, because the upstream firm can invest in R&D to lower cost and as a consequence the price.

If the downstream firm decides to integrate with an upstream supplier, the integrated firm controls the price on the downstream market and the investment on the upstream market.[15] In that case, the objective and state dynamic for the integrated firm are

$$\begin{aligned} &\max_{q_i, I_i} \left\{ \Pi_i^d = \int_0^\infty e^{-rt} [p_i q_i - C_i(q_i, K_i, K_j) - R_i(I_i)]\, dt \right\} \\ &s.t. : \dot{K}_i = I_i - \delta K_i, \quad K_i(0) = K_0. \end{aligned} \tag{2.2.7}$$

where $0 < r \leq 1$ is the discount rate.

2.2.4 Linear quadratic game and Markov perfect equilibrium

Although the number of players is different in all three vertical scenarios, the differential game in each setting has a similar structure. The innovating firms face an infinite-time horizon linear quadratic game. It is assumed that firms use Markovian strategies and by using a dynamic programming approach, the value functions of the players have to be determined.

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[13] Thus, Integration eliminates the effect of double marginalization.

[14] The argumentation follows the industrial organization literature whereas in the marketing literature the supplier is a Stackelberg leader and the retailer is the follower.

[15] The input good is transfered for free ($w = 0$).

Still, it is well known that the value functions of the players are linear quadratic and the strategies are linear in the state. In addition to the linear quadratic structure, the game is also autonomous, i.e. time does not occur in the instantaneous payoff and state dynamics. As a consequence, the coefficients of the value functions are constant and stationary Markovian feedback strategies exist.[16] For that reason, the value functions for all three vertical structures can be given by

$$\begin{aligned} V_1 &= \frac{1}{2}B_1K_1^2 + \frac{1}{2}B_2K_2^2 + B_3K_1K_2 + B_4K_1 + B_5K_2 + B_6 \\ V_2 &= \frac{1}{2}D_1K_1^2 + \frac{1}{2}D_2K_2^2 + D_3K_1K_2 + D_4K_1 + D_5K_2 + D_6 \end{aligned} \tag{2.2.8}$$

where V_i is the value function of the innovating firm in the supply chain. Depending on the vertical structure, this can be an independent upstream firm or an integrated firm. Each players value function includes six coefficients. The system is nonlinear and its solution is not unique. The solution, which satisfies the condition of global stability of the steady state, is determined by numerical calculation. The resulting partial derivatives are

$$\begin{aligned} \frac{\partial V_1}{\partial K_1} &= B_1K_1 + B_3K_2 + B_4 \\ \frac{\partial V_1}{\partial K_2} &= B_2K_2 + B_3K_1 + B_5 \\ \frac{\partial V_2}{\partial K_1} &= D_1K_1 + D_3K_2 + D_4 \\ \frac{\partial V_2}{\partial K_2} &= D_2K_2 + D_3K_1 + D_5. \end{aligned} \tag{2.2.9}$$

If the innovating firms have chosen their optimal investment strategy, the system of state dynamics can be rewritten in the following way

$$\begin{pmatrix} \dot{K}_1 \\ \dot{K}_2 \end{pmatrix} = \begin{pmatrix} \frac{B1-\gamma\delta}{\gamma} & \frac{B3}{\gamma} \\ \frac{D3}{\gamma} & \frac{D2-\gamma\delta}{\gamma} \end{pmatrix} \begin{pmatrix} K_1 \\ K_2 \end{pmatrix} + \begin{pmatrix} B_4 \\ D_5 \end{pmatrix} \tag{2.2.10}$$

The following proposition characterizes the globally asymptotically stable Markov perfect equilibrium.

Proposition 2.2.1 *If the determinant of the matrix in equation* (2.2.10) *is unequal to zero and the following condition holds:*

$$B_1 + D_2 \pm \sqrt{(B_1 - D_2)^2 + 4B_3D_3} < 2\gamma\delta, \tag{2.2.11}$$

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[16] For more details about linear quadratic game and Markovian strategies see Dockner et al. (2000).

then the equilibrium steady state (K_1^{ss}, K_2^{ss}) *is globally asymptotically stable.*

Proof: The eigenvalues of the matrix in equation (2.2.10) are

$$\lambda_{1,2} = \frac{B_1 + D_2 \pm \sqrt{(B_1 - D_1)^2 + 4B_3D_3} - 2\gamma\delta}{2\gamma}. \tag{2.2.12}$$

If the eigenvalues are negative, the system is locally asymptotically stable. Due to the linear quadratic structure, it is even globally asymptotically stable.
Following Dockner et al. (2000), the transversality condition for the given infinite time horizon problem has to be satisfied

$$\lim_{t\to\infty} e^{-rt} V(K_i(t), K_j(t), t) \leq 0.$$

If V is bounded from below and the system globally asymptotically stable, the transversality condition holds.

2.3 Cournot Feedback Equilibrium

2.3.1 No Integration

No integration takes place. The downstream firms are faced with Cournot competition with differentiated products. The sum of the resulting Nash outputs determine the demand for the upstream market and the upstream firms produce the homogeneous input for the downstream industry. They maximize their optimization problems subject to the dynamics and the given demand of the downstream industry.[17] The competition on the upstream market is strong because of the homogeneous inputs, but the producers can benefit from each other due to the spillovers on their market. The spillover parameter β and differentiation degree s will be varied to figure out the "competition effect" in the downstream section and the "spillover effect" in the upstream section. In this setting both effects are separated on their respective markets. In the two other vertical structures these effects will mix up, because integrated firms act on both markets.

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[17] Assuming the downstream firm i buys his input from upstream firm i, the inputs are transfered on-to-one to the downstream section.

Downstream Market

The maximization problem of the downstream firms is a static problem because of the independence of any state dynamic.

$$\max_{q_i} \Pi_i^d = [A - q_i - sq_j - w]q_i \tag{2.3.1}$$

The first order condition and the reply function yield to the Cournot Nash equilibrium

$$q_i^N = \frac{(2-s)(A-w)}{4-s^2} = \frac{A-w}{s+2} \tag{2.3.2}$$

Upstream Market

The downstream demand for the input good is the sum of equilibrium outputs and therefore, the inverse demand for the upstream market is

$$w = A - (\frac{s+2}{2})(q_1 + q_2). \tag{2.3.3}$$

The upstream firms compete in Cournot competition and are faced to the following maximization problem

$$\begin{aligned} &\max_{q_i, I_i} \int_0^\infty e^{-rt}[(w - c_i + K_i + \beta K_j)q_i - \frac{\gamma}{2}(I_i)^2]\, dt \\ &\text{s.t.} \quad \dot{K}_i = I_i - \delta K_i, \quad K_i(0) = K_0 = 0, \quad I_i, q_i, K_i \geq 0, \quad i = 1,2 \end{aligned} \tag{2.3.4}$$

In this noncooperative Cournot feedback game, the firms select independently their output q_i and their R&D investments. To determine the feedback strategies, the Hamilton-Jacobi-Bellman equation for two state variables is used.

$$\begin{aligned} rV_i(K_i, K_j) = \max_{q_i, I_i} \Bigg[\Big(A - \left(\frac{s+2}{2}\right)(q_i + q_j) - c_i + K_i + \beta K_j\Big) q_i \\ - \frac{\gamma}{2}(I_i)^2 + \frac{\partial V_i(\cdot)}{\partial K_i}(I_i - \delta K_i) + \frac{\partial V_i(\cdot)}{\partial K_j}(\phi_j - \delta K_j) \Bigg] \end{aligned} \tag{2.3.5}$$

Proposition 2.3.1 *The firms' Cournot feedback equilibrium output and R&D investment strategies in the absence of integration are given by*

$$\begin{aligned} q_i^C &= \frac{2(A - 2c_i + c_j + (2-\beta)K_i + (2\beta - 1)K_j)}{3(s+2)} \qquad i,j = 1,2,\ i \neq j \\ I_1^C &= \frac{1}{\gamma}(B_1 K_1 + B_3 K_2 + B_4) \\ I_2^C &= \frac{1}{\gamma}(D_2 K_2 + D_3 K_1 + D_5) \end{aligned} \tag{2.3.6}$$

where the coefficients B_i and D_i solve the system of equation given by the Hamilton-Jacobi-Bellman equation. The value functions are quadratic and symmetric according to (2.2.8).

Proof: See section A.1.

Results for No Integration

In the absence of integration, the competition effect created by s and the spillover effect created by β can be separated strictly. First, higher knowledge spillovers decrease the steady state investment incentives because higher spillover create a negative externality. This effect is independent of the intensity of competition. The higher the spillover the lower the investment. The spillover effect can be seen in the left picture of Figure 2.2. However, it does not mean that high spillover result in lower knowledge stock, because the higher spillover compensates the lower effort. Overall, on one hand spillover lower R&D activity, but raises knowledge flow of the whole industry.

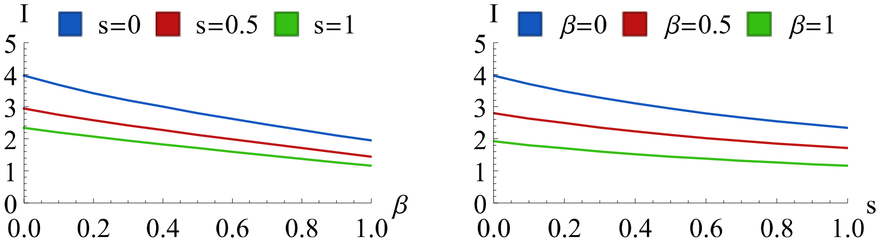

Figure 2.2: Steady state investment in the absence of integration

Second, stronger competition reduces reduces R&D investment. The stronger the competition on the downstream market the lower is the input demand for the upstream industry, and lower demand comes to lower incentives for the upstream firms to invest in process R&D. The competition effect for different spillovers can be seen in right picture of Figure 2.2.

2.3.2 Partial Integration

Asymmetric integration describes the vertical structure of an integrated supply chain, one downstream firm is integrated with one upstream firm, and an independent supply chain.[18]

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[18] W.l.o.g. assume downstream firm 1 is integrated with upstream firm 1.

Due to the integration, upstream firm 2 is not competing on the upstream market and supplies only downstream firm 2. No foreclosure occurs because of the presence of an independent supply chain with one firm on each market. Similar to the first problem (No Integration) the differentiation degree and spillover degree will be varied to see how both effects change steady state investment in a modified vertical structure. The integrated firm, called F_1, is associated with the remaining independent firms in different ways. The output of the downstream section depends on the intensity of competition, therefore s, which affects the output of the independent downstream firm. The upstream section, or R&D section, is affected by the knowledge spillover which has an effect on the investment incentives of the independent upstream firm. In contrast to No Integration both effects are mixed because of the supply chain of the independent firms. Thus, all sections of the market are connected and the connection can be seen in the second picture of Figure (b).

Downstream Market

With Cournot competition on the downstream market, the integrated firm F_1 produces the input himself and the output of firm 2 is the demand for the remaining monopolist on the upstream market. The independent downstream firm maximizes the following static problem

$$\max_{q_2} \Pi_2^d = (p_2 - w_2)q_2, \tag{2.3.7}$$

because of the absence of state dynamics and is price taker for the wholesale price w_2. The FOC yields to the best reply function

$$q_2 = \frac{A - sq_1 - w_2}{2}. \tag{2.3.8}$$

Due to vertical integration, F_1 owns the upstream and downstream division and controls both the production (q_1) and the R&D investment (I_1). Hence, the maximization problem is as follows

$$\begin{aligned} &\max_{q_1, I_1} \int_0^\infty e^{-rt}[A - q_1 - sq_2 - c_1 + K_1 + \beta K_2)q_1 - \gamma(I_1)^2]\, dt \\ &\text{s.t.} \quad \dot{K}_1 = I_1 - \delta K_1, \quad K_1(0) = K_0. \end{aligned} \tag{2.3.9}$$

Similar to the No Integration case, the Hamilton-Jacobi-Bellmann (HJB) equation is used to determine the Cournot feedback strategy.[19]

$$\begin{aligned} rV_1(K_1,K_2) = \max_{I_1,q_1} \Bigg[&(A - q_1 - sq_2 - c_1 + K_1 + \beta K_2)q_1 \\ &- \frac{\gamma}{2}(I_1)^2 + \frac{\partial V_1}{\partial K_1}(I_1 - \delta K_1) + \frac{\partial V_1}{\partial K_2}(\phi_2 - \delta K_2)\Bigg] \end{aligned} \tag{2.3.10}$$

Upstream Market

Equation (2.3.8) determines the demand for the upstream market depending on w_2 and by transformation the inverse demand for the independent upstream firm is

$$w_2 = A - sq_1 - 2q_2. \tag{2.3.11}$$

As a consequence, the monopolist on the upstream market has to solve

$$\begin{aligned} &\max_{q_2,I_2} \int_0^\infty e^{-rt}[(A - sq_1 - 2q_2 - c_2 + K_2 + \beta K_1)q_2 - \frac{\gamma}{2}(I_2)^2]\, dt \\ &\text{s.t.} \quad \dot{K}_i = I_i - \delta K_i, \quad K_i(0) = K_0, \quad i = 1, 2\,, \end{aligned} \tag{2.3.12}$$

which results in the HJB equation of the upstream monopolist

$$\begin{aligned} rV_2(K_1,K_2) = \max_{I_2,q_2} \Bigg[&(A - sq_1 - 2q_2 - c_2 + \beta K_1 + K_2)q_2 \\ &- \frac{\gamma}{2}(I_2)^2 + \frac{\partial V_2}{\partial K_2}(I_2 - \delta K_2) + \frac{\partial V_2}{\partial K_1}(\phi_1 - \delta K_1)\Bigg]. \end{aligned} \tag{2.3.13}$$

The solution of both dynamic problems provides the feedback strategies for Partial Integration which is stated in the following proposition.

Proposition 2.3.2 *The firms' Cournot feedback equilibrium output and R&D investment strategies for Partial Integration are given by*

$$\begin{aligned} q_1^c &= \frac{(4-s)A - 4c_1 + sc_2 + (4 - s\beta)K_1 + (4\beta - s)K_2}{8 - s^2} \\ q_2^c &= \frac{(2-s)A - 2c_2 + sc_1 + (2\beta - s)K_1 + (2 - s\beta)K_2}{8 - s^2} \\ I_1^c &= \frac{1}{\gamma}(B_1K_1 + B_3K_2 + B_4) \\ I_2^c &= \frac{1}{\gamma}(D_2K_2 + D_3K_1 + D_5) \end{aligned} \tag{2.3.14}$$

---

[19] q_2 is the output decision of the downstream firm and depends on the price w_2. Thus, the maximization problem of the integrated firm F_1 is also related to the dynamic maximization problem of upstream firm 2.

and where the coefficients B_i and D_i solve the system of equation given by the Hamilton-Jacobi-Bellman equation and the value functions are quadratic according to equation (2.2.8).

Proof: See section A.2.

Results for Partial Integration

With an integrated firm, competition and spillover effect can not be separated anymore and both effects depend on each other. First, the spillover effect depends on the intensity of competition. With low competition, knowledge spillover encourage R&D investment but with strong competition, knowledge discourages R&D activity. The reason for this contrary effects can be explained by the interaction parameters and the vertical structure. If the competition is weak, the R&D sections care less about the transfered knowledge to the rival because the lowered cost does not harm them on the downstream market.[20] If the competition is strong, R&D incentives are reduced because investment harm the firms on the downstream market. For example, the knowledge of the integrated firm is transferred to the independent upstream firm and results in a lower wholesale price. The lower the wholesale price the better the positioning of the independent upstream firm and this is all the worse if the competition is strong.[21] In the end, investment on the upstream market has a harming effect on the downstream market. In summary, knowledge spillover encourage investment if competition is low and discourage investment with strong competition.

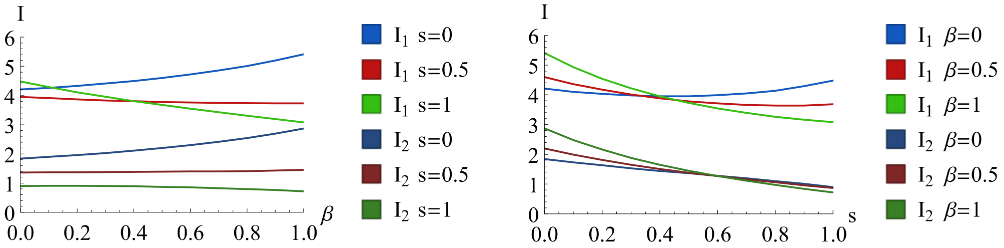

Figure 2.3: Steady state investment for Partial Integration

Second, the competition effect looks different depending on firm and degree of spillover. For the independent upstream firm, stronger competition lowers investment incentives. Due to

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[20] Just a size effect for $s = 0 :\ \beta \nearrow \Rightarrow\ c_i \searrow \Rightarrow\ q_i \nearrow \Rightarrow$ incentives to invest $\nearrow$.

[21] This is effect is also the other way around, but without the wholesale price because F_1 is integrated.

the integration, F_1 is more efficient and has a stronger market position because F_1 can sell their products directly to the consumers and do not have to pay a wholesale price. If the competition intensity increases, the independent downstream firm reduces their demand and therefore reduces investment incentives of the supplier. For the integrated firm the investment curve has a U-shaped form for low spillover and is decreasing for high spillover. An explanation for the U-shape and the relation of competition, spillover and U-shaped investment incentives have been discussed by Kopel (2009) and Sacco and Schmutzler (2011). The declining investment is a result of the high spillover. If the spillovers are high, R&D investment harms the firms on the downstream market the stronger the competition is. Finally, both firms invest on different levels. Buehler and Schmutzler (2008) call this *"intimidation effect"*: If a vertical integration takes place the integrating firm increase investment whereas the independent firm decreases investment. This effect will be discussed in section 2.4.1.

2.3.3 Full Integration

With Full integration both downstream firms are integrated with one upstream firm and produce the input on their own. Two acquisitions of the upstream firms eliminate the competition on the upstream market and reduces the model to a differentiated market with knowledge spillover.[22]

Setting

In contrast to the No-Integration case, where both upstream upstream firms compete with each other, and the Partial-Integration case, where one monopolist is on the upstream market, no firm is left on the upstream market. Both R&D firms are fully integrated and each supply chain is controlled by one player. Competition takes place only on the downstream market and the maximization problem for both integrated firms can be described with a differential game between two firms. As a result and in contrast to the previous cases, knowledge spillover and competition intensity affect each other on the same market.

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[22] This setting is similar to the model in Breton et al. (2004), but differs in cost structure, spillover and parameters.

Each upstream firms is integrated with a supplier.[23] Production and the R&D investment are controlled by one player. The input good is produced on the upstream market and transfered one-to-one to the downstream market, where the integrated firms compete with each other. The maximization problem can be described as follows

$$\begin{aligned} &\max_{q_i, I_i} \left\{ \Pi_i^d = \int_0^\infty e^{-rt}[p_i - c_i + K_i + \beta K_j]q_i - \frac{\gamma}{2}(I_i)^2]\, dt \right\} \\ &s.t.: \dot{K}_i = I_i - \delta K_i, \quad K_i(0) = K_0 \qquad i,j = 1,2;\ i \neq j. \end{aligned} \tag{2.3.15}$$

Similar to the previous cases, the HJB equation is used to determine the Cournot feedback strategies

$$\begin{aligned} rV_i(K_i, K_j) = \max_{I_i, q_i} \Bigg[&(A - q_i - sq_j - c_i + K_i + \beta K_j)q_i \\ &- \frac{\gamma}{2}(I_i)^2 + \frac{\partial V_i}{\partial K_i}(I_i - \delta K_i) + \frac{\partial V_i}{\partial K_j}(\phi_j - \delta K_j) \Bigg]. \end{aligned} \tag{2.3.16}$$

Differentiating the right-hand side w.r.t. q_i and I_i and equating to zero leads to the following equilibrium.

