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Price Effects on Compound Commodities*

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Abstract

We examine the effect of simultaneous price changes on the total demand for a group of goods, which we call a compound commodity. Specifically, we consider unit and proportional cost components (e.g., taxes, transportation costs) imposed on compound commodities. If the unit cost is positive, then the proportional cost raises the relative price of the more expensive good, and thus induces substitution towards the less expensive good within this group. Then, the substitution effect of the proportional cost for a compound commodity is non-negative if and only if the compound commodity and the other goods are, on average, not strongly substitutable.

Keywords: Commodity taxation; demand for a group of goods; Giffen goods; Slutsky equation; unit and proportional costs

JEL classification: *D*01; *D*11; *H*20

I. Introduction

In the classic demand theory, it is accepted as fundamentally true that the substitution effect of an increase in the price of a good always decreases the demand for that good. However, in practice, price changes frequently occur for a group of goods rather than for a single good. Therefore, it might be useful to analyze the substitution effects caused by simultaneous price changes for a group of goods. In particular, by considering two types of simultaneous, parallel price changes, we find that the above property for the demand of a single good does not generally hold for the demand for a group of goods in economically meaningful situations. Thus, we face a serious aggregation problem.

Clearly, aggregation is a significant and long discussed issue in economics, and it can be traced back to Hicks (1939), at least. When

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the total expenditure on a group of goods can be treated as a single good, this group is referred to as a composite commodity – a result well-known as the Hicks composite commodity theorem. The fundamental condition for this theorem to hold is that the prices of these goods change proportionally in such a way that the relative prices within this group are kept constant.[1] Yet, non-proportional price changes are, as we shall argue, a common phenomenon, even if such changes result from common cost components included in all of these prices. Furthermore, little is known about how the total demand (rather than the expenditure) for a group of goods is affected by simultaneous price changes, especially by those that alter relative prices. Here, the total demand for a group of goods represents the sum of all demands for goods that are similar and are measured by the same unit. We can think of this group of goods as different varieties of a particular good (e.g., brands of cigarettes), which can be naturally aggregated over these varieties. This type of aggregation over similar goods or varieties of some general good is omnipresent in science (e.g., in the work of economists and statisticians, as well as in real life): cheese, meat, fish, transport, accommodation, jewelry, etc., are all typically referred to as single goods but actually represent aggregates. In this paper, we refer to such a commodity that is composed of similar goods or varieties of some general good as a compound commodity. Here, in order to acknowledge the omnipresence of compound commodities and the fact that their components are typically subject to simultaneous price changes, we explore the effects of parallel, non-proportional price changes on the total demand for a group of goods, or on the demand for a compound commodity.

The basic idea of our arguments is as follows. A per unit cost added to the prices of two goods decreases the relative price of the more expensive good and hence leads to a relative increase in the compensated demand for that good. This observation was first made by Alchian and Allen (1964, pp. 74–75). Then, holding the unit cost unchanged, a proportional cost added to both net prices increases the relative price of the more expensive good and hence leads to a relative decrease in the compensated demand for that good.[2] This implication was suggested and tested by Hummels and Skiba (2004). In the literature on the Alchian–Allen theorem, both cost components are variously specified: the common unit cost component

[1]There are many studies that have formalized, generalized, and tested the composite commodity theorem (e.g., Samuelson, 1947; Katzner, 1970; Green, 1976; Deaton and Muellbauer, 1980; Carter, 1995; Lewbel, 1996; Moro, 2001; Davis, 2003).

[2]We focus on the case where the proportional cost is applied to the net price, but we can equally think of a proportional cost applied to the gross price (i.e., to the net price plus the unit cost). In some cases (e.g., commodity taxation), the latter specification seems to be applicable, and for this reason we briefly discuss the latter specification in Section V.

is interpreted as a per unit tax, a transportation fee, a wage (opportunity cost of leisure), etc., whereas the common proportional cost component is interpreted as an ad valorem tax, an iceberg transportation cost, a markup rate, etc.[3]

While these substitution effects for relative demand provide valuable insights, the substitution effects induced by changes in the common unit and proportional cost components for the demand for a compound commodity also have important implications. It is already known that the effect of an increase in the unit cost on the compensated demand for a compound commodity (i.e., the substitution effect of a unit cost for a compound commodity) is non-positive (see Silberberg and Suen, 2001, pp. 335–336). In this paper, we show that the effect of an increase in a proportional cost component on the compensated demand for a compound commodity (i.e., the substitution effect of a proportional cost for a compound commodity) is basically opposite to that of a unit cost. In particular, both substitution effects are unambiguously opposed when a compound commodity and the other goods are, on average, not strongly substitutable, in a sense that will become clear in Proposition 3.

This general implication also has practical relevance for tax policies, for example. Governments often aim at reducing total consumption of demerit goods, such as cigarettes and alcohol, or environmentally harmful products,[4] or at increasing total consumption of merit goods, such as sporting activities, or consumption of cultural goods.[5] We show that the choice of the type of a tax rate – a unit tax (specific tax) or an ad valorem tax – is crucial in this respect: while an increase in the unit tax decreases the compensated demand for a compound commodity, an increase in the ad valorem tax increases the compensated demand for a compound commodity. Consequently, provided that income effects can be neglected, a government aiming at reducing (or increasing) the demand for a compound commodity always has a suitable tax instrument at hand, assuming that both unit taxes and ad valorem taxes are institutionally feasible.

The positive substitution effect of the proportional cost is surprising at first sight, but it is in fact quite intuitive. Suppose that there are only two goods, both of which are subject to unit and proportional costs. Then,

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[3]Further details and development on this topic can be found in, for example, Minagawa and Upmann (2015).

