Dynamics and Heterogeneity of Subjective Stock Market Expectations

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Discussion Paper No. 157

May 28, 2019

Collaborative Research Center Transregio 190 | www.rationality-and-competition.de
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Dynamics and heterogeneity of subjective stock market expectations*

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March 31, 2019

Abstract: Between 2004 and 2016, we elicited individuals' subjective expectations of stock market returns in a Dutch internet panel at bi-annual intervals. In this paper, we develop a panel data model with a finite mixture of expectation types who differ in how they use past stock market returns to form current stock market expectations. The model allows for rounding in the probabilistic responses and for observed and unobserved heterogeneity at several levels. We estimate the type distribution in the population and find evidence for considerable heterogeneity in expectation types and meaningful variation over time, in particular during the financial crisis of 2008/09.

JEL codes: D12, D84, G11

Keywords: expectations, stock markets, financial crisis, mixture models, surveys

* We would like to thank Charles Bellemare, Luc Bissonnette, Tobias Klein, Charles Manski, and Arthur van Soest as well as seminar participants at Tilburg University, Université Laval, the University of Passau, the 2018 Workshop on Subjective Expectations and Probabilities in Economics in Munich, the 2018 European Meeting of the Econometric Society in Cologne, and the 2018 Annual Meeting of the Verein für Socialpolitik in Freiburg for helpful comments. Rossmann gratefully acknowledges financial support through the Elite Network of Bavaria. We also acknowledge support by the Deutsche Forschungsgemeinschaft through CRC TRR 190. The views expressed in this paper are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank (DNB).

1 Introduction

Subjective expectations are crucial in all individual decisions where outcomes only materialize in the future and are subject to uncertainty. These include decisions regarding education, health, insurance, and household finance. One might argue that such intertemporal decisions are among the most important ones individuals make. In order to understand the determinants of subjective expectations and their role in decision-making, they must be measured at the individual level. Since the early 1990s, researchers have elicited subjective expectations of individuals in large-scale surveys (e. g., Dominitz and Manski, 1997).[1] For example, data on subjective stock market expectations contributed to the understanding of the stock market participation puzzle; see Dominitz and Manski (2007, 2011); Hudomiet et al. (2011); Hurd et al. (2011); Hurd and Rohwedder (2012), *inter alia*. These papers document substantial heterogeneity of subjective expectations in the population and show that individuals' expectations predict their stock-market decisions. However, less is known about how individuals form and adjust their expectations.

This paper reports on findings from a study that collected data on subjective stock market expectations over a twelve-year period in a large, representative internet panel in the Netherlands. Expectations were elicited using a probabilistic format and refer to the one-year ahead rate of return of the Amsterdam Stock Exchange index (AEX), with four questions on gains and losses, respectively. We thus have, for each respondent and each interview date, eight responses that correspond to well-defined points on the subjective distribution of expected one-year rates of return. Results from the first two surveys, conducted in 2004 and 2006, are reported in Hurd et al. (2011); that paper documents substantial heterogeneity in stock market expectations.

The present paper uses a much longer panel with data from follow-up surveys conducted in 2008, 2009, 2010, 2012, 2014, and 2016. These data are unique because they cover subjective expectations elicited with the same probabilistic format over a period of twelve years which includes the 2008/09 financial market crisis. As an important innovation in the econometric methodology for the analysis of subjective expectations, we propose a panel data model with

[1] For comprehensive overviews of the measurement and analysis of subjective expectations, see Hurd (2009) and Manski (2004, 2018).

a finite mixture of expectation types who differ in how they use past stock market returns to form current stock market expectations, following ideas by Dominitz and Manski (2011). The model allows for rounding in the probabilistic responses and for observed and unobserved heterogeneity at several levels.

We argue that individuals may differ in how they use past stock market returns when forming stock market expectations. Dominitz and Manski (2011) suggest that the population can be described by three latent expectation types. The first type (Random Walk, RW) believes that returns are independent and identically distributed (i.i.d.) over time and – given this belief – uses the long-run historical average return to predict returns. Type two (Mean Reversion, MR) believes that recent stock market changes will be reversed in the near future and type three (Persistence, P) believes that recent stock market changes will persist into the near future.

In this paper, we build on this insight and develop a panel data model with a finite mixture of three distinct expectation types who are allowed to differ in how they use the recent stock market performance to form expectations. Since individual type membership is not directly observed in the data, we model type probabilities and allow them to depend on both observed and unobserved individual-specific heterogeneity. The inclusion of year-fixed effects in the model also allows us to study the dynamics of the sample type distribution throughout the financial crisis. Our model includes two additional features. First, we specify an entire reporting model for subjective stock market expectations to capture different rounding patterns of individuals. Second, our model provides a sophisticated method to take use of the very detailed information on individuals' stock market expectations, i.e. all eight points on individuals' c.d.f. of expected one-year rates of return. The entire model is then estimated jointly to avoid selection bias (Kleinjans and van Soest, 2014).

Our results suggest that the population may indeed be described by three distinct expectation types who differ in how they use the past one-year AEX return to form expectations. While the first type does not seem to use this return, the second and third type do so in a negative and positive manner, respectively. These findings are in line with the Dominitz and Manski (2011) interpretation and allow us to label the three expectation types (RW,MR,P), as defined above. The implied distribution of these three types in the sample is given by (0.60,0.19,0.21), suggesting that most individuals do not use the past one-year AEX return when forming their expectations.

Further analysis reveals the existence of substantial individual-specific heterogeneity in the type probabilities. For example, females are significantly more likely to be type MR or type P than males. Similarly, highly educated respondents are more likely to be type RW. We also find evidence for the importance to account for unobserved factors. The model identifies several significant correlations between the individual effects, implying that, for example, individuals who are more likely to be type MR are also more likely to be type P.

We also use the coefficients of the year-fixed effects to predict the dynamics of the expectation type distribution in the sample. The results suggest that in years unaffected by the 2008 financial crisis, the type distribution is very similar. However, after the onset of the crisis, there is a substantial increase in the MR type share, which is followed by a large increase in the P type share. Both effects are, however, shown to be temporary, resulting in a 2016 type distribution which is close to the pre-crisis distributions of 2004 and 2006.

Moreover, our model confirms substantial heterogeneity in individuals' reported stock market expectations, as often found in the literature (cf. Dominitz and Manski, 2007; Hudomiet et al., 2011; Hurd et al., 2011). For example, males and more educated respondents have on average higher expectations than females and less educated respondents. Heterogeneity with respect to observable characteristics can also be found in our rounding model. Males tend to round less often than females; this also holds for young and highly educated respondents. Again, unobserved heterogeneity is found to be an additional important factor to account for. While we find evidence for different rounding behavior between questions on more or less extreme stock market changes, we find no differences between the gain and loss domain.

Our paper is related to several strands of the literature. Substantively, we add to the analysis of heterogeneity in subjective expectations, both with respect to the level and to updating of beliefs, specifically in the domain of stock market returns (Dominitz and Manski, 2007, 2011; Hudomiet et al., 2011; Hurd et al., 2011; Ameriks et al., 2018). We also extend the econometric toolkit for the analysis of subjective expectations data, in two directions. First, we embed the discrete type classification for belief updating proposed by Dominitz and Manski (2011) in a panel model. Second, we enrich this panel model by a response model that allows for nonresponse and rounding, inspired by Manski and Molinari (2010) and building on the paramteric framework of Kleinjans and van Soest (2014). Our paper is also related to current research on response behavior in probabilistic expectations questions (Giustinelli et al., 2018) and the econometric modeling of stock market beliefs by Drerup et al. (2017) and von Gaudecker and Wogrolly (2018).

The remainder of this paper is organized as follows. We first describe our data and present basic descriptive analyses (Section 2). We then introduce our panel data model in Section 3 and present the results in Section 4. Robustness analyses are discussed in Section 5, while Section 6 concludes.

2 Data

The study was conducted using the CentER Panel, a household panel administrated by CentERdata at the University of Tilburg. About 2,000 Dutch households are interviewed online every spring in 2004, 2006, 2008, 2009, 2010, 2012, 2014 and 2016, making a total of eight waves (see Figure 1). While the majority of respondents participated right away, others who did not were contacted again three or four weeks later.

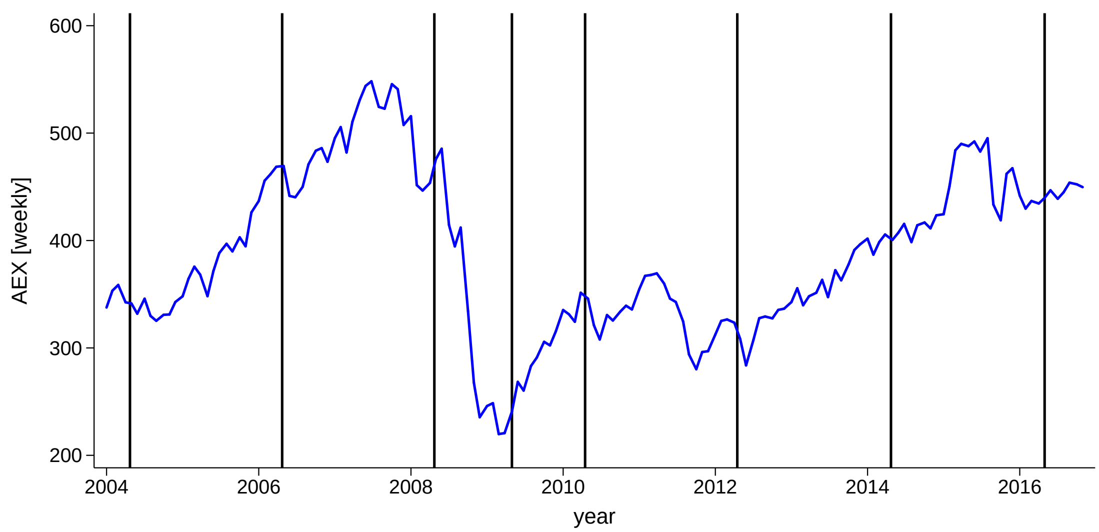


Figure 1: Amsterdam Stock Market Index (AEX) and spring interviews (vertical lines)

Most importantly, the questionnaire contains detailed probabilistic expectation questions on the stock market over a one-year horizon. The questions start with a short introduction explaining that the respondent has to imagine that she unexpectedly received 10,000 Euro from a rich relative and is thinking of putting the money into a mutual fund invested in "blue chip" stocks (like those in the Amsterdam AEX stock market index). We then ask for the chances that an investment in a broad investment fund will generate gains of more than 0%, 10%, 20% and 30%, as well as losses of more than 0%, 10%, 20% and 30% percent, for a total of eight questions. The four questions within each sequence (gain and loss) are always presented with increasing absolute threshold returns, but the gain and loss sequences are presented in random order. The wording of the first question in the gain sequence reads as follows:

> *Suppose you put the 10,000 Euro in the stock mutual fund and left it in for one year. What are the chances that you would make money where 0 means absolutely no chance and 100 means absolutely certain; that is what are the chances that in a year your investment would be worth more than 10,000 Euro?*

The other questions in this sequence use a very similar wording, with different numbers and adjusted to reflect the gain and loss sequence where appropriate. Moreover, the questionnaire in some years also contains questions on stock market experience, knowledge of average long-term returns for investment in risky and safe assets, and past trading history. For more detailed information, we refer the reader to our earlier paper (Hurd et al., 2011).

Households from the CentER Panel also participate in the annual DNB Household Survey (DHS), formerly known as the CentER Savings Survey, which has two major advantages. First, we are able to merge our data with very detailed background information from the DHS. Second, since probabilistic questions have repeatedly been asked both in the DHS and in special purpose surveys run in the CentER panel, members of the panel are well acquainted with this question format.

Overall, our (unbalanced) panel dataset contains 5,718 individuals who are observed in up to eight waves between 2004 and 2016, resulting in a total of 16,565 observations. Panel A of Table 1 displays standard summary statistics for the eight stock market expectation questions.[2] Overall, the respondents are quite pessimistic regarding the future stock market performance, confirming findings from earlier literature (Dominitz and Manski, 2007; Hurd

[2] Item non-response rates for subjective probability questions on the stock market are typically higher than for expectation questions in other domains (see, for example, Hurd, 2009). However, item non-response rates in our data are very similar to those from other surveys that include questions on stock market expectations, such as the Health and Retirement Survey (Kleinjans and van Soest, 2014).

et al., 2011). The average subjective probability that the stock market will make any loss (Loss > 0%) is 40.9% and therefore almost as high as the average subjective probability that the stock market will make any gain (42.7%). For questions on more extreme changes in the stock market (gains and losses of more than 10, 20 and 30 percent), respondents assign on average even more probability mass to negative events than to positive events. For example, respondents report an average chance of 7.3% for the Gain > 30% question, compared to an average chance of 14.9% for the Loss > 30% question.

Panel B of Table 1 concentrates on three past returns of the Amsterdam Exchange index (AEX), which respondents experienced prior to their interview. Specifically, we use the respondents' interview week to calculate the experienced returns for one year, one month and one week, respectively.[3] On average, respondents experienced a one-year return of three percent prior to their interview date. However, these returns are also quite volatile, with a standard deviation of roughly 27 percent and a minimum (maximum) return of −48 (+46) percent in April 2009 (April 2010). The experienced returns over shorter periods are naturally smaller in magnitude, but on average positive.

Panel C of Table 1 describes our sample regarding several socio-economic dummy variables. Overall, there are slightly fewer females than males in the sample. One in three respondents is younger than 45 years, while one in four respondents is 65 or older. One third of the respondents completed no more than primary school or prevocational training (low education), while another third completed higher vocational training or university education (high education). The average household has 0.7 children. Our measure of risk aversion is based on a measure developed by Barsky et al. (1997), which asks if respondents prefer their current income above a gamble with equal probabilities on a 33% worse lifetime income and a doubling of the income. Using this methodology, the majority of respondents are classified as risk averse. Unfortunately, this question is not asked in 2006, 2008 and 2009, leading to a substantial reduction in number of observations. In addition, respondents are asked whether they agree on that – generally speaking – *Most people can be trusted* rather than *One has to be very careful with other people*. Overall, 58% of the respondents agree on the former. Again, this question has not been asked in 2009.

[3] Since the large majority of respondents are interviewed in the same week, the variation in these returns is mainly temporal rather than cross-sectional.

