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Meta-Search and Market Concentration

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Metasearch and market concentration

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Abstract

Competing intermediaries search on behalf of consumers among a large number of horizontally differentiated sellers. Consumers either pick the best deal offered by a random intermediary, or compare the intermediaries. A higher number of deal finders has the direct effect of decreasing their search effort, but also increases the incentives for consumers to become informed. A higher share of informed consumers in turns increases the search effort of deal finders, so that the sign of the total effect is ambiguous. If the total effect of lower concentration is to increase search effort, it always decreases the price offered by sellers.

1 Introduction

Consumers often rely on intermediaries to help them find the product that suits the best to their needs. In the case of online intermediaries, it is easy - yet costly - for consumers to compare the different recommendations received and pick the best offer. A natural question on this market is whether consumers benefit from having a large numbers of intermediaries at disposal. More precisely, could limiting entry or, to the contrary, mergers of intermediaries increase consumer welfare?

In this paper, I set up a model where a large number of consumers want to buy one unit of a product in a market with a large number of horizontally differentiated sellers. Several competing intermediaries, called deal finders, search on behalf of consumers to find the best deal for them. Lower concentration generates a tradeoff between search intensity by the competing intermediaries and the incentives for consumers to become informed. On the one hand, higher market concentration benefits consumers, as each deal finder provides more search effort. On the other hand,

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more precise deal finders decrease the incentives for consumers to compare their options. This in turns decreases the incentives for deal finders to provide search efforts, but also the incentives for sellers to offer low prices for their products. The reason behind this last effect is that, for a given choosiness of deal finders, the demand from uninformed consumers is less elastic.

A deal finder can be an individual recruitment agency hired to search for job candidates, a real estate agency searching for prospective tenants (for the owner) or properties (for the tenants), an insurance broker, or one of the many "deal finding websites" on the Internet. The question of whether free entry should be granted on these markets is a long-lasting debate. A typical argument to limit entry is that low market concentration can decrease the quality of advice as consumers are unable to perfectly screen the quality of these intermediaries.[1] This point is however much less obvious on the Internet, where consumers are only a few clicks away from comparing their options. Moreover, digital markets seems to often converge towards very concentrated structures making the question of excessive entry less relevant.[2] For instance, in 2015 in the US, Expedia[3] (Expedia.com, tripadvisor.com, orbitz.com, hotels.com, venere.com, trivago.com,...) and Priceline[4] (priceline.com, kayak.com, booking.com,...) controlled 95 percent of the online travel-marketplace after a number of successful fusions and acquisitions.[5]

Innovation in search quality is an essential part of the competition in advice markets. To keep the travel example, Andrew Warner of Expedia reports in a 2014 interview[6] that "for a standard trip from LA to New York, Expedia has 65,000,000,000 different combinations of travel for each consumer - given variations in flight times, airlines, car rentals, hotels, offers." Being able to use consumer data to provide the best personalized advice (and beat competitors) is thus a huge and rather costly challenge, with Expedia claiming to spend £500 million yearly in R&D. Warner described the objective of such investment as being able to do more than mechanically answering a query and providing the cheapest price. Today's competition in the online travel industry is thus

[1]For instance in the UK, from 1977 to 2005, the Insurance Broker Registration Act limited entry on the market. Similarly, most US states require a special license to be a recognized broker.

[2]Malik (2015) summarizes the dynamic of concentration on digital markets as consisting of three phases: "The first is when there is a new idea, product, service, or technology dreamed up by a clever person or group of people. For a brief while, that idea becomes popular, which leads to the emergence of dozens of imitators, funded in part by the venture community. Most of these companies die. When the dust settles, there are one or two or three players left standing."

[3]http://www.expediainc.com/expedia-brands/

[4]http://ir.pricelinegroup.com/

[5]See for instance Sun, sea and surfing, The Economist, June 21, 2014 ; Competition is shaking up the online travel market, Forbes, January 5, 2015 and Expedia and Orbitz are merging. Here's what it means for you, Cecila Kang and Brian Fung, The Washington Post, September 16, 2015.

[6]"Expedia is investing billions in data to create personalized travel-graphs", Derek du Preez, March 24, 2014, diginomica.com

largely based on being able to provide a good individual match to a specific consumer.[7]

Perhaps the main argument in support of market concentration in such a market is that competition among deal finders resembles an all-pay-auction, where each sale benefits one deal finder only, but all bear the cost of providing the search technology. A typical result of all-pay auctions is that individual investments decrease in the number of bidders (see for instance Baye *et al.*, 1996), implying that the quality of individual advice should decrease with the number of deal finders active on the market. In this paper, I show that this argument is only valid if one takes as granted the behaviour of consumers. The only reason deal finders compete in offering advice of higher quality is to attract those informed buyers that compare their options. If lower market concentration increases the incentives for consumers to do so, it may as well increase the individual search quality offered by deal finders.

I assume that competing deal finders search (at a cost) for the best product to recommend to a specific consumer. I model the search effort of deal finders as a linear random sequential search within a distribution of deals, in the tradition of Wolinsky (1986) and Anderson and Renault (1999). I show in Appendix C that the effects I identify are similar when I use a model of non-sequential search à la Burdett and Judd (1983). Deal finders derive revenue from the sales they make possible. Consumers are of two (endogenous) types. Some are "savvy" and pick the best deal among all the deal finders. Some are "non-savvy" and take the best deal offered by a deal finder chosen at random. I borrow this dichotomy from a literature started by Varian (1980) to study price dispersion. I thus study a market for "meta search," where consumers choose among deal finders searching on their behalf. I show in Appendix D that my results also hold when consumers bear a linear cost of observing an additional deal finder.

I first derive a classic result from this literature: the existence of search externalities (Armstrong, 2015). The savvy consumers protect the non-savvy, as deal finders cannot discriminate among types, so that fiercer competition for the savvy types make all consumers better off. I then study the impact of market concentration on the expected price paid by consumers and the expected quality of advice offered. More deal finders on the market have a direct negative effect on the individual search efforts. Hence, for a given share of savvy types, higher concentration can be beneficial in the sense that it protects the less informed consumers by increasing the search effort of each individual deal finder. This effect is not similar for savvy consumers, as those benefit from the larger choice offered by an increase in the number of deal finders. Lower concentration

[7] On this topic, see "Expedia Thinks It Can Help You Find the Dream Vacation You Didn't Know You Wanted", Drake Bennet, Bloomberg Business Week, February 25, 2016 and "How Expedia, Hopper and Skyscanner Use Big Data to Find You the Cheapest Airfares", Isabel Thottam, Paste Magazine, January 16, 2017

therefore has the indirect effect that savviness matters more, hence increasing the incentives to become informed, partly internalizing the search externality. This indirect effect can outweigh the direct one, so that lower concentration may actually benefit all consumers. As shown in appendix D, this is not a mechanical consequence of the higher number of deal finders at disposal of savvy consumers. Indeed, as the marginal benefit from observing an additional deal finder is higher when these are less choosy, savviness matters more in the presence of more deal finders even if the choice is whether or not to observe a single additional one.

Both the number and the choosiness of deal finders have an effect on the symmetric equilibrium price offered by the sellers. If lower market concentration makes deal finders more choosy, sellers unambiguously decrease their price. If it makes deal finders less choosy, the effect is ambiguous. On the one hand, sellers have an incentive to increase the price because the price elasticity of the demand from non-savvy consumers decreases in absolute value. On the other hand, the share of the most elastic segment of the market (savvy consumers) in the demand increases, giving sellers incentives to decrease prices.

I make the assumption that the revenue of deal finders depends linearly on the volume of sales. This is the case for instance if they are financed by selling information about buyers on a competitive market for advertising, or if they collect fixed commissions. In practice, deal finders are financed in various ways. Some charge a fixed amount of "administrative fees," others get rewarded by a commission paid by either the buyer or seller (that can be fixed per purchase or per-click, or proportional to the value of the purchase), and finally a part of the revenue of online deal finders comes from advertisement on the website, and from gathering information on the consumers and selling by-products. These sources of revenue are however constrained by the fact that buyers always have the possibility of bypassing the deal finder that made the recommendation in order to directly buy from the sellers. While it would certainly be interesting to compare the different pricing possibilities and the type of contracts allowed between deal finders and firms,[8] I take the agnostic option to model a more general relationship between sales and revenues. I also assume that sellers do not pay to be listed on a specific website, so that deal finders search among all existing sellers.