[4]For example, the World Health Organization Framework Convention on Tobacco Control requires its parties to implement tax and price policies to contribute to health objectives aimed at reducing tobacco consumption (World Health Organization, 2015). From this perspective, the total quantity of cigarettes (rather than the quantity for a particular brand of cigarettes) consumed by an individual represents the relevant quantity.

[5]The concept of (de)merit goods was introduced by Musgrave (1959, pp. 13–14).

because an increase in the proportional cost increases the relative price of the more expensive good, a consumer substitutes the less expensive good for the more expensive good. However, because of the exchange rate, the consumer then gives up one unit of the more expensive good in exchange for receiving more than one unit of the less expensive good. Therefore, this substitution leads to an increase in the sum of the compensated demands for the two goods.

In a second step, we extend the analysis by taking into account income effects. We show that as unit and proportional cost components induce basically opposite substitution effects, the presence of income effects strengthens the negative effect of the unit cost component, but mitigates the positive effect of the proportional cost component, provided that the compound commodity is a normal good. We also demonstrate that the effects of both the unit cost and the proportional cost on the demand for a compound commodity can be additively decomposed into a substitution and an income effect. Thus, this can be written in the form of the familiar Slutsky equation. We have two versions of the Slutsky equation for a compound commodity.

This result sheds new light on the law of demand, especially on the Giffen phenomenon.[6] In this field, it is common to treat a good with varieties (e.g., high-quality apples and low-quality apples) as a single, aggregate commodity (apples), and then to consider price effects on this aggregate commodity. In this case, the Giffen phenomenon can arise only when the commodity is an inferior good. However, this analysis disregards the intra-group substitution among varieties. Our analysis shows that if we take account of such an interaction, inferiority is no longer necessary for a commodity (with varieties) to have an upward-sloping demand curve. Thus, the concept of the Slutsky equation for a compound commodity can revise the traditional understanding of price effects.

To illustrate this, consider the following example. Following the approach of Hildenbrand (1983), Baruch and Kannai (2002) provided a case where a necessary condition for violating the law of demand (i.e., a condition that the average income effect term is negative) is satisfied; in particular, they found that in Japan in the 1980s, shochu, a Japanese distilled alcoholic beverage, meets this condition (i.e., shochu might be a Giffen good in the standard sense).[7] Yet, because shochu is a generic commodity with varieties, it is natural to consider simultaneous price changes for the varieties of shochu, unless we consider one variety of shochu only. With several varieties of shochu, it matters whether, for each pair of those

[6]For a survey on the law of demand, see Jerison and Quah (2008).
[7]For a recent study on Giffen goods, see also Jensen and Miller (2008).

varieties, the relative price of a more expensive shochu for a less expensive shochu increases or decreases.

We can apply this shochu example in the framework of commodity taxation. Given the plausible assumption that alcohol is a complement to leisure, the untaxed good, our results show that the substitution effect of an increase in an ad valorem tax for shochu is positive.[8] Then, together with the empirical inferiority of shochu, mentioned above, it follows that the demand curve for shochu must be upward-sloping when price changes are caused by a common ad valorem tax. This result can be applied to any inferior good (with varieties) complementary to leisure, and thus it seems to be helpful both for an explanation of the Giffen phenomenon and for the design of tax policies.

As a conclusion, we suggest the importance and necessity of taking into account intra-group substitution (i.e., substitution between varieties of a generic commodity), as it can lead to qualitatively different implications from the conventional analysis of price effects where those varieties are treated as a single (aggregate) commodity.

The rest of this paper is organized as follows. In Section II, we set up the model. Then, in Sections III and IV, we analyze the substitution effects of both the unit cost and the proportional cost on the demand for a compound commodity, and the income effects, respectively. In Section V, we apply these results to commodity taxation. Finally, we conclude in Section VI.

II. Setting

Consider the standard expenditure minimization problem with n goods. We denote the commodity vector by $\boldsymbol{x} \equiv (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n_+$. Suppose that the first k goods, collected in the vector $\boldsymbol{x}_k \equiv (x_1, \ldots, x_k)$, are categorized into a fixed group of goods (all other $n - k$ goods are collected in the vector $\boldsymbol{x}_{-k} \equiv (x_{k+1}, \ldots, x_n)$). We can think of the group $\{1, \ldots, k\}$ as a set of similar goods, the quantities of which can all be expressed in the same unit of measurement; for example, different varieties of the same basic good can be suitably aggregated in this way. Accordingly, we can refer to such a group of goods as a compound commodity.[9] All goods of this group are subject to a unit cost $t \geqq 0$ and a proportional cost $\tau > 0$.[10] Let $p_i > 0$, $i \in \{1, \ldots, n\}$, be the (net) prices of the goods, and

[8] See Proposition 3 and Corollary 3 for the conditions for this claim to hold.

[9] As the case of $k = 1$ is trivial, we focus only on the case of $k \geqq 2$. For convenience, we speak of a compound commodity even in the case of $k = n$, where all goods are categorized into a single group.

[10] Although we simply call τ a proportional cost, it is in fact a factor of proportional cost, and an additional cost is imposed on the group of goods only when $\tau > 1$.