Table 1: Summary statistics

	Mean	SD	Min	Max	Observations
A: Stock market expectations [%]					
Gain > 0%	42.70	27.03	0	100	13,940
Gain > 10%	22.77	21.65	0	100	13,666
Gain > 20%	12.76	16.77	0	100	13,569
Gain > 30%	7.31	13.39	0	100	13,510
Loss > 0%	40.91	25.55	0	100	13,936
Loss > 10%	29.55	25.64	0	100	13,541
Loss > 20%	20.40	23.64	0	100	13,417
Loss > 30%	14.94	22.42	0	100	13,281
B: AEX returns [%]					
One-year return	3.18	27.05	-48.91	46.65	16,565
One-month return	0.68	5.39	-11.31	10.08	16,565
One-week return	0.18	2.43	-4.23	4.02	16,565
C: Covariates [0/1]					
Female	0.47	0.50	0	1	16,554
Age > 64	0.25	0.44	0	1	16,565
Age < 45	0.33	0.47	0	1	16,565
Low education	0.30	0.46	0	1	16,547
High education	0.39	0.49	0	1	16,547
Partner	0.77	0.42	0	1	16,565
HH income: 1st quartile	0.25	0.43	0	1	16,565
HH income: 2nd quartile	0.25	0.43	0	1	16,565
HH income: 3rd quartile	0.25	0.43	0	1	16,565
HH income: 4th quartile	0.25	0.43	0	1	16,565
No. children in HH [#]	0.70	1.06	0	7	16,565
Riskaverse	0.84	0.36	0	1	9,856
Trust in other people	0.58	0.49	0	1	13,682

Notes: Sample consists of 5,718 individuals who are observed in up to eight waves between 2004 and 2016, $N = 16,565$. Varying number of observations due to item nonresponse. The dummy variable "Trust in other people" is not available in 2009, "Riskaverse" not in 2006, 2008 and 2009. For details see text.

Respondents' stock market expectations display considerable variation over time. Figure 2 displays average stock market expectations over time for each of the eight expectations questions. In general, expectations seem to follow the business cycle. The financial crisis in 2009 as well as the subsequent European sovereign debt crisis in 2012, coincide with dips in expectations in the gain domain (left panel) and peaks in the loss domain (right panel). Similarly, expectations are more optimistic during the boom of 2006. Overall, the largest changes in mean expectations can be found in the Gain > 0% and Loss > 0% question. If individuals are asked about more extreme events (gains and losses of more than 10, 20 or 30 percent), average expectations display considerably less variation over time. For example, the average probability that the AEX will increase by more than 30 percent in the next year varies only between 5.5% (in 2012) and 9% in (2014). For questions on the probability of more extreme loss events, there is more variation over time; in 2009, respondents shifted their entire distribution by roughly five percentage points upwards. Interestingly, the sovereign debt crisis in 2012 did not have such an effect. Overall, it seems as if during the financial crisis respondents systematically shifted probability mass to negative outcomes of the distribution. Additional descriptive analyses are reported in Appendix A.

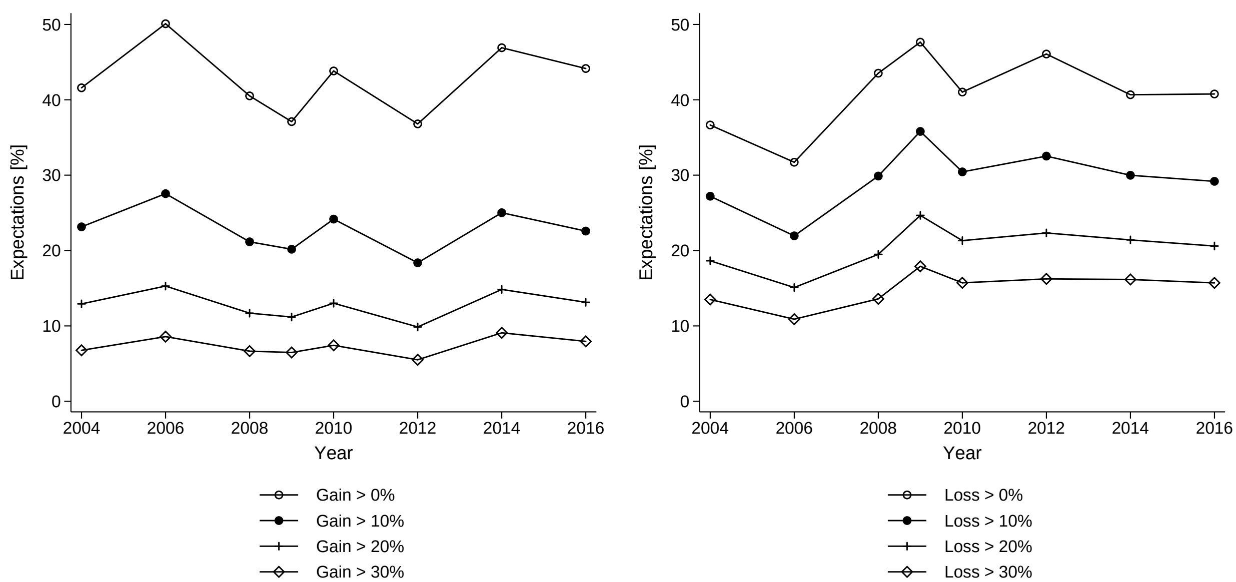


Figure 2: Descriptive time-series of subjective stock market expectations

3 Model

3.1 Modeling the subjective mean

Following Dominitz and Manski (2011), we assume that the population can be described by three latent expectation types who differ in how they use past stock market returns to form their stock market expectations. The first type (Random Walk, RW) believes that returns are independent and identically distributed (i.i.d.) over time and – given this belief – uses the long-run historical average return to predict returns. Type two (Mean Reversion, MR) believes that recent stock market changes will be reversed in the near future and type three (Persistence, P) believes that recent stock market changes will persist into the near future. Note that the literature also refers to type P as "Momentum" type (see, for example, Armona et al., 2018).

Suppose that the (latent) mean μ^*_{itk} of the subjective year-ahead stock market return distribution of respondent i of type k in period t can be described by

$$\mu^*_{itk} = \alpha_i^{Mu} + \boldsymbol{x}_{it}\boldsymbol{\beta} + f_k(\mathcal{R}_t), \qquad k = 1, 2, 3 \tag{1}$$

where α_i^{Mu} is a respondent-specific, unobserved effect and $\boldsymbol{x}_{it}$ is a vector of potentially time-varying covariates including a constant. $f_k(\mathcal{R}_t)$ is an expectation type-specific function of the history of past stock market returns at period t, $\mathcal{R}_t$. This function captures how an individual of expectation type k processes past stock market information. In Equation 1, expectation types differ in $f_k(\mathcal{R}_t)$ only, while other influences on μ^*_{itk} are assumed to be the same across expectation types. While $f_k(\mathcal{R}_t)$ may in principle contain any past return, which respondents experienced prior to their interviews, our model assumes that individuals particularly focus on the past one-year AEX return in period t (r_t). This seems natural, because respondents are also asked about their one-year ahead stock market expectations. The past one-year AEX return should therefore be particularly salient.[4] We assume that the function $f_k(\cdot)$ takes the following linear form:

$$f_k(\mathcal{R}_t) = f_k(r_t) = \gamma_k r_t, \qquad k = 1, 2, 3 \tag{2}$$

[4] As a robustness check, we also estimate the model for other return lags in Section 5.

This specification allows individuals to differ in their expectation type by different return coefficients γ_k. While the three return coefficients will later be unrestricted in the econometric model, the insights from Dominitz and Manski (2011) yield the following sign predictions:

- $k = 1$, Random Walk type. For these individuals, the return coefficient should be equal to zero, as they do not use the past one-year stock market to predict future returns, but rather focus on the long-run historical average return ($\gamma_1 = 0$).
- $k = 2$, Mean Reversion type. These individuals believe that recent stock market changes will be reversed in the near future, implying that the return coefficient should be negative ($\gamma_2 < 0$).
- $k = 3$, Persistence type. These individuals believe that recent stock market changes will persist into the near future, implying that the return coefficient should be positive ($\gamma_3 > 0$).

Since individuals' expectation type cannot be observed in the data, we use a standard random effects multinomial logit model with three possible outcomes to model individual type probabilities. Applying standard assumptions, these type probabilities are given by

$$P(D_{it} = k | \boldsymbol{x}_{it}, \alpha_i^2, \alpha_i^3) = \frac{\exp(\alpha_i^k + \tau_t^k + \boldsymbol{x}_{it}\boldsymbol{\beta}^k)}{\sum_{j=1}^{3} \exp(\alpha_i^j + \tau_t^j + \boldsymbol{x}_{it}\boldsymbol{\beta}^j)}, \qquad k = 1, 2, 3 \tag{3}$$

where α_i^k is an individual-specific unobserved effect for type k and τ_t^k are type-specific time effects. Without loss of generality, α_i^1, τ_t^1 and $\boldsymbol{\beta}^1$ are normalized to zero. Note that the type probabilities are allowed to depend on a vector of covariates $\boldsymbol{x}_{it}$ and that by construction the three type probabilities for a given individual i in period t sum up to one.

3.2 Construction of subjective means from survey responses

As discussed earlier, our survey respondents are presented with a total of eight questions on the future performance of the stock market. Specifically, respondents are asked about the following eight subjective probabilities:

$$p_{its} = \begin{cases} P(z > \delta_s) \quad \text{for } \delta_s \in \{0, 0.1, 0.2, 0.3\}, \qquad s = 1, 2, 3, 4 \\ P(z < \delta_s) \quad \text{for } \delta_s \in \{0, -0.1, -0.2, -0.3\}, \quad s = 5, 6, 7, 8 \end{cases} \tag{4}$$

where p_{its} is the reported probability of respondent i in period t to question s that the future one-year stock market return (z) will be greater or smaller than some threshold δ_s. Importantly, these eight answers refer to well-defined points on individuals' subjective cumulative distribution function (c.d.f.). We follow the literature and assume that the one-year stock market returns roughly follow a normal distribution, allowing us to calculate a parametric counterpart $\tilde{p}_{its}$ (for a given subjective unobserved mean μ^*_{itk} and standard deviation σ^*_k) to every survey answer:[5]

$$\tilde{p}_{itks} = \begin{cases} P(z > \delta_s) = \Phi\left(\frac{\mu^*_{itk} - \delta_s}{\sigma^*_k}\right) \quad \text{for } \delta_s \in \{0, 0.1, 0.2, 0.3\}, \qquad s = 1, 2, 3, 4 \\ P(z < \delta_s) = \Phi\left(\frac{\delta_s - \mu^*_{itk}}{\sigma^*_k}\right) \quad \text{for } \delta_s \in \{0, -0.1, -0.2, -0.3\}, \quad s = 5, 6, 7, 8 \end{cases} \tag{5}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function (c.d.f.) of the standard normal distribution. The existence of up to eight points on individuals' subjective distribution function actually over-identifies the model, as the normal distribution only depends on two parameters. While in Hurd et al. (2011) we estimate the parameters by non-linear least squares, our model estimates the two parameters by maximum likelihood. Specifically, we assume that respondents report unbiased expectations for all eight questions, i.e.

$$p_{itks} = \hat{p}_{itks} + u_{itks} \tag{6}$$

where u_{its} is normal with mean zero and variance σ^2 for all s: $u_{itks} \sim N(0, \sigma^2)$. For tractability reasons, we assume the variance of the error term to be identical across survey questions. However, an extension of our model could allow these variances to differ across questions, capturing potential differences between the gain and loss domain.

[5] Other studies that use reported points on individuals' normal subjective distribution function to calculate individual-level means and standard deviations include, amongst others, Dominitz and Manski (2007) who exactly identify the two parameters, Hurd et al. (2011) who use non-linear least squares and Bellemare et al. (2012) who approximate the distribution non-parametrically using splines.

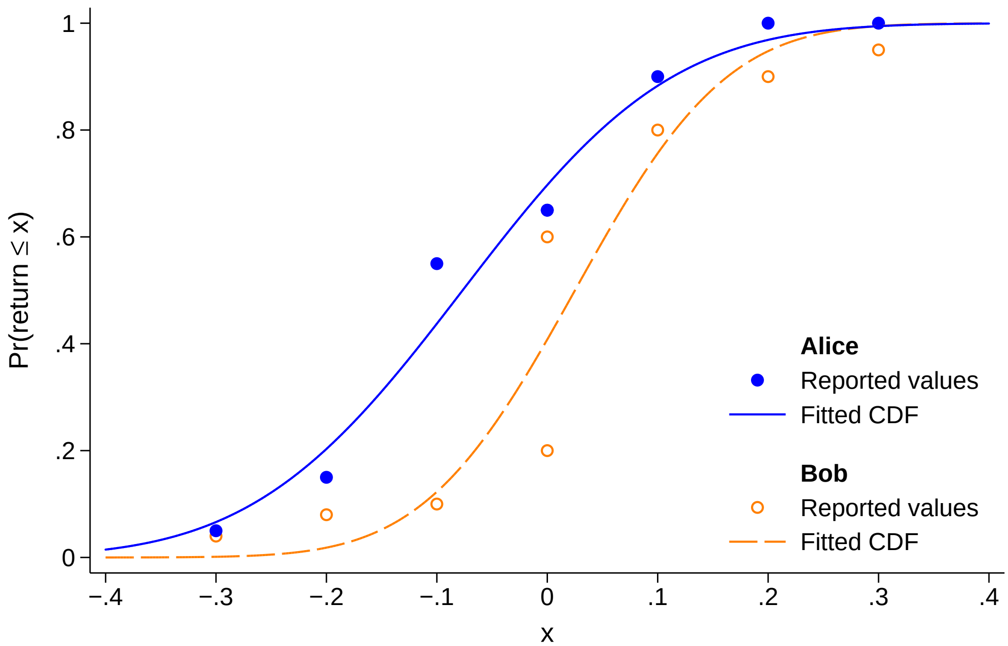


Figure 3: Fitting individual CDFs through the reported survey responses

Figure 3 illustrates how an individual's eight survey responses can be used to estimate the latent mean and standard deviation of her subjective distribution of future stock market returns. Consider two individuals, Alice and Bob. Alice (Bob) reports a 55% (10%) chance probability that the stock market will *decrease* by more than ten percent; for the probability that the stock market will *increase* by more than ten percent, the reported probabilities are 100%-90%=10% (Alice) and 100%-80%=20% (Bob). The fact that Bob's c.d.f. is shifted to the right indicates that his expectations regarding the future stock market performance are higher than those of Alice. Fitting a normal c.d.f. through these points by maximum likelihood (solid and dashed lines) yields estimated means of 2.34% for Bobs' c.d.f. and -8.60% for Alice. The mixture model we develop in this paper uses these ideas by averaging over the reported answers and allowing the parameters of the c.d.f. to differ across expectation types and to depend on socio-economic characteristics.