Related literature:

This paper relates to the literature on advice and delegated search. In the literature on delegated

[8] For instance, a recent settlement for the hotel industry between booking.com and several European countries rules out the possibility for the deal finders to force hotels not to offer a lower price for direct bookings (Booking.com in European settlement over hotel prices, Malcolm Moore and Adam Thomson, Financial Times, April 21, 2015). This question has been studied recently by Edelman and Wright (2015).

search, it relates to Lewis (2012), and even more closely to Ulbricht (2016). I share with the latter the assumption that the relationship between the buyer and the deal finders combines a problem of moral hazard (the effort of the deal finder) and of asymmetric information (the fact that the deal finders know the market better). By adding competition on the side of the deal finders, and studying different types of buyers, I have two related yet different problems. The moral hazard problem is partly alleviated by competition among intermediaries. The asymmetric information is modified by the fact that buyers have different and privately known information.

I consider a world were, instead of firms competing for the attention of a customer, to be included in their consideration set (Eliaz and Spiegler, 2011a), consumers rely on intermediaries to make them a recommendation. In the advice market, most of the focus has been on a single intermediary. For instance, Armstrong and Zhou (2011) study a large number of possible transactions between sellers of a product and the adviser choosing how to present the information to consumers. The question of competition among advisers has been discussed in an extension of Inderst and Ottaviani (2012), who study financial advice and compare the case of competitive advisors to the one of a monopolist. Competition among two search engines is also studied in Section 5 of de Cornière (2016), in a two-sided framework were search engines compete in order to attract both consumers and advertisers by auctioning "keywords". In a related model, Eliaz and Spiegler (2011b) study competition among two search engines. None of these papers however study specific investment to be made by the advisers to improve the quality of their advice. Another difference is that I compare duopoly to more intense competition. The case of a monopoly would be trivial in my model as such a deal finder would provide no effort and sellers would offer the monopoly price.

I present the setup of the model in the next Section. Section 3 solves the equilibrium effort of deal finders for a given equilibrium price, by assuming a fixed share of savvy consumers. I allow this share to be endogenous in Section 4. I then endogenize the prices offered by sellers in Section 5, and characterize the welfare effect of market concentration on consumer welfare. I allow for heterogeneous costs of becoming savvy in Section 6, and consider the entry decisions of deal finders in Section 7. Section 8 concludes.

2 Model setup

A mass 1 of consumers wants to buy a single unit of a particular product which is supplied by a large number of sellers at a marginal production cost of zero. Building on the specification of Anderson and Renault (1999), each consumer i has tastes described by a conditional utility function of the form

$$u_{i,j}(p_j) = v - p_j - \varepsilon_{i,j}, \tag{1}$$

if she buys product j at price p_j. The intrinsic valuation of the product v is assumed to be sufficiently high for each consumer to always buy. The parameter $\varepsilon_{i,j}$ is the realization of a random variable with log-concave probability density function $f(\varepsilon)$, cumulative density function $F(\varepsilon)$ and support over $[0,b]$, with $b > 0$. The distribution of ε is common knowledge. The assumption of log-concavity applies to most commonly used density functions (see Caplin and Nalebuff, 1991 and Anderson and Renault, 1999), and is necessary to obtain the results on the symmetric equilibrium price in Section 5. The random component ε represents the (exogenous) distance between a particular version of the product j and the ideal product given the taste of buyer i. I denote this parameter as the mismatch value.

Between the consumers and the sellers are a number $N \geq 2$ of identical intermediaries, called deal finders. I start by taking N as given and study the entry decision of deal finders in Section 7. Suppose that any deal finder receiving a query from a consumer of type i can sequentially sample sellers by each time incurring a linear search cost s to discover a price p_j and mismatch value $\varepsilon_{i,j}$. A deal finder that sampled q sellers thus bears a total cost of qs. Assume that deal finders generate revenue from a competitive market for advertisers, bidding for the information on buyers gathered by successful deal finders. As all consumers buy exactly one unit, the willingness to pay for this information is not influenced by the search behaviour of deal finders. I normalize the willingness of advertisers to pay for the information extracted from consumers to 1, so that a consumer buying from a deal finder generates a revenue of 1 for this deal finder.

There are a number of scenarios one might consider. However, perhaps the simplest to start with is to suppose that, following a query, the N deal finders simultaneously search for deals, and when they find a satisfactory deal $p + \varepsilon$ they advertise it to the consumer. This assumption can be taken literally in the case of physical intermediaries exerting an effort to answer a customer's request. In the case of a website, this can be interpreted as the investment in building the right algorithms and search environment to be able to deal with specific preferences. I show in Appendix C that the assumption that search is sequential is not crucial to my results, as the effects are similar if instead deal finders decide ex-ante to carry a specific number of searches, as in Burdett and Judd (1983). A share σ of "savvy" consumers makes N simultaneous queries, observes N deals, and chooses to buy from the deal finder offering the best one. Each remaining consumer makes only one query to a deal finder picked at random, and receives the best quote of this deal finder. The savviness of a consumer is unobservable to the deal finder, so that she does not know for whom she is competing at the time of the query. I start by taking σ as given, and then study the incentives to become "savvy."

As in Anderson and Renault (1999), I focus on a symmetric solution where each seller offers an (endogenous) identical price p. I study the price offered by sellers in Section 5 and show that such an equilibrium exists. Hence, at equilibrium, deals only vary by their mismatch value $\varepsilon_{i,j}$. I also look for a symmetric solution where deal finders keep searching for a deal until finding a mismatch value ε below some threshold w. As a tie-breaking rule, I assume that deal finders search when indifferent. If all deal finders follow this strategy, the probability that a given deal finder with mismatch value $\varepsilon < w$ provides the best deal to a savvy consumer is $(\frac{F(w)-F(\varepsilon)}{F(w)})^{N-1}$. If its $N-1$ rivals follow the above strategy, if a deal finder has found a product with mismatch value ε, its expected revenue abstracting from search and entry costs is

$$\pi(\varepsilon) = \sigma \left(\frac{F(w) - F(\varepsilon)}{F(w)} \right)^{N-1} + \frac{1-\sigma}{N}. \quad (2)$$

The first part is the demand from savvy consumers multiplied by the probability of offering the best deal among the N queries they made. The second part is the non-savvy consumers who randomly picked the deal finder and made only one query. The expected search cost to be paid by a firm in order to find a mismatch value below w is equal to $\frac{s}{F(w)}$ (this is a general property of a geometric distribution).[9]

To summarize, the timing of the game is as follows:

1. Sellers simultaneously set their price p_j (I focus on a symmetric equilibrium price p). Buyers simultaneously choose whether to send a query to a deal finder chosen at random, or to send a query to all deal finders at cost c (the equilibrium share of informed consumers is σ).

2. Deal finders sequentially search for each query they received until they find a mismatch value below their optimal cutoff value (I focus on a symmetric equilibrium cutoff "w") and buyers accept the best deal out of all the queries they made.

All players choose the strategy that maximizes their utility given their expectation of other players'

[9] As I assume deal finders search a discrete number of times within a large number of sellers, the deal finders search within independent and identically distributed deals. With a more limited selection of sellers, I would have to consider overlapping suggestions by deal finders to savvy consumers, therefore limiting the incentives to become savvy.

strategies. I focus on symmetric equilibria, so that players best respond to the expected share of savvy consumers σ, equilibrium price p, as well as to the expected cutoff value w of deal finders. As I assume a large number of sellers, an individual price deviation $p_j \neq p$ only affects the expected profit of seller j. I look for subgame perfect equilibria and solve the game by backward induction. Hence, I start by solving for the equilibrium symmetric cutoff w. Then I study the endogenous share of informed consumer σ and the symmetric equilibrium price p.

3 Equilibrium search

Standard search theory indicates that for a symmetric price p the optimal threshold mismatch value w must satisfy

$$s = \int_0^w (\pi(\varepsilon) - \pi(w)) f(\varepsilon) d\varepsilon, \tag{3}$$

so that the following result holds.

