Essays on Brain Drain and Tax Evasion in a Growth Context

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1.1 Introduction

The recognition of human capital as a growth engine in the theoretical and empirical growth literature raised the question on the statistical significance of human capital flight (brain drain) from less industrialized countries and its influence on growth. The brain drain rates (the proportion of working individuals above 25 with tertiary education working abroad) estimated by Docquier and Marfouk (2004) in 2000 were not negligible, for instance, from Croatia (29,4%), Bosnia and Herzegovina (28.6%), Macedonia (20.9%), Serbia and Montenegro (17.4%), Slovakia (15.3%), Romania (14.1%), Greece (14%). These countries export prevailingly high-educated individuals despite the migration of some low-skilled migrants.

The influence of brain drain on growth is still a contradictory issue. From on hand, brain drain is *ex–post* seen as detrimental to growth of the sending country as it decreases the level of human capital (see Miyagiwa (1991), Haque and Kim (1995), Wong and Yip (1999)). From the other hand, brain drain from the source country *may* exert *ex–ante* a positive effect on human capital formation and growth. The intuition in the work of Mountford (1997), Vidal (1998), Stark *et al.* (1997), Beine *et al.* (2001) is as follows: due to the prospect of migration agents have an incentive to invest in education; still, because not all leave the source country, a probability exists that the level of the average human capital increases and stimulates growth at home (the so–called *brain gain* theory).

The assumption on the occupational choice of the brain drain literature rests on the setting that agents in the domestic country remain either low–skilled or become skilled workers, whose earnings are safe. In this respect the brain drain literature omits two growth relevant factors, which may further influence its results. Those are the existence of (i) risk–taking (skilled) entrepreneurship and (ii) risk in the occupational choice of educated workers. Uncertainty in the employment generating (skilled) entrepreneurship and the educational decision of workers could

ex–ante decrease the share of those willing to invest in human capital. These two factors could be especially important for testing the robustness of the traditional *brain gain* theory, which does not exclude the existence of a favorable effect of skilled outmigration on growth.

Our incentive to extend the *brain gain* analysis with the topic of uncertainty in occupational choice has empirical and theoretical grounds as well. (Risk–taking) entrepreneurship has been emphasized as a growth engine empirically (see Audretsch and Thurik (2000), Audretsch *et al.* (2002), and Carree *et al.* (2002)) and theoretically (see Romer (1990), Chou and Shy (1991), Grossman and Helpman (1991), Schmitz (1989), Aghion and Howitt (1992), Iyigun and Owen (1999), Clemens and Heinemann (2006), Clemens (2008), and Clemens and Heinemann (2009)). Our motivation to model entrepreneurs as skilled agents stems additionally from the recent empirical observation that human capital of entrepreneurs is a favorable prerequisite for the establishment and growth of a firm (see van Praag and van Stel (2011), Kim *et al.* (2006), Taye (2006)). Risk in human capital investment (of workers), on the other hand, has been highlighted as an economic factor decreasing educational investment in the theoretical discussion by Levhari and Weiss (1974), Rillaers (1998), Krebs (2003). By incorporating the occupational choice under risk in the probabilistic *brain gain* theory, we look for answers to the following questions: (i) Does skilled outmigration erode entrepreneurship and eventually decrease human capital accumulation? (ii) Do risk measures reduce the incentive of risk averse and risk–bearing skilled agents to invest in human capital and in this way decrease the likelihood that brain gain takes place? (iii) What is the quantitative impact of skilled migration and risk in occupational choice on growth and welfare? We discuss these issues in what follows.

The unexplored relationship between occupational choice under risk and brain drain in the context a two–period overlapping generations endogenous growth model with human capital accumulation is the core issue of our analysis. The model draws on the work of Kanbur (1979), and Clemens (2008) for occupational choice under risk and Beine *et al.* (2001) for human capital accumulation and probabilistic brain drain. The domestic economy consists of two sectors: a traditional and a modern sector both producing an identical consumption good but by a different technology. At the beginning of life *ex–ante* homogeneous and risk–averse agents simultaneously make a human capital decision and an occupational choice entailing either certain or uncertain income. An agent may decide either to become high–skilled by investing educational time in one's human capital or to remain low–skilled both periods. Low–skilled workers obtain sure income in the traditional sector, educated become skilled workers or entrepreneurs (who employ skilled workers) in the modern sector. High–skilled workers are *ex–ante* unaware of their labor productivities, while entrepreneurs experience variation in their profits due to

a technology shock, which is beyond their control. Some high–skilled workers are randomly selected to work abroad for a higher foreign wage at the beginning of the second period. In equilibrium the expected utility of *ex–ante* homogeneous and risk averse agents is equal due to the expected utility arbitrage argument, which determines the distribution of individuals across occupations.

Our assumption that would–be entrepreneurs are home bound (they do not migrate and do not establish a business abroad, i.e. migrants are employed as skilled workers) could be justified by the empirical evidence that the average propensity to set up a business with more than 20 workers in the OECD countries is lower for foreigners than for natives according to OECD (2010) in Table 1.3 (an exception to this rule among the countries in Western Europe is the UK with foreigners, who are more prone to set up firms than natives). This empirical evidence makes us believe that migrants abroad take up an occupation rather as a worker than an entrepreneur. Moreover, van Praag (2009) shows that the perception of status[1] is correlated with the probability of opting for entrepreneurship, which is evidence for our assumption that it is skilled workers (and not agents determined to develop their own business) who migrate abroad. Growth in our model depends positively on the share of those willing to obtain education. The decision on education, on the other hand, is tantamount to willingness to take on risk because skilled workers are subject to risk in their earnings stemming from *ex–ante* unknown labor productivity, while entrepreneurs obtain income dependent on a technology shock. The educational decision of skilled workers, moreover, is influenced by the probability of migration. That is how the interplay between brain drain and occupational choice under risk determines the economic development of the domestic country.

According to our theoretical model, a higher brain drain rate biases the occupational choice of agents away from entrepreneurship and may increase skilled workers' employment if the share of skilled wage earners remaining at home is relatively low (lower than 36% for a brain drain probability of 30% and lower than 56.9% for a brain drain probability of 5% and under the assumption of a foreign wage, which is 4.8 time higher than the domestic skilled labor income). A higher gap between the earnings of skilled workers in the domestic country and abroad decreases entrepreneurship but increases skilled workers' employment. A larger risk measure in the occupational choice of both entrepreneurs and skilled wage earners reduces their respective shares. As a consequence of the equilibrium occupational choice, growth improves with a larger skilled migration probability only if the share of skilled wage earners in the domestic coun-

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[1]Notice that we drop the issue of status in the risky occupational choice of entrepreneurs. For a discussion on this topic see Clemens (2006)

try improves, but unambiguously rises with a higher wedge between skilled workers' earnings at home and abroad. On the other hand, a higher risk measure in the occupational choice of skilled employees decreases human capital accumulation, while a higher risk measure in entrepreneurial profits has an ambiguous effect on growth. Our calibration shows that in terms of growth and welfare the brain drain probability has a stronger (positive) effect than the gap in skilled wages between the foreign and the domestic country in the short and the long run. Larger risk in the occupational choice of skilled workers' exhibits a stronger (negative) impact on growth and, therefore, long–term welfare compared to risk of entrepreneurial profits. However, larger risk in entrepreneurial profits has a much more pronounced negative effect on short–term welfare compared to the risk in skilled workers' earnings.

This paper is divided as follows: In Section 1.2 the general assumptions of the model are presented. In Section 1.3 we specify the market equilibrium. In Section 1.4 and 1.5 we conduct a sensitivity analysis and calibration. In Section 1.6 we perform a welfare analysis. Section 1.7 is devoted to the conclusion.

1.2 The Model

1.2.1 The Household Sector

Ex–ante homogeneous and risk averse agents live for two periods in the framework of an overlapping generations model. Each individual is endowed with one unit of labor, which he inelastically supplies to the market. The population is normalized to one, which means that each generation is equal to 0.5 and there is no population growth.

At the beginning of life agents make an educational and occupational choice. An agent may remain uneducated both periods. Those who decide to gain human capital work as low–skilled when young and spend some time in education ν, which is predetermined by the government. At the second period the educated agent may become either a skilled worker or an entrepreneur. Following Yakita (2003) we assume that low–skilled (young and middle–aged) living at period t obtain a base wage $\overline{w}_t$ augmented by the low–skilled human capital $h_t^l = h_{t-1}\gamma_t$, where $\gamma_t > 0$ is the knowledge spillover (determined endogenously), while h_{t-1} is the average human capital of the previous period. Total low–skilled human capital is in practice inherited for free.

A skilled worker i born at period t obtains the base wage $\overline{w}_{t+1}$ augmented by *ex–ante* unknown abilities a_i and the human capital h_{t+1}^w when middle–aged at period $t + 1$. Skilled workers' abilities a follow a lognormal distribution with mean and variance equal to $E[\ln a] = \mu_a$, $Var[\ln a] = \sigma_a^2$.

An entrepreneur j born at period t obtains profit π_{jt+1} when middle–aged at period $t+1$, which is influenced by a production idiosyncratic shock θ following a lognormal distribution with mean and variance equal to $E[\ln\theta] = \mu_\theta$, $Var[\ln\theta] = \sigma_\theta^2$. Let the profit per entrepreneur's human capital $\overline{\pi}_{jt+1}$ be augmented by the entrepreneurial human capital h_{t+1}^e. Moreover, we assume that the risk in skilled workers' earnings is lower than the technological risk in entrepreneurs' profits $\sigma_a < \sigma_\theta$ in order to replicate the empirical observation that entrepreneurs' income is riskier than workers' wages. The correlation between a and θ is zero.

The winning tickets to leave the home country in the second period are distributed randomly only among educated workers. The probability that a skilled worker is selected to go abroad is equal to p^a. If a skilled worker obtains the winning ticket to work abroad, one receives a foreign wage which is a share $\phi^f > 1$ of skilled workers' productivity in the domestic country[2], i.e. even abroad the agent obtains earnings in correspondence to one's abilities. Firm owners do not migrate abroad. Throughout this paper we use the index w for skilled workers, e for entrepreneurs, l for low–skilled workers, f for the foreign economy and t for a period if not otherwise stated.

An individual born at t who decides to invest in education and become a high–skilled worker at period $t + 1$ consumes c_t^w in the first and c_{it+1}^w in the second period if employed as a skilled worker at home and c_{it+1}^f if one is abroad. The expected lifetime utility of a skilled worker i with abilities a_i who stands a probability of migration p^a at period $t + 1$ is,

$$V_{i,t,t+1}^w = \ln(c_t^w) + \beta(1 - p^a)\ln(c_{it+1}^w) + \beta p^a \ln(c_{it+1}^f) \qquad 0 < \beta < 1 \qquad (1.1)$$

where β is the discount factor of future utility of consumption. The first and second period consumption is exactly equal to the obtained income at the respective period due to the lack of a capital market,

$$c_t^w = (1 - \nu)\overline{w}_t h_t^l \qquad (1.2)$$

$$c_{it+1}^w = a_i h_{t+1}^w \overline{w}_{t+1} \qquad (1.3)$$

$$c_{it+1}^f = \phi^f a_i h_{t+1}^w \overline{w}_{t+1} \qquad (1.4)$$

The human capital of skilled workers and entrepreneurs is linear in educational time ν with $A > 0$,

$$h_{t+1}^k = (1 + A\nu)h_t \qquad k \in \{w, e\} \qquad (1.5)$$

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[2]Our assumption on the foreign wage is a simplification of the postulation of Beine *et al.* (2001), who set the income growth abroad after education to be higher than the income growth at home.

Because entrepreneurs and skilled workers have to spend the same time ν in school (which is determined by the government), their human capital across periods is equal, i.e. $h_t^w = h_t^e \; \forall t$. Still, we keep the indices e and w for tractability.

An agent born at t who decides at the beginning of one's life to gain human capital and to become an entrepreneur at period $t+1$ consumes in both periods, c_t^e and c_{jt+1}^e. The welfare of an entrepreneur is then,

$$V_{j,t,t+1}^e = \ln(c_t^e) + \beta \ln(c_{jt+1}^e) \tag{1.6}$$

Total income is spent on consumption at the respective period,

$$c_t^e = (1 - \nu)\overline{w}_t h_t^l \tag{1.7}$$

$$c_{jt+1}^e = h_{t+1}^e \overline{\pi}_{jt+1} \tag{1.8}$$

Low–skilled born at t consume in both periods their total income and have, therefore, welfare equal to,

$$V_{t,t+1}^l = \ln(c_t^l) + \beta \ln(c_{t+1}^l) \tag{1.9}$$

$$c_t^l = \overline{w}_t h_t^l \tag{1.10}$$

$$c_{t+1}^l = \overline{w}_{t+1} h_{t+1}^l \tag{1.11}$$

1.2.2 Production Sectors

There are two sectors in the economy operating in a perfectly competitive market (traditional and modern) both producing an identical consumption good but by a different technology. The price of the good is normalized to 1 in both sectors for simplicity. In the traditional sector the production is linear in the human capital of low–skilled labor H_t^l,

$$Q_t = B H_t^l \tag{1.12}$$

where $B > 0$ is a productivity parameter, $H_t^l = h_t^l L_t^l$ with L_t^l equal to the demanded share of low–skilled labor in the traditional sector.

The modern sector, on the other hand, comprises p_t^e skilled entrepreneurs who demand skilled workers. The technology in the modern sector is identical for each entrepreneur and assumes the following form,

$$f_{jt} = \theta_{jt}(h_t^e)^{1-\alpha}(h_t^w L_{jt}^w)^\alpha \qquad 0 < \alpha < 1. \tag{1.13}$$

where the subscript j stands for a firm j; L_{jt}^w signifies the demanded skilled workers by entrepreneur j; θ_{jt} is a lognormally distributed idiosyncratic technology shock at period t, which is

non–diversifiable and uncorrelated across entrepreneurs; we normalize $\mu_\theta = \frac{-\sigma_\theta^2}{2(1-\alpha)}$ and $\mu_a = \frac{-\sigma_a^2}{2}$ to avoid size effects stemming from the means of the statistic distributions.[3]

We assume that entrepreneurs employ skilled labor when the technology shock θ_{jt} has realized so that they do not have to lay off skilled labor force in case of bad realization of the shock. We further assume that agents cannot switch between professions, which ensures lack of dynamics in the labor supply decision.

1.3 Market Equilibrium

1.3.1 Labor Market Equilibrium in the Traditional Sector

The profit of the representative firm in the traditional sector,

$$\Pi_t = BH_t^l - \overline{w}_t h_t^l L_t^l \tag{1.14}$$

is zero because low–skilled labor is the only input of production and it is remunerated according to its marginal productivity. The optimization of the profit with respect to the low–skilled labor L_t^l implies that the base wage is exactly equal to the productivity parameter in the traditional sector,

$$\overline{w}_t = B \tag{1.15}$$

Because in the labor market equilibrium the supply of low–skilled must be equal to the demand for low–skilled, it should hold that

$$L_t^l = 1 - p_t^w - p_t^e - M_t \tag{1.16}$$

where it can be shown that $M_t = \frac{p^a}{1-p^a} p_t^w$ is the equilibrium share of migrants with p_t^w equal to the share of skilled workers remaining in the domestic country.

1.3.2 Labor Market Equilibrium in the Modern Sector

As the production of an entrepreneur exhibits decreasing returns to scale with respect to the human capital of skilled workers, each entrepreneur obtains a profit (managerial wage) after the wage bill is paid. The profit (managerial wage) of an entrepreneur is defined as,

$$\pi_{jt} = \theta_{jt}(h_t^e)^{1-\alpha}(h_t^w L_{jt}^w)^\alpha - h_t^w L_{jt}^w \overline{w}_t \tag{1.17}$$

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[3]Notice that by this assumption we postulate that $E(a) = 1$ and $E(\theta) = exp(-\alpha/(1-\alpha)\sigma_\theta^2/2)$, i.e. it is possible that $E(a) \gtrless E(\theta)$, but what matters for the mean preserving spread of expected profits and expected skilled wages (as we will see later) is that $E(a) = E(\theta^{\frac{1}{1-\alpha}}) = 1$.

The average labor costs in the modern sector are independent of the moments of the lognormal distribution. This result stems from the normalization of the mean of skilled workers' abilities, which we undertook in section 1.2.2. Entrepreneur j maximizes the profit (managerial wage) in the modern sector with respect to skilled labor, which yields the skilled labor demand of firm j,

$$L_{jt}^w = \left(\frac{\alpha\theta_{jt}}{\overline{w}_t}\right)^{\frac{1}{1-\alpha}} \tag{1.18}$$

Because the share of skilled workers' income in the production of a firm (if we consider the optimal demand for high–skilled workers) is,

$$\alpha f_{jt} = h_t^w L_{jt}^w \overline{w}_t \tag{1.19}$$

the profit (managerial wage) of an entrepreneur should be defined as,

$$\pi_{jt} = (1-\alpha)\theta_{jt}(h_t^e)^{1-\alpha}(h_t^w L_{jt}^w)^\alpha \tag{1.20}$$

Plugging (1.18) into (1.20), we obtain,

$$\pi_{jt} = (1-\alpha)\theta_{jt}^{\frac{1}{1-\alpha}} h_t^e \left(\frac{\alpha}{\overline{w}_t}\right)^{\frac{\alpha}{1-\alpha}} \tag{1.21}$$

Now it remains to determine the base skilled wage in (1.21) in order to find the profit of entrepreneur j. We use the individual demand for skilled workers (1.18) to obtain the aggregate demand for skilled workers.

$$\left(\frac{\alpha}{\overline{w}_t}\right)^{\frac{1}{1-\alpha}} \int_{\theta\in\Theta} \int_0^{p_t^e} \theta_{jt}^{\frac{1}{1-\alpha}} f(\theta)d\theta dj = p_t^w \tag{1.22}$$

Because of the normalization of μ_θ in Section 1.2.2, the labor market equilibrium for high–skilled is independent of the moments belonging to $f(\theta)$,

$$\left(\frac{\alpha}{\overline{w}_t}\right)^{\frac{1}{1-\alpha}} p_t^e = p_t^w \tag{1.23}$$

Then the base wage, which should be equal to the base wage in the traditional sector in equation (1.15), can be derived after we rearrange equation (1.23),

$$\frac{\alpha(p_t^e)^{1-\alpha}}{(p_t^w)^{1-\alpha}} = \overline{w}_t \tag{1.24}$$

and, moreover, we have that the expected skilled wage is equal to

$$\frac{\alpha(p_t^e)^{1-\alpha}}{(p_t^w)^{1-\alpha}} h_t^w = \overline{w}_t h_t^w \tag{1.25}$$

The expected high–skilled wage (1.25) increases with the share of entrepreneurs p_t^e and the human capital of skilled wage earners h_t^w but decreases with the share of skilled workers p_t^w.

If we plug equation (1.24) in (1.21), we obtain that the profit (managerial wage) is,

$$\pi_{jt} = (1-\alpha)\theta_{jt}^{\frac{1}{1-\alpha}} h_t^e (p_t^w)^\alpha (p_t^e)^{-\alpha} \tag{1.26}$$

The entrepreneur's income (1.26) decreases with the share of entrepreneurs p_t^e but increases with their human capital h_t^e and the share of skilled workers p_t^w.

The expected profit (managerial wage) after integrating equation (1.26) over the realization of the technology shock in the modern sector is defined as,

$$E(\pi_{jt}) = (1-\alpha)h_t^e (p_t^w)^\alpha (p_t^e)^{-\alpha} \tag{1.27}$$

The base profit (the profit (1.26) per entrepreneur's human capital unit) is,

$$\bar{\pi}_{jt} = (1-\alpha)\theta_{jt}^{\frac{1}{1-\alpha}} (p_t^w)^\alpha (p_t^e)^{-\alpha} \tag{1.28}$$

The base expected profit or managerial wage (integrating (1.28) over the technology shock in the modern sector) is defined as,

$$\bar{\pi}_t = (1-\alpha)(p_t^w)^\alpha (p_t^e)^{-\alpha} \tag{1.29}$$

1.3.3 Total Output

The output in the modern sector is summation of the income of entrepreneurs and skilled workers. Given equations (1.25) and (1.26), we come up with total production in the modern sector equal to,

$$Y_t = p_t^w \bar{w}_t h_t^w \int_{a\in\bar{A}} a_i f(a) da + p_t^e \int_{\theta\in\Theta} \pi_{jt} f(\theta) d\theta$$
$$Y_t = (p_t^w h_t^w)^\alpha (h_t^e p_t^e)^{1-\alpha} \tag{1.30}$$

The output in the traditional sector is summation of the income of low–skilled workers,

$$Q_t = \bar{w}_t h_t^l \left(1 - \frac{p_t^w}{1-p^a} - p_t^e\right) \tag{1.31}$$

Because the modern and traditional sector produce identical goods, we let the share of low–skilled income to total income in the economy be constant over time and equal to b in line with the neoclassical growth theory,

$$\frac{Q_t}{Q_t + Y_t} = b \tag{1.32}$$

If we rearrange (1.32) and apply (1.30) and (1.31), we obtain a relationship between the low–skilled income and high–skilled income defined by the endogenous variables in the model,

$$\overline{w}_t h_t^l \Big(1 - \frac{p_t^w}{1 - p^a} - p_t^e\Big) = \frac{b}{(1 - b)} (p_t^w h_t^w)^\alpha (h_t^e p_t^e)^{1-\alpha} \tag{1.33}$$

Until now we defined the endogenous income variables (the skilled and low–skilled wage, the entrepreneur's profit, total income in the traditional and modern sector) in terms of the share of entrepreneurs p_t^e and skilled workers p_t^w in the population and γ_t, but we have not yet determined how agents select occupations. This is our next step.

1.3.4 Equilibrium Occupational Choice

Proposition 1.1 *In equilibrium ex–ante homogeneous and risk averse individuals should be ex–ante indifferent between an occupation of a low–skilled worker, who obtains safe income, and a risk–bearing high–skilled individual. The indifference of economic agents between an occupation under risk and non–risky profession, even with a strict preference for migration of high–skilled workers, defines endogenously the distribution of individuals across occupations in the domestic country.*

The *ex–ante* indifference of agents for choosing an occupation with risky or certain income stems from the fact that all individuals are risk averse and *ex–ante* homogeneous. If individuals were not *ex–ante* homogeneous and had advantage over the others in exercising a profession, they would self–select into it. If *ex–ante* homogeneous agents were not indifferent to exercise an occupation, because it brought higher utility, they would choose the profession with the highest welfare, which would lead to the extinction of a specific occupation. Therefore,

$$E(V_{j,t,t+1}^e) = \int_{\theta \in \Theta} \Big[\ln((1 - \nu)\overline{w}_t h_t^l) + \beta \ln((1 + A\nu)h_t \overline{\pi}_{jt+1}) \Big] f(\theta) d\theta =$$

$$E(V_{i,t,t+1}^w) = \int_{a \in \overline{A}} \Big[\ln((1 - \nu)\overline{w}_t h_t^l) + \beta \ln(a_i(1 + A\nu)h_t \overline{w}_{t+1}\phi^{p^a}) \Big] f(a) da =$$

$$V_{t,t+1}^l = \ln(\overline{w}_t h_t^l) + \beta \ln(\overline{w}_{t+1} h_{t+1}^l)$$

The above integration boils down to finding the mean of $E(\ln a_i^\beta)$ and $E(\ln \theta_j^{\frac{\beta}{1-\alpha}})$,

$$E(\ln a_i^\beta) = -\beta \frac{\sigma_a^2}{2} \qquad E(\ln \theta_j^{\frac{\beta}{1-\alpha}}) = -\beta \frac{\sigma_\theta^2}{2(1 - \alpha)^2}$$

We substitute for the definitions of the expected utilities, use the normalization of μ_a and μ_θ that we did in Section 1.2.2 as well as equation (1.33) and obtain a system of three equations

and three unknowns (p_t^w, p_t^e, γ_t),

$$\ln\left[((1-\nu)\overline{w}_t h_t^l)(h_t^w \overline{w}_{t+1})^\beta exp\left(-\frac{\beta\sigma_a^2}{2}\right)(\phi^f)^{p^a\beta}\right]$$
$$= \ln\left[((1-\nu)\overline{w}_t h_t^l)(h_{t+1}^e \overline{\pi}_{t+1})^\beta exp\left(-\frac{\beta\sigma_\theta^2}{2(1-\alpha)^2}\right)\right] \tag{1.34}$$

$$\ln\left[((1-\nu)\overline{w}_t h_t^l)(h_{t+1}^w \overline{w}_{t+1})^\beta exp\left(-\beta\frac{\sigma_a^2}{2}\right)(\phi^f)^{p^a\beta}\right] = \ln\left[(\overline{w}_t h_t^l)(\overline{w}_{t+1} h_{t+1}^l)^\beta\right] \tag{1.35}$$

$$\overline{w}_t h_t^l\left(1 - \frac{p_t^w}{1-p^a} - p_t^e\right) = \frac{b}{(1-b)}(p_t^w h_t^w)^\alpha (h_t^e p_t^e)^{1-\alpha} \tag{1.36}$$

Proposition 1.2 *The equilibrium share of entrepreneurs, (low–)skilled workers, migrants and the human capital spillover are constant over time and equal to[4],*

$$p^e = \frac{F}{b(1-p^a) + N + F} \tag{1.37}$$

$$p^w = \frac{(1-p^a)N}{b(1-p^a) + N + F} \tag{1.38}$$

$$M = \frac{p^a N}{b(1-p^a) + N + F} \tag{1.39}$$

$$L^l = \frac{b(1-p^a)}{b(1-p^a) + N + F} \tag{1.40}$$

$$\gamma = (1-\nu)^{\frac{1}{\beta}}(\phi^f)^{p^a} exp\left(-\frac{\sigma_a^2}{2}\right)(1 + A\nu) \tag{1.41}$$

where $F = (1-\alpha)(1-b)(1-p^a)(1-\nu)^{\frac{1}{\beta}} exp(-\frac{\sigma_\theta^2}{2(1-\alpha)^2})$, $N = \alpha(1-b)(\phi^f)^{p^a} exp(-\frac{\sigma_a^2}{2})(1-\nu)^{\frac{1}{\beta}}$

Proof: See Appendix 1.9.1

Corollary 1.1 *The base wage* $\overline{w}_t$ *and the expected base income of entrepreneurs* $\overline{\pi}_t$ *are constant over time.*

Proof. The base wage $\overline{w}_t$ and the expected base profit $\overline{\pi}_t$ should be constant if the distribution of agents across professions is constant. ∎

Proposition 1.3 *Risk averse and skilled agents (entrepreneurs and skilled workers) obtain a risk and skill premium over the certain low–skilled wage because of higher labor productivity and as reimbursement for facing uncertainty in income. The risk premium that entrepreneurs gain is higher than the risk premium of skilled workers as firm owners bear more risk, i.e.* $\sigma_\theta^2 > \sigma_a^2$ *by assumption.*

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[4]The share of low–skilled could be calculated by the assumption that the population share is normalized to one, while $M = \frac{p^a}{1-p^a}p^w$. From now on we leave the time subindex for the variables which are proved to be constant.

Proof. The expected risk premium of entrepreneurs over the expected skilled wage ϕ_t,

$$\phi_t = E(\pi_{jt}) - \overline{w}h_t^w$$

$$\phi_t = (1-\alpha)\Big(\frac{p^w}{p^e}\Big)^\alpha h_t^e\Big[1 - exp\Big(-\frac{1}{2}\Big(\frac{\sigma_\theta^2}{(1-\alpha)^2} - \sigma_a^2\Big)\Big)\frac{1}{(\phi^f)^{p^a}}\Big] \qquad (1.42)$$

is always positive because of the assumptions that $\phi^f > 1$, $0 < \alpha < 1$ and $\sigma_\theta^2 > \sigma_a^2$,

$$(\phi^f)^{p^a} - exp\Big(-\frac{1}{2}\Big(\frac{\sigma_\theta^2}{(1-\alpha)^2} - \sigma_a^2\Big)\Big) > 0 \Leftrightarrow p^a \underbrace{\ln\phi^f}_{>0} > -\frac{1}{2}\underbrace{\Big(\frac{\sigma_\theta^2}{(1-\alpha)^2} - \sigma_a^2\Big)}_{>0}$$

The risk and skill premium of skilled workers over the sure income of low–skilled ϕ'_t,

$$\phi'_t = \overline{w}h_t^w - \overline{w}h_{t-1}\gamma$$

$$\phi'_t = \overline{w}h_{t-1}[(1 + A\nu) - \gamma] \qquad (1.43)$$

is always positive under the assumption that $1 + A\nu > \gamma$. Therefore, the expected risk and skill premium of entrepreneurs over the low–skilled income is positive under the same assumption.

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1.3.5 Income Distribution and Growth

Having made the assumption in equation (1.33) and with the help of equations (1.25) and (1.27), we can determine the income distribution in terms of income shares. The income share of low–skilled workers is, as we assumed, equal to b. Therefore, the income shares of high–skilled agents, entrepreneurs, and high–skilled workers are respectively,

$$\frac{Y_t}{Q_t + Y_t} = (1-b) \qquad \frac{E(\pi_{jt})p^e}{Q_t + Y_t} = (1-b)(1-\alpha) \qquad \frac{\overline{w}h_t^w p^w}{Q_t + Y_t} = (1-b)\alpha \qquad (1.44)$$

In line with the neoclassical growth theory, income shares are constant. Entrepreneurs' income and high–skilled workers' earnings are lognormally distributed because of the presence of the lognormally distributed idiosyncratic technological shock and skilled workers' abilities respectively. Low–skilled workers obtain a uniform low–skilled wage. As we already showed, entrepreneurs and skilled workers obtain a risk and skill premium over the safe low–skilled wage because they bear occupational risk and are more productive after education. Expected entrepreneurs' profits are higher than expected skilled workers' wages because entrepreneurs experience a higher variance in their income distribution by assumption.

It remains to determine only the growth of average human capital in the economy. The average human capital in the economy is equal to the human capital which is accumulated by skilled

individuals and the human capital which is inherited for free by low–skilled. Therefore, the growth rate g^h in the economy is,

$$1 + g^h = \frac{p^w(1 + A\nu)}{1 - M} + \frac{p^e(1 + A\nu)}{1 - M} + \frac{\left(1 - \frac{p^w}{1 - p^a} - p^e\right)\gamma}{1 - M} \qquad (1.45)$$

After substituting for the equilibrium occupational choice and rearranging, we obtain for g^h,

$$1 + g^h = (1 + A\nu)\frac{p^w}{1 - M}\left(\frac{1 - \alpha}{\alpha(\phi^f)^{p^a}}exp\left(-\frac{1}{2}\left(\frac{\sigma_\theta^2}{(1 - \alpha)^2} - \sigma_a^2\right)\right) + \frac{\alpha(1 - b) + b}{\alpha(1 - b)}\right) \qquad (1.46)$$

where $p^e/p^w = \frac{1 - \alpha}{\alpha(\phi^f)^{p^a}}exp\left(-\frac{1}{2}\left(\frac{\sigma_\theta^2}{(1 - \alpha)^2} - \sigma_a^2\right)\right)$, $M = p^a p^w/(1 - p^a)$ and the equilibrium share of skilled workers p^w in (1.46) has been already defined in the previous subsection.

1.4 Sensitivity Analysis

In this section we provide a sensitivity analysis of the equilibrium occupational shares, the growth equation and the income variables with respect to the probability of migration p^a, the wedge between the skilled wage at home and abroad ϕ^f, and the risk measures σ_a and σ_θ.

Proposition 1.4

(i) The influence of p^a on p^w for $p^w \lessgtr \frac{(1 - p^a)^2 \ln \phi^f}{1 + (1 - p^a)\ln \phi^f}$, and the influence of p^a on p^e and L^l is,[5]

$$\frac{\partial p^w}{\partial p^a} \gtrless 0 \qquad \frac{\partial p^e}{\partial p^a} < 0 \qquad \frac{\partial L^l}{\partial p^a} < 0$$

(ii) The influence of ϕ^f on p^w, p^e, and L^l is,

$$\frac{\partial p^w}{\partial \phi^f} > 0 \qquad \frac{\partial p^e}{\partial \phi^f} < 0 \qquad \frac{\partial L^l}{\partial \phi^f} < 0$$

Proof: See Appendix 1.9.2.

As we see from Proposition 1.4, an increase in the probability of skilled workers' migration improves the share of skilled workers remaining at home only if the equilibrium share of skilled workers in the domestic country is relatively small. This result can be explained as follows: from one hand, a higher skilled migration probability increases the expected welfare of skilled workers (for a constant expected skilled wage), which attracts more agents in education; from the other hand, the competition between skilled agents remaining in the domestic country rises and the expected skilled wage and, therefore, utility fall with higher p^a, reducing the share of native agents willing to become skilled employees; third the negative impact of p^a on p^w can

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[5]The structure $\frac{\partial y}{\partial x} \lessgtr 0$ if $a \gtrless b$ is to read: $\frac{\partial y}{\partial x} < 0$ if $a > b$ and $\frac{\partial y}{\partial x} > 0$ if $a < b$ in all propositions

be strengthened by the *ex–post* decline in the share of domestic educated workers due to skilled outmigration. Only if the first effect is stronger than the others, a larger brain drain rate leads to a higher share of skilled wage earners (net of migration). For the positive effect of p^a on p^w to take place, the equilibrium share of skilled workers should not exceed 56.9% if $p^a = 0.05$, 44.5% if $p^a = 0.2$, and 36.6% if $p^a = 0.3$ for a value of $\phi^f = 4.8$, which we use in our calibration.[6] These threshold values of p^w are far beyond the empirically observed shares of skilled workers in Eastern Europe. An increase in p^a, on the other hand, decreases the equilibrium share of entrepreneurs and low-skilled workers. Entrepreneurship falls with p^a because the relatively less risky foreign wage is attractive enough for risk-averse agents to switch away from risk-bearing firm ownership to the end of migration. The share of low-skilled individuals decreases with p^a because agents perceive the opportunity to invest in human capital, which is a prerequisite to earn a higher foreign wage abroad, as more rewarding despite the presence of risk in skilled earnings.

A higher wedge in the payment between skilled workers' income at home and abroad ϕ^f has a clear positive effect on the equilibrium share of skilled workers remaining at home but a negative influence on the share of entrepreneurs and low-skilled individuals. This effect is a natural consequence of the improvement of skilled workers' welfare due to a rise in ϕ^f.

Proposition 1.5

(i) The influence of the risk measure σ_a on p^w, p^e, and L^l is,

$$\frac{\partial p^w}{\partial \sigma_a} < 0 \qquad \frac{\partial p^e}{\partial \sigma_a} > 0 \qquad \frac{\partial L^l}{\partial \sigma_a} > 0$$

(ii) The influence of the risk measure σ_θ on p^w, p^e, and L^l is,

$$\frac{\partial p^w}{\partial \sigma_\theta} > 0 \qquad \frac{\partial p^e}{\partial \sigma_\theta} < 0 \qquad \frac{\partial L^l}{\partial \sigma_\theta} > 0$$

Proof: See Appendix 1.9.2.

As we see from Proposition 1.5, a higher risk measure σ_a reduces the equilibrium share of risk-averse skilled workers. The equilibrium share of entrepreneurs and low-skilled workers increases with σ_a due to risk aversion. In the same token, the risk measure σ_θ has a negative effect on the share of entrepreneurs and a positive effect on the share of skilled and low-skilled workers due to risk aversion.

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[6] For higher ϕ^f the threshold p^w improves.

Proposition 1.6 *The influence of p^a, ϕ^f, and σ_a as well as σ_θ on growth g^h is,*

$$(i) \quad \frac{\partial(1+g^h)}{\partial p^a} > 0 \qquad if \qquad p^w < \frac{(1-p^a)^2 \ln \phi^f}{1+(1-p^a)\ln \phi^f}$$

$$(ii) \quad \frac{\partial(1+g^h)}{\partial \phi^f} > 0$$

$$(iii) \quad \frac{\partial(1+g^h)}{\partial \sigma_a} < 0$$

$$(iv) \quad \frac{\partial(1+g^h)}{\partial \sigma_\theta} \gtrless 0$$

Proof: See Appendix 1.9.3.

In general all parameters which decrease the share of low–skilled agents lead to a rise in the accumulation of human capital. This effect can be explained with the inferiority of human capital which low–skilled possess compared to the knowledge of skilled workers and entrepreneurs. In other words, due to the assumption that $1 + A\nu > \gamma$, low–skilled do not contribute to the human capital accumulation as much as skilled agents. Therefore, the influence of brain drain on growth is positive only if p^a induces a higher equilibrium share of domestic skilled workers by decreasing the share of low–skilled. This condition is fulfilled if $p^w < \frac{(1-p^a)^2 \ln \phi^f}{1+(1-p^a)\ln \phi^f}$. It should be noticed that the share of entrepreneurs falls but this is only to the advantage of the share of skilled workers at home (who have equal human capital as entrepreneurs). On the other hand, brain drain has an ambiguous impact on growth if p^w declines with higher p^a because the effect of p^a on the share of skilled migrants M remains unclear.

A rise in the wedge between the payment of skilled wage earners at home and abroad ϕ^f leads to a stronger growth rate because it makes agents switch away from the low–skilled profession to a profession as skilled workers. Even though entrepreneurship declines, it is to the advantage of the share of skilled wage earners at home, who attain the same level of education. In other words, the falling entrepreneurship rate does not exercise any effect on growth as long as p^w increases overproportionally (i.e. if more agents from the low–skilled sector decide to become skilled wage earners in the domestic country). Moreover, a loss in the population share results due to a rising share of skilled migrants, which boosts additionally the accumulation of average human capital.

A higher risk in skilled workers' earnings σ_a makes more individuals opt for a profession as entrepreneurs but also low–skilled, which implies that the total share of skilled agents remaining at home falls. As a result, the growth rate declines, also due to the falling share of skilled migrants, which reduces the human capital per person in society (all other things being constant). A higher risk in entrepreneurial profits σ_θ leads to a rising share of skilled workers but also low–skilled out of the pool of entrepreneurs. In this way the total share of domestic skilled

falls, which would reduce human capital accumulation if there were no other effects on growth. However, a higher share of skilled employees at home is accompanied with a higher share of migrants, which enhances the *average* human capital of agents (all other things being constant) through the positive effect of a declining population on growth.

Proposition 1.7

(*i*) *The influence of p^a, ϕ^f σ_a and σ_θ on the short–run low–skilled income is,*

$$\frac{\partial(\overline{w}h_t^l)}{\partial p^a} > 0 \qquad \frac{\partial(\overline{w}h_t^l)}{\partial \phi^f} > 0 \qquad \frac{\partial(\overline{w}h_t^l)}{\partial \sigma_a} < 0 \qquad \frac{\partial(\overline{w}h_t^l)}{\partial \sigma_\theta} < 0$$

(*ii*) *The influence of p^a, ϕ^f σ_a and σ_θ on the short–run expected high–skilled wage is,*

$$\frac{\partial(\overline{w}h_t^w)}{\partial p^a} < 0 \qquad \frac{\partial(\overline{w}h_t^w)}{\partial \phi^f} < 0 \qquad \frac{\partial(\overline{w}h_t^w)}{\partial \sigma_a} > 0 \qquad \frac{\partial(\overline{w}h_t^w)}{\partial \sigma_\theta} < 0$$

(*iii*) *The influence of p^a, ϕ^f σ_a and σ_θ on the short–run expected entrepreneurial profit is,*

$$\frac{\partial E(\pi_{jt})}{\partial p^a} > 0 \qquad \frac{\partial E(\pi_{jt})}{\partial \phi^f} > 0 \qquad \frac{\partial E(\pi_{jt})}{\partial \sigma_a} < 0 \qquad \frac{\partial E(\pi_{jt})}{\partial \sigma_\theta} > 0$$

Proof: See Appendix 1.9.4.

Proposition 1.7 makes statements about the short–run development of the income of economic agents, i.e. before the parameters' changes are incorporated in growth. In the short run a higher probability of skilled migration decreases the entrepreneurs–skilled workers ratio, which leads to a fall in the expected skilled wage (via the base wage) and to a rise in expected profits (via the expected profit per efficiency labor unit). This outcome is in response to the stronger competition on the labor market for skilled workers and the weaker competition among entrepreneurs. The low–skilled wage grows due to an improving spillover γ despite the decline in the base wage induced by higher p^a.