3.3 Rounding

The literature on subjective expectations has shown that individuals' survey responses are subject to rounding (cf. Manski and Molinari, 2010; Kleinjans and van Soest, 2014). Figure

4 plots the response distribution for two expectation questions, pooled across years. Clearly, there is evidence for considerable heaping at multiples of five and ten percent.[6] In the left panel (Gain > 0%), only 573 out of 13,940 respondents (4.1%) report a probability that is not a multiple of five. It is thus quite likely that at least some individuals do not report their true subjective probabilities p_{its}, but rather some rounded value. The same applies to the right panel, where respondents are asked about the probability that the stock market will increase by at least 20 percent (Gain > 20%). Here, the crude share of responses which are not multiples of five increases to 18% (2,449 out of 13,569 respondents). In addition, fewer respondents report a probability of 50% compared to the left graph.

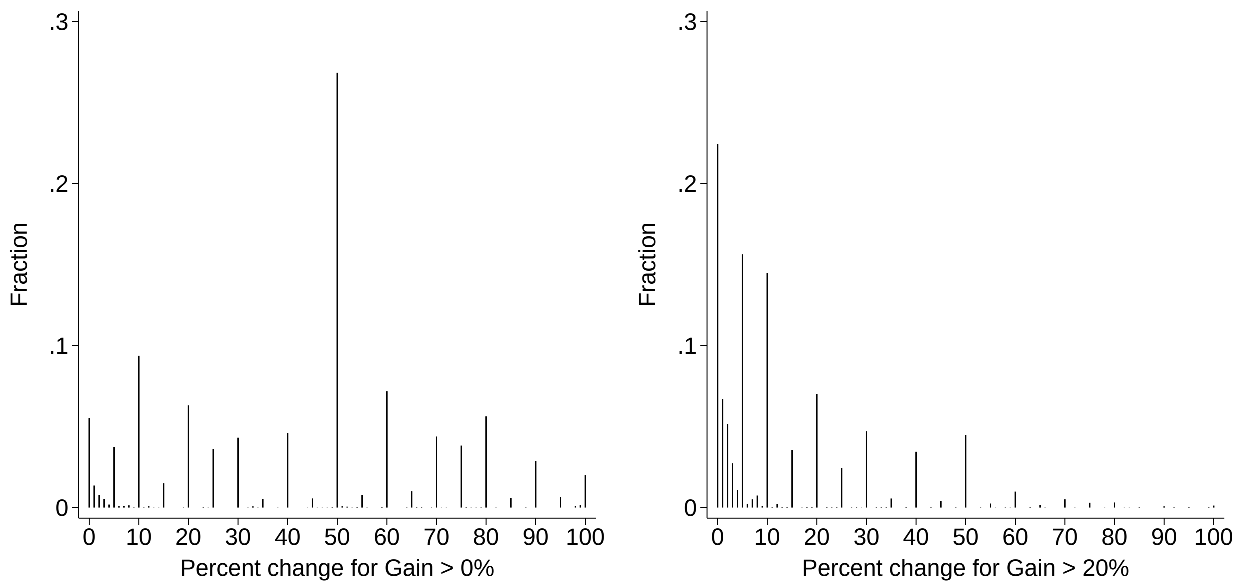


Figure 4: Response distribution for the Gain > 0% and Gain > 20% questions

Note that previous research often assumes that rounding patterns in probabilistic expectation questions are constant across domains. For example, using data from the Health and Retirement Survey (HRS), Giustinelli et al. (2018) make this assumption for different domains, such as health, personal finances and economic conditions. Our model, in contrast, assumes that rounding behavior is question-specific, even though the eight questions refer to the same domain, namely stock market performance. The difference may be explained by the fact that the objective (retrospectively "correct") answers to the eight probability questions differ in

[6] In addition, individuals also seem to round more in the center of the distribution than in the tails (cf. Giustinelli et al., 2018). For tractability reasons, however, we abstract from this phenomenon in our model.

magnitude (independent of expectation type). This might not be true in other settings, such as in Giustinelli et al. (2018), where the objective probabilities might be closer (even though across different domains). Moreover, question-specific rounding may also be more able to explain differences in rounding patterns, such as depicted in Figure 4.

To model individuals' rounding behavior, we adjust a model by Kleinjans and van Soest (2014) and argue that the population can be described by the following three (latent) rounding types:[7]

$R_{its} = 1$ (type R1):	the subjective probability is rounded to a multiple of 1 percent
$R_{its} = 2$ (type R5):	the subjective probability is rounded to a multiple of 5 percent
$R_{its} = 3$ (type R50):	the subjective probability is rounded to a multiple of 50 percent

where R_{its} represents the rounding type of individual i in period t to question s. Obviously, the three rounding types are increasing in their extent of rounding. As respondents only report integers, type R1 does in fact not round her expectations. Type R5 always rounds to the next multiple of five percent, while type R50 displays the strongest versions of rounding. She always rounds to the next multiple of 50 percent, which is equivalent to reporting 0, 50 or 100 percent.

Similar to the expectation types earlier, rounding types are generally unobserved in the data. For example, consider an individual who reports a subjective probability of 50% to the question of a positive stock market return (Gain > 0%). Clearly, her answer is consistent with all three rounding types. In contrast, a reported probability of 70% would identify her as either rounding type 1 or 2, while a reported probability of 18% exactly identifies her to be rounding type 1. This illustrates that individuals' rounding type is only partially revealed in the data.

Again, we use a finite mixture approach to model individual rounding type probabilities. Similar to Kleinjans and van Soest (2014), we model rounding behavior in a standard random

[7] Our model could easily be extended to include additional rounding types who round to multiples of ten or twenty percent, or another type who uses 50 percent answers to express epistemic uncertainty rather than an actual probability of 50 percent (cf. Bruine de Bruin et al., 2000). However, as we allow the rounding types to vary across survey questions, each additional rounding type comes with a substantial increase in the number of parameters to be estimated. To reduce the computational burden, we thus restrict the number of rounding types to three.

effects ordered probit model with three possible categories. Applying standard assumptions, the rounding type probabilities are given by

$$P(R_{its} = r|\boldsymbol{x}_{it}, \alpha_i^R) = \Phi\left(m_{sr} - \alpha_i^R - \boldsymbol{x}_{it}\boldsymbol{\beta}^R\right) - \Phi\left(m_{sr-1} - \alpha_i^R - \boldsymbol{x}_{it}\boldsymbol{\beta}^R\right), \qquad r = 1, 2, 3 \quad (7)$$

where α_i^R is a respondent-specific time-constant unobserved random effect that drives rounding behavior and $\boldsymbol{x}_{it}$ is a vector of potentially time-varying covariates (not including a constant). m_{sr} are the cut-off parameters for question s with the usual normalization $m_{s0} = -\infty$ and $m_{s3} = \infty$. For tractability reasons, we assume that differences in the rounding types probabilities across questions stem from the cut-offs only. We thus assume that the effect of the covariates and the random individual-specific effect on the rounding type probabilities are constant across the eight survey questions.

3.4 Distributional assumptions and likelihood function

In general, the likelihood function of the model depends on the unobserved individual random effects $\alpha_i^{Mu}, \alpha_i^2, \alpha_i^3$ and α_i^R, which we will denote by the vector of unobserved heterogeneity $\boldsymbol{\alpha}$. Using the assumptions from the previous sections, the likelihood function conditional on the unobserved heterogeneity, L^c, is given by:

$$L^c(\alpha_i^{Mu}, \alpha_i^2, \alpha_i^3, \alpha_i^R) = \prod_{i=1}^{N}\prod_{t=1}^{T} L_{it}^c(\alpha_i^{Mu}, \alpha_i^2, \alpha_i^3, \alpha_i^R) \quad (8)$$

where

$$L_{it}^c(\alpha_i^{Mu}, \alpha_i^2, \alpha_i^3, \alpha_i^R) = \sum_{k=1}^{3} P\left(D_{it} = k|\boldsymbol{x}_{it}, \alpha_i^2, \alpha_i^3\right) \cdot \prod_{s=1}^{8} L_{itks}^c(\alpha_i^{Mu}, \alpha_i^R). \quad (9)$$

The conditional likelihood contribution $L_{itks}^c(\alpha_i^{Mu}, \alpha_i^R)$ depends on the reported probabilities p_{its} as follows:

For $p_{its} \in \{0\%, 1\%, 2\%, ..., 100\%\}$ and $p_{its} \notin \{0\%, 5\%, 10\%, ..., 100\%\}$ (Rounding type 1):

$$L_{itks}^c(\alpha_i^{Mu}, \alpha_i^R) = P\left(R_{its} = 1|\boldsymbol{x}_{it}, \alpha_i^R\right)\left[\Phi\left(\frac{p_{its}+0.005-\tilde{p}_{itks}}{\sigma_u}|\boldsymbol{x}_{it}, r_t, \alpha_i^{Mu}\right) - \Phi\left(\frac{p_{its}-0.005-\tilde{p}_{itks}}{\sigma_u}|\boldsymbol{x}_{it}, r_t, \alpha_i^{Mu}\right)\right]$$

For $p_{its} \in \{0\%, 5\%, 10\%, ..., 100\%\}$ and $p_{its} \notin \{0\%, 50\%, 100\%\}$ (Rounding type 1 or 2):

$$\begin{aligned}L^c_{itks}(\alpha_i^{Mu}, \alpha_i^R) =& P\left(R_{its} = 1|\boldsymbol{x}_{it}, \alpha_i^R\right)\left[\Phi\left(\frac{p_{its}+0.005-\tilde{p}_{itks}}{\sigma_u}|\boldsymbol{x}_{it}, r_t, \alpha_i^{Mu}\right) - \Phi\left(\frac{p_{its}-0.005-\tilde{p}_{itks}}{\sigma_u}|\boldsymbol{x}_{it}, r_t, \alpha_i^{Mu}\right)\right] + \\ &+P\left(R_{its} = 2|\boldsymbol{x}_{it}, \alpha_i^R\right)\left[\Phi\left(\frac{p_{its}+0.025-\tilde{p}_{itks}}{\sigma_u}|\boldsymbol{x}_{it}, r_t, \alpha_i^{Mu}\right) - \Phi\left(\frac{p_{its}-0.025-\tilde{p}_{itks}}{\sigma_u}|\boldsymbol{x}_{it}, r_t, \alpha_i^{Mu}\right)\right]\end{aligned}$$

For $p_{its} \in \{0\%, 50\%, 100\%\}$ (Rounding type 1, 2 or 3):

$$\begin{aligned}L^c_{itks}(\alpha_i^{Mu}, \alpha_i^R) =& P\left(R_{its} = 1|\boldsymbol{x}_{it}, \alpha_i^R\right)\left[\Phi\left(\frac{p_{its}+0.005-\tilde{p}_{itks}}{\sigma_u}|\boldsymbol{x}_{it}, r_t, \alpha_i^{Mu}\right) - \Phi\left(\frac{p_{its}-0.005-\tilde{p}_{itks}}{\sigma_u}|\boldsymbol{x}_{it}, r_t, \alpha_i^{Mu}\right)\right] + \\ &+P\left(R_{its} = 2|\boldsymbol{x}_{it}, \alpha_i^R\right)\left[\Phi\left(\frac{p_{its}+0.025-\tilde{p}_{itks}}{\sigma_u}|\boldsymbol{x}_{it}, r_t, \alpha_i^{Mu}\right) - \Phi\left(\frac{p_{its}-0.025-\tilde{p}_{itks}}{\sigma_u}|\boldsymbol{x}_{it}, r_t, \alpha_i^{Mu}\right)\right] + \\ &+P\left(R_{its} = 3|\boldsymbol{x}_{it}, \alpha_i^R\right)\left[\Phi\left(\frac{p_{its}+0.250-\tilde{p}_{itks}}{\sigma_u}|\boldsymbol{x}_{it}, r_t, \alpha_i^{Mu}\right) - \Phi\left(\frac{p_{its}-0.250-\tilde{p}_{itks}}{\sigma_u}|\boldsymbol{x}_{it}, r_t, \alpha_i^{Mu}\right)\right]\end{aligned}$$

where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function (c.d.f.). i, t, k and s index respondents, periods, expectation types and survey questions, respectively. Recall that the (observed) reported subjective probabilities are denoted by p_{its}, while their parametric counterparts are denoted by $\tilde{p}_{itks}$. Note also that the likelihood function is written for a respondent who participates in every period and answers all eight probability questions. For the estimation, if a respondent does not answer one particular question, her likelihood contribution (L^c_{itks}) is replaced by one. Similarly, if a respondent did not participate in one particular period, her likelihood contribution for this period (L^c_{it}) is also replaced by one.

The unconditional likelihood function (L) can be derived by integrating out the individual effects:

$$L = \prod_{i=1}^{N} \int_{\mathbb{R}^4} \prod_{t=1}^{T} L^c_{it}(\alpha_i^{Mu}, \alpha_i^2, \alpha_i^3, \alpha_i^R) f(\boldsymbol{\alpha}) d\boldsymbol{\alpha}. \tag{10}$$

To avoid numerical integration in four dimensions, we use Maximum Simulated Likelihood (MSL) and replace the integral by a simulated mean. The simulated sample likelihood (SL) is then given by

$$SL = \prod_{i=1}^{N} \frac{1}{Q} \sum_{q=1}^{Q} \prod_{t=1}^{T} L_{it}^{c}(\alpha_{iq}^{Mu}, \alpha_{iq}^{2}, \alpha_{iq}^{3}, \alpha_{iq}^{R}) \tag{11}$$

where $\alpha_{iq}^{Mu}, \alpha_{iq}^{2}, \alpha_{iq}^{3}, \alpha_{iq}^{R}$ are simulated random effects for a given draw q. For the estimation, we assume that the unobserved heterogeneity follows a multivariate normal distribution with mean zero and arbitrary variance covariance matrix $\mathbf{\Sigma}$:

$$\boldsymbol{\alpha} \sim N(\mathbf{0}, \mathbf{\Sigma}). \tag{12}$$

Applying a Cholesky decomposition of $\mathbf{\Sigma}$, yields a positive semi-definite lower diagonal matrix $\mathbf{L}$ such that $\mathbf{\Sigma} = \mathbf{L}\mathbf{L}'$. For a given draw q, the unobserved heterogeneity is then calculated by $\boldsymbol{\alpha} = \mathbf{L}\boldsymbol{\tau}$, where $\boldsymbol{\tau}$ contains simulated vectors of the independent standard normal distribution. We follow Train (2003) and use draws from Halton sequences to obtain our independent standard normal variables $\boldsymbol{\tau}$.