let $\boldsymbol{p} \equiv (p_1, \ldots, p_n)$ denote the corresponding price vector. Then, the gross consumer prices are given by $q_i \equiv \tau p_i + t$ for $i \in \{1, \ldots, k\}$ and $q_i \equiv p_i$ for $i \in \{k+1, \ldots, n\}$, or in vector notation, $\boldsymbol{q}_k \equiv (q_1, \ldots, q_k) \equiv \tau \boldsymbol{p}_k + t\mathbf{1}$ and $\boldsymbol{q}_{-k} \equiv (q_{k+1}, \ldots, q_n) \equiv \boldsymbol{p}_{-k}$, where $\mathbf{1}$ denotes the all-ones vector $(1, \ldots, 1)$ of suitable length. (Similarly, we write $\mathbf{0}$ to denote $(0, \ldots, 0)$.) Using these definitions, we write the net and gross price vectors as $\boldsymbol{p} \equiv (\boldsymbol{p}_k, \boldsymbol{p}_{-k})$ and $\boldsymbol{q} \equiv (\boldsymbol{q}_k, \boldsymbol{q}_{-k})$, respectively. Assume that a consumer has continuous, locally non-satiated, and strictly convex preferences, represented by a utility function $u : \mathbb{R}^n_+ \to \mathbb{R}$. We then denote the compensated (or Hicksian) demand function, as a function of the prices $\boldsymbol{q}$ and the utility level v, by $\boldsymbol{x}^*(\boldsymbol{q}, v)$, and we define the compensated demand function for the compound commodity as $z^*(\boldsymbol{q}, v) \equiv \sum_{i=1}^{k} x_i^*(\boldsymbol{q}, v) \equiv \boldsymbol{x}_k^*(\boldsymbol{q}, v) \cdot \mathbf{1}$, where the dot $(\cdot)$ is used to indicate the (Euclidian) inner product. Subsequently, we focus on interior solutions, assuming the continuous differentiability of the compensated demand functions.

We are interested in the effects of the unit cost t and the proportional cost τ on the compensated demand for the compound commodity, $z^*[\boldsymbol{q}(t, \tau), v]$. Our interest is thus different from the composite commodity theorem of Hicks (1939) where a "composite commodity" refers to the total expenditure on a group of commodities, $\boldsymbol{p}_k \cdot \boldsymbol{x}_k$ (in our notation, and provided that $t = 0$).[11] To avoid confusion, we use the term "compound commodity" for $\sum_{i=1}^{k} x_i \equiv \boldsymbol{x}_k \cdot \mathbf{1}$, as opposed to, but in analogy with, the notion of a composite commodity.

III. Substitution Effects

We define the expenditure function by $E(\boldsymbol{q}, v) \equiv \boldsymbol{q} \cdot \boldsymbol{x}^*(\boldsymbol{q}, v)$, or as a function of the unit cost t, the proportional cost τ, the prices of the $n-k$ non-grouped commodities $\boldsymbol{q}_{-k}$, and the utility level v by $\hat{E}(t, \tau, \boldsymbol{q}_{-k}, v) \equiv E(\tau \boldsymbol{p}_k + t\mathbf{1}, \boldsymbol{p}_{-k}, v)$.[12] Similarly, we rewrite the compensated demand function as $\hat{\boldsymbol{x}}^*(t, \tau, \boldsymbol{q}_{-k}, v)$ with $\hat{x}_i^*(t, \tau, \boldsymbol{q}_{-k}, v) \equiv x_i^*(\tau \boldsymbol{p}_k + t\mathbf{1}, \boldsymbol{p}_{-k}, v)$. Thus, we denote the compensated demand function for the compound commodity as $\hat{z}^*(t, \tau, \boldsymbol{q}_{-k}, v) \equiv \hat{\boldsymbol{x}}_k^*(t, \tau, \boldsymbol{q}_{-k}, v) \cdot \mathbf{1}$, and we denote the total net (or "before tax") expenditure on the compound commodity as the function $\hat{e}^*(t, \tau, \boldsymbol{q}_{-k}, v) \equiv \boldsymbol{p}_k \cdot \hat{\boldsymbol{x}}_k^*(t, \tau, \boldsymbol{q}_{-k}, v)$. Then, the function $\hat{E}$ has the same properties as the expenditure function E has.[13]

[11] For this theorem, see the literature listed in footnote 1, and also Silberberg and Suen (2001, pp. 332–335).

[12] For convenience, we drop the net prices $(p_1, \ldots, p_k)$ as explicit arguments. Also, in view of Propositions 1 and 2 and Remark 1, this practice turns out to be quite natural.

[13] For the standard properties of the expenditure function, see, for example, Propositions 3.E.2 and 3.G.1 in Mas-Colell *et al*. (1995).

Proposition 1. *The expenditure function* $\hat{E}$ *has the following properties: (a)* $\hat{E}(t, \tau, \boldsymbol{q}_{-k}, v)$ *is non-decreasing in* $(t, \tau, \boldsymbol{q}_{-k})$*; (b)* $\hat{E}(t, \tau, \boldsymbol{q}_{-k}, v)$ *is homogeneous of degree* 1 *in* $(t, \tau, \boldsymbol{q}_{-k})$*; (c)* $\hat{E}(t, \tau, \boldsymbol{q}_{-k}, v)$ *is concave in* $(t, \tau, \boldsymbol{q}_{-k})$*; (d)* $\partial \hat{E}(t, \tau, \boldsymbol{q}_{-k}, v)/\partial t = \hat{z}^*(t, \tau, \boldsymbol{q}_{-k}, v)$, $\partial \hat{E}(t, \tau, \boldsymbol{q}_{-k}, v)/\partial \tau = \hat{e}^*(t, \tau, \boldsymbol{q}_{-k}, v)$*, and* $\partial \hat{E}(t, \tau, \boldsymbol{q}_{-k}, v)/\partial q_j = \hat{x}_j^*(t, \tau, \boldsymbol{q}_{-k}, v), \quad j = k+1, \ldots, n$.

