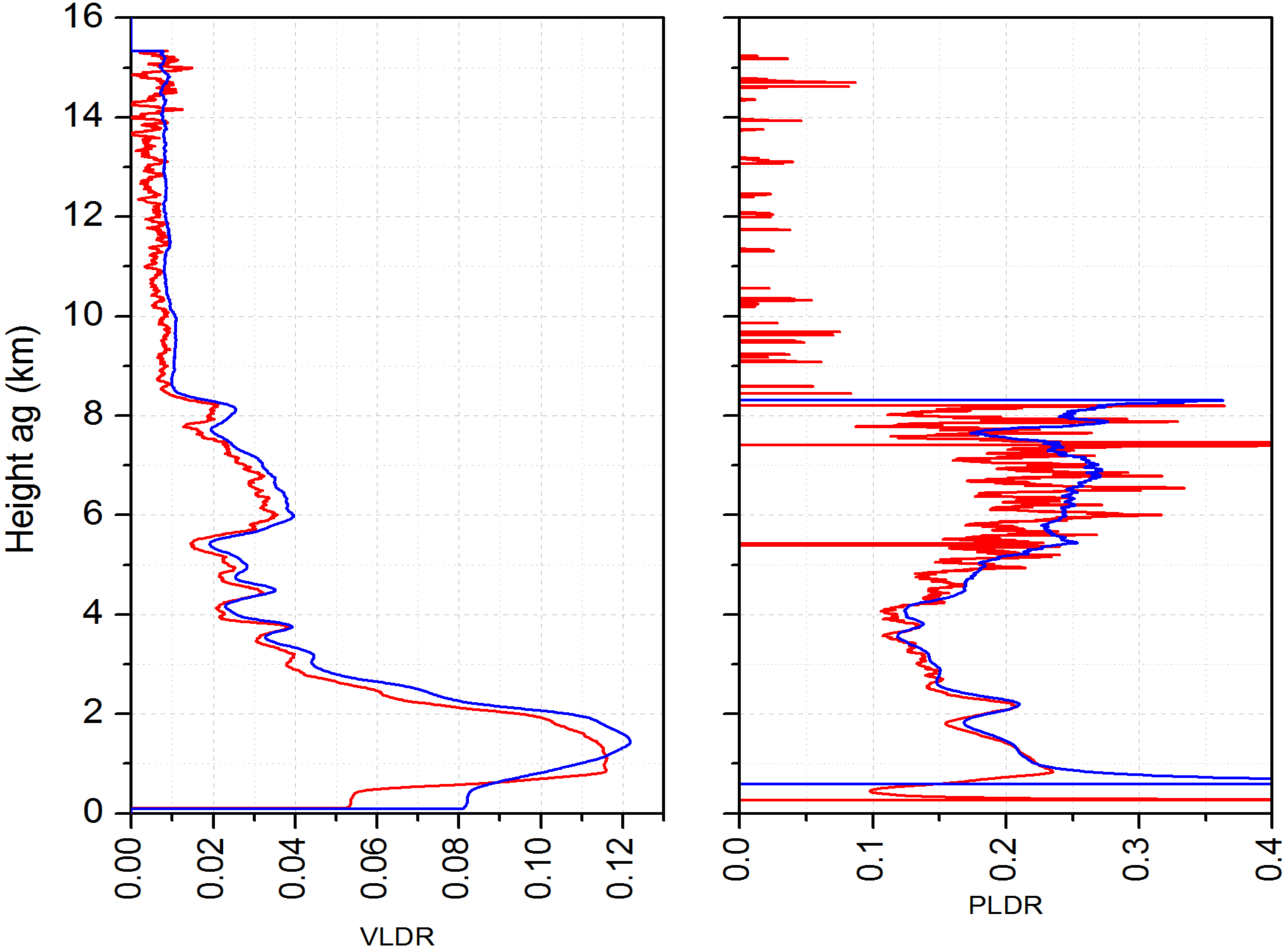
Polarizing lidars and the instrument function

Volker Freudenthaler

Meteorological Institute, Ludwig-Maximilians-University
Munich, Germany

volker.freudenthaler@lmu.de

Comparison of the VLDR and PLDR of two lidar systems



Lidar equation – Linear depolarisation ratio (LDR)

Lidar equation
$$P(r)=\frac{C}{r^2}\beta(r)\exp\left[-2\int_0^{r'}\beta(r)LR(r)dr'\right]+background(r)$$

$$=\frac{C}{r^2}\beta(r)T^2(0,r)+background(r)$$

P(r) lidar power incident on receiver from range *r*

Problem: absolute calibration => *C*

Solution: relative calibration with reference value (Klett, ...)

Lidar equation – Linear depolarisation ratio (LDR)

Lidar equation $P(r) = \frac{C}{r^2} \beta(r) T^2(0,r)$

Separation of explicit r-dependence

Polarisation measurements

$$I_\parallel(r) = Filter_\parallel\{P(r)\} = Filter_\parallel\{\beta(r)\} \times \frac{C}{r^2} T^2(0,r)$$

$$I_\perp(r) = Filter_\perp\{P(r)\} = Filter_\perp\{\beta(r)\} \times \frac{C}{r^2} T^2(0,r)$$

=> Linear depolarisation ratio δ, LDR *

$$\delta(r) = \frac{I_\perp(r)}{I_\parallel(r)} = \frac{Filter_\perp\{\beta(r)\}}{Filter_\parallel\{\beta(r)\}} \qquad = \xcancel{\frac{\beta_\perp(r)}{\beta_\parallel(r)}}$$

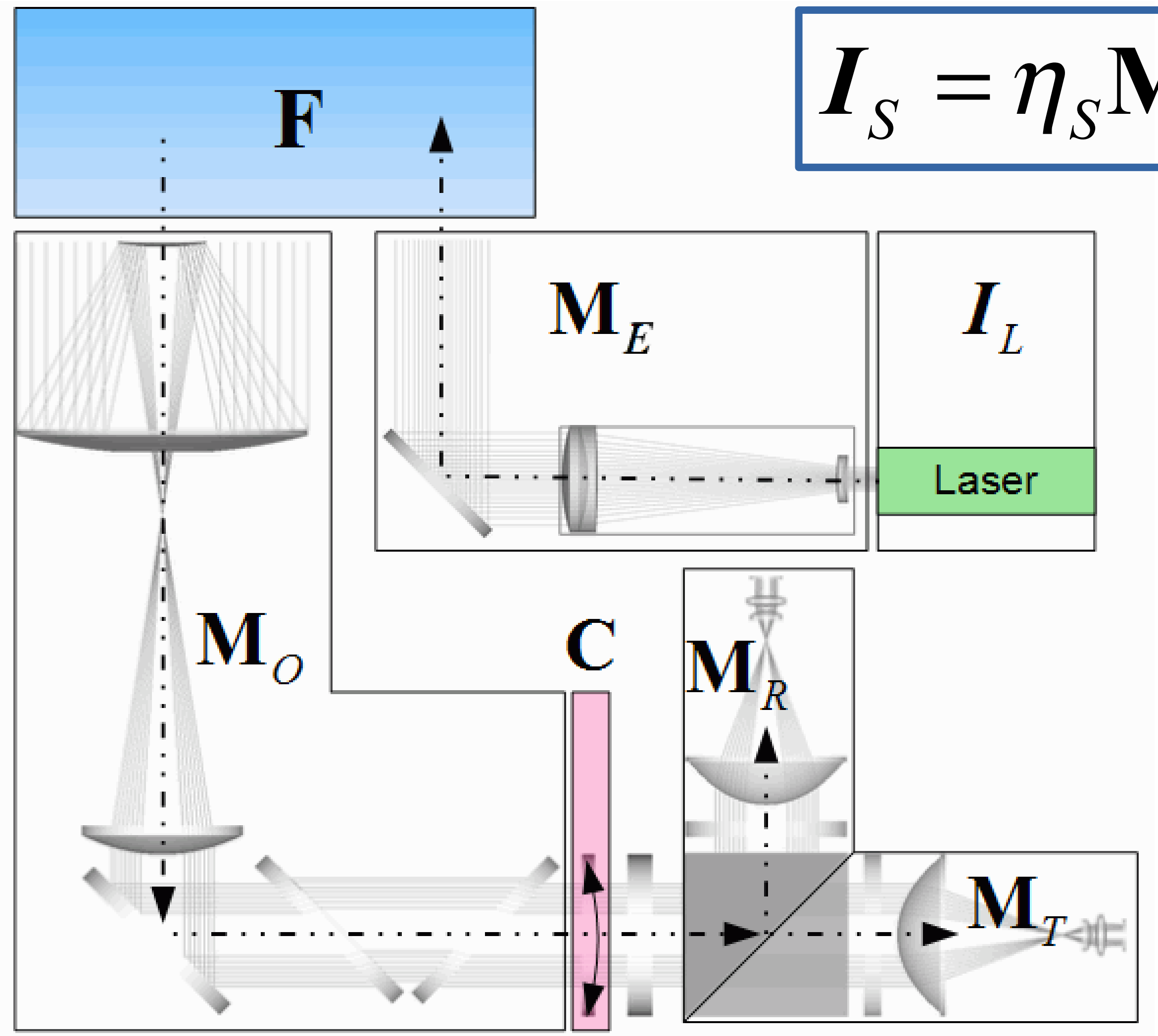
$I_{s,p}(r)$ = signals recorded with data acquisition in channels *s* and *p*

---

* *Gimmestad, G. G.: Reexamination of depolarization in lidar measurements, Appl. Opt., 47(21), 3795–3802, 2008.*

Typical lidar setup => The Model

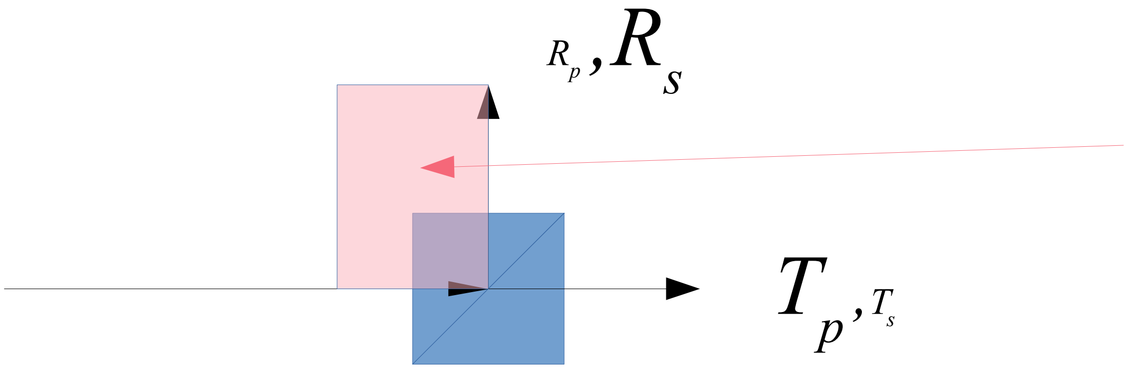
Stokes vectors describe the state of polarisation of a parallel light beam
Müller matrices describe the transformation by optical media



$$I_S = \eta_S \mathbf{M}_S \mathbf{C} \mathbf{M}_O \mathbf{F} \mathbf{M}_E I_L$$

Freudenthaler, V. About the effects of polarising optics on lidar signals and the Δ90-calibration Atmos. Meas. Tech., 9, 4181-4255, 2016.

p- & s-polarisation with respect to laser or incidence plane?

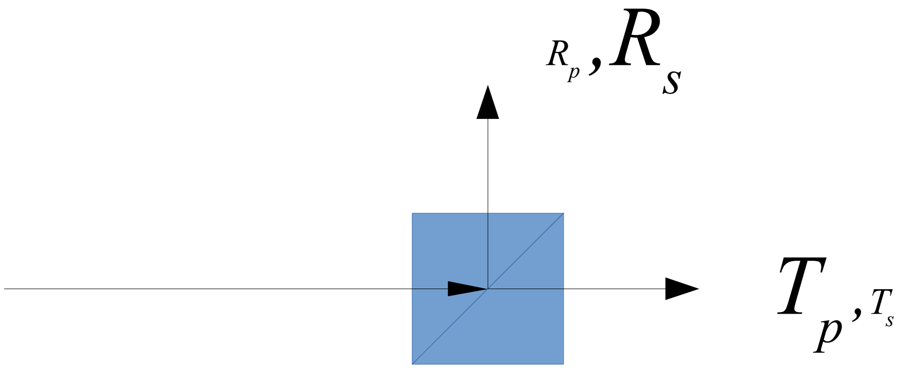


ideal polarsing beamsplitter (cube)

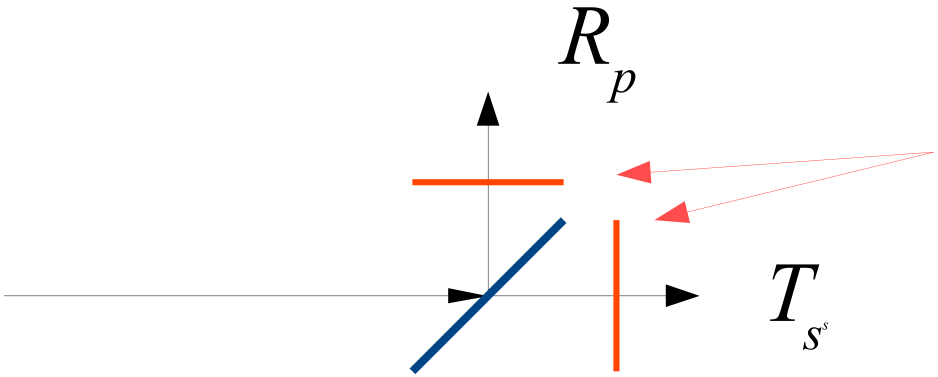
p and s polarization are
defined with respect to the
plane of incidence
for each optical element

R intensity reflectance
T intensity transmittance

p- & s-polarisation with respect to laser or incidence plane?



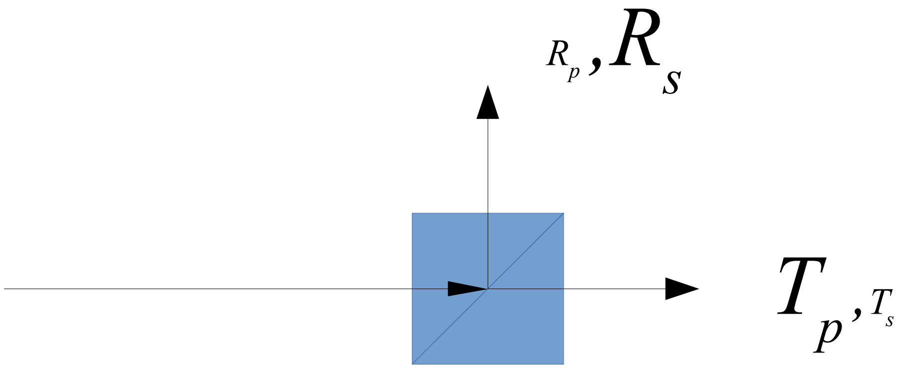
ideal polarsing beamsplitter (cube)



ideal polarisation filters

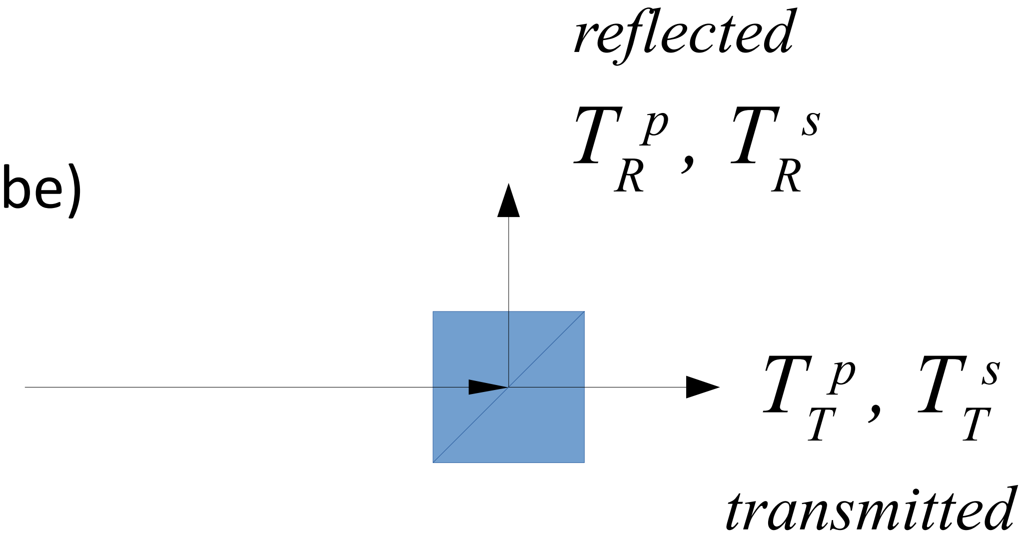
50/50 beamsplitter

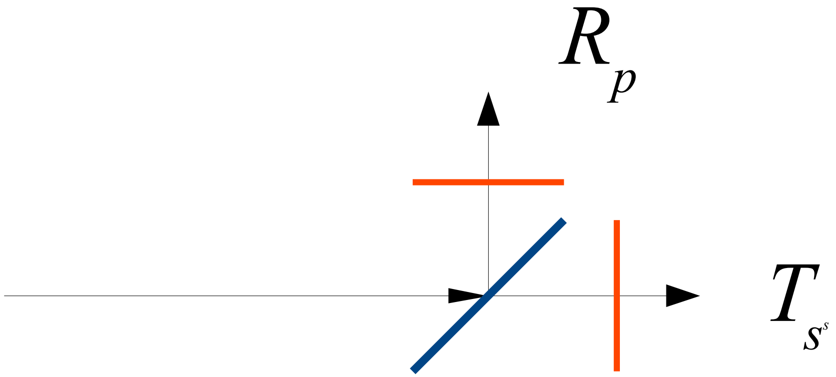
p- & s-polarisation with respect to laser or incidence plane?



ideal polarsing beamsplitter (cube)

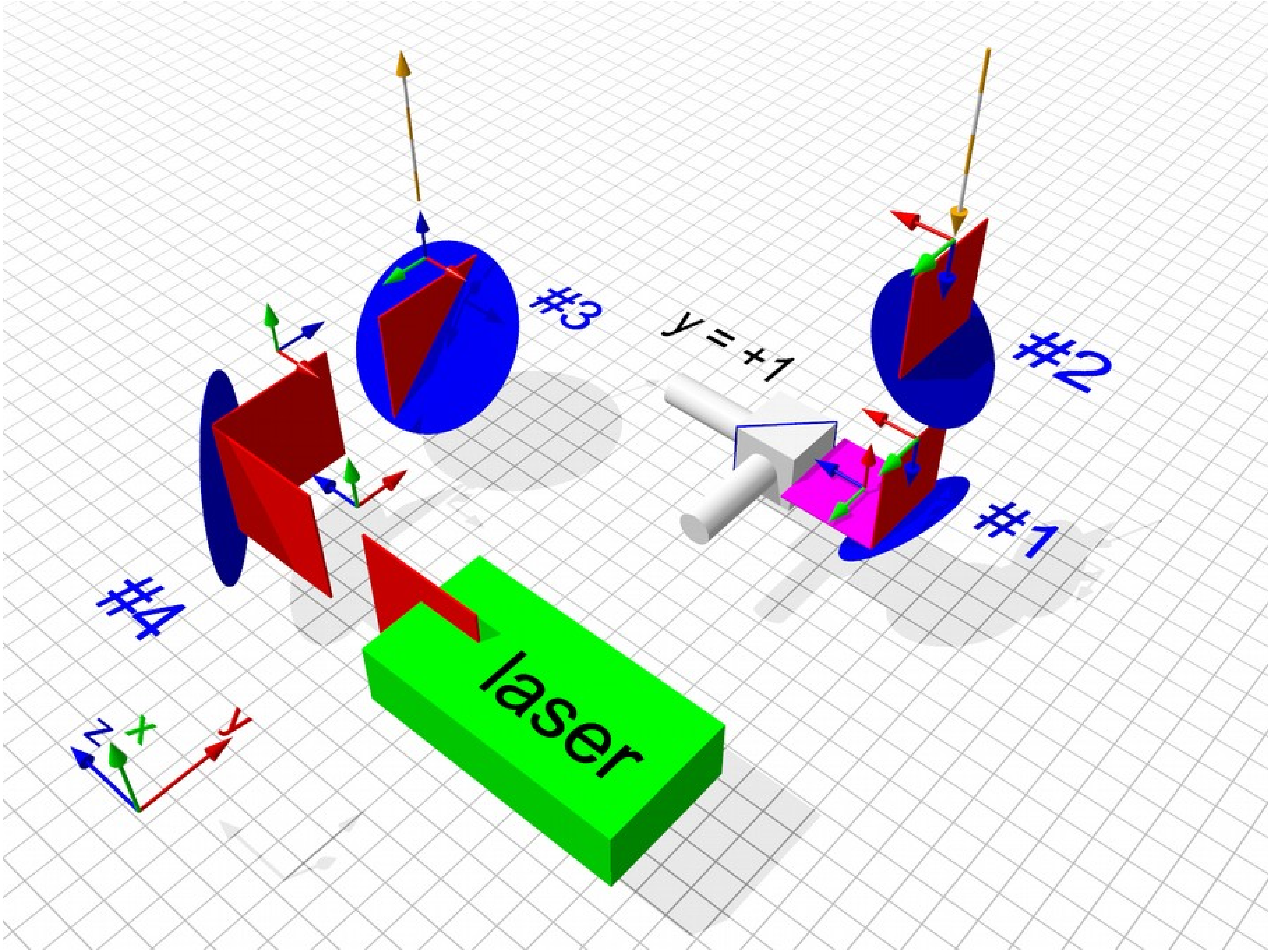
more general





50/50 beamsplitter

Magenta reference plane defines rotation angles and parameter y = ±1



Retarding Diattenuator

$$\mathbf{M}_T = \frac{1}{2}\begin{pmatrix} T_T^p + T_T^s & T_T^p - T_T^s & 0 & 0 \\ T_T^p - T_T^s & T_T^p + T_T^s & 0 & 0 \\ 0 & 0 & 2\sqrt{T_T^p T_T^s}\cos\mathit{\Delta}_T & 2\sqrt{T_T^p T_T^s}\sin\mathit{\Delta}_T \\ 0 & 0 & -2\sqrt{T_T^p T_T^s}\sin\mathit{\Delta}_T & 2\sqrt{T_T^p T_T^s}\cos\mathit{\Delta}_T \end{pmatrix} = T_T \begin{pmatrix} 1 & D_T & 0 & 0 \\ D_T & 1 & 0 & 0 \\ 0 & 0 & Z_T \mathrm{c}_T & Z_T \mathrm{s}_T \\ 0 & 0 & -Z_T \mathrm{s}_T & Z_T \mathrm{c}_T \end{pmatrix}$$

$$T_T = \frac{T_T^p + T_T^s}{2},\; D_T = \frac{T_T^p - T_T^s}{T_T^p + T_T^s},\; Z_T = \frac{2\sqrt{T_T^p T_T^s}}{T_T^p + T_T^s} = \sqrt{1 - D_T^2},\; \mathrm{c}_T = \cos\mathit{\Delta}_T,\; \mathrm{s}_T = \sin\mathit{\Delta}_T,\; \mathit{\Delta}_T = \varphi_T^p - \varphi_T^s$$

Retarding Diattenuator

$$\mathbf{M}_T = \frac{1}{2}\begin{pmatrix} T_T^p + T_T^s & T_T^p - T_T^s & 0 & 0 \\ T_T^p - T_T^s & T_T^p + T_T^s & 0 & 0 \\ 0 & 0 & 2\sqrt{T_T^p T_T^s}\cos\Delta_T & 2\sqrt{T_T^p T_T^s}\sin\Delta_T \\ 0 & 0 & -2\sqrt{T_T^p T_T^s}\sin\Delta_T & 2\sqrt{T_T^p T_T^s}\cos\Delta_T \end{pmatrix} = T_T\begin{pmatrix} 1 & D_T & 0 & 0 \\ D_T & 1 & 0 & 0 \\ 0 & 0 & Z_T \mathrm{c}_T & Z_T \mathrm{s}_T \\ 0 & 0 & -Z_T \mathrm{s}_T & Z_T \mathrm{c}_T \end{pmatrix} =$$

$$= T_T\begin{pmatrix} 1 & D_T & 0 & 0 \\ D_T & 1 & 0 & 0 \\ 0 & 0 & Z_T & 0 \\ 0 & 0 & 0 & Z_T \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \mathrm{c}_T & \mathrm{s}_T \\ 0 & 0 & -\mathrm{s}_T & \mathrm{c}_T \end{pmatrix}$$

linear diattenuator   retarder

$$T_T = \frac{T_T^p + T_T^s}{2},\ D_T = \frac{T_T^p - T_T^s}{T_T^p + T_T^s},\ Z_T = \frac{2\sqrt{T_T^p T_T^s}}{T_T^p + T_T^s} = \sqrt{1 - D_T^2},\ \mathrm{c}_T = \cos\Delta_T,\ \mathrm{s}_T = \sin\Delta_T,\ \Delta_T = \varphi_T^p - \varphi_T^s$$

Retarding Diattenuator

$$\mathbf{M}_T = \frac{1}{2}\begin{pmatrix} T_T^p + T_T^s & T_T^p - T_T^s & 0 & 0 \\ T_T^p - T_T^s & T_T^p + T_T^s & 0 & 0 \\ 0 & 0 & 2\sqrt{T_T^p T_T^s}\cos\Delta_T & 2\sqrt{T_T^p T_T^s}\sin\Delta_T \\ 0 & 0 & -2\sqrt{T_T^p T_T^s}\sin\Delta_T & 2\sqrt{T_T^p T_T^s}\cos\Delta_T \end{pmatrix} = T_T \begin{pmatrix} 1 & D_T & 0 & 0 \\ D_T & 1 & 0 & 0 \\ 0 & 0 & Z_T \mathrm{c}_T & Z_T \mathrm{s}_T \\ 0 & 0 & -Z_T \mathrm{s}_T & Z_T \mathrm{c}_T \end{pmatrix} =$$

$$= T_T \begin{pmatrix} 1 & D_T & 0 & 0 \\ D_T & 1 & 0 & 0 \\ 0 & 0 & Z_T & 0 \\ 0 & 0 & 0 & Z_T \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \mathrm{c}_T & \mathrm{s}_T \\ 0 & 0 & -\mathrm{s}_T & \mathrm{c}_T \end{pmatrix}$$

linear diattenuator   retarder

$$T_T = \frac{T_T^p + T_T^s}{2},\; D_T = \frac{T_T^p - T_T^s}{T_T^p + T_T^s},\; Z_T = \frac{2\sqrt{T_T^p T_T^s}}{T_T^p + T_T^s} = \sqrt{1 - D_T^2},\; \mathrm{c}_T = \cos\Delta_T,\; \mathrm{s}_T = \sin\Delta_T,\; \Delta_T = \varphi_T^p - \varphi_T^s$$

Retarding linear diattenuator mirror

$$\mathbf{M}_T = T_T \begin{pmatrix} 1 & D_T & 0 & 0 \\ D_T & 1 & 0 & 0 \\ 0 & 0 & Z_T \mathrm{c}_T & Z_T \mathrm{s}_T \\ 0 & 0 & -Z_T \mathrm{s}_T & Z_T \mathrm{c}_T \end{pmatrix}$$

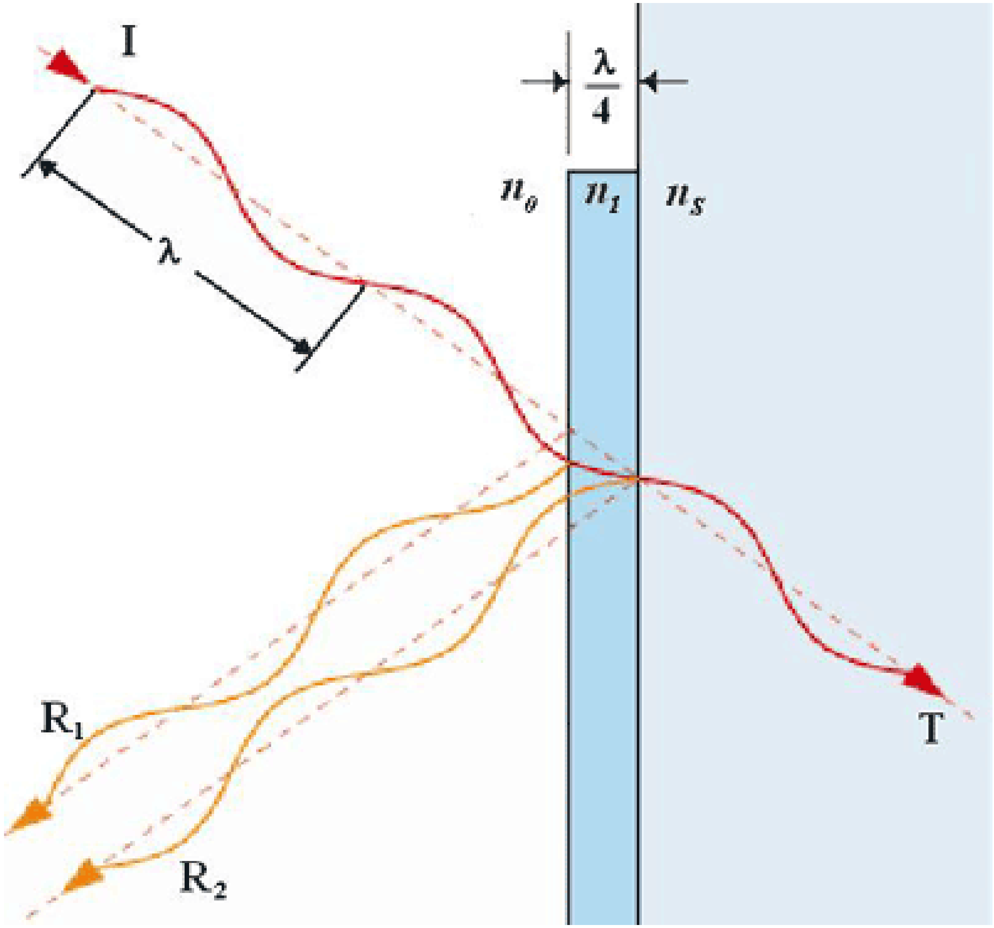
mirror   linear diattenuator

$$\mathbf{M}_R = T_R \begin{pmatrix} 1 & D_R & 0 & 0 \\ D_R & 1 & 0 & 0 \\ 0 & 0 & -Z_R \mathrm{c}_R & -Z_R \mathrm{s}_R \\ 0 & 0 & Z_R \mathrm{s}_R & -Z_R \mathrm{c}_R \end{pmatrix} = T_R \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & D_R & 0 & 0 \\ D_R & 1 & 0 & 0 \\ 0 & 0 & Z_R \mathrm{c}_R & Z_R \mathrm{s}_R \\ 0 & 0 & -Z_R \mathrm{s}_R & Z_R \mathrm{c}_R \end{pmatrix}$$

$$T_T = \frac{T_T^p + T_T^s}{2}, \; D_T = \frac{T_T^p - T_T^s}{T_T^p + T_T^s}, \; Z_T = \frac{2\sqrt{T_T^p T_T^s}}{T_T^p + T_T^s} = \sqrt{1 - D_T^2}, \; \mathrm{c}_T = \cos \Delta_T, \; \mathrm{s}_T = \sin \Delta_T, \; \Delta_T = \varphi_T^p - \varphi_T^s$$

