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Labor Market Segmentation and Efficient Bargaining in a Macroeconomic Model

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Abstract

This paper studies the implications of a segmented labor market with efficient wage–employment bargaining on the internal labor market and a competitive external labor market on the temporary equilibrium of a closed monetary macroeconomy of the AS–AD type with government activity, fiat money, and expectations. Workers have identical preferences, those on the internal labor market are represented by a labor union. There is no mobility between the labor markets.

Union power measured by the share of the production surplus allotted to the union and union density measured by the fraction of workers who are union members impact the functional income distribution, but neither affect the individual employment levels nor the aggregate employment level and the aggregate supply function. The wage on the internal labor market is above the wage on the external labor market if and only if the profit share of total revenue is smaller than under a fully competitive labor market.

Unique temporary equilibria exist for all combinations of union power and union density. The paper provides a complete comparative-statics analysis showing in particular a negative price effect of union power and a positive price effect of union density. In general, the effects of union power and union density on any equilibrium value are usually of opposite signs. Single-labor-market models with a fully competitive or a fully unionized labor market are special or limiting cases of the segmented-labor-market model.

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Contents

1 Introduction

Centralized bargaining between a producer (or a producers' association) and a labor union is a common feature of (Western) economies and a typically observed procedure to determine wage levels and/or working conditions including working hours. However, only a fraction of workers are actually organized in unions; on average in OECD countries, one out of six workers are a union member, with country-specific values ranging from above 92% in Iceland to less than 5% in Estonia (see OECD 2017, Chapter 4, also for data on other countries and a time series).[1] Therefore, centralized bargaining usually coexists with other forms of labor market interaction, i.e. the labor market is *segmented*. As Taubman & Wachter (1986) already point out, the segmented labor market (SLM) approach addresses numerous aspects of allocative issues and institutional design, mainly departing from the observation of a dualism between a firm-specific *internal labor market* with high-wage "good" jobs and an *external labor market* with low-wage "bad" jobs. In their seminal work, Doeringer & Piore (1971) were the first to describe the dichotomy of the labor market. The terminology primary and secondary sector, which is also used by Doeringer & Piore (1971) and many others, is intentionally avoided here to allow for a different meaning of the sectors of the economy. Within this literature, McDonald & Solow (1985) present the first formal model of the labor market consisting of a unionized and a nonunionized market segment. They extend McDonald & Solow (1981) who model a fully unionized labor market, see also the expositions in the textbooks by Booth (1995) and Landmann & Jerger (1999). Both papers, however, are partial-equilibrium models of the labor market(s), thus not allowing for the analysis of cross-market effects such as the interplay of different earning schemes on consumption and on savings.

Other contributions published at the same time as McDonald & Solow (1985), which also employ segmented labor markets in a partial-equilibrium framework tend either to use an insider–outsider approach which gives those already employed market power or to use efficiency wages that provide a rationale for why a producer voluntarily pays "more than necessary", i.e. a wage above the market-clearing level at which there is excess labor supply. Lindbeck & Snower (1986) argue nonformally that wage differentials can be attributed to costly labor turnover and thus in order to avoid these costs incumbent workers (the insiders) are employed at a wage which is above the market-clearing one. It is excess labor supply (from outsiders) that results in unemployment. Bulow & Summers (1986) combine the efficiency-wage (or shirking) model and SLM model, explaining involuntary unemployment by workers queuing for primary-sector jobs. Gottfries & McCormick (1995) obtain a similar result in a model in which primary-sector employment requires firm-specific training. Since firms are imperfectly informed about individual productivity before the training is completed – they receive only a signal –, a secondary-sector job is taken as a bad sign. Therefore, firms never hire workers from these jobs with the result that some workers rather choose voluntary unemployment in the primary sector than employment in the secondary sector. A further aspect dealt with in the literature is the effects of a minimum wage on wage determination in a growth model with a segmented labor market (see for instance Flaschel & Greiner 2011).

The main objective of this paper is to explore the cross-market effects within a closed monetary

[1] Union coverage, i.e. the share of workers who are employed under a centralized contract, is usually significantly higher than union density; approximately twice as high on average (OECD 2017). Cahuc & Zylberberg (2004, Chapter 7) explain this by legal and institutional issues such as the possibility for nonunion members to free ride on the bargaining agreement in some countries or the legal obligation to engage in collective bargaining in most French firms.

macroeconomy with a segmented labor market. Choosing the standard AS–AD model which is presented in great detail in Böhm (2017) as a general framework, this paper extends the model with a fully unionized labor market as presented in Böhm (2017) and in Böhm & Claas (2012) to one with a fully unionized internal labor market and a competitive external labor market, creating heterogeneity within labor supply through union membership. In the short run, i.e. within a given period, union membership is fixed so that neither union members may leave the union nor nonunionized workers join.[2] The setup with one aggregate firm is maintained, i.e. this single firm is active on both labor market segments. The drivers of this model are union density (the share of unionized workers to all workers), which measures the relative size of the labor markets, and the bargaining power of the union on the internal labor market. This paper will discover the channel(s) through which union density and union power operate; these could be the aggregate supply function and the functional income distribution which feeds into the aggregate demand function.

While the question concerning supply-side and demand-side effects is only relevant for the general equilibrium, the following questions will be addressed both on the partial-equilibrium and on the general-equilibrium level: (1) whether the firm receives higher profits than under a fully competitive labor market due to additional choice, (2) whether a unionized worker receives a higher wage and/or works less than a nonunionized worker due to union power exercised on the internal labor market, i.e. *before* the firm's activity on the external labor market, and (3) whether the external labor market can dry out under certain conditions.

The rest of this paper is organized as follows: Section 2 sets up the model with a fully unionized internal labor market and a competitive external labor market and obtains the partial equilibria on both labor markets. In Section 3, the model is closed, the full general equilibrium is derived, and its comparative-statics properties are analyzed. Section 4 compares the temporary equilibrium under a segmented labor market to the ones with a fully competitive labor market and with a fully unionized labor market while Section 5 concludes.

2 The Economy with a Segmented Labor Market

Consider a monetary macroeconomy of the AS–AD type (see Böhm 2017, for a presentation of the AS–AD model in its standard form and several variations) in discrete time with overlapping generations of heterogeneous consumers, with one (aggregate) firm, and with a government/central bank. Economic activity takes place in three sectors – public, consumption, and production – and on four markets – commodity, internal labor and external labor, and money. Consumers are either shareholders or workers; shareholders and workers differ in their consumption–savings behavior. Workers have identical preferences, but only some are members of a labor union which creates additional heterogeneity within labor supply. While union members supply labor exclusively to the internal labor market which is governed by an efficient bargaining procedure between the union and the firm, all other workers supply their labor to the competitive external labor market only.[3]

[2] This is discrimination in the sense of Cahuc & Zylberberg (2004, Chapter 5).

[3] In line with Katz (1988) who concludes that workers move across markets "only gradually" and Dickens & Lang (1988) who "cast serious doubts on workers' ability to choose their sector of employment", this paper deals with the implications of an exogenously given union membership structure on the temporary equilibria of the economy; workers do not join or leave the union *within a given period*. The dynamics of union membership (i.e. of coalition formation in game-theoretic wording) are to be analyzed as part of the

The interaction between the agents in the economy is described in detail in the next section.

2.1 The Sectors of the Economy and the Union

The production sector consists of one[4] profit-maximizing firm which produces a single commodity using a sufficiently smooth, Inada-type production function

$$F : \mathbb{R}_+ \to \mathbb{R}_+, \qquad z \mapsto F(z)$$

with labor z as the only input. The firm receives labor $z_{\text{ilm}} \geq 0$ from a unionized internal labor market and demands labor $z_{\text{elm}} \geq 0$ on a competitive external labor market; labor from the two sources are perfect substitutes. Wage payments are the only costs incurring. Let $p > 0$ denote the (nominal) market price, and let $w_{\text{ilm}}, w_{\text{elm}} > 0$ denote the different wage rates. Then profits are given by

$$\Pi_{\text{slm}}(p, w_{\text{ilm}}, w_{\text{elm}}, z_{\text{ilm}}, z_{\text{elm}}) := pF(z_{\text{ilm}} + z_{\text{elm}}) - w_{\text{ilm}}z_{\text{ilm}} - w_{\text{elm}}z_{\text{elm}}, \tag{1}$$

which is the difference between nominal returns from production and the wage sums.[5] The firm acts as a price taker on the competitive commodity market and on the external labor market. Thus, it takes the commodity price p and the wage w_{elm} paid on the external labor market as parametrically given.

The Public Sector

The public sector consists of a government and a central bank. The government demands the amount $g \geq 0$ of the produced commodity, which is purchased at market price and which is used to provide public goods to the economy. These public goods do not induce any marginal effects by the agents in the economy. In order to finance its spendings, the government levies proportional taxes on profit income (τ_π) and on wage income (τ_w), with $0 \leq \tau_\pi \leq 1$ and $0 \leq \tau_w \leq 1$.

The government parameters g, τ_w, and τ_π are assumed to be parametrically given[6] so that – in general – the government's budget is not balanced. According to the deficit/surplus of the budget, the central bank creates/destroys fiat money which is held by consumers and which is the only intertemporal store of value in the economy. No interest is paid on savings so that the amount of money holdings $M \geq 0$ by consumers at the beginning of a given period are equal to the amount of savings at the end of the previous period.

The Consumption Sector

The consumption sector consists of overlapping generations of three types of consumers: shareholders, unionized workers, and nonunionized workers. All consumers live for two consecutive

intertemporal evolution of the economy.

[4]The generalization to several homogeneous firms which are organized in a producers association is straightforward.

[5]Since the profit function is different from one with only one type of labor input, the subscript "slm" (short for segmented labor market) is added. The same procedure is applied to other functions not coinciding with their counterparts in the model with a single labor market.

[6]For this reason, g, τ_w, and τ_π are suppressed throughout this paper to simplify notation.

periods and only receive income during the first period so that their second-period consumption has to be financed entirely by savings. In each generation, there are n_s homogeneous shareholders and n_w workers with identical preferences. Shareholders do not work while workers only receive wage income.[7] A fraction γ of the workers, $0 < \gamma < 1$, are union members, i.e. there are γn_w homogeneous unionized workers; the remaining $(1-\gamma)n_w$ homogeneous workers are nonunionized. The number γ denotes *union density*, i.e. the proportion of workers who are union members. All consumers take the commodity price $p > 0$ as given and share a common point forecast $p^e > 0$ for next period's commodity price.

In each period, the young shareholders earn the firm's net profits, which is their only source of income. Due to a homothetic utility function, their propensity of consumption $0 \le c(\theta^e) \le 1$ is a function of the expected rate of inflation $\theta^e := p^e/p$ only. The shareholders save the remainder of their net income in the form of money to be consumed in the second period of their lifes.

Every young worker supplies labor ℓ at a wage w and entirely saves the net wage income $(1-\tau_w)w\ell$ in the form of money. A young worker's consumption–labor decision is based on the additively separable utility function

$$u : \mathbb{R}^2_+ \to \mathbb{R}, \qquad u(\ell, c^e) := c^e - v(\ell),$$

which is the difference of the planned ("expected") consumption in next period c^e and the disutility from labor $v(\ell)$.[8] The disutility function $v(\ell)$ is assumed to be strictly increasing, convex, and satisfying $v(0) = v'(0) = 0$.[9] A worker thus faces the optimization problem

$$\max_{(\ell, c^e)\in\mathbb{R}^2_+} \left\{ c^e - v(\ell) \,\middle|\, (1-\tau_w)w\ell = p^e c^e \right\}$$

which is subject to the intertemporal budget constraint $(1-\tau_w)w\ell = p^e c^e$. Straightforward calculations show that the worker's individual labor supply is given by

$$\ell \overset{!}{=} (v')^{-1}\left((1-\tau_w)\frac{w}{p^e}\right)$$

which is strictly increasing in the expected net real wage $(1-\tau_w)w/p^e$. Since a worker achieves a utility of $u(0,0)$ by not working, any feasible wage–labor supply pair (w, ℓ) must satisfy the individual participation constraint

$$u(0,0) = 0 \le u\left(\ell, (1-\tau_w)\frac{w}{p^e}\ell\right)$$

or

$$(1-\tau_w)\frac{w}{p^e} \ge \frac{v(\ell)}{\ell}.$$

Under competitive conditions, aggregating the labor supply of all n_w workers yields the *aggregate competitive labor supply function*

$$N_{\text{com}}\left(\frac{w}{p^e}\right) := n_w (v')^{-1}\left((1-\tau_w)\frac{w}{p^e}\right)$$

[7] These assumptions are made for the ease of exposition only.

[8] Using a homothetic utility function instead would allow for consumption in both periods.

[9] Restricting the domain of v to the compact interval from zero to some finite maximal labor supply level would not change the way this model is solved as long as the production capacity exceeds the minimal aggregate demand.

which is an increasing and invertible function of the expected net real wage w/p^e. Assuming equal treatment of all workers, the *inverse aggregate competitive labor supply function* is given by

$$S_{\text{com}}(L) \coloneqq \frac{1}{1-\tau_w} v'\left(\frac{L}{n_w}\right)$$

which is the expected real wage workers need to be paid to work the amount L, i.e. to work L/n_w each. This function also is strictly monotonically increasing and invertible. Similarly, the *aggregate participation constraint*, which denotes the minimal expected real wage workers need to be paid to supply L altogether or L/n_w each, is given by

$$S_{\text{res}}(L) \coloneqq \frac{1}{1-\tau_w} \frac{v(L/n_w)}{L/n_w};$$

the subscript "res" indicates that $p^e S_{\text{res}}(L)$ is the *workers' reservation wage* function. Because of the convexity of v, the function S_{res} is strictly monotonically increasing and invertible.

Although all workers have identical preferences implying that their individual supply behavior and their participation constraints are the same, their labor is supplied on two distinct markets and therefore feeds into two distinct aggregates. Since the γn_w unionized workers on the internal labor market (ilm)[10] do not necessarily receive a wage payment at a competitive level, their market behavior is characterized by the minimal wage $p^e S_{\text{res}}(L_{\text{ilm}}/\gamma)$ they need to be paid in order to supply the total amount of labor L_{ilm} or L_{ilm}/n_w each. The labor supply of the $(1-\gamma)n_w$ nonunionized workers on the competitive external labor market (elm)[11] is proportional to the competitive labor supply of all workers, i.e. their aggregate competitive labor supply at a wage w_{elm} is $(1-\gamma)N_{\text{com}}(w_{\text{elm}}/p^e)$. Accordingly, their inverse aggregate competitive labor supply is $S_{\text{com}}(L_{\text{elm}}/(1-\gamma))$ which is a function of the employment level L_{elm}.

The Union

It is assumed that the unionized workers' joint labor supply L_{ilm} is controlled by one labor union which maximizes the *aggregate excess wage bill*, i.e. the wage sum paid above the workers' reservation wage $p^e S_{\text{res}}(L_{\text{ilm}}/\gamma)$.[12] Therefore, the union's objective function is given by

$$\Omega_{\text{slm}} : \mathbb{R}^3_+ \times (0,1) \to \mathbb{R}, \quad \Omega_{\text{slm}}(p^e, w_{\text{ilm}}, L_{\text{ilm}}, \gamma) \coloneqq w_{\text{ilm}} L_{\text{ilm}} - p^e S_{\text{res}}\left(\frac{L_{\text{ilm}}}{\gamma}\right) L_{\text{ilm}} \qquad (2)$$

where w_{ilm} is the wage paid to union members. In contrast to the model with a single labor market, the excess wage bill (2) depends additionally on the union density parameter γ.

[10] The subscript "ilm" is used to identify variables and functions which are exclusive to the internal labor market.

[11] Whenever variables and functions refer to the external labor market, they carry the subscript "elm" to distinguish them from their counterparts on the internal labor market.

[12] Farber (1986) presents and discusses a number of union objectives, in particular their relation to the workers' individual utility function. The approach taken here is more specific than the one by, e.g., McDonald & Solow (1985) who rather consider the difference between the indirect utilities from the bargaining wage and the reservation wage. However, for every fixed employment level, the "objective is consistent with each worker having a linear within-period utility of income", as Card, Devicenti & Maida (2014) report. Furthermore, assuming the excess wage bill as the union's objective allows for aggregating over heterogeneous workers.

2.2 Clearing of the Labor Markets

The labor market is assumed to be segmented into an internal, fully unionized labor market and an external, competitive labor market which are cleared sequentially: First, on the internal labor market, the firm and the union efficiently bargain over the wage rate $w_{\text{ilm}} > 0$ paid to all union members *and* the aggregate employment level $L_{\text{ilm}} \geq 0$ simultaneously. Second, on the external labor market, the firm is allowed to demand additional labor L_{elm} at the wage $w_{\text{elm}} > 0$. It is assumed that the producer only hires external workers *in addition* to its internal workforce, i.e. the firm may not threaten to replace the internal workers by externals.[13] This implies that the bargaining parties have to agree on a positive outcome before the firm may enter the external labor market.

The firm's two-stage profit maximization problem is solved by backward induction using subgame consistency. Therefore, as a first step, the firm's second-stage labor demand is determined for a given first-stage bargaining outcome. Then, as a second step, the bargaining problem in the first stage is solved subject to the firm's second-stage response to the bargaining. Finally, the wage on the competitive external labor market is determined endogenously.

The External Labor Market

Consider a positive bargaining outcome $(w_{\text{ilm}}, L_{\text{ilm}}) \gg 0$,[14] which has been reached in the first stage, and, since the external labor market is a competitive market on which the firm acts as a price taker, let a commodity price $p > 0$ and a wage w_{elm} paid on the external labor market be given. The firm, knowing this data, demands labor on the external market L_{elm} such that it maximizes its profit function, i.e.

$$\begin{aligned} &\max_{L_{\text{elm}} \geq 0} \Big\{ \Pi_{\text{slm}}(p, w_{\text{ilm}}, w_{\text{elm}}, L_{\text{ilm}}, L_{\text{elm}}) \Big\} \\ = &\max_{L_{\text{elm}} \geq 0} \Big\{ pF(L_{\text{ilm}} + L_{\text{elm}}) - w_{\text{ilm}} L_{\text{ilm}} - w_{\text{elm}} L_{\text{elm}} \Big\}. \end{aligned} \tag{3}$$

In spite of the fact that the wage bill $w_{\text{ilm}} L_{\text{ilm}}$ and the wage paid to the unionized workers w_{ilm} enter the profit function, neither of the two affects the firm's external labor demand function: An interior solution to the maximization problem (3) solves $L_{\text{elm}} \overset{!}{=} (F')^{-1}(w_{\text{elm}}/p) - L_{\text{ilm}}$; because of the strict concavity of the profit function, the firm's best response function is given by

$$L_{\text{elm}} = \max \left\{ (F')^{-1} \left(\frac{w_{\text{elm}}}{p} \right) - L_{\text{ilm}}, 0 \right\} =: h \left(\frac{w_{\text{elm}}}{p}, L_{\text{ilm}} \right). \tag{4}$$

Since $h_{\text{com}}(w/p) := (F')^{-1}(w/p)$ is the labor demand function of a profit-maximizing firm on a competitive market with profit function $\Pi(p, w, z) := pF(z) - wz$, the term $(F')^{-1}(w_{\text{elm}}/p) - L_{\text{ilm}} = h_{\text{com}}(w_{\text{elm}}/p) - L_{\text{ilm}}$ denotes the difference between the internal employment level and the profit-maximizing labor demand. If such a gap exists, the firm fills it with external workers; otherwise, no external workers are hired.