Lemma 1 *In a symmetric equilibrium, deal finders search until they find a mismatch value lower or equal to w, with* $\frac{\partial w}{\partial s} \geq 0$, $\frac{\partial w}{\partial \sigma} \leq 0$ *and* $\frac{\partial w}{\partial N} \geq 0$.[10]

Proof. Rewriting (3) by using (2), it is easy to show that if there is an interior solution w solves

$$s = \sigma \int_0^w \left(\frac{F(w) - F(\varepsilon)}{F(w)} \right)^{N-1} f(\varepsilon) d\varepsilon, \tag{4}$$

by using the fact that $\pi(w) = \frac{1-\sigma}{N}$. It is straightforward that the left-hand side increases with s while the right-hand side increases with σ and w.[11] There exists no corner solution $w = 0$ as $s > 0$, and there exists a corner solution $w = 1$ if and only if $s \geq \sigma \int_0^b (\frac{1-F(\varepsilon)}{F(\varepsilon)})^{N-1} f(\varepsilon) d\varepsilon$. ■

As we would expect, the threshold mismatch value increases with the search cost s. The threshold w also necessarily increases with the number of deal finders, so that for a given share of savvy consumers σ a deal finder becomes less choosy when it faces more rivals. This is a direct consequence of the fact that, for a given symmetric search strategy of the competitors, the marginal benefit of an additional search is lower when the number of deal finders is higher. Through the paper, I focus on cases where $w < b$, so that the delegated search problem has an interior solution.

[10] In order to save space I use the derivative and partial derivative symbol with respect to N even if N is a discrete variable. For instance, in this case, a more rigorous notation would be $w(N+1, \sigma, s) > w(N, \sigma, s)$, $\forall N \geq 2$.

[11] To see that the right-hand side increases with w, consider $w' > w$. The right-hand side of (4) becomes $\sigma \left(\int_0^w (\frac{F(w')-F(\varepsilon)}{F(\varepsilon)})^{N-1} f(\varepsilon) d\varepsilon + \int_w^{w'} (\frac{F(w')-F(\varepsilon)}{F(\varepsilon)})^{N-1} f(\varepsilon) d\varepsilon \right)$. As $F(w') > F(w)$, $\int_0^w (\frac{F(w')-F(\varepsilon)}{F(\varepsilon)})^{N-1} f(\varepsilon) d\varepsilon > \int_0^w (\frac{F(w)-F(\varepsilon)}{F(\varepsilon)})^{N-1} f(\varepsilon) d\varepsilon$ and $\int_w^{w'} (\frac{F(w')-F(\varepsilon)}{F(\varepsilon)})^{N-1} f(\varepsilon) d\varepsilon > 0$.

Example 1 *To illustrate, if ε is uniformly distributed on $[0,1]$, then if there is an interior solution,*

$$w = \frac{sN}{\sigma}, \tag{5}$$

where N is the number of deal finders on the market, and σ the endogenous share of savvy consumers.

For later reference, I need to define $\eta_{i,j} = \varepsilon_{i,j} + p - p_j$ for a firm j setting a different price than the equilibrium p. Define $\phi(\eta)$ the distribution of $\eta_{i,j}$ so that, at the symmetric equilibrium price $\phi(\eta) = f(\varepsilon)$. Off the equilibrium path, (4) rewrites

$$s = \sigma \int_0^w \left(\frac{\Phi(w) - \Phi(\eta)}{\Phi(\eta)} \right)^{N-1} \phi(\eta) d\eta, \tag{6}$$

with $\Phi(x) = \int_0^x \phi(y) dy$.

In order to understand the direct effect of market concentration, it is useful to see how mismatches are affected when the share of savvy consumers is assumed to be exogenous. I first derive a result on the direct effect of market concentration on the utility of non-savvy consumers.

Proposition 1 *For a given symmetric market price p, if the share of savvy consumers is exogenous, a non-savvy consumer receives an expected utility u^{ns}, that is decreasing in the search cost of deal finders (s), increasing in the share of savvy types (σ) and decreasing in the number of deal finders (N).*

Proof. Using the result from Lemma 1, it is easy to show that the expected mismatch of a non-savvy consumer ε_{ns} is equal to the expected value of a random draw over the interval $[0,w]$,

$$\varepsilon^{ns} = \int_0^w \varepsilon \frac{f(\varepsilon)}{F(w)} d\varepsilon, \tag{7}$$

with $\frac{\partial \varepsilon^{ns}}{\partial s} \geq 0$, $\frac{\partial \varepsilon^{ns}}{\partial \sigma} \leq 0$ and $\frac{\partial \varepsilon^{ns}}{\partial N} \geq 0$. Hence, as the expected utility of a non-savvy type is given by $u^{ns} = v - p - \varepsilon^{ns}$, the proposition follows. ■

This first proposition starts by recovering the classic search externalities, as the share of savvy types benefits the non-savvy types. The intuition is that the higher the share of savvy types, the more the deal finders compete for them (and search), and the individual efforts of deal finders also benefit the non-savvy. This Proposition also conveys the direct effect of lower market concentration (higher N) on the deals received by the non-savvy types. Because, when there are more deal finders, each deal finder searches with lower intensity (Lemma 1), a consumer buying from a deal finder chosen at random receives deals of lower quality when there is more competition. The qualifier that the share of savvy types is exogenous is however crucial, as by taking σ as given, I only capture the direct effect of market concentration on equilibrium deals.

Example 2 *To illustrate, if ε is uniformly distributed on $[0,1]$, (7) yields*

$$\varepsilon^{ns} = \frac{sN}{2\sigma}. \tag{8}$$

I can now turn to the expected mismatch for savvy consumers.

Proposition 2 *For a given symmetric market price p, if the share of savvy consumers is exogenous, a savvy consumer receives an expected utility u^s, that is decreasing in the search cost of deal finders (s) and increasing in the share of savvy types (σ). The impact of the number of deal finders (N) is ambiguous, as the presence of more deal finders increases u^s if and only if*

$$-\frac{\partial \varepsilon^s}{\partial N} \geq \frac{\partial \varepsilon^s}{\partial w}\frac{\partial w}{\partial N}, \tag{9}$$

where ε^s is the expected mismatch value received by a savvy consumer.

The proof is in Appendix. The difference between the savvy and non-savvy consumers comes from the fact that the impact of market concentration N on the expected mismatch value of a savvy type ε^s is ambiguous. The right-hand side of (9) represents the effect that a higher number of deal finders makes each deal finder less choosy (higher w). The left-hand side represents the effect that it also increases the number of options a savvy consumer can choose from.

Example 3 *If ε is uniformly distributed on $[0,1]$, the overall direct effect is that more firms make the savvy type worse off as ε^s simplifies to*

$$\varepsilon^s = \frac{sN}{\sigma(1+N)}. \tag{10}$$

Two well-documented consequences follow immediately from the observation of the two mismatch values. First, savvy consumers always have a better deal than non-savvy ones. Second, savvy consumers "protect" the others by decreasing the mismatch value received by all consumers.

4 Equilibrium share of savvy types

I can now solve for the equilibrium share of savvy consumers σ by studying an initial stage where consumers simultaneously choose whether or not to become "savvy", at a constant cost c. I show in Section 6 that this assumption is not innocuous, as allowing for different consumers to have different costs of becoming savvy may revert the results. I show in Appendix D that the assumption that consumers observe either one or all the deal finders for a fixed cost is not crucial to the results. The reason is that, even for linear search costs, the benefit from requesting a quote from an additional deal finder is always higher if those are less choosy. Define

$$\Delta = \varepsilon^{ns} - \varepsilon^{s}, \tag{11}$$

as the expected premium (in terms of expected mismatch) paid by uninformed consumers. If there exists an interior solution, the equilibrium share σ is found by solving

$$\Delta(\sigma) = c, \tag{12}$$

else $\sigma = 0$ if $\Delta(0) \leq c$ and $\sigma = 1$ if $\Delta(1) \geq c$.

It is therefore possible to identify the impact of market concentration on the share of savvy consumers.

Proposition 3 *The equilibrium share of savvy consumers is increasing in the number of deal finders (N). For a given equilibrium price and endogenous share of savvy consumers σ, lower market concentration makes deal finders more choosy if and only if*

$$\frac{\partial w}{\partial N} \leq -\frac{\partial w}{\partial \sigma}\frac{\partial \sigma}{\partial N}. \tag{13}$$

The proof is in Appendix. This Proposition describes the second (indirect) effect of market concentration. The presence of more deal finders increases the difference between the best deal a savvy and a non-savvy consumer observe. It thus becomes more interesting for a consumer to invest in being informed. This allows to write condition (13), deal finders become more choosy if the effect described in Lemma 1 is dominated by the indirect effect of higher rates of savviness.

