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Abstract

This paper examines a dynamic incumbent-entrant framework with stochastic evolution of the (inverse) demand, in which both the optimal timing of the investments and the capacity choices are explicitly considered. We find that the incumbent invests earlier than the entrant and that entry deterrence is achieved through timing rather than through overinvestment. This is because the incumbent invests earlier and in a smaller amount compared to a scenario without potential entry. If, on the other hand, the capacity size is exogenously given, the investment order changes and the entrant invests before the incumbent does.

Keywords: Incumbent/Entrant, Capacity choice, Investment under Uncertainty, Oligopoly, Real-Option Games

JEL classification: C73, D92, L13

1 Introduction

Starting with the seminal paper by Spence (1977) the choice of production capacity as an instrument for entry deterrence has been extensively studied in the literature. In a standard two-stage set-up, where the incumbent chooses its capacity before the potential competitor decides about entry, entry deterrence is achieved by the incumbent through overinvestment and leads to eternal absence of the competitor from the market. After installing a sufficiently large capacity by the incumbent, the potential entrant finds the market not profitable enough to undertake an investment. In a dynamic setting, where the demand evolves over time (with a positive trend), however, it cannot be expected that potential entrants are perpetually deterred from the market. Hence, the question arises how the investment behavior of the incumbent is affected by the threat of entry in such a setting.

This paper considers a dynamic model where both an incumbent and an entrant have the option to acquire once some (additional) production capacity. Both firms are free to choose the size of their installment, which is assumed to be irreversible and is fully used in the market competition. As a first result, we find that under general conditions the incumbent is most eager to undertake the investment first. In this way the incumbent accomplishes that it delays the investment of the entrant and it extends its monopoly period. The entrant reacts by waiting with investment until demand has become sufficiently large.

A second important result is that entry deterrence is not achieved via overinvestment, but via timing. The threat of entry makes the incumbent invest sooner in order to precede investment of the entrant. Since the incumbent's investment increases the quantity on the market, the output price is reduced, which in turn reduces the profitability of entering this market, and thus delays entry. Furthermore, where large parts of the literature find that a monopolist sets a smaller capacity than a (potential) duopolist facing a threat of entry, we find the opposite result. Since the incumbent invests early, i.e. in a market with a still relatively small demand, it pursues a small capacity expansion. In the absence of an entry threat the monopolist would wait for a market with a higher demand and invest in a larger capacity. In other words, when deterring entry, timing is of greater importance than overinvesting.

A crucial aspect of these results is that the size of the investment is flexible. Considering a variant of our model in which investment sizes are fixed, the incumbent no longer has the possibility to undertake a small investment in a small market in order to preempt the entrant. Interestingly, we find that in such a setting the investment order is reversed; the entrant undertakes an investment first. The reason is that in this situation, where the investment size and thus investment costs are equal, the entrant, which does not suffer from cannibalization, has a larger incentive to invest. Being able to choose the investment size is thus of key importance for making preemption optimal for the incumbent.

In a competitive set-up, the total net welfare as a result of investment is smaller than in a set-up where a social welfare optimizer chooses the investment moment and investment size. Our study implies that policies, aiming to closen the welfare gap between these two settings, include the intention to delay investment. The introduction of, e.g., a license would contribute to such a policy. The incurred lump-sum cost induces firms to delay investment. Resultingly, a larger capacity is installed which, in turn, contributes to an increase in total welfare.

The results sketched above contribute to two main streams of literature, namely to the analysis of entry deterrence strategies and to strategic real option theory. Based on early contributions by Spence (1977) and Dixit (1980), a rich literature has explored the rationale behind entry deterrence in two-stage games under a variety of assumptions about the mode of post-entry competition between firms. Whereas most of this literature is based on deterministic models, Perrakis and Walskett (1983) show that key insights about optimality of deterrence respectively accommodation might change qualitatively if it is assumed that demand is stochastic and uncertain for the firms at the time of investment. In more recent contributions to this stream of literature Maskin (1999) and Swinney *et al.* (2011) highlight that high demand uncertainty makes entry deterrence less attractive and fosters the use of accommodation strategies by incumbents. Robles (2011) develops a two-period game where demand is deterministic and increasing between the two periods. He characterizes conditions under which incumbents build capacities, which are partly idle in the first period, in order to deter other firms from the market. Our main contribution relative to these papers is not only that we address the role of investment timing for potential entry deterrence, but also that we consider a stochastically evolving market environment.

Early dynamic models of entry deterrence, like Spence (1979) or Fudenberg and Tirole (1983), focused on the dynamics of (irreversible) capacity build-up in static market environments, if investment is bounded from above. A key insight in this literature is that, in addition to equilibria which essentially correspond to a Stackelberg equilibrium with the incumbent as leader, there exist Markov-perfect equilibria in which the incumbent can strategically deter the follower from investing, thereby weakening competition. This is due to the initial asymmetry and the dynamic build-up of capacity. More recently Boyer *et al.* (2004) studied entry deterrence in a dynamic setting with price competition and a stochastically evolving willingness to pay consumers. They assume that firms can invest repeatedly, where the size of each investment is fixed, and point out that in such a setting an important effect of investment is the delay of the competitor's investment. It is shown that different types of equilibria might arise in such a setting. In spite of the usual logic associated with preemption under price competition, in some of these equilibria, firms acquire positive rents. Concerning the timing of investment Boyer *et al.* (2004) show that in their setting (under certain conditions) the incentives for preemption are smaller for the incumbent than for the challenger with lower capacity. A similar setting with Cournot competition is studied in Boyer *et al.* (2012). It is shown that competition induces too early first investment relative to the social optimum and that the smaller firm invests first. The market environment considered in Boyer *et al.* (2004, 2012) is closely related to our setup. However, the assumption of fixed investment units crucially distinguishes these studies from our approach, where both timing and investment size are chosen by the firms. We find that the endogeneity of investment size is crucial and leads to qualitatively different insights compared to settings with fixed investment size. Also, due to the consideration of Cournot competition, investment in Boyer *et al.* (2012) has considerably less commitment power compared to the setup we consider. A main focus of Boyer *et al.* (2004, 2012), as well as of recent studies by Besanko and Doraszelski (2004) and Besanko *et al.* (2010) dealing with (partly) reversible capacity investments in oligopolistic markets with stochastically evolving demand, is the long run industry structure that emerges. Considering only one investment option for each firm, our paper does not address this issue, but rather focuses on entry deterrence in the early phase of an industry with evolving demand. Steg (2012) considers a dynamic oligopoly setting with stochastically evolving demand, in which timing is endogeneous and investment size is fully flexible. Analyzing Open-Loop Nash Equilibria of the game he shows that investment is always done by the smallest firm in the market. Our paper obtains that in Markov-Perfect-Equilbira this conclusion of Steg (2012) no longer holds.

The main insight of our analysis that the incumbent invests prior to the entrant can be seen to follow the logic to "eat your own lunch before someone else does" (Deutschman, (1994)). This logic has been, among others, explored in Nault and Vandenbosch (1996) in the framework of a model, where firms endogenously choose the time to launch a new product generation. Apart from the fact that their paper does not explicitly deal with capacity investment, the key difference to our approach is that the type of expansion as such is fixed and the size of the expansion cannot be chosen by the firms.

Our paper extends, in the second place, the literature on strategic real option models, where firms have to decide about investing in a stochastic oligopolistic environment. Early work includes Smets (1991) and Grenadier (1996). Like most of the papers in this field, the investment decision only involves the timing of investment. However, we study a problem where firms are free to choose their capacity levels as well. Within a strategic real options framework, investment decisions involving both capacity choice and timing have first been considered by Huisman and Kort (2015). They study this problem for two symmetric entrants on a new market. Our paper differs from their analysis by considering an incumbent-entrant framework, in which one of the players has an initial capacity.

Consistent with the existing literature on real options games we consider a setting where each firm has a single investment option. This assumption corresponds to a scenario in which investment is lumpy and allows us to derive explicit characterizations of both the timing and the size of investment in equilibrium. Whereas our analysis is already innovative also from a technical perspective, fully characterizing the timing and size of several lumpy investments in such a setting seems infeasible.

This paper is organized in the following way. Section 2 explains the model and discusses its assumptions. Section 3.1 looks at the case of exogenous firms roles, i.e. an individual firm knows beforehand whether it will be the first or second investor. Then the other firm can choose to invest at the same time or later. This is followed by Section 3.2 studying the game when endogenizing investment roles, i.e. both firms are allowed to become the first investor. Section 3.3 then studies the case of a fixed investment size. Section 4 focuses on the size of the incumbent's investment relative to that of the entrant and to an incumbent's investment without entry threat. Robustness checks are performed in Section 5 and Section 6 considers the problem from the point of view of the social planner. The paper is concluded in Section 7. Three appendices provide all proofs as well as numerical robustness checks and analyses of model extensions.

2 The Model

Consider an industry setting with two firms. One firm is actively producing and the other firm is a potential entrant. The first firm is the incumbent and is denoted as firm I. The potential entrant is denoted as firm E. Both firms have a one-off investment opportunity. For firm I this means an expansion of its current capacity and for the entrant an investment means starting up production and entering this market. Both firms are assumed to be rational, risk neutral and value maximizing. The inverse demand function on this market is multiplicative[1] and equals

$$p(t) = x(t)(1 - \eta Q(t)),$$

where $p(t)$ is the output price, $Q(t)$ equals the total aggregate quantity made available at time t and η is a fixed price sensitivity parameter. The exogenous shock process $x(t)$ follows a geometric Brownian motion, i.e.

$$\mathrm{d}x(t) = \alpha x(t)\mathrm{d}t + \sigma x \mathrm{d}z(t).$$

Here α and $\sigma > 0$ are the trend and volatility parameters and $z(t)$ is a Wiener process.[2] Although from an economic perspective the consideration of an expanding market (i.e. $\alpha > 0$) seems most relevant in our framework, formally no assumption about the sign of α is required to carry out our analysis. Discounting takes place under a fixed positive rate $r > \alpha$. The investment costs are linearly related to the investment size, where the marginal cost parameter equals δ. The inverse demand function is chosen to be in line with Huisman and Kort (2015), giving a linear relation between the production size and the output price. This relation is also used by e.g. Pindyck (1988), He and Pindyck (1992), Aguerrevere (2003) and Wu (2007). As of now, the denotation of time t shall be omitted to simplify notation. In this model firms are committed to produce the amount their capacity allows. This assumption is widely used in the literature on capacity constrained oligopolies (e.g. Deneckere *et al.* (1997), Chod and Rudi (2005), Anand and Girotra (2007), Goyal and Netessine (2007) and Huisman and Kort (2015)). For example, Goyal and Netessine (2007) argue that firms may find it difficult to produce below capacity due to fixed costs associated with, for example, labor, commitments to suppliers, and production ramp-up.

The investment comprises two decisions: timing and capacity size. The game is solved backwards, first determining the reaction curve of the firm investing last and then determining the optimal strategy of the firm that invests first. In this way all subgame perfect equilibria are determined.[3]

Initial capacity size

The incumbent is currently active on the market with initial capacity q_{1I}. In principle, the parameter q_{1I} can take any value. However, in parts of the following analysis we will consider scenarios where the size of initial capital is determined according to the optimal investment level of the incumbent under the assumption

[1] In Section 5.2 the robustness of our results will be tested by analyzing a different demand function.

[2] Throughout the paper we will refer to the current value of the process $x(t)$ as X.

[3] Since a rigorous formulation of the Markovian strategy profiles corresponding to the equilibria characterized in this paper induces a heavy notational and technical load without providing additional economic insights, we refrain from presenting them here. Riedel and Steg (2014) provide an approach for a rigorous foundation of preemption-type equilibria in stochastic timing games based on the original ideas of extended mixed strategies by Fudenberg and Tirole (1985). This approach could be applied to formulate strategy profiles underlying our equilibria.

that no future investment will be made by any firm.[4] We refer to such an initial capacity as the myopic investment level q_{1I}^{myop}. Straightforward calculations (see Huisman and Kort (2015)) give

$$q_{1I}^{myop} = \frac{1}{\eta(\beta+1)}.$$

In any case, we analyze equilibria with $q_{1I} \leq q_{1I}^{myop}$, since it is unreasonable to assume any larger initial capacity.

3 Equilibrium Analysis

In this section we characterize the investment behavior in the unique subgame perfect equilibrium of the game described above. Employing the standard terminology in timing games (see, e.g., Fudenberg and Tirole (1985)), the first investor is called the leader and the second investor is called the follower. As a first step in our analysis, the next section derives optimal size and timing of investments of the two firms if investment roles are given, i.e. it is ex-ante determined which of the two firms invests first. We first derive the optimal decisions of the follower. Next, the leader's strategies are studied. Section 3.2 considers the case of endogenous firm roles, where both firms are allowed to invest first. In this part, the results about optimal behavior and the corresponding value functions of the two firms under fixed investment roles are employed to determine which of the firms will be the investment leader. Finally, in the last part of this section we contrast the obtained results with equilibrium behavior in a setting where the size of investment is fixed.

3.1 Exogenous firm roles

Assuming that the sequence of investments is fixed, we denote the following firm as firm F and similarly, the leading firm as firm L. The follower's and leader's initial capacities are denoted by q_{1F} and q_{1L} respectively. Capacity expansion is done by installing additional quantities q_{2F} and q_{2L}. We distinguish between two cases. First, the incumbent takes the role of the leader and the entrant takes the role of the follower, with $q_{1L} = q_{1I}$, $q_{1F} = 0$, $q_{2L} = q_{2I}$ and $q_{2F} = q_E$. Second, the entrant undertakes an investment before the incumbent expands and we have $q_{1L} = 0$, $q_{1F} = q_{1I}$, $q_{2L} = q_E$ and $q_{2F} = q_{2I}$. In this section, both cases are analyzed simultaneously.

Follower's decision

Consider the situation where one firm, the leader, has already invested. Suppose the market has grown sufficiently large for the follower to undertake an investment, i.e. the current value of the process x is

[4] Implicitly we thereby assume that the monopoly investment trigger has been reached in the past inducing the positive investment by the firm.

sufficiently large. One then obtains the following value function reflecting the follower's expected payoff,

$$V_F(X, q_{1L}, q_{1F}, q_{2L}, q_{2F}) = \mathbb{E}\left[\int_{t=0}^{\infty} (q_{1F} + q_{2F}) p(t) e^{-rt} \mathrm{d}t - \delta q_{2F} \;\middle|\; x(0) = X\right]$$
$$= \frac{X}{r - \alpha}(q_{1F} + q_{2F})(1 - \eta(q_{1L} + q_{1F} + q_{2L} + q_{2F})) - \delta q_{2F}.$$

The follower's value function consists of two terms. The expected discounted cash inflow stream resulting from selling goods on the market is reflected by the first term. The involved costs, when making the investment, are captured by the second term. The optimal size of the investment, q_{2F}^{opt}, is found by optimizing the value function.

To determine the optimal moment of investment we derive the investment threshold $X_F^*(q_{1L}, q_{1F}, q_{2L})$. Investment takes place at the moment the stochastic process x reaches this level for the first time (see, e.g., Dixit and Pindyck (1994)). Thereto, one first needs the value function of the follower before it invests. Standard calculations presented in the appendix show that

$$V_F(X, q_{1L}, q_{1F}, q_{2L}) = \frac{\delta}{\beta - 1}\left(\frac{X}{X_F^*}\right)^{\beta} q_{2F}^{opt}(X_F^*, q_{1L}, q_{1F}, q_{2L}) + \frac{X}{r - \alpha} q_{1F}(1 - \eta(q_{1L} + q_{1F} + q_{2L})),$$

where

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}. \tag{1}$$

Due to the assumption that $r > \alpha$ we have $\beta > 1$. The value function F_F consists of two terms. The second term represents the current profit stream. In case the incumbent is follower, this stream is positive with $q_{1F} = q_{1I}$. When the entrant is the follower one has $q_{1F} = 0$ leading to zero current profits. The first term is the current value of the option to invest.