[13] If the firm was allowed to replace all internal workers by externals, the wage paid on the internal labor market would always be lower than the one paid on the external market. The implications of this assumption are discussed in Appendix 6.3.

[14] The notation "$\gg 0$" is used to indicate that all components of a vector are positive.

The Bargaining Problem on the Internal Labor Market

On the internal labor market, the firm and the union efficiently bargain over the employment level L_{ilm} and the wage w_{ilm} in order to maximize their nominal payoffs, i.e. the firm's profit function (1) and the union's excess wage bill function (2). Since both parties implicitly assume that the outcome of the bargaining has no influence on the equilibrium wage on external market and the equilibrium price on the commodity market, they take a commodity price $p > 0$, a point forecast for next period's commodity price $p^e > 0$, and a wage $w_{\text{elm}} > 0$ paid on the external labor market as given. However, they take the firm's best-response function on the external labor market $L_{\text{elm}} = h(w_{\text{elm}}/p, L_{\text{ilm}})$ into account, i.e. they are fully aware of the consequences of the bargaining to the firm's subgame-perfect labor demand on the external labor market. Therefore, the two objective functions are given by

$$\Pi_{\text{slm}}\left(p, w_{\text{ilm}}, w_{\text{elm}}, L_{\text{ilm}}, h\left(\frac{w_{\text{elm}}}{p}, L_{\text{ilm}}\right)\right)$$
$$= pF\left(L_{\text{ilm}} + h\left(\frac{w_{\text{elm}}}{p}, L_{\text{ilm}}\right)\right) - w_{\text{ilm}}L_{\text{ilm}} - w_{\text{elm}}h\left(\frac{w_{\text{elm}}}{p}, L_{\text{ilm}}\right)$$

and

$$\Omega_{\text{slm}}(p^e, w_{\text{ilm}}, L_{\text{ilm}}, \gamma) = w_{\text{ilm}}L_{\text{ilm}} - p^e S_{\text{res}}\left(\frac{L_{\text{ilm}}}{\gamma}\right) L_{\text{ilm}}.$$

A failure of the bargaining would lead to a complete shutdown of production, i.e. $L_{\text{ilm}} = 0$ *and* $L_{\text{elm}} = 0$, which would imply that the payoff levels for both agents were zero. In particular, since unionized workers are not able to sell their labor on the external market, this would imply zero utility for each union member which is below the utility level the nonunionized workers receive from supplying labor to the external labor market.

Let

$$\mathcal{B}(p^e, p, w_{\text{elm}}, \gamma) := \bigcup_{w_{\text{ilm}}, L_{\text{ilm}} \geq 0} \left\{ \begin{pmatrix} \Pi_{\text{slm}}\left(p, w_{\text{ilm}}, w_{\text{elm}}, L_{\text{ilm}}, h\left(\frac{w_{\text{elm}}}{p}, L_{\text{ilm}}\right)\right) \\ \Omega_{\text{slm}}(p^e, w_{\text{ilm}}, L_{\text{ilm}}, \gamma) \end{pmatrix} \right\} \tag{5}$$

denote the set of payoffs which can be supported by wage–employment pairs $(w_{\text{ilm}}, L_{\text{ilm}})$. Then, for $(p^e, p, w_{\text{elm}}, \gamma)$ given, the *bargaining problem* of the producer and the union is given by the pair

$$\left(\mathcal{B}(p^e, p, w_{\text{elm}}, \gamma), \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right)$$

where the set $\mathcal{B}(p^e, p, w_{\text{elm}}, \gamma)$ is called the *feasible set* of the bargaining (or just *bargaining set*) and the point $\binom{0}{0} \in \mathcal{B}(p^e, p, w_{\text{elm}}, \gamma)$ is the so-called *status-quo point.* The status-quo point is the outcome that would be reached if the bargaining failed.

In general, an (asymmetric) *bargaining solution* is a payoff vector which satisfies the following four properties: feasiblity, Pareto efficiency, individual rationality, and invariance of scale and translation.

The Employment Levels

The set of efficient employment–wage pairs on the internal labor market, the *Pareto curve* or *contract curve*, is depicted in Figure 1 for two given levels of the external-labor-market

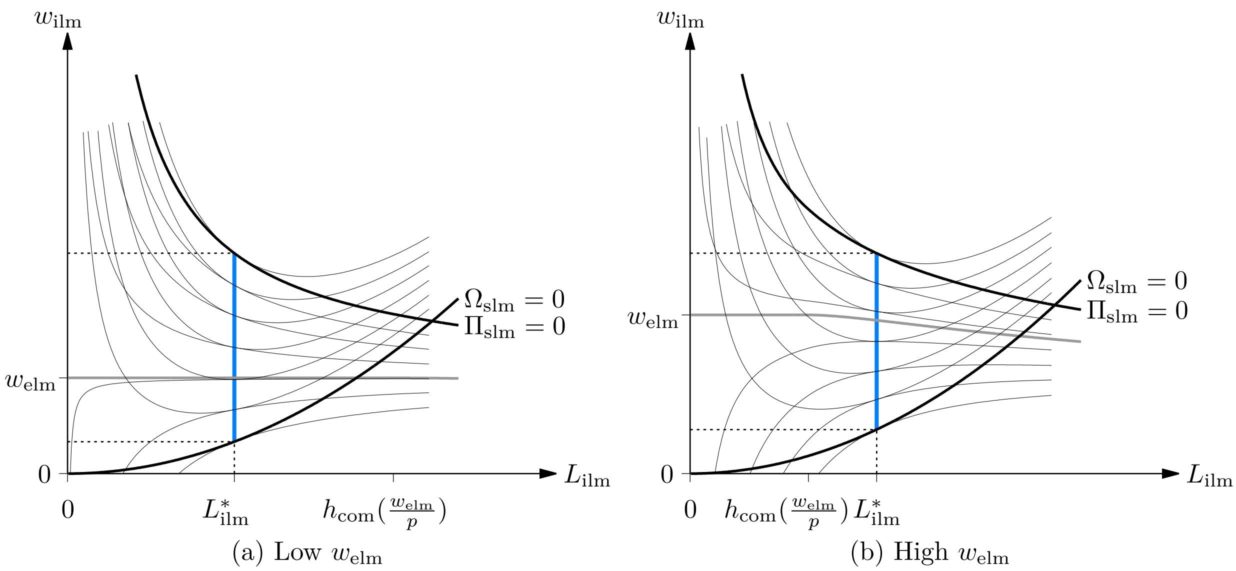


Figure 1: The employment level and the wage on the internal labor market; p, p^e, γ, w_{elm} given

wage w_{elm}.[15] Each point on the contract curve is the tangency point of one iso-profit and one iso-excess wage bill curve (the thin lines). Since both utility functions are linear in the wage bill $w_{ilm}L_{ilm}$, the marginal effects of a wage change are of same size, but of opposite sign. Therefore, the contract curve is a vertical line segment in the space of employment and wages (the bold blue line), i.e. the efficient employment level L^*_{ilm} is independent of the internal-labor market wage level. The panels of the figure present two scenarios which correspond to positive labor demand on the external labor market (Figure 1(a); w_{elm} low) and zero labor demand for external labor (Figure 1(b); w_{elm} high). Note that the level curves of the excess wage bill are not affected through the change of the wage w_{elm} while the family of isoprofit curves are differently shaped for $L_{ilm} < h_{com}(w_{elm}/p)$. Most importantly, all isoprofit curves above the gray line are strictly monotonically decreasing while the isoprofit curves below the gray line have a unique maximum. The gray line, which is the isoprofit curve for the profit level under a single, fully competitive labor market with wage w_{elm}, is flat for $L_{ilm} < h_{com}(w_{elm}/p)$ and decreasing for $L_{ilm} > h_{com}(w_{elm}/p)$.

Table 1: Standard parametrization

A	B	C	$\tau_\pi = \tau_w$	λ	γ	M	g	p^e	c	n_w
1	0.6	0.5	0.68	0.5	0.8	0.33	0.86	1	0.5	1

Due to the linearity of the utility functions in the wage bill $w_{ilm}L_{ilm}$, the producer and the union are risk-neutral and transfer payoffs linearly through the wage bill. In other words, the bargaining problem for any given employment level L_{ilm} – in particular the efficient one – is a zero-sum game between the two agents. In such situations, the bargaining set is a halfspace

[15] All diagrams in this paper are drawn to scale using an isoelastic production function $F(z) = A\,z^B/B$ with $A > 0$ and $0 < B < 1$, a logarithmic intertemporal utility function $\log c_0 + \delta \log c^e$ with $\delta \geq 0$, and an isoelastic disutility from labor $v(\ell) = C(C+1)^{-1}\ell^{1+1/C}$ with $0 < C < 1$. This implies a constant propensity of consumption of young shareholders $c \equiv (1+\delta)^{-1}$ and an isoelastic reservation wage function $S_{res}(L) = C(C+1)^{-1}(1-\tau_w)^{-1}(L/n_w)^{1/C}$. The parametrization given in Table 1 is used if not indicated otherwise.

with slope minus one, and the locus of its boundary is determined by the *joint surplus*

$$\Pi_{\text{slm}}\left(p, w_{\text{ilm}}, w_{\text{elm}}, L_{\text{ilm}}, h\left(\frac{w_{\text{elm}}}{p}, L_{\text{ilm}}\right)\right) + \Omega_{\text{slm}}(p^e, w_{\text{ilm}}, L_{\text{ilm}}, \gamma)$$
$$= pF\left(L_{\text{ilm}} + h\left(\frac{w_{\text{elm}}}{p}, L_{\text{ilm}}\right)\right) - w_{\text{elm}} h\left(\frac{w_{\text{elm}}}{p}, L_{\text{ilm}}\right) - p^e S_{\text{res}}\left(\frac{L_{\text{ilm}}}{\gamma}\right) L_{\text{ilm}},$$

which is independent of the bargaining wage w_{ilm}. More precisely, for $(p^e, p, w_{\text{elm}}, \gamma)$ given, the boundary of the halfspace is the linear function $\Pi_{\text{slm}} = pF(L_{\text{ilm}} + h(w_{\text{elm}}/p, L_{\text{ilm}})) - w_{\text{elm}} h(w_{\text{elm}}/p, L_{\text{ilm}}) - p^e S_{\text{res}}(L_{\text{ilm}}/\gamma) - \Omega_{\text{slm}}$.

The property that, for every employment level L_{ilm}, all bargaining sets are halfspaces with a mutual slope allows for a criterion to choose the employment level L_{ilm} independently of the bargaining wage w_{ilm}: Since every bargaining solution is Pareto efficient, the employment level L_{ilm} is chosen jointly at the level which maximizes the joint surplus, i.e.

$$h_{\text{ilm}}\left(\frac{w_{\text{elm}}}{p}, \frac{p^e}{p}, \gamma\right) := \arg\max_{L_{\text{ilm}} \geq 0} \left\{ pF\left(L_{\text{ilm}} + h\left(\frac{w_{\text{elm}}}{p}, L_{\text{ilm}}\right)\right) - w_{\text{elm}} h\left(\frac{w_{\text{elm}}}{p}, L_{\text{ilm}}\right) - p^e S_{\text{res}}\left(\frac{L_{\text{ilm}}}{\gamma}\right) L_{\text{ilm}} \right\}. \quad (6)$$

The joint surplus can be seen as the cake to be shared in the bargaining. Thus, choosing the employment level L_{ilm} determines the *size* of the cake to be split and the bargaining wage w_{ilm} determines *how* to split the surplus between the firm and the union.

The function describing the employment level under efficient bargaining has an explicit formulation which is subject of the following lemma.

Lemma 2.1. *The solution of the maximization problem* (6)*, i.e. the efficient-bargaining employment level* L_{ilm}*, is given by the function*

$$h_{\text{ilm}}\left(\frac{w_{\text{elm}}}{p}, \frac{p^e}{p}, \gamma\right) = \begin{cases} \gamma N_{\text{com}}\left(\frac{w_{\text{elm}}/p}{p^e/p}\right), & \text{if } h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right) \geq \gamma N_{\text{com}}\left(\frac{w_{\text{elm}}/p}{p^e/p}\right) \\ h_{\text{eff}}\left(\frac{p^e}{p}, \gamma\right), & \text{otherwise} \end{cases}$$
$$= \min\left\{\gamma N_{\text{com}}\left(\frac{w_{\text{elm}}/p}{p^e/p}\right), h_{\text{eff}}\left(\frac{p^e}{p}, \gamma\right)\right\}, \quad (7)$$

where $h_{\text{eff}}(p^e/p, \gamma)$ *is the solution of* $pF'(L_{\text{ilm}}) \stackrel{!}{=} p^e S_{\text{com}}(L_{\text{ilm}}/\gamma)$*.*

Proof. See Appendix 6.1. □

The two branches of $h_{\text{ilm}}(w_{\text{elm}}/p, p^e/p, \gamma)$ reflect two scenarios, depending on whether the firm's demand for labor on the external market is positive or not: In the first scenario, the employment level $\gamma N_{\text{com}}(w_{\text{elm}}/p^e)$ is the same level the unionized workers would supply at wage w_{elm} under competitive conditions. Therefore, it makes no difference for a worker which labor market he is attached to as long as the same wage is paid on both markets; note that unionized workers receive higher utility if the wage on the internal labor market is higher than the one on the competitive market. In the second scenario, in which the firm does not demand labor on the external labor market, the firm and the union behave as if there was no external market. This situation is the same as the one described in Böhm & Claas (2014), with the labor supply of γn_w workers. Due to the fact that the number of workers depends on union density γ, the employment level $h_{\text{eff}}(p^e/p, \gamma)$ depends on γ.

For $(p^e, p) \gg 0$ and $0 < \gamma < 1$ given, the employment level $h_{\text{ilm}}(w_{\text{elm}}/p, p^e/p, \gamma)$ can be read off horizontally in Figure 2. The figure illustrates that the firm's labor input comes from both markets if the wage on the external labor market w_{elm} is sufficiently low, namely below $pF'(h_{\text{eff}}(p^e/p, \gamma))$ whereas the external labor market dries out for high wages, i.e. above $pF'(h_{\text{eff}}(p^e/p, \gamma))$, so that the firm and the union are in a classical efficient-bargaining situation.

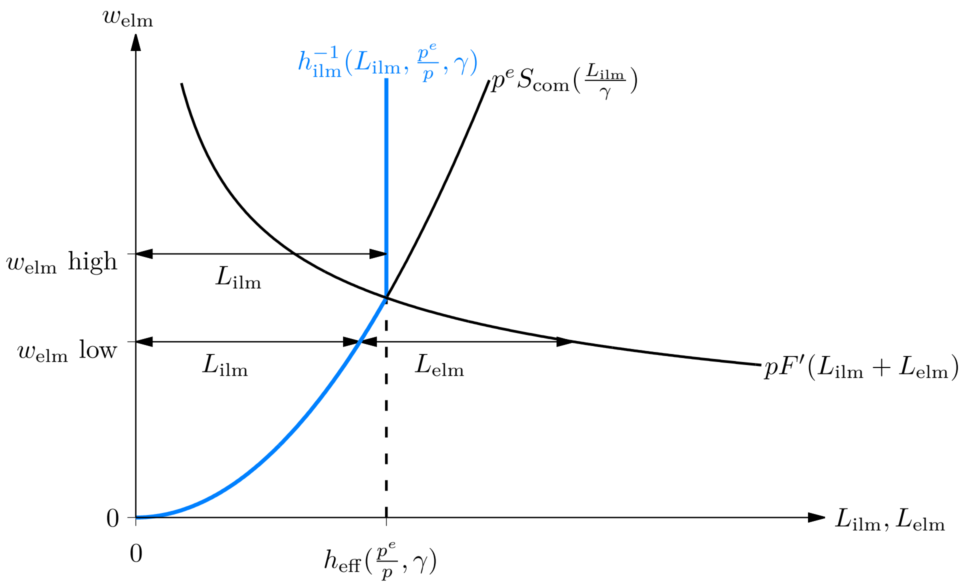


Figure 2: The employment levels and the wage on the external labor market; p, p^e, γ given

Inserting the employment level function (7) into the firm's best response function (4) yields the firm's labor demand function on the competitive market as a function of the real competitive wage w_{elm}/p, the expected rate of inflation p^e/p, and union density γ.

Corollary 2.1. *The firm's labor demand on the competitive external labor market is given by the function*

$$\begin{aligned} L_{\text{elm}} &= h\left(\frac{w_{\text{elm}}}{p}, h_{\text{ilm}}\left(\frac{w_{\text{elm}}}{p}, \frac{p^e}{p}, \gamma\right)\right) \\ &= \max\left\{h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right) - h_{\text{ilm}}\left(\frac{w_{\text{elm}}}{p}, \frac{p^e}{p}, \gamma\right), 0\right\} \\ &= \max\left\{h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right) - \gamma N_{\text{com}}\left(\frac{w_{\text{elm}}/p}{p^e/p}\right), 0\right\} =: h_{\text{elm}}\left(\frac{w_{\text{elm}}}{p}, \frac{p^e}{p}, \gamma\right). \end{aligned} \tag{8}$$

In particular, the employment level functions (7) and (8) depend on the wage on the external labor market w_{elm} and on the good's price on the commodity market p. Before their equilibrium values are determined endogenously, the wage function on the internal labor market is calculated.