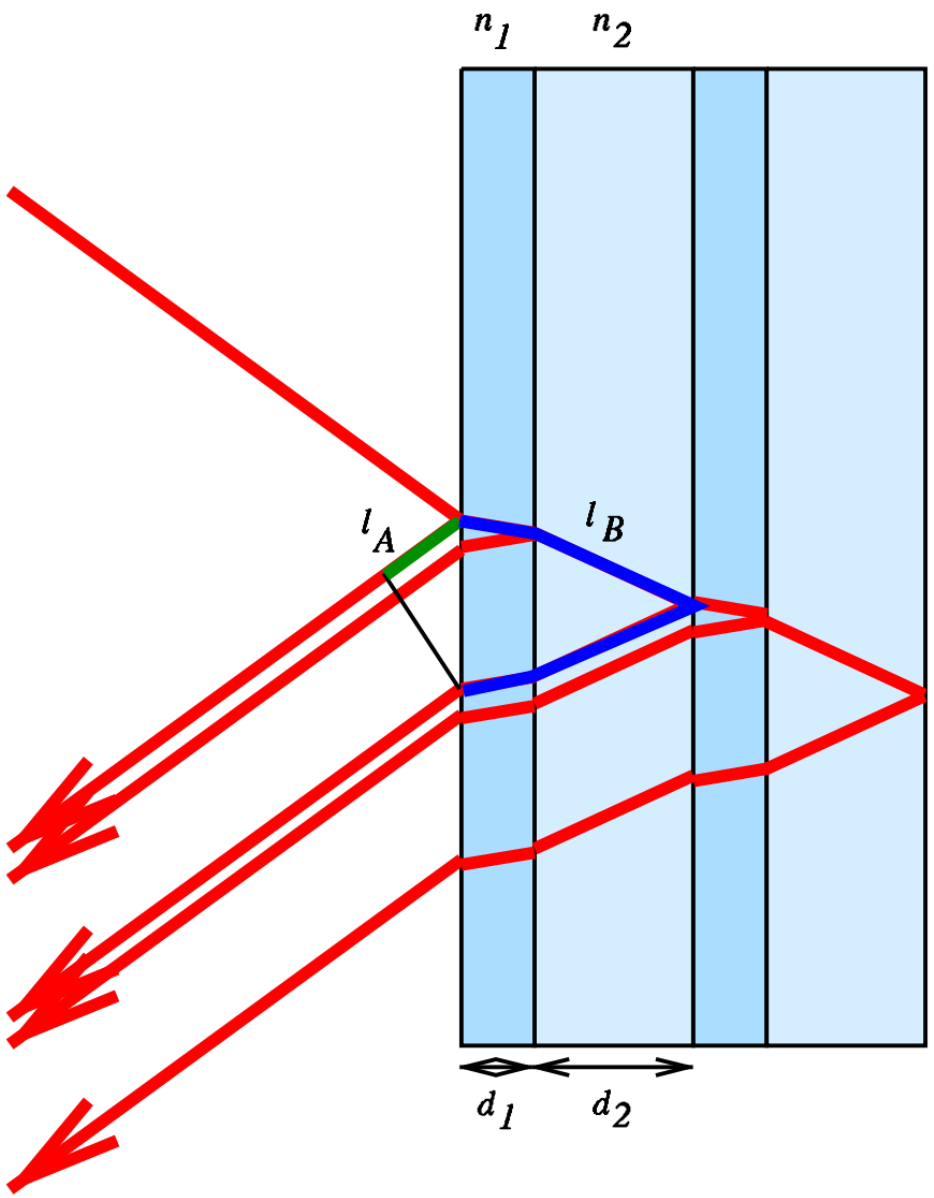
$$T_R = \frac{T_R^p + T_R^s}{2}, \; D_R = \frac{T_R^p - T_R^s}{T_R^p + T_R^s}, \; Z_R = \frac{2\sqrt{T_R^p T_R^s}}{T_R^p + T_R^s} = \sqrt{1 - D_R^2}, \; \mathrm{c}_R = \cos \Delta_R, \; \mathrm{s}_R = \sin \Delta_R, \; \Delta_R = \varphi_R^p - \varphi_R^s$$

Quarter wave coating



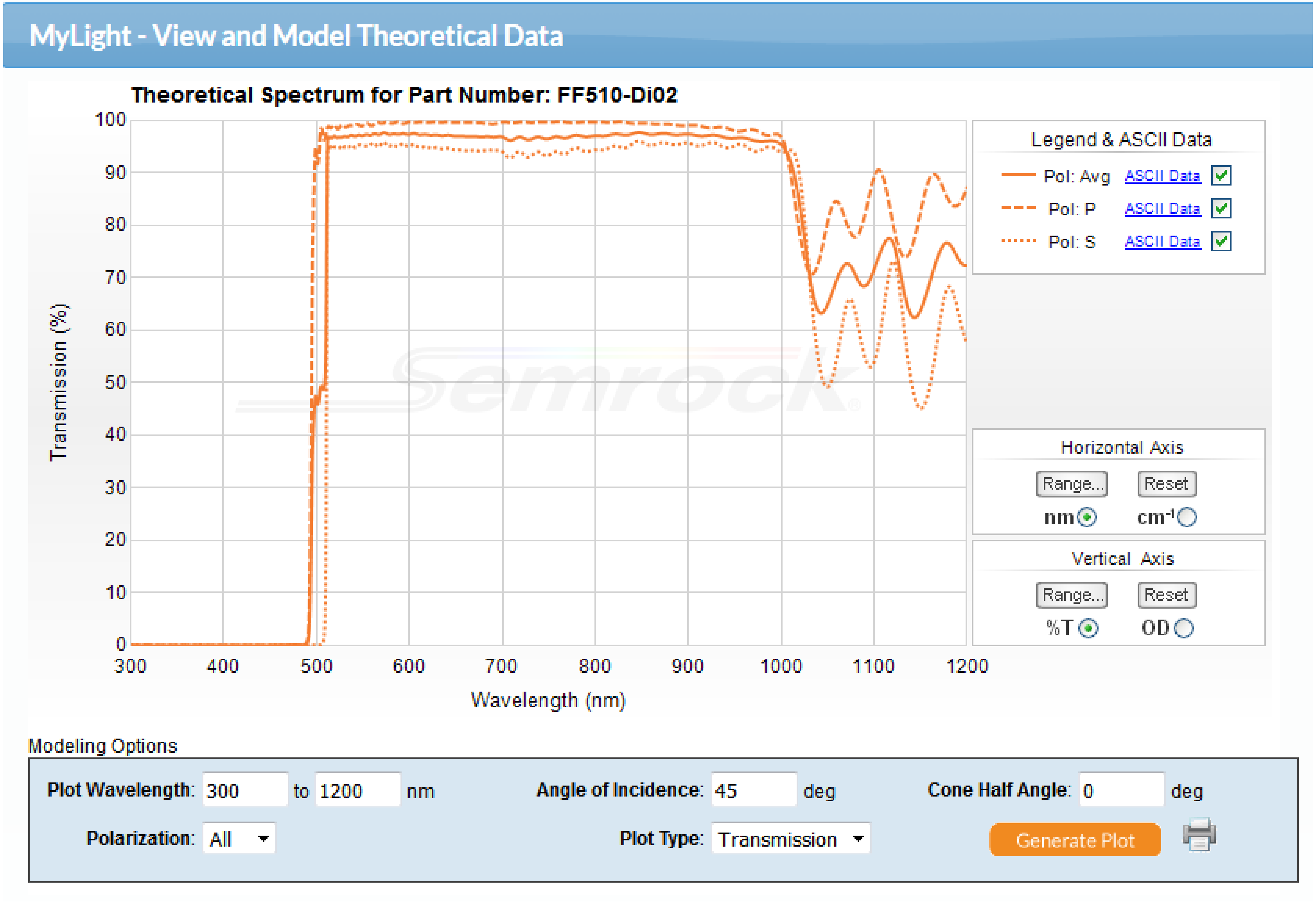
source: JEFF BLAKE and RICHARD PAYNTON, Choosing optical coatings for medical displays

Coating with more layers



source: Dielectric mirror, http://en.wikipedia.org/w/index.php?title=Dielectric_mirror&oldid=544165129

Coatings Semrock beamsplitter



Modeling Options

Plot Wavelength: 300 to 1200 nm

Angle of Incidence: 45 deg

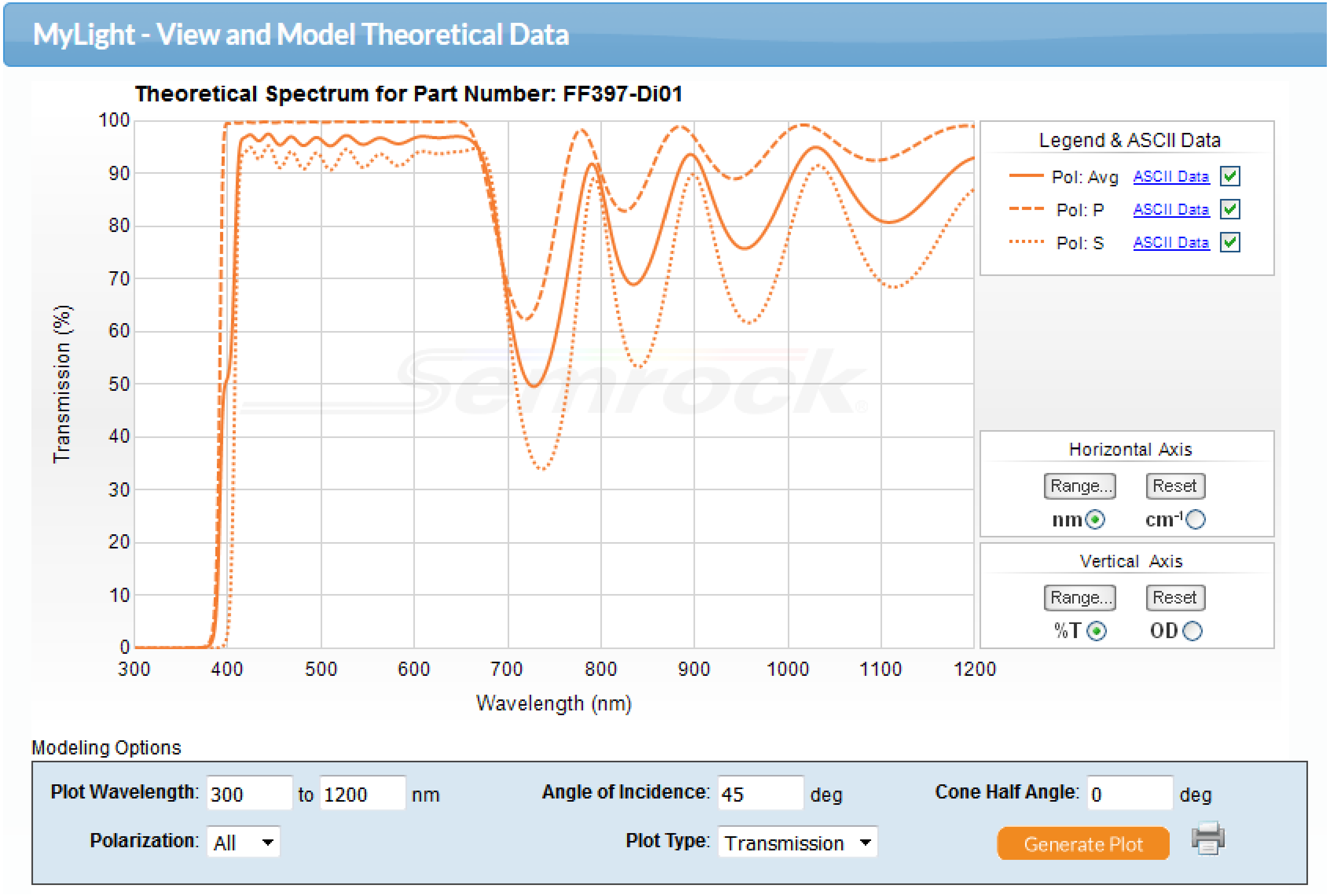
Cone Half Angle: 0 deg

Polarization: All

Plot Type: Transmission

https://www.semrock.com/filters.aspx

Coatings Semrock beamsplitter



MyLight - View and Model Theoretical Data

Theoretical Spectrum for Part Number: FF397-Di01

Modeling Options

Plot Wavelength: 300 to 1200 nm

Angle of Incidence: 45 deg

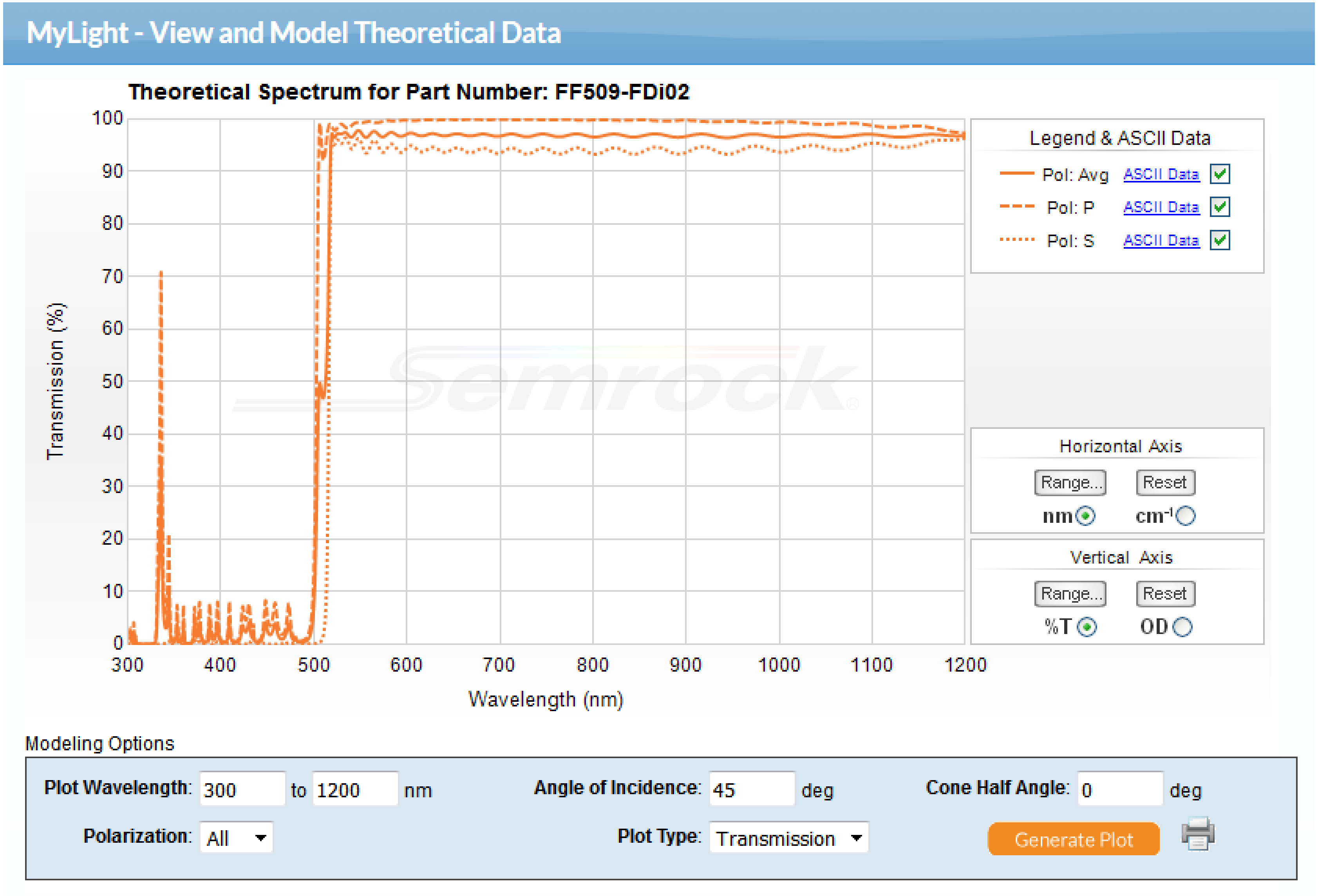
Cone Half Angle: 0 deg

Polarization: All

Plot Type: Transmission

https://www.semrock.com/filters.aspx

Coatings Semrock beamsplitter



MyLight - View and Model Theoretical Data

Theoretical Spectrum for Part Number: FF509-FDi02

Modeling Options

Plot Wavelength: 300 to 1200 nm

Angle of Incidence: 45 deg

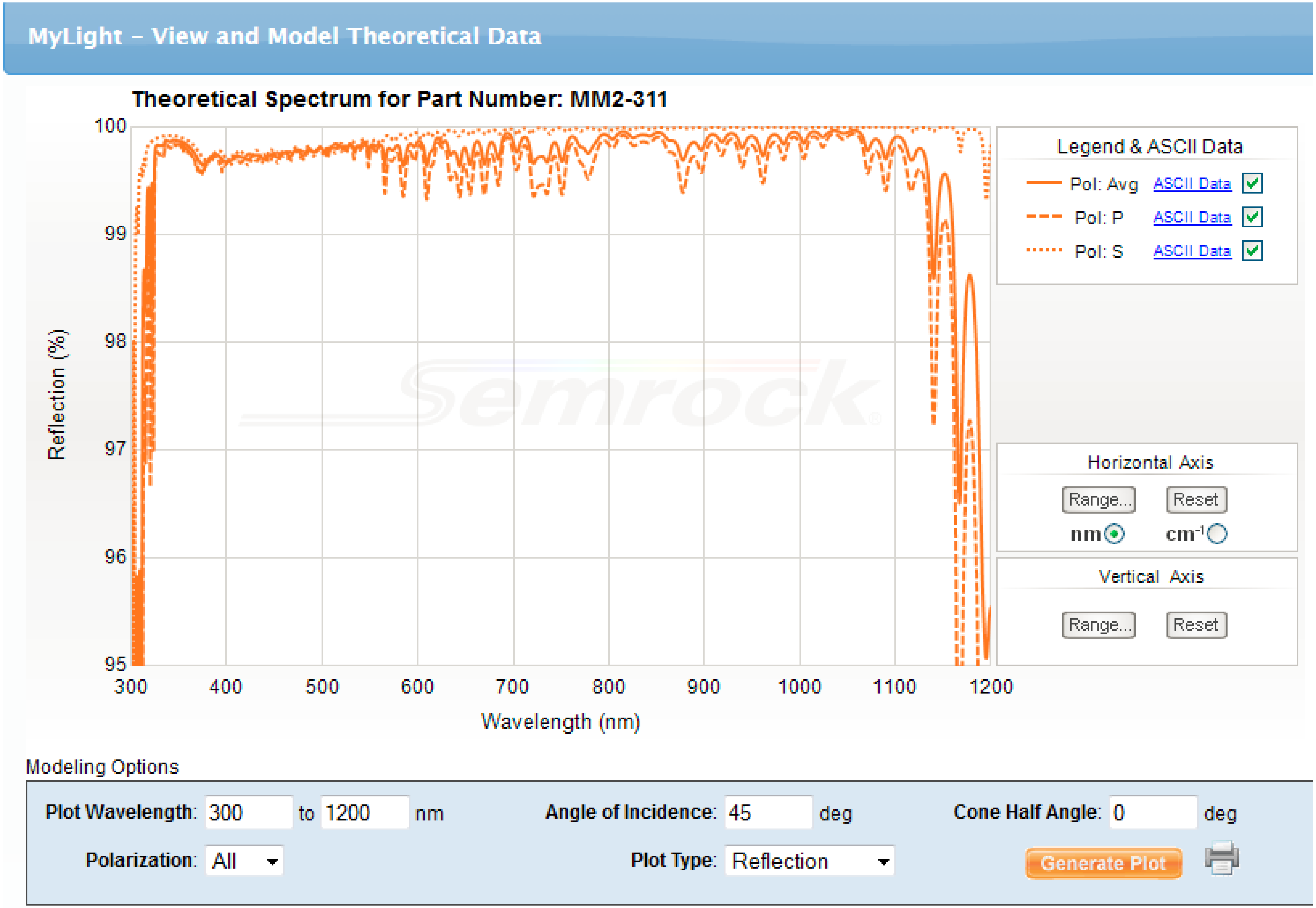
Cone Half Angle: 0 deg

Polarization: All

Plot Type: Transmission

https://www.semrock.com/filters.aspx

Coatings Semrock MaxMirror Reflection



Plot Wavelength: 300 to 1200 nm

Angle of Incidence: 45 deg

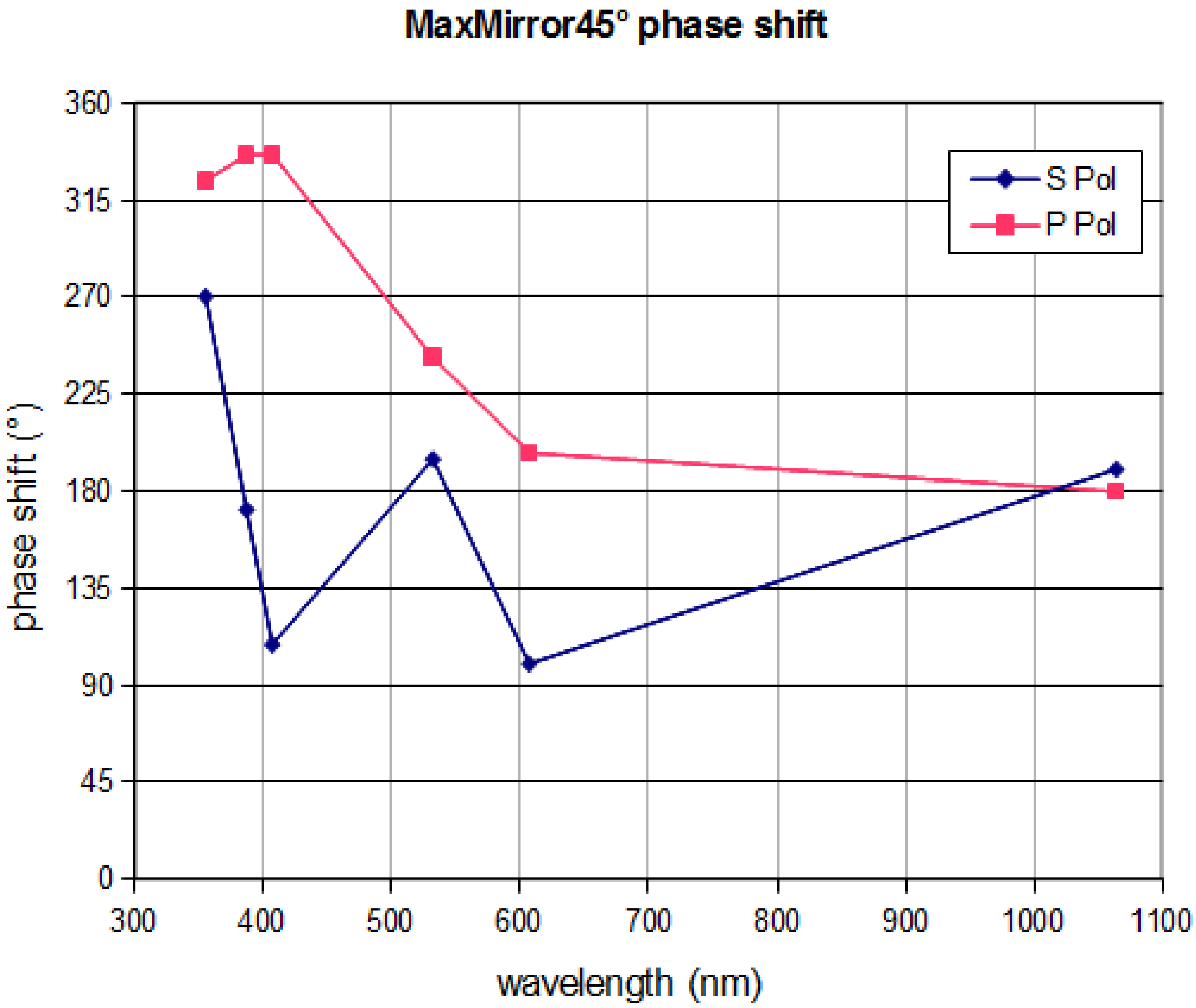
Cone Half Angle: 0 deg

Polarization: All

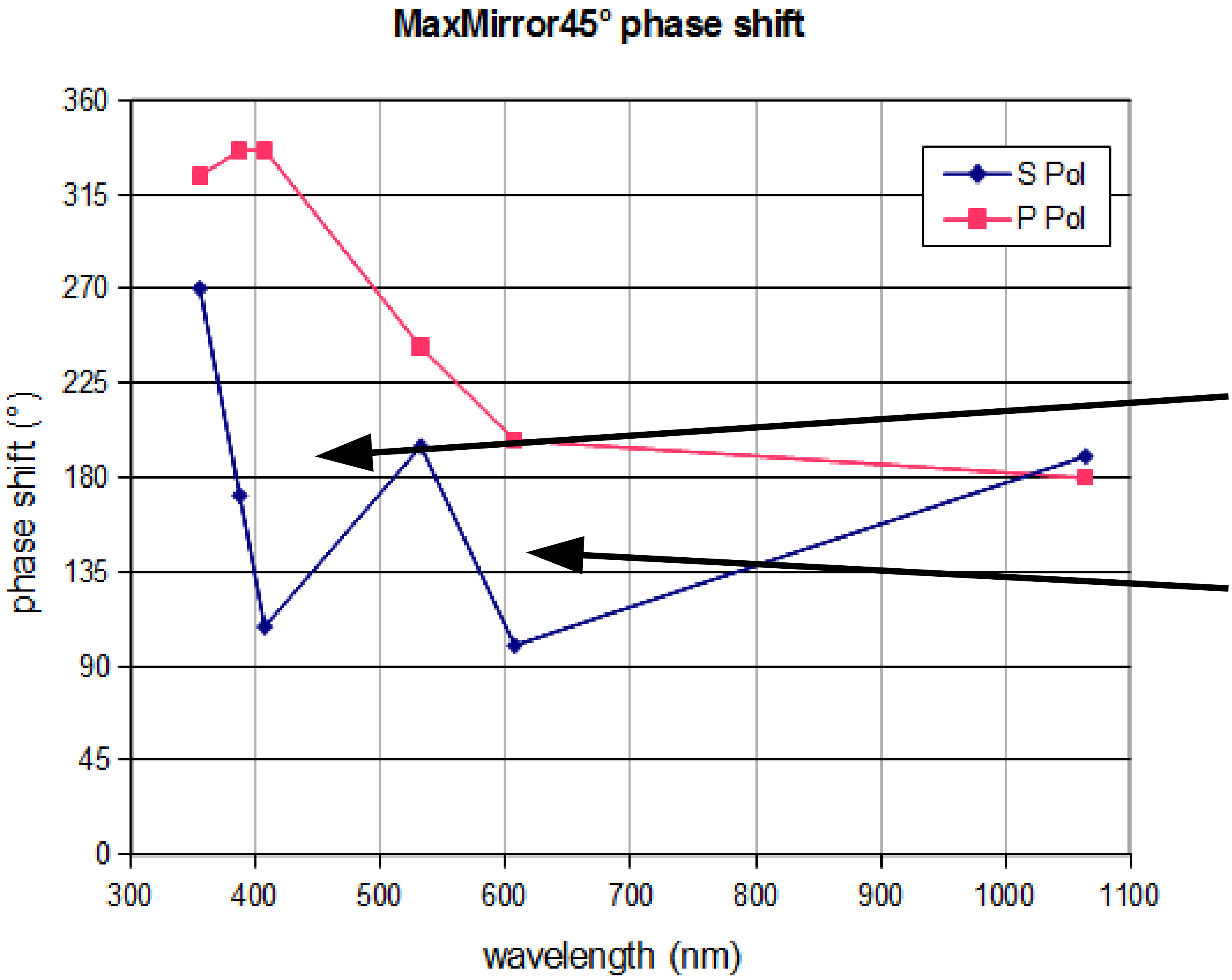
Plot Type: Reflection

http://www.semrock.com/FilterDetails.aspx?id=MM2-311-25

Coatings Semrock MaxMirror Phase shift



Coatings Semrock MaxMirror Phase shift



half wave plate

quarter wave plate

Rotated retarding linear diattenuator

$$\mathbf{M}_O(\phi)=\mathbf{R}(\phi)\mathbf{M}_O\mathbf{R}(-\phi)=$$

$$=T_O\begin{pmatrix}1&0&0&0\\0&c_{2\phi}&-s_{2\phi}&0\\0&s_{2\phi}&c_{2\phi}&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}1&D_O&0&0\\D_O&1&0&0\\0&0&Z_Oc_O&Z_Os_O\\0&0&-Z_Os_O&Z_Oc_O\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&c_{2\phi}&s_{2\phi}&0\\0&-s_{2\phi}&c_{2\phi}&0\\0&0&0&1\end{pmatrix}=$$

$$=T_O\begin{pmatrix}1&c_{2\phi}D_O&s_{2\phi}D_O&0\\c_{2\phi}D_O&1-s_{2\phi}{}^2W_O&s_{2\phi}c_{2\phi}W_O&-s_{2\phi}Z_Os_O\\s_{2\phi}D_O&s_{2\phi}c_{2\phi}W_O&1-c_{2\phi}{}^2W_O&c_{2\phi}Z_Os_O\\0&s_{2\phi}Z_Os_O&-c_{2\phi}Z_Os_O&Z_Oc_O\end{pmatrix}$$

$$c_{2\phi}=\cos 2\phi, s_{2\phi}=\sin 2\phi, c_O=\cos\Delta_O, s_O=\sin\Delta_O,\quad Z_O\equiv\sqrt{1-D_O{}^2},\quad W_O=1-Z_Oc_O$$

Two rotated retarding linear diattenuator

$$\mathbf{M}_O = T_O \begin{pmatrix} 1 & D_O & 0 & 0 \\ D_O & 1 & 0 & 0 \\ 0 & 0 & Z_O \mathrm{c}_O & Z_O \mathrm{s}_O \\ 0 & 0 & -Z_O \mathrm{s}_O & Z_O \mathrm{c}_O \end{pmatrix}$$

$$\mathbf{M}_O(\phi) = T_O \begin{pmatrix} 1 & \mathrm{c}_{2\phi} D_O & \mathrm{s}_{2\phi} D_O & 0 \\ \mathrm{c}_{2\phi} D_O & 1 - \mathrm{s}_{2\phi}{}^2 W_O & \mathrm{s}_{2\phi} \mathrm{c}_{2\phi} W_O & -\mathrm{s}_{2\phi} Z_O \mathrm{s}_O \\ \mathrm{s}_{2\phi} D_O & \mathrm{s}_{2\phi} \mathrm{c}_{2\phi} W_O & 1 - \mathrm{c}_{2\phi}{}^2 W_O & \mathrm{c}_{2\phi} Z_O \mathrm{s}_O \\ 0 & \mathrm{s}_{2\phi} Z_O \mathrm{s}_O & -\mathrm{c}_{2\phi} Z_O \mathrm{s}_O & Z_O \mathrm{c}_O \end{pmatrix}$$

$$\forall \left(\mathbf{M}_S, \mathbf{M}_O \right) \in \text{retarding linear diattenuator}$$

$$\mathbf{M}_S \mathbf{M}_O \in \text{retarding linear diattenuator}$$

But if $\mathbf{M}_S, \mathbf{M}_O$ are rotated individually:

$$\mathbf{M}_S(\theta)\, \mathbf{M}_O(\phi) \notin \text{rotated retarding linear diattenuator}$$

Müller matrix for atmospheric backscatter of randomly oriented particles

$$\beta \Rightarrow \mathbf{F} = \begin{pmatrix} F_{11} & 0 & 0 & 0 \\ 0 & F_{22} & 0 & 0 \\ 0 & 0 & -F_{22} & 0 \\ 0 & 0 & 0 & F_{44} \end{pmatrix} = F_{11} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & -a & 0 \\ 0 & 0 & 0 & 1-2a \end{pmatrix}$$

with polarisation parameter $a = \frac{F_{22}}{F_{11}} = 1 - d$

depolarisation parameter d

e.g.: *Mishchenko, M. I., Hovenier, J. W.: Depolarization of light backscattered by randomly oriented nonspherical particles, Opt. Lett., 20(12), 1356–1358, 1995.*

SVLE for atmospheric backscatter of randomly oriented particles

Stokes vector of linearly polarised light

$$\boldsymbol{I}_S = \mathbf{F}\boldsymbol{I}_L = F_{11}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & -a & 0 \\ 0 & 0 & 0 & 1-2a \end{pmatrix} I_L \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = F_{11} I_L \begin{pmatrix} 1 \\ a \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} I_{\parallel} + I_{\perp} \\ I_{\parallel} - I_{\perp} \\ 0 \\ 0 \end{pmatrix}$$

$$\delta = \frac{I_{\perp}}{I_{\parallel}} = \frac{1-a}{1+a} = \frac{F_{11} - F_{22}}{F_{11} + F_{22}} \quad \Rightarrow \quad a = \frac{1-\delta}{1+\delta}$$

$I_{\parallel}, I_{\perp}$ traditional terminology: parallel and perpendicular to the laser polarisation

SVLE : Stokes Vector Lidar Equation

Hayman, M. and Thayer, J. P.: General description of polarization in lidar using Stokes vectors and polar decomposition of Mueller matrices, J. Opt. Soc. Am. A, 29(4), 400–409, doi:10.1364/JOSAA.29.000400, 2012.