The following proposition characterizes the follower's optimal investment strategy.

Proposition 1 *For small current values X of the stochastic demand process the follower waits until the process $x(t)$ reaches the investment trigger X_F^* to install q_{2F}^* and for $X \geq X_F^*$ the firm invests immediately. As a result, the follower's value function is given by*

$$V_F(X, q_{1L}, q_{1F}, q_{2L}) = \begin{cases} \frac{\delta}{\beta-1}\left(\frac{X}{X_F^*}\right)^{\beta} q_{2F}^* + \frac{X}{r-\alpha} q_{1F}(1 - \eta(q_{1L} + q_{1F} + q_{2L})) & \text{if } X < X_F^*, \\ \frac{X}{r-\alpha}(q_{1F} + q_{2F}^{opt})(1 - \eta(q_{1L} + q_{1F} + q_{2L} + q_{2F}^{opt})) - \delta q_{2F} & \text{if } X \geq X_F^*, \end{cases} \tag{2}$$

where the optimal capacity level for the follower q_{2F}^{opt} and the investment trigger X_F^* are defined by

$$q_{2F}^{opt}(X, q_{1L}, q_{1F}, q_{2L}) = \frac{1}{2\eta}\left(1 - \eta(q_{1L} + 2q_{1F} + q_{2L}) - \frac{\delta(r - \alpha)}{X}\right), \tag{3}$$

$$X_F^*(q_{1L}, q_{1F}, q_{2L}) = \frac{\beta + 1}{\beta - 1} \frac{\delta(r - \alpha)}{1 - \eta(q_{1L} + 2q_{1F} + q_{2L})}. \tag{4}$$

The follower's capacity in case the follower invests at the investment trigger equals

$$q_{2F}^* = q_{2F}^{opt}(X_F^*, q_{1L}, q_{1F}, q_{2L}) = \frac{1 - \eta(q_{1L} + 2q_{1F} + q_{2L})}{\eta(\beta + 1)}. \tag{5}$$

Leader's decision

As can be inferred from equation (4), there is a positive relation between the leader's investment quantity q_{2L} and the follower's investment threshold. The leader can thus delay the follower's investment by setting q_{2L} in such a way that the follower's trigger X_F^* exceeds the current value of x. To that extend, there exists a $\hat{q}_{2L}$ such that for $q_{2L} > \hat{q}_{2L}$ it holds that $X < X_F^*$. From equation (4) one obtains

$$\hat{q}_{2L}(X, q_{1L}, q_{1F}) = \frac{1}{\eta}\left[1 - \eta(q_{1L} + 2q_{1F}) - \frac{\delta(\beta+1)(r-\alpha)}{(\beta-1)X}\right].$$

For $q_{2L} > \hat{q}_{2L}$ the follower invests later, which means that by choosing a sufficiently high investment q_{2L} the leader can delay the investment of the follower. In case the incumbent is the leader, we have that the incumbent is a monopolist as long as $X < X_F^*$, and as soon as x hits X_F^* a duopoly arises, since at that point the entrant undertakes an investment. Hence, this strategy of the incumbent corresponds to entry deterrence. If the leader chooses $q_{2L} \leq \hat{q}_{2L}$ then the follower's investment occurs immediately and the follower chooses a capacity given by (5). In case the incumbent is the leader such behavior corresponds to an entry accommodation strategy. Without specifying whether the leader is the incumbent or the entrant we refer to the leader's choice of $q_{2L} > \hat{q}_{2L}$ as *delaying the follower* and to the opposite case of $q_{2L} \leq \hat{q}_{2L}$ as *inducing immediate follower investment*. In what follows, the implication of both strategies are examined and then the leader's payoffs under these strategies are compared.

Delaying the follower

Straightforward calculations yield that the value function of the leader under this strategy, denoted by $V_L^{det}(X, q_{1L}, q_{1F}, q_{2L})$, is given by[5]

$$\begin{aligned}
V_L^{det} &= \mathbb{E}\left[\int_{t=0}^{t_F}(q_{1L}+q_{2L})x(t)(1-\eta(q_{1L}+q_{1F}+q_{2L}))e^{-rt}\mathrm{d}t\right.\\
&\quad \left. + \int_{t=t_F}^{\infty}(q_{1L}+q_{2L})x(t)(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}^*))e^{-rt}\mathrm{d}t \middle| x(0)=X\right] - \delta q_{2L}\\
&= \frac{X}{r-\alpha}(q_{1L}+q_{2L})(1-\eta(q_{1L}+q_{1F}+q_{2L}) - \eta q_{2F}^*(q_{1L}+q_{2L})\frac{X_F^*}{r-\alpha}\left(\frac{X}{X_F^*}\right)^{\beta} - \delta q_{2L}\\
&= \frac{X}{r-\alpha}(q_{1L}+q_{2L})(1-\eta(q_{1L}+q_{1F}+q_{2L})) - \frac{\delta}{\beta-1}(q_{1L}+q_{2L})\left(\frac{X}{X_F^*}\right)^{\beta} - \delta q_{2L}.
\end{aligned}$$

This value function consists of three parts. The first integral denotes the expected discounted revenue stream obtained by the leader before the follower has invested. Then, at the (stochastic) time $t_F \geq 0$ the follower decides to make an investment, where, $t_F = \inf\{t \geq 0 \mid x(t) \geq X_F^*\}$, and the second integral reflects the leader's expected discounted revenue stream from that moment on. The third term is the investment

[5]Since, as we will show later, in equilibrium the incumbent becomes the leader, we prefer to comply the denotation associated with this strategy with the deterrence strategy and we will hence use *det* to signify the strategy where the leader delays the follower's investment. In slight abuse of notation we will, in what follows, express the leader's value function sometimes as an explicit function of the second investment of the leader, q_{2L}. Whenever no such argument is given the value function under the optimal choice of q_{2L} is considered.

outlay. The expression in the last line can then by interpreted as the expected revenue stream in case the follower will never invest minus the (negative) adjustment of the cash flow stream from the moment the second firm makes an investment, followed by the investment costs. The second term includes the stochastic discount factor $\mathbb{E}\left[e^{-rt_F}\right] = \left(\frac{X}{X_F^*}\right)^\beta$, where again t_F is the time of investment of the follower.

Recalling that $\hat{q}_{2L}(X, q_{1L}, q_{1F})$ denotes the minimal investment size of the leader to make the follower invest later, the optimal investment size under this strategy is given by

$$q_L^{det}(X, q_{1L}, q_{1F}) = \text{argmax}\{V_L^{det}(X, q_{1L}, q_{1F}, q_{2L}) \mid q_{2L} \geq \max\{0, \hat{q}_{2L}\}\}. \tag{6}$$

If X is so small that $q_L^{det}(X, q_{1L}, q_{1F}) = 0$, no investment is made at this point. The corresponding lower bound for X, under which investment is not considered feasible, is given by

$$X_1^{det} = \max\{X \mid q_L^{det}(X, q_{1L}, q_{1F}) = 0\}.$$

Furthermore, considering the lower bound $\hat{q}_{2L}$ needed to make the follower invest later, one can infer that for some values of X the optimum does not lie in the interior of the feasible region, i.e. for some X one has $q_L^{det} = \hat{q}_{2L}$, leading to immediate investment by the follower. Thus, for these scenarios delaying cannot be optimal for the leader. The following proposition, which characterizes the optimal leader strategy while delaying the follower shows that there exists an upper bound X_2^{det} such that $q_L^{det} > \hat{q}_{2L}$ if and only if $X < X_2^{det}$. For $X \geq X_2^{det}$ it would be too costly for the leader to delay the follower's investment, since demand is so large that the incentive for the follower to invest at the same time as the leader is very high.

Proposition 2 *There exist unique values $0 < X_1^{det}(q_{1L}, q_{1F}) < X_2^{det}(q_{1L}, q_{1F})$ such that $q_L^{det}(X, q_{1L}, q_{1F}) > \max\{0, \hat{q}_{2L}(X, q_{1L}, q_{1F})\}$ if and only if $X \in (X_1^{det}, X_2^{det})$. Furthermore, for sufficiently small q_{1L} there exists a pair (q_L^{det*}, X_L^{det}) with $X_L^{det} \in (X_1^{det}, X_2^{det})$ satisfying $q_L^{det*} = q_L^{det}(X_L^{det}, q_{1L}, q_{1F})$ and*

$$X_L^{det} = \frac{\beta}{\beta - 1} \frac{\delta(r - \alpha)}{1 - 2\eta q_{1L} - \eta q_{1F} - \eta q_L^{det*}}, \tag{7}$$

such that under the delaying follower investment strategy,

(i) for $X \geq X_L^{det}$ the leader immediately invests $q_L^{det}(X, q_{1L}, q_{1F})$ and the value function of the leader is given by

$$V_L^{det}(X, q_{1L}, q_{1F}) = \frac{X}{r - \alpha}(q_{1L} + q_L^{det})(1 - \eta(q_{1L} + q_{1F} + q_L^{det})) - \frac{\delta}{\beta - 1}(q_{1L} + q_L^{det}) \left(\frac{X}{X_F^*}\right)^\beta - \delta q_L^{det}; \tag{8}$$

(ii) for $X < X_L^{det}$ the leader invests q_L^{det*} at the moment x reaches the investment threshold value X_L^{det}. The value function before investment is given by

$$F_L^{det}(X, q_{1L}, q_{1F}) = \frac{X}{r - \alpha} q_{1L}(1 - \eta(q_{1L} + q_{1F})) + \left(\frac{X}{X_L^{det}}\right)^\beta \frac{\delta q_L^{det*}}{\beta - 1} - (q_{1L} + q_L^{det*}) \left(\frac{X}{X_F^*}\right)^\beta \frac{\delta}{\beta - 1}. \tag{9}$$

The intuition for the observation, that a threshold X_L^{det}, at which the leader invests, exists only if the initial capacity size of the leader is sufficiently small, is straightforward. In case the initial capacity of the leader is large, it is optimal for the leader to abstain from any further investment, since this also blocks any further investment of the follower[6] and allows the leader to sell the quantity corresponding to its current capacity at a larger price. Although the proof of Proposition 2 assumes that q_{1L} is small, numerical analysis indicates that the range of values of q_{1L}, for which the threshold X_L^{det} exists and the leader therefore eventually invests, is typically of substantial size. Clearly, the assumption of fixed investment roles is crucial for the observation that the leader can block the follower by not investing. With endogenous investment roles delaying own investment does not block investments of the competitor and hence preemption is a crucial issue. This is analyzed in Section 3.2.

Inducing immediate follower investment

If the leader chooses a capacity below $\hat{q}_{2L}(X, q_{1L}, q_{1F})$, it induces immediate investment by the follower but nevertheless acts as Stackelberg capacity leader. The value function contains two terms, the expected discounted revenue stream resulting from investment and the investment cost,[7]

$$\begin{aligned} V_L^{acc}(X, q_{1L}, q_{1F}, q_{2L}) &= \mathbb{E}\left[\int_{t=0}^{\infty} (q_{1L} + q_{2L}) x(t) (1 - \eta(q_{1L} + q_{1F} + q_{2L} + q_{2F}^{opt})) e^{-rt} \mathrm{d}t \middle| x(0) = X \right] - \delta q_{2L} \\ &= \frac{X}{r - \alpha} (q_{1L} + q_{2L})(1 - \eta(q_{1L} + q_{1F} + q_{2L} + q_{2F}^{opt})) - \delta q_{2L}. \end{aligned}$$

The firm chooses its capacity $q_L^{acc}(X, q_{1L}, q_{1F})$ in such a way that it optimizes $V_L^{acc}(X, q_{1L}, q_{1F}, q_{2L})$, given the restriction $q_L^{acc} \leq \hat{q}_{2L}$. The latter makes that this strategy is restricted to a certain region of x. When the shock process attains a relatively large value, the optimal quantity q_L^{acc} meets the restriction $q_L^{acc} \leq \hat{q}_{2L}$. However, for small values of x, the market is too small for two firms to invest at the same time and one observes that $q_L^{acc} > \hat{q}_{2L}$. Therefore, there exists a $X_0^{acc}(q_{1L}, q_{1F})$ such that for $X < X_0^{acc}$ simultaneous investment will not occur, since the optimal investment of the leader is sufficiently large to delay the follower's investment.[8] Furthermore, similarly to when delaying the follower, making the follower investment immediately requires that the optimal investment level q_L^{acc} is strictly positive. We obtain from the first order condition of maximizing V_L^{acc} the investment level

$$q_L^{acc}(X, q_{1L}) = \frac{1}{2\eta} \left[1 - 2\eta q_{1L} - \frac{\delta(r - \alpha)}{X} \right]. \tag{10}$$

The optimal investment size of the leader does not depend on the initial capacity of the follower. In fact it corresponds to the Stackelberg leader capacity level, where it turns out that the Stackelberg leader quantity equals the quantity of the monopolist. The following proposition presents the inducing immediate follower investment strategy.

Proposition 3 *The inducing immediate follower investment strategy is feasible for $X > X_1^{acc}$, where*

$$X_1^{acc} = \max\left\{ \frac{\beta + 3}{\beta - 1} \frac{\delta(r - \alpha)}{1 - 4\eta q_{1F}} \;,\; \frac{\delta(r - \alpha)}{1 - 2\eta q_{1L}} \right\}. \tag{11}$$

---

[6]Note that we are in the exogenous firm roles case where the follower is only allowed to invest after the leader has chosen to do so.

[7]The superscript *acc* refers to the accommodation strategy that arises here, when the investment leader is the incumbent.

[8]Note that investing $q_L^{acc} = \hat{q}_{2L}$ is never optimal: $V_L^{acc}(\hat{q}_{2L}) = V_L^{det}(\hat{q}_{2L}) < V_L^{det}(q_L^{det})$.

Furthermore, for sufficiently small q_{1L} *there exists a pair* (q_L^{acc*}, X_L^{acc}) *with* $X_L^{acc} > \frac{\delta(r-\alpha)}{1-2\eta q_{1L}}$ *satisfying* $q_L^{acc*} = q_L^{acc}(X_L^{acc}, q_{1L})$ *and*

$$X_L^{acc} = \frac{\delta(r-\alpha)\beta}{\beta-1} \frac{q_L^{acc*} - q_{1L}}{(q_L^{acc*} - q_{1L})(1-\eta q_{1L}) - \eta q_L^{acc*}(q_L^{acc*} + q_{1L})}, \tag{12}$$

such that under the inducing immediate follower investment strategy

(i) for $X \geq X_L^{acc}$ *the leader immediately invests* $q_L^{acc}(X, q_{1L}, q_{1F})$ *and the value function of the leader is given by*

$$V_L^{acc}(X, q_{1L}, q_{1F}) = \frac{X}{r-\alpha} \tfrac{1}{2}(q_{1L} + q_L^{acc})(1 - \eta(q_{1L} + q_L^{acc})) - \tfrac{1}{2}\delta(q_L^{acc} - q_{1L}); \tag{13}$$

(ii) for $X < X_L^{acc}$ *the leader invests* q_L^{acc*} *at the moment* x *reaches the investment threshold value* X_L^{acc}. *The value function before investment is given by*

$$F_L^{acc}(X, q_{1L}, q_{1F}) = \frac{X}{r-\alpha} q_{1L}(1 - \eta(q_{1L} + q_{1F})) + \left(\frac{X}{X_L^{acc}}\right)^{\beta} \frac{\delta q_L^{acc*}}{\beta - 1}. \tag{14}$$

Optimal leader strategy

The characterization of the leader's optimal behavior under the delaying follower investment respectively inducing immediate follower investment strategy, allows us to derive the optimal strategy of the leader.

Proposition 4 *There is an interval of positive length* $[X_1^{acc}, X_2^{det}]$ *on which both the delaying follower investment strategy and the inducing immediate follower investment strategy are feasible for the leader. For* $X < X_1^{acc}$ *the leader delays the follower's investment and for* $X > X_2^{det}$ *the firms invest simultaneously. Moreover, there exists an* $\hat{X} \in (X_1^{acc}, X_2^{det})$ *such that the delaying follower investment strategy is always optimal for* $X < \hat{X}$.