The Wage on the Internal Labor Market

Under the assumption that a failure of the union–firm bargaining would result in a complete shutdown of production, both agents would receive a payoff of zero if no agreement could be found. Since this payoff level can be reached unilaterally by either agent, the individual rationality of the bargaining parties implies two participation constraints. In the case of the firm, it is

$$\begin{aligned}
& 0 \overset{!}{\leq} \Pi_{\mathrm{slm}}\left(p, w_{\mathrm{ilm}}, w_{\mathrm{elm}}, L_{\mathrm{ilm}}, h\left(\frac{w_{\mathrm{elm}}}{p}, L_{\mathrm{ilm}}\right)\right) \\
\iff \quad & 0 \overset{!}{\leq} pF\left(L_{\mathrm{ilm}} + h\left(\frac{w_{\mathrm{elm}}}{p}, L_{\mathrm{ilm}}\right)\right) - w_{\mathrm{ilm}} L_{\mathrm{ilm}} - w_{\mathrm{elm}} h\left(\frac{w_{\mathrm{elm}}}{p}, L_{\mathrm{ilm}}\right) \\
\iff \quad & w_{\mathrm{ilm}} \overset{!}{\leq} \frac{pF(L_{\mathrm{ilm}} + h(\frac{w_{\mathrm{elm}}}{p}, L_{\mathrm{ilm}})) - w_{\mathrm{elm}} h(\frac{w_{\mathrm{elm}}}{p}, L_{\mathrm{ilm}})}{L_{\mathrm{ilm}}} =: W_{\Pi_{\mathrm{slm}}}(p, w_{\mathrm{elm}}, L_{\mathrm{ilm}}).
\end{aligned}$$

The union's participation constraint is

$$\begin{aligned}
& 0 \overset{!}{\leq} \Omega_{\mathrm{slm}}(p^e, w_{\mathrm{ilm}}, L_{\mathrm{ilm}}, \gamma) \\
\iff \quad & 0 \overset{!}{\leq} w_{\mathrm{ilm}} L_{\mathrm{ilm}} - p^e S_{\mathrm{res}}\left(\frac{L_{\mathrm{ilm}}}{\gamma}\right) L_{\mathrm{ilm}} \\
\iff \quad & w_{\mathrm{ilm}} \overset{!}{\geq} p^e S_{\mathrm{res}}\left(\frac{L_{\mathrm{ilm}}}{\gamma}\right) =: W_{\Omega_{\mathrm{slm}}}(p^e, L_{\mathrm{ilm}}, \gamma).
\end{aligned}$$

The functions $W_{\Pi_{\mathrm{slm}}}(p, w_{\mathrm{elm}}, L_{\mathrm{ilm}})$ and $W_{\Omega_{\mathrm{slm}}}(p^e, L_{\mathrm{ilm}}, \gamma)$ denote the maximal and the minimal wage at which an agent agrees on a bargaining, i.e. the *firm's* and the *union's reservation wage functions*. Both reservation wage functions are continuous and homogeneous of degree 1 in (p, w_{elm}). While $W_{\Omega_{\mathrm{slm}}}$ is differentiable, $W_{\Pi_{\mathrm{slm}}}$ has a kink at $h_{\mathrm{com}}(w_{\mathrm{elm}}/p) = L_{\mathrm{ilm}}$.

Note that the range of individually rational bargaining wages

$$W_{\Omega_{\mathrm{slm}}}(p^e, L_{\mathrm{ilm}}, \gamma) \overset{!}{\leq} w_{\mathrm{ilm}} \overset{!}{\leq} W_{\Pi_{\mathrm{slm}}}(p, w_{\mathrm{elm}}, L_{\mathrm{ilm}})$$

is nonempty if and only if

$$\begin{aligned}
0 & \overset{!}{\leq} \left(W_{\Pi_{\mathrm{slm}}}(p, w_{\mathrm{elm}}, L_{\mathrm{ilm}}) - W_{\Omega_{\mathrm{slm}}}(p^e, L_{\mathrm{ilm}}, \gamma)\right) L_{\mathrm{ilm}} \\
& = pF\left(L_{\mathrm{ilm}} + h\left(\frac{w_{\mathrm{elm}}}{p}, L_{\mathrm{ilm}}\right)\right) - w_{\mathrm{elm}} h\left(\frac{w_{\mathrm{elm}}}{p}, L_{\mathrm{ilm}}\right) - p^e S_{\mathrm{res}}\left(\frac{L_{\mathrm{ilm}}}{\gamma}\right) L_{\mathrm{ilm}},
\end{aligned}$$

i.e. if and only if the joint surplus is positive. In particular, this is the case for $L_{\mathrm{ilm}} = h_{\mathrm{ilm}}(w_{\mathrm{elm}}/p, p^e/p, \gamma)$ at which the joint surplus is maximal.

Let $0 \leq \lambda \leq 1$ denote the (relative) *bargaining power of the union* and let the commodity price p, its next period forecast p^e, the competitive wage w_{elm}, and union density γ be given. Furthermore, consider some L_{ilm} such that $W_{\Omega_{\mathrm{slm}}}(p^e, L_{\mathrm{ilm}}, \gamma) < W_{\Pi_{\mathrm{slm}}}(p, w_{\mathrm{elm}}, L_{\mathrm{ilm}})$. Due to the specific structure of the bargaining problem,[16] the (asymmetric) bargaining solution to the firm's and the union's bargaining problem is given by the convex combination with coefficient λ of the "corner points" of the bargaining set at which the entire surplus $(W_{\Pi_{\text{slm}}}(p, w_{\text{elm}}, L_{\text{ilm}}) - W_{\Omega_{\text{slm}}}(p^e, L_{\text{ilm}}, \gamma))L_{\text{ilm}}$ is allotted to one party, i.e.

$$\Big(W_{\Pi_{\text{slm}}}(p, w_{\text{elm}}, L_{\text{ilm}}) - W_{\Omega_{\text{slm}}}(p^e, L_{\text{ilm}}, \gamma)\Big)L_{\text{ilm}} \begin{pmatrix} 1-\lambda \\ \lambda \end{pmatrix}. \tag{9}$$

The bargaining solution (9) is induced by the bargaining wage function

$$w_{\text{ilm}} = \lambda W_{\Pi_{\text{slm}}}(p, w_{\text{elm}}, L_{\text{ilm}}) + (1-\lambda) W_{\Omega_{\text{slm}}}(p^e, L_{\text{ilm}}, \gamma) \tag{10}$$

which is the convex combination of the two reservation wage functions. The bargaining solution is a function which is homogeneous of degree one in (p, p^e, w_{elm}) because of the homogeneity of degree one in (p, p^e, w_{elm}) of both reservation wage functions. Recall that the union's reservation wage $W_{\Omega_{\text{slm}}}(p^e, L_{\text{ilm}}, \gamma) = p^e S_{\text{res}}(L_{\text{ilm}}/\gamma)$ is less than the inverse labor supply $p^e S_{\text{com}}(L_{\text{ilm}}/\gamma)$. Therefore, for a range of small levels of λ, i.e. for a "weak" union, the bargaining wage w_{ilm} is below the wage that would be paid under full competition at the employment level L_{ilm}/γ.

The Wage on the External Labor Market

For a given commodity price p and its next-period forecast p^e, any equilibrium competitive wage $w_{\text{elm}} > 0$ forces the firm's labor demand on the competitive external labor market $h_{\text{elm}}(w_{\text{elm}}/p, p^e/p, \gamma) > 0$ to be positive because the competitive external labor supply $(1-\gamma)N_{\text{com}}(w_{\text{elm}}/p^e) > 0$ is positive. Therefore, an equilibrium competitive wage w_{elm} needs to solve

$$h_{\text{elm}}\left(\frac{w_{\text{elm}}}{p}, \frac{p^e}{p}, \gamma\right) = h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right) - \gamma N_{\text{com}}\left(\frac{w_{\text{elm}}}{p^e}\right) \stackrel{!}{=} (1-\gamma) N_{\text{com}}\left(\frac{w_{\text{elm}}}{p^e}\right) \tag{11}$$

which holds true if and only if

$$h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right) \stackrel{!}{=} N_{\text{com}}\left(\frac{w_{\text{elm}}}{p^e}\right). \tag{12}$$

The first and remarkable observation is that this equation is independent of union density γ as well as the number of unionized workers γn_w. However, the most astonishing property of this equation is that it is identical to the market clearing condition as under full competition. This implies that for any pair of the commodity price and its forecast for the following period $(p, p^e) \gg 0$, the wage on the competitive labor market is the same as the wage in a fully competitive labor market. The solution of (12) with respect to the real wage w_{elm}/p is given by the implicitly defined function $W_{\text{elm}} : \mathbb{R}_+ \to \mathbb{R}_+$ which is a function of the expected inflation p^e/p. Then

$$W_{\text{elm}}\left(\frac{p^e}{p}\right) \equiv W_{\text{com}}\left(\frac{p^e}{p}\right). \tag{13}$$

[16] Since both bargaining agents are risk-neutral, the bargaining set is a halfspace in payoff space, and the contract curve is vertical in the employment–wage space. In such situations, the generalized Zeuthen solution which coincides with the generalized Nash solution yet requiring less properties, is applicable. If the union was more risk-averse than the firm, the contract curve would be upward-sloping which implies that the bargaining solution (and the employment–wage pairs inducing it) would depend on the solution concept chosen; see Gerber & Upmann (2006) for a discussion.

The function W_{elm} inherits all properties from W_{com}, in particular, W_{elm} is strictly monotonically increasing and invertible with the explicitly given inverse

$$W_{\text{elm}}^{-1}\left(\frac{w_{\text{elm}}}{p}\right) = \frac{\frac{w_{\text{elm}}}{p}}{S_{\text{com}}\left(h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right)\right)}. \tag{14}$$

Since the elasticity[17] of W_{elm}^{-1}

$$E_{W_{\text{elm}}^{-1}}\left(\frac{w_{\text{elm}}}{p}\right) = 1 - E_{S_{\text{com}}}\left(h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right)\right) E_{h_{\text{com}}}\left(\frac{w_{\text{elm}}}{p}\right) > 1,$$

is greater than unity, $0 < E_{W_{\text{elm}}}(\frac{p^e}{p}) < 1$ holds true which implies that the real wage function W_{elm} is usually strictly concave in expected inflation p^e/p. Since the firm's demand function on the external labor market is always positive in equilibrium, substituting $W_{\text{elm}}(p^e/p)$ into the function of the employment level under bargaining (7) and the external labor demand function (8) yields

$$h_{\text{ilm}}\left(W_{\text{elm}}\left(\frac{p^e}{p}\right), \frac{p^e}{p}, \gamma\right) \equiv \gamma h_{\text{com}}\left(W_{\text{elm}}\left(\frac{p^e}{p}\right)\right) \tag{15}$$

and

$$h_{\text{elm}}\left(W_{\text{elm}}\left(\frac{p^e}{p}\right), \frac{p^e}{p}, \gamma\right) \equiv (1-\gamma) h_{\text{com}}\left(W_{\text{elm}}\left(\frac{p^e}{p}\right)\right). \tag{16}$$

This equation shows that, in equilibrium, the employment level on the external labor market is always positve, i.e. that the competitive market cannot dry out due to some configuration on the bargaining market. Adding up the employment levels on the two labor markets yields that the level of equilibrium aggregate employment is given by

$$h_{\text{ilm}}\left(W_{\text{elm}}\left(\frac{p^e}{p}\right), \frac{p^e}{p}, \gamma\right) + h_{\text{elm}}\left(W_{\text{elm}}\left(\frac{p^e}{p}\right), \frac{p^e}{p}, \gamma\right) \equiv h_{\text{com}}\left(W_{\text{elm}}\left(\frac{p^e}{p}\right)\right), \tag{17}$$

which also is the aggregate employment level for a fully competitive labor market. Therefore, the aggregate employment level is independent of union density γ. The employment levels on the two markets are proportional to the aggregate employment level; the proportion of the bargaining employment level being union density. Regardless of union membership, the individual labor supply of each worker is $h_{\text{com}}(W_{\text{elm}}(p^e/p))/n_w$ so that workers only differ by the wage payment they receive and not by their employment levels.

The wage on the internal labor market is obtained by inserting the employment level function (15) and the competitive wage function $w_{\text{elm}} = pW_{\text{elm}}(p^e/p)$ into the bargaining wage function (10). By taking advantage of homogeneity, the *real*-wage function on the internal labor market is described by a function $W_{\text{ilm}} : \mathbb{R}_+ \times [0,1] \times (0,1) \to \mathbb{R}_+$ of the expected inflation p^e/p, union power λ, and union density γ.

Now, the real wage under bargaining is obtained as a function $W_{\text{ilm}} : \mathbb{R}_+ \times [0,1] \times (0,1) \to \mathbb{R}_+$ of the expected inflation p^e/p, union power λ, and union density γ by inserting the employment

[17] The elasticity of a differentiable function f at x with $f(x) \neq 0$ is defined as $E_f(x) := f'(x)x/f(x)$.

level function (15) and the competitive wage function $w_{\text{elm}} = pW_{\text{elm}}(p^e/p)$ into the bargaining wage function (10). It is given by

$$\begin{aligned} W_{\text{ilm}}\left(\frac{p^e}{p}, \lambda, \gamma\right) &:= \lambda W_{\Pi_{\text{slm}}}\left(1, W_{\text{elm}}\left(\frac{p^e}{p}\right), \gamma h_{\text{com}}\left(W_{\text{elm}}\left(\frac{p^e}{p}\right)\right)\right) \\ &\qquad + (1-\lambda) W_{\Omega_{\text{slm}}}\left(\frac{p^e}{p}, \gamma h_{\text{com}}\left(W_{\text{elm}}\left(\frac{p^e}{p}\right)\right), \gamma\right). \end{aligned} \tag{18}$$

Let $L = h_{\text{com}}(W_{\text{elm}}(p^e/p))$ denote the aggregate employment level and write the real wage on the competitive external labor market $W_{\text{elm}}(p^e/p) = h_{\text{com}}^{-1}(L) = F'(L)$ as the marginal product of production. Then, writing the firm's reservation wage function as a markup over of the average product $F(L)/L$

$$\begin{aligned} & W_{\Pi_{\text{slm}}}\left(1, W_{\text{elm}}\left(\frac{p^e}{p}\right), \gamma h_{\text{com}}\left(W_{\text{elm}}\left(\frac{p^e}{p}\right)\right)\right) = W_{\Pi_{\text{slm}}}\left(1, F'(L), \gamma L\right) \\ &= \frac{F(L) - (1-\gamma)F'(L)L}{\gamma L} = \frac{1-(1-\gamma)F'(L)L/F(L)}{\gamma}\frac{F(L)}{L} \\ &= \frac{1-(1-\gamma)E_F(L)}{\gamma}\frac{F(L)}{L} \end{aligned}$$

shows that the markup depends on the elasticity of production $E_F(L)$ and union density γ. Using $S_{\text{com}}(L)/S_{\text{res}}(L) = E_v(L/n_w) = E_{S_{\text{res}}}(L) + 1$ shows that the union's reservation wage function

$$\begin{aligned} W_{\Omega_{\text{slm}}}\left(\frac{p^e}{p}, \gamma h_{\text{com}}\left(W_{\text{elm}}\left(\frac{p^e}{p}\right)\right), \gamma\right) &= W_{\Omega_{\text{slm}}}\left(\frac{p^e}{p}, \gamma L, \gamma\right) = \frac{p^e}{p} S_{\text{res}}(L) \\ &= \frac{p^e}{p}\frac{S_{\text{com}}(L)}{E_{S_{\text{res}}}(L)+1} \overset{(14)}{=} \frac{F'(L)}{E_{S_{\text{res}}}(L)+1} = \frac{E_F(L)}{E_{S_{\text{res}}}(L)+1}\frac{F(L)}{L}, \end{aligned}$$

with $L = h_{\text{com}}(W_{\text{elm}}(p^e/p))$, is a markdown of the average product. Combining these two results yields

$$W_{\text{ilm}}\left(\frac{p^e}{p}, \lambda, \gamma\right) = \left(\lambda \frac{1-(1-\gamma)E_F(L)}{\gamma} + (1-\lambda)\frac{E_F(L)}{E_{S_{\text{res}}}(L)+1}\right)\frac{F(L)}{L} \tag{19}$$

with $L = h_{\text{com}}(W_{\text{elm}}(p^e/p))$. Similarly, the union's reservation wage function can be written as a markup over the labor supply function under fully competitive conditions

$$W_{\text{ilm}}\left(\frac{p^e}{p}, \lambda, \gamma\right) = \left(\lambda \frac{1-(1-\gamma)E_F(L)}{\gamma E_F(L)} + (1-\lambda)\frac{1}{E_{S_{\text{res}}}(L)+1}\right)\frac{p^e}{p} S_{\text{com}}(L) \tag{20}$$

with $L = h_{\text{com}}(W_{\text{elm}}(p^e/p))$. Note that

$$E_{W_{\text{ilm}}}(\lambda) \in (0,1) \qquad \text{and} \qquad E_{W_{\text{ilm}}}(\gamma) \in (-1,0)$$

while the partial derivative with respect to the expected rate of inflation cannot be signed in general. If the effects stemming from the elasticities are sufficiently small, W_{ilm} is strictly monotonically increasing in θ^e.

2.3 The Functional Income Distribution

On both labor markets, wages and employment levels can be expressed as functions of the expected inflation p^e/p and the union parameters λ and γ, see equations (18), (15), and (16). It is informative to compute the factor shares of the different types of income, i.e. wage and profit income. To this end, let $L = h_{\text{com}}(W_{\text{elm}}(p^e/p))$ denote the aggregate employment level so that the efficient and the competitive levels of employment are given by $L_{\text{ilm}} = \gamma L$ and $L_{\text{elm}} = (1-\gamma)L > 0$, and the corresponding wages are $w_{\text{ilm}} = pW_{\text{ilm}}(p^e/p, \lambda, \gamma)$ and $w_{\text{elm}} = pW_{\text{elm}}(p^e/p)$. Thus, the profit share of total revenue is given by

$$\begin{aligned}\frac{pF(L) - w_{\text{ilm}}L_{\text{ilm}} - w_{\text{elm}}L_{\text{elm}}}{pF(L)} &= 1 - \lambda\Big(1 - (1-\gamma)E_F(L)\Big) - (1-\lambda)\gamma\frac{E_F(L)}{E_{S_{\text{res}}}(L)+1} \\ &\quad - (1-\gamma)E_F(L) \\ &= (1-\lambda)\left(1 - E_F(L)\left(1 - \gamma\frac{E_{S_{\text{res}}}(L)}{E_{S_{\text{res}}}(L)+1}\right)\right) \in [0,1]. \quad (21)\end{aligned}$$

Similarly, the wage share of total revenue[18] is given by

$$\begin{aligned}&\frac{w_{\text{ilm}}L_{\text{ilm}} + w_{\text{elm}}L_{\text{elm}}}{pF(L)} \\ &= \lambda\Big(1 - (1-\gamma)E_F(L)\Big) + (1-\lambda)\gamma\frac{E_F(L)}{E_{S_{\text{res}}}(L)+1} + (1-\gamma)E_F(L) \\ &= 1 - (1-\lambda)\left(1 - E_F(L)\left(1 - \gamma\frac{E_{S_{\text{res}}}(L)}{E_{S_{\text{res}}}(L)+1}\right)\right) \in [0,1].\end{aligned}$$

In general, because of $L = h_{\text{com}}(W_{\text{elm}}(p^e/p))$, the shares of profits and wages on total revenue can be expressed as a function of the expected rate of inflation p^e/p, of union power λ, of and union density γ. If F and S_{res} are isoelastic functions in labor, the wage share of total revenue is constant in the expected rate of inflation and only depends on the union parameters λ and γ.

