ESSAYS IN ECONOMIC AND POLITICAL CHOICE

Thesis by

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PREFACE

This thesis is a collection of essays about choice. It examines the determinants and outcomes of economic and political actions in different arenas. Whether the analysis concerns the determinants and outcomes of the different strategies of a water reservoir manager, the centralized management of an oil field, a voter or a party, there is a unifying theme which runs through all of these. This theme is the "rationality" of choice. In this thesis rationality is interpreted to mean that actions and strategies are chosen in order to maximize certain economic or political indices which are related to the objectives of the actors.

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ABSTRACT

This thesis is a collection of essays. Each essay is a whole chapter. I have prepared the following separate abstracts for each chapter.

Chapter 1

CHANCE CONSTRAINED DYNAMIC PROGRAMMING MODEL OF WATER RESERVOIR WITH JOINT PRODUCTS

This chapter presents two models for reservoir management. Model 1 is a dynamic programming formulation which only allows exporting and importing of water to correct excesses or deficiencies caused by the optimizing decisions. It shows that if the one-period profit is a function of the water releases, water stock, and the physical capacity of the reservoir, then optimal decisions regarding water releases and capacity exist and are unique both for the N-period and the infinite period problems. Moreover, the model shows that if the profit function is separable and linear in the releases, then the optimal decision rule is linear and the long-run distribution for the stock of water exists and is a simple form. The model in the Appendix of this chapter is a deterministic equivalent chance constraint formulation in which an approximation for the long-run distribution of the water stock is determined.

Chapters 2 and 3

ON THE RETROSPECTIVE EFFECT OF ECONOMIC CONDITIONS IN CONGRESSIONAL ELECTIONS: A SIMULTANEOUS EQUATION MODEL APPROACH

In these chapters a simultaneous equation model is employed to investigate the relative effects of: 1) economic conditions, 2) incumbency, and 3) recognition of the presidential party's candidate on the dual decisions of the individual to participate and vote in congressional elections. My finding is decidedly negative regarding the effect of economic conditions on both turnout and voting for the presidential party. I have, however, established the relative effects of both incumbency and recognition.

Chapter 4

THE DECLINE OF COMPETITION IN CONGRESSIONAL ELECTIONS: MAYHEW MAY STILL BE RIGHT

Several theories have been advanced to explain the reduction in the number of competitive congressional districts during the past decade. Among these is Mayhew's theory, which attributes the reduction to the increasing control of campaign resources by incumbents. Ferejohn presents evidence which casts doubt on Mayhew's thesis. In this chapter, Ferejohn's evidence is examined within the framework of a simultaneous equation model. I conclude that Mayhew's thesis, although bloodied by Ferejohn's attack, is still very much alive.

Chapter 5

LEFTIST IDEOLOGICAL SHIFTS IN ARAB CONTEMPORARY POLITICS: A SPATIAL THEORY APPROACH

This chapter presents a formal model of some aspects of leftist ideological shifts in contemporary Arab politics. In particular it focuses on the effects of information costs and the cost of ideological vagueness on the competitive parties of the left. A spatial model is used to examine analytically an observation originally advanced by the Baath Arab Socialist party. This observation states that both the cost of vagueness and the inability to meet the high information costs inherent to an articulate ideology may have been the factors which caused the adoption of Marxism Leninism by some leftist groups. Certain reasonable assumptions generated results consistent with the Baath observation.

Chapter 6

THE EFFECT OF A RANDOM PLANNING HORIZON ON PRODUCTION AND INVESTMENT FOR PETROLEUM RESERVOIR -- A NOTE ON KULLER'S AND CUMMING'S MODEL

There have been several formulations of models for crude oil production which tried to identify the elements of user cost and show their effect on production and investment decisions. In this chapter, previous results are extended by incorporating the uncertainty regarding the date of arrival of the backstop technology in the model. This uncertainty adds a new element to the user cost identified previously and is shown to affect the production and investment decisions.

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CHAPTER 1

CHANCE CONSTRAINED DYNAMIC PROGRAMMING MODEL OF WATER RESERVOIR WITH JOINT PRODUCTS

1.1 Introduction

Historically, rivers have been the focus of many international conflicts, especially in the arid and semi-arid areas of the world. However, within a particular river basin, water was relatively abundant and there was generally enough to meet the various needs of the basin's population. Water usage was limited mainly to human consumption and irrigation.

The growth in population and rising level of industrialization in many arid and semi-arid parts of the world are increasing the demands for water. However, no corresponding change in the world supply of river water occurred. It has become a scarce resource, and active planning for water utilization is under way.

An important aspect of this planning is the distribution of the benefits of the rivers over time and among uses and users. Increasingly the construction of large reservoirs is becoming the vehicle to achieve and integrate these diverse objectives. Very few reservoirs are normally dedicated to achieve a single objective, at least as far as their "feasibility studies" are concerned. Invariably, irrigation, power generation, flood control and recreation are among the objectives listed for any dam project. That does not mean there is no hierarchy imposed on these objectives by the planner. In fact, there may exist one or two prime objectives. The absence of explicit statements on this hierarchy has become a political expedient to appease the various groups affected by the construction of the dam. Model builders have reflected this hierarchy by directly including some variables in the objective function and others are formulated as constraints.

Some of these constraints are "soft," in the sense that they could be violated at a cost. This cost is dictated by the demand of the planner for these constraints to hold. The following analysis will focus on irrigation and power generation with soft constraints on the stock of water in the reservoir. These soft constraints reflect a trade-off between flood control and recreation purposes on the one hand and salinity control in the downstream on the other.

There are two design considerations in the process of reservoir construction: 1) the optimal reservoir size, and 2) the optimal operating rule of the reservoir. Although many scholars [12, 15] have previously pointed out that the two considerations cannot be separated, many attempt to separate the dual decisions of optimal size and optimal operating policy. The model in this paper will recognize the "jointness" of the decisions and treat them in a unified manner within the framework of dynamic programming.

An often neglected aspect in the design of impounding reservoirs in arid and semi-arid regions where evaporation losses are significant is the trade-off between two opposing considerations:

1. There are benefits from assuring a more regular flow of water and hence a "better" distribution of the river benefit over time and among users and uses.

2. There are also costs imposed by the evaporation of the impounded water in the reservoir. These costs are significant. As Quirk and Burness point out [22] for a minor river such as the Colorado with an annual mean runoff of 13.5 million acre-feet per year, evaporation losses from existing reservoirs have already reached as high as 1.5 million acre-feet per year.

To produce an outflow pattern satisfying a given economic objective, the preceding trade-off is taken into consideration in ascertaining the relationship between the hydrology of a stream and the optimal decision rule. The optimal size of the reservoir which is consistent with the chosen operating rule will be derived. Moreover, the long-run distribution of the water stock in the reservoir when the profit function from the reservoir operation has a special form will be derived.

Uncertainty will be revealed as the single most important factor affecting the optimal design and operation of a reservoir. Formally, this uncertainty may be reflected in the objective function, the constraints, or both. Consider the situation where the reservoir manager is maximizing an n-period downstream profit function $\pi(y)$ of water releases $y = (y_1, y_2, \ldots, y_n)$. This maximization is subject to non-negativity and minimum pool level (R) constraints in every period i of the form:

$$r_i(x_{i-1} - y_i) + e_i \geq R \qquad (1)$$

where x_{i-1} is the stock of water at the start of period i,

y_i is the release at the start of period i (before e_i is observed),

e_i is the stochastic runoff in period i with known probability density function f_e, and

$1 - r_i$ is the evaporation loss in period i.

We can re-arrange (1) as follows:

$$r_i y_i \leq r_i x_{i-1} + e_i - R \qquad \forall\, i,\ i=1,\ldots,n$$

or in matrix form

$Ay \leq b$ where b_i is a function of the random variable e_i.

Thus the problem becomes that of:

$$\text{Max } \pi(y) \tag{2}$$

$$\text{Subject to } Ay \leq b = f(e) \tag{3}$$

$$y \geq 0 \tag{4}$$

where e, y, b: $n \cdot 1$ and A: $n \cdot n$.

There is a possibility that optimal decisions will lead to violation of the constraints because of very high or very low values of e. This is the basic problem posed by the nature of the random constraints.

At least three different types of characterizations are available in the optimization literature to cope with the random nature of the constraints. First, there is the penalty function approach [27] which introduces penalties for violating the random constraints. This is accomplished by adding the expected penalty costs to the objective function. For example, if there is a constant penalty cost $C_j > 0$ per unit violation of the jth constraint $a_j y \leq b_j$, and the violation of the constraint has a finite probability density function $\psi(z)$, then the total expected penalty cost is $CE[\psi(b - Ay)]$. The modified problem then becomes

$$\text{Max } \pi(y) - CE[\psi(b - Ay)], \text{ subject to } y \geq 0. \tag{5}$$

This method is actually related to two-stage programming under uncertainty [8].

The model in this paper uses a dynamic programming approach in conjunction with a penalty function. The penalty is a convex increasing function of the magnitude of the violations. These penalty costs differ from the fixed accounting costs employed by Askew [2, 3]. Accounting costs are never actually intended to be paid, but are merely devices to ensure optimal behavior by the management. Penalty costs in this model, however, are actually economic costs imposed on the manager to correct for stock deficiency or surplus which results from his decisions and the random flow of the river.

Second, there is the truncated distribution approach which interprets the inequalities $a_i'y \leq b_i$ $(i = 1,2,\ldots,m)$ as a truncation of the probability distribution of b_i. For example, Sengupta [28] uses the χ^2 distribution for a truncated normal.

Thirdly, there is the chance constrained characterization [5], [6] which puts a reliability interpretation on the constraint, such as

$$\text{prob } (b_i \geq a_i'y_i) \geq \lambda_i, \qquad 0 \leq \lambda_i \leq 1,\ i = 1,\ldots,m \tag{6}$$

by preassigning reliability (tolerance) measures λ_i up to which constraint violations are permitted. The λ_i can be varied parametrically to account for the different reliability levels. Alternatively, a reliability term can be added to the objective function and can be solved for an optimal set of λ_i's [24]. For example, the problem could be characterized as:

$$\text{Max } U(y,\lambda) = w_1 \pi(y) + w_2 \sum_{i=1}^{m} \log \lambda_i \tag{7}$$

$$\text{Subject to } y \geq 0,\ 0 \leq \lambda_i \leq 1,\ 0 \leq w_j \leq 1 \tag{8}$$

$$\text{and} \quad 1 - F_i(a_i'y) \geq \lambda_i \quad \forall\, i, i=1,2,.,.,.,m \tag{9}$$

where F_i is the cumulative distribution function of the random variable and w_j, $j = 1,2$ are weighting factors.

In the first version, where λ_i's are not derived optimally, the chance constraint is reduced to an equivalent deterministic constraint [6] by the use of the marginal distribution function of b_i: $\phi(b_i)$. The existence of a fractile $\overline{b}_i$ such that

$$P(b_i \geq a_i'y) \geq \lambda_i \quad \Longleftrightarrow \quad \overline{b}_i(1 - \lambda_i) \geq a_i'y \tag{10}$$

makes this reduction possible. To facilitate this transformation in the reservoir models, the optimal decision rule is restricted to the class of linear functions [16, 17, 19]. Additionally, it is sometimes assumed that the random variable is distributed normally or truncated normal at zero [7, 28].

Linear Decision-Rule and Chance Constraint

Essentially the linear decision rule is a device to facilitate the transformation of chance constraints into equivalent deterministic forms while avoiding a difficult convolution problem [9]. To illustrate this, consider the situation where, at any period p the starting stock of water is x_{p-1}, and the inflow and discharge is e_p and y_p, respectively. Then the continuity equation, assuming no evaporation losses, is

$$x_p = x_{p-1} + e_p - y_p. \tag{11}$$

The deterministic equivalent for a chance constraint of the form $P(x_p \leq x^u) \geq \alpha_1$, cannot be determined since the probability distribution of x_p is unknown even if the distribution of e_p is known. The linear decision rule, first used by Revelle et al. [23], defines x_p and y_p in terms of e_p by postulating that the optimal decision rule is of the form

$$y_p = x_{p-1} - a_p \quad \text{where } a_p \text{ is a decision variable.} \tag{12}$$

Since, from the continuity equation, $x_p = x_{p-1} + e_p - y_p$ then

$$x_p = e_p + a_p \tag{13}$$

and

$$y_p = e_{p-1} + a_{p-1} - a_p. \tag{14}$$

Since the distribution of e_p is known and a_p is a deterministic decision variable, (13) and (14) define the distribution of x_p and y_p. Hence, deterministic equivalents for the chance constraints:

$$P(x_p \leq x^u) \geq \alpha_1 \tag{15}$$

or

$$P(y_p \geq \bar{y}) \geq \alpha_2 \tag{16}$$

can be found.

There exists no guarantee that the linear decision rule is actually optimal among all possible classes of bounded functions. Moreover, the deterministic equivalent approach to the solution of chance constrained problems suffers from a number of shortcomings:

1. The continuity equation, used to develop the deterministic equivalent for the chance constraints and the steady state distribution of the stocks, ignores the overspill. The overspill occurs because the constraints may be violated in these models.

2. The net return function in these models does not reflect the probability that the constraints could be violated. Violation could occur as a result of the optimizing program, yet the net return is not affected. This condition raises a question regarding the incentive structure in these kinds of models.

3. The ad hoc specification of the reliability levels(λ_i's) raises objections from many planners and politicians. No decision-maker would risk making an explicit statement on reliability. The problem of the choice of the weight w_i given to the reliability term in the composite objective function persists, even if the choice of λ_i's is included within the optimizing framework, as in the model shown in (7-9). Nevertheless, chance constrained models are an important and prevalent class of water resource formulations. The model in the appendix serves as an introduction to this class, and its shortcomings shall be pointed out. However, the inclusion of this model is more than a mere expediency to the current literature on water resources, in that it assumes no linear decision rule for optimal policies. Moreover, it allows a bound to be developed on the long-run distribution of water stock, using the Chebychef inequality. This bound is developed without assuming a linear profit function. Such an assumption is necessary to get similar results in the dynamic programming framework, as will be presented in this paper.