Extensive numerical exploration shows that the threshold $\hat{X}$ is unique and therefore separates the parts of the state-space where the delaying follower investment strategy respectively inducing immediate follower investment strategy is optimal, i.e. there exists a $\hat{X} \in (X_1^{acc}, X_2^{det})$ such that for $X < \hat{X}$ it is optimal for the leader to delay the follower's investment, whereas inducing immediate investment is optimal for $X \geq \hat{X}$. Furthermore, we find that $\hat{X} > \max\{X_L^{det}, X_L^{acc}\}$, which implies that the leader optimally waits in the region $0 \leq X < X_L^{det}$ and invests $q_L^{det}(X, q_{1L}, q_{1F})$ in the region $X_L^{det} \leq X < \hat{X}$, thereby delaying investment by the follower. For $X \geq \hat{X}$ it is optimal for the leader to immediately invest $q_L^{acc}(X, q_{1L}, q_{1F})$, which triggers an immediate investment of the follower. Figure 1 illustrates these findings.[9] The value function of the leader is therefore given by

$$V_L(X, q_{1L}, q_{1F}) = \begin{cases} F_L^{det}(X, q_{1L}, q_{1F}) & \text{if } X \in (0, X_L^{det}), \\ V_L^{det}(X, q_{1L}, q_{1F}) & \text{if } X \in [X_L^{det}, \hat{X}), \\ V_L^{acc}(X, q_{1L}, q_{1F}) & \text{if } X \in [\hat{X}, \infty). \end{cases} \tag{15}$$

[9] All examples in this paper use the following parametrization: $\alpha = 0.02,\ r = 0.1,\ \sigma = 0.1,\ \eta = 0.1,\ \delta = 1000,\ q_{1I} = \frac{1}{\eta(\beta+1)}$.

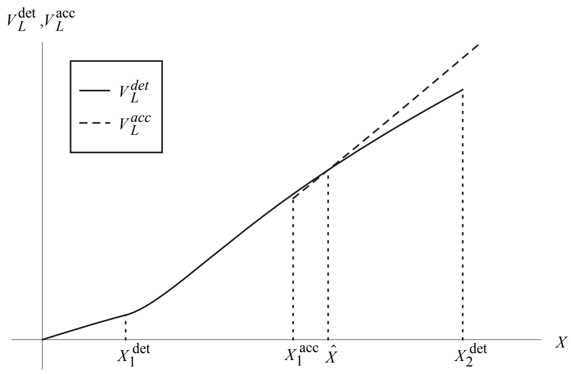
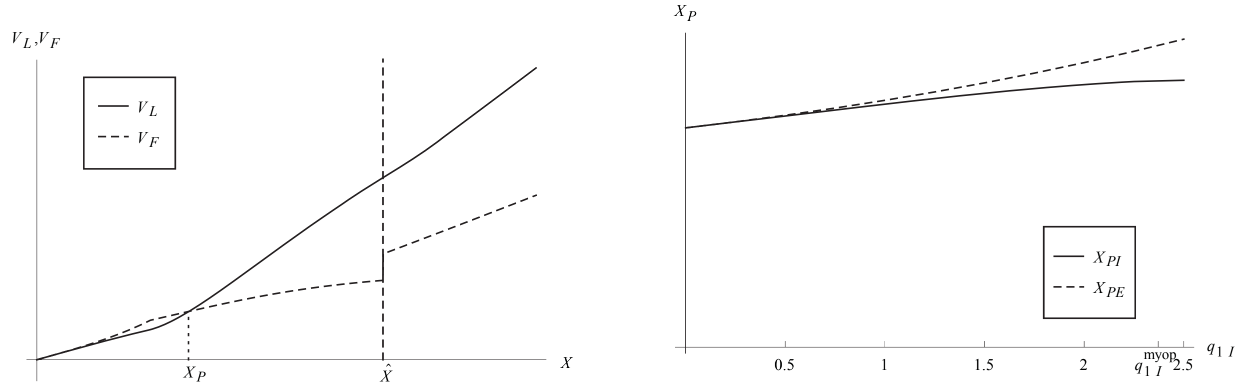


Figure 1: The leader's value functions while delaying the follower (solid) and while inducing immediate follower investment (dashed).

Assuming $x(0)$ to be sufficiently small, our analysis implies that for exogenous firm roles the leader waits until $x(t)$ reaches X_L^{det} and invests. Then the follower waits until $x(t)$ reaches X_F^*, at which point in time the follower invests.

3.2 Endogenous firm roles

Based on the results of the previous section we can now examine the equilibrium behavior if the investment order is not fixed ex-ante and both firms are allowed to invest first. To characterize the firms' optimal behavior we need to consider the value functions of a firm if it acts as leader respectively follower. Figure 2a shows the two value functions for a firm depending on the current value X of the state variable. The solid curve corresponds to the outcome if the considered firm takes the leader role, where the payoff of immediate investment is depicted. If the firm takes the position of the follower, one arrives at the dashed curve, corresponding to (2). For small values of X investment is not profitable. Then no firm wants to invest first, which is why the follower curve lies above the leader curve. For larger values, though, each firm wants to be the first investor. The curves are qualitatively similar for the incumbent and the entrant firm. This means that when X is large enough both firms prefer to become the leader. To prevent that the competitor undertakes an investment first, thereby making the firm end up with the follower value instead of the higher leader value, it is best to preempt the other firm by making an investment just a bit earlier. As this strategy is optimal to both firms, investment is made as early as possible, provided the leader's payoff exceeds the follower's payoff. Hence, assuming that the initial value of the process $x(t)$ is smaller than X_P, as given in Figure 2a, the first moment for a firm to invest, that is, when investment as a leader becomes worth-while, is at the lowest value of X for which the leader curve no longer yields a smaller value than the follower curve. This point is called the preemption point X_P. Formally, the preemption points of the incumbent and



(a) Value functions that correspond to becoming leader (solid) and follower (dashed). The follower is delayed for $X < \hat{X}$ and invests immediately for $X \geq \hat{X}$.

(b) Preemption points X_{PI} (solid) and X_{PE} (dashed) with free capacity choice for different values of q_{1I}.

Figure 2: Preemption points.

entrant are defined in the following way,

$$X_{PI} = \min\{X > 0 \mid V_L^{det}(X, q_{1I}, 0) = V_F(X, 0, q_{1I}, q_L^{det}(X, 0, q_{1I}))\},$$
$$X_{PE} = \min\{X > 0 \mid V_L^{det}(X, 0, q_{1I}) = V_F(X, q_{1I}, 0, q_L^{det}(X, q_{1I}, 0))\}.$$

Since $q_{1I} > 0$, the two firms are asymmetric and therefore their preemption points do not coincide. Clearly, for a firm, of which its preemption point is below that of the competitor, it can never be an equilibrium strategy to choose an investment trigger above the competitor's preemption point. If the firm would choose such a large trigger the opponent's best response would imply that the firm ends up as follower, and therefore with a smaller value compared to what it can gain as leader (see Figure 2a). If, furthermore, the optimal trigger X_L^{det} under the delaying follower investment strategy of that firm is larger than the opponent's preemption point, then the firm has no incentives to invest before the opponent's preemption point is reached. In such a situation it constitutes equilibrium behavior for the firm with the lower preemption point to set its investment trigger to the opponent's preemption point and to invest an amount which delays the opponent's investment. Following its optimal strategy the opponent chooses the follower's investment trigger and invests once this trigger is reached. Such an equilibrium is referred to as a preemption equilibrium and the following proposition shows that at least for appropriate initial capacity of the incumbent no other types of subgame-perfect equilibria exist in the considered game.

Proposition 5 *Assume that the initial capacity of the incumbent is sufficiently close to* q_{1I}^{myop}*. Then, preemptive investment constitutes a unique subgame perfect Nash equilibrium.*

In order to clarify which firm acts as leader in the preemption equilibrium, we depict in Figure 2b the preemption points of the incumbent and the entrant for values of q_{1I} in the entire relevant range $[0, q_{1I}^{myop}]$.

It can be clearly seen that the preemption point of the incumbent is below that of the entrant. Furthermore, it is easy to check that the leader's investment trigger under the delaying follower investment strategy, if it is finite, is generically much larger than the entrant's preemption point (see also Lemma 1 in Appendix A).[10] Together, these two observations establish that the incumbent acts as leader in the preemption equilibrium. Hence, for a sufficiently small value of $x(0)$ it is optimal for the incumbent to wait initially and to invest the amount q_L^{det} just as the process x reaches the preemption point of the entrant. The investment is chosen in a way to delay the investment of the entrant and therefore it is an instrument of entry deterrence. Using this strategy the incumbent can delay the entry of its opponent till the trigger $X_F^*(q_{1I}, 0, q_{2I})$ is reached by $x(t)$.

Example 1 *Considering, as an illustrative example, the case* $q_{1I} = q_{1I}^{myop}$*, which under our default parametrization yields* $q_{1I} = 2.37$*, we obtain* $X_{PI} = 134$ *and* $X_{PE} = 167$*. This means that for* $X < 134$ *both firms prefer to wait, for* $134 \leq X < 167$ *the incumbent prefers to be leader and the entrant prefers to wait and for* $X \geq 167$ *both want to invest. The investment trigger* X_{LI}^{det} *is not finite in this situation since the incumbent would not undertake an investment in the case of exogenous firm roles (see Lemma 1 in the Appendix). Hence, the only reason the incumbent invests at* $X = X_{PE} = 167$ *is strategic. Due to its investment the entrant's investment trigger is set to* $X_F^* = 208$ *and till* $x(t)$ *reaches this level the incumbent stays a monopolist.*

To understand this result one must realize that any investment reduces the output price, since this price is negatively related with the total market output. Investment by the entrant thus reduces the incumbent's value. It is then better for the incumbent to cannibalize than let the entrant reduce the price. To do so, the incumbent installs a small capacity level: small in order not to make the cannibalization effect too large, but large enough to delay investment of the entrant. To conclude, the incumbent installs a small additional capacity with the aim to protect its demand, and to prolong the period where it can profit from its monopoly position. The entrant will invest later when demand is higher so it can set a larger quantity on the market. This leads to the result that the incumbent invests first and expands to delay a large investment by the entrant. The latter waits until the state variable hits the follower's investment threshold.

3.3 Fixed Capacity

In order to highlight the importance of the endogenous choice of investment size for our main finding that the incumbent invests prior to the entrant, in this section, we consider a scenario where the size of investment is fixed. Apart from improving our understanding of the role of endogenous investment size, the main motivation for considering a scenario with fixed investment is that for industries where expansion has to be typically carried out in fixed units, for example the establishment of an additional laboratory in the pharmaceutical industry, the assumption of a fixed investment size seems more appropriate than that of

[10] As elaborated in Section 5.1 these two inequalities do not depend on the particular parametrization of the model chosen here but stay intact over a large range of relevant parameter settings.

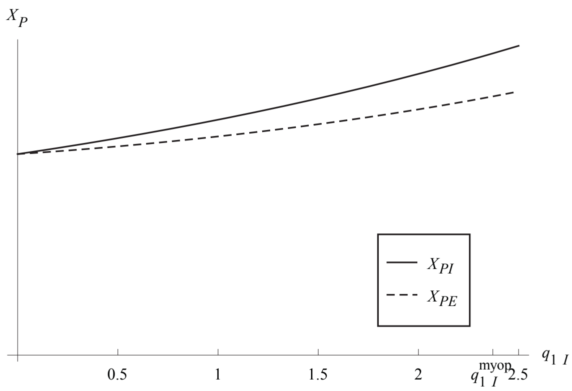


Figure 3: Preemption triggers X_{PI} (solid) and X_{PE} (dashed) with fixed capacity for different values of q_{1I} with $K = 2.5$.

complete flexibility in the size of investment. This section shows that whether investment size is exogenous or endogenous is indeed crucial for the emerging investment order.

Consider the model presented above, but assume investment size is fixed such that $q_{2I} = q_E = K$. The incumbent's value functions as leader (under the deterrence strategy) and follower are then similar to what was found previously,

$$\begin{aligned}
V_{LI}^{det}(X, q_{1I}, 0, K) &= \frac{X}{r-\alpha}(q_{1I} + K)(1 - \eta(q_{1I} + K)) - \frac{X_{FE}^*}{r-\alpha}\eta K(q_{1I} + K)\left(\frac{X}{X_{FE}^*}\right)^\beta - \delta K, \\
V_{FI}(X, q_{1I}, 0, K) &= \frac{X}{r-\alpha}(q_{1I} + K)(1 - \eta(q_{1I} + 2K)) - \delta K, \\
F_{FI}(X, q_{1I}, 0, K) &= A_{FI}^{det} X^\beta + \frac{X}{r-\alpha} q_{1I}(1 - \eta(q_{1I} + K)),
\end{aligned}$$

where $X_{FE}^* = \frac{\beta}{\beta-1}\frac{\delta(r-\alpha)}{1-\eta(q_{1I}+2K)}$ and $X_{FI}^*(q_{1I}, 0, K) = \frac{\beta}{\beta-1}\frac{\delta(r-\alpha)}{1-2\eta(q_{1I}+K)}$ are the investment triggers of the entrant and the incumbent as follower, and $A_{FI}^{det}(q_{1I}, 0, K) = \frac{\delta K}{\beta-1}\left(X_{FI}^*\right)^{-\beta}$. In a similar way one can determine the value functions of the entrant.

Next, one can calculate the preemption points. In Section 3.2 it was shown that under endogenous choice of the investment size the incumbent invests first, where it expands by an adequate amount such that the entrant's investment is temporarily hold off. Figure 3 shows the preemption points for the model presented in this section, i.e. where investment size is fixed. The relative position of the curves has changed compared to Figure 2b, which depicts the case with endogenous investment size: the entrant's curve now lies below the incumbent's curve, signifying that in this model the entrant precedes the incumbent in undertaking an investment. Thus, the entrant takes the leader role and the incumbent becomes follower.

If firms are free to choose the size of their installment, the incumbent has the largest incentive to invest first, for it can undertake a small investment in order to delay a large investment by the entrant. When fixing capacity for both firms at an equal level, this no longer applies: since capacity size is fixed, the incumbent cannot make a small investment to delay a large investment by the follower. Then the incentive to invest is higher for the entrant, since it does not suffer from cannibalization. As a result, the incumbent is more eager

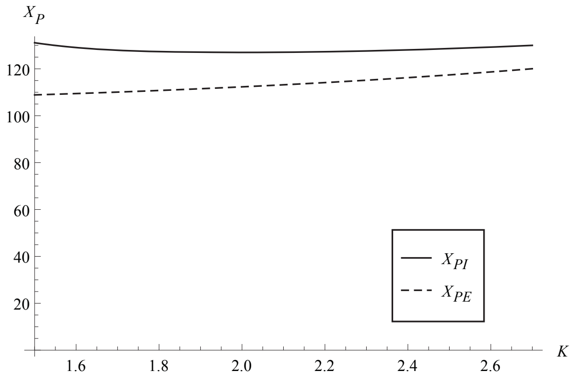


Figure 4: Preemption triggers X_{PI} (solid) and X_{PE} (dashed) with fixed capacity for different values of K with $q_{1I} = q_{1I}^{myop}$.

to delay its own investment and the entrant is the investment leader. Figure 4 shows that the observation that the entrant invests first when capacity size is fixed, is robust with respect to changes in the size of investment K.

4 Overinvestment and Market Leadership

4.1 Overinvestment

In the literature on entry deterrence incumbents mainly deter entrants by means of overinvestment (e.g., Spence (1979) and Dixit (1980)). That is, by building large capacities on the market, it becomes unprofitable for other firms to enter this market. Static entry deterrence models suggest that, apart from cases where markets are blocked (e.g. due to high entry costs), the quantity put on the market under an entry threat exceeds the amount that would be optimal for the firm in case that there is no potential entrant. This section investigates whether this notion of overinvestment also applies in the dynamic stochastic market framework presented in Section 3.