Multiplying the profit share (21) by total revenue $pF(L)$ to obtain equilibrium profits

$$(1-\lambda)\left(1 - E_F(L)\left(1 - \gamma\frac{E_{S_{\text{res}}}(L)}{E_{S_{\text{res}}}(L)+1}\right)\right)pF(L), \qquad L = h_{\text{com}}\left(W_{\text{elm}}\left(\frac{p^e}{p}\right)\right) \quad (22)$$

easily shows that the profit can be higher or lower than

$$\Big(1 - E_F(L)\Big)pF(L), \qquad L = h_{\text{com}}\left(W_{\text{elm}}\left(\frac{p^e}{p}\right)\right),$$

depending on the levels of the union parameters λ and γ. Since the aggregate employment level in the model with a fully competitive labor market and in the present model are identical, the levels of bargaining power λ and union density γ determine in which setting the firm's profit is higher.

[18] The wage shares for unionized and nonunionized workers can be computed separately.

2.4 The Rate of Underemployment

Define the *rate of underemployment* as the relative gap between the desired labor supply of all workers at their individual wages and the actual employment level as

$$\begin{aligned}
&U_{\mathrm{slm}}\left(L_{\mathrm{ilm}}, L_{\mathrm{elm}}, \frac{w_{\mathrm{ilm}}}{p^e}, \frac{w_{\mathrm{elm}}}{p^e}\right) \\
&:= \frac{\gamma N_{\mathrm{com}}(w_{\mathrm{ilm}}/p^e) + (1-\gamma) N_{\mathrm{com}}(w_{\mathrm{elm}}/p^e) - L_{\mathrm{ilm}} - L_{\mathrm{elm}}}{\gamma N_{\mathrm{com}}(w_{\mathrm{ilm}}/p^e) + (1-\gamma) N_{\mathrm{com}}(w_{\mathrm{elm}}/p^e)} \\
&= 1 - \frac{L_{\mathrm{ilm}} + L_{\mathrm{elm}}}{\gamma N_{\mathrm{com}}(w_{\mathrm{ilm}}/p^e) + (1-\gamma) N_{\mathrm{com}}(w_{\mathrm{elm}}/p^e)}.
\end{aligned} \tag{23}$$

The rate of underemployment is a measure of the *aggregate* labor market. Thus, if union density decreases (increases), the impact, which the difference between the unionized workers' desired and actual labor supply exerts on U_{slm}, is reduced (increased). In the limiting cases, the rate of underemployment approaches the one of a single competitive labor market ($\gamma \to 0$) or a fully unionized labor market ($\gamma \to 1$).

Let the commodity price $p > 0$ and its next period expectation $p^e > 0$ be given and consider the situation when the labor markets are in equilibrium. Then, because of (12), $N_{\mathrm{com}}(w_{\mathrm{elm}}/p^e) = h_{\mathrm{com}}(w_{\mathrm{elm}}/p) =: L$ so that

$$U_{\mathrm{slm}}\left(\gamma L, (1-\gamma)L, \frac{w_{\mathrm{ilm}}}{p^e}, \frac{w_{\mathrm{elm}}}{p^e}\right) = 1 - \frac{L}{\gamma N_{\mathrm{com}}(w_{\mathrm{ilm}}/p^e) + (1-\gamma)L}. \tag{24}$$

Evaluating (14) at $w_{\mathrm{ilm}}/p = F'(L)$ yields that $p^e/p = W_{\mathrm{elm}}^{-1}(F'(L))$ so that

$$\frac{w_{\mathrm{ilm}}}{p^e} = \frac{W_{\mathrm{ilm}}(p^e/p, \lambda, \gamma)}{p^e/p} = \frac{W_{\mathrm{ilm}}(W_{\mathrm{elm}}^{-1}(F'(L)), \lambda, \gamma)}{W_{\mathrm{elm}}^{-1}(F'(L))}$$

shows that U_{slm} can be written in terms of L (or, equivalently, expected inflation p^e/p) and the union parameters λ and γ.

If the production function F, the reservation wage function S_{res}, and the competitive labor supply function N_{com} are isoelastic with elasticities B, $1/C$, and C, the aggregate employment level L cancels out because of (20) so that the rate of underemployment (24) becomes

$$\begin{aligned}
U_{\mathrm{slm}}\left(\gamma L, (1-\gamma)L, \frac{w_{\mathrm{ilm}}}{p^e}, \frac{w_{\mathrm{elm}}}{p^e}\right) &= 1 - \frac{1}{\gamma\left(\lambda \frac{1-(1-\gamma)B}{\gamma B} + (1-\lambda)\frac{C}{C+1}\right)^C + (1-\gamma)} \\
&= 1 - \frac{1}{1 + \gamma\left(\left(\frac{C}{C+1} + \lambda\left(\frac{1-B}{\gamma B} + \frac{1}{C+1}\right)\right)^C - 1\right)}
\end{aligned} \tag{25}$$

which depends only on the elasticities and on the union parameters λ and γ, i.e. it is independent of the expected inflation p^e/p. While the function (25) is increasing in union power λ, the effects of an increase of union density γ are ambiguous.

3 The Temporary Equilibrium with a Segmented Labor Market

In the previous section, the employment levels and wages have been determined as functions of the commodity price $p > 0$, its next period forecast $p^e > 0$, the union's bargaining power $0 \leq \lambda \leq 1$, and union density $0 < \gamma < 1$. These are the prerequisites to close the model, to determine the temporary equilibrium – in particular the equilibrium price –, and its properties. In addition to the parameters p^e, λ, and γ, aggregate money holdings $M \geq 0$ which are held by old consumers are the data at the beginning of an arbitrary period. To facilitate notation, let $m := M/p$ and $\theta^e := p^e/p$ denote real money balances and the expected inflation.

3.1 Aggregate Supply and Aggregate Demand

Although the employment levels on the two labor markets each depend on union density γ, the aggregate employment level $L = L_{\text{ilm}} + L_{\text{elm}} = h_{\text{com}}(W_{\text{elm}}(\theta^e))$ is independent of γ, as shown in equation (17). The level of bargaining power λ affects the bargaining wage only, but neither the employment levels on the labor markets nor the aggregate employment level. Therefore, the *aggregate commodity supply function*, i.e. the output the firm produces from the labor input $L = h_{\text{com}}(W_{\text{elm}}(\theta^e))$, is a function of the expected inflation θ^e alone. It is defined as

$$AS_{\text{slm}} : \mathbb{R}_{++} \to \mathbb{R}_{++}, \quad AS_{\text{slm}}\left(\theta^e\right) := F\left(h_{\text{com}}\left(W_{\text{elm}}\left(\theta^e\right)\right)\right). \tag{26}$$

The aggregate commodity supply function is a strictly monotonically decreasing function in expected inflation with an explicitly given inverse

$$AS^{-1}_{\text{slm}}(y) = W^{-1}_{\text{elm}}\left(F'\left(F^{-1}(y)\right)\right) = \frac{F'\left(F^{-1}(y)\right)}{S_{\text{com}}\left(F^{-1}(y)\right)}.$$

Since the aggregate supply function is strictly decreasing in $\theta^e = p^e/p$, it is strictly monotonically increasing in the commodity price p. Equation (13), which states that the wage functions W_{elm} (the wage on the external labor market) and W_{com} (the wage as under a fully competitive labor market) are identical, implies that the aggregate supply functions under a segmented labor market AS_{slm} and under a fully competitive labor market AS_{com} are identical, i.e.

$$AS_{\text{slm}}(\theta^e) \equiv AS_{\text{com}}(\theta^e).$$

Therefore, compared to a single, fully competitive labor market, the labor market specification (and, in particular, the different wages paid on the labor markets) induces no change to the supply side of the economy.

Due to the overlapping-generations structure of consumers, young consumers face a consumption–savings decision. Because of the specific assumptions made, only the propensity $0 \leq c(\theta^e) \leq 1$ of the net profit income is consumed by the young shareholders while the wage income is saved entirely. Thus, aggregate real demand y is the sum of real money balances m, public demand g, and the demand by the young shareholders which, as seen in (21), is proportional to y. Therefore, given money balances M, a price expection p^e, the bargaining weight λ, and union density γ, as well as the public demand g and the tax rate τ_π on profit income, income consistency implies that the aggregate demand y has to solve

$$y = \frac{M}{p} + g + c\left(\frac{p^e}{p}\right)(1-\tau_\pi)(1-\lambda)\left(1 - E_F(L) + \gamma\frac{E_F(L)E_{S_{\text{res}}}(L)}{E_{S_{\text{res}}}(L)+1}\right)y, \tag{27}$$

with $L = h_{\text{com}}(W_{\text{elm}}(p^e/p))$.[19]

Define the *demand multiplier* with respect to real money balances $m = M/p$ and government demand g as a function $\tilde{c}_{\text{slm}} : \mathbb{R}_+ \times [0,1] \times (0,1) \to [0,1]$ of expected inflation $\theta^e = p^e/p$, union power λ, and union density γ by

$$\tilde{c}_{\text{slm}}(\theta^e, \lambda, \gamma) := 1 - c(\theta^e)(1-\tau_\pi)(1-\lambda)\left(1 - E_F(L) + \gamma \frac{E_F(L) E_{S_{\text{res}}}(L)}{E_{S_{\text{res}}}(L) + 1}\right),$$

with $L = h_{\text{com}}(W_{\text{elm}}(\theta^e))$ so that (27) becomes $m + g = \tilde{c}_{\text{slm}}(\theta^e, \lambda, \gamma) y$.

The demand multiplier is strictly monotonically increasing in bargaining power λ and strictly monotonically decreasing in union density γ; the effect of expected inflation cannot be signed in general. Then, the *income-consistent aggregate demand function* solving (27) for y is given by

$$D_{\text{slm}} : \mathbb{R}^2_+ \times [0,1] \times (0,1) \to \mathbb{R}_+, \qquad D_{\text{slm}}(m, \theta^e, \lambda, \gamma) := \frac{m+g}{\tilde{c}_{\text{slm}}(\theta^e, \lambda, \gamma)} \tag{28}$$

which is strictly monotonically increasing in real money balances m, strictly decreasing in bargaining power λ, and strictly increasing in union density γ; the effect of price expectations θ^e is ambiguous. Note that the effects of union power and union density cannot be signed generally if young workers consumed as well.

If F and S_{res} are isoelastic functions in labor with elasticities B and $1/C$, with B and C in the unit interval, then the *aggregate demand function* is given by

$$D_{\text{slm}}(m, \theta^e, \lambda, \gamma) = \frac{m+g}{1 - c(\theta^e)(1-\tau_\pi)(1-\lambda)(1 - B + \gamma \frac{B}{C+1})}.$$

If the propensity to consume $c' \geq 0$ increases in expected inflation, the aggregate demand function is monotonically increasing in expected inflation θ^e and strictly decreasing in nominal prices p, i.e.

$$\frac{\mathrm{d}D_{\text{slm}}\left(\frac{M}{p}, \frac{p^e}{p}, \lambda, \gamma\right)}{\mathrm{d}p} = -\frac{m}{p}\frac{\partial D_{\text{slm}}(m, \theta^e, \lambda, \gamma)}{\partial m} - -\frac{\theta^e}{p}\frac{\partial D_{\text{slm}}(m, \theta^e, \lambda, \gamma)}{\partial \theta^e} < 0.$$

3.2 The Price Law

For the remainder of this paper, assume that $\partial \tilde{c}_{\text{slm}}/\partial \theta^e \leq 0$. Then, the aggregate demand function (28) is strictly decreasing in nominal prices p whereas the aggregate supply function (26) is strictly increasing in nominal prices p.

For any tuple $(M, p^e, \lambda, \gamma) \in \mathcal{X} := \mathbb{R}^2_+ \times [0,1] \times (0,1)$, the *temporary equilibrium* of the economy is given by a price $p \geq 0$ which clears the commodity market, i.e.

$$AS_{\text{slm}}\left(\frac{p^e}{p}\right) = D_{\text{slm}}\left(\frac{M}{p}, \frac{p^e}{p}, \lambda, \gamma\right). \tag{29}$$

Lemma 3.1. *Let the aggregate supply function AS_{slm} be globally invertible with $AS'_{\text{slm}}(\theta^e) < 0$, and assume that $\partial D_{\text{slm}}/\partial m > 0$, $\partial D_{\text{slm}}/\partial \theta^e \geq 0$ hold. Then, for every $(M, p^e, \lambda, \gamma) \in \mathcal{X}$, there exists a unique positive temporary equilibrium price $p \geq 0$ solving the above stated market-clearing equation* (29)*.*

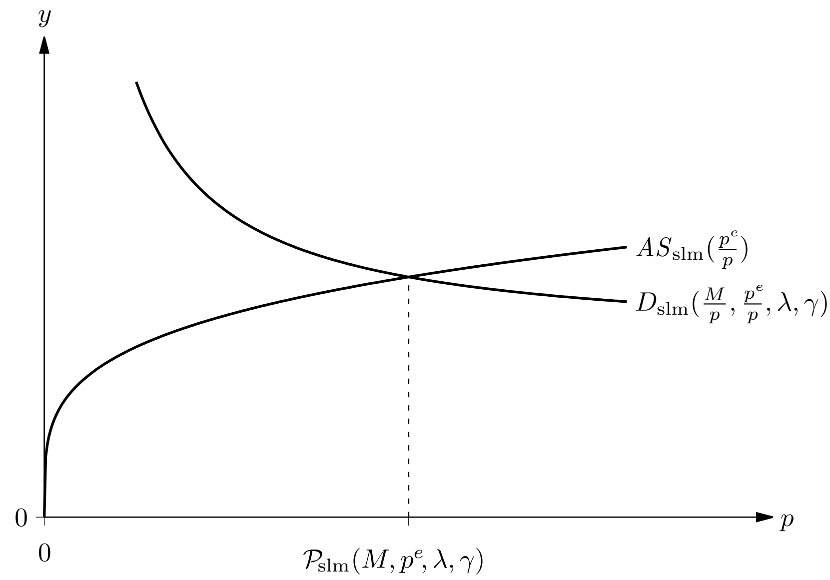


Figure 3: The temporary equilibrium price

Figure 3 shows the aggregate supply–aggregate demand diagram of the commodity market.

of Lemma 3.1. Since the aggregate supply function is invertible, it has full range so that there is excess demand for small prices while supply exceeds demand for high prices; because of the continuity of the excess demand function, the market-clearing price exists. Its uniqueness follows from the monotonicity of the supply and demand functions. □

Let the temporary equilibrium price be denoted by the *price law*

$$\mathcal{P}_{\text{slm}} : \mathcal{X} \to \mathbb{R}_+, \qquad p = \mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma).$$

The price law is a time-invariant mapping of the state space $\mathcal{X}$ into the positive reals. It is homogeneous of degree 1 in $(M, p^e) \gg 0$ for any given $(\lambda, \gamma) \in [0, 1] \times (0, 1)$. Applying the implicit function theorem on the excess demand function $D_{\text{slm}}(M/p, p^e/p, \lambda, \gamma) - AS_{\text{slm}}(p^e/p)$ yields

$$\frac{\partial \mathcal{P}_{\text{slm}}}{\partial M} = \frac{\frac{1}{\mathcal{P}_{\text{slm}}} \frac{\partial D_{\text{slm}}}{\partial m}}{-\frac{p^e}{\mathcal{P}_{\text{slm}}{}^2} AS'_{\text{slm}} + \frac{M}{\mathcal{P}_{\text{slm}}{}^2} \frac{\partial D_{\text{slm}}}{\partial m} + \frac{p^e}{\mathcal{P}_{\text{slm}}{}^2} \frac{\partial D_{\text{slm}}}{\partial \theta^e}} > 0$$

with an elasticity

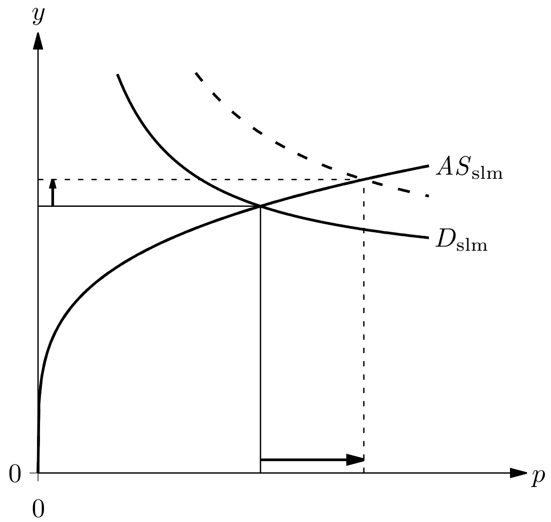
$$0 < E_{\mathcal{P}_{\text{slm}}}(M) = \frac{E_{D_{\text{slm}}}(m)}{-E_{AS_{\text{slm}}}(\theta^e) + E_{D_{\text{slm}}}(m) + E_{D_{\text{slm}}}(\theta^e)} < 1.$$

Because of the homogeneity of the price law, $0 < E_{\mathcal{P}_{\text{slm}}}(p^e) < 1$ and, in particular, $\partial \mathcal{P}_{\text{slm}} / \partial p^e > 0$ hold true. The positive effects of money balances and price expectations on the equilibrium price are depicted in Figure 4. Similar calculations yield

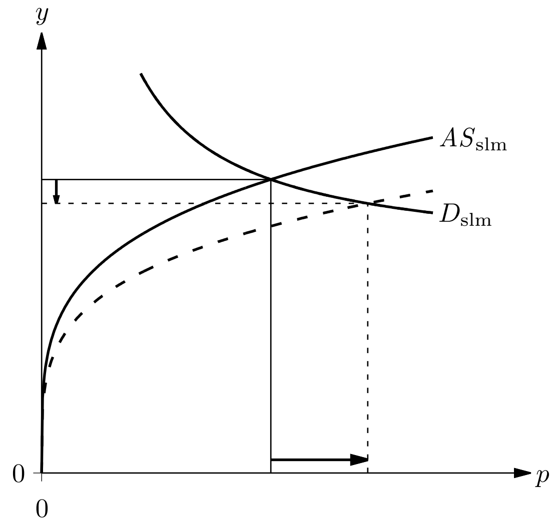
$$\frac{\partial \mathcal{P}_{\text{slm}}}{\partial \lambda} < 0 \qquad \text{and} \qquad \frac{\partial \mathcal{P}_{\text{slm}}}{\partial \gamma} > 0.$$

Figure 5 displays how the union parameters λ and γ affect the temporary equilibrium. The left panel of Figure 5 shows the ranges of prices and aggregate output for all values of bargaining

[19] If young workers consumed as well, terms consisting of their net propensity of consumption times their according wage shares were added.