Proposition 2.3.3 *The firms' Cournot feedback equilibrium output and R&D investment strategies for Full Integration are given by*

$$\begin{aligned} q_i^c &= \frac{(2-s)A - 2c_i + sc_j + (2 - \beta s)K_i + (2\beta - s)K_j}{4 - s^2} \qquad i,j = 1,2,\ i \neq j \\ I_1^C &= \frac{1}{\gamma} B_1 K_1 + B_3 K_j + B_4 \\ I_2^C &= \frac{1}{\gamma} D_2 K_2 + B_3 K_2 + D_5, \end{aligned} \tag{2.3.17}$$

where the coefficients B_i and D_i solve the system of equation given by the Hamilton-Jacobi-Bellman equation.The value functions are quadratic and symmetric. The structure is shown in equation 2.2.8.

Proof: See section A.2.

Results for Full Integration

Similar to Partial Integration, competition and spillover effect depend on each other. If competition intensity is low, spillover increase investment incentives whereas for high intensity

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[23] Assume downstream firm i is integrated with supplier i.

spillover decrease incentives. The left picture of Figure 2.4 displays the spillover effect. The competition effect behaves also in the same way to partial integration where investment has U-shape without spillover and is declining for higher spillover (see the right picture of Figure 2.4). The difference to Partial Integration is shown in section 2.4.

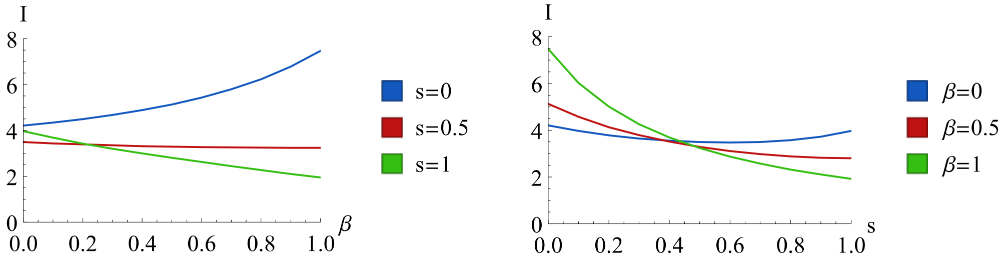

Figure 2.4: Competition and spillover effect for Full Integration

2.4 Comparing the vertical structures

In this section the different investment strategies for the three vertical structures are compared. Besides investment, the profitability of an integration is analyzed. Assuming no price has to be paid to merge the upstream and downstream firm, an integration is profitable if the profit of the integrated firm is higher than the aggregated profits of the independent firm without integration. As a final point, consumer surplus and welfare are presented for each vertical structure. Table 2.2 shows the endogenous integration decisions of the upstream and downstream firms.

		SC_2			
		separate		integrate	
SC_1	separate	$(V_{U_1} + V_{D_1})^{NI}$	$(V_{U_2} + V_{D_2})^{NI}$	$(V_{U_1} + V_{D_1})^{PI}$	V_2^{PI}
	integrate	V_1^{PI}	$(V_{U_2} + V_{D_2})^{PI}$	V_1^{FI}	V_2^{FI}

Table 2.2: Supply chain (SC) long-run profits (V_1, V_2)

2.4.1 Integration effect

In the absence of R&D investments, the overall effect of vertical integration depends on the trade off between higher efficiency of production and a lower intensity of competition. The higher efficiency of production results from an elimination of markups in the supply chain. The lower intensity of competition on the upstream market, as a result of the integrated supplier, leads to higher wholesale prices for other supply chains. Which effect prevails and if a vertical integration increases welfare depends on each single case or rather each parameter setting.[24]

By the addition of R&D investments, the results differ in a static and dynamic case. Buehler and Schmutzler (2008) show that vertical integration enhances the investment incentives for the integrating firm and decreases investment for the independent firm due to an *”intimidation effect”*. In the present model existing spillover between the R&D sections, either independent supplier or integrated supplier, have an effect on incentives. Overall, the integrating firm increases the investment, but the extent depends on the intensity of competition and spillover. The higher investment results from higher efficiency and sales of the integrated firm. Partial integration is always negative for the independent supplier, but can be beneficial for independent or integrated downstream firm to competition and spillover. The next two subsections provide further details of spillover and competition effect.

2.4.2 Spillover effect

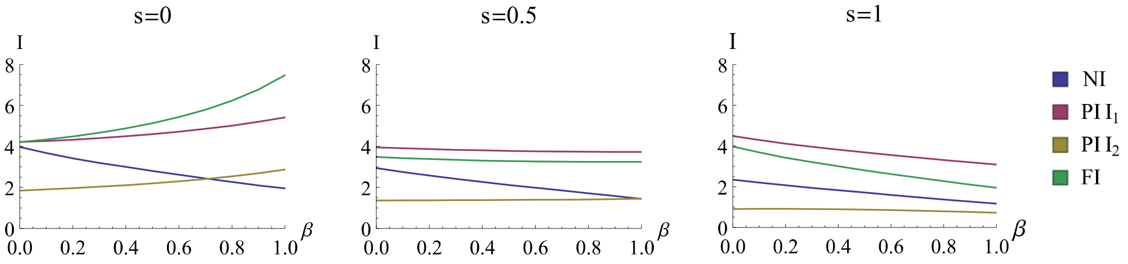

Figure 2.5: Steady state investment and spillover effect

The spillover effect depends on the vertical structure. Without integration, both markets

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[24] See Buehler and Jaeger (2002), p.140-144.

are separated and no interaction of both parameters exists. A more competitive downstream market lowers the demand of the input good and therefore investment incentives are generally lower. The intensity of competition is always the same (homogeneous good) and an increasing spillover has two effects. On the one hand the spillover increases knowledge in the industry, but on the other hand firms reduce investment.[25] Figure 2.5 reveals just the latter effect and as a consequence higher spillover lower investment independent of the competition on the downstream market. With one or two vertical integrations, both parameters interact with each other. If one firm is integrated, the integrated firm invests more than the independent upstream firm (see section 2.4.1). An increasing spillover is supportive if the final good differentiated but is harmful for more similar goods. The explanation for this contrary spillover effect results from the vertical structure. The integrated firm is related to upstream monopolist via spillover and investment of the integrated firm reduces marginal costs of the monopolist. The lowered costs are passed on to the independent upstream firm via wholesale price and the impact for the integrated firm depends on the degree of differentiation (s) of the final products.[26] As a result, the vertical structure generates some kind of feedback for own investment via spillover, supply chain and product differentiation. With Full Integration, the upstream market is broken up and the integrated firms compete on the same market. If the competition intensity is low, spillover are encouraging for investment wheres high intensity is discouraging for investment. The incentives are similar to Partial Integration except that both supply chains are integrated and the negative feedback is not relaxed by an independent supply chain.

2.4.3 Competition effect

Figure 2.6 displays the competition effect for all three vertical structures. Without integration, more similar final products are always discouraging for upstream firms to invest due to lower demand of the downstream industry. For partial integration, the investment curve

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[25] Overall, the knowledge for increasing spillover has an inverted U-shape.

[26] The line of argument works also the other way around with the wholesale of the integrated firm is equal to zero. In general, the three firms are connected with each other and each decision (output or investment) has an effect on the other two firms.

of the integrated firm shifts from an U-shape to a declining form with higher spillover. The independent upstream firm reduces always investment for a lower product differentiation independent of the degree of spillover. The curves of both full integrated firms are U-shaped without spillover and declining for high spillover.

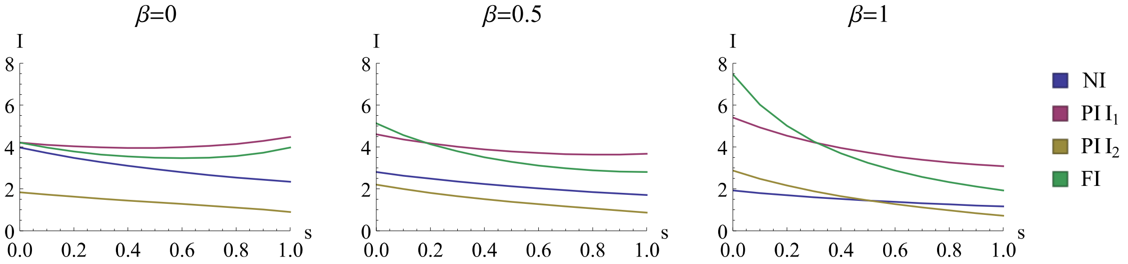

Figure 2.6: Steady state investment and competition effect

2.4.4 Profitability

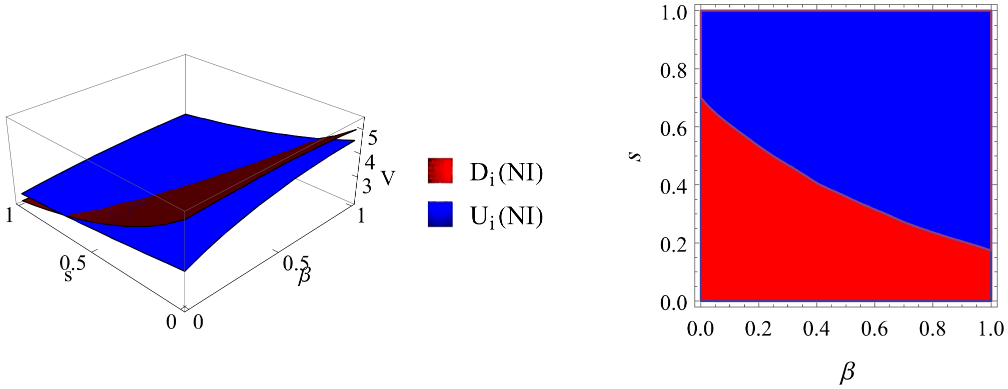

Figure 2.7: Initial profits for upstream firms (blue) and downstream firms (red)

To see the impact of a vertical integration, it is reasonable to start with the initial profits of the up- and downstream firm. Figure 2.7 shows the initial profits of an upstream firm (blue) and a downstream firm. In the following treatment a vertical integration is profitable if the profit of the integrated firm is higher than the profit of the aggregated independent firms.

Due to the integration, the integrated firm has higher incentives to invest in R&D because the effort is not weakened by a supply chain and the lower cost have an direct effect on

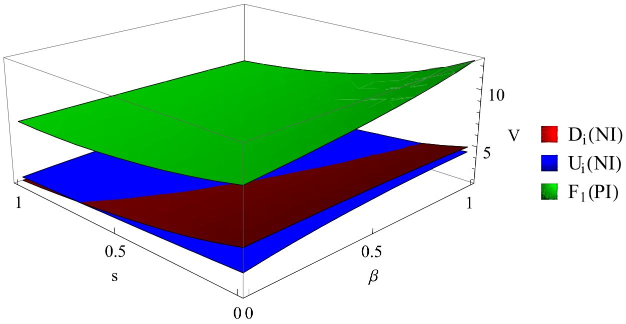

Figure 2.8: Integration effect for the integrating firm

the success on the downstream market. The profit of the integrated firm is increasing in β and decreasing in s. In conclusion, the profit of the integrated firm is higher than the profit of the two independent firms (Figure 2.8). However, can the integration also be beneficial for the two independent firms? Thanks to the spillover higher investment of the integrated firm can lower the cost of the monopolist on the upstream market that can result in a lower wholesale price for the independent upstream firm. In contrast to this positive effect, the integration changes the duopoly on the upstream market into a monopoly which results in higher wholesale prices. The central question is which effect has a bigger impact? Figure 2.9 displays the initial profits and the profits of the independent firms with Partial Integration. The integration is always costly for the independent downstream firm which is shown in the left picture by the gray graph. Still, both pictures show an integration can be profitable for the upstream monopolist if the products are differentiated and the spillover is high.

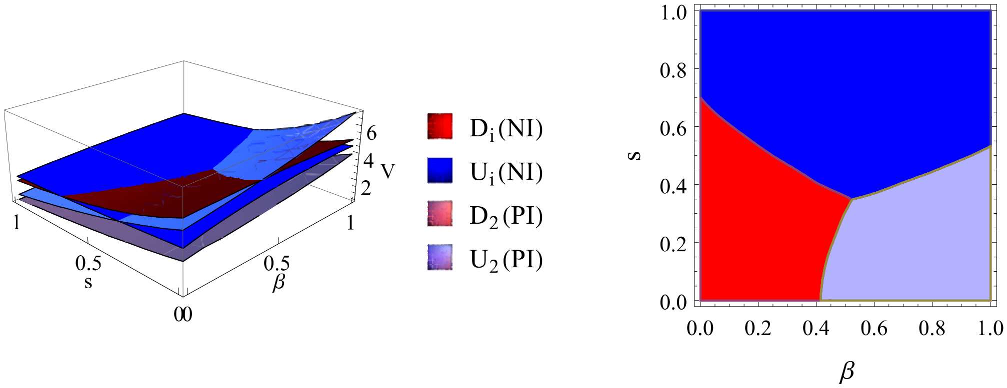

Figure 2.9: Integration effect for the independent supply chain

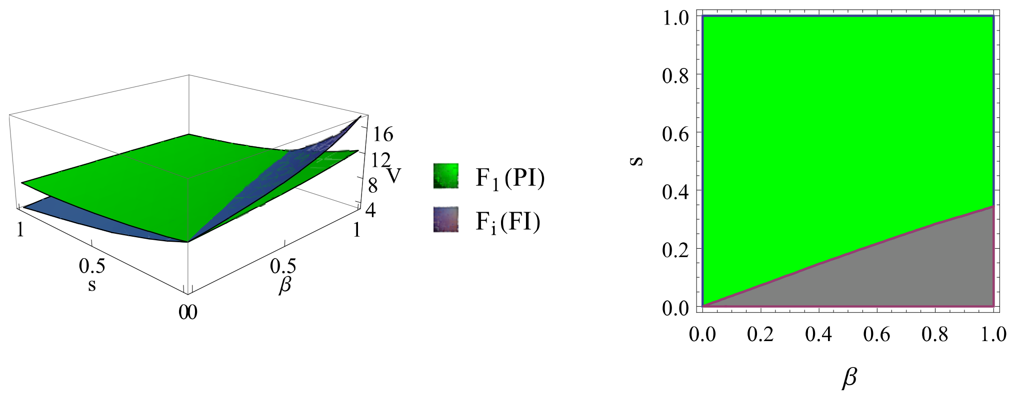

Figure 2.10: Full Integration for F_1

The profitability of Full Integration for the already integrated firm F_1 can be seen in Figure 2.10. As a result of the eliminated markup the second integrated firm invests more in R&D which is profitable for the already integrated firm if competition is low and spillover are high. Therefore, an integration of a competing independent supply chain can be beneficial or unfavorable for the integrated firm depending on the parameters. Figure 2.11 shows the profits of the independent supply chain with Partial Integration compared to the profits of the integrating firm with Full Integration. Obviously, the integrated firm generates higher profits as the two independent firms but also higher profits as the sum of both profits. In sum, Full integration is profitable for the already integrated firm for differentiated goods and higher spillover and is always profitable the independent up- and downstream firm.

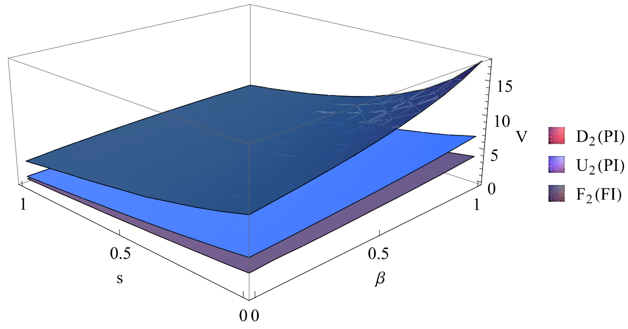

Figure 2.11: Full Integration for F_2

2.4.5 Consumer surplus and Welfare

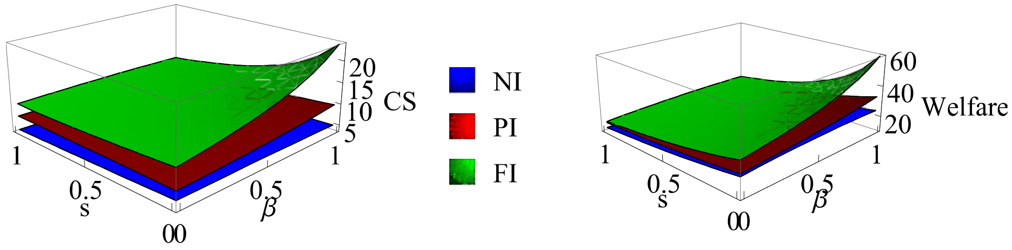

Figure 2.12: Consumer surplus and Welfare

Are vertical integration profitable for the consumers and the welfare? The left picture of Figure 2.12 shows the consumer surplus for No Integration(blue), Partial Integration(red) and Full Integration(green). A vertical integration is always beneficial for the consumers because it eliminates double marginalization and increases investment incentives of the integrating firm. As consequence of the strict preference for consumers and profitability for the integrating firm, the welfare is maximized for Full Integration.

2.4.6 Intertemporal strategic effect

Instead of static setting, a differential game is used to analyze long-run or intertemporal strategic effects. To separate the steady state K_i^{ss} into an instant and intertemporal effect, proceed as follows:

The general formulation for the HJB equation for an infinite time horizon differential game with two state variables K_i, K_j and the own strategy ϕ_j is

$$rV_i = F_i(K_i, K_j, \phi_i) + \frac{\partial V_i}{\partial K_i}(\phi_i - \delta K_i) + \frac{\partial V_i}{\partial K_j}(\phi_j - \delta K_j).$$

Differentiation of the above equation with respect to K_i results in

$$\frac{r\partial V_i}{\partial K_i} = \frac{\partial F_i}{\partial K_i} - \delta\frac{\partial V_i}{\partial K_i} + \frac{\partial V_i}{\partial K_j}\frac{\partial \phi_j}{\partial K_i}$$

$$\Leftrightarrow \quad \frac{\partial V_i}{\partial K_i} = \frac{1}{r+\delta}\left(\frac{\partial F_i}{\partial K_i} + \frac{\partial V_i}{\partial K_j}\frac{\partial \phi_j}{\partial K_i}\right) \tag{2.4.1}$$

The structure of the optimal investment strategy for each vertical structures in the steady state is known.[27]

$$\text{maximization problem:} \quad I_i^* = \frac{1}{\gamma}\frac{\partial V_i}{\partial K_i}$$

$$\text{steady state condition:} \quad \dot{K}_i = 0 \quad \Rightarrow \quad I_i^{ss} = \delta K_i^{ss}$$

Combining both conditions leads to the steady state stock, which is different for all three cases because of the vertical structure.

$$\Rightarrow K_i^{ss} = \frac{1}{\delta\gamma}\frac{\partial V_i(K_i^{ss}, K_j^{ss})}{\partial K_i} \tag{2.4.2}$$

By using equation (2.4.1) and (2.4.2), it is possible to split the steady state into a direct, which occurs also in a static game, and an intertemporal effect. Taking into account the known partial derivative for each player, the equation can be written as follows

$$\begin{aligned} K_1^{ss} &= \frac{1}{\delta\gamma}\left(\frac{\partial F_1(K_1^{ss}, K_2^{ss}}{\partial K_1} + (B_2 K_2^{ss} + B_3 K_1^{ss} + B_5)\frac{D_3}{\gamma}\right) \\ K_2^{ss} &= \frac{1}{\delta\gamma}\left(\underbrace{\frac{\partial F_2(K_1^{ss}, K_2^{ss}}{\partial K_2}}_{\text{direct effect}} + \underbrace{(D_1 K_1^{ss} + D_3 K_1^{ss} + D_4)\frac{B_3}{\gamma}}_{\text{intertemporal strategic effect(ISE)}}\right) \end{aligned} \tag{2.4.3}$$

In the absence of integration and with Full Integration both equations are symmetric, but for Partial Integration the equations are not symmetric. The intertemporal strategic effect (ISE) just occurs in differential games and is lacking in a static setting. As a consequence, firms gain additional knowledge.