Proof: See the Appendix. □

Using Proposition 1, we can derive fundamental properties of the compensated demand for a compound commodity, which correspond to the properties of the compensated demand for a single good.[14] To this end, we define the substitution matrix by

$$\boldsymbol{S}(t, \tau, \boldsymbol{q}_{-k}, v) \equiv \begin{pmatrix} \dfrac{\partial \hat{z}^*}{\partial t} & \dfrac{\partial \hat{z}^*}{\partial \tau} & \dfrac{\partial \hat{z}^*}{\partial q_{k+1}} & \cdots & \dfrac{\partial \hat{z}^*}{\partial q_n} \\ \dfrac{\partial \hat{e}^*}{\partial t} & \dfrac{\partial \hat{e}^*}{\partial \tau} & \dfrac{\partial \hat{e}^*}{\partial q_{k+1}} & \cdots & \dfrac{\partial \hat{e}^*}{\partial q_n} \\ \dfrac{\partial \hat{x}_{k+1}^*}{\partial t} & \dfrac{\partial \hat{x}_{k+1}^*}{\partial \tau} & \dfrac{\partial \hat{x}_{k+1}^*}{\partial q_{k+1}} & \cdots & \dfrac{\partial \hat{x}_{k+1}^*}{\partial q_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial \hat{x}_n^*}{\partial t} & \dfrac{\partial \hat{x}_n^*}{\partial \tau} & \dfrac{\partial \hat{x}_n^*}{\partial q_{k+1}} & \cdots & \dfrac{\partial \hat{x}_n^*}{\partial q_n} \end{pmatrix}. \quad (1)$$

Proposition 2. *Demand for the compound commodity and the associated substitution matrix have the following properties: (a)* $\hat{z}^*(t, \tau, \boldsymbol{q}_{-k}, v)$ *and* $\hat{e}^*(t, \tau, \boldsymbol{q}_{-k}, v)$ *are homogeneous of degree* 0 *in* $(t, \tau, \boldsymbol{q}_{-k})$*; (b)* $\boldsymbol{S}(t, \tau, \boldsymbol{q}_{-k}, v)$ *is symmetric; (c)* $\boldsymbol{S}(t, \tau, \boldsymbol{q}_{-k}, v)$ *is negative semidefinite; (d)* $\boldsymbol{S}(t, \tau, \boldsymbol{q}_{-k}, v)\,(t, \tau, \boldsymbol{q}_{-k}) = \boldsymbol{0}$.

Proof: (a) This part follows from Proposition 1(b) and (d). (b) For the $(1, 2)$- and the $(2, 1)$-elements of $\boldsymbol{S}$, we use Proposition 1(d) and Young's theorem to obtain $\partial \hat{z}^*/\partial \tau = \partial^2 \hat{E}/\partial \tau \partial t = \partial^2 \hat{E}/\partial t \partial \tau = \partial \hat{e}^*/\partial t$; and by similar steps we show the symmetry of the rest of $\boldsymbol{S}$. (c) This follows from Proposition 1(c) and (d). (d) Given Proposition 2(a) and the fact that $\hat{x}_i^*(t, \tau, \boldsymbol{q}_{-k}, v)$ is homogeneous of degree 0 in $(t, \tau, \boldsymbol{q}_{-k})$, applying Euler's theorem yields the desired result. □

Remark 1. *Propositions 1 and 2 suggest that our model amounts to a model with* $2 + n - k$ *goods, with quantities z, e, and* $\boldsymbol{x}_{-k} \equiv (x_{k+1}, \ldots, x_n)$*, and prices*

[14]For the standard properties of the compensated demand for a single good, see, for example, Propositions 3.E.3(i) and 3.G.2 in Mas-Colell *et al*. (1995).

t, τ, and $\boldsymbol{q}_{-k}$, respectively. This is because the function $\hat{E}$, which can also be expressed as

$$\hat{E}(t,\tau,\boldsymbol{q}_{-k},v) \equiv t\hat{z}^{*}(t,\tau,\boldsymbol{q}_{-k},v) + \tau\hat{e}^{*}(t,\tau,\boldsymbol{q}_{-k},v) + \boldsymbol{q}_{-k}\cdot\hat{\boldsymbol{x}}^{*}_{-k}(t,\tau,\boldsymbol{q}_{-k},v),$$

is an expenditure function.

We see from Proposition 2(c) that the diagonal entries of the substitution matrix $\boldsymbol{S}$ are non-positive. This means, in particular, that the substitution effect of the unit cost component t on the compound commodity is non-positive (i.e., $\partial\hat{z}^{*}/\partial t \leqq 0$). This corresponds to the result presented in Silberberg and Suen (2001, pp. 335–336).

Proposition 2(d) yields the relationship between the substitution effects of the unit cost component and of the proportional cost component.

Corollary 1. *The Hicks' "third law" for a compound commodity is*

$$\frac{\partial\hat{z}^{*}(t,\tau,\boldsymbol{q}_{-k},v)}{\partial t}t + \frac{\partial\hat{z}^{*}(t,\tau,\boldsymbol{q}_{-k},v)}{\partial\tau}\tau + \sum_{j=k+1}^{n}\frac{\partial\hat{z}^{*}(t,\tau,\boldsymbol{q}_{-k},v)}{\partial q_j}q_j = 0. \qquad (2)$$

Assume that $t > 0$. Define the elasticity of the compensated demand for the compound commodity with respect to the unit cost t to be

$$\varepsilon^{*}_{zt} \equiv \frac{t}{\hat{z}^{*}(t,\tau,\boldsymbol{q}_{-k},v)}\frac{\partial\hat{z}^{*}(t,\tau,\boldsymbol{q}_{-k},v)}{\partial t},$$

and define the elasticity of the compensated demand for the compound commodity with respect to the proportional cost τ to be

$$\varepsilon^{*}_{z\tau} \equiv \frac{\tau}{\hat{z}^{*}(t,\tau,\boldsymbol{q}_{-k},v)}\frac{\partial\hat{z}^{*}(t,\tau,\boldsymbol{q}_{-k},v)}{\partial\tau}.$$

In addition, defining the elasticity of the compensated demand for the compound commodity with respect to the gross price q_j to be

$$\varepsilon^{*}_{zj} \equiv \frac{q_j}{\hat{z}^{*}(t,\tau,\boldsymbol{q}_{-k},v)}\frac{\partial\hat{z}^{*}(t,\tau,\boldsymbol{q}_{-k},v)}{\partial q_j},$$

we also obtain an elasticity version of Corollary 1.