From Proposition 1 and 2, it is also the case that $\frac{\partial(\varepsilon^{ns}-\varepsilon^{s})}{\partial \sigma} < 0$, so that the incentives to become informed decrease when the number of informed consumers increase. This is a pretty standard intuition, as the protection of non-savvy consumers increases when the number of savvy consumers increases.

Example 4 *If ε is uniformly distributed between $[0,1]$, the premium for being informed rewrites $\Delta = \frac{sN(N-1)}{2\sigma(N+1)}$ and* (12) *solves*

$$\sigma = \frac{sN(N-1)}{2c(N+1)}, \tag{14}$$

if σ has an interior solution (if $c > \frac{sN(N-1)}{2(N+1)}$) and $\sigma = 1$ else. Plugging (14) *into* (5) *yields*

$$w = \frac{2c(N+1)}{N-1}, \tag{15}$$

if σ has an interior solution and $w = sN$ if $\sigma = 1$. We thus observe that the entry of new deal finders decreases the equilibrium mismatch received by all consumers once the information decision of consumers is made endogenous. The direct effect documented in (4) *is that each deal finder becomes less choosy when an additional competitor enters the market. However, the indirect effect is that the entry of new deal finders increases the incentives to become informed, because the gap between the best deal obtained by savvy and non-savvy consumers increases. Hence, the increase in the share of savvy consumers more than compensates the direct effect, so that entry actually makes deal finders more choosy.*

I illustrate the different effects on Figure 1. On the left panel, I take the share of savvy types σ as given. We see that for a given σ, a higher number of deal finders increases the mismatch value received by non-savvy consumers and (slightly) increases the mismatch value received by the savvy types. This is the direct effect of deal finders becoming less choosy. When the number of deal finders is equal to $N = 10$, these just pick one seller at random, so that a non-savvy customer receives an expected mismatch value of 0.5. For more deal finders, there is no interior solution for w. I do not study this case as it would imply making further assumption about deal finder and non-savvy consumers. In particular, if deal finders need at least one price quote in order to attract the non-savvy types, they may still benefit from searching once up to a certain point. We also observe that the difference between the two expected mismatches Δ increases with the number of deal finders N. On the right panel, I allow for the share of savvy types σ to be endogenous. Because the difference between the expected mismatch value received by a savvy and a non-savvy type increases with N, more and more consumers choose to become savvy. In this case, the effect of higher rates of savviness is to decrease the expected mismatch value received by both types of consumers in equilibrium. Hence, in this example, market concentration is bad for consumers

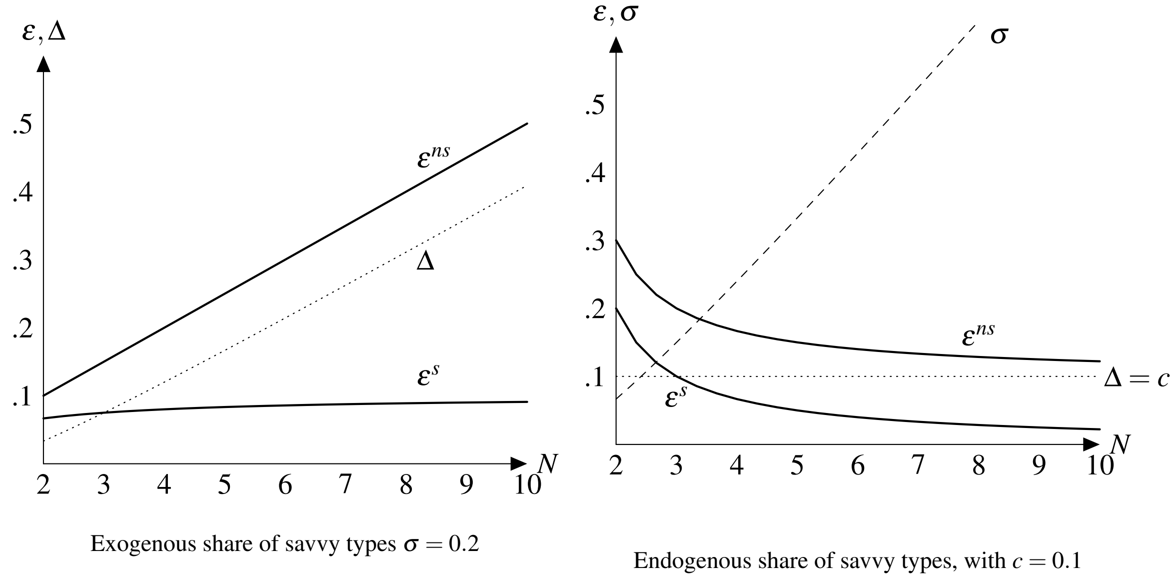


Figure 1: The impact of market concentration on expected mismatch, with $s = 0,02$, $F(\varepsilon) = \varepsilon$.

as even if a smaller number of deal finders has the direct positive effect of making each of them more choosy, it also has the indirect effect of making consumers less informed, thus allowing deal finders to become less choosy.

5 Symmetric equilibrium price

To close the model, I now turn to the equilibrium price offered by a large number of sellers M. Assuming this form of monopolistic competition simplifies a lot the analysis, as it implies that no two deal finders will recommend an identical deal to a given consumer. As will be made clear below, this is however not the driving force behind the results. What matters is that the demand of a savvy consumer is more elastic to a change in the price of a given seller than the one of a non-savvy consumer.

I assume a symmetric market price p, and study the optimal price p_i chosen by a seller i. A symmetric equilibrium is thus a situation in which, for each firm, the optimal $p_i = p$. The first part of the demand comes from non-savvy consumers. The expected demand a seller i receives from a non-savvy consumer is given by

$$D^{ns}(p_i, p, N) = F(w + p - p_i) \frac{\sum_{j=0}^{M-1}(1-F(w))^j}{M}. \tag{16}$$

This expression corresponds to the probability of being selected by a deal finder following the search strategy defined in (6), with i being the only firm off the price equilibrium path, so that the distribution of η for all other firms is $\phi(\eta) = f(\varepsilon)$. As deal finders are selected at random by non-savvy consumers, this is equivalent to N times the probability of being selected by each of the deal finders, divided by the probability that a non-savvy consumer picks a deal finder N. The first part $F(w+p-p_i)$ is the probability of offering a mismatch below w for a specific consumer, that can be alleviated by offering a different price than the market. The second part is the probability to either be selected first, or after unsuccessful searches by the deal finder. As we assume M to be very large, this could be approximated to $D^{ns} \approx \frac{F(w+p-p_i)}{MF(w)}$.

Now denote by $r(j)$ the probability that a random draw between $p_i - p$ and w be lower than j random draws between 0 and w,

$$r(j) = \int_0^{w+p-p_i} \frac{f(\varepsilon)}{F(w+p-p_i)} \left(\frac{F(w) - F(\varepsilon + p - p_i)}{F(w)} \right)^j d\varepsilon, \tag{17}$$

The expected demand a seller i receives from a savvy consumer is given by,

$$D^s(p_i, p, N) = ND^{ns} r(N-1), \tag{18}$$

which corresponds N times the probability of being selected by the a deal finder, multiplied by the probability to offer a better deal than the $N-1$ other selected sellers.

The expected demand for a given seller i is thus

$$D(p_i, p, N) = (1-\sigma)D^{ns} + \sigma D^s. \tag{19}$$

As the expected profit of a firm is equal to $p_i D(p_i, p, n)$, it follows that the equilibrium symmetric price p posted by sellers solves

$$p = \frac{-D(p,p,N)}{D_{p_i}(p_i,p,N)}, \tag{20}$$

where $D(p,p,n) = \frac{1}{M}$ as all firms post an identical price in equilibrium. It is thus possible to chacterize the impact of market concentration on the equilibrium price p.