The increase in the gap between the foreign and domestic skilled wage decreases the expected income of skilled workers but increases the expected earnings of entrepreneurs because of improvement in the skilled workers–entrepreneurs ratio. Higher ϕ^f makes low–skilled income rise due to a larger spillover of human capital γ despite the fall in the base wage.

Skilled workers' risk in earnings σ_a decreases the competition among the falling share of educated wage earners and raises it for the growing share of entrepreneurs, which drives expected skilled wages up and leads to a fall in expected entrepreneurs' profits. The income of low–skilled also falls due to a lower spillover γ although the base wage grows.

Higher entrepreneurial risk σ_θ makes some agents deviate to the relatively safer professions of a skilled or low–skilled worker, which increases the expected profits of entrepreneurs (who experience lower competition) and decreases the nominal income of all workers (due to the fall in the base wage in response to stronger competition on the labor market).

Proposition 1.8 *The long–run influence of p^a, ϕ^f, and the risk measures σ_a and σ_θ on the long–term income of economic agents depends on the impact of p^a, ϕ^f, σ_a and σ_θ on growth g^h.*

Proof: See Appendix 1.9.4.

Proposition 1.8 considers what is the effect of p^a, ϕ^f, σ_a and σ_θ on the long–term income of individuals. While in the short term the sensitivity of income with respect to p^a, ϕ^f, σ_a and σ_θ is determined by the distribution of agents among occupations, in the long term the growth of average human capital is decisive for the influence of p^a, ϕ^f, σ_a and σ_θ on the income variables. This can be explained as follows: the direction of change in earnings in response to a change of a parameter depends on the base income (which is related to the distribution of agents across professions) and on the growth of average human capital. In the long run a parameter change will be incorporated in the growth of average human capital so many times so that the growth effect will dominate the short run effect of altered occupational choice. In this way we can conclude that long–term earnings will be completely dependent on the influence of the parameters on growth.

1.5 Calibration

We calibrate the model by defining the parameters: α, b, β, σ_a, σ_θ, A, p^a and ϕ^f to match empirical data for the benchmark model (BM). We assume that at period $t = 0$ the average human capital is $h_0 = 1$. Furthermore, human capital grows at an annual growth rate of 0.025, which implies that if an agent lives for 60 years (we exclude the retirement), and each period is 30 years, we come up with the value $A = 6.03$ in the growth equation.

We choose $\beta = 0.74$, which implies that the annual discount factor of expected utility for 30 years is 0.99. The educational time ν is set at 0.5, which reflects that agents have to invest 15 years in education in the first period of their life before they become skilled.

The value of b and α are defined to correspond to an empirically observable entrepreneurial income share. For $b = 0.5$ and $\alpha = 0.6$ we obtain that the entrepreneurial income share is equal to 20%, which is close to the quoted profit share in the economic literature (See Clemens (2008), Clemens and Heinemann (2009)).

The measures of risk σ_θ (0.67) and σ_a (0.4) are chosen to target a skilled workers' share of approximately 18% and a high–skilled entrepreneurial rate of 3%. These calibration targets imply that the total share of educated is around 21% in line with empirical data by *Eurostat* (See Appendix 1.9.5). We obtain the targeted value of 3% entrepreneurship rate as follow: (i) first, we consider that the share of skilled entrepreneurs out of total entrepreneurs (according

to Morris (2011) who uses data from the Global Entrepreneurship Monitor (GEM)) is approximately 41.5%,[7] (ii) second, the average entrepreneurship rate in the Eastern European countries is approximately 7%–10% according to Xavier *et al.* (2012) performing their analysis also with data of GEM (depending on whether Latvia and Estonia are respectively excluded or not). This implies that the share of (skilled) entrepreneurs is around 3-4%. Nevertheless, we target an entrepreneurship rate of 3% to mimic the relatively higher job creation of fewer (skilled) entrepreneurs according to Morris (2011).

For the baseline model under brain drain we set the probability of skilled outflow at 5%, which corresponds to the emigration probability of high–skilled from Eastern Europe according to Docquier and Marfouk (2004) (See Appendix 1.9.5 for brain drain rates of separate countries). For ϕ^f we assume that it is equal to 4.8, which reflects the observed gap in the average high–skilled remuneration in Eastern Europe vs. Western and Northern Europe among skilled workers according to *Eurostat* (See Appendix 1.9.5). Based on our calculations with data from *Eurostat*, the average annual remuneration of high–skilled employees in Bulgaria, Greece, Hungary, Poland, Romania, Slovakia for the period 2003 till 2008 is approximately 8300 euro per year, while the average annual remuneration for the same period of high–skilled employees in Austria, Denmark, Finland, Germany, Great Britain, Spain, and Sweden is around 40 000 euro, which implies $\phi^f = 4.8$.

By the resulting specifications we obtain a skill premium equal to approximately 2.56 in the benchmark model. It is higher than the observed skill premium of the Czech Republic, Slovakia, Bulgaria and Romania but lower than the observed skill differential in remuneration in Slovenia and Cyprus according to Freeman and Oostendorp (2000) (See Appendix 1.9.5).

The sensitivity analysis of the parameter p^a as well as ϕ^f, σ_a and σ_θ are shown in Table 1.1. We present the calibration analysis of higher p^a (which is increased stepwise by 100% compared to the benchmark in order to replicate empirically observed brain drain rates), ϕ^f (which is increased by 50% and 100%), σ_a and σ_θ (both increased by 25% and 50% to reflect the observed trends for the share of skilled workers and entrepreneurs). We compare alternative steady states given changes in the parameters of interest and do not explore a transition from one equilibrium occupational choice to the other. We report the income variables in Table 1.1 for $t = 1$ (notice that $h_0 = 1$) so that the low–skilled income, depending on the average human capital from the previous period, is defined.

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[7]The formula for calculation is the sum of high–growth (0.04) and moderate growth entrepreneurship rate (0.06) out of total entrepreneurship plus the low–growth share of entrepreneurship, who are educated $0.04 + 0.06 + 0.9 \cdot 35\% = 41.5\%$, where 35% is the share of skilled entrepreneurs attaining low–growth in their firms. We also assume that all agents with high–growth and low–growth firms are skilled.

Table 1.1: *Calibration Results for p^a, ϕ^f, σ_a, σ_θ*

	BM	p_1^a	p_2^a	p_3^a	ϕ_1^f	ϕ_2^f	$\sigma_{a,1}$	$\sigma_{a,2}$	$\sigma_{\theta,1}$	$\sigma_{\theta,2}$
p^w	0.1826	0.1923	0.2017	0.2107	0.1856	0.1878	0.1761	0.1683	0.1857	0.1873
p^e	0.03	0.0292	0.0283	0.0273	0.0299	0.0298	0.0302	0.0305	0.0138	0.0053
L^l	0.7778	0.7572	0.7344	0.7093	0.7747	0.7726	0.7844	0.7923	0.7907	0.7976
γ	1.5711	1.6993	1.8379	1.9879	1.6033	1.6265	1.502	1.4216	1.5711	1.5711
g^h	0.025	0.027	0.029	0.031	0.0255	0.0259	0.0238	0.0224	0.0244	0.0242
$E(\pi_{jt})$	4.7482	4.977	5.2168	5.4682	4.8063	4.848	4.6217	4.4717	7.6234	13.5974
$\overline{w}h_t^w$	1.1694	1.1333	1.0983	1.0644	1.16	1.1533	1.1907	1.2172	0.8529	0.5799
$\overline{w}h_t^l$	0.4576	0.4797	0.5028	0.527	0.4632	0.4672	0.4454	0.431	0.3338	0.2269
$\frac{1+A\nu}{\gamma}$	2.5555	2.3627	2.1845	2.0197	2.5042	2.4685	2.6731	2.8243	2.5555	2.5555

Parameters calibrated as follows: $p_1^a = 0.1$, $p_2^a = 0.15$, $p_3^a = 0.2$, $\phi_1^f = 7.2$, $\phi_2^f = 9.6$, $\sigma_{a,1} = 0.5$, $\sigma_{a,2} = 0.6$, $\sigma_{\theta,1} = 0.8375$, $\sigma_{\theta,2} = 1.005$

As it is evident in Table 1.1, doubling the benchmark brain drain rate from 0.05 to 0.1, we obtain that the share of skilled workers increases by 5.3% (from 0.1826 to 0.1923), while if brain drain rises from 0.05 to 0.2, the share of skilled workers at home improves by 15.4% (from 0.1826 to 0.2107). This result stems from the fact that risk–bearing entrepreneurs and low–skilled workers are disadvantaged in terms of expected utility compared to skilled workers with a growing probability to obtain a higher foreign expected wage abroad. As a result, growth increases to 0.027 when the benchmark brain drain reaches a value of 0.1 and to 0.031 when the skilled outmigration becomes as high as 0.2. Due to the increase in competition among skilled workers, their expected nominal wage falls by 3.1% (from 1.1694 to 1.1333) if p^a reaches 0.1 and by 8.98% (from 1.1694 to 1.0644) if p^a reaches 0.2. The expected profit of entrepreneurs goes up due to the lower competition among entrepreneurs by 4.8% (from 4.7482 to 4.977) when the skilled outmigration rises to 0.1 and by 15.2% (from 4.7482 to 5.4682) when the skilled outmigration attains a value of 0.2. Low–skilled, who are fewer, are allowed to inherit a higher share of the average human capital from the previous period γ, and their total income increases although their base wage, which is pegged to the base wage of skilled agents, falls. Similar in magnitude to the reaction of the expected profit to changes in p^a is the response of the short–run low–skilled wage to changes in p^a.

A higher wedge between the remuneration of educated at home and abroad ϕ^f improves the expected utility of skilled agents, whose share increases, at the cost of the population of entrepreneurs and low–skilled. Doubling ϕ^f from 4.8 to 9.6, we obtain a rise in p^w equal to 2.8% (from 0.1826 to 0.1878). Because ϕ^f increases the incentive of low–skilled to obtain education, growth rises to 0.0259 when ϕ^f improves to 9.6. As a consequence of the lower competition

between entrepreneurs, expected profits go up by approximately 2.1% (from 4.7482 to 4.848) when ϕ^f attains 9.6. The expected skilled wages fall by 1.38% (from 1.1694 to 1.1533) in the same case because of more intense competition, while low–skilled income improves by 2.1% (from 0.4576 to 0.4672) due to rising γ despite a declining base wage.

A higher risk measure σ_a in Table 1.1 decreases the incentive effect of individuals to accumulate human capital: some prefer employment as low–skilled or entrepreneurs. The fall in p^w is equal to 7.8% (from 0.1826 to 0.1683) when σ_a rises by 50%. Growth rate falls to 0.0224 in this case because the economy ends up with more low–skilled agents. The weaker competition among skilled workers makes the expected skilled wage go up by 4.1% (from 1.1694 to 1.2172) when σ_a reaches 0.6. Entrepreneurs' expected profits decrease by 5.8% (from 4.7482 to 4.4717) in the same case, while the low–skilled wage decreases by 5.8% (from 0.4576 to 0.431) because of a lower spillover of human capital accumulation γ (in spite of a higher base wage).

Higher σ_θ discourages agents to become entrepreneurs due to risk aversion. They switch away from entrepreneurship to occupations as skilled and low–skilled workers. Entrepreneurship falls by 82.3% (from 0.03 to 0.0053) when σ_θ rises to 1.005. The increase in the share of skilled workers is not high enough to compensate for the fall in entrepreneurship, which would lead to higher growth, because some agents prefer to obtain an occupation in the low–skilled sector. That is why the growth rate falls to 0.0242 with σ_θ increasing by 50%. Entrepreneurs obtain higher expected profits, which rise by a factor of 2.86 (from 4.7482 to 13.5974) for an increase in σ_θ to 1.005 because of the lower competition. In the same case the expected skilled wage declines by 50.4% (from 1.1694 to 0.5799) due to higher competition on the skilled labor market. Low–skilled wages fall (because of a decline in the base wage) by 50.4% (from 0.4576 to 0.2269) if σ_θ rises by 50%.

Having discussed the size effects of p^a, ϕ^f, σ_a and σ_θ on the occupational choice and the short–run income, we can conclude the following: First, brain drain has a stronger positive influence on the endogenous variables compared to the wedge in skilled earnings between the foreign and the domestic country; Second, the risk in skilled workers' earnings has a larger (negative) influence on growth than the risk in entrepreneurs' profits. Third, the risk in entrepreneurs' profits leads to stronger changes in the income of all professions compared to the risk in skilled workers earnings. As will see later, this causes the dominance of σ_θ in terms of short term welfare compared to σ_a.

1.6 Welfare Analysis

In this section we perform a welfare analysis by determining the welfare of young agents $V_{y,t}$, agents who have reached their middle age: $V_{e,t}$ (entrepreneurs), $V_{sw,t}$ (skilled workers), $V_{lsw,t}$ (low–skilled workers) and migrants $V_{m,t}$ at period t. V_t^d is the sum of the lifetime utilities of all domestic individuals at a specific point, while $\tilde{V}_t^d = V_t^d/(1-M)$ is the average welfare of the population remaining at home at the same period, M being the share of migrants as we earlier mentioned (the index d stands for the domestic country). V_t measures not only the utility of the domestic population but also considers the lifetime welfare of migrants abroad. The average welfare of all agents $\tilde{V}_t$ is equal to total welfare V_t because the population share is normalized to one.

We report $\tilde{V}_t^d$, which is important for government's decision making, as well as $\tilde{V}_t$, which is essential for the decision making of the social planner. We estimate the average domestic welfare of the remaining individuals in the source country (and not total domestic welfare) in order to avoid understatement of the welfare measure due to the fall in the population share in response to a rising brain drain probability.[8] For the *ex–post* welfare analysis we need the following definitions,

$$V_t = 0.5V_{y,t} + p_{t-1}^e V_{e,t} + p_{t-1}^w V_{sw,t} + (0.5 - p_{t-1}^e - p_{t-1}^w - M_{t-1})V_{lsw,t} + M_{t-1}V_{m,t}$$

$$V_t^d = 0.5V_{y,t} + p_{t-1}^e V_{e,t} + p_{t-1}^w V_{sw,t} + (0.5 - p_{t-1}^e - p_{t-1}^w - M_{t-1})V_{lsw,t}$$

$$V_{y,t} = E(V_{j,t,t+1}^e) = E(V_{i,t,t+1}^w) = V_{t,t+1}^l$$

$$V_{e,t} = \int_{\theta \in \Theta} \ln(\pi_{jt})f(\theta)d\theta$$

$$V_{sw,t} = \int_{a \in \overline{A}} \ln(\overline{w}h_t^w a_i)f(a)d(a)$$

$$V_{m,t} = \int_{a \in \overline{A}} \ln(\overline{w}h_t^w \phi^f a_i)f(a)d(a)$$

$$V_{lsw,t} = \ln(\overline{w}h_{t-1}\gamma)$$

We perform the welfare analysis from period $t = 1$ till period $t = 4$, i.e. we look at the influence of the parameters on the welfare of the individuals for the next 120 years with $t = 1$ being the period of an initial parameter change ($h_0 = 1$). Following Soares (2008), we define our social welfare function for each period and do not give weight to future generations in order to make our results independent of the welfare weight of next generations.

It is already known that the influence of a parameter on long–run welfare in an overlapping

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[8]This problem does not exist when we consider total welfare, which comprises of the welfare of the domestic population and migrants.

generations setting is determined by the impact of the same parameter on growth (see Yakita (2004), Wong and Yip (1999)). This is a normal consequence of the fact that welfare depends positively on growth and that the further in future welfare is computed, the higher is the change in welfare due to the accumulation of the new parameter over many periods through growth. From here we can expect that the long–term influence of p^a, ϕ^f, σ_a and σ_θ on (domestic) welfare is reciprocal in sign to the impact of p^a, ϕ^f, σ_a and σ_θ on growth.

Alternative values of p^a, ϕ^f, σ_a and σ_θ change the lifetime utility of *young agents in the period of the introduction of a new parameter* $t = 1$ because it has an impact on the occupational choice (and, therefore, future remuneration and growth). Alternative values of p^a also influence *the welfare of middle–aged in the same period* although agents are not allowed to switch occupations and γ is constant. A rise in p^a leads to a fall in the share of middle–aged skilled workers, which decreases unexpectedly the skilled labor supply and leads to adjustment in the base wages and the base profit in $t = 1$. Higher ϕ^f raises the welfare of middle–aged migrants in $t = 1$, while p^a raises the total utility of migrants in $t = 1$ because of the sudden increase in their share M (for constant p^w) during the same period. Moreover, higher σ_a or higher σ_θ reduces respectively the expected utility of young and middle–aged skilled workers or young and middle–aged entrepreneurs in all periods due to their risk aversion.

The theoretical value of *ex–post* welfare is equal to the *ex–ante* expected welfare of the agents who incur risk in their occupational choice. All young agents have the same *ex–post* expected utility by the equilibrium occupational choice condition. We keep the occupational choice of middle–aged equal to the occupational choice of the previous period (which is the benchmark occupational choice for $t = 1$). This is presented by the subindex $t - 1$ in the share of middle–aged p^e_{t-1}, p^w_{t-1} and M_{t-1} belonging to the welfare equations above, which implies that the decision on an occupation is taken at period $t - 1$. The welfare of migrants is not explicitly shown in the following analysis but can be easily deducted from Table 1.2.

We follow the calibration analysis and assume that the starting point in the economy is $t = 0$, while we consider the realization of the income variables at period $t = 1$. This allows for determination of the human capital of low–skilled. We increase (stepwise) the brain drain rate and the gap in skilled remuneration at home and abroad by 100%. We do not report $p^a = 0.15$, as its influence on welfare compared to $p^a = 0.1$ and 0.2 is qualitatively the same. Risk measures in the sensitivity analysis are raised by 25% and 50%.

As we see from Table 1.2 in $t = 1$ (the period of the parameter change with occupational choice of middle–aged fixed to the benchmark), higher brain drain results in higher expected welfare of young agents. This can be explained with the positive effect of higher expected growth trig-

Table 1.2: *Transitional Welfare Analysis with Respect to p^a, ϕ^f, σ_a, σ_θ*

	BM	p_1^a	p_2^a	ϕ_1^f	$\sigma_{a,1}$	$\sigma_{a,2}$	$\sigma_{\theta,1}$	$\sigma_{\theta,2}$
$V_{y,t=1}$	-0.8127	-0.7574	-0.6619	-0.7973	-0.8327	-0.8571	-1.0462	-1.3317
$V_{em,t=1}$	0.155	0.0917	0.0211	0.155	0.155	0.155	-0.6341	-1.5986
$V_{sw,t=1}$	0.0765	0.1187	0.1658	0.0765	0.0315	-0.0235	0.0765	0.0765
$V_{lsw,t=1}$	-0.7817	-0.7396	-0.6925	-0.7817	-0.7817	-0.7817	-0.7817	-0.7817
$\tilde{V}_{t=1}^d$	-2.0507	-2.0432	-2.0356	-2.043	-2.1062	-2.1741	-2.9654	-4.0833
$\tilde{V}_{t=1}$	-2.0944	-1.9763	-1.7992	-2.0801	-2.1944	-2.3166	-3.0003	-4.1075
$V_{y,t=2}$	0.4747	0.5494	0.749	0.5303	0.4025	0.3139	-0.0859	-0.7633
$V_{em,t=2}$	0.8949	0.9126	0.9779	0.9157	0.8679	0.8349	0.5792	0.1935
$V_{sw,t=2}$	0.8164	0.7558	0.6642	0.8026	0.7894	0.7564	0.5008	0.115
$V_{lsw,t=2}$	-0.0418	-0.024	0.0412	-0.021	-0.0688	-0.1018	-0.3575	-0.7432
$\tilde{V}_{t=2}^d$	-1.0344	-1.0075	-0.9266	-0.9985	-1.1269	-1.2406	-2.295	-3.8225
$\tilde{V}_{t=2}$	-1.0808	-1.0146	-0.8359	-1.0375	-1.2185	-1.3875	-2.3316	-3.8474
$V_{y,t=3}$	1.7622	1.9397	2.3415	1.8633	1.6304	1.4687	1.1745	0.4827
$V_{em,t=3}$	1.6348	1.7116	1.8931	1.6817	1.5736	1.4985	1.3036	0.9096
$V_{sw,t=3}$	1.5563	1.5548	1.5794	1.5687	1.4951	1.4201	1.2252	0.8311
$V_{lsw,t=3}$	0.6981	0.775	0.9564	0.7451	0.6369	0.5619	0.3669	-0.0271
$\tilde{V}_{t=3}^d$	-0.0181	0.0936	0.3461	0.0539	-0.1577	-0.3292	-1.3	-2.8388
$\tilde{V}_{t=3}$	-0.0671	0.08	0.4179	0.012	-0.2517	-0.4782	-1.3392	-2.8663
$V_{y,t=4}$	3.0496	3.33	3.9339	3.1963	2.8583	2.6236	2.435	1.7287
$V_{em,t=4}$	2.3747	2.5107	2.8083	2.4478	2.2793	2.1622	2.028	1.6257
$V_{sw,t=4}$	2.2962	2.3538	2.4946	2.3348	2.2008	2.0838	1.9496	1.5472
$V_{lsw,t=4}$	1.438	1.574	1.8716	1.5112	1.3426	1.2255	1.0913	0.689
$\tilde{V}_{t=4}^d$	0.9982	1.1947	1.6187	1.1063	0.8116	0.5822	-0.3049	-1.8551
$\tilde{V}_{t=4}$	0.9465	1.1747	1.6717	1.0616	0.7152	0.431	-0.3468	-1.8853

Parameters calibrated as follows: $p_1^a = 0.1$, $p_2^a = 0.2$, $\phi_1^f = 9.6$, $\sigma_{a,1} = 0.5$, $\sigma_{a,2} = 0.6$, $\sigma_{\theta,1} = 0.8375$,
$\sigma_{\theta,2} = 1.005$

gered by a rising share of skilled workers in the economy. In the first period the only losers are middle–aged entrepreneurs because they experience a sudden fall in the skilled labor force, which drives skilled earnings and middle–aged skilled workers' welfare up and reduces expected profits and the welfare of firm owners. Low–skilled earnings and the welfare of low–skilled middle–aged surge due to a rise in the base skilled wage, which is pegged to the low–skilled base wage. All in all, average welfare at $t = 1$ rises because of the prevailing positive effect of higher brain drain on the expected welfare of young, and middle–aged workers. In the second period after the introduction of a higher migration probability, middle–aged skilled workers experience lower welfare because the positive effect which higher growth has on their earnings is dominated by the negative effect of stronger competition. In the third period middle–aged skilled work-ers' payment recovers from the negative effect of competition if brain drain is relatively strong

($p^a = 0.2$), for relatively small skilled outmigration $p^a = 0.1$ skilled workers' remuneration and welfare remain lower than the benchmark also in this period. In the fourth period all agents in society are winners compared to the benchmark due to improving g^h. Average (domestic) welfare in all periods exceeds the benchmark social utility.

A rise in the wedge in payment of educated between the domestic and the foreign country induces higher welfare for the young agents in the same period because of improved human capital accumulation. The income and welfare of (low)–skilled middle–aged agents in the first period remains constant. Middle–aged skilled workers become temporarily losers in $t = 2$ because of the increased competition in spite of the improvement in human capital accumulation. Average (domestic) welfare exceeds the benchmark in every period due to improvement in growth.

Higher risk in the occupational choice of skilled workers σ_a has a negative impact on the expected welfare of young individuals in $t \geq 1$ reflected by a reduction in the accumulation of human capital and disutility due to higher skilled income variance. The welfare of middle–aged entrepreneurs and middle–aged low–skilled workers in the first period remains the same because the occupational choice for middle–aged is fixed. The welfare of middle–aged skilled workers falls as they experience higher variance in the income distribution although their base wages are constant in $t = 1$. All in all, average (domestic) welfare deteriorates in the first period. This trend is preserved next periods due to dropping human capital accumulation. The expected utility of middle–aged skilled workers also decreases in all periods after $t = 1$ because of declining growth and because the disutility in response to a higher risk measure dominates the positive effect of lower competition on skilled wages.

In the period of the introduction of a higher risk measure in the occupational choice of entrepreneurs σ_θ, young agents experience a decline in expected welfare due to a declining growth rate and disutility due to a rise in the volatility of entrepreneurs' profits. The same is valid for $t > 1$. The welfare of all middle–aged workers in $t = 1$ remains the same. Entrepreneurs experience a decline in utility in $t = 1$ because of the higher risk in income they incur. The lower competition among entrepreneurs for $t > 1$ driving their expected profits up is not strong enough to result in higher welfare because agents are risk averse and experience disutility due to the presence of a larger risk measure in their profits. Average (domestic) welfare in $t = 1$ falls. The same applies for all periods after period $t = 1$, which can be explained with declining human capital accumulation making agents worse off.

According to Table 1.2, an increase in the entrepreneurial technological risk has a stronger (negative) impact on social utility compared to a rise in the occupational choice risk of skilled workers in the short run. This result can be explained with the relatively larger effect which

σ_θ exerts on the income distribution. In the very long run, however, the impact of a larger risk measure in the human capital of skilled workers is expected to exert a stronger (negative) effect on welfare because of its stronger negative effect on growth. The (positive) impact of higher brain drain on $\tilde{V}_t$ and $\tilde{V}_t^d$ is on average larger than the impact of ϕ^f, which is valid for the short and the long run.

1.7 Conclusion

In this paper we investigated the relationship between occupational choice under risk and brain drain in the context a two–period overlapping generations endogenous growth model with human capital accumulation. The model draws on the work of Kanbur (1979), and Clemens (2008) for occupational choice under risk and Beine *et al.* (2001) for probabilistic brain drain. At the beginning of life *ex–ante* homogeneous and risk–averse agents decide among occupations as low–skilled in the traditional sector, entrepreneurs or skilled workers in the modern sector. High–skilled workers, who are allowed to migrate, experience variation in their income due to *ex–ante* unknown labor productivities (at home and abroad), while entrepreneurs are subject to a technology shock, which is beyond their control. The equilibrium occupational choice is dictated by equality in the utilities of all agents. Growth depends on the share of agents, who decide to take up a profession as skilled (either skilled workers or entrepreneurs) and remain in the domestic country. That is how brain drain and risk in the occupational choice of educated play a vital role in human capital accumulation.

Theory predicts that higher brain drain rates bias the occupational choice of individuals away from entrepreneurship, but the total share of educated agents rises if the proportion of skilled workers in the population net of skilled migration is relatively low (lower than 36% for a brain drain probability of 30% and lower than 56.9% for a brain drain probability of 5% and for domestic skilled wage, which is 4.8 time lower than the foreign one). Entrepreneurship becomes a less attractive occupation for a higher wedge between the skilled earnings at home and abroad, but the share of educated workers improves. The occupational risk in entrepreneurship and skilled workers' employment makes agents deviate respectively to alternative employment.

Growth increases in response to a larger brain drain rate if the share of skilled workers in the domestic country improves. Human capital accumulation rises in the wedge between the skilled wage in the domestic and the foreign country and falls in the risk of skilled workers' earnings but behaves ambiguously with respect to the technological risk of entrepreneurial profits.

Our calibration shows that brain drain has a stronger (positive) effect on growth and welfare than the gap in skilled earnings between the foreign and the domestic country in the short and

the long run. The risk measure in the occupational choice of skilled workers leads to a higher decline in growth than the risk in entrepreneurs' profits, so the former has a stronger negative effect on welfare in the long run. However, an increase in the risk in entrepreneurs' profits has a stronger (negative) effect on social utility in the short run compared to the risk in skilled workers' earnings.

Given the unfavorable effect of a higher risk measure on welfare, the construction of an insurance system for skilled agents may be considered as an extension of the model. Another omission in the probabilistic brain drain theory is the assumption that young agents cannot borrow when they make their educational decision. Nevertheless, post socialist Eastern European countries have experienced a gradual opening of the credit markets although borrowing constraints still exist. In this respect it is relevant to examine the impact of (probabilistic) brain drain for different levels of borrowing constraints on economic development and welfare.

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1.9 Appendix

1.9.1 Proof of Proposition 1.2

By equating $E(V_{i,t,t+1}^w) = E(V_{j,t,t+1}^e)$, we obtain a relationship between p_t^e and p_t^w,

$$\frac{p_t^e}{p_t^w} = \frac{1-\alpha}{\alpha(\phi^f)^{p^a}} exp\Big(-\frac{\sigma_\theta^2}{2(1-\alpha)^2} + \frac{\sigma_a^2}{2}\Big)$$

By equating $E(V_{i,t,t+1}^w) = V_{t,t+1}^l$, we obtain the human capital spillover γ_t,

$$\gamma_t = (1-\nu)^{\frac{1}{\beta}}(\phi^f)^{p^a}(1+A\nu)exp\Big(-\frac{\sigma_a^2}{2}\Big)$$

By applying additionally the equilibrium condition (1.33) and the assumption that the population share is equal to one, we obtain the equilibrium values of p^w, p^e, L^l and γ, cited by Proposition 1.2, which are constant over time.

1.9.2 Proof of Proposition 1.4 and 1.5

Dividing the numerator and denominator of the equilibrium value of p^w by $1 - p^a$, we obtain,

$$p^{w'} = \frac{\alpha(1-b)(\phi^f)^{p^a}exp(-\frac{\sigma_a^2}{2})(1-\nu)^{\frac{1}{\beta}}}{b + \frac{\alpha(1-b)}{1-p^a}(\phi^f)^{p^a}exp(-\frac{\sigma_a^2}{2})(1-\nu)^{\frac{1}{\beta}} + (1-\alpha)(1-b)(1-\nu)^{\frac{1}{\beta}}exp(-\frac{\sigma_\theta^2}{2(1-\alpha)^2})}$$

$$\frac{\partial p^{w'}}{\partial p^a} = p^{w'}\ln\phi^f - \frac{(p^{w'})^2}{\alpha(1-b)exp(-\frac{\sigma_a^2}{2})(\phi^f)^{p^a}(1-\nu)^{\frac{1}{\beta}}}$$

$$\left[\frac{\alpha(1-b)}{(1-p^a)^2}exp\left(-\frac{\sigma_a^2}{2}\right)(\phi^f)^{p^a}(1-\nu)^{\frac{1}{\beta}} + \frac{\alpha(1-b)}{1-p^a}exp\left(-\frac{\sigma_a^2}{2}\right)(\phi^f)^{p^a}(1-\nu)^{\frac{1}{\beta}}\ln\phi^f\right]$$

$$\frac{\partial p^{w'}}{\partial p^a} = p^{w'}\ln\phi^f - (p^{w'})^2\left[\frac{1}{(1-p^a)^2} + \frac{1}{1-p^a}\ln\phi^f\right]$$

The above expression is unambiguously positive if

$$\ln\phi^f - p^{w'}\left(\frac{1}{(1-p^a)^2} + \frac{\ln\phi^f}{1-p^a}\right) > 0 \Leftrightarrow p^{w'} < \frac{(1-p^a)^2\ln\phi^f}{1+(1-p^a)\ln\phi^f}$$

and negative if

$$\ln\phi^f - p^w\left(\frac{1}{(1-p^a)^2} + \frac{1}{1-p^a}\ln\phi^f\right) < 0 \Leftrightarrow p^{w'} > \frac{(1-p^a)^2\ln\phi^f}{1+(1-p^a)\ln\phi^f}$$

Notice that $p^{w'} = p^w$. Dividing the numerator and denominator of the equilibrium value of p^w by $(\phi^f)^{p^a}$, we obtain

$$p^{w''} = \frac{\alpha(1-b)(1-p^a)(1-\nu)^{\frac{1}{\beta}}exp(-\frac{\sigma_a^2}{2})}{\frac{b(1-p^a)}{(\phi^f)^{p^a}} + \alpha(1-b)exp(-\frac{\sigma_a^2}{2})(1-\nu)^{\frac{1}{\beta}} + \frac{1-\alpha}{(\phi^f)^{p^a}}(1-b)(1-p^a)(1-\nu)^{\frac{1}{\beta}}exp(-\frac{\sigma_\theta^2}{2(1-\alpha)^2})}$$

$$\frac{\partial p^{w''}}{\partial \phi^f} = \frac{(p^{w''})^2 p^a}{\alpha(1-b)(1-\nu)^{\frac{1}{\beta}}exp(-\frac{\sigma_a^2}{2})}\left[\frac{b}{(\phi^f)^{1+p^a}} + \frac{(1-b)}{(\phi^f)^{p^a+1}}(1-\alpha)(1-\nu)^{\frac{1}{\beta}}exp\left(-\frac{\sigma_\theta^2}{2(1-\alpha)^2}\right)\right] > 0$$

Dividing the numerator and denominator of the equilibrium value of p^w by $exp(-\frac{\sigma_a^2}{2})$, we obtain,

$$p^{w'''} = \frac{\alpha(1-b)(1-p^a)(\phi^f)^{p^a}(1-\nu)^{\frac{1}{\beta}}}{\frac{b(1-p^a)}{exp(-\frac{\sigma_a^2}{2})} + \alpha(1-b)(\phi^f)^{p^a}(1-\nu)^{\frac{1}{\beta}} + (1-\alpha)(1-b)(1-p^a)(1-\nu)^{\frac{1}{\beta}}\frac{exp(-\frac{\sigma_\theta^2}{2(1-\alpha)^2})}{exp(-\frac{\sigma_a^2}{2})}}$$

$$\frac{\partial p^{w'''}}{\partial \sigma_a} = -\frac{(p^{w'''})^2}{\alpha(1-b)(\phi^f)^{p^a}(1-\nu)^{\frac{1}{\beta}}}\sigma_a\left[\frac{b}{exp(-\frac{\sigma_a^2}{2})} + (1-\alpha)(1-b)(1-\nu)^{\frac{1}{\beta}}\frac{exp(-\frac{\sigma_\theta^2}{2(1-\alpha)^2})}{exp(-\frac{\sigma_a^2}{2})}\right] < 0$$

$$\frac{\partial p^w}{\partial \sigma_\theta} = \frac{(p^w)^2}{\alpha(\phi^f)^{p^a}exp(-\frac{\sigma_a^2}{2})}exp\Big(-\frac{\sigma_\theta^2}{2(1-\alpha)^2}\Big)\frac{1}{(1-\alpha)}\sigma_\theta > 0$$

Dividing the numerator and denominator of the equilibrium value of p^e by $1 - p^a$, which results in $p^{e'}$, we obtain,

$$p^{e'} = \frac{(1-\alpha)(1-b)(1-\nu)^{\frac{1}{\beta}}exp(-\frac{\sigma_\theta^2}{2(1-\alpha)^2})}{b + \frac{\alpha}{1-p^a}(1-b)(\phi^f)^{p^a}exp(-\frac{\sigma_a^2}{2})(1-\nu)^{\frac{1}{\beta}} + (1-\alpha)(1-b)(1-\nu)^{\frac{1}{\beta}}exp(-\frac{\sigma_\theta^2}{2(1-\alpha)^2})}$$

$$\frac{\partial p^{e'}}{\partial p^a} = -\frac{(p^{e'})^2}{(1-\alpha)exp(-\frac{\sigma_\theta^2}{2(1-\alpha)^2})}\frac{\alpha exp(-\frac{\sigma_a^2}{2})(\phi^f)^{p^a}}{1-p^a}\Big(\ln\phi^f + \frac{1}{1-p^a}\Big) < 0$$

$$\frac{\partial p^e}{\partial \phi^f} = -\frac{(p^e)^2}{(1-\alpha)(1-p^a)exp(-\frac{\sigma_\theta^2}{2(1-\alpha)^2})}\Big[\alpha exp\Big(-\frac{\sigma_a^2}{2}\Big)p^a\phi^{p^a-1}\Big] < 0$$

$$\frac{\partial p^e}{\partial \sigma_a} = -\frac{(p^e)^2}{(1-\alpha)(1-p^a)exp(-\frac{\sigma_\theta^2}{2(1-\alpha)^2})}\Big[-\alpha exp\Big(-\frac{\sigma_a^2}{2}\Big)\phi^{p^a}\sigma_a\Big] > 0$$

Dividing the numerator and denominator of the equilibrium value of p^e by $exp(-\frac{\sigma_\theta^2}{2(1-\alpha)^2})$, which results in $p^{e''}$, we obtain,

$$p^{e''} = \frac{(1-\alpha)(1-b)(1-p^a)(1-\nu)^{\frac{1}{\beta}}}{\frac{b(1-p^a)}{exp(-\frac{\sigma_\theta^2}{2(1-\alpha)^2})} + \alpha(1-b)(\phi^f)^{p^a}\frac{exp(-\frac{\sigma_a^2}{2})}{exp(-\frac{\sigma_\theta^2}{2(1-\alpha)^2})}(1-\nu)^{\frac{1}{\beta}} + (1-p^a)(1-\alpha)(1-b)(1-\nu)^{\frac{1}{\beta}}}$$

$$\frac{\partial p^{e''}}{\partial \sigma_\theta} = -\frac{(p^{e''})^2}{(1-\alpha)(1-b)(1-p^a)(1-\nu)^{\frac{1}{\beta}}}\frac{1}{(1-\alpha)^2}\sigma_\theta\Big[\frac{b(1-p^a)}{exp(-\frac{\sigma_\theta^2}{2(1-\alpha)^2})}$$
$$+ \alpha(1-b)(1-\nu)^{\frac{1}{\beta}}(\phi^f)^{p^a}\frac{exp(-\frac{\sigma_a^2}{2})}{exp(-\frac{\sigma_\theta^2}{2(1-\alpha)^2})}\Big] < 0$$

The sensitivity analysis for L^l with respect to p^a, ϕ^f, σ_a, and σ_θ can be conducted in the same manner.

1.9.3 Proof of Proposition 1.6

The impact of p^a on growth g^h for $\frac{\partial p^w}{\partial p^a} > 0$ is defined as,

$$\frac{\partial(1+g^h)}{\partial p^a} = \underbrace{\frac{\partial(p^w+p^e)}{\partial p^a}}_{>0}\frac{(1+A\nu)}{1-M} + \underbrace{\frac{\partial L^l}{\partial p^a}}_{<0}\frac{\gamma}{1-M} + \frac{L^l}{1-M}\underbrace{\frac{\partial\gamma}{\partial p^a}}_{>0} + \frac{(1+g^h)}{(1-M)^2}\underbrace{\frac{\partial M}{\partial p^a}}_{>0}$$

It is easier to see that the share of skilled agents $p^e + p^w$ increases in case of $\frac{\partial p^w}{\partial p^a} > 0$ or $p^w < \frac{(1-p^a)^2\ln\phi^f}{1+(1-p^a)\ln\phi^f}$ as *Proposition 5* claims, which can be explained as follows: although it is true that some entrepreneurs will be also attracted to become skilled workers, i.e. p^e declines,

the total effect on p^w will be positive because of the rising share skilled workers coming from the pool of low–skilled. Moreover, the human capital of both entrepreneurs and skilled workers is the same, so switching to another skilled profession among skilled does not have any influence on the total value of their human capital. Notice that $L^l \gamma = \frac{b}{(1-b)\alpha}(1 + A\nu)p^w$, which implies that for $\frac{\partial p^w}{\partial p^a} > 0$ the impact of p^a on growth is eventually defined as,

$$\frac{\partial(1 + g^h)}{\partial p^a} = \underbrace{\frac{\partial(p^w + p^e)}{\partial p^a}}_{>0} \frac{(1 + A\nu)}{1 - M} + \underbrace{\frac{\partial(L^l \gamma)}{\partial p^a}}_{>0} \frac{1}{1 - M} + \frac{(1 + g^h)}{(1 - M)^2} \underbrace{\frac{\partial M}{\partial p^a}}_{>0} > 0$$

The effect of p^a on growth is ambiguous for $\frac{\partial p^w}{\partial p^a} < 0$, i.e. if $p^w > \frac{(1-p^a)^2 \ln \phi^f}{1+(1-p^a)\ln \phi^f}$ as *Proposition 5* claims because the differential $\frac{\partial M}{\partial p^a}$ is ambiguous.