Corollary 2. *The Hicks' "third law" for a compound commodity in elasticity form is*

$$\varepsilon^{*}_{zt} + \varepsilon^{*}_{z\tau} + \sum_{j=k+1}^{n}\varepsilon^{*}_{zj} = 0. \qquad (3)$$

Now we are ready to state our main result.

Proposition 3. *The substitution effect of the proportional cost component* τ *on the compound commodity is non-negative (i.e.,* $\varepsilon^*_{z\tau} \geqq 0$*), if and only if the compound commodity and the other* $n - k$ *goods are, on average, not strongly substitutable (i.e.,* $\sum_{j=k+1}^{n} \varepsilon^*_{zj} \leqq -\varepsilon^*_{zt}$*). In particular, the latter condition always holds when all goods are subject to the unit and proportional cost components (i.e.,* $k = n$*).*

Proof: From Corollary 2, we have $\varepsilon^*_{z\tau} \geqq 0$ if and only if $\varepsilon^*_{zt} + \sum_{j=k+1}^{n} \varepsilon^*_{zj} \leqq 0$. In particular, when all goods are subject to the unit and proportional cost components (i.e., $k = n$), the summation $\sum_{j=k+1}^{n} \varepsilon^*_{zj}$ vanishes, and hence the latter condition always holds. □

Corollary 3. *Suppose that the unit cost component* t *is zero. Then, we have* $\varepsilon^*_{z\tau} \geqq 0$ *if and only if* $\sum_{j=k+1}^{n} \varepsilon^*_{zj} \leqq 0$*. In particular, we have* $\varepsilon^*_{z\tau} = 0$ *for the case of* $k = n$*.*

According to Proposition 3, the substitution effect of the unit cost t is opposite to the substitution effect of the proportional cost τ if the compound commodity and the other goods are, on average, not strongly substitutable. These results imply, for example, that a higher unit cost component results in a lower demand for the compound commodity, while a higher proportional cost component results in a higher demand for the compound commodity.

The positive substitution effect of the proportional cost τ seems to be a paradox at first glance, but it is not. It is in fact quite intuitive. To see this, suppose that $k = n = 2$ and that $p_1 > p_2$. Then, an increase in t reduces the relative price of the more expensive good, q_1/q_2, while an increase in τ raises this price, provided that $t > 0$:[15]

$$\frac{\partial}{\partial t}\frac{q_1}{q_2}(t,\tau) = -\frac{\tau(p_1 - p_2)}{(q_2)^2} < 0, \qquad \frac{\partial}{\partial \tau}\frac{q_1}{q_2}(t,\tau) = \frac{t(p_1 - p_2)}{(q_2)^2} > 0. \tag{4}$$

Consequently, an increase in τ makes the consumer substitute good 2 for good 1. However, because good 1 is now more expensive than good 2, implying that the marginal rate of substitution at the original optimal choice was greater than 1, the consumer requires more than one unit of good 2 to be compensated for sacrificing one unit of good 1. Therefore, this substitution leads to an increase in the sum of the compensated demands

[15]Note that when $k = n$, the substitution effects of the unit cost t and of the proportional cost τ on the compound commodity both vanish, either if the unit cost is zero or if the net prices p_i are all identical. This is simply because in these cases neither cost component affects relative prices.

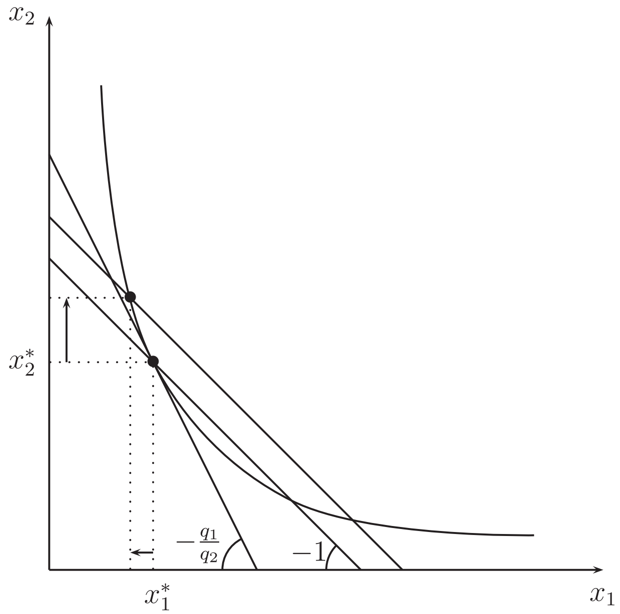

Fig. 1. Effects of an increase in the relative price q_1/q_2 (> 1) on the compensated demand (x_1^*, x_2^*)

for goods 1 and 2. From a geometric point of view, as the budget line (and thus the indifference curve at the optimal choice) is steeper than the iso-demand ($x_1 + x_2$ = constant) line, an increase in q_1/q_2, caused by an increase in τ, leads to a North–West shift of (x_1^*, x_2^*) above the iso-demand line passing through the original optimal choice – that is, a shift on to a higher iso-demand line (see Figure 1).

IV. Income Effects

Consider the standard utility maximization problem. We denote the ordinary (or Marshallian) demand function, as a function of the prices $\boldsymbol{q}$ and the (money) income $I > 0$, by $\boldsymbol{x}^o(\boldsymbol{q}, I)$, and we define the ordinary demand function for the compound commodity as $z^o(\boldsymbol{q}, I) \equiv \boldsymbol{x}_k^o(\boldsymbol{q}, I) \cdot \mathbf{1}$.