Proposition 4 *The equilibrium symmetric price offered by sellers p is always decreasing in the number of deal finders N if lower market concentration makes deal finders more choosy ($\frac{dw}{dN} < 0$). Else, it is decreasing in the number of deal finders N if and only if*

$$D^s_{p_i}(p,p,N)\frac{d\sigma}{dN} + D^{ns}_{p_i}(p,p,N)\frac{d(1-\sigma)}{dN} + (1-\sigma)\frac{dD^{ns}_{p_i}(p,p,N)}{dN} + \sigma\frac{dD^s_{p_i}(p,p,N)}{dN} \leq 0. \tag{21}$$

The proof is in Appendix. Firms have an incentive to offer lower prices if deal finders become more choosy. They also want to lower the price if savvy consumers observe more options, and if the share of savvy consumers increases, as all these three effects make demand more elastic. Hence, the symmetric price equilibrium increases when the number of deal finders increases only if deal finders become less choosy, and if this effect dominates the two others.

Formally, the impact of N can be decomposed into several effects. The sum of the first two terms in is always negative, as $\frac{d\sigma}{dN} \geq 0$ (Proposition 3) and $D^s_{p_i}(p,p,N) < D^{ns}_{p_i}(p,p,N)$ (the "savvy" segment of the market is more elastic than the non-savvy one). The third term is negative if and only if $\frac{dw}{dN} \leq 0$, as the loss in demand when p_i increases is higher if the deal finders are more selective (lower w). Finally, the last term is always negative if $\frac{dw}{dN} \leq 0$ for identical reasons, but it can also be negative with $\frac{dw}{dN} \geq 0$ if the pro-competitive effect of having consumers comparing more options is more important than the anti-competitive effect of deal finders being less selective. One can observe that if the number of deal finders is very high ($n \to \infty$) and if all consumers are savvy ($\sigma \to 1$), how choosy the deal finders are has no impact on the equilibrium price, and this price converges to the Perloff-Salop model (see Proposition 1 in Anderson and Renault, 1999).

The result derives from the fact that, if deal finders become more choosy, sellers have to offer better prices on all segments of the market, in order to have a chance of being selected. The consequence is one of a virtuous (vicious) circle: the more choosy the deal finders are, the more a seller wants to provide a low price. However, there are other reasons that could lead to lower prices when concentration on the market for deal finders decreases. The first one is that fewer consumers pick a deal finder at random, so that the competitive segment of the market matters more to sellers. The second is that the competitive segment becomes even more competitive as savvy consumers have more options to pick from.

Example 5 *If ε is uniformly distributed between $[0,1]$, the equilibrium price solves*[12]

$$p = \frac{w}{1+(N-1)\sigma}, \tag{22}$$

with w and σ defined in (14) *and* (15)*. We immediately see that the equilibrium price always decreases with N, as w decreases with N and σ increases with N. Using the equilibrium values for these parameters, if σ has an interior solution,* (22) *simplifies to*

$$p = \frac{4c^2(1+N)^2}{2c(N^2-1)+(N-1)^3 Ns}. \tag{23}$$

[12] See Appendix for the computations, just below the proof of Proposition 4.

In this particular case, we see that the equilibrium price decreases with the number of deal finders, for two reasons. First, more deal finders increase the share of savvy consumers, making each deal finder more choosy. Second, a higher share of savvy consumers increases the price elasticity of demand for a given seller. Note that equation (23) *only represents the case where* σ *has an interior solution, for* $c > \frac{sN(N-1)}{2(N+1)}$ *(see Example 4). When* c *becomes smaller, the price does not converge to zero as suggested by* (23)*, but to* $p = s$ *- as from* (22) *and* (5)*,* $p = \frac{sN}{\sigma(1+(N-1)\sigma)}$*.*

I can now come back to the initial research question, to know whether market concentration benefits consumers' welfare. As all consumers are ex-ante identical, and as consumers choose to become savvy up to the point where

$$u^s - c = u^{ns}, \tag{24}$$

all consumers have an identical expected surplus. As I have assumed the utility to be quasi-linear so that all payments are directly subtracted from the utility, the expected surplus of each consumer is equal to

$$u = u^s - c = u^{ns} = v - p - \varepsilon^{ns}. \tag{25}$$

Thus, it is enough to characterize the expected surplus received by a non-savvy consumer in equilibrium in order to understand the welfare effect of market concentration.

Proposition 5 *Lower market concentration always increases consumers' welfare if*

$$\frac{\partial w}{\partial N} \leq -\frac{\partial w}{\partial \sigma}\frac{\partial \sigma}{\partial N}. \tag{26}$$

Else, it increases consumers' welfare if and only if

$$\frac{d\varepsilon^{ns}}{dN} \leq -\frac{dp}{dN}. \tag{27}$$

The Proof is in Appendix. The first part of the proposition derives from Lemma 1 (the impact of the number of deal finders and the share of savvy consumers on w), and Proposition 3 (the impact of the number of deal finders on the equilibrium share of savvy types). As the expected mismatch received by non-savvy consumers is a function of w by Proposition 1, and as by Proposition 4 prices decrease with w, equation (26) is sufficient to characterize a positive welfare effect of lower market concentration. The negative effect of more competition is a higher mismatch for a given

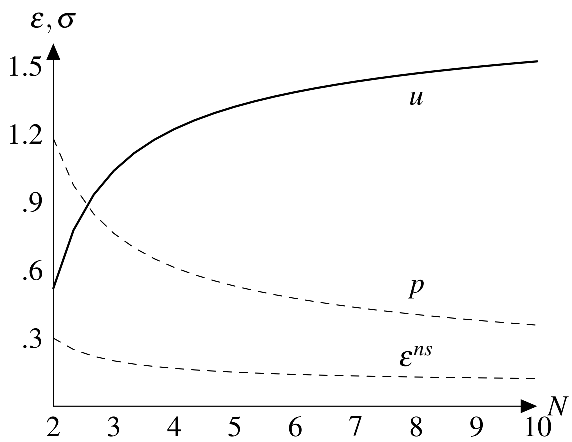


Figure 2: equilibrium price and welfare, with $s = 0,02$, $F(\varepsilon) = \varepsilon$, $c = 0.1$, $v = 2$.

share of savvy consumers. The positive effect is a higher share of savvy consumers, therefore decreasing the mismatch and the price. If the positive effect of competition is more important lower market concentration always benefits consumers, as from Proposition 4, more choosy deal finders always leads to lower prices. The second part derives from Proposition 4, as lower market concentration, even if it makes deal finders less choosy, can still make consumers better off it the prices decrease sufficiently. In that case, consumers trade off lower prices with higher mismatches.

Using the uniform distribution $F(\varepsilon) = \varepsilon$ and the same parameters as Figure 1, Figure 2 represents the sum of the mismatch and the price effect on the equilibrium welfare. The dashed lines represent the expected mismatch of a non-savvy consumer and the expected price (both decreasing with N). The solid line represents the expected welfare u, equal to $v - p - \varepsilon^{ns}$, and is increasing with N. In the uniform case, the signs of the effect of concentration on price and mismatch are identical, but it is striking that, at least for the highest levels of market concentration, the most important impact on consumer welfare is not so much the search effort by deal finders, but the price competition among sellers. What really benefits consumers is the externality generated by more consumers observing more than one product, even more so than the fact that deal finders are more selective.

As the aim of the paper is to understand the impact of market concentration, it is useful to show that mergers are actually a strategy firms would like to pursue in my model.

Lemma 2 *All other things held equal a merger always increases the joint profit of two firms if these are allowed to present themselves as independent deal finders and share search results.*

The Proof is in Appendix. This implies that the result in Proposition 4 may also hint to the conditions under which sellers have an incentive to oppose more concentration in the market for deal finders: if $\frac{dw}{dN} < 0$, a merger makes deal finders less selective. Hence $\frac{dp}{dN} < 0$, a merger makes sellers offer less competitive prices. Thus, it is precisely when the sellers oppose a merger between two deal finders that the merger may actually benefit consumers.