1.2: A Dynamic Programming Model with Penalty Function

In this model the penalty function approach is utilized to account for the possibility of violating the constraints within a dynamic programming framework. These penalty costs are more than "accounting" costs used to insure that the dam manager takes the imposed "soft" constraints into consideration in arriving at his decision rule [2], [3]. They are costs actually paid by the dam manager for importing or exporting water to compensate for violating the constraints.

Although no a priori form for the optimal decision rule is imposed, it will become evident that this formulation implies a simple "one part" decision rule with "predictable" characteristics. Further, the linear decision rule, which implies constant optimal stock policy, will be shown as a consequence of certain restrictions on the form of the profit function.

It will also be shown that the long-run distribution for the stock of water in the reservoir exists and can be derived when the linear decision rule applies. This formulation will not suffer from the shortcomings of the chance constraints-deterministic equivalent approach. Moreover, the analysis will be expanded to include profits from the generation of electricity directly in the profit function and will be shown to affect the optimal policy and the optimal reservoir size.

The Objective Function

The manager of the reservoir is maximizing at every period, a concave objective function of the form $\pi(y,x-p)$ where $\pi_{12} \geq 0$, y is the release at the start of the period and x is the stock at the start of the period. The first argument of the objective function, y, reflects the payoff to agricultural downstream users from releases and the second argument reflects the payoff from power generated by the electrostatic head provided by the stock of water after releasing y.

The objective function, π, may be interpreted in various ways:

1. In a socialist economy, π might be the criterion function provided by the central planners. π is, then, the expected net social revenue which equals the expected revenue minus expected cost, all inputs and outputs being evaluated at prices set by the planners. The manager carries on the maximization procedure treating these prices as parameters. Under ideal conditions, the prices for inputs and outputs set by the central planners would be prices consistent with a Pareto optimum. In this special case, the optimality rule derived from the maximization procedure is also optimal from the point of view of welfare maximization. Under more realistic conditions, the criterion function simply reflects the central planners' evaluation of all the alternatives in the economy.

2. In a private economy operating under the appropriative doctrine, property rights to water are held by users of water. A possible situation is one in which the reservoir manager is instructed to operate so as to maximize aggregate expected profits, $E\pi$, of down stream users where $\pi = \{\sum_i \pi_i + \pi_E\}$ and π_i is the profit of downstream user i and π_E is the profit from power generation. Such a scenario is approximately the situation for the Colorado river where down-

stream users hold appropriative rights to the water in the reservoir and the Bureau of Reclamation operate the reservoir system for them under rules that derive from the Supreme Court decision in the famous Arizona v. California case (1963). Note that maximization of aggregate profits of downstream users is generally inconsistent with Pareto optimality, particularly when there is market power, e.g., as in the case of the Imperial Valley Irrigation District, a major force in the winter fruit farm market of the U.S., and the largest user of Colorado River water. The situation gets worse if there are externalities in the agricultural and power markets or if there are other imperfections in these markets.

3. π may be interpreted as the payoff in terms of social welfare associated with operating the reservoir. In this case π is the total expected surplus which equals expected consumer's surplus plus expected producers' surplus. The use of total expected surplus involves the usual difficulties of partial equilibrium welfare economics. Such problems include the need to use compensated demand curves, aggregating areas under demand curves over all consumers, the interactions with other markets and the like. There are further complications posed by the multi-periods nature of the problem: the lack of contingent claims markets to internalize uncertainty with respect to prices of future inputs and outputs means that we have the added problem of dealing with expectations involving diverse subjective probability distributions. Finally, if there are many reservoirs taking only the output of this particular reservoir into consideration in measuring social welfare

is inappropriate. In this case, we are only considering the output of a part of the industry rather than the whole. This leads to special problems of measuring consumers' surplus.

4. π may be the utility of the reservoir manager over profits from the operation of the dam. The concavity of π introduces risk aversion directly in the analysis. No social welfare argument may be made from this interpretation unless we consider this reservoir as part of a competitive market and no imperfections in any input and output markets. In this case, the usual classical welfare arguments applies to the economy and efficiency and unbiasedness are assured.

We pointed out that interpretations of the properties and results in this chapter differ according to how π is being interpreted. However, we shall show that from a technical point of view all that is needed to derive the formal results are concavity and/or linearity, and separability and/or the nonnegatively of the second mixed partials of the objective function.

The Model

The manager of the reservoir is maximizing at every stage[1] p, $1 \leq p \leq n$, a profit function $\pi(y_p, x_p - y_p)$, concave in both its arguments such that $\pi_{12} \geq 0$. The maximization is subject to an upper constraint x^u and a lower constraint x^m on the reservoir storage level.

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1 Following dynamic programming tradition, p is counted in reverse order from the terminal point.

The optimization is conducted as follows:

1. The manager observes the reservoir level, x_p, at the start of the period.

2. He calculates the optimal release in the period y_p^*, $0 \leq y_p^*(x_p) \leq x_p$, taking into consideration the following factors:

   a. The one period objective function[2] $[\pi(y_p, x_p - y_p) - c(\bar{x})]$ where $c(\bar{x})$ is the strictly convex annualized cost of construction;

   b. The costs of violating the upper and lower constraints $c_1(z)$ and $c_2(z)$. Each cost is related, respectively, to the cost of disposing or importing water to compensate for excesses or deficiencies in water storage;

   c. The probability distribution of the inflow $\phi_e(\cdot)$;

   d. The evaporation rate k.

3. He implements the optimizing decision y_p^* by releasing water from the reservoir.

4. Toward the end of the period p, the manager has enough information to observe the inflow e_p. Then he makes the following decisions:

   If, as a result of his decision, the water level in the reservoir falls below x^m, he imports water at the cost of $c_2(z)$ to make up for the deficiency (z). He then starts

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[2] When the profit function is separable, it will be expressed as: $g(y) + h(x)$.

period $(p+1)$ with a water stock equal to x^m. $c_2(z)$ is assumed to be a convex and increasing function of z with $c_2(0) = 0$ and $c_2'(0)$ is finite and positive. This assumption holds whether a constant or increasing net price for water is assumed. The case of rising net price of water is being considered to account for the increasing difficulty of importing larger amounts of water from further locations. See Figure 1. If, however, the optimizing decision results in water stock exceeding x^u, the manager disposes of the excess water (z) at the cost of $c_1(z)$. He then starts the next period $(p + 1)$ with a water stock equal to x^u. $c_1(z)$ is assumed to be a convex function of z with $c_1(0) = 0$ and $c_1'(0)$ finite. This is consistent with a situation where the export price of water net of transportation cost is constant or decreasing because of the increasing difficulty of marketing larger quantities of water. The net export price may eventually be negative. See Figure 2. The sequence of events and decisions are illustrated in Figure 3.

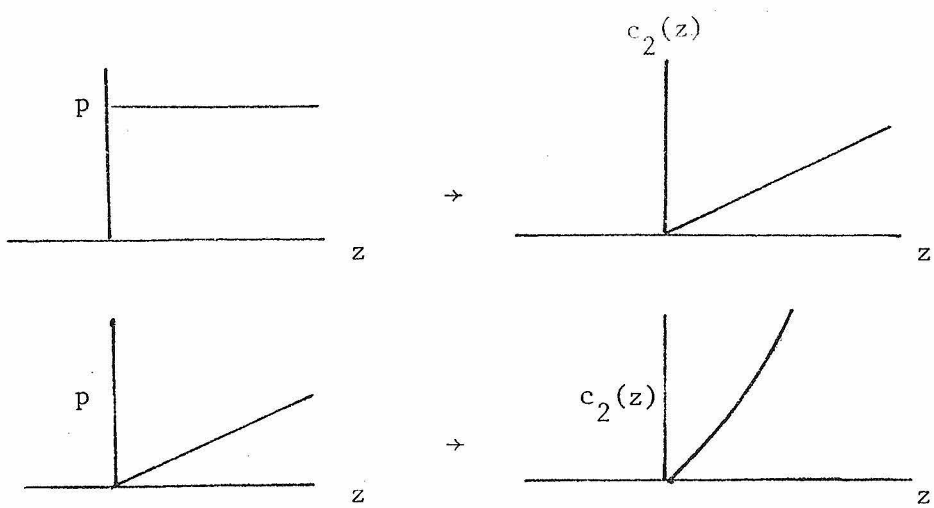

Figure 1

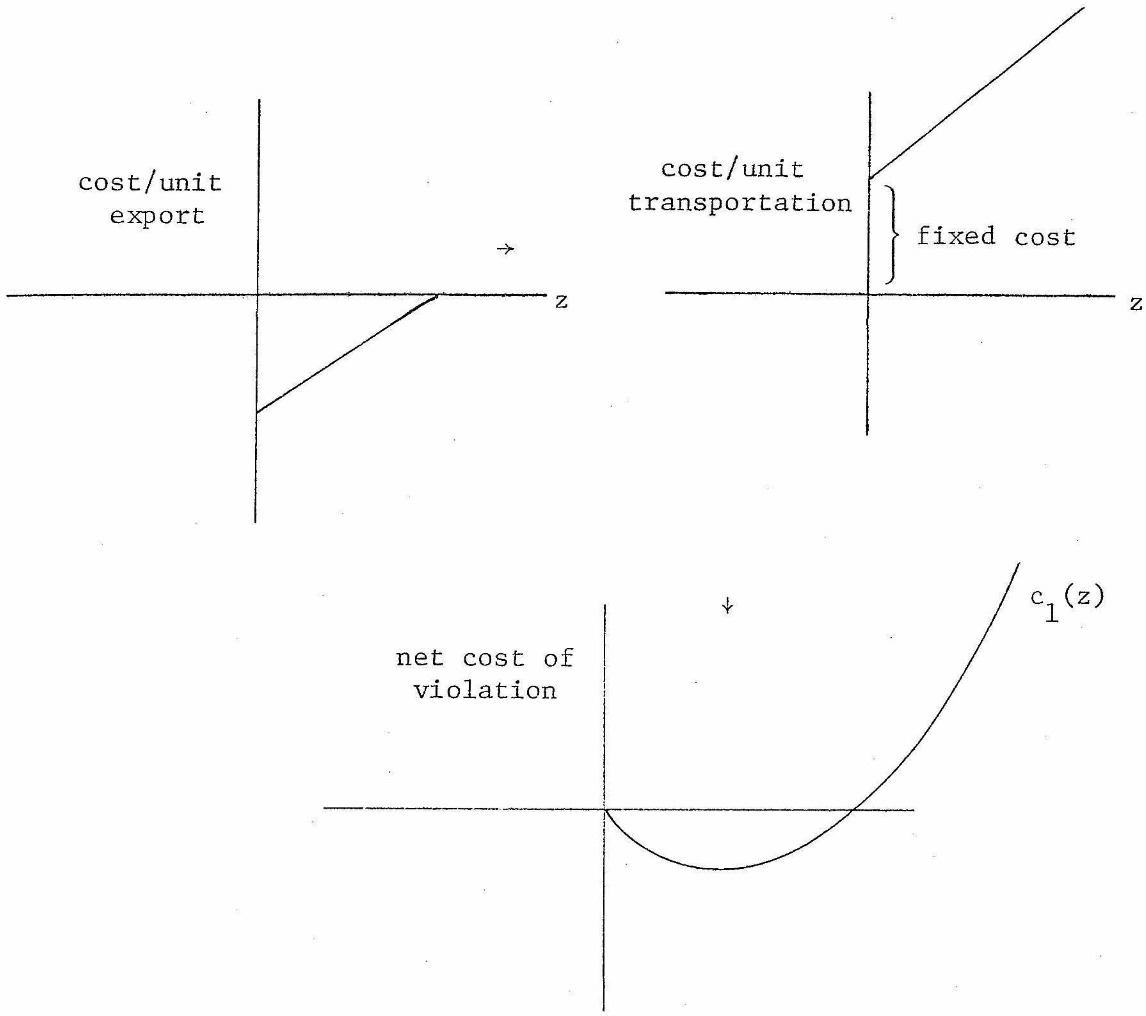
cost/unit
export
z
cost/unit
transportation
fixed cost
z
net cost of
violation
c1(z)

Figure 2

The Continuity Equation

This is the mass balance equation for water in any period p, $1 \le p \le n$, and is given by:

$$x_{p-1} = r(x_p - y_p) + e_p + i_p - m_p \qquad (17)$$

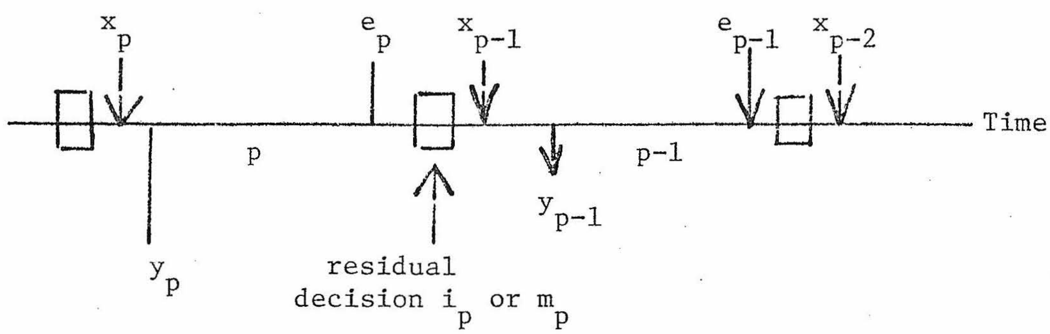

Figure 3

where: r = 1 - k and k is the constant evaporation rate, i_p is the amount of imported water, m_p is the amount of exported water and x_p is the level of the reservoir at the end of stage (p + 1) and after implementing the importing and exporting decisions. Or, equivalently x_p is the water stock at the start of period p.