Overinvestment is defined as the difference between the quantity an incumbent sets on the market when there exists a threat of entry and the quantity it would set when this threat would not be present. In other words, the incumbent's expansion in the duopoly setting as presented in the previous section is compared to the incumbent's expansion in case it is a monopolist forever. To this end, the monopolist's model is presented and analyzed.

The value function of the monopolist, at the moment of investment, is given by

$$\begin{aligned} V_M(X, q_1) &= \mathbb{E}\left[\int_0^{\infty} (q_1 + q_2)X(t)(1 - \eta(q_1 + q_2))e^{-rt}\mathrm{d}t \;\middle|\; x(0) = X\right] - \delta q_2 \\ &= \frac{X}{r - \alpha}(q_1 + q_2)(1 - \eta(q_1 + q_2)) - \delta q_2, \end{aligned}$$

	With Entry Threat		Without Entry Threat	
	Trigger	Expansion	Trigger	Expansion
q_{1I}	X_{PE}	q_L^{det*}	X_L^{mon}	q_2^{mon*}
0	123	1.59	152	2.37
0.5	129	1.26	169	2.13
1	136	0.97	190	1.90
1.5	145	0.71	217	1.66
2	157	0.48	254	1.42
2.5	171	0.29	304	1.19

Table 1: Expansions made by the incumbent with and without entry threat for different values of q_{1I}.

in which q_1 is the initial capacity and q_2 corresponds to the capacity acquired by investment. Maximizing the monopolist's value function leads to the optimal capacity expansion size,

$$q_2^{mon}(X, q_1) = \max\left\{\frac{1}{2\eta}\left(1 - 2\eta q_1 - \frac{\delta(r-\alpha)}{X}\right), 0\right\}.$$

Hence, one obtains,

$$V_M(X, q_1) = \begin{cases} \frac{X}{r-\alpha} q_1(1-\eta q_1) + \left(\frac{X}{X_M^*}\right)^\beta \frac{\delta}{\beta-1} q_2^{mon*} & \text{if } X < X_M^*, \\ \frac{(X(1-2\eta q_1) - \delta(r-\alpha))^2}{4\eta(r-\alpha)X} + \frac{X q_1(1-\eta q_1)}{r-\alpha} & \text{if } X \geq X_M^*, \end{cases}$$

where β is defined as in (1). The optimal moment of expansion is defined as the value of x for which the option to wait no longer yields a larger value than immediate investment. Standard analysis shows that for the expansion the optimal threshold and size equal

$$\begin{aligned} X_M^* &= \frac{\beta+1}{\beta-1}\frac{\delta(r-\alpha)}{1-2\eta q_1}, \\ q_2^{mon*} &= \frac{1-2\eta q_1}{\eta(\beta+1)}. \end{aligned}$$

To measure overinvestment, the difference between q_L^{det*} and q_2^{mon*} needs to be considered. Table 1 illustrates this difference for our standard parameter setting. In this table, the optimal investment moment and the optimal investment size are given for different values of the initial investment size, both with (first pair of columns) and without (second pair of columns) threat of an entrant. Overinvestment would occur if $q_L^{det*} > q_2^{mon*}$. However, the table illustrates the opposite. To explain this, one must realize that the investment threshold values of the monopolist are higher than the ones of the incumbent in a duopoly setting. The incumbent, by all force, prefers to keep its monopoly position as long as possible and thereto it delays investment of the entrant by preempting the entrant's preferred investment moment. This leads to an investment in a market that is still small at the moment of investment. For this reason the capacity investment of the firm is small as well. The monopolist, however, has the flexibility to wait for a price that

	Incumbent (Leader)		Entrant (Follower)	
q_{1I}	Trigger	Total Capac.	Trigger	Capacity
0	123	1.59	180	1.99
0.5	129	1.76	185	1.95
1	136	1.97	190	1.90
1.5	145	2.21	195	1.85
2	157	2.48	202	1.78
2.5	171	2.79	211	1.71

Table 2: Total capacities of the incumbent and the entrant for different values of q_{1I}.

has grown to a considerable level before investing. We conclude that, under consideration of endogenous timing as well as endogenous investment size, entry deterrence is not so much about the size but more about the timing of the investment.

4.2 Market leadership

When studying industry evolution and entry deterrence, a crucial issue is the question under which circumstances early incumbents in an industry are able to maintain their market leadership as the market grows. This section illustrates that in our considered setting the incumbent does not necessarily maintain its market leader position after the entrant's investment. Intuitively, a larger initial capacity level has two contradictory effects on the expansions. First, a larger initial capacity, makes the expansion size decrease, for the cannibalization effect is larger for the firm already owning a larger capital stock. Second, since investment is delayed, a larger market is observed at the moment of investment, which gives an incentive to increase investment size. The former effect, however, is dominant and one observes that a larger initial capacity makes the size of the expansion decrease, see e.g. Table 1. The incumbent's total capacity, however, increases when the initial market's output size is larger.

Table 2 shows the final capacity level of the incumbent firm (third column) and the entrant (fifth column). As one can observe, for a small initial capacity level the entrant becomes the market leader. However, when the incumbent starts with a sufficiently large capacity level, it keeps its position as market leader after the second firm's entry. Market leadership thus depends on the initial market size.

In a framework with two potential entrants, i.e. no firm prossesses an initial capacity, Huisman and Kort (2015) point out that market leadership is dependent on uncertainty. In particular, they show that for large demand uncertainty the first investor becomes market leader, while the second investor will invest in a larger capacity when the demand uncertainty is low. Combining this with our findings, implies that market leadership depends on both initial capacity and demand uncertainty. Denote by q_{1I}^{ML} the value of the initial capacity for which the total incumbent's capacity equals the amount set by the entrant. As

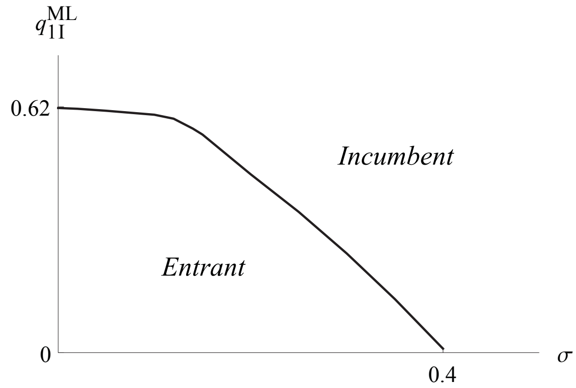


Figure 5: Market leader regions for different σ.

illustrated in Figure 5, q_{1I}^{ML} decreases when uncertainty increases. Larger uncertainty makes the incumbent delay investment, which results in a larger expansion investment, making it market leader for smaller values of q_{1I} relative to the case of smaller uncertainty. In this figure one can clearly observe for which combinations of the initial capacity level and the uncertainty level the incumbent is market leader and in which region the entrant becomes market leader. In a very uncertain economic environment, the incumbent always becomes market leader. However, for small uncertainty levels a certain range $q_{1I} \in [0, q_{1I}^{ML})$ exists for which the entrant ends up with the largest market share.

5 Robustness

In this section two types of robustness checks are performed in order to verify the validity of our results. First, the effects of changes of parameter values is studied. It is shown that the investment order remains the same for a large range of parameter values. Second, we impose a different demand structure and show that also this does not change our qualitative conclusions.

5.1 Parameter variations

In order to inspect the effect of changes in parameter values on the investment order, the difference between the two preemption points as well as the difference between the incumbent's investment threshold and the entrant's preemption trigger is shown for a variation of all parameters in Appendix B. This makes clear that the insight that the incumbent invests first to delay the entrant's investment is very robust. There is a single exception, which occurs when the sensitivity of the market clearing price with respect to the supplied quantity (η) is very small or when there is an almost negligible size of the incumbent's initial capacity. In such a setting the entrant's preemption trigger might fall below the one of the incumbent. The trade-off between the initial capacity and the sensitivity parameter is depicted in Figure 6. This figure shows the two regions where either of the firms invest first. The curve in between depicts all values of η and q_{1L} for which

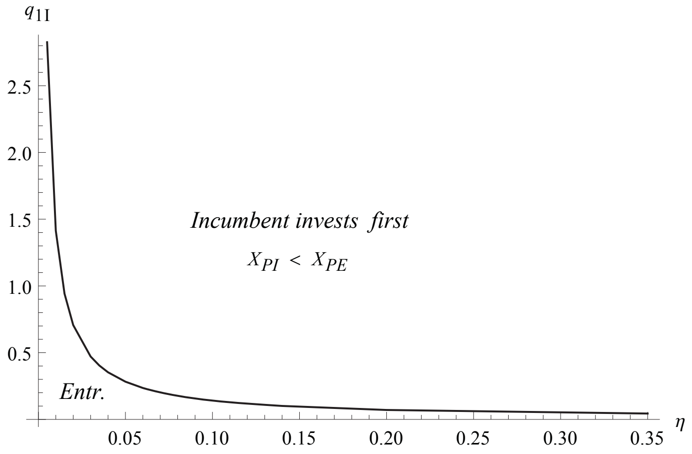


Figure 6: Regions where the incumbent invests first (upper right) and where the entrant invests first.

both firms' preemption triggers are identical. We see that the incumbent invests first, except for a small region close to both axes where the entrant is the first investor. In fact, it holds that for $\eta \cdot q_{1L} > 0.01413$ the incumbent is leader and the entrant invests first for $\eta \cdot q_{1L} < 0.01413$. Intuition behind this result is that for the situation where η and q_{1L} are small the cannibalization effect is small. The incentives to preempt the entrant vanish the moment there is almost nothing to protect.

5.2 Additive demand structure

One characteristic of the multiplicative demand function, as chosen in this paper, is that the market size is bounded. In particular, price is only positive when market quantity is lower than $\frac{1}{\eta}$. In order to check that this property of the inverse demand is not crucial for our results, we carry out the same analysis as in

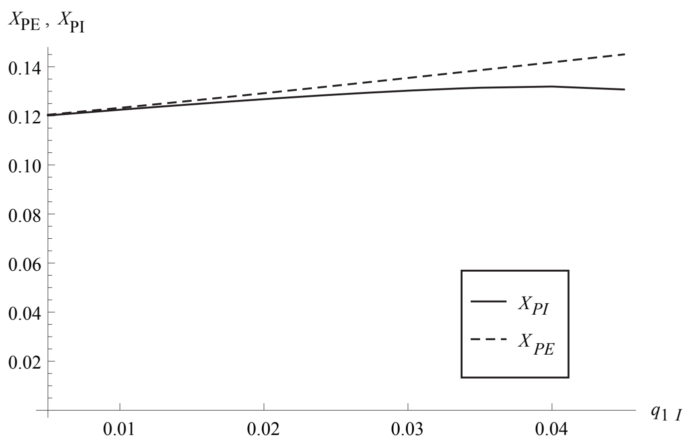
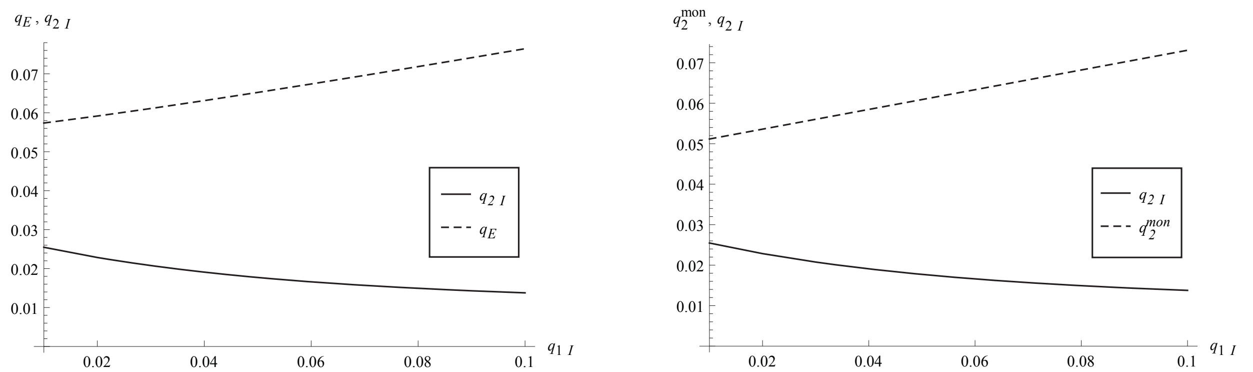


Figure 7: Preemption triggers X_{PI} (solid) and X_{PE} (dashed) under additive demand.



(a) Investments of the incumbent (solid) and the entrant (dashed).

(b) Investments of the incumbent (solid) and the monopolist (dashed).

Figure 8: Investment size for different values of q_{1I} under additive demand.

Section 3 for an additive demand structure:

$$p(t) = x(t) - \eta Q(t).$$

Details of the analysis are provided in Appendix C. Similar to the findings before, it is found that the incumbent preempts the entrant and acquires a smaller capacity. For different values of the initial capacity level, Figure 7 shows the resulting preemption triggers.[11] The initial capacity level under myopic investment equals $q_{1I}^{myop} = 0.0487$. In Appendix C also the sensitivity of the results with respect to parameter changes is tested. It is confirmed that for large sets of parameter values the incumbent always undertakes an investment first. Figure 8 shows the installed capacities of the incumbent, entrant and monopolist, illustrating that also the results about underinvestment in the presence of an entry-threat, discussed above, remain valid. We conclude that the results under additive demand are qualitatively the same as those under multiplicative demand.

6 Welfare analysis

To conclude our analysis we examine how the behavior emerging in equilibrium compares to the outcome chosen by a social planner interested in maximizing welfare. For the sake of comparability with the equilibrium analysis we restrict the number of investments of the social planner to the total number of investments by both firms in the scenario with competition. Therefore, we consider a social planner that has the option to invest twice in an existing market. It optimizes total welfare, being the sum of the total expected consumer surplus and expected producer surplus. It is easy to see that in our setting from the perspective of

[11] The parameter values are chosen differently in this example, for a different inverse demand function requires a different parametrization. Analogous to Boonman (2014) we set $\alpha = 0.01,\ r = 0.1,\ \sigma = 0.05,\ \eta = 0.5$, and $\delta = 1$.

consumer surplus, total industry profits and welfare, it is irrelevant whether these investments are made by the same or two different firms. The analysis is analogous to Section 3.1: first the final investment moment is determined, together with the optimal investment size. Next the investment moment of the first investment is determined, along with the corresponding capacity size. Concerning the final investment, the output price after investment equals $p(t) = x(t)(1 - \eta(q_0 + q_1 + q_2))$. The total expected consumer surplus after the final investment then equals

$$CS_2(X, q_0, q_1, q_2) = \mathbb{E}\left[\int_{t=0}^{\infty}\int_{P=p(t)}^{X} D(P)\ \mathrm{d}P\ e^{-rt}\ \mathrm{d}t \ \middle|\ x(0) = X\right] = \frac{X\eta(q_0 + q_1 + q_2)^2}{2(r - \alpha)},$$

where $D(P) = \frac{1}{\eta}\left(1 - \frac{P}{X}\right)$. The expected producer surplus equals the expected discounted net revenue stream minus the investment outlay,

$$PS_2(X, q_1, q_2, q_3) = \frac{X}{r - \alpha}(q_0 + q_1 + q_2)(1 - \eta(q_0 + q_1 + q_2)) - \delta q_2 - \frac{X}{r - \alpha}(q_0 + q_1)(1 - \eta(q_0 + q_1)).$$

Expected welfare after the second investment is then obtained by adding the two together, leading to

$$W_2(X, q_0, q_1, q_2) = \frac{X}{r - \alpha} q_2(1 - \eta(q_0 + q_1 + \tfrac{1}{2}q_2)) - \delta q_2.$$

We find that, for an equal level of q_1 cq. q_{2I}, the investment moment of the final investment equals the investment moment of the follower, but the resulting investment size is twice as large for the case of the social planner,

$$\begin{aligned} X_2^W &= \frac{\beta + 1}{\beta - 1}\frac{\delta(r - \alpha)}{1 - \eta(q_0 + q_1)} \\ q_2^W &= 2\frac{1 - \eta(q_0 + q_1)}{\eta(\beta + 1)}. \end{aligned}$$

In a similar way, one can determine the optimal moment of the first investment,

$$X_1^W = \frac{\beta}{\beta - 1}\frac{\delta(r - \alpha)}{1 - \eta q_0 - \frac{1}{2}\eta q_1^W},$$

where the optimal investment size of the first investment q_1^W is implicitly determined by solving the following equation for q_1,

$$1 - \frac{\beta\eta q_1}{2(1 - \eta(q_0 + q_1))} - 2\left(\frac{\beta}{\beta + 1}\frac{1 - \eta(q_0 + q_1)}{1 - \eta q_0 - \frac{1}{2}\eta q_1}\right)^{\beta} = 0.$$

The maximal expected welfare before any investment is given by

$$W^W(X, q_0) = \frac{X}{r - \alpha} q_0(1 - \tfrac{1}{2}\eta q_0) + \left(\frac{X}{X_1^W}\right)^{\beta}\frac{\delta}{\beta - 1}q_1^W + \left(\frac{X}{X_2^W}\right)^{\beta}\frac{\delta}{\beta - 1}q_2^W.$$

and can be divided into two components. The first part, consisting of one term, reflects the accumulated discounted welfare stream resulting from the initial capacity level. The second part consists of two terms, reflecting the value of the investment options. Let us denote the first term as $W_0^W(X, q_0)$ and the sum of the final two terms $W_{opt}^W(X, q_0)$. Then we can rewrite $W^W(X, q_0)$ as $W_0^W(X, q_0) + W_{opt}^W(X, q_0)$.