6 Heterogenous costs of savviness

The assumption of an homogenous cost of information c is crucial to my results. Assume for instance that the ability for a consumer to become savvy depends on a parameter θ, randomly drawn from a uniform distribution over $[0,1]$, so that the cost for a consumer i to become savvy is equal to $c(\theta_i) = \gamma\theta_i$. This means that the most able consumer has no cost of becoming informed, that the least able has a cost c of becoming informed, and that the cost of acquiring information is linear in the ability. For a given expected value of the mismatch differential between informed and uninformed consumers Δ, if a consumer of type j with $\theta_j > \theta_i$ prefers to become savvy, a consumer of type i also prefers to become savvy. Consumers can thus be ranked by their ability to acquire information, so that the cost for the σth consumer to become savvy is $c(\sigma) = \gamma\sigma$. This also implies that the more consumers become savvy, the higher the expected difference between the mismatch received by a savvy and a non-savvy consumer in equilibrium.

While Propositions 3 and 4 still hold, Proposition 5 does not as the expected payoff is now different for each consumer. Condition (27) however allows characterizing some Pareto improvements. If the condition is satisfied, all consumers are better off with more competition. However, if it is not satisfied, lower competition could make non-savvy types better off while making some savvy types worse off. It is possible that when σ increases savvy consumers are made better off, but those who cannot afford becoming savvy are worse off. If even the non-savvy consumers are made better off however, it means that more competition on the market for deal finders is Pareto improving. The intuition is relatively straightforward, as allowing for heterogeneous costs of savviness is an intermediary case between ex-ante identical consumers and assuming an exogenous share of σ.

In the uniform case, I can rewrite (12) as

$$\sigma = \sqrt{\frac{sN(N-1)}{2\gamma(N+1)}}, \tag{28}$$

where the share of savvy consumers still increases with N, but the increase is slower due to the marginally increasing cost of becoming informed. Plugging (28) into (5) yields

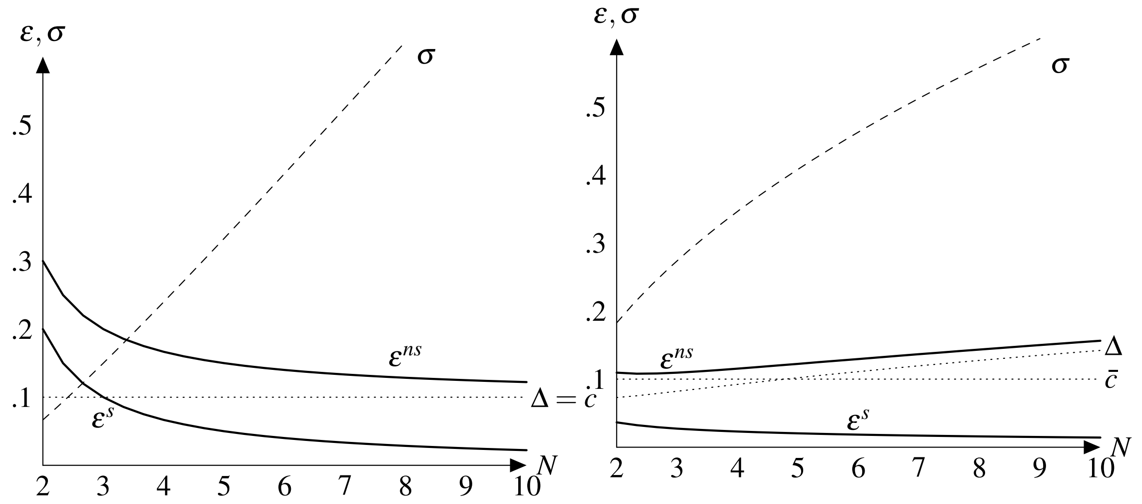
$$w = \frac{sN\sqrt{2\gamma(N+1)}}{\sqrt{sN(N-1)}}, \tag{29}$$

which can be shown to be increasing in N. This means that when market concentration decreases, (i) the share of consumers choosing to be informed increases, (ii) the expected mismatch received by informed consumers decreases, but (iii) the expected mismatch received by the remaining uninformed consumers increases. The difference between (15) and (29) is the existence of a distributional impact of the level of concentration on the market for deal finders. A key assumption for more competition on the market for deal finders to be Pareto improving for consumers is that the cost of becoming informed does not vary too much among consumers.

I illustrate this idea on Figure 3, by comparing the case studied in the Figure 1 with $c = 0.1$, to a cost function $c(\theta) = 0.2\theta$, so that as θ is drawn from a uniform distribution on $[0,1]$, the average cost of becoming savvy is identical in both examples, $\bar{c} = 0.1$. The left panel is just the right panel of Figure 1. On the right panel, we see the impact of heterogeneous costs of savviness. When a small number of firms are active on the market, the expected mismatch received by both types of consumers is pretty close, and is lower than on the left panel for both types as some consumers have almost no cost of being savvy. When the number of deal finders increases, the share of savvy consumers increases. The impact of N on σ however quickly becomes insufficient to make non-savvy consumers better off. Hence, in this case market concentration does not have a uniform impact on all consumers. The less able consumers, with the highest cost of becoming savvy γ, benefit from mergers until the number of deal finders is equal to $N = 3$, while the most able consumers always prefer a higher number of deal finders.

7 Endogenous entry

Until now, I have taken as exogenous the number of deal finders on the market. It is however possible to solve the model by making entry endogenous. Assume now that a deal finder enters the market at a fixed cost α until expected profit equals zero. It is easy to show that N is fully determined by c, α and s. In equilibrium, all symmetric deal finders get a share $1/N$ of the customers. Hence, the expected profit of a deal finder including search and entry costs is



Endogenous share of savvy types, with $c = 0.1$

Endogenous share of savvy types, with $\gamma = 0.2$, $c = 0.2\theta$.

Figure 3: The importance of an homogenous cost of savviness.

$$\pi = \frac{1}{N} - \frac{s}{F(w)} - \alpha, \tag{30}$$

and the equilibrium number of deal finders is

$$N = \lfloor \frac{F(w)}{s - \alpha F(w)} \rfloor, \tag{31}$$

where $\lfloor x \rfloor$ is the highest integer smaller than x. It follows that N is - unsurprisingly - decreasing in α, allowing us to fully characterize the equilibrium. Assuming identical cost for consumers to be informed c, a lower entry cost for deal finders increases entry, but not in a linear way. Indeed, as shown in (15), a consequence of entry in this case is that deal finders invest more in search as they become more choosy. Hence, entry decreases the market share of each firm and increases search costs.

Example 6 *If I assume ε to be uniformly distributed on $[0,1]$, I find by replacing w by its value found in* (15)*, the equilibrium number of deal finders N as*

$$N = \lfloor \frac{\sqrt{(-2c\alpha + 2c + s)^2 - 8c(-2c\alpha - s)} - 2c\alpha + 2c + s}{2(2c\alpha + s)} \rfloor. \tag{32}$$

I represent this example on Figure 4 (in Appendix), with identical parameter values as in the right panel of Figure 1. For a given level of effort, dividing by 2 the cost of entry would double the number of firms. Here, it is not the case as more entry implies higher costs.

8 Conclusions

The present paper puts together the incentives deal finders have to invest in search with the incentives consumers have to become informed and the incentives for sellers to offer low prices. This conjunction leads to two opposite effects of the impact of market concentration on the search behaviour of deal finders. The first effect is that more competition decreases the incentives for deal finders to invest in search. The second effect is that more competition increases the share of consumers choosing to become informed. These two effects alone do not suffice to characterize the welfare impact of competition, as they also influence the price offered by sellers. In particular, even if more competition on the market for deal finders makes these intermediaries less choosy, sellers may still decrease their price, and this effect may compensate the negative effect of higher horizontal mismatches.

More generally, this paper aims at contributing to the general debate about the impact of the multiplication of sources of information available on the Internet. The main message from this study of deal finders is that by ignoring the indirect effect of market concentration on consumer education one might draw incorrect conclusions overestimating the benefits from an economy with a limited number of (presumably) high quality source.

There are several ways in which this model could be extended. A first one would be to develop further where the revenue of deal finders come from, by modelling an explicit price relationship between these platforms, buyers, and sellers, as a two-sided market. A second one would be to explicitly model the choice consumers make of whether to use deal finders or to directly buy from the sellers. This would imply for consumers to balance the cost of observing the result of deal finders with the cost of themselves linearly search for deals. A third one would be to consider ordered search among deal finders, and to allow those to compete (for instance by advertising) for prominence.