In what follows, we assume the continuous differentiability of the ordinary demand functions, and we analyze the effects of the unit cost t and of the proportional cost τ on the ordinary demand for the compound commodity, $z^o(\boldsymbol{q}(t, \tau), I)$. Similarly to the case of the compensated demand function, we rewrite the ordinary demand function as $\hat{\boldsymbol{x}}^o(t, \tau, \boldsymbol{q}_{-k}, I)$ with

$$\hat{x}_i^o(t, \tau, \boldsymbol{q}_{-k}, I) \equiv x_i^o(\tau \boldsymbol{p}_k + t\mathbf{1}, \boldsymbol{p}_{-k}, I).$$

Using this, we write the ordinary demand function for the compound commodity as

$$\hat{z}^o(t, \tau, \boldsymbol{q}_{-k}, I) \equiv \hat{\boldsymbol{x}}_k^o(t, \tau, \boldsymbol{q}_{-k}, I) \cdot \mathbf{1}.$$

Moreover, we denote the level of the ordinary demand for the compound commodity by $z^o \equiv \boldsymbol{x}_k^o \cdot \mathbf{1}$ and the level of total net (or "before tax") expenditure on the compound commodity by $e^o \equiv \boldsymbol{p}_k \cdot \boldsymbol{x}_k^o$.

Proposition 4. *The Slutsky equations for a compound commodity are*

$$\frac{\partial \hat{z}^o(t, \tau, \boldsymbol{q}_{-k}, I)}{\partial t} = \frac{\partial \hat{z}^*(t, \tau, \boldsymbol{q}_{-k}, v)}{\partial t} - \frac{\partial \hat{z}^o(t, \tau, \boldsymbol{q}_{-k}, I)}{\partial I} z^o, \quad (5)$$

$$\frac{\partial \hat{z}^o(t, \tau, \boldsymbol{q}_{-k}, I)}{\partial \tau} = \frac{\partial \hat{z}^*(t, \tau, \boldsymbol{q}_{-k}, v)}{\partial \tau} - \frac{\partial \hat{z}^o(t, \tau, \boldsymbol{q}_{-k}, I)}{\partial I} e^o. \quad (6)$$

Proof: From a familiar duality result, we obtain the identity

$$\hat{z}^*(t, \tau, \boldsymbol{q}_{-k}, v) \equiv \hat{z}^o[t, \tau, \boldsymbol{q}_{-k}, \hat{E}(t, \tau, \boldsymbol{q}_{-k}, v)].$$

Differentiating both sides of this identity with respect to t yields

$$\frac{\partial \hat{z}^*}{\partial t} = \frac{\partial \hat{z}^o}{\partial t} + \frac{\partial \hat{z}^o}{\partial I}\frac{\partial \hat{E}}{\partial t}.$$

Applying Proposition 1(d) and using the above identity, we obtain the first equation. Similar steps yield the second equation. □

Remark 2. *Suppose that the consumer has a positive endowment in at least one good, that is, $I \equiv \boldsymbol{q} \cdot \boldsymbol{\omega} + m$, where ω_i are non-negative initial endowments for the n goods and m is a positive money income. Then, Proposition 4 can be rewritten to accommodate endowment effects in addition to income effects.*[16]

As the substitution effect of the unit cost is non-positive by Proposition 2, Proposition 4 shows that if the ordinary demand for the compound commodity reacts positively to an increase in income – that is, if the compound commodity is a normal good (or if the considered commodities are, on average, normal goods) – the total effect of the unit cost is negative. In this sense, the substitution effect of the unit cost is reinforced if we take income effects into account.

If, however, the ordinary demand for the compound commodity reacts negatively to an increase in income – that is, if the compound commodity

[16]For the Slutsky equation with endowment effects, see, for example, Cornwall (1984, p. 749).

is inferior (or if the considered commodities are, on average, inferior goods – the total effect can be either negative or positive. In the latter case, the compound commodity can be regarded as a Giffen good.[17]

Basically, the presence of income effects does not reinforce but mitigates the effects of the proportional cost component characterized by Corollaries 1 and 2, provided that the compound commodity is a normal good. Because a proportional cost component might have a positive effect on the compensated demand for the compound commodity (Proposition 3), and higher costs reduce the income, the income effect counteracts the substitution effect if the compound commodity is a normal good. Thus, the total effect can be positive even when the compound commodity is a normal good. This suggests that inferiority is not necessary for a compound commodity to have an upward-sloping demand curve. Only if the compound commodity is an inferior good does the income effect work in parallel with the substitution effect, provided that the latter is positive.

Assume that $t > 0$. Define the elasticity of the ordinary demand for the compound commodity with respect to the unit cost t to be

$$\varepsilon_{zt} \equiv \frac{t}{\hat{z}^o(t, \tau, \boldsymbol{q}_{-k}, I)} \frac{\partial \hat{z}^o(t, \tau, \boldsymbol{q}_{-k}, I)}{\partial t}),$$

and the income elasticity of the ordinary demand for the compound commodity to be

$$\varepsilon_{zI} \equiv \frac{I}{z^o(\boldsymbol{q}, I)} \frac{\partial z^o(\boldsymbol{q}, I)}{\partial I}.$$

Moreover, define the share of the unit cost for the compound commodity in income to be $\eta_{zt} \equiv tz^o/I$. Similarly, define the elasticity of the ordinary demand for the compound commodity with respect to the proportional cost τ to be

$$\varepsilon_{z\tau} \equiv \frac{\tau}{\hat{z}^o(t, \tau, \boldsymbol{q}_{-k}, I)} \frac{\partial \hat{z}^o(t, \tau, \boldsymbol{q}_{-k}, I)}{\partial \tau}.$$