Similar to the previous results, the intertemporal strategic effect (ISE) is analyzed for $\beta \in [0,1]$ and $s \in [0,1]$ for each vertical structure. Figure 2.13 pictures the results. The spillover β and the competition in combination with the vertical structure are essential for the long-run investment or knowledge stock. For example, in the absence of integration and low competition (s small) the ISE is high for low spillover but low for high spillover whereas for Partial Integration it is the other way around. Thus, the ISE is connected to the vertical structure. For all three vertical structures the ISE is close to zero if the products are almost perfect substitutes and knowledge spillover are high. If the ISE is small, the differential

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[27] See subsections of chapter 2.3

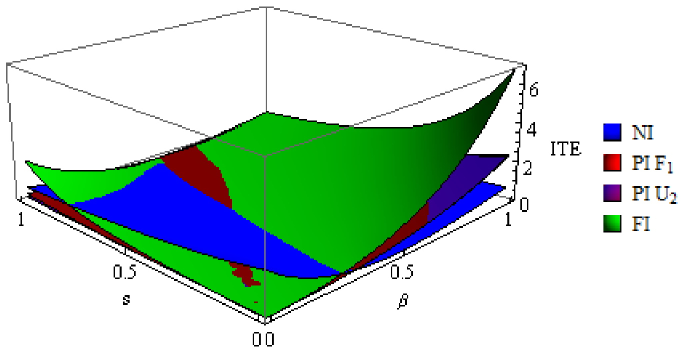

Figure 2.13: Overall intertemporal strategic effect

game does not differ much from a static setting. To sum up, the extent of the intertemporal strategic effect depends on the vertical structure, spillover and intensity of competition.

2.5 Conclusion

In this paper a differential game is used to show different aspects of vertical integration. First, vertical integration changes the market structure by eliminating one firm off a market (upstream market) which affects the remaining up- and downstream firms and their supply chain. The integrating firm increases his efficiency by eliminating double marginalization which results in higher investment efforts. The integration is harmful for an independent downstream firm because of the higher efficiency and lower costs of the integrated firm. In contrast, if the innovating sections are connected with a knowledge spillover, the integration can be profitable for an independent upstream firm for differentiated products and higher spillover. In this case the higher investment of the integrated firm combined with higher spillover outweighs the lower demand of the independent downstream firm. An integration of the remaining supply chain is always profitable because of the eliminated double marginalization and raises R&D investment. The already integrated firm can profit as well, if the spillover and product differentiation are high. All in all, the integration effect, or higher efficiency, is so strong that every vertical integration is profitable for the integrating firm and the consumers.

Nevertheless, the model has some restrictions and future research could extent the model in different aspects. First, in contrast to Buehler and Schmutzler (2008) the model neglects integration costs and market size. They show that the relation of both aspects is crucial for the equilibrium. Depending on both parameters, all three vertical structures can be an equilibrium outcome. The present model uses just one market size and integration is not costly. Second, the suppliers produce homogeneous inputs. For the initial setting it is assumed that the supply of the upstream markets satisfies the demand of the downstream market but not how units each supplier delivers to each downstream firm.[28] Lambertini and Rossini (2003) assumes that the inputs are different and the demand of the input of two suppliers depends on the substitutability or product differentiation of the final products. Therefore, the downstream firms purchase the inputs of each supplier. This is a possible extension to the present model and it would be also interesting to analyze if an integrated firm has incentives supply their competitor. Their model is analyzed for Cournot and Betrand competition which can result, depending on product differentiation, in different equilibrium outcomes for the vertical structure.

Finally, the integration takes place perfectly without costs and a loss of efficiency. In general, the adaption process is not so easy because the production process has to be adjusted to the new firm. This topic is related to Lambertini and Rossini (2008) and Ishii (2004) where spillover occur between a upstream and downstream firm. These spillover would vanish with a vertical integration and therefore separation could be more efficient.

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[28] To simplify the model and due to a symmetric setting, supplier i satisfies demand of downstream firm i.

Chapter 3

Corporate Social Responsibility (CSR)

3.1 Introduction CSR

The impact of *Corporate Social Responsibility (CSR)* for a firm's competitive position has gained greater significance in the past ten years (e.g. KPMG, 2011; Ernst & Young, 2011; Economist Intelligence Unit, 2007)). Firms have explored possibilities for CSR activities to gain a competitive advantage, to choose a strategic level of CSR (e.g. Baron, 2001; McWilliams and Siegel, 2001, 2011; Fernandez-Kranz and Santalo, 2010). Due to media coverage of the social efforts (Zyglidopoulos et al., 2012), the public perception of CSR has increased as well. In addition to the discussion in the scientific literature, CSR is debated in practice as well. For example, *The Economist* has published lengthy special reports on CSR in January 2005 and January 2008. Practitioners link CSR to the "honorable merchant" who wants *"long-term economic success without harming the interests of the society"*. Accordingly, firms do not act honorable due to moral or altruism but to *"be in business tomorrow"* (Dercks (2013), p. 6). As a result, firms have to take CSR into account to generate long-term profits. Therefore, we try to answer questions about long-run profitability. First, can a socially responsible firm outperform a non-social competitor? Second, can social behavior be a dominant strategy?

In spite of extensive discussions about corporate social responsibility (CSR) and the rising interest from firms, consumers and stakeholders, a *"general consensus as to what activities are included under the CSR umbrella has not emerged"* (Servaes and Tamayo (2012), p.2). Baron (2001) argues that *"corporate social responsibility is an ill- and incompletely defined concept"*(p. 9). The European Commission (2001) defines CSR as a *"concept whereby companies integrate social and environmental concerns in their business operations and their interactions with their stakeholders on a voluntary basis"* and *"being social responsible means not only fulfilling legal expectations, but also going beyond compliance and investing 'more' into human capital, the environment and the relations with stakeholders"*(p. 6). A very similar definition was proposed by the World business council for sustainable development (2003): *"CSR is the commitment of a business to contribute to sustainable economic development, working with employees, their families, the local community and society at large to improve their quality of life"*(p. 5). To sum up, corporate social responsibility (CSR) is a sustainable voluntary effort to accomplish social, economic or environmental goals. Based on this definition, our model shows the incentives for CSR and the impact for the consumers in a competitive market.

Due to the imprecise definition of CSR, multiple theoretical approaches exist to model social behavior. For example, Alves and Santos-Pinto (2008) provide a theory of corporate social responsibility in a Cournot duopoly where firms can donate a certain amount of money per each unit sold to social projects to be socially responsible.

Another stream of literature models social behavior from a stakeholder management perspective with a focus on consumers. Consequently, a the firm's objective is given by its profit and a fraction of consumer surplus. This approach was applied to delegation models by Goering (2007) and extended by Kopel and Brand (2012, 2013). Furthermore, Goering (2012) argues that *"this definition of CSR implies the firm is willing to accept less profits to act in a more social concerned manner"* (p. 144). Therefore, CSR is costly. In a mixed market with a private for-profit firm, a public firm and a "commercial" non-profit organization (NPO) which differ in their respective objective function, Goering (2008) finds that the technical efficiency, or production costs of each firm, are crucial in determining whether social welfare rises or falls as the NPO places more weight on the consumer surplus. Besides market competition, this approach has also been applied to supply chain management. Goering (2012) analyzes a bilateral monopoly where either the manufacturer with a two-part tariff or the retailer can be socially concerned. Brand and Grothe (2013) extend the paper such that both firms can be socially concerned. In a further model, Brand and Grothe (2014) analyze a bilateral monopoly with an imperfectly coordinated marketing channel. There they show, that firms' social concern results in a Pareto-improvement due to a weakened double marginalization problem.

Several papers highlight the strategic use of CSR where *"CSR attributes are like any other attributes a firm offers"* (McWilliams and Siegel (2001), p. 125) to improve the performance of the firm. They discuss a general supply and demand model and figure out which parameters are positive or negative related to CSR efforts. Baron (2001) uses strategic CSR as firms' reaction to social concerns where they can donate an additional amount for environmental issues for each unit sold. Firms react to a threat of activists who want to change the production practices of the firm. A first-mover advantage of CSR activities for a socially responsible firm and the necessary conditions are examined by Kopel (2009).

Complementary to these static models, different authors consider the long-term effect of strategic CSR and whether it is sustainable. For example, Lundgren (2011) investigates an optimal control problem of a socially responsible firm where CSR investment has a positive effect on the goodwill stock. The dynamic approach is carried on by Wirl (2013) and Wirl et al. (2013). The latter asks why CSR waves can be observed. Starting with an intertemporal optimization problem of a social firm, they add interaction with competitors which is essential: If CSR is profitable for one firm, it may be profitable for the competitors as well. Strategic interactions w.r.t to CSR activities are the focus of Wirl (2013). He investigates the differences of open-loop and Markov perfect Nash equilibria with the cooperation as a benchmark. Becchetti et al. (2012) combine the idea of Hotelling (1929) and dynamic CSR investment. A profit maximizing firm faces a non-profit socially responsible competitor. The location model of Hotelling represents a *"ethical segment"* $[0, 1]$ where consumers are heterogeneous towards social and environmental features of the final good. The non-profit social firm is located at one and the monopolist can decide endogenously about its location or degree of social responsibility which increases the production costs but long-run CSR investment is more efficient. Therefore, social behavior is a response to the competition of a non-profit firm.

The strategic CSR approach is not without problems. For example, Baron (2001) argues: *"A firm motivated only by profits may adopt a practice labeled as socially responsible because it increases the demand for its product. This strategic CSR is simply profit-maximization strategy motivated by self interest and not by conception of corporate social responsibility."* (p. 9). Nevertheless, we pick up the idea about endogenous CSR investment, additional marginal costs and competition and use CSR investment as a positive signal to advance the goodwill of consumers. Based on the idea of Nerlove and Arrow (1962), each firm can make costly investment in order to raise its own goodwill (reputation or brand equity). The higher the own goodwill stock the higher are the current sales. In a duopoly, both firms must decide initially if they want to invest in a socially responsible or non-socially responsible way. We want to examine the following points:

- What differentiates socially responsible strategies from other investment strategies? Motivated by practical examples, we give a definition of socially responsible investment.
- Does socially responsible behavior lead to an economic advantage? Furthermore, are socially responsible firms able to outperform a non-social competitor?
- Which combination of firm strategies is optimal for consumers by means of consumer surplus?

Initially, two firms decide if they want to invest socially responsible or not over an infinite time horizon. Both investments create goodwill of consumers but in contrast to non-social investment, social investment raises the production costs as well. However, social investment is more sustainable which results in a lower depreciation of the goodwill stock. Referring to the literature, we use CSR as strategic investment. Within a differential game we derive the Markov Perfect Equilibria (MPE) given their initial decision about social behavior. The results show that the initial decision of the firms depends on the relation of additional cost from social investment and the sustainability of the goodwill stock. Overall, either both firms invest socially or none. If business stealing attracts consumers from the competitor, the firms may end up in a prisoner's dilemma. Consumers' point of view deviates from firms' point of view, but could prevent firms from the prisoner's dilemma under certain circumstances.

Our paper is organized as follows. Section 3.2 presents the framework. Afterwards, we show equilibria and consumer surplus in section 3.3. The last section of our paper concludes and discusses future research.

3.2 Framework

Two firms compete with differentiated products and decide initially about their firm label. They can either spend their money to create a product which is produced socially responsible, like "fair-trade" products, and use their social label to attract new consumers or choose advertising to grab the consumers attention for a product without a socially concerned production process. The term "social" or "socially responsible/concerned" indicates that a firm cares about social or environmental problems during the production process. Social behavior in our model meets the following criteria which are in line with the literature about corporate social responsibility. First, social responsibility is used as an investment strategy (CSR investment) that creates a social label for a product (e.g. "fair-trade" label) or reputation for the firm. A CSR investment enhances the quality of the product and differentiates (vertically) the product from its competitors. Second, CSR investment is a *"visible strategy"* to its stakeholders. Visibility is important to use it strategically and to change the beliefs of firms, customers and other stakeholders.[1] Therefore, CSR investment is a mix of different strategies (quality improvement, advertising etc.) to build up reputation. For example, firms may decide to change their range of products to "fair-trade"-products or pay their workers higher wages compared to their competitors with the intention of creating a long-run competitive advantage in order to maximize profits.[2]

Advertising as a non-social strategy can build up reputation as well. What is the difference between a non-social and social strategy? We define a CSR strategy as a long-term and sustainable investment compared to a non-social strategy. If a firm sells "fair-trade" products or pays higher wages, the short-run production costs go up and CSR effort results in a short-run disadvantage. In contrast to a non-social strategy, CSR investment is more sustainable due

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[1] Without visibility the consumers would not recognize the social products or projects. We follow Kopel (2009) and use similar arguments.

[2] In contrast to some works about corporate social responsibility, firm's objective is to maximize profits.

to the visibility of the product. Consumers reward CSR effort in the future with a price premium and the short-run disadvantage can convert into a long-run advantage.[3] To analyze short- and long-term effects, the problem is modeled in a differential game where we analyze incentives towards social and non-social investment.

Similar to Singh and Vives (1984) consumers are able to purchase two differentiated products and maximize their respective utility function

$$U = \alpha(q_1+q_2)+(mG_1+\beta(G_1-G_2))q_1+(mG_2+\beta(G_2-G_1))q_2-0.5(q_1^2+q_2^2)-sq_1q_2. \quad (3.2.1)$$

G_i represents the additional utility created by firm i with social or non-social investment. Maximization with respect to the products subject to a budget constraint yields the inverse demand system

$$\begin{aligned} p_1 &= \alpha - q_1 - sq_2 + \quad mG_1 \quad + \ \beta(G_1 - G_2) \\ p_2 &= \alpha - q_2 - sq_1 + \underbrace{mG_2}_{\text{market extension}} + \underbrace{\beta(G_2 - G_1)}_{\text{business stealing}} . \end{aligned}$$

mG_i indicates the market extension or the new consumers induced by the goodwill stock of firm i whereas β shows the impact of business stealing when firms have different goodwill stocks.[4] Moreover, the inverse demand p_i depends on the reservation price α, the quantity q_i and the degree of product differentiation s.

$$p_i = \alpha - q_i - sq_j + \underbrace{(m+\beta)}_{=:d} G_i - \beta G_j, \quad i = 1,2; i \neq j \quad (3.2.2)$$

Thus, the ratio $0 \leq \frac{\beta}{d} \leq 1$ indicates the effect of market extension and business stealing. The firms can build up the reputation G_i, which creates a price premium by social or non-social investment.

Initially, firms have to decide endogenously about their image: Are they social responsible or not? Firms cannot switch their strategy afterwards. The next section specifies the maximization problems for a non-social and a socially responsible firm.

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[3] In our model, the reward by the market respectively by the consumers is a long term advantage and is covered by a lower depreciation rate of the goodwill stock.

[4] Business stealing just occurs when the firms choose different strategies.

3.2.1 Profit maximizing firm (PMF)

A firm without social investments is called profit maximizing firm (PMF) from now on. In contrast to a socially responsible firm (SRF), the PMF tries to increase the reputation of its product by standard advertising instruments.[5] In line with the differential game literature, more precisely capital accumulation games (Dockner et al. (2000), chapter 9), the reputation is modeled as a goodwill stock and investment I raises the reputation or goodwill respectively $\dot{G}_i(t) = I(t) - \delta G_i(t)$. Nerlove and Arrow (1962) use advertising as their investment strategy and analyze a dynamic optimization problem for one firm. Multiple authors picked up the idea of *"advertising goodwill models"* and extended Nerlove and Arrow (1962) to an oligopolistic market or different dynamics.[6] Advertising A is used to describe a non-social investment and to improve the reputation of the PMF. Consequently, the maximization problem of the PMF is

$$\begin{aligned} &\max_{p_i, A_i} \Pi_i = \int_0^\infty e^{-rt}[(p_i - c_i))q_i - \frac{\gamma}{2}A_i^2]\, dt \\ &\text{s.t. } \dot{G}_i^{PM} = A_i - \delta_i^{PM} G_i^{PM}, \quad G_i^{PM}(0) = G_0 \end{aligned} \tag{3.2.3}$$

where $\frac{\gamma}{2}$ is the cost parameter for advertising efforts.

3.2.2 Socially responsible firm (SRF)

Social investment causes R&D costs and production costs as well.[7] R&D, even though it causes costs, is necessary to find possibilities for CSR investment and to publish them afterwards. The argumentation of increasing production costs is in line with social behavior. For example, if a firm pays their employees higher wages compared to rival firms to create a social image, the additional wage raises the production costs of the product. "Fair-trade"

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[5] In our setting, the socially responsible firm is profit maximizer as well but takes social or environmental issues into account to increase its reputation.

[6] An overview of advertising goodwill models can be found in Jøgensen and Zaccour (2004), chapter 3.5 or Dockner et al. (2000), chapter 11.

[7] This property is similar to De Giovanni (2011) where investment of the manufacturer improves the product quality. In contrast to our model, De Giovanni (2011) examines a supply chain where the goodwill is the sum of quality improvement $d(t)$ by the manufacturers and advertising effort $A(t)$ of the retailer $\dot{G} = \epsilon A(t) + \gamma d(t) - \delta G(t)$.

products are a second example for this argumentation. Due to the fair trade, the retailer pays a higher wholesale price which results in higher costs. If a firms wants to change its product range to "fair-trade" products, it has to take higher costs into account. Nevertheless, a socially responsible firm (SRF) is still profit maximizing but willing to sacrifice short-run profits for future compensation. The advantage of CSR investment is its sustainability which appears in a lower depreciation rate ($\delta_{SR} < \delta_{PM}$) for the goodwill stock.[8] This can be a result of word-of-mouth effects of the employees or the visibility of the social product (social label). Overall, the maximization problem of a socially responsible firm is

$$\begin{aligned} &\max_{p_i, CSR_i} \Pi_i = \int_0^\infty e^{-rt}[(p_i - (c_i + w\, CSR_i)))q_i - \frac{\gamma}{2}(CSR_i)^2]\, dt \\ &s.t.\ \dot{G}_i^{SR} = CSR_i - \delta_i^{SR} G_i^{SR}, \quad G_i^{SR}(0) = G_0, \qquad w \geq 0,\ \delta_i^{SR} \leq \delta_i^{PM} \end{aligned} \tag{3.2.4}$$

where w indicates the impact of CSR investment on production costs.
Therefore, firms have to examine the market conditions and have to decide if they rather act socially responsible or not. Assuming a non-cooperative mode of play, Table 3.1 summarizes the possible outcomes and value functions for the two player game. Each player can be socially responsible (SR) or profit maximizing (PM).

		Firm 2	
		PM	SR
Firm 1	PM	V_1^{PM} V_2^{PM}	V_1^{PM} V_2^{SR}
	SR	V_1^{SR} V_2^{PM}	V_1^{SR} V_2^{SR}

Table 3.1: Value functions (V_1, V_2) depending on social commitment

For all possible outcomes, the setting is an infinite horizon linear quadratic (goodwill or capital accumulation) game. The players have two control variables, the price $p_i \geq c_i$ and their investment I or CSR, to impact sales and their respective goodwill stocks $G_i^j(t) \geq 0, i =$

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[8] The depreciation rates indicate aging or forgetting effects. The underlying argumentation suggests that social investment is more sustainable due to the visibility of the social behavior.