	Duopoly						Social Planner							
q_0	X_P	q_L	X_F	q_F	Q^{duop}	W^{duop}_{opt}	X^W_1	q^W_1	X^W_2	q^W_2	Q^W	W^W_{opt}	$\frac{W^{duop}_{opt}}{W^W_{opt}}$	$\frac{Q^{duop}}{Q^W}$
0	123	1.59	180	1.99	3.58	0.3058	135	2.84	212	3.40	6.24	0.3790	0.8069	0.5737
0.5	129	1.26	185	1.95	3.71	0.2258	142	2.70	224	3.23	6.43	0.3052	0.7397	0.5770
1	136	0.97	190	1.90	3.87	0.1647	150	2.55	236	3.06	6.61	0.2423	0.6798	0.5855
1.5	145	0.71	195	1.85	4.06	0.1179	159	2.41	250	2.89	6.80	0.1899	0.6208	0.5971
2	157	0.48	202	1.78	4.26	0.0815	169	2.27	266	2.72	6.99	0.1469	0.5551	0.6094
2.5	171	0.29	211	1.71	4.50	0.0565	180	2.13	283	2.55	7.18	0.1126	0.5020	0.6267

Table 3: Welfare implications of the initial capacity under duopoly and the social optimum.

Tabel 3 shows the investment triggers, the correspondig capacities and the resulting surpluses for both the social planner and the duopoly. The table also shows the accumulated capacities Q^{duop} and Q^W. The first observation is that the first and second investment moment of the social planner are later than the investment moments in the duopoly model. Moreover, the resulting capacities are larger in the case of a welfare maximizer. So, the preemption effect combined with the cannibalization effect forces the incumbent to invest too soon and acquire a small capacity. The entrant also invests sooner compared to the second investment of the social planner and this is because the incumbent invests in a smaller capacity. The social planner is more interested in larger quantities than profit maximizing firms because firms do not internalize the additional consumer surplus generated by a capacity increase.

Table 3 can also be used to study the effect of the initial incumbent's capacity size on the welfare. For both the duopoly model and for the social planner model, additional welfare gained by investing drops when the initial capacity is larger. Intuitively, the larger the old market, the lower the marginal value of an additional unit of capital. Additionally, a larger initial capacity is equivalent to a more severe cannibalization effect. The result that additional welfare in the duopoly is more affected by an increase in the initial capacity can be explained by the presence of competition that marginalizes surplus as a result of protective behavior towards the firms' own profit. The social planner delays investment relative to the market outcome, because it is not affected by a potential entrant's willingness to invest soon.

The insight that under (potential) competition investment occurs too early and in too small amounts compared to the social optimum gives rise to potential policy implications of our analysis. In particular, policies leading to later investment by the firms, thereby inducing larger long term capacities and output quantities would lead to a welfare improvement. Introducing a license on building capacity would, e.g., contribute to the desired objective. Licensing requires that upon investment the firms incur a lump-sum cost. Since investing becomes more expensive, firms will delay their investment moment. This cost has no direct influence on the investment amount, however, since the firms invest later, the realized investment size will increase.

7 Conclusions

The main message of this paper is that the interaction between timing and size of investment plays a crucial role in the strategy of an incumbent facing the threat of entry in a dynamic market environment. Where entry deterrence is generally understood to ward off entrants by overinvesting, we find that entry is delayed by accelerating the investment. This induces an investment, which is smaller than that of an incumbent in a comparable market without an entry threat. This implication of our analysis is well suited to explain the empirical observations reported in Leach *et al.* (2013). These authors show that, contrary to the predictions of the standard entry deterrence literature, the entry threat generated by the deregulation of the U.S. telecommunication industry did not result in an increase of capacity investments by incumbents. As the telecommunications industry in this period clearly has the characteristics of an expanding market, it fits well with the setup of our model. Therefore, our insight that in the presence of choices about both timing and size the incumbent's investment should be smaller than without an entry threat, provides a clear theoretical guidance for understanding these empirical observations. Also our result that, depending on whether investment size is flexible or fixed, the incumbent respectively the entrant invests first, is not only a new insight in the theoretical literature, but also gives rise to potentially testable empirical implications.

The model could be extended in different ways. In particular, it is important to examine the implications of the existence of multiple incumbents and/or potential entrants for the main insights of our analysis. Furthermore, the consideration of multiple investment options, although technically very challenging, is an important further step to gain a better understanding of the evolution of industries characterized by irreversible investments. Finally, the implications of innovating firms, either adopting new technologies or performing R&D themselves, for capacity dynamics could be studied as an extension of the setting considered here.

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Appendix A: Proofs

Proof of Proposition 1 The follower's value function with respect to the shock process x can be divided into two regions. For x sufficiently large the firm invests, that is for $x \geq X_F^*$, this region is called the stopping region. The complementary region is called the continuation region, for these values the firm waits (see e.g. Dixit and Pindyck (1994)). In the stopping region the firm realizes the following accumulated and discounted expected profits $V_F(X, q_{1L}, q_{1F}, q_{2L}, q_{2F})$ at the investment moment,

$$\begin{aligned}
V_F &= \mathbb{E}\left[\int_{t=0}^{\infty} x(t)(q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F})e^{-rt}\mathrm{d}t \;\Big|\; x(0)=X\right] - \delta q_{2F} \\
&= (q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F})\mathbb{E}\left[\int_{t=0}^{\infty} x(t)e^{-rt}\mathrm{d}t \;\Big|\; x(0)=X\right] - \delta q_{2F} \\
&= (q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F})\int_{t=0}^{\infty} x(0)e^{(\alpha-r)t}\mathrm{d}t - \delta q_{2F} \\
&= \frac{X}{r-\alpha}(q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}) - \delta q_{2F}.
\end{aligned}$$

The firm chooses capacity such that it maximizes its profits, thereto,

$$\begin{aligned}
\frac{\partial V_F}{\partial q_{2F}} &= \frac{X}{r-\alpha}\left[1-\eta(q_{1L}+2q_{1F}+q_{2L})-2\eta q_{2F})\right]-\delta \\
&= 0 \quad \Leftrightarrow \\
q_{2F}(X, q_{1L}, q_{1F}, q_{2L}) &= \frac{1}{2\eta}\left[1-\eta(q_{1L}+2q_{1F}+q_{2L}) - \frac{\delta(r-\alpha)}{X}\right].
\end{aligned}$$

The second order conditions reassure us that this is indeed a maximum, $-2\eta\frac{X}{r-\alpha} < 0$.

In the continuation region it is optimal for the firm the delay investment, for waiting yields a larger value than investment. The waiting value is embodied by the option value. The function F_F, following standard real options analysis (see e.g. Dixit and Pindyck (1994)), equals the sum two terms reflecting the value of waiting and the value of current production,

$$F_F(X, q_{1L}, q_{1F}, q_{2L}) = A_F X^{\beta} + \frac{X}{r-\alpha}q_{1F}(1-\eta(q_{1L}+q_{1F}+q_{2L})),$$

where β is the positive root following from,

$$\sigma^2\beta^2 + (2\alpha-\sigma^2)\beta = 2r \Leftrightarrow \beta = \tfrac{1}{2} - \tfrac{\alpha}{\sigma^2} + \sqrt{\left(\tfrac{1}{2}-\tfrac{\alpha}{\sigma^2}\right)^2 + \tfrac{2r}{\sigma^2}}.$$

The investment trigger and the value of the parameter $A_F(q_{1L}, q_{1F}, q_{2L})$ can be found by applying the value matching and smooth pasting conditions,

$$\begin{aligned}
\frac{X}{r-\alpha}(q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F})) - \delta q_{2F} &= A_F X^{\beta} + \frac{X}{r-\alpha}q_{1F}(1-\eta(q_{1L}+q_{1F}+q_{2L})), \\
\frac{1}{r-\alpha}(q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F})) &= A_F\beta X^{\beta-1} + \frac{1}{r-\alpha}q_{1F}(1-\eta(q_{1L}+q_{1F}+q_{2L})).
\end{aligned}$$

Together they make

$$\frac{X}{r-\alpha}(q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}))\left(1-\frac{1}{\beta}\right) - \frac{X}{r-\alpha}q_{1F}(1-\eta(q_{1L}+q_{1F}+q_{2L}))\left(1-\frac{1}{\beta}\right) = \delta q_{2F},$$

which leads to

$$X_F = \frac{\beta}{\beta-1}\frac{\delta(r-\alpha)}{1-\eta(q_{1L}+2q_{1F}+q_{2L}+q_{2F})}.$$

Plugging in q_{2F} leads to

$$X_F^*(q_{1L}, q_{1F}, q_{1L}) = \frac{\beta+1}{\beta-1}\frac{\delta(r-\alpha)}{1-\eta(q_{1L}+2q_{1F}+q_{2L})}.$$

Moreover,

$$\begin{aligned} A_F(X_F^*)^\beta &= \frac{X_F^*}{r-\alpha}q_{2F}^*(1-\eta(q_{1L}+2q_{1F}+q_{2L}+q_{2F}^*)) - \delta q_{2F}^* \\ &= \frac{\delta q_{2F}^*}{\beta-1}. \end{aligned}$$

Rewring leads to equation

$$A_F(q_{1L}, q_{1F}, q_{2L}) = \frac{\delta^2(r-\alpha)}{\eta(\beta-1)^2}\left(\frac{1-\eta(q_{1L}+2q_{1F}+q_{2L})}{\delta(r-\alpha)}\frac{\beta-1}{\beta+1}\right)^{\beta+1}.$$

□

Proof of Proposition 2 The region with respect to X where the leader delays the follower's investment is bounded from below and above by X_1^{det} and X_2^{det}. In this region the leader optimizes capacity at investment. The optimal amount can be found by solving the first order condition,

$$\frac{\partial V_L^{det}}{\partial q_{2L}} = \frac{X}{r-\alpha}[1-\eta(2q_{1L}+q_{1F}+2q_{2L})] - \frac{\delta}{\beta-1}\left(\frac{X}{X_F^*}\right)^\beta\left[1-\frac{\eta\beta(q_{1L}+q_{2L})}{1-\eta(q_{1L}+2q_{1F}+q_{2L})}\right] - \delta. \tag{16}$$

Differentiating both the left hand side and the right hand side with respect to X leads to

$$\frac{\partial q_L^{det}}{\partial X} = \frac{1-\eta(2q_{1L}+q_{1F}+2q_L^{det}) - \frac{\beta}{\beta+1}\left(\frac{X}{X_F^*}\right)^{\beta-1}(1-2\eta q_{1F}-(1+\beta)\eta(q_{1L}+q_L^{det}))}{X\left[2-\frac{\beta}{\beta+1}\left(\frac{X}{X_F^*}\right)^{\beta-1}\left(1-\frac{1-\eta(2q_{1F}+\beta q_{1L}+\beta q_L^{det})}{1-\eta(2q_{1F}+q_{1L}+q_L^{det})}\right)\right]} > 0.$$

The value of X_2^{det} then follows from first inserting X_F^* in the first order conditions,

$$\frac{\partial V_L^{det}}{\partial q_{2L}} = \frac{\delta}{\beta-1}\frac{1-2\eta q_{1L}+\eta(\beta-1)q_{1F}-2\eta q_{2L}}{1-\eta(q_{1L}+2q_{1F}+q_{2L})} = 0 \Leftrightarrow q_{2L}^R = \frac{1}{2\eta}(1-2\eta q_{1L}+(\beta-1)\eta q_{1F}). \tag{17}$$

Hence, by plugging the latter expression into X_F^* one obtains X_2^{det},

$$X_2^{det} = X_F^*(q_{1L}, q_{1F}, q_{2L}^R) = \frac{\beta+1}{\beta-1}\frac{2\delta(r-\alpha)}{1-(\beta+3)\eta q_{1F}}$$

The conditions determining X_1^{det} also follow from the first order conditions by setting $q_{2L} = 0$. To show that there exists a unique point X_1^{det}, it is sufficient to do the following. Define $\psi(X) = \frac{\partial V_L^{det}}{\partial q_{2L}}|_{q_{2L}=0}$, this function dictates the first order conditions for the value of X yielding zero capacity,

$$\psi(X) = \frac{X}{r-\alpha}[1-\eta(2q_{1L}+q_{1F})] - \frac{\delta}{\beta-1}\left(\frac{\beta-1}{\beta+1}\frac{1-\eta q_{1L}-2\eta q_{1F}}{\delta(r-\alpha)}X\right)^\beta\left[1-\frac{\eta\beta q_{1L}}{1-\eta q_{1L}-2\eta q_{1F}}\right] - \delta.$$

Then,

$$\begin{aligned} \psi(0) &= -\delta < 0, \\ \psi(X_F^*) &= \frac{\delta}{\beta-1}\frac{1-2\eta q_{1L}+(\beta-1)\eta q_{1F}}{1-\eta q_{1L}-2\eta q_{1F}}, \\ \psi'(X) &= \frac{1-\eta(2q_{1L}+q_{1F})}{r-\alpha}\left[1-\frac{\beta}{\beta+1}\left(\frac{X}{X_F^*}\Big|_{q_{2L}=0}\right)^{\beta-1}\right] + \frac{\beta}{\beta+1}\left(\frac{X}{X_F^*}\Big|_{q_{2L}=0}\right)^{\beta-1}\frac{\eta q_{1F}+(\beta-1)\eta q_{1L}}{r-\alpha}. \end{aligned}$$

From (17) it follows that $\psi(X_F^*) > 0$. Since $\psi'(X) > 0$ one can conclude that, according to the Mean Value theorem, there exists a unique $X_1^{det} \in (0, X_F^*)$ such that $q_L^{det}(X_1^{det}, q_{1L}, q_{1F}) = 0$.