If (a) $i_p > 0$ then $m_p = 0$, $x_{p-1} = x^m$

and $i_p = x^m - rx_p - e_p + ry_p$. (18)

If (b) $m_p > 0$ then $i_p = 0$, $x_{p-1} = x^u$

and $m_p = rx_p + e_p - ry_p - x^u$. (19)

If (c) $i_p = m_p = 0$ then $x_{p-1} = rx_p + e_p - ry_p$. (20)

A concave salvage value function of the terminal stock of water $v(x)$ will be added to account for the concern of the planners for future generations. $v(x)$ will also prevent the use of water to the point where its marginal profitability is zero. Moreover, it will be assumed that the manager does not import or export water unless he must. In the static case, this implies that the marginal salvage value at x^u must not be less than the marginal benefit from exporting water $v'(x^u) \geq -c_1'(0)$. It also means that the marginal salvage value of water at x^m must not be greater than the marginal cost of importing water $v'(x^m) \leq c_2'(0)$. Clearly, if these conditions do not hold, exporting and importing water becomes profitable and should be included in the optimizing framework of the problem. Whether to import or export water, in this model, is merely a residual decision taken at the end of each period.

The lines of this analysis will follow the traditional methods employed by dynamic programming formulations [12]. First, the existence and uniqueness of the solution for $p = 1$ and $p = 2$ and the concavity of the expected net discounted revenue functions will be established. This will pave the way for an inductive proof for the existence and uniqueness of the solution to the n-period problem. Next, it will be shown that for an infinite period problem the sequence of the expected net discounted revenue function converges under the assumed regularity conditions. This establishes the existence and uniqueness of the solution for the infinite period problem. Finally, maximization of the n-period expected discounted net revenue function will define the optimal size of the reservoir.

The One Period Problem

Let $f_1(x_1)$ be the expected revenue from the release of an optimal quantity of water including revenue from the hydroelectric operation of the reservoir. Let

$$f_0(x_0) = v(x_0) \tag{21}$$

where v(x) is the concave salvage value function indicating the worth of the terminal stock of water to the future generations.

Define

$$f_1(x_1,\bar{x}) = \max_{0 \le y_1 \le x_1} G_1(y_1, x_1 - y_1; \bar{x}) \tag{22}$$

$$x^m \le x_1 \le x^u$$

where

$$G_1(y_1, x_1 - y_1, \bar{x}) = [\pi(y_1, x_1 - y_1) + \beta \int_0^{x^m - rx_1 + ry_1} v(x^m)\, \phi_e de$$

$$+ \beta \int_{x^m - rx_1 + ry_1}^{x^u - rx_1 + ry_1} v(rx_1 + e - ry_1)\phi_e de \quad + \beta \int_{x^u - rx_1 + ry_1}^{\infty} v(x^u)\, \phi\, de$$

$$- \beta \int_{x^u - rx_1 + ry_1}^{\infty} c_1(rx_1 + e - ry_1 - x^u)\, \phi_e de$$

$$- \beta \int_0^{x^m - rx_1 + ry_1} c_2(x^m - rx_1 - e + ry_1)\, \phi_e de] - c(\bar{x}). \tag{23}$$

We have the following proposition:

Proposition 1

If a) $\pi(y,x-y)$ is concave in the first argument and strictly concave in the second and $\pi_{12} \geq 0$;

b) $c_1(z)$, $c_2(z)$ are convex and $c_1(0) = c_2(0) = 0$;

c) $v'(x^u) \geq c_1'(0)$;

d) $v'(x^m) \leq c_2'(0)$;

where the primes denote the derivatives of the functions with respect to the arguments then :

1) there exists a unique interior maximum $y_1^*(x_1)$ for $G_1(y_1, x_1 - y_1)$;

2) $0 \leq \dfrac{dy_1^*}{dx_1} \leq 1$

Moreover if

e) $x^u = g(\bar{x})$, $x^m = h(\bar{x})$, and

$0 \leq h' \leq g' \leq r$

then 3) $-1 < \dfrac{dy_1^*}{d\bar{x}} \leq 0$.

Proof of 1):

From (23) we have:

$$\frac{dG_1}{dy_1} = \pi_1 - \pi_2 - \beta r \int_0^{x^m - rx_1 + ry_1} c_2'(x^m - rx_1 + ry_1 - e)\phi_e de$$

$$- \beta r \int_{x^m - rx_1 + ry_1}^{x^u - rx_1 + ry_1} v'(rx_1 + e - ry_1)\phi_e de + \beta r \int_{x^u - rx_1 + ry_1}^{\infty} c_1'(rx_1 + e - ry_1 - x^u)\phi_e de \tag{24}$$

The primed functions denote their derivatives and all functions are parameterized by $\bar{x}$. The optimal release policy[3] $y_1^*(x_1)$ is defined by

$$\frac{dG_1}{dy_1} = 0 \tag{25}$$

Second Order Conditions:

To show that $y_1^*(x_1)$ is a regular maximum, observe that[4]

$$\begin{aligned}\frac{d^2G_1}{dy_1^2} &= \pi_{11} - 2\pi_{12} + \pi_{22} - \beta r^2[v'(x^u) + c_1'(0)]\,\phi_e(x^u - rx_1 + ry_1) \\ &-\beta r^2[c_2'(0) - v'(x^m)]\phi_e(x^m - rx_1 + ry_1) - \beta r^2 \int_{x^u - rx_1 + ry_1}^{\infty} c_1''(rx_1 + e - ry_1 - x^u)\phi_e de \\ &-\beta r^2 \int_0^{x^m - rx_1 + ry_1} c_2''(x^m - rx_1 + ry_1 - e)\phi_e de + \beta r^2 \int_{x^m - rx_1 + ry_1}^{x^u - rx_1 + ry_1} v''(x^m - rx_1 + ry_1 - e)\phi_e de.\end{aligned} \tag{26}$$

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[3] (24) illustrates the effect of incorporating the stock of water in the profit function. Consider Case 1: $\pi = g(y) + h(x - y)$, g and h are concave; Case 2: $\pi = g(y)$. Then the expression of (24) in Case 1 is less than that of Case 2 by $h' > 0$. Since the g functions are identical in the two cases then

$$\left.\frac{dG_1}{dy_1}\right|_{\text{Case 1}} < \left.\frac{dG_1}{dy_1}\right|_{\text{Case 2}} \quad \text{everywhere.}$$

This implies that $\left. y_1^* \right|_{\text{Case 1}} < \left. y^* \right|_{\text{Case 2}}$. However, when the profit function is separable: $\pi = g(y) + h(x)$, the optimal release policy will not be effected when the g functions are identical.

[4] Consider the two cases in footnote 2. We have

$$\left.\frac{d^2G_1}{dy_1^2}\right|_{\text{Case 1}} < \left.\frac{d^2G_1}{dy_1^2}\right|_{\text{Case 2}}$$

However, when the profit function is separable: $g(y) + h(x)$, the second partial will be the same provided all the g functions are identical.

We have c_1'', $c_2'' \geq 0$ by convexity; $v'' \leq 0$, $\pi_{11} \leq 0$, $\pi_{22} < 0$ and $\pi_{12} \geq 0$ by assumption. Also if, as we have reasonably argued before,

$$v'(x^u) \geq -c_1'(0) \quad (27)$$

and

$$v'(x^m) \leq c_2'(0) \quad (28)$$

then

$$\frac{d^2G_1}{dy_1^2} < 0. \quad (29)$$

Therefore, $y_1^*(x_1)$ is a regular maximum.

Assumption (27) implies that the marginal salvage value of the stock of water at x^u, at the terminal time, must not be less than the net marginal benefit from exporting water. This must be the case if the interest of the future generation (represented by the terminal stock) is to be safeguarded against profitable water export. Assumption (28) states that the marginal salvage value of the stock of water at x^m, at the terminal time, must not be greater than the net marginal cost of importing water. This relationship is reasonable if the planner is not pushed to import water beyond $x = x^m$. The important assumption in both (27) and (28) is that the manager does not import or export water unless he must. This is because (27) and (28) also imply that $c_1'(0) \leq v'(x_1) \leq c_2'(0)$, $\forall x_1$, $x^m \leq x_1 \leq x^u$, which means that it is not profitable to engage in importing or exporting water in the permissible region of x_1.

Notice that it does not matter whether $c_1'(0)$ is positive or negative provided that (27) holds.

To show that $y_1^*(x_1)$ is an interior maximum, it is sufficient to show:

$$\frac{dG_1}{dy_1}(0,x_1) > 0 \tag{30}$$

and

$$\frac{dG_1}{dy_1}(x_1,0) < 0 . \tag{31}$$

Or from (24):

$$\frac{dG_1}{dy_1}(0,x_1) = \pi_1(0,x_1) - \pi_2(0,x_1) - \beta r \int_0^{x^m - rx_1} c_2'(x^m - rx_1 - e)\phi_e de$$
$$-\beta r \int_{x^m - rx_1}^{x^u - rx_1} v'(rx_1 + e_1)\phi_e de + \beta r \int_{x^u - rx_1}^{\infty} c_1'(rx_1 + e - x^u)\phi_e de > 0 \tag{32}$$

$$\forall \quad x_1, \; x^m \leq x_1 \leq x^u \quad .$$

This is trivially satisfied if $\pi(y, x-y)$ is a neoclassical function, which implies that $\lim_{y_1 \to 0} \pi_1(y_1, x_1 - y_1) \to \infty$ and $\lim_{y_1 \to x_1} \pi_2(y_1, x_1 - y_1) \to \infty$. Generally, however, the assumption that (30) and (31) are satisfied is reasonable in terms of an intuitive economic argument. This is demonstrated by rearranging the terms of (32) as follows:

$$\pi_1(0,x_1) - \beta r \int_0^{x^m - rx_1} c_2'(x^m - rx_1 - e)\phi_e de \geq \pi_2(0,x_1)$$

$$- \beta r \int_{x^u - rx_1}^{\infty} c_1'(rx_1 + e \quad x^u)\phi_e de + \beta r \int_{x^m - rx_1}^{x^u - rx_1} v'(vx_1 + e)\phi_e de. \qquad (33)$$

The economic interpretation of (33) is that the net marginal profitability of releasing water exceeds that of storing it at any stock of water between x^m and x^u, providing water release is zero. Even at $x_1 = x^m$, this must be true if large scale damage to the downstream users is to be avoided. To see that this interpretation is correct, we have to remember that:

$$c_2'(z) \begin{cases} > 0 & \forall \; z \geq 0 \\ = 0 & \forall \; z < 0 \end{cases} \qquad (34)$$

also

$$c_1'(z) \begin{cases} > 0 & \forall \; z \geq 0 \\ = 0 & \forall \; z < 0 \end{cases} \qquad (35)$$

and

$$v'(z) \begin{cases} > 0 & \forall \; x^m \leq z \leq x^u \\ = 0 & \text{otherwise.} \end{cases} \qquad (36)$$

This means that (33) can be rewritten and the limits of integration changed as follows:

$$\pi_1(0,x_1) - \beta r \underset{e}{E} \{c_2'(z)\} \geq \pi_2(0,x_1) - \beta r \underset{e}{E} \{c_1'(z)\}$$

$$+ \beta r \underset{e}{E} \{v'(z)\}. \qquad (37)$$

Equation (37) is essentially what the previous economic interpretation asserts. One might notice the peculiar range of the salvage value function, but this range facilitates a smooth induction argument. It can be clarified by reinterpreting the salvage value function as follows:

$$V(z) = \begin{cases} v(x^m) - c_2(z) & \forall \ z, \ z < x^m \\ v(z) & \forall \ z, \ x^m \leq z \leq x^u \\ v(x^u) - c_1(z) & \forall \ z, \ z > x^u \end{cases} \qquad (38)$$

where v(z) is defined as before. Thus, (37) can be rewritten as follows:

$$\pi_1(0,x_1) \geq \pi_2(0,x_1) + \underset{e}{E}\{V(z)\}. \qquad (39)$$

which is a formalization of the preceding argument.

On the other hand, (31) implies that

$$\pi_2(x_1,0) - \beta r \int_{x^u}^{\infty} c_1'(e - x^u) + \beta r \int_{x^m}^{x^u} v'(e)\phi_e de > \pi_1(x_1,0)$$

$$-\beta r \int_0^{x^m} c_2'(x^m - e)\phi_e de. \qquad (40)$$

This is true if π is a neoclassical profit function. Also, using the previous argument, this is equivalent to either:

$$\pi_2(x_1,0) - \beta r \underset{e}{E}\{c_1'(z)\} + \beta r \underset{e}{E}\{v'(z)\} > \pi_1(x_1,0)$$

$$- \beta r \underset{e}{E}\{c_2'(z)\}$$

or

$$\pi_2(x_1,0) + \beta r \underset{e}{E}\{V(z)\} > \pi_1(x_1,0) \quad . \tag{41}$$

(41) states that the marginal profitability of storing the last unit of water exceeds that of releasing it, assuming all x_1 is released.