$1, 2,\ j \in \{PM, SR\}$. Due to the complexity of the system when one or both firms select social investments, the Markov perfect equilibria are analyzed numerically in section 3.3.[9]

3.3 Numerical calculation

To calculate profits, equilibria and consumer surplus, we fix the following parameter setting

$$\alpha = 1, c_i = 0.1, s = 0.4, \delta^{PM} = 0.4, \gamma = 20, r = 0.07,$$

and create an array of different values for the social depreciation rate (δ^{SR}) and the impact of CSR on production costs w. The array for the numerical calculation is

$$w \in [0, 0.9], \delta^{SR} \in [0.3, 0.4],$$

where the step size for w is 0.1 and for δ^{SR} is 0.01.[10] Finally, we calculate the $w - \delta^{SR}$-array for different values of the market extension-business stealing ratio $\frac{\beta}{d}$ with a fixed d and different values for β:

$$d = 1 \quad \text{and} \quad \beta = 0.1, 0.5, 0.9.$$

3.3.1 Stability and transversality condition

Independent of the choice of the firms w.r.t. their types, the underlying system is a time autonomous linear quadratic game with two linear differential equations

$$\left.\begin{aligned} \dot{G}_1^j &= I_1 - \delta_1^j G_1^j \\ \dot{G}_2^j &= I_2 - \delta_2^j G_2^j \end{aligned}\right\} \quad \text{with } I_i, G_i \geq 0 \tag{3.3.1}$$

where $I_i \in \{A_i, CSR_i\}$ and $j \in \{PM, SR\}$. If we substitute I_i with the optimal investment following from the HJB equations (see appendix B.2.1, B.2.2 and B.2.3), the system can be

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[9] The best reply functions for each case can be found in appendix B.2.1 to B.2.3.

[10] Due to the setting, the CSR investments are not defined for the case (SRF, SRF) and $w = 0$. When no additional costs w occur, it is always better to be socially responsible. Thus, we calculate the profits for $w = 0.001$ instead of $w = 0$ to create the array and restrict our observations and statements to the case $w > 0$.

rewritten as

$$\begin{pmatrix} \dot{G}_1 \\ \dot{G}_2 \end{pmatrix} = \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix} \begin{pmatrix} G_1 \\ G_2 \end{pmatrix} + \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} \tag{3.3.2}$$

If $|K| \neq 0$, the system has a unique fixed point (G_1^{ss}, G_2^{ss}). Only equilibrium values who are globally asymptotically stable are allowed for numerical calculation. If the fixed point is globally asymptotically stable, the transversality condition, where V_i is bounded from below and

$$\lim_{t \to \infty} e^{-rt} V(G_i(t), G_j(t), t) \leq 0, \tag{3.3.3}$$

is satisfied as well.

3.3.2 Profits and equilibria

Given the previous parameter setting, we calculate the overall profits and resulting equilibria for different degrees of business stealing β. If β is small (or equal to zero), goodwill stocks are almost independent and consumers pay a price premium for both products. If β is large, the consumers just pay a price premium for the firm with the higher goodwill stock and the firm with the lower stock looses consumers to its competitor. Figure 3.1 displays the value functions with a low impact of business stealing $\beta = 0.1$. The green graph of Figure 3.1 shows the profits for the symmetric setting (PMF,PMF) and the orange graph for (SRF,SRF). The profits for the PMF and SRF in an asymmetric setting (PMF,SRF) are shown with the blue and red graph respectively. The profits for $\beta = 0.5$ and $\beta = 0.9$ are

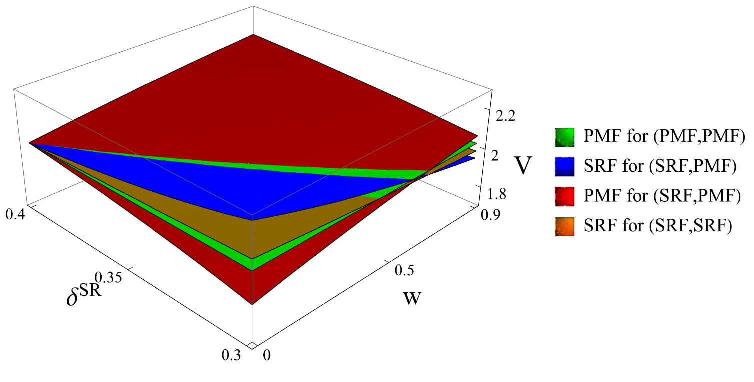


Figure 3.1: Profits for $\beta = 0.1$

calculated similarly and can be found in the appendix B.3. The resulting equilibria for different degrees of business stealing can be observed in Figure 3.2. The dark line indicates two coinciding thresholds. The first threshold indicates the incentive of a PMF to switch to social investment in a symmetric profit maximizing setting (PMF,PMF). If the w-δ^{SR} combination is below the threshold, profits for a SRF competing with a PMF is superior to competition between PMFs. Consequently, PMFs have incentives to deviate from a symmetric setting (PMF,PMF) to an asymmetric setting (SRF,PMF). For points above the threshold firms favor non-social investment because social investment is too expensive in the short-run or not sustainable enough or both. The second threshold illustrates incentives for firms to deviate from an asymmetric setting towards the symmetric social setting. If the w-δ^{SR} point is below the threshold, a firm goes for a symmetric social setting (SRF,SRF) instead of being a PMF in an asymmetric setting (PMF,SRF) and the other way around if the point is above the threshold. Since both thresholds coincide, two equilibria emerge. A symmetric equilibrium with two PMFs (PMF,PMF) exists if the w-δ^{SR} combination is above the thresholds whereas for points below the threshold (SRF,SRF) is an equilibrium.

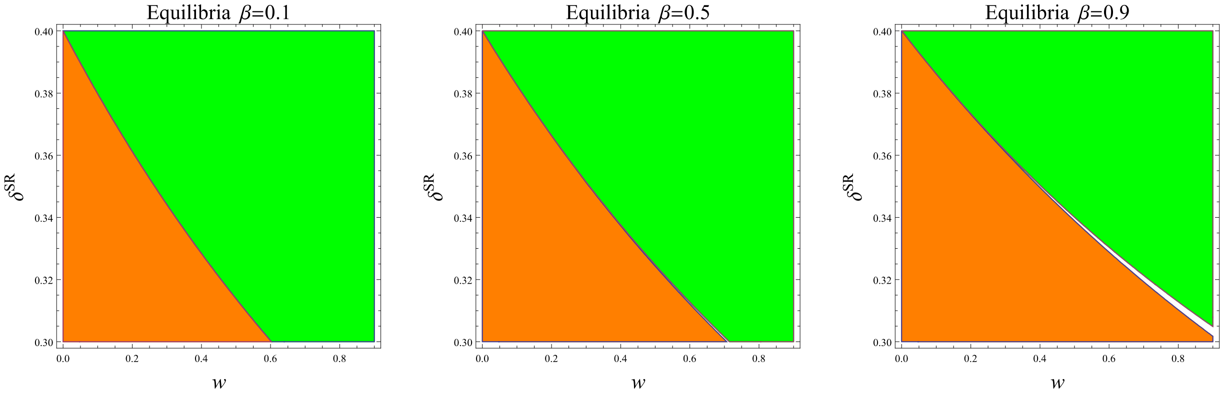


Figure 3.2: Equilibria with respect to β

The second picture of Figure 3.2 presents the effect of an increase in business stealing, meaning, a more competitive environment (business stealing) where a higher goodwill stock distracts consumers from the competitor if both firms choose different strategies. Contrary to the initial case, both thresholds move upwards. Due to the more competitive environment, both firms have higher incentives to take advantage of the social investment and competitors must counter the deviation as well.

In a highly competitive environment with high business stealing effects $\beta = 0.9$, the thresholds move further upwards and enlarge the area of the social equilibrium. For this setting, it can be observed that both thresholds do not coincide for a low depreciation rate δ^{SR} and a high cost parameter w. As a result, for parameters in the white area two equilibria coexist where either two firms are profit maximizing firms or both are socially responsible.
Overall, only symmetric equilibria occur where both firms are either socially responsible or profit maximizing firms. Nevertheless, it is reasonable to analyze which of the two equilibria the firms prefer. More generally, are the equilibria the best possible outcome or would the firms choose other strategies if they were able to coordinate their commitment about social behavior? The following subsections cover this question and the consumers point of view.

3.3.3 Prisoner's dilemma

The profits and equilibria in a competitive market when firm act non-cooperatively were shown in the previous section. It remains to be seen if the equilibria are the preferred outcome when firms can collude their social decision but set prices and investment independently to maximize their profits. The following results arise from the bimatrix game, shown in Figure 3.1, when firms can coordinate their decision about social behavior. Figure 3.3 reveals the results for different intensities of business stealing.

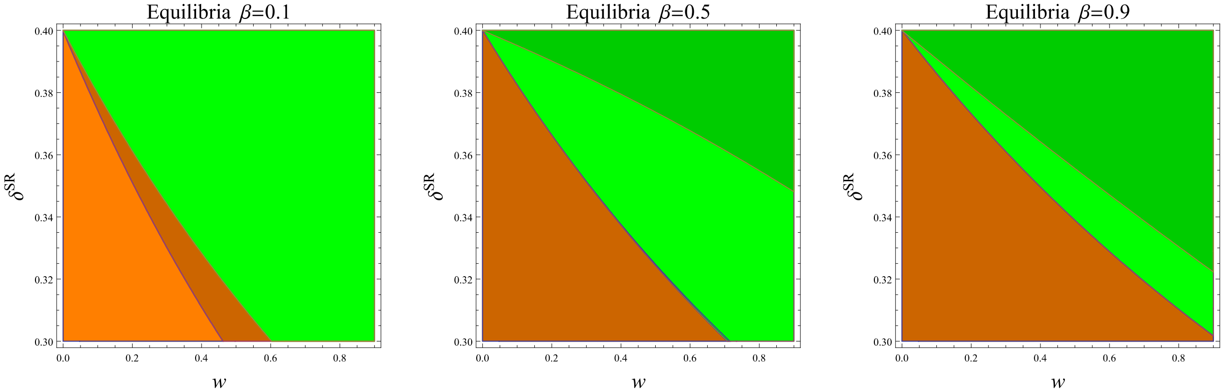


Figure 3.3: Collusive outcome and prisoner's dilemma

It can be seen in Figure 3.3 that a collusive setup with respect to the decision about social behavior deviates from the non-cooperative setup. The darker colors indicate that the equilibria do not coincide with the preferred outcome. Meaning, in a cooperative setting with respect to the social decision both firms are better off if they do not choose the equilibrium strategy (social or non-social). Consequently, non-cooperative behavior results in a "prisoner's dilemma" for both firms which intensifies for higher business stealing effects.

3.3.4 Consumer surplus and contrast to equilibrium

Since CSR is always related to consumers or welfare it is important to analyze the consumers surplus and which setting consumers prefer. As stated in the introduction, Goering (2008) brought up the idea to model social behavior by the objective $V = \pi + \theta CS$. Comparing consumer surplus and equilibria from competition, we might get an idea how the results would deviate if the consumers have some voting power about the social behavior of the firm. Figure 3.4 displays the CS for $\beta = 0.1$ and Figure 3.5 shows the preferred outcome when consumer can pick the type of firm. The CS calculated for $\beta = 0.5$ and $\beta = 0.9$ can be found in the appendix B.3. Depending on the impact of business stealing, consumers want social, non-social or a mix of both as preferred setting. The color purple (yellow) expresses: both firms should be social (non-social) from the consumers' point of view. It is colored cyan if both firms should choose different strategies. Whether the consumers' point of view deviates from the firms' point of view depends on the cost parameters and the impact of business stealing. Figure 3.6 compares the scenarios where firms and consumers coincide or deviate. The following insights can be observed:

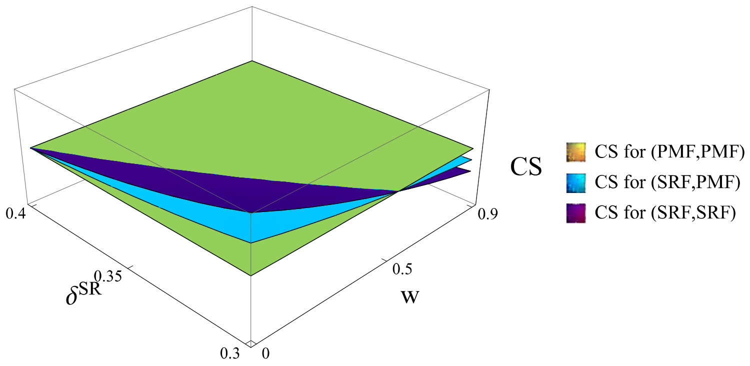


Figure 3.4: Consumer surplus for $\beta = 0.1$

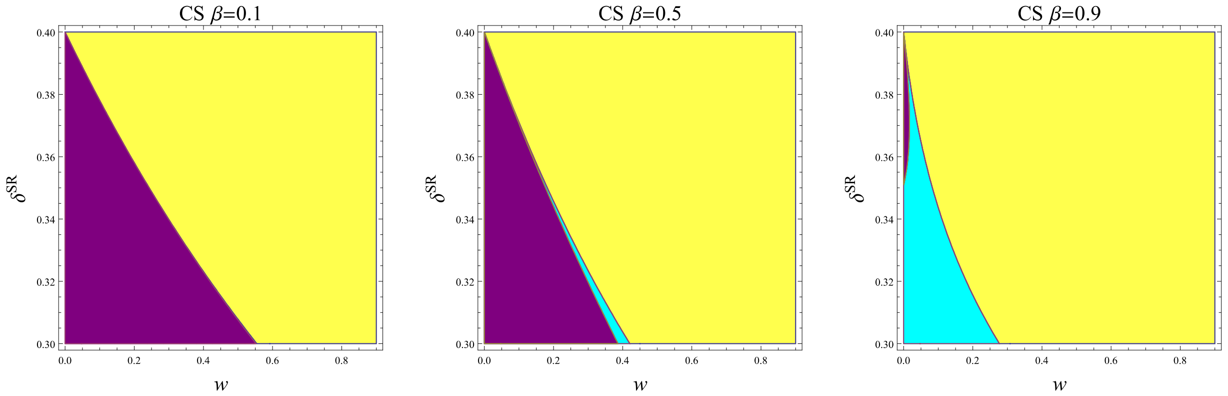
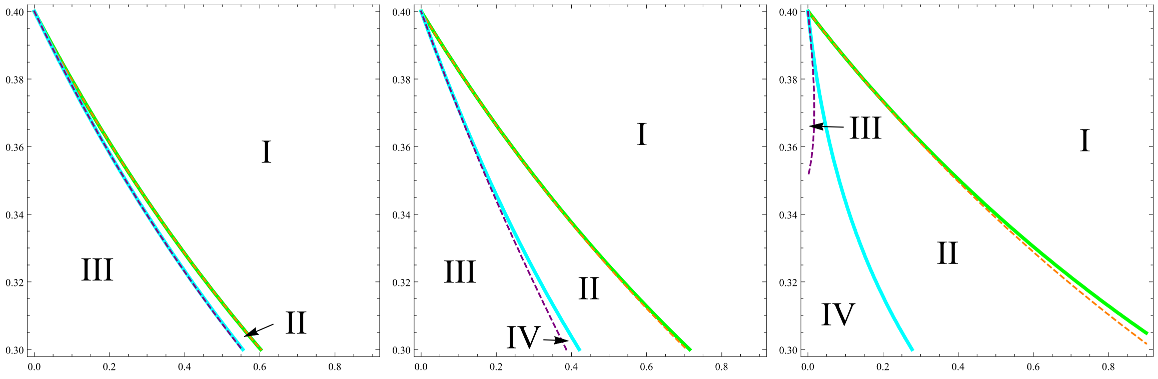


Figure 3.5: Consumer surplus



(a) $\beta = 0.1$ (b) $\beta = 0.5$ (c) $\beta = 0.9$

Figure 3.6: Comparing equilibrium outcome and consumer preferences

(I) Consumers prefer non-social firms which corresponds to the equilibrium.

(II) Each firm invests in social projects whereas consumers wish they would not. Although both deviate, the firms should listen to consumers because that would avoid the prisoner's dilemma. An integration of the consumers point of view is beneficial for the consumers and the firms as well. This includes also the case when both symmetric equilibria are possible.[11]

(III) Both firms are socially concerned which is also the best market structure for the consumers.

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[11] In contrast to our point of view, where CSR is social investment, another stream of literature incorporates a fraction of consumer surplus into firm's objective. Here, we try to link both approaches.

(IV) In contrast to the socially concerned firms, the consumers prefer two different types of firms. If the firms use different investment strategies, the socially concerned firm would be better off and the non-social firm would receive less profit.

Overall, the different point of views lead to the following insights. First, although the firms are socially responsible, it is not in the best interest of the consumers. Socially responsible investments are not beneficial for consumers (**(II)** and **(IV)**). Second, for some setups the prisoner's dilemma can be avoided if the firms take the consumers' point of view into account. It is beneficial for consumers and firms if investment is non-social (**(II)**). Business stealing creates or reinforces both effects.

3.4 Concluding remarks

The purpose of this work was to establish socially responsible investment strategies and to differentiate them from non-social strategies, such as advertising, to draw the attention of the consumers by assembling goodwill of the consumers. Initially, firms have to decide if they want to be labeled as social or non-social firm. A social firm has to invest in social projects which, compared to non-social investment, are more sustainable but increases the production costs as well. Afterwards, firms choose their prices and respective strategic investments non-cooperatively in a duopoly with differentiated products. Consequently, either symmetric firms, both are socially concerned or profit maximizing, or asymmetric firms, one socially responsible and one profit maximizing, compete in the market. The Markovian strategies and profits are derived numerically for the three possible outcomes.

The results show that the ratio of lower social depreciation rate δ^{SR} and additional costs w is crucial for social incentives and the threshold is increasing in β. Although the thresholds do not necessarily coincide for symmetric or asymmetric market structure, we finish in a symmetric social or non-social equilibrium. In addition, business stealing creates a prisoner's dilemma for the firms: They end up in a symmetric social (non-social) equilibrium but favor a symmetric non-social (social) equilibrium. With low business stealing, $\beta = 0.1$, the firms desire the equilibrium outcome but the prisoner's dilemma intensifies with an increasing business stealing effect. In contrast to the firms, the consumers would choose a different market structure. The higher β the less preferable is social investment for consumers. The results are in line with the works of Wirl et al. (2013) and Wirl (2013) where both firms are symmetric by assumption. Even if the firms can decide endogenously about their social behavior, the firms choose the same strategy depending on the market parameters. If one firm has an incentive to implement a socially responsible strategy, the competitor has an incentive to do so as well. As a result, the duopolistic setting confirms the results of Wirl et al. (2013) and Wirl (2013) that either all or no firm is socially responsible and no asymmetric equilibrium with social and non-social firms occur.

The observations can be summarized as follows: First, supported by practical examples, we define social investment as a more sustainable but also a more costly strategy. Second, the preferred investment depends on the market parameters and no asymmetric equilibrium occur. If one firm has an incentive for a different investment strategy, the competitor has an incentive as well. Third, business stealing creates a prisoner's dilemma for the firms. Taking the consumers' point of view into account may help to avoid the dilemma and therefore, can be beneficial for firms and consumers. Fourth, the higher the business stealing effect the more the consumers' preferred market structure and equilibrium market structure differ.

In our setting, we assume that the firms decide about their behavior once - either to be profit-maximizing or socially concerned - but are not able to switch from one to another. Therefore, further research could analyze the effects of a switching behavior. In addition, our analysis is concentrated on a simultaneous strategy choice concerning the firms' strategies. In line with Kopel (2009) an extension of our model would be the analysis of the sequential strategy choice of the two firms which would be possible within a multi-mode game approach with monopoly and duopoly periods.