The impact of ϕ^f on growth is in the same toke equal to,

$$\frac{\partial(1 + g^h)}{\partial \phi^f} = \underbrace{\frac{\partial(p^w + p^e)}{\partial \phi^f}}_{>0} \frac{(1 + A\nu)}{1 - M} + \underbrace{\frac{\partial(L^l \gamma)}{\partial \phi^f}}_{>0} \frac{1}{1 - M} + \frac{(1 + g^h)}{(1 - M)^2} \underbrace{\frac{M}{\phi^f}}_{>0} > 0$$

The effect of σ_a on growth is,

$$\frac{\partial(1 + g^h)}{\partial \sigma_a} = \underbrace{\frac{\partial(p^w + p^e)}{\partial \sigma_a}}_{<0} \frac{(1 + A\nu)}{1 - M} + \underbrace{\frac{\partial(L^l \gamma)}{\partial \sigma_a}}_{<0} \frac{1}{1 - M} + \frac{(1 + g^h)}{(1 - M)^2} \underbrace{\frac{\partial M}{\partial \sigma_a}}_{<0} < 0$$

The effect of σ_θ on growth is,

$$\frac{\partial(1 + g^h)}{\partial \sigma_\theta} = \underbrace{\frac{\partial(p^w + p^e)}{\partial \sigma_\theta}}_{<0} \frac{(1 + A\nu)}{1 - M} + \underbrace{\frac{\partial L^l}{\partial \sigma_\theta}}_{>0} \frac{\gamma}{1 - M} + \frac{(1 + g^h)}{(1 - M)^2} \underbrace{\frac{\partial M}{\partial \sigma_\theta}}_{>0} \gtrless 0$$

The total share of skilled agents $p^w + p^e$ declines, as some entrepreneurs decide to switch to the low–skilled profession (the share of skilled workers increases out of the pool of entrepreneurs). Although this redistribution of agents across professions implies that there will be more agents who carry less human capital to the accumulation function, $1 + A\nu > \gamma$, the impact of σ_θ on growth is inconclusive because higher σ_θ implies a higher share of skilled migrants, which boosts growth through the population effect of declining $1 - M$.

1.9.4 Proof of Proposition 1.7 and 1.8

The short–run income variables for $h_0 = 1$ and $t = 1$ are defined as follows,

$$\overline{w}h_0\gamma = \alpha^\alpha(1 - \alpha)^{1-\alpha}exp\Big(-\frac{1}{(1 - \alpha)}\frac{\sigma_\theta^2}{2} - \frac{\alpha\sigma_a^2}{2}\Big)(1 - \nu)^{\frac{1}{\beta}}(1 + A\nu)(\phi^f)^{\alpha p^a}$$

$$\overline{w}h_1^w = \alpha^\alpha(1 - \alpha)^{1-\alpha}exp\Big(-\frac{1}{(1 - \alpha)}\frac{\sigma_\theta^2}{2} + \frac{(1 - \alpha)\sigma_a^2}{2}\Big)\frac{1}{(\phi^f)^{p^a(1-\alpha)}}(1 + A\nu)$$

$$E(\pi_{j1}) = \alpha^\alpha(1 - \alpha)^{1-\alpha}exp\Big(\frac{\alpha}{(1 - \alpha)^2}\frac{\sigma_\theta^2}{2} - \frac{\alpha}{2}\sigma_a^2\Big)(\phi^f)^{\alpha p^a}(1 + A\nu)$$

from where the short run influence of p^a, ϕ^f, σ_a, and σ_θ on the above income variables is,

$$\frac{\partial(\overline{w}h_0\gamma)}{\partial p^a} = \overline{w}h_0\gamma\alpha\ln\phi^f > 0 \qquad\qquad \frac{\partial(\overline{w}h_0\gamma)}{\partial\phi^f} = \overline{w}h_0\gamma\frac{p^a\alpha}{\phi^f} > 0$$

$$\frac{\partial(\overline{w}h_0\gamma)}{\partial\sigma_a} = -\overline{w}h_0\gamma\alpha\sigma_a < 0 \qquad\qquad \frac{\partial(\overline{w}h_0\gamma)}{\partial\sigma_\theta} = -\overline{w}h_0\gamma\frac{1}{(1-\alpha)}\sigma_\theta < 0$$

$$\frac{\partial(\overline{w}h_1^w)}{\partial p^a} = -\overline{w}h_1^w(1-\alpha)\ln\phi^f < 0 \qquad\qquad \frac{\partial(\overline{w}h_1^w)}{\partial\phi^f} = -\overline{w}h_1^w\frac{(1-\alpha)p^a}{\phi^f} < 0$$

$$\frac{\partial(\overline{w}h_1^w)}{\partial\sigma_a} = \overline{w}h_1^w(1-\alpha)\sigma_a > 0 \qquad\qquad \frac{\partial(\overline{w}h_1^w)}{\partial\sigma_\theta} = -\overline{w}h_1^w\frac{1}{(1-\alpha)}\sigma_\theta < 0$$

$$\frac{\partial E(\pi_{j1})}{\partial p^a} = E(\pi_{j1})\alpha\ln\phi^f > 0 \qquad\qquad \frac{\partial E(\pi_{j1})}{\partial\phi^f} = E(\pi_{j1})\frac{\alpha p^a}{\phi^f} > 0$$

$$\frac{\partial E(\pi_{j1})}{\partial\sigma_a} = -E(\pi_{j1})\alpha\sigma_a < 0 \qquad\qquad \frac{\partial E(\pi_{j1})}{\partial\sigma_\theta} = E(\pi_{j1})\frac{\alpha}{(1-\alpha)^2}\sigma_\theta > 0$$

The long–term equilibrium wages and profits in the model for $h_0 = 1$ are,

$$\overline{w}h_{t-1}\gamma = \alpha^\alpha(1-\alpha)^{1-\alpha}exp\Big(-\frac{1}{(1-\alpha)}\frac{\sigma_\theta^2}{2} - \frac{\alpha\sigma_a^2}{2}\Big)(1-\nu)^{\frac{1}{\beta}}(1+A\nu)(\phi^f)^{\alpha p^a}(1+g^h)^{t-1}$$

$$\overline{w}h_t^w = \alpha^\alpha(1-\alpha)^{1-\alpha}exp\Big(-\frac{1}{(1-\alpha)}\frac{\sigma_\theta^2}{2} + \frac{(1-\alpha)\sigma_a^2}{2}\Big)\frac{1}{(\phi^f)^{p^a(1-\alpha)}}(1+A\nu)(1+g^h)^{t-1}$$

$$E(\pi_{jt}) = \alpha^\alpha(1-\alpha)^{1-\alpha}exp\Big(\frac{\alpha}{(1-\alpha)^2}\frac{\sigma_\theta^2}{2} - \frac{\alpha}{2}\sigma_a^2\Big)(\phi^f)^{\alpha p^a}(1+A\nu)(1+g^h)^{t-1}$$

In the long–run $t \longrightarrow \infty$ the impact of p^a, ϕ^f, σ_a, and σ_θ (designated with x below) on the income variables depends not only on the short–run changes in occupational choice but also on the growth rate g^h,

$$\frac{\partial(\overline{w}h_{t-1}\gamma)}{\partial x} = \frac{\partial\overline{w}}{\partial x}h_{t-1}\gamma + \overline{w}\frac{\partial\gamma}{\partial x}h_{t-1} + \overline{w}\gamma(t-1)(1+g^h)^{t-2}\frac{\partial(1+g^h)}{\partial x}$$

$$\frac{\partial(\overline{w}h_t^w)}{\partial x} = \frac{\partial\overline{w}}{\partial x}h_t^w + \overline{w}(t-1)(1+g^h)^{t-2}\frac{\partial(1+g^h)}{\partial x}(1+A\nu)$$

$$\frac{\partial E(\pi_{jt})}{\partial x} = \frac{\partial\overline{\pi}}{\partial x}h_t^e + \overline{\pi}(t-1)(1+g^h)^{t-2}\frac{\partial(1+g^h)}{\partial x}(1+A\nu)$$

1.9.5 Empirical Data

Table 1.3: *Measures of Wage Structure in Selected Eastern European States in 1988-92*

	p90/p10
Bulgaria	1.88
Cyprus	3.43
CzechRepublic	1.61
Romania	1.88
Slovakia	1.61
Slovenia	2.93
Yugoslavia	2.40

Source: Freeman and Oostendorp (2000)

Table 1.4: *Annual Earnings of Employees in Full–time Jobs in Selected European States*

	2003	2004	2005	2006	2007	2008	*Average*
Bulgaria	2039.5	2205.5	2436.9	2699.7	3205.8	4082.7	2778.35
Hungary	8298.3	8928.9	9346.4	9183.1	9731.7	–	9097.68
Greece	19410.4	–	–	–	–	–	19410.4
Poland	–	7334.3	–	–	–	–	7334.3
Romania	–	3018.5	3996.8	4860.2	6512.2	7443.1	4305.13
Slovakia	5090.3	6033.4	6828.5	7744.6	9431.7	–	7025.7
Austria	38952	39101	40819	41653	42956.9	44363	41307.48
Denmark	49401.9	51076.9	52031.2	52803.7	52767.3	55248.7	52221.62
Finnland	31679	32950	34249	34891	36616	38775	34860
Germany	42900	43700	44300	44863	46000	47300	44843.83
Spain	–	22965.1	23556	25124.7	25073.1	–	24179.73
Sweden	35805.4	34602.8	35224.4	36122.2	37806.8	37837.5	36233.18
UK	41614.8	44131.1	–	47282.7	48478.8	–	45376.85

Source: Eurostat

Online Source Path: In the database of *Eurostat* under Population and Social Conditions, Labor Market, Earnings, Gross earnings - annual data, Average annual gross earnings by occupation; *Employees*: ISCO1: Legislators, senior officials and managers; ISCO2 Professionals; ISCO3 Technicians and associate professionals; ISCO4 Clerks; ISCO5 Service and shop and market sales workers; Last accessed on 1.12.2013.

Table 1.5: *Brain Drain Rates (%) of Total Migrants from Selected Eastern European States*

	1990	2000
Bulgaria	2.7	5.8
Bosnia&Herzegovina	–	28.6
Croatia	–	29.4
CzechRepublic	–	9.9
Hungary	13.5	12.1
Greece	18.9	14.0
Latvia	–	10.2
Lathuania	–	11.8
Macedonia	–	20.9
Poland	12.8	12.3
Romania	11.1	14.1
Serbia&Montenegro	–	17.4
Slovakia	–	15.3
Slovenia	–	11.0

Source: Docquier and Marfouk (2004)

Table 1.6: *Share of Skilled (%) in Population from Selected Eastern European States (age 15-64)*

	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Bulgaria	17.7	17.9	17.8	18.2	18.5	18.9	19.2	19.4	20.1	20.7
CzechRepublic	10.0	10.4	11.0	11.4	11.6	12.4	13.4	14.5	15.9	17.0
Hungary	13.1	14.2	14.5	15.0	15.4	16.4	16.9	17.2	18.1	19.0
Greece	15.7	17.6	17.7	18.7	19.2	19.8	20.0	21.0	22.3	23.0
Poland	11.6	12.8	13.9	14.9	15.7	16.5	18.1	19.8	20.7	21.5
Romania	7.9	8.7	9.1	9.6	9.9	10.7	11.2	11.9	13.0	13.6
Slovakia	9.6	10.4	11.4	11.9	11.9	12.3	13.4	15.1	16.5	17.0
Slovenia	14.4	15.7	16.7	17.8	18.5	19.0	19.6	20.2	21.6	23.0
Latvia	15.1	16.7	17.1	17.6	18.8	21.0	21.7	22.5	23.6	25.1
Lithuania	19.8	21.2	22.0	22.4	24.1	25.4	25.5	27.0	27.9	28.8

Source: Eurostat

Online Source Path: In the Database of *Eurostat* under Population and Social Conditions $\Rightarrow$ Education and training $\Rightarrow$ Educational attainment, outcomes and returns of education $\Rightarrow$ Main indicators on education attainment: time series and regional data $\Rightarrow$ Persons with a given education attainment, by age and sex (%) $\Rightarrow$ Persons with tertiary education attainment by age and sex (%); Last accessed on 1.12.2013.

Table 1.7: *Entrepreneurship Share of Firms Owners according to Education and Firms' Growth*

Entrepreneur's Educational Attainment	*HG*	*MG*	*LG*
College	31%	29%	23%
Post − graduate	23%	22%	12%
Total	54%	51%	35%

Source: Morris (2011), Data based on GEM Adult Population Surveys 2006-2010

High Growth (HG), Moderate Growth (MG) and Low–Growth (LG) Firms

HG: above 20% per year, MG: from 5% or 20% per year LG: below 5% per year

Table 1.8: *Entrepreneurship Rate in Population of Selected Eastern European States*

	Enterpr. Rate %
Bosnia&Herzegovina	7.5
Croatia	8
Estonia	13.5
Latvia	14.5
Hungary	9
Greece	6
Macedonia	7.5
Poland	9
Romania	9
Slovakia	6.5
Slovenia	5

Source: Xavier et al. (2012)

Entrepreneurship Rate: % of adult population (18–64 years) involved in entrepreneurship in 2012

2 Brain Drain, Borrowing Constraints, and Endogenous Growth in an Economy with Perfect Physical Capital Mobility

2.1 Introduction

The theoretical literature is still not unanimous on what is the impact of brain drain on the economic development of the source country. While Miyagiwa (1991), Haque and Kim (1995), Wong and Yip (1999) argue that brain drain decreases growth because *ex–post* the level of human capital declines, there are authors such as Mountford (1997), Vidal (1998), Stark *et al.* (1997), and Beine *et al.* (2001), who support the idea that brain drain *may* increase growth for a relatively small skilled migration rate because it stimulates *ex–ante* the investment in human capital (the so–called *brain gain* theory). One of the merits of our work in this respect is that we test the robustness of the *brain gain* theory by relaxing its assumption that agents in the domestic country are *completely* excluded from the credit market when they invest in human capital. In reality borrowing is allowed although it may be constrained, which is an additional channel to influence the investment in human capital when brain drain takes place.

The theoretical and empirical significance of borrowing constraints for educational attainment, on the other hand, has already been verified. Buiter and Kletzer (1992), Galor and Zeira (1993), Ljungqvist (1993), De Gregorio (1996), Christou (2001) theoretically emphasize the unfavorable impact of (exogenously imposed) constrained borrowing on education.[1] De Gregorio (1996) and Flug *et al.* (1998) show that borrowing constraints exert a negative effect on secondary education, while Mimoun (2008) finds a positive impact of higher borrowing on secondary and tertiary enrollment in education in aggregate terms. Vandenberghe (2007) estimates that parental income increases the likelihood of agents to attend high education in Poland (33% rise in parental income leads to an increase in university enrollment of 3%) as well as in Hungary (33% rise

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[1]For models relating human capital investment to endogenous borrowing constraints see De la Croix and Michel (2007), Andolfatto and Gervais (2006).

in parental income leads to an increase in university enrollment of 20%, which, nevertheless, should be treated with caution because of the small sample size).

The empirical relevance of brain drain, on the other hand, has already been tested by Docquier and Marfouk (2004). They estimate significantly high levels of skilled outmigration from Eastern European countries. In 2000 the proportion of working individuals above 25 with tertiary education from some European transition countries, which are already part of the EU, such as Croatia, Slovakia, Romania, Greece, Poland, Hungary, Lithuania, Latvia, working abroad ranges from 10% to 30%. These countries exhibit predominantly higher outmigration rate of skilled rather than low–skilled individuals.

Intuitively, probabilistic brain drain and liberalized borrowing constraints should be complements in inducing higher educational investment. As we shall see later, however, the *brain gain* effect does not hold in case of relatively liberalized borrowing constraints, because agents do not have an incentive to invest in education in response to rising skilled outmigration if they already avail of some economic resources. In such a setting we are additionally able to define the aggregate savings rate and the impact of brain drain and credit market liberalization on it. The investigation of this relationship has never been attempted in the brain drain literature because of the assumption that individuals are deprived of physical capital. Moreover, we are interested in the welfare effects which result in response to changes in the brain drain phenomenon and the relaxation of borrowing.

Our model is embedded in the literature on human capital accumulation within a three–period overlapping generations model and probabilistic brain drain (see Beine et al. (2001)) as well as borrowing constraints in an environment of perfect physical capital mobility (see De Gregorio (1996)). Young obtain income as low–skilled augmented by an exogenous borrowing constraint and invest in optimal educational time. At the end of the first period some educated are randomly selected to go abroad and work for a higher wage. Middle–aged agents earn income, which is augmented by their human capital, repay the credit and save. Old agents consume their savings and do not work. Migrants follow the same pattern of life when they are middle–aged and old. We postulate full enforcement of credit contracts for migrants and domestic agents. Young and middle–aged individuals at home work in the production sector supplying their labor inelastically. The production sector additionally uses physical capital. Because physical capital is perfectly mobile, the domestic interest rate is equal to the international interest rate. Growth is triggered by human capital accumulation, which reacts to changes in the brain drain parameters and the credit constraint.

Our theoretical model predicts that the educational time (which is the engine of human cap-

ital growth) increases with a higher brain drain probability and a higher wedge between the remuneration of skilled at home and abroad only if agents are relatively patient with respect to current consumption (i.e. if the intertemporal elasticity of substitution is relatively high). Moreover, the intertemporal elasticity of substitution must outweigh the effects of the borrowing constraint. This implies that credit constraints play a vital role in the decision to invest in human capital when migration chances or the difference in skilled payment in the domestic and the foreign country improve(s). More relaxed borrowing constraints increase the investment in education and growth only if the intertemporal elasticity of substitution is lower than or equal to one.

The influence of a higher brain drain rate on the aggregate savings rate is ambiguous even if brain drain is beneficial for growth. A higher wedge between the earnings of skilled in the foreign and the domestic country leads to a higher aggregate savings rate only if the growth is influenced positively and vice versa. On the other hand, the aggregate savings rate falls unambiguously when the growth rate is negatively influenced by more relaxed borrowing constraints. In all other cases (including the case in which growth rises), the influence of higher credit relaxation on the aggregate savings rate is ambiguous.

According to the calibration, beneficial brain drain is evident only if borrowing constraints are relatively tight (or at most 74% of current disposable income). This result overlaps with the earlier *brain gain* literature. In case of more liberal borrowing constraints (for the credit share out of disposable income exceeding 74%), we observe growth deteriorating brain drain. The aggregate savings rate falls with a rise in the skilled outmigration probability regardless of the level of borrowing constraints. All agents in the economy win in case of a larger brain drain rate if borrowing is relatively constrained. In an environment of relatively relaxed borrowing constraints, the sum of the discounted utilities of all (domestic) agents is higher than the benchmark in the short run, but middle–aged and old aged remain losers. The long–term impact of brain drain in this case is expected to be negative.

According to our calibration, a higher wedge between skilled earnings in the domestic and the foreign country increases the optimal time in education, growth, savings and welfare compared to the benchmark case independent of the severity of the borrowing constraints.

More relaxed borrowing constraints in our calibration raise the optimal investment in education and growth. The aggregate savings rate, on the other hand, decreases. Domestic welfare gain is observed in all periods although one generation (when middle–aged and old) face a temporary fall in utility a period after the introduction of more liberal borrowing constraints.

We would like to stress that the afore–mentioned results are obtained under the assumption

of perfect physical capital mobility. This postulation has the property that savings decisions are not determined by the growth of physical capital. The latter stems from the assumption that the interest rate is exogenous, which pins down a value for the ratio of physical capital to average human capital in the profit optimization of the representative firm, i.e. on the balanced growth path physical capital grows at the rate of human capital. Alternatively, we can assume that physical capital is immobile, which would imply that labor is more mobile than capital. There is not much empirical support for this setting. Moreover, free movement of physical capital and labor is one of the targets of the EU. That is why we calibrate our model targeting the economies of EU Eastern European countries.

This paper is divided as follows: In Section 2.2 the general assumptions of the model are presented. In Section 2.3 we specify the market equilibrium. In Section 2.4 and 2.5 we conduct a sensitivity analysis and calibration of the endogenous variables of interest. In Section 2.6 we perform a welfare analysis. Section 2.7 is devoted to the conclusion.

2.2 The Model

2.2.1 Intertemporal Optimization of Households

Homogeneous and risk averse agents live for three periods in the framework of an overlapping generations model. Each individual supplies inelastically one unit of labor to the market. Still, only young and middle–aged agents work on the labor market, old individuals are retired. Each generation is normalized to one, which means that the population equals 3. There is no population growth.

Young agents born in period t decide on the optimal share of time ν_t in education, work as low–skilled and earn income $\Psi(1 - \nu_t)\overline{w}_t h_t \varphi$ in the first period, which comprises the base wage $\overline{w}_t$ and the human capital $h_t \varphi$ with $0 < \varphi < 1$. We assume that young agents inherit part φ of the average human capital of skilled middle–aged h_t from the same period.[2] Moreover, young are allowed to borrow a share $\Psi \geq 1$ of their income from the capital market at a constant interest rate equal to r. Ψ stands for the severity of the borrowing constraints with $\Psi = 1$ implying that credit markets are completely absent and with $\Psi \longrightarrow \infty$ meaning they are completely liberalized. By conditioning the amount of credit on earnings, we reflect an empirical observation that borrowing depends on current income. Total income and debt are used for consumption (education is free of charge). Agents incur only the opportunity cost of

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[2]This assumption helps us model the life–cycle profile of earnings with young earning the lowest income compared to middle-aged and old at a certain period.

not being employed during education, represented by $\Psi \nu_t \overline{w}_t h_t \varphi$. After having gained human capital, individuals are selected with a probability p^a to migrate abroad and work for a foreign wage, which is $\phi^f > 1$ times higher than the domestic wage $\overline{w}_{t+1} h_{t+1}$ in the second period of their life.[3] Middle–aged agents (at home or abroad) have to repay their debt from the first period equal to $(\Psi - 1)(1 - \nu_t)\overline{w}_t h_t \varphi R$ where $R = 1 + r$ and to make their optimal savings decision in the domestic (s_{t+1}) and in the foreign country (s^f_{t+1}). We assume full enforcement of credit contracts for migrants. In the third period all agents (abroad or at home) consume the savings they have made from the second period.

The consumption stream of an agent born in t who stays in the domestic country over one's lifetime is: $c_{y,t}$, $c_{m,t+1}$, $c_{o,t+2}$, where the subindex y stands for young, m for middle–aged and o for old throughout this paper. Individuals who manage to migrate in the second period consume $c^f_{m,t+1}$ and $c^f_{o,t+2}$ when middle–aged and old respectively. In the whole paper we use the index f to refer to variables connected with the foreign economy. Agents also weight present to future consumption by a factor $0 < \beta < 1$. Given the usual CES utility function with σ being the intertemporal elasticity of substitution, the expected lifetime utility of a young worker born at period t (where E is the expectation operator), who faces a probability of migration p^a at period $t + 1$ is,

$$E(V_{t,t+1,t+2}) = \frac{(c_{y,t})^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + \beta(1 - p^a)\frac{(c_{m,t+1})^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}$$
$$+ \beta^2(1 - p^a)\frac{(c_{o,t+2})^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + \beta p^a \frac{(c^f_{m,t+1})^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + \beta^2 p^a \frac{(c^f_{o,t+2})^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} \tag{2.1}$$

with budget constraints,

$$c_{y,t} = \Psi(1 - \nu_t)\overline{w}_t h_t \varphi \tag{2.2}$$

$$c_{m,t+1} = h_{t+1}\overline{w}_{t+1} - (\Psi - 1)(1 - \nu_t)\overline{w}_t h_t \varphi R - s_{t+1} \tag{2.3}$$

$$c^f_{m,t+1} = \phi^f h_{t+1}\overline{w}_{t+1} - (\Psi - 1)(1 - \nu_t)\overline{w}_t h_t \varphi R - s^f_{t+1} \tag{2.4}$$

$$c_{o,t+2} = s_{t+1}R \tag{2.5}$$

$$c^f_{o,t+2} = s^f_{t+1}R \tag{2.6}$$

The first period constraint holds with an equality and agents are not assumed to make savings, because the borrowing constraint is binding. The human capital of a skilled middle–aged worker is linear in educational time ν_t,

$$h_{t+1} = (1 + A\nu_t)h_t \qquad A > 0 \tag{2.7}$$

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[3]We follow Beine *et al.* (2001) in modeling the foreign wage by simplifying his assumption that educated experience higher income growth abroad compared to domestic agents.

and h_t is the average human capital of skilled middle–aged agents in the domestic country. The optimization problem of an agent who lives at period t, $t+1$ and $t+2$ and may migrate abroad constitutes a decision on the educational time and savings. The optimality of these decisions is ensured by the first order conditions,

$$\frac{\partial E(V_{t,t+1,t+2})}{\partial s_{t+1}} = 0 \qquad \frac{\partial E(V_{t,t+1,t+2})}{\partial s_{t+1}^f} = 0 \qquad \frac{\partial E(V_{t,t+1,t+2})}{\partial \nu_t} = 0$$

Maximizing the expected utility with respect to savings (at home and abroad) as well as educational time, we obtain that

$$s_{t+1}^* = \frac{h_{t+1}\overline{w}_{t+1} - (\Psi-1)(1-\nu)\overline{w}_t h_t \varphi R}{1 + \beta^{-\sigma} R^{1-\sigma}} \tag{2.8}$$

$$s_{t+1}^{f*} = \frac{h_{t+1}\overline{w}_{t+1}\phi^f - (\Psi-1)(1-\nu)\overline{w}_t h_t \varphi R}{1 + \beta^{-\sigma} R^{1-\sigma}} \tag{2.9}$$

$$\frac{(\Psi\overline{w}_t\varphi)^{(1-\frac{1}{\sigma})}}{(1-\nu_t^*)^{\frac{1}{\sigma}}} = (1-p^a)N \frac{(\overline{w}_{t+1}A + \overline{w}_t(\Psi-1)R\varphi)}{(\overline{w}_{t+1}(1 + A\nu_t^*) - \overline{w}_t(\Psi-1)R(1-\nu_t^*)\varphi)^{\frac{1}{\sigma}}}$$

$$+ p^a N \frac{(\phi^f A\overline{w}_{t+1} + \overline{w}_t(\Psi-1)R\varphi)}{(\overline{w}_{t+1}(1 + A\nu_t^*)\phi^f - \overline{w}_t(\Psi-1)R(1-\nu_t^*)\varphi)^{\frac{1}{\sigma}}} \tag{2.10}$$

where $N = \frac{(1+\beta^{-\sigma}R^{1-\sigma})^{\frac{1}{\sigma}}\beta^2}{R^{\frac{1}{\sigma}-1}}$. The asterisk implies optimality. As it is evident from the above result, savings at home and abroad are equal to a constant share of the net income earned in the second period of agent's life. For now we assume that equation (2.10) has a solution $\nu_t^* \in (0,1)$. In equilibrium, as we will see later, it really has. Because we postulate that there is no difference in the abilities of agents to accumulate human capital, educational time does not differ among individuals. That is why there is no difference between the average and the individual level of human capital of skilled. The same applies for the savings and consumption decisions of agents.

In the end, we would like to discuss the case in which the borrowing constraint is binding. That is why it is worth considering the model without credit market frictions. In this case the optimal educational time maximizes human wealth. The expected budget constraint is defined as,

$$(1-\nu_t)\overline{w}_t h_t \varphi + \frac{(1-p^a)h_{t+1}\overline{w}_{t+1}}{R} + \frac{p^a \phi^f h_{t+1}\overline{w}_{t+1}}{R}$$

$$- c_{yt} - (1-p^a)\frac{c_{mt+1}}{R} - (1-p^a)\frac{c_{ot+2}}{R^2} - p^a\frac{c_{mt+1}^f}{R} - p^a\frac{c_{ot+2}^f}{R^2} = 0 \tag{2.11}$$

Maximizing the budget constraint with respect to the educational time ν_t results in two possible solutions: agents will be either investing $\nu_t = 1$ or not investing in education $\nu_t = 0$. This discrete educational choice stems from the linearity of the educational time in the human capital accumulation function. For agents to invest in education with $\nu_t = 1$, it is sufficient that

$$\frac{\overline{w}_{t+1}}{\overline{w}_t\varphi}\frac{A}{R}(1 - p^a + p^a\phi^f) > 1 \tag{2.12}$$

This condition holds for values of the parameters which we use in our calibration ($r = 2.243$, $p^a \in (0.05 - 0.2)$, $\phi^f \in (4.8 - 9.6)$, $\varphi = 0.4$ and $A = 1.83$)[4]. Notice that in equilibrium the base wage is constant.

2.2.2 Production Sector

The production sector operates in an environment of perfect competition to produce the final output. The price of the good is normalized to 1 for simplicity. The production function is,

$$Q_t = BK_t^\alpha (H_t)^{1-\alpha} \qquad 0 < \alpha < 1 \tag{2.13}$$

where $B > 0$ is a productivity parameter, K_t is physical capital, while H_t is total human capital. Total human capital H_t at period t is equal to the raw labor l_t multiplied with the average human capital $\overline{h}_t$, i.e. $H_t = l_t \overline{h}_t$, or is defined as $H_t = (1 - p^a)h_t + \varphi h_t$ where the average human capital is $\overline{h}_t = h_t(1 - p^a + \varphi)/l_t$. The representative firm maximizes its profit

$$\pi_t = BK_t^\alpha (H_t)^{1-\alpha} - RK_t - \overline{w}_t \overline{h}_t l_t \tag{2.14}$$

by deciding on the optimal share of physical capital K_t to be borrowed and raw labor l_t to be employed, from where we obtain that

$$\overline{w}_t = \frac{B(1-\alpha)}{l_t^\alpha} \left(\frac{K_t}{\overline{h}_t} \right)^\alpha \tag{2.15}$$

$$R = B\alpha l_t^{(1-\alpha)} \left(\frac{\overline{h}_t}{K_t} \right)^{1-\alpha} \tag{2.16}$$

with R containing the interest rate net of depreciation. Because of the assumption of full employment (of young and skilled middle–aged) and the inelastic labor supply, the raw labor is equal to $l_t = 2 - p^a$ and is constant over time. Moreover, the ratio of physical to average human capital $K_t/\overline{h}_t$ can be determined immediately from (2.16) because R is exogenously defined and equals the return on K abroad due to our assumption that there are no barriers to physical capital flow. Therefore, we can also obtain an endogenous value for the base wage $\overline{w}_t$, which turns out to be constant over time for a constant interest rate.[5] The equilibrium value of $\overline{w}$ (after substituting for $K_t/\overline{h}_t$) is independent of the labor supply but declines in R.

2.3 Market Equilibrium

In this section we define the equilibrium growth in human capital, and wealth as an intermediate step in determining the (domestic) aggregate savings rate, which we address as a savings rate

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[4]See Section 2.5 for the choice of parameters' values.

[5]From now on we drop the time indices of the endogenous variables which are proved to be constant.

in the rest of the paper for simplicity. Because of the exclusion of physical capital, no brain drain paper has derived the level of the savings rate. Our work is innovative in this respect.

2.3.1 Equilibrium Growth

We have from the production side of the economy that $\overline{w}_t = \overline{w}_{t+1} = \overline{w}$, which implies that the equilibrium educational time (2.10) can be rewritten as

$$
\frac{(\Psi\varphi)^{(1-\frac{1}{\sigma})}}{(1-\nu^*)^{\frac{1}{\sigma}}} = \frac{(1-p^a)N(A+(\Psi-1)R\varphi)}{((1+A\nu^*)-(\Psi-1)R(1-\nu^*)\varphi)^{\frac{1}{\sigma}}} + \frac{p^a N(\phi^f A+(\Psi-1)R\varphi)}{((1+A\nu^*)\phi^f-(\Psi-1)R(1-\nu^*)\varphi)^{\frac{1}{\sigma}}} \tag{2.17}
$$

with $N = \frac{(1+\beta^{-\sigma}R^{1-\sigma})^{\frac{1}{\sigma}}\beta^2}{R^{\frac{1}{\sigma}-1}}$. Therefore, ν^{*6} is constant over time because it depends only on the parameters of the model. It can be proved that ν^* has a solution in the range $\in (0,1)$. These results are summarized in the following Proposition.

Proposition 2.1 *The optimal educational time ν^* is constant and lies in the range $(0,1)$.*

Proof: See Appendix 2.9.1

The accumulation of the average human capital h_t of skilled middle–aged over time is,

$$
g^h = A\nu^* \tag{2.18}
$$

while the growth of average human capital g^a,

$$
1 + g^a = \frac{\overline{h}_{t+1}}{\overline{h}_t} = \frac{h_{t+1}(1-p^a+\varphi)}{2-p^a} \bigg/ \frac{h_t(1-p^a+\varphi)}{2-p^a} = \frac{h_{t+1}}{h_t} = 1 + g^h \tag{2.19}
$$

is equal to the growth of the human capital of middle–aged skilled agents, according to the balanced growth condition (2.19). The equilibrium growth rate depends exclusively on the educational time and, therefore, grows at a constant rate. These results are summarized in the following Proposition.

Proposition 2.2 *The growth rate of average human capital $A\nu^*$ is constant.*

2.3.2 Equilibrium Wealth, Savings and (Domestic) Aggregate Savings Rate

Domestic wealth at period t is equal to the sum of the dissaved income of young, who take a credit, and the saved income of middle–aged who stay at home as well as the credit repayment

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[6]We drop the time index for ν_t^*.

of emigrants. Old do not save, that is why they are excluded from the calculations of the wealth function. Given the above specification, the wealth function is defined as,

$$W_t = (1 - p^a)s_t^* + (1 - \Psi)(1 - \nu^*)\overline{w}h_t\varphi + p^a(\Psi - 1)(1 - \nu^*)\overline{w}h_{t-1}\varphi R \qquad (2.20)$$

The wealth function contains also the credit market equilibrium by which middle–aged lend some of their savings to young and to the representative firm. Lending to young (contrary to lending to the representative firm) constitutes a loss to W_t because it does not create economic value via production. Substituting for the definition of s_t^* in the wealth function, we obtain,

$$W_t = (1 - p^a)\frac{h_t\overline{w} - (\Psi - 1)(1 - \nu^*)\overline{w}h_{t-1}R\varphi}{1 + \beta^{-\sigma}R^{1-\sigma}}$$
$$+ (1 - \Psi)(1 - \nu^*)\overline{w}h_t\varphi + p^a(\Psi - 1)(1 - \nu^*)\overline{w}h_{t-1}R\varphi \qquad (2.21)$$

W_t is a function of the wage income $\overline{w}_t\overline{h}_t$, i.e. $W_t = \overline{w}\overline{h}_t\overline{f}$, where

$$\overline{f} = \frac{2 - p^a}{(1 - p^a + \varphi)}\left[\frac{1 - p^a}{1 + \beta^{-\sigma}R^{1-\sigma}} - \frac{(1 - p^a)(\Psi - 1)(1 - \nu^*)R\varphi}{1 + \beta^{-\sigma}R^{1-\sigma}}\frac{1}{1 + g^h}\right.$$
$$\left. + (1 - \Psi)(1 - \nu^*)\varphi + \frac{p^a(\Psi - 1)(1 - \nu^*)R\varphi}{1 + g^h}\right] \qquad (2.22)$$

and $h_t/\overline{h}_t = (2 - p^a)/(1 - p^a + \varphi)$. Total domestic savings, on the other hand, are measured by the growth rate of wealth reflecting an intergenerational aspect, i.e. $W_{t+1} - W_t = S_{t+1}$. Given that the wealth and total domestic savings increase in equilibrium by the growth of human capital, we obtain that $(1 + g^h)W_t - W_t = (1 + g^h)S_t$, i.e. $S_t = \frac{g^h}{1+g^h}W_t$. Substituting for the definitions of the wealth function and the savings function, it is evident that

$$S_t = \frac{g^h}{1 + g^h}\left[(1 - p^a)\frac{h_t\overline{w} - (\Psi - 1)(1 - \nu^*)\overline{w}h_{t-1}\varphi R}{1 + \beta^{-\sigma}R^{1-\sigma}}\right.$$
$$\left. + (1 - \Psi)(1 - \nu^*)\overline{w}h_t\varphi + p^a(\Psi - 1)(1 - \nu^*)\overline{w}h_{t-1}\varphi R\right] \qquad (2.23)$$

The savings rate, on the other hand, is equal by definition to $s = S_t/Q_t$, while $\overline{w}\overline{h}_t l = (1 - \alpha)Q_t$. Therefore, we obtain for the savings rate that

$$s = \frac{g^h}{1 + g^h}\frac{1 - \alpha}{(1 - p^a + \varphi)}f \qquad (2.24)$$

where

$$f = \frac{1 - p^a}{1 + \beta^{-\sigma}R^{1-\sigma}} - \frac{(1 - p^a)(\Psi - 1)(1 - \nu^*)R\varphi}{1 + \beta^{-\sigma}R^{1-\sigma}}\frac{1}{1 + g^h}$$
$$+ (1 - \Psi)(1 - \nu^*)\varphi + \frac{p^a(\Psi - 1)(1 - \nu^*)R\varphi}{(1 + g^h)} \qquad (2.25)$$

2.4 Sensitivity Analysis

In this section we provide a theoretical sensitivity analysis of the growth equation, and the savings rate with respect to the probability of migration p^a, the wedge between the skilled wage at home and abroad ϕ^f, and the borrowing constraint Ψ. We aim at answering the questions: (i) What is the relationship between the brain drain phenomenon and growth when individuals are subject to different levels of credit constraints? (ii) How does more liberal borrowing influence human capital accumulation when brain drain is present? (iii) What is the reaction of the savings rate to changes in the brain drain parameters and the degree of borrowing? Because the growth of average human capital depends on the educational time only, the conditions which determine the influence of p^a, ϕ^f and Ψ on ν^* are the same as the conditions which define the impact of these parameters on g^h. Before we begin conducting the sensitivity analysis, we establish a positive relationship between the growth rate and the savings rate as well as the educational time and the savings rate.

Proposition 2.3 *If $p^a \leq \frac{1}{2 + \beta^{-\sigma} R^{1-\sigma}}$, the relationship between the savings rate s and growth g^h, or the savings rate s and optimal educational time ν^* is positive.*[7]

Proof: See Appendix 2.9.2.

If we replace the parameters in the above condition with values which we use in our calibration ($\beta = 0.74$, $r = 2.243$, $\sigma = 1.4286$), we obtain that for any brain drain rate lower than 34.1%, higher growth and higher educational time lead to a higher savings rate. Our main findings from the sensitivity analysis under the validity of Proposition 2.3 are summarized in the next propositions.

Proposition 2.4 *The influence of the probability of skilled migration p^a on ν^*, g^h, and s (under the assumption of Proposition 2.3) is* [8],

$$\frac{\partial \nu^*}{\partial p^a} \gtreqless 0 \quad if \quad \sigma \gtreqless \ln\left[\frac{(1 + A\nu^*)\phi^f - (\Psi - 1)R(1 - \nu^*)\varphi}{1 + A\nu^* - (\Psi - 1)R(1 - \nu^*)\varphi}\right] \Big/ \ln\left[\frac{A\phi^f + (\Psi - 1)R\varphi}{A + (\Psi - 1)R\varphi}\right]$$

$$\frac{\partial g^h}{\partial p^a} \gtreqless 0 \quad if \quad \sigma \gtreqless \ln\left[\frac{(1 + A\nu^*)\phi^f - (\Psi - 1)R(1 - \nu^*)\varphi}{1 + A\nu^* - (\Psi - 1)R(1 - \nu^*)\varphi}\right] \Big/ \ln\left[\frac{A\phi^f + (\Psi - 1)R\varphi}{A + (\Psi - 1)R\varphi}\right]$$

$$\frac{\partial s}{\partial p^a} \gtreqless 0$$

Proof: See Appendix 2.9.3.

As Proposition 2.4 shows, a higher probability of skilled emigration can lead to higher optimal

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[7]For the empirical relevance of the relationship between the growth and the savings rate, see Carroll and Weil (1994).