Stokes vector, intensity measurements

The Stokes vector ***S*** is defined with
six intensity (flux) measurements
using ideal polarization analyzers in front of a radiometer

I_H	horizontal linear polarizer (0°)
I_V	vertical linear polarizer (90°)
I_{45}	linear polarizer at 45°
I_{135}	linear polarizer at 135°
I_{RC}	right circular polarizer
I_{LC}	left circular polarizer

$$\boldsymbol{S} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} I_H + I_V \\ I_H - I_V \\ I_{45} - I_{135} \\ I_{RC} - I_{LC} \end{pmatrix}$$

SVLE for atmospheric backscatter of randomly oriented particles

backscatter from atmosphere

$$\boldsymbol{I}_S = \mathbf{F}\boldsymbol{I}_L = F_{11}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & -a & 0 \\ 0 & 0 & 0 & 1-2a \end{pmatrix} I_L \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = F_{11} I_L \begin{pmatrix} 1 \\ a \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} I_{\|} + I_{\perp} \\ I_{\|} - I_{\perp} \\ 0 \\ 0 \end{pmatrix}$$

$$\delta = \frac{I_{\perp}}{I_{\|}} = \frac{1-a}{1+a} = \frac{F_{11} - F_{22}}{F_{11} + F_{22}} \quad \Rightarrow \quad a = \frac{1-\delta}{1+\delta}$$

$I_{\|}, I_{\perp}$ traditional terminology: parallel and perpendicular to the laser polarisation

SVLE with instrument filter – polarisation lidar

polarisation filter (splitter)

$$\boldsymbol{I}_S = \eta_S \mathbf{M}_S \mathbf{F}\boldsymbol{I}_L = \eta_S T_S \begin{pmatrix} 1 & D_S & 0 & 0 \\ D_S & 1 & 0 & 0 \\ 0 & 0 & Z_S \mathrm{c}_S & Z_S \mathrm{s}_S \\ 0 & 0 & -Z_S \mathrm{s}_S & Z_S \mathrm{c}_S \end{pmatrix} F_{11} I_L \begin{pmatrix} 1 \\ a \\ 0 \\ 0 \end{pmatrix} = \eta_S T_S F_{11} I_L \begin{pmatrix} 1 + D_S a \\ D_S + a \\ 0 \\ 0 \end{pmatrix}$$

$$I_S = \eta_S T_S F_{11} I_L (1 + D_S a)$$

$$-1 \le D_S \le +1$$

measured signal with single channel detection

$$I_S(r) r^2 = C' (1 + D_S a) \beta(r) T^2(0,r)$$

$$P(r) r^2 = C \qquad \beta(r) T^2(0,r)$$ lidar equation

$$\frac{I_R}{I_T} = \frac{\eta_R T_R}{\eta_T T_T} \frac{1 + D_R a}{1 + D_T a} = \frac{\eta_R}{\eta_T} \frac{\left(T_R^p + T_R^s \delta\right)}{\left(T_T^p + T_T^s \delta\right)}$$

two channel linear polarisation detection

Calibration factor η and calibration factor correction K

$$\frac{I_R}{I_T} = \frac{\eta_R T_R}{\eta_T T_T} \times \frac{1 + D_R a}{1 + D_T a} = \eta \times K(a)$$

cross-talk correction *K*

Freudenthaler, V. About the effects of polarising optics on lidar signals and the Δ90-calibration Atmos. Meas. Tech., 9, 4181-4255, 2016.

Particle linear depolarisation ratio δ_p , PLDR

$$\delta_p = \frac{(1+\delta_m)\delta_v R - (1+\delta_v)\delta_m}{(1+\delta_m)R - (1+\delta_v)} = \delta_v \frac{\frac{1+\delta_m}{1+\delta_v} R - \frac{\delta_m}{\delta_v}}{\frac{1+\delta_m}{1+\delta_v} R - 1}$$

$$R = \frac{\beta_m + \beta_p}{\beta_m}$$

δ_m	molecular linear depolarisation ratio MLDR (Rayleigh)
δ_v	volume linear depolarisation ratio VLDR
δ_p	particle linear depolarisation ratio PLDR
β_m	molecular backscatter coefficient (Rayleigh)
β_p	particle backscatter coefficient
R	backscatter ratio

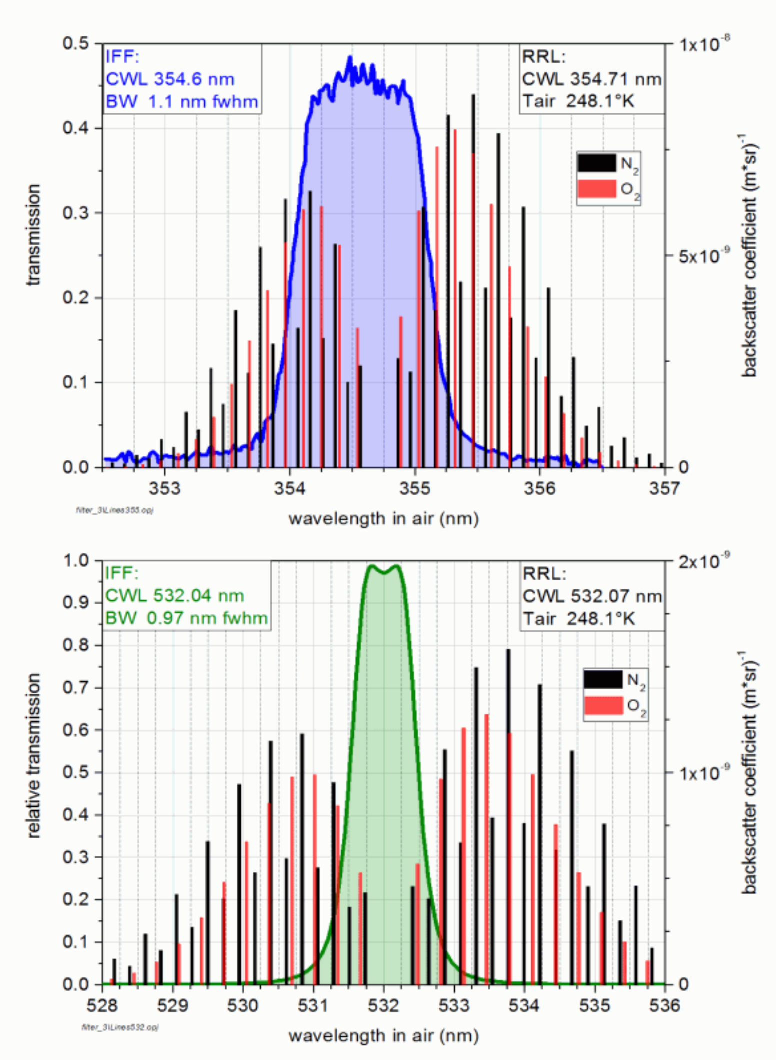
Biele, J.; Beyerle, G. & Baumgarten, G.; Polarization Lidar: Correction of instrumental effects, Opt. Express, OSA, 2000, 7, 427-435

Particle linear depolarisation ratio δ_p , PLDR

$$R \to 1 \Rightarrow \begin{cases} \frac{1+\delta_m}{1+\delta_v} R \to 1 \\ \frac{\delta_m}{\delta_v} \to 1 \end{cases} \Rightarrow \qquad \delta_p = \delta_v \frac{\frac{1+\delta_m}{1+\delta_v} R - \frac{\delta_m}{\delta_v}}{\frac{1+\delta_m}{1+\delta_v} R - 1} \to \frac{0}{0}$$

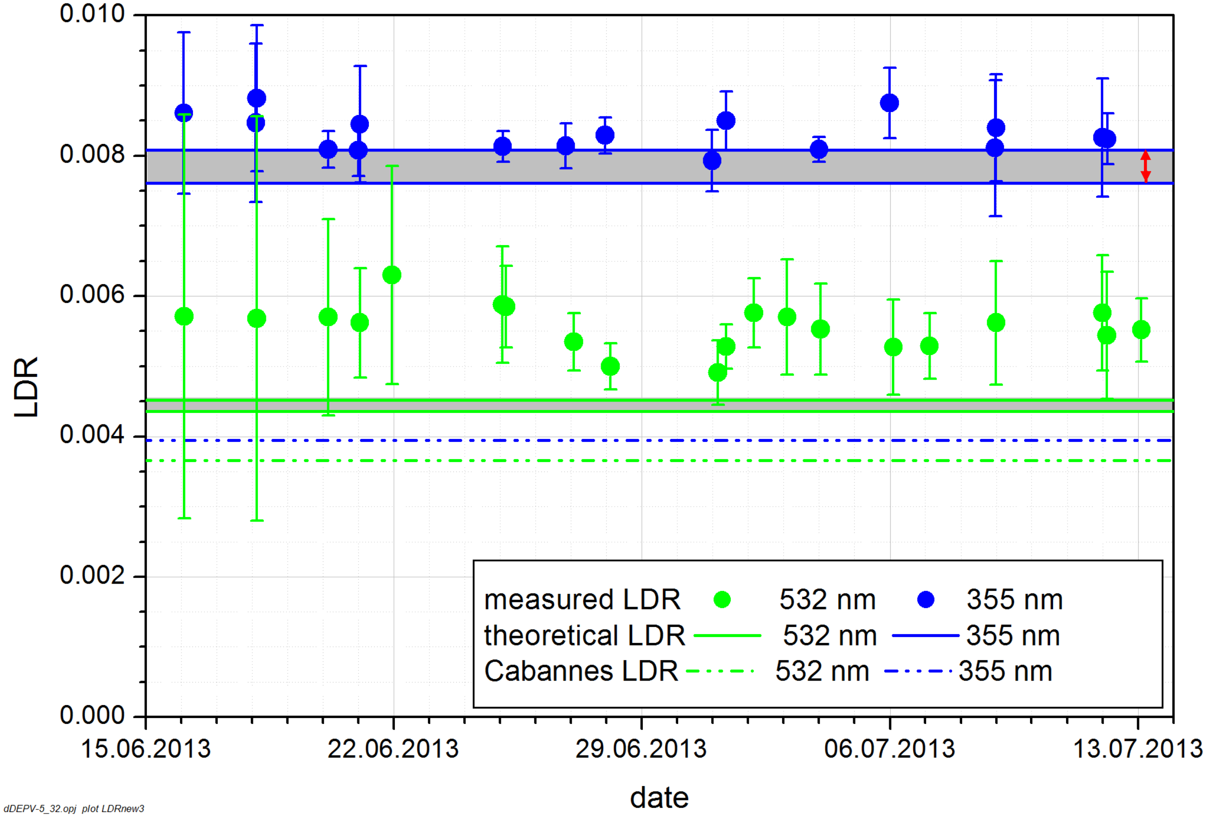
for small backscatter ratios R (little aerosol)
the exact value of δm *becomes very important*

measured / theoretical molecular LDR – POLIS-6 – 355 / 532 nm –



uncertain theoretical LDR due to uncertainties in

- laser wavelength
- Rot. Raman Lines in interference filter bandwidth



➡ Laser polarisation must be cleaner than deviation of measurements from theory

source: Freudenthaler et al., 27th ILRC 2015, Accuracy of linear depolaristion ratios in clear air ranges measured with POLIS-6 at 355 and 532 nm. http://dx.doi.org/10.1051/epjconf/201611925013

Rayleigh calibration: molecular linear depolarisation ratio (mLDR, RLDR)

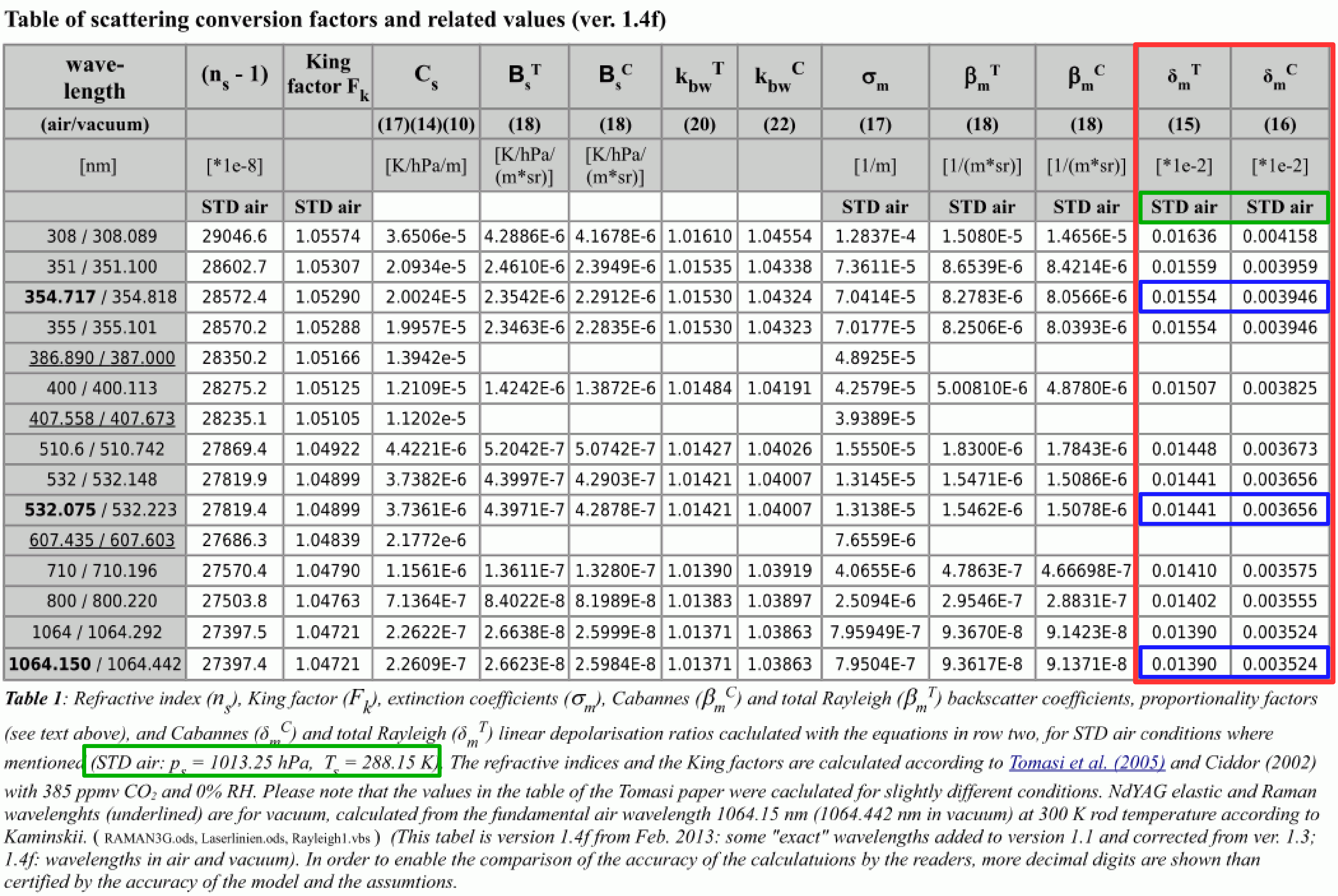


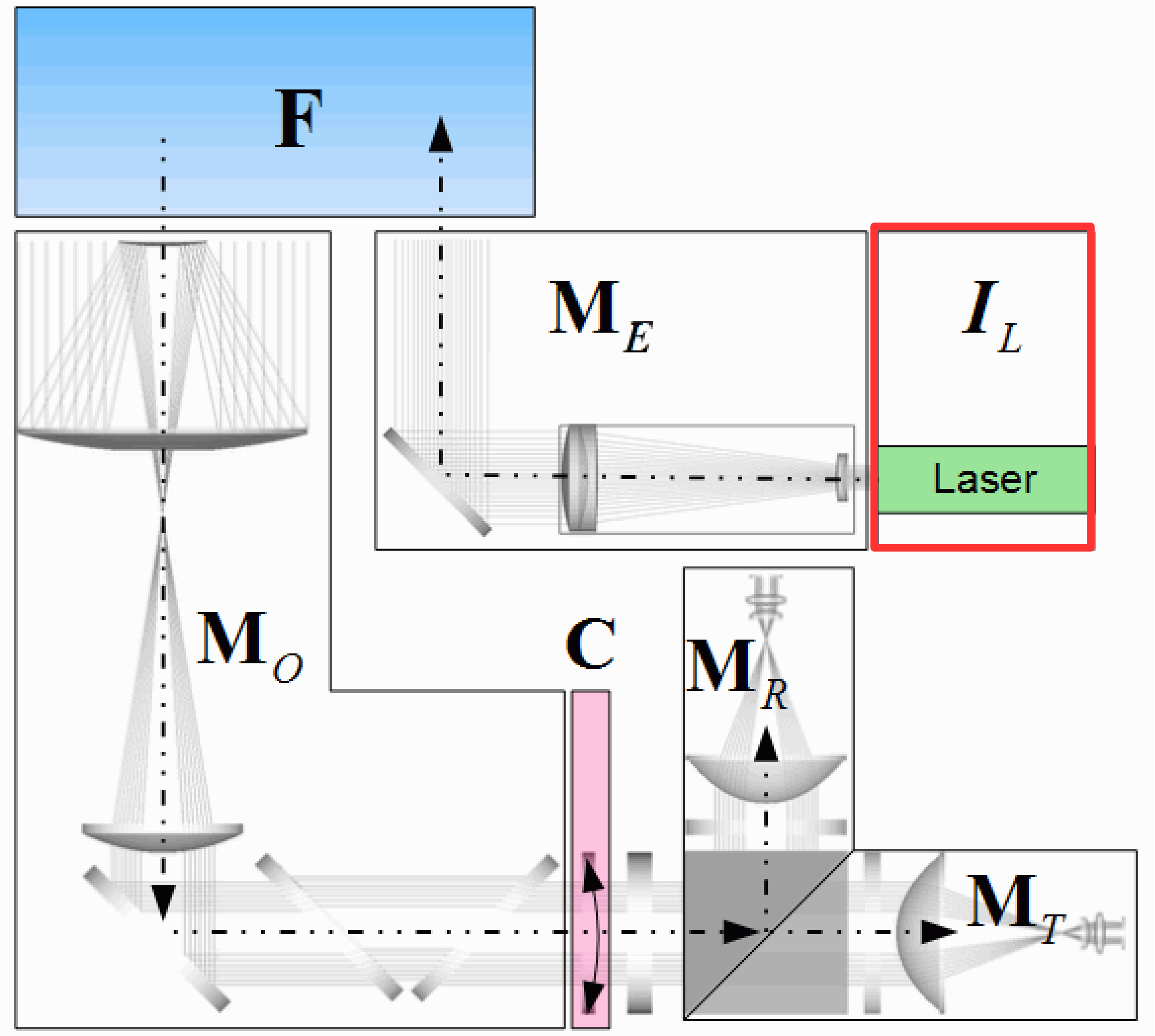
Table of scattering conversion factors and related values (ver. 1.4f)

wave-length	$(n_s - 1)$	King factor F_k	C_s	B_s^T	B_s^C	k_{bw}^T	k_{bw}^C	σ_m	β_m^T	β_m^C	δ_m^T	δ_m^C
(air/vacuum)			(17)(14)(10)	(18)	(18)	(20)	(22)	(17)	(18)	(18)	(15)	(16)
[nm]	[*1e-8]		[K/hPa/m]	[K/hPa/(m*sr)]	[K/hPa/(m*sr)]			[1/m]	[1/(m*sr)]	[1/(m*sr)]	[*1e-2]	[*1e-2]
	STD air	STD air						STD air	STD air	STD air	STD air	STD air
308 / 308.089	29046.6	1.05574	3.6506e-5	4.2886E-6	4.1678E-6	1.01610	1.04554	1.2837E-4	1.5080E-5	1.4656E-5	0.01636	0.004158
351 / 351.100	28602.7	1.05307	2.0934e-5	2.4610E-6	2.3949E-6	1.01535	1.04338	7.3611E-5	8.6539E-6	8.4214E-6	0.01559	0.003959
354.717 / 354.818	28572.4	1.05290	2.0024E-5	2.3542E-6	2.2912E-6	1.01530	1.04324	7.0414E-5	8.2783E-6	8.0566E-6	0.01554	0.003946
355 / 355.101	28570.2	1.05288	1.9957E-5	2.3463E-6	2.2835E-6	1.01530	1.04323	7.0177E-5	8.2506E-6	8.0393E-6	0.01554	0.003946
386.890 / 387.000	28350.2	1.05166	1.3942e-5					4.8925E-5				
400 / 400.113	28275.2	1.05125	1.2109E-5	1.4242E-6	1.3872E-6	1.01484	1.04191	4.2579E-5	5.00810E-6	4.8780E-6	0.01507	0.003825
407.558 / 407.673	28235.1	1.05105	1.1202e-5					3.9389E-5				
510.6 / 510.742	27869.4	1.04922	4.4221E-6	5.2042E-7	5.0742E-7	1.01427	1.04026	1.5550E-5	1.8300E-6	1.7843E-6	0.01448	0.003673
532 / 532.148	27819.9	1.04899	3.7382E-6	4.3997E-7	4.2903E-7	1.01421	1.04007	1.3145E-5	1.5471E-6	1.5086E-6	0.01441	0.003656
532.075 / 532.223	27819.4	1.04899	3.7361E-6	4.3971E-7	4.2878E-7	1.01421	1.04007	1.3138E-5	1.5462E-6	1.5078E-6	0.01441	0.003656
607.435 / 607.603	27686.3	1.04839	2.1772e-6					7.6559E-6				
710 / 710.196	27570.4	1.04790	1.1561E-6	1.3611E-7	1.3280E-7	1.01390	1.03919	4.0655E-6	4.7863E-7	4.66698E-7	0.01410	0.003575
800 / 800.220	27503.8	1.04763	7.1364E-7	8.4022E-8	8.1989E-8	1.01383	1.03897	2.5094E-6	2.9546E-7	2.8831E-7	0.01402	0.003555
1064 / 1064.292	27397.5	1.04721	2.2622E-7	2.6638E-8	2.5999E-8	1.01371	1.03863	7.95949E-7	9.3670E-8	9.1423E-8	0.01390	0.003524
1064.150 / 1064.442	27397.4	1.04721	2.2609E-7	2.6623E-8	2.5984E-8	1.01371	1.03863	7.9504E-7	9.3617E-8	9.1371E-8	0.01390	0.003524

***Table 1**: Refractive index (n_s), King factor (F_k), extinction coefficients (σ_m), Cabannes (β_m^C) and total Rayleigh (β_m^T) backscatter coefficients, proportionality factors (see text above), and Cabannes (δ_m^C) and total Rayleigh (δ_m^T) linear depolarisation ratios caclulated with the equations in row two, for STD air conditions where mentioned (STD air: p_s = 1013.25 hPa, T_s = 288.15 K). The refractive indices and the King factors are calculated according to Tomasi et al. (2005) and Ciddor (2002) with 385 ppmv CO_2 and 0% RH. Please note that the values in the table of the Tomasi paper were caclulated for slightly different conditions. NdYAG elastic and Raman wavelenghts (underlined) are for vacuum, calculated from the fundamental air wavelength 1064.15 nm (1064.442 nm in vacuum) at 300 K rod temperature according to Kaminskii.* (RAMAN3G.ods, Laserlinien.ods, Rayleigh1.vbs) *(This tabel is version 1.4f from Feb. 2013: some "exact" wavelengths added to version 1.1 and corrected from ver. 1.3; 1.4f: wavelengths in air and vacuum). In order to enable the comparison of the accuracy of the calculatuions by the readers, more decimal digits are shown than certified by the accuracy of the model and the assumtions.*

http://www.meteo.physik.uni-muenchen.de/~stlidar/earlinet_asos/Rayleigh_scattering/Rayleigh_coefficients.pdf