4 Results

4.1 Heterogeneity in expectations, types and rounding

We apply our model to data from the CentER panel, as described in Section 2. We generally present the results from three different model specifications, while several alternative specifications are discussed as robustness checks in Section 5. The first specification fits a model with constants and the three (unrestricted) return coefficients only, while the second specification adds several socio-economic covariates and in the random effects multinomial logit model also year fixed effects. The third specification adds sign restrictions to the three return coefficients, i.e. we enforce $\gamma_1 = 0$, $\gamma_2 < 0$ and $\gamma_3 > 0$, corresponding to the expectation types Random Walk (RW), Mean Reversion (MR) and Persistence (P), respectively. For the entire analysis, it is important to keep in mind that our model is estimated jointly, even though we report the model estimates – for illustrative reasons – in several tables.

Table 2 reports estimated coefficients for the subjective mean model (Equation 1). As shown in column 1, the model identifies three distinct expectation types whose return coefficients for the past year AEX return differ in both sign and magnitude. Specifically, the three

Table 2: Model for the mean of the subjective distributions

	(1) Constants only		(2) Full model		(3) Restricted return coeff.	
γ_1: Return coeff. Cl1	0.0271***	[0.0015]	0.0286***	[0.0014]		
γ_2: Return coeff. Cl2	-0.5774***	[0.0167]	-0.5890***	[0.0166]	-0.5964***	[0.0185]
γ_3: Return coeff. Cl3	0.6576***	[0.0125]	0.7143***	[0.0127]	0.6154***	[0.0104]
Female			-0.0354***	[0.0023]	-0.0203***	[0.0024]
Age >64			-0.0014	[0.0016]	-0.0016	[0.0018]
Age <45			-0.0057***	[0.0017]	-0.0023	[0.0017]
Low education			-0.0103***	[0.0025]	-0.0257***	[0.0027]
High education			0.0134***	[0.0023]	0.0075***	[0.0028]
Partner			-0.0022	[0.0020]	0.0006	[0.0027]
HH income: 1st quart.			-0.0033	[0.0021]	-0.0000	[0.0027]
HH income: 2nd quart.			-0.0027	[0.0019]	-0.0013	[0.0022]
HH income: 3rd quart.			-0.0020	[0.0015]	0.0003	[0.0018]
No. children in HH			0.0007	[0.0008]	-0.0005	[0.0008]
Constant	-0.0386***	[0.0015]	-0.0114***	[0.0030]	-0.0128***	[0.0037]
σ_1^*	0.1176***	[0.0007]	0.1185***	[0.0007]	0.1167***	[0.0007]
σ_2^*	0.5445***	[0.0084]	0.5533***	[0.0087]	0.5767***	[0.0095]
σ_3^*	0.2775***	[0.0038]	0.2791***	[0.0037]	0.2601***	[0.0032]
σ_{CDFfit}	0.1597***	[0.0004]	0.1602***	[0.0004]	0.1596***	[0.0004]
LogLik	-332,714.34		-331,725.45		-331,997.03	
AIC	665,500.68		663,630.90		664,172.05	
Observations	14,282		14,264		14,264	

Notes: Table displays results for the subjective means model (Equation 1) as well as the type-specific estimates for the subjective standard deviations. For details see text. Standard errors in brackets; *** p <0.01, ** p <0.05, * p <0.1.

(unrestricted) return coefficients $(\gamma_1, \gamma_2, \gamma_3)$ are given by $(0.027, -0.577, 0.658)$. While the estimate for type two is significantly negative, the type three estimate is significantly positive and of similar magnitude. The third estimate, for example, suggests that a one percentage point increase in the past year AEX return increases the expected mean return for the year ahead of type three by roughly 0.658 percentage points, *ceteris paribus*. For type three, higher past year returns of the AEX are therefore associated with higher expectations for the year ahead. Similarly, for type two, past year AEX returns are associated with lower stock market expectations. Even though the estimated coefficient for the first type (γ_1) is significantly positive, its magnitude is – compared to the other two – considerably smaller, differing by a factor of 20 and in fact being close to zero. In approximation, we therefore argue that type one does not use the past year AEX return when forming her expectations. Using our earlier definition of expectation types, we therefore label the three expectations types (1,2,3) as (RW,MR,P).

Adding several socio-economic variables to the model leaves the three return coefficients almost unchanged (specification 2). We find, however, that stock market expectations – here summarized by the mean of the expected return distribution – vary substantially across individuals. Similar to findings from previous literature, males and more educated respondents have on average higher expectations than females and less educated respondents (cf. Dominitz and Manski, 2007, 2011; Hudomiet et al., 2011; Hurd et al., 2011).

In our third specification, we add sign restrictions to our return coefficients. Specifically, we restrict the first return coefficient (RW type) to be exactly zero. As the return coefficients of the second and third type are already negative and positive, the sign restrictions for those are actually non-binding. Overall, the estimates of the restricted model are almost identical to those of the unrestricted model, strengthening our interpretation of the three different expectation types.

The bottom part of Table 2 reports estimates for the type-specific standard deviations (σ_k^*) of the subjective return distributions. Throughout all specifications, the smallest dispersion in expectations can be found for type 1 (RW). Its estimate suggests a standard deviation of 0.12 for the expected year ahead return distribution. As one would expect, the return distribution is more volatile for the other two expectation types. While the distribution of type P has an estimated standard deviation of about 0.27, the estimate for type MR is equal to 0.55. This is in line with our interpretation that RW types base their expectations on the historical average return, while the other two types focus on recent changes and are thus subject to higher volatility. Last, Table 2 also reports the estimated standard deviation of the error term in Equation 6, σ_{CDFfit}, which is assumed to be constant across the eight probability questions.

We next turn to the estimates of the random effects multinomial logit model for the expectation type probabilities (Equation 3), which are reported in Table 3. Recall that the omitted category is type 1 (RW). Clearly, there is evidence for substantial heterogeneity in the type

Table 3: Random effects multinomial logit model for the expectation types

	(1) Constants only		(2) Full model		(3) Restricted return coeff.	
Class 2 (Mean Reversion)						
Female			0.6120***	[0.0713]	0.3394***	[0.0715]
Age >64			-0.5586***	[0.0901]	-0.5734***	[0.0936]
Age <45			0.6899***	[0.0747]	0.7835***	[0.0766]
Low education			0.0084	[0.0860]	0.2229**	[0.0889]
High education			-0.6089***	[0.0816]	-0.5621***	[0.0870]
Partner			0.2717***	[0.0878]	0.1975**	[0.0944]
HH income: 1st quart.			0.6510***	[0.1035]	0.5987***	[0.1070]
HH income: 2nd quart.			0.3570***	[0.0942]	0.3336***	[0.0974]
HH income: 3rd quart.			0.2764***	[0.0891]	0.2305**	[0.0919]
No. children in HH			-0.0152	[0.0333]	0.0011	[0.0338]
Constant	-1.3143***	[0.0421]	-1.9978***	[0.1467]	-1.9628***	[0.1543]
Class 3 (Persistence)						
Female			0.4790***	[0.0709]	0.2216***	[0.0661]
Age >64			-0.2835***	[0.0841]	-0.2922***	[0.0809]
Age <45			0.5440***	[0.0763]	0.5227***	[0.0733]
Low education			0.0870	[0.0872]	0.2687***	[0.0834]
High education			-0.3230***	[0.0811]	-0.2241***	[0.0795]
Partner			0.3157***	[0.0878]	0.2552***	[0.0879]
HH income: 1st quart.			0.4740***	[0.1036]	0.3926***	[0.1008]
HH income: 2nd quart.			0.2834***	[0.0934]	0.2189**	[0.0900]
HH income: 3rd quart.			0.2743***	[0.0867]	0.2341***	[0.0833]
No. children in HH			-0.0362	[0.0342]	-0.0154	[0.0322]
Constant	-1.1492***	[0.0444]	-2.2920***	[0.1532]	-2.0169***	[0.1497]
Implied Cl1 share	0.59		0.62		0.60	
Implied Cl2 share	0.20		0.19		0.18	
Implied Cl3 share	0.21		0.19		0.21	
LogLik	-332,714.34		-331,725.45		-331,997.03	
AIC	665,500.68		663,630.90		664,172.05	
Observations	14,282		14,264		14,264	

Notes: Table displays results for the random effects multinomial logit model for the expectation types (Equation 3). Baseline type 1 (Random Walk) is omitted. Specifications 2 and 3 also include year fixed effects. For details see text. Standard errors in brackets; *** p <0.01, ** p <0.05, * p <0.1.

probabilities. For example, females are significantly more likely to be type 2 (MR) or type 3 (P) than males. Highly educated respondents are more likely to be type 1 (RW). One interpretation could be that men and more educated respondents are usually found to be more informed about the stock market, hence more likely to have some information about the historical average return and therefore more likely to be type RW. The estimates also suggest the existence of an age gradient: younger respondents are more likely to be type 2 (MR) and type 3 (P), compared to older respondents. Overall, there is evidence for substantial heterogeneity in the expectation type probabilities.

Moreover, the model estimates can also be used to predict (unconditional) individual type probabilities.[8] The bottom part of Table 3 reports aggregated type probabilities, which can be interpreted as the (unconditional) sample distribution of expectation types. With only minor differences across specifications, our estimates suggest that the distribution of expectation types (RW,MR,P) is roughly (0.60,0.19,0.21). This indicates that most answers are actually in line with a RW type interpretation, while fewer responses are in line with type MR or type P. A detailed discussion of these type shares is presented in Section 4.2.

Table 4 reports coefficients of the random effects ordered probit model for the three rounding types R1, R5 and R50 (Equation 7). Recall that a higher rounding type is associated with a higher degree of rounding. Similar to Kleinjans and van Soest (2014), we find evidence for heterogeneity across the population. Males tend to round less often than females, as well as younger and highly educated people. In contrast, income is not associated with rounding behavior. The sixteen cut-off coefficients (two for each of the eight probability questions) are not reported, but used in order to determine the individual rounding type probabilities. Similar to the expectation type shares, we average the individual rounding type probabilities to predict the rounding type distribution in the population. Figure 5 displays these rounding type shares for each of the eight probability question. Clearly, there is evidence for less rounding in questions on more extreme outcomes. While less than five percent of respondents are estimated to provide exact answers in the questions Gain $> 0\%$ and Loss $< 0\%$, almost 30 percent of respondents do so for the questions Gain $> 30\%$ and Loss $< 30\%$. Interestingly, there seems to be no difference between the gain and the loss domain. In fact, the shares are almost identical.

Last, Table 5 reports the variances and the correlations of the four random individual effects,

[8] The (unconditional) individual type probabilities are based on Equation 3, with the true parameter vectors τ_t^k and $\boldsymbol{\beta}_k$ being replaced by their respective estimates $\hat{\tau}_t^k$ and $\hat{\boldsymbol{\beta}}_k$ and the individual effects α^k being integrated out by simulation. Specifically, we use 71 draws from Halton sequences and simulate the normal individual effects with mean zero and a variance-covariance matrix which is given by the estimate of $\hat{\boldsymbol{\Sigma}}$.

Table 4: Random effects ordinal probit model for rounding types

	(1) Constants only	(2) Full model		(3) Restricted return coeff.	
Female		0.0848***	[0.0229]	-0.0154	[0.0209]
Age >64		0.0081	[0.0195]	0.0233	[0.0203]
Age <45		-0.0871***	[0.0192]	-0.0515***	[0.0188]
Low education		-0.0537**	[0.0261]	-0.0048	[0.0247]
High education		-0.0651***	[0.0227]	-0.0989***	[0.0248]
Partner		0.0317	[0.0243]	0.0005	[0.0268]
HH income: 1st quart.		0.0303	[0.0250]	-0.0294	[0.0275]
HH income: 2nd quart.		0.0270	[0.0217]	-0.0270	[0.0238]
HH income: 3rd quart.		-0.0201	[0.0194]	-0.0316	[0.0200]
No. children in HH		0.0123	[0.0086]	0.0113	[0.0090]
LogLik	-332,714.34	-331,725.45		-331,997.03	
AIC	665,500.68	663,630.90		664,172.05	
Observations	14,282	14,264		14,264	

Notes: Table displays results for the random effects ordinal probit model for the rounding types (Equation 7). Dependent variable is equal to 1 if the respondent does not round (R1), 2 if the respondent rounds to the next multiple of five (R5) and 3 if the respondent rounds to the next multiple of 50 (R50). Question type-specific cut-off parameters are not reported. For details see text. Standard errors in brackets; *** p <0.01, ** p <0.05, * p <0.1.

which are derived from the estimated entries of the Cholesky matrix $\hat{\mathbf{L}}$. All four individual effects have in fact a variance significantly different from zero. In addition, their correlations are also significantly different from zero. The correlation between α^R and α^2 is, for example, significantly positive. This indicates that individuals who are more likely to round are also more likely to be of type 2 (MR). Applying the same logic, we also find that individuals who are more likely to round are also more likely to be of type 3 (P). The other interpretations are similar, but less intuitive.