[8]The structure $\frac{\partial y}{\partial x} \lesseqgtr 0$ if $a \gtreqless b$ is to read: $\frac{\partial y}{\partial x} < 0$ if $a > b$ and $\frac{\partial y}{\partial x} > 0$ if $a < b$ in all propositions

educational time only if agents are eager to shift consumption to the future (if the intertemporal elasticity of substitution is relatively high). The preference for future consumption mitigates the decline in utility due to foregone earnings for education when young. We call this phenomenon the incentive effect to accumulate human capital in response to p^a. If σ is relatively small, the optimal educational time decreases in p^a, which can be explained as follows: agents who are unwilling to postpone consumption will prefer to decrease their educational time in order to counterbalance the positive effect of p^a on their expected utility (all other things being constant). This is the disincentive effect to accumulate human capital in response to p^a. Moreover, for the influence of p^a on ν^* to be defined, the intertemporal elasticity of substitution is compared to a function which depends on the borrowing constraints. This proves that borrowing constraints do play a role in the educational choice of individuals when brain drain increases. In other words, for different values of Ψ, either the incentive or disincentive effect to accumulate human capital due to higher p^a may take place.

A rise in the skilled outmigration probability has an ambiguous impact on s. This is due to the fact that, from one hand, a higher brain drain rate implies a lower share of savers and thus a lower savings rate at home, but, from the other hand, a higher share of debtors from abroad and, therefore, a higher savings rate. Due to this trade–off the influence of p^a on s remains ambiguous even if g^h and ν^* unambiguously rise or decrease in response to a higher brain drain rate.

Proposition 2.5 *The influence of the wedge between skilled earnings at home and abroad ϕ^f on ν^*, g^h, and s (under the assumption in Proposition 2.3) is*

$$\frac{\partial \nu^*}{\partial \phi^f} \gtrless 0 \quad if \quad \sigma \gtrless \frac{(A\phi^f + (\Psi - 1)R\varphi)(1 + A\nu^*)}{((1 + A\nu^*)\phi^f - (\Psi - 1)(1 - \nu^*)R\varphi)A}$$

$$\frac{\partial g^h}{\partial \phi^f} \gtrless 0 \quad if \quad \sigma \gtrless \frac{(A\phi^f + (\Psi - 1)R\varphi)(1 + A\nu^*)}{((1 + A\nu^*)\phi^f - (\Psi - 1)(1 - \nu^*)R\varphi)A}$$

$$\frac{\partial s}{\partial \phi^f} \gtrless 0 \quad if \quad \sigma \gtrless \frac{(A\phi^f + (\Psi - 1)R\varphi)(1 + A\nu^*)}{((1 + A\nu^*)\phi^f - (\Psi - 1)(1 - \nu^*)R\varphi)A}$$

Proof: See Appendix 2.9.4.

Proposition 2.5 discusses the influence of the gap in skilled payment at home and abroad on the endogenous variables of interest. An increase in ϕ^f, similar to the influence of p^a, leads to higher educational time only if agents are willing to shift relatively more consumption to the future, i.e. in this case they are ready to forego more earnings for education in the present period. As before, the intertemporal elasticity of substitution should be measurably higher than a function containing the borrowing constraints, which implies that credit market frictions also play a role in determining the relationship between ν^*, g^h, s and ϕ^f. If σ is relatively small, an

increase in ϕ^f reduces the investment in human capital because agents counterbalance the rise in their expected utility due to higher ϕ^f (all other things being constant) by adjusting their educational time downwards.

Higher ϕ^f ensures a higher savings rate if the educational time and growth increase in response to ϕ^f due to Proposition 2.3. This is satisfied for a relatively high intertemporal elasticity of substitution σ. The opposite is valid if σ is low enough.

Proposition 2.6 *The influence of the borrowing constraint Ψ on ν^*, g^h, and s (under the assumption in Proposition 2.3) is*

$$\frac{\partial \nu^*}{\partial \Psi} > 0 \quad if \quad \sigma \leq 1$$
$$\frac{\partial g^h}{\partial \Psi} > 0 \quad if \quad \sigma \leq 1$$
$$\frac{\partial s}{\partial \Psi} < 0 \quad if \quad \varepsilon_\Psi^{\nu*} < 0$$

(ε_x^y is the elasticity of y with respect to x).

Proof: See Appendix 2.9.5.

As we see from Proposition 2.6, more relaxed borrowing constraints increase the investment in human capital only if agents put more weight on present consumption $\sigma \leq 1$.[9] This condition can be explained as follows: stronger credit relaxation raises first period consumption and decreases second period consumption (all other things being constant). Agents who are impatient and prefer to consume immediately experience an increase in utility in response to more relaxed borrowing constraints because their sacrifice in consumption for education is rebalanced by higher borrowing. Agents who are relatively eager to postpone consumption to the future experience disutility in response to higher borrowing due to the rise in the relatively less preferred first period consumption (all other things being constant). That is why the optimal educational decision in response to more extensive borrowing of agents who are relatively patient with respect to consumption is unclear.

More relaxed borrowing constraints decrease the savings rate if the optimal educational time decreases in response to higher Ψ (because of Proposition 2.3). In all other cases (even for improving growth), the relationship between Ψ and s is ambiguous because borrowing promotes dissaving of young agents.

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[9]In his original paper De Gregorio (1996) uses a lognormal utility function and comes to the conclusion that more relaxed borrowing constraints lead to higher educational investment.

2.5 Calibration

We calibrate the model by defining the parameters: β, σ, ϕ^f, p^a, B, φ, α, r, and A for the benchmark model (BM) with $\Psi = 1$. We assume that human capital grows at an annual growth rate of 2.5%, and an agent lives for 30 years each period, which implies that the productivity parameter in the human capital production function is equal to $A = 1.83$.

For ϕ^f we assume that it is equal to 4.8, which reflects the observed wedge between the average high–skilled remuneration in Eastern Europe vs. Western and Northern Europe according to *Eurostat* (See Appendix 1.9.5). For the baseline model we set the probability of skilled outflow at 5%, which corresponds to the emigration probability of high–skilled from Eastern Europe according to Docquier and Marfouk (2004) (See Appendix 1.9.5 for the brain drain rates of some EU Eastern European countries).

The interest rate at which debt is repaid $r = 2.243$ corresponds to an annual risk–free interest rate of approximately $r^a = 4\%$. According to the ECB (See Appendix 2.9.6), the annual interest rate of ten year government bonds for February 2013 of some transition countries lies in the range from 2.01 for the Czech Republic to 5.72 for Romania and 6.29 for Hungary. Although private debt is related to higher interest rates, the yield of government bonds is an appropriate measure for r in our model because of the assumption that agents cannot default on their credit. For β we choose a value of 0.74, which is derived from the assumption that the annual discount factor of future utility is 0.99 and $0.99^{30} = 0.74$. We set $\sigma = 1.4286$ by which we assume that young spend around half of their time in education.

Following Freeman and Oostendorp (2000), who estimate skill differentials in earnings, we assume that $\varphi = 0.4$, which defines an implicit skill premium between skilled (middle–aged) and low–skilled (young). B is normalized to 1. The elasticity of physical capital in the production function α is assumed as usual to be equal to 0.3.

By calibrating the model with the already discussed values, we end up with a benchmark aggregate savings rate equal to around 14%, which is rather on the lower bound of the empirically observed aggregate savings rates in some countries of Eastern Europe (see Appendix 2.9.6). In fact, we can attain higher s by increasing the interest rate. However, larger R would imply the presence of risk on the credit market, which our model excludes by assumption. If agents stand a default probability, the optimal educational time may increase due to the probability of avoiding the credit costs. Moreover, the aggregate savings rate could improve because it is positively related to growth and to the income of natives net of debt costs. This is true if the fall in migrants' credit repayment, augmenting domestic wealth, is relatively small.

The sensitivity analysis of the parameter p^a as well as ϕ^f, and Ψ are shown in Table 2.1. We

Table 2.1: *Calibration Results for p^a, ϕ^f and Ψ*

	$BM(\Psi = 1.0)$	$p^a = 0.10$	$p^a = 0.15$	$p^a = 0.20$	$\phi^f = 7.2$	$\phi^f = 9.6$
ν^*	0.6009	0.613	0.6244	0.6354	0.6051	0.6084
g^h	0.025	0.0254	0.0257	0.026	0.0252	0.0253
s	0.1412	0.1402	0.1389	0.1373	0.1416	0.142
	$BM(\Psi = 1.2)$	$p^a = 0.10$	$p^a = 0.15$	$p^a = 0.20$	$\phi^f = 7.2$	$\phi^f = 9.6$
ν^*	0.6579	0.6642	0.6704	0.6764	0.6606	0.6627
g^h	0.0267	0.0269	0.027	0.0272	0.0268	0.0268
s	0.1325	0.1316	0.1305	0.1291	0.1328	0.1332
	$BM(\Psi = 1.4)$	$p^a = 0.10$	$p^a = 0.15$	$p^a = 0.20$	$\phi^f = 7.2$	$\phi^f = 9.6$
ν^*	0.7008	0.7038	0.7068	0.7097	0.7026	0.7041
g^h	0.0279	0.028	0.0281	0.0281	0.0279	0.028
s	0.1253	0.1244	0.1234	0.1222	0.1256	0.1259
	$BM(\Psi = 1.6)$	$p^a = 0.10$	$p^a = 0.15$	$p^a = 0.20$	$\phi^f = 7.2$	$\phi^f = 9.6$
ν^*	0.7342	0.7352	0.7362	0.7372	0.7354	0.7365
g^h	0.0288	0.0288	0.0288	0.0289	0.0288	0.0289
s	0.1194	0.1185	0.1175	0.1164	0.1197	0.1199
	$BM(\Psi = 1.74)$	$p^a = 0.10$	$p^a = 0.15$	$p^a = 0.20$	$\phi^f = 7.2$	$\phi^f = 9.6$
ν^*	0.7535	0.7535	0.7536	0.7536	0.7545	0.7553
g^h	0.0293	0.0293	0.0293	0.0293	0.0293	0.0294
s	0.1159	0.115	0.114	0.1129	0.1162	0.1164
	$BM(\Psi = 1.75)$	$p^a = 0.10$	$p^a = 0.15$	$p^a = 0.20$	$\phi^f = 7.2$	$\phi^f = 9.6$
ν^*	0.7548	0.7547	0.7547	0.7547	0.7557	0.7566
g^h	0.0293	0.0293	0.0293	0.0293	0.0294	0.0294
s	0.1157	0.1148	0.1138	0.1127	0.1159	0.1161
	$BM(\Psi = 1.8)$	$p^a = 0.10$	$p^a = 0.15$	$p^a = 0.20$	$\phi^f = 7.2$	$\phi^f = 9.6$
ν^*	0.7609	0.7606	0.7604	0.7601	0.7618	0.7627
g^h	0.0295	0.0295	0.0295	0.0295	0.0295	0.0296
s	0.1146	0.1136	0.1126	0.1115	0.1148	0.115
	$BM(\Psi = 2)$	$p^a = 0.10$	$p^a = 0.15$	$p^a = 0.20$	$\phi^f = 7.2$	$\phi^f = 9.6$
ν^*	0.7828	0.7817	0.7806	0.7794	0.7834	0.784
g^h	0.0301	0.03	0.03	0.03	0.0301	0.0301
s	0.1105	0.1094	0.1084	0.1073	0.1106	0.1108

report a stepwise increase in the brain drain rate by 100% in correspondence to the observed empirical values of p^a in the range of (0.05-0.2). The gap in skilled remuneration at home and abroad is raised by 50% and 100%. We check the predictions of the model for Ψ equal from 1.0 to 2.0 within a step of 0.2, i.e. the share of credit out of disposable income in the first period lies between 0% to 100% in the calibration. We explicitly show the calibration results for $\Psi = 1.74$ and $\Psi = 1.75$ (although this spoils the steps in the calibration analysis), as for $\Psi \geq 1.75$ the impact of p^a on ν^* and g^h reverses its sign compared to the case where $\Psi \leq 1.74$.

In an opened economy where borrowing is relatively constrained $\Psi \leqslant 1.74$, a higher brain drain

rate makes agents invest more in education. This statement overlaps with the traditional result, on which the *brain gain* theory is based. Accordingly, the higher skilled migration probability increases growth. Raising the skilled migration probability from $p^a = 0.05$ to $p^a = 0.1$ or $p^a = 0.2$ ($\Psi = 1.2$) increases the annual growth from 0.0267 to 0.0269 or 0.0272 respectively. A major difference in our results in case of more liberal borrowing constraints $\Psi > 1.74$ is the negative influence of the higher brain drain probability on the educational incentives. This can be explained with the disincentive effect of agents to accumulate human capital when they avail of income for immediate consumption. The positive impact of p^a on g^h becomes *weaker* as borrowing constraints get more relaxed ($\Psi < 1.75$) due to a declining incentive to invest in education. The fall in growth for $\Psi = 1.8$ in response to higher p^a is almost negligible, while for $\Psi = 2.0$ g^h declines from 0.0301 to 0.03 when p^a reaches 0.1 or 0.2. This can be explained with the *stronger* (negative) influence of p^a on g^h at more relaxed borrowing constraints ($\Psi > 1.75$). The latter is a result of the propensity of agents to reduce their educational investment in response to p^a for relatively liberalized borrowing constraints.

Raising the brain drain probability (from $p^a = 0.05$ to $p^a = 0.2$) leads to a lower savings rate independent of the level of the borrowing constraints. The argument for this is as follows: from one hand, higher p^a leads to capital import (because more migrants supply capital) but also larger growth at least for $\Psi \leq 1.74$, which contributes to a higher savings rate; from the other hand, higher p^a leads to a lower population share of savers, which decreases the potential for savings. The positive effect of higher p^a on s (even for $\Psi \leq 1.74$) is lower than the negative effect of p^a on s so that the savings rate falls. The savings rate declines in response to higher $p^a = 0.2$ by 0.34 percentage points (from 0.1325 to 0.1291) when $\Psi = 1.2$, by 0.30 percentage points (from 0.1194 to 0.1164) when $\Psi = 1.6$ and by 0.32 percentage points (from 0.1105 to 0.1073) when $\Psi = 2$. If p^a rises to 0.1, the savings rate falls by a smaller degree by 0.09 percentage points (from 0.1325 to 0.1316) when $\Psi = 1.2$, by 0.09 percentage points (from 0.1194 to 0.1185) when $\Psi = 1.6$ and by 0.11 percentage points (from 0.1105 to 0.1094) when $\Psi = 2.0$. A higher gap in skilled earnings at home and abroad ϕ^f improves human capital accumulation independent of the borrowing constraints' severity because agents invest more in education to the end of migration. The higher educational incentive (and the higher growth) are sufficient conditions in this case for the savings rate to rise. Still, the change in the growth rate and the savings rate is almost negligible if ϕ^f increases to 7.2 and to 9.6. The positive effect of ϕ^f on g^h becomes *weaker* as borrowing relaxation rises, which implies that the impact of ϕ^f on g^h should also revert its sign at relatively large Ψ.

According to Table 2.1, more relaxed borrowing constraints (higher Ψ) increase growth by 0.38

percentage points (from 0.025 to 0.0288) when Ψ rises from 1.0 to 1.6 and by 0.13 percentage points (from 0.0288 to 0.0301) when the magnitude of the credit relaxation improves from $\Psi = 1.6$ to $\Psi = 2.0$ for a brain drain rate of 0.05. In the same case ($p^a = 0.05$), the savings rate falls by 2.18 percentage points (from 0.1412 to 0.1194) when Ψ rises from 1.0 to 1.6 and by 0.89 percentage points (from 0.1194 to 0.1105) when Ψ rises from 1.6 to 2.0 because of the augmented borrowing. Similar in quality is the impact of credit market liberalization on g^h and s when $p^a > 0.05$.

2.6 Welfare Analysis

In this section we perform a welfare analysis by determining the welfare of young agents $V_{y,t}$, agents who have reached their middle and old age in the domestic country, defined respectively by $V_{m,t}$ and $V_{o,t}$ at period t. V_t^d is the sum of the lifetime utilities of all domestic individuals at a specific point in time t (the index d stands for the domestic country). V_t measures not only the utility of the domestic population but also considers the lifetime welfare of migrants abroad at period t, where $V_{m,t}^f$, $V_{o,t}^f$ is the welfare of middle–aged and old agents abroad respectively, (the index f stands for the foreign country as explained earlier).

We report the average domestic welfare $\tilde{V}_t^d = V_t^d/n$ (n is the share of the remaining population), which is important for government's decision making, as well as average welfare $\tilde{V}_t = V_t/3$, which is essential for the decision making of the social planner. We calculate the average domestic welfare of the remaining individuals in the source country (and not total domestic welfare) in order to avoid understatement of the welfare measure due to the fall in the population share caused by brain drain.[10] For the welfare analysis we need the following definitions,

$$V_t = V_{y,t} + (1 - p^a)V_{m,t} + (1 - p^a)V_{o,t} + p^a V_{m,t}^f + p^a V_{o,t}^f$$

$$V_t^d = V_{y,t} + (1 - p^a)V_{m,t} + (1 - p^a)V_{o,t}$$

$$V_{y,t} = U(c_{y,t}) + (1 - p^a)\beta U(c_{m,t+1}) + (1 - p^a)\beta^2 U(c_{o,t+1}) + p^a \beta U(c_{m,t+1}^f) + p^a \beta^2 U(c_{o,t+2}^f)$$

$$V_{m,t} = U(c_{m,t}) + \beta U(c_{o,t+1})$$

$$V_{o,t} = U(c_{o,t})$$

$$V_{m,t}^f = U(c_{m,t}^f) + \beta U(c_{o,t+1}^f)$$

$$V_{o,t}^f = U(c_{o,t}^f)$$

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[10]This problem does not exist when we consider total welfare, which comprises of the welfare of the domestic population and migrants. Still, we report the average welfare of total population including migrants for comparability with the average domestic welfare.

Our welfare analysis is performed from period $t = 0$ till period $t = 4$ ($h_0 = 1$) for the next 150 years, whereas we assume that in $t = 0$ a parameter change takes place. As proposed by Soares (2008), the social welfare function does not discount the welfare of next generations.

We expect that the long–term influence of p^a, ϕ^f and Ψ on (domestic) welfare is in accordance with the impact of p^a, ϕ^f and Ψ on growth. This results stems from the fact that the effect of a parameter on long–run welfare in an overlapping generations setting is determined by the impact of the same parameter on growth (see Yakita (2004), Wong and Yip (1999)).

A change in p^a, ϕ^f and Ψ alters the lifetime utility of young agents in the period of alternative values of p^a, ϕ^f and Ψ ($t = 0$). However, p^a, ϕ^f and Ψ do not influence the already established individual welfare of middle–aged and old agents in the domestic country in the same period (notice that wages are not affected by labor supply). Nevertheless, in $t = 0$ the total welfare of middle–aged suddenly changes in response to p^a because a larger share of them may migrate and remain in the foreign country. On the other hand, rising ϕ^f in $t = 0$ alters the income of middle–aged migrants abroad although it does not have any impact on the income of middle–aged agents at home. That is why for $t = 0$ we include the utility of middle–aged agents in the welfare analysis of p^a and ϕ^f. We also consider the utility of middle–aged (for $t = 0$) in the welfare analysis of Ψ for comparability with the welfare analysis of p^a and ϕ^f although Ψ does not exert any effect on the individual or total welfare of middle–aged domestic agents or middle–aged migrants in this period.

For $t = 0$ we have that $V^d = V_y + (1 - p^a)V_m$ and $n = 2 - p^a$. For all other periods, $V^d = V_y + (1 - p^a)V_m + (1 - p^a)V_o$ and $n = 3 - 2p^a$. The welfare of migrants is not explicitly shown in the following analysis but can be easily deducted from Table 2.2 and Table 2.3.

We report our welfare analysis in Table 2.2 and 2.3 assuming a benchmark with $\Psi = 1.2$ and $\Psi = 2.0$ respectively. Here we find what is the impact of p^a, ϕ^f and Ψ on individual welfare and average (domestic) welfare. We increase (stepwise) the brain drain rate and the gap in skilled remuneration at home and abroad by 100%. We do not report $p^a = 0.15$ as its influence on welfare compared to $p^a = 0.1$ and 0.2 is qualitatively the same. Credit market relaxation represented by $\Psi = 1.3$ and $\Psi = 1.4$ (with a benchmark of $\Psi = 1.2$ in Table 2.2) and by $\Psi = 2.5$ and 3.0 (with a benchmark of $\Psi = 2.0$ in Table 2.3) constitutes respectively a 50% and 100% increase in the credit share out of income defined as $(\Psi_1 - \Psi_0)/(\Psi_0 - 1)$.

Table 2.2: *Transitional Welfare Analysis of BM ($\Psi = 1.2$) with Respect to p^a, ϕ^f and Ψ*

	BM	p_1^a	p_2^a	ϕ_1^f	Ψ_1	Ψ_2
$V_{y,t=0}$	4.769	4.8755	5.0888	4.8337	4.7721	4.7748
$V_{m,t=0}$	3.6498	3.6498	3.6498	3.6498	3.6498	3.6498
$\tilde{V}_{t=0}^d$	4.2238	4.2949	4.4493	4.257	4.2254	4.2267
$\tilde{V}_{t=0}$	4.2661	4.3761	4.5963	4.333	4.2677	4.2691
$V_{y,t=1}$	6.0448	6.1895	6.4798	6.1343	6.083	6.1161
$V_{m,t=1}$	4.6263	4.6354	4.6528	4.6333	4.6232	4.6206
$V_{o,t=1}$	2.6988	2.6988	2.6988	2.6988	2.6988	2.6988
$\tilde{V}_{t=1}^d$	4.484	4.568	4.7543	4.5172	4.4962	4.5067
$\tilde{V}_{t=1}$	4.5325	4.6598	4.9148	4.611	4.5449	4.5555
$V_{y,t=2}$	7.6619	7.8577	8.2511	7.7848	7.754	7.8341
$V_{m,t=2}$	5.8639	5.8847	5.9247	5.8799	5.8932	5.9185
$V_{o,t=2}$	3.4208	3.4276	3.4405	3.426	3.4186	3.4166
$\tilde{V}_{t=2}^d$	5.6836	5.7996	6.0551	5.7329	5.7242	5.7595
$\tilde{V}_{t=2}$	5.7451	5.9162	6.2593	5.8519	5.7864	5.8222
$V_{y,t=3}$	9.7117	9.9755	10.5065	9.8794	9.884	10.0347
$V_{m,t=3}$	7.4327	7.4708	7.5442	7.462	7.5121	7.5811
$V_{o,t=3}$	4.336	4.3514	4.3809	4.3478	4.3576	4.3764
$\tilde{V}_{t=3}^d$	7.2041	7.3627	7.7102	7.2754	7.2966	7.3773
$\tilde{V}_{t=3}$	7.2821	7.5107	7.9703	7.4265	7.3758	7.4577
$V_{y,t=4}$	12.3099	12.6642	13.3784	12.5376	12.5991	12.8535
$V_{m,t=4}$	9.4211	9.4843	9.6064	9.4697	9.5756	9.7106
$V_{o,t=4}$	5.496	5.5242	5.5784	5.5177	5.5547	5.6057
$\tilde{V}_{t=4}^d$	9.1314	9.3471	9.8178	9.233	9.301	9.4497
$\tilde{V}_{t=4}$	9.2302	9.535	10.149	9.4246	9.402	9.5526

Parameters calibrated as follows: $p_1^a = 0.1$, $p_2^a = 0.2$, $\phi_1^f = 9.6$, $\Psi_1 = 1.3$, $\Psi_2 = 1.4$

Table 2.3: *Transitional Welfare Analysis of BM ($\Psi = 2.0$) with Respect to p^a, ϕ^f and Ψ*

	BM	p_1^a	p_2^a	ϕ_1^f	Ψ_1	Ψ_2
$V_{y,t=0}$	4.7855	4.899	5.1262	4.8529	4.7905	4.7939
$V_{m,t=0}$	3.5312	3.5312	3.5312	3.5312	3.5312	3.5312
$\tilde{V}_{t=0}^d$	4.1744	4.2511	4.4173	4.209	4.177	4.1787
$\tilde{V}_{t=0}$	4.2171	4.3326	4.5638	4.2856	4.2196	4.2213
$V_{y,t=1}$	6.248	6.3947	6.6877	6.3378	6.3112	6.3539
$V_{m,t=1}$	4.6104	4.6077	4.6023	4.6133	4.6056	4.6024
$V_{o,t=1}$	2.6111	2.6111	2.6111	2.6111	2.6111	2.6111
$\tilde{V}_{t=1}^d$	4.5201	4.6041	4.7917	4.552	4.5404	4.554
$\tilde{V}_{t=1}$	4.5699	4.6981	4.9542	4.6484	4.5904	4.6043
$V_{y,t=2}$	8.1576	8.347	8.725	8.277	8.3146	8.4214
$V_{m,t=2}$	6.0194	6.0145	6.0042	6.0249	6.0676	6.1
$V_{o,t=2}$	3.4091	3.4071	3.4031	3.4112	3.4055	3.4032
$\tilde{V}_{t=2}^d$	5.9016	6.0094	6.2503	5.9453	5.9704	6.0171
$\tilde{V}_{t=2}$	5.9666	6.1321	6.4624	6.0711	6.0365	6.084
$V_{y,t=3}$	10.6508	10.8953	11.3827	10.8097	10.9539	11.1617
$V_{m,t=3}$	7.8591	7.8507	7.8332	7.8684	7.9936	8.085
$V_{o,t=3}$	4.451	4.4473	4.4397	4.455	4.4866	4.5106
$\tilde{V}_{t=3}^d$	7.7053	7.8441	8.1543	7.7644	7.8656	7.975
$\tilde{V}_{t=3}$	7.7902	8.0042	8.431	7.9287	7.9527	8.0637
$V_{y,t=4}$	13.9059	14.2216	14.8501	14.1173	14.4311	14.7937
$V_{m,t=4}$	10.2611	10.2474	10.2194	10.276	10.5311	10.7158
$V_{o,t=4}$	5.8113	5.8051	5.7922	5.8182	5.9108	5.9783
$\tilde{V}_{t=4}^d$	10.0602	10.2389	10.6382	10.1402	10.3624	10.57
$\tilde{V}_{t=4}$	10.1711	10.4478	10.9993	10.3548	10.4772	10.6876

Parameters calibrated as follows: $p_1^a = 0.1$, $p_2^a = 0.2$, $\phi_1^f = 9.6$, $\Psi_1 = 2.4$, $\Psi_2 = 3$

As it is evident from Table 2.2, an increase of the brain drain rate for relatively tight borrowing constraints $\Psi = 1.2$ raises the utility of the domestic population in all periods across all generations as there are no losers in society. This result can be explained with the positive impact of a higher brain drain rate on the investment in human capital in an environment with constrained borrowing. If, to the contrary, the credit market is relatively liberalized (as in Table 2.3 with $\Psi = 2.0$), the (domestic) welfare improvement in response to higher skilled migration is caused solely by an increase in young agents' expected utility. Losers in this case are middle–aged from $t = 1$ onwards and old–aged from $t = 2$ onwards as their consumption falls in response to deteriorating growth. Average short term welfare improves but it is not as large as the welfare in response to higher p^a with relatively constrained borrowing. This temporary gain in utility is expected to lose importance in the long run because of the negative impact of p^a on growth when credit constraints are relatively liberal.

A rise in the wedge of skilled earnings (Table 2.2 and 2.3) between the foreign and the domestic country independent of the severity of borrowing constraints increases the welfare of all generations. This can be explained by the positive incentive effect which ϕ^f exerts on human capital accumulation. There are no losers in any period in society if ϕ^f goes up to 9.6.

More liberal borrowing constraints in Table 2.2 ($\Psi = 1.3$ or $\Psi = 1.4$) make young agents better–off but decrease the lifetime utility of middle–aged (period $t = 1$) and old (period $t = 2$) due to higher credit costs. Middle–aged profit at period $t = 2$, while old at period $t = 3$ from the increasing growth. All in all, the lifetime utility of the (domestic) population in all periods rises due to more liberal credit distribution when Ψ reaches 1.3 or 1.4. The same results are obtained in Table 2.3 where credit market liberalization (for Ψ rising to 2.5 or 3.0) also leads to welfare improvement.

2.7 Conclusion

We build a three–period overlapping generations model with human capital accumulation subject to binding borrowing constraints in an environment of perfect physical capital mobility (drawing on De Gregorio (1996)) and probabilistic brain drain (based on Beine *et al.* (2001)). On the consumer side there are three types of agents: young, middle–aged and old. Young agents invest optimal time in education and obtain additional income by borrowing, which is constrained. Middle–aged individuals who remain at home earn income in correspondence to their human capital, repay their debt and save. Savings are used for consumption in the third period. Some educated are randomly singled out to leave the domestic country and work for a higher foreign wage abroad (following the same lifetime pattern as at home). We postulate

full enforcement of credit markets for migrants and natives. On the production side agents (with the exception of old) supply inelastically their labor. Physical capital, which is perfectly mobile, is used additionally as an input. Human capital investment, which is the triggering growth factor, is influenced by borrowing constraints and skilled outmigration.

Our theoretical model shows that an increase in the skilled migration probability and the wedge between skilled earnings at home and abroad could raise the optimal educational time and growth only if agents are relatively eager to consume in the future (i.e the intertemporal elasticity of substitution σ is high enough). Moreover, the intertemporal elasticity of substitution must outweigh the effects of the borrowing constraint, which confirms the importance of credit constraints for the educational decision of agents when brain drain takes place. The aggregate savings rate reacts ambiguously to changes in the outmigration of skilled agents. A sufficient condition for the increase in the aggregate savings rate in response to a higher wedge between the remuneration of educated at home and abroad is that growth rises. More relaxed borrowing constraints increase growth only if agents are more prone to consume in the present $\sigma \leq 1$. The aggregate savings rate declines with the relaxation of the borrowing constraints if growth decreases. In all other cases the aggregate savings rate's behavior in response to more relaxed borrowing constraints is ambiguous.

According to our calibration, educational time increases in response to a higher brain drain probability only in an environment of tighter borrowing constraints (or if the share of credit out of disposable income is lower than or equal to 74%, $\Psi \leq 1.74$). The opposite is valid when borrowing constraints are relaxed (or if $\Psi > 1.74$). The aggregate savings rate declines due to a higher brain drain probability independent of the severity of borrowing constraints. The average (domestic) welfare is also higher than the benchmark case in response to a rising brain drain rate independent of the degree of credit market liberalization in the short run. Nevertheless, the immediate losers from a stronger brain drain probability in an setting with relatively relaxed borrowing constraints are middle–aged and old–aged. The long–term impact of brain drain on (domestic) social utility with relatively relaxed borrowing should be negative. A higher wedge between the skilled wage in the foreign and the domestic country leads to higher human capital accumulation, a higher aggregate savings rate and higher welfare compared to the benchmark case. More relaxed borrowing constraints are good for growth but decrease the aggregate savings rate. Credit market liberalization enhances average (domestic) welfare in all periods although a generation of middle–aged and later old loses temporarily in terms of welfare immediately after the introduction of more liberal borrowing constraints.

We simplified our analysis by assuming the lack of a government sector. The existence of a

government with its taxation policies very often influences the flow of capital and investors' decisions. The model could then be extended to describe the interaction between government taxation, physical capital flow in a setting of human capital accumulation and brain drain.

2.8 References

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2.9 Appendix

2.9.1 Proof of Proposition 2.1

Let ν from (2.17) be defined as,

$$\nu = 1 - (\Psi\varphi)^{\sigma-1} \Big/ \Big[\frac{(1-p^a)(A + (\Psi-1)R\varphi)N}{((1+A\nu) - (\Psi-1)R(1-\nu)\varphi)^{\frac{1}{\sigma}}} + \frac{p^a(\phi^f A + (\Psi-1)R\varphi)N}{((1+A\nu)\phi^f - (\Psi-1)R(1-\nu)\varphi)^{\frac{1}{\sigma}}} \Big]^{\sigma}$$

and ν in the above equation is $\nu \equiv G(\nu)$, where we prove that $0 < G(\nu) < 1$. First, for $\nu > 1$ in (2.17), while the RHS will be positive, the same cannot be said about the LHS unless $\sigma = 1/2$. Second, for $\nu < 0$ in (2.17), the LHS of (2.17) will be positive, while the RHS of (2.17) is not positive unless $\sigma = 1/2$. To see this, assume that $1 + A\nu = D\nu$, where $D = \frac{1+A\nu}{\nu} > 0$ similar to the assumption on $A > 0$. If (2.17) has a solution where $\nu < 0$, for D to remain always positive, it must hold that $1 + A\nu < 0$, which implies that the RHS of (2.17) is not positive unless $\sigma = 1/2$. If ν is close to one, the LHS of (2.17) will tend to infinity, which cannot be said about the RHS of (2.17). For $\nu = 0$ in (2.17) there is a constraint $\tilde{\Psi}$, which completely

disallows agents to invest in education. This case is not interesting for our model. That is why we assume that $\Psi < \tilde{\Psi}$. Therefore, $0 < G(\nu) < 1$.

Furthermore, assume that $P(\nu) \equiv \nu - G(\nu)$. Because $0 < G(\nu) < 1$, it must hold that $P(0) < 0$ and $P(1) > 0$ (Notice that $\nu = 1$ or $\nu = 0$ are not supposed to be roots of (2.17)), which implies that there exists at least one equilibrium value of ν in ν–$P(\nu)$ plane.

Next, we prove that ν is also unique. By the definition of $P(\nu)$ it follows that its derivative with respect to ν is equal to, $P'(\nu) = 1 - G'(\nu)$. Because $G'(\nu)$ is always negative, it must be that $P'(\nu)$ is always positive. Assuming that there are more than one equilibrium in (2.17), we can apply the mean value theorem. If ν_3 is found in the area of (ν_2, ν_1) where $\nu_2 > \nu_1$, the mean value theorem implies that $P'(\nu_3) = \frac{P(\nu_2) - P(\nu_1)}{\nu_2 - \nu_1}$. Given that $P(\nu_1) = 0$ and $P(\nu_2) = 0$ (if ν_1 and ν_2 are roots, $P(.)$ should be always zero by definition), it should hold that $P'(\nu_3) = 0$. Nevertheless, we proved that $P'(\nu)$ is always positive, by which we have reached a contradiction. Therefore, the equilibrium value of ν in (2.17) is unique and equal to ν^*.

2.9.2 Proof of Proposition 2.3

Differentiating f with respect to g^h,

$$\frac{\partial f}{\partial g^h} = \frac{(1 - p^a)(\Psi - 1)(1 - \nu^*)R\varphi}{(1 + \beta^{-\sigma}R^{1-\sigma})(1 + g^h)^2} - \frac{p^a(\Psi - 1)(1 - \nu^*)R\varphi}{(1 + g^h)^2}$$

Differentiating f with respect to ν^*,

$$\frac{\partial f}{\partial \nu^*} = \frac{(1 - p^a)(\Psi - 1)R\varphi}{(1 + \beta^{-\sigma}R^{1-\sigma})(1 + g^h)} - \underbrace{(1 - \Psi)}_{<0}\varphi - \frac{p^a(\Psi - 1)R\varphi}{(1 + g^h)}$$

Both derivatives are positive if

$$\frac{1 - p^a}{1 + \beta^{-\sigma}R^{1-\sigma}} \geq p^a$$

from where Proposition 2.3 is derived.

2.9.3 Proof of Proposition 2.4

We define F from the condition for optimal educational time, so we obtain,

$$F = -\frac{(\Psi\varphi)^{(1 - \frac{1}{\sigma})}}{(1 - \nu^*)^{\frac{1}{\sigma}}} + \frac{(1 - p^a)N(A + (\Psi - 1)R\varphi)}{((1 + A\nu^*) - (\Psi - 1)R(1 - \nu^*)\varphi)^{\frac{1}{\sigma}}}$$
$$+ \frac{p^a N(\phi^f A + (\Psi - 1)R\varphi)}{((1 + A\nu^*)\phi^f - (\Psi - 1)R(1 - \nu^*)\varphi)^{\frac{1}{\sigma}}} = 0$$

By applying the implicit function theorem using (2.17) (where $F_z = \frac{\partial F}{\partial z}$), we obtain for the influence of p^a on ν^* the following,

$$\frac{\partial \nu^*}{\partial p^a} = -\frac{F_{p^a}}{F_{\nu^*}}$$

It is obvious that $F_{\nu^*} < 0$. As far as the derivate of F with respect to p^a is concerned,

$$F_{p^a} = -\frac{(A + (\Psi - 1)R\varphi)N}{(1 + A\nu^* - (\Psi - 1)(1 - \nu^*)R\varphi)^{\frac{1}{\sigma}}} + \frac{(A\phi^f + (\Psi - 1)R\varphi)N}{((1 + A\nu^*)\phi^f - (\Psi - 1)(1 - \nu^*)R\varphi)^{\frac{1}{\sigma}}}$$

Simplifying the above expression,

$$F_{p^a} = -\frac{((1 + A\nu^*)\phi^f - (\Psi - 1)(1 - \nu^*)R\varphi)^{\frac{1}{\sigma}}(A + (\Psi - 1)R\varphi)}{(1 + A\nu^* - (\Psi - 1)(1 - \nu^*)R\varphi)^{\frac{1}{\sigma}}((1 + A\nu^*)\phi^f - (\Psi - 1)(1 - \nu^*)R\varphi)^{\frac{1}{\sigma}}}N$$
$$+ \frac{(1 + A\nu^* - (\Psi - 1)(1 - \nu^*)R\varphi)^{\frac{1}{\sigma}}(A\phi^f + (\Psi - 1)R\varphi)}{((1 + A\nu^*)\phi^f - (\Psi - 1)(1 - \nu^*)R\varphi)^{\frac{1}{\sigma}}((1 + A\nu^*) - (\Psi - 1)(1 - \nu^*)R\varphi)^{\frac{1}{\sigma}}}N$$

The sign of F_{p^a} depends on the value of

$$\tilde{F}_{p^a} = -((1 + A\nu^*)\phi^f - (\Psi - 1)(1 - \nu^*)R\varphi)^{\frac{1}{\sigma}}(A + (\Psi - 1)R\varphi)$$
$$+ (1 + A\nu^* - (\Psi - 1)(1 - \nu^*)R\varphi)^{\frac{1}{\sigma}}(A\phi^f + (\Psi - 1)R\varphi)$$

Therefore, $F_{p^a} > 0$ and $\frac{\partial \nu^*}{\partial p^a} = -\frac{F_{p^a}}{F_\nu} > 0$ if

$$\sigma > \ln\left[\frac{(1 + A\nu^*)\phi^f - (\Psi - 1)R(1 - \nu^*)\varphi}{1 + A\nu^* - (\Psi - 1)R(1 - \nu^*)\varphi}\right] \Big/ \ln\left[\frac{A\phi^f + (\Psi - 1)R\varphi}{A + (\Psi - 1)R\varphi}\right]$$

and the other way around. The same condition determines the relationship between growth and brain drain. In the end, we discuss the relationship between the savings rate and the brain drain probability. The direct effect of p^a on f,

$$\frac{\partial f}{\partial p^a} = -\frac{1 + g^h - (\Psi - 1)(1 - \nu^*)R\varphi}{(1 + \beta^{-\sigma}R^{1-\sigma})(1 + g^h)} + \frac{(\Psi - 1)(1 - \nu^*)R\varphi}{(1 + g^h)} \gtrless 0$$

is ambiguous. Therefore, the influence of p^a on s is also ambiguous,

$$\frac{\partial s}{\partial p^a} = \underbrace{\frac{\partial s}{\partial p^a}}_{>0} + \underbrace{\frac{\partial s}{\partial g^h}\frac{\partial g^h}{\partial p^a}}_{>0} + \underbrace{\frac{\partial s}{\partial f}\frac{\partial f}{\partial g^h}\frac{\partial g^h}{\partial p^a}}_{>0 \quad >0} + \underbrace{\frac{\partial s}{\partial f}\frac{\partial f}{\partial \nu^*}\frac{\partial \nu^*}{\partial p^a}}_{>0 \quad >0} + \underbrace{\frac{\partial s}{\partial f}\frac{\partial f}{\partial p^a}}_{>0 \quad \gtrless 0}$$

2.9.4 Proof of Proposition 2.5

By applying the implicit function theorem using (2.17), we obtain for the influence of ϕ^f on ν^*,

$$\frac{\partial \nu^*}{\partial \phi^f} = -\frac{F_{\phi^f}}{F_{\nu^*}}$$

The derivate of F with respect to ϕ^f is,

$$F_{\phi^f} = p^a N \frac{A}{((1 + A\nu^*)\phi^f - (\Psi - 1)(1 - \nu^*)R\varphi)^{\frac{1}{\sigma}}}$$

$$- \frac{1}{\sigma} p^a N \frac{A\phi^f + (\Psi - 1)R\varphi}{((1 + A\nu^*)\phi^f - (\Psi - 1)(1 - \nu^*)R\varphi)^{\frac{1}{\sigma}+1}}(1 + A\nu^*)$$

The sign of F_{ϕ^f} depends on the value of

$$\tilde{F}_{\phi^f} = ((1 + A\nu^*)\phi^f - (\Psi - 1)(1 - \nu^*)R\varphi)A - \frac{1}{\sigma}(A\phi^f + (\Psi - 1)R\varphi)(1 + A\nu^*)$$

Therefore, $F_{\phi^f} > 0$ and $\frac{\partial \nu^*}{\partial p^a} = -\frac{F_{\phi^f}}{F_\nu} > 0$ if

$$\sigma > \frac{(A\phi^f + (\Psi - 1)R\varphi)(1 + A\nu^*)}{((1 + A\nu^*)\phi^f - (\Psi - 1)(1 - \nu^*)R\varphi)A}$$

and the other way around. The conditions which lead to a change in the optimal educational time in case of a rise in ϕ^f lead to a parallel change in growth. In the end, we discuss the relationship between the savings rate and the wedge between skilled payment in the foreign and the domestic country. For the partial derivative of s with respect to ϕ^f, we obtain,

$$\frac{\partial s}{\partial \phi^f} = \underbrace{\frac{\partial s}{\partial g^h} \frac{\partial g^h}{\partial \phi^f}}_{>0} + \underbrace{\frac{\partial s}{\partial f} \frac{\partial f}{\partial g^h}}_{>0} \underbrace{\frac{\partial g^h}{\partial \phi^f}}_{>0} + \underbrace{\frac{\partial s}{\partial f} \frac{\partial f}{\partial \nu^*}}_{>0} \underbrace{\frac{\partial \nu^*}{\partial \phi^f}}_{>0}$$

The influence of ϕ^f on the educational time and growth is decisive for the impact of ϕ^f on the savings rate.