Laser - polarisation



laser

divergence

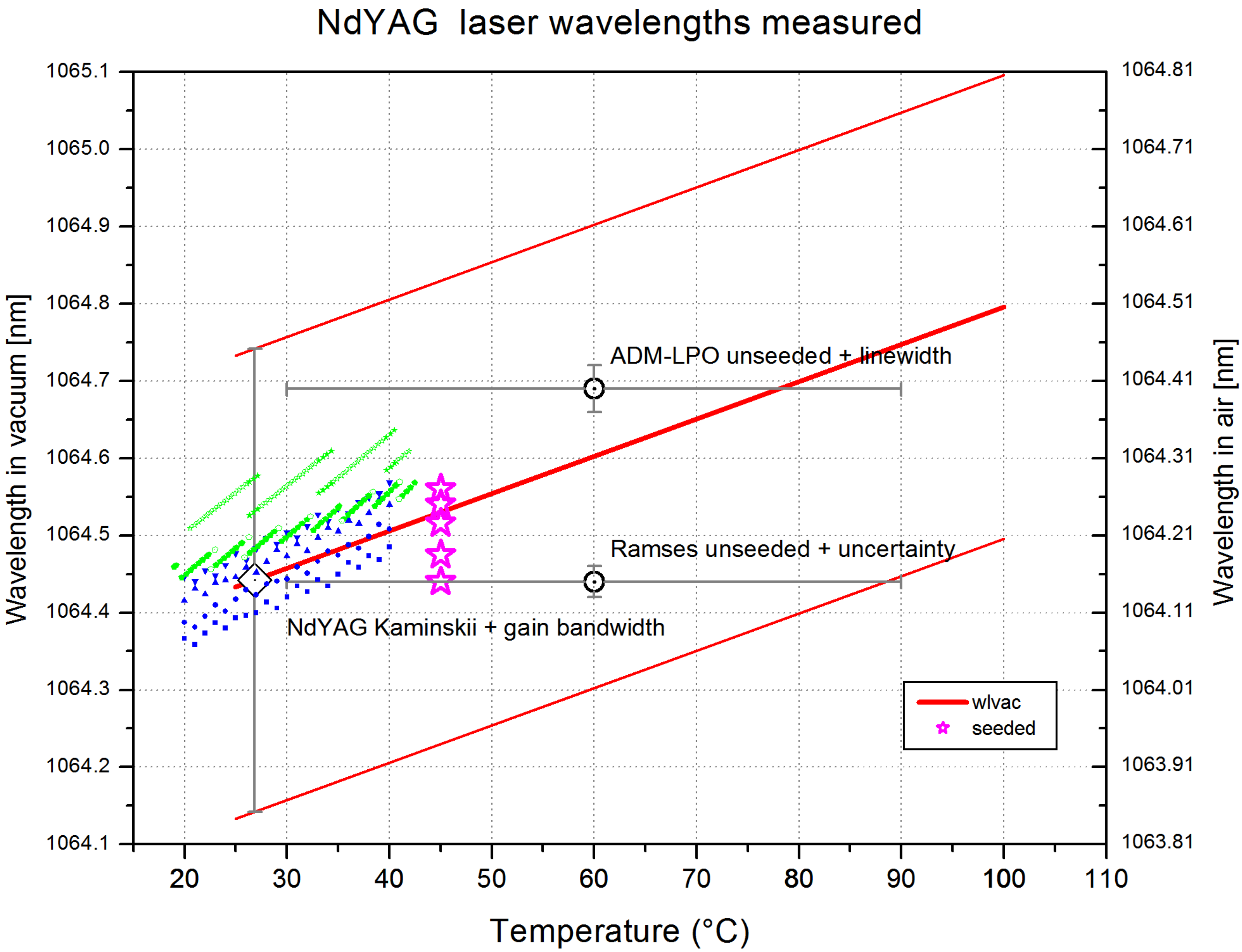
pointing / jitter

wavelength

polarisation
  state of polarisation
  orientation

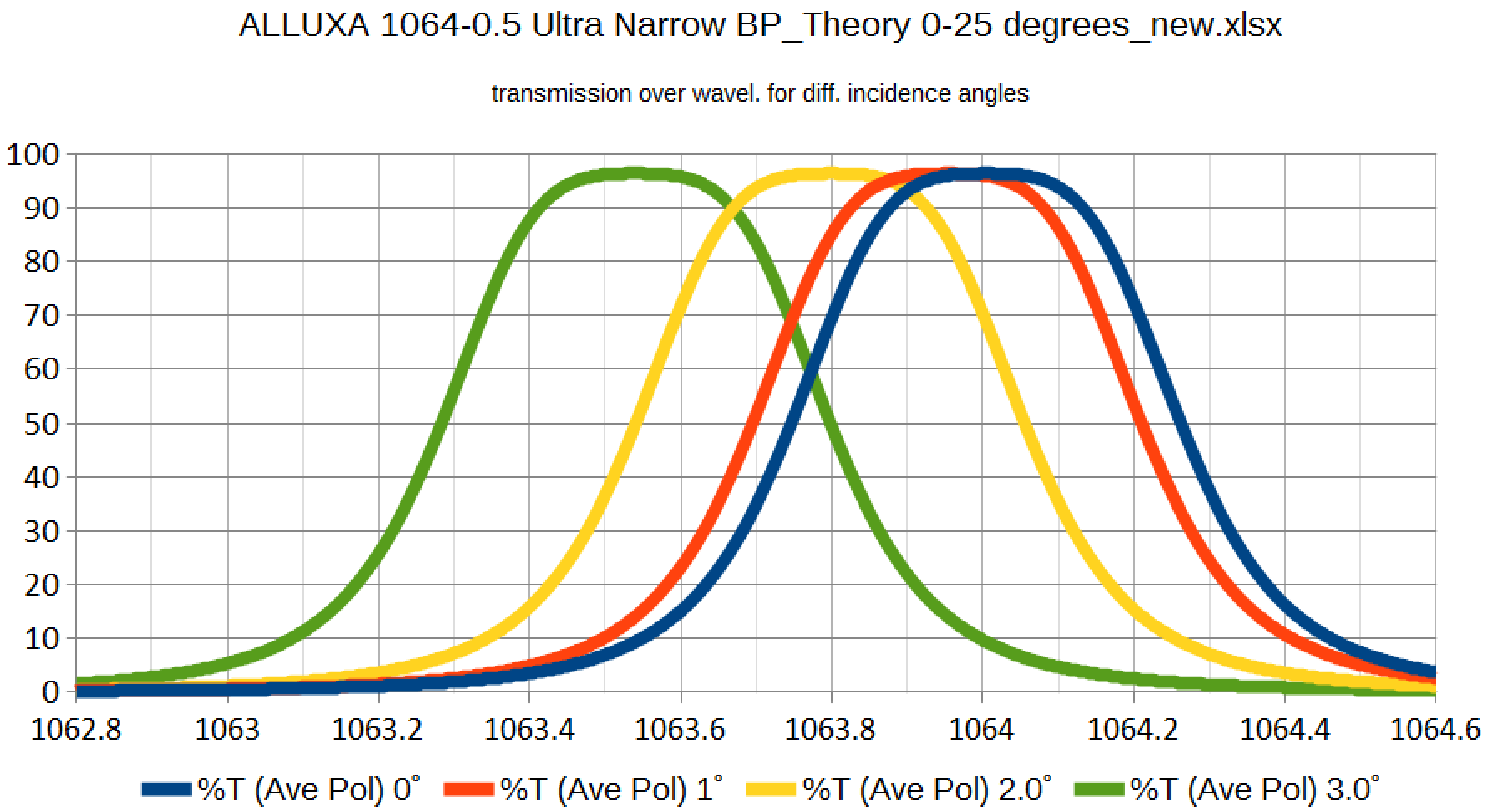
temporal / thermal stability

Laser – wavelengths measured and theory

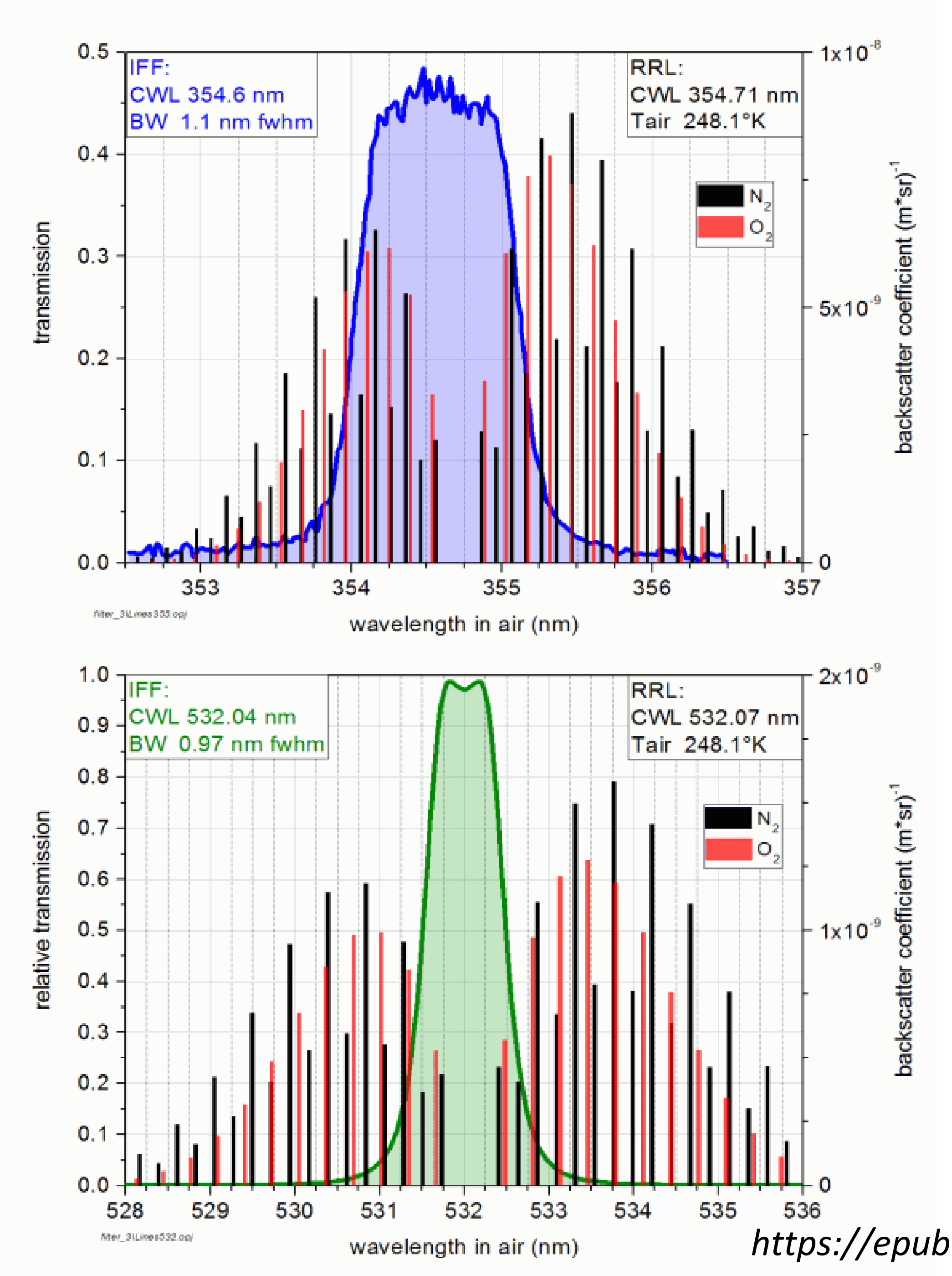


source: http://www.meteo.physik.uni-muenchen.de/~stlidar/earlinet_asos/Rayleigh_scattering/Rayleigh_coefficients.pdf

Laser – wavelength – interference filter - centre wavelength over incidence angle



Laser – wavelength – interference filter bandwidth – rotational Raman lines (RRL)



laser

divergence

pointing / jitter

wavelength

polarisation
  state of polarisation
  orientation

temporal / thermal stability

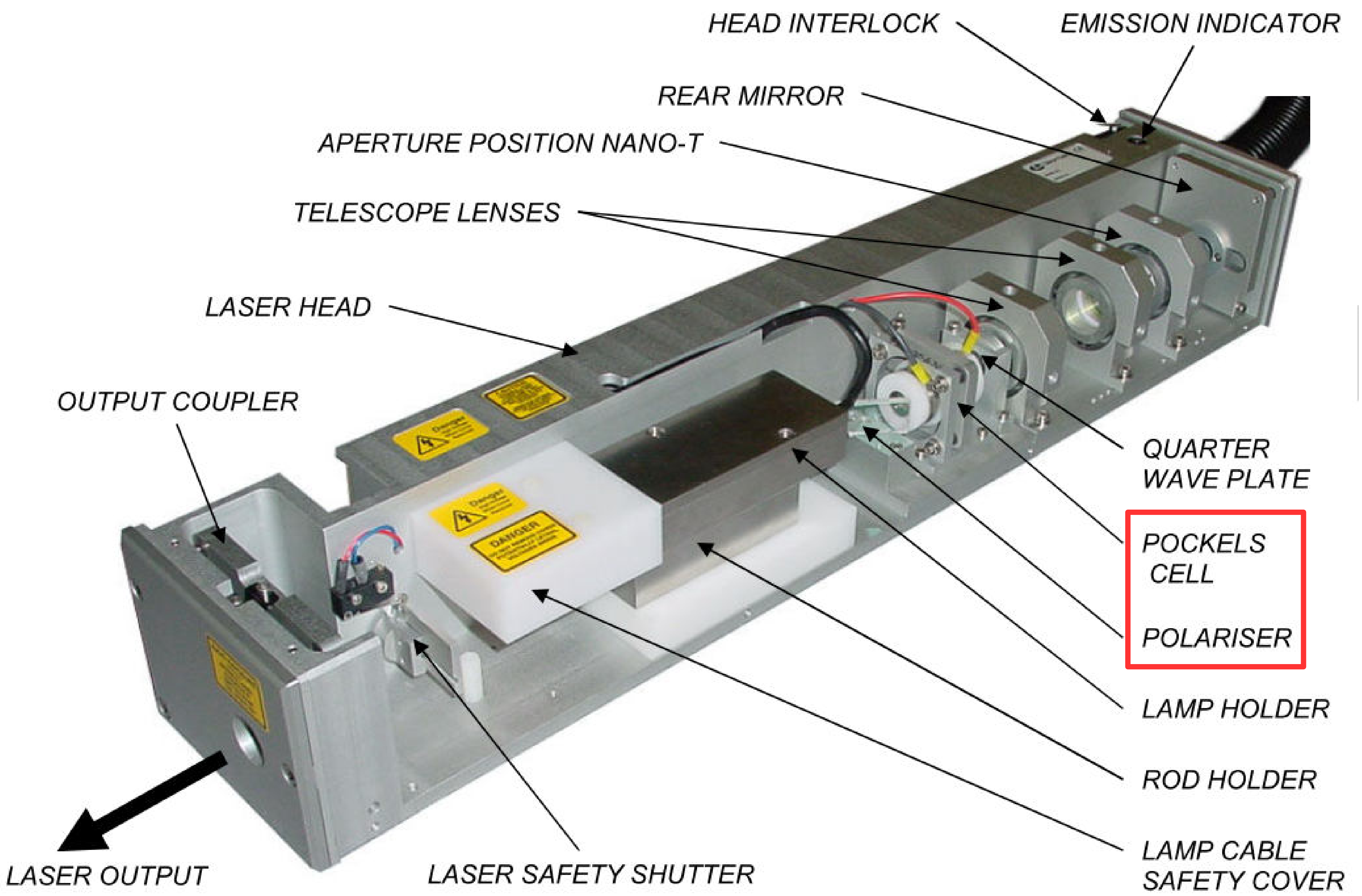
https://epub.ub.uni-muenchen.de/24942/index.html

theoretically measured molecular LDR

The combination of
   laser wavelength
   IF-filter center wavelength
   IF-filter bandwidth
   IF-filter incidence angle

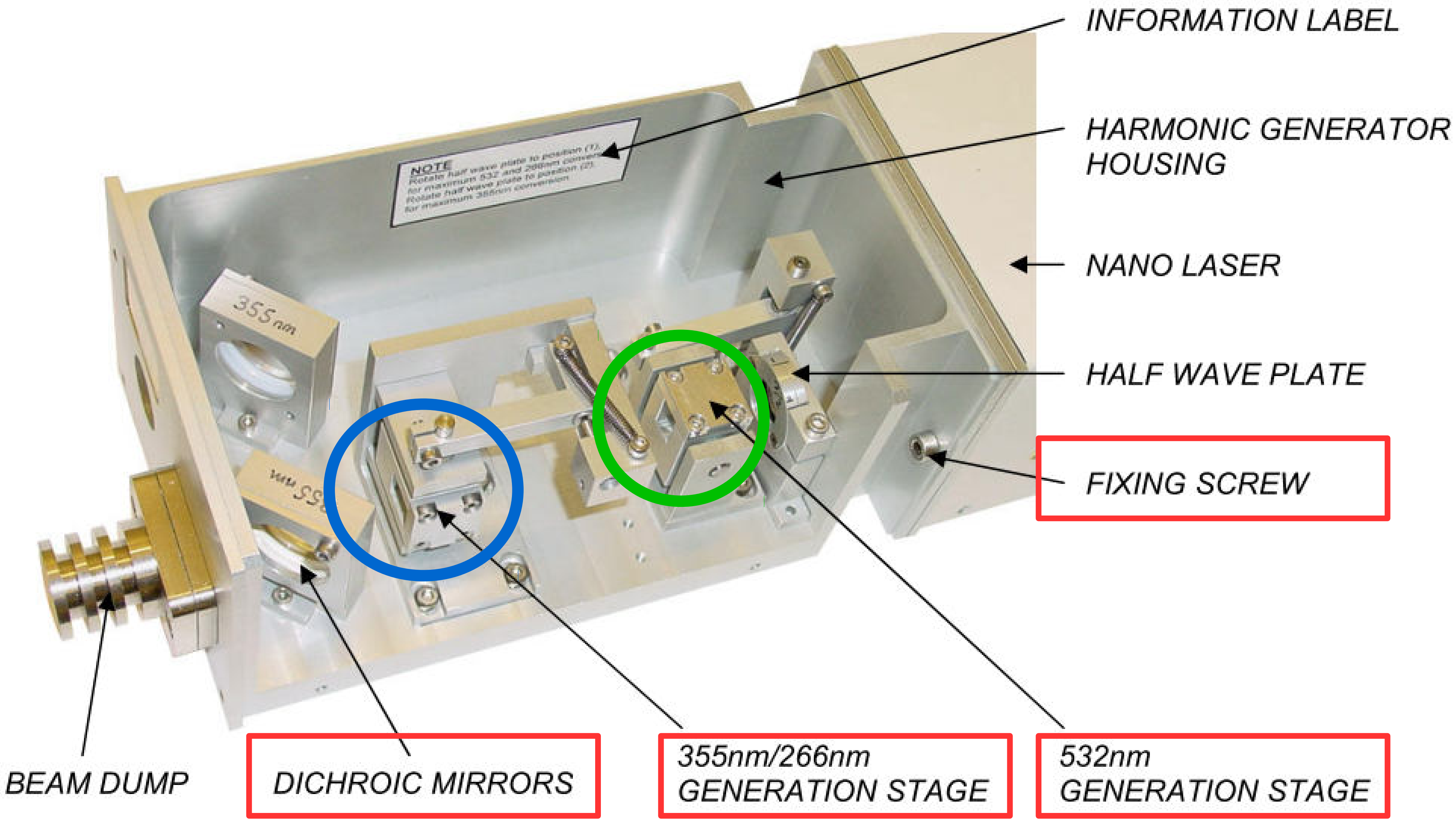
determines the theoretically measured molecular LDR

Laser – polarisation - orientation



source: http://www.litronlasers.com/pages/nano_series.html

Laser – polarisation – orientation – SHG / THG / beam separation



source: http://www.litronlasers.com/pages/nano_series.html

Laser - polarisation

Linear polariser in the resonator should clean the 1064 polarisation,

- but NdYAG rod birefringence can decrease
  the DOLP (<u>D</u>egree <u>O</u>f <u>L</u>inear <u>P</u>olarisation) of 1064 nm

- SHG and THG only convert light in certain polarisation planes
  => DOLP of 355 should be very clean
  => DOLP of 532 could be decreased by THG
  => DOLP of the residual 1064 less than original

- Harmonic beam separators can decrease the DOLP

see also: https://en.wikipedia.org/wiki/Second-harmonic_generation
https://www.rp-photonics.com/frequency_doubling.html

Laser – polarisation – 532 nm Surelite II, Continuum

Giuseppe d'Amico 2006:

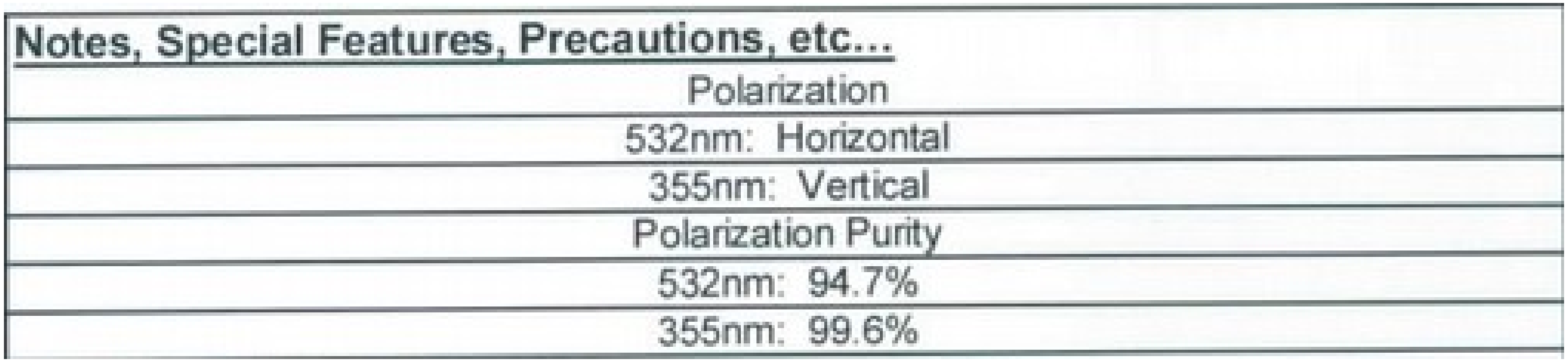
Results from Continuum USA.

The measurements were done using a laser Surelite II - 10Hz with a SHG crystal of Type I and II. (so at 532 nm; Giuseppe)

*Using both crystals, the energy of the vertical component of polarization was 3 Watts and the energy of the horizontal component was 2 mWatts corresponding to **polarization purity of about 99.93%**.*

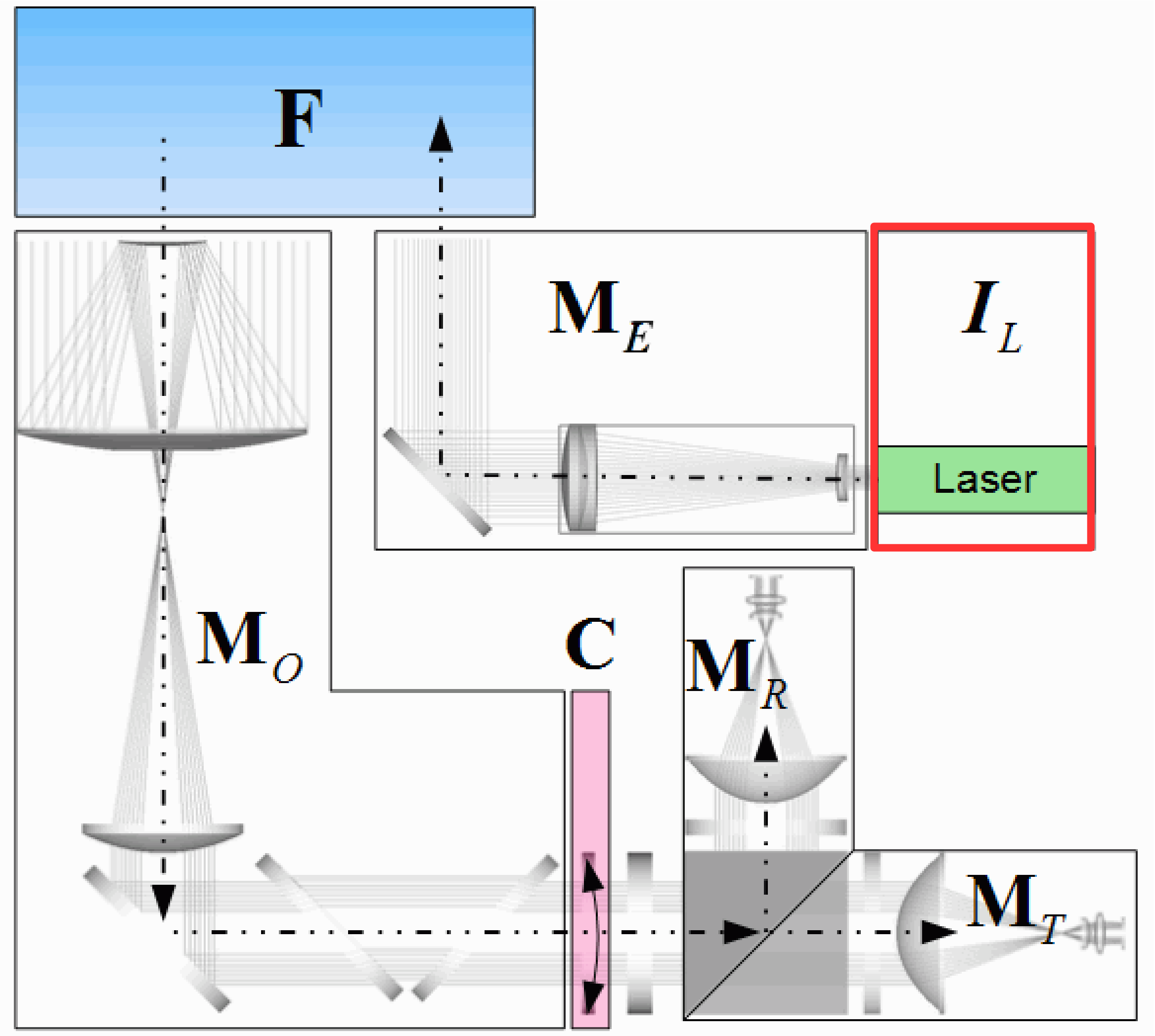
=> *LDR = 0.002 / 3 = 0.00067*

Laser – Quantel CFR 200 ? Polarisation Purity



Notes, Special Features, Precautions, etc...
Polarization
532nm: Horizontal
355nm: Vertical
Polarization Purity
532nm: 94.7%
355nm: 99.6%

Laser – temporal stability



laser

divergence

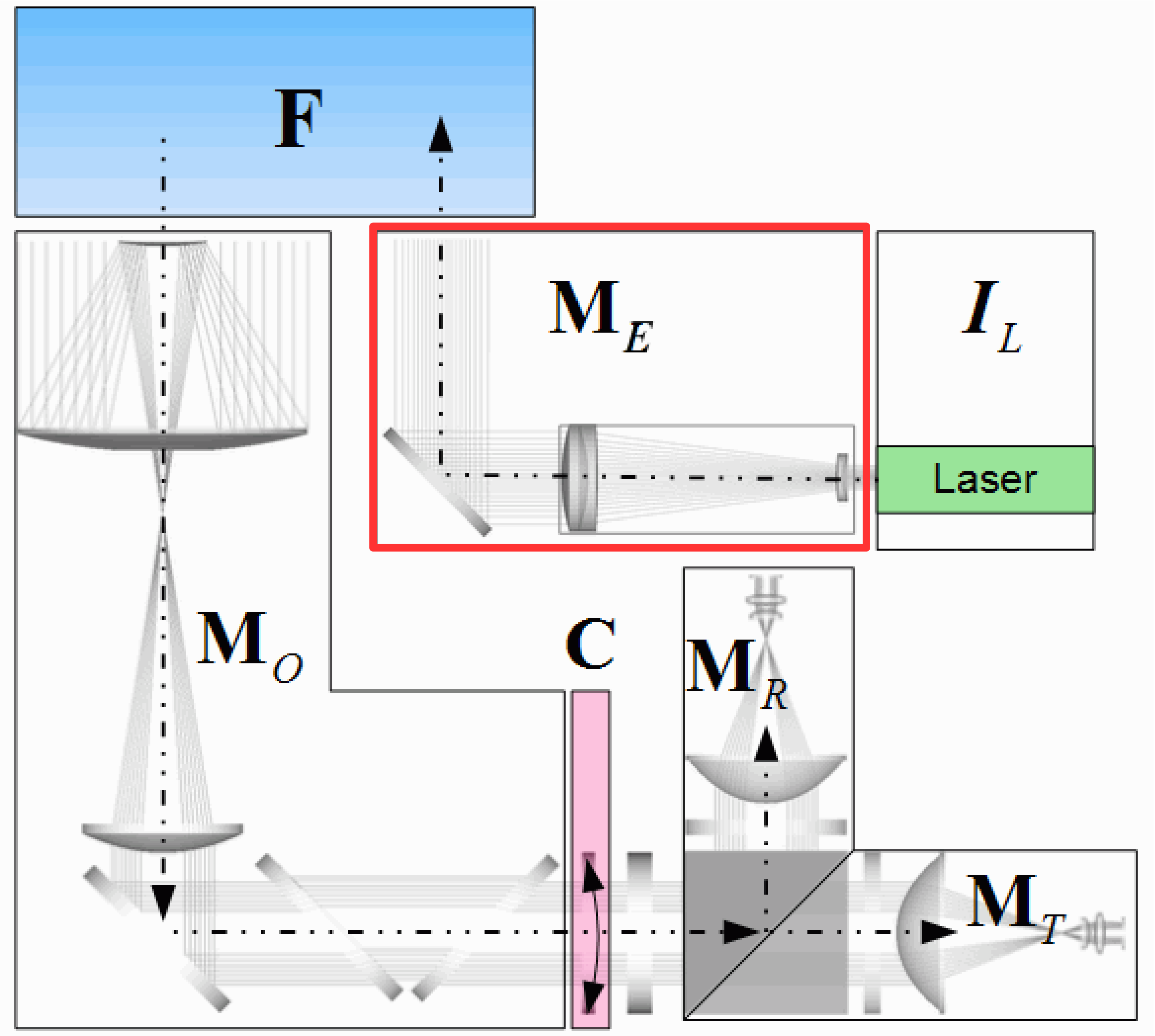
pointing / jitter

wavelength

polarisation
state of polarisation
orientation

temporal / thermal stability

Typical lidar setup – emitter and steering optics



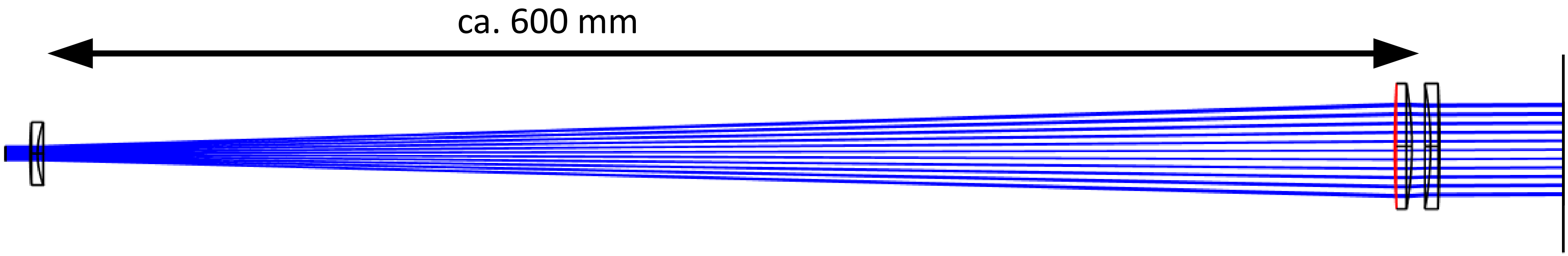
emitter and steering optics

wavelength dependence
  focal length => divergence
  transmission
  polarisation
  birefringence

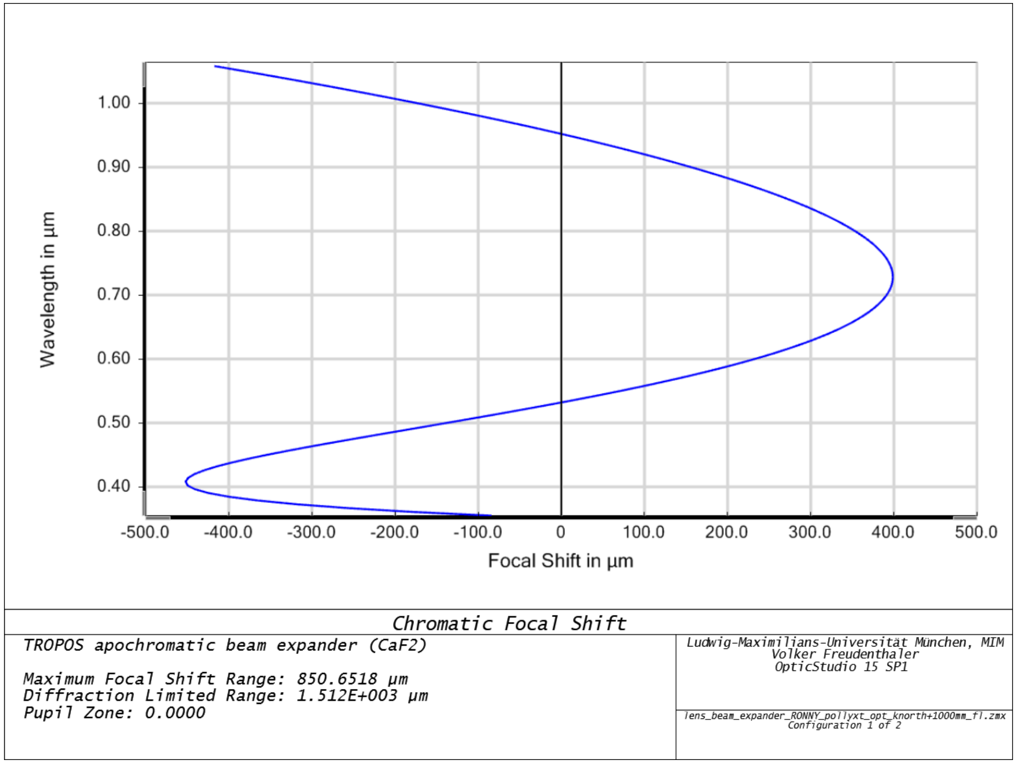
alignment
  accuracy
  stability
  alignment control

polarisation
  orientation
  flatness

Emitter optics – TROPOS beam expander 6.5x (with CaF2 lens) apochromat



Engelmann, R., et al., The Automated Multiwavelength Raman, Polarization, and Water-Vapor Lidar Polly XT : The NeXT Generation, AMT, 2016. http://www.atmos-meas-tech.net/9/1767/2016/amt-9-1767-2016.html

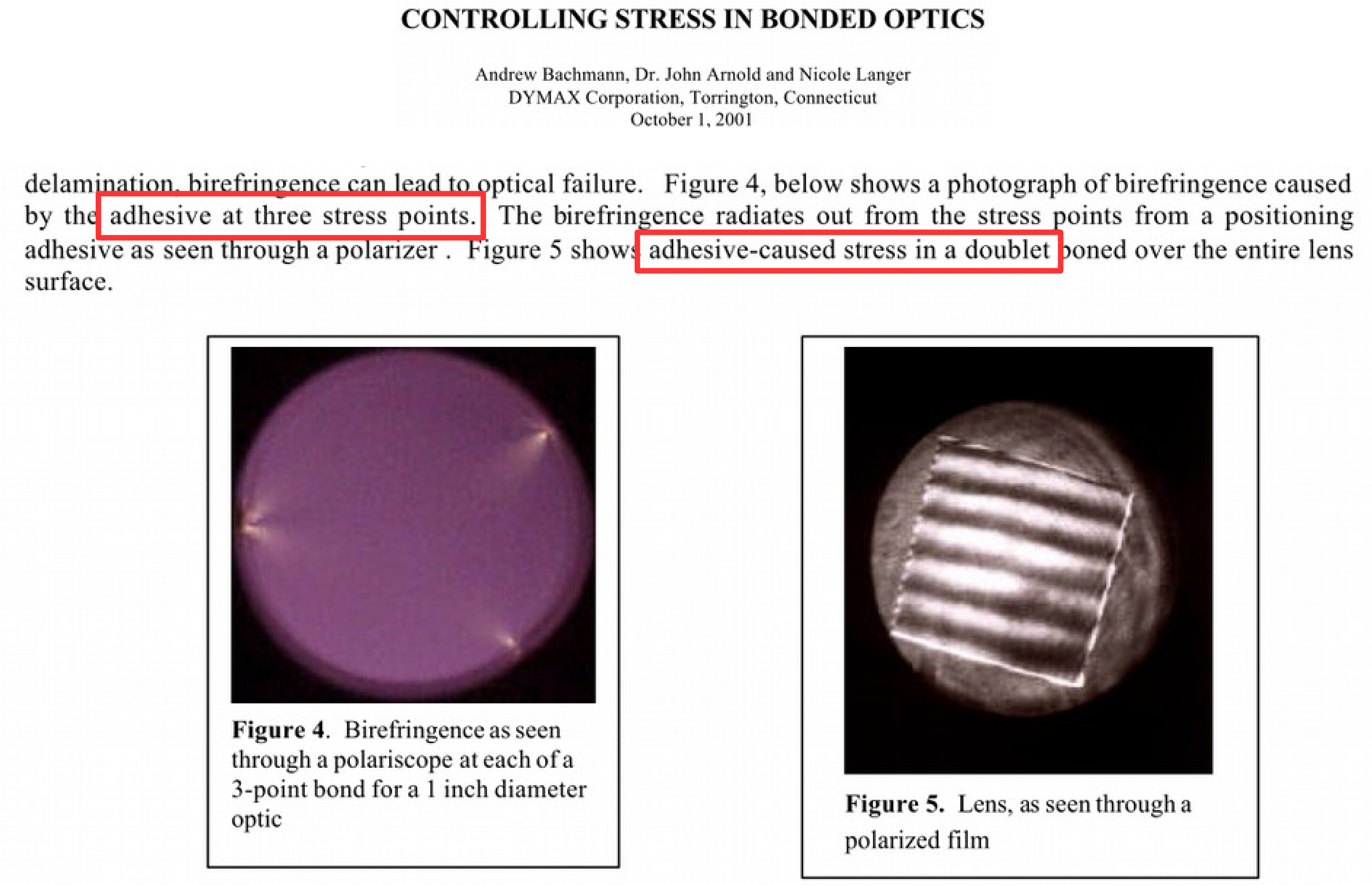


```
Chromatic Focal Shift
TROPOS apochromatic beam expander (CaF2)

Maximum Focal Shift Range: 850.6518 µm
Diffraction Limited Range: 1.512E+003 µm
Pupil Zone: 0.0000

Ludwig-Maximilians-Universität München, MIM
Volker Freudenthaler
OpticStudio 15 SP1

lens_beam_expander_RONNY_pollyxt_opt_knorth+1000mm_fl.zmx
Configuration 1 of 2
```