[17]When $k = n$, at least one element of the (finite) series $S_i \equiv (\partial/\partial I)\sum_{j=1}^{i} x_j^o$, $i = 1, 2, \ldots, n$ must be positive. This can be verified as follows. Suppose that $p_1 \geqq p_2 \geqq \cdots \geqq p_n$. Set $\Delta p_i \equiv \tau(p_i - p_{i+1})$, $i = 1, 2, \ldots, n-1$. Then, q_i ($i = 1, 2, \ldots, n-1$) can be expressed as $\sum_{j=i}^{n-1} \Delta p_j + q_n$. Now, differentiating both sides of the budget constraint $\sum_{i=1}^{n} q_i x_i^o \equiv I$ with respect to I yields $\sum_{i=1}^{n} q_i(\partial x_i^o/\partial I) = 1$, which can be written as $\sum_{i=1}^{n-1} \Delta p_i \sum_{j=1}^{i}(\partial x_j^o/\partial I) + q_n \sum_{j=1}^{n}(\partial x_j^o/\partial I) = 1$. As $\Delta p_i \geqq 0$ and $q_n > 0$, all of $\sum_{j=1}^{i}(\partial x_j^o/\partial I)$, $i = 1, 2, \ldots, n$ cannot be non-positive. This means that when $k = n = 2$, if the sum of the ordinary demands for goods 1 and 2 reacts negatively to an increase in income, then the more expensive good cannot be an inferior good.

Moreover, defining the (gross) share of the proportional cost for the compound commodity in income by $\eta_{z\tau} \equiv \tau e^o / I$,[18] we obtain the formulae of Proposition 4 in elasticity form.

Corollary 4. *The Slutsky equations for a compound commodity in elasticity form are*

$$\varepsilon_{zt} = \varepsilon^*_{zt} - \eta_{zt}\,\varepsilon_{zI}, \qquad \varepsilon_{z\tau} = \varepsilon^*_{z\tau} - \eta_{z\tau}\,\varepsilon_{zI}. \tag{7}$$

These equations suggest that the difference between the uncompensated and compensated elasticities of the demand for a compound commodity with respect to each cost will be small, *ceteris paribus*, if either the share of each cost for the compound commodity in income is small,[19] or if the income elasticity of the ordinary demand for the compound commodity is small.

V. Application

There is a conventional wisdom in economics that taxing a good reduces its consumption (see Cournot, 1838, Chapter VI); more specifically, the substitution effect of an increase in the price of a good, caused by an increase in taxes, always decreases the demand for that good (see Stiglitz, 2000, Chapter 19). While it might be believed that the standard tax effects on a single good carry over to the case of a group of closely related goods, our results in the previous sections suggest that such a presumption is misguided.

To see this, consider excise (or commodity) taxes on specific goods such as gasoline, cigarettes, alcohol, etc. Most of these goods have different varieties (e.g., grades of gasoline, brands of cigarettes, qualities of wine, etc.), and these varieties are, from an economic perspective, closely related in consumption (usually they are close substitutes); also, they are typically measured in the same units, and can thus easily be subsumed. For these reasons, different varieties are commonly treated in the same way by tax laws, and are therefore subject to the same tax rates. Accordingly, changes in the rules of taxation (i.e., in the tax rates) affect all of those varieties in parallel. Then, by specifying the unit and proportional cost components in our price specification $q_i = \tau p_i + t$ as unit and ad valorem taxes respectively, we can see that all the results in the previous sections directly apply.

[18]The term "gross" means that the numerator τe^o includes the net expenditure on the compound commodity. See also footnote 10.
[19]This is analogous to a common argument of a vanishing magnitude of the income term in the Slutsky equation. For this argument, see also Vives (1987).

The tax specification is applicable, for example, in the case of cigarettes in low- and middle-income countries where the base for ad valorem excise tax is the manufacturer's/distributor's price (World Health Organization, 2015, p. 110).[20] Yet, in some other countries, the base of ad valorem excise tax is the retail price (including a unit excise tax). Then, the tax specification can be expressed in our notation as $q_i = \tau(p_i + t)$. In this case, the substitution effect of the ad valorem tax on the compound commodity coincides with that in the special case where there is no common unit tax in our original tax specification.[21] Thus, for this alternative tax specification, Corollary 3 applies. The substitution effect of the ad valorem tax on the compound commodity is non-negative if and only if the sum of the substitution effects of all other goods on the compound commodity is non-positive; in particular, the substitution effect of the ad valorem tax is zero when all goods are subject to the ad valorem taxes. While the latter implication that ad valorem taxes induce no substitution effects is well known in economics (see also footnote 15), the former implication seems to be unrecognized even though it might be significant, for example, for designing tax policies for goods complementary to leisure (e.g., shochu, as mentioned in the Introduction).

VI. Conclusion

If a single good is subject to both unit and proportional costs, increases in the unit cost and in the proportional cost both lead to an increase in the relative price of that good; therefore, the own-substitution effects of the unit cost and of the proportional cost are both non-positive. However, if a group of goods (i.e., a compound commodity) is subject to both unit and proportional costs, these cost components affect relative prices differently: for any given pair of those goods, an increase in the unit (respectively, proportional) cost decreases (respectively, increases) the relative price of a more expensive good. Accordingly, the substitution effects of the unit cost and of the proportional cost for the compound commodity can have conflicting implications.

As the substitution effect of the unit cost for a compound commodity is non-positive, the compound commodity has the property of a single good; hence, regarding the compound commodity as a single (aggregate)

[20]See the price specification of cigarettes, given by equation (2) in World Health Organization (2015, p. 109), in which an ad valorem excise tax is applied to the price excluding a unit excise tax, as is ours. The World Health Organization (2015, p. 84) also reports that 94 countries impose a uniform tax on all tobacco products without variations in rates.

[21]This is because we can write $q_i = \tau p'_i$ by setting that $p'_i \equiv p_i + t$.

commodity is justified, as argued by Silberberg and Suen (2001). However, as the substitution effect of the proportional cost for a compound commodity can be non-negative, a compound commodity generally does not have the properties of a single good. Therefore, when variations in proportional cost components are considered, a compound commodity cannot simply (i.e., without qualification) be treated as a single (aggregate) commodity.