4.2 Expectation type shares and the financial crisis

The estimated values of the model parameters can be used to predict individual unconditional type probabilities as well as posterior probabilities, i.e. conditional on the reported expectations. Figure 6 plots the sample distribution of the unconditional type probabilities based on the results of the full model (Table 2, specification 2). The posterior probabilities

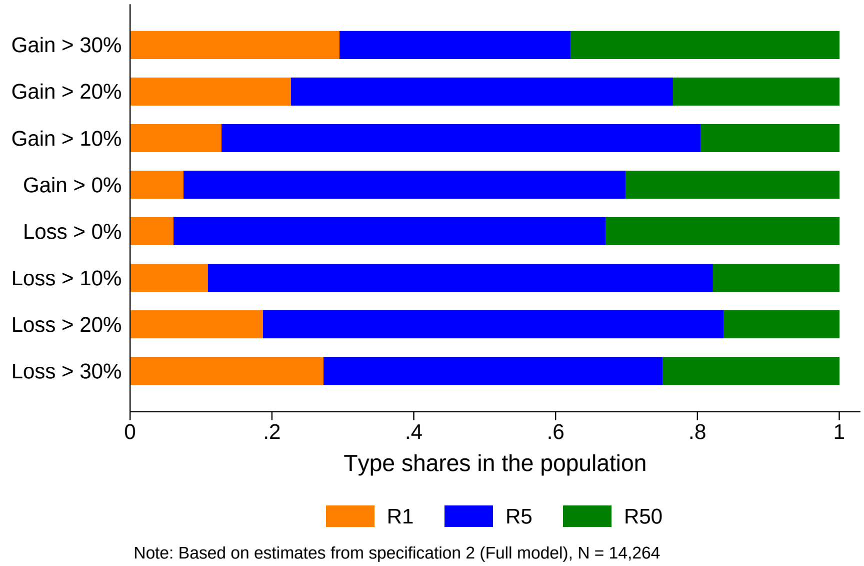


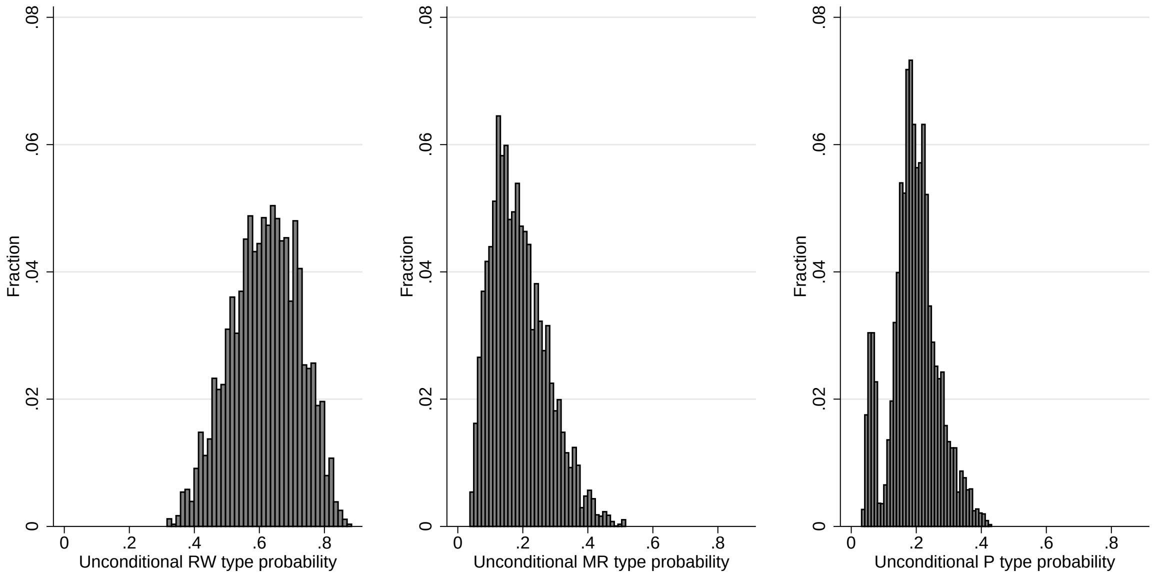
Figure 5: Rounding type distribution in the sample across questions for the model with covariates

Table 5: Variances and correlations of the individual effects

	(1) Constants only		(2) Full model		(3) Restricted return coeff.	
Variances						
$V(\alpha^{Mu})$	0.0087***	[0.0002]	0.0071***	[0.0002]	0.0088***	[0.0003]
$V(\alpha^2)$	2.8354***	[0.1777]	2.2242***	[0.1592]	2.6355***	[0.1771]
$V(\alpha^3)$	2.2625***	[0.1559]	1.8099***	[0.1380]	1.6872***	[0.1330]
$V(\alpha^R)$	0.6991***	[0.0180]	0.7345***	[0.0186]	0.7307***	[0.0188]
Correlations						
$Corr(\alpha^{Mu}, \alpha^2)$	-0.9274***	[0.0100]	-0.9926***	[0.0071]	-0.8771***	[0.0147]
$Corr(\alpha^{Mu}, \alpha^3)$	-0.9809***	[0.0054]	-0.9997***	[0.0012]	-0.9658***	[0.0083]
$Corr(\alpha^{Mu}, \alpha^R)$	-0.1483***	[0.0140]	-0.3222***	[0.0121]	-0.2227***	[0.0187]
$Corr(\alpha^2, \alpha^3)$	0.9824***	[0.0054]	0.9948***	[0.0040]	0.9708***	[0.0080]
$Corr(\alpha^2, \alpha^R)$	0.5072***	[0.0240]	0.3239***	[0.0122]	0.6635***	[0.0224]
$Corr(\alpha^3, \alpha^R)$	0.3377***	[0.0278]	0.3229***	[0.0123]	0.4663***	[0.0306]
Observations	14,282		14,264		14,264	

Notes: This table reports estimates for the variances of the four random effects and their correlations. α^{Mu} denotes the random effect in Equation 1. α^2 and α^3 are the random effects in the multinomial logit model for the expectation type probabilities, where α^1 is normalized to zero (Equation 3). α^R denotes the random effect in the ordered probit model for the rounding type probabilities (Equation 7). Standard errors in brackets; *** p <0.01, ** p <0.05, * p <0.1.

(not shown), indicates that our model classifies respondents reasonably well: 11,022 out of 14,264 respondents (77%) are as good as uniquely classified (i.e. with a posterior probability of more than 90%) as either type RW, MR or P.



Note: Based on estimates from specification 2 (Full model), N = 14,264

Figure 6: Distributions of implied unconditional type probabilities

The upper panel of Figure 6 shows the unconditional sample contribution of type probabilities, averaged over individuals and time. The means of the three distributions are given by (0.60,0.19,0.21) for expectation types (RW,MR,P), as already reported in Table 3. This is somewhat in contrast to the findings by Dominitz and Manski (2011) who find that most individuals are found to be type P. Specifically, using a simple ordinal criterion, the authors find the type distribution to be (0.27,0.32,0.41) for survey participants of the Michigan Survey of Consumers. We present two potential reasons for this difference.

First, our model assumes that type RW puts zero weight on recent stock market changes, because she uses the long-run historical average return rather than short-run fluctuations to form expectations. There may, however, also be other reasons not to use recent stock market returns when forming expectations. For example, respondents may form expectations intuitively or rely on heuristics (Drerup et al., 2017). The return coefficient of these respondents will also be zero. Unfortunately, these respondents are observationally equivalent to true RW types, explaining a higher RW share, compared to Dominitz and Manski (2011).

Second, while Dominitz and Manski (2011) base their classification on one probabilistic question only (Gain > 0%), we base the analysis on the entire distribution of future stock market returns, which is identified by the responses to the eight probability questions. As presented in Appendix B, applying their original methodology to our Gain > 0% question yields an average type distribution of (0.30, 0.26, 0.44). This implied type distribution is extremely close to their original finding using data on the S&P 500 index and from the Michigan Survey of Consumers between 2002 and 2004. However, applying the same methodology to questions on larger changes in the stock market, such as Gain > 30%, yields a share distribution which is actually very close to our results. In particular, the type distributions based on questions of more extreme changes in the stock market, such as Gain > 20% or Loss > 20%, yields substantially higher RW type shares. For more details, see Appendix B.

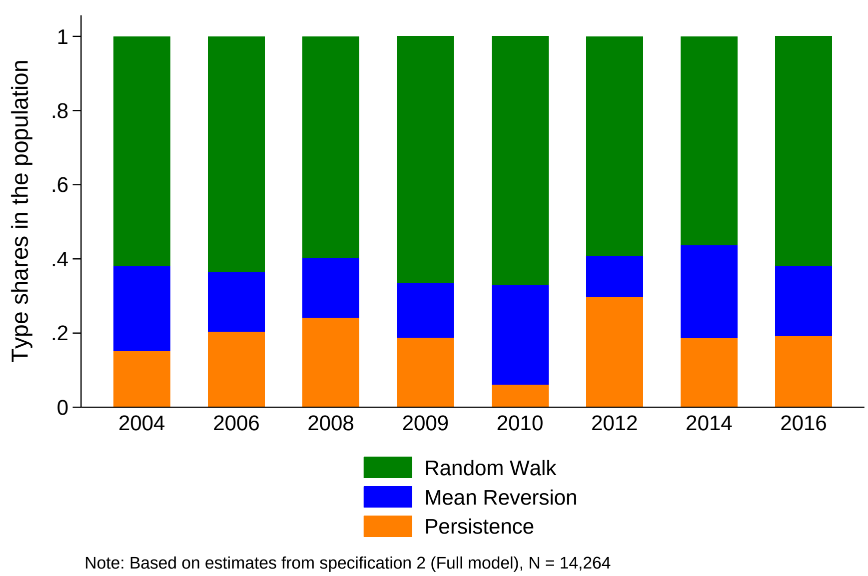


Figure 7: Expectation type distribution in the sample across years

Next, the inclusion of year-fixed effects in the random effects multinomial logit model for the type probabilities allows us to predict year-specific type distributions. As our sample covers the period between 2004 and 2016, we are able to analyze the effect of the 2008/09 financial crisis on the type distributions. Figure 7 plots the conditional type distribution over time, again based on the results of the full model (Table 2, specification 2); the graph for alternative specifications looks very similar. Clearly, there is evidence for variation over time. In years not affected by the financial crisis (2004, 06, 08, 14, 16) the type distribution looks similar.[9] In addition, there is little change at the onset of the financial crisis in 2009. In 2010, however, the MR share increases substantially. Two years later, the MR share drops again and is replace by a substantial increase in the P share. After 2012, the effect of the financial crisis seems to level off and the type share distribution returns to levels, which are similar to those of 2004. We therefore conclude that the effects of the financial crisis on the expectation type distribution were only temporary.

5 Robustness

This section provides several robustness checks to variations in methodology and sample size. To reduce the computational burden, the specifications are estimated under the sign restrictions of the three return coefficients ($\gamma_1 = 0$,$\gamma_2 < 0$,$\gamma_3 > 0$) for the three expectations types (RW,MR,P). The results can thus be compared to the estimates from specification 3 in Tables 2, 3 and 4, respectively. The corresponding tables are presented in Appendix C.

Monotonicity of probability responses. Similar to other surveys, some respondents in our data report expectations which clearly violate basic laws of probabilities. For example, they report a higher chance that the stock market will increase by 20 percent than that the stock market will increase by 10 percent, clearly violating monotonicity. These respondents can actually be included in our main model, because we require monotonicity only at the aggregate, but not at the individual level.[10] Overall, roughly 20 percent of the observations violate (weak) monotonicity at least once. Excluding those from the estimation (Tables C1, C2 and C3), however, leaves the results unchanged. The (absolute) magnitude of the return coefficients decreases slightly, while the associations with the covariates as well as the implied type share distribution remain almost identical.

Answering all eight probability questions. We also estimate one specification that restricts the sample to respondents who answer all eight probability questions, resulting in a nine percent drop in the number of observations (Tables C1, C2 and C3). All the estimates are extremely close to our main specification, including the return coefficients. This ensures, in particular,

[9] Note that interviews are conducted in April and May of 2008. Since the financial crisis hit the Netherlands in June 2008, the first wave affected in our data is the 2009 wave. See also Figure 1.

[10] As shown in Figure 2, aggregate monotonicity in our data set is fulfilled at any point in time.

that our finding that respondents round less when asked about more extreme changes in the stock market is not driven by the fact that some respondents only answer the questions Gain > 0% or Loss > 0%, potentially because the follow-up questions are too difficult for them to understand.

50/50 answers and epistemic uncertainty. Bruine de Bruin et al. (2000) show that some respondents use 50/50 answers to express uncertainty rather than an actual probability of 50% (epistemic uncertainty). In principle, our model could also include another reporting type (next to R1, R5 and R50) which reports 50% to express uncertainty. However, in order to not further increase the complexity of our model, we rather estimate a specification which excludes all observations where at least one of the eight probability questions is answered with "50%" (Tables C1, C2 and C3). This almost halves our number of observations to 7,353. Surprisingly, the absolute magnitude of the return coefficients increases by a factor of five. More importantly, however, the sign of the return coefficients and thus the interpretation of our expectation types remains the same. The (RW,MR,P) type distribution is given by (0.82,0.08,0.10), thus predicting a considerably higher share of RW types.

Short-run returns. Our model assumes that respondents put a particular focus on the past one-year AEX return when forming their expectations. This assumption seems rather plausible, because respondents are also asked about their one-year ahead expectations. However, we also estimate the model under the assumption that respondents focus on the past one-month and one-week return (Tables C4, C5 and C6). Again, the magnitude of the return coefficients increases substantially, which can, however, be explained by the smaller magnitude of the short-term returns, as shown in Table 1. More importantly, the implied type share distributions for the one-month and the one-week return are given by (0.67,0.14,0.19) and (0.65,0.14,0.21), respectively, and thus almost identical to our main specifications. The associations with the covariates is extremely similar to the main findings, the only exception being that the covariates seem to be less associated with individuals' rounding behavior.

Risk aversion and trust. We are also interested in how economic preferences, such as risk aversion and general trust in other people, affect type probabilities and expectations per se. Unfortunately, both variables have not been asked in all waves (cf. Section 2), leading to a substantial reduction in sample size (Tables C7, C8 and C9). Including both preference variables in the model (specification 1) shows that risk averse individuals have, on average, lower stock market expectations and are more likely to round. Risk aversion is, however, not related to individual expectation type probabilities. In contrast, individuals with higher levels of trust are more likely to be type RW than type MR or P. In addition, they also have higher expectations and are less likely to round. The magnitude of the type P return coefficient increases as well as the sample share of type RW (at the expense of type P). Both effects are shown to be driven by the reduction in sample size rather than by the inclusion of both economic preferences (specification 2).

6 Conclusion

This paper introduced a panel data model with a finite mixture of different expectation types who differ in how they take past returns into account when forming expectations. Such response types are not naturally given, and one could think of alternative definitions. We follow Dominitz and Manski (2011) and estimate the model for three expectations types that are governed by random walk, mean revision, and persistence updating, respectively. We find that most respondents report expectations which are in line with a random-walk interpretation, while fewer answers are consistent with mean reversion or persistence updating. We find evidence for considerable heterogeneity in the type membership, which is predicted by observable characteristics, and also considerable variation over time.

We believe that our approach could be extended in several directions. Conceptually, it would be straightforward to add additional expectations types, even though they are not naturally given and it is unclear what would be gained from such an exercise. From a more technical perspective, the finite mixture model might get more unstable if too many types are added. One could also try to make the rounding model more realistic, for instance by adding additional types as in Kleinjans and van Soest (2014) or by using ideas developed in Giustinelli et al. (2018).