Proof of 2):

In this section, the effect on the optimal release policy of a parametric change in the starting stock of water x_1 or in the physical capacity of the reservoir $\bar{x}$ will be investigated. Differentiating the first order conditions (eq. 24) with respect to x_1 gives:

$$(\pi_{11} - \pi_{21})\frac{dy_1^*}{dx_1} = \beta r^2 \left(-1 + \frac{dy_1^*}{dx_1} \right) \Big[\int_0^{x^m - rx_1 + ry_1} c_2''(x^m - rx_1 + ry_1 - e)\phi_e de$$

$$+ c_2'(0)\,\phi(x^m - rx_1 + ry_1) - \int_{x^m - rx_1 + ry_1}^{x^u - rx_1 + ry_1} v''(rx_1 + e - ry_1)\phi_e de$$

$$+ v'(x^u)\phi(x^u - rx_1 + ry_1) - v'(x^m)\phi(x^m - rx_1 + ry_1)$$

$$+ \int_{x^u - rx_1 + ry_1}^{\infty} c_1''(rx_1 + e - ry_1 - x^u)\phi_e de + c_1'(0)\phi(x^u - rx_1 + ry_1)$$

$$+ \frac{\pi_{12} - \pi_{22}}{\beta r^2} \Big] \quad . \tag{42}$$

or, equivalently[5]

$$\frac{dy_1^*}{dx_1} = \frac{\dfrac{d^2G_1}{dy_1^2} - (\pi_{11} - \pi_{21})}{\dfrac{d^2G_1}{dy_1^2}} \quad . \tag{43}$$

From (31) we have $\dfrac{d^2G_1}{dy_1^2} < \pi_{11} - \pi_{21} \leq 0.$ (44)

Therefore, (43) implies that $0 < \dfrac{dy_1^*}{dx_1} \leq 1.$ (45)

Proof of 3):

$$\text{Let } x^u = g(\bar{x}) \text{ and } x^m = h(\bar{x}), \text{ such that } 0 \leq h' \leq g' \leq r \tag{46}$$

Differentiating (23) with respect to $\bar{x}$ and using (26), we have[6]

$$\frac{dy_1^*}{d\bar{x}} \cdot \frac{d^2G_1}{dy_1^2} = \beta rh' \int c_2''\phi_e de + \beta rh'[c_2'(0) - v(h(\bar{x}))]\phi + \beta rg' \int c_1''\phi_e de$$
$$+ \beta rg'[c_1'(0) + v'(g(\bar{x}))]\phi \tag{47}$$

---

[5]Consider the two cases of footnote 3:

$$\left.\frac{dy_1^*}{dx_1}\right|_{\text{Case 2}} = 1 - \frac{g''}{\left.\dfrac{d^2G_1}{dy_1^2}\right|_{\text{Case 2}}}$$

$$\left.\frac{dy_1^*}{dx_1}\right|_{\text{Case 1}} = 1 - \frac{g''(1 - \frac{dy}{d(x-y)})}{\left.\dfrac{d^2G_1}{dy_1^2}\right|_{\text{Case 1}}} = \left.\frac{dy_1^*}{dx_1}\right|_{\text{Case 2}} + g''\frac{\frac{dy}{d(x-y)}}{\left.\dfrac{d^2G_1}{dy_1^2}\right|_{\text{Case 1}}}$$

From footnote 3: $\left.\dfrac{d^2G_1}{dy_1^2}\right|_{\text{Case 1}} < \left.\dfrac{d^2G_1}{dy_1^2}\right|_{\text{Case 2}}$ therefore $\left.\dfrac{dy_1^*}{dx_1}\right|_{\text{Case 2}} > \left.\dfrac{dy_1^*}{dx_1}\right|_{\text{Case 1}}$, only if $\dfrac{dy}{d(x-y)}$ is unambiguously negative otherwise it is ambiguous.

[6]To keep the expressions simple, we shall drop the arguments of the functions and the integral limits in such expressions whenever it is unambiguous to do so.

From (27), (28), and the convexity of c_1 and c_2, the right-hand side of (47) is positive, which implies that $\frac{dy_1^*}{d\bar{x}}$ is negative. Moreover, for each term in the right-hand side of (47), there is a corresponding term in the expression of $\frac{d^2G_1}{dy_1^2}$ with opposite sign and weight equal to either $\frac{r}{h'}$ or $\frac{r}{g'}$, which, by assumption, are greater than 1. Thus, comparing the expression on the right-hand side of (47) with the expression of $\frac{d^2G_1}{dy_1^2}$ we conclude that

$$-1 < \frac{dy_1^*}{d\bar{x}} \leq 0. \tag{48}$$

(End of Proof of Proposition 1)

This result has been obtained by placing some restrictions on the derivatives of h and g; these are $0 \leq h' \leq r$, and $0 \leq g' \leq r$. These assumptions will be justified on the following basis:

a) the non-negativity restriction on $g'(\bar{x})$ is reasonable. This is because increasing the physical capacity of the reservoir, for the same inflow and hydrology of the river basin, offers the opportunity to increase $x^u = g(\bar{x})$ and hence, the hydroelectric power potential of the reservoir. This increase in x^u must not be greater than 1 in order to avoid decreasing the designed free board capacity $(\bar{x} - x^u)$ of the reservoir. To illustrate further, consider the case where $g' = \alpha$ (α is a constant), and the inflow in the period before last brings the total storage to $\bar{x}$. The storage after evaporation in this case is $r\bar{x}$. Hence, if $g' = \alpha \geq r$ or $\alpha\bar{x} \geq r\bar{x}$ which means that $x^u \geq r\bar{x}$, then there is no need to export water under all conditions where $x_1 \leq \bar{x}$. That is, the natural process of evaporation under these conditions provides an automatic excess water disposal. Such a situation is imaginary and will not be considered any further. Thus, it seems reasonable to accept the assumption that g' is bounded in the range $0 \leq g' < r$.

b) The non-negativity of $h'(x)$ is more straightforward. This is because the minimum pool requirement $x^m = h(\bar{x})$ is dictated by the minimum hydrostatic head required for the operation of a particular turbine on one hand and the salinity control on the other. Neither of these requirements is affected negatively by the increase in the physical capacity of the reservoir. x^m can be expected to stay constant or increase slightly to account for the increase in salinity brought about by a larger stock of water. Moreover, increasing $\bar{x}$ is expected to weaken the overall constraints on the system. Hence, the control volume $x^c = x^u - x^m$ is expected to increase. Therefore, $g' \geq h'$. However, by the previous discussion in (a), $g' \leq r$, which implies that $0 \leq h' \leq g' \leq r$. In the previous sections, it has been argued that the assumptions responsible for our seemingly counterintuitive results, $-1 \leq \frac{dy_1^*}{d\bar{x}} \leq 0$, are reasonable. The meaning of the result itself follows. Given the same inflow and river basin hydrology and starting with the same stock of water x, the increase in the physical capacity of the reservoir $\bar{x}$ has resulted in:

1) weakening the upper constraints x^u,

2) strengthening the lower constraints x^m.

This situation leads to a reduction in risk of having excess water and an increase in risk of having to import water, which can only lead to a reduction in the optimal release policy $y^*(x_1)$.

Lemma 1

If $y_1^*(x_1)$ exists and is unique and $\pi_{11} = \pi_{12}$ or both identically vanish, then the optimal release rule is linear of the form $y_1^*(x_1) = x_1 - a_1$, where a_1 is a constant dictated by the hydrology of the stream, the size of the reservoir, and the specific form of the profit function.

Proof:

From (43), if $\pi_{11} = \pi_{12} \equiv 0$, then $\frac{dy_1^*}{dx_1} = 1$ and

$$y_1^*(x_1) = x_1 - a \qquad (49)$$

a is a constant dictated by the hydrology of the river basin, the size of the reservoir and the specific form of π.

Thus, the celebrated linear decision rule, used so often in chance constraint models, emerges as the optimizing decision rule when a specific form of the profit function π is used in this model.

Considering the interpretation given to π earlier:

1. In the first case, where π is the expected net social revenue from operating the reservoir, π_{12} may be zero if either the second mixed partials of expected social revenue and expected social cost functions are identically equal, as evaluated by the central planners, or that both second mixed partials vanishes. The latter case may be argued on the bases that there is no reason for marginal expected cost to be affected by a change in the water head left in the reservoir after the release and used for power generation. Moreover, $\pi_{11} \equiv 0$ if either the second partials with respect to the releases of the expected social revenue and expected social cost functions are identically equal or if both partials vanishes. The latter is consistent with a situation where both functions are characterized by fixed proportion and there is a perfectly competitive market for agricultural products.
2. In the second case, where π reflect the aggregate expected profit of downstream users who own the water in the reservoir, π_{12} may be zero if the marginal profitability in agriculture is unaffected by a change in the stock of water which remains after the release. Moreover $\pi_{11} \equiv 0$, if the production function of the downstream farmers is characterized by fixed proportions and that farmers sell their product in perfectly competitive market.
3. In the third case, where π is the total surplus, since we are talking about the areas under compensated demand curves conditions such as

$\pi_{12} \equiv 0$ if the utility is separable and $\pi_{11} \equiv 0$ if the marginal utility from the payoff which arises from release is linear. This case arises if risk neutrality with respect to uncertainty in agricultural prevails.

Proposition 2

a) If assumptions (a) - (d) in proposition 1 hold, then the expected return $f_1(x_1;\bar{x})$ has the following characteristics:

1) $\frac{df_1}{dx_1} = \pi_1\{y_1^*, x_1 - y_1^*\}$;

2) $f_1(x_1;\bar{x})$ is strictly concave in x_1.

b) If assumption (e) in proposition 1 holds, and

c) if $x^u = g(\bar{x})$, g is concave, and

d) if $x^m = h(\bar{x})$ = constant, then

3) $f_1(x_1,\bar{x})$ is strictly concave in $\bar{x}$.

Proof of 1):

The existence and uniqueness of the solution to the two period problem depends on the nature of the expected net return function in the last period f_1. Therefore, in the following, the concavity of f_1 with respect to x_1 and, under some assumptions, with respect to $\bar{x}$, shall be shown. From (25) we have

$$f_1(x_1;\bar{x}) = \pi(y_1^*, x_1 - y_1^*) + \beta \int_0^{x^m - rx_1 + ry_1^*} v(x^m)\phi_e de$$

$$+ \beta \int_{x^m - rx_1 + ry_1^*}^{x^u - rx_1 + ry_1^*} v(rx_1 + e - ry_1^*)\phi_e de$$

$$+ \beta \int_{x^u - rx_1 + ry_1^*}^{\infty} v(x^u)\phi_e de - \beta \int_{x^u - rx_1 + ry_1^*}^{\infty} c_1(rx_1 + e - ry_1^* - x^u)\phi_e de$$

$$- \beta \int_0^{x^m - rx_1 + ry_1^*} c_2(x^m - rx_1 - e + ry_1^*)\phi_e de - c(\bar{x}) \quad . \qquad (50)$$

Therefore,

$$\frac{df_1}{dx_1} = \pi_1 \frac{dy_1^*}{dx_1} + (-1 + \frac{dy_1^*}{dx_1}) [-\beta r \int_{x^m - rx_1 + ry_1^*}^{x^u - rx_1 + ry_1^*} v'(rx_1 + e - ry_1^*)\phi_e de$$

$$+ \beta r \int_{x^u - rx_1 + ry_1^*}^{\infty} c_1'(rx_1 + e - ry_1^* - x^u)\phi_e de$$

$$- \beta r \int_0^{x^m - rx_1 + ry_1^*} c_2'(x^m - rx_1 - e + ry_1^*)\phi_e de - \pi_2] \quad . \qquad (51)$$

However, from (24), the bracketed term in (51) equals $(- \pi_1)$, then

$$\frac{df_1}{dx_1} = \pi_1(y^*_1, x_1 - y^*_1) \quad . \qquad (52)$$

Proof of 2).

Differentiating (52) with respect to x_1, we have

$$\frac{d^2 f_1}{dx_1^2} = (\pi_{11} - \pi_{12}) \frac{dy_1^*}{dx_1} + \pi_{12}. \tag{53}$$

Also, from (43)

$$\frac{dy_1^*}{dx_1} = \frac{\frac{d^2 G_1}{dy_1^2} - (\pi_{11} - \pi_{21})}{\frac{d^2 G_1}{dy_1^2}} .$$

Substituting for $\frac{dy_1^*}{dx_1}$ in eq. (53), we have

$$\frac{d^2 G_1}{dy_1^2} \cdot \frac{d^2 f_1}{dx_1^2} = [\pi_{11} \frac{d^2 G_1}{dy_1^2} - (\pi_{11} - \pi_{21})^2]. \tag{54}$$

However, (23) shows that

$$\frac{d^2 G_1}{dy_1^2} < \pi_{11} - \pi_{12} < 0 , \tag{55}$$

or equivalently,

$$\left| \frac{d^2 G_1}{dy_1^2} \right| > \pi_{11} - \pi_{12} .$$

Also since $\pi_{12} \geq 0$, then

$$\pi_{11} > \pi_{11} - \pi_{12} .$$

Therefore

$$\pi_{11} \frac{d^2G_1}{dy_1^2} > (\pi_{11} - \pi_{21})^2 \quad .$$

And, hence,

$$\frac{d^2f_1}{dx_1^2} < 0 \tag{56}$$

That is, f_1 is strictly concave in x_1.

Proof of 3):

Also from (24), substituting for $x^u = g(\bar{x})$ and $x^m = h(\bar{x})$ and differentiating with respect to $\bar{x}$, we have

$$\frac{df_1}{d\bar{x}} = -c'(\bar{x}) - \beta h'(\bar{x}) \int_0^{h(\bar{x})-rx_1+ry_1^*} c_2'(h(\bar{x}) - rx_1 - e + ry_1^*)\phi_e de$$

$$+ \beta g'(\bar{x}) \int_{g(\bar{x})-rx_1+ry_1^*}^{\infty} c_1'(rx_1 + e - ry_1^* - g(x))\phi_e de$$

$$+ \beta h'(\bar{x}) \int_0^{h(\bar{x}) - rx_1 + ry_1^*} v'(h(\bar{x})\phi_e de + \beta g'(\bar{x}) \int_{g(\bar{x}) - rx_1 + ry_1^*}^{\infty} v'(g(\bar{x})\phi_e de. \tag{57}$$

In particular if $h' = 0$, then

$$\frac{df_1}{d\bar{x}} = -c'(\bar{x}) + \beta g'(\bar{x}) \int_{g(\bar{x}) - rx_1 + ry^*_1}^{\infty} c'\phi_e de + \beta g'(\bar{x}) \int_{g(\bar{x}) - rx_1 + ry_1^*}^{\infty} v'(g(\bar{x})\phi_e de. \tag{58}$$

In general, however,

$$\frac{d^2 f_1}{d\bar{x}^2} = c'' - \beta h'' \int c_2' \phi_e de - \beta h'(h' + r \frac{dy_1^*}{d\bar{x}}) \int c_2'' \phi_e de$$

$$+ \beta h'(h' + r \frac{dy_1^*}{d\bar{x}})(v'(h) - c_2'(0))\phi(h - rx_1 + ry_1^*)$$

$$+ \beta g'' \int c_1' \phi_e de$$

$$- \beta g'(g' + r \frac{dy_1^*}{d\bar{x}}) \int c_1'' \phi_e de - \beta g'(g' + r \frac{dy_1^*}{d\bar{x}})(v'(g)$$

$$+ c_1'(0)\phi(g - rx_1 + ry_1^*)$$

$$+ \beta h'' \int v' \phi_e de + [h']^2 \int v''(\phi_e de$$

$$+ \beta g'' \int v' \phi_e de + [g']^2 \int v'' \phi_e de. \qquad (59)$$

It can be shown that $g' + r \frac{dy_1^*}{d\bar{x}} > 0$ while $h' + r \frac{dy_1^*}{d\bar{x}}$ is ambiguous, which makes the sign of $\frac{d^2 f_1}{d\bar{x}^2}$ indeterminate. However, it is obvious that $\frac{d^2 f_1}{d\bar{x}^2} < 0$, under assumption that x^m is a constant. Thus, under plausible assumptions, f_1 is shown to be strictly concave in $\bar{x}$ (as well as x_1).[7]

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[7] $\frac{d^2 f_1}{dx^2}$ is also negative if $h' = g' =$ constant.