The value before investment is similar to the one for the follower,

$$F_L^{det}(X, q_{1L}, q_{1F}) = A_L^{det} X^\beta + \frac{X}{r-\alpha} q_{1L}(1 - \eta(q_{1L} + q_{1F})).$$

Applying the value matching and smooth pasting conditions gives (7) and

$$A_L^{det} = (X_L^{det})^{-\beta} \frac{\delta q_L^{det}}{\beta - 1} - \frac{\delta}{\beta - 1}(q_{1L} + q_L^{det})\,(X_F^*)^{-\beta}.$$

In order to show that for sufficiently small q_{1L} there exists a pair (X_L^{det}, q_L^{det*}) satisfying (7) and the first order condition for the leader, we insert (7) into (16). We treat the following two cases separately. First we look as the scenario where the incumbent is the investment leader. Then, $q_{1F} = 0$ and one obtains the equivalent condition

$$\beta\left(1 - \frac{1 - \eta(2q_{1L} + 2q_{2L})}{1 - \eta(2q_{1L} + q_{2L})}\right) = 1 - \frac{1 - \eta(\beta+1)(q_{1L} + q_{2L})}{1 - \eta(q_{1L} + q_{2L})}\left(\frac{\beta}{\beta+1}\right)^\beta. \tag{18}$$

After rewriting this equation, one could similarly say that it is required that $H(q_{2L}) = 0$, where,

$$H(q_{2L}) = 1 - \eta(q_{1L} + q_{2L}) - (1 - \eta(\beta+1)(q_{1L} + q_{2L}))\left(\frac{\beta}{\beta+1}\right)^\beta + \beta\left[\frac{1 - \eta(2q_{1L} + 2q_{2L})}{1 - \eta(2q_{1L} + q_{2L})} - 1\right](1 - \eta(q_{1L} + q_{2L})).$$

Since,

$$H(0) = 1 - \eta q_{1L} - (1 - \eta(\beta+1)q_{1L})\left(\frac{\beta}{\beta+1}\right)^\beta > (1 - \eta q_{1L})\left(1 - \left(\frac{\beta}{\beta+1}\right)^\beta\right) > 0,$$

$$H\left(\frac{1 - 2\eta q_{1L}}{2\eta}\right) = \frac{1}{2}(\beta - 1)\left(\left(\frac{\beta}{\beta+1}\right)^\beta - 1\right) < 0, \; and$$

$$\frac{\partial H}{\partial q_{2L}}(q_{2L}) = \underbrace{-1 + (\beta+1)\left(\frac{\beta}{\beta+1}\right)^\beta}_{<0} - \frac{\beta\eta q_{2L}}{1 - \eta(2q_{1L} + q_{2L})}\underbrace{\left[\frac{1 - \eta(q_{1L} + q_{2L})}{1 - \eta(2q_{1L} + q_{2L})} - 1\right]}_{>0} < 0,$$

it can be concluded that, according to the Mean Value Theorem, there exists a q_{2L} on the interval $\left(0, \frac{1-2\eta q_{1L}}{2\eta}\right)$, such that $H(q_{2L}) = 0$. This value is denoted by q_L^{det*}. Earlier we showed that q_L^{det} in an increasing function. Then, since $\frac{1}{2\eta}(1 - 2\eta q_{1L}) = q_{2L}^R$, it follows that $X_L^{det} > X_1^{det}$ and $X_L^{det} < X_2^{det}$.

In a similar way one can prove this for the scenario where the entrant is leader. Here one shows that for $q_{2L} = 0$ the function H takes a positive value, while for $q_{2L} = \frac{1}{2\eta} < q_{2L}^R$ the function becomes negative. □

Proof of Proposition 3 The leader's value function under the inducing immediate follower investment strategy is determined in the same way as before. In this case, however, one needs to substitute the follower's capacity (3) to obtain equation (13). The leader chooses capacity such that it optimizes the value function,

$$\frac{\partial V_L^{acc}}{\partial q_L^2} = \frac{X}{2(r-\alpha)}[1 - 2\eta(q_{1L} + q_{2L})] - \frac{1}{2}\delta \Leftrightarrow q_L^{acc}(X, q_{1L}, q_{1F}) = \frac{1}{2\eta}\left[1 - 2\eta q_{1L} - \frac{\delta(r-\alpha)}{X}\right].$$

It is easily checked that $q_L^{acc} \geq 0$ if and only if $1 - 2\eta q_{1L} \geq \frac{\delta(r-\alpha)}{X}$, i.e. if

$$X \geq \frac{\delta(r-\alpha)}{1 - 2\eta q_{1L}}.$$

The second order conditions again make sure that we obtain a maximum, $-2\eta\frac{X}{r-\alpha} < 0$. Solving $q_L^{acc} = \hat{q}_{2L}$ leads to the first term in equation (11), i.e.,

$$q_L^{acc} = \frac{1}{2\eta}\left[1 - 2\eta q_{1L} - \frac{\delta(r-\alpha)}{X}\right] = \frac{1}{\eta}\left[1 - 2\eta q_{1F} - \eta q_{1L} - \frac{\delta(\beta+1)(r-\alpha)}{(\beta-1)X}\right] = \hat{q}_{2L}$$

$$\Leftrightarrow$$

$$1 - 4\eta q_{1F} = \frac{\delta(r-\alpha)(\beta+3)}{(\beta-1)X} \Leftrightarrow X_0^{acc} = \frac{\beta+3}{\beta-1}\frac{\delta(r-\alpha)}{1-4\eta q_{1F}}.$$

Assuming again a value function of the form

$$F_L^{acc}(X, q_{1L}, q_{1F}) = A_L^{acc} X^\beta + \frac{X}{r-\alpha} q_{1L}(1 - \eta(q_{1L} + q_{1F}))$$

one can apply the value matching and smooth pasting conditions. Using $q_{1L} \cdot q_{1F} = 0$, to simplify the term for $X_L^{acc}(q_{1L}, q_{1F}, q_{2L})$ resulting from these two conditions one ends up with (12). Moreover,

$$\begin{aligned} A_L^{acc} \cdot (X_L^{acc})^\beta &= \frac{X_L^{det}}{r-\alpha}\left[(q_{1L} + q_L^{acc})(1 - \eta(q_{1L} + q_{1F} + q_L^{acc} + q_{2F}^*)) - q_{1L}(1 - \eta(q_{1L} + q_{1F}))\right] - \delta q_L^{acc} \\ &= \frac{\delta\beta}{\beta-1} q_L^{acc} - \delta q_L^{acc} = \frac{\delta q_L^{acc}}{\beta-1}. \end{aligned}$$

To show existence of (q_L^{acc*}, X_L^{acc}) for sufficiently small q_{1L} we insert $q_{1L} = 0$ and (12) into the equation $q_L^{acc*} = q_L^{acc}(X_L^{acc}, q_{1L})$. Solving for q_{2L} gives $(q_L^{acc*} = \frac{1}{3\beta-1} > 0$. Therefore, by continuity, we have $X_L^{acc} > \frac{\delta(r-\alpha)}{1-2\eta q_{1L}}$ for sufficiently small q_{1L}. □

Proof of Proposition 4 We observe that

$$\frac{X_1^{acc}}{X_2^{det}} = \underbrace{\frac{\beta+3}{2\beta+2}}_{<1}\underbrace{\frac{1-(3+\beta)\eta q_{1F}}{1-4\eta q_{1F}}}_{<1} < 1,$$

and therefore $X_1^{acc} < X_2^{det}$. Furthermore, for $X = X_1^{acc}$ we have by definition $q_L^{acc} = \hat{q}_{2L}$ (ignoring the trivial case where $q_L^{acc} = 0$) and, due to $X_1^{acc} < X_2^{det}$, $q_L^{det} > \hat{q}_{2L}$. Since q_L^{det} is the maximizer of V_L^{det}, this yields

$$V_L^{det}(X_1^{acc}, q_{1L}, q_{1F}, q_L^{det}) > V_L^{det}(X_1^{acc}, q_{1L}, q_{1F}, q_L^{acc}) = V_L^{acc}(X_1^{acc}, q_{1L}, q_{1F}, q_L^{acc}),$$

where the last equality follows from the observation that at $q_{2L} = q_L^{acc} = \hat{q}_{2L}$ the value functions both investment strategies coincide. Similarly, we obtain for $X = X_2^{det}$

$$V_L^{acc}(X_1^{det}, q_{1L}, q_{1F}, q_L^{acc}) > V_L^{acc}(X_1^{det}, q_{1L}, q_{1F}, q_L^{det}) = V_L^{det}(X_1^{det}, q_{1L}, q_{1F}, q_L^{det}),$$

because $q_L^{det} = \hat{q}_{2L}$. Since the delaying follower investment strategy is feasible for $X \in [X_1^{det}, X_2^{det}]$ and the inducing immediate follower investment strategy for $X \in [X_1^{acc}, \infty)$ we conclude that for $X \leq X_1^{acc}$ the leader optimally delays the follower's investment and for $X \geq X_2^{det}$ the follower is enforced to invest immediately. □

Proof of Proposition 5 Existence of the preemption equilibrium follows from the arguments given in the text. In order to show that no other subgame-perfect equilibria exist, we first note that in this type of games, apart from preemption equilibria, potentially also sequential investment, simultaneous investment and joint investment might arise as equilibrium behavior. Here we follow the terminology as in Pawlina and Kort (2006). Sequential equilibria would imply that one firm is investing strictly before the preemption point of the competitor, which has already been ruled out for this setting. The remaining two types of equilibria involve tacit collusion among the firms. When firms decide to collude, they wait for the market to expand, that is, wait for a larger value of X, before investment is undertaken together at the same time. One can discriminate two types of collusion, distinguished by the order in which firms determine their capacity size. In the first type, one firm is Stackelberg capacity leader and decides upon the amount first where subsequently the second firm makes an immediate investment. The second investor sets its capacity after the first firm decided upon its investment scale. This type is called simultaneous investment. The second type, referred to as joint investment, is the category where there is no colluded investment order. Firms simultaneously decide upon capacities, leading to a Cournot type of equilibrium. The following Lemmas 2 and 3 rule out the existence of simultaneous investment and joint investment equilibria.

Lemma 1 *Assume that $q_{1I} = q_{1I}^{myop}$. Then for the incumbent the leader's investment threshold $X_L^{det}(q_{1I}^{myop}, 0)$ does not exist. Hence, it is optimal for the incumbent to delay investment as much as possible and to invest just before the entrant's preemption point X_{PE}.*

Proof of Lemma 1 We rewrite equation (9) as

$$F_L^{det}(X, q_{1L}, q_{1F}) = \frac{X}{r-\alpha} q_{1L}(1 - \eta(q_{1L} + q_{1F})) + A_L^{det} X^{\beta}.$$

Then A_L^{det} reflects the net gain from investment. Let X_L^{det} and X_F^* be defined as in equations (7) and (4). Let $q_{1L} = q_{1I} = q_{1I}^{myop} = \frac{1}{\eta(\beta+1)}$ and $q_{1F} = 0$. Then,

$$\frac{X_L^{det}}{X_F^*} = \frac{\beta - 1 + \frac{1}{\beta+1} - \beta\eta q_{2I}^{det}}{\beta - 1 - (\beta+1)\eta q_{2I}^{det}} > 1.$$

Furthermore,

$$\begin{aligned} A_L^{det} &= (X_L^{det})^{-\beta} \frac{\delta q_{2I}^{det}}{\beta - 1} - \frac{\delta}{\beta - 1}(q_{1I}^{myop} + q_{2I}^{det})(X_F^*)^{-\beta} \\ &= \frac{\delta}{\beta - 1}\left[q_{2I}^{det}\left[\left(\frac{1}{X_L^{det}}\right)^{\beta} - \left(\frac{1}{X_F^*}\right)^{\beta}\right] - q_{1I}^{myop}\left(\frac{1}{X_F^*}\right)^{\beta}\right] \\ &< 0. \end{aligned}$$

This means that investment decreases the incumbent's payoff and the incumbent would never choose this strategy as a leader, if investment roles were exogenously determined. Hence, X_L^{det} does not exist and under endogenous investment roles it is optimal for the incumbent to delay investment as long as possible without jeopardizing the role as leader. □

Lemma 2 *Simultaneous investment does not yield an equilibrium.*

Proof of Lemma 2 For the resulting value functions, the curves in Figure 2a should be considered. Here, the Stackelberg leader utilizes the inducing immediate follower investment strategy, denoted by *acc*. As a result, the competitor receives the follower value, being smaller than the leader value. For this reason neither of the firms would prefer to be a follower in the outcome and they would, consequently, preempt each other in taking the leader role. This forces the firms to end up in the region where the leader delays the follower's investment and the sole resulting equilbrium is the preemptive equilibrium where the follower prefers to wait rather than invest at the same time. Hence, simultaneous investment is not an equilibrium. □

Lemma 3 *Joint investment does not yield and equilibrium.*

Proof of Lemma 3 Let $J(X, q_{1L}, q_{1F}, q_{2L}, q_{2F})$ be the firm value for joint investment, then,

$$J = \frac{X}{r-\alpha}(q_{1L}+q_{2L})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F})) - \delta q_{2L}.$$

Optimal capacities equal,

$$\begin{aligned} q_{2L}^{join} &= \frac{1}{3\eta}\left(1-\frac{\delta(r-\alpha)}{X}\right) - q_{1L}, \\ q_{2F}^{join} &= \frac{1}{3\eta}\left(1-\frac{\delta(r-\alpha)}{X}\right) - q_{1F}. \end{aligned}$$

This leads to,

$$\begin{aligned} V_L^{acc}(X, q_{1L}, q_{1F}, q_L^{acc}, q_{2F}^{*}) &= \frac{X}{r-\alpha}\frac{1}{8\eta}\left(1-\frac{\delta(r-\alpha)}{X}\right)^2 + \delta q_{1L} \\ J(X, q_{1L}, q_{1F}, q_{2L}^{join}, q_{2F}^{join}) &= \frac{X}{r-\alpha}\frac{1}{9\eta}\left(1-\frac{\delta(r-\alpha)}{X}\right)^2 + \delta q_{1L} \end{aligned}$$

Hence, it holds that $V_L^{acc} > J$. This is sufficient to show that joint investment does not yield an equilibrium. Intuition behind this result is that when firms are leader they can set a larger capacity which leads to a higher payoff. □

This concludes the proof of the Proposition.

Appendix B: Robustness

Robustness of the preemption equilibrium

In Figures 9 and Figure 10 we show the differences in preemption points $(X_{PE} - X_{PI})$ for variations of all model parameters in a relevant range. This is done for both $q_{1I} = q_{1I}^{myop}$ and $q_{1I} = 0.5$. Similarly, the difference between the leaders investment trigger under the delaying follower investment strategy under the entrants preemption point $(X_L^{det} - X_{PE})$ is shown for the same parameter variations and $q_{1I} = 0.5$ in Figure

11. For $q_{1I} = q_{1I}^{myop}$ the investment trigger X_L^{det} does not exist and hence the incumbent does not have an incentive to invest before the entrants preemtpion point is reached (see Lemma 1 in Appendix A). These figures confirm the claim that, apart from the hardly relevant case where η is extremely small (discussed in Section 5.1), under all parameter variations the preemption equilibrium with the incumbent as leader exists.