Consequently, when we consider the effects of simultaneous price changes on a compound commodity, it is imperative to inspect the induced changes in the relative prices. Ignoring this inspection and simply analyzing the compound commodity as if it were a single (aggregate) commodity might lead to seriously flawed conclusions and interpretations. Actually, price changes affect the relative prices even if these prices share a common cost component: a change in the common cost components, such as taxes or transportation costs, modifies relative prices. In particular, changes in unit or ad valorem taxes imposed on all varieties of a generic commodity affect the relative prices among these varieties. Then, the properties for the demand of a single good do not generically hold for the demand for the compound commodity. Thus, we should be careful about aggregation when analyzing price effects on a generic commodity with varieties.

Appendix

Proof of Proposition 1:

(a) Suppose that $t'' \geqq t'$, $\tau'' \geqq \tau'$, and $\boldsymbol{q}''_{-k} \geqq \boldsymbol{q}'_{-k}$.[22] Letting $\boldsymbol{q}'_k \equiv \tau' \boldsymbol{p}_k + t'\mathbf{1}$ and $\boldsymbol{q}''_k \equiv \tau'' \boldsymbol{p}_k + t''\mathbf{1}$, we obtain $\boldsymbol{q}''_k \geqq \boldsymbol{q}'_k$, and thus $\boldsymbol{q}'' \geqq \boldsymbol{q}'$. Then,

$$\begin{aligned} \hat{E}(t'', \tau'', \boldsymbol{q}''_{-k}, v) &\equiv \boldsymbol{q}'' \cdot \boldsymbol{x}^*(\boldsymbol{q}'', v) \geqq \boldsymbol{q}' \cdot \boldsymbol{x}^*(\boldsymbol{q}'', v) \geqq \boldsymbol{q}' \cdot \boldsymbol{x}^*(\boldsymbol{q}', v) \\ &\equiv \hat{E}(t', \tau', \boldsymbol{q}'_{-k}, v), \end{aligned}$$

where the first inequality follows from $\boldsymbol{q}'' \geqq \boldsymbol{q}'$; and the second, from expenditure minimization.

(b) Let $\theta > 0$. Using $\theta \boldsymbol{q} \equiv \theta\tau\boldsymbol{p} + \theta t\mathbf{1}$, we obtain

$$\hat{E}(\theta t, \theta\tau, \theta\boldsymbol{q}_{-k}, v) = \theta\boldsymbol{q} \cdot \boldsymbol{x}^*(\theta\boldsymbol{q}, v) = \theta\boldsymbol{q} \cdot \boldsymbol{x}^*(\boldsymbol{q}, v) = \theta\hat{E}(t, \tau, \boldsymbol{q}_{-k}, v),$$

where the second equality follows from the homogeneity of degree zero of Hicksian demands.

(c) Set $t^\theta \equiv \theta t' + (1-\theta)t''$, $\tau^\theta \equiv \theta\tau' + (1-\theta)\tau''$, and $\boldsymbol{q}^\theta_{-k} \equiv \theta\boldsymbol{q}'_{-k} + (1-\theta)\boldsymbol{q}''_{-k}$ where $\theta \in [0,1]$. Letting $\boldsymbol{q}^\theta_k \equiv \tau^\theta \boldsymbol{p}_k + t^\theta \mathbf{1}$ and using the notations

[22] We write $\boldsymbol{q}''_{-k} \geqq \boldsymbol{q}'_{-k}$ whenever $q''_j \geqq q'_j$, $j = k+1, \ldots, n$.

$\boldsymbol{q}'_k$ and $\boldsymbol{q}''_k$ as in (a), we obtain $\boldsymbol{q}^\theta_k \equiv \theta\boldsymbol{q}'_k + (1-\theta)\boldsymbol{q}''_k$. By expenditure minimization, we have

$$\hat{E}(t', \tau', \boldsymbol{q}'_{-k}, v) \leqq \boldsymbol{q}' \cdot \boldsymbol{x}^*(\boldsymbol{q}^\theta, v)$$

and

$$\hat{E}(t'', \tau'', \boldsymbol{q}''_{-k}, v) \leqq \boldsymbol{q}'' \cdot \boldsymbol{x}^*(\boldsymbol{q}^\theta, v).$$

Thus,

$$\theta\hat{E}(t', \tau', \boldsymbol{q}'_{-k}, v) + (1-\theta)\hat{E}(t'', \tau'', \boldsymbol{q}''_{-k}, v) \leqq \theta\boldsymbol{q}' \cdot \boldsymbol{x}^*(\boldsymbol{q}^\theta, v)$$
$$+(1-\theta)\boldsymbol{q}'' \cdot \boldsymbol{x}^*(\boldsymbol{q}^\theta, v) = \boldsymbol{q}^\theta \cdot \boldsymbol{x}^*(\boldsymbol{q}^\theta, v) = \hat{E}(t^\theta, \tau^\theta, \boldsymbol{q}^\theta_{-k}, v).$$

(d) The Lagrangian for the expenditure minimization problem is

$$L(\boldsymbol{x}, \lambda; t, \tau, \boldsymbol{q}_{-k}, v) \equiv \boldsymbol{q} \cdot \boldsymbol{x} + \lambda[v - u(\boldsymbol{x})].$$

Applying the envelope theorem, we obtain

$$\frac{\partial \hat{E}}{\partial t} = \frac{\partial L}{\partial t} = \hat{\boldsymbol{x}}^*_k \cdot \mathbf{1} = \hat{z}^*$$

and

$$\frac{\partial \hat{E}}{\partial \tau} = \frac{\partial L}{\partial \tau} = \boldsymbol{p}_k \cdot \hat{\boldsymbol{x}}^*_k = \hat{e}^*.$$

□

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