Proposition 3

Under the assumptions of proposition 2, there exists a unique optimal size $\bar{x}_1^*$ for the reservoir which maximizes $f_1(x_1,\bar{x})$.

Proof :

If it is assumed, as in the first model, that $\exists\, y_0$ such that $\pi_1(y_0) = 0$ and that $\exists\, x_0 \ni v'(x_0) = 0$, then $\bar{x}$ is bounded by 0 and $y_0 + x_0$. This implies that f_1 is defined on a compact set $0 \leq \bar{x} \leq y_0 + x_0$. If the assumptions of proposition 2 hold, then f_1 is a strictly concave function in $\bar{x}$ defined on a compact set. Therefore, it must have a unique maximum $\bar{x}_1^*$.

This ends the analysis of the one-period problem. It appears that the inclusion of the water stock in the profit function, although it affected the optimal policy and size of the reservoir, did not make substantial difference to the technical conditions needed to get the usual inventory dynamic programming results. Inspecting (33) and (40), the conditions which insure interior maximum, enhance this observation. Certainly, for a neoclassical profit function, the finite terms $\pi_2(0,x_1)$ and $\pi_2(x_1,0)$ do not make either of the inequalities (33) and (40) more stringent or relaxed. For any other concave function, the inclusion of the water stock makes (33) more stringent while relaxing (40). Thus, the concavity of π with respect to the water stock and that $\pi_{12} \geq 0$ are all the additional requirements needed to get the usual inventory dynamic programming results.

Summary of the One-Period Problem

It has been shown that a unique solution $y_1^*(x_1)$ for the functional equation (1) exists and is unique if

1) $v'(x^u) \geq -c_1'(0)$

2) $v'(x^m) \leq c_2'(0)$.

Moreover, it has been shown that

a) $0 < \frac{dy_1^*}{dx_1} \leq 1$

b) f_1 is strictly concave in x_1.

Furthermore, it has been shown that if

3) $x^u = g(\bar{x})$,

4) $x^m = h(\bar{x}) \qquad 0 \leq h' \leq g' \leq r$

then

c) $-1 < \frac{dy_1^*}{d\bar{x}} \leq 0$.

In particular, if

5) $h' = 0$ and g is concave

then

d) f_1 is strictly concave in $\bar{x}$; and

e) $f_1(\bar{x})$ has a unique maximum, $\bar{x}_1^*$, provided $\bar{x}$ is bounded above.

The Two Period Horizon

In this case, the continuity equation is[8]

$$x_1 = r(x_2 - y_2) + e + i_2 - m_2 \quad . \tag{60}$$

Define

$$f_2(x_2) = \underset{0 \le y_2 \le x_2}{\text{Max}} \; G_2(y_2, x_2 - y_2)$$

where

$$G_2(y_2, x_2 - y_2) = \underset{0 \le y_2 \le x_2}{\text{Max}} \; [\pi(y_2, x_2 - y_2) + \beta \int_0^{x^m - rx_2 + ry_2} f_1(x^m)\phi_e de \tag{61}$$

$$+ \beta \int_{x^u - rx_2 + ry_2}^{\infty} f_1(x^u)\phi_e de$$

$$+ \beta \int_{x^m - rx_2 + ry_2}^{x^u - rx_2 + ry_2} f_1(rx_2 + e - ry_2)\phi_e de - \beta \int_{x^u - rx_2 + ry_2}^{\infty} c_1(rx_2 + e - ry_2 - x^u)\phi_e de$$

$$- \beta \int_0^{x^m - rx_2 + ry_2} c_2(x^m - rx_2 - e + ry_2)\phi_e de \; - c(\overline{x}),$$

---

[8] Assume that at the start of every period the manager knows the actual inflow. However, he only knows the probability distribution of the inflow for future periods. Then a redefinition of terms and a relabeling of periods leaves the analysis intact. For example, in the two period case, x_2 is the starting stock of water, after observing e_2 and correcting for deficiencies or surplus in the previous period. Therefore, $x_1 = r(x_2 - y_2) + e_1 + i_2 - m_2$. Relabelling e_i by e_{i+1} gives

$x_1 = r(x_2 - y_2) + e_2 + i_2 - m_2$, which is the original continuity

equation. However, it must be noted that since e_T is now known with certainty, the decision in the last period is deterministic, not stochastic.

Proposition 4

If assumptions (a - d) in proposition 1 are accepted, the following results hold:

1) There exists a unique interior maximum $y_2^*(x_2)$,

2) $y_2^*(x) \leq y_1^*(x)$,

3) $0 < \frac{dy_2^*}{dx_2} \leq 1$,

4) Further, if $\pi_{11} = \pi_{12} \equiv 0$ then the optimal release rule takes the form

$$y_2^*(x_2) = x_2 - a_2, \quad \forall x_2, \; x^m \leq x_2 \leq x^u .$$

a_2 is a constant dictated by the hydrology of the stream, the size of the reservoir and the specific form of the profit function π.

Moreover, if assumption (3) in proposition 1 holds then,

5) $-1 < \frac{dy_2^*}{d\bar{x}} \leq 0.$

Proof of 1):

When (61) is compared with (22), the two expressions for the optimal return function in the one period and the two period case are identical except that f_1 replaces v_1 wherever v_1 occurs in expression (22). Moreover, since both v_1 and f_1 are concave, it can be verified that under identical assumptions, all the qualitative results of the one period problem also hold in the two period case. In particular:

$$\frac{dG_2}{dy_2} = \pi_1(y_2, x_2 - y_2) - \pi_2(y_2, x_1 - y_2) \tag{62}$$

$$- \beta r \int_0^{x^m - rx_2 + ry_2} c_2'(x^m - rx_2 + ry_2 - e)\, \phi_e de$$

$$+ \beta r \int_{x^u - rx_2 + ry_2}^{\infty} c_1'(rx_2 + e - ry_2 - x^u)\phi_e de$$

$$- \beta r \int_{x^m - rx_2 + ry_2}^{x^u - rx_2 + ry_2} f_1'(rx_2 + e - ry_2)\phi_e de \quad .$$

The primes denote the derivatives of the functions with respect to their arguments. Thus, $y_2^*(x_2)$ is defined by $\frac{dG_2}{dy_2} = 0$. Similarly,

$$\frac{d^2G_2}{dy_2^2} = \pi_{11} - 2\pi_{12} + \pi_{23} - \beta r^2[f_1'(x^u + c_1'(0)]\phi_e(x^u - rx_1 + ry_1)$$

$$- \beta r^2[c_2'(0) - f_1'(x^m)]\phi_e(x^m - rx_1 + ry_1)$$

$$- \beta r^2 \int_{x^u - rx_1 + ry_1}^{\infty} c_1''(rx_1 + e - ry_1 - x^u)\phi_e de$$

$$- \beta r^2 \int_0^{x^m - rx_1 + ry_1} c_2''(x^m - rx_1 + ry_1 - e)\, \phi_e de$$

$$+ \beta r^2 \int_{x^m - rx_1 + ry_1}^{x^u - rx_1 + ry_1} f_1''(x^m - rx_1 + ry_1 - e)\, \phi_e de \quad . \tag{63}$$

Moreover, at x^u, the benefit from releasing the last unit of $y_1^*(x_1)$ must exceed the marginal benefit from exporting water. If this is not the case, then it becomes profitable to export water rather than release it to downstream users. Hence,

$$f_1'(x^u) = \pi_1(y_1^*(x^u), x^u - y_1^*(x^u)) \geq -c_1'(0). \qquad (64)$$

Similarly at x^m, once $y_1^*(x^m)$ is released, the marginal benefit from releasing an extra unit of water must be less than the marginal cost of violating the lower constraint (the price of water import). If this is not the case, it becomes profitable to import water and release it to downstream users. Therefore,

$$f_1'(x^m) = \pi_1\{y_1^*(x^m),\ x_1 - y_1^*(x^m)\} \leq c_1'(0). \qquad (65)$$

These conditions motivate the same economic behavior as that in (32) and (33) ; it is not profitable to engage in importing or exporting water in the permissable range of x_1, $x^m \leq x_1 \leq x^u$. However, from (63) , these conditions imply

$$\frac{d^2G_2}{dy_2^2} < 0. \qquad (66)$$

Thus, $y_2^*(x_1)$ is a regular maximum. Moreover, it can be shown that the relations $\frac{dG_2(0,x_1)}{dy_2} > 0$ and $\frac{dG_2}{dy_2}(x_2,0) < 0$ hold, and are based on the same economic arguments presented in the one period case. Hence, $y_2^*(x_2)$ is an interior maximum.

Proof of 2):

To prove that $y_2^*(x) \leq y_1^*(x)$, notice that if $v(x) = 0$, then from (24) and (62), $\frac{dG_2}{dy} < \frac{dG_1}{dy}$ everywhere and hence $y_2^*(x) < y_1^*(x)$. See Figure 4.

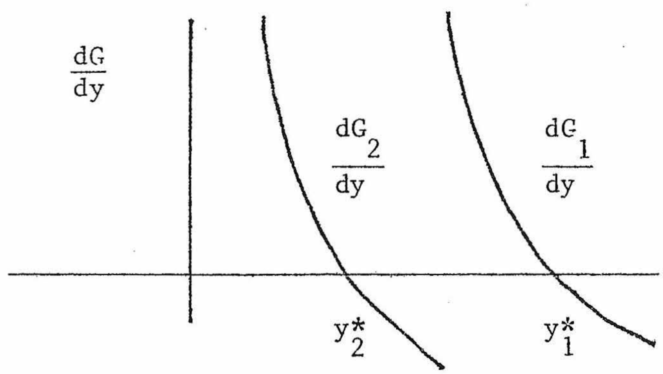

Figure 4

In general, however, if $\frac{dG_1}{dy}$ is evaluated at y_2^*, it can be proven that

$\left.\frac{dG_1}{dy}\right|_{y_2 = y_2^*} > 0$. Since $\frac{d^2G_i}{dy^2} < 0$, i = 1,2 everywhere and $\frac{dG_i}{dy} = 0$ has only one solution, the following inequality must hold:

$$y_1^*(x) > y_2^*(x). \qquad (67)$$

See Figure 4. This result has already been implied by the previous analysis, where it has been shown that

$$0 < \frac{dy_1^*}{dx_1} \leq 1 \quad \forall\, x,\ x^m \leq x_1 \leq x^u.$$

Hence, it is economical to release some of the unit increase in initial storage rather than retaining the entire storage increase. Therefore, the marginal expected return from releasing some of the unit increase in initial storage and storing the rest must exceed the marginal expected increase in salvage value due to the storage of the whole unit increase,

$$f_1'(x_1) > v'(x_1), \quad \forall x_1, \quad x^m \le x_1 \le x^u \quad .$$

Hence,

$$\int_{x^m-rx+ry}^{x^u-rx+ry} f_1'(rx + e - ry)\phi_e de > \int_{x^m-rx+ry}^{x^u-rx+ry} v'(rx + e - ry)\phi_e de. \quad (68)$$

(68) holds because the arguments of both f_1' and v' lie in the interval x^m to x^u for the specific range of the random variable e defined by the limits of integration.

Comparing the first order conditions in the one period and the two period cases, the previous argument implies that $y_2^*(x) \le y_1^*(x)$.

Proof of 3):

From (62), it is found that

$$\frac{d^2G_2}{dy_2^2}\frac{dy_2^*}{dx_2} = \frac{d^2G_2}{dy_2^2} - (\pi_{11} - \pi_{21}) \quad . \quad (69)$$

It follows that

$$0 < \frac{dy_2^*}{dx_1} \le 1. \quad (70)$$

Proof of 4):

Notice that if $\pi_{11} = \pi_{21} \equiv 0$, then

$$\frac{dy_2^*}{dx_2} = 1 \quad (71)$$

and

$$y_2^* = x_2 - a_2, \quad \forall\, x_2, \quad x^m \le x_2 \le x^u \quad . \tag{72}$$

a_2 is dictated by the hydrology of the river basin, the size of the reservoir and the specific form of the profit function.

Proof of 5):

If $x^u = g(\bar{x})$, $x^m = h(\bar{x})$, $0 \le h' \le g' \le r$, then

$$-1 < \frac{dy_2^*}{d\bar{x}} \le 0. \tag{73}$$

Proposition 5

Under the assumptions (a - e) of proposition 1:

1) $\frac{df_2}{dx_2} = \pi_2\{y_2^*(x_2), x_2 - y_2(x_2)\}$

2) f_2 is strictly concave in x_2.

3) If, in addition $x^u = g(\bar{x})$, $x^m = h(\bar{x})$, g is concave and h is a constant, then f_2 is strictly concave in $\bar{x}$.

4) $f_2(x) > f_1(x)$.

Proof of 1):

Differentiating (61), we have

$$\frac{df_2}{dx_2} = \pi_1 \frac{dy_2^*}{dx_2} + (1 - \frac{dy_2^*}{dx_2})\pi_1 = \pi_1(y_2^*, x_2 - y_2^*) \quad . \tag{74}$$

Since $\frac{df_1}{dx_1} = \pi_1(y_1^*(x_1), x_1 - y_1^*(x_1))$ and $y_1^*(x) > y_2^*(x)$, then by the concavity of π, $\frac{df_{2(x)}}{dx} > \frac{df_{1(x)}}{dx}$.   (75)

Proof of 2):

From (74) and (69),

$$\frac{d^2 f_2}{dx_2} = \frac{1}{\frac{d^2 G_2}{dy_2^*}} \left[\pi_{11} \frac{d^2 G_2}{dy_2^2} - (\pi_{11} - \pi_{21})^2\right] \tag{76}$$

This can be shown to be negative, in a manner similar to that employed in the one period case. Thus, f_2 is concave in x_2.