2.9.5 Proof of Proposition 2.6

By applying the implicit function theorem using (2.17), we obtain for the influence of Ψ on ν^*,

$$\frac{\partial \nu^*}{\partial \Psi} = -\frac{F_\Psi}{F_{\nu^*}}$$

The derivate of F with respect to Ψ is,

$$F_\Psi > 0 \qquad \text{if} \qquad \frac{1}{\sigma} \geq 1$$

The condition $\frac{1}{\sigma} - 1 \geq 0$ stems form the differentiation of the first term in F with respect to Ψ. Because all other terms in F differentiated with respect to Ψ are positive, only the intertemporal elasticity of substitution determines straightforward the relationship between Ψ and ν^*. The latter implies that $\frac{\partial \nu^*}{\partial \Psi} = -\frac{F_\Psi}{F_{\nu^*}} > 0$ if $\frac{1}{\sigma} \geq 1$ or $\sigma \leq 1$. For the partial derivative of s on Ψ, we obtain

$$\frac{\partial s}{\partial \Psi} = \underbrace{\frac{\partial s}{\partial g^h} \frac{\partial g^h}{\partial \Psi}}_{>0} + \underbrace{\frac{\partial s}{\partial f} \frac{\partial f}{\partial g^h}}_{>0} \underbrace{\frac{\partial g^h}{\partial \Psi}}_{>0} + \underbrace{\frac{\partial s}{\partial f} \frac{\partial f}{\partial \nu^*}}_{>0} \underbrace{\frac{\partial \nu^*}{\partial \Psi}}_{>0} + \underbrace{\frac{\partial s}{\partial f}}_{>0} \underbrace{\frac{\partial f}{\partial \Psi}}_{<0}$$

where

$$\frac{\partial f}{\partial \Psi} = \frac{(1-\nu^*)R}{1+g^h}\varphi \underbrace{\left[-\frac{(1-p^a)}{1+\beta^{-\sigma}R^{1-\sigma}} + p^a \right]}_{<0} - (1-\nu^*)\varphi < 0$$

Therefore, the influence of the borrowing constraints on savings will be negative if growth decreases in response to relaxation of the borrowing constraints and is ambiguous otherwise.

2.9.6 Empirical Data

Table 2.4: *Aggregate Savings Rates from Selected Eastern European States*

	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	*Av.*
Bulgaria	13.2	12.6	10.8	11.6	10.8	11.3	12.5	14.6	14.4	31.3	27.9	28.1	28.7	28.9	15.2
CzechRepublic	26.9	27.8	28.2	27.0	25.8	28.0	29.2	30.7	32.5	31.3	27.9	28.1	28.7	28.9	28.6
Greece	12.4	11.4	11.6	10.2	13.9	14.1	12.1	13.2	12.6	9.5	7.1	8.3	8.0	8.6	10.9
Hungary	23.6	23.7	24.2	22.8	19.8	22.3	22.4	23.2	23.3	24.0	22.8	25.2	26.2	25.4	23.5
Poland	19.3	18.4	17.1	15.2	16.1	17.7	18.5	19.2	21.6	19.9	20.4	19.8	20.9	21.0	18.9
Romania	11.2	14.3	14.9	16.0	14.3	13.2	12.6	14.7	16.9	18.2	19.3	19.8	21.6	21.9	16.3
Slovakia	23.3	23.4	21.5	21.8	22.7	23.6	24.2	24.0	26.7	25.3	19.1	22.4	24.5	25.7	23.4
Slovenia	23.5	24.0	24.2	25.1	25.2	26.2	26.8	28.4	30.2	28.7	24.0	22.1	21.7	22.3	25.2

Source: World Bank (World Development Indicators),

Last accessed on 1.12.2013

Table 2.5: *Annual Interest Rates of Government Bonds with Maturities Close to Ten Years in Selected Eastern European States*

	05.12	06.12	07.12	08.12	09.12	10.12	11.12	12.12	01.12	02.12
Bulgaria	5.11	5.07	4.87	4.28	3.80	3.39	3.22	3.44	3.27	3.25
CzechRepublic	3.31	3.11	2.60	2.38	2.37	2.24	1.92	1.92	1.96	2.01
Hungary	8.33	8.30	7.56	7.36	7.28	6.94	6.87	6.44	6.23	6.29
Latvia	5.15	5.07	4.67	4.45	3.92	3.52	3.32	3.24	3.21	3.22
Lithuania	5.30	4.96	4.82	4.84	4.53	4.32	4.11	4.00	3.97	4.06
Poland	5.41	5.24	4.99	4.88	4.85	4.57	4.18	3.88	3.91	3.99
Romania	6.50	6.68	6.52	6.84	6.54	6.85	6.84	6.65	5.90	5.72

Source: http://www.ecb.europa.eu/stats/money/long/html/index.en.html,

Last accessed on 1.12.2013

3 The Shadow Economy, and Risky Human Capital Accumulation in an Environment of Productive Government Spending, and Public Education

3.1 Introduction

The question about the economic rationale of income underreporting has been first raised by Allingham and Sandmo (1972). Their work is established in a utility optimization setting, where an agent decides how much income to report to authorities given a probability of paying a penalty if caught underreporting. The model by Allingham and Sandmo (1972) does not explain why people report so much income to tax authorities given the empirically observed low probability of detection. Since then economic theorists have come up with other factors, which could explain individuals' propensity to pay taxes such as tax morale, perception of inequity, intrinsic motivation, tax structure, trust in government, preference for specific public goods etc. (see Andreoni *et al.* (1998) and Alm (2012) for a review of studies).

In aggregate terms tax evasion has found its place in the economic research dealing with the shadow economy. The interest of economic theorists in the shadow economy has empirical grounds. The share of the informal sector out of official income has a significant part, for instance, in European transition countries. Bühn and Schneider (2012) show that the average share of the informal sector out of GNP for 1999-2007 in some EU Eastern European countries such as the Slovak Republic is around 18%, in Hungary is 24.4%, followed by Poland with 27.2%, the Czech Republic with 32% and Bulgaria with 35.3%.

Although the term *shadow economy* does not incorporate tax evasion only[1], the macroeconomic theory on growth and the informal sector has established itself as a disciple of the microeconomic

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[1]Agents or firms which work in the shadow economy may pay their taxes but still do not meet specific standards of production, which makes them a part of the *informal economy*.

theory on tax evasion. The models relating growth to the shadow economy, which examine the impact of alternative government policies (tax rates, penalty rate, audit rate) on the share of the informal sector and economic development, have been mostly embedded in a physical capital accumulation setting with at most a positive government externality on production. Loayza (1996) explains the existence of an informal sector with the presence of a too large income tax rate. He additionally investigates the impact of taxation on growth in an economy whose production is augmented by government spending congested by the informal sector. Lin and Yang (2001) disprove that the impact of higher income taxation on the size of the informal economy is negative (a proposition by Allingham and Sandmo (1972)) modeling a dynamic portfolio choice with AK production where the infinitely lived representative agent additionally draws utility from a public good. Ihrig and Moe (2004) find that the income tax rate is the best policy to reduce the informal sector if the shadow production decreases over time due to the assumption that it is labor intensive. Chen (2003) investigates the impact of the penalty and the audit rate and corruption costs in an economy with government externality on production, where the representative agent chooses its optimal tax evasion, while the government sets the optimal tax rate given tax evasion. Peñalosa and Turnovsky (2005) explore the optimal distribution of taxation among capital and labor if an informal sector exists. Turnovsky and Basher (2009) extend the analysis of the optimality of policies aiming to decrease the informal sector by including not only the effectiveness of labor and capital taxation but tax enforcement policies as well.

A significant omission of the previous literature on the shadow economy in a growth context is the disregard for (i) risk in the formal sector (represented here by uncertainty in the realization of human capital of skilled) and (ii) public financement of the risky enterprise to gain human capital (represented here by public education). The impact of risk in human capital accumulation is vital for the share of the shadow economy if risk averse agents are able to choose an occupation with less risk, for instance, in the low–skilled informal sector (incurring only a penalty cost if detected). In this context it is relevant to investigate the qualitative and quantitative effect of the risk measure on the informal sector and economic development and to compare it with the impact of taxation and the usually cited tax enforcement policies on these measures. This could answer the question whether the introduction of an insurance mechanism in the formal economy could be wished or not. The inclusion of public education as a growth factor enriches, on the other hand, the arsenal of policies, which a government may use when dealing with the shadow sector. Because we expect that a tax rate may not always has the wished favorable effect on economic development, the inclusion of a policy which is targeted to enhance growth (such as the share of government expenditure on public education) can be

a useful means to stimulate growth (if necessary), when the tax rate (or other policies) fight successfully the informal sector but cause a decline in economic development. Both the risk in human capital and public education have been highlighted as relevant to growth. Risky human capital accumulation has been a subject of the work of Levhari and Weiss (1974), Rillaers (1998), Krebs (2003), who confirm the negative impact of uncertainty on the investment in education theoretically. On the other hand, Eckstein and Zilcha (1994), Glomm and Ravikumar (1992), Su (2004), Blankenau (2005) model public education as a positive externality on human capital accumulation,[2] while Blankenau *et al.* (2007) verify empirically this relationship.

Our paper is embedded in the literature on occupational choice under risk and risky investment in public education in a two–period overlapping generations endogenous growth model with a shadow sector and productive government spending in production. We follow the work of Kanbur (1979)[3] for modeling the occupational choice under risk, Glomm and Ravikumar (1992) for developing risky human capital accumulation with public education, and Loayza (1996) for the construction of the shadow sector with a congestion effect on productive government expenditure. In this model we are able to determine (i) the impact of the informal sector on growth and long–term welfare, (ii) the effectiveness of the usually cited government policies (tax rate, penalty rate, audit rate) but also government spending on education in combating the shadow economy, (iii) the effect of the risk in the accumulation of human capital on the unofficial economy, and (iv) the impact of these policies, their combinations, and the risk measure on growth and welfare.

The economy consists of two sectors: formal and informal both producing an identical consumption good but by a different technology. At the beginning of life *ex–ante* homogeneous and risk–averse agents make a human capital decision and an occupational choice. An agent may decide to become high–skilled by spending certain time in education or to remain low–skilled both periods. High–skilled work in the formal sector and have the advantage of earning higher average wages but face occupational risk due to *ex–ante* unknown abilities in gaining human capital. Low–skilled who work in the official sector obtain a sure low–skilled wage. Low–skilled may work in the informal sector in the second period but face the probability of paying a penalty fee if caught. We follow the empirical evidence that agents do not have an objective idea about the audit probability (see Andreoni *et al.* (1998) for a review on simulation studies) and assume that the choice to work in the informal sector depends on the individual perception of the audit incidence. We model the shadow economy workers as low–skilled due

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[2]Blankenau and Simpson (2004) support an alternative view.

[3]Kanbur (1979) endogenizes the occupational choice of agents, who have the opportunity to become either workers or risk–bearing entrepreneurs.

to the empirical evidence that the share of low–skilled workers who evade taxes is higher than the share of high–skilled who do so. For instance, Pedersen (2003) shows for Denmark, Norway, Sweden, Germany and Great Britain (p. 66) that the share of blue collar workers who evade taxes is higher than the share of white collar workers who do so. Unfortunately, no evidence is available for Eastern European countries in this respect. In equilibrium the expected utility of a skilled worker is equal to the expected utility of a low–skilled wage earner in the formal sector. This determines the distribution of individuals across occupations in the formal sector. Low–skilled in the shadow economy self–select on the basis of a relatively optimistic idea about the audit rate. Growth in our model depends positively on the educational expenditure, which is financed by the taxes levied in the formal sector together with the penalty fees gathered from the informal sector. This implies that human capital accumulation hinges indirectly on the occupational choice of economic agents, who choose to be either taxpayers or not.

We show that reduction of the shadow economy leads to improvement in growth (and, therefore, long–term welfare). According to our theoretical results, a decrease in the shadow economy can be attained through a lower tax rate, a higher audit rate, a higher penalty rate and a lower risk measure. According to our calibration, a decline in the share of the informal production (without an unfavorable impact on growth) can be reached most effectively by lower taxation in combination with a higher share of government expenditure on education, followed by a higher penalty. A lower risk measure ranks last in this respect. In terms of short–run welfare, a lower risk measure attains the highest value of social utility, followed by a higher penalty rate and a higher audit rate. However, in terms of long–term welfare, the penalty rate is superior compared to the other policies. The risk measure ranks last in this respect. Although a lower risk measure does not cause a substantial fall in the informal sector, it is the only parameter, which does not create a trade–off between short–term and long–term welfare on aggregate and individual level. This should make policy makers consider the introduction of an insurance mechanism in the formal skilled labor market as an alternative approach to influence the unofficial economy. On the other hand, a government which is interested in reducing the share of the informal sector as effectively as possible should use a policy mix of lower income taxation and higher government expenditure on public education. This approach to decrease the unofficial sector will be accompanied, however, with a short–run welfare loss.

This paper is divided as follows: Section 3.2 presents the assumptions of the model, while Section 3.3 the market equilibrium, Section 3.4 the sensitivity analysis, Section 3.5 the calibration, Section 3.6 the transitional welfare analysis, Section 3.8 in the paper draws a conclusion.

3.2 The Model

3.2.1 The Household Sector

Ex–ante homogeneous and risk averse agents live for two periods in the framework of an overlapping generations model. Each individual is endowed with one unit of labor, which he inelastically supplies to the market. The population is normalized to one, which means that each generation is equal to 0.5 and there is no population growth.

The occupational and educational choice made in the first period is irreversible. Those who want to invest in education work as formal low–skilled when young and as formal skilled when middle–aged. Those who do not invest in education find employment in the formal sector both periods or may work in the informal sector but only when middle–aged. Low–skilled individuals can take a decision to switch to the informal sector after obtaining information about the penalty rate (through personal contacts) while working as (formal) low–skilled when young and forming a subjective perception of the audit rate in the first period of their life. Low–skilled in education are excluded from this process.[4] Although we do not include the participation of young low–skilled in the informal economy by assumption, our model describes between 70% and 75% of the share of agents employed in the informal sector according to Pedersen (2003).[5] Individuals born at period t who decide to obtain education invest time ν and work as low–skilled for the base wage in the formal sector $\overline{w}_t^l$ augmented by the low–skilled human capital h_t^l when young. Indices l, w, and s stand respectively for variables related to low–skilled workers, skilled workers and shadow sector workers in the whole paper. Once educated, skilled worker i obtains the base wage $\overline{w}_{t+1}^w$ augmented by one's human capital h_{it+1} in the second period. As agents differ with respect to their innate learning abilities (*ex–ante* unknown to them), they gain different levels of human capital. The lifetime utility of skilled worker i at period t and $t+1$ with c_t^w and $c_{i,t+1}^w$ measuring the consumption level is,

$$V_{i,t,t+1}^w = \ln(c_t^w) + b\ln(c_{i,t+1}^w) \tag{3.1}$$

$$c_t^w = (1-\tau)(1-\nu)\overline{w}_t^l h_t^l \tag{3.2}$$

$$c_{i,t+1}^w = (1-\tau)h_{i,t+1}\overline{w}_{t+1}^w \tag{3.3}$$

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[4]Alternatively, we can assume that those in education may also switch to the low–skilled informal sector after the first period, but this phenomenon will imply brain waste in the informal sector, for which no evidence is present.

[5]This can be calculated with the help of Table 3.3 in Pedersen (2003) as follow: $\frac{p_i * Ob_i}{p * Ob}$, where p_i is the percentage of agents at specific age i who evade taxes, while Ob_i is the total share of individuals interviewed at age i. p and Ob relate to the total population of interviewees.

with $0 < b < 1$ being the utility discount factor. Due to the lack of a capital market, the first and second period consumption is exactly equal to the obtained income at the respective period. The income of those willing to become skilled is reduced at the tax rate τ in both periods as they work in the formal sector. Following Yakita (2003), we assume that low–skilled workers' human capital (in the formal and informal production) equals a share of the average human capital of skilled workers $0 < \varphi < 1$. The human capital of skilled workers, on the other hand, grows according to,

$$h_{i,t+1} = a_i D \nu^\xi S_t^\phi h_t^{1-\phi} \qquad 0 < \xi, \phi < 1 \tag{3.4}$$

where S_t stands for the quality of public education, measured by the mean government income for education in total population at period t (similar to Glomm and Ravikumar (1992)), h_t is the average human capital of skilled workers at period t, a_i is an uninsurable risk (*ex–ante* unknown abilities), which follows a lognormal distribution, $a_i \sim f(-\frac{\sigma^2}{2}, \sigma^2)$.[6] The educational expenditure is exogenous to the problem of the agent but it serves as a positive externality to the production of new human capital. The educational time ν for high–skilled is also determined by the government. The government expenditure on education (or quality of public education) is not congested by a higher share of students. We find support for this assumption in the ambiguous relationship between students' performance and a class size. For instance, Borland *et al.* (2005) argue empirically that this relationship is non–monotone and there is an optimal class size, below which students' attainment deteriorates. Denny and Oppedisano (2013) find positive significant effects, and insignificant, but positive effects of a larger class on students' performance in Great Britain and the USA respectively. However, this outcome is not replicated in the studies of De Paola *et al.* (2013), and De Giorgi *et al.* (2012).

Agents who remain low–skilled in the formal sector in both periods of life t and $t + 1$ spend total net income on consumption c_t^l and c_{t+1}^l. Low–skilled who work in the formal sector pay taxes at the rate τ as well. Their welfare is equal to,

$$V_{t,t+1}^l = \ln(c_t^l) + b \ln(c_{t+1}^l) \tag{3.5}$$

$$c_t^l = (1-\tau)\overline{w}_t^l h_t^l \tag{3.6}$$

$$c_{t+1}^l = (1-\tau)\overline{w}_{t+1}^l h_{t+1}^l \tag{3.7}$$

We assume that only some low–skilled may work in the informal sector in the second period and they are employed as formal workers when young. A low–skilled worker born at period t who earns a living in the shadow economy as middle–aged obtains the base wage $\overline{w}_{t+1}^s$ augmented by

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[6]The normalization of the expected value of the ability shock $E(a)$ helps us avoid size effects from the mean of the ability distribution $f(a)$.

the low–skilled human capital h_{t+1}^l and faces the probability of being caught p^c and pays in this case a penalty at a rate equal to ρ. Agents in the informal sector misperceive the real probability of audit by attaching an individual perception weight ψ to p^c. The subjective weight ψ of the real probability to be caught in the informal sector is distributed uniformly within the range $[\underline{\psi}, \overline{\psi},]$, where $0 \leq \underline{\psi} < \overline{\psi} \leq 1/p^c$.[7] A low–skilled worker j with a subjective probability of being detected $\psi_j p^c$, who spends total income on consumption c_t^s when young, c_{t+1}^{sc} (if caught working in the shadow sector), c_{t+1}^s (if not caught working in the informal sector) when middle–aged, has welfare equal to,

$$V_{j,t,t+1}^s = \ln(c_t^s) + b(1 - p^c \psi_j) \ln(c_{t+1}^s) + b p^c \psi_j \ln(c_{t+1}^{sc}) \tag{3.8}$$

$$c_t^s = (1 - \tau)\overline{w}_t^l h_t^l \tag{3.9}$$

$$c_{t+1}^s = h_{t+1}^l \overline{w}_{t+1}^s \tag{3.10}$$

$$c_{t+1}^{sc} = (1 - \rho)h_{t+1}^l \overline{w}_{t+1}^s \tag{3.11}$$

The assumption on ψ implies that there are agents in the economy who may consider that the probability of detection in the shadow economy is higher or lower than the objective probability. We show later that in equilibrium agents will naturally self–select into the shadow or the formal sector on the basis of their perception of the probability of detection.

The penalty rate ρ is not a penalty over the evaded taxes due to the tendency of the government to understate the evaded taxation. This is a relevant assumption for the shadow sector, which either has no access to or does not want to use the bank system in order to avoid potential tractability from the authorities. We assume that (i) $\rho > \tau$ to mimic the empirically observed higher punishment rate compared to the tax rate, and that (ii) $\tau > p^c \rho$, which corresponds to the empirically observed lower expected loss rate compared to the tax rate in case of tax evasion.

3.2.2 Production Sectors

There are two sectors in the economy (formal and informal economy) working in a perfect competition setting, both producing an identical consumption good but by a different technology. The price of the good is normalized for simplicity to 1 in both sectors.

The formal sector employs skilled and some low–skilled agents, who further bring along human capital. The total human capital of skilled and low–skilled is equal to $H_t^w = L_t^w h_t$, and $H_t^l = L_t h_t^l$ with L_t^w and L_t measuring respectively the share of skilled agents and low–skilled

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[7]The upper bound on $\overline{\psi}$ implies that agents cannot misperceive the actual audit probability to be more than 100%.

at period t in the formal production. Following Loayza (1996), we model a positive externality in the production function of the formal sector, represented by the government expenditure on infrastructure G_t (including legislature, police, etc.), which is congested by the total production of goods $Y_t + Q_t$, where Q_t is the production in the shadow economy. The elasticity β measures the productivity of public services to private services. The production function is,

$$Y_t = A\Big(\frac{G_t}{Y_t + Q_t}\Big)^\beta (H_t^w)^\alpha (H_t^l)^{1-\alpha} \qquad 0 < \alpha, \beta < 1 \tag{3.12}$$

where $A > 0$ is a productivity parameter.

The shadow economy, on the other hand, works only with the human capital of low–skilled agents. The production is linear in their human capital $H_t^s = L_t^s h_t^l$ with L_t^s being the share of agents working in the informal sector. The elasticity η measures the productivity of public services to private services in the informal economy. Government spending on infrastructure is congested by total production. The shadow economy does not have full access to the public goods provided by the government, that is why the spillover effect of government expenditure is smaller compared to the formal economy. This assumption is reflected by $0 < \delta < 1$. The production function of the informal economy has the following form,

$$Q_t = B\Big(\frac{\delta G_t}{Y_t + Q_t}\Big)^\eta H_t^s \qquad 0 < \eta < 1 \tag{3.13}$$

where $B > 0$ is a productivity parameter.

3.2.3 The Government

The government collects taxes from all workers in the formal economy charging a linear tax rate τ on income, and fines at rate ρ on earnings from the shadow economy in order to spend them on infrastructure G_t and education S_t by incurring no debt. For a balanced budget the government expenditure equals the taxes and fees that are levied,

$$S_t + G_t = \tau Y_t + p^c \rho Q_t \tag{3.14}$$

If $0 < d < 1$ measures the relative importance of education to infrastructure in the government choice, and the share of the shadow economy out of the formal economy is γ_t, i.e. $Q_t = \gamma_t Y_t$, the distribution of government expenditure in terms of official income can be viewed as,

$$S_t = dY_t(\tau + p^c \rho \gamma_t) \tag{3.15}$$

$$G_t = (1 - d)Y_t(\tau + p^c \rho \gamma_t) \tag{3.16}$$

3.3 Market Equilibrium

3.3.1 Labor Market Equilibrium

The profit of the representative firm in the formal sector

$$\Pi_t = A\Big(\frac{G_t}{Y_t + Q_t}\Big)^\beta (H_t^w)^\alpha (H_t^l)^{1-\alpha} - \overline{w}_t^w h_t L_t^w - \overline{w}_t^l h_t^l L_t \tag{3.17}$$

is maximized with respect to the share of skilled L_t^w and low–skilled (young and middle–aged) L_t in the formal economy. The equilibrium base wages of skilled and low–skilled are then respectively defined as,

$$\overline{w}_t^w = \alpha A\Big(\frac{(1-d)(\tau + p^c \rho \gamma_t)}{1 + \gamma_t}\Big)^\beta \Big(\frac{\varphi L_t}{L_t^w}\Big)^{1-\alpha} \tag{3.18}$$

$$\overline{w}_t^l = (1-\alpha)A\Big(\frac{(1-d)(\tau + p^c \rho \gamma_t)}{1 + \gamma_t}\Big)^\beta \Big(\frac{L_t^w}{\varphi L_t}\Big)^\alpha \tag{3.19}$$

Notice that the human capital of low–skilled is a share of the average human capital of skilled workers, $h_t^l = \varphi h_t$.

The profit of the representative firm in the informal sector,

$$\Pi_t^s = B\Big(\frac{\delta G_t}{Y_t + Q_t}\Big)^\eta H_t^s - \overline{w}_t^s h_t^l L_t^s \tag{3.20}$$

is maximized with respect to the share of low–skilled workers L_t^s in the shadow economy. The base wage of low–skilled agents in the informal sector is then,

$$\overline{w}_t^s = B\Big(\frac{\delta(1-d)(\tau + p^c \rho \gamma_t)}{(1 + \gamma_t)}\Big)^\eta \tag{3.21}$$

Because in the labor market equilibrium the supply of skilled and low–skilled must be equal to the demand for (low–)skilled and due to the normalization of the population to one, it should hold that

$$L_t = 1 - L_t^w - L_t^s = 1/2 + L_t^l \tag{3.22}$$

where L_t^l is the population share of low–skilled middle–aged in the formal sector.

3.3.2 Equilibrium Occupational Choice

Proposition 3.1 *(First stage decision process) In equilibrium homogeneous and risk averse individuals should be ex–ante indifferent between working as a low–skilled worker in the formal sector, who obtains a sure low–skilled wage, and a high–skilled worker in the same sector, who bears occupational risk stemming from human capital investment. This condition determines occupational choice of agents in the formal sector.*

Proposition 3.2 *(Second stage decision process) The distribution of low–skilled agents across the formal and informal economy is determined by the perceived probability of detection. Those low–skilled who perceive the probability of detection as relatively small self–select into the informal sector. Those low–skilled who perceive the probability of detection as relatively high self–select into the formal sector.*

The above propositions are based on the simplifying assumption that the decision process concerning the occupational choice is divided in two stages. In the first stage agents decide between a profession as skilled or low–skilled in the formal sector. In the second stage those who prefer to remain low–skilled have the opportunity to choose employment in the formal sector or the informal sector. In other words, an agent cannot *ex–ante* compare the utilities of all professions. This result is a consequence of the assumption that young low–skilled who are not in education obtain information about the penalty rate and form a subjective perception of the audit rate only when they start working in the first period (for instance through personal contacts).

The propositions determine the distribution of agents across sectors. Because the agents who decide to invest in education face risk in terms of human capital accumulation, an individual should be *ex–ante* indifferent between being a high–skilled earner obtaining an *ex–ante* risky wage and the sure income of a low–skilled worker in the formal sector. If this condition is not satisfied, agents will have an incentive to deviate from the occupation which brings them lower (expected) utility.

If an agent decides to remain low–skilled, he may choose also to migrate to the informal sector. Low–skilled agents who perceive that the objective probability to be detected in the informal sector is relatively small will choose to work in the shadow economy. Low–skilled agents who overweight the objective probability to be caught in the shadow sector will prefer staying in the formal sector. This condition implies that there is a low–skilled agent with a threshold perception of the objective probability of detection ψ^*, who will be indifferent to work in the formal and informal sector. All other low–skilled will have a specific preference to work either in the shadow economy or the formal economy.

The first and the second proposition can be summarized in a system of equations. In order to solve for the four unknowns L_t^w, L_t^s, L_t^l, γ_t, we use the condition that the population is normalized to one and equate additionally the net wage of low–skilled in the formal and informal sector. The last condition ensures that low–skilled who work in the formal and informal economy will not have an incentive to choose their occupation due to a difference in work payment but only due to a subjective perception of the audit rate. The resulting system of equations is presented

below,

$$E(V_{i,t,t+1}^w) = V_{t,t+1}^l \qquad V_{j,t+1}^s(\psi_j^*) = V_{t+1}^l \qquad \overline{w}_t^s = \overline{w}_t^l \qquad 1/2 - L_t^w - L_t^l = L_t^s$$

The expected utility of skilled workers (E is the expectation operator) can be easily calculated by summing over the risk distribution, which boils down to finding the mean of $E(\ln a_i^b)$ equal to $b\mu$ or $-b\frac{\sigma^2}{2}$ after we normalize μ. The solution of the system of equations is summarized in the following proposition.

Proposition 3.3 *The share of agents across occupations and sectors, as well as the share of the informal sector are constant over time and equal to[8],*

$$L^s = \frac{M-1}{2M + 2(1+M)\frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}}} \qquad L^w = \frac{1-L^s}{1+M} \qquad L^l = \frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}}L^s \qquad \gamma = \frac{L^s}{L}(1-\alpha)$$

with the following constants,

$$M = \frac{1-\alpha}{\alpha}\frac{exp\left(\frac{\sigma^2}{2}\right)}{(1-\nu)^{\frac{1}{b}}} > 1 \qquad \psi^* = \frac{1}{p^c}\frac{\ln(1-\tau)}{\ln(1-\rho)}$$

Proof: See Appendix 3.10.1.

Corollary 3.1 *Base wages $\overline{w}^w$, $\overline{w}^l$, $\overline{w}^s$ are also constant.*

Proof. As a consequence of the outcome that the occupational choice is constant over time, $\overline{w}^w$, $\overline{w}^l$, $\overline{w}^s$ do not change over time. ∎

The next proposition determines the risk and skill premium of agents who are educated and/or bear risk over the low–skilled certain remuneration in the formal sector. A sufficient condition is derived which guarantees that educated obtain a higher skill and risk premium than informal low–skilled, who, nevertheless, may suffer an income loss if they are detected to work in the shadow economy. The skill and risk premia are related to the *ex–post* realization of income and do not include the *ex–ante* perception of earnings, which informal workers have.

Proposition 3.4 *Risk averse high–skilled workers obtain (i) a skill and risk premium over the certain low–skilled wage in the formal sector because of a higher labor productivity and as reimbursement for facing uncertainty in the accumulation of human capital (ii) a skill and risk premium over the low–skilled wage in the informal sector if they bear more uncertainty in income than shadow economy workers. Risk averse low–skilled workers in the informal economy obtain a risk premium over the certain low–skilled wage because of the uncertainty of being detected working in the shadow economy. The risk and skill premia expressed as ratios are constant over time.*

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[8]We leave the subindex t for the variables which are proved to be constant.

Proof. The proposition above shows that agents who are skilled and/or face risky income require a skill and/or risk premium. The skill and risk premium pr^w that skilled workers require over the low–skilled wage in the formal sector can be defined as a ratio between the expected wage of a high–skilled worker and the wage of a low–skilled worker in the formal sector. The skill and risk premium pr^w should be higher than one and is defined as,

$$pr^w = \frac{(1-\tau)\overline{w}^w h_t}{(1-\tau)\overline{w}^l h_t^l} \implies pr^w = \frac{\alpha L}{(1-\alpha)L^w}$$

When we use the equilibrium occupational choice, it follows that

$$pr^w = \frac{exp\left(\frac{\sigma^2}{2}\right)}{(1-\nu)^{\frac{1}{b}}} \tag{3.23}$$

Because $exp\left(\frac{\sigma^2}{2}\right) > (1-\nu)^{\frac{1}{b}}$, due to $\frac{\sigma^2}{2} > 1/b \underbrace{\ln(1-\nu)}_{<0}$, $pr^w > 1$ is always true.

The skill and risk premium $pr^{w'}$, which skilled obtain over the expected income of low–skilled in the informal economy, is defined as,

$$pr^{w'} = \frac{(1-\tau)\overline{w}^w h_t}{(1-p^c)\overline{w}^s h_t^l + p^c(1-\rho)\overline{w}^s h_t^l} = \frac{1-\tau}{1-p^c\rho} \frac{exp\left(\frac{\sigma^2}{2}\right)}{(1-\nu)^{\frac{1}{b}}}$$

$pr^{w'} > 1$ if human capital risk is strong enough, i.e. $exp\left(\frac{\sigma^2}{2}\right) > (1-\nu)^{\frac{1}{b}} \frac{1-p^c\rho}{1-\tau}$.

The risk premium of low–skilled working in the informal sector pr^s vs. formal employment is,

$$pr^s = \frac{(1-p^c)\overline{w}^s h_t^l + p^c(1-\rho)\overline{w}^s h_t^l}{(1-\tau)\overline{w}^l h_t^l} = \frac{1-p^c\rho}{1-\tau} \tag{3.24}$$

The risk premium $pr^s > 1$ for $\tau > p^c\rho$, which is an assumption of the model. ∎

3.3.3 Income Distribution and Growth

The income shares of the formal sector, of skilled workers, low–skilled formal and low–skilled informal workers out of total production are respectively,

$$\frac{Y_t}{Q_t + Y_t} = \frac{1}{1+\gamma} \quad \frac{\overline{w}^w h_t L^w}{Q_t + Y_t} = \frac{\alpha}{1+\gamma} \quad \frac{\overline{w}^l h_t^l L}{Q_t + Y_t} = \frac{1-\alpha}{1+\gamma} \quad \frac{\overline{w}^s h_t^l L^s}{Q_t + Y_t} = \frac{\gamma}{1+\gamma} \tag{3.25}$$

In line with the neoclassical growth theory, income shares are constant. Skilled workers' income is lognormally distributed because of the presence of the uncertainty in human capital investment. Low–skilled workers in both sectors obtain low–skilled wages. As we already showed, skilled workers obtain a risk and skill premium in contrast to low–skilled agents working in the formal sector because risk averse skilled individuals bear occupational risk and are more productive than low–skilled. Risk averse low–skilled workers in the shadow economy require a risk

premium over the sure low–skilled wage in the formal sector because they face the probability of paying a penalty fee for working in the informal sector. Skilled workers receive a higher skill and risk premium than low–skilled in the informal sector if the risk in human capital accumulation is large enough.

The average human capital in the economy, on the other hand, is equal to the human capital which is accumulated by skilled individuals and the human capital which is inherited for free by low–skilled. Therefore, the growth g_{t+1}^a of the average human capital h_{t+1}^a for period $t+1$ is determined as,

$$1 + g_{t+1}^a = \frac{h_{t+1}^a}{h_t^a} = \frac{L^w h_{t+1} + \varphi(1 - L^w)h_{t+1}}{L^w h_t + \varphi(1 - L^w)h_t} = \frac{L^w + \varphi(1 - L^w)}{L^w + \varphi(1 - L^w)}(1 + g_{t+1}^h)$$

where g_{t+1}^h is the growth of average skilled workers' human capital, which is defined as $1 + g_{t+1}^h = D(\nu)^\xi \frac{S_t^\phi}{h_t^\phi}$. Therefore, the growth of average human capital and average skilled human capital are equal because the occupational choice is constant over time. We can substitute for the educational expenditure, for the income in the formal sector and for the equilibrium occupational choice to obtain g_{t+1}^h,

$$1 + g_{t+1}^h = D(\nu)^\xi \left[dA \frac{(1-d)^\beta(\tau + p^c \rho \gamma)^{1+\beta}}{(1+\gamma)^\beta \gamma} \left(\frac{1}{M} \right)^\alpha (\varphi)^{1-\alpha} \frac{(1-\alpha)(M-1)}{2M + 2(1+M)\frac{\bar{\psi} - \psi^*}{\psi^* - \underline{\psi}}} \right]^\phi \qquad (3.26)$$

The growth rate of average skilled workers' human capital is constant (because the distribution of agents across professions does not change over time) and equal to $g_{t+1}^h = g^h$. It is then easy to see that $g_{t+1}^a = g^a = g^h$ (a balanced growth condition).

3.4 Sensitivity Analysis

Because we already determined the occupational choice of agents in society, i.e. L^w, L^l, L^s and the share of the shadow sector γ in the economy, individual income and growth, it is now possible to conduct a sensitivity analysis of these variables with respect to the government policies τ, ρ, p^c, d and the risk measure σ. This is a necessary step if the shadow economy exerts a negative effect on growth (and long–term welfare), what we prove later in this chapter. We aim, furthermore, at determining which policies are effective in reducing the share of the shadow economy without causing lower human capital accumulation. We investigate the effect of the risk measure in this respect as well. The implications are summarized in what follows.

Proposition 3.5

(i) The influence of the tax rate τ on the occupational choice and γ is,

$$\frac{\partial L^s}{\partial \tau} > 0 \qquad \frac{\partial L^w}{\partial \tau} < 0 \qquad \frac{\partial L^l}{\partial \tau} < 0 \qquad \frac{\partial \gamma}{\partial \tau} > 0$$

(ii) The influence of the penalty rate ρ on the occupational choice and γ is,

$$\frac{\partial L^s}{\partial \rho} < 0 \qquad \frac{\partial L^w}{\partial \rho} > 0 \qquad \frac{\partial L^l}{\partial \rho} > 0 \qquad \frac{\partial \gamma}{\partial \rho} < 0$$

(iii) The influence of the audit rate p^c on the occupational choice and γ is,

$$\frac{\partial L^s}{\partial p^c} < 0 \qquad \frac{\partial L^w}{\partial p^c} > 0 \qquad \frac{\partial L^l}{\partial p^c} > 0 \qquad \frac{\partial \gamma}{\partial p^c} < 0$$

(iv) The influence of the share of government expenditure on education d on the occupational choice and γ is,

$$\frac{\partial L^s}{\partial d} = 0 \qquad \frac{\partial L^w}{\partial d} = 0 \qquad \frac{\partial L^l}{\partial d} = 0 \qquad \frac{\partial \gamma}{\partial d} = 0$$

(v) The influence of the risk measure σ on the occupational choice and γ is,

$$\frac{\partial L^s}{\partial \sigma} > 0 \qquad \frac{\partial L^w}{\partial \sigma} < 0 \qquad \frac{\partial L^l}{\partial \sigma} > 0 \qquad \frac{\partial \gamma}{\partial \sigma} > 0$$

Proof: See Appendix 3.10.2.

A higher tax rate τ increases the costs of employment in the formal sector so that the share of skilled and low–skilled in the formal economy decreases to the advantage of a higher population of informal workers. As a consequence, the share of the shadow economy out of official income jumps. A higher penalty rate ρ or an audit rate p^c has an unfavorable effect on the expected utility of those employed in the shadow economy, which results in a larger share of skilled and low–skilled agents in the formal economy and a smaller share of informal workers in the population. As a result, the share of the shadow economy out of official income falls. The share of government spending on education d has no impact on the occupational choice of economic agents, because government investment in education is a public good. As a consequence, the share of the informal sector remains unaffected. A higher risk measure σ in human capital accumulation reduces the share of professionals due to risk aversion and improves the share of low–skilled in both sectors. This leads to a higher share of the informal sector.