From a substantive perspective, the model might be used to study the determinants of heterogeneity in expectations formation, for instance by conditioning type membership on experiences individuals made over their life, as for example in Malmendier and Nagel (2011), Malmendier et al. (2017) or Rossmann (2019).

Appendix

A Additional descriptive analyses

In the following sections, we provide additional descriptive analyses of our data. While Figure 2 reported the cross-sectional means for the eight probability questions, we also report the cross-sectional standard deviations in Figure A1. Note that this measure is often used in the literature to measure disagreement among respondents and thus uncertainty (cf. Zarnowitz and Lambros, 1987; Bachmann et al., 2013).

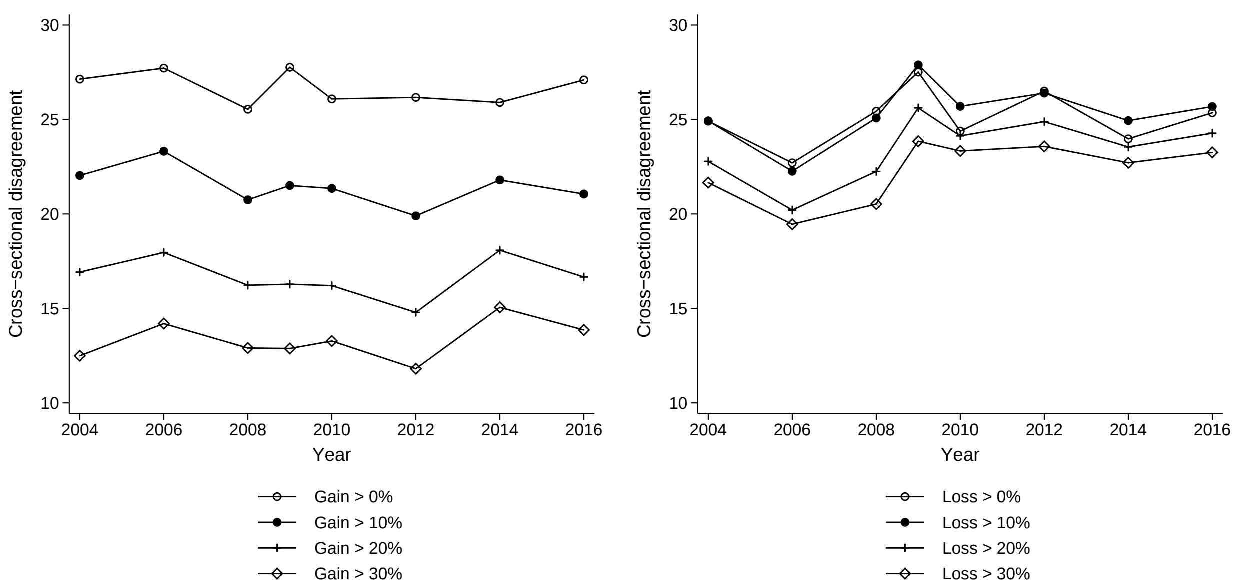


Figure A1: Cross-sectional disagreement of expectations over time.

Overall, there are two striking differences between the gain and loss domain. First, the level of disagreement decreases if respondents are asked about more extreme changes in the AEX in the gain domain, but not in the loss domain. For example, the cross-sectional average standard deviation for the question Gain > 30% is only 13 percent, compared to 22 percent for the question Loss > 30%. In contrast, disagreement levels for the questions Gain > 0% and Loss > 0% are similarly high. This difference may be driven by the fact that many respondents agree on a zero percent chance for large gains, but they agree less on a zero percent chance for large losses. Second, our data supports the argument from the literature that respondents' disagreement may be used as an indicator for uncertainty, but only for questions in the loss domain. While in the loss domain, there is indeed a stark increase during the financial crisis in 2009, this increase is less pronounced or even absent for questions in the gain domain. The correlations between the four disagreement measures in the loss domain vary between 0.65 and 0.97. In contrast, the disagreement measures in the gain domain are rather uncorrelated, with correlation ranging between 0.03 (Gain > 0% and Gain > 30%) and 0.87 (Gain > 20% and Gain > 30%). These findings indicate that uncertainty measures based on questions in a loss framing might be more appropriate than from questions in a gain framing.

Another indicator for a stark difference between the gain and the loss domain can be found when looking at the within-respondent variation over time, for each of the eight survey questions. In particular, we are interested in how strongly respondents change their expectations between periods. We therefore estimate respondent-specific (sample) standard deviations of answers across periods for each of the eight questions separately. For clarification consider the following example. Respondent A (B) is observed in four (two) periods. The corresponding responses for the question on a positive stock market return (Gain > 0%) are given by (70, 80, 60, 60) and (80, 80), respectively. The within-respondent (sample) standard deviation across periods for the question on positive stock market returns would then be 9.57 for respondent A and zero for respondent B. For each respondent, we calculate the standard deviation across periods for all of the eight expectations questions.[11]

Table A1 displays summary statistics for our measure of within-respondent disagreement. Again, the largest adjustments are made for questions on any gain or any loss. The more extreme the outcome of the question gets, the less volatile are the answers to that particular question. More interestingly, however, is the difference between the gain and loss domain. While there is almost no difference for the question on any gain or loss, the picture changes when we look at questions on larger gains and losses. Here, answers in the loss domain are considerably more volatile than in the gain domain. For the questions on gains and losses

[11] Note that individuals have to be observed at least twice in order to calculates the (sample) standard deviation.

of more than 30 percent, the difference in average standard deviation amounts to roughly five percentage points (12.06% versus 6.73%). In line with previous evidence, it seems that respondents tend to adjust their expectations more in the loss domain than the gain domain.

Table A1: Summary statistics for within-respondent disagreement (across years).

	Mean	p25	p50	p75	Min	Max	N
Gains							
Gain > 0%	18.58	9.57	18.35	26.15	0	70.71	2,783
Gain > 10%	14.80	7.07	13.45	21.21	0	70.71	2,732
Gain > 20%	10.03	2.89	7.07	15.00	0	67.18	2,709
Gain > 30%	6.73	0.71	3.21	9.06	0	70.71	2,701
Losses							
Loss > 0%	18.49	9.57	17.56	25.32	0	70.71	2,778
Loss > 10%	17.43	7.07	15.12	24.75	0	70.71	2,705
Loss > 20%	14.34	4.35	10.61	21.21	0	70.71	2,686
Loss > 30%	12.06	2.19	6.83	19.24	0	70.71	2,665

Notes: This table reports summary statistics for the within-respondent disagreement, i.e. sample standard deviation, across periods for each of the eight probabilistic questions on stock market returns. The across-period standard deviation is only defined if the respondent answers the question in at least two periods. For details see text.

B Ordinal methodology by Dominitz and Manski (2011)

Using the same definitions for the (RW,MR,P) expectation types as we do in the present paper, Dominitz and Manski (2011) propose an ordinal methodology to classify respondents. They argue that expectations of a given respondent are consistent with the RW type if and only if expectations hardly change between waves. Similarly, if a respondent adjusts her expectations by more than a certain cut-off, she can be classified as MR or P type, depending on the adjustment's direction and the recent short-term stock market performance.

For clarification, consider the following example. A respondent is interviewed on her stock market expectations in 2004 and 2006 – a period in which the AEX index increased almost monotonically (see Figure 1) and more importantly, the one-year return in 2006 was higher than the one-year return in 2004. If the respondent was a RW type, she would hardly adjust her expectations in 2006, as the long-run historical average return will only be marginally affected by those two additional years. In contrast, a P type would positively adjust her 2004 expectations, because she beliefs the (positive) recent stock market performance to persist into the near future. Similarly, if she was a MR type, she would lower her expectations in 2006. Note that this simple methodology uniquely classifies respondents into one of the three expectation types, while our panel data model avoids this classification by assigning individual probabilities for each of the three types.

Following Dominitz and Manski (2011), we measure recent stock market performance by the difference in the past one-year stock market returns between two waves and choose a cut-off of five percentage points. We apply this methodology to all eight probability questions on the stock market for every respondent who is observed in at least two subsequent waves. The results are summarized in Figure B1. Focusing on the question of a positive stock market return (Gain $> 0\%$), we get a type distribution of (0.29,0.26,0.45), which is extremely close to the findings by Dominitz and Manski (2011) using data from the Michigan Survey of Consumers and data on the S&P 500 index. However, this distribution differs somewhat to the results from our panel data model, which suggest a higher share of RW types. Potential reasons are discussed in Section 4.2. However, when applying the same methodology to questions on larger gains increases the share of RW types almost monotonically. In fact, responses to the question Gain $> 30\%$ imply a type distribution of (0.73,0.11,0.16) and therefore an even higher share of RW types as suggested in our model. Interestingly, at least in terms of the implied type distribution there seems to be absolutely no difference between the gain and the loss domain. Both the levels and the monotonic increase of the RW types are similar for both domains.

Similarly, also increasing the ad-hoc cut-off of five percentage points increases by definition the share of RW respondents, and can thus confirm our findings.

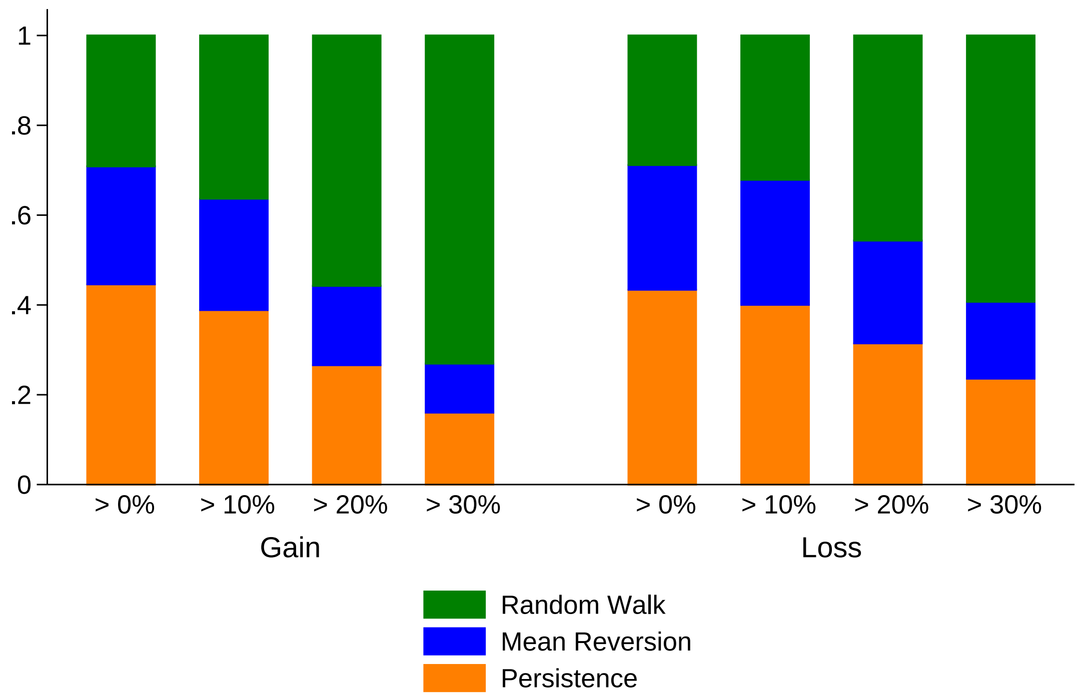


Figure B1: Type distributions with ordinal Dominitz and Manski (2011) criterion

C Additional Figures and Tables

Table C1: Model for the mean of the subjective distributions – robustness with respect to expectation response behavior

	(1) Only monotonic answers		(2) Eight probability questions		(3) Drop 50s	
γ_2: Return coeff. Cl2	-0.3200***	[0.0105]	-0.5801***	[0.0178]	-2.6640***	[0.1525]
γ_3: Return coeff. Cl3	0.4779***	[0.0080]	0.5898***	[0.0099]	2.5035***	[0.1108]
Female	-0.0116***	[0.0020]	-0.0229***	[0.0023]	-0.0128***	[0.0017]
Age >64	0.0003	[0.0015]	-0.0034**	[0.0016]	-0.0027*	[0.0016]
Age <45	0.0042***	[0.0016]	-0.0028	[0.0018]	0.0044***	[0.0017]
Low education	-0.0088***	[0.0022]	-0.0130***	[0.0024]	-0.0086***	[0.0020]
High education	0.0130***	[0.0021]	0.0060**	[0.0025]	0.0012	[0.0019]
Partner	0.0006	[0.0019]	-0.0026	[0.0031]	-0.0011	[0.0019]
HH income: 1st quart.	-0.0072***	[0.0022]	-0.0051**	[0.0026]	-0.0066***	[0.0022]
HH income: 2nd quart.	-0.0072***	[0.0019]	-0.0063***	[0.0020]	-0.0068***	[0.0019]
HH income: 3rd quart.	-0.0039***	[0.0015]	-0.0015	[0.0017]	-0.0032*	[0.0017]
No. children in HH	0.0000	[0.0009]	-0.0006	[0.0008]	-0.0027***	[0.0007]
Constant	-0.0166***	[0.0029]	-0.0058	[0.0048]	0.0188***	[0.0027]
σ_1^*	0.1026***	[0.0007]	0.1162***	[0.0007]	0.1063***	[0.0008]
σ_2^*	0.3963***	[0.0053]	0.5831***	[0.0098]	0.6436***	[0.0337]
σ_3^*	0.1894***	[0.0022]	0.2573***	[0.0031]	0.4237***	[0.0151]
σ_{CDFfit}	0.1401***	[0.0004]	0.1573***	[0.0004]	0.1777***	[0.0006]
LogLik	-252,684.91		-316,029.07		-182,140.62	
AIC	505,547.81		632,236.13		364,459.24	
Observations	11,402		12,973		7,353	

Notes: This table re-estimates the main model using only respondents who do not violate basic laws of probabilities (specification 1), who answer all eight expectations questions on the stock market performance (specification 2) and who do not report a probability of 50 percent to a specific expectation question (specification 3). Model uses sign restrictions for the return coefficients ($\gamma_1 = 0$, $\gamma_2 < 0$, $\gamma_3 > 0$). Table displays results for the subjective means model (Equation 1) as well as the type-specific estimates for the subjective standard deviations. For details see text. Standard errors in brackets; *** p <0.01, ** p <0.05, * p <0.1.