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Appendix A: Proofs

Proof of Proposition 2

Proof. Savvy consumers obtain an expected mismatch equal to the expected value of the minimum of N random draws between 0 and w. The probability density of a random draw over the interval $[0,w]$ is $g(\varepsilon) = \frac{f(\varepsilon)}{F(w)}$, with cumulative density $G(\varepsilon) = \frac{F(\varepsilon)}{F(w)}$ (the probability density function is therefore specific to a given value of w). The expected value of the first order statistic of N independent draws of $g(\varepsilon)$ is given by the standard formula

$$\varepsilon^s = \int_0^w N(1 - G(\varepsilon))^{N-1} \varepsilon g(\varepsilon) d\varepsilon. \tag{33}$$

Using the definition of $g(\varepsilon)$, this rewrites

$$\varepsilon^s = \int_0^w N(1 - \frac{F(\varepsilon)}{F(w)})^{N-1} \varepsilon \frac{f(\varepsilon)}{F(w)} d\varepsilon. \tag{34}$$

The impact of s and σ are clear, as these variables only affect w. For a given value of w, the marginal impact of N is to decrease ε^s. However, as from Lemma 1, $\frac{\partial w}{\partial N} > 0$, the overall effect is ambiguous. By totally differentiating $\varepsilon^s(N, P(N))$, the expression in the Proposition follows. ■

Proof of Proposition 3

Proof. The first part of the proof follows directly from the properties of the first order statistic. For any expected value of the first order statistic of k random draws, $X(1,k)$, over an interval it is always true that $X(1,k) < X(1,k')$ if and only if $k' < k$. For a given number of deal finders, it is trivial that the larger the interval of the draws $[0,w]$, the higher the expected absolute gain from observing more draws. As $\frac{\partial w}{\partial N} > 0$, it therefore follows that $\frac{\partial \Delta}{\partial N} > 0$, and thus $\frac{\partial \sigma}{\partial N} \geq 0$. For the second part, using Lemma 1, and writing $w(N,\sigma(N))$ so that σ is endogenous, equation (13) is just the condition $\frac{dw(N,\sigma(N))}{dN} < 0$, where we know from Lemma 1 that $\frac{\partial w}{\partial N} > 0$ and $\frac{\partial w}{\partial \sigma} < 0$, and it follows from $\frac{\partial \Delta}{\partial N} > 0$ that $\frac{\partial \sigma}{\partial N} \geq 0$. ■

Proof of Proposition 4

Proof. It is easy to show that, for M sufficiently large,

$$D^{ns}_{p_i} = \frac{-f(w)}{MF(w)}, \tag{35}$$

so that with $\sigma = 0$, as f is log-concave, the unique symmetric equilibrium price would be $p = \frac{F(w)}{f(w)}$. For $D^s_{p_i}$ the expression is less straightforward, as it takes the derivative of D^{ns} multiplied by the probability of being smaller than the first order statistic (also log-concave, as from Chen *et al.*, 2009) of $N-1$ draws over $[0,w]$, $r(N-1)$. This demand has been described in (18) as

$$D^s = ND^{ns}r(N-1). \tag{36}$$

At equilibrium, $D^s = D^{ns} = \frac{1}{M}$. Define $\beta = Nr(N-1)$, so that $D^s = D^{ns}\beta$. At equilibrium $p_i = p$, for $D^s = D^{ns}$ to hold, it must be that $\beta = 1$. We can thus differentiate D_s as,

$$D^s_{p_i} = D^{ns}_{p_i} + D^{ns}\frac{d\beta}{dp_i}. \tag{37}$$

As $D^{ns} = \frac{1}{M}$, it is enough to show that $\frac{d\beta}{dp_i} < 0$ in order to show that $D^s_{p_i} < D^{ns}_{p_i}$. This result is straightforward as $\frac{dr(j)}{dp_i} < 0, \forall j > 0$. ■

In the special case of the uniform distribution, with M sufficiently large, the demand can be rewritten as

$$D = D^{ns}((1-\sigma) + \sigma Nr(N-1)). \tag{38}$$

Denoting $\beta = (1-\sigma) + \sigma N r(N-1)$, I find

$$\begin{aligned} D_{p_i} &= D^{ns}_{p_i} + \frac{1}{M}\frac{d\beta}{dp_i} \\ &= -\frac{1}{Mw} - \frac{1}{Mw}(N-1)\sigma, \end{aligned} \tag{39}$$

so that

$$\begin{aligned} p &= -\frac{\frac{1}{M}}{-D_{p_i}} \\ &= \frac{w}{1+(N-1)\sigma}. \end{aligned} \tag{40}$$

Proof of Proposition 5

Proof. In order to prove the first part of the Proposition, I need to assess the impact of N on u^{ns} for a given value of p. Using Lemma 1 and Propositions 1 and 3, it is possible to rewrite (7) as

$$u^{ns}(w) = v - p - \varepsilon^{ns}(N, \sigma(N)). \tag{41}$$

Hence, totally differentiating ε^{ns} with respect to N yields

$$\frac{d\varepsilon^{ns}}{dN} = \frac{\partial \varepsilon^{ns}}{\partial w}\left(\frac{\partial w}{\partial N} + \frac{\partial w}{\partial \sigma}\frac{\partial \sigma}{\partial N}\right), \tag{42}$$

with $\frac{\partial \varepsilon^{ns}}{\partial w} > 0$, $\frac{\partial w}{\partial N} > 0$, $\frac{\partial W}{\partial \sigma} < 0$ and $\frac{\partial \sigma}{\partial N} > 0$. As by Proposition 1, $\frac{d\varepsilon^{ns}}{dw} > 0$, and as by Proposition 4 $\frac{dp}{dN} < 0$ if $\frac{dw}{dN} < 0$, the first part of the Proposition follows. The second part is simply a rewriting of the derivative of the utility function with respect to N. ■

Proof of Lemma 2

Proof. Consider two firms 1 and 2, both searching until they find a mismatch below w. The expected profit in equilibrium, including search costs of the two separate firms is

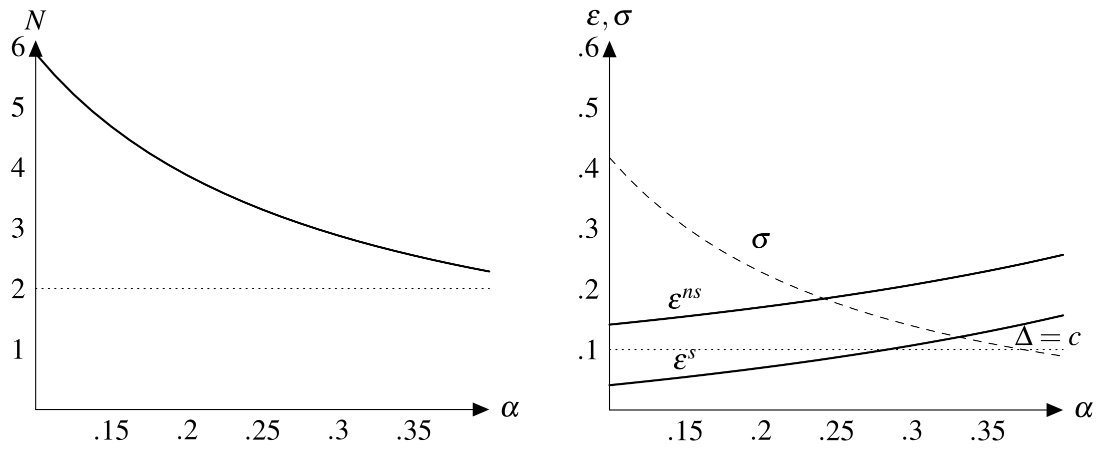
$$\Pi_1 + \Pi_2 = 2\sigma\left(\frac{F(w) - F(\varepsilon)}{F(w)}\right)^{N-1} + 2\frac{1-\sigma}{N} - 2\frac{s}{F(w)} \tag{43}$$

If the deal finders merge but keep two separated firms, they keep an identical share of non-savvy consumers. To show that a merger is beneficial, I assume the two deal finders continue to search independently until they find a mismatch below w, but put their search effort in common so that they offer the smallest of the two draws. The probability that this draw is smaller than the best deal found by the $N-2$ other firms is $(\frac{F(w)-F(\varepsilon)}{F(w)})^{N-2}$, so that the joint profit of the two merging firms becomes

$$\Pi_1' + \Pi_2' = 2\sigma(\frac{F(w)-F(\varepsilon)}{F(w)})^{N-2} + 2\frac{1-\sigma}{N} - 2\frac{s}{F(w)} > \Pi_1 + \Pi_2. \tag{44}$$

This search behaviour is however not optimal, as the search intensity of the joint entity would need to maximize the search behaviour when facing $N-2$ competitors (more choosy than a single firm facing $N-1$ competitors, but less choosy that two independent firms). ■

Appendix B: Figure 4, endogenous entry of deal finders



Number of deal finders in equilibrium

Equilibrium expected mismatch and share of savvy types.