Big problem:
CaF2 and MgF2 lenses are birefringent

Other problems:
- residual wavelength dependence
- 3 lambda AR-coating
- glass solarisation

Stress birefringence



CONTROLLING STRESS IN BONDED OPTICS

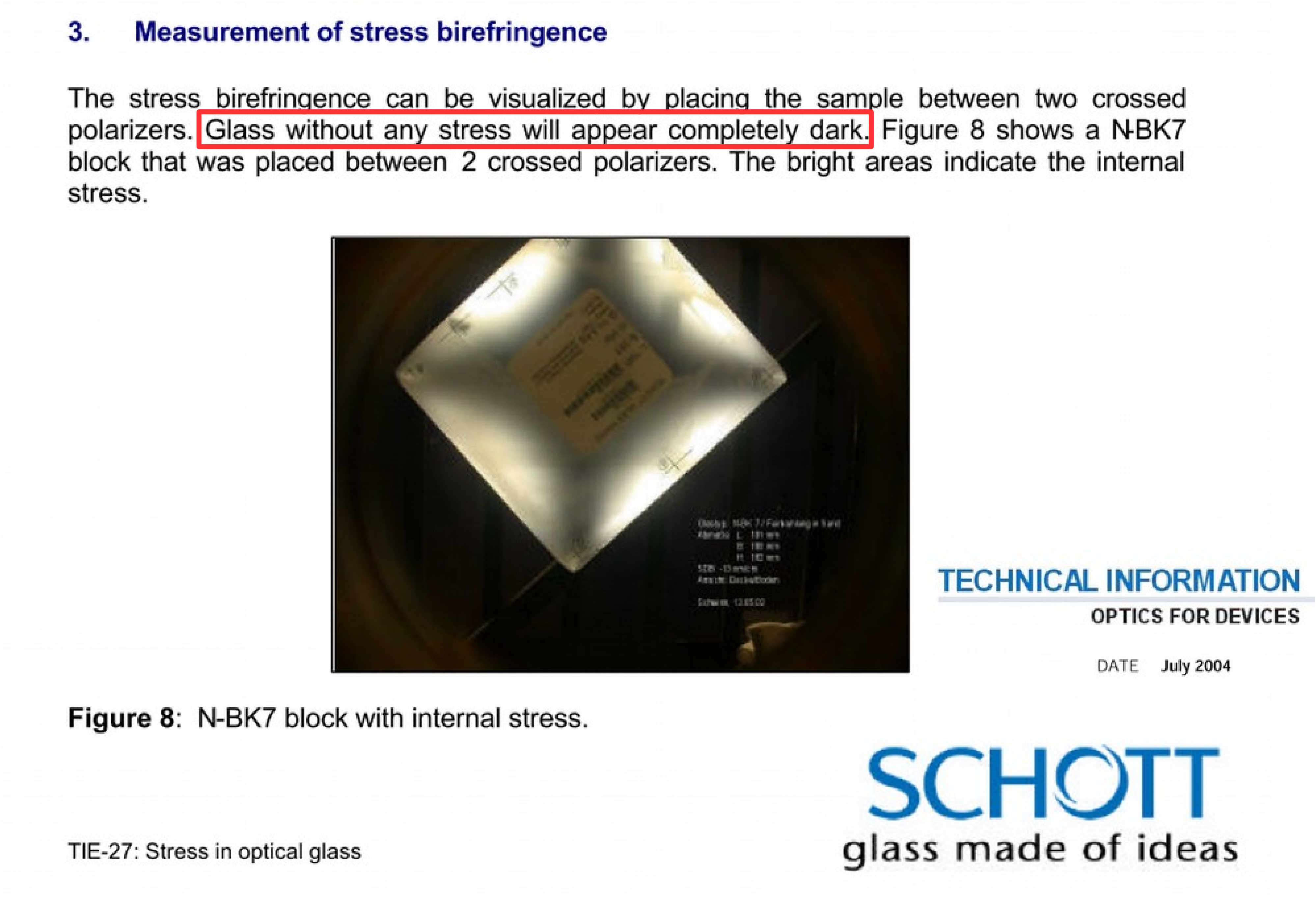
Andrew Bachmann, Dr. John Arnold and Nicole Langer
DYMAX Corporation, Torrington, Connecticut
October 1, 2001

delamination, birefringence can lead to optical failure. Figure 4, below shows a photograph of birefringence caused by the adhesive at three stress points. The birefringence radiates out from the stress points from a positioning adhesive as seen through a polarizer . Figure 5 shows adhesive-caused stress in a doublet boned over the entire lens surface.

Figure 4. Birefringence as seen through a polariscope at each of a 3-point bond for a 1 inch diameter optic

Figure 5. Lens, as seen through a polarized film

Stress birefringence



3. Measurement of stress birefringence

The stress birefringence can be visualized by placing the sample between two crossed polarizers. Glass without any stress will appear completely dark. Figure 8 shows a N-BK7 block that was placed between 2 crossed polarizers. The bright areas indicate the internal stress.

Figure 8: N-BK7 block with internal stress.

TECHNICAL INFORMATION
OPTICS FOR DEVICES

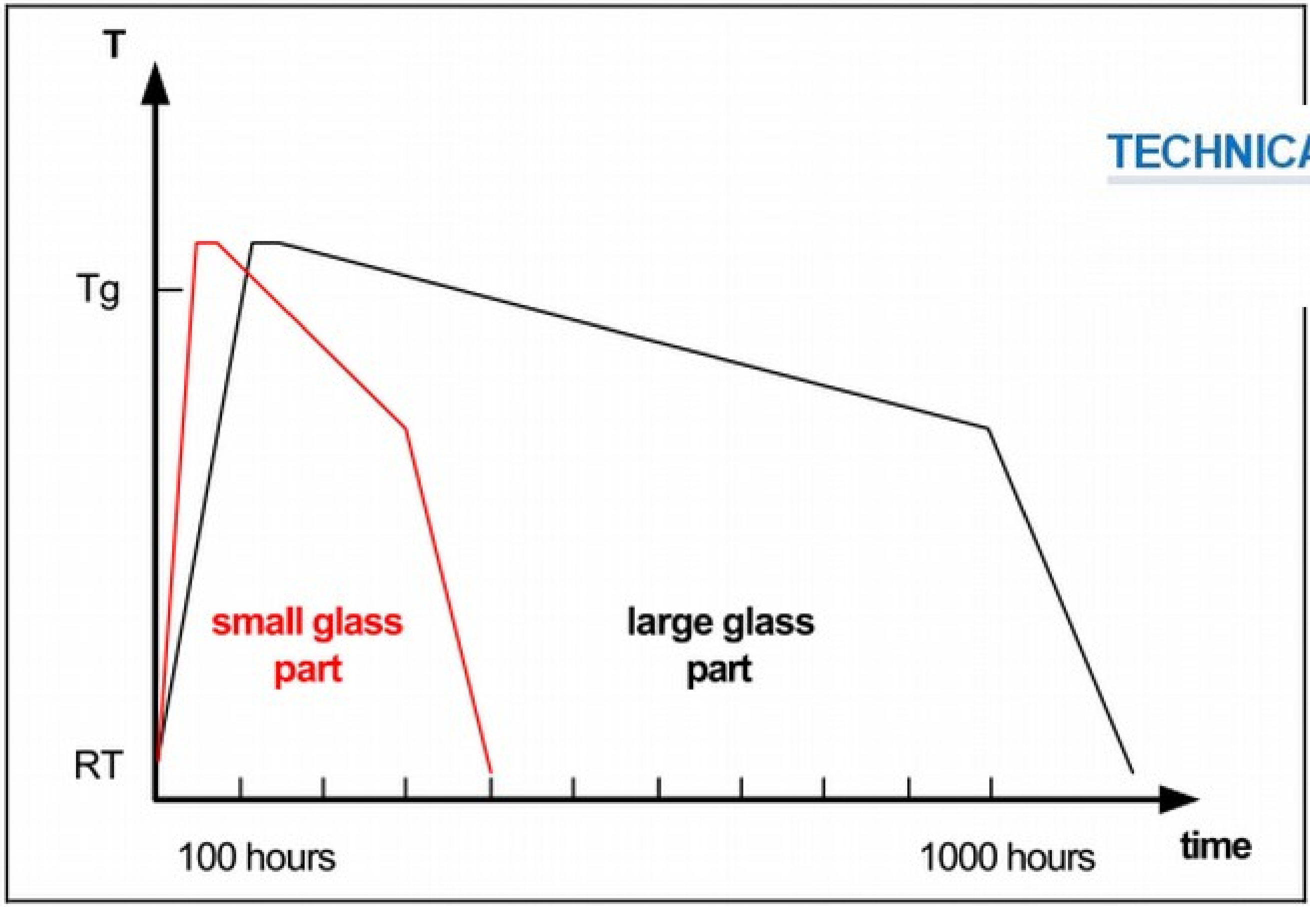
DATE July 2004

SCHOTT
glass made of ideas

TIE-27: Stress in optical glass

Stress birefringence

Figure 2 shows typical annealing temperature curves. In the first step the glass is heated up to a temperature above Tg. After keeping the temperature constant for a certain period of time the fine annealing starts. The glass is then cooled down very slowly to temperatures below Tg. At temperatures far below Tg the cooling rate will be increased. The annealing time strongly depends on the volume / bulkiness of the glass. Large sized glass parts need significantly longer annealing times.



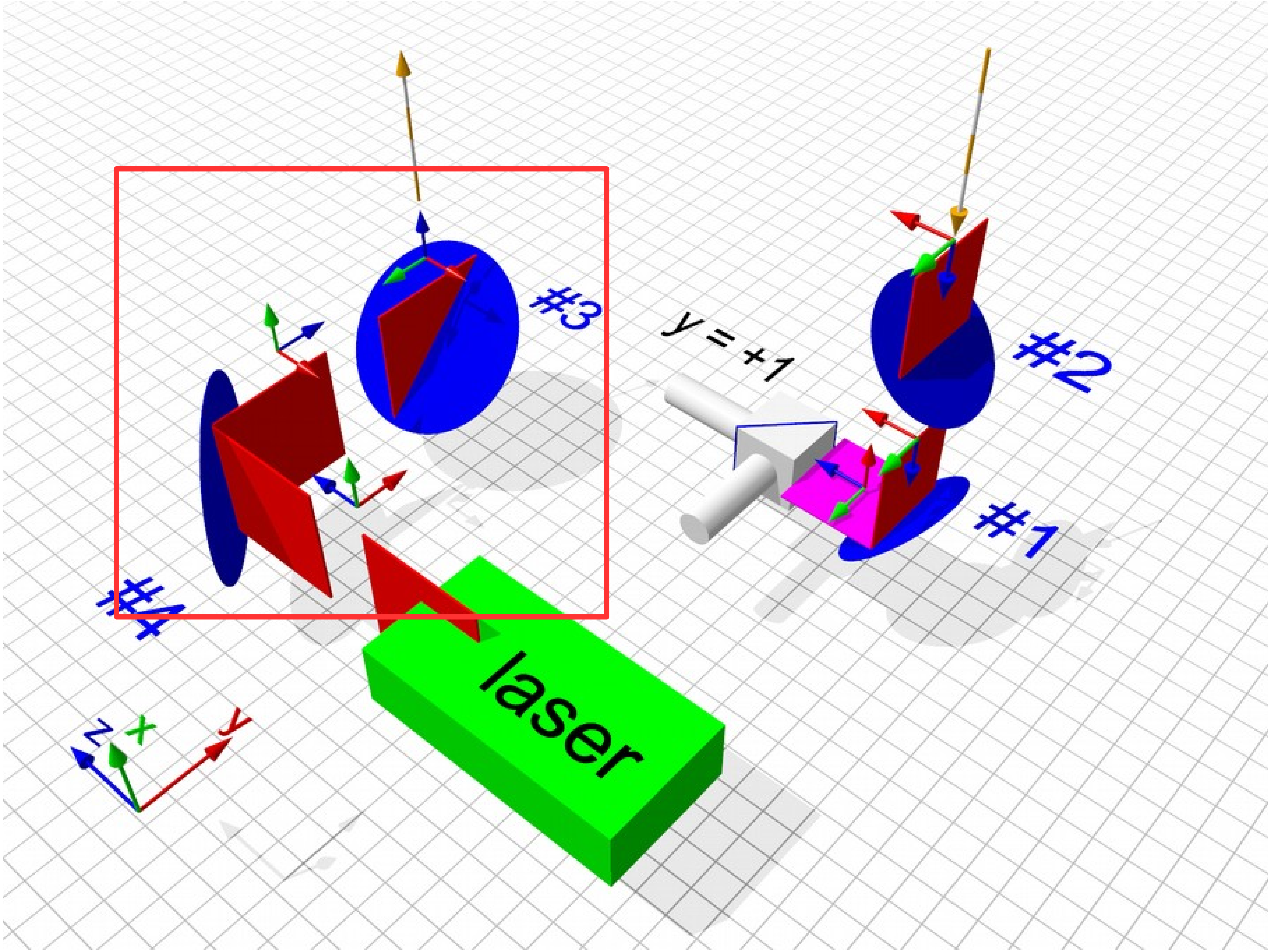
TECHNICAL INFORMATION

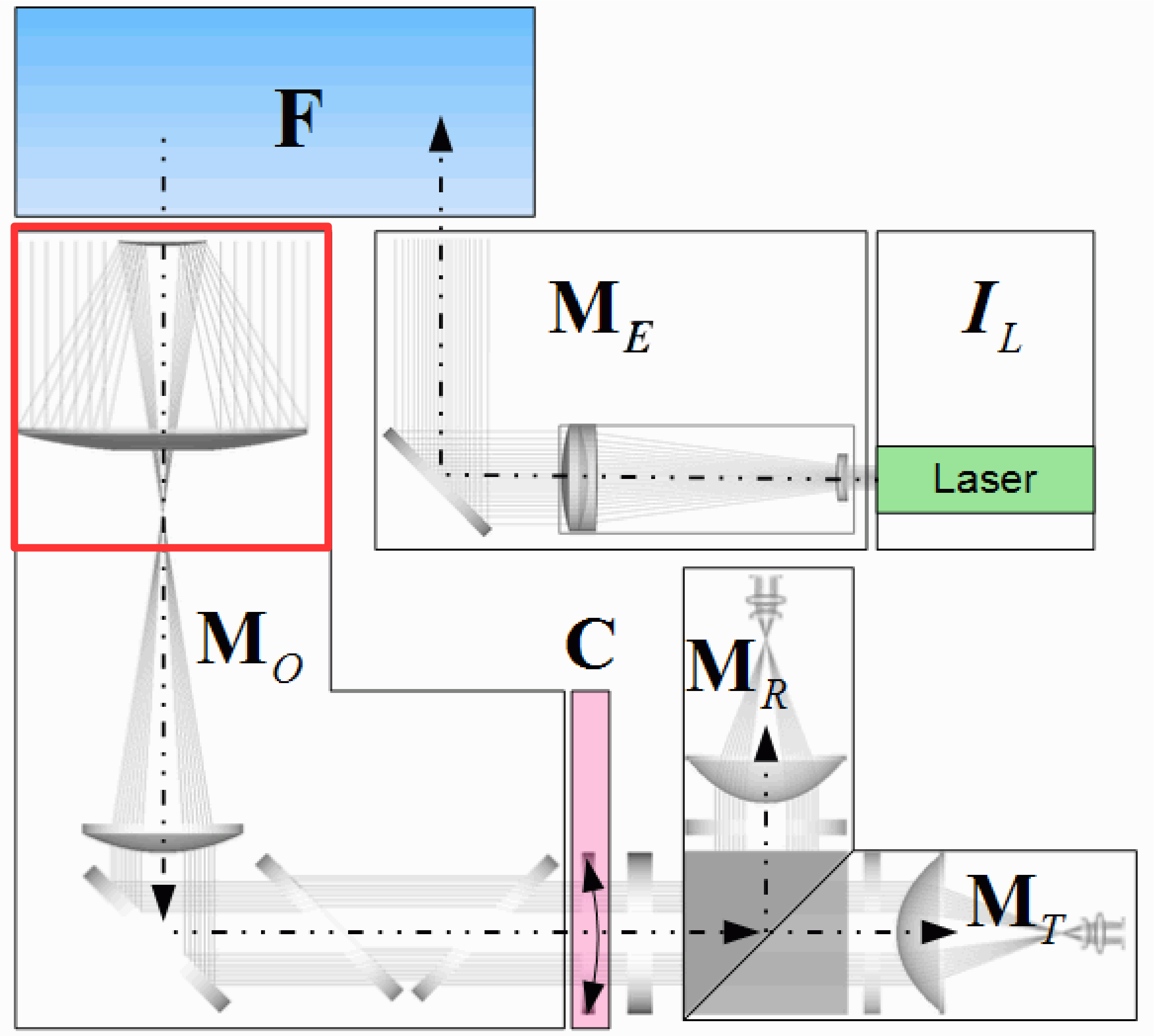
OPTICS FOR DEVICES

DATE July 2004

Figure 2: Schematic representation of the annealing temperature as a function of time.

Crossed steering mirrors compensate diattenuation and retardation perfectly





receiving optics
telescope

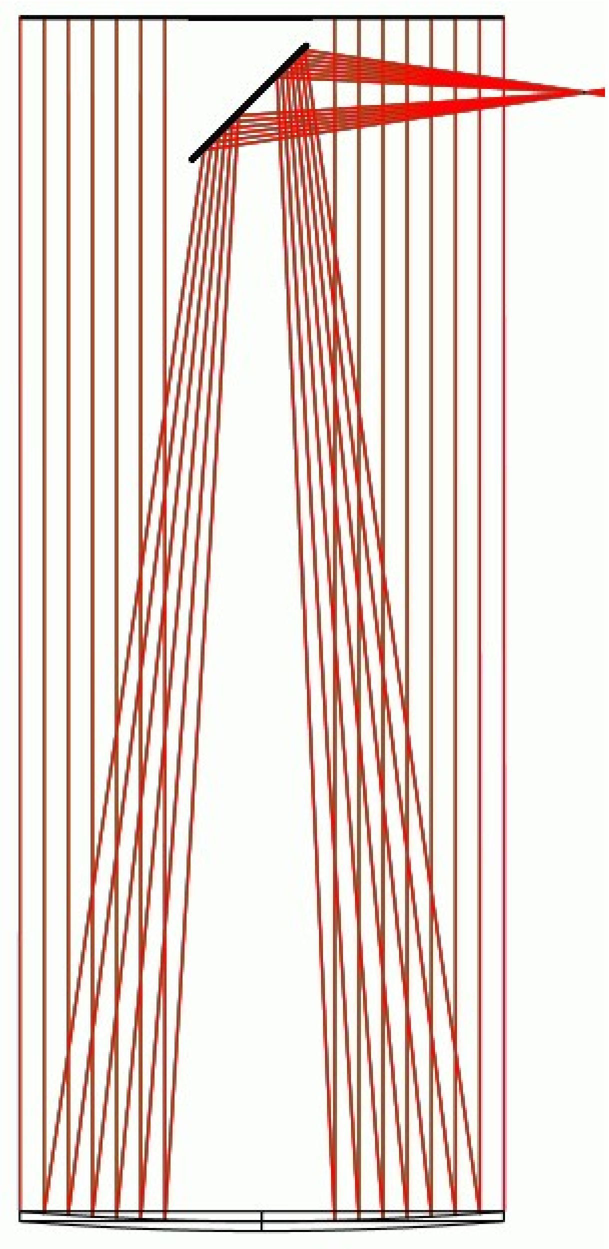
focal length, diameter

alignment stability

Newton telescope
90° mirror depolarisation

field of view

Newton 90° mirror



receiving optics
telescope

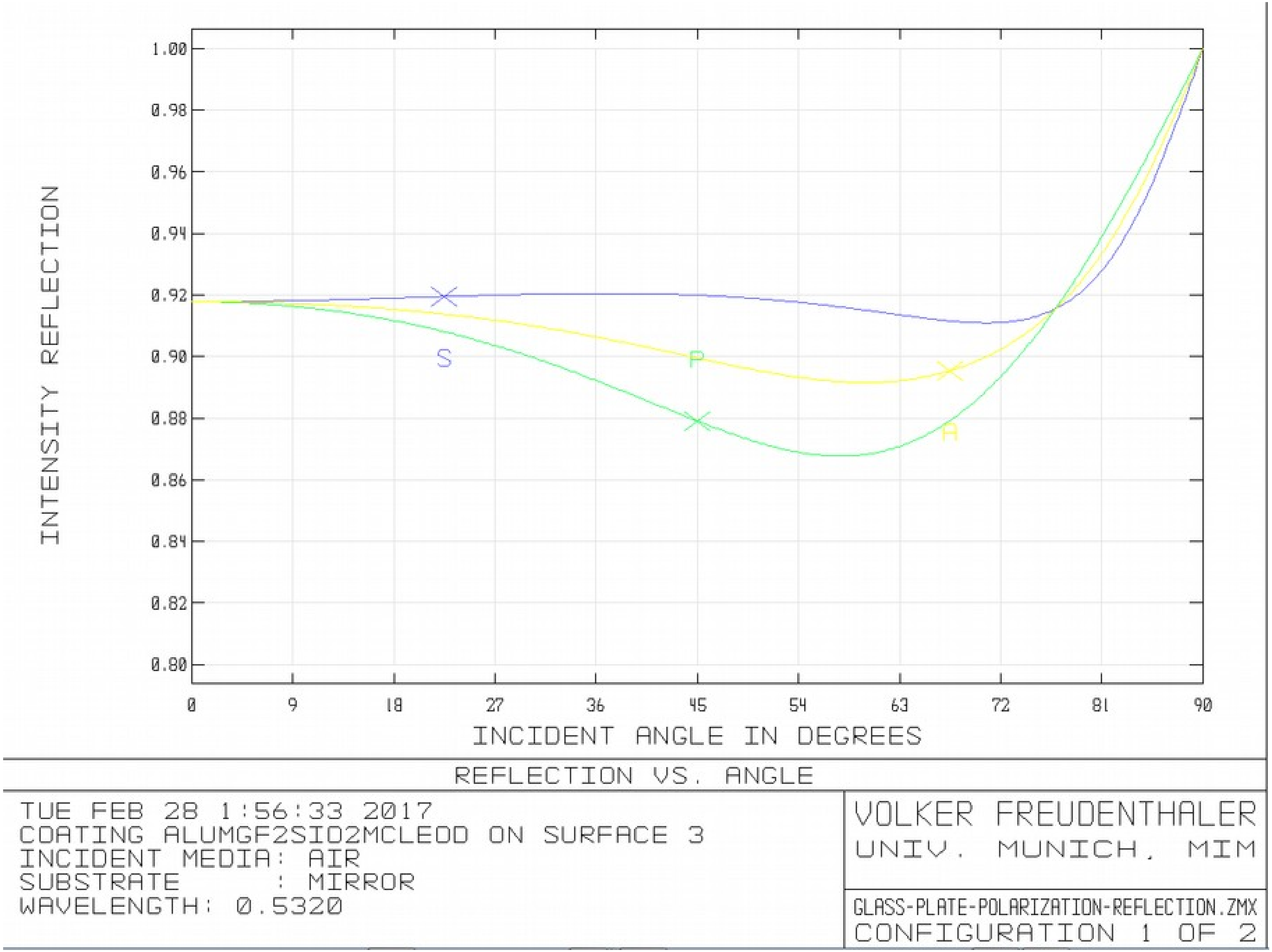
focal length, diameter

alignment stability

Newton telescope
  90° mirror depolarisation

field of view

Newton 90° mirror, Aluminium + MgF2 + SiO2 coating reflection vs. angle



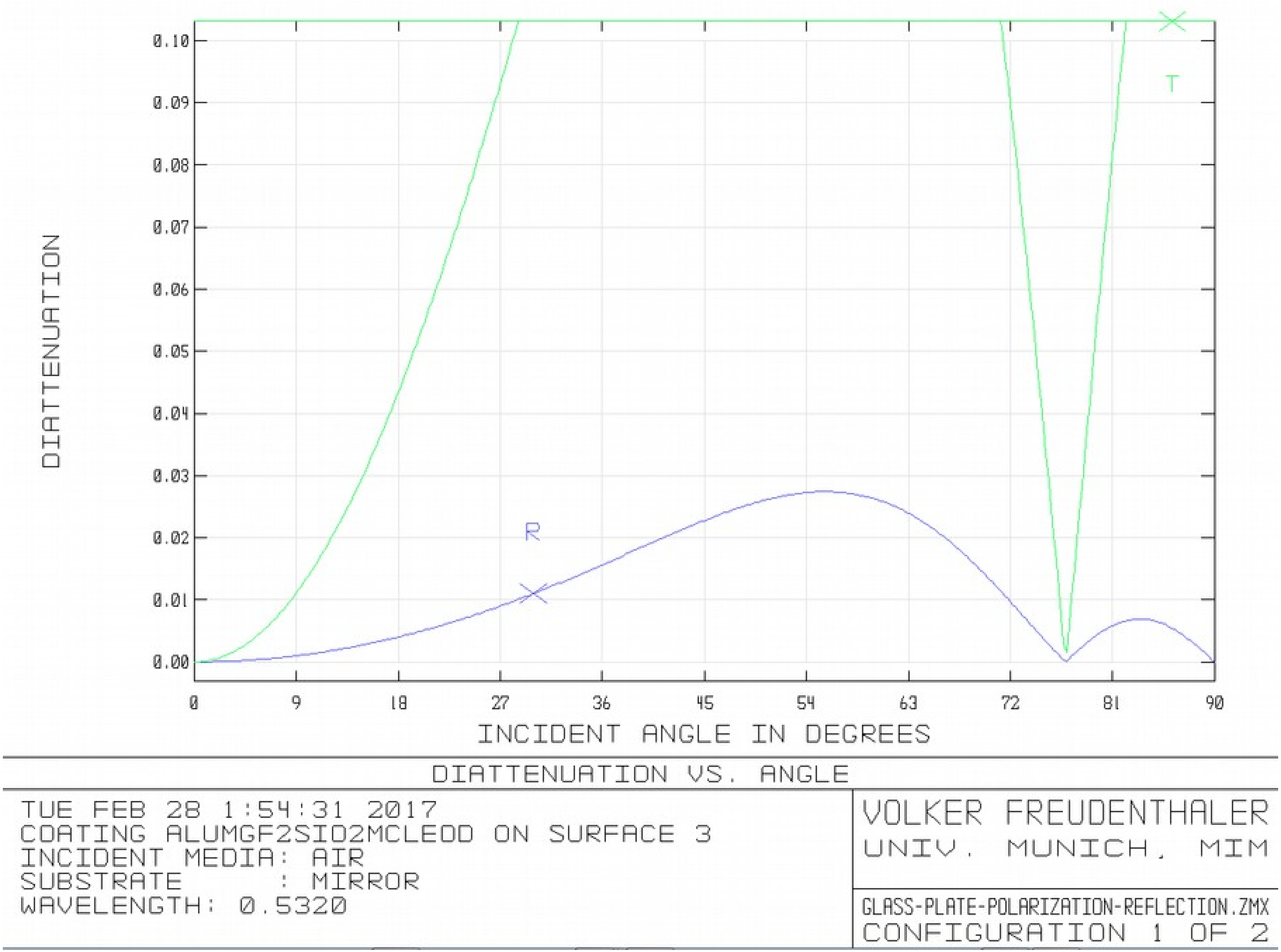
REFLECTION VS. ANGLE

TUE FEB 28 1:56:33 2017
COATING ALUMGF2SIO2MCLEOD ON SURFACE 3
INCIDENT MEDIA: AIR
SUBSTRATE : MIRROR
WAVELENGTH: 0.5320

VOLKER FREUDENTHALER
UNIV. MUNICH, MIM

GLASS-PLATE-POLARIZATION-REFLECTION.ZMX
CONFIGURATION 1 OF 2

Newton 90° mirror, Aluminium + MgF2 + SiO2 coating   reflection vs. angle



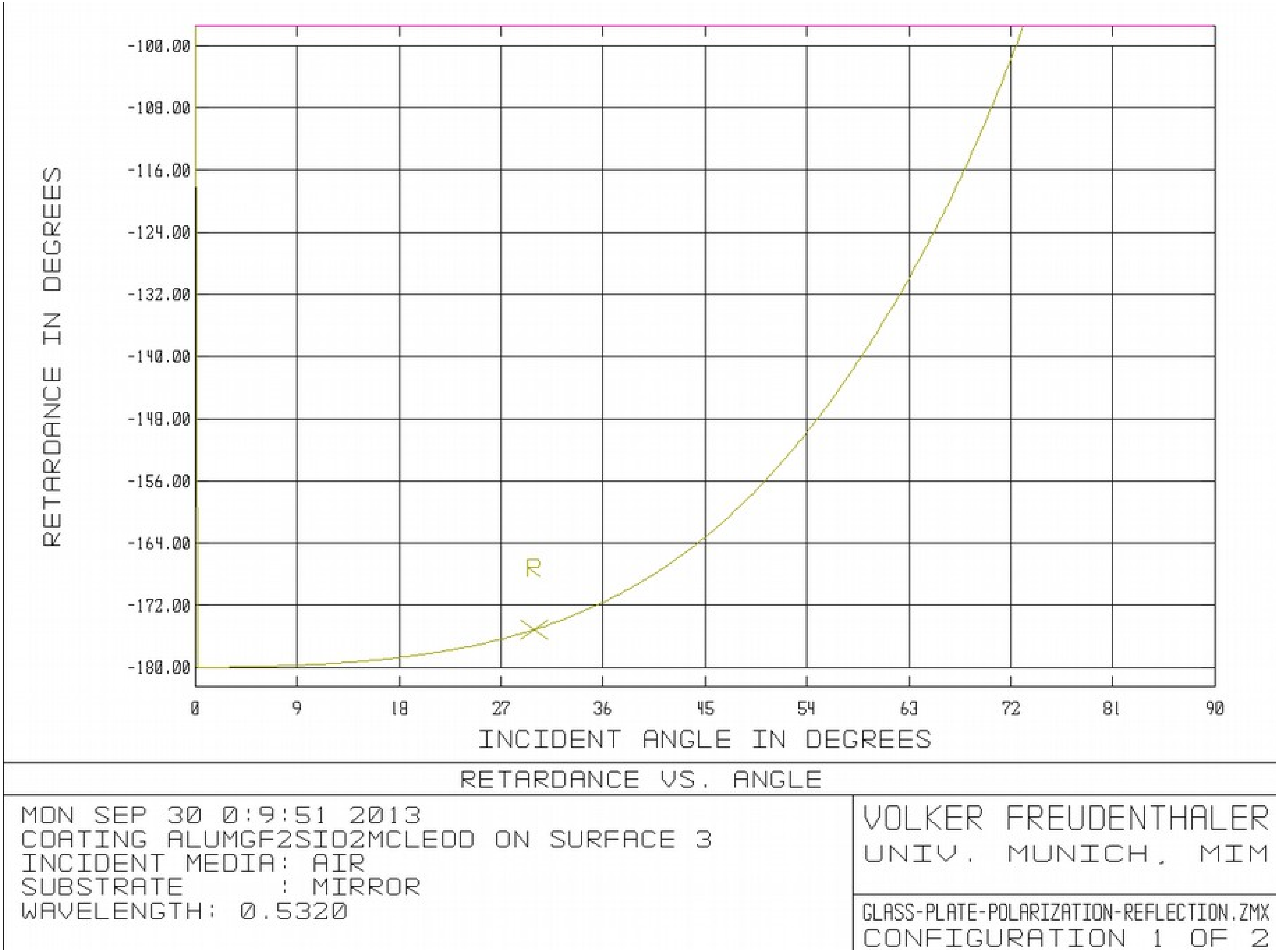
DIATTENUATION VS. ANGLE

TUE FEB 28 1:54:31 2017
COATING ALUMGF2SIO2MCLEOD ON SURFACE 3
INCIDENT MEDIA: AIR
SUBSTRATE : MIRROR
WAVELENGTH: 0.5320

VOLKER FREUDENTHALER
UNIV. MUNICH, MIM

GLASS-PLATE-POLARIZATION-REFLECTION.ZMX
CONFIGURATION 1 OF 2

Newton 90° mirror, Aluminium + MgF2 + SiO2 coating   retardance vs. angle



RETARDANCE VS. ANGLE

MON SEP 30 0:9:51 2013
COATING ALUMGF2SIO2MCLEOD ON SURFACE 3
INCIDENT MEDIA: AIR
SUBSTRATE   : MIRROR
WAVELENGTH: 0.5320

VOLKER FREUDENTHALER
UNIV. MUNICH, MIM

GLASS-PLATE-POLARIZATION-REFLECTION.ZMX
CONFIGURATION 1 OF 2

Newton 90° mirror diattenuation and retardance

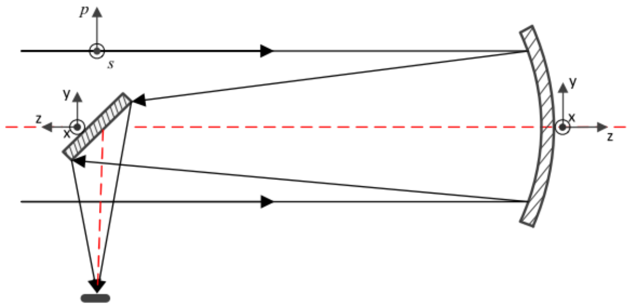


Fig. 1. Sketch of a polarized ray path through a Newton telescope.