Chapter 4

Software Platform Competition

4.1 Motivation

Although open source development accompanies computer development, the emergence of portable and web-enabled devices (e.g. notebook, smartphone, tablet) has supported open source development. As a result, open source software (OSS) has gained a significant market share in several software markets (operating system, internet browser, office suite, applications etc.). Depending on the market, either proprietary or open source software dominate the market.[1] As an example, Figure 4.1 shows the market share of the operating system on mobile and desktop platforms. The desktop market is dominated by a proprietary software (Windows) whereas the majority of users in the mobile sector use an open source platform (Android). What are the reasons for each high market share? What are the conditions for a strong diffusion of a software? To the best of my knowledge, no model has covered the case when a closed source firm and an open source firm enter the market at different times.

In general, innovators gain a first-mover advantage and are able to extract monopolistic profits. As a result, innovating firms prevent competitors from their discoveries, either by not disclosing their innovations or by means of a patent.

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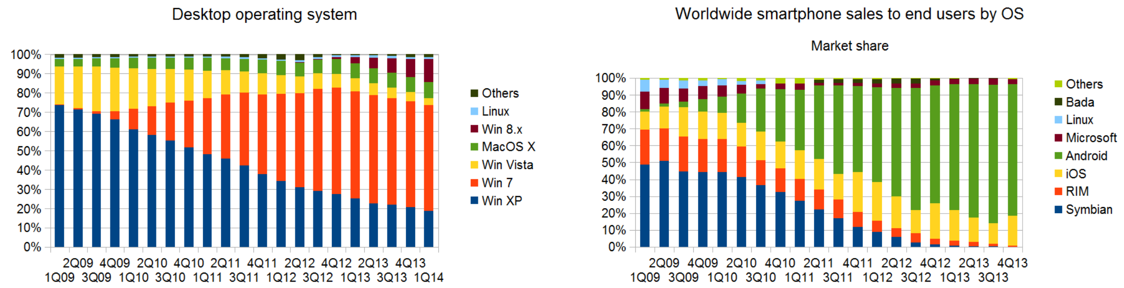
[1] Proprietary software is developed by a commercial firm and sold to consumers for a price larger than zero.



(a) Source: StatCounter (April 2014)

(b) Source: Gartner (February 2014)

Figure 4.1: Market share of desktop platforms and sales of mobile platforms

4.1.1 Literature review

Since the emergence of open source software, research tries to explain the individual user motivation to contribute to open source products and the commercial motivation to contribute or support open source development. Lerner and Tirole (2001, 2002, 2005) discuss the motivation of programmers and commercial firms to engage in open source projects. In addition to programmer incentives, Schiff (2002) studies also business models used by profit maximizing firms. In contrast to user incentives, the main drivers for firms to contribute to open source are selling complementary services, building greater innovative capability and cost reduction through open sourcing to an external community (Andersen-Gott et al. (2012)). Firms are willing to support open source development if it creates a compatibility between the open source product and the commercial product of the firms, because compatibility increases the willingness of consumers to pay for the commercial product (Mustonen (2005)). Kort and Zaccour (2011) continue with this topic and examine the strategic interactions of open source software and complementary products in a duopoly framework. If the software market is competitive and the complementary product market is less competitive, firms are willing to open the source code. Open source (OS) software is not restricted to compatibility to commercial products, it can be necessary to sell commercial products if the OS software is an operating system (Linux, Android). Some papers focus on competition between open source and proprietary systems but assume that open source firms realize no profits (Casadesus-

Masanell and Ghemawat (2006), Economides and Katsamakas (2006)). However, studies by Bonaccorsi et al. (2006); Bonaccorsi and Rossi (2006) show that firms use *"hybrid business models in the open source software industry"* to enter the market and are profit maximizing firms. Even if they do not owe the OS software, interested commercial groups support the OS movement (Haruvy et al. (2005), p. 198). Similarly, Lanzi (2009) studies duopolistic competition between proprietary and open source software and how learning and adaption costs of open source software for consumers affect diffusion of the open source software. Although proprietary and open source systems compete, the OS system is non-profitable as well.[2] In several works Haruvy et al. (2005, 2008a, 2008b) examine OS development in a competitive environment. They show that user involvement and compatibility between rival products have a positive effect on profits and that open source development can arise from competitive climate with two firms. Even though firms decide endogenously about their business model (open or closed source), they argue that a firm has no incentive to invest in open source software if the competitor closes its source code. A common component is shared and developed in an open source project to increase each demand. If *"one firm chooses to close its code, the two source codes for the common component are no longer shared"* and it simplifies to the closed source case.[3] As a result, either both firms open or close their source code depending on cost parameters. Contrary, Bonaccorsi et al. (2006) show empirically that firms have adopted to an environment dominated by incumbent standards by choosing hybrid business models with proprietary and OS software. The coexistence of open source and proprietary firms is confirmed by Llanes and De Elejalde (2013) when firms sell packages composed of a primary and a complementary good. Even when firms decide endogenously about their software type, prices and innovation, coexistence can arise as an equilibrium. They determine the conditions for the coexistence for both software types but they exclude user innovation to software quality. The incentive to change the business model from open source to closed source are studied by Lambardi (2009) and Caulkins et al. (2013). The former considers how innovation investment in a software duopoly is affected by the fact that

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[2] More precisely, a profit-oriented firm competes with the OS community that develops for free and wants to maximize the users of the OS software.

[3] Closing the source code and taking the code from the competitor to incorporate to its common component without opening its own source code can not be an equilibrium (see Haruvy et al. (2005), p. 205)

one firm is, or might become, open source whereas the latter considers the timing to open the source code.

4.1.2 Research idea

Comparable to Llanes and De Elejalde (2013), an open source and a closed source firm sell packages of software (operating system) and a complementary good (hardware). Initially, the closed source firm acts as a monopolist and the open source software firm enters at time $\tau > 0$. Users can contribute to the quality of the open source product similar to Haruvy et al. (2005, 2008a, 2008b). How does the timing and the market structure affect the diffusion process of the open source software product? Within a multi-mode differential game the following research questions are central.

- What are the conditions for successful market penetration of an operating system (compare Windows/Linux and iOS/Android)?
- What is the optimal behavior of a software incumbent with respect to prices and investment when threatened by an OS firm entry? Is market deterrence possible?
- Do OS firms contribute more or less to innovation $I^{OS} \gtrless I^{PS}$? Do user contributions substitute or complement R&D efforts by firms?

Firms sell packages composed of a primary good (the operating system) and a complementary good (hardware).[4] Subsequently, open source users can contribute to the quality of the open source software. Within a multi-mode differential game where a proprietary software monopolist is threatened by an open source entrant in mode 1 (monopoly mode). The entry depends on an exogenously given switching rate λ and can occur at any time. After the entry, the system switches to mode 2 (duopoly mode) where the proprietary and open source firm compete in prices in a differentiated market. The results show that the anticipation of an entry leads to lower prices and higher sales in the monopoly period. The higher the expected level of competition, the lower the prices in the monopoly mode. Additionally, the proprietary software firm has higher investment incentives to increase its market power if a

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[4] Underlying assumption: The consumer sticks to using the installed operating system.

quality advantage creates business stealing and lower incentives in case of positive quality externalities occur. The threat of user contributions lowers the monopoly price further. In general, the objective of this paper is to investigate the conditions to establish an open source operating system while the market entry is delayed. Section 4.2 shows the details of the multi-mode differential game. The numerical results are presented in section 4.3. The focus of the numerical calculation is set on the impact of entry, product differentiation and durability. Finally, section 4.4 summarizes the main implications of the analysis and presents topics for future research.

4.2 Framework

4.2.1 Business model and incentives

Despite a recent discussion about open source software, an essential definition is lacking in the literature. Schiff (2002) states very general that *"open source software can be modified, extended, adapted, and incorporated into other programs by other programmers, without paying a fee to any previous contributers to the software"* (p. 67). While picking up early contributions from Raymond (1999a,b) and Schiff (2002), Haruvy et al. (2008b) is able to identify four open source business models: the market positioner model, the compatible hardware model, the service and support model and the information model. In addition, Haruvy et al. (2008b) define open source as *"an active tool for improving the quality of the software or platform"* and development is *"less expensive and more rapid in the open source model"* (p. 30).

The subsequent model uses a competitive entry model similar to Eliashberg and Jeuland (1986), but with the extension of quality competition. Firms sell packages of software and compatible hardware to consumers. A proprietary software firm sells its product immediately and the open source software competitor enters the market after a period τ. The open source (OS) firm uses the market positioner business model to create a higher market penetration rate, to establish its brand and to compensate a delayed entry. The benefit of open source software are no costs in contrast to additional fees for buying the proprietary software from the software incumbent. Additionally, the development of the open source software is accelerated due to the contributions of the users, but less users are available due to the delayed entry.

4.2.2 Market structure

In general, consumers purchase the product of the proprietary or the open source software firm which maximizes their utility. Both firms compete with each other and try to generate a high market share for their products (software platforms). Inspired by market share models, both products are similar to durable goods to the effect that consumers decide about their preferred product (platform) and buy it once. As a consequence, the market size is shrinking over time.

Demand dynamics: In the absence of quality competition, both firms sell differentiated products, which can be illustrated by the following standard linear inverse demand function

$$p_i = A - q_i - sq_j$$

where $0 < s < 1$ is the degree of horizontal differentiation. The number of consumers who want to buy the respective software is represented by the reservation price A.[5] Hence, the market size is indirectly described by A because the higher the reservation price, the more consumers are willing to purchase either product. For this reason, the total market size for both products is given by $\frac{2}{1+s}A$ if both products have the same reservation price ($A_1 = A_2 = A$) and in the absence of quality. The smaller the differentiation, the smaller the initial market. The consumers buy their preferred platform depending on software quality and market price and stick to their system afterwards. Similar to durable goods and renewable resources, the market size is a stock, which decreases generally with the number of sales. The market size is renewable because of an external growth rate and firm investments. Nevertheless, the market size is unmodified[6] by the entry and determines substantially price and investment incentives. The focus of this model is on the diffusion process of proprietary and

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[5] A is the reservation price in the absence of quality competition.

[6] The entry of an open source competitor does not expand the overall market size. Instead, the remaining consumers are divided equally between both firms.

open source software. For this reason, the consumers make their decisions about their preferred system and stick with their decisions afterwards. As a consequence, the market size adjusts with respect to the sales of each firm and the resulting demand dynamic is

$$\dot{A} = -\frac{1+s}{2}(D_1(p_1, p_2) + D_2(p_1, p_2)) + \delta^A A,\ A_0 > 0. \tag{4.2.1}$$

δ^A represents the external growth rate of the market. $\frac{1+s}{2}$ is the inverse of the total market size and the dynamic of A expresses the market saturation caused by the diffusion process of the products. Consumers purchase their preferred software and stick to their product from now on.

Network: After buying the open source product, consumers can contribute to the quality of the open source product subsequently.[7] The disadvantage of the delayed entry for the open source software firm can be compensated by user network contributions. The network of open source user N is generated by sales and the exit rate δ^N of the network.

$$\dot{N} = D_2(p_1, p_2) - \delta^N N \tag{4.2.2}$$

Quality: In contrast to horizontal differentiation, the firms have to build up the quality K_i endogenously by investment I_i and according to their respective state equation $\dot{K}_i$. The quality of the proprietary software K_1 relies just on the investment of its firm I_1 whereas the quality of the open source software depends on firm investment I_2 and network contributions θN. For this reason, the quality and network dynamics are

$$\begin{aligned} \dot{K}_1 &= I_1 - \delta^K K_1 \\ \dot{K}_2 &= \theta N + I_2 - \delta^K K_2. \end{aligned} \tag{4.2.3}$$

Each investment I_i causes R&D costs $\frac{\gamma}{2} I_i^2$.

Utility function: Overall, both products are vertically and horizontally differentiated. The respective quantity of each product is q_i. The parameter $0 < s < 1$ indicates the

---

[7] By assumption, only consumers who prefer the opens source product are willing to contribute to the quality.

horizontal differentiation and is exogenously given. K_i illustrates the quality of each product. The significance of the quality regarding the respective utility function

$$U(q_1, q_2) = (A+d(K_1-\beta K_2))q_1+(A+d(K_2-\beta K_1))q_2-0.5q_1^2-0.5q_2^2-sq_1q_2), \quad (4.2.4)$$

is measured by d. β shows the competitive effect of the quality. If β is smaller than zero, quality externalities influence the demand positively, whereas a β larger than zero creates business stealing. Subsequently, the respective demand functions for each firm are

$$\begin{aligned} D_1(p_1, p_2) &= \frac{(1-s)A - p_1 + sp_2 + d[(1+s\beta)K_1 - (s+\beta)K_2]}{1-s^2} \\ D_2(p_1, p_2) &= \frac{(1-s)A - p_2 + sp_1 + d[(1+s\beta)K_2 - (s+\beta)K_1]}{1-s^2}. \end{aligned} \quad (4.2.5)$$

Timing: To return to the initial motivation, a firm takes the first-mover advantage to generate monopoly profit and closes the source code to prevent an entry or make an entry as hard as possible. Motivated by different software markets (desktop platforms, mobile platforms (cell phone and tablets)), the proprietary software firm has a first-mover advantage (Windows, iOS) and is able to make a monopoly profit until the open source software firm (Linux, Android) enters the market. The different entry times are designed by a multi-mode game with a monopoly and a duopoly mode (see section 4.2.3). During the monopoly mode, only the proprietary software firm sells its product at the market and the demand structure (4.2.5) and market dynamics (4.2.1) adjust to

$$\begin{aligned} D_1(p_1) &= A - p_1 + dK_1 \\ \dot{A} &= -D_1(p_1) + \delta^A A. \end{aligned} \quad (4.2.6)$$

Initially at time $t = 0$, the proprietary software firm F_1 is a monopolist on the market. At time τ an open source competitor F_2 enters the market with its differentiated product and both firms compete in prices.[8] Eliashberg and Jeuland (1986) differentiate between a myopic, non-myopic and surprised monopolist, who does not foresee the entry of the

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[8] Generally, open source software is for free and firms earn money with complementary services, but in this model firms sell bundles of soft- and hardware. Open source software is characterized by a delayed entry and user contributions.

competitor. A myopic monopolist discounts the duopoly period whereas the non-myopic monopolist predicts the entry time perfectly. In contrast to their three approaches, the OS firm entry can happen at any time and the proprietary software firm maximizes its expected profit with the conditional probability $\lambda(t)$ that the OS firm enters the market given the entry has not occurred so far. The entry time is denoted by τ and is exponentially distributed with switching rate[9] $\lambda(t) > 0$. The switching rate is defined as

$$\lambda(t) = \lim_{t \to \infty} \frac{1}{\Delta} Prob\{m(t+\Delta) = m_2 \mid m(t) = m_1\} \tag{4.2.7}$$

where m_1 is the monopoly mode with the proprietary software firm and m_2 the duopoly mode with both firms. In order to focus on market and quality dynamics for the analysis, the exponential distribution of entry is assumed to be exogenous. Nevertheless, different switching rates are used to analyze the relation between expected entry time and the control variables.

4.2.3 Objectives and dynamics for each firm

The proprietary software firm chooses its price and investment in order to maximize its discounted profits over an infinite time horizon. Denoting the discount rate by $r > 0$ and the marginal costs by c_i, the objective function of the proprietary software firm is given by

$$\max_{p_1, I_1} \Pi_1 = \mathbb{E}\left[\int_0^{\tau} e^{-rt}[(p_1 - c_1)D_1(p_1) - \frac{\gamma}{2} I_1^1]\, dt + \int_{\tau}^{\infty} e^{-rt}[(p_1 - c_1)D_1(p_1, p_2) - \frac{\gamma}{2} I_1^2]\, dt\right].$$

The quality stock K_1 has a depreciation rate of δ^k. The respective dynamics for the proprietary software firm are

$$\begin{array}{rll}
\textbf{Monopoly period } (m_1)\textbf{:} & \dot{A} = -D_1(p_1) + \delta^A A, & A(0) > 0 \\
& \dot{K}_1 = I_1 - \delta^K K_1, & K_1(0) = 0, K_1(\tau) > 0 \\
\textbf{Duopoly period } (m_2)\textbf{:} & \dot{A} = -\frac{1+s}{2}(D_1 + D_2) + \delta^A A, & A(\tau) > 0 \\
& \dot{K}_1 = I_1 - \delta^K K_1, & K_1(\tau) > 0 \\
& \dot{N} = D_2(p_1, p_2) - \delta^N N, & N(\tau) = 0 \\
& \dot{K}_2 = \theta N + I_2 - \delta^K K_2, & K_2(\tau) = 0.
\end{array}$$

---

[9] The system switches from the monopoly mode to the duopoly mode due to the entry of the OS firm.

The entrant uses open source software due to the absence of license fees to the proprietary software firm and the development is faster thanks to user contributions. The OS software complements the core product and the firm supports the development of the software. The open source firm enters the market at time τ and competes in a duopoly (m_2) with the proprietary software firm. The OS firm selects prices and investment to maximize its discounted profit as well.

$$\max_{p_2, I_2} \Pi_2 = \int_{\tau}^{\infty} e^{-rt}[(p_2 - c_2)D_2(p_1, p_2) - \frac{\gamma}{2} I_2^2] \, dt$$

$$\begin{aligned} \dot{A} &= -\frac{1+s}{2}(D_1 + D_2) + \delta^A A, & A(\tau) > 0 \\ \dot{N} &= D_2(p_1, p_2) - \delta^N N, & N(\tau) = 0 \\ \dot{K}_2 &= \theta N + I_2 - \delta^K K_2, & K_2(\tau) = 0 \end{aligned}$$

The quality of the open source product depends on user contributions and firm investment since *"successful open source software development has become structured, and oftentimes funded by interested commercial groups"* (Haruvy et al. (2005), p. 198).
Assuming firms use stationary Markovian feedback strategies, a Markov Perfect Equilibrium is calculated numerically.

4.2.4 Timing of the model and characterization of the Markov Perfect Equilibria

To understand the steps of this piecewise deterministic differential game, the game can be divided into two periods. In the monopoly period

$$\begin{aligned} \max_{p_1, I_1} \Pi_1 &= \int_0^{\tau} e^{-rt}[(p_1 - c_1)D_1(p_1) - \frac{\gamma}{2} I_1^2] \, dt + e^{-r\tau} V_1(\bullet, m_2) \\ \dot{A} &= -D_1(p_1) + \delta^A A, \quad A(0) > 0 \\ \dot{K}_1 &= I_1 - \delta K_1, \quad K_1(0) = 0, K_1(\tau) > 0, \end{aligned}$$

the proprietary software firm is a monopolist, but has to take into account an entrant at time τ.