Table C2: Random effects multinomial logit model for the expectation types – robustness with respect to expectation response behavior

	(1) Only monotonic answers		(2) Eight probability questions		(3) Drop 50s	
Class 2 (Mean Reversion)						
Female	0.2432***	[0.0741]	0.4152***	[0.0701]	0.3926***	[0.1211]
Age >64	-0.7801***	[0.1037]	-0.5538***	[0.0962]	-0.7351***	[0.1750]
Age <45	0.7960***	[0.0814]	0.7650***	[0.0770]	0.3207**	[0.1383]
Low education	-0.2180**	[0.0963]	0.1743**	[0.0871]	0.2227	[0.1491]
High education	-0.5892***	[0.0854]	-0.4323***	[0.0831]	-0.5208***	[0.1497]
Partner	0.1961**	[0.0957]	0.3562***	[0.0961]	0.1558	[0.1617]
HH income: 1st quart.	0.6028***	[0.1150]	0.7227***	[0.1071]	0.5837***	[0.1910]
HH income: 2nd quart.	0.4355***	[0.1042]	0.3988***	[0.0974]	0.2023	[0.1784]
HH income: 3rd quart.	0.2446**	[0.0968]	0.2924***	[0.0920]	0.1128	[0.1685]
No. children in HH	-0.0120	[0.0383]	-0.0019	[0.0338]	0.0272	[0.0627]
Constant	-1.9476***	[0.1641]	-2.2393***	[0.1578]	-3.0667***	[0.2629]
Class 3 (Persistence)						
Female	0.1131	[0.0696]	0.2447***	[0.0657]	0.3960***	[0.1065]
Age >64	-0.3372***	[0.0891]	-0.2542***	[0.0820]	-0.3940***	[0.1414]
Age <45	0.4864***	[0.0801]	0.5218***	[0.0742]	0.5666***	[0.1250]
Low education	-0.0611	[0.0910]	0.1590*	[0.0831]	0.4030***	[0.1338]
High education	-0.2007**	[0.0812]	-0.1505*	[0.0779]	-0.2417*	[0.1337]
Partner	0.1835**	[0.0909]	0.3406***	[0.0916]	0.1361	[0.1424]
HH income: 1st quart.	0.4292***	[0.1089]	0.4950***	[0.1022]	0.5767***	[0.1725]
HH income: 2nd quart.	0.2640***	[0.0982]	0.2865***	[0.0904]	0.2559	[0.1590]
HH income: 3rd quart.	0.1960**	[0.0908]	0.2647***	[0.0839]	0.1313	[0.1494]
No. children in HH	-0.0087	[0.0370]	-0.0091	[0.0326]	0.0182	[0.0563]
Constant	-1.7239***	[0.1603]	-2.1845***	[0.1580]	-4.6496***	[0.3118]
Implied Cl1 share	0.59		0.61		0.82	
Implied Cl2 share	0.19		0.17		0.08	
Implied Cl3 share	0.22		0.22		0.10	
LogLik	-252,684.91		-316,029.07		-182,140.62	
AIC	505,547.81		632,236.13		364,459.24	
Observations	11,402		12,973		7,353	

Notes: This table re-estimates the main model using only respondents who do not violate basic laws of probabilities (specification 1), who answer all eight expectations questions on the stock market performance (specification 2) and who do not report a probability of 50 percent to a specific expectation question (specification 3). Model uses sign restrictions for the return coefficients ($\gamma_1 = 0$, $\gamma_2 < 0$, $\gamma_3 > 0$). Table displays results for the random effects multinomial logit model for the expectation types (Equation 3). Baseline type 1 (Random Walk) is omitted. Specifications 2 and 3 also include year fixed effects. For details see text. Standard errors in brackets; *** p <0.01, ** p <0.05, * p <0.1.

Table C3: Random effects ordinal probit model for rounding types – robustness with respect to expectation response behavior

	(1) Only monotonic answers		(2) Eight probability questions		(3) Drop 50s	
Female	-0.0016	[0.0240]	-0.0345*	[0.0202]	-0.0447	[0.0309]
Age >64	0.0081	[0.0210]	0.0505***	[0.0188]	0.0193	[0.0291]
Age <45	-0.0840***	[0.0217]	-0.0689***	[0.0189]	-0.1739***	[0.0298]
Low education	-0.0782***	[0.0297]	0.0537**	[0.0240]	0.0292	[0.0373]
High education	-0.1737***	[0.0259]	0.0092	[0.0227]	0.0329	[0.0370]
Partner	0.0724***	[0.0276]	0.0870***	[0.0241]	0.0326	[0.0342]
HH income: 1st quart.	0.0098	[0.0311]	0.0418*	[0.0253]	-0.0008	[0.0397]
HH income: 2nd quart.	0.0515*	[0.0269]	0.0262	[0.0214]	0.0260	[0.0342]
HH income: 3rd quart.	-0.0067	[0.0218]	-0.0025	[0.0190]	0.0010	[0.0307]
No. children in HH	0.0150	[0.0125]	0.0134	[0.0091]	0.0037	[0.0142]
LogLik	-252,684.91		-316,029.07		-182,140.62	
AIC	505,547.81		632,236.13		364,459.24	
Observations	11,402		12,973		7,353	

Notes: This table re-estimates the main model using only respondents who do not violate basic laws of probabilities (specification 1), who answer all eight expectations questions on the stock market performance (specification 2) and who do not report a probability of 50 percent to a specific expectation question (specification 3). Model uses sign restrictions for the return coefficients ($\gamma_1 = 0$, $\gamma_2 < 0$, $\gamma_3 > 0$). Table displays results for the random effects ordinal probit model for the rounding types (Equation 7). Dependent variable is equal to 1 if the respondent does not round (R1), 2 if the respondent rounds to the next multiple of five (R5) and 3 if the respondent rounds to the next multiple of 50 (R50). Question type-specific cut-off parameters are not reported. For details see text. Standard errors in brackets; *** p <0.01, ** p <0.05, * p <0.1.

Table C4: Model for the mean of the subjective distributions – robustness with respect to alternative returns

	(1) One-year return (main)		(2) One-month return		(3) One-week return	
γ_2: Return coeff. Cl2	-0.5964***	[0.0185]	-4.7453***	[0.1125]	-8.9345***	[0.2305]
γ_3: Return coeff. Cl3	0.6154***	[0.0104]	2.6610***	[0.0614]	6.7803***	[0.1311]
Female	-0.0203***	[0.0024]	-0.0187***	[0.0025]	-0.0212***	[0.0022]
Age >64	-0.0016	[0.0018]	-0.0035**	[0.0015]	-0.0003	[0.0016]
Age <45	-0.0023	[0.0017]	-0.0012	[0.0016]	-0.0007	[0.0016]
Low education	-0.0257***	[0.0027]	-0.0119***	[0.0025]	-0.0142***	[0.0031]
High education	0.0075***	[0.0028]	0.0049*	[0.0026]	0.0059**	[0.0026]
Partner	0.0006	[0.0027]	-0.0041**	[0.0020]	-0.0019	[0.0022]
HH income: 1st quart.	-0.0000	[0.0027]	-0.0057***	[0.0022]	-0.0045*	[0.0024]
HH income: 2nd quart.	-0.0013	[0.0022]	-0.0065***	[0.0019]	-0.0049**	[0.0020]
HH income: 3rd quart.	0.0003	[0.0018]	-0.0033**	[0.0016]	-0.0037**	[0.0017]
No. children in HH	-0.0005	[0.0008]	-0.0022***	[0.0008]	-0.0019**	[0.0008]
Constant	-0.0128***	[0.0037]	0.0031	[0.0034]	-0.0011	[0.0040]
σ_1^*	0.1167***	[0.0007]	0.1221***	[0.0007]	0.1194***	[0.0007]
σ_2^*	0.5767***	[0.0095]	0.3258***	[0.0055]	0.3394***	[0.0067]
σ_3^*	0.2601***	[0.0032]	0.5175***	[0.0082]	0.4516***	[0.0066]
σ_{CDFfit}	0.1596***	[0.0004]	0.1617***	[0.0004]	0.1600***	[0.0004]
LogLik	-331,997.03		-332,450.63		-331,660.91	
AIC	664,172.05		665,079.26		663,499.82	
Observations	14,264		14,264		14,264	

Notes: This table re-estimates the main model using different AEX returns. Specifications 1, 2 and 3 focus on the one-year, one-month and one-week AEX return, respectively. Model uses sign restrictions for the return coefficients ($\gamma_1 = 0$, $\gamma_2 < 0$, $\gamma_3 > 0$). Table displays results for the subjective means model (Equation 1) as well as the type-specific estimates for the subjective standard deviations. For details see text. Standard errors in brackets; *** p <0.01, ** p <0.05, * p <0.1.

Table C5: Random effects multinomial logit model for the expectation types – robustness with respect to alternative returns

	(1) One-year return (main)		(2) One-month return		(3) One-week return	
Class 2 (Mean Reversion)						
Female	0.3394***	[0.0715]	0.2945***	[0.0754]	0.4291***	[0.0761]
Age >64	-0.5734***	[0.0936]	-0.1697*	[0.0912]	-0.2188**	[0.0935]
Age <45	0.7835***	[0.0766]	0.3002***	[0.0852]	0.4295***	[0.0841]
Low education	0.2229**	[0.0889]	0.1784**	[0.0905]	0.2455**	[0.0978]
High education	-0.5621***	[0.0870]	-0.3865***	[0.0913]	-0.3411***	[0.0914]
Partner	0.1975**	[0.0944]	0.3286***	[0.0972]	0.2382**	[0.0992]
HH income: 1st quart.	0.5987***	[0.1070]	0.4760***	[0.1125]	0.4315***	[0.1141]
HH income: 2nd quart.	0.3336***	[0.0974]	0.1765*	[0.1041]	0.0931	[0.1052]
HH income: 3rd quart.	0.2305**	[0.0919]	0.1261	[0.0984]	0.1598	[0.0973]
No. children in HH	0.0011	[0.0338]	-0.0021	[0.0376]	0.0240	[0.0371]
Constant	-1.9628***	[0.1543]	-2.5656***	[0.1670]	-2.0716***	[0.1683]
Class 3 (Persistence)						
Female	0.2216***	[0.0661]	0.4165***	[0.0674]	0.3229***	[0.0655]
Age >64	-0.2922***	[0.0809]	-0.4785***	[0.0854]	-0.4331***	[0.0816]
Age <45	0.5227***	[0.0733]	0.8155***	[0.0700]	0.8097***	[0.0695]
Low education	0.2687***	[0.0834]	0.0790	[0.0800]	0.1326	[0.0839]
High education	-0.2241***	[0.0795]	-0.4224***	[0.0790]	-0.4575***	[0.0771]
Partner	0.2552***	[0.0879]	0.2496***	[0.0839]	0.2795***	[0.0836]
HH income: 1st quart.	0.3926***	[0.1008]	0.5211***	[0.0977]	0.5601***	[0.0969]
HH income: 2nd quart.	0.2189**	[0.0900]	0.2745***	[0.0884]	0.3583***	[0.0873]
HH income: 3rd quart.	0.2341***	[0.0833]	0.2464***	[0.0827]	0.2300***	[0.0821]
No. children in HH	-0.0154	[0.0322]	-0.0115	[0.0314]	-0.0230	[0.0311]
Constant	-2.0169***	[0.1497]	-2.2951***	[0.1469]	-2.2874***	[0.1546]
Implied Cl1 share	0.60		0.67		0.65	
Implied Cl2 share	0.18		0.14		0.14	
Implied Cl3 share	0.21		0.19		0.21	
LogLik	-331,997.03		-332,450.63		-331,660.91	
AIC	664,172.05		665,079.26		663,499.82	
Observations	14,264		14,264		14,264	

Notes: This table re-estimates the main model using different AEX returns. Specifications 1, 2 and 3 focus on the one-year, one-month and one-week AEX return, respectively. Model uses sign restrictions for the return coefficients ($\gamma_1 = 0$, $\gamma_2 < 0$, $\gamma_3 > 0$). Table displays results for the random effects multinomial logit model for the expectation types (Equation 3). Baseline type 1 (Random Walk) is omitted. Specifications 2 and 3 also include year fixed effects. For details see text. Standard errors in brackets; *** p <0.01, ** p <0.05, * p <0.1.

Table C6: Random effects ordinal probit model for rounding types – robustness with respect to alternative returns

	(1) One-year return (main)		(2) One-month return		(3) One-week return	
Female	-0.0154	[0.0209]	0.0359	[0.0234]	0.0251	[0.0230]
Age >64	0.0233	[0.0203]	0.0272	[0.0233]	0.0269	[0.0197]
Age <45	-0.0515***	[0.0188]	-0.0466**	[0.0193]	-0.0401**	[0.0192]
Low education	-0.0048	[0.0247]	-0.0386	[0.0267]	-0.0303	[0.0261]
High education	-0.0989***	[0.0248]	-0.0671**	[0.0293]	-0.0752***	[0.0257]
Partner	0.0005	[0.0268]	0.0497*	[0.0269]	0.0373	[0.0265]
HH income: 1st quart.	-0.0294	[0.0275]	0.0236	[0.0288]	0.0393	[0.0282]
HH income: 2nd quart.	-0.0270	[0.0238]	0.0201	[0.0263]	0.0242	[0.0237]
HH income: 3rd quart.	-0.0316	[0.0200]	-0.0093	[0.0202]	-0.0076	[0.0203]
No. children in HH	0.0113	[0.0090]	0.0108	[0.0094]	0.0114	[0.0090]
LogLik	-331,997.03		-332,450.63		-331,660.91	
AIC	664,172.05		665,079.26		663,499.82	
Observations	14,264		14,264		14,264	

Notes: This table re-estimates the main model using different AEX returns. Specifications 1, 2 and 3 focus on the one-year, one-month and one-week AEX return, respectively. Model uses sign restrictions for the return coefficients ($\gamma_1 = 0$, $\gamma_2 < 0$, $\gamma_3 > 0$). Table displays results for the random effects ordinal probit model for the rounding types (Equation 7). Dependent variable is equal to 1 if the respondent does not round (R1), 2 if the respondent rounds to the next multiple of five (R5) and 3 if the respondent rounds to the next multiple of 50 (R50). Question type-specific cut-off parameters are not reported. For details see text. Standard errors in brackets; *** p <0.01, ** p <0.05, * p <0.1.