392 APPLIED OPTICS / Vol. 54, No. 3 / 20 January 2015

considered: raw aluminium coatings

$$\mathbf{M}_N = \begin{pmatrix} 0.8482 & 0.0236 & -0.002 & 0 \\ 0.0236 & 0.8481 & -0.0016 & -0.0147 \\ -0.002 & -0.0016 & 0.8293 & -0.1765 \\ 0 & 0.0147 & 0.1765 & 0.8292 \end{pmatrix}. \quad (24)$$

<u>own calculations:</u>

- confirm above values
- diattenuation ~ 0.03
- retardance ~ 13°
- LDR   0.0003

aluminium + MgF2 coating reduces effect

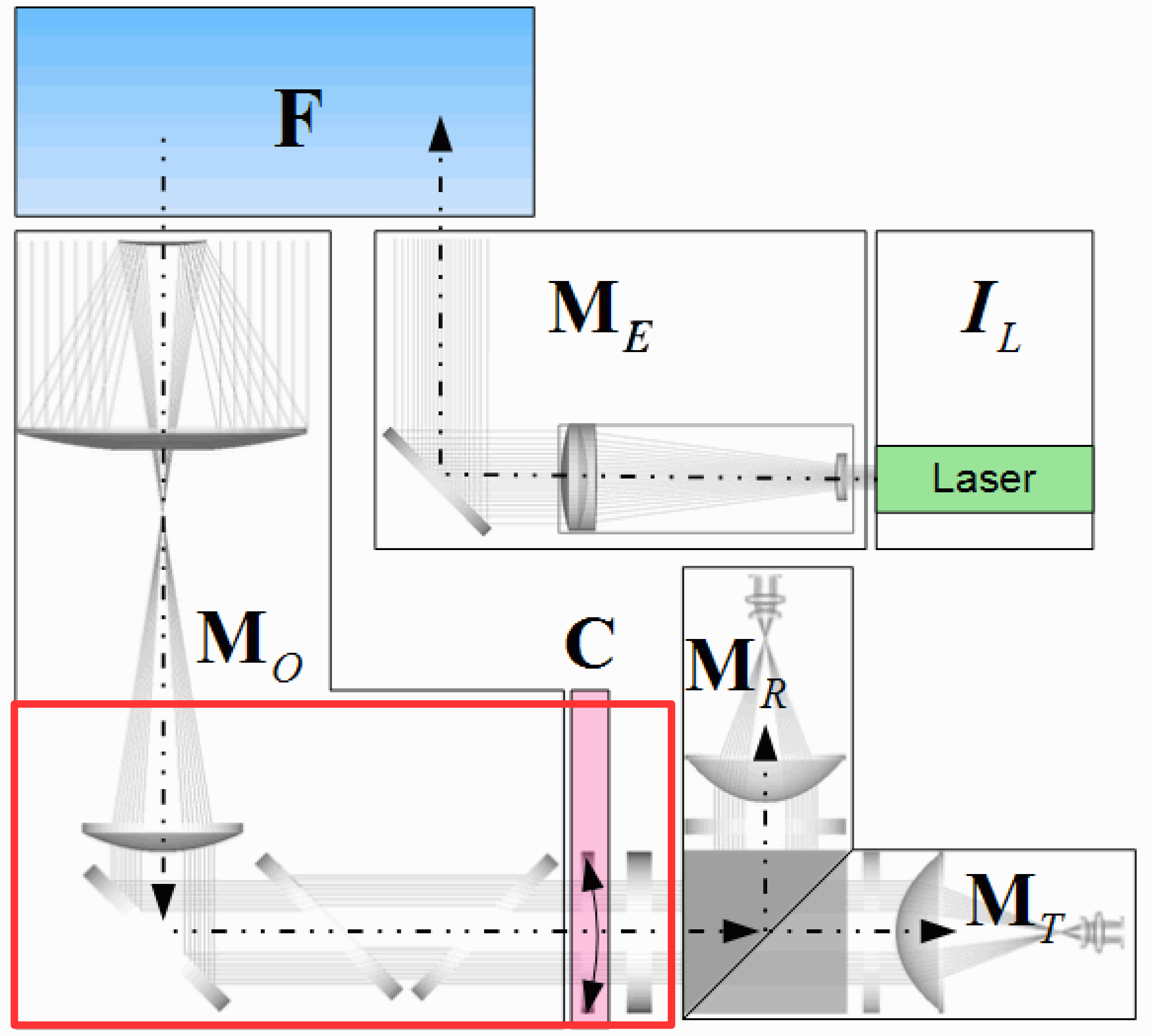
- diattenuation ~ 0.001
- retardance ~ 7°

own measurements

- no diattenuation
- retardance ~ 30°

Di, et. al., Polarization analysis and corrections of different telescopes in polarization lidar, Appl. Opt., OSA, 2015, 54, 389-397

Typical lidar setup – receiver optics



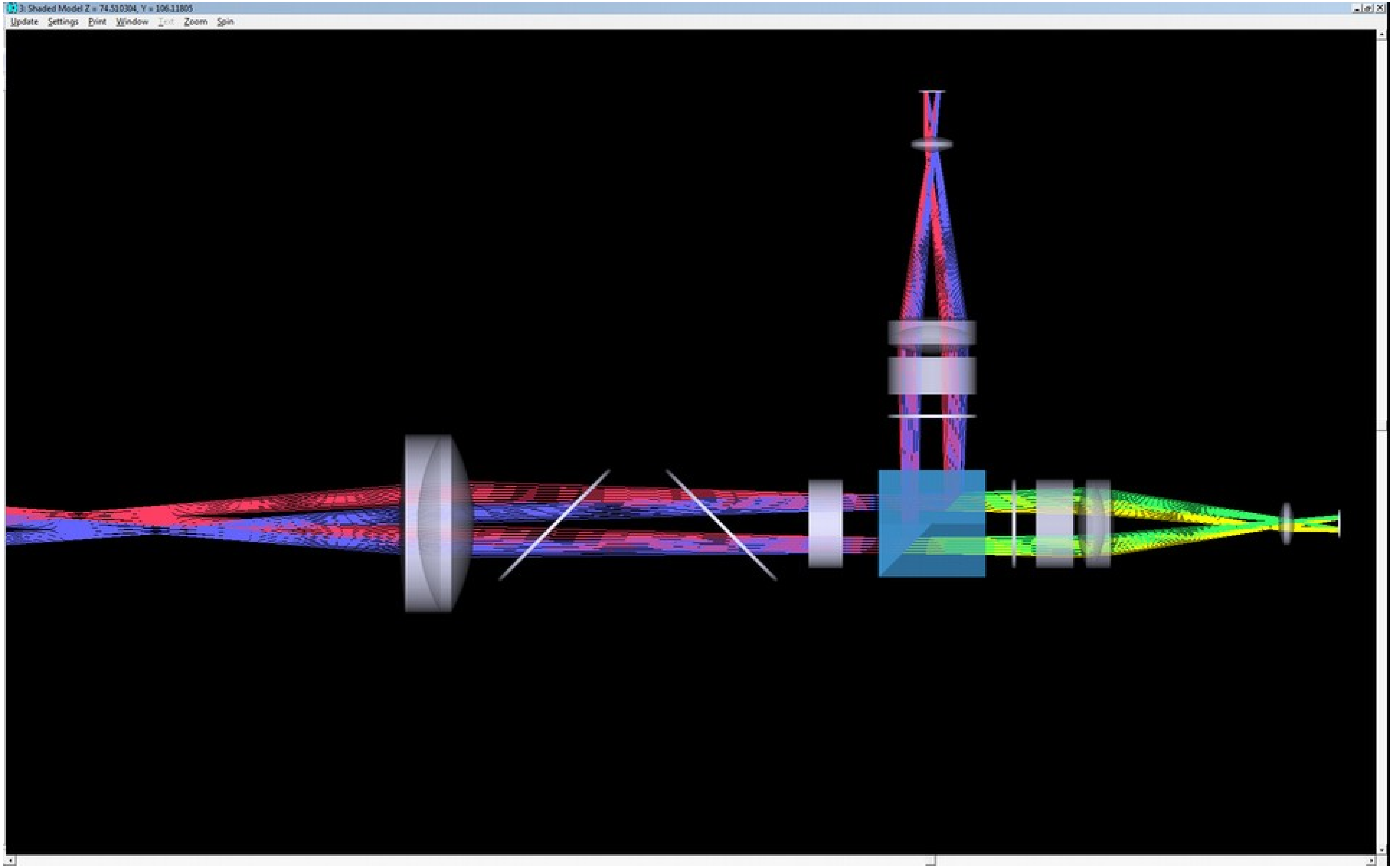
receiving optics
beamsplitters and filters

focal length of collimator
=> beam divergence
=> beam diameter

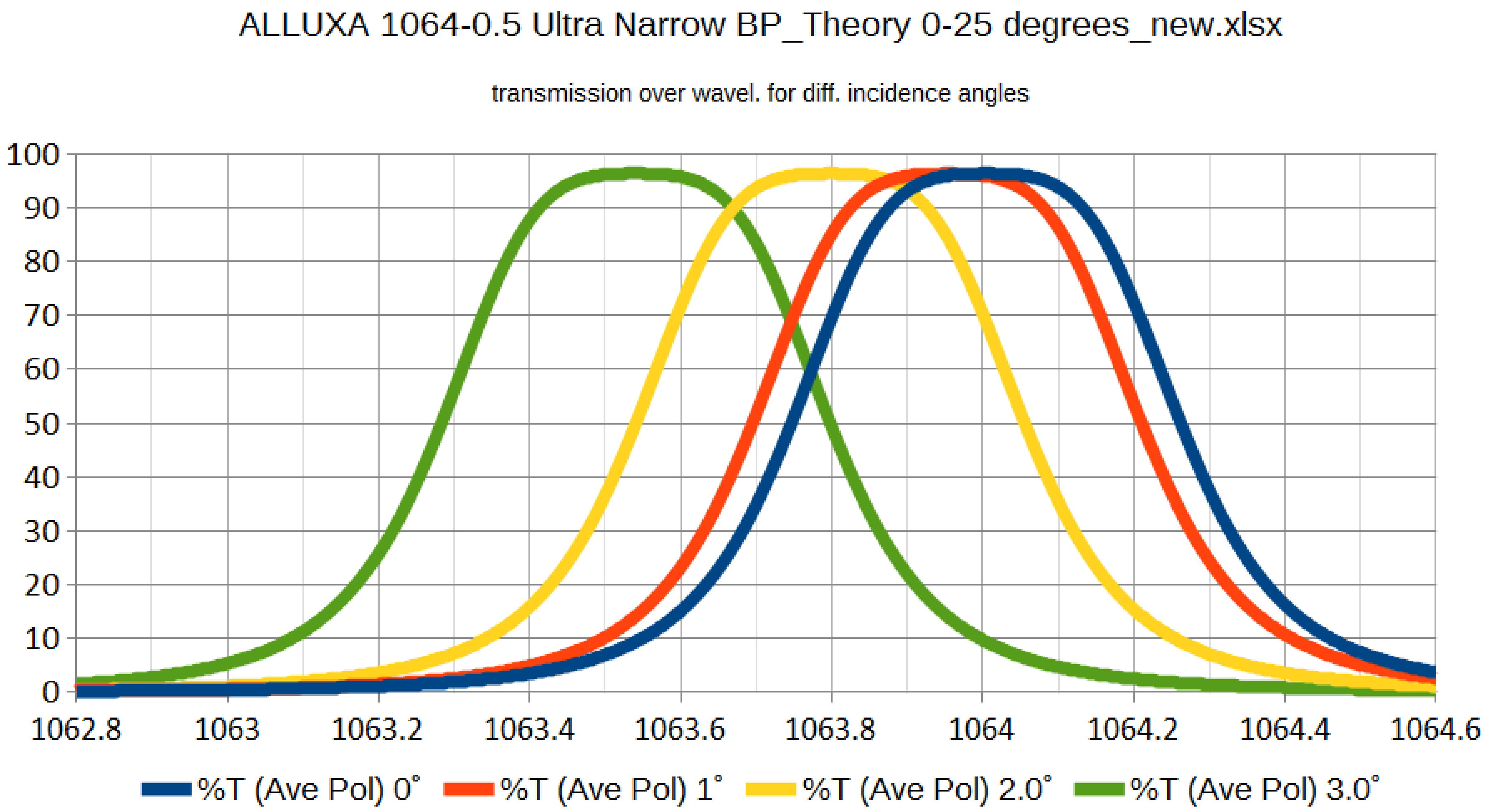
accpetance angles of
beamsplitters and
interference filters

polarisation problemes
diattenuation
retardance

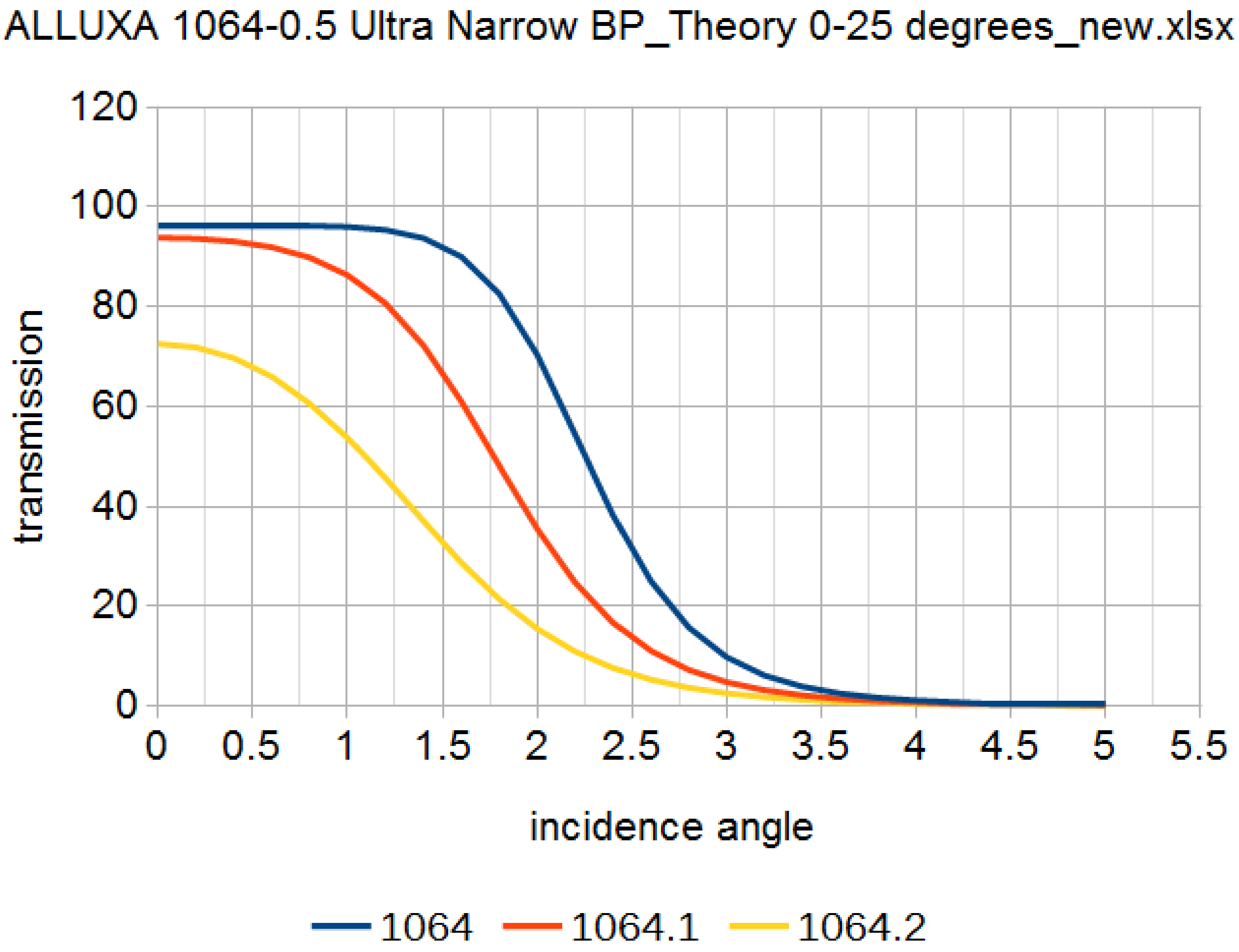
Effect of optical elements can be described by Müller matrices



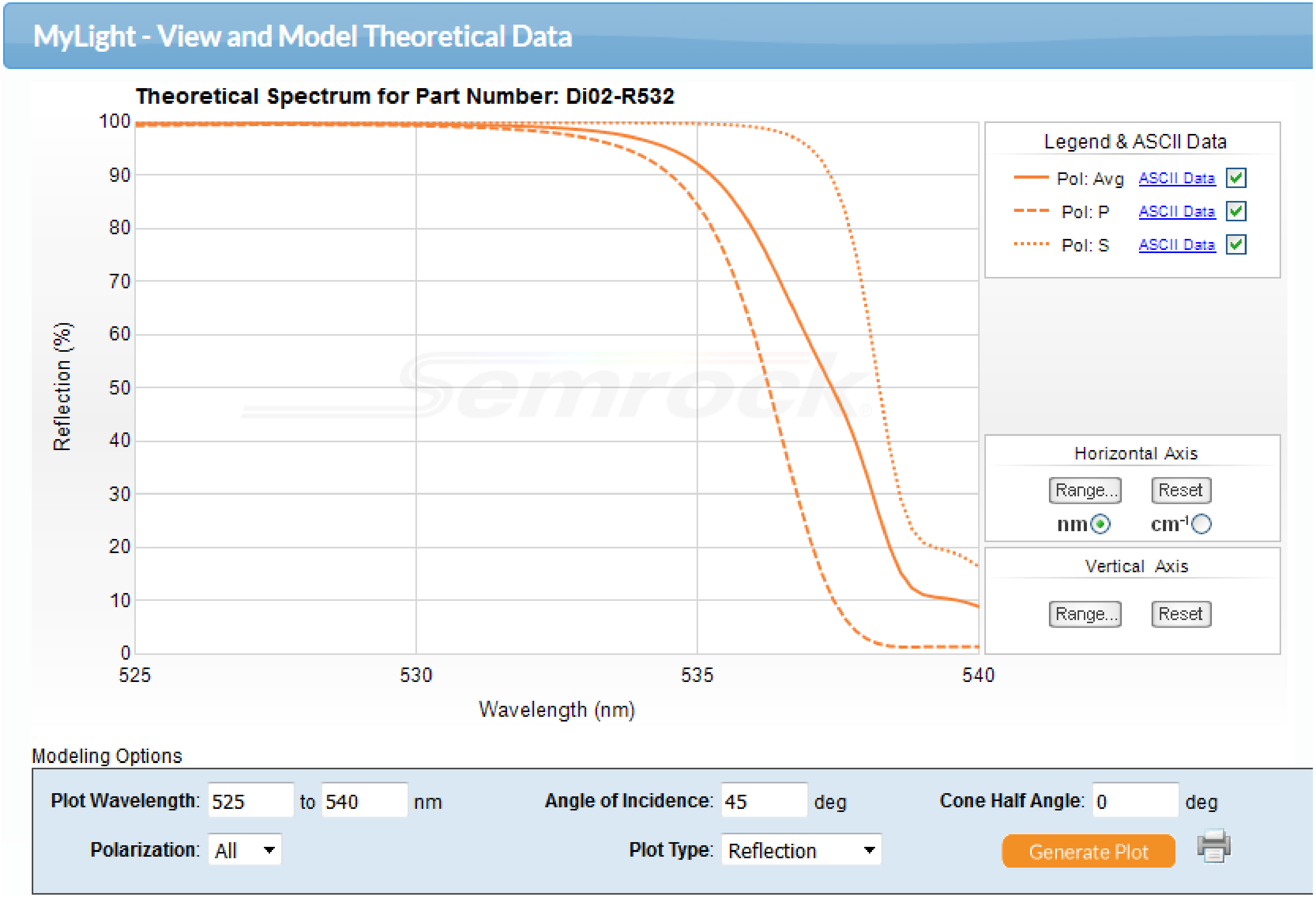
Laser – wavelength – interference filter - centre wavelength over incidence angle