The duopoly can be formulated as

$$
\begin{aligned}
\max_{p_1,I_1} \Pi_1 &= \int_{\tau}^{\infty} e^{-rt}[(p_1-c_1)D_1(p_1,p_2) - \frac{\gamma}{2}I_1^2]\,dt \\
\max_{p_2,I_2} \Pi_2 &= \int_{\tau}^{\infty} e^{-rt}[(p_2-c_2)D_2(p_1,p_2) - \frac{\gamma}{2}I_2^2]\,dt \\
\dot{A} &= -\frac{(1+s)}{2}(D_1(p_1,p_2)+D_2(p_1,p_2)) + \delta^A A, \qquad A(\tau) > 0 \\
\dot{N} &= D_2(p_1,p_2) - \delta^N N, \qquad N(\tau) = 0 \\
\dot{K}_1 &= I_1 - \delta^k K_1, \qquad K_1(\tau) > 0 \\
\dot{K}_2 &= \theta N + I_2 - \delta^k K_2, \qquad K_2(\tau) = 0.
\end{aligned}
$$

To characterize the Markov perfect equilibria of the underlying game, a multi-mode game (see Dockner et al. (2000), chap. 8) with a monopoly and duopoly mode is used. Due to the linear quadratic structure of the game, the structure of the value functions are known and the dynamic programming approach is reasonable. The value functions of the players depend on the state variables and the mode as well. The Hamilton-Jacobi-Bellman (HJB) equations are different in the monopoly and duopoly mode because of the absence of a competitor ($s = K_2 = 0$). The HJB equation for the proprietary software firm in the monopoly mode (m_1) is

$$
\begin{aligned}
rV_1(A,0,K_1,0,m_1) = \max_{p_1,I_1} \Big\{ &[(p_1-c_1)D_1(p_1) - \frac{\gamma}{2}I_1^2] + \frac{\partial V_1(\bullet,m_1)}{\partial A}(-D_1(p_1)+\delta^A A) \\
&+ \frac{\partial V_1(\bullet,m_1)}{\partial K_1}(I_1-\delta^k K_1) + \lambda(V_1(\bullet,m_2)-V_1(\bullet,m_1)) \Big\}. \quad (4.2.8)
\end{aligned}
$$

The expression in the first brackets indicates the instantaneous profit of selling proprietary software. The next two terms express the effect of changes of the demand and quality. The last term shows the value function effect of the entry by an open software competitor (i.e. the potential jumps from mode m_1 (monopoly) to mode m_2 (duopoly)) where λ is the switching rate. If the open source firm enters the market, both firms compete with differentiated products and their respective HJB equations are

$$\begin{aligned} rV_i(A,N,K_1,K_2,m_2) =&\max_{p_i,I_i}\Bigg\{[(p_i-c_i)D_i(p_i,p_j^*)-\frac{\gamma}{2}I_i^2] \\ &+\frac{\partial V_i(\bullet,m_2)}{\partial A}(-0.5(1+s)(D_i(p_i,p_j^*)+D_j(p_j^*,p_i))+\delta^A A) \\ &+\frac{\partial V_i(\bullet,m_2)}{\partial K_i}(I_i-\delta^k K_i)+\frac{\partial V_i(\bullet,m_2)}{\partial N}(D_2(p_i,p_j^*)-\delta^N N) \\ &+\frac{\partial V_i(\bullet,m_2)}{\partial K_j}(I_j^*-\delta^k K_j)\Bigg\}, \quad \text{for} \quad i,j=1,2,i\neq j. \end{aligned} \tag{4.2.9}$$

To verify the feedback strategies for both modes, the multi-mode game is solved backward. The open source firm has already entered the market in the duopoly mode and the feedback strategies can be determined. The first order conditions from (4.2.9) provide the Markovian investment and pricing strategies for both firms in mode 2:

$$\begin{aligned} p_i^*(A,N,K_1,K_2,m_2) =& \frac{1}{4-s^2}\Bigg((2-s-s^2)A+2c_i+sc_j \\ &+d[(2-s^2+s\beta)K_i-(s+\beta-s^2\beta)K_j] \\ &+(1-s^2)\frac{\partial V_i(\bullet,m_2)}{\partial A}+2s\frac{\partial V_i(\bullet,m_2)}{\partial N} \\ &+\frac{s(1-s^2)}{2}\frac{\partial V_j(\bullet,m_2)}{\partial A}-s\frac{\partial V_j(\bullet,m_2)}{\partial N}\Bigg) \\ I_i^*(A,N,K_1,K_2,m_2) =& \frac{1}{\gamma}\frac{\partial V_i(\bullet,m_2)}{\partial K_i}, \quad i=1,2,i\neq j. \end{aligned} \tag{4.2.10}$$

In the monopoly mode the game consists of two state variables (A_1,K_1) whereas in the duopoly mode the firms have to take four states (A,N,K_1,K_2) into account. Due to the anticipated entry, the maximization problem of the proprietary software firm still consists of all four states in the monopoly mode but the feedback strategies just depend on two states (A,K_1). The first order conditions from (4.2.8) yield to the following feedback strategies in mode 1:

$$\begin{aligned} p_1^*(A,K_1,m_1) &= \frac{1}{2}\Bigg(A+c_1+dK_1+\frac{\partial V_1(\bullet,m_1)}{\partial A}\Bigg) \\ I_1^*(A,K_1,m_1) &= \frac{1}{\gamma}\frac{\partial V_1(\bullet,m_1)}{\partial K_1}. \end{aligned} \tag{4.2.11}$$

Thanks to the linear quadratic structure of the game, the following value functions solve the

HJB equations (4.2.8) and (4.2.9)

$$\begin{aligned} V_1(A,0,K_1,0,m_1) =& f_1^1 A^2 + f_2^1 K_1^2 + f_3^1 A K_1 + f_4^1 A + f_5^1 K_1 + f_6^1 \\ V_i(A,N,K_1,K_2,m_2) =& f_{1i}^2 A^2 + f_{2i}^2 N^2 + f_{3i}^2 K_1^2 + f_{4i}^2 K_2^2 \\ &+ f_{5i}^2 A\, N + f_{6i}^2 A\, K_1 + f_{7i}^2 A\, K_2 + f_{8i}^2 N K_1 + f_{9i}^2 N\, K_2 + f_{10i}^2 K_1 K_2 \\ &+ f_{11i}^2 A + f_{12i}^2 N + f_{13i}^2 K_1 + f_{14i}^2 K_2 + f_{15i}^2 \quad \text{for} \quad i,j = 1,2, i \neq j. \end{aligned} \tag{4.2.12}$$

In order to calculate the coefficients of the respective value functions, solve the HJB equations for mode 2 and use $V_1(m_2)$ to solve mode 1 afterwards. The Markovian strategies $p_i^*(A,N,K_1,K_2,m_2)$ and $I_i^*(A,N,K_1,K_2,m_2)$ for mode 2 (duopoly mode) are used for the HJB equation (4.2.9) and determine the f_{ni}^2 for $i,j = 1,2, i \neq j$ and $n = 1,..,15$ numerically. Subsequently, using the Markovian strategies for mode 1 (monopoly mode) $p_1^*(A,N,K_1,K_2,m_1)$, $I_1^*(A,N,K_1,K_2,m_1)$, the switching rate λ and $V_1(m_2)$ solves HJB equation (4.2.8).

4.2.5 Stability

The Markovian price and investment strategies lead to a linear dynamical system

$$\begin{pmatrix} \dot{A} \\ \dot{N} \\ \dot{K}_1 \\ \dot{K}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44}. \end{pmatrix}}_{H} \begin{pmatrix} A \\ N \\ K_1 \\ K_2 \end{pmatrix} + \begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{pmatrix} \tag{4.2.13}$$

The system is globally asymptotically stable iff all the eigenvalues of the matrix H have negative real parts. The numerical analysis is restricted to (globally)[10] asymptotically stable settings such that the transversality condition

$$\lim_{t\to\infty} e^{-rt} V_i(A(t),N(t),K_1(t),K_2(t),t) \leq 0 \quad i = 1,2, \tag{4.2.14}$$

is satisfied.

---

[10] Due to the linear quadratic structure of the game.

4.3 Economic Analysis-Numerical calculation

4.3.1 Mode 1: Anticipation effect

Market competition

To focus on pricing incentives and entry effects on pricing, quality is disregarded in a first step and the model can be simplified to one control p_i and one state A. In the absence of quality effects, investment incentives are equal to zero and users are unable to contribute to the quality.[11] The resulting Markovian strategies $p_1(A, m_1)$ for mode 1 and $p_i(A, m_2)$ for mode 2 just depend on the stock A. Figure 4.2 and Figure 4.3 show the impact of intensity of competition and switching rate on pricing strategies.[12] Even though the entry occurs earlier or later as expected, the price effect is obvious: The earlier the expected entry and the higher the expected competition, the lower is the price of software incumbent. The lower price results in faster shrinking market, which is shown by the reservation price A. It is disadvantageous for an open source business model because less users are available, who could contribute to the open source software quality.

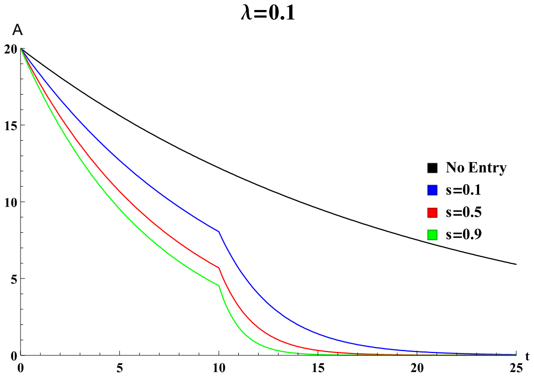

Figure 4.2: Competition effect for pricing

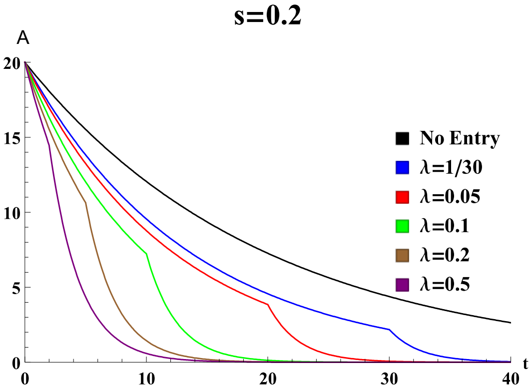

Figure 4.3: Entry effect for pricing

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[11] If the quality does not affect the demand ($d = 0$), pricing is the only control and the market size A is the only stock to take into account.

[12] For the graphical illustration it is assumed that the actual entry time is given by the expected value $\mathbb{E}(\tau) = \frac{1}{\lambda}$.

Quality competition

If firms compete additionally with their quality according to (4.2.5), the anticipation of entry also influences the investment incentives of the proprietary software firm in the monopoly. Figure 4.4 shows that the expected entry time increases investment of the proprietary software firm in the monopoly mode. The higher the probability of an entrant, the higher the investment to improve its market position. The investment incentives for quality competition depend crucially on β. If $\beta < 0$ and positive quality externalities exist, the proprietary software firm has less incentives because the OS firm benefits immediately from the knowledge stock K_1 of the proprietary software firm accumulated in the monopoly mode. Whereas business stealing $\beta > 0$ creates higher incentives in the monopoly mode because it improves the market position of the proprietary software firm and lowers the amount of consumers who can contribute to the OS software. The quality competition is presented in Figure 4.5. Lastly, the network effect measures how many users contribute to the OS software. The impact on pricing and investment incentives in the monopoly mode is demonstrated in Figure 4.6. The network effect decreases prices and increases investment incentives. The higher the network contribution in the duopoly mode, the lower the prices of the proprietary software firm in the monopoly mode (see Figure 4.6 (a)). Lower prices lead to less consumers who can contribute to the OS software in the duopoly such that the proprietary software firm keeps its quality advantage. Contrary to pricing incentives, expected network contributions lead to higher investments in the monopoly mode (see Figure 4.6 (b)). Higher investments bring higher quality and higher prices, which hinders the OS firm to build up a network and to use the contributions. As a result, the proprietary software firm uses both controls in the monopoly

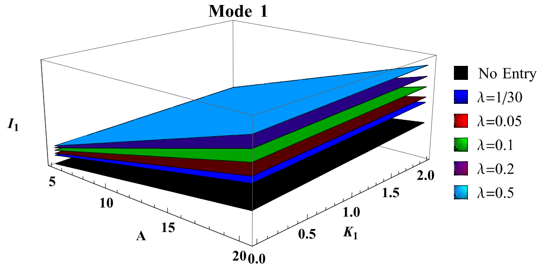


Figure 4.4: Entry effect λ

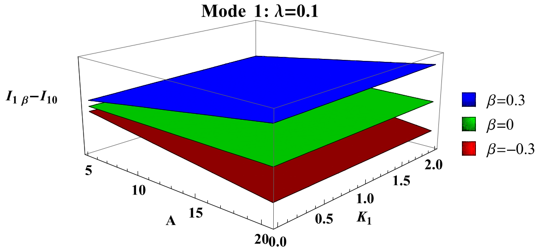
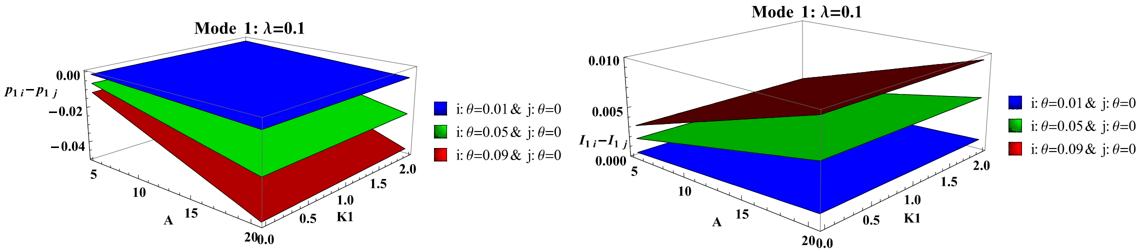


Figure 4.5: Quality competition effect β



(a) Pricing (b) Investment

Figure 4.6: Network effect without quality competition ($\beta = 0$)

mode to complicate the diffusion of the OS software.

4.3.2 Mode 2: Impact of market structure on incentives

The previous section showed the influence of entry, network contributions and quality competition on pricing and investment incentives in the monopoly. Subsequently, these effects are analyzed for the duopoly mode, meaning after the entry of the OS firm.

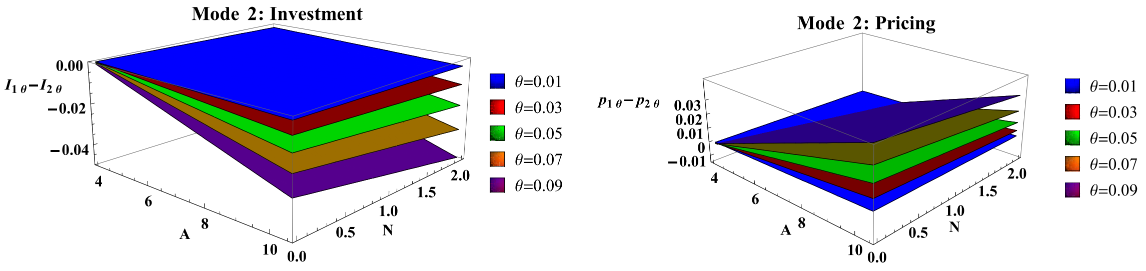
Due to the four states the Markovian strategies are a mapping $\phi_i : \mathbb{R}^4_+ \to \mathbb{R}$ with the following structure

$$\begin{aligned} I_i^*(A, N, K_1, K_2, m_2) &= b_{i1}A + b_{i2}N + b_{i3}K_1 + b_{i4}K_2 + b_{i5} \\ p_i^*(A, N, K_1, K_2, m_2) &= b_{i6}A + b_{i7}N + b_{i8}K_1 + b_{i9}K_2 + b_{i10}, \end{aligned} \tag{4.3.1}$$

where the parameters b_{ij} depend on the numerical solution of the HJB equation and, therefore, on the network effect θ and quality competition β. In order to examine both effects, the analysis is restricted to the A-N-state space because both firms react differently to both states. The investment incentives I_i and pricing incentives p_i are always positively correlated with K_i and negatively correlated with K_j with $i, j = 1, 2$ $i \neq j$ as shown in Appendix C.1 and Appendix C.2.

Network

At first, how do network contributions affect investment incentives of the proprietary and the OS software firm? Figure 4.7 displays the significance of the network effect for investment



(a) Investment incentives

(b) Pricing incentives

Figure 4.7: Incentive differences for $\beta = 0$

and pricing for both firms. Figure 4.7 (a) shows the investment difference of both firms with respect to the $A - N -$ space and neglected quality stocks (assuming $K_1 = K_2 = 0$).[13] The investment difference is negative. In other words, the open source software firm has higher incentives than the proprietary software firm.[14] The higher the network contributions θ, the higher the difference. The open source firm has higher incentives because in addition to higher quality and sales users contribute afterwards. Generally, the open source firm also sets a lower price compared to the proprietary software firm (see Figure 4.7 (b)) to create higher sales and add network contributions.[15]

From a consumer's point of view it is more interesting if the competition between a proprietary and open source software firm generates higher investment and, as a result, higher quality products for the consumers. Are network contributions and investment substitutes or complements? Figure 4.8 shows the overall investment consisting of firms' investment if user contribute minus investment without contribution $(I_{1,\theta} + I_{2,\theta} - (I_{1,0} + I_{2,0}))$. All surfaces are positive and, thus, network contributions bring out higher overall investment of the software industry. The higher the network contributions θ, the higher the overall investment. Therefore, the open source business model has a positive effect on overall investment. Consumers benefit from lower prices and higher investment.

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[13] See Appendix C.1 for further details

[14] Actually, the investment of the proprietary software is higher due to the higher quality $K_1 > K_2 = 0$ when the open source software firm enters the market.

[15] The actual prices of both firms depend crucially on parameter setting, qualities K_i, K_j and the dynamics.

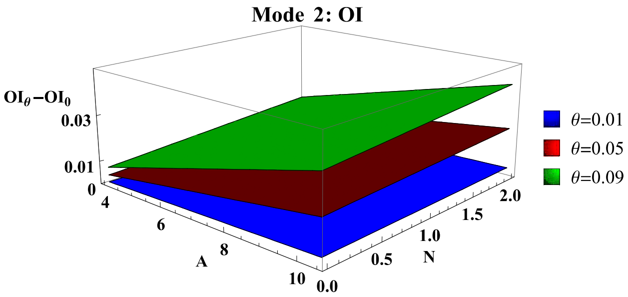

Figure 4.8: Overall Investment: $I_{1,\theta} + I_{2,\theta}$

Quality competition

To demonstrate the effect of quality competition (business stealing or positive quality externalities), the network effect is neglected ($\theta = 0$). In addition and similar to the network effect, assume similar to the previous section $K_i = K_j = 0$ (see Appendix C.2 for more details). The impact of quality competition β after the entry is shown in Figure 4.9. The investment for both firms I_i are increasing in β with respect to A.[16] This is in line with results for the monopoly mode where the investment $I_1(A, N, K_1, K_2, m_1)$ was also increasing in β. In the monopoly mode, positive quality externalities $\beta < 0$ lower incentives because the open source software firm would benefit immediately from the accumulated quality K_1 of the proprietary software firm when a market entry occurs. Business stealing $\beta > 0$ gives

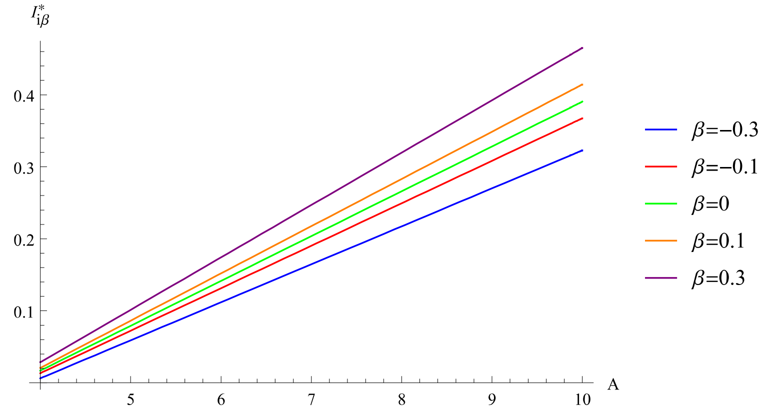

Figure 4.9: Quality competition

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[16] The actual investments have to be adjusted due to the stocks K_i and K_j.

the proprietary software firm a competitive advantage ($K_2(\tau) - \beta K_1(\tau) < 0$) when the open source firm enters the market.