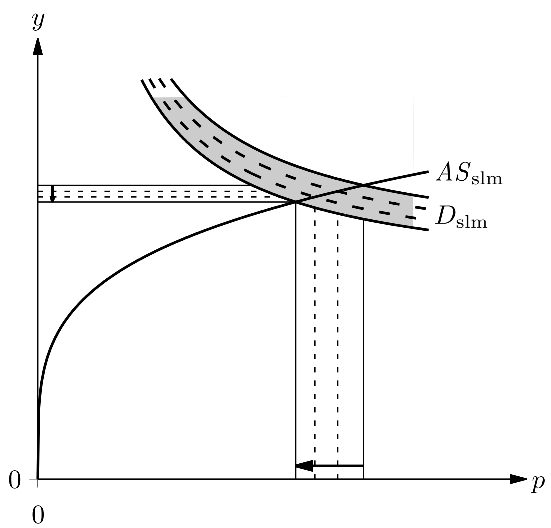


(a) Increase of money balances M

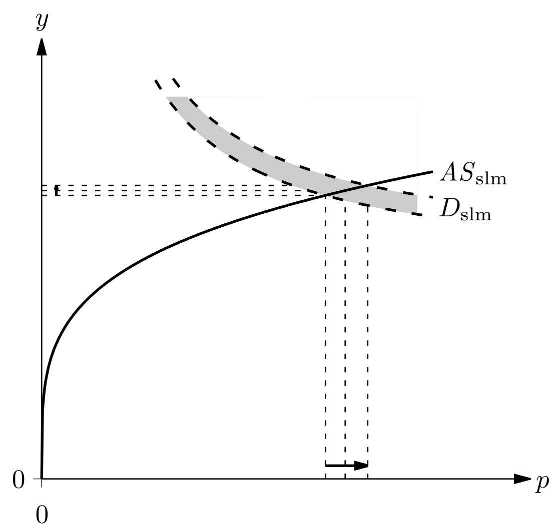


(b) Increase of p^e (for $\partial D_{\text{slm}}/\partial \theta^e = 0$)

Figure 4: Comparative-statics effects of money balances and price expectations



(a) Range of equilibria for $\lambda \in [0, 1]$; γ given



(b) Range of equilibria for $\gamma \in (0, 1)$; $\lambda = 0$

Figure 5: Comparative-statics effects of union power and union density

power $\lambda \in [0,1]$, for given money balances M, price expectations p^e, and for a given degree of unionization γ. The right panel of Figure 5 depicts the ranges of prices and aggregate output for all values of union density $\gamma \in (0,1)$ and for given money balances M, price expectations p^e, and union power $\lambda = 0$. Bigger levels of union power would result in smaller changes of the demand multiplier $\tilde{c}_{\text{slm}}$ and thus in smaller ranges of the equilibrium values. Both figures indicate that there is a nonlinear feedback from the union parameters on the equilibrium price.

3.3 The Equilibrium Mappings

The equilibrium price induces the equilibrium wages, employment levels, and the aggregate output level. These mappings are called the wage laws, the employment laws, and the output law which are all functions on the state space $\mathcal{X}$ and which are defined as follows:

$$\mathcal{W}_{\text{ilm}}(M,p^e,\lambda,\gamma) := \mathcal{P}_{\text{slm}}(M,p^e,\lambda,\gamma) W_{\text{ilm}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M,p^e,\lambda,\gamma)},\lambda,\gamma\right)$$

$$\mathcal{W}_{\text{elm}}(M,p^e,\lambda,\gamma) := \mathcal{P}_{\text{slm}}(M,p^e,\lambda,\gamma) W_{\text{elm}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M,p^e,\lambda,\gamma)}\right)$$

$$\mathcal{L}_{\text{ilm}}(M,p^e,\lambda,\gamma) := h_{\text{ilm}}\left(\frac{\mathcal{W}_{\text{elm}}(M,p^e,\lambda,\gamma)}{\mathcal{P}_{\text{slm}}(M,p^e,\lambda,\gamma)},\frac{p^e}{\mathcal{P}_{\text{slm}}(M,p^e,\lambda,\gamma)},\gamma\right)$$

$$\mathcal{L}_{\text{elm}}(M,p^e,\lambda,\gamma) := h_{\text{elm}}\left(\frac{\mathcal{W}_{\text{elm}}(M,p^e,\lambda,\gamma)}{\mathcal{P}_{\text{slm}}(M,p^e,\lambda,\gamma)},\frac{p^e}{\mathcal{P}_{\text{slm}}(M,p^e,\lambda,\gamma)},\gamma\right)$$

$$\mathcal{Y}_{\text{slm}}(M,p^e,\lambda,\gamma) := AS_{\text{slm}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M,p^e,\lambda,\gamma)}\right)$$

Furthermore, the aggregate-employment law and the average-wage law can be defined as follows:

$$\begin{aligned}
&\mathcal{L}_{\text{slm}}(M,p^e,\lambda,\gamma)\\
&:= \mathcal{L}_{\text{ilm}}(M,p^e,\lambda,\gamma) + \mathcal{L}_{\text{elm}}(M,p^e,\lambda,\gamma) = h_{\text{com}}\left(W_{\text{elm}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M,p^e,\lambda,\gamma)}\right)\right)
\end{aligned}$$

$$\begin{aligned}
&\mathcal{W}_{\text{slm}}(M,p^e,\lambda,\gamma)\\
&:= \frac{\mathcal{W}_{\text{ilm}}(M,p^e,\lambda,\gamma)\mathcal{L}_{\text{ilm}}(M,p^e,\lambda,\gamma) + \mathcal{W}_{\text{elm}}(M,p^e,\lambda,\gamma)\mathcal{L}_{\text{elm}}(M,p^e,\lambda,\gamma)}{\mathcal{L}_{\text{slm}}(M,p^e,\lambda,\gamma)}\\
&= \gamma\mathcal{W}_{\text{ilm}}(M,p^e,\lambda,\gamma) + (1-\gamma)\mathcal{W}_{\text{elm}}(M,p^e,\lambda,\gamma).
\end{aligned}$$

Exploiting the structure of the bargaining solution (9), the payoffs laws

$$\boldsymbol{\Omega}_{\text{slm}}(M,p^e,\lambda,\gamma) := \Omega_{\text{slm}}(p^e,\mathcal{W}_{\text{ilm}}(M,p^e,\lambda,\gamma),\mathcal{L}_{\text{ilm}}(M,p^e,\lambda,\gamma),\gamma)$$

$$\boldsymbol{\Pi}_{\text{slm}}(M,p^e,\lambda,\gamma) := \begin{cases} \frac{1-\lambda}{\lambda}\boldsymbol{\Omega}_{\text{slm}}(M,p^e,\lambda,\gamma), & \text{if } 0 < \lambda \leq 1 \\ 0, & \text{if } \lambda = 0 \end{cases} \tag{30}$$

are defined in the same fashion, and the law of the rate of underemployment is

$$\mathcal{U}_{\text{slm}}(M, p^e, \lambda, \gamma) := U_{\text{slm}}\left(\mathcal{L}_{\text{ilm}}(M, p^e, \lambda, \gamma), \mathcal{L}_{\text{elm}}(M, p^e, \lambda, \gamma), \frac{\mathcal{W}_{\text{ilm}}(M, p^e, \lambda, \gamma)}{p^e}, \frac{\mathcal{W}_{\text{elm}}(M, p^e, \lambda, \gamma)}{p^e}\right).$$

For any given pair $(\lambda, \gamma) \in [0,1] \times (0,1)$, the wage laws are homogeneous of degree 1 in $(M, p^e) \gg 0$, and the employment and output laws are homogeneous of degree 0 in $(M, p^e) \gg 0$. Thus, the payoff laws are homogeneous of degree 1 in $(M, p^e) \gg 0$ as well while the law of the rate of underemployment is of degree 0 in $(M, p^e) \gg 0$.

The comparative-statics effects are derived in Appendix 6.1 and their results are summarized in the Table 2. The effects of the union parameters λ and γ on equilibrium prices and employment

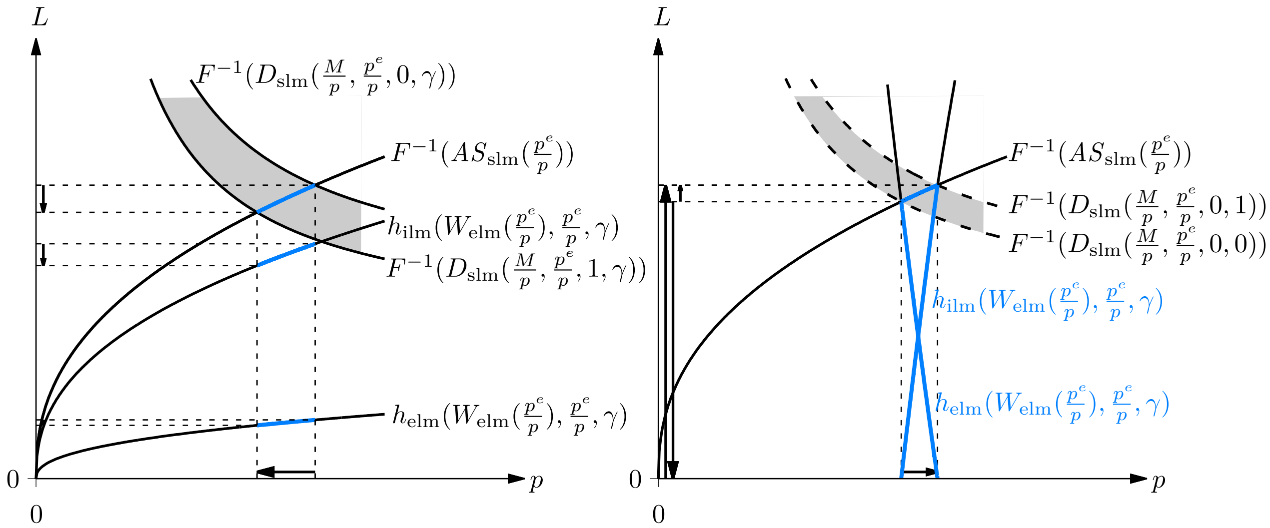
Table 2: Summary of comparative-statics effects. Parentheses indicate that further assumptions such as isoelastic specifications required.

	M	p^e	λ	γ
$\mathcal{P}_{\text{slm}}$	$+$	$+$	$-$	$+$
$\mathcal{W}_{\text{ilm}}$	$(+)$	$(+)$	**?**	?
$\mathcal{W}_{\text{elm}}$	$+$	$+$	$-$	$+$
$\mathcal{W}_{\text{slm}}$	$(+)$	$(+)$	**?**	?
$\mathcal{L}_{\text{ilm}}$	$+$	$-$	$-$	$+$
$\mathcal{L}_{\text{elm}}$	$+$	$-$	$-$	**?**
$\mathcal{L}_{\text{slm}}$	$+$	$-$	$-$	$+$
$\mathcal{Y}_{\text{slm}}$	$+$	$-$	$-$	$+$
$\mathbf{\Pi}_{\text{slm}}$	$(+)$	$(+)$	$(-)$	$(+)$
$\mathbf{\Omega}_{\text{slm}}$	$(+)$	$(+)$	**?**	$(+)$
$\mathcal{U}_{\text{slm}}$	(0)	(0)	$(+)$	**?**

levels as well as on equilibrium payoffs are demonstrated in Figure 6 and Figure 7.

Figure 6(a) depicts the equilibrium employment levels on both labor markets (the bold blue lines) as well as their aggregate. It shows that all equilibrium employment levels are always positive and that they are strictly decreasing and proportional in union power λ. For a small size of the respective workforce (i.e. γ close to zero or one), the dispersion of the employment level on the respective market is small. The ranges of the equilibrium levels of employment with respect to union density γ which are plotted in Figure 6(b) differ in two regards from the ones depicted in the first panel: First, the employment level on the internal market increases in union density whereas the employment level on the external market usually decreases.[20] Second, since the amount of labor traded on the internal labor market or on the external labor market approaches zero if γ approaches zero or one, the dispersion of employment levels is relatively large in union density.

[20]Counterexamples can be constructed for very high levels of public consumption g and very flat production functions.



(a) Range of equilibria for $\lambda \in [0, 1]$; γ given

(b) Range of equilibria for $\gamma \in (0, 1)$; $\lambda = 0$

Figure 6: Ranges of prices and employment

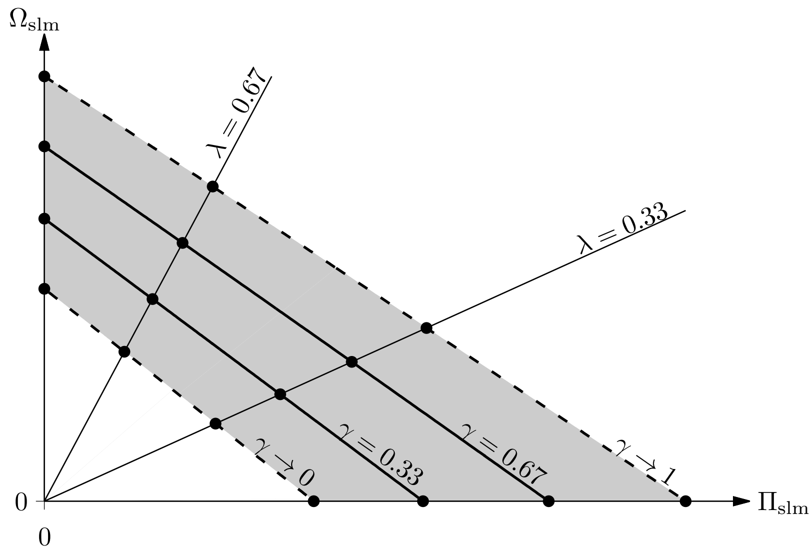


Figure 7: Ranges of payoffs λ resp. γ from zero to one

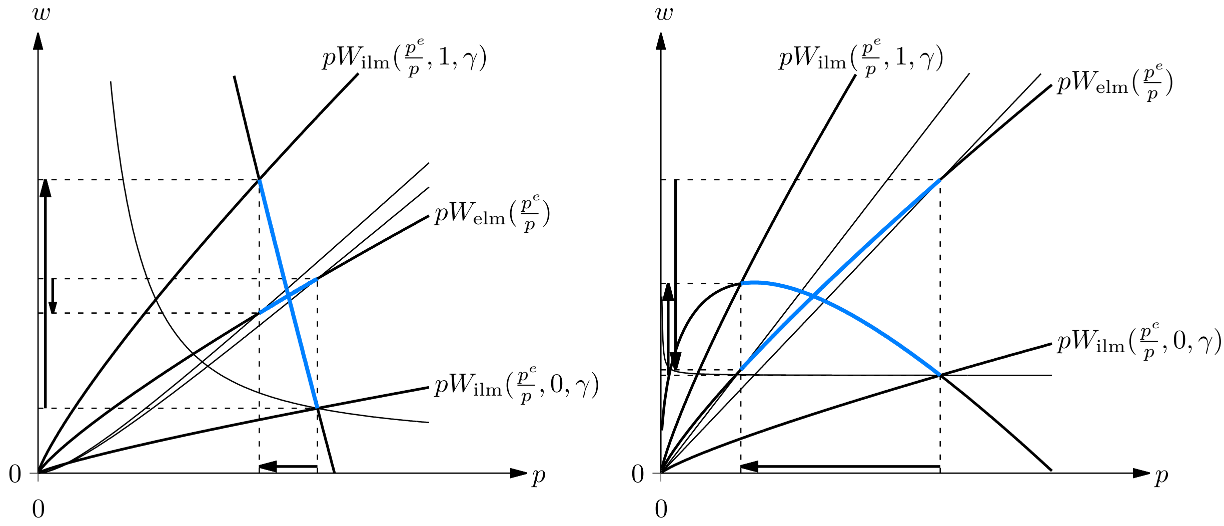


Figure 8: Ranges of prices and wages; λ from 0 to 1

The effects of union power λ and union density γ on the equilibrium payoffs are illustrated in Figure 7. The shaded area contains all possible pairs of equilibrium profits $\mathbf{\Pi}_{\text{slm}}$ and equilibrium union utility $\mathbf{\Omega}_{\text{slm}}$ for $0 \leq \lambda \leq 1$ and $0 < \gamma < 1$. For given λ, all equilibria are located on a ray through the origin with slope $\lambda/(1-\lambda)$; the boundary cases $\lambda = 0$ and $\lambda = 1$ coincide with segments of the axes.[21] The value $\lambda/(1-\lambda)$ is the ratio of the relative bargaining powers of the two agents and the ratio of their shares of the joint surplus. Given γ, an increase of λ therefore results in a counterclockwise rotation of the ray which causes profits to decrease and union utility to usually increase, i.e. a change in the distribution of the joint surplus. Due to the positive effect of union density γ on the joint surplus, an increase of γ increases both profits and union utility, maintaining the same sharing ratio $\lambda/(1-\lambda)$.