Proof of 3):

Also, if $x^u = g(\bar{x})$, and $x^m = h(\bar{x})$, then

$$\frac{df_2}{d\bar{x}} = \beta h'(\bar{x}) \int_0^{h(\bar{x})-rx_2+ry_2^*} f_1'(h(\bar{x}))\phi_e de + \beta g'(\bar{x}) \int_{g(\bar{x})-rx_2+ry_2^*}^{\infty} f_1'(g(\bar{x}))\phi_e de$$

$$+ \beta g'(\bar{x}) \int_{g(\bar{x})-rx_2+ry_2^*}^{\infty} c_1'(rx_2 + e - ry_2^* - g(\bar{x}))\phi_e de$$

$$- \beta h'(\bar{x}) \int_0^{h(\bar{x})-rx_2+ry_2^*} c_2'(h(\bar{x}) - rx_2 - e + ry_2^*)\phi_e de - c'(\bar{x}). \tag{77}$$

Therefore, it follows that

$$\frac{d^2 f_2}{d\bar{x}^2} = \beta h'(h' + r \frac{dy_2^*}{d\bar{x}}) [f_1'(h(\bar{x})) - c_2'(0)]\phi(h(\bar{x}) - rx_1 + ry_1^*)$$

$$- \beta g'(g' + r \frac{dy_2^*}{d\bar{x}}) [f_1'(g(\bar{x})) + c_1'(0)]\phi(g(\bar{x}) - rx_1 + ry_1^*)$$

$$-\beta h'' \int c_2'\phi_e de - \beta h'(h' + r \frac{dy_1^*}{d\bar{x}}) \int c_2''\phi_e de$$

$$+ \beta g'' \int c_1' \phi_e de - \beta g' (g' + r \frac{dy_1^*}{d\bar{x}}) \int c_1'' \phi_e de$$

$$+ \beta h'' \int f_1' \phi_e de + \beta [h']^2 \int f_1'' \phi_e de$$

$$+ \beta g'' \int f_1' \phi_e de + \beta [g']^2 \int f_1'' \phi_e de. \qquad (78)$$

In particular, if $h' = 0$ and g is concave, and since $f_1'(g(\bar{x})) + c_1'(0) \geq 0$ and $f_1'(h(\bar{x})) - c_2'(0) \leq 0$, we conclude that

$$\frac{d^2 f_2}{d\bar{x}^2} < 0 \qquad (79)$$

i.e. f_2 is strictly concave in $\bar{x}$.

Proof of 4):

f_1 and f_2 can be rewritten as follows:

$$f_1 = \underset{y_1}{\text{Max}} [\pi + E\{V\}]$$

$$f_2 = \underset{y_2}{\text{Max}} [\pi + E\{f_1\}]$$

where V is defined as in (38). An equivalent expression for f_2 is

$$f_2 = \underset{y_2}{\text{Max}} [\pi + \underset{y_1}{\text{Max}} \{\pi\} + \underset{y_1}{\text{Max}} E\{V\}]$$

$$= \underset{y_2}{\text{Max}} [f_1 + \underset{y_1}{\text{Max}} \{\pi\}]$$

hence

$$f_2 > f_1 \quad . \qquad (80)$$

Proposition 6

Under the assumption of proposition 3,

1) $\exists$ a unique optimal size $\bar{x}_2^*$ for the reservoir which maximizes the total expected return $f_2(\bar{x})$;

2) $\bar{x}_2^* > \bar{x}_1^*$.

Proof of 1):

In this case, $\bar{x}_2$ is bounded below by 0 and above by $2y_0 + x_0$. Thus, $f_2(\bar{x})$ is defined on a compact set. However, $f_2(\bar{x})$, under the assumptions of proposition 4, is strictly concave. Thus, $f_2(\bar{x})$ must posess a unique maximum $\bar{x}_2^*$ on its convex and compact domain.

Proof of 2):

x_2^* and $\bar{x}_1^*$ are defined by $\frac{df_2}{d\bar{x}} = 0$ and $\frac{df_1}{d\bar{x}} = 0$ respectively.

or, equivalently by

$$-c'(\bar{x}) + \beta g'(\bar{x}) \int_{g(\bar{x})-rx_2+ry_2^*}^{\infty} c_1'(rx_2 + e - ry_2^* - g(\bar{x})\phi_e de$$

$$+ \beta g'(\bar{x}) \int_{g(\bar{x})-rx_2+ry_2^*}^{\infty} f_1'(g(\bar{x}))\phi_e de = 0 \quad , \tag{81}$$

and

$$-c'(\bar{x}) + \beta g'(\bar{x}) \int_{g(\bar{x})-rx_1+ry_1^*}^{\infty} c_1'(rx_1 + e - ry_1^* - g(\bar{x}))\phi_e de$$

$$+ \beta g'(\bar{x}) \int_{g(\bar{x})-rx_1+ry_1^*}^{\infty} v'(g(\bar{x}))\phi_e de = 0. \tag{82}$$

Since $y_2^*(x) < y_1^*(x)$, therefore

$$c_1'(rx + e - ry_2^* - g(\bar{x})) > c_1'(rx + e - ry_1^* - g(\bar{x}))$$

and

$$g(\bar{x}) - rx + ry_2^* < g(\bar{x}) - rx + ry_1^* .$$

As a result

$$\int_{g(\bar{x})-rx+ry_2^*}^{\infty} c_1'(rx + e - ry_2^* - g(\bar{x}))\phi_e de \quad \int_{g(\bar{x})-rx+ry_1^*}^{\infty} c_1'(rx + e - ry_1^* - g(\bar{x}))\phi_e de.$$

Also, since

$$f_1'(g(\bar{x})) > v'(g(\bar{x}))$$

then

$$\int_{g(\bar{x})-rx+ry_2^*}^{\infty} f_1'(g(\bar{x}))\phi_e de > \int_{g(\bar{x})-rx+ry_1^*}^{\infty} v'(g(\bar{x}))\phi_e de.$$

From (81) and (82), these results imply that

$$\frac{df_2(\bar{x}^*)}{d\bar{x}} > \frac{df_1(\bar{x}^*)}{d\bar{x}} \quad \forall\ \bar{x}^*; \quad 0 \leq \bar{x}^* \leq 2y_0 + x_0 \tag{83}$$

and hence

$$\bar{x}_2^* > \bar{x}_1^*. \tag{84}$$

The n Period Problem

For an arbitrary n, the continuity equation is given by

$$x_{n-1} = r(x_n - y_n) + e_n + i_n - m_n .$$

Define

$$f_n(x_n) = \text{Max } G_n(y_n, x_n - y_n)$$

where

$$G_n(y_n, x_n - y_n) = \pi(y_n, x_n - y_n) + \beta \int_0^{x^m - rx_n + ry_n} f_{n-1}(x^m)\phi_e de$$

$$+ \beta \int_{x^u - rx_n + ry_n}^{\infty} f_{n-1}(x^u)\phi_e de + \beta \int_{x^m - rx_n + ry_n}^{x^u - rx_n + ry_n} f_{n-1}(rx_n + e - ry_n)\phi_e de$$

$$- \beta \int_{x^u - rx_n + ry_n}^{\infty} c_1(rx_n + e - ry_n - x^u)\phi_e de$$

$$- \beta \int_0^{x^m - rx_n + ry_n} c_2(x^m - rx_n - e + ry_n)\phi_e de - c(\bar{x}). \tag{85}$$

Then $y_n^*(x_n)$ is defined by $\frac{dG_2}{dy_2} = 0$ or equivalently by

$$\pi_1(y_n, x_n - y_n) - \pi_2(y_n, x_n - y_n) = \beta r \int_{x^u - rx_n + ry_n}^{\infty} c_1'(rx_n + e - ry_n$$

$$- x^u)\phi_e de - \beta r \int_0^{x^m - rx_n + ry_n} c_2'(x^m - rx_n + ry_n - e)\phi_e de$$

$$- \beta r \int_{x^m - rx_n + ry_n}^{x^u - rx_n + ry_n} f_{n-1}'(rx_n + e - ry_n)\phi_e de. \tag{86}$$

Using a straightforward induction argument [31], the following propositions can be proven:

Proposition 7

If assumptions (a - d) in proposition 1 hold, then

1) $\exists$ a unique interior maximum $y_n^*(x_n)$

2) $0 < \frac{dy_n^*}{dx_n} \leq 1$

3) $y_n^*(x) \leq y_{n-1}^*(x)$

4) If $\pi_{11} = \pi_{12} \equiv 0$, then the optimal release rule is of the form

$$y_n^*(x_n) = x_n - a_n$$

where a_n is a constant dictated by the hydrology of the river basin, the size of the reservoir, and the specific form of the profit function.

Proposition 8

Under the assumptions of proposition 1 (a - e), if g is concave and h is a constant, then

1) f_n is strictly concave in x_n and $\bar{x}$

2) $\frac{df_n}{dx_n} = \pi_2(y_n^*(x_n), x_n - y_n^*(x_n))$

3) $f'_n > f'_{n-1}$.

Proposition 9

Under the assumptions of proposition 3:

1) $\exists$ a unique optimal size $\bar{x}_n^*$ for the reservoir which maximizes

the total expected return $f_n(\bar{x})$

2) $\bar{x}^*_n > \bar{x}^*_{n-1}$.

The Infinite Stage Process

In this section the following functonal equation will be discussed.

$$f(x) = \underset{0\le y\le x}{\text{Max}} \Big[\pi(y,x-y) + \beta \int_0^{x^m-rx+ry} f(x^m)\phi_e de + \beta \int_{x^u-rx+ry}^{\infty} f(x^u)\phi_e de$$

$$- \beta \int_{x^u-rx+ry}^{\infty} c_1(rx + e - ry - x^u)\phi_e de + \beta \int_{x^m-rx+ry}^{x^u-rx+ry} f(rx + e - ry)\phi_e de$$

$$- \beta \int_0^{x^m-rx+ry} c_2(x^m - rx - e - ry)\phi_e de - c(\bar{x})\Big]. \qquad (87)$$

Proposition 10

There is a unique solution to (85) which is bounded for x in any finite real interval. This solution, f(x), is continuous and concave.

The proof of this proposition is well known and follows closely the development given in Bellman [31]. Define the sequence $\{f_n(x)\}$ as follows:

$$f_{n+1}(x) = \underset{0\le y\le x}{\text{Max}} G(y, x - y, f_n). \qquad n = 0,1,2,...$$

where $f_0(x) = v(x)$ and $f_0(x)$ is continuous over $x \ge 0$. Then it can be shown that $\lim_{n\to\infty} f_n(x) = f(x)$ exists for $x \ge 0$ and is the solution of $f(x) = \underset{0\le y\le x}{\text{Max}}\, G(y,x,f)$. Moreover, the convergence of $f_n(x)$ is uniform.

Therefore, since each function in the sequence is continuous and concave, f(x) is continuous. To show the similarity of (87) to the problem discussed by Bellman [31], the following theorem is stated:

Bellman's Theorem

The functional equation

$$f(x) = \underset{y \geq x}{\text{Min}} \; [k(y - x) + z \; [\int_y^\infty p(s-y)\phi(s)ds + f(0) \int_y^\infty \phi(s)ds$$
$$+ \int_0^y f(y - s)\phi(s)ds]]$$

has a unique solution which is bounded for x contained in any finite interval. The solution f(x) is continuous. Assumptions:

K(y-x) and P(x-y) are convex.

Proposition 11

In the case of an infinite planning horizon and under the assumption that $y_n^*(x)$ exists and is unique for any arbitrary n:

1) there exists a unique optimal policy y*(x) where

$$y_n^*(x) \to y^*(x), \qquad x^m \leq x \leq x^u$$

2) $0 < \frac{dy^*}{dx} \leq 1$

3) $-1 \leq \frac{d\overline{y}^*}{d\overline{x}} < 0.$

Proof:

Since for any arbitrary n, we have $x^m \leq x_n \leq x^u$; it follows that

$y_n^*(x_n)$ has an upper bound equal to x_n and a lower bound equal to 0. It has also been shown that the sequence $\{y_n^*\}$ is a non-decreasing sequence such that

$$y_1^*(x) \geq y_2^*(x) \geq y_3^*(x) \geq \ldots.$$

Since each y_1^* is bounded below, $y_n^*(x)$ converges to $y^*(x)$ [26], where $y^*(x)$ is the solution of

$$\pi_1(y, x - y) - \pi_2(y, x - y) - \beta r \int_0^{x^m - rx + ry} c_2'(x^m - rx + ry - e)\phi_e de$$

$$+ \beta r \int_{x^u - rx + ry}^{\infty} c_1'(rx + e - ry - x^u)\phi_e de - \beta r \int_{x^m - rx + ry}^{x^u - rx + ry} f'(rx + e - ry)\phi_e de = 0. \quad (88)$$

The proof of the comparative statics results in the infinite stage process is similar to the proof previously outlined for the two period case.

Proposition 12

There exists a unique optimal size $\bar{x}^*$ for the reservoir which maximizes $f(x; \bar{x})$.

Proof:

Since the assumptions of the model make each member of the sequence $\{f_n(\bar{x})\}$ concave, $f(\bar{x})$ is also concave. The next step is to prove that $\bar{x}$ is bounded. Assume as before that there exists y_0 such that $\pi_1(y_0) = 0$ and $\bar{x}_0$ such that $\pi_2(\bar{x}_0) = 0$. The discounted gross revenue realised must be less than the gross revenue when the reservoir is always operating at y_0, because of the cost of importing and exporting water. Thus,

$$\text{realized gross revenue} \leq \frac{\pi(y_0, x_0)}{(1-r)} \qquad \forall \; \bar{x} .$$

Define $\bar{\bar{x}}$ by $\dfrac{c(\bar{\bar{x}})}{(1-r)} = \dfrac{\pi(y_0, x_0)}{(1-r)}$;

then

$$\text{realized gross revenue} \leq \frac{c(\bar{\bar{x}})}{(1-r)} \quad . \qquad \forall \; \bar{x} .$$

That is if $\bar{x} > \bar{\bar{x}}$, then the realized net revenue must be negative and hence $\bar{\bar{x}}$ bounds $\bar{x}$.