4.3.3 Value Functions

Finally, this section demonstrates the meaning of quality competition, entry time and network contributions on the value function of the two firms. Given the initial stocks in the monopoly mode $A(0) = 20$ and $K_1(0) = 0$ as well as the initial stocks in the duopoly mode $N(\tau) = 0$ and $K_2(\tau) = 0$, the expected value for each firm at $t = 0$ can be calculated. Figure 4.10 displays quality competition whereas Figure 4.11 shoes the effect of network competition. The value function of the proprietary software $V_1(m_1)$ firm is decreasing in λ whereas the value function of the open source firm $V_2(m_1)$ is increasing in λ: The proprietary software firm wants to prevent an entry whereas the open source firm wants to enter as soon as possible. Compared to the entry effect, the effect of quality competition and network contributions is less substantial for the proprietary software firm because the main revenue is generated in the monopoly period. Nevertheless, network contributions increase the revenue of the open source firm and lower the revenue of the proprietary software firm. In contrast to the network effect, quality competition decreases the revenues of both firm. In sum, network contributions are always valuable for the open source firm, but due to a lot of positive quality externalities the proprietary software firm can benefit as well.

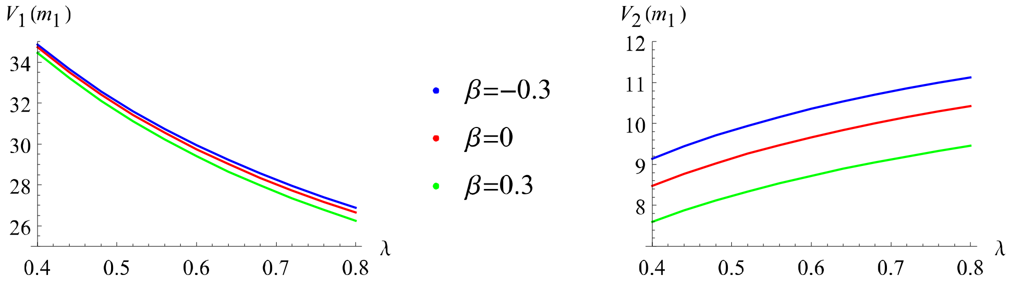

Figure 4.10: Quality competition

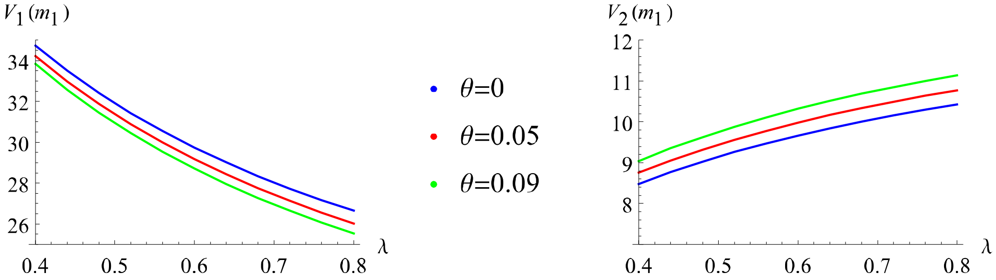

Figure 4.11: Network effect

4.4 Conclusion

Empirical data for the desktop and mobile platforms showed that a proprietary software had a first mover advantage but an open source software competitor entered subsequently. The proprietary software (Windows) dictates the desktop market whereas the mobile market is dominated by the open source software (Android). The purpose of this model was to analyze the influence of an open source threat on pricing and investment incentives if the proprietary software firm has a first mover advantage. In mode 1, the proprietary software firm acts monopolistically but is threatened by an open source competitor. The system switches to the duopoly mode after the entry of the OS firm. The uncertainty of the entry and the expected market structure influence the proprietary software firm and the results show the strategic effects.

Anticipation effect: The earlier the entry of open source competitor, the lower the price and the higher the investment of the proprietary software firm in the monopoly period. The higher the expected intensity of competition, the lower the monopoly price, the higher the initial profits and the steeper the slope of the profit function of the proprietary software firm.

Network effect: The open source firm increases its investment and lowers prices with respect to network contributions. In comparison to the open source firm, the proprietary software firm sets higher prices and invests less. In mode 2, user contributions complement firm investments and overall innovation activity is higher thanks to user contributions.

Quality competition: Business stealing increases investment incentives and positive quality externalities lower incentives in the monopoly and duopoly period.

In sum, the threat of an open source competitor brings out a stronger diffusion rate of the proprietary software bundle with higher quality such that the development of the OS software is as difficult as possible. Less consumers are able to contribute to the OS software and higher quality gives a competitive advantage.
To resume the initial questions, the answers are as follows:

- The entry time and product differentiation determine essentially the success or failure of an successful software market penetration.
- The software incumbent lowers prices to complicate market entry. If both firms compete in software quality after the entry, the incumbent invests more initially. The incumbent invests less if positive quality externalities exist.
- OS firms invest more than proprietary software firms and their investments complement user contributions.

Given these insights and getting back to the initial example, the software dominance of desktop and mobile platforms can be explained. Microsoft was able to build a high rate of diffusion by bundling their Windows platform with computer sales. Consumers could not buy hardware and software separately. Consumers got used to the system and additional software was developed which improved the quality of Windows. Contrary, Linux was not bundled and user contributions were less efficient[17] compared to Android, because of the complexity of an operating system source code. Thus, Linux diffusion rate was too low and user contributions were insufficient to create a higher market share. In contrast to desktop platforms, Android achieves a significant market share. Due to an early entry and substantial user contribution[18], Google was able to establish its software in the mobile and tablet market despite of a delayed entry and a dominant competitor Apple with its high quality product.

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[17] User contributions can be additional applications or software source code. Both contributions improve the quality, but source code contributions are less profitable and more complex.

[18] Users have personal and monetary incentives to contribute to the source code or developing additional applications.

Android retailers (Samsung, HTC, LG etc.) were able to achieve a high diffusion rate with a full range from low-end to high-end Android products whereas Apple just offered high-priced products.

The model provided some beneficial results about market entry, quality competition and network contributions. Nevertheless, some points are still open for discussion and future research. First, both firms sell bundles of hardware and software for a price $p_i > 0$. Generally, open source software is for free and firms sell complementary products to realize profit (for instance see Haruvy et al. (2005, 2008a, 2008b), Kort and Zaccour (2011), Caulkins et al. (2013)). In order to focus on the diffusion process, the model is simplified to bundled products with two controls and four states. However, the accumulated user network for proprietary and open source software can serve as a complementary product where firms can sell additional applications or sell mobile advertisement. Second, the supply chain management is managed by one firm. A look at the mobile sector reveals that different vertical structures compete with each other. Vertical integrated firms (Apple uses its own iOS) compete with independent firms, which either use open source (Samsung, HTC etc. use Android) or develop their own operating system (Nokia, Samsung etc. use Windows Phone). In contrast to the retailers, the operating system firm makes profit by advertisement and additional applications. Therefore, the supplier and retailer have conflicting diffusion incentives. The pricing incentives and development efforts in different vertical structures should be compared in separated models. Third, both firms sell one product and their cost structures (production costs and R&D costs) are symmetric. The setup does not cover multi-product firms. For example, should the OS firm establish a low quality product to create a network and develop a high quality product afterwards? Finally, the model examines how an open source firm competes with a proprietary software firm, but contrary, what entry barriers can be created by an open source firm? Are other firms able to enter the market if development is accelerated by user contributions? Are competitors excluded from using the open source platform? What are economic policy implications from an open source dominance?

All these questions are topics for future research in separated settings.

Chapter 5

Final Conclusion

The interaction of structural changes and operational decisions were the objective of this research project. How do theses market parameters influence profitability or consumer surplus?

Chapter 2 showed that vertical integration is always profitable for the integrating firms and Full Integration is the equilibrium of the Bimatrix Game. It can be profitable for competing downstream firms if knowledge spillover are high and products are differentiated. Vertical integration resolves the double marginalization and increases the investment incentive of the integrating firm. If competing firms increase or decrease investments, depends on market structure and knowledge spillover. Furthermore, the intertemporal strategic effect differs between all three market structure (no integration, partial integration, full integration). Consumers benefit from each vertical integration.

Chapter 3 shows that social responsibility depends on the relation of additional production costs and sustainability. The threshold is related to market extension and business stealing $0 \leq \frac{\beta}{d} \leq 1$. The higher this relation, the more are firms willing to invest in social products. However, this business stealing creates a prisoner's dilemma: Both firms invest in the less profitable product. In addition, the higher business stealing, the more deviate consumer preferences and equilibrium outcome.

Chapter 4 examines the impact of an market entry in the software industry. An open source entrant competes with an proprietary software incumbent. Customer of the open source product contribute to the quality afterwards. Due to the open source threat, the software incumbent lowers his prices to generate higher profits before the entry. He increases investment if the product quality improves his market position. After the entry, the open source software firm chooses lower prices and higher investment compared to the proprietary software firm. Overall, the open source business model decreases prices and increases investment.

Structural changes and operational decisions affect each other, but their effects and the preferences depend on the point of view. Vertical integration was beneficial for all participating firms and consumers. Still, the preferences about socially responsible products can differ highly between firms and consumers. Finally, the open source business model improves investment and lowers prices, but is problematic for profits.

Appendix A

Appendix Chapter 2

A.1 Proof of Proposition 2.3.1

Differentiating the right-hand side of (2.3.5) w.r.t. q_i and equating to zero leads to the best reply functions

$$q_i = \frac{A - c_i + K_i + \beta K_j}{s+2} - \frac{1}{2} q_j \qquad i, j = 1, 2\ i \neq j$$

The best reply functions and differentiating (2.3.5) w.r.t. I_i yield to the equilibrium strategies[1]

$$\begin{aligned} q_i^c &= \frac{2(A - 2c_i + c_j + (2-\beta)K_i + (2\beta - 1)K_j)}{3(s+2)} \\ I_1^c &= \frac{B_1 K_1 + B_3 K_j + B_4}{\gamma} \\ I_2^c &= \frac{D_2 K_2 + B_3 K_2 + D_5}{\gamma} \end{aligned} \tag{A.1.1}$$

Substituting the outputs and investments we get the following HJB

$$\begin{aligned} rV_i(K_i, K_j) = \Bigg[&\big(A - ((s+2)/2)(q_i^c + q_j^c) - c_i + K_i + \beta K_j\big) q_i^c - \frac{1}{2\gamma} \left(I_i^c\right)^2 \\ &+ \frac{\partial V_i(\cdot)}{\partial K_i} \left(I_i^c - \delta K_i\right) + \frac{\partial V_i(\cdot)}{\partial K_j} \left(I_j^c - \delta K_j\right) \Bigg] \end{aligned} \tag{A.1.2}$$

where q_i^c is given by (A.1.1) and the partial derivatives by (2.2.9). Comparing coefficients of the quadratic polynomial on the left and on the right hand side of the two HJB equations

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[1] Due to the linear quadratic structure, the optimal strategies are linear w.r.t. the state variables.

gives the set of 12 equations and 12 unknowns:

$$\begin{aligned}
\frac{r}{2}B_1 &= \frac{2(\beta-2)^2}{9(2+s)} + \frac{B_1^2}{2\gamma} + \frac{B_3 D_3}{\gamma} - \delta B_1 \\
\frac{r}{2}B_2 &= \frac{2(1-2\beta)^2}{9(2+s)} + \frac{B_3^2}{2\gamma} + \frac{B_2 D_2}{\gamma} - \delta B_2 \\
rB_3 &= \frac{4(5\beta-2\beta^2-2)}{9(2+s)} + \frac{B_1 B_3 + B_2 D_3 + B_3 D_2}{\gamma} - 2\delta B_3 \\
rB_4 &= \frac{4(A-2c_1+c_2)(2-\beta)}{9(2+s)} + \frac{B_1 B_4 + B_3 D_5 + B_5 D_3}{\gamma} - \delta B_4 \\
rB_5 &= \frac{4(A-2c_1+c_2)(2\beta-1)}{9(2+s)} + \frac{B_3 B_4 + B_2 D_5 + B_5 D_2}{\gamma} - \delta B_5 \\
rB_6 &= \frac{2(A-2c_1+c_2)^2}{9(2+s)} + \frac{B_4^2}{2\gamma} + \frac{B_5 D_5}{\gamma}
\end{aligned}$$

$$\begin{aligned}
\frac{r}{2}D_1 &= \frac{2(1-2\beta)^2}{9(2+s)} + \frac{D_3^2}{2\gamma} + \frac{B_1 D_1}{\gamma} - \delta D_1 \\
\frac{r}{2}D_2 &= \frac{2(\beta-2)^2}{9(2+s)} + \frac{D_2^2}{2\gamma} + \frac{B_3 D_3}{\gamma} - \delta D_2 \\
rD_3 &= \frac{4(5\beta-2\beta^2-2)}{9(2+s)} + \frac{D_2 D_3 + B_1 D_3 + B_3 D_1}{\gamma} - 2\delta D_3 \\
rD_4 &= \frac{4(A-2c_2+c_1)(2\beta-1)}{9(2+s)} + \frac{D_3 D_5 + B_1 D_4 + B_4 D_1}{\gamma} - \delta D_4 \\
rD_5 &= \frac{4(A-2c_2+c_1)(2-\beta)}{9(2+s)} + \frac{D_2 D_5 + B_3 D_4 + B_4 D_3}{\gamma} - \delta D_5 \\
rD_6 &= \frac{2(A-2c_2+c-1)^2}{9(2+s)} + \frac{D_5^2}{2\gamma} + \frac{B_4 D_4}{\gamma}
\end{aligned}$$

A.2 Proof of Proposition 2.3.2

Differentiating the right-hand side of (2.3.10) and (2.3.13) w.r.t. q_i and I_i and equating to zero leads to the equilibrium strategies

$$\begin{aligned}
q_1^c &= \frac{(4-s)A - 4c_1 + sc_2 + (4-s\beta)K_1 + (4\beta-s)K_2}{8-s^2} \\
q_2^c &= \frac{(2-s)A - 2c_2 + sc_1 + (2\beta-s)K_1 + (2-s\beta)K_2}{8-s^2} \\
I_1 &= \frac{B_1 K_1 + B_3 K_2 + B_4}{\gamma} \\
I_2 &= \frac{D_2 K_2 + D_3 K_1 + D_5}{\gamma}
\end{aligned} \tag{A.2.1}$$

Substituting the optimal outputs and investments the HJB for each player is

$$\begin{aligned} rV_1(K_1,K_2) &= \Bigg[\big(A - q_1^c - sq_2^c - c_1 + K_1 + \beta K_2\big)q_1^c - \frac{1}{2\gamma}\left(I_1^c\right)^2 \\ &\qquad + \frac{\partial V_1(\cdot)}{\partial K_1}\left(I_1^c - \delta K_1\right) + \frac{\partial V_1(\cdot)}{\partial K_2}\left(I_2^c - \delta K_2\right)\Bigg] \\ rV_2(K_1,K_2) &= \Bigg[\big(A - sq_1^c - 2q_2^c - c_2 + K_2 + \beta K_1\big)q_2^c - \frac{1}{2\gamma}\left(I_2^c\right)^2 \\ &\qquad + \frac{\partial V_2(\cdot)}{\partial K_2}\left(I_2^c - \delta K_2\right) + \frac{\partial V_2(\cdot)}{\partial K_1}\left(I_1^c - \delta K_1\right)\Bigg] \end{aligned} \tag{A.2.2}$$

where q_i^c and I_i^c are given by (A.2.1) and the partial derivatives by (2.2.9). Comparing coefficients of the quadratic polynomial on the left and on the right hand side of the two HJB equations for Partial Integration gives the set of 12 equations and 12 unknowns:

$$\begin{aligned} \frac{r}{2}B_1 &= \frac{(s\beta-4)^2}{(s^2-8)^2} + \frac{B_1^2}{2\gamma} + \frac{B_3D_3}{\gamma} - \delta B_1 \\ \frac{r}{2}B_2 &= \frac{(s-4\beta)^2}{(s^2-8)^2} + \frac{B_3^2}{2\gamma} + \frac{B_2D_2}{\gamma} - \delta B_2 \\ rB_3 &= \frac{2(4\beta-s)(4-\beta s)}{(s^2-8)^2} + \frac{B_1B_3 + B_2D_3 + B_3D_2}{\gamma} - 2\delta B_3 \\ rB_4 &= \frac{2(4-\beta s)((4-s)A - 4c_1 + sc_2)}{(s^2-8)^2} + \frac{B_1B_4 + B_3D_5 + B_5D_3}{\gamma} - \delta B_4 \\ rB_5 &= \frac{2(4\beta-s)((4-s)A - 4c_1 + sc_2)}{(s^2-8)^2} + \frac{B_3B_4 + B_2D_5 + B_5D_2}{\gamma} - \delta B_5 \\ rB_6 &= \frac{((4-s)A - 4c_1 + sc_2)^2}{(s^2-8)^2} + \frac{B_4^2}{2\gamma} + \frac{B_5D_5}{\gamma} \end{aligned}$$

$$\begin{aligned} \frac{r}{2}D_1 &= \frac{2(2\beta-s)^2}{(s^2-8)^2} + \frac{D_3^2}{2\gamma} + \frac{B_1D_1}{\gamma} - \delta D_1 \\ \frac{r}{2}D_2 &= \frac{2(2-\beta s)^2}{(s^2-8)^2} + \frac{D_2^2}{2\gamma} + \frac{B_3D_3}{\gamma} - \delta D_2 \\ rD_3 &= \frac{4(2\beta-s)(2-\beta s)}{(s^2-8)^2} + \frac{D_2D_3 + B_1D_3 + B_3D_1}{\gamma} - 2\delta D_3 \\ rD_4 &= \frac{4(2\beta-s)((2-s)A - 2c_2 + sc_1)}{(s^2-8)^2} + \frac{D_3D_5 + B_1D_4 + B_4D_1}{\gamma} - \delta D_4 \\ rD_5 &= \frac{4(2-\beta s)((2-s)A - 2c_2 + sc_1)}{(s^2-8)^2} + \frac{D_2D_5 + B_3D_4 + B_4D_3}{\gamma} - \delta D_5 \\ rD_6 &= \frac{2((2-s)A - 2c_2 + sc_1)^2}{(s^2-8)^2} + \frac{D_5^2}{2\gamma} + \frac{B_4D_4}{\gamma} \end{aligned}$$

A.3 Proof of Proposition 2.3.3

Differentiating the right-hand side of (2.3.16) w.r.t. q_i and I_i and equating to zero leads to the equilibrium strategies

$$\begin{aligned}
q_i^c &= \frac{(2-s)A - 2c_i + sc_j + (2-\beta s)K_i + (2\beta - s)K_j}{4-s^2} \qquad i,j = 1,2,\ i \neq j \\
I_1^C &= \frac{1}{\gamma} B_1 K_1 + B_3 K_j + B_4 \qquad \text{(A.3.1)} \\
I_2^C &= \frac{1}{\gamma} D_2 K_2 + B_3 K_2 + D_5
\end{aligned}$$