Figure 8 displays the different effects of union power λ on prices and wages in equilibrium. Both panels show that, in equilibrium, the wage on the competitive external labor market $\mathcal{W}_{\text{elm}}$ is reduced by an increase of union power due to the negative effect of λ on the equilibrium price $\mathcal{P}_{\text{slm}}$. However, the effect of union power on the wage on the internal labor market is ambiguous: Figure 8(a) depicts a situation in which an increase of λ leads to a rising wage on the internal labor market whereas Figure 8(b) indicates that $\mathcal{W}_{\text{ilm}}$ can be decreasing in λ for some parametrizations, in particular ones with high government activity. For each parametrization, there exists one level of union power for which the wages on the two labor markets coincide, implying that the ranges of wages overlap necessarily and are never disjoint. The wage law $\mathcal{W}_{\text{ilm}}$ is above $\mathcal{W}_{\text{elm}}$ if and only if λ is greater than the level of union power for which the wages on the two labor markets coincide. The dispersion of wages on the external labor market can be smaller than on the internal market (cf. Figure 8(a)) or bigger (cf. Figure 8(b)). Figure 8(b) also shows that the effect of union power λ on the average of the two wage laws $\mathcal{W}_{\text{slm}}$ is ambiguous because it is increasing for low levels of λ and decreasing for λ close to one.

If the wage function W_{ilm} is increasing in expected inflation, the real wage paid on the internal labor market $\mathcal{W}_{\text{ilm}}/\mathcal{P}_{\text{slm}} = W_{\text{ilm}}(p^e/\mathcal{P}_{\text{slm}})$ is increasing in λ so that a negative influence of λ on the wage $\mathcal{W}_{\text{ilm}}$ can only occur under a dominating price effect $\partial\mathcal{P}_{\text{slm}}/\partial\lambda$. Whenever a

[21]This property immediately follows from (9) or from (30).

negative wage effect as shown in Figure 8(b) occurs for wages above the competitive level, i.e. $\mathcal{W}_{\text{ilm}} > p^e S_{\text{com}}(\mathcal{L}_{\text{ilm}}/\gamma)$, the excess wage bill $\mathbf{\Omega}_{\text{slm}}$ is decreasing in λ.

4 Comparison with Economies with Single Labor Markets

As shown above, the condition (11) which defines a market-clearing wage on the external labor market can be reformulated as

$$h_{\text{com}}\left(\frac{w}{p}\right) \overset{!}{=} N_{\text{com}}\left(\frac{w}{p^e}\right).$$

This is exactly the labor-market-clearing condition under the absence of a union, i.e. under a fully competitive labor market. Therefore, the real wage clearing the external labor market must be the same as the real wage clearing a fully competitive labor market, i.e.

$$W_{\text{elm}}(\theta^e) \equiv W_{\text{com}}(\theta^e), \tag{31}$$

which immediately implies that the aggregate supply functions are identical, i.e.

$$AS_{\text{slm}}(\theta^e) \equiv AS_{\text{com}}(\theta^e). \tag{32}$$

In order to compare the equilibria under labor market segmentation and under single labor markets, first consider the level of bargaining power $\lambda_{\text{nat}} : \mathbb{R}_+ \times (0,1) \to [0,1]$,

$$\lambda_{\text{nat}}(\theta^e,\gamma) := 1 - \frac{1 - E_F(L)}{1 - E_F(L) + \gamma\frac{E_F(L)E_{S_{\text{res}}}(L)}{E_{S_{\text{res}}}(L)+1}}, \qquad \text{with } L = h_{\text{com}}(W_{\text{com}}(\theta^e)).$$

By construction, the identity of the wage functions (31) induces the following three properties: First, under labor market segmentation, the wages on the internal and on the external labor market coincide, i.e.

$$W_{\text{ilm}}(\theta^e, \lambda_{\text{nat}}(\theta^e,\gamma),\gamma) \overset{(19)}{\equiv} W_{\text{elm}}(\theta^e)$$

so that, second, the profit share of total revenue (21) under labor market segmentation is the same as the profit share under a fully competitive single labor market, i.e.

$$(1 - \lambda_{\text{nat}}(\theta^e,\gamma))\left(1 - E_F(L) + \gamma\frac{E_F(L)E_{S_{\text{res}}}(L)}{E_{S_{\text{res}}}(L)+1}\right), \quad \text{with } L = h_{\text{com}}(W_{\text{com}}(\theta^e))$$

$$\iff \quad 1 - E_F(L), \quad \text{with } L = h_{\text{com}}(W_{\text{com}}(\theta^e)),$$

and, finally, the aggregate demand functions under labor market segmentation and under full competition are identical, i.e.

$$D_{\text{slm}}(m,\theta^e,\lambda_{\text{nat}}(\theta^e,\gamma),\gamma) \equiv D_{\text{com}}(m,\theta^e). \tag{33}$$

The level of bargaining power $\lambda = \lambda_{\text{nat}}(\theta^e,\gamma)$ therefore can be interpreted as the "natural" level of bargaining power at which the (partial-equilibrium) wage on the internal labor market and the (partial-equilibrium) aggregate employment level are the same.

Evaluating $\lambda_{\text{nat}}(p^e/p, \gamma)$ at an equilibrium price yields an equilibrium notion of the natural level of bargaining power: Let $\lambda_{\text{com}} : \mathbb{R}^2_+ \times (0,1) \to [0,1]$ be defined as the level of λ_{nat} at the equilibrium price $\mathcal{P}_{\text{com}}(M, p^e)$ under a fully competitive labor market, i.e.

$$\lambda_{\text{com}}(M, p^e, \gamma) := \lambda_{\text{nat}}\left(\frac{p^e}{\mathcal{P}_{\text{com}}(M, p^e)}, \gamma\right).$$

Note that, if F and S_{res} are isoelastic functions with elasticities B and $1/C$, then the two levels of bargaining power are identical, i.e.

$$\lambda_{\text{com}}(M, p^e, \gamma) \equiv \frac{\gamma\frac{B}{C+1}}{1 - B + \gamma\frac{B}{C+1}} \equiv \lambda_{\text{nat}}(\theta^e, \gamma).$$

Using property (33), and evaluating the demand functions at the price $p = \mathcal{P}_{\text{com}}(p^e, M)$ yields

$$D_{\text{slm}}\left(\frac{M}{\mathcal{P}_{\text{com}}(M, p^e)}, \frac{p^e}{\mathcal{P}_{\text{com}}(M, p^e)}, \lambda_{\text{com}}(M, p^e, \gamma), \gamma\right) \equiv D_{\text{com}}\left(\frac{M}{\mathcal{P}_{\text{com}}(M, p^e)}, \frac{p^e}{\mathcal{P}_{\text{com}}(M, p^e)}\right).$$

This, in combination with the identity of the aggregate supply functions (32), implies that

$$\mathcal{P}_{\text{slm}}\Big(M, p^e, \lambda_{\text{com}}(M, p^e, \gamma), \gamma\Big) \equiv \mathcal{P}_{\text{com}}(M, p^e)$$

has to hold. Therefore, the model with a single competitive labor market is the special case $\lambda = \lambda_{\text{com}}(M, p^e, \gamma)$ of the model with a segmented labor market, i.e. the function $\lambda_{\text{com}}(M, p^e, \gamma)$ is the *equilibrium* condition to ensure the same equilibrium price in both models. Straightforward calculations show that also the aggregate levels of the real variables as well as all wages coincide in equilibrium, i.e.

$$\mathcal{Y}_{\text{com}}(M, p^e) \equiv \mathcal{Y}_{\text{slm}}\Big(M, p^e, \lambda_{\text{com}}(M, p^e, \gamma), \gamma\Big),$$

$$\mathcal{L}_{\text{com}}(M, p^e) \equiv \mathcal{L}_{\text{slm}}\Big(M, p^e, \lambda_{\text{com}}(M, p^e, \gamma), \gamma\Big),$$

$$\begin{aligned}\mathcal{W}_{\text{com}}(M, p^e) &\equiv \mathcal{W}_{\text{slm}}\Big(M, p^e, \lambda_{\text{com}}(M, p^e, \gamma), \gamma\Big)\\ &\equiv \mathcal{W}_{\text{ilm}}\Big(M, p^e, \lambda_{\text{com}}(M, p^e, \gamma), \gamma\Big)\\ &\equiv \mathcal{W}_{\text{elm}}\Big(M, p^e, \lambda_{\text{com}}(M, p^e, \gamma), \gamma\Big).\end{aligned}$$

As before, union density γ determines the relative sizes of the two labor markets, i.e.

$$\mathcal{L}_{\text{ilm}}\Big(M, p^e, \lambda_{\text{com}}(M, p^e, \gamma), \gamma\Big) \equiv \gamma\mathcal{L}_{\text{com}}(M, p^e),$$

$$\mathcal{L}_{\text{elm}}\Big(M, p^e, \lambda_{\text{com}}(M, p^e, \gamma), \gamma\Big) \equiv (1-\gamma)\mathcal{L}_{\text{com}}(M, p^e).$$

Because of $\mathcal{W}_{\text{ilm}}(M, p^e, \lambda_{\text{com}}(M, p^e, \gamma), \gamma) \equiv \mathcal{W}_{\text{elm}}(M, p^e, \lambda_{\text{com}}(M, p^e, \gamma), \gamma)$, the equilibrium rate of underemployment

$$\mathcal{U}_{\text{slm}}(M, p^e, \lambda_{\text{com}}(M, p^e, \gamma), \gamma) = 0$$

is zero.

The difference between $\lambda_{\text{com}}(M, p^e, \gamma)$ and $\lambda_{\text{nat}}(\theta^e, \gamma)$ is that λ_{com} is a general-equilibrium mapping whereas λ_{nat} is a benchmark for analyzing the partial equilibrium on the internal (unionized) labor market.

Similarly, because of

$$\lim_{\gamma\to 1} D_{\text{slm}}(m, \theta^e, \lambda, \gamma) = D_{\text{eff}}(m, \theta^e, \lambda) \qquad \text{for all } (m, \theta^e, \lambda)$$

and $AS_{\text{slm}} = AS_{\text{eff}}$, the model with a single labor market governed by efficient bargaining is the limiting case $\gamma \to 1$ of the present model:

$$\begin{aligned}
\mathcal{P}_{\text{eff}}(M, p^e, \lambda) &\equiv \lim_{\gamma\to 1} \mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma),\\
\mathcal{Y}_{\text{eff}}(M, p^e, \lambda) &\equiv \lim_{\gamma\to 1} \mathcal{Y}_{\text{slm}}(M, p^e, \lambda, \gamma),\\
\mathcal{L}_{\text{eff}}(M p^e, \lambda) &\equiv \lim_{\gamma\to 1} \mathcal{L}_{\text{slm}}(M, p^e, \lambda, \gamma),\\
\mathcal{W}_{\text{eff}}(M, p^e, \lambda) &\equiv \lim_{\gamma\to 1} \mathcal{W}_{\text{slm}}(M, p^e, \lambda, \gamma) \equiv \lim_{\gamma\to 1} \mathcal{W}_{\text{ilm}}(M, p^e, \lambda, \gamma).
\end{aligned}$$

Also, since $\lim_{\gamma\to 1} \mathcal{W}_{\text{elm}}(M, p^e, \lambda, \gamma) \equiv \mathcal{W}_{\text{com}}(M, p^e)$ is finite for given (M, p^e, λ), the rate of underemployment approaches the one under a fully unionized labor market

$$\begin{aligned}
&\lim_{\gamma\to 1} \mathcal{U}_{\text{slm}}(M, p^e, \lambda, \gamma)\\
&\overset{(23)}{=} \lim_{\gamma\to 1} 1 - \frac{\mathcal{L}_{\text{slm}}(M, p^e, \lambda, \gamma)}{\gamma N_{\text{com}}(\mathcal{W}_{\text{ilm}}(M, p^e, \lambda, \gamma)/p^e) + (1-\gamma) N_{\text{com}}(\mathcal{W}_{\text{elm}}(M, p^e, \lambda, \gamma)/p^e)}\\
&= 1 - \frac{\mathcal{L}_{\text{slm}}(M, p^e, \lambda)}{N_{\text{com}}(\mathcal{W}_{\text{ilm}}(M, p^e, \lambda)/p^e)} = 1 - \frac{\mathcal{L}_{\text{eff}}(M, p^e, \lambda)}{N_{\text{com}}(\mathcal{W}_{\text{eff}}(M, p^e, \lambda)/p^e)} = \mathcal{U}_{\text{eff}}(M, p^e, \lambda)
\end{aligned}$$

if union density γ approaches one.

5 Summary and Extensions

This paper has presented a monetary macroeconomy of the AS–AD type with a segmented labor market (SLM) with efficient union–firm bargaining on the internal labor market and a competitive external labor market which extends Böhm & Claas (2012) and Böhm (2017). The splitting of the labor market has led to an additional state variable, namely union density γ. In most cases, the effects of γ are opposite to the effects of the union's relative bargaining power λ. Temporary equilibria uniquely exist under the same general set of assumptions as under the related approaches. The qualitative results are structurally the same, with the only exception being the equilibrium wage on the external labor market which decreases in the union's bargaining power λ.

The union parameters λ and γ directly affect the wage on the internal labor market and therefore lead to wage differentiation between the two labor markets. Furthermore, they drive the functional income distribution which feeds into the aggregate demand function. However, since the employment levels on the two labor markets are proportional to the respective group sizes, the individual employment levels are the same for all workers. Thus, the individual employment levels are independent of the union parameters. For the same reason, both the aggregate employment level and the aggregate supply function under labor market segmentation are independent of λ and γ so that all equilibrium effects on the output and employment levels are solely induced by the price effects.

Whenever the union is "weak", i.e. if its bargaining power λ is sufficiently close to zero, the equilibrium wage paid to union members is below the wage on the competitive external labor market. While this result is similar to the model with a fully unionized single labor market, it is important to note that the comparison remains a hypothetical one under a single labor market with a single wage. Here, the result of union bargaining yields an outcome which in any respect – wage, wage sum, utility – leaves union members worse off than the nonunionized workers. This heavily depends on the assumption that there is no mobility between the labor markets within a period. For a similar reason, a temporary equilibrium with a bargaining wage exceeding the competitive wage due to a "strong" union can only be sustained because the nonunionized workers are excluded from joining the union and benefiting from its bargaining power.

Furthermore, both related single-labor-market models are special or limiting cases of the model with a segmented labor market.

This paper does not provide an analysis of the intertemporal evolution of the economy under perfect-foresight price expectations and an endogenous savings behavior of consumers. For given union parameters λ and γ, all results derived in Böhm & Claas (2012) extend to the SLM setup. Therefore, the properties of the dynamical systems in nominal and in real terms will be structurally the same as under a single labor market because the only difference is the multiplier of the aggregate demand function which is a constant in this case. In particular, it is clear that

- *monetary stationary states* will not exist generically, and if they exist, they will not be unique;
- in *intensity form*, the one-dimensional system of real money balances will have two positive fixed points for wide ranges of parametrizations; and
- convergence of the dynamical system in intensity form does not necessarily imply that the *growth rates* of money holdings and prices converge, i.e. the two-dimensional monetary system does not always converge to a so-called balanced path even if the associated system in real terms converges.

However, the eigenvalues of these dynamical systems depend on both union parameters so that the bifurcation analysis should become richer.

The full dynamics of the economy, i.e. when union power and union density do not remain constant over time, lead to a number of open questions:

- How do the union parameters adapt over time, i.e. based on which information or signals do these values react? In particular, which measures of inequality such as wage differentials lead to joining or leaving the union? What role does the governement play?
- Do stationary states of the economy exist? If yes, are they unique? Are they asymptoti-

cally stable? What is the impact of the adjustment of the union parameters on stability? Can there be stationary states with wage differentials or with nonzero rates of underemployment/overemployment?

- Do union power and union density move in opposite directions? If this is the case, the nonmonotonicity of the time-one map could be the source of cycles.

An analysis addressing the above questions goes beyond the scope of this paper and should be carried out separately. Due to the higher dimensionality of the problem outlined above, it remains a challenging but very interesting issue.

The present paper has assumed that the union and the firm – even if they meet repeatedly for negotiations – behave *myopically*, i.e. they ignore all intertemporal aspects of the bargaining although, even under constant union parameters, one would expect that the bargaining outcome in one period would affect future bargaining positions. This reasoning immediately leads to the question whether or under which conditions a sequence of equilibria of an economy with agents who have long-term objectives and who are engaged in periodic efficient bargaining could be intertemporally optimal.[22] It remains an unsolved issue whether deviations from myopic equilibria are achieved by, for example, not fully exercising bargaining power or by monetary transfers. Furthermore, the effects of these mechanisms on the evolution of union density as well as the feedback from changes of union density on the other state variables need to be understood.

The SLM framework could be extended to allow for heterogeneous firms with a joint external labor market for all firms and with additional individual internal labor markets for some of the firms. This extension is interesting because the competition on the external labor market could yield equilibria at which firms with access to two labor markets only demand from their internal labor markets, i.e. with zero demand for labor from the competitive external labor market. Furthermore, strategic union behavior could be analyzed by contrasting one union facing all firms on internal labor markets to independent unions, i.e. one on each internal labor market. However, it is highly questionable whether any of these modifications remain to be as tractable and comparable to the single-labor-market models as the model presented here.

In further research, the efficient-bargaining procedure on the internal labor market should be replaced by right-to-manage wage bargaining which is often used in the literature. Under right-to-manage wage bargaining, the producer is allowed to choose the employment level unilaterally after both parties have bargained over the wage successfully. The single-labor-market case has been studied in Böhm & Claas (2014). There it is shown that leaving the choice of the employment level to the employer's discretion leads to combinations of employment levels and wages at which the wage sum is independent of union power. Under labor market segmentation, however, the total wage sum most likely depends on union parameters since the firm's best-response function on the external labor market is independent of the wage on the internal labor market. Without explicitly solving for the partial equilibrium on the labor markets, it could even be that the firm's demand function for external labor implied that the total labor demand function – and thus the aggregate supply function – was independent of any union parameter. Due to these structural differences with the single-labor-market models and the efficient-bargaining SLM model, a right-to-manage SLM model is a highly interesting object to study.

[22] The only paper so far which deals with subgame-perfect temporary equilibria of a macroeconomy with intertemporarily optimizing agents and with bargaining is Selten & Güth (1982) which analyzes a real (i.e. nonmonetary) multiplier–accelerator-type economy with (Nash) wage bargaining.