Before we address the question about the impact of the government policies and the risk measure on growth, we investigate the relationship between the share of the informal sector and human capital accumulation. By doing this, we want to explore whether economic policy directed against the informal sector is justified or not.

Corollary 3.2 *The impact of the share of the informal sector γ on growth g^h is negative.*

Proof: See Appendix 3.10.3.

The negative relationship between γ and g^h determined by the corollary can be explained

as follows: from one hand, a larger share of the shadow economy crowds out more intensively government investment in infrastructure and implies lower employment in the tax–paying formal sector, which decreases growth; from the other hand, a larger informal sector is associated with a higher government revenue from penalty fees (all other things being constant), which supports higher government expenditure on infrastructure (per unit of production) and, therefore, growth. Because the afore–mentioned negative effect of γ on human capital accumulation is dominant, higher γ leads to lower g^h. This outcome justifies government intervention directed to reduce the share of the informal sector in terms of alternative policies of τ, ρ, p^c, d. We address also the question about the impact of σ for comparison reasons.

Proposition 3.6 *The influence of the tax rate τ, the penalty rate ρ, the audit rate p^c, the government budget share on education d, and the risk measure σ on growth is[9],*

$$(i) \quad \frac{\partial(1+g^h)}{\partial\tau} \gtrless 0$$

$$(ii) \quad \frac{\partial(1+g^h)}{\partial\rho} > 0 \qquad if \qquad \beta > \frac{p^c\rho(1+\gamma)}{\tau - p^c\rho}$$

$$(iii) \quad \frac{\partial(1+g^h)}{\partial p^c} > 0 \qquad if \qquad \beta > \frac{p^c\rho(1+\gamma)}{\tau - p^c\rho}$$

$$(iv) \quad \frac{\partial(1+g^h)}{\partial d} \gtrless 0 \qquad if \qquad d \lessgtr \frac{1}{1+\beta}$$

$$(v) \quad \frac{\partial(1+g^h)}{\partial\sigma} \lessgtr 0$$

Proof: See Appendix 3.10.4.

As the above proposition states, the impact of the tax rate τ on growth is ambiguous. The tax rate has a direct (positive) impact on human capital accumulation through the tax revenue per unit of production and indirect (positive) influence on g^h through the penalty fees revenue per unit of production (for growing γ) but indirect (negative) impact on growth through the rising share of the informal sector, which congests the productivity of government expenditure on infrastructure. The ambiguity of the indirect relationship between g^h and τ is additionally reinforced by the fact that rising τ decreases the employment in the formal sector, which is the main tax payer.

A higher penalty rate ρ or an audit rate p^c has a positive effect on human capital accumulation only if β is large enough. Both policies, similar to the tax rate, exert a direct positive effect on the government revenue per unit of production (here out of fees, all other things being constant), and contrary to the tax rate, decrease the share of the shadow economy. A reduction in the

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[9]The structure $\frac{\partial y}{\partial x} \lessgtr 0$ if $a \gtrless b$ is to read: $\frac{\partial y}{\partial x} < 0$ if $a > b$ and $\frac{\partial y}{\partial x} > 0$ if $a < b$ in all propositions

informal sector implies, from on hand, a decrease in the congestion effect on infrastructure, which augments human capital accumulation, but also a fall in the share of government revenue out of penalty fees per unit of production depending on the size of the informal sector. Additionally, lower γ implies improvement of the employment in the formal sector paying taxes. This makes the relationship between g^h, p^c and ρ inconclusive also in this case.

An increase in the educational expenditure share d leads to a higher growth rate independent of the share of the shadow economy if d is small enough. In fact, with empirically admissible values for $\beta \in [0.03 - 0.39]$ (see calibration) and $d \in [10\% - 20\%]$ this condition holds. The latter can be explained with the nature of this policy. While the government expenditure on infrastructure has a consumption character (and a relatively low elasticity in g^h), the spending on education has an investment character (and a relatively high elasticity in g^h).

A higher risk measure in the earnings of skilled σ has an ambiguous impact on growth. The share of the informal sector jumps, which, from one hand, reduces the positive externality of the government spending per unit of production through more intensive congestion but increases government revenues out of penalties per unit of production, from the other hand. Moreover, a higher risk measure decreases the employment in the tax–paying formal sector, which contributes additionally to a fall in growth. This implies that the impact of σ on human capital accumulation is not straightforward.

In the end we can conclude that it is possible to apply even the tax rate (or the risk measure) in reducing the share of the shadow economy. Because a lower tax rate or a lower risk measure reduces γ unambiguously, but the impact of a lower tax rate/risk measure on growth is not straightforward, it is possible to increase the share of the government spending on education d in order to counteract the presumably negative effect of τ and σ on growth.

Proposition 3.7 *The influence of the tax rate τ, the penalty rate ρ, the audit rate p^c, the government budget share on education d, and the risk measure σ on wages in the short run is,*

$$
\begin{aligned}
&(i) && \frac{\partial(\overline{w}^w h_t)}{\partial \tau} \lessgtr 0 && \frac{\partial(\overline{w}^l h_t^l)}{\partial \tau} \lessgtr 0 \\
&(ii) && \frac{\partial(\overline{w}^w h_t)}{\partial \rho} > 0 && \frac{\partial(\overline{w}^l h_t^l)}{\partial \rho} > 0 \\
&(iii) && \frac{\partial(\overline{w}^w h_t)}{\partial p^c} > 0 && \frac{\partial(\overline{w}^l h_t^l)}{\partial p^c} > 0 \\
&(iv) && \frac{\partial(\overline{w}^w h_t)}{\partial d} < 0 && \frac{\partial(\overline{w}^l h_t^l)}{\partial d} < 0 \\
&(v) && \frac{\partial(\overline{w}^w h_t)}{\partial \sigma} \lessgtr 0 && \frac{\partial(\overline{w}^l h_t^l)}{\partial \sigma} < 0
\end{aligned}
$$

Proof: See Appendix 3.10.5.

Proposition 3.7 shows how the income variables react to changes in the policy instruments

and to the risk measure in the short run, i.e. before a parameter change is incorporated in growth. A tax increase raises the government revenue out of taxes and penalties per unit of total production (directly and indirectly by improvement in γ), which supports a higher positive government externality on production and drives wages up. Nevertheless, a higher congestion effect on production results due to a rise in the shadow sector in response to higher τ, which drives wages down. This trade–off is reflected in the ambiguous relationship between wages and the tax rate. Competition does not play a role in the formation of both wages in response to τ, because a rise in τ reduces simultaneously the share of skilled and low–skilled in the formal sector, so their ratio M, vital for the base wage, stays constant.

A higher penalty rate or an audit rate improves the government externality on production unambiguously. First, both policy measures decrease the shadow economy sector and the congestion effect of γ falls. Second, both policies raise the share of levied penalty revenue per unit of total production (all other things being constant) and dominate the negative impact that lower γ has on the government levies from penalty fees per unit of total production. Therefore, p^c and ρ have a positive impact on labor productivity in the short run. Competition does not play a role in the formation of both wages in response to p^c and ρ, because both policies increase simultaneously the share of skilled and low–skilled in the formal sector.

A higher share of government spending on education has a negative effect on the base wage as higher d reduces the expenditure share on infrastructure. That is why higher d makes wages decline in the short run.

A higher risk measure makes agents deviate from the risky skilled profession to low–skilled employment in the formal and informal sector due to risk aversion. The share of the informal sector rises. The lower supply of skilled labor and the higher levies out of penalties per unit of total production due to rising γ, which drives skilled wages up, counteract the higher share of informal sector, which crowds out the positive externality of government expenditure on production, and drives skilled wages down. Therefore, the impact of a higher risk measure on skilled labor productivity is ambiguous in the short run. The negative effect of a larger supply of low–skilled labor on low–skilled wages combined with the congestion effect of rising γ on production exceeds the positive effect of higher penalty levies per unit of total production on low–skilled wages due to larger γ. Therefore, low–skilled labor productivity falls in the short run in response to larger σ.

Proposition 3.8 *The long–run influence of τ, p^c, ρ, d and the risk measure σ on the long–term income of economic agents depends on the impact of τ, p^c, ρ, d and σ on growth g^h.*

Proof: See Appendix 3.10.6.

Proposition 3.8 discusses the effect of τ, p^c, ρ, d and σ on the long–term income of individuals. While in the short term policies' changes and changes in σ influence the income variables through occupational choice and the government externality on workers' productivity, in the long term the growth of average human capital is vital for the effect of τ, p^c, ρ, d and σ on the income variables. In the long–run a parameter change is incorporated in growth so many times so that the growth effect dominates the short run effect of altered occupational choice and government externality on workers' productivity.

3.5 Calibration

We calibrate the model by defining the parameters: α, β, η, b, $\overline{\psi}$, $\underline{\psi}$, σ, τ, ρ, p^c, d, A, D, ϕ, ξ, φ and ν to match empirical data on the real economies of European transition countries for the benchmark model (BM).[10] We assume that at period $t = 0$ the average human capital of skilled is $h_0 = 1$. Under the postulation of an annual growth rate of 2.5%, and a period of 30 years we can determine D in the human capital accumulation function ($D = 6.26$). We set the elasticity of educational time and the elasticity of educational expenditure in the human capital accumulation function respectively equal to $\xi = 0.8$ and $\phi = 0.1$ following Glomm and Ravikumar (1998). The share of time spent on education ν is set equal to 0.5 to reflect the fact that agents who live 30 years during their first period spend around 15 years for education. The weight of government expenditure on education d in most countries in Eastern Europe (the Czech Republic, the Slovak Republic, Poland, Romania, Bulgaria) is approximately 10-15% of their budget according to the World Bank for the period 2007–2010 (see Appendix 3.10.7). We choose the lowest value (10%) to be able to investigate increases in the share of government expenditure on education. The spillover φ is set equal to 0.4. It does not influence the skill premium, but it has an indirect impact on human capital accumulation via the government spending on education, which can be corrected by adjusting D.

The values of $\alpha = 0.5$, $\sigma = 0.52$, $\underline{\psi} = 0$, and $\overline{\psi} = 7$ are defined to correspond to an empirically observable share of high–skilled workers in the formal economies, the share of informal workers and the share of the shadow economy in some European transition countries. We target a share of skilled individuals of around 18% in the population according to data by *Eurostat* (See Appendix 1.9.5) and a value of 27% for the share of the shadow economy out of official income in compliance with Bühn and Schneider (2012) (see Appendix 3.10.7). The share of unofficial

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[10]It is not necessary to define B and δ because of the assumption that low–skilled base wages in the formal and informal sector are equal.

workers out of official labor lies between the average estimate of 9% for France in 1997 and 1998 and the maximum estimate of 48% for Italy in 1997 and 1998 according to Schneider (2000) (see Appendix 3.10.7). Unfortunately, no recent estimates about the share of participants in the shadow economy are available for European transition countries, so we just put up with the share of informal labor $\frac{L^s}{L+L^w}$, which our base model predicts (around 40%).

For the baseline model we further assume that $\tau = 0.3$, which corresponds to the largest tax rate of personal income and social insurance contributions in Eastern Europe paid by workers according to OECD (2011) (see Appendix 3.10.7). By choosing the largest tax rate, we are interested in finding out how a general tax fall influences the share of the shadow economy, economic development and later welfare. The probability of a tax audit is set at $p^c = 0.089$, following Fullerton and Karayannis (1994). We set ρ to be equal to 0.47 so that it does not contradict earlier targets. The value of ρ implies that the penalty rate is 57% higher than the tax rate, which is close to the value that Fullerton and Karayannis (1994) use in their paper.

We choose b to correspond to an annual discount factor of 0.99 of the utility of future consumption, which implies that $b = 0.99^{30} = 0.74$. A is normalized to 1. According to the literature review of Glomm and Ravikumar (1997), the empirical elasticity of public goods in production varies between 0.03 and 0.39. We conduct our calibration analysis with the average value $\beta = \eta = 0.2$.

In Table 3.1 we report the qualitative and quantitative impact of τ, p^c, ρ, d and σ and combinations of τ, d on the occupational choice variables,[11] the share of the shadow economy, growth and the income variables. The income variables refer to period $t = 1$, the starting period is $t = 0$. We report the income variables one period after the starting period to investigate the combined effect of growth and the government externality on labor productivity when a policy changes. Nevertheless, the base income ($\overline{w}^w$, $\overline{w}^l$) is also shown. The results in Table 3.1 reflect alternative steady states, when certain policies dominate.

We report only those policies which reduce the share of the shadow economy and endeavor to find which do this most effectively. That is why we show (i) changes in the tax rate τ and σ below the benchmark by reducing the benchmark stepwise by 25% and (ii) a stepwise 25% rise in p^c, ρ and additionally d (which increases by 25%, 50% and 100%). By the sensitivity analysis of d we try to determine whether a lower tax rate, which may lead theoretically to a lower share of the shadow economy, can be combined with higher d so that a lower share of the shadow economy is attained without hurting human capital accumulation. The same question is not posed with respect to the interaction of a lower risk measure σ and higher d because lower σ

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[11]$L^w + L + L^s$ may not sum exactly to one due to approximation errors.

results in higher growth according to the calibration.

As Table 3.1 shows, a decrease in the tax rate τ makes the occupation as a shadow worker a

Table 3.1: *Calibration Results for τ, p^c, ρ, d, σ and Policy Combinations*

	BM	τ_1	τ_2	p_1^c	p_2^c	ρ_1	ρ_2	...
L^w	0.1819	0.2264	0.2069	0.1998	0.2107	0.2067	0.2218	...
L	0.5313	0.6612	0.6042	0.5835	0.6154	0.6037	0.6478	...
L^s	0.2869	0.1124	0.1889	0.2167	0.1739	0.1896	0.1305	...
γ	0.27	0.085	0.1563	0.1857	0.1413	0.1571	0.1007	...
g^h	0.025	0.0229	0.0243	0.0253	0.0255	0.0254	0.0257	...
$\overline{w}^w$	0.3995	0.3579	0.3836	0.4046	0.4075	0.4062	0.4097	...
$\overline{w}^l$	0.3419	0.3063	0.3284	0.3463	0.3487	0.3476	0.3507	...
$\overline{w}^l h_t^l$	0.2866	0.2419	0.2696	0.2932	0.297	0.2953	0.3001	...
$\overline{w}^w h_t$	0.8371	0.7065	0.7874	0.8565	0.8675	0.8627	0.8766	...

Parameters calibrated as follows: $\tau_1 = 0.15$, $\tau_2 = 0.225$ $p_1^c = 0.1112$, $p_2^c = 0.1335$, $\rho_1 = 0.5875$,

$$\rho_2 = 0.7050$$

Calibration Results for τ, p^c, ρ, d, σ and Policy Combinations (cont'd)

	...	d_1	d_2	d_3	σ_1	σ_2	τ_1, d_2	τ_1, d_3
L^w	...	0.1819	0.1819	0.1819	0.2006	0.1926	0.2264	0.2264
L	...	0.5313	0.5313	0.5313	0.5294	0.5302	0.6612	0.6612
L^s	...	0.2869	0.2869	0.2869	0.27	0.2772	0.1124	0.1124
γ	...	0.27	0.27	0.27	0.255	0.2614	0.085	0.085
g^h	...	0.0257	0.0263	0.0273	0.0251	0.0251	0.0243	0.0252
$\overline{w}^w$	...	0.3972	0.3949	0.3902	0.3805	0.3883	0.3538	0.3496
$\overline{w}^l$	...	0.34	0.338	0.3339	0.3604	0.3526	0.3029	0.2992
$\overline{w}^l h_t^l$	...	0.2912	0.2947	0.2993	0.3035	0.2963	0.2487	0.2526
$\overline{w}^w h_t$	...	0.8507	0.8608	0.8742	0.8011	0.8159	0.7265	0.7378

Parameters calibrated as follows: $d_1 = 0.125$, $d_2 = 0.15$, $d_3 = 0.2$, $\sigma_1 = 0.26$, $\sigma_2 = 0.39$,

$$\tau_1 = 0.15$$

less attractive option, which increases the share of low–skilled and skilled agents in the formal economy. As a result, the share of the shadow economy decreases from 0.27 to 0.085 if τ falls to 0.15. A disadvantage of this policy is that it reduces growth by 0.21 percentage points (from 0.025 to 0.0229). Lower government externality on infrastructure and lower human capital accumulation drive both skilled wages and low–skilled wages down by approximately 16% (to 0.7065 and to 0.2419 respectively) for τ declining by 50%.

A higher penalty rate ρ or an audit rate p^c improves the share of skilled and low–skilled workers in the official sector. A rise in p^c and ρ by 50% leads to a fall in the share of the shadow sector respectively to 0.1413 and to 0.1007. Growth rises stronger in case of higher ρ (to 2.57%)

than p^c (to 2.55%) in these cases. Wages improve because of the positive spillover effect of the government externality on base income $\overline{w}^l$ and $\overline{w}^w$ and improved human capital accumulation. The surge in skilled and low–skilled remuneration for 50% larger p^c (amounting for both wages to approximately 3.6% increase to $\overline{w}^w h_t = 0.8675$ and $\overline{w}^l h_t^l = 0.297$) is lower than the positive effect which 50% larger ρ has on (low–)skilled labor productivity (which is around 4.7% increase to $\overline{w}^w h_t = 0.8766$ and $\overline{w}^l h_t^l = 0.3001$).

A higher government expenditure share on education d does not create an incentive for agents to change occupations, because educational expenditure is a public good available to all and low–skilled are entitled to a share of skilled agents' human capital. That is why the share of the informal sector remains unaffected. Still, growth rises as high as 2.63% if the government invests in education 15% of its revenue out of taxation and penalties. Skilled wages rise to 0.8608, while low–skilled to 0.2947 for d going up to 0.15 (which constitutes improvement of around 2.8% for both) due to enhanced human capital accumulation in spite of the fall in base wages.

A lower risk measure σ leads to a higher skilled labor supply to the disadvantage of the pool of low–skilled in the formal and the informal economy. The shadow economy share falls to 0.255 if σ declines by 50%. Growth improves to 2.51% in the same case. The higher competition among skilled workers in the formal sector makes high–skilled wages decrease by approximately 4.3% to 0.8011 for a decline in σ of 50% despite higher human capital accumulation. Low–skilled wages improve by around 5.9% to 0.3035 in the same case because of lower competition among low–skilled (triggering higher base income) and a higher growth rate.

The policy mixes τ, d presented in Table 3.1 are selected among combinations of τ, d which could lead to the strongest decline in the shadow economy in response to lower τ. If τ decreases by 15% and d rises by 50% or 100%, the resulting occupational choice and the share of informal sector ($\gamma = 0.085$) are solely dictated by τ. Because the positive impact of higher d on g^h is stronger than the negative influence of lower τ on g^h, growth improves to 2.52% in case of realization of the policy mix $\tau = 0.15, d = 0.2$. The short–run impact of the policy mix ($\tau = 0.15, d = 0.2$) leads to a fall of (low)–skilled wages by 11.9% (to $\overline{w}^w h_t = 0.7378$, $\overline{w}^l h_t^l = 0.2526$) as growth is not strong enough to compensate for the fall in $\overline{w}^l$, $\overline{w}^w$.

To sum up, the most effective policy in combating the share of the unofficial economy without an adverse impact on growth is the policy mix of a lower tax rate τ and a relatively high share of government expenditure on education d, followed by an increase in the penalty rate ρ, and the audit rate p^c. A fall in the risk measure σ ranks last in decreasing the share of the informal sector without causing an unfavorable effect on long–term economic development.

3.6 Welfare Analysis

For the welfare analysis we calculate the *ex–post* welfare of young agents who work later in the formal sector $V_{y,t}$ (addressed also as formal young), young who participate later in the informal sector $V_{y,t}^s$ (addressed also as informal young), middle–aged skilled $V_{m,t}^w$, middle–aged formal low–skilled $V_{m,t}^l$, and middle–aged informal low–skilled workers $V_{m,t}^s$ at period t. The welfare of young, who become skilled workers or low–skilled in the formal sector, is equal due to the equilibrium occupational choice. The index y stands for young, m for middle–aged. We define V_t as the sum of lifetime utilities of all domestic individuals at time t. For the *ex–post* welfare,

$$V_t = L_t^w V_{y,t} + L_t^l V_{y,t} + L_t^s V_{y,t}^s + L_{t-1}^w V_{m,t}^w + L_{t-1}^l V_{m,t}^l + L_{t-1}^s V_{m,t}^s$$

$$V_{y,t} = U(c_t^w) + \int_{a \in \bar{A}} \beta U(c_{i,t+1}^w) f(a) da = U(c_t^l) + \beta U(c_{t+1}^l)$$

$$V_{y,t}^s = U(c_t^s) + (1 - p^c)\beta U(c_{t+1}^s) + p^c \beta U(c_{t+1}^{sc})$$

$$V_{m,t}^w = \int_{a \in \bar{A}} U(c_{i,t}^w) f(a) da$$

$$V_{m,t}^s = (1 - p^c)U(c_t^s) + p^c U(c_t^{sc})$$

$$V_{m,t}^l = U(c_t^l)$$

We perform a welfare analysis from period $t = 0$ till period $t = 4$, i.e. we look at the influence of the policies and the risk measure on the welfare of the individuals for the next 150 years, given that the parameter change takes place at period $t = 0$ ($h_0 = 1$). In accordance with Soares (2008), we weight equally the welfare of present and future generations.

We expect that the long–term influence of p^c, ρ, τ, d, σ on welfare is reciprocal in sign to p^c, ρ, τ, d, σ on growth. This is a natural consequence of the prevailing growth effect of any policy in the long term in an overlapping generations model (see Yakita (2004), Wong and Yip (1999)). A change in p^c, ρ, τ, d, σ alters the lifetime utility of young agents with the introduction of a new parameter ($t = 0$) through a new distribution of young across professions, new wages and human capital accumulation. In case of alternative values of σ, young agents in $t \geq 0$ who remain formal experience additionally declining/increasing utility due to risk aversion. The lifetime utility change of middle–aged in $t = 0$ in response to different government policies except for σ is reflected only in the adjustment of base wages. This outcome is put down to the fact that the occupational choice remains fixed. The latter is presented by the subindex $t - 1$ in the share of middle–aged L_{t-1}^w, L_{t-1}^l and L_{t-1}^s in the welfare equations above, which implies that the decision on an occupation is taken at period $t - 1$. An analogical interpretation applies for L_t^w, L_t^l and L_t^s. Alternative values of σ in $t \geq 0$ cause an additional effect on the utility of middle–aged skilled due to the assumption of risk aversion.

Table 3.2: *Transitional Welfare Analysis with Respect to τ, p^c, ρ*

	BM	τ_1	τ_2	p_1^c	p_2^c	ρ_1	ρ_2
$V_{y,t=0}$	-3.5349	-3.4541	-3.4582	-3.5162	-3.5049	-3.5108	-3.4972
$V_{y,t=0}^s$	-3.3128	-3.3756	-3.3114	-3.3045	-3.3037	-3.3052	-3.3136
$V_{m,t=0}^w$	-1.4095	-1.3468	-1.3629	-1.4077	-1.4059	-1.4077	-1.4059
$V_{m,t=0}^s$	-2.046	-2.1775	-2.1011	-2.0583	-2.0707	-2.0665	-2.0946
$V_{m,t=0}^l$	-2.3462	-2.2835	-2.2995	-2.3444	-2.3426	-2.3444	-2.3426
$V_{t=0}$	-2.731	-2.7698	-2.7345	-2.7426	-2.751	-2.7492	-2.765
$V_{y,t=1}$	-2.2476	-2.2492	-2.1924	-2.2003	-2.1737	-2.1853	-2.152
$V_{y,t=1}^s$	-2.0255	-2.1707	-2.0456	-1.9886	-1.9725	-1.9797	-1.9684
$V_{m,t=1}^w$	-0.6697	-0.6452	-0.6291	-0.6468	-0.634	-0.6396	-0.6235
$V_{m,t=1}^s$	-1.3062	-1.4758	-1.3674	-1.2974	-1.2988	-1.2984	-1.3122
$V_{m,t=1}^l$	-1.6064	-1.5818	-1.5658	-1.5835	-1.5707	-1.5763	-1.5602
$V_{t=1}$	-1.7174	-1.7873	-1.7274	-1.7051	-1.6993	-1.7028	-1.6973
$V_{y,t=2}$	-0.9604	-1.0659	-0.9413	-0.8954	-0.8588	-0.8747	-0.8286
$V_{y,t=2}^s$	-0.7382	-0.9874	-0.7945	-0.6837	-0.6576	-0.669	-0.6451
$V_{m,t=2}^w$	0.0701	0.0349	0.0899	0.1031	0.1217	0.1137	0.137
$V_{m,t=2}^s$	-0.5664	-0.7958	-0.6484	-0.5475	-0.543	-0.5452	-0.5516
$V_{m,t=2}^l$	-0.8666	-0.9018	-0.8468	-0.8335	-0.815	-0.823	-0.7997
$V_{t=2}$	-0.7039	-0.8557	-0.7423	-0.6777	-0.664	-0.6708	-0.6553
$V_{y,t=3}$	0.3269	0.1174	0.3099	0.4095	0.4561	0.436	0.4948
$V_{y,t=3}^s$	0.549	0.1958	0.4567	0.6212	0.6573	0.6416	0.6783
$V_{m,t=3}^w$	0.8099	0.7149	0.809	0.8531	0.8774	0.8669	0.8976
$V_{m,t=3}^s$	0.1734	-0.1157	0.0707	0.2025	0.2127	0.2081	0.2089
$V_{m,t=3}^l$	-0.1267	-0.2218	-0.1277	-0.0836	-0.0593	-0.0698	-0.0391
$V_{t=3}$	0.3097	0.076	0.2428	0.3498	0.3714	0.3612	0.3867
$V_{y,t=4}$	1.6142	1.3007	1.561	1.7145	1.7711	1.7467	1.8182
$V_{y,t=4}^s$	1.8363	1.3791	1.7078	1.9262	1.9723	1.9523	2.0017
$V_{m,t=4}^w$	1.5498	1.395	1.528	1.6031	1.6331	1.6202	1.6582
$V_{m,t=4}^s$	0.9132	0.5643	0.7897	0.9524	0.9684	0.9614	0.9695
$V_{m,t=4}^l$	0.6131	0.4583	0.5913	0.6664	0.6965	0.6835	0.7215
$V_{t=4}$	1.3232	1.0077	1.2279	1.3772	1.4067	1.3931	1.4286

Parameters calibrated as follows: $\tau_1 = 0.15$, $\tau_2 = 0.225$, $p_1^c = 0.1112$, $p_2^c = 0.1335$, $\rho_1 = 0.5875$,

$\rho_2 = 0.7050$

Table 3.3: *Transitional Welfare Analysis with Respect to d and σ and Policy Combinations*

	BM	d_1	d_2	d_3	σ_1	σ_2	τ_1, d_2	τ_1, d_3
$V_{y,t=0}$	-3.5349	-3.5286	-3.5256	-3.5263	-3.4924	-3.5101	-3.4448	-3.4455
$V_{y,t=0}^s$	-3.3128	-3.3065	-3.3035	-3.3042	-3.2703	-3.288	-3.3664	-3.3671
$V_{m,t=0}^w$	-1.4095	-1.4151	-1.4209	-1.4331	-1.3081	-1.3504	-1.3583	-1.3704
$V_{m,t=0}^s$	-2.046	-2.0517	-2.0574	-2.0696	-2.046	-2.046	-2.189	-2.2011
$V_{m,t=0}^l$	-2.3462	-2.3518	-2.3576	-2.3697	-2.3462	-2.3462	-2.295	-2.3071
$V_{t=0}$	-2.731	-2.7306	-2.732	-2.7385	-2.6121	-2.6616	-2.7709	-2.7773
$V_{y,t=1}$	-2.2476	-2.2035	-2.1698	-2.1226	-2.1443	-2.1874	-2.1713	-2.1241
$V_{y,t=1}^s$	-2.0255	-1.9814	-1.9477	-1.9004	-1.9222	-1.9653	-2.0929	-2.0456
$V_{m,t=1}^w$	-0.6697	-0.6536	-0.6417	-0.6263	-0.6123	-0.6362	-0.6172	-0.6018
$V_{m,t=1}^s$	-1.3062	-1.2901	-1.2782	-1.2628	-1.2488	-1.2727	-1.4479	-1.4324
$V_{m,t=1}^l$	-1.6064	-1.5903	-1.5784	-1.563	-1.549	-1.5729	-1.5539	-1.5384
$V_{t=1}$	-1.7174	-1.6873	-1.6645	-1.6332	-1.5447	-1.6164	-1.7344	-1.7031
$V_{y,t=2}$	-0.9604	-0.8784	-0.814	-0.7188	-0.8488	-0.8953	-0.9195	-0.8243
$V_{y,t=2}^s$	-0.7382	-0.6562	-0.5918	-0.4967	-0.6267	-0.6732	-0.8411	-0.7459
$V_{m,t=2}^w$	0.0701	0.108	0.1375	0.1805	0.1322	0.1063	0.1023	0.1452
$V_{m,t=2}^s$	-0.5664	-0.5285	-0.499	-0.456	-0.5043	-0.5302	-0.7284	-0.6854
$V_{m,t=2}^l$	-0.8666	-0.8287	-0.7992	-0.7562	-0.8045	-0.8304	-0.8344	-0.7914
$V_{t=2}$	-0.7039	-0.6439	-0.597	-0.5279	-0.5247	-0.5991	-0.7488	-0.6797
$V_{y,t=3}$	0.3269	0.4468	0.5419	0.685	0.4467	0.3968	0.3323	0.4755
$V_{y,t=3}^s$	0.549	0.6689	0.764	0.9071	0.6688	0.6189	0.4108	0.5539
$V_{m,t=3}^w$	0.8099	0.8696	0.9167	0.9873	0.8768	0.8489	0.8217	0.8923
$V_{m,t=3}^s$	0.1734	0.233	0.2802	0.3507	0.2403	0.2124	-0.009	0.0616
$V_{m,t=3}^l$	-0.1267	-0.0671	-0.02	0.0506	-0.0599	-0.0878	-0.115	-0.0444
$V_{t=3}$	0.3097	0.3994	0.4705	0.5774	0.4953	0.4183	0.2369	0.3437
$V_{y,t=4}$	1.6142	1.7719	1.8977	2.0888	1.7422	1.6888	1.5842	1.7753
$V_{y,t=4}^s$	1.8363	1.994	2.1198	2.3109	1.9643	1.911	1.6626	1.8537
$V_{m,t=4}^w$	1.5498	1.6311	1.6959	1.794	1.6213	1.5915	1.5412	1.6393
$V_{m,t=4}^s$	0.9132	0.9946	1.0594	1.1575	0.9848	0.955	0.7105	0.8086
$V_{m,t=4}^l$	0.6131	0.6944	0.7592	0.8573	0.6846	0.6548	0.6045	0.7026
$V_{t=4}$	1.3232	1.4427	1.5381	1.6827	1.5153	1.4356	1.2225	1.3671

Parameters calibrated as follows: $d_1 = 0.125$, $d_2 = 0.15$, $d_3 = 0.2$ $\sigma_1 = 0.26$, $\sigma_2 = 0.39$, $\tau_1 = 0.15$

We follow the calibration of the previous section and conduct a welfare analysis by investigating what is the impact of a 25% stepwise decline in τ or σ, and a 25% stepwise improvement in ρ or p^c. We are interested in what is the individual influence of a 25% stepwise increase in d on welfare in order to be able to draw a conclusion about the effect of the policy mix of lower taxation and higher government investment in public education on social utility.

As Table 3.2 shows if the tax rate τ falls to 0.15 or 0.225 at $t = 0$, formal middle–aged and formal young workers are better–off as they are exempted from taxation (despite a decline in the base wage in $t = 0$ for constant γ and a fall in growth in the expected utility of formal young). Even young agents who become later shadow workers profit in $t = 0$ for a relatively mild decline in τ to 0.225. If the tax rate declines to 0.15, the welfare of formal middle–aged skilled and formal middle–aged low–skilled remains higher than the benchmark in $t = 1$. The individual utility of all agents after $t = 1$ drops in the same case. For τ declining to 0.225, formal middle–aged and formal young workers experience higher welfare in $t = 1$ and $t = 2$, but afterwards their utility also is reduced and no more winners in society can be found. The decrease in welfare in all periods in response to lower τ is due to a declining positive government externality on income combined with a declining growth rate.

A higher audit probability p^c improves the welfare of all agents with the exception of middle–aged informal workers in $t = 0$. This outcome can be explained as follows: a higher probability of detection p^c stimulates human capital accumulation and improves the base wages $\overline{w}^l$ and $\overline{w}^s$ of future generations as it discourages participation in the informal sector. This also leads to a rise in the welfare of young in $t = 0$. In $t = 0$ labor productivities ($\overline{w}^l$ and $\overline{w}^s$) rise due to improvement in the penalty revenue per production unit (for constant γ). As the utility from a higher base wage (for constant γ in $t = 0$) is lower than the disutility of being detected with a higher probability, shadow economy middle–aged workers in $t = 0$ are worse off in contrast to all other middle–aged workers in the same period. As a consequence, total utility falls in $t = 0$, but it remains larger than the benchmark in all other periods.

A 25% higher penalty rate ρ similar to the audit rate is accompanied by a fall in the utility of middle–aged shadow economy workers and total social utility in $t = 0$ and improvement in overall social welfare after this period. If, however, ρ increases by 50%, informal middle–aged and informal young workers in $t = 0$ are worse off, which results in lower first period welfare. The latter can be explained with the prevailing effect of the disutility of paying a higher penalty rate in $t = 0$. In $t = 1$ in the same case, the only losers are middle–aged shadow economy workers. Nevertheless, higher total welfare in all periods after $t = 0$ is observed in case of $\rho = 0.7050$. This can be explained with the favorable impact of ρ on human capital accumulation and labor

productivities.

Higher d (Table 3.3) leads to higher welfare of young in $t = 0$ due to higher human capital accumulation despite a fall in $\overline{w}^l$ and $\overline{w}^w$. The decline in base income results in lower welfare of all middle–aged agents in the same period. If the increase in d is relatively small (25%), social welfare is beyond the benchmark at period $t = 0$. If d rises by 50% or higher, total utility in $t = 0$ declines. Welfare gain is observed in all periods after $t = 0$ irrespective of d because there are no losers in society from $t = 1$ onwards. This can be explained with the prevailing positive effect that d exerts on growth, which compensates for the permanent fall in $\overline{w}^s$ and $\overline{w}^l$.

Lower risk in skilled earnings σ improves the expected welfare of formal young individuals in $t \geqslant 0$ directly via the higher utility of a lower income volatility (due to risk aversion) and indirectly via increased human capital accumulation, which raises also social welfare in $t = 0$. The positive indirect effect of lower σ on g^h boosts as well the utility of young who become later informal workers for $t \geqslant 0$. Middle–aged low–skilled at $t = 0$ do not experience any welfare change because of the assumption that the occupational choice stays constant. Middle–aged skilled workers are better–off in response to lower variance of the income distribution due to risk aversion. This is true also for periods beyond $t = 0$ for middle–aged skilled workers in case of higher σ although higher competition is supposed to drive their expected wages and utility down. Welfare gain is observed in all periods as there are no losers in any period. This is also partly due to higher human capital accumulation.

A combination of a lower tax $\tau = 0.15$ and higher $d = 0.2$ leads to a short–term welfare loss due to the fall in the utility of informal middle–aged workers and young shadow workers in $t = 0$. The welfare fall of middle–aged informal and young informal agents is determined by the decline in the base wage in $t = 0$. While young informal workers take advantage of a higher growth rate after $t = 2$, the utility of informal middle–aged is unable to recover over the considered period due to the prevailing unfavorable impact of the policy mix $\tau = 0.15, d = 0.2$ on the base income. The welfare of formal young in $t \geq 0$ rises because of the expectation of higher growth and tax exemption (despite a fall in future base wages). Middle–aged formal workers are exempted from taxation and benefit additionally from rising base income (for constant γ), which makes their (expected) utility go up in $t = 0$. The favorable effect of tax exemption on gross income is strengthened by a positive growth effect after $t = 0$, which drives up further formal middle–aged individuals' welfare. The overall welfare improves after $t = 0$. As expected, the policy mix $\tau = 0.15, d = 0.15$ leads to gradual deterioration of welfare. This is to be explained with the falling human capital accumulation, which makes the policy mix $\tau = 0.15, d = 0.15$ ineligible to combat the shadow economy.

In terms of absolute welfare, according to Table 3.2 and 3.3, a decline in the risk measure (also compared to the rest of the policies which reduce γ) leads to the strongest rise in welfare in the short run (till the end of period $t = 4$), followed by the penalty rate and the audit rate. A policy mix of a larger tax rate and a larger share of government expenditure on education ranks last in this respect. In the very long run, we expect that the penalty rate has the strongest welfare rise due to its superior impact on growth, followed by the audit rate p^c, a combination of τ, d (for relatively high d) and the risk measure σ. A lower risk measure, compared to the other government policies which decrease the share of the informal economy, is the only alternative, which does not create a trade–off between short term and long term social welfare. Therefore, an insurance system for skilled in the formal economy may, nevertheless, turn out to be a favorable way to fight the unofficial economy.

3.7 Comparison with Related Literature

Our results can be compared most successfully with the work of Ihrig and Moe (2004), Chen (2003), and Turnovsky and Basher (2009). Contrary to our model, Ihrig and Moe (2004) find that the existence of a shadow economy does not necessarily lead to an efficiency loss. This is mainly due to their assumption that the unofficial sector employs only labor and does not grow over time.

The superiority of the income tax rate in fighting the informal sector has been already verified by Ihrig and Moe (2004) (but not by Turnovsky and Basher (2009) for the labor income tax). Ihrig and Moe (2004), however, find that a decline in the tax rate leads to an increase in the standard of living, a result, which we cannot replicate due to the assumption that production is augmented by government expenditure on infrastructure. Contrary to our model, Turnovsky and Basher (2009) find that a decrease in the labor tax rate leads to a short–term welfare loss and a long–term welfare gain. This result is due to the lack of a government expenditure on infrastructure in their production function.

Similar to your work, Ihrig and Moe (2004), Chen (2003), Turnovsky and Basher (2009) find that the penalty rate and the audit rate have a positive impact on growth. Our theoretical model predicts that the effect of the tax enforcement policies on growth depends on the elasticity of government expenditure on infrastructure in the production function. Chen (2003), in the same spirit, argues that the effectiveness of the audit rate and the penalty rate to influence growth is strengthened by a larger government externality in production. In terms of welfare, Turnovsky and Basher (2009) find that an increase in auditing may lead to a short–term welfare loss, which we can also replicate in our model.

3.8 Conclusion

This paper is embedded in the literature on occupational choice under risk and risky investment in public education in a two–period overlapping generations endogenous growth model with a shadow sector and productive government spending. We follow the work of Kanbur (1979) for occupational choice under risk, Glomm and Ravikumar (1992) for developing risky human capital accumulation with public eduction, and Loayza (1996) for the construction of the shadow sector with a congestion effect on productive government expenditure in production. *Ex–ante* homogeneous and risk–averse agents are free to determine their education (skilled vs. low–skilled) and sector of employment (informal vs. formal) at the beginning of their life. Formal skilled bear risk in earnings due to *ex–ante* uninsurable abilities to accumulate human capital. Formal low–skilled are exempted from uncertainty in income. Informal low–skilled are subject to the risk of being detected evading taxes and misperceive the probability of the audit rate. The equilibrium occupational choice in the formal sector is determined by the equality of the (expected) utility of formal skilled and formal low–skilled workers. Low–skilled in the shadow economy self–select on the basis of a relatively optimistic idea about the audit rate. Growth depends positively on the government expenditure on education, which is financed by taxes levied in the formal sector together with penalty fees gathered from the informal sector, so it is related to the occupational choice of economic agents.

We show that a decrease in the share of the unofficial income improves growth (and long–term welfare). According to our theoretical analysis, a fall in the shadow economy can result in response to a lower tax rate, a higher audit rate, a higher penalty rate or a lower risk measure. Nevertheless, the impact of the policies and the risk measure on human capital accumulation (and long–term welfare) is inconclusive. According to our calibration, the policies in combating the shadow sector without adverse consequences for growth ranked with respect to their effectiveness are the combination of lower taxation and a higher share of government expenditure on education, the penalty rate, the audit rate, and at last the risk measure. A lower risk measure attains the highest welfare in the short run, followed by a higher penalty rate. Nevertheless, in terms of long–term welfare, the penalty rate ranks first, the audit rate taking up the next position. The risk measure is inferior to all other policies in this respect.