The Long Term Distribution and the Case of the Linear Decision Rule

The process we are dealing with is represented by the continuity equation

$$x_{p-1} = rx_p - ry^* + e_p + i_p - m_p .$$

This is a discrete time, continuous state Markov process. Therefore the usual "ergodic theorem" could not be employed to find the long-run distribution of the water stock.

In this section it is shown that the long-run distribution exists and can be derived for a special class of objective functions. This class of functions corresponds to the case when π_{11} and $\pi_{12} \equiv 0$.

Proposition 13

If the assumptions of proposition 12 hold, and if $\pi_{11} = \pi_{12} \equiv 0$, then there exists a long run distribution for the water stock in the reservoir given by

$$P(x = x^m) = \Phi(x^m + ra)$$

$$P(x = x^u) = 1 - \Phi(x^u + ra)$$

and $$x \sim \phi(x + ra) \text{ for } x,\ x^m < x < x^u$$

where a is a constant.

Proof:

We have seen that

$$i_p > 0 \Leftrightarrow m_p = 0 \text{ or } rx_p - ry^*(x_p) + e_p < x^m$$

$$\Leftrightarrow e_p < x^m - rx_p + ry^*(x_p).^{9} \qquad (89)$$

Moreover, we have seen that separability and linearity of π implies a linear decision rule of the form

$$y^*(x_p) = x_p - a \quad ^{10} \qquad (90)$$

Then, from (89) and substituting for $y^*(x_p)$ from (90), we have

$$i_p > 0 \Leftrightarrow e_p < x^m - ra. \qquad (91)$$

Therefore, it follows that

$$P(i_p > 0) = \Phi(x^m - ra) \cdot \qquad (92)$$

---

[9] This is Iff statement, because importing and exporting actions are not optimizing decisions, but rather a penalty imposed by the stochastic nature of the inflow to correct for deficiencies or surpluses after the decisions are taken.

[10] Notice that a is the same from period to period only in the long run for the infinite planning horizon case. However, in the finite case $y_p^*(x_p) = x_p - a_p$.

In a similar fashion, it is possible to show that

$$P(m_p > 0) = 1 - \Phi(x^u - ra) \tag{93}$$

and

$$P(i_p = 0, m_p = 0) = \Phi(x^u - ra) - \Phi(x^m - ra). \tag{94}$$

However, we know that

$$P(i > 0) = P(x_{p-1} = x^m), \; P(m_p > 0) = P(x_{p-1} = x^u) \tag{95}$$

and

$$P(i_p = 0, m_p = 0) = P(x^m < x_{p-1} < x^u)$$

Therefore,

$$P(x_{p-1} = x^m) = \Phi(x^m - ra) \tag{96}$$

$$P(x_{p-1} = x^u) = 1 - \Phi(x^u - ra) \tag{97}$$

and x_{p-1} is distributed as $\phi_e(x_{p-1} - ra) \quad x^m \leq x_p \leq x^u$. (98)

(96-98) show that the distribution of x_{p-1} has two mass points at x^u and x^m and is continuously distributed with $\phi_e(x_{p-1} - ra)$ in the range of (x^m, x^u). That is, the distribution of x is given by

$$\left.\begin{array}{ll} \Phi(x^m+ra) & \text{at } x = x^m \\ \phi(x+ra) & x^m < x < x^u \\ 1 - \Phi(x^u+ra) & x = x^u \end{array}\right\} \tag{99}$$

and

$$E(x) = x^m\Phi(x^m - ra) + x^u \{1 - \Phi(x^u - ra)\} + \int_{x^m}^{x^u} x \, (x - ra)dx.$$

The expression above could only be evaluated if a specific form for the profit function is postulated. It is also necessary to simulate the dynamic program for a large number of periods p until

$$(a_p - a_{p-1}) \to 0 \ .$$

Using the simulated value of a and postulating a specific form for the inflow distribution (e.g. log-normal or χ^2) after calibrating with actual data, the solution is found by: (1) select the optimal policy, given a particular physical size of the reservoir $\bar{x}$ (i.e. $y^* = x - a(\bar{x})$); (2) obtain the optimal size of the reservoir $\bar{x}^*$. The selection of $\bar{x}^*$ defines a exactly; therefore, the distribution of x is determined and so is E(x).

1.3 Conclusion and Summary

It has been demonstrated that chance constrained programming can be incorporated within the usual dynamic programming formulation by transforming the chance constraints into a penalty function that is added to the criterion function to be maximized. Moreover, it has been found that allowing for importing and exporting of water from the reservoir provides an economic rational for the penalty function and provides acceptable economic interpretation to the technical requirements for the solution of the maximization problem. Allowing for evaporation losses, the manager of the reservoir maximizes a criterion function which reflects benefits from water releases to agriculture and from the water stock in the reservoir for power generation. Within the chance constrained dynamic programming, the manager solves for the dual problem of optimal operating policy and optimal size of the reservoir. The procedure of maximization is similar to that of two-step programming in that water import and export is considered a residual decision to correct for the violation of the constraints. Specifically, it does not pay to engage in importing or exporting water unless violation of the constraints occur as a result of implementing the optimal policy. These conditions, together with concavity of the criterion function and convexity of the penalty function, are found sufficient to get all the usual dynamic programming results, such as the existence, uniqueness, monotonicity, and convergence of the optimal policy. It has also been demonstrated that the usual dynamic programming results extend to the optimal size of the reservoir under these and some other plausible conditions. Assuming the criterion function to be separable and linear in water releases, the optimal operation policy is found to be linear. Moreover, under this condition, it has been demonstrated that the long-run distribution of the water stock in the reservoir exists and is derived. Finally, another model is presented in the Appendix which incorporates the chance constrained problem into a planning model by finding a deterministic equivalent to the chance constraints. It has been demonstrated, that for an infinite sized reservoir, the optimal operating policy exists and is unique. Moreover, a formula for the long-run distribution of the water stock is derived and some bounds on the expected value are developed.

Appendix

A Chance Constrained Model: Deterministic Equivalent Approach

Consider a reservoir of infinite size, the problem is to maximize over a T period planning horizon a net discounted benefit function subject to chance constraints. Formally:

$$\max_{\substack{0 \le y_p \le y_{max} \\ p=1,\ldots,T}} \sum_{p=1}^{T} \beta^{p-1} \pi(y_p) \tag{1}$$

$$\text{Subject to } P(x_p \le x^u) \ge \alpha_1 \quad , \forall\, p = 1,2,\ldots,T \tag{2}$$

$$P(x_p \ge x^m) \ge \alpha_2 \quad , \forall\, p = 1,2,\ldots,T \tag{3}$$

$$x_p = rx_{p-1} \pm e_p - ry_p \ge 0 \quad \forall\, p = 1,2,\ldots,T \tag{4}$$

where x^u is the usable capacity, fixed by law to provide for flood control or some other considerations. x^m is the minimum head required for power generation. Alternatively, x^m can be determined by environmental considerations such as wildlife preservation or, perhaps more importantly, salinity control downstream. α_i is the maximum tolerance level associated with the ith constraint and x_p is the storage level at the end of period p (measured from the start of the planning period). y_p and e_p are the release and inflow in period p, respectively. β is an appropriate discount rate. Finally,

$$r = 1 - k, \quad 0 < r < 1 \tag{5}$$

where k is the percentage evaporation from the reservoir. For simplicity, the salvage value function at the end of the horizon is assumed to be zero. $\pi(y_p)$ is a strictly concave profit function such that

$$\pi(y_p) = 0 \quad \Leftrightarrow \quad y_p = 0 \text{ or } y_p = y_{max}.$$

It is assumed that $\frac{\partial \pi}{\partial y_p}$ at 0 and y_{max} are finite, and that there exists y_0, $0 < y_0 < y_{max}$ such that $y_p < y_0 \implies \frac{\partial \pi}{\partial y_p} > 0$, $y_p > y_0 \implies \frac{\partial \pi}{\partial y_p} < 0$, $y_p = y_0 \implies \frac{\partial \pi}{\partial y_p} = 0$, as shown in Figure 1. e_p is assumed independent and identically distributed with mean μ and variance σ.

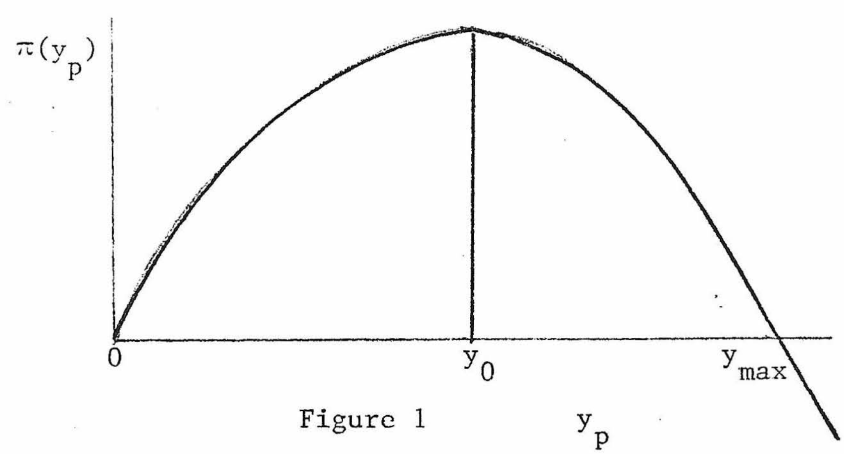

Figure 1

In the following, the deterministic equivalent of the stochastic problem is found, using the method developed by Charnes and Cooper [5]. The deterministic problem is then solved for the optimal policy (y_1^*, y_2^*, ..., y_T^*) over the planning horizon. Next the implication of this policy is examined within the original random context of the problem. In particular, the effect of this deterministic policy on the distribution of the stock of water is investigated when the planning horizon is extended indefinitely and the random setting of the problem is restored. This method has some problems which will be mentioned later. Finally, the distribution of the water stock, developed here, is only an approximation, as will be explained in detail below.

A Proposition

There exists a unique optimal solution $y_1^*, y_2^*, \ldots, y_T^*$ to the reduced equivalent deterministic planning problem of the original change constraint of equations (1) - (4) if $(x^u - x^m) \geq (\frac{1}{\sqrt{\beta_1}} - \frac{1}{\sqrt{\alpha_2}}) \frac{\sigma}{\sqrt{(2-k)}}$.

The implementation of this policy yields a family of approximate long term distributions for the water stock in the reservoir given by

$\psi^j(\mu_{xp}^j, \sigma_{xp}^2)$ where

$$x^m - \frac{\sigma}{\sqrt{k\alpha_2(2-k)}} \leq \mu_{xp}^j \leq x^u - \frac{\sigma}{\sqrt{k\beta_1(2-k)}}, \ \forall \ j.$$

Proof

The deterministic equivalents for the chance constraints will be developed first. Consider (2): $P(x_p \leq x^u) \geq \alpha_1$ or equivalently

$$P(x_p \geq x^u) \leq \beta_1, \quad \text{where } \beta_1 = 1 - \alpha_1. \tag{6}$$

But from the continuity equation, we have $x_p = rx_{p-1} + e_p - ry^*_p$ where y^*_p is the optimal release in period p. Hence,

$$x_p = r^p x_0 - \sum_{i=1}^{p} r^{p-i+1} y^*_i + \sum_{i=1}^{p} r^{p-i} e_i. \tag{7}$$

Or,

$$x_p = r^p x_0 - y^*(p) + E_p \tag{8}$$

where

$$y^*(p) = \sum_{i=1}^{p} r^{p-i+1} y^*_i, \tag{9}$$

$$E_p = \sum_{i=1}^{p} r^{p-i} e_i. \tag{10}$$

Then

$$E_p \sim g(\mu_p, \sigma_p) \tag{11}$$

where

$$\mu_p = \frac{\mu(1 - r^p)}{1-r} \tag{12}$$

$$\sigma_p^2 = \frac{\sigma^2(1 - r^{2p})}{1 - r^2}. \tag{13}$$

Thus from (6) we have

$$P(r^p x_0 - y^*(p) + E_p \geq x^u) \leq \beta_1$$

or, equivalently

$$P\left(\frac{x^u - r^p x_0 + y^*(p) - \mu_p}{\sigma_p} - \frac{E_p - \mu_p}{\sigma_p}\right) \leq \beta_1. \tag{14}$$

Define K_{β_1} by $$P(K_{\beta_1} \le \frac{E_p - \mu_p}{\sigma_p}) = \beta_1. \tag{15}$$

Then (14) implies

$$K_{\beta_1} \le \frac{x^u - r^p x_0 + y^*(p) - \mu_p}{\sigma_p} \tag{16}$$

However, by Chebychef's inequality,

$$P(K_{\beta_1} \le \frac{E_p - \mu_p}{\sigma_p}) \le \frac{1}{K^2 \beta_1} .$$

Therefore, (15) $\Longrightarrow$ $\beta_1 \le \frac{1}{K^2_{\beta_1}} \Longrightarrow K_{\beta_1} \le \frac{1}{\sqrt{\beta_1}}$ (17)

Substitution in (16) for $K_{\beta_1} = \frac{1}{\sqrt{\beta_1}}$ we have

$$x^u - r^p x_0 + \sum_{i=1}^{p} r^{p-i+1} y_i^* - \mu_p - \frac{\sigma_p}{\sqrt{\beta_1}} \ge 0 . \tag{18}$$

This is a more stringent constraint than the original deterministic equivalent constraint which would have resulted from using the actual distribution of e_p rather than the Chebychef bound. Alternatively, sharper bounds such as Markov, or special case bounds [4] could be used to develop deterministic equivalents for the chance constraints in this problem.

Similarly, the equivalent deterministic form for (3) is found to be

$$x^m - r^p x_0 + \sum_{i=1}^{p} r^{p-i+1} y_i^* - \mu_p - \frac{\sigma_p}{\sqrt{\alpha_2}} \le 0 \quad . \tag{19}$$

Thus the problem is transformed into

$$\max_{\substack{0 \leq y_p \leq y_{max} \\ p=1,2,\ldots,T}} \sum_{p=1}^{T} \beta^{p-1} \pi(y_p) \tag{20}$$

subject to (18) and (19).