6 Appendix

6.1 Proofs

6.1.1 Proof of Lemma 2.1

Let p, p^e, and w_{elm} be given. Show that

$$h_{\mathrm{ilm}}\left(\frac{w_{\mathrm{elm}}}{p}, \frac{p^e}{p}, \gamma\right) = \min\left\{\gamma N_{\mathrm{com}}\left(\frac{w_{\mathrm{elm}}/p}{p^e/p}\right), h_{\mathrm{eff}}\left(\frac{p^e}{p}, \gamma\right)\right\}$$

maximizes

$$\max_{L_{\mathrm{ilm}}\geq 0}\left\{pF\left(L_{\mathrm{ilm}} + h\left(\frac{w_{\mathrm{elm}}}{p}, L_{\mathrm{ilm}}\right)\right) - w_{\mathrm{elm}}h\left(\frac{w_{\mathrm{elm}}}{p}, L_{\mathrm{ilm}}\right) - p^e S_{\mathrm{res}}\left(\frac{L_{\mathrm{ilm}}}{\gamma}\right)L_{\mathrm{ilm}}\right\}.$$

Proof. Consider $L_{\mathrm{ilm}} < h_{\mathrm{com}}(w_{\mathrm{elm}}/p)$ and $L_{\mathrm{ilm}} > h_{\mathrm{com}}(w_{\mathrm{elm}}/p)$ separately. Because of

$$\frac{\mathrm{d}}{\mathrm{d}L_{\mathrm{ilm}}}S_{\mathrm{res}}\left(\frac{L_{\mathrm{ilm}}}{\gamma}\right)L_{\mathrm{ilm}} = \left(E_{S_{\mathrm{res}}}\left(\frac{L_{\mathrm{ilm}}}{\gamma}\right) + 1\right)S_{\mathrm{res}}\left(\frac{L_{\mathrm{ilm}}}{\gamma}\right) = S_{\mathrm{com}}\left(\frac{L_{\mathrm{ilm}}}{\gamma}\right),$$

the interior solutions to the respective first-order conditions are

$$L_{\mathrm{ilm}} \overset{!}{=} \gamma N_{\mathrm{com}}\left(\frac{w_{\mathrm{elm}}}{p^e}\right).$$

for $L_{\mathrm{ilm}} < h_{\mathrm{com}}(w_{\mathrm{elm}}/p)$ and

$$L_{\mathrm{ilm}} \overset{!}{=} h_{\mathrm{eff}}\left(\frac{p^e}{p}, \gamma\right)$$

for $L_{\mathrm{ilm}} > h_{\mathrm{com}}(w_{\mathrm{elm}}/p)$ which leads to

$$L_{\mathrm{ilm}} \overset{!}{=} \min\left\{\gamma N_{\mathrm{com}}\left(\frac{w_{\mathrm{elm}}}{p^e}\right), h_{\mathrm{com}}\left(\frac{w_{\mathrm{elm}}}{p}\right)\right\}$$

and

$$L_{\mathrm{ilm}} \overset{!}{=} \max\left\{h_{\mathrm{eff}}\left(\frac{p^e}{p}, \gamma\right), h_{\mathrm{com}}\left(\frac{w_{\mathrm{elm}}}{p}\right)\right\}.$$

Comparing the objective function at 0 and at $\gamma N_{\mathrm{com}}\left(\frac{w_{\mathrm{elm}}}{p^e}\right)$

$$\begin{aligned} &pF\left(h_{\mathrm{com}}\left(\frac{w_{\mathrm{elm}}}{p}\right)\right) - w_{\mathrm{elm}}h_{\mathrm{com}}\left(\frac{w_{\mathrm{elm}}}{p}\right) \\ &\quad + \underbrace{\left(w_{\mathrm{elm}} - p^e S_{\mathrm{res}}\left(S_{\mathrm{com}}^{-1}\left(\frac{w_{\mathrm{elm}}}{p^e}\right)\right)\right)}_{>0}\gamma N_{\mathrm{com}}\left(\frac{w_{\mathrm{elm}}}{p^e}\right) \\ &> pF\left(h_{\mathrm{com}}\left(\frac{w_{\mathrm{elm}}}{p}\right)\right) - w_{\mathrm{elm}}h_{\mathrm{com}}\left(\frac{w_{\mathrm{elm}}}{p}\right) \end{aligned}$$

rules out the boundary solution.

Since

$$\gamma N_{\text{com}}\left(\frac{w_{\text{elm}}}{p^e}\right) \gtrless h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right)$$

is equivalent to

$$\frac{p^e}{p} \lessgtr \frac{\frac{w_{\text{elm}}}{p}}{S_{\text{com}}(\frac{1}{\gamma}h_{\text{com}}(\frac{w_{\text{elm}}}{p}))} = \frac{F'(h_{\text{com}}(\frac{w_{\text{elm}}}{p}))}{S_{\text{com}}(\frac{1}{\gamma}h_{\text{com}}(\frac{w_{\text{elm}}}{p}))} = h_{\text{eff}}^{-1}\left(h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right), \gamma\right),$$

i.e.

$$h_{\text{eff}}\left(\frac{p^e}{p}, \gamma\right) \gtrless h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right),$$

the two cases can be combined to

$$\arg\max_{L_{\text{ilm}}}\left\{pF\left(L_{\text{ilm}} + h\left(\frac{w_{\text{elm}}}{p}, L_{\text{ilm}}\right)\right) - w_{\text{elm}}h\left(\frac{w_{\text{elm}}}{p}, L_{\text{ilm}}\right) - p^e S_{\text{res}}\left(\frac{L_{\text{ilm}}}{\gamma}\right)L_{\text{ilm}}\right\}$$

$$= \min\left\{\gamma N_{\text{com}}\left(\frac{w_{\text{elm}}/p}{p^e/p}\right), h_{\text{eff}}\left(\frac{p^e}{p}, \gamma\right)\right\} = h_{\text{ilm}}\left(\frac{w_{\text{elm}}}{p}, \frac{p^e}{p}, \gamma\right).$$

which completes the proof. □

6.1.2 Comparative Statics

The property $0 < E_{W_{\text{elm}}}(\frac{p^e}{p}) < 1$ implies that

$$E_{\mathcal{W}_{\text{elm}}}(M) = E_{\mathcal{P}_{\text{slm}}}(M)\left(1 - E_{W_{\text{elm}}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma)}\right)\right) \in (0, 1),$$

$$E_{\mathcal{W}_{\text{elm}}}(p^e) = E_{\mathcal{P}_{\text{slm}}}(p^e) + E_{W_{\text{elm}}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma)}\right)(1 - E_{\mathcal{P}_{\text{slm}}}(p^e))$$

$$= 1 + \left(E_{W_{\text{elm}}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma)}\right) - 1\right)(1 - E_{\mathcal{P}_{\text{slm}}}(p^e)) \in (0, 1),$$

$$E_{\mathcal{W}_{\text{elm}}}(\lambda) = E_{\mathcal{P}_{\text{slm}}}(\lambda)\left(1 - E_{W_{\text{elm}}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma)}\right)\right) < 0,$$

$$E_{\mathcal{W}_{\text{elm}}}(\gamma) = E_{\mathcal{P}_{\text{slm}}}(\gamma)\left(1 - E_{W_{\text{elm}}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma)}\right)\right) > 0.$$

Because of $\mathcal{L}_{\text{slm}}(M, p^e, \lambda, \gamma) \equiv S_{\text{com}}^{-1}(\frac{\mathcal{W}_{\text{elm}}(M,p^e,\lambda,\gamma)}{p^e})$

$$E_{\mathcal{L}_{\text{slm}}}(M) = E_{S_{\text{com}}^{-1}}\left(\frac{\mathcal{W}_{\text{elm}}(M, p^e, \lambda, \gamma)}{p^e}\right)E_{\mathcal{W}_{\text{elm}}}(M) > 0,$$

$$E_{\mathcal{L}_{\text{slm}}}(p^e) = E_{S_{\text{com}}^{-1}}\left(\frac{\mathcal{W}_{\text{elm}}(M, p^e, \lambda, \gamma)}{p^e}\right)(E_{\mathcal{W}_{\text{elm}}}(p^e) - 1) < 0,$$

$$E_{\mathcal{L}_{\text{slm}}}(\lambda) = E_{S_{\text{com}}^{-1}}\left(\frac{\mathcal{W}_{\text{elm}}(M, p^e, \lambda, \gamma)}{p^e}\right)E_{\mathcal{W}_{\text{elm}}}(\lambda) < 0,$$

$$E_{\mathcal{L}_{\text{slm}}}(\gamma) = E_{S_{\text{com}}^{-1}}\left(\frac{\mathcal{W}_{\text{elm}}(M, p^e, \lambda, \gamma)}{p^e}\right)E_{\mathcal{W}_{\text{elm}}}(\gamma) > 0$$

Because of $\mathcal{L}_{\text{ilm}}(M, p^e, \lambda, \gamma) \equiv \gamma \mathcal{L}_{\text{slm}}(M, p^e, \lambda, \gamma)$

$$\begin{aligned} E_{\mathcal{L}_{\text{ilm}}}(M) &= E_{\mathcal{L}_{\text{slm}}}(M) > 0, \\ E_{\mathcal{L}_{\text{ilm}}}(p^e) &= E_{\mathcal{L}_{\text{slm}}}(p^e) < 0, \\ E_{\mathcal{L}_{\text{ilm}}}(\lambda) &= E_{\mathcal{L}_{\text{slm}}}(\lambda) < 0, \\ E_{\mathcal{L}_{\text{ilm}}}(\gamma) &= 1 + E_{\mathcal{L}_{\text{slm}}}(\gamma) > 1 \end{aligned}$$

and, because of $\mathcal{L}_{\text{elm}}(M, p^e, \lambda, \gamma) \equiv (1-\gamma)\mathcal{L}_{\text{slm}}(M, p^e, \lambda, \gamma)$,

$$E_{\mathcal{L}_{\text{elm}}}(M) > 0, \qquad E_{\mathcal{L}_{\text{elm}}}(p^e) < 0, \qquad E_{\mathcal{L}_{\text{elm}}}(\lambda) < 0.$$

Since the elasticity of $1-\gamma$ becomes arbitrarily negative as λ approaches one, $E_{\mathcal{L}_{\text{elm}}}(\gamma) < 0$ is negative for λ sufficiently close to one, but could be positive for γ "small".

Consider $\mathcal{Y}_{\text{slm}}(M, p^e, \lambda, \gamma) \equiv AS_{\text{slm}}(\frac{p^e}{\mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma)})$. Then

$$\begin{aligned} E_{\mathcal{Y}_{\text{slm}}}(M) &= -E_{AS_{\text{slm}}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma)}\right) E_{\mathcal{P}_{\text{slm}}}(M) > 0 \\ E_{\mathcal{Y}_{\text{slm}}}(p^e) &= E_{AS_{\text{slm}}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma)}\right) (1 - E_{\mathcal{P}_{\text{slm}}}(p^e)) < 0 \\ E_{\mathcal{Y}_{\text{slm}}}(\lambda) &= -E_{AS_{\text{slm}}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma)}\right) E_{\mathcal{P}_{\text{slm}}}(\lambda) < 0 \\ E_{\mathcal{Y}_{\text{slm}}}(\gamma) &= -E_{AS_{\text{slm}}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma)}\right) E_{\mathcal{P}_{\text{slm}}}(\gamma) > 0 \end{aligned}$$

Finally consider

$$\mathcal{W}_{\text{ilm}}(M, p^e, \lambda, \gamma) = \mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma) W_{\text{ilm}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma)}, \lambda, \gamma\right).$$

Since all partial derivatives depend on $E_{W_{\text{ilm}}}(\theta^e)$, which cannot be signed in general, no global statements can be made. If $E_{W_{\text{ilm}}}(\theta^e) \in (0,1)$,[23] then

$$\begin{aligned} E_{\mathcal{W}_{\text{ilm}}}(M) &= E_{\mathcal{P}_{\text{slm}}}(M)\left(1 - E_{W_{\text{ilm}}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma)}\right)\right) \in (0,1) \\ E_{\mathcal{W}_{\text{ilm}}}(p^e) &= E_{\mathcal{P}_{\text{slm}}}(p^e) + E_{W_{\text{ilm}}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma)}\right)(1 - E_{\mathcal{P}_{\text{slm}}}(p^e)) \\ &= 1 + \left(E_{W_{\text{ilm}}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma)}\right) - 1\right)(1 - E_{\mathcal{P}_{\text{slm}}}(p^e)) \in (0,1) \end{aligned}$$

Because of $\operatorname{sgn} E_{W_{\text{ilm}}}(\lambda) = -\operatorname{sgn} E_{\mathcal{P}_{\text{slm}}}(\lambda)$ and $\operatorname{sgn} E_{W_{\text{ilm}}}(\gamma) = -\operatorname{sgn} E_{\mathcal{P}_{\text{slm}}}(\gamma)$, the elasticities

[23] If F and S_{res} are isoelastic with elasticities B and $1/C$, then $E_{W_{\text{ilm}}}(\theta^e) = \frac{C(1-B)}{C(1-B)+1} \in (0,1)$.

$E_{\mathcal{W}_{\text{ilm}}}(\lambda)$ and $E_{\mathcal{W}_{\text{ilm}}}(\gamma)$ cannot be signed; however, the following bounds can be established.

$$\begin{aligned}E_{\mathcal{W}_{\text{ilm}}}(\lambda) &= E_{\mathcal{P}_{\text{slm}}}(\lambda)\left(1 - E_{W_{\text{ilm}}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M,p^e,\lambda,\gamma)}\right)\right) + E_{W_{\text{ilm}}}(\lambda) < E_{W_{\text{ilm}}}(\lambda)\\ &= 1 - \frac{\gamma\frac{E_F(L)}{E_{S_{\text{res}}}(L)+1}}{\lambda(1-(1-\gamma)E_F(L)) + (1-\lambda)\gamma\frac{E_F(L)}{E_{S_{\text{res}}}(L)+1}}, \quad L = \mathcal{L}_{\text{slm}}(M,p^e,\lambda,\gamma)\\ &< 1\end{aligned}$$

$$\begin{aligned}E_{\mathcal{W}_{\text{ilm}}}(\gamma) &= E_{\mathcal{P}_{\text{slm}}}(\gamma)\left(1 - E_{W_{\text{ilm}}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M,p^e,\lambda,\gamma)}\right)\right) + E_{W_{\text{ilm}}}(\gamma) > E_{W_{\text{ilm}}}(\lambda)\\ &= -\frac{\lambda(1-E_F(L))}{\lambda(1-E_F(L)) + \gamma E_F(L)\left(\lambda + \frac{1-\lambda}{E_{S_{\text{res}}}(L)+1}\right)}, \qquad L = \mathcal{L}_{\text{slm}}(M,p^e,\lambda,\gamma)\\ &> -1\end{aligned}$$

Because of $\mathcal{W}_{\text{elm}}(M,p^e,\lambda,\gamma) \equiv F'(L) \equiv E_F(L)\frac{F(L)}{L}$, with $L = \mathcal{L}_{\text{slm}}(M,p^e,\lambda,\gamma)$, the wage law for the internal labor market can be decomposed into a markup and the wage law for the external market, i.e.

$$\begin{aligned}&\mathcal{W}_{\text{ilm}}(M,p^e,\lambda,\gamma)\\ &\qquad \overset{(19)}{=} \left(\lambda\frac{1-(1-\gamma)E_F(L)}{\gamma E_F(L)} + (1-\lambda)\frac{1}{E_{S_{\text{res}}}(L)+1}\right)\mathcal{W}_{\text{elm}}(M,p^e,\lambda,\gamma).\end{aligned}$$

Rearranging terms in the markup yields

$$\begin{aligned}&\mathcal{W}_{\text{ilm}}(M,p^e,\lambda,\gamma)\\ &\overset{(19)}{=} \left(\lambda\underbrace{\left(\frac{1-E_F(L)}{\gamma E_F(L)} + \frac{E_{S_{\text{res}}}(L)}{E_{S_{\text{res}}}(L)+1}\right)}_{>0} + \frac{1}{E_{S_{\text{res}}}(L)+1}\right)\mathcal{W}_{\text{elm}}(M,p^e,\lambda,\gamma)\\ &= \left(\frac{1}{\gamma}\underbrace{\lambda\frac{1-E_F(L)}{E_F(L)}}_{>0} + \frac{\lambda E_{S_{\text{res}}}(L)+1}{E_{S_{\text{res}}}(L)+1}\right)\mathcal{W}_{\text{elm}}(M,p^e,\lambda,\gamma)\end{aligned} \tag{34}$$

which shows that the effects of the union parameters λ and γ on the markup are opposite to the effects on the wage law for the external market.

Plugging (34) into the equilibrium average wage $\mathcal{W}_{\text{slm}}(M,p^e,\lambda,\gamma) \equiv \gamma\mathcal{W}_{\text{ilm}}(M,p^e,\lambda,\gamma) + (1-\gamma)\mathcal{W}_{\text{elm}}(M,p^e,\lambda,\gamma)$ yields

$$\begin{aligned}&\mathcal{W}_{\text{slm}}(M,p^e,\lambda,\gamma)\\ &= \left(\lambda\underbrace{\left(\frac{1-E_F(L)}{E_F(L)} + \gamma\frac{E_{S_{\text{res}}}(L)}{E_{S_{\text{res}}}(L)+1}\right)}_{>0} - \gamma\frac{E_{S_{\text{res}}}(L)}{E_{S_{\text{res}}}(L)+1} + 1\right)\mathcal{W}_{\text{elm}}(M,p^e,\lambda,\gamma)\\ &= \left(\lambda\frac{1-E_F(L)}{E_F(L)} - \gamma\underbrace{(1-\lambda)\frac{E_{S_{\text{res}}}(L)}{E_{S_{\text{res}}}(L)+1}}_{>0} + 1\right)\mathcal{W}_{\text{elm}}(M,p^e,\lambda,\gamma),\end{aligned}$$

which on the one hand implies that

$$\frac{\partial \mathcal{W}_{\text{slm}}}{\partial M} = \frac{\partial \mathcal{W}_{\text{elm}}}{\partial M} \in (0,1), \qquad \frac{\partial \mathcal{W}_{\text{slm}}}{\partial p^e} = \frac{\partial \mathcal{W}_{\text{elm}}}{\partial p^e} \in (0,1)$$

if $E_{W_{\text{ilm}}}(\theta^e) \in (0,1)$, but, on the other hand, that both union parameters move the markup and the wage law for the external market in opposite directions so that no general statement can be made.