Substituting the outputs and investments we get the following HJB

$$\begin{aligned}
rV_i(K_i, K_j) = \Bigg[\big(A - q_i - sq_j - c_i + K_i + \beta K_j\big) q_i^c - \frac{1}{2\gamma} \left(I_i^c\right)^2 \\
+ \frac{\partial V_i(\cdot)}{\partial K_i} \left(I_i^c - \delta K_i\right) + \frac{\partial V_i(\cdot)}{\partial K_j} \left(I_j^c - \delta K_j\right) \Bigg] \qquad \text{(A.3.2)}
\end{aligned}$$

where q_i^c is given by (A.3.1) and the partial derivatives by (2.2.9). Comparing coefficients of the quadratic polynomial on the left and on the right hand side of the two HJB equations for Full Integration gives the set of 12 equations and 12 unknowns:

$$\frac{r}{2}B_1 = \frac{(2-\beta s)^2}{(4-s^2)^2} + \frac{B_1^2}{2\gamma} + \frac{B_3D_3}{\gamma} - \delta B_1$$
$$\frac{r}{2}B_2 = \frac{(2\beta - s)^2}{(4-s^2)^2} + \frac{B_3^2}{2\gamma} + \frac{B_2D_2}{\gamma} - \delta B_2$$
$$rB_3 = \frac{2(2\beta - s)(2-\beta s)}{(4-s^2)^2} + \frac{B_1B_3 + B_2D_3 + B_3D_2}{\gamma} - 2\delta B_3$$
$$rB_4 = \frac{2(2-\beta s)((2-s)A - 2c_1 + sc_2)}{(4-s^2)^2} + \frac{B_1B_4 + B_3D_5 + B_5D_3}{\gamma} - \delta B_4$$
$$rB_5 = \frac{2(2\beta - s)((2-s)A - 2c_1 + sc_2)}{(4-s^2)^2} + \frac{B_3B_4 + B_2D_5 + B_5D_2}{\gamma} - \delta B_5$$
$$rB_6 = \frac{((2-s)A - 2c_1 + sc_2)^2}{(4-s^2)^2} + \frac{B_4^2}{2\gamma} + \frac{B_5D_5}{\gamma}$$

$$\frac{r}{2}D_1 = \frac{(2\beta - s)^2}{(4-s^2)^2} + \frac{D_3^2}{2\gamma} + \frac{B_1D_1}{\gamma} - \delta D_1$$
$$\frac{r}{2}D_2 = \frac{(2-\beta s)^2}{(4-s^2)^2} + \frac{D_2^2}{2\gamma} + \frac{B_3D_3}{\gamma} - \delta D_2$$
$$rD_3 = \frac{2(2\beta - s)(2-\beta s)}{(4-s^2)^2} + \frac{D_2D_3 + B_1D_3 + B_3D_1}{\gamma} - 2\delta D_3$$
$$rD_4 = \frac{2(2\beta - s)((2-s)A - 2c_2 + sc_1)}{(4-s^2)^2} + \frac{D_3D_5 + B_1D_4 + B_4D_1}{\gamma} - \delta D_4$$
$$rD_5 = \frac{2(2-\beta s)((2-s)A - 2c_2 + sc_1)}{(4-s^2)^2} + \frac{D_2D_5 + B_3D_4 + B_4D_3}{\gamma} - \delta D_5$$
$$rD_6 = \frac{((2-s)A - 2c_2 + sc_1)^2}{(4-s^2)^2} + \frac{D_5^2}{2\gamma} + \frac{B_4D_4}{\gamma}$$

Appendix B

Appendix Chapter 3

B.1 Hamilton-Jacobi-Bellman equation (HJB)

For all maximization problems we calculate stationary Markov perfect equilibria and use the HJB equation to find the equilibrium price and investment strategies. To apply the HJB approach, it is necessary to find a bounded and continuously differentiable value function $V_i(G_i, G_j)$ with $i, j = 1, 2; i \neq j$ which satisfies the HJB equations (B.2.1), (B.2.3) and (B.2.5). For all three scenarios we have a linear quadratic game (see Dockner et al. (2000) chapter 7.1 for further details) and, therefore, a quadratic value function

$$V_i = \frac{1}{2}k_{1i}G_i^2 + \frac{1}{2}k_{2i}G_j^2 + k_{3i}G_iG_j + k_{4i}G_i + k_{5i}G_j + k_{6i} \quad i, j = 1, 2; i \neq j, \tag{B.1.1}$$

solves the HJB equation. The unknown parameters k_{il} with $l = 1, .., 6$ and $i, j = 1, 2; i \neq j$ have to be determined for each scenario. Comparing the coefficients of the left-hand side and the right-hand side of the HJB equation leads to a non-linear equation system which is solved numerically.

B.2 Feedback strategies

B.2.1 Two profit maximizing firms

If both firms decide not to use social responsible investment, their respective Hamilton-Jacobi-Bellman equations for their optimization problems are

$$\begin{aligned} rV_i(G_i^{PM}, G_j^{PM}) = \max_{p_i, A_i} \Bigg\{ &[(p_i - c_i)q_i - \frac{\gamma}{2}A_i^2] \\ &+ \frac{\partial V_i(\cdot)}{\partial G_i^{PM}}(A_i - \delta_i^{PM} G_i^{PM}) + \frac{\partial V_i(\cdot)}{\partial G_j^{PM}}(A_j - \delta_j^{PM} G_j^{PM}) \Bigg\}, \quad i = 1,2; i \neq j \end{aligned} \tag{B.2.1}$$

where q_i are the demand functions following from equation (3.2.2). The coefficients k_{il} with $l = 1,..,6$ are calculated numerically. Differentiating the right hand side w.r.t. p_i and A_i and equating to zero leads to symmetric Markov perfect strategies

$$\begin{aligned} A_i &= \frac{1}{\gamma}\frac{\partial V_i(\cdot)}{\partial G_i^{PM}} \\ p_i &= 0.5((1-s)\alpha + d(G_i^{PM} - sG_j^{PM}) + \beta(sG_i^{PM} - G_j^{PM}) + c_i + sp_j), \quad i,j = 1,2; i \neq j. \end{aligned} \tag{B.2.2}$$

The strategies are linear in the state thanks to the linear-quadratic structure. Substituting the prices p_i and investments A_i in equation (B.2.1) results in an equation where just the coefficients k_{il} are unknown. Comparing the coefficients of the left-hand side and the right-hand side of the HJB equation leads to a nonlinear equation system with twelve unknown coefficients (k_{il} with $l = 1,..,6$ and $i,j = 1,2; i \neq j$) and twelve equations which is solved numerically.

B.2.2 A socially concerned firm challenging a non-social competitor

The setting is different if both firms choose different investment strategies

$$\begin{aligned} rV_i(G_i^{SR}, G_j^{PM}) = \max_{p_i, CSR_i} & \Big\{ [(p_i - (c_i + w\ CSR_i)))q_i - \frac{\gamma}{2} CSR_i^2] \\ & + \frac{\partial V_i(\cdot)}{\partial G_i}(CSR_i - \delta_i^{SR} G_i^{SR}) + \frac{\partial V_i(\cdot)}{\partial G_j}(A_j - \delta_j^{PM} G_j^{PM}) \Big\} \\ rV_j(G_i^{SR}, G_j^{PM}) = \max_{p_j, A_j} & \Big\{ [(p_j - c_j)q_j - \frac{\gamma}{2} A_j^2] \\ & + \frac{\partial V_j(\cdot)}{\partial G_j^{PM}}(A_j - \delta_j^{PM} G_j^{PM}) + \frac{\partial V_j(\cdot)}{\partial G_i^{SR}}(CSR_i - \delta_i^{SR} G_j^{SR}) \Big\}, \quad i \neq j. \end{aligned} \tag{B.2.3}$$

Differentiating the right-hand side w.r.t. p_i, A_i respectively CSR_i and equating to zero, the resulting best-reply functions are

$$\begin{aligned} CSR_i &= \frac{1}{\gamma}\left(\frac{\partial V_i}{\partial G_i^{SR}} + \frac{s(\alpha + dG_j^{PM} - \beta G_i^{SR} - p_j) - (\alpha + dG_i^{SR} - \beta G_j^{PM} - p_i)w}{1 - s^2} \right) \\ p_i &= 0.5((1-s)\alpha + d(G_i^{SR} - sG_j^{PM}) + \beta(sG_i^{SR} - G_j^{PM}) + sp_j + c_i + wCSR) \\ A_j &= \frac{1}{\gamma}\frac{\partial V_j}{\partial G_j^{PM}} \\ p_j &= 0.5((1-s)\alpha + d(G_j^{PM} - sG_i^{SR}) + \beta(sG_j^{PM} - G_i^{SR}) + sp_i + c_j), \quad i = 1, 2; i \neq j. \end{aligned} \tag{B.2.4}$$

where G_i^{SR} is the stock of a socially responsible firm and G_j^{PM} of a non-social firm. Solving for the Markovian strategies and substituting in (B.2.3) leads to the HJB equation with the unknown coefficients k_{il}. Similar to the previous case, we calculate the k_{il} numerically.

Due to the interaction of price and investment if firms choose socially responsible strategies we restrict the displaying to the best reply functions instead of the equilibrium solution for the asymmetric case and the following symmetric social case as well. Nevertheless, the strategies are linear in the state because of the linear-quadratic structure of the game.

B.2.3 Two socially concerned firms

If both firms want a social image, their respective HJB equations are

$$\begin{aligned} rV_i(G_i^{SR}, G_j^{SR}) = \max_{p_i, CSR_i} \Big\{ & [(p_i - (c_i + w\, CSR_i))q_i - \frac{\gamma}{2} CSR_i^2] \\ & + \frac{\partial V_i(\cdot)}{\partial G_i^{SR}}(CSR_i - \delta_i^{SR} G_i^{SR}) + \frac{\partial V_i(\cdot)}{\partial G_j^{SR}}(CSR_j - \delta_j^{SR} G_j^{SR}) \Big\}, \quad i = 1,2; i \neq j \end{aligned} \tag{B.2.5}$$

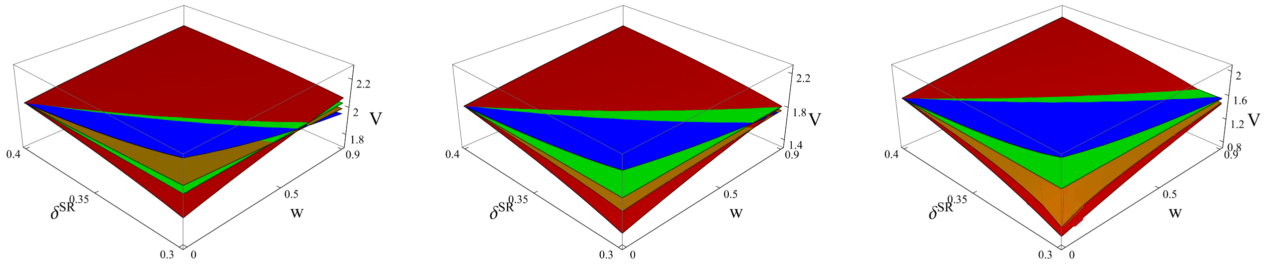
The best reply functions (B.2.6) follow the first order conditions.

$$\begin{aligned} CSR_i &= \frac{1}{\gamma}\left(\frac{\partial V_i}{\partial G_i^{SR}} + \frac{s(\alpha + dG_j^{SR} - \beta G_i^{SR} - p_j) - (\alpha + dG_i^{SR} - \beta G_j^{SR} - p_i)w}{1 - s^2} \right) \\ p_i &= 0.5((1-s)\alpha + d(G_i^{SR} - sG_j^{SR}) + \beta(sG_i^{SR} - G_j^{SR}) + sp_j + c_i + wCSR_i, \\ & \qquad i = 1,2; i \neq j. \end{aligned} \tag{B.2.6}$$

The coefficients k_{il} of the value function (B.1.1) have to be determined numerically by the equation system which follows from comparing the right-hand side and the left-hand side of the HJB equation.

B.3 Graphical illustration of profits and consumer surplus

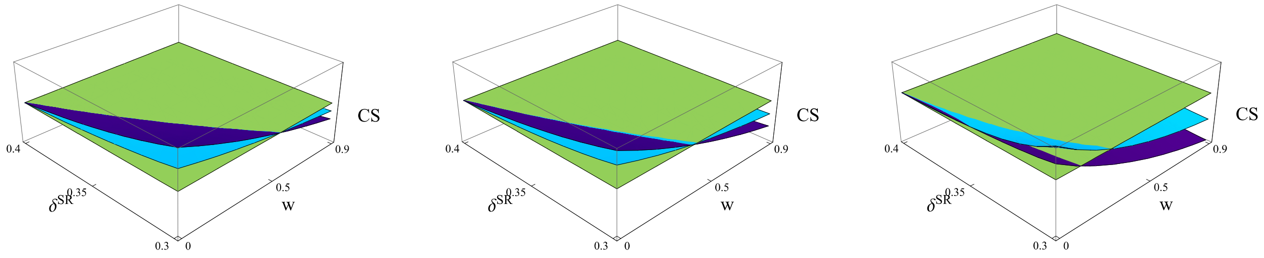
The discounted profits over the time horizon t to ∞ are displayed below.



(a) $\beta = 0.1$ (b) $\beta = 0.5$ (c) $\beta = 0.9$

Figure B.1: Two PMFs (green), asymmetric case with a SRF (blue) and a PMF (red), two SRFs (orange)

Figure B.2 shows the overall consumer surplus for each scenario with different degrees of business stealing.



(a) $\beta = 0.1$ (b) $\beta = 0.5$ (c) $\beta = 0.9$

Figure B.2: CS for two non-social firms (yellow), asymmetric firms (cyan), two socially concerned firms (purple)

Appendix C

Appendix Chapter 4

C.1 Network effect with respect to quality stocks

Because of graphical illustration, the analysis in section 4.3.2 was restricted to the $A-N$-state space. Nevertheless, the Markovian strategies are a 4-dimensional mapping $\phi_i(A, N, K_i, K_j)$ (see equation (4.3.1)) and this section shows how the parameters with respect to the knowledge stocks K_i and K_j adjust if network contributions θ or quality competition β are modified.

Figure C.1 displays how the parameter b_{i3} and b_{i4} in equation (4.3.1) adjust with respect to the quality stocks. Investment incentives are positive with respect to the own stock and negative with respect to the stock of the competitor. Network contributions affect slightly positively the investment with respect to its own stock and negatively with respect to the competitive stock.

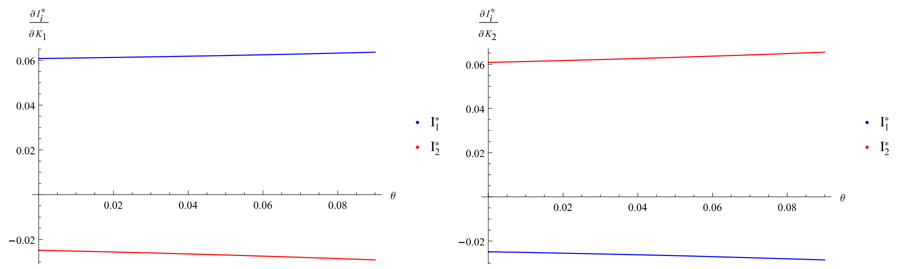


Figure C.1: Investment incentives with respect to quality stocks

Network contributions have almost no effect on the parameters b_{i8} and b_{i8} of the pricing strategies (see equation (4.3.1)), shown in Figure C.2.

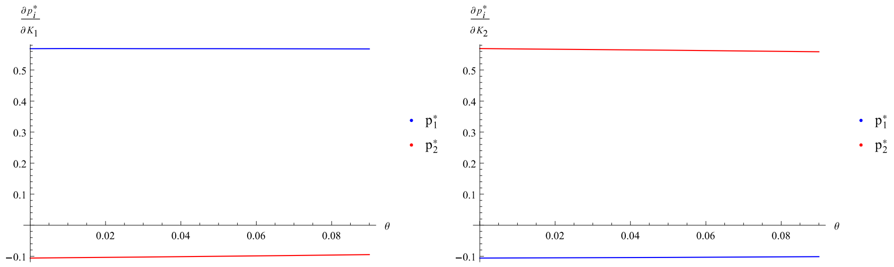


Figure C.2: Pricing incentives with respect to quality stocks

Overall, the network contributions do highly affect the parameters of the quality stocks. The network contributions effect is more obvious in the $A - N-$space which is presented in section 4.3.2.

C.2 Quality competition with respect to quality stocks

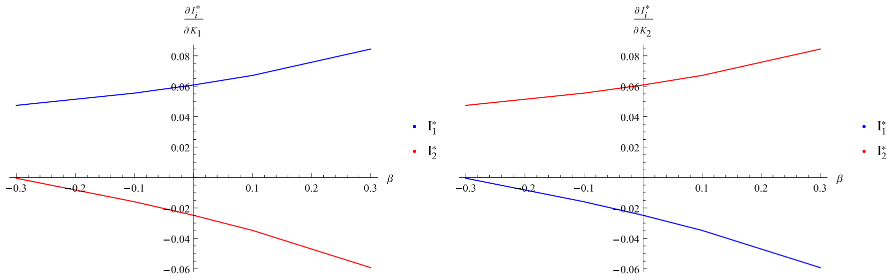


Figure C.3: Investment incentives w.r.t. quality stocks
($\frac{\partial I}{\partial N} = 0$ because of $\theta = 0$)

Similar to the previous section and the network affect, this sections shows how the parameters of the Markovian strategies adjust if quality competition intensifies. A positive externality ($\beta < 0$) lowers my investment incentives whereas quality competition ($\beta > 0$) enhances its investment incentives with respect to its own stock K_i and vice versa with respect to the competitive stock K_j (see Figure C.3). The pricing incentives adjust to variations of quality competition similar to investment incentives (see Figure C.4).

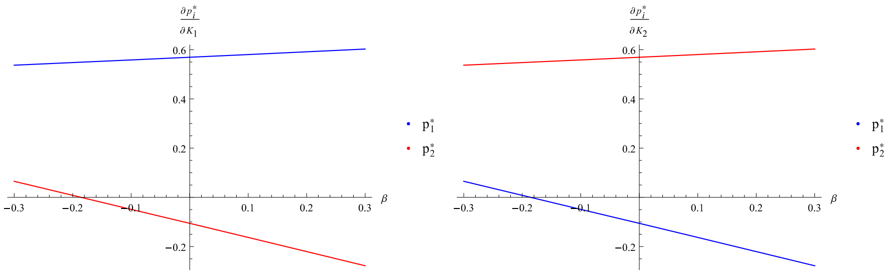


Figure C.4: Pricing incentives w.r.t. quality stocks

Similar to section 4.3.2, intensified quality competition leads to higher investments and lower prices.

C.3 Trajectories for a specific setup

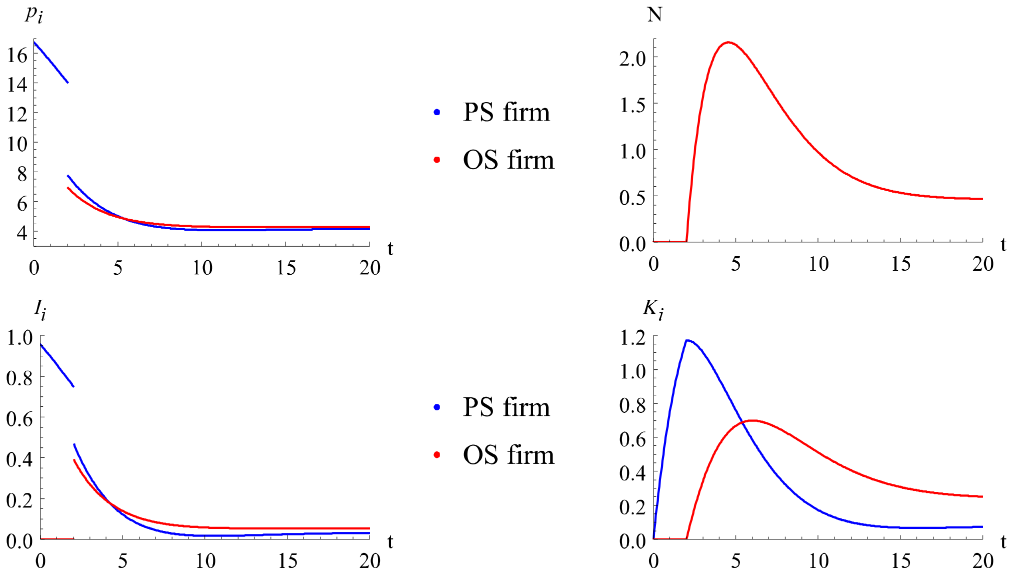


Figure C.5: Trajectories for $\theta = 0.09$ and $\beta = 0$

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