Interference filter - transmission over incidence angle



Coatings Semrock beamsplitter



Modeling Options

Plot Wavelength: 525 to 540 nm

Angle of Incidence: 45 deg

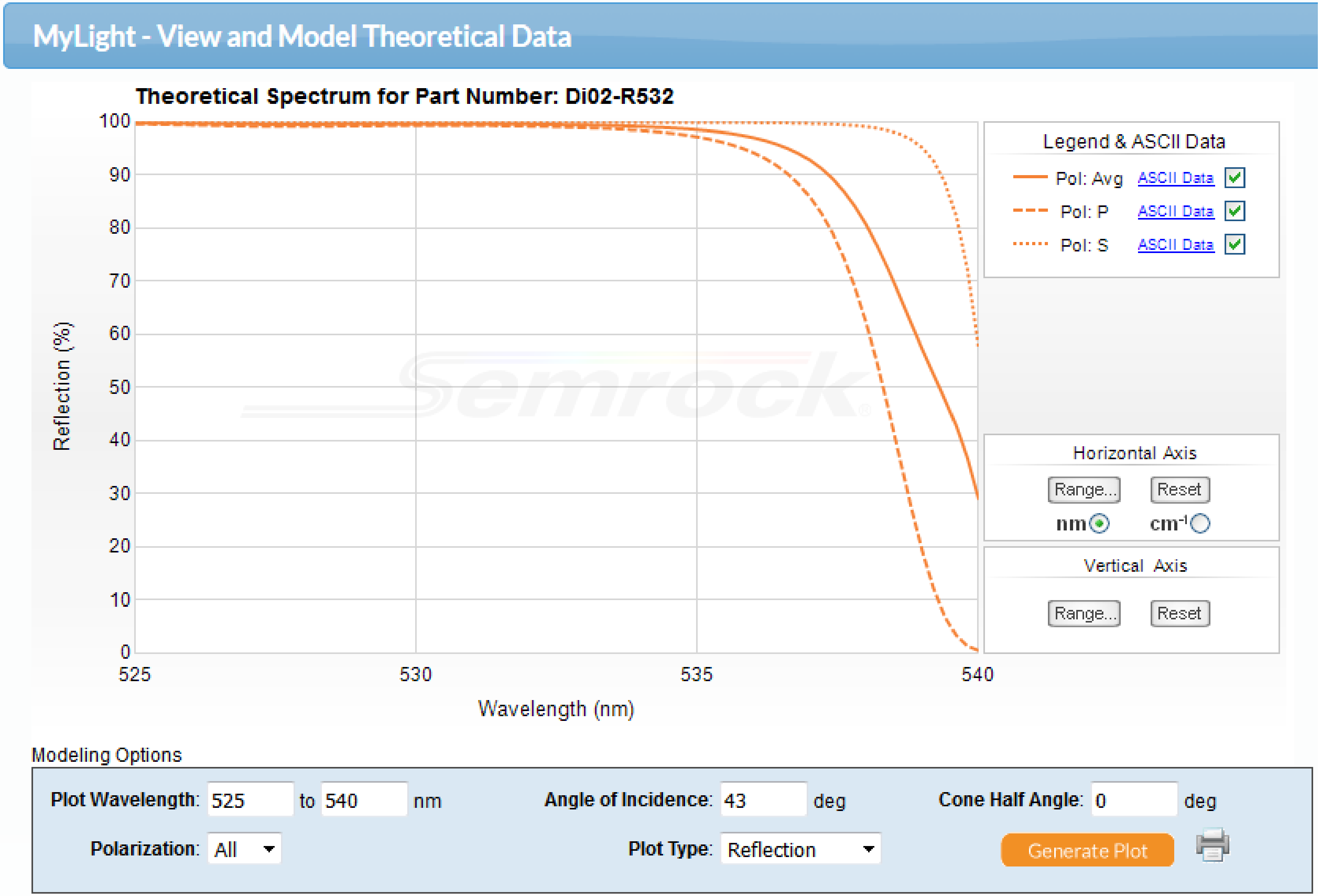
Cone Half Angle: 0 deg

Polarization: All

Plot Type: Reflection

https://www.semrock.com/filters.aspx

Coatings Semrock beamsplitter



Modeling Options

Plot Wavelength: 525 to 540 nm

Angle of Incidence: 43 deg

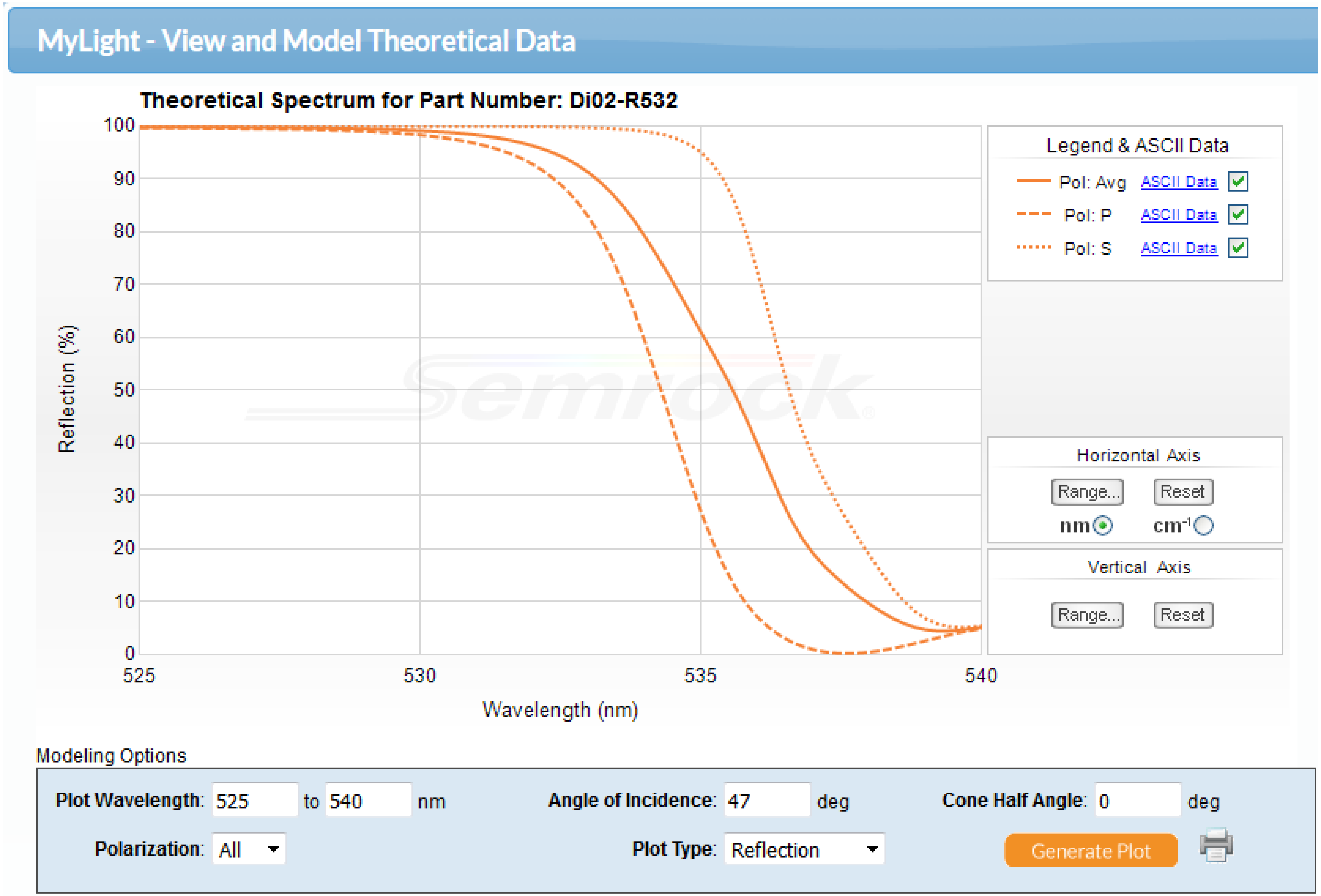
Cone Half Angle: 0 deg

Polarization: All

Plot Type: Reflection

https://www.semrock.com/filters.aspx

Coatings Semrock beamsplitter



MyLight - View and Model Theoretical Data

Theoretical Spectrum for Part Number: Di02-R532

Modeling Options

Plot Wavelength: 525 to 540 nm

Angle of Incidence: 47 deg

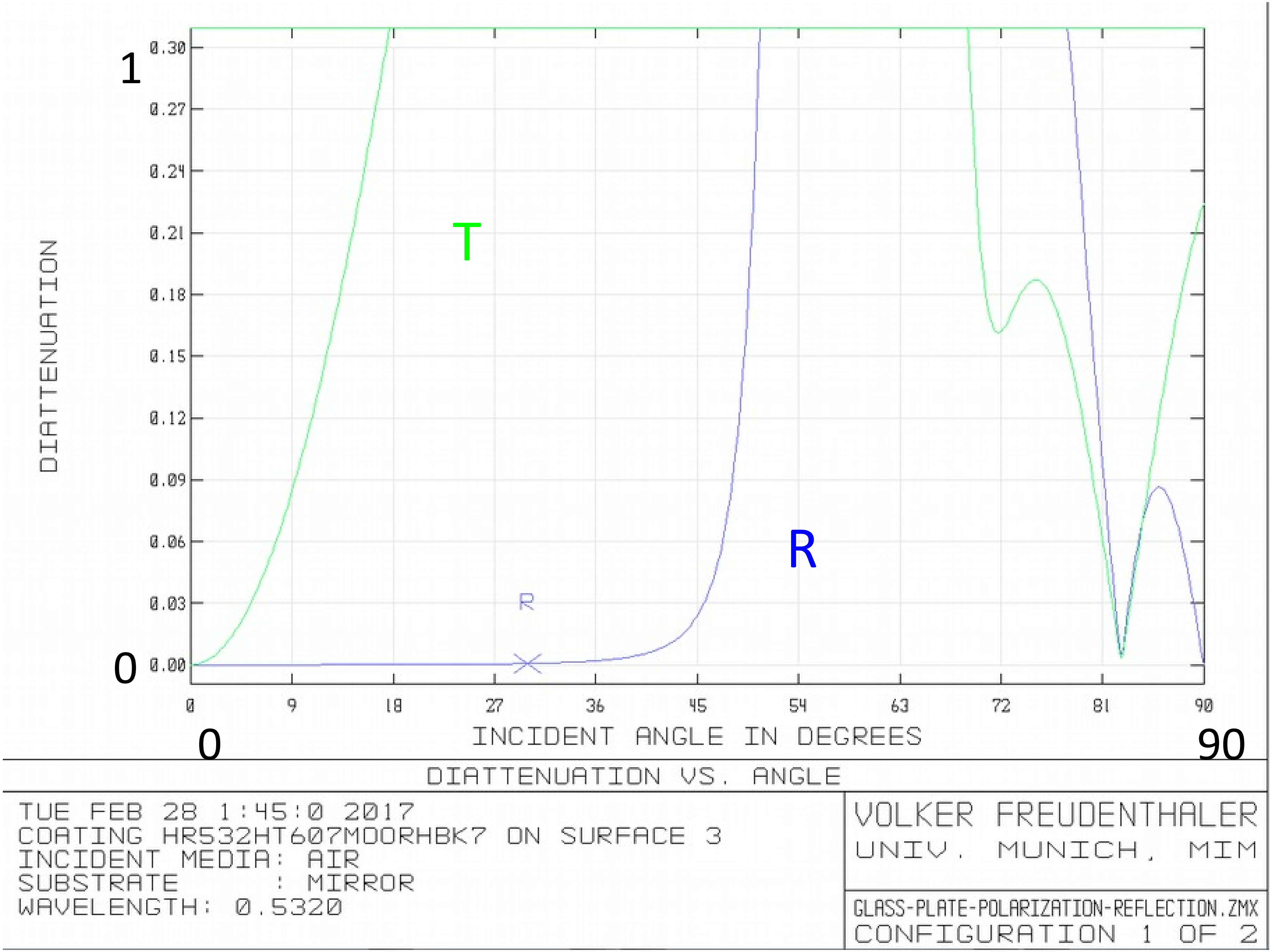
Cone Half Angle: 0 deg

Polarization: All

Plot Type: Reflection

https://www.semrock.com/filters.aspx

Diattenuation of HR532 HT607 beamsplitter coating over incident angle



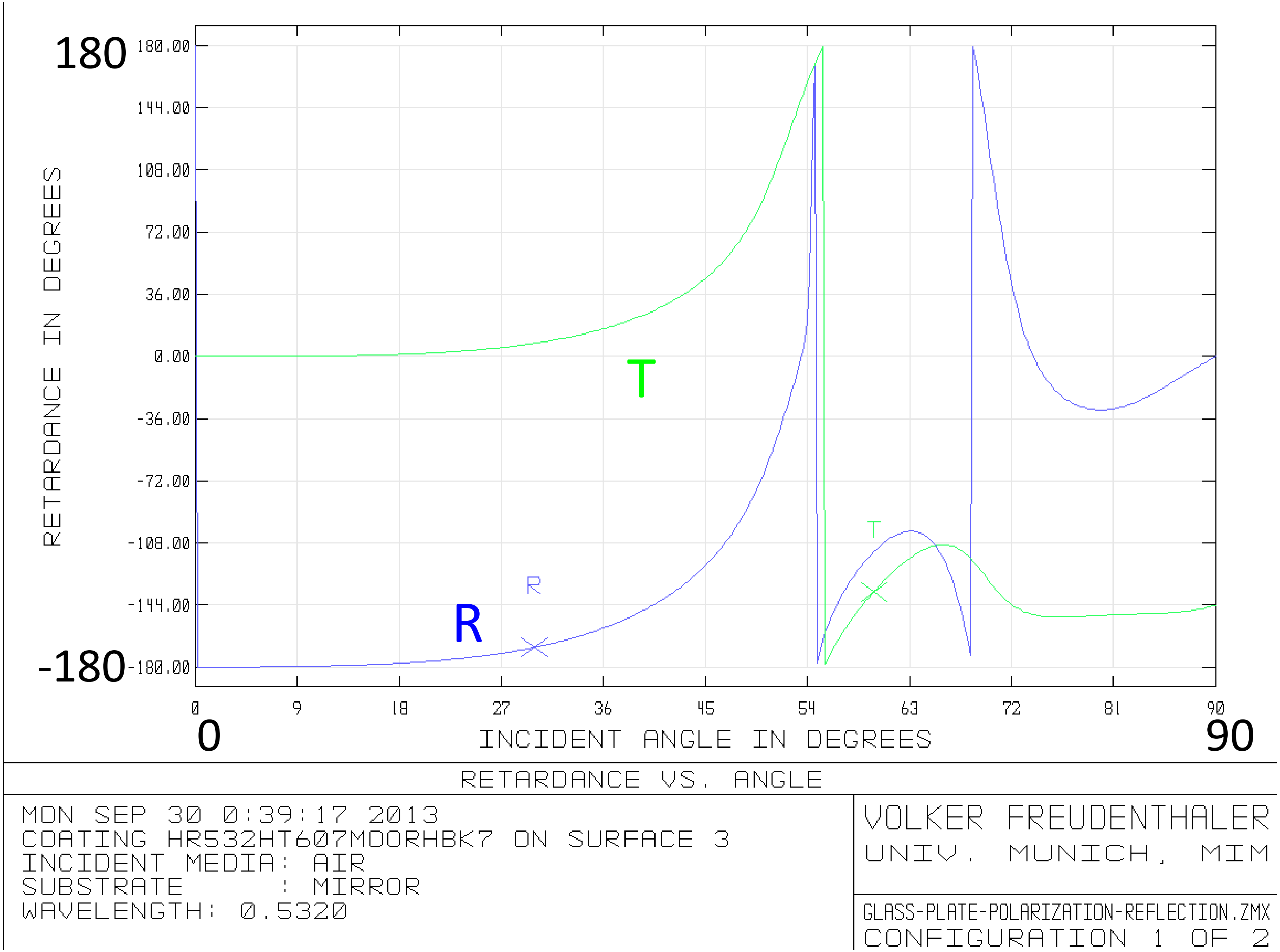
DIATTENUATION VS. ANGLE

TUE FEB 28 1:45:0 2017
COATING HR532HT607MDORHBK7 ON SURFACE 3
INCIDENT MEDIA: AIR
SUBSTRATE : MIRROR
WAVELENGTH: 0.5320

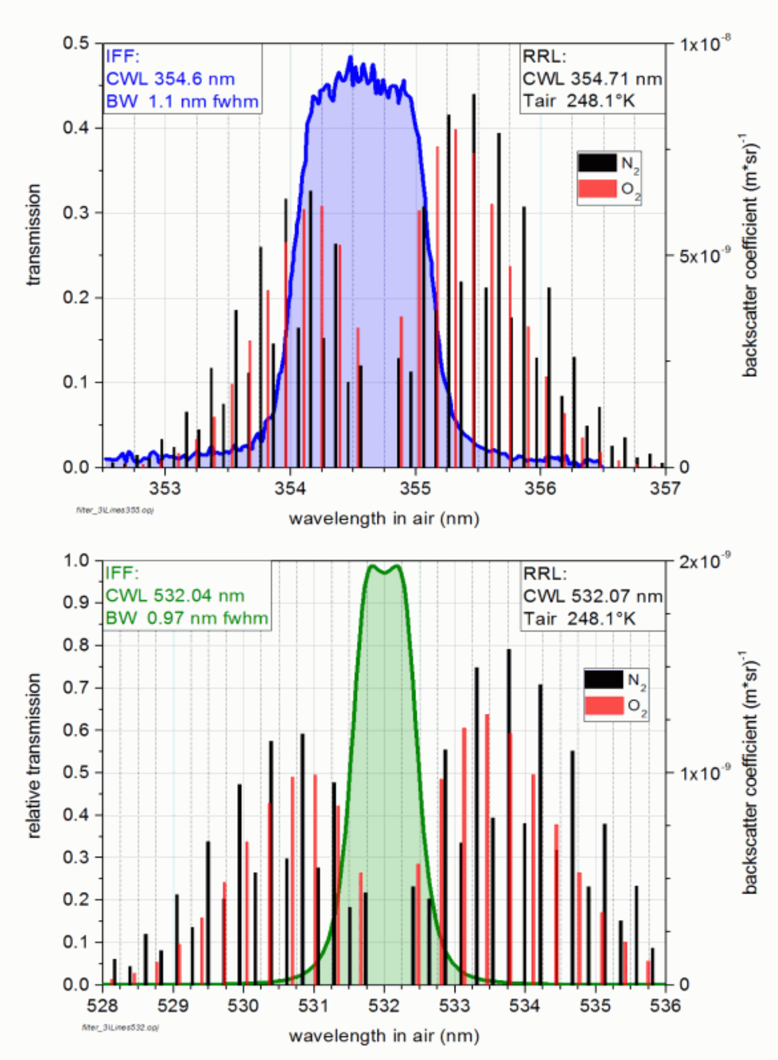
VOLKER FREUDENTHALER
UNIV. MUNICH, MIM

GLASS-PLATE-POLARIZATION-REFLECTION.ZMX
CONFIGURATION 1 OF 2

Retardance of HR532 HT607 beamsplitter coating over incident angle

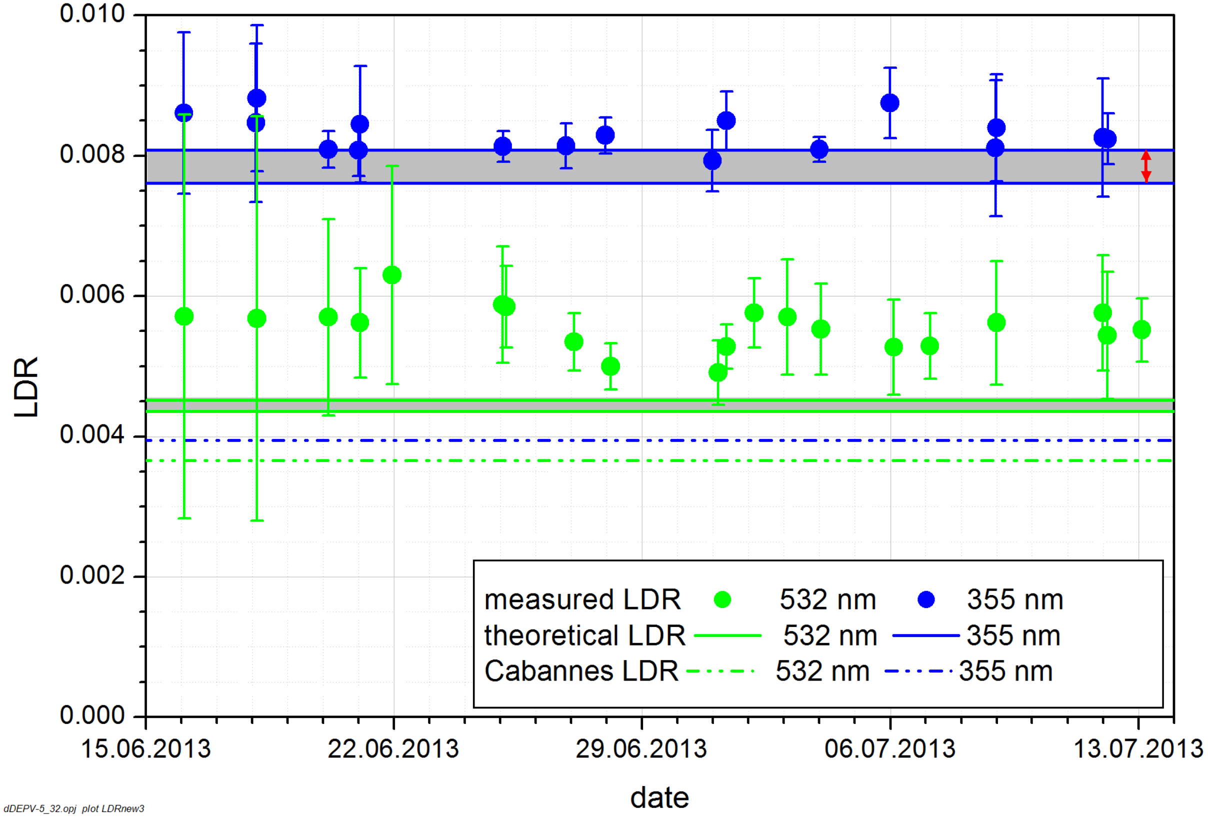


Laser – polarisation – POLIS-6 – 355 / 532 nm – measured / theoretical Rayleigh LDR



uncertain theoretical LDR due to uncertainties in

- laser wavelength
- Rot. Raman Lines in interference filter bandwidth



source: Freudenthaler et al., 27th ILRC 2015, Accuracy of linear depolaristion ratios in clear air ranges measured with POLIS-6 at 355 and 532 nm. *http://dx.doi.org/10.1051/epjconf/201611925013*

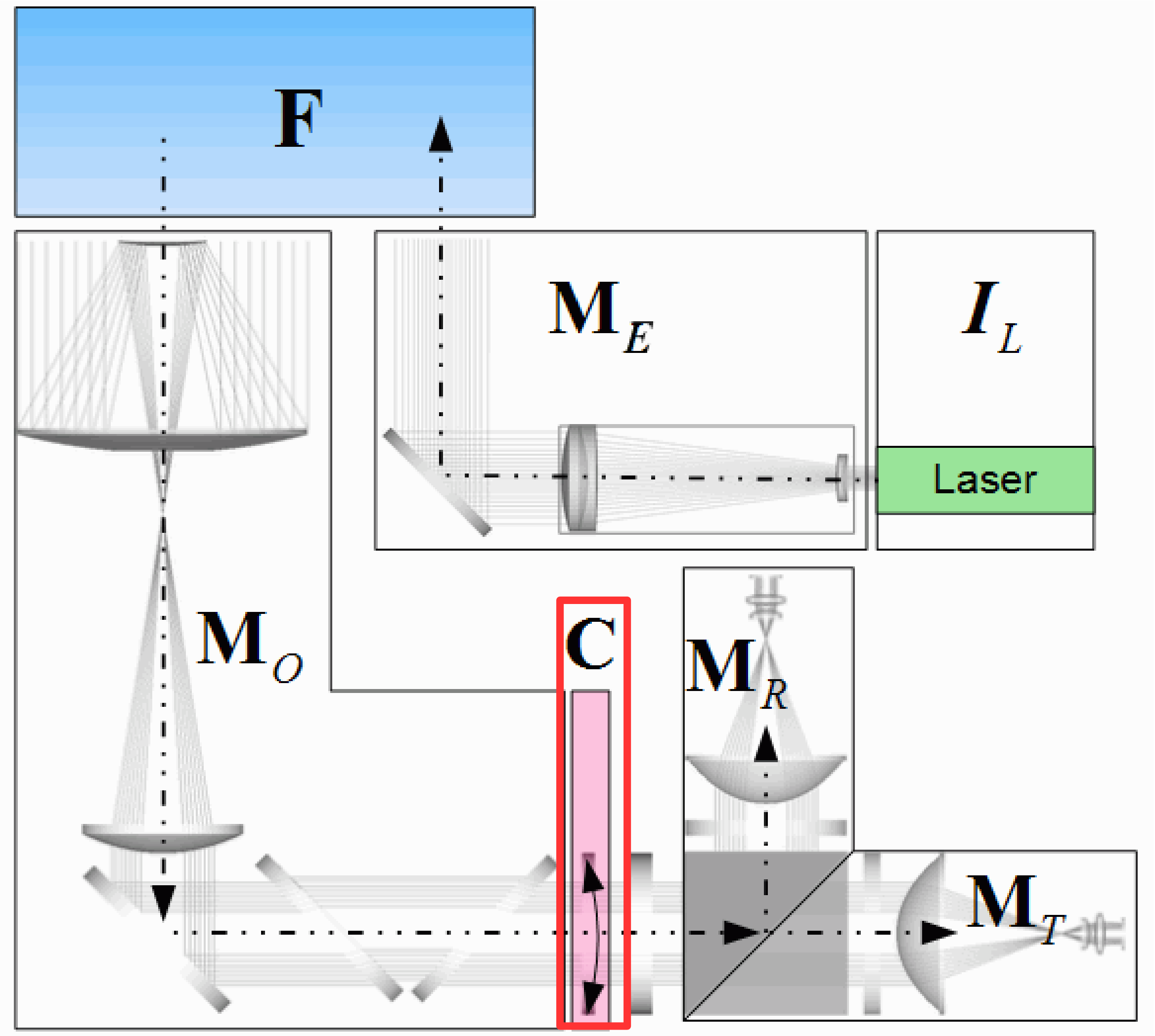
theoretically measured molecular LDR

The combination of
  laser wavelength
  IF-filter center wavelength
  IF-filter bandwidth
  IF-filter incidence angle

determines the theoretically measured molecular LDR

Typical lidar setup – polarisation calibrator

polarisation calibrator



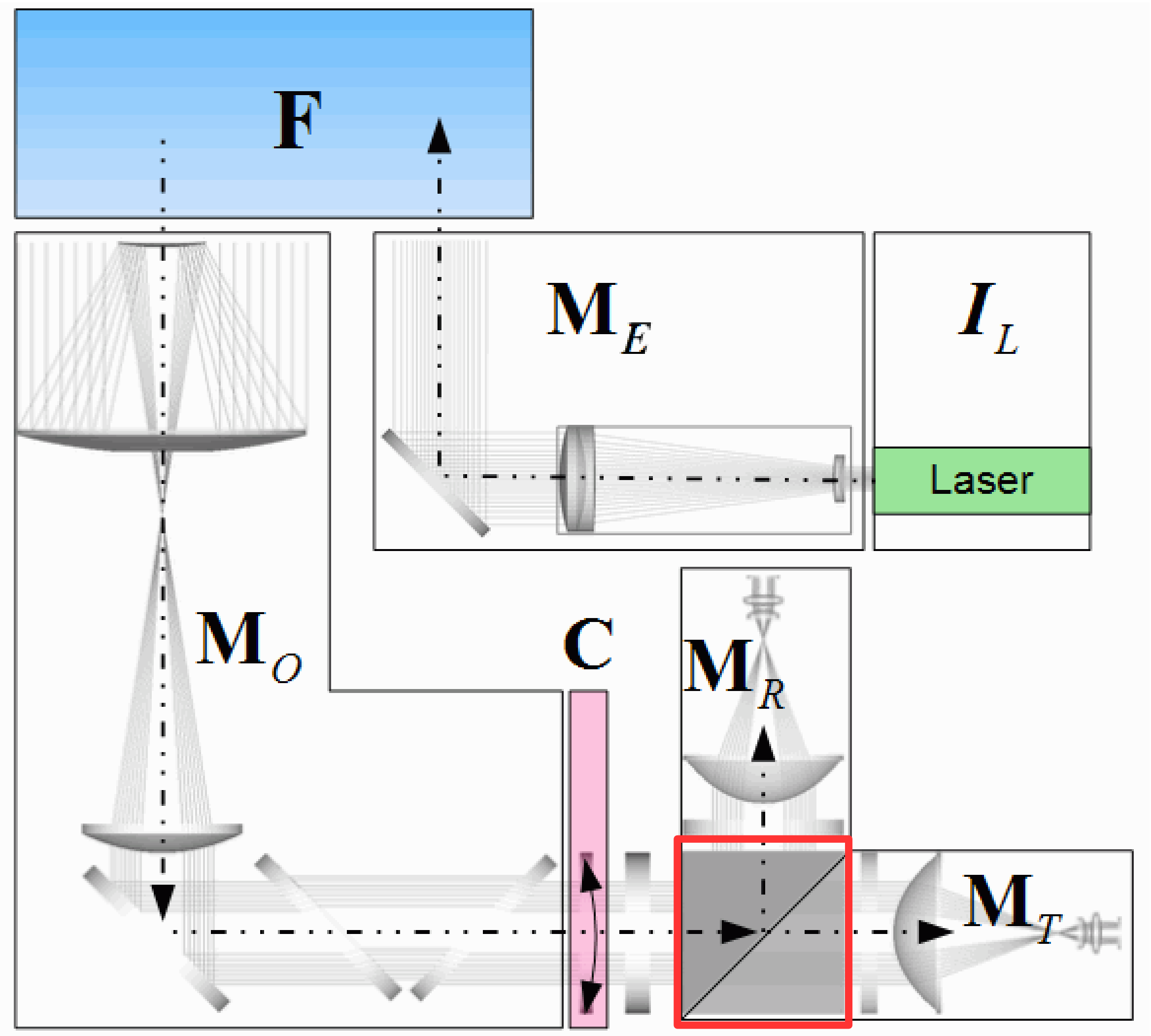
- error of η due to non-ideal linear polarizer calibrator with extinction ratio ρ

  $\rho = 10^{-5}$ => $\Delta\eta/\eta = 1.3\%$

  $\rho = 10^{-4}$ => $\Delta\eta/\eta = 8\%$

- Δ 90-calibration reduces the calibration error dur to rotational misalignment by orders of magnitude

Typical lidar setup – polarising beam splitter



polarising beam splitter

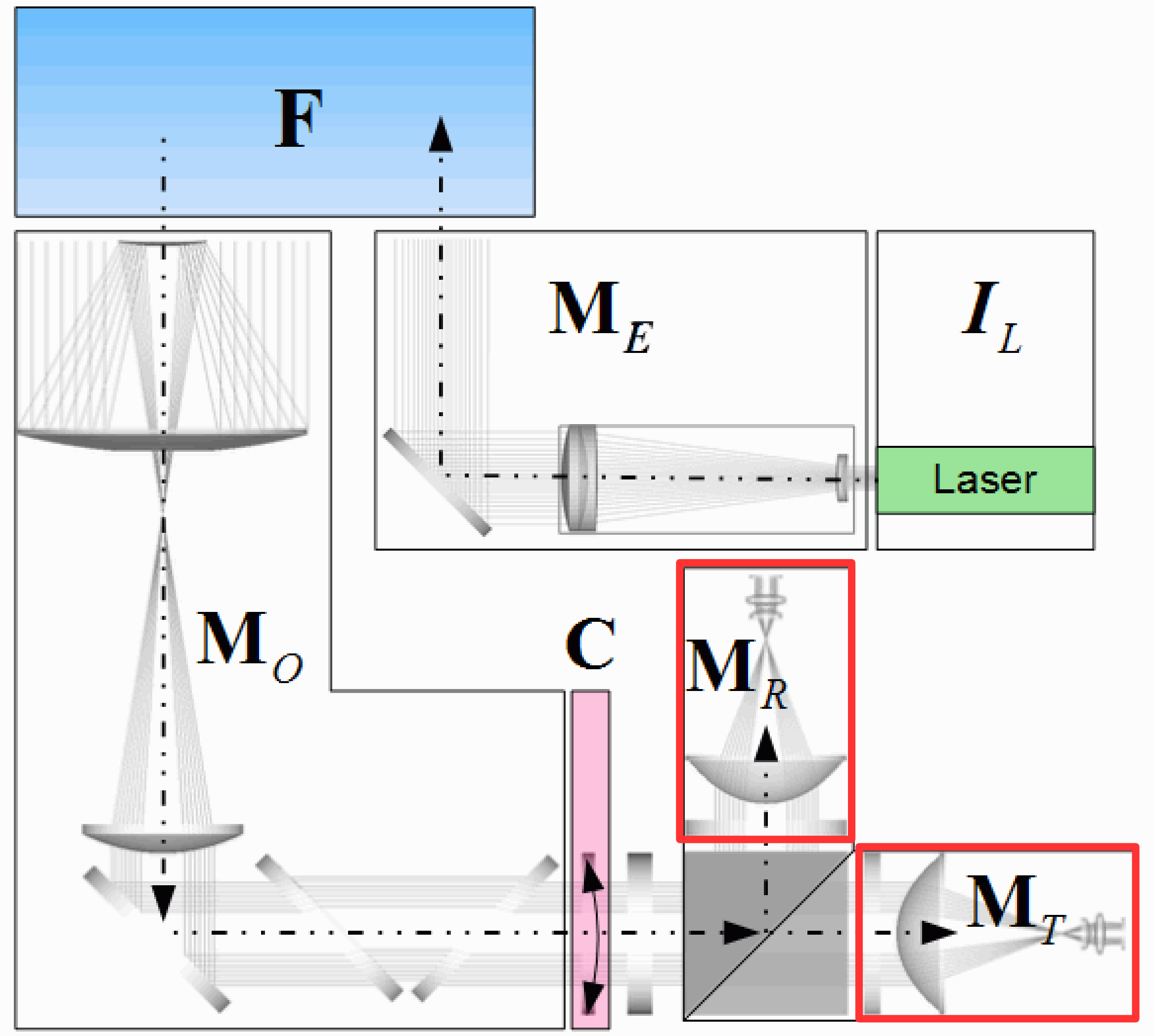
acceptance angles

extinction ratio

cleaning the cross talk with polarizing sheet filters ($\rho = 10^{-3}$ is sufficient) removes many problems!

Typical lidar setup – detector and optics

detectors and optics



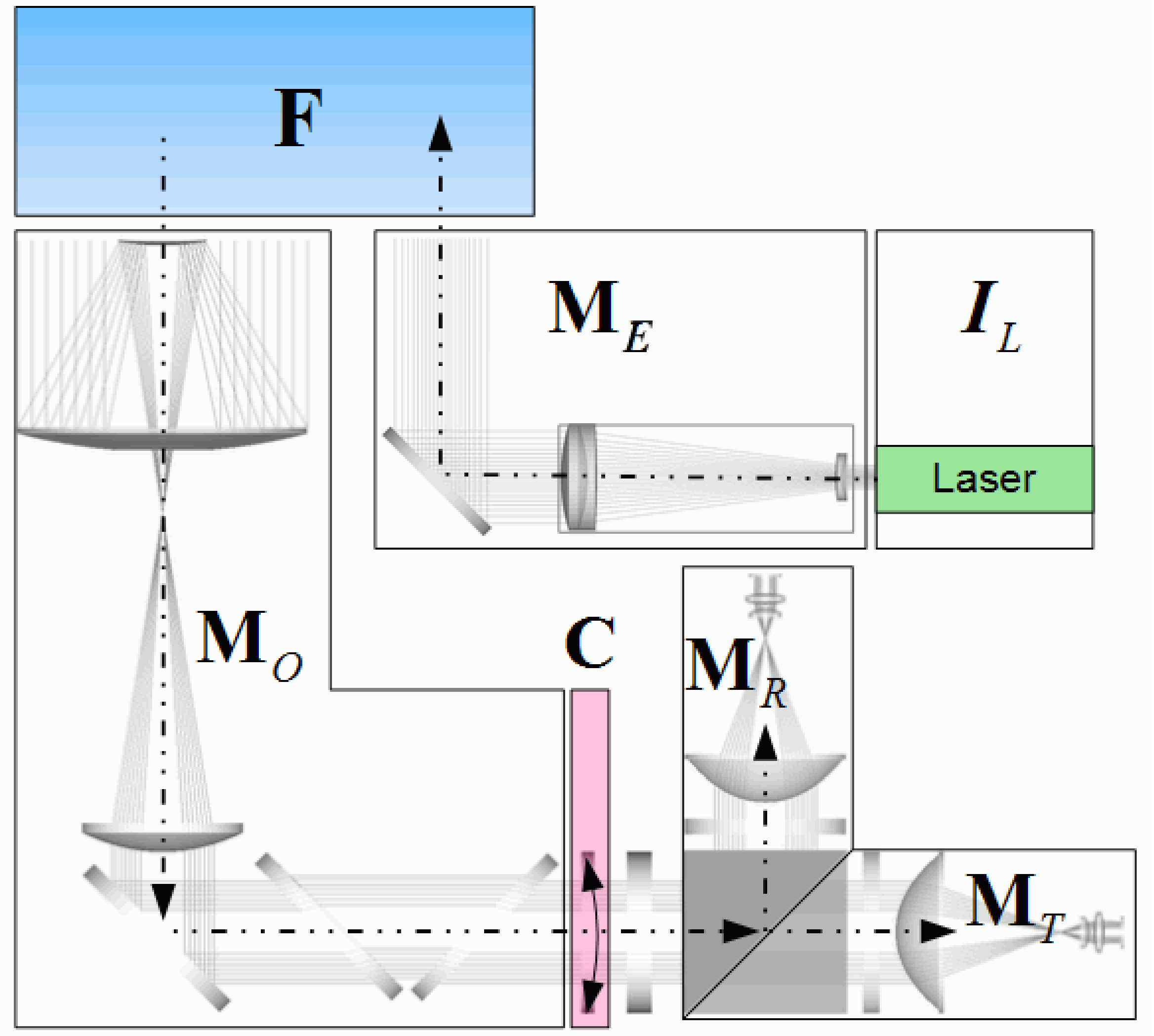
PMT
	homogeneity of the sensitivity

APD
	small diameter

eyepiece
	=> telescope imaging

neutral density filters
	adjust signal level (LICEL)

Mathematical description – assumptions/limitations



randomly oriented aerosol

modules consist of
  n *π/2* - rotated
  retarding linear diattenuators

  without depolarization

Mathematical description – SVLE parameters

$$\boldsymbol{I}_S = \eta_S \ \mathbf{M}_S \ \mathbf{R}_y \ \mathbf{R}_\varepsilon \ \mathbf{M}_O \ \mathbf{F} \ \mathbf{M}_E \ \boldsymbol{I}_L$$

$\eta_{R,T}$	$T_{R,T}$			T_O	F_{11}	T_E	I_L	unpolarised transmission
					a		b	polarisation parameter / degree of linear polarization
	$D_{R,T}$			D_O		D_E		linear diattenuation parameter
	$\rho_{R,T}$							extinction ratio of cleaning pol-filters
				Δ_O		Δ_E		retardation
	$\phi_{R,T}$		ε	γ		β	α	module rotation

22 independent parameters

Mathematical description – GHK- summary

$$\boldsymbol{I}_S = \eta_S \mathbf{M}_S \mathbf{R}_\mathrm{y} \mathbf{R}_\varepsilon \mathbf{M}_O \mathbf{F} \mathbf{M}_E \boldsymbol{I}_L$$

$$\boxed{I_S = \eta_S T_S T_O F_{11} T_E I_L \left(G_S + a H_S \right)}$$

$$G_S = \left(1 + \mathrm{y} D_S D_O \mathrm{c}_{2\gamma+2\varepsilon} \right) i_E - \mathrm{y} D_S Z_O \mathrm{s}_O \mathrm{s}_{2\gamma+2\varepsilon} v_E$$

$$H_S = D_O \left(\mathrm{c}_{2\gamma} q_E - \mathrm{s}_{2\gamma} u_E \right) + \mathrm{y} D_S \left[\left(\mathrm{c}_{2\varepsilon} q_E + \mathrm{s}_{2\varepsilon} u_E \right) - \mathrm{s}_{2\gamma+2\varepsilon} \left\{ W_O \left(\mathrm{s}_{2\gamma} q_E + \mathrm{c}_{2\gamma} u_E \right) - 2 Z_O \mathrm{s}_O v_E \right\} \right]$$