Because of (22), the profit law is

$$\begin{aligned} &\mathbf{\Pi}_{\text{slm}}(M, p^e, \lambda, \gamma) \\ &= (1-\lambda)\left(1 - E_F(L)\left(1 - \gamma\frac{E_{S_{\text{res}}}(L)}{E_{S_{\text{res}}}(L)+1}\right)\right)\mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma)\mathcal{Y}_{\text{slm}}(M, p^e, \lambda, \gamma), \\ &\qquad L = \boldsymbol{\mathcal{L}}_{\text{slm}}(M, p^e, \lambda, \gamma). \end{aligned}$$

Therefore, if F and S_{res} are isoelastic, if $c(\theta^e)$ is constant, and if $\lambda < 1$,

$$\begin{aligned} E_{\mathbf{\Pi}_{\text{slm}}}(M) &= E_{\mathcal{P}_{\text{slm}}}(M) + E_{\mathcal{Y}_{\text{slm}}}(M) > 0 \\ E_{\mathbf{\Pi}_{\text{slm}}}(p^e) &= E_{\mathcal{P}_{\text{slm}}}(p^e) + E_{\mathcal{Y}_{\text{slm}}}(p^e) \\ &= E_{\mathcal{P}_{\text{slm}}}(p^e) + E_{AS_{\text{slm}}}\left(\frac{p^e}{\mathcal{P}_{\text{slm}}(M, p^e, \lambda, \gamma)}\right)(1 - E_{\mathcal{P}_{\text{slm}}}(p^e)) \\ &= E_{\mathcal{P}_{\text{slm}}}(p^e)\,(1 - E_{AS_{\text{slm}}}(\theta^e)) + E_{AS_{\text{slm}}}(\theta^e) \\ &= \frac{-E_{AS_{\text{slm}}}(\theta^e)}{-E_{AS_{\text{slm}}}(\theta^e) + E_D(m)}\,(1 - E_{AS_{\text{slm}}}(\theta^e)) + E_{AS_{\text{slm}}}(\theta^e) \\ &> \frac{-E_{AS_{\text{slm}}}(\theta^e)}{-E_{AS_{\text{slm}}}(\theta^e) + 1}\,(1 - E_{AS_{\text{slm}}}(\theta^e)) + E_{AS_{\text{slm}}}(\theta^e) = 0 \\ E_{\mathbf{\Pi}_{\text{slm}}}(\lambda) &= \frac{-\lambda}{1-\lambda} + E_{\mathcal{P}_{\text{slm}}}(\lambda) + E_{\mathcal{Y}_{\text{slm}}}(\lambda) < 0 \\ E_{\mathbf{\Pi}_{\text{slm}}}(\gamma) &= \frac{\gamma\frac{E_F(L)E_{S_{\text{res}}}(L)}{E_{S_{\text{res}}}(L)+1}}{1 - E_F(L) + \gamma\frac{E_F(L)E_{S_{\text{res}}}(L)}{E_{S_{\text{res}}}(L)+1}} + E_{\mathcal{P}_{\text{slm}}}(\gamma) + E_{\mathcal{Y}_{\text{slm}}}(\gamma) > 0 \end{aligned}$$

For $\lambda > 0$, the union's payoff law is $\mathbf{\Omega}_{\text{slm}}(M, p^e, \lambda, \gamma) = \frac{\lambda}{1-\lambda}\mathbf{\Pi}_{\text{slm}}(M, p^e, \lambda, \gamma)$ which implies

$$E_{\mathbf{\Omega}_{\text{slm}}}(M) = E_{\mathbf{\Pi}_{\text{slm}}}(M) > 0, \qquad E_{\mathbf{\Omega}_{\text{slm}}}(p^e) = E_{\mathbf{\Pi}_{\text{slm}}}(p^e) > 0, \qquad \text{and}$$

$$E_{\mathbf{\Omega}_{\text{slm}}}(\gamma) = E_{\mathbf{\Pi}_{\text{slm}}}(\gamma) > 0.$$

Once again due to opposing effects, the direction of an increase of union power λ cannot be signed in general:

$$E_{\mathbf{\Omega}_{\text{slm}}}(\lambda) = 1 + E_{\mathcal{P}_{\text{slm}}}(\lambda) + E_{\mathcal{Y}_{\text{slm}}}(\lambda).$$

6.2 A Parametric Example

Assume

- production function: $F(z) = \frac{A}{B} z^B$ with $A > 0$, $0 < B < 1$
- shareholder's utility: $\log c_0 + \delta \log c^e$ with $\delta > 0$; implies $c \equiv \frac{1}{\delta+1}$
- disutility from labor: $v(\ell) = \frac{C}{C+1} \ell^{1+\frac{1}{C}}$ with $0 < C < 1$

This implies

- $h_{\text{com}}(\alpha) = (\frac{A}{\alpha})^{\frac{1}{1-B}}$
- $S_{\text{res}}(\frac{L_{\text{ilm}}}{\gamma}) = \frac{C}{C+1} \frac{1}{1-\tau_w} (\frac{L_{\text{ilm}}}{\gamma n_w})^{1/C}$, $S_{\text{com}}(\frac{L_{\text{ilm}}}{\gamma}) = \frac{1}{1-\tau_w} (\frac{L_{\text{ilm}}}{\gamma n_w})^{1/C}$
- $S_{\text{res}}^{-1}(\frac{\alpha}{\theta^e}) = n_w((1-\tau_w) \frac{C+1}{C} \frac{\alpha}{\theta^e})^C$, $S_{\text{com}}^{-1}(\frac{\alpha}{\theta^e}) = n_w((1-\tau_w) \frac{\alpha}{\theta^e})^C = N_{\text{com}}(\frac{\alpha}{\theta^e})$
- $h_{\text{eff}}(\theta^e, \gamma) = (1-\tau_w)^{\frac{C}{C(1-B)+1}} A^{\frac{C}{C(1-B)+1}} (\gamma n_w)^{\frac{1}{C(1-B)+1}} (\theta^e)^{-\frac{C}{C(1-B)+1}}$,
  $h_{\text{eff}}^{-1}(L_{\text{ilm}}, \gamma) = \frac{F'(L_{\text{ilm}})}{S_{\text{com}}(L_{\text{ilm}}/\gamma)}$
- $\theta^e = W_{\text{elm}}^{-1}(\alpha_{\text{elm}}) = \frac{\alpha_{\text{elm}}}{S_{\text{com}}(h_{\text{com}}(\alpha_{\text{elm}}))} = (1-\tau_w) n_w^{1/C} A^{-\frac{1}{C(1-B)}} \alpha_{\text{elm}}^{\frac{C(1-B)+1}{C(1-B)}}$
- $\alpha_{\text{elm}} = W_{\text{elm}}(\theta^e) = (1-\tau_w)^{-\frac{C(1-B)}{C(1-B)+1}} n_w^{-\frac{1-B}{C(1-B)+1}} A^{\frac{1}{C(1-B)+1}} (\theta^e)^{\frac{C(1-B)}{C(1-B)+1}}$
- $\alpha_{\text{ilm}} = W_{\text{ilm}}(\theta^e, \lambda, \gamma) = \frac{A}{B\gamma} \Big(h_{\text{com}}(W_{\text{elm}}(\theta^e))\Big)^{B-1} \Big(\lambda(1-(1-\gamma)B) + (1-\lambda)\gamma \frac{BC}{C+1}\Big)$
- $AS_{\text{slm}}(\theta^e) = F(h_{\text{com}}(W_{\text{elm}}(\theta^e))) = A^{\frac{C+1}{C(1-B)+1}} \frac{1}{B} n_w^{\frac{B}{C(1-B)+1}} (\frac{\theta^e}{1-\tau_w})^{-\frac{CB}{C(1-B)+1}}$
- $D_{\text{slm}}(m, \lambda, \gamma) = \frac{m+g}{\tilde{c}_{\text{slm}}(\lambda,\gamma)} = \frac{m+g}{1-c(1-\tau_\pi)(1-\lambda)(1-B+\gamma\frac{B}{C+1})}$
- $\lambda_{\text{com}}(\gamma) = \frac{\gamma \frac{B}{C+1}}{1-B+\gamma\frac{B}{C+1}}$
- $0 < E_{D_{\text{slm}}}(m) = \frac{m}{m+g} \leq 1$
- $0 < E_{\tilde{c}_{\text{slm}}}(\lambda) = \frac{c(1-\tau_\pi)(1-\lambda)(1-B+\gamma\frac{B}{C+1})}{1-c(1-\tau_\pi)(1-\lambda)(1-B+\gamma\frac{B}{C+1})} \leq \frac{(1-\lambda)(1-B+\gamma\frac{B}{C+1})}{1-(1-\lambda)(1-B+\gamma\frac{B}{C+1})} < 1$
- $E_{D_{\text{slm}}}(\lambda) = -E_{\tilde{c}_{\text{slm}}}(\lambda)$, $-1 < E_{D_{\text{slm}}}(\lambda) < 0$

6.3 The Powerful Producer

In this section, it is assumed that the firm is able to demand on the competitive market alone, i.e. to threaten to hire no unionized worker at all ($L_{\text{ilm}} = 0$). Therefore, being active on two labor markets has to yield a profit which has to be at least as high as the one the firm would obtain on the competitive market alone, i.e. $pF(h_{\text{com}}(w_{\text{elm}}/p)) - w_{\text{elm}} h_{\text{com}}(w_{\text{elm}}/p)$. This is the firm's status quo; the status quo of the union remains zero. The bargaining problem is therefore given by

$$\left(\mathcal{B}(p^e, p, w_{\text{elm}}, \gamma), \begin{pmatrix} pF(h_{\text{com}}(\frac{w_{\text{elm}}}{p})) - w_{\text{elm}} h_{\text{com}}(\frac{w_{\text{elm}}}{p}) \\ 0 \end{pmatrix}\right) \tag{35}$$

where $\mathcal{B}(p^e, p, w_{\text{elm}}, \gamma)$ is the same bargaining set as defined in (5).

Since the bargaining set is unchanged, the bargaining parties agree on the same employment level $L_{\text{ilm}} = h_{\text{ilm}}(w_{\text{elm}}/p, p^e/p, \gamma)$ at which the feasible set is maximal. This implies that the firm's labor demand on the competitive market is given by $L_{\text{elm}} = h_{\text{elm}}(w_{\text{elm}}/p, p^e/p, \gamma)$.

Because of the different status quo, the firm's reservation wage is different. It is given by

$$\begin{aligned} w_{\text{ilm}} &\overset{!}{\leq} \frac{pF\left(L_{\text{ilm}}+L_{\text{elm}}\right)-w_{\text{elm}}L_{\text{elm}}-\left(pF\left(h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right)\right)-w_{\text{elm}}h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right)\right)}{L_{\text{ilm}}} \\ &= \frac{pF\left(L_{\text{ilm}}+h\left(\frac{w_{\text{elm}}}{p},L_{\text{ilm}}\right)\right)-w_{\text{elm}}h\left(\frac{w_{\text{elm}}}{p},L_{\text{ilm}}\right)}{L_{\text{ilm}}} \\ &\qquad -\frac{\left(pF\left(h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right)\right)-w_{\text{elm}}h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right)\right)}{L_{\text{ilm}}} \\ &=: W_{\Pi}(p,w_{\text{elm}},L_{\text{ilm}}). \end{aligned}$$

To simplify notation, those functions which are altered due to the firm's different status-quo level, are not marked by an additional or different subscript. Note that

$$W_{\Pi}(p,w_{\text{elm}},L_{\text{ilm}})=\begin{cases} w_{\text{elm}}, & \text{if } h(\frac{w_{\text{elm}}}{p},L_{\text{ilm}})>0 \\ \frac{pF(L_{\text{ilm}})-\left(pF\left(h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right)\right)-w_{\text{elm}}h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right)\right)}{L_{\text{ilm}}}, & \text{if } h(\frac{w_{\text{elm}}}{p},L_{\text{ilm}})=0 \end{cases}$$

i.e. the wage paid to the union members for $h(w_{\text{elm}}/p,L_{\text{ilm}})=h_{\text{com}}(w_{\text{elm}}/p)-L_{\text{ilm}}\geq 0$ is at most as high as the one paid under competition. This asymmetry guarantees an autonomous rent to the firm whereas unionized workers are committed to supply their individual shares of labor according to the aggregate level the union has sold to the firm.

The bargaining solution to the bargaining problem (35) remains structurally unchanged, i.e.

$$\begin{aligned} &\begin{pmatrix} \Pi_{\text{slm}}\left(p,.,w_{\text{elm}},L_{\text{ilm}},h\left(\frac{w_{\text{elm}}}{p},L_{\text{ilm}}\right)\right) \\ \Omega_{\text{slm}}(p^e,.,L_{\text{ilm}},\gamma) \end{pmatrix} \\ &= \left(W_{\Pi}(p,w_{\text{elm}},L_{\text{ilm}})-W_{\Omega_{\text{slm}}}(p^e,L_{\text{ilm}},\gamma)\right)\begin{pmatrix}1-\lambda \\ \lambda\end{pmatrix}. \end{aligned}$$

For a given employment level L_{ilm} such that $W_{\Omega_{\text{slm}}}(p^e,L_{\text{ilm}},\gamma)<W_{\Pi}(p,w_{\text{elm}},L_{\text{ilm}})$, the wage solving the firm's and the union's bargaining problem is given by the convex combination of the two reservation wage functions, i.e.

$$\begin{aligned} w_{\text{ilm}} &= \lambda W_{\Pi}(p,w_{\text{elm}},L_{\text{ilm}})+(1-\lambda)W_{\Omega_{\text{slm}}}(p^e,L_{\text{ilm}},\gamma) \\ &= \begin{cases} \lambda w_{\text{elm}}+(1-\lambda)p^e S_{\text{res}}\left(\frac{L_{\text{ilm}}}{\gamma}\right), & \text{if } h\left(\frac{w_{\text{elm}}}{p},L_{\text{ilm}}\right)>0 \\ \lambda\frac{pF(L_{\text{ilm}})-\left(pF\left(h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right)\right)-w_{\text{elm}}h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right)\right)}{L_{\text{ilm}}}+(1-\lambda)p^e S_{\text{res}}\left(\frac{L_{\text{ilm}}}{\gamma}\right), & \\ & \text{if } h\left(\frac{w_{\text{elm}}}{p},L_{\text{ilm}}\right)=0 \end{cases} \end{aligned}$$

where $0\leq\lambda\leq 1$ denotes the union's relative bargaining power.

As before, the clearing of the labor market implies that $w_{\text{elm}}/p=W_{\text{elm}}(p^e/p)\equiv W_{\text{com}}(p^e/p)$ as well as

$$L_{\text{ilm}}=\gamma h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right)=\gamma N_{\text{com}}\left(\frac{w_{\text{elm}}}{p^e}\right) \quad\text{and}\quad L_{\text{elm}}=(1-\gamma)h_{\text{com}}\left(\frac{w_{\text{elm}}}{p}\right)>0.$$

Let the aggregate employment level be denoted by L, i.e. $L := h_{\text{com}}(w_{\text{elm}}/p) = h_{\text{com}}(W_{\text{elm}}(p^e/p))$, which implies $L_{\text{ilm}} = \gamma L$ and $L_{\text{elm}} = (1-\gamma)L$. Then, the profit share of total revenue is

$$\begin{aligned}
&\frac{pF(L_{\text{ilm}} + L_{\text{elm}}) - w_{\text{ilm}}L_{\text{ilm}} - w_{\text{elm}}L_{\text{elm}}}{pF(L_{\text{ilm}} + L_{\text{elm}})} \\
&= 1 - \frac{\lambda\gamma w_{\text{elm}}L + (1-\lambda)\gamma p^e S_{\text{res}}(L)L + w_{\text{elm}}(1-\gamma)L}{pF(L)} \\
&= 1 - \frac{w_{\text{elm}}L}{pF(L)} - (1-\lambda)\gamma\frac{w_{\text{elm}}L}{pF(L)}\left(\frac{p^e S_{\text{res}}(L)}{w_{\text{elm}}} - 1\right) \\
&= 1 - E_F(L) - (1-\lambda)\gamma\frac{E_F(L)E_{S_{\text{res}}}(L)}{E_{S_{\text{res}}}(L)+1}
\end{aligned}$$

so that

$$\tilde{c}_{\text{slm}}\left(\frac{p^e}{p}, \lambda, \gamma\right) := 1 - c\left(\frac{p^e}{p}\right)(1-\tau_\pi)\left(1 - E_F(L) + (1-\lambda)\gamma\frac{E_F(L)E_{S_{\text{res}}}(L)}{E_{S_{\text{res}}}(L)+1}\right)$$

with $L = h_{\text{com}}(W_{\text{elm}}(p^e/p))$. The aggregate demand function is therefore given by

$$\begin{aligned}
D_{\text{slm}}(m, \theta^e, \lambda, \gamma) &:= \frac{m+g}{\tilde{c}_{\text{slm}}(\theta^e, \lambda, \gamma)} \\
&= \frac{m+g}{1 - c(\theta^e)(1-\tau_\pi)\left(1 - E_F(L) + (1-\lambda)\gamma\frac{E_F(L)E_{S_{\text{res}}}(L)}{E_{S_{\text{res}}}(L)+1}\right)},
\end{aligned}$$

with $L = h_{\text{com}}(W_{\text{elm}}(\theta^e))$.

The change of the reservation wage function induces a different demand multiplier $\tilde{c}_{\text{slm}}(\theta^e, \lambda, \gamma)$, but neither new nor different structural properties of the aggregate demand function. Since the aggregate supply function $AS_{\text{slm}}(\theta^e)$ remains unchanged, the temporary equilibrium uniquely exists under the same set of assumptions as in the previously treated case and displays the same qualitative properties.

The competitive equilibrium is the special case $\lambda_{\text{com}} = 1$.

Under the isoelastic specifications, aggregate demand function is given by

$$D_{\text{slm}}(m, \lambda, \gamma) = \frac{m+g}{\tilde{c}_{\text{slm}}(\lambda, \gamma)} = \frac{m+g}{1 - c(1-\tau_\pi)(1 - B + (1-\lambda)\gamma\frac{B}{C+1})}$$

with

$$\begin{aligned}
E_{\tilde{c}_{\text{slm}}}(\lambda) &= \frac{c(1-\tau_\pi)\lambda\gamma\frac{B}{C+1}}{1 - c(1-\tau_\pi)(1 - B + (1-\lambda)\gamma\frac{B}{C+1})} \\
&\leq \frac{\lambda\frac{B}{C+1}}{1 - (1 - B + (1-\lambda)\frac{B}{C+1})} = \frac{\lambda}{C+\lambda} < 1.
\end{aligned}$$

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