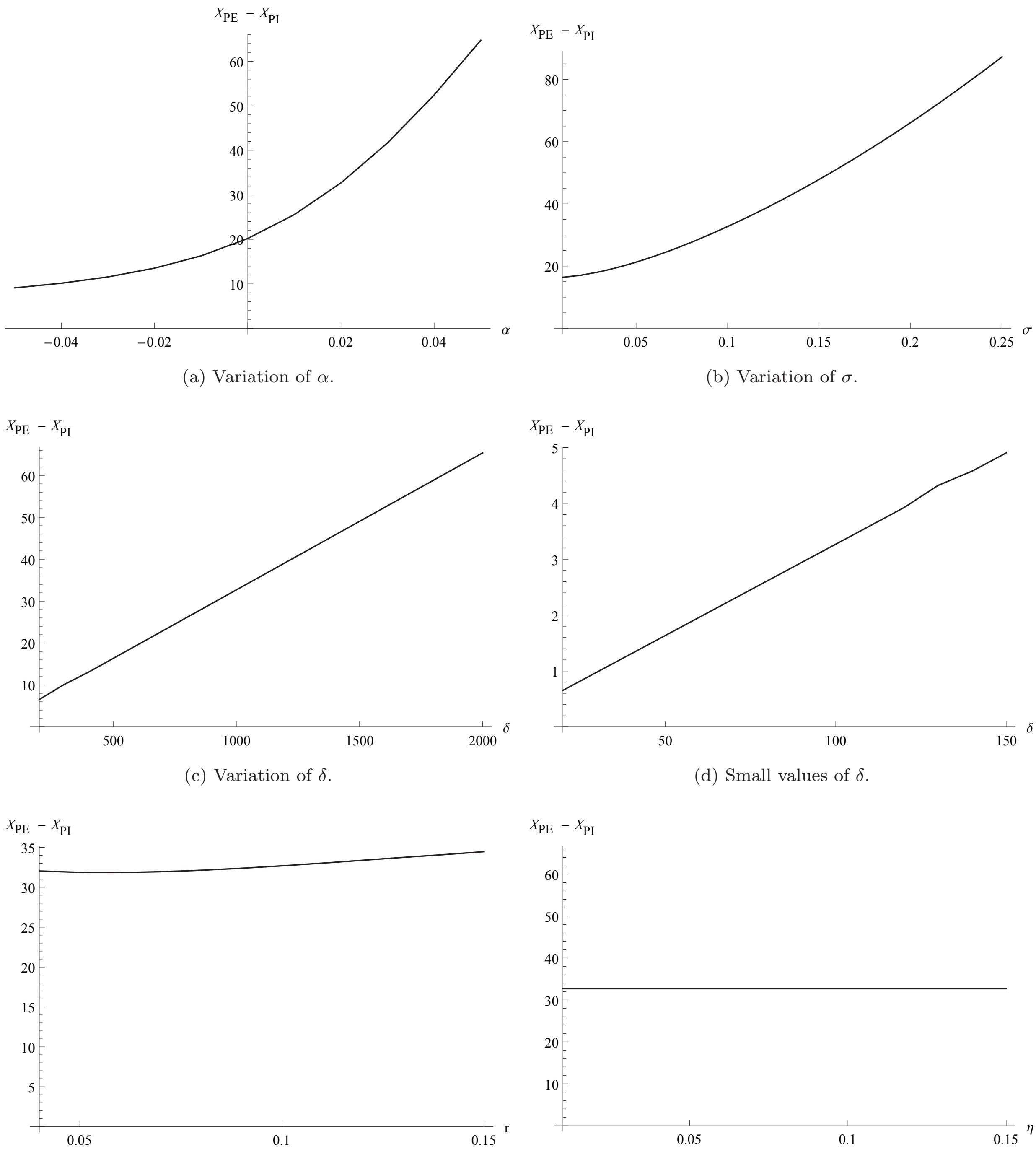
Sensitivity analysis

The aim of this section is to briefly study the effect of the model parameters on the equilibrium. In this model there are six parameters to be taken a closer look at. First of all, the sensitivity parameter η capturing the negative relation between prices and output. The second parameter is the fixed discount rate r. Then, the drift parameter α and the volatility parameter σ reflecting the market's uncertainty, both present in the geometric Brownian motion describing the state variable's path. Subsequently, we have the marginal investment cost δ.

	η	r	α	δ	σ
X_{PE} $(q_{1I} = q_{1I}^{myop})$	0	+	-/+	+	+
q_{2I}^{det} $(q_{1I} = q_{1I}^{myop})$	-	+/-	+/-	0	+/-
X_{PE} (q_{1I} fixed)	+	+	-	+	+
q_{2I}^{det} (q_{1I} fixed)	-	-	+	0	+

Table 4: Effect of an increase in parameter values on triggers and capacities

When η increases the output q_{2I} decreases exactly canceling out the increase in η, i.e. the product $\eta \cdot q_{2I}$ remains constant. Similarly $\eta \cdot q_{1I}^{myop}$ and $\eta \cdot q_E$ remain constant. In this way, when assuming $q_{1I} = q_{1I}^{myop}$, neither the investment threshold X_L^{det}, nor the preemption trigger are affected by an increase in η. However, when one assumes q_{1I} to be fixed, triggers are affected. An increase in η means an increase in ηq_{1I} and resultingly a decrease in the price, which, hence, makes firms delay investment. Nevertheless, the total effect on the investment size is negative, considering the different effects. When discounting is done under a higher rate, one values future revenues relatively less and one becomes more concerned about current profits. If the interest rate increases, one prefers current profits to be higher and therefore delays investment. In the first place, this increases the myopic capacity size on the initial market. In the second place, since there are two effects that influence the optimal investment size for the expansion - i.e. delaying increases the capacity level, but a larger old market decreases it - it is found that the change is ambiguous. For small r the installment increases, but for relatively large r it decreases. When one fixes the initial capacity, the effect of the old market dominantly influences the capacity leading to decreasing installments. As standard in literature, the drift parameter has an opposite effect: a larger α makes firms invest earlier. The main line of reasoning is the same, when the drift parameter increases. Market demand, and therefore profits, are expected to increase more rapidly; one is then prepared to invest earlier to meet the same expectations concerning expected



(a) Variation of α.

(b) Variation of σ.

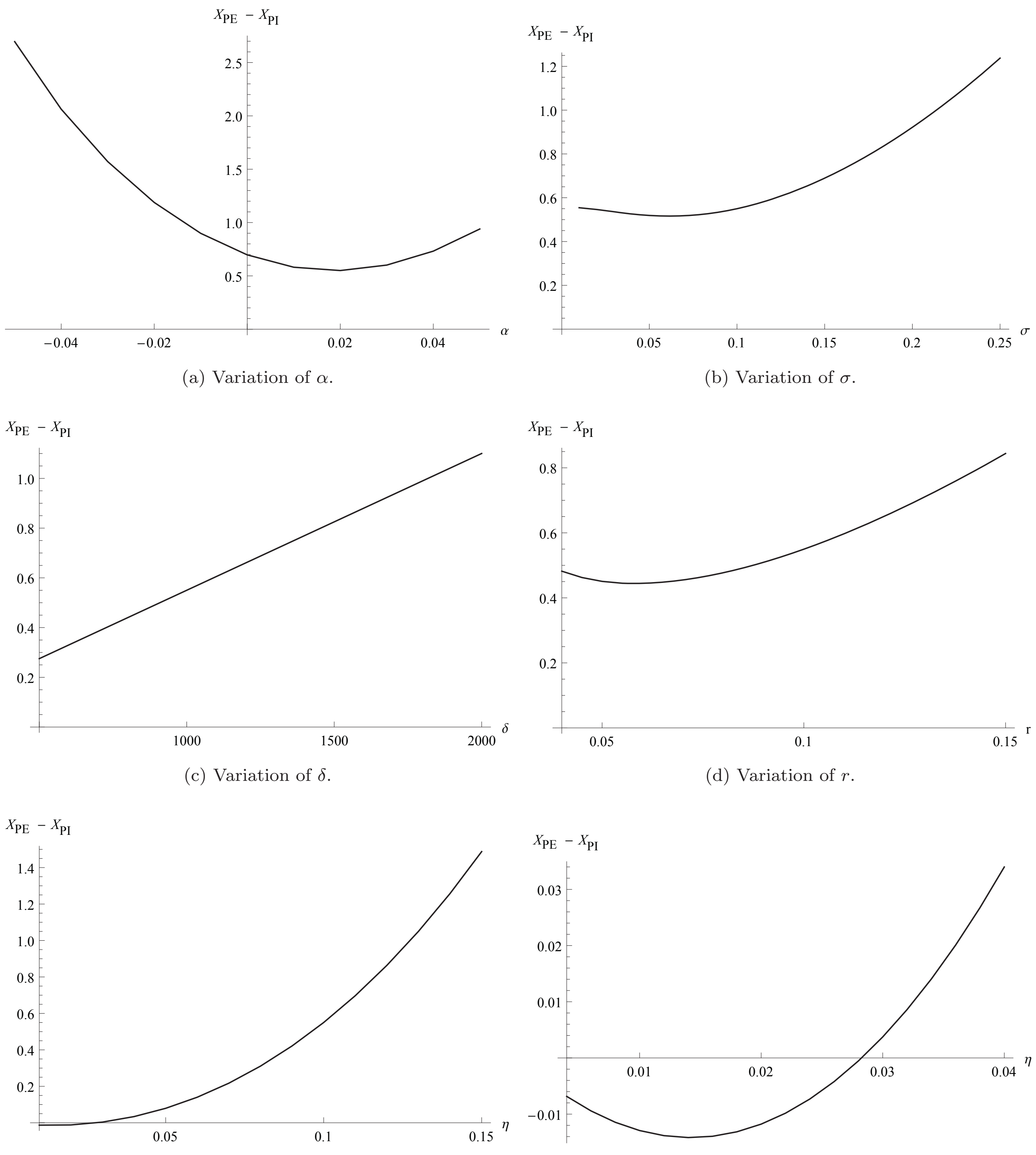
(c) Variation of δ.

(d) Small values of δ.

(e) Variation of r.

(f) Variation of η.

Figure 9: Difference between preemption points with $q_{1I} = q_{1I}^{myop}$.



(a) Variation of α.

(b) Variation of σ.

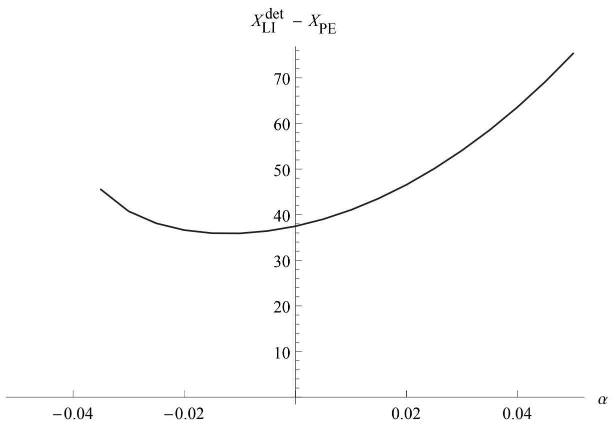
(c) Variation of δ.

(d) Variation of r.

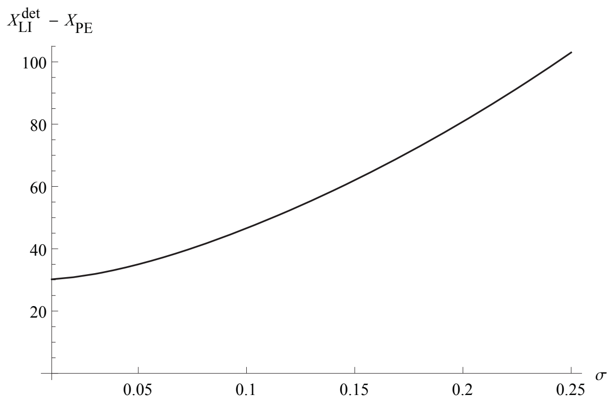
(e) Variation of η.

(f) Small values of η.

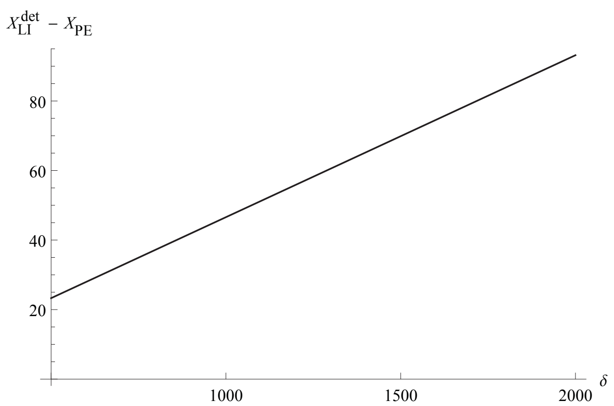
Figure 10: Difference between preemption points with $q_{1I} = 0.5$.



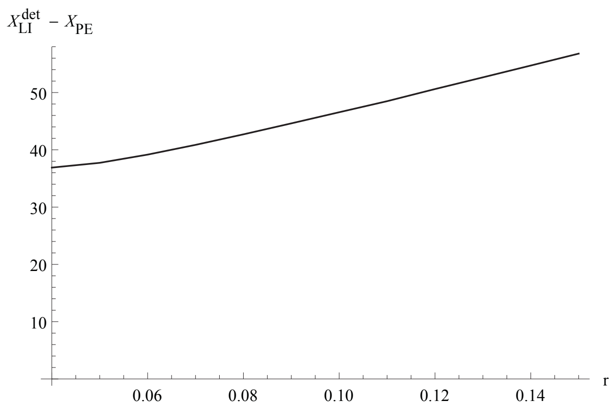
(a) Variation of α.



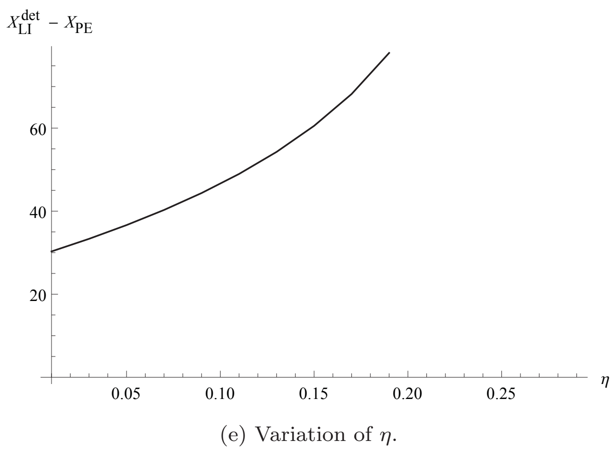
(b) Variation of σ.



(c) Variation of δ.



(d) Variation of r.



(e) Variation of η.

Figure 11: Difference between the incumbent's investment triggers and the entrant's preemption point for $q_{1I} = 0.5$.

revenues. Nevertheless, when the initial capacity also changes under a change in parameter values,[12] a second effect comes in, similar to the analysis of η: a larger drift increases the initial capacity which leads to a delay of the investment. The effect on the optimal capacity is similar to the effect of r when the initial capacity is determined endogenously as q_{1I}^{myop}, but is, as expected, opposite to r when fixing it. The marginal investment cost has a positive effect on the investment trigger. When investing becomes more expensive, firms prefer to wait for a market where a larger output is required in order to meet the larger costs. The optimal capacities, both when fixing the initial market size and taking it myopically, are not affected. Finally, in a more uncertain market, i.e. a larger σ, future realizations become more important. Waiting gives more information. This leads to the decision to wait for a higher price, in other words, the firm is only prepared to invest for a larger value of x. This leads to an increase in the optimal capacity size. However, as in the case of r and α, the effect is ambiguous when assuming a myopic initial market size.

Appendix C: Model Extensions

Fixed capacity

Suppose $X = X_{PE}$, then one can show that

$$V_{LE}^{det} - F_{FE}^{det} = \left[\frac{X}{r-\alpha}K(1-\eta K-\eta q_{1I}) - \delta K\right]\left[1 - \frac{\frac{(X_{FE}^*)^{1-\beta}}{r-\alpha}K(\eta q_{1I}+\eta K) + \frac{\delta K}{\beta-1}(\frac{1}{X_{FI}^*})^\beta}{\frac{(X_{FI}^*)^{1-\beta}}{r-\alpha}\eta K^2 + \frac{\delta K}{\beta-1}(\frac{1}{X_{FE}^*})^\beta}\right].$$

One can conclude, if

$$f_1(q_{1I}, K) = (1-\eta q_{1I} - 2\eta K)^{\beta-1}(1-\eta q_{1I}(\beta+1) - \eta K(\beta+2))$$
$$>$$
$$f_2(q_{1I}, K) = (1-2\eta q_{1I} - 2\eta K)^{\beta-1}(1-2\eta q_{1I} - \eta K(\beta+2)),$$

then $V_{LI}^{det}(X_{PE}) > F_{FI}(X_{PE})$ and as a result $X_{PI} < X_{PE}$.