Figure 4: Endogenous entry, with $c = 0.1$, $s = 0.02$

Appendix C: Non-sequential search

In this Appendix, I consider a non-sequential variant of the model. Instead of linearly searching until they find a deal below a threshold value w, I assume deal finders simultaneously choose the number of sellers they sample, in the tradition of Burdett and Judd (1983). Denote by q the symmetric equilibrium number of sellers sampled by a deal finder, and assume a symmetric equilibrium price p, the expected profit of deal finder i sampling q_i prices is

$$\pi(q_i, q) = \sigma\left(\int_0^b f_{(1),q_i}(\varepsilon)(1-F_{(1),q}(\varepsilon))^{N-1} d\varepsilon\right) + \frac{1-\sigma}{N}, \tag{45}$$

where $f_{(1),x}(\varepsilon)$ is the density of the first order statistic of x independent draws with individual density $f(\varepsilon)$, and similarly $F_{(1),x}$ is the cumulative density. The first part of the profit is thus the probability that the smallest of q_i random draws be lower than the smallest of q random draws, times the $N-1$ other deal finders, multiplied by the share of savvy consumers σ. The second part is identical to the sequential model, and represents the fact that a share $1-\sigma$ of non-savvy consumers choose the best of the q deals offered by a deal finder chosen at random. Using the properties of the order statistics, this expression rewrites:

$$\pi(q_i,q) = \sigma\left(\int_0^b q_i f(\varepsilon)(1-F(\varepsilon))^{q_i-1+q(N-1)}d\varepsilon\right) + \frac{1-\sigma}{N}. \tag{46}$$

The symmetric equilibrium q is such that

$$\frac{d\pi(q_i,q)}{q_i} = s, \tag{47}$$

for all deal finders i. As q is an integer, a continuous value of q has to be interpreted as a mixed strategy. It is straightforward that for a given q the marginal benefit of an additional search is decreasing in q_i. As in Lemma 1, a simple inspection of (46) and (47) shows that $\frac{\partial q}{\partial s} < 0$ (if the marginal cost increases, the marginal benefit must also increase). It is also clear that $\frac{\partial q}{\partial \sigma} > 0$, as σ directly multiplies the marginal benefit of an additional search, and $\frac{\partial q}{\partial N} < 0$, as N only enters the expression $(1-F(\varepsilon))^{q_i-1+q(N-1)}$.

The expected mismatch obtained by a non-savvy consumer is the minimum of q random draws by one deal finder,

$$\varepsilon^{ns} = \int_0^b q(1-F(\varepsilon))^{q-1}\varepsilon f(\varepsilon)d\varepsilon, \tag{48}$$

so that $\frac{\partial \varepsilon^{ns}}{\partial q} < 0$, and Proposition 1 holds. The expected mismatch obtained by a savvy consumer is the minimum of q random draws by N deal finders,

$$\varepsilon^{s} = \int_0^b Nq(1-F(\varepsilon))^{Nq-1}\varepsilon f(\varepsilon)d\varepsilon, \tag{49}$$

with$\frac{\partial \varepsilon^{s}}{\partial N} < 0$, and $\frac{\partial \varepsilon^{s}}{\partial q} < 0$. Hence, as $\frac{\partial q}{\partial N} < 0$, the sign of $\frac{d\varepsilon^{s}}{dN}$ is ambiguous. As in Proposition 2, the presence of an additional deal finder increases u^s if and only if

$$-\frac{\partial \varepsilon^{s}}{\partial N} \geq \frac{\partial \varepsilon^{s}}{\partial q}\frac{\partial q}{\partial N}. \tag{50}$$

From (48) and (49) it follows directly that $\frac{\partial \Delta}{\partial N} > 0$, so that Proposition 3 also holds.

Switching to the sellers' side, the demand from non-savvy consumers is

$$D^{ns}(p_i, p, N) = \frac{q}{M} \int_0^b f(\varepsilon)(1 - F(\varepsilon - p + p_i))^{q-1} d\varepsilon, \tag{51}$$

the probability of being selected by each deal finder, of offering the best deal among the q random draws of this deal finder, and the probability that each deal finder is chosen at random by a consumer. The demand from savvy consumers is

$$D^{s}(p_i, p, N) = \frac{Nq}{M} \int_0^b f(\varepsilon)(1 - F(\varepsilon - p + p_i))^{Nq-1} d\varepsilon, \tag{52}$$

the probability of offering the best deal among Nq random independent draws. The demand for a given seller is $D = \sigma D^s + (1 - \sigma) D^{ns}$, and the profit is $p_i D(p, p_i, N)$. As, at a symmetric price equilibrium $D^s = D^{ns} = \frac{1}{M}$, D^s is more elastic. Thus, following a similar reasoning as for the sequential search, even if a higher value of N yields a lower value of q, it is possible that p decreases with N if

$$D^s_{p_i}(p,p,N)\frac{d\sigma}{dN} + D^{ns}_{p_i}(p,p,N)\frac{d(1-\sigma)}{dN} + (1-\sigma)\frac{dD^{ns}_{p_i}(p,p,N)}{dN} + \sigma\frac{dD^{s}_{p_i}(p,p,N)}{dN} \leq 0, \tag{53}$$

because $\frac{d\sigma}{dN} > 0$.

Appendix D: linear information cost

Consider a variant of the model where instead of observing either all or one deal finder, consumers bear a linear search cost c to (non sequentially) observe an additional finder. In line with Burdett and Judd (1983), for an equilibrium where some - but not all - consumers choose to observe only one deal finder to exist, I can focus on equilibria where consumers mix between observing 1 or 2 deal finders (because the marginal benefit of an additional observation is decreasing in the number of observations).

If there is a share σ of consumers observing 2 deal finders, $\pi(\varepsilon)$ becomes

$$\pi(\varepsilon) = \frac{1-\sigma}{N} + \frac{2\sigma}{N}\frac{F(w) - F(\varepsilon)}{F(w)}, \tag{54}$$

so that in the second stage w solves

$$s = \frac{2\sigma}{N} \int_0^w \frac{F(w) - F(\varepsilon)}{F(w)} f(\varepsilon) d\varepsilon, \tag{55}$$

with identical properties as in the “Varian” setting. In the first stage, σ solves

$$c = \int_0^w \varepsilon g(\varepsilon) d\varepsilon - \int_0^w 2(1 - G(\varepsilon)) \varepsilon g(\varepsilon) d\varepsilon, \tag{56}$$

with $g(\varepsilon) = \frac{f(\varepsilon)}{F(w)}$ and w from (55). As in the Varian setting, the difference increases with w, so that the indirect effect of higher N is to increase σ. For such a mixed strategy to be an equilibrium, c must not be too low, as consumers must strictly prefer to observe 2 deal finders over 3,

$$c > \int_0^w 2(1 - G(\varepsilon)) \varepsilon g(\varepsilon) d\varepsilon - \int_0^w 3(1 - G(\varepsilon))^2 \varepsilon g(\varepsilon) d\varepsilon. \tag{57}$$

Finally, the price solves a similar problem as in the Varian case, with as only difference

$$D^s = 2D^{ns} r(2). \tag{58}$$

Hence, as in the Varian case, higher σ increases the demand elasticity even for a given w. In the special case of uniformally distributed ε over $[0,1]$, the results are such that lower market concentration makes all consumers better off, by making deal finders as choosy but inducing lower prices

$$w(\sigma) = \frac{sN}{\sigma} \tag{59}$$

$$\sigma = \frac{Ns}{6c} \tag{60}$$

$$w^* = 6c \tag{61}$$

$$p = \frac{w}{1+\sigma} = 6c + \frac{6c^2}{Ns}. \tag{62}$$