Mathematical description – GHK- summary

$$\boldsymbol{I}_S = \eta_S \mathbf{M}_S \mathbf{R}_y \mathbf{R}_\varepsilon \mathbf{M}_O \mathbf{F} \mathbf{M}_E \boldsymbol{I}_L$$

$$\boxed{I_S = \eta_S T_S T_O F_{11} T_E I_L \left(G_S + aH_S\right)}$$

$$G_S = \left(1 + \mathrm{y} D_S D_O \mathrm{c}_{2\gamma+2\varepsilon}\right) i_E - \mathrm{y} D_S Z_O \mathrm{s}_O \mathrm{s}_{2\gamma+2\varepsilon} v_E$$

$$H_S = D_O\left(\mathrm{c}_{2\gamma} q_E - \mathrm{s}_{2\gamma} u_E\right) + \mathrm{y} D_S\left[\left(\mathrm{c}_{2\varepsilon} q_E + \mathrm{s}_{2\varepsilon} u_E\right) - \mathrm{s}_{2\gamma+2\varepsilon}\left\{W_O\left(\mathrm{s}_{2\gamma} q_E + \mathrm{c}_{2\gamma} u_E\right) - 2 Z_O \mathrm{s}_O v_E\right\}\right]$$

$$\frac{\boldsymbol{I}_{in}(\beta,\alpha)}{T_E I_L} = \frac{\mathbf{M}_E(\beta)\left|\boldsymbol{I}_L(\alpha)\right\rangle}{T_E I_L} =$$

$$= \left| i_E \quad q_E \quad u_E \quad v_E \right\rangle = \left| \begin{matrix} i_L + D_E\left(\mathrm{c}_{2\alpha-2\beta} q_L - \mathrm{s}_{2\alpha-2\beta} u_L\right) \\ \mathrm{c}_{2\beta} D_E i_L + \left(\mathrm{c}_{2\alpha} q_L - \mathrm{s}_{2\alpha} u_L\right) + \mathrm{s}_{2\beta}\left[W_E\left(\mathrm{s}_{2\alpha-2\beta} q_L + \mathrm{c}_{2\alpha-2\beta} u_L\right) - Z_E \mathrm{s}_E v_L\right] \\ \mathrm{s}_{2\beta} D_E i_L + \left(\mathrm{s}_{2\alpha} q_L + \mathrm{c}_{2\alpha} u_L\right) - \mathrm{c}_{2\beta}\left[W_E\left(\mathrm{s}_{2\alpha-2\beta} q_L + \mathrm{c}_{2\alpha-2\beta} u_L\right) - Z_E \mathrm{s}_E v_L\right] \\ -Z_E \mathrm{s}_E\left(\mathrm{s}_{2\alpha-2\beta} q_L + \mathrm{c}_{2\alpha-2\beta} u_L\right) + Z_E \mathrm{c}_E v_L \end{matrix} \right\rangle$$

Mathematical description – GHK- summary

$$\boldsymbol{I}_S = \eta_S \mathbf{M}_S \mathbf{R}_{\mathrm{y}} \mathbf{R}_\varepsilon \mathbf{M}_O \mathbf{F} \mathbf{M}_E \boldsymbol{I}_L$$

$$\boxed{I_S = \eta_S T_S T_O F_{11} T_E I_L \left(G_S + a H_S \right)}$$

$$G_S = \left(1 + \mathrm{y} D_S D_O \mathrm{c}_{2\gamma+2\varepsilon}\right) i_E - \mathrm{y} D_S Z_O \mathrm{s}_O \mathrm{s}_{2\gamma+2\varepsilon} v_E$$

$$H_S = D_O \left(\mathrm{c}_{2\gamma} q_E - \mathrm{s}_{2\gamma} u_E \right) + \mathrm{y} D_S \left[\left(\mathrm{c}_{2\varepsilon} q_E + \mathrm{s}_{2\varepsilon} u_E \right) - \mathrm{s}_{2\gamma+2\varepsilon} \left\{ W_O \left(\mathrm{s}_{2\gamma} q_E + \mathrm{c}_{2\gamma} u_E \right) - 2 Z_O \mathrm{s}_O v_E \right\} \right]$$

$$\delta^* = \frac{1}{\eta} \frac{I_R}{I_T} = \frac{G_R + d H_R}{G_T + d H_T}, \qquad \eta = \frac{\eta_R T_R}{\eta_T T_T}, \qquad a = \frac{\delta^* G_T - G_R}{H_R - \delta^* H_T}$$

$$\boxed{\begin{gathered} \delta = \frac{1-a}{1+a} = \frac{\delta^* \left(G_T + H_T \right) - \left(G_R + H_R \right)}{\left(G_R - H_R \right) - \delta^* \left(G_T - H_T \right)} \\ \\ F_{11} \propto \eta H_R I_T - H_T I_R \end{gathered}}$$

Mathematical description – Calibration

$$\boldsymbol{I}_S = \eta_S \mathbf{M}_S \mathbf{R}_y \mathbf{C}(\varepsilon) \mathbf{M}_O \mathbf{F} \mathbf{M}_E \boldsymbol{I}_L =$$

$$I_S = \eta_S \langle \mathbf{A}_S | \mathbf{C}(\mathrm{x}45^\circ + \varepsilon) | \boldsymbol{I}_{in} \rangle$$

gain ratio Theory Measurement

$$\eta^*(\mathrm{x}45^\circ + \varepsilon) = \frac{\eta_R}{\eta_T} \frac{\langle \mathbf{A}_R | \mathbf{C}(\mathrm{x}45^\circ + \varepsilon) | \boldsymbol{I}_{in} \rangle}{\langle \mathbf{A}_T | \mathbf{C}(\mathrm{x}45^\circ + \varepsilon) | \boldsymbol{I}_{in} \rangle} = \frac{I_R(\mathrm{x}45^\circ + \varepsilon)}{I_T(\mathrm{x}45^\circ + \varepsilon)}$$

we need calibration factor $\eta = \frac{\eta_R T_R}{\eta_T T_T}$

we don't know: $\frac{\eta_R}{\eta_T}$ but we know: $\frac{T_T}{T_R}$

$\Rightarrow$

$$\boxed{K = \frac{\eta^*}{\eta} = \frac{T_T}{T_R} \frac{\langle \mathbf{A}_R | \mathbf{C}(\mathrm{x}45^\circ + \varepsilon) | \boldsymbol{I}_{in} \rangle}{\langle \mathbf{A}_T | \mathbf{C}(\mathrm{x}45^\circ + \varepsilon) | \boldsymbol{I}_{in} \rangle}}$$

analytically derived correction factor K for the measured gain ratio η^*

Δ90-calibration

$$\eta^*_{\Delta 90} \equiv \sqrt{\eta^*\left(+45^\circ+\varepsilon\right)\eta^*\left(-45^\circ+\varepsilon\right)} = \sqrt{\frac{I_R\left(+45^\circ+\varepsilon\right)}{I_T\left(+45^\circ+\varepsilon\right)}\cdot\frac{I_R\left(-45^\circ+\varepsilon\right)}{I_T\left(-45^\circ+\varepsilon\right)}}$$

e.g. Calibration with an ideal linear polariser before the receiving optics

$$\text{with } D_P = 1, D_T = +1, D_R = -1, \gamma = 0 \Rightarrow$$

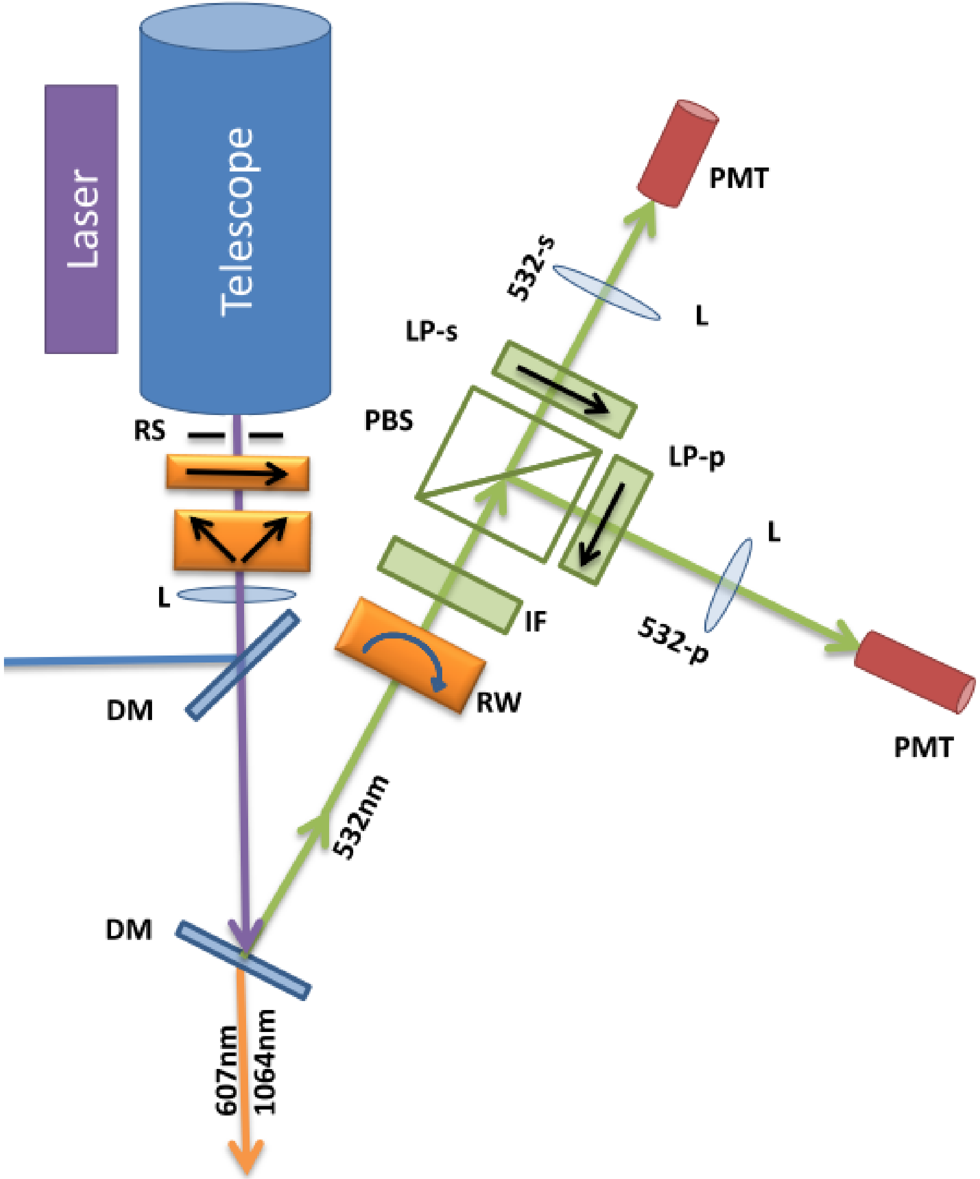
$$\frac{\eta^*\left(\mathrm{x}45^\circ+\varepsilon\right)}{\eta} = \frac{1-\mathrm{y}D_O}{1+\mathrm{y}D_O}\frac{1+\mathrm{xys}_{2\varepsilon}}{1-\mathrm{xys}_{2\varepsilon}}$$

$$\frac{\eta^*_{\Delta 90}}{\eta} = \frac{1-\mathrm{y}D_O}{1+\mathrm{y}D_O}$$

rotation ε error vanishes [AMT, 9, 4181–4255, 2016 Eq.(138)]

POLIS-6 ±45 rotation of the receiver optics => Δ90-calibration

Determination of receiver optis diattenuation with two calibrations



Amodeo et al., ACTRIS 3rd Joint WP2-WP20 workshop, Limassol, 26-29 November 2013

Determination of receiver optis diattenuation with two calibrations

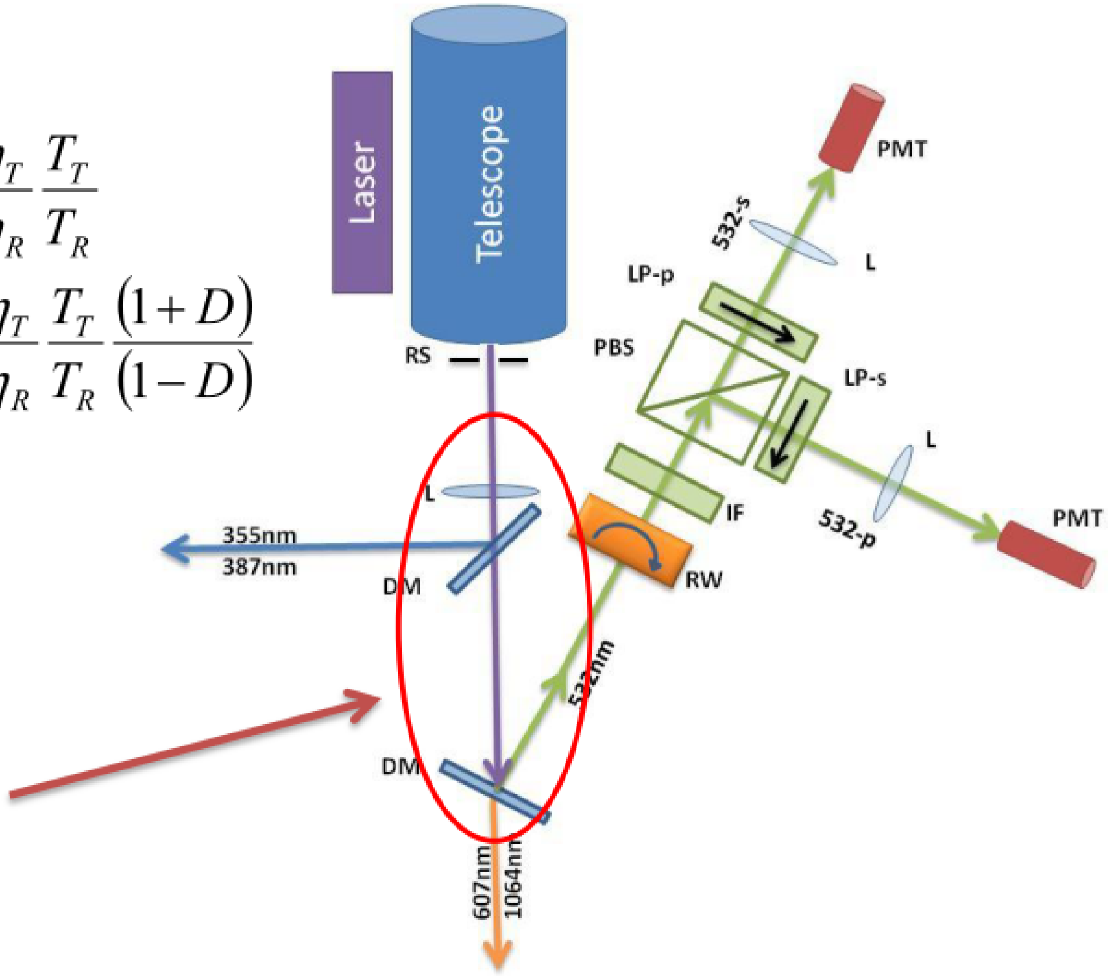
DIATTENUATION CALCULATION

$$\eta_1 = \sqrt{\eta^*(+45°)\cdot\eta^*(+45°)} = \frac{\eta_T}{\eta_R}\frac{T_T}{T_R}$$

$$\eta_2 = \sqrt{\eta^*(+45°)\cdot\eta^*(+45°)} = \frac{\eta_T}{\eta_R}\frac{T_T}{T_R}\frac{(1+D)}{(1-D)}$$

D: diattenuation

$$\boxed{D = \frac{\left(\frac{\eta_1}{\eta_2}-1\right)}{\left(\frac{\eta_1}{\eta_2}+1\right)} = -0.055 \pm 0.003}$$



Determination of GHK-parameters and error calculation

$$\boldsymbol{I}_S = \eta_S \ \mathbf{M}_S \ \mathbf{R}_y \ \mathbf{R}_\varepsilon \ \mathbf{M}_O \ \mathbf{F} \ \mathbf{M}_E \ \boldsymbol{I}_L$$

$\eta_{R,T}$	$T_{R,T}$			T_O	F_{11}	T_E	I_L	unpolarised transmission
					a		b	polarisation parameter / degree of linear polarization
	$D_{R,T}$			D_O		D_E		linear diattenuation parameter
	$\rho_{R,T}$							extinction ratio of cleaning pol-filters
				Δ_O		Δ_E		retardation
	$\phi_{R,T}$		ε	γ		β	α	module rotation

22 independent parameters + calibrator parameters (depending on type)

difficult to manage by hand (without errors)

=>

Freudenthaler, V., 2017:

Open source Python code for polarization related error analysis of aerosol lidar signals

https://bitbucket.org/iannis_b/atmospheric_lidar_ghk

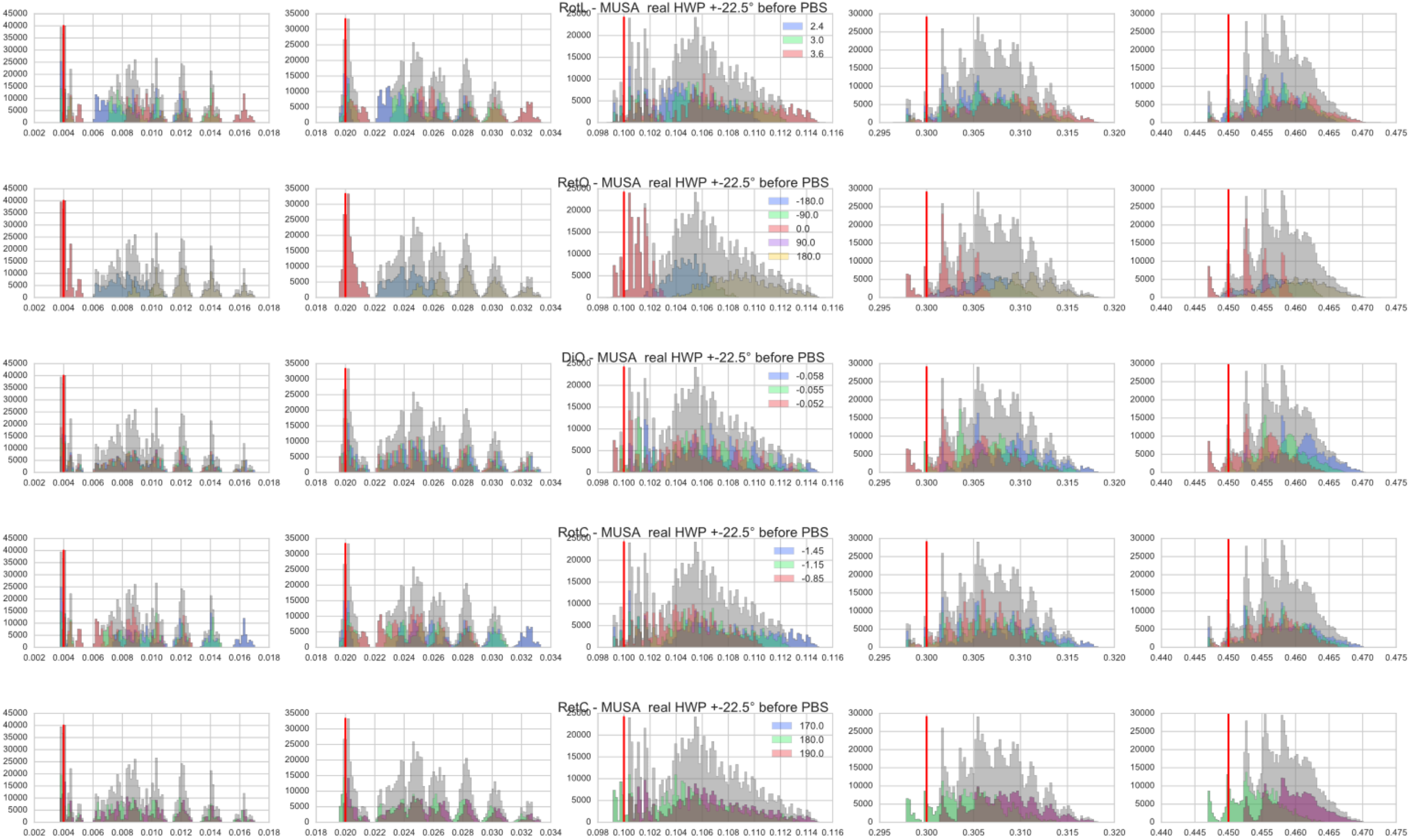
atmospheric_lidar_ghk   input file   example

```
# Do you want to calculate the errors? If not, just the GHK-parameters are determined.
Error_Calc = True

# Header to identify the lidar system
#   MUSA configuration http://www.atmos-meas-tech-discuss.net/amt-2015-339/amt-2015-339.pdf Table 5, 532 xcg xpg
EID = "po"                 # Earlinet station ID
LID = "MUSA"    # Additional lidar ID (short descriptive text)
print("    Lidar system :", EID, ", ", LID)

# --- IL Laser IL and +-Uncertainty
DOLP, dDOLP, nDOLP = 1.0, 0.00,  0    #degree of linear polarization; default 1
RotL, dRotL, nRotL    = 3.0, 0.6,     1     #alpha; rotation of laser polarization in degrees; default 0

# --- ME Emitter and +-Uncertainty
DiE, dDiE, nDiE = 0.,      0.00,      0     # Diattenuation
TiE        = 1.       # Unpolarized transmittance
RetE, dRetE, nRetE    = 0.,      180.0,     0     # Retardance in degrees
RotE, dRotE, nRotE    = 0.,      0.0, 0     # beta: Rotation of optical element in degrees

# --- MO Receiver Optics including telescope
DiO,  dDiO, nDiO      = -0.055, 0.003,     1
TiO                   = 0.9
RetO, dRetO, nRetO    = 0.,      180.0,     2
RotO, dRotO, nRotO    = 0.,      0.1, 0     #gamma

# +++++ PBS MT Transmitting path defined with TS, TP, PolFilter extinction ratio ERaT, and +-Uncertainty
# --- PBS
TP,   dTP, nTP        = 0.95,     0.01,     1
TS,   dTS, nTS        = 0.001, 0.001, 1
RetT, dRetT, nRetT = 0.,   180.,0 # Retardance in degrees
#   --- Pol.Filter behind transmitted path of PBS
ERaT, dERaT, nERaT = 0.001, 0.001,    1 # Extinction ratio
RotaT, dRotaT, nRotaT = 0.,     3., 1 # Rotation of the pol.-filter in degrees
TiT = 0.5 * (TP + TS)
DiT = (TP-TS)/(TP+TS)
DaT = (1-ERaT)/(1+ERaT)
TaT = 0.5*(1+ERaT)
```

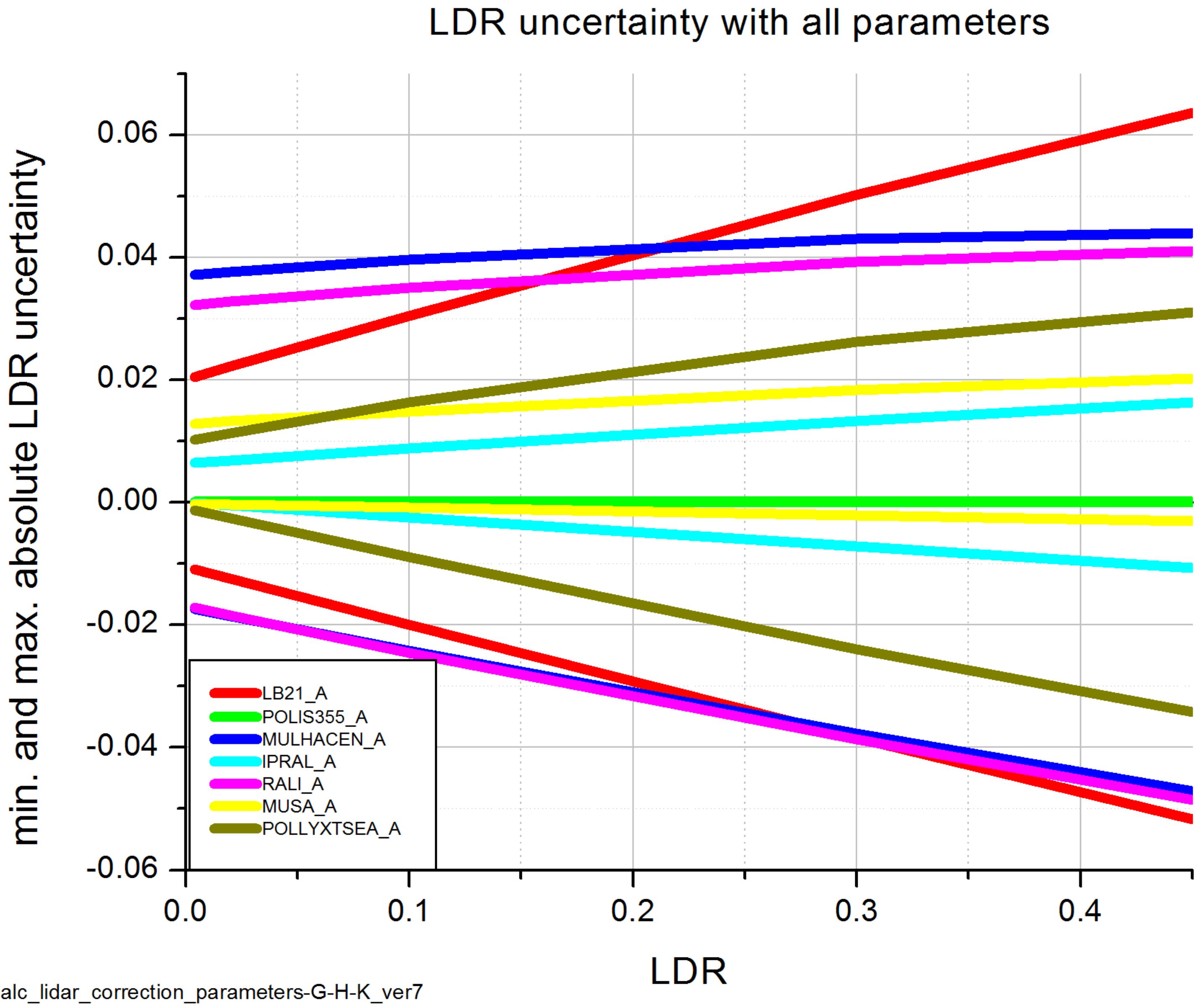
atmospheric_lidar_ghk   input file   example   continued

```
# --- Parallel signal detected in the transmitted channel => Y = 1, or in the reflected channel => Y = -1
Y = -1.
# --- Calibrator Location
LocC = 4 #location of calibrator: 1 = behind laser; 2 = behind emitter; 3 = before receiver; 4 = before PBS
# --- Calibrator Type used; defined by matrix values below
# Type of calibrator: 1 = mechanical rotator; 2 = hwp rotator (fixed retardation); 3 = linear polarizer;
# 4 = qwp; 5 = circular polarizer; 6 = real HWP calibration +-22.5°
TypeC = 6
# --- MC Calibrator
if TypeC == 1:  #mechanical rotator
     DiC, dDiC, nDiC = 0.,      0.,   0
     TiC = 1.
     RetC, dRetC, nRetC    = 0.,      0.,   0
     RotC, dRotC, nRotC    = -2.3, 0.1,    1    #constant calibrator offset epsilon
     # Rotation error without calibrator: if False, then epsilon = 0 for normal measurements
     RotationErrorEpsilonForNormalMeasurements = True #    is in general True for TypeC == 1 calibrator
elif TypeC == 2:   # HWP rotator
     DiC, dDiC, nDiC = 0.,      0.,   0
     TiC = 1.
     RetC, dRetC, nRetC    = 180., 0.,     0
     #NOTE: use here twice the HWP-rotation-angle
     RotC, dRotC, nRotC    = -2.3, 0.1,    1    #constant calibrator offset epsilon
     RotationErrorEpsilonForNormalMeasurements = True #    is in general True for TypeC == 2 calibrator
elif TypeC == 3:   # linear polarizer calibrator
     DiC, dDiC, nDiC = 1.0,     0.,   0    # ideal 1.0
     TiC = 0.5  # ideal 0.5
     RetC, dRetC, nRetC    = 0.,      0.,   0
     RotC, dRotC, nRotC    = 0.0,     0.1,  1    #constant calibrator offset epsilon
     RotationErrorEpsilonForNormalMeasurements = False    #    is  False for TypeC == 3 calibrator
elif TypeC == 4:   # QWP calibrator
     DiC, dDiC, nDiC = 0.0,     0.,   0    # ideal 1.0
     TiC = 1.0  # ideal 0.5
     RetC, dRetC, nRetC    = 90.,     0.,   0
     RotC, dRotC, nRotC    = 0.0,     0.1,  1    #constant calibrator offset epsilon
     RotationErrorEpsilonForNormalMeasurements = False    #    is  False for TypeC == 4 calibrator
elif TypeC == 6:   # real half-wave plate calibration at +-22.5°  => rotated_diattenuator_X22x5deg.odt
     DiC, dDiC, nDiC = 0.,      0.,   0
```

atmospheric_lidar_ghk   output   (ANACONDA Spyder => IPhython console)



atmospheric_lidar_ghk output (ANACONDA Spyder => IPhython console)



calc_lidar_correction_parameters-G-H-K_ver7

Errors in Atmos. Meas. Tech., 9, 4181–4255, 2016

- Eq. (116) reads
$$\frac{\eta^{*}_{\Delta 90}}{\eta}=\sqrt{\frac{\left(1+\mathrm{y}D_O D_R\right)^2 i_E{}^2-\left(D_O+\mathrm{y}D_R\right)^2\left(q_E \mathrm{s}_{2\varepsilon}-\mathrm{h}u_E \mathrm{c}_{2\varepsilon}\right)^2}{\left(1+\mathrm{y}D_O D_T\right)^2 i_E{}^2-\left(D_O+\mathrm{y}D_T\right)^2\left(q_E \mathrm{s}_{2\varepsilon}-\mathrm{h}u_E \mathrm{c}_{2\varepsilon}\right)^2}}$$
Correct is:
$$\frac{\eta^{*}_{\Delta 90}}{\eta}=\sqrt{\frac{\left(1+\mathrm{y}D_O D_R\right)^2 i_E{}^2-a^2\left(D_O+\mathrm{y}D_R\right)^2\left(q_E \mathrm{s}_{2\varepsilon}-\mathrm{h}u_E \mathrm{c}_{2\varepsilon}\right)^2}{\left(1+\mathrm{y}D_O D_T\right)^2 i_E{}^2-a^2\left(D_O+\mathrm{y}D_T\right)^2\left(q_E \mathrm{s}_{2\varepsilon}-\mathrm{h}u_E \mathrm{c}_{2\varepsilon}\right)^2}}$$

- Supplement S.1, first paragraph, line 10 ff reads: "*Light polarised with its E-vector on the x-axis, i.e. parallel to the incident plane of the PBS in Fig. 7,...*"
Correct is: "*Light polarised with its E-vector on the x-axis, i.e. parallel to the incident plane of the PBS in Fig. 7a,...*"

- Supplement S.1, first paragraph, line 14 reads: "*…. which means that the incident plane in Fig. 7 is the x-z-plane).*"
Correct is: "*…. which means that the incident plane in Fig. 7a is the x-z-plane).*"

- Supplement S.1, Fig. 8 caption reads: " *Reflection of a Stokes vector.*"
Correct is " *Reflection of an E-vector.*"