Moreover, our analysis shows that while government policies may lead to substantial changes in the share of the informal sector, and economic development, they are all related to some trade–off in welfare in the short run vs. the long run. This result does not apply for a decline in the risk measure. Given the positive impact of lower risk on growth, the individual and overall utility, a government may consider it worthwhile to implement an insurance mechanisms for

skilled agents working in the formal economy. If, on the other hand, the government target is to attain the smallest share of the unofficial economy, it should apply a policy mix of lower taxation and higher investment in education. This approach will cause, however, a temporal loss in social welfare.

For future research the present work can be extended to incorporate physical capital accumulation as well. The interplay between physical capital accumulation, human capital accumulation and occupational choice under risk with alternative taxation on physical capital can give a new insight on further government policies aiming to reduce the shadow sector (if necessary). Moreover, given the significantly positive effect of a lower risk measure on welfare, which simultaneously reduces the share of the informal sector, the construction of an insurance system for skilled agents may also be considered.

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3.10 Appendix

3.10.1 Proof of Proposition 3.3

Simplifying the condition $E(V^w_{i,t,t+1}) = V^l_{t,t+1}$, we obtain,

$$\frac{L_t}{L^w_t} = \frac{1-\alpha}{\alpha} \frac{exp\left(\frac{\sigma^2}{2}\right)}{(1-\nu)^{1/b}}$$

Simplifying the condition $V^s_{i,t+1} = V^l_{t+1}$, and using that $\overline{w}^s_{t+1} = \overline{w}^l_{t+1}$, we have,

$$\ln(1-\tau) = \psi^* p^c \ln(1-\rho)$$

The perceived risk weight by the agent who is indifferent between participating in the shadow economy and the formal sector ψ^* in equilibrium is equal to,

$$\psi^* = \frac{1}{p^c} \frac{\ln(1-\tau)}{\ln(1-\rho)}$$

A middle–aged low–skilled agent with perceived risk weight higher than ψ^* will participate in the formal economy, while the agents with a lower risk perception will prefer to work in the

formal sector. This implies that the share of the middle–aged low–skilled labor force working in the formal sector could be determined as follow,

$$\frac{L_t^l}{L_t^l + L_t^s} = \int_{\psi^*}^{\overline{\psi}} f(\psi)d\psi = \frac{\overline{\psi} - \psi^*}{\overline{\psi} - \underline{\psi}} \implies L_t^l = \frac{\overline{\psi} - \psi^*}{\psi^* - \underline{\psi}} L_t^s$$

We use the relationship between low–skilled in the formal and informal sector and the normalization of the population to one, $L_t = 1 - L_t^s - L_t^w$ to obtain,

$$\frac{L_t}{L_t^w} = \frac{1-\alpha}{\alpha} \frac{exp\left(\frac{\sigma^2}{2}\right)}{(1-\nu)^{1/b}} \implies \frac{1 - L_t^s - L_t^w}{L_t^w} = \frac{1-\alpha}{\alpha} \frac{exp\left(\frac{\sigma^2}{2}\right)}{(1-\nu)^{1/b}} \implies L_t^w = \frac{1 - L_t^s}{1 + M}$$

where $M = \frac{1-\alpha}{\alpha} \frac{exp\left(\frac{\sigma^2}{2}\right)}{(1-\nu)^{1/b}} > 1$ because $L_t > L_t^w$ by assumption. With the derived relationship between L_t^l and L_t^s, as well as L_t^w with L_t^s, and, moreover, $L_t^l = 0.5 - L_t^s - L_t^w$, we obtain that the share of agents across occupations is constant because L_t^s is constant and equal to,

$$L^s = \frac{M - 1}{2M + 2(1 + M)\frac{\overline{\psi} - \psi^*}{\psi^* - \underline{\psi}}}$$

The share of the shadow economy out of the formal economy is by definition,

$$\gamma_t = \frac{Q_t}{Y_t} = \frac{B\left(\frac{\delta(1-d)(\tau + p^c \rho \gamma_t)}{(1+\gamma_t)}\right)^\eta H_t^s}{A\left(\frac{(1-d)(\tau + p^c \rho \gamma_t)}{1+\gamma_t}\right)^\beta (H_t^w)^\alpha (H_t^l)^{1-\alpha}} \implies \gamma_t \frac{H_t^l}{H_t^s} = \frac{B\left(\frac{\delta(1-d)(\tau + p^c \rho \gamma_t)}{(1+\gamma_t)}\right)^\eta}{A\left(\frac{(1-d)(\tau + p^c \rho \gamma_t)}{1+\gamma_t}\right)^\beta \frac{(H_t^w)^\alpha}{(H_t^l)^\alpha}}$$

On the other hand, we know that $\overline{w}^s = \overline{w}^l$, therefore,

$$\frac{B\left(\frac{\delta(1-d)(\tau + p^c \rho \gamma_t)}{(1+\gamma_t)}\right)^\eta}{(1-\alpha)A\left(\frac{(1-d)(\tau + p^c \rho \gamma_t)}{1+\gamma_t}\right)^\beta \left(\frac{H_t^w}{H_t^l}\right)^\alpha} = 1 \implies \gamma = \frac{L^s}{L}(1 - \alpha)$$

Because $\overline{w}^l = \overline{w}^s$, we could determine the sector efficiency parameter B endogenously for given A from the following condition,

$$B\left(\frac{\delta(1 - d)(\tau + p^c \rho \gamma)}{(1 + \gamma)}\right)^\eta = (1 - \alpha)A\left(\frac{(1 - d)(\tau + p^c \rho \gamma)}{1 + \gamma}\right)^\beta \left(\frac{H_t^w}{H_t^l}\right)^\alpha$$

Let $A = 1$, then B for given δ is,

$$B = \frac{1 - \alpha}{\delta^\eta}\left(\frac{(1 - d)(\tau + p^c \rho \gamma)}{1 + \gamma}\right)^{\beta - \eta}\left(\frac{L^w}{\varphi L}\right)^\alpha$$

3.10.2 Proof of Proposition 3.5

The sensitivity analysis of ψ^* for $0 < \rho, \tau < 0$ is,

$$\frac{\partial \psi^*}{\partial \tau} = \frac{1}{p^c} \frac{1}{(1 - \tau)\ln(1 - \rho)}(-1) > 0$$

$$\frac{\partial \psi^*}{\partial \rho} = \frac{1}{p^c} \frac{\ln(1 - \tau)}{(\ln(1 - \rho))^2} \frac{1}{(1 - \rho)} < 0$$

$$\frac{\partial \psi^*}{\partial p^c} = \frac{1}{(p^c)^2} \frac{\ln(1 - \tau)}{\ln(1 - \rho)}(-1) < 0$$

The influence of τ, ρ, p^c and σ on the share of low–skilled working in the shadow economy is,

$$\frac{\partial L^s}{\partial \tau} = 2(1+M) \underbrace{\frac{\partial \frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}}}{\partial \psi^*}}_{<0} \underbrace{\frac{\partial \psi^*}{\partial \tau}}_{>0} (-1) \frac{(M-1)}{(2M+2(1+M)\frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}})^2} > 0$$

$$\frac{\partial L^s}{\partial \rho} = 2(1+M) \underbrace{\frac{\partial \frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}}}{\partial \psi^*}}_{<0} \underbrace{\frac{\partial \psi^*}{\partial \rho}}_{<0} (-1) \frac{(M-1)}{(2M+2(1+M)\frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}})^2} < 0$$

$$\frac{\partial L^s}{\partial p^c} = 2(1+M) \underbrace{\frac{\partial \frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}}}{\partial \psi^*}}_{<0} \underbrace{\frac{\partial \psi^*}{\partial p^c}}_{<0} (-1) \frac{(M-1)}{(2M+2(1+M)\frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}})^2} < 0$$

$$\frac{\partial L^s}{\partial \sigma} = \frac{L^s}{M-1} \underbrace{\frac{\partial M}{\partial \sigma}}_{>0} \underbrace{[1-2L^s-2L^l]}_{>0} > 0 \quad \text{because} \quad \frac{1}{2} > L^l + L^s$$

The influence of τ, ρ, p^c and σ on the share of skilled is,

$$\frac{\partial L^w}{\partial \tau} = -\frac{1}{1+M} \underbrace{\frac{\partial L^s}{\partial \tau}}_{>0} < 0 \qquad \frac{\partial L^w}{\partial \rho} = -\frac{1}{1+M} \underbrace{\frac{\partial L^s}{\partial \rho}}_{<0} > 0$$

$$\frac{\partial L^w}{\partial p^c} = -\frac{1}{1+M} \underbrace{\frac{\partial L^s}{\partial p^c}}_{<0} > 0 \qquad \frac{\partial L^w}{\partial \sigma} = -\frac{(1-L^s)}{(1+M)^2} \underbrace{\frac{\partial M}{\partial \sigma}}_{>0} - \frac{1}{1+M} \underbrace{\frac{\partial L^s}{\partial \sigma}}_{>0} < 0$$

The influence of τ, ρ, p^c and σ on the share of low–skilled working in the formal economy is,

$$\frac{\partial L^l}{\partial \tau} = \underbrace{\frac{\partial \frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}}}{\partial \psi^*}}_{<0} \underbrace{\frac{\partial \psi^*}{\partial \tau}}_{>0} L^s \Big[1 - L^s \frac{2(1+M)}{M-1}\frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}}\Big] < 0$$

$$\frac{\partial L^l}{\partial \rho} = \underbrace{\frac{\partial \frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}}}{\partial \psi^*}}_{<0} \underbrace{\frac{\partial \psi^*}{\partial \rho}}_{<0} L^s \Big[1 - L^s \frac{2(1+M)}{M-1}\frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}}\Big] > 0$$

$$\frac{\partial L^l}{\partial p^c} = \underbrace{\frac{\partial \frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}}}{\partial \psi^*}}_{<0} \underbrace{\frac{\partial \psi^*}{\partial p^c}}_{<0} L^s \Big[1 - L^s \frac{2(1+M)}{M-1}\frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}}\Big] > 0$$

$$\frac{\partial L^l}{\partial \sigma} = \frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}} \underbrace{\frac{\partial L^s}{\partial \sigma}}_{>0} > 0$$

Notice that $1 - L^s \frac{2(1+M)}{M-1}\frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}} > 0$ can be transformed into $1 > \frac{2(1+M)\frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}}}{2M+2(1+M)\frac{\overline{\psi}-\psi^*}{\psi^*-\underline{\psi}}}$ (when we substitute for L^s), which is always true. The influence of τ, ρ, p^c and σ on the share of the shadow economy is,

$$\frac{\partial \gamma}{\partial \tau} = \frac{1-\alpha}{L} \underbrace{\frac{\partial L^s}{\partial \tau}}_{>0} - (1-\alpha)\frac{L^s}{(L)^2} \underbrace{\frac{\partial L^l}{\partial \tau}}_{<0} > 0 \qquad \frac{\partial \gamma}{\partial \rho} = \frac{1-\alpha}{L} \underbrace{\frac{\partial L^s}{\partial \rho}}_{<0} - (1-\alpha)\frac{L^s}{(L)^2} \underbrace{\frac{\partial L^l}{\partial \rho}}_{>0} < 0$$

$$\frac{\partial \gamma}{\partial p^c} = \frac{1-\alpha}{L}\underbrace{\frac{\partial L^s}{\partial p^c}}_{<0} - (1-\alpha)\frac{L^s}{(L)^2}\underbrace{\frac{\partial L^l}{\partial p^c}}_{>0} < 0 \qquad \frac{\partial \gamma}{\partial \sigma} = \underbrace{\frac{\partial L^s}{\partial \sigma}}_{>0}\frac{1-\alpha}{L}\underbrace{\left[1 - \frac{L^l}{L}\right]}_{>0} > 0$$

3.10.3 Proof of Corollary 3.2

By differentiating the growth rate with respect to γ, we obtain,

$$\frac{\partial(1+g^h)}{\partial \gamma} = (1+g^h)\phi\left[\frac{(1+\beta)p^c\rho}{\tau + p^c\rho\gamma} - \frac{\beta}{1+\gamma} - \frac{1}{\gamma}\right]$$

$$\frac{\partial(1+g^h)}{\partial \gamma} = \frac{(1+g^h)\phi}{(\tau + p^c\rho\gamma)(1+\gamma)\gamma}(\beta\gamma(p^c\rho - \tau) - \tau(1+\gamma)) < 0$$

The relationship between the share of the shadow economy and the growth is negative because $\tau > p^c\rho$ by assumption.

3.10.4 Proof of Proposition 3.6

The impact of τ on growth is,

$$\frac{(1+g^h)}{\partial \tau} = (1+g^h)\frac{\phi}{\tau + p^c\rho\gamma}\underbrace{\left[\varepsilon_\tau^{L^s}\gamma\frac{\beta(p^c\rho - \tau) + p^c\rho(1+\gamma)}{\tau(1+\gamma)} + (1+\beta)\right]}_{\gtrless 0}$$

$$- (1-\alpha)\frac{L^s}{(L)^2}\underbrace{\frac{\partial L^l}{\partial \tau}}_{<0}\underbrace{\frac{\phi\left[\beta\gamma(p^c\rho - \tau) - \tau(1+\gamma)\right]}{(\tau + p^c\rho\gamma)(1+\gamma)\gamma}}_{<0} \gtrless 0$$

The influence of the penalty rate and the audit rate on growth is,

$$\frac{(1+g^h)}{\partial \rho} = (1+g^h)\Big[\phi\underbrace{\frac{\partial L^s}{\partial \rho}}_{<0}\underbrace{\gamma\frac{\beta(p^c\rho - \tau) + p^c\rho(1+\gamma)}{L^s(\tau + p^c\rho\gamma)(1+\gamma)}}_{<0}$$

$$- \frac{(1-\alpha)L^s}{(L)^2}\underbrace{\frac{\partial L^l}{\partial \rho}}_{>0}\underbrace{\frac{\phi\left[\beta\gamma(p^c\rho - \tau) - \tau(1+\gamma)\right]}{(\tau + p^c\rho\gamma)(1+\gamma)\gamma}}_{<0} + \frac{\phi(1+\beta)}{\tau + p^c\rho\gamma}p^c\gamma\Big] > 0$$

$$\text{and} \quad \frac{(1+g^h)}{\partial p^c} = (1+g^h)\Big[\phi\underbrace{\frac{\partial L^s}{\partial p^c}}_{<0}\underbrace{\gamma\frac{\beta(p^c\rho - \tau) + p^c\rho(1+\gamma)}{L^s(\tau + p^c\rho\gamma)(1+\gamma)}}_{<0}$$

$$- \frac{(1-\alpha)L^s}{(L)^2}\underbrace{\frac{\partial L^l}{\partial p^c}}_{>0}\underbrace{\frac{\phi\left[\beta\gamma(p^c\rho - \tau) - \tau(1+\gamma)\right]}{(\tau + p^c\rho\gamma)(1+\gamma)\gamma}}_{<0} + \frac{\phi(1+\beta)}{\tau + p^c\rho\gamma}\rho\gamma\Big] > 0$$

$$\text{if} \quad \gamma\frac{\beta(p^c\rho - \tau) + p^c\rho(1+\gamma)}{L^s(\tau + p^c\rho\gamma)(1+\gamma)} < 0 \quad \text{which is true if} \quad \beta > \frac{p^c\rho(1+\gamma)}{\tau - p^c\rho}$$

The influence of risk on growth is defined as follows,

$$\frac{(1+g^h)}{\partial\sigma} = (1+g^h)\Big[\phi\underbrace{\frac{\partial L^s}{\partial\sigma}}_{>0}\gamma\underbrace{\frac{\beta(p^c\rho-\tau)+p^c\rho(1+\gamma)}{L^s(\tau+p^c\rho\gamma)(1+\gamma)}}_{\gtrless 0}$$

$$-\frac{(1-\alpha)L^s}{(L)^2}\underbrace{\frac{\partial L^l}{\partial\sigma}}_{>0}\underbrace{\frac{\phi\big[\beta\gamma(p^c\rho-\tau)-\tau(1+\gamma)\big]}{(\tau+p^c\rho\gamma)(1+\gamma)\gamma}}_{<0}-\frac{\phi\alpha}{M}\underbrace{\frac{\partial M}{\partial\sigma}}_{>0}\Big]\lessgtr 0$$

For the impact of educational expenditure share on growth, we obtain,

$$\frac{(1+g^h)}{\partial d} = (1+g^h)\frac{\phi(1-d-\beta d)}{d(1-d)} > 0 \qquad \text{if} \qquad d < \frac{1}{1+\beta}$$

and the other way around.

3.10.5 Proof of Proposition 3.7

The short–run income variables for $h_0 = 1$ are defined as follows,

$$\overline{w}^w h_0 = \alpha A\Big(\frac{(1-d)(\tau+p^c\rho\gamma)}{1+\gamma}\Big)^\beta(\varphi M)^{1-\alpha}$$

$$\overline{w}^l h_0^l = (1-\alpha)A\Big(\frac{(1-d)(\tau+p^c\rho\gamma)}{1+\gamma}\Big)^\beta(\varphi M)^{-\alpha}\varphi$$

The skilled wage is influenced ambiguously by τ,

$$\frac{\partial(\overline{w}^w h_0)}{\partial\tau} = \overline{w}^w h_0\Big[\underbrace{\frac{\partial\gamma}{\partial\tau}}_{>0}\beta\underbrace{\Big[\frac{p^c\rho-\tau}{(\tau+p^c\rho\gamma)(1+\gamma)}\Big]}_{<0}+\frac{\beta}{\tau+p^c\rho\gamma}\Big]\lessgtr 0$$

The same is valid for the low–skilled wage. The skilled wage is influenced positively by an increase in ρ and p^c,

$$\frac{\partial(\overline{w}^w h_0)}{\partial p^c} = \overline{w}^w h_0\Big[\underbrace{\frac{\partial\gamma}{\partial p^c}}_{<0}\beta\underbrace{\Big[\frac{p^c\rho-\tau}{(\tau+p^c\rho\gamma)(1+\gamma)}\Big]}_{<0}+\frac{\beta\rho\gamma}{\tau+p^c\rho\gamma}\Big] > 0$$

$$\frac{\partial(\overline{w}^w h_0)}{\partial\rho} = \overline{w}^w h_0\Big[\underbrace{\frac{\partial\gamma}{\partial\rho}}_{<0}\beta\underbrace{\Big[\frac{p^c\rho-\tau}{(\tau+p^c\rho\gamma)(1+\gamma)}\Big]}_{<0}+\frac{\beta p^c\gamma}{\tau+p^c\rho\gamma}\Big] > 0$$

The same applies for the low–skilled wage. The influence of σ on the high–skilled wage is,

$$\frac{\partial(\overline{w}^w h_0)}{\partial\sigma} = \overline{w}^w h_0\Big[\underbrace{\frac{\partial\gamma}{\partial\sigma}}_{>0}\beta\underbrace{\Big[\frac{p^c\rho-\tau}{(\tau+p^c\rho\gamma)(1+\gamma)}\Big]}_{<0}+\frac{1-\alpha}{M}\underbrace{\frac{\partial M}{\partial\sigma}}_{>0}\Big]\lessgtr 0$$

The influence of σ on the low–skilled wage is,

$$\frac{\partial(\overline{w}^l h_0^l)}{\partial\sigma} = \overline{w}^l h_0^l\Big[\underbrace{\frac{\partial\gamma}{\partial\sigma}}_{>0}\beta\underbrace{\Big[\frac{p^c\rho-\tau}{(\tau+p^c\rho\gamma)(1+\gamma)}\Big]}_{<0}-\frac{\alpha}{M}\underbrace{\frac{\partial M}{\partial\sigma}}_{>0}\Big] < 0$$

The high–skilled wage is influenced positively by higher d,

$$\frac{\partial(\overline{w}^w h_0)}{\partial d} = -\frac{\beta}{1-d}(\overline{w}^w h_0) < 0$$

The same proof can be derived for the low–skilled wage.

3.10.6 Proof of Proposition 3.8

The long–term equilibrium wages in the model for $h_0 = 1$ are,

$$\overline{w}^w h_t = \alpha A \Big(\frac{(1-d)(\tau + p^c \rho \gamma)}{1+\gamma} \Big)^\beta (\varphi M)^{1-\alpha} (1+g^h)^t$$

$$\overline{w}^l h_t^l = (1-\alpha) A \Big(\frac{(1-d)(\tau + p^c \rho \gamma)}{1+\gamma} \Big)^\beta (\varphi M)^{-\alpha} (1+g^h)^t \varphi$$

In the long–run $t \longrightarrow \infty$ the impact of τ, p^c, ρ, d, and σ on the income variables depends not only on the short–run changes in occupational choice and the government externality on production but also on the growth rate g^h. This can be easily seen if we differentiate $\overline{w}^l h_t^l$ and $\overline{w}^w h_t$ with respect τ, p^c, ρ, d, and σ (which we here substitute for x in order to simplify our illustration),

$$\frac{\partial (\overline{w}^l h_t^l)}{\partial x} = \frac{\partial \overline{w}^l}{\partial x} h_t^l + t \overline{w}^l (1+g^h)^{t-1} \frac{\partial (1+g^h)}{\partial x} \varphi$$

$$\frac{\partial (\overline{w}^w h_t)}{\partial x} = \frac{\partial \overline{w}^w}{\partial x} h_t + t \overline{w}^w (1+g^h)^{t-1} \frac{\partial (1+g^h)}{\partial x}$$

3.10.7 Empirical Data

Table 3.4: *Shadow Economy Share (%) out of GNP from Selected Eastern European States*

	1999	2000	2001	2002	2003	2004	2005	2006	2007	*Av.*
Bulgaria	37.3	36.9	36.6	36.1	35.6	34.9	34.1	33.5	32.7	35.3
Bosnia&Herzegovina	34.3	34.1	34.0	33.9	33.5	33.6	33.2	32.9	32.8	33.6
CzechRepublic	33.8	33.4	33.2	32.6	32.1	31.7	31.3	30.8	30.4	32.1
Hungary	25.4	25.1	24.8	24.5	24.4	24.1	24.0	23.7	23.7	24.4
Greece	28.5	28.7	28.2	28.0	27.4	27.1	26.9	26.4	26.5	27.5
Macedonia	39.0	38.2	39.1	38.9	38.4	37.4	36.9	36.0	34.9	37.6
Poland	27.7	27.6	27.7	27.7	27.5	27.3	26.9	26.4	26.0	27.2
Romania	34.3	34.4	33.7	33.5	32.8	32.0	31.7	30.7	30.2	32.6
Slovakia	18.9	18.9	18.8	18.6	18.3	18.1	17.6	17.2	16.8	18.1
Slovenia	27.3	27.1	26.7	26.6	26.4	26.2	25.8	25.3	24.7	26.2
Latvia	30.8	30.5	30.1	29.8	29.4	29.0	28.4	27.7	27.2	29.2
Lithuania	33.8	33.7	33.3	32.8	32.0	31.7	31.0	30.4	29.7	32.0

Source: Bühn and Schneider (2012)

Table 3.5: *Informal Labor Force/Sector as % of Official Labor Force/Sector from Selected West-ern European States*

	Year	%Inf. Labor	%Inf. Sector
Austria	97/98	16.0	8.93
Denmark	1994	15.4	17.6
France	1997/98	6.0 − 12.0	14.9
Germany	1997/98	19.0 − 23.0	14.7
Italy	1997/98	30.0 − 48.0	27.3
Spain	1997/98	11.5 − 32.3	23.1
Sweden	1997/98	19.8	19.8

Source: Schneider (2000)

Table 3.6: *Income Tax and Employees' Social Security Contributions for 2010 as % of Gross Earnings from Selected Eastern European States*

	Total	Income Tax	SSC
Slovenia	33.1	11.0	22.1
Greece	18.8	2.8	16.0
CzechRepublic	22.5	11.5	11.0
Poland	24.6	6.7	17.8
Hungary	31.2	14.2	17.0
Estonia	19.4	16.6	2.8
SlovakRepublic	21.5	8.1	13.4

Source: OECD (2011)

Table 3.7: *Government Expenditure on Education as % of Total Government Spending from Selected Eastern European States*

	2008	2009	2010
Bulgaria	12.3	11.3	10.8
CzechRepublic	9.5	9.8	9.7
Hungary	10.4	10.0	9.8
Poland	11.8	11.4	11.4
Romania		10.3	
Slovakia	10.3	9.8	10.6
Estonia	14.2	13.5	14.0
Latvia	14.7	12.8	11.3
Lithuania	13.1	12.9	13.2
Slovenia	11.8	11.6	11.4

Source: The World Bank

http://data.worldbank.org/indicator/SE.XPD.TOTL.GB.ZS, Last accessed on 1.12.2013

4 Pollution, Environmental Tax Evasion, and Corruption in an Endogenous Growth Model

4.1 Introduction

Environmental regulation (in terms of government abatement activities sponsored with pollution taxation[1]) has been highlighted extensively in economic discussions (see Marrewijk *et al.* (1993), Nielsen *et al.* (1995), Smulders and Gradus (1996), Bovenberg and de Mooij (1994,1997), Chen *et al.* (2003), Vondra and Zagler (2004)). Neverthless, some authors have questioned the immediate implementability of environmental regulation due to corruption. As Fredriksson and Svensson (2003), Pellegrini and Gerlagh (2006) and Woods (2008) show empirically, environmental policy stringency can be easily decreased in the presence of corruption.

Inspired by previous research, our work builds an endogenous growth model on environmental policy and corruption by additionally allowing for environmental regulation incompliance (here, environmental tax incompliance). Biswas *et al.* (2011) explore the existence of a relationship between low-scale pollution, environmental regulation with the existence of an unofficial sector and corruption. They find that the informal sector (involved in filthy production) in some developing countries leads to higher pollution and that the interaction between the shadow economy and corruption also plays a role in local environmental degradation. Although our model does not exclusively assume that the shadow economy is the sole culprit in creating pollution, we show that a shadow economy evading environmental taxation used for abatement activities contributes to pollution. We oppose the conventional notion that environmental taxation is hard to evade. We argue that the latter is possible because of the presence of corruption. Corruption and the informal sector in our model reinforce each other (to a degree where corruption costs are small enough). This assumption is backed up empirically by Dreher and Schneider (2010)

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[1]In fact, there is a gradual trend of switching from distortionary (capital and labor) taxation to environmental taxation (see Eurostat (2009)), which is justified by the double dividend hypothesis, so not all environmental taxes are spent on abatement activities.

for low–income countries[2].

Our interest in the relationship between environmental policy, corruption and the shadow economy has an empirical background as well. According to CPI (Corruption Perception Index) for 2012, the level of perceived corruption in most European transition economies has a scale lower than 60 points[3] in Poland (58), Hungary (55), the Czech Republic (49), Romania (44), and Greece (36). Bühn and Schneider (2012) show, on the other hand, that the average share of the shadow economy out of GNP for 1999-2007 in some EU Eastern European countries ranges from 18% in the Slovak Republic to 35.3% in Bulgaria.

The literature on tax evasion and growth has been mostly focused on finding the impact of taxation and tax enforcement policies on economic development and welfare (see Loayza (1996), Lin and Yang (2001), Ihrig and Moe (2004), Peñalosa and Turnovsky (2005), Turnovsky and Basher (2009)). On the other hand, economists are not unanimous on the relationship between corruption and growth, taking account of the fact that corruption may (i) divert resources from productive investment (see Shleifer and Vishny (1993), Mauro (1995, 2004), Ehrlich and Lui (1999)) but also (ii) alleviate administrative borders (see Barreto (2000)). In this respect Sarte (2000), Blackburn *et al.* (2010), Barreto and Alm (2003), Chen (2003) examine the interplay between income tax evasion and corruption. While Sarte (2000) and Blackburn *et al.* (2010) focus on the consequences of this interaction for growth and welfare, Barreto and Alm (2003) investigate the optimal taxation policy (on income vs. consumption) in the presence of corruption. Chen (2003), on the other hand, determines the impact of tax enforcement policies reducing *income tax evasion* in the presence of *corruption* on economic development. In our paper we relate *tax evasion and corruption to environmental quality*. In particular, we are interested in: What is the impact of green tax evasion on pollution, growth and welfare? Which government policies maximize environmental quality when environmental tax incompliance and corruption are present? Which government policies decreasing pollution are favorable to growth and welfare in the same context and under what conditions? What is the impact of corruption costs on environmental quality, growth and welfare? In our work we try to find answers to the afore–mentioned questions.

Our model combines the literature on pollution in a capital accumulation context, (environmental) tax evasion with corruption (*á la* Chen (2003)) considering two types of production functions: with pollution externalities (*á la* Smulders and Gradus (1996)) and without pollu-

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[2]For an alternative view of the relationship between corruption and the informal sector, see Dreher *et al.* (2009) and Friedman *et al.* (2000).

[3]According to CPI, the countries with the lowest level of corruption perception are the ones with the highest scores. The scale of CPI ranges from 0 to 100.

tion externalities (*á la* Gradus and Smulders (1993)). Pollution is modeled as a flow variable, which depends positively on the economic activity represented by capital stock and negatively on abatement. The representative firm pays a pollution tax and decides on the optimal share of environmental tax compliance by incurring additionally corruption costs. In our setting the firm cooperates with corrupt officials in evading environmental taxation. Corruption costs do not contribute to output. The government (non-corrupt officials) enforces penalties on the evaded environmental taxes. The environmental taxes and penalties are spent on abatement activities. Growth is induced by capital accumulation (of the infinitely lived household) net of pollution taxation and corruption costs.

We find that green taxation compliance increases environmental quality, capital accumulation, and welfare. Environmental quality is positively influenced by higher green taxation as long as the environmental tax compliance rate is stronger than 42.3% (for a penalty rate equal to 1.5 and an audit rate of 0.089). A higher audit rate as well as a higher penalty rate and higher corruption costs lead unambiguously to lower pollution, higher growth and larger welfare. Higher pollution taxation has an ambiguous impact on capital accumulation in the model with pollution externalities in production, a negative effect on growth in the model without pollution externalities on production and an ambiguous influence on welfare in both models. These results imply that the best policies which a government may apply in order to ensure higher environmental quality when corrupted officials coordinate with environmental tax evaders is a larger audit rate or a larger penalty rate. Alternatively, the government may choose to fight corruption.

This paper is organized as follows: Section 4.2 presents the model with pollution externalities in production and the accompanying assumptions on the representative household, the output and the government sector; Section 4.3 presents the market equilibrium; Section 4.4 implements a sensitivity analysis of the government policies and corruption costs, while Section 4.5 does the same exercise but in a model without pollution externalities in production; Section 4.6 makes a conclusion.

4.2 The Model with Pollution Externalities in Production

4.2.1 The Production Sector

As in the work by Rebelo (1991), we assume that the production function has constant returns to scale with respect to capital, which in this case can be interpreted as physical and human capital altogether. Furthermore, the production function is negatively influenced by pollution P similar to Smulders and Gradus (1996) (with the elasticity of pollution in the production

function equal to γ). Given these assumptions, the production function has the form,[4]

$$Y = AKP^{-\gamma} \qquad A > 0 \qquad 0 < \gamma < 1 \qquad (4.1)$$

Following Gradus and Smulders (1993)[5], we assume that pollution is triggered by capital K, but reduced by abatement activities undertaken by the government G,

$$P = \frac{K}{G} \qquad \text{if} \qquad P < \overline{P} \qquad (4.2)$$

Moreover, sustainability requires that pollution is lower than the pollution level $\overline{P}$, at which the economy goes extinct due to lower environmental quality. Alternatively, we could assume that pollution is a stock variable. However, as Smulders and Gradus (1996) show, the steady state of an endogenous growth model related to pollution, which can be modeled either as a stock or as a flow variable, remains the same for a constant depreciation rate although an endogenous growth model with pollution as a stock variable leads to some transitional dynamics.

The representative firm works in a perfectly competitive market where the price of the produced good is normalized to 1. It hires capital services from the household and pays interest on it. The firm has to pay pollution income to the government τP, where τ is the pollution tax. The firm transfers $0 < \beta < 1$ of the due taxes to non–corrupt authorities, while $1 - \beta$ is the share of environmental taxation that the firm evades. Following Chen (2003), we assume that the firm pays corruption costs $PH(1 - \beta)^2$ to corrupt government officials, which are increasing in the share of pollution evaded $1 - \beta$, with $H > 0$ being a corruption cost parameter. The corruption costs embody an efficiency loss as they are not an income source for any sector of the economy. The firm takes the risk of being caught not paying environmental taxes by non–corrupt officials with a probability p. In this case it has to pay a penalty rate $\pi > 1$ over the evaded environmental taxation.

As already mentioned, we assume that the relationship between corruption and environmental tax evasion is complementary, which implies that the representative firm cooperates with corrupt officials when evading taxes. Alternatively, we could postulate that the firm goes underground to avoid higher bribery costs. Still, as Dreher and Schneider (2010) claim, small companies are relatively less accessible to corrupt officials than big corporations, moreover, big corporations are simultaneously big (environmental) tax payers. That is why we choose to model the interaction between corruption and the environmental tax evasion rather as cooperative than competitive.

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[4]For simplicity the time index is omitted.

[5]In their work the elasticity of pollution with respect to capital to abatement is normalized to one in the calibration.

Given the above assumptions, the firm, whose expected profit is defined as,

$$\Pi = AKP^{-\gamma} - rK - P(\beta\tau + (1-\beta)\pi p\tau + H(1-\beta)^2) \qquad (4.3)$$

has to choose its optimal capital stock K and the optimal green tax compliance rate β. By setting $\frac{\partial \Pi}{\partial K} = 0$ and $\frac{\partial \Pi}{\partial \beta} = 0$, we obtain that

$$\beta = 1 - \frac{\overline{\tau}(1 - \pi p)}{2h} \qquad (4.4)$$

$$r = AP^{-\gamma} - P(\beta\overline{\tau} + (1-\beta)\pi p\overline{\tau} + h(1-\beta)^2) \qquad (4.5)$$

where $\tau/K = \overline{\tau}$ is the detrended tax rate, $h = H/K$ is the detrended corruption cost parameter. It should be noticed that $1 - \pi p > 0$ so that the expected value of a unit tax evaded is always positive. The pollution flow in the production function is perceived as an externality by the representative firm, which is why pollution in the production function is not considered in the optimization problem. The specification of pollution and corruption costs additionally implies that it is always optimal for the firm to engage in tax evasion paying corruption costs instead of submitting the total amount of due pollution taxes $P\tau$.

Furthermore, we can conclude from (4.4) that the environmental tax compliance β increases in response to a lower tax $\overline{\tau}$, a higher penalty rate π, a higher audit rate p and higher corruption costs represented by an increase in the detrended parameter h. The influence of $\overline{\tau}, p, \pi, h$ on the interest rate and growth will be discussed later.

4.2.2 The Government

The government (here non–corrupt officials) collects taxes from the production sector in order to invest them in abatement activities G. Its budget is balanced every instant, so

$$G = \tau P(\beta + (1-\beta)\pi p) \qquad (4.6)$$

We assume that the execution of tax audits is for free. The government sets the tax τ, (from which the detrended tax $\overline{\tau}$ results), the audit and the penalty rate, p and π. We assume that corruption costs h are exogenous to the decision making of the government.

4.2.3 The Household Sector

An infinitely lived household derives positive utility out of consumption C, while pollution P influences negatively his welfare. Following Smulders and Gradus (1996), we assume that the

preference for environment quality θ is constant over time. The utility function has the form,

$$U(C,P) = \begin{cases} \frac{(CP^{-\theta})^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} & \text{if } \sigma \neq 1, \sigma > 0, 1 > \theta > 0 \\ \ln C - \theta \ln P & \text{if } \sigma = 1, 1 > \theta > 0 \end{cases} \tag{4.7}$$

σ is the intertemporal elasticity of substitution and $U_C > 0$, $U_{CC} < 0$, $U_P < 0$ and $U_{PP} < 0$, $U_{CP} < 0$. The representative household decides on the optimal growth of consumption and savings each instant by optimizing its infinite utility function,

$$\int_0^\infty U(C,P)e^{-\rho t}dt \tag{4.8}$$

subject to the budget constraint with income from renting capital services K at the interest rate r,

$$\dot{K} = rK - C \tag{4.9}$$

given the initial level of capital stock $K_0 = 1$ and assuming that pollution is external to its optimization problem. The resulting Hamiltonian function has the form,

$$H = \frac{(CP^{-\theta})^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}e^{-\rho t} + \lambda(rK - C) \tag{4.10}$$

For simplicity we neglect depreciation. The subjective discount rate ρ is constant over time, λ is the shadow price of capital. By applying the usual optimality conditions, $\frac{\partial H}{\partial C} = 0$ and $-\frac{\partial H}{\partial K} = \dot{\lambda}$, we obtain together with the transversality condition,

$$\frac{\dot{C}}{C} = \sigma(r - \rho) + \theta(1-\sigma)\frac{\dot{P}}{P} \tag{4.11}$$

$$\lim_{t \to \infty} K(t)\lambda(t) = 0 \tag{4.12}$$

Equation (4.11) is the Euler equation, which shows that consumption growth does not depend only on the difference between the interest rate r and the preference rate ρ but also on pollution growth. Equation (4.12) is the transversality condition, which bounds the growth rate.

4.3 Market Equilibrium

Definition 4.1 *A general market equilibrium is a set of allocations $\{\beta, \{C,K\}_{t=0}^\infty\}$ and prices $\{r\}$ such that for the given prices and fiscal policy $\{\{\tau\}_{t=0}^\infty, \pi, p\}$: (i) $\{C,K\}_{t=0}^\infty$ maximizes household welfare (4.8) subject to the budget constraint (4.9), taking pollution $\{P\}$ and K_0 as given (ii) $\{\beta, \{K\}_{t=0}^\infty\}$ are chosen in a manner to maximize the representative firm's profit (4.3) taking pollution $\{P\}$ in the production function as given, (iii) the government budget*

constraint (4.6) *is balanced each instant and (iv)* $\{\{C, K, G, H\}_{t=0}^{\infty}, P, \beta\}$ *obey the aggregate resources constraint,*

$$\dot{K} = Y - C - G - PH(1 - \beta)^2 \tag{4.13}$$

As we assumed earlier, corruption costs are assumed to lead to an efficiency loss, that is why they reduce the value of total output. The model lacks transitional dynamics and its competitive equilibrium on the balanced growth path is characterized by the following conditions:

1. C, K, G, H, Y, τ grow at one and the same rate g. Therefore, $\bar{\tau}$, h and P are constant.

2. The sustainable level of equilibrium pollution P has the form,

$$P = \sqrt{\frac{1}{\bar{\tau}(\beta + (1 - \beta)\pi p)}} < \overline{P} \tag{4.14}$$

3. Equilibrium government spending on abatement is equal to,

$$G = K\sqrt{\bar{\tau}(\beta + (1 - \beta)\pi p)} \tag{4.15}$$

4. The growth rate g is constant and defined as,

$$g = \frac{\dot{C}}{C} = \sigma(AP^{-\gamma} - P(\beta\bar{\tau} + (1 - \beta)\pi p\bar{\tau} + h(1 - \beta)^2) - \rho) \tag{4.16}$$

5. Consumption to capital ratio μ or initial consumption $C_0 = \mu K_0$ is constant,

$$\mu = (1 - \sigma)(AP^{-\gamma} - P(\beta\bar{\tau} + (1 - \beta)\pi p\bar{\tau} + h(1 - \beta)^2)) + \sigma\rho \tag{4.17}$$

6. Welfare $U(C, P)$ converges if $\rho - \frac{(\sigma - 1)}{\sigma}g > 0$ and it is equal to,

$$U(C, P) = \frac{(P^{-\theta}K_0)^{1 - \frac{1}{\sigma}}\mu^{\frac{-1}{\sigma}}}{\left(1 - \frac{1}{\sigma}\right)} \tag{4.18}$$

4.4 Sensitivity Analysis

In this section we discuss the role of the environmental policy, corruption costs and the tax compliance for the endogenous variables $P, g, U(C, P)$. The government invests in abatement activities in order to keep the pollution level sustainable, to correct the market failure of not internalized pollution externalities from the production function and satisfy the preference of the representative household for environmental quality. In this respect, we are interested in whether reduction in pollution tax evasion contributes to higher environment quality, faster economic development and higher welfare; and if so, which policies are effective in attaining lower pollution tax incompliance without harming growth and welfare. The question about the impact of corruption costs on $P, g, U(C, P)$ is addressed as well.