Additive demand

Here, we first shortly summarize all the obtained propositions. Then, we will show some graphs to check the robustness of the results.

Proposition 6 *Let the current value of the stochastic demand process be denoted by X, and let the initial production capacity be denoted by q_{1L} and q_{1F} respectively for the leader and the follower. Let the capacities associated with the investments be denoted by q_{2L} and q_{2F} respectively for the leader and the follower. Then the value function of the follower can be partitioned into two regions: for small X the firm waits until it*

[12] Note that, since the initial capacity equals the myopic investment level, i.e. $q_{1I} = \frac{1}{\eta(\beta+1)}$, its level depends on the other parameter values.

reaches the investment trigger X_F^* *and for* $X \geq X_F^*$ *the firm invests immediately. As a result, the follower's value function* $V_F^*(X, q_{1L}, q_{1F}, q_{2L}, q_{2F})$ *is given by*

$$V_F^* = \begin{cases} A_F X^{\beta} + q_{1F}\left(\frac{X}{r-\alpha} - \frac{\eta}{r}(q_{1L} + q_{1F} + q_{2L})\right) & \text{if } X < X_F^*, \\ (q_{1F} + q_{2F})\left(\frac{X}{r-\alpha} - \frac{\eta}{r}(q_{1L} + q_{1F} + q_{2L} + q_{2F})\right) - \delta q_{2F} & \text{if } X \geq X_F^*, \end{cases}$$

where the optimal capacity level for the follower q_{2F}^**, the investment trigger* X_F^* *and* A_F *are defined by*

$$q_{2F}^*(X, q_{1L}, q_{1F}, q_{2L}) = \frac{r}{2\eta}\left(\frac{X}{r-\alpha} - \delta\right) - \tfrac{1}{2}(q_{1L} + 2q_{1F} + q_{2L}), \tag{19}$$

$$X_F^*(q_{1L}, q_{1F}, q_{2L}) = (\eta(q_{1L} + 2q_{1F} + q_{2L}) + \delta r)\frac{\beta(r-\alpha)}{r(\beta-2)}, \tag{20}$$

$$A_F = \frac{\eta(q_{1L} + 2q_{1F} + q_{2L}) + \delta r}{\eta(\beta-2)\beta(r-\alpha)}(X_F^*)^{1-\beta}. \tag{21}$$

The follower's capacity in case the follower invests at the investment trigger equals

$$q_{2F}^*(X_F^*, q_{1L}, q_{1F}, q_{2L}) = \frac{\eta(q_{1L} + 2q_{1F} + q_{2L}) + \delta r}{\eta(\beta-2)}.$$

Proposition 7 *Let the production capacities be defined as in Proposition 6 and let the current value of the shock process be defined as* X*. Then the delaying follower investment strategy leads to value function* $V_L^{det}(X, q_{1L}, q_{1F}, q_{2L})$*,*

$$V_L^{det} = (q_{1L} + q_{2L})\left(\frac{X}{r-\alpha} - \frac{\eta(q_{1L} + q_{1F} + q_{2L})}{r}\right) - (q_{1L} + q_{2L})\frac{\eta(q_{1L} + 2q_{1F} + q_{2L}) + \delta r}{r(\beta-2)}\left(\frac{X}{X_F^*}\right)^{\beta} - \delta q_{2L},$$

where X_F^* *is defined as equation (20).*

For large initial values of X *the leader invests immediately and chooses optimal capacity*

$$q_L^{det}(X, q_{1L}, q_{1F}) = \text{argmax}\{V_L^{det}(X, q_{1L}, q_{1F}, q_{2L}) \mid q_{2L} > \hat{q}_{2L}\}, \tag{22}$$

where,

$$\hat{q}_{2L}(X, q_{1L}, q_{1F}) = \frac{r}{\eta}\left[\frac{X(\beta-2)}{\beta(r-\alpha)} - \frac{\delta}{r}\right] - (q_{1L} + 2q_{1F}).$$

Delaying the follower's investment is considered for $X \in (X_1^{det}, X_2^{det})$*, where*

$$X_1^{det} = \{X \mid q_2^{det}(X, q_{1L}, q_{1F}) = 0\},$$
$$X_2^{det} = \{X \mid q_2^{det}(X, q_{1L}, q_{1F}) = \hat{q}_{2L}(X, q_{1L}, q_{1F})\}.$$

For low initial values of x*, that is* $x(0) < X_L^{det}$*, the leader invests at the moment* x *reaches the investment threshold value* X_L^{det}*. The value of the investment threshold and the associated capacity level* q_L^{det} *are determined as the solution of the set of equations determined by equation (22) and*

$$X_L^{det}(q_{1L}, q_{1F}, q_{2L}) = \left[\frac{\eta}{r}(2q_{1L} + q_{1F} + q_{2L}^*) + \delta\right]\frac{\beta(r-\alpha)}{\beta-1}.$$

The value function before investment is defined as

$$F_L^{det}(X, q_{1L}, q_{1F}, q_L^{det}) = q_{1L}\left(\frac{X}{r-\alpha} - \frac{\eta(q_{1L} + q_{1F})}{r}\right) + \left(\frac{X}{X_L^{det}}\right)^{\beta}\frac{\delta q_L^{det}}{\beta-1} - (q_{1L} + q_L^{det})\left(\frac{X}{X_F^*}\right)^{\beta}\frac{\delta}{\beta-1}.$$

Proposition 8 *Let the production capacities be defined as in Proposition 6 and let the current value of the shock process be defined as* X*. Then the inducing immediate follower investment strategy is considered for* $X \in (X_1^{acc}, \infty)$*, where,*

$$X_1^{acc} = \max\left\{ \frac{\beta(r-\alpha)}{r(\beta-4)}(4\eta q_{1F} + \delta),\ \delta(r-\alpha) \right\}.$$

Inducing immediate follower investment leads to value function $V_L^{acc}(X, q_{1L}, q_{1F}, q_{2L})$*,*

$$\begin{aligned} V_L^{acc} &= (q_{1L} + q_{2L})\left(\frac{X}{r-\alpha} - \frac{\eta(q_{1L} + q_{1F} + q_{2L} + q_{2F}^*)}{r}\right) - \delta q_{2L}, \\ &= (q_{1L} + q_{2L})\tfrac{1}{2}\left(\frac{X}{r-\alpha} - \frac{\eta(q_{1L} + q_{2L})}{r}\right) - \tfrac{1}{2}\delta(q_{2L} - q_{1L}). \end{aligned}$$

For large initial values of X *the leader invests immediately and chooses optimal capacity*

$$q_L^{acc}(X, q_{1L}) = \frac{r}{2\eta}\left(\frac{X}{r-\alpha} - \delta\right) - q_{1L}. \tag{23}$$

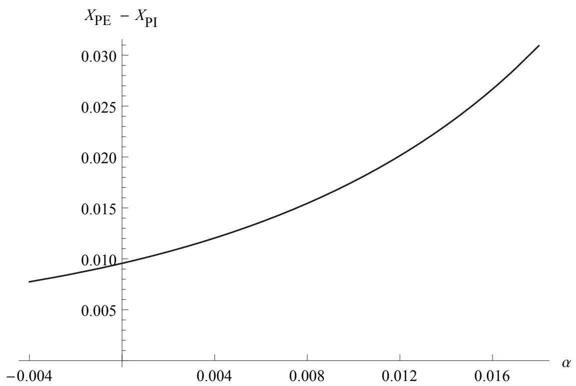
For low values of $x(0) = X$*, that is* $X < X_L^{acc}$*, the leader will invest when* x *reaches investment threshold value* X_L^{acc}*. The value of the investment threshold and the associated capacity level* q_L^{acc} *are determined as the solution of the set of equations determined by equation (23) and*

$$X_L^{acc}(q_{1L}, q_{1F}, q_{2L}) = \frac{\eta q_{2L}(q_{2L} + 2q_{1L}) + \delta r q_{2L}}{q_{2L} - q_{1L}} \frac{2\beta(r-\alpha)}{r(\beta-1)}.$$

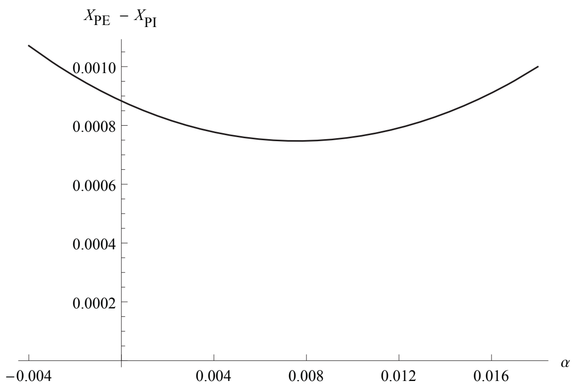
The value function before investment is defined as

$$F_L^{acc}(X, q_{1L}, q_{1F}) = q_{1L}\left(\frac{X}{r-\alpha} - \frac{\eta(q_{1L} + q_{1F})}{r}\right) + \left(\frac{X}{X_L^{acc}}\right)^{\beta} \frac{\delta q_L^{acc}}{\beta - 1}.$$

The following figures show how the preemption points change under a change in parameter values for the model with additive demand.

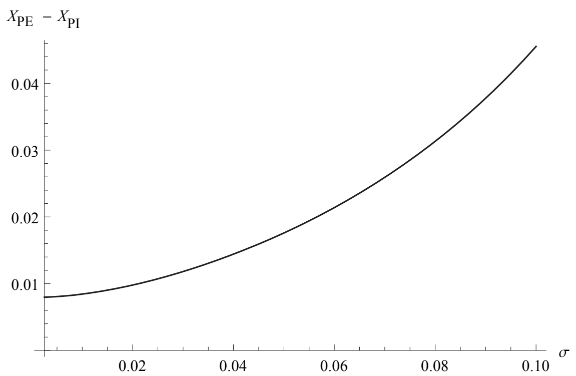


(a) Difference between preemption points with $q_{1I} = q_{1I}^{myop}$.

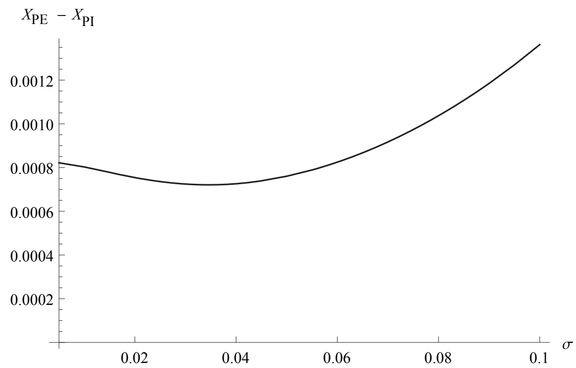


(b) Difference between preemption points with $q_{1I} = 0.01$.

Figure 12: Difference between preemption points for different values of α.

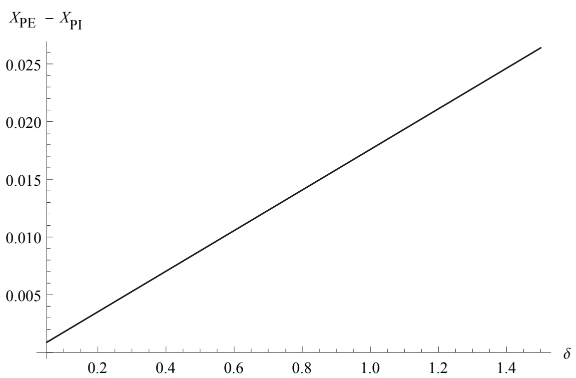


(a) Difference between preemption points with $q_{1I} = q_{1I}^{myop}$.

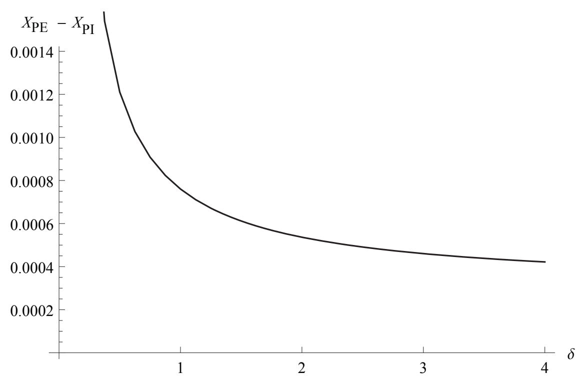


(b) Difference between preemption points with $q_{1I} = 0.01$.

Figure 13: Difference between preemption points for different values of σ.

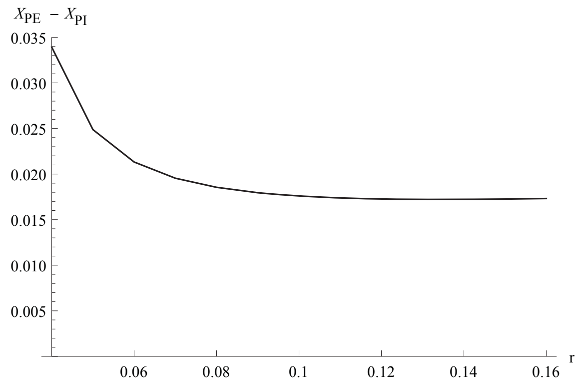


(a) Difference between preemption points with $q_{1I} = q_{1I}^{myop}$.

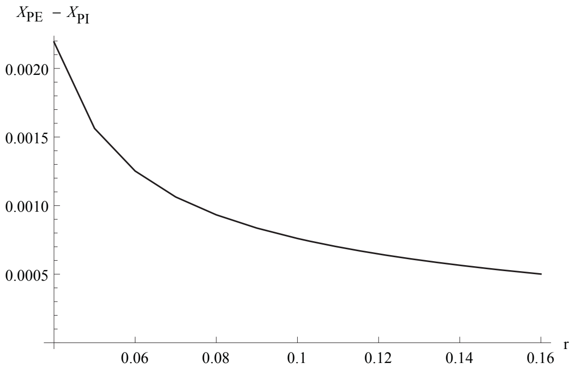


(b) Difference between preemption points with $q_{1I} = 0.01$.

Figure 14: Difference between preemption points for different values of δ.

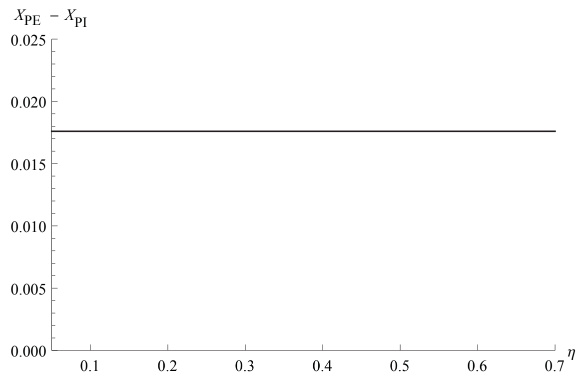


(a) Difference between preemption points with $q_{1I} = q_{1I}^{myop}$.

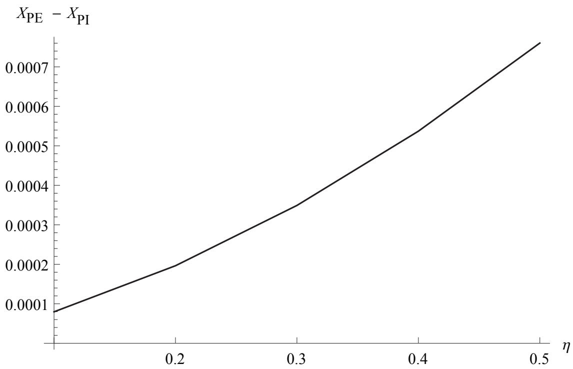


(b) Difference between preemption points with $q_{1I} = 0.01$.

Figure 15: Difference between preemption points for different values of r.



(a) Difference between preemption points with $q_{1I} = q_{1I}^{myop}$.



(b) Difference between preemption points with $q_{1I} = 0.01$.

Figure 16: Difference between preemption points for different values of η.