Table C7: Model for the mean of the subjective distributions – robustness with respect to excluding survey years

	(1) Add preferences (no 2006,08,09)		(2) no 2006-2009		(3) Pre crisis (2004-2009)		(4) Post crisis (2010-2016)	
γ_2: Return coeff. Cl2	-0.7671***	[0.0251]	-0.7825***	[0.0245]	-0.9229***	[0.0503]	-0.5147***	[0.0225]
γ_3: Return coeff. Cl3	1.8050***	[0.0510]	1.8180***	[0.0534]	0.5141***	[0.0123]	0.6296***	[0.0176]
Female	-0.0128***	[0.0022]	-0.0116***	[0.0021]	-0.0165***	[0.0029]	-0.0165***	[0.0029]
Age >64	-0.0009	[0.0018]	-0.0001	[0.0018]	0.0024	[0.0031]	0.0081***	[0.0025]
Age <45	0.0023	[0.0020]	0.0001	[0.0019]	0.0015	[0.0030]	-0.0063**	[0.0029]
Low education	-0.0081***	[0.0027]	-0.0133***	[0.0027]	-0.0034	[0.0040]	-0.0206***	[0.0037]
High education	0.0088***	[0.0025]	0.0070***	[0.0024]	0.0119***	[0.0034]	0.0106***	[0.0032]
Partner	-0.0044*	[0.0023]	0.0026	[0.0023]	0.0059*	[0.0035]	0.0052*	[0.0030]
HH income: 1st quart.	-0.0036	[0.0026]	-0.0016	[0.0027]	-0.0034	[0.0039]	0.0067*	[0.0035]
HH income: 2nd quart.	0.0005	[0.0022]	0.0012	[0.0022]	-0.0074**	[0.0034]	0.0038	[0.0026]
HH income: 3rd quart.	-0.0016	[0.0019]	0.0008	[0.0019]	-0.0006	[0.0029]	0.0036	[0.0023]
No. children in HH	-0.0011	[0.0009]	-0.0003	[0.0009]	-0.0035***	[0.0013]	-0.0004	[0.0011]
Riskaverse	-0.0054***	[0.0018]						
Trust in other people	0.0065***	[0.0016]						
Constant	-0.0006	[0.0040]	-0.0050	[0.0035]	-0.0206***	[0.0057]	-0.0311***	[0.0048]
σ_1^*	0.1259***	[0.0009]	0.1249***	[0.0009]	0.1019***	[0.0012]	0.1219***	[0.0009]
σ_2^*	0.5365***	[0.0108]	0.5422***	[0.0104]	0.5302***	[0.0136]	0.6366***	[0.0155]
σ_3^*	0.3677***	[0.0090]	0.3649***	[0.0091]	0.2354***	[0.0055]	0.2917***	[0.0047]
σ_{CDFfit}	0.1589***	[0.0005]	0.1599***	[0.0005]	0.1536***	[0.0006]	0.1577***	[0.0005]
LogLik	-192,546.36		-211,164.38		-127,162.12		-203,456.82	
AIC	385,274.72		422,494.77		254,482.25		407,079.64	
Observations	8,339		9,214		5,356		8,908	

Notes: This table re-estimates the main model adding economic preferences as explanatory variables (specification 1) and excluding several survey years (specifications 2, 3 and 4). Model uses sign restrictions for the return coefficients ($\gamma_1 = 0$, $\gamma_2 < 0$, $\gamma_3 > 0$). Table displays results for the subjective means model (Equation 1) as well as the type-specific estimates for the subjective standard deviations. For details see text. Standard errors in brackets; *** p <0.01, ** p <0.05, * p <0.1.

Table C8: Random effects multinomial logit model for the expectation types – robustness with respect to excluding survey years

	(1) Add preferences (no 2006,08,09)		(2) no 2006-2009		(3) Pre crisis (2004-2009)		(4) Post crisis (2010-2016)	
Class 2 (Mean Reversion)								
Female	0.3852***	[0.0817]	0.3539***	[0.0770]	0.3730***	[0.1089]	0.3223***	[0.0912]
Age >64	-0.4853***	[0.1096]	-0.5584***	[0.1051]	-0.6795***	[0.1569]	-0.7778***	[0.1211]
Age <45	0.7078***	[0.0918]	0.7937***	[0.0867]	0.6297***	[0.1251]	0.7783***	[0.1022]
Low education	-0.1005	[0.1049]	-0.0068	[0.0991]	0.0643	[0.1364]	-0.0086	[0.1156]
High education	-0.5366***	[0.0971]	-0.5694***	[0.0923]	-0.5886***	[0.1360]	-0.5368***	[0.1052]
Partner	0.2295**	[0.1088]	0.2151**	[0.1023]	0.0863	[0.1463]	0.1770	[0.1205]
HH income: 1st quart.	0.5101***	[0.1269]	0.5322***	[0.1202]	0.7835***	[0.1721]	0.3734***	[0.1417]
HH income: 2nd quart.	0.2700**	[0.1132]	0.2655**	[0.1087]	0.4695***	[0.1608]	0.3112**	[0.1247]
HH income: 3rd quart.	0.1371	[0.1078]	0.1288	[0.1034]	0.3222**	[0.1530]	0.1648	[0.1180]
No. children in HH	-0.0244	[0.0407]	-0.0390	[0.0382]	0.0497	[0.0536]	0.0059	[0.0446]
Riskaverse	-0.0542	[0.1021]						
Trust in other people	-0.4113***	[0.0775]						
Constant	-1.6766***	[0.1967]	-2.0529***	[0.1640]	-1.7181***	[0.2235]	-2.4283***	[0.2025]
Class 3 (Persistence)								
Female	0.3106***	[0.0999]	0.2506***	[0.0938]	0.2416**	[0.1018]	0.1904**	[0.0833]
Age >64	-0.1083	[0.1274]	-0.1334	[0.1208]	-0.4836***	[0.1360]	-0.3461***	[0.1029]
Age <45	0.6714***	[0.1199]	0.6520***	[0.1127]	0.7028***	[0.1162]	0.5116***	[0.0978]
Low education	0.1069	[0.1254]	0.1471	[0.1178]	0.0566	[0.1338]	0.1336	[0.1063]
High education	-0.5708***	[0.1227]	-0.6074***	[0.1150]	-0.1512	[0.1232]	-0.2883***	[0.0985]
Partner	0.3291**	[0.1358]	0.2825**	[0.1274]	0.0391	[0.1352]	0.2711**	[0.1110]
HH income: 1st quart.	0.3779**	[0.1623]	0.3985***	[0.1523]	0.4123**	[0.1609]	0.2736**	[0.1319]
HH income: 2nd quart.	0.2041	[0.1426]	0.1785	[0.1365]	0.4294***	[0.1435]	0.0569	[0.1171]
HH income: 3rd quart.	0.1995	[0.1361]	0.1422	[0.1303]	0.0920	[0.1383]	0.1678	[0.1071]
No. children in HH	-0.0447	[0.0531]	-0.0314	[0.0494]	0.0074	[0.0509]	0.0071	[0.0428]
Riskaverse	0.0546	[0.1326]						
Trust in other people	-0.4445***	[0.0978]						
Constant	-3.5395***	[0.2717]	-3.6616***	[0.2311]	-1.4377***	[0.2173]	-1.7795***	[0.1800]
Implied Cl1 share	0.70		0.70		0.55		0.63	
Implied Cl2 share	0.20		0.20		0.16		0.17	
Implied Cl3 share	0.10		0.10		0.29		0.20	
LogLik	-192,546.36		-211,164.38		-127,162.12		-203,456.82	
AIC	385,274.72		422,494.77		254,482.25		407,079.64	
Observations	8,339		9,214		5,356		8,908	

Notes: This table re-estimates the main model adding economic preferences as explanatory variables (specification 1) and excluding several survey years (specifications 2, 3 and 4). Model uses sign restrictions for the return coefficients ($\gamma_1 = 0$, $\gamma_2 < 0$, $\gamma_3 > 0$). Table displays results for the random effects multinomial logit model for the expectation types (Equation 3). Baseline type 1 (Random Walk) is omitted. Specifications 2 and 3 also include year fixed effects. For details see text. Standard errors in brackets; *** p <0.01, ** p <0.05, * p <0.1.

Table C9: Random effects ordinal probit model for rounding types – robustness with respect to excluding survey years

	(1) Add preferences (no 2006,08,09)		(2) no 2006-2009		(3) Pre crisis (2004-2009)		(4) Post crisis (2010-2016)	
Female	-0.0262	[0.0269]	-0.0178	[0.0262]	0.0551*	[0.0287]	-0.0134	[0.0274]
Age >64	0.0286	[0.0252]	0.0190	[0.0244]	-0.0697**	[0.0354]	-0.0327	[0.0259]
Age <45	-0.1039***	[0.0258]	-0.0370	[0.0241]	-0.0951***	[0.0303]	-0.1300***	[0.0267]
Low education	-0.0256	[0.0330]	-0.0442	[0.0320]	0.0271	[0.0374]	-0.0928***	[0.0352]
High education	-0.0652**	[0.0310]	-0.0890***	[0.0310]	-0.1111***	[0.0339]	-0.0256	[0.0297]
Partner	0.0408	[0.0304]	0.0574**	[0.0282]	-0.1454***	[0.0358]	0.0730**	[0.0330]
HH income: 1st quart.	0.0046	[0.0347]	0.0017	[0.0335]	-0.0537	[0.0409]	-0.0089	[0.0371]
HH income: 2nd quart.	0.0344	[0.0299]	0.0325	[0.0300]	-0.0378	[0.0367]	0.0250	[0.0333]
HH income: 3rd quart.	-0.0338	[0.0266]	-0.0170	[0.0259]	0.0269	[0.0328]	-0.0445	[0.0284]
No. children in HH	0.0014	[0.0123]	-0.0168	[0.0117]	0.0335**	[0.0135]	0.0205	[0.0134]
Riskaverse	0.0859***	[0.0237]						
Trust in other people	-0.0898***	[0.0201]						
LogLik	-192,546.36		-211,164.38		-127,162.12		-203,456.82	
AIC	385,274.72		422,494.77		254,482.25		407,079.64	
Observations	8,339		9,214		5,356		8,908	

Notes: This table re-estimates the main model adding economic preferences as explanatory variables (specification 1) and excluding several survey years (specifications 2, 3 and 4). Model uses sign restrictions for the return coefficients ($\gamma_1 = 0$, $\gamma_2 < 0$, $\gamma_3 > 0$). Table displays results for the random effects ordinal probit model for the rounding types (Equation 7). Dependent variable is equal to 1 if the respondent does not round (R1), 2 if the respondent rounds to the next multiple of five (R5) and 3 if the respondent rounds to the next multiple of 50 (R50). Question type-specific cut-off parameters are not reported. For details see text. Standard errors in brackets; *** p <0.01, ** p <0.05, * p <0.1.

References

Ameriks, J., G. Kézdi, M. Lee, and M. D. Shapiro (2018). Heterogeneity in expectations, risk tolerance, and household stock shares: The attenuation puzzle. *NBER Working Paper, No. 25269*.

Armona, L., A. Fuster, and B. Zafar (2018). Home price expectations and behaviour: Evidence from a randomized information experiment. *The Review of Economic Studies, forthcoming*.

Bachmann, R., S. Elstner, and E. R. Sims (2013). Uncertainty and economic activity: Evidence from business survey data. *American Economic Journal: Macroeconomics 5*(2), 217–49.

Barsky, R. B., F. T. Juster, M. S. Kimball, and M. D. Shapiro (1997). Preference parameters and behavioral heterogeneity: An experimental approach in the Health and Retirement Study. *Quarterly Journal of Economics 112*(2), 537–579.

Bellemare, C., L. Bissonnette, and S. Kröger (2012). Flexible approximation of subjective expectations using probability questions. *Journal of Business & Economic Statistics 30*(1), 125–131.

Bruine de Bruin, W., B. Fischhoff, S. G. Millstein, and B. L. Halpern-Felsher (2000). Verbal and numerical expressions of probability: "It's a fifty-fifty chance". *Organizational Behavior and Human Decision Processes 81*(1), 115–131.

Dominitz, J. and C. F. Manski (1997). Using expectations data to study subjective income expectations. *Journal of the American Statistical Association 92*, 855–867.

Dominitz, J. and C. F. Manski (2007). Expected equity returns and portfolio choice: Evidence from the Health and Retirement Study. *Journal of the European Economic Association 5*(2-3), 369–379.

Dominitz, J. and C. F. Manski (2011). Measuring and interpreting expectations of equity returns. *Journal of Applied Econometrics 26*(3), 352–370.

Drerup, T., B. Enke, and H.-M. Von Gaudecker (2017). The precision of subjective data and the explanatory power of economic models. *Journal of Econometrics 200*(2), 378–389.

Giustinelli, P., C. F. Manski, and F. Molinari (2018). Tail and center rounding of probabilistic expectations in the Health and Retirement Study. *NBER Working Paper, No. 24559*.

Hudomiet, P., G. Kézdi, and R. J. Willis (2011). Stock market crash and expectations of American households. *Journal of Applied Econometrics 26*(3), 393–415.

Hurd, M., M. van Rooij, and J. K. Winter (2011). Stock market expectations of Dutch households. *Journal of Applied Econometrics 26*(3), 416–436.

Hurd, M. D. (2009). Subjective probabilities in household surveys. *Annual Review of Economics 1*(1), 543–562.

Hurd, M. D. and S. Rohwedder (2012). Stock price expectations and stock trading. NBER Working Paper No. 17973.

Kleinjans, K. J. and A. van Soest (2014). Rounding, focal point answers and nonresponse to subjective probability questions. *Journal of Applied Econometrics 29*(4), 567–585.

Malmendier, U. and S. Nagel (2011). Depression babies: Do macroeconomic experiences affect risk taking? *The Quarterly Journal of Economics 126*(1), 373–416.

Malmendier, U., S. Nagel, and Z. Yan (2017). The making of hawks and doves: Inflation experiences on the FOMC. *NBER Working Paper, No. 23228*.

Manski, C. F. (2004). Measuring expectations. *Econometrica 72*(5), 1329–1376.

Manski, C. F. (2018). Survey measurement of probabilistic macroeconomic expectations: progress and promise. *NBER Macroeconomics Annual 32*(1), 411–471.

Manski, C. F. and F. Molinari (2010). Rounding probabilistic expectations in surveys. *Journal of Business & Economic Statistics 28*(2), 219–231.

Rossmann, T. (2019). Does experience shape subjective expectations? *mimeo*.

Train, K. E. (2003). *Discrete choice methods with simulation.* Cambridge University Press.

von Gaudecker, H.-M. and A. Wogrolly (2018). The dynamics of households' stock market expectations. *mimeo*.

Zarnowitz, V. and L. A. Lambros (1987). Consensus and uncertainty in economic prediction. *Journal of Political Economy 95*(3), 591–621.