Proposition 4.2 *Higher environmental tax compliance β results in lower pollution, higher growth, as well as larger welfare.*

Proof: See Appendix 4.8.1

A higher environmental tax compliance rate β decreases pollution (all other things being constant). This stems from the fact that β increases the detrended tax base $\bar{\tau}(\beta + (1-\beta)\pi p)$, which is spent immediately on abatement activities. Higher environmental compliance also stimulates growth by decreasing the pollution level, which results in higher capital productivity and lower green taxation costs. Because β is optimally chosen by the firm, corruption costs equalize expected pollution costs, so there are no further influence of β on growth. Welfare improves due to higher β and there is no trade–off between C_0 and g^h because of the assumption of constant returns to capital services in the production function (where pollution is not perceived as a negative externality on production).

Proposition 4.3 *A higher pollution tax $\bar{\tau}$ has a positive effect on environmental quality as long as the compliance rate β is strong enough, $\beta > \frac{1-2\pi p}{2(1-\pi p)}$ and vice versa. A higher audit rate p, a higher penalty rate π and higher corruption costs h lead unambiguously to lower pollution.*

Proof: See Appendix 4.8.1

Proposition 4.3 determines the relationship between policy changes as well as corruption costs and environmental quality. The impact of a larger audit rate and a penalty rate is straightforward. This should be put down to the fact that these policies influence the detrended tax base used for abatement not only directly, but also indirectly via β. Because the direct effect of these policies on the detrended tax base is positive (all other things being constant) and by the indirect effect, larger β is attained, π and p have a favorable impact on environmental quality. The same outcome applies for higher corruption costs h. In contrast to π, p and h, the pollution tax $\bar{\tau}$ reduces the compliance rate β. That's why the influence of a higher tax on pollution is positive only if β is large enough. However, assuming empirically observed values for $p = 0.089$ and $\pi = 1.5$ as in the work of Fullerton and Karayannis (1994), we obtain that the relationship between $\bar{\tau}$ and environmental quality is also positive if the green tax compliance rate is at least higher than 42.3%. This threshold β is far below the observed average share of the formal economy in EU Eastern European states according to Bühn and Schneider (2012), which makes also the green tax eligible for a policy aiming to decrease pollution (at least in countries of EU Eastern Europe).

Proposition 4.4 *Growth is positively influenced by π, p and h and behaves ambiguously with respect to $\bar{\tau}$.*

Proof: See Appendix 4.8.1

The impact of $\bar{\tau}$ on growth is ambiguous. This can be explained as follows: the indirect impact of the green tax on growth is positive only if it leads to lower pollution i.e. if $\beta > \frac{1-2\pi p}{2(1-\pi p)}$ holds. However, the direct influence of τ on g via the taxation costs is negative, which makes the relationship between the pollution tax and capital accumulation unclear. The penalty rate and the audit rate create the same trade–off between their indirect (via P and β) and direct effect on g. However, their positive influence on growth is stronger, and, therefore, higher π and p^c cause higher g. Larger corruption costs increase the environmental tax compliance rate and environmental quality, which boosts growth, but also represent an efficiency loss to the economy because no agent obtains income out of corruption. Nevertheless, the former effect is more pronounced than the latter, which is why higher corruption costs also bring about higher growth.

Proposition 4.5 *Larger π, p, and h result in higher welfare because they promote better environmental quality and higher capital accumulation. The impact of $\bar{\tau}$ on utility is ambiguous.*

See Appendix 4.8.1

According to Proposition 4.5, a trade–off in welfare between initial consumption and consumption growth in case of changes in π, p and h is not observed. This condition implies that in response to a policy change or corruption costs the representative household will experience higher utility only if growth and environmental quality are positively affected. Therefore, welfare will behave favorably with respect to π, p and h. As we already alluded, this result stems from the assumption that capital, which is the sole input factor in production perceived by the firm, is linear in output. Due to the inconclusive relationship between the green tax and environmental quality, as well as the green tax and growth, the impact of $\bar{\tau}$ on welfare is not clear.

4.5 The Model without Pollution Externalities in Production

In this section we change the previous model by postulating a production function without pollution externalities *á la* Gradus and Smulders (1993), equal to $Y = AK$ but preserve the rest of the assumptions. The representative firm operating in a competitive market (the price of the good is set to one) chooses the optimal green tax compliance rate β and the optimal investment in capital K by maximizing its expected profit,

$$\Pi = AK - rK - P(\beta\tau + (1-\beta)\pi p\tau + H(1-\beta)^2) \tag{4.19}$$

with respect to β, i.e. $\frac{\partial \Pi}{\partial \beta} = 0$ and K, i.e. $\frac{\partial \Pi}{\partial K} = 0$, from where we obtain that

$$\beta = 1 - \frac{\overline{\tau}(1 - \pi p)}{2h} \tag{4.20}$$

$$r = A - P(\beta \overline{\tau} + (1 - \beta)\pi p \overline{\tau} + h(1 - \beta)^2) \tag{4.21}$$

The government tax and penalty levies are spent on abatement activities G as before, so

$$G = \tau P(\beta + (1 - \beta)\pi p) \tag{4.22}$$

where pollution is a flow variable equal to,

$$P = \frac{K}{G} < \overline{P} \tag{4.23}$$

The household optimizes its infinite utility function,

$$\int_0^\infty U(C, P)e^{-\rho t}dt \quad \text{with} \quad U(C, P) = \begin{cases} \frac{(CP^{-\theta})^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} & \text{if } \sigma \neq 1, \sigma > 0, 1 > \theta > 0 \\ \ln C - \theta \ln P & \text{if } \sigma = 1, 1 > \theta > 0 \end{cases} \tag{4.24}$$

and $U_C > 0$, $U_{CC} < 0$, $U_P < 0$, $U_{PP} < 0$, $U_{CP} < 0$ subject to the budget constraint,

$$\dot{K} = rK - C \tag{4.25}$$

The Hamiltonian function reads as,

$$H = \frac{(CP^{-\theta})^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}e^{-\rho t} + \lambda(rK - C) \tag{4.26}$$

Depreciation is neglected. By choosing the optimal share of consumption and savings out of income ($\frac{\partial H}{\partial C} = 0$ and $-\frac{\partial H}{\partial K} = \dot{\lambda}$), taking the pollution flow in the utility function as exogenous and $K_0 = 1$, the growth rate of household's consumption and the transversality condition are defined as,

$$\frac{\dot{C}}{C} = \sigma(r - \rho) + \theta(1 - \sigma)\frac{\dot{P}}{P} \tag{4.27}$$

$$\lim_{t \to \infty} K(t)\lambda(t) = 0 \tag{4.28}$$

In equilibrium corruption costs are sunk, which is why they embody a loss to total output in the aggregate resources constraint,

$$\dot{K} = Y - C - G - PH(1 - \beta)^2 \tag{4.29}$$

The only equilibrium variables which change compared to the model with pollution externalities in production are the equilibrium growth rate,

$$g = A - P(\beta \overline{\tau} + (1 - \beta)\pi p \overline{\tau} + h(1 - \beta)^2) \tag{4.30}$$

and the consumption to capital ratio,

$$\mu = (1-\sigma)(A - P(\beta\bar{\tau} + (1-\beta)\pi p\bar{\tau} + h(1-\beta)^2)) + \sigma\rho \tag{4.31}$$

Statements $2, 3, 6$ in the model with pollution externalities in production are valid in the model without pollution externalities in production. As there is a necessity for combating pollution due to the sustainability requirement and the preference for clean environment of the representative household also in this model, we investigate the impact of green tax evasion on pollution, economic development and social utility and the effectiveness of alternative government policies and corruption costs in combating tax evasion, if necessary, in enhancing environment quality and promoting growth and welfare.

Proposition 4.6 *Higher environmental tax compliance β leads to lower pollution, and higher growth. Welfare improves in response to larger environmental tax compliance.*

Proof: See Appendix 4.8.2

Proposition 4.6 attains results similar to the previous model confirming the positive impact of larger environmental tax compliance on economic development and welfare. This can be explained with the positive effect which higher β exerts on environmental quality and in this way also on the amount of (declining) environmental taxes. Welfare depends on initial consumption, but the differential of the welfare function with respect β does not exhibit a trade–off between C_0 and g^h similar to the model with pollution externalities in production. Therefore, the growth effect and the pollution effect of larger β play the decisive role in determining the response of utility with respect to β.

Proposition 4.7 *A higher pollution tax $\bar{\tau}$ has a positive effect on environmental quality as long as the compliance rate β is strong enough, $\beta > \frac{1-2\pi p}{2(1-\pi p)}$ and vice versa. A higher audit rate p, a higher penalty rate π and larger corruption costs h lead unambiguously to lower pollution.*

Proof: See Appendix 4.8.2

Proposition 4.7 determines the impact of government policies and corruption costs on pollution. As before, the tax rate $\bar{\tau}$ has an ambiguous effect on P (due to the negative relationship between $\bar{\tau}$ and β) in contrast to the other government policies and corruption costs. However, for empirical values of p and π we already discussed ($p = 0.089$ and $\pi = 1.5$), $\bar{\tau}$ has a positive effect on environmental quality as long as the green tax compliance rate is at least higher than 42.3%.

Proposition 4.8 *Growth is negatively influenced by $\bar{\tau}$ and positively influenced by π, p and h.*

Proof: See Appendix 4.8.2

Proposition 4.9 *Changes in welfare in response to π, p, h are solely dictated by changes in growth and pollution. The impact of $\bar{\tau}$ on welfare is ambiguous.*

Proof: See Appendix 4.8.2

Propositions 4.8 and 4.9 determine the impact of government policies and corruption costs on capital accumulation and welfare. In contrast to the model with pollution externalities in production, a higher green tax here has an unfavorable impact on growth. The effect of $\bar{\tau}$ on g can be explained with the lower positive direct effect of $\bar{\tau}$ once pollution does not augment output. Although a higher green tax has a positive indirect effect on g in reducing pollution taxation via P and β (at least for $\beta > \frac{1-2\pi p}{2(1-\pi p)}$), its direct negative impact on g via the pollution taxation costs is stronger and capital accumulation falls in response to $\bar{\tau}$. The influence of π and p^c on growth is at first glance contradictory as well: first, these policies stimulate g indirectly through the environmental tax compliance, which reduces taxation related to environmental quality; second, both of them lead simultaneously to a rise in expected (taxation enforcement) costs. Because the first effect is higher than the second one, a larger audit rate or a larger penalty rate leads to higher capital accumulation. Higher corruption costs also have a favorable effect on capital accumulation similar to the model without pollution externalities in production. Also here, the positive effect which h exerts on growth via higher environmental tax compliance and better quality of environment is larger than its negative effect on g due to increased efficiency costs.

For welfare to rise in response to p^c, ρ, h, and $\bar{\tau}$, it is necessary that growth and environmental quality are affected positively. This is the case for larger values of the tax enforcement policies (p and π) as well as corruption costs. The household's utility behaves ambiguously with respect to $\bar{\tau}$ because of the contradictory impact of a larger green tax on environmental policy and growth. The welfare effects with respect to government policies and corruption costs are not dictated by the level of initial consumption (in addition to pollution and growth). This result stems from the assumption of constant returns to scale of capital services in production.

4.6 Conclusion

This work combines the literature of capital accumulation and environmental tax evasion and corruption (see Chen (2003)) considering two types of production functions: with pollution

externalities and without pollution externalities (similar to Smulders and Gradus (1996) and Gradus and Smulders (1993)). Pollution is a flow variable depending positively on capital stock and negatively on abatement undertaken by the government. The representative firm cooperates with corrupt officials in the evasion of environmental taxes. Corruption costs are sunk costs. If caught by non-corrupt authorities, the firm pays a penalty on the evaded amount of environmental taxes. Non-corrupt authorities have several regulatory instruments: the environmental tax, the penalty rate and the audit rate. We assume that corruption costs are exogenous to government decision–making. Growth is generated by capital accumulation of the infinitely lived household net of pollution taxation and corruption costs.

We find that environmental tax compliance decreases pollution and increases growth and welfare. Environmental quality rises in response to higher corruption costs, a higher penalty rate and a higher audit rate, while the impact of the pollution tax is positive only if the propensity to pay green taxes is at least higher than 42.3% (for a penalty rate equal to 1.5 and an audit rate of 0.089). The influence of corruption costs, the penalty and the audit rate on growth and welfare in both models is positive. The impact of the pollution tax on growth, on the other hand, is ambiguous in the model with pollution externalities in production and negative in the model without pollution externalities in production. A larger pollution tax stands in an ambiguous relationship with welfare in both models. Therefore, the best policy response of a government maintaining environmental quality when green tax evasion and corruption are complements is (i) enforcement of a larger audit rate, a larger penalty rate or (ii) reducing corruption.

We already highlighted that the welfare effects in response to stricter environment policies or corruption costs are dictated solely by growth, with no divergent impact of initial consumption on utility due to the assumption of AK production function with constants returns to capital (with or without pollution externalities). It is interesting then for future research to test the robustness of our analysis with respect to a production function which is subject to increasing returns including labor similar to Romer (1986) (with or without pollution externalities).

4.7 References

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4.8 Appendix

4.8.1 The Model with Pollution Externalities in Production

Proof. The impact of β on the endogenous variables P, g, μ, $U(.)$ is,

$$\frac{\partial P}{\partial \beta} = -\frac{P}{2(\beta + (1-\beta)\pi p)}(1 - \pi p) < 0$$

$$\frac{\partial g}{\partial \beta} = \gamma \frac{\sigma A P^{-\gamma}}{2(\beta + (1-\beta)\pi p)}(1 - \pi p) + \frac{\sigma P}{2(\beta + (1-\beta)\pi p)}(1 - \pi p)B > 0$$

$$\frac{\partial U(C,P)}{\partial \beta} = -\frac{\theta}{P}(P^{-\theta}K_0)^{1-\frac{1}{\sigma}}\mu^{\frac{-1}{\sigma}}\underbrace{\frac{\partial P}{\partial \beta}}_{<0} + \frac{1}{\mu}\frac{(P^{-\theta}K_0)^{1-\frac{1}{\sigma}}}{1-\sigma}\mu^{\frac{-1}{\sigma}}\underbrace{\frac{\partial \mu}{\partial \beta}}_{\gtrless 0}$$

The latter can be rewritten as,

$$\frac{\partial U(C,P)}{\partial \beta} = -\frac{\theta}{P}(P^{-\theta}K_0)^{1-\frac{1}{\sigma}}\mu^{\frac{-1}{\sigma}}\underbrace{\frac{\partial P}{\partial \beta}}_{<0} + \frac{1}{\mu}\frac{(P^{-\theta}K_0)^{1-\frac{1}{\sigma}}}{\sigma}\mu^{\frac{-1}{\sigma}}\underbrace{\frac{\partial g}{\partial \beta}}_{>0} > 0$$

where $B = \beta\overline{\tau} + (1-\beta)\pi p\overline{\tau} + h(1-\beta)^2 = \overline{\tau} - \frac{\overline{\tau}^2(1-\pi p)^2}{4h}$ ∎

Proof. The influence of $\overline{\tau}$ on the endogenous variables P, g, $U(.)$ is,

$$\frac{\partial P}{\partial \overline{\tau}} = -\frac{P}{2\overline{\tau}(\beta + (1-\beta)\pi p)}\left[1 - \frac{\overline{\tau}(1-\pi p)^2}{h}\right] \Longrightarrow$$

$$\frac{\partial P}{\partial \overline{\tau}} > 0 \quad \text{if} \quad \beta < \frac{1-2\pi p}{2(1-\pi p)}; \qquad \frac{\partial P}{\partial \overline{\tau}} < 0 \quad \text{if} \quad \beta > \frac{1-2\pi p}{2(1-\pi p)}$$

$$\frac{\partial g}{\partial \overline{\tau}} = \sigma\frac{P}{2(\beta\overline{\tau} + (1-\beta)\overline{\tau}\pi p)}\underbrace{\frac{\partial A'}{\partial \overline{\tau}}}_{\gtrless 0}\left[A\gamma P^{-\gamma-1} + B\right] - \sigma P\underbrace{\frac{\partial B}{\partial \overline{\tau}}}_{>0} \Longrightarrow$$

$$\frac{\partial g}{\partial \overline{\tau}} \gtrless 0 \quad \text{if} \quad \frac{\partial A'}{\partial \overline{\tau}} > 0 \quad \text{and} \quad A\gamma P^{-\gamma-1} + B \gtrless \frac{2B\varepsilon_{\overline{\tau}}^B}{\varepsilon_{\overline{\tau}}^{A'}}$$

$$\frac{\partial g}{\partial \overline{\tau}} < 0 \quad \text{if} \quad \frac{\partial A'}{\partial \overline{\tau}} < 0$$

$$\frac{\partial U(C,P)}{\partial \overline{\tau}} = -\frac{\theta}{P}(P^{-\theta}K_0)^{1-\frac{1}{\sigma}}\mu^{\frac{-1}{\sigma}}\underbrace{\frac{\partial P}{\partial \overline{\tau}}}_{\lessgtr 0} + \frac{1}{\mu}\frac{(P^{-\theta}K_0)^{1-\frac{1}{\sigma}}}{\sigma}\mu^{\frac{-1}{\sigma}}\underbrace{\frac{\partial g}{\partial \overline{\tau}}}_{\gtrless 0}$$

where $A' = \overline{\tau}\beta + \overline{\tau}(1-\beta)\pi p = \overline{\tau} - \frac{\overline{\tau}^2(1-\pi p)^2}{2h}$, $\varepsilon_x^y = \frac{\partial y}{\partial x}\frac{x}{y}$ and $\frac{\varepsilon_{\overline{\tau}}^B}{\varepsilon_{\overline{\tau}}^{A'}} = \frac{(2h-\overline{\tau}(1-\pi p)^2)^2}{(4h-\overline{\tau}(1-\pi p)^2)(h-\overline{\tau}(1-\pi p)^2)} > 1$ for $\varepsilon_{\overline{\tau}}^{A'} = \frac{2(h-\overline{\tau}(1-\pi p)^2)}{2h-\overline{\tau}(1-\pi p)^2} > 0$ or $h > \overline{\tau}(1-\pi p)^2$ which can be transformed into $\beta > \frac{1-2\pi p}{2(1-\pi p)}$; $\varepsilon_{\overline{\tau}}^{A'} = \frac{2(h-\overline{\tau}(1-\pi p)^2)}{2h-\overline{\tau}(1-\pi p)^2} < 0$ for $h < \overline{\tau}(1-\pi p)^2$ which can be transformed into $\beta < \frac{1-2\pi p}{2(1-\pi p)}$; $\varepsilon_{\overline{\tau}}^B = \frac{2(2h-\overline{\tau}(1-\pi p)^2)}{(4h-\overline{\tau}(1-\pi p)^2)} > 0$ ∎

Proof. The influence of π on the endogenous variables P, g, $U(.)$ is,

$$\frac{\partial P}{\partial \pi} = -\frac{P}{2\bar{\tau}(\beta + (1-\beta)\pi p)} \frac{\bar{\tau}^2(1-\pi p)p}{h} < 0$$

$$\frac{\partial g}{\partial \pi} = \sigma \frac{P}{2(\beta\bar{\tau} + (1-\beta)\bar{\tau}\pi p)} \underbrace{\frac{\partial A'}{\partial \pi}}_{>0} \left[A\gamma P^{-\gamma-1} + B\right] - \sigma P \underbrace{\frac{\partial B}{\partial \pi}}_{>0} \implies$$

$$\frac{\partial g}{\partial \pi} \lessgtr 0 \quad \text{if} \quad A\gamma P^{-\gamma-1} + B \lessgtr \frac{2B\varepsilon_\pi^B}{\varepsilon_\pi^{A'}} \quad \text{with} \quad \frac{\varepsilon_\pi^B}{\varepsilon_\pi^{A'}} = \frac{2h - \bar{\tau}(1-\pi p)^2}{4h - \bar{\tau}(1-\pi p)^2}$$

$$\frac{\partial g}{\partial \pi} < 0 \quad \text{if} \quad P < \left[\frac{2B}{A\gamma}\right]^{\frac{-1}{1+\gamma}} \left[\frac{2h - \bar{\tau}(1-\pi p)^2}{4h - \bar{\tau}(1-\pi p)^2} - \frac{1}{2}\right]^{\frac{-1}{1+\gamma}}$$

where

$$\frac{2h - \bar{\tau}(1-\pi p)^2}{4h - \bar{\tau}(1-\pi p)^2} < \frac{1}{2}$$

Because P cannot be negative, it must hold that $\frac{\partial g}{\partial \pi} > 0$.

$$\frac{\partial U(C,P)}{\partial \pi} = -\frac{\theta}{P}(P^{-\theta}K_0)^{1-\frac{1}{\sigma}} \mu^{\frac{-1}{\sigma}} \underbrace{\frac{\partial P}{\partial \pi}}_{<0} + \frac{1}{\mu} \frac{(P^{-\theta}K_0)^{1-\frac{1}{\sigma}}}{\sigma} \mu^{\frac{-1}{\sigma}} \underbrace{\frac{\partial g}{\partial \pi}}_{>0} > 0$$

Notice that $\varepsilon_\pi^B = \frac{2\bar{\tau}p\pi(1-\pi p)}{(4h - \bar{\tau}(1-\pi p)^2)} > 0$, $\varepsilon_\pi^{A'} = \frac{2\bar{\tau}p\pi(1-\pi p)}{(2h - \bar{\tau}(1-\pi p)^2)} > 0$. The impact of p and h on g, and welfare can be derived analogically. ∎

4.8.2 The Model without Pollution Externalities in Production

Proof. The impact of β on the endogenous variables P, g, $U(.)$ is as follows,

$$\frac{\partial P}{\partial \beta} = -\frac{P}{2(\beta + (1-\beta)\pi p)}(1 - \pi p) < 0$$

$$\frac{\partial g}{\partial \beta} = \frac{\sigma P}{2(\beta + (1-\beta)\pi p)}(1 - \pi p)B > 0$$

$$\frac{\partial U(C,P)}{\partial \beta} = -\frac{\theta}{P}(P^{-\theta}K_0)^{1-\frac{1}{\sigma}} \mu^{\frac{-1}{\sigma}} \underbrace{\frac{\partial P}{\partial \beta}}_{<0} + \frac{1}{\mu} \frac{(P^{-\theta}K_0)^{1-\frac{1}{\sigma}}}{\sigma} \mu^{\frac{-1}{\sigma}} \underbrace{\frac{\partial g}{\partial \beta}}_{>0} > 0$$

where $B = \beta\bar{\tau} + (1-\beta)\pi p\bar{\tau} + h(1-\beta)^2 = \bar{\tau} - \frac{\bar{\tau}^2(1-\pi p)^2}{4h}$ ∎

Proof. The impact of $\bar{\tau}$ on the endogenous variables P, g, $U(.)$ is,

$$\frac{\partial P}{\partial \bar{\tau}} = -\frac{P}{2\bar{\tau}(\beta + (1-\beta)\pi p)}\left[1 - \frac{\bar{\tau}(1-\pi p)^2}{h}\right] \implies$$

$$\frac{\partial P}{\partial \bar{\tau}} > 0 \quad \text{if} \quad \beta < \frac{1 - 2\pi p}{2(1 - \pi p)}; \qquad \frac{\partial P}{\partial \bar{\tau}} < 0 \quad \text{if} \quad \beta > \frac{1 - 2\pi p}{2(1 - \pi p)}$$

$$\frac{\partial g}{\partial \bar{\tau}} = \sigma \frac{P}{2(\beta\bar{\tau} + (1-\beta)\bar{\tau}\pi p)} \underbrace{\frac{\partial A'}{\partial \bar{\tau}}}_{\gtrless 0} B - \sigma P \underbrace{\frac{\partial B}{\partial \bar{\tau}}}_{>0} \implies$$

$$\frac{\partial g}{\partial \bar{\tau}} > 0 \quad \text{if} \quad \frac{\partial A'}{\partial \bar{\tau}} > 0 \quad \text{and} \quad \frac{1}{2} > \frac{\varepsilon_{\bar{\tau}}^B}{\varepsilon_{\bar{\tau}}^{A'}}$$

However,

$$\frac{\varepsilon_{\overline{\tau}}^{B}}{\varepsilon_{\overline{\tau}}^{A'}} = \frac{(2h - \overline{\tau}(1 - \pi p)^2)^2}{(4h - \overline{\tau}(1 - \pi p)^2)(h - \overline{\tau}(1 - \pi p)^2)} > 1$$

Therefore, it must hold that $\frac{\partial g}{\partial \overline{\tau}} < 0$ for $\varepsilon_{\overline{\tau}}^{A'} > 0$, which is equivalent to $h > \overline{\tau}(1 - \pi p)^2$ or $\beta > \frac{1 - 2\pi p}{2(1 - \pi p)}$. If $\frac{\partial A'}{\partial \overline{\tau}} < 0$ or $\varepsilon_{\overline{\tau}}^{A'} < 0$

$$\frac{\partial g}{\partial \overline{\tau}} = \sigma \frac{P}{2(\beta \overline{\tau} + (1 - \beta)\overline{\tau}\pi p)} \underbrace{\frac{\partial A'}{\partial \overline{\tau}}}_{<0} B - \sigma P \underbrace{\frac{\partial B}{\partial \overline{\tau}}}_{>0} < 0$$

Therefore, the impact of $\overline{\tau}$ on g is always negative.

$$\frac{\partial U(C, P)}{\partial \overline{\tau}} = -\frac{\theta}{P}(P^{-\theta}K_0)^{1 - \frac{1}{\sigma}}\mu^{\frac{-1}{\sigma}}\underbrace{\frac{\partial P}{\partial \overline{\tau}}}_{\gtrless 0} + \frac{1}{\mu}\frac{(P^{-\theta}K_0)^{1 - \frac{1}{\sigma}}}{\sigma}\mu^{\frac{-1}{\sigma}}\underbrace{\frac{\partial g}{\partial \overline{\tau}}}_{<0}$$

where $A' = \overline{\tau}\beta + \overline{\tau}(1 - \beta)\pi p = \overline{\tau} - \frac{\overline{\tau}^2(1 - \pi p)^2}{2h}$, and $\varepsilon_x^y = \frac{\partial y}{\partial x}\frac{x}{y}$, $\varepsilon_{\overline{\tau}}^{A'} = \frac{2(h - \overline{\tau}(1 - \pi p)^2)}{2h - \overline{\tau}(1 - \pi p)^2} > 0$ for $h > \overline{\tau}(1 - \pi p)^2$ or $\beta > \frac{1 - 2\pi p}{2(1 - \pi p)}$; $\varepsilon_{\overline{\tau}}^{A'} = \frac{2(h - \overline{\tau}(1 - \pi p)^2)}{2h - \overline{\tau}(1 - \pi p)^2} < 0$ for $h < \overline{\tau}(1 - \pi p)^2$ or $\beta < \frac{1 - 2\pi p}{2(1 - \pi p)}$; $\varepsilon_{\overline{\tau}}^{B} = \frac{2(2h - \overline{\tau}(1 - \pi p)^2)}{(4h - \overline{\tau}(1 - \pi p)^2)} > 0$ ∎

Proof. The influence of π on the endogenous variables P, g, $U(.)$ is,

$$\frac{\partial P}{\partial \pi} = -\frac{P}{2\overline{\tau}(\beta + (1 - \beta)\pi p)}\frac{\overline{\tau}^2(1 - \pi p)p}{h} < 0$$

$$\frac{\partial g}{\partial \pi} = \sigma \frac{P}{2(\beta \overline{\tau} + (1 - \beta)\overline{\tau}\pi p)}\underbrace{\frac{\partial A'}{\partial \pi}}_{>0}B - \sigma P \underbrace{\frac{\partial B}{\partial \pi}}_{>0} \Longrightarrow$$

$$\frac{\partial g}{\partial \pi} > 0 \qquad \text{if} \qquad \frac{1}{2} > \frac{\varepsilon_\pi^B}{\varepsilon_\pi^{A'}}$$

which is always true because

$$\frac{\varepsilon_\pi^B}{\varepsilon_\pi^{A'}} = \frac{2h - \overline{\tau}(1 - \pi p)^2}{4h - \overline{\tau}(1 - \pi p)^2} < \frac{1}{2}$$

$$\frac{\partial U(C, P)}{\partial \pi} = -\frac{\theta}{P}(P^{-\theta}K_0)^{1 - \frac{1}{\sigma}}\mu^{\frac{-1}{\sigma}}\underbrace{\frac{\partial P}{\partial \pi}}_{<0} + \frac{1}{\mu}\frac{(P^{-\theta}K_0)^{1 - \frac{1}{\sigma}}}{\sigma}\mu^{\frac{-1}{\sigma}}\underbrace{\frac{\partial g}{\partial \pi}}_{>0} > 0$$

Moreover, $\varepsilon_\pi^B = \frac{2\overline{\tau}p\pi(1 - \pi p)}{(4h - \overline{\tau}(1 - \pi p)^2)} > 0$, $\varepsilon_\pi^{A'} = \frac{2\overline{\tau}p\pi(1 - \pi p)}{(2h - \overline{\tau}(1 - \pi p)^2)} > 0$. The influence of p and h on the endogenous variables P, g, $U(.)$ can be derived analogically. ∎

5 Extended Summary

In the first part of this dissertation we discuss the interaction of brain drain, growth and welfare under the assumption of risky occupational choice (*Brain Drain, Occupational Choice under Risk, and Endogenous Growth*) or the existence of credit constraints (*Brain Drain, Borrowing Constraints, and Endogenous Growth in an Economy with Perfect Physical Capital Mobility*). In both cases we test the robustness of the *brain gain* theory, which claims that brain drain may result in higher human capital accumulation because the prospect of migration may *ex–ante* create an educational incentive.

The value of our paper *Brain Drain, Occupational Choice under Risk, and Endogenous Growth* is the introduction of the endogenous risky occupational choice of entrepreneurs and skilled workers vs. sure low–skilled employment within a probabilistic brain drain model. The traditional brain drain literature is based on the assumption that agents in the domestic country are employed as (educated) workers and are not subject to risk–taking. In this way the conventional brain drain theory downsizes the importance of risk in the occupational choice of skilled workers, which has an impact on the decision to invest in education (being a prerequisite for skilled migration). Moreover, (risk–taking) entrepreneurship has also been assumed away as a production augmenting factor although its impact on economic development is verified as relevant. In this respect, brain drain may influence entrepreneurship via changes in the occupational choice of agents and changes in the entrepreneurial production due to alteration in the supply of skilled labor.

Our first paper is embedded within a two–period overlapping generations endogenous growth model based on the work of Kanbur (1979) and Clemens (2008) for occupational choice under risk and Beine *et al.* (2001) for the construction of probabilistic skilled migration. The economy consists of two sectors operating in perfect competition: a traditional sector (employing low–skilled) and a modern skilled sector (with entrepreneurs hiring skilled workers). Agents may invest in education when young. While skilled are subject to risk in earnings due to *ex–ante* unknown abilities in human capital (for educated workers) or a technology shock (for entrepreneurs), low–skilled obtain safe income. Skilled workers are randomly singled out

to work abroad for a higher wage (depending on their *ex–ante* unknown labor productivities), which defines the brain drain phenomenon. Because agents choose an occupation and make an education decision under the veil of ignorance, their (expected) utility is equal in equilibrium, which defines the distribution of individuals across occupations. The accumulation of average human capital hinges on the human capital of skilled remaining at home and the human capital of low–skilled, i.e. on the level of risk in occupational choice and the brain drain rate.

The calibration with economic targets of Eastern European countries shows that a higher *brain drain* rate has a positive effect on growth because it attracts more agents into the employment as skilled workers despite a fall in entrepreneurship. In this case average welfare in all periods is beyond the benchmark because brain gain takes place. Still, entrepreneurs are temporally worse off due to a fall in skilled labor force, while skilled workers have to recover later from lower expected utility due to higher competition induced by a larger brain drain rate. The *wedge* in skilled wages at home and abroad leads to a decrease in entrepreneurship, but always stimulates growth and results in higher welfare because it increases the total share of skilled agents. Nevertheless, skilled workers have to suffer transitionally lower welfare due to higher competition in response to a rise in the gap in skilled earnings at home and abroad. Brain drain has a stronger positive effect on growth and social utility in the short and the long term compared to the gap in skilled earnings at home and abroad. Growth decreases in response to higher levels of *occupational risk* because it leads to a fall in the share of skilled. The (negative) impact of larger risk in skilled wages on growth and, therefore, long–term welfare is much more stronger that the effect of the technological shock. However, larger risk in entrepreneurs' profits leads to stronger changes in the income of agents immediately after its introduction and has a more pronounced (negative) effect on short–term welfare compared to the risk in skilled workers' earnings.

Our second work *Brain Drain, Borrowing Constraints, and Endogenous Growth in an Economy with Perfect Physical Capital Mobility* concentrates on the impact of probabilistic brain drain on growth and welfare for different levels of credit market liberalization. We challenge the traditional *brain gain* theory once borrowing constraints are relaxed. Another merit of this paper is that it draws conclusions about the behavior of the aggregate savings rate with respect to brain drain, which is innovative for the brain drain literature.

Our second paper builds a three–period overlapping generations model with human capital accumulation subject to binding borrowing constraints in a setting of perfect physical capital mobility (*à la* De Gregorio (1996)) and probabilistic brain drain (*à la* Beine *et al.* (2001)). Young agents invest optimal time in education and borrow but are subject to a credit con-

straint. Middle–aged work as high–skilled and save repaying their debt. Old consume savings made when middle–aged. Some educated individuals may migrate in the second period of their life and earn a higher wage abroad. Agents supply inelastically their labor and capital is borrowed as an input by the firm, which operates in a setting of perfect competition. Human capital accumulation, which is the triggering growth factor, depends on the optimal investment in education determined by skilled migration and the stringency of borrowing constraints. According to the calibration with economic targets of Eastern European countries, once credit exceeds the threshold 74% of current income, a higher migration probability reduces the optimal educational time due to the availability of economic resources. Welfare is beyond the benchmark in the short run (as the expected utility of young improves in response to a rising probability to gain more income abroad) although middle–aged and old–aged suffer a loss in utility due to declining human capital accumulation. In this case, social utility, however, falls in the long run. A higher *wedge* in skilled wages at home and abroad leads to higher human capital accumulation, and higher social utility compared to the benchmark case. More relaxed *borrowing constraints* enhance growth and (domestic) welfare monotonically in all periods although a generation of middle–aged, and later old agents have to experience a temporary welfare loss due to increased credit costs. The aggregate savings rate declines in response to a higher brain drain probability independent of the tightness of borrowing due to the lower share of native agents, who accumulate physical capital. The aggregate savings rate increases with the gap of skilled earnings at home and abroad due to stronger growth. Further credit market liberalization leads to a fall in the aggregate savings rate because credit distribution implies dissaving of credit takers.

In the second part of the dissertation, we are interested to find what is the impact of the shadow economy on growth and welfare in the presence of risk in the formal sector when the government can invest in infrastructure and public education (*The Shadow Economy, and Risky Human Capital Accumulation in an Environment of Productive Government Spending, and Public Education*), or the influence of pollution tax evasion on environment quality, growth and welfare when corruption exists (*Pollution, Environmental Tax Evasion, and Corruption in an Endogenous Growth Model*). We investigate, furthermore, alternative approaches, which (if necessary) reduce the share of the informal sector or green tax evasion.

In *The Shadow Economy, and Risky Human Capital Accumulation in an Environment of Productive Government Spending, and Public Education*, we follow Glomm and Ravikumar (1992) for developing risky human capital accumulation with public education, and Loayza (1996) for the construction of the formal and informal sector and the congestion mechanism of public services by private services in production. The value of this paper is that it considers the presence

of risk in the occupational choice of growth–driving skilled agents and suggest that uncertainty in skilled workers' earnings (in the formal sector) could be one of the reason for the existence of the shadow economy. In this way we are able to compare the impact of a declining risk measure and the usually cited policies (penalty, audit, tax rate), which are claimed to influence the share of the unofficial sector. Moreover, we allow the government to be able to invest in public education to enrich the number of policies, of which the state avails to act against the informal sector (if necessary).

The model is a two–period overlapping generations model with human capital accumulation. When young, agents decide on education and on employment in one of the production sectors (formal or informal, both operating in a perfectly competitive market). High–skilled work only in the formal sector when middle–aged. They obtain a skill and risk premium over the safe low–skilled wage in the official sector because of their education and due to *ex–ante* unknown abilities to accumulate human capital. Low–skilled may decide to switch to the informal sector in the second period but are subject to the probability of paying a penalty if caught. In equilibrium the expected utilities of skilled and low–skilled in the official sector are equal, which defines the distribution of agents across occupations in the formal sector. The decision of low–skilled on employment in the shadow economy is based on their relatively optimistic perception of the audit rate. Growth in this model depends positively on the government expenditure on public education, which hinges on the occupational choice of economic agents choosing to be taxpayers or tax evaders.

We show that the share of the unofficial economy is detrimental to growth and any policy to reduce it is justified. According to the calibration with Eastern European countries, this can be attained most effectively by a lower *tax rate* with a higher *share of government expenditure on education* (to counteract the negative impact of lower taxation on growth), a higher *penalty rate* or a higher *audit rate* (ordered by the magnitude of their impact on the share of the shadow sector). These policies are connected with a short–term welfare loss. An exception to this rule is a lower *risk measure*. It attains the highest rise in utility in the short run and does not create a trade–off between short–run and long–run social utility although it induces the smallest decline in the share of the unofficial sector compared to the rest of the government policies. Therefore, a government which is interested to reduce the share of the informal economy, without hindering economic development or accepting a trade–off in individual and aggregate welfare, should find a mechanism to insure high–skilled agents in the formal sector. If a government, on the other hand, aims at attaining the lowest possible share of the shadow economy, it should decrease the *tax rate* and counterbalance this effect with a rise in the *share of educational expenditure*. This

policy mix, however, leads to a short–run welfare loss.

Our aim in the paper *Pollution, Environmental Tax Evasion, and Corruption in an Endogenous Growth Model* is to determine a relationship between environmental tax evasion, growth and welfare in the presence of corruption (based on the model of Chen (2003)) and explore the effectiveness of government policies and corruption reduction in combating green tax evasion (if necessary), when production does not exhibit or does exhibit pollution externalities (*à la* Gradus and Smulders (1993) as well as Smulders and Gradus (1996)). This paper is innovative in connecting pollution tax evasion and corruption to environmental quality in a growth and welfare context.

The representative firm operating in a perfectly competitive market cooperates with corrupt officials in the evasion of environmental taxes by incurring additionally corruption costs. Corruption costs are sunk. The firm stands a possibility to be caught evading pollution taxes by non–corrupt authorities and in this case it pays a penalty on the evaded amount of environmental taxes. Non-corrupt government representatives are entitled to decide on the level of the environmental tax, the penalty rate and the audit rate. Pollution is modeled as a flow variable depending positively on abatement undertaken by government authorities and negatively on economic activity represented by the level of capital stock. Growth in the model is triggered by capital accumulation of the infinitely lived household net of pollution taxation and corruption costs.

We show that higher environmental tax compliance leads to lower pollution, higher growth, and higher welfare in both models. Pollution falls unambiguously in response to higher *corruption costs*, a higher *penalty rate* and a higher *audit rate*, while the *green tax* is favorable to environmental quality if the propensity to pay green taxes is at least higher than 42.3% (for a penalty rate equal to 1.5 and an audit rate of 0.089). The influence of larger *corruption costs*, a larger *penalty* or a larger *audit rate* on growth and welfare in both models is positive. The impact of a higher *pollution tax* on growth in the model with pollution externalities in production is ambiguous, while in the model without pollution externalities in production it is negative. The impact of a higher *pollution tax on* welfare is inconclusive in both models. These results imply that a government which would like to enhance environmental quality, when environmental tax evasion with corruption takes place, has to strengthen tax enforcement policies or curb corruption rather than apply green taxation.