Note that (18) and (19) can be rewritten as

$$y_p \geq \frac{1}{r} \left[-x^u + r^p x_0 - \sum_{i=1}^{p-1} r^{p-i+1} y_i^* + \mu_p + \frac{\sigma_p}{\sqrt{\beta_1}} \right] \tag{21}$$

$$y_p \leq \frac{1}{r} \left[-x^m + r^p x_0 - \sum_{i=1}^{p-1} r^{p-i+1} y_i^* + \mu_p + \frac{\sigma_p}{\sqrt{\alpha_2}} \right] . \tag{22}$$

The solution will be determined next. The Langrangian for the problem in (20-21) is given by:

$$L = \sum_{p=1}^{T} \beta^{p-1} \{\pi(y_p) - C(\bar{x})\} + \sum_{p=1}^{T} \lambda_{1p} \left[-x^u + r^p x_0 - \sum_{i=1}^{p} y_i^* r^{p-i+1} + \mu_p + \frac{\sigma_p}{\sqrt{\beta_1}} \right] + \sum_{p=1}^{T} \lambda_{2p} \left[x^m - r^p x_0 + \sum_{i=1}^{p} y_i^* r^{p-i+1} - \mu_p - \frac{\sigma_p}{\sqrt{\alpha_2}} \right]. \tag{23}$$

Ignoring the nonnegativity constraints on the y's, the first order conditions are given by

$$\beta^{p-1} \frac{\partial \pi}{\partial y_p} - \sum_{i=p}^{T} (\lambda_{2i} - \lambda_{1i}) \sigma^{i-p} = 0 . \tag{24}$$

$\forall p,\ p = 1, 2, \ldots, T.$

This is the usual marginality condition; the discounted marginal benefit from a particular choice of water release y* must be equal to the total discounted marginal cost which results from that choice. The other first order conditions are:

$$-x^u + r^p x_0 - \sum_{i=1}^{p} y_i^* r^{p-i+1} + \mu_p + \frac{\sigma_p}{\sqrt{\beta_1}} \leq 0 \tag{25}$$

(strict inequality implies $\lambda_{1p}^* = 0$);

$$-x^m - r^p x_0 + \sum_{i=1}^{p} y_i^* r^{p-i+1} - \mu_p - \frac{\sigma_p}{\sqrt{\alpha_2}} \leq 0 \tag{26}$$

(strict inequality implies $\lambda_{2p}^* = 0$);

and $y_p^*, \lambda_{1p}, \lambda_{2p}^* \geq 0$. (27)

Differentiating the first order condition (24) with respect to y_p:

$$\frac{d^2L}{dy_p^2} = \beta^{p-1} \frac{d^2\pi}{dy_p^2} \tag{28}$$

But $\frac{d^2\pi}{dy_p^2} < 0$ by strict concavity of π,

therefore $\frac{d^2L}{dy_p^2} < 0$. (29)

Thus the solution to (24), y_p^*, is unique.

Denoting the right-hand side of (21) and (22) by $\underline{y}_p$ and $\bar{\bar{y}}_p$ respectively, it follows that

$$y_p \geq \underline{y}_p = \frac{1}{r}[-x^u + r^p x_0 - \sum_{i=1}^{p-1} y_i^* r^{p-i+1} + \mu_p + \frac{\sigma_p}{\sqrt{\beta_1}}] \tag{30}$$

$$y_p \leq \bar{\bar{y}}_p = \frac{1}{r}[-x^\mu + r^p x_0 - \sum_{i=1}^{p} y_i^* r^{p-i+1} + \mu_p + \frac{\sigma_p}{\sqrt{\alpha_2}}] \tag{31}$$

and

$$y_{max} \geq \bar{\bar{y}}_p \geq y_p^* \geq \underline{y}_p \geq 0 \iff (x^u - x^m) \geq \sigma_p(\frac{1}{\sqrt{\beta_1}} - \frac{1}{\sqrt{\alpha_2}}). \tag{32}$$

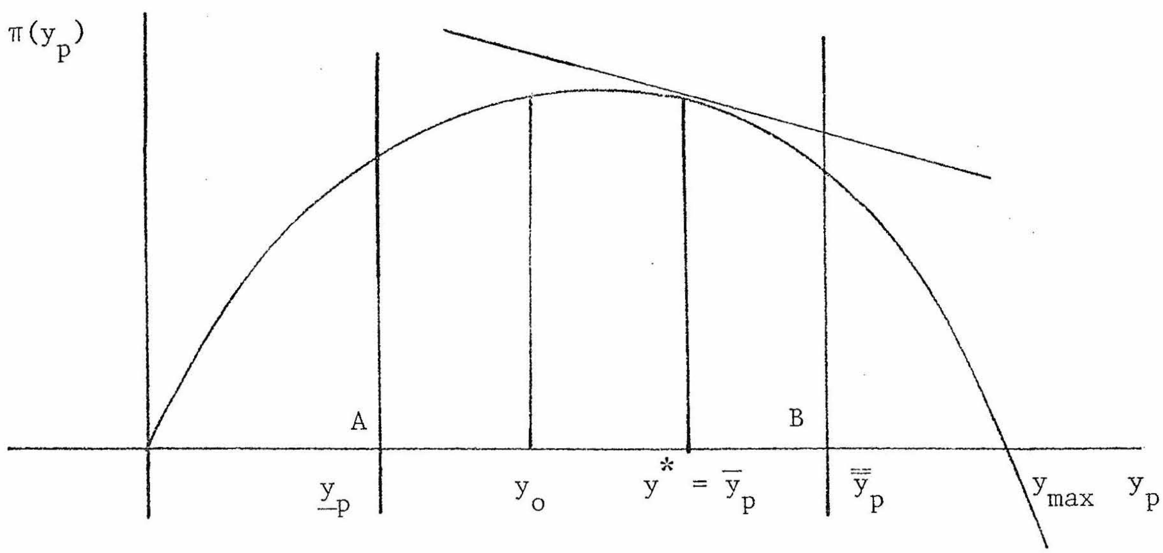

Figure 2

In this case, y_p^* lies in the closed convex interval {AB} in Figure 2. On the other hand, if the choice of α_1 and β_1 is such that

$$x^u - x^m < \sigma_p\ (\frac{1}{\sqrt{\beta_1}} - \frac{1}{\sqrt{\alpha_1}}). \tag{33}$$

Then (21) and (22) cannot hold simultaneously.

Let $\bar{y}_p$ denote the solution to (24). Thus,

$$y_p^* = \begin{cases} \bar{\bar{y}}_p & \text{if } \lambda_{2p}^* > 0 \\ \bar{y}_p & \text{if neither } \lambda_{1p}^*, \lambda_{2p}^* > 0 \\ \underline{y}_p & \text{if } \lambda_{1p}^* > 0 \end{cases} \tag{34}$$

Figure 3 illustrates the nature of the solution of (24).

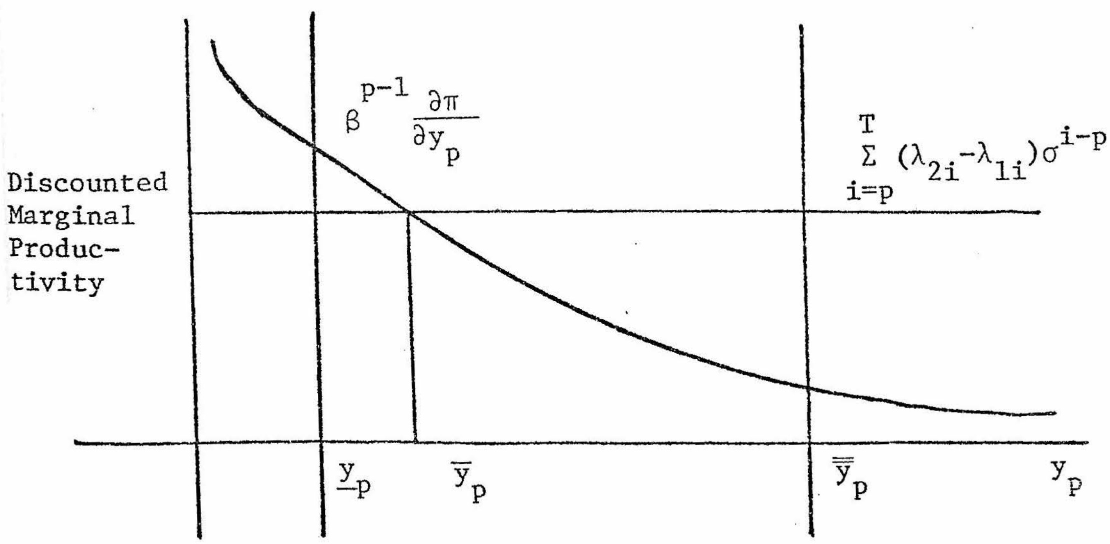

Figure 3

The Long-Run Distribution of x_p

For an infinite size reservoir, the probability of a spillover is zero. Moreover, if μ is large and we start with $x_0 = x^u$, the probability of empty reservoir is, also, very small.

From (12) and (13) as p is increased, $r^p x_0 \to 0$, $\mu_p \to \mu/k$ and

$$\sigma_p^2 \to \frac{\sigma^2}{k(2-k)} \tag{35}$$

Hence from (7)

$$x_p \to \psi(\mu/k - \sum_{i=1}^{p} r^{p-i+1} y_i^*, \frac{\sigma^2}{k(2-k)}). \tag{36}$$

However, if (32) holds $(x^u - x^m) \geq \sigma_p(\frac{1}{\sqrt{\beta_1}} - \frac{1}{\sqrt{\alpha_2}})$. That is, when the "adjusted" variability of the stream flow is small in comparison with the usable capacity,

$$\underline{y}_i \leq y_i^* \leq \overline{\overline{y}}_i, \qquad \forall\, i = 1, 2, \ldots, p. \tag{37}$$

Hence,

$$\sum_{i=1}^{p} r^{p-i+1}\underline{y}_i \le \sum_{i=1}^{p} r^{p-i+1} y_i^* \le \sum_{i=1}^{p} r^{p-i+1}\bar{\bar{y}}_i \quad . \tag{38}$$

From (25) and (26) we have

$$\sum_{i=1}^{p} r^{p-i+1}\underline{y}_i = -x^u + r^p x_0 + \mu_p + \frac{\sigma_p}{\sqrt{\beta_1}} \tag{39}$$

and

$$\sum_{i=1}^{p} r^{p-i+1}\bar{\bar{y}}_i = -x^m + r^p x_0 + \mu_p + \frac{\sigma_p}{\sqrt{\alpha_2}} \quad . \tag{40}$$

When $p \to \infty$ then $\mu_p \to \frac{\mu}{k}$, $\sigma_p \to \frac{\sigma}{\sqrt{k(2-k)}}$ and $r^p x_0 \to 0$.

Therefore,

$$\sum_{i=1}^{p} r^{p-i+1}\underline{y}_i \to -x^u + \frac{\mu}{k} + \frac{\sigma}{\sqrt{k\beta_2(2-k)}} \tag{41}$$

and

$$\sum_{i=1}^{p} r^{p-i+1}\bar{\bar{y}}_i \to -x^m + \frac{\mu}{k} + \frac{\sigma}{\sqrt{k\alpha_2(2-k)}} \quad . \tag{42}$$

Hence,

$$-x^u + \frac{\mu}{k} + \frac{\sigma}{\sqrt{k\beta_1(2-k)}} \le \sum_{i=1}^{p} r^{p-i+1} y_i^* \le -x^m + \frac{\mu}{k} + \frac{\sigma}{\sqrt{k\alpha_2(2-k)}} \; . \tag{43}$$

Thus, the long term distribution of x_p belongs to a class of distribution functions $\psi_j(\mu_{xp}^j, \sigma_{xp}^2)$ where

$$\sigma_{xp}^2 = \frac{\sigma^2}{k(2-k)} \tag{44}$$

and μ^j_{xp} is bounded as follows

$$x^m - \frac{\sigma}{\sqrt{k\alpha_2(2-k)}} \leq \mu^j_{xp} \leq x^u - \frac{\sigma}{\sqrt{k\beta_1(2-k)}} . \tag{45}$$

Notice that there exist β_1 small enough so that

$$x^m - \frac{\sigma}{\sqrt{k\alpha_2(2-k)}} = x^u - \frac{\sigma}{\sqrt{k\beta_1(2-k)}} . \tag{46}$$

In this case,

$$\mu_{xp} \to x^u - \frac{\sigma}{\sqrt{k\beta_1(2-k)}} . \tag{47}$$

In general, however, (47) holds if: 1) the value of r is large enough, and 2) the nature of the solutions y^*_i, which is bounded above, makes the sequence $s_p = \sum_{i=1}^{p} r^{p-i+1} y^*_i$ a nondecreasing sequence. In this case, $s_p \to \bar{s}$ [30] and

$$x_p \to \psi(\mu_{xp}, \sigma^2_{xp}) , \tag{48}$$

where σ^2_{xp} is given by (44).

(This ends the proof of the proposition).

In this model, treating water release as a deterministic decision variable facilitated the transformation of the chance constraints into deterministic form. This was done without assuming an a priori specific form for the decision rule. Moreover, an approximation for the long-run distribution of the stock of water in the reservoir was derived that provided reasonable bounds for the expected value of the distribution.

REFERENCES

1. Askew, A.J. "Optimum Reservoir Operating Policies and the Imposition of a Reliability Constraint." Water Resources Research, Vol. 10, No. 1 (February 1974): 51-56.

2. ________. "Chance-Constrained Dynamic Programming and the Optimization of Water Resource Systems." Water Resources Research, Vol. 10, No. 6 (December 1974): 1099-1105.

3. ________. "Use of Risk Premiums in Chance-Constrained Dynamic Programming." Water Resources Research, Vol. 11, No. 6 (December 1975): 862-866.

4. Barlow, R.E. and Marshall, A.W.. "Bounds for Distributions with Monotone Hazard Rate." Annal of Mathematical Statistics 35: 1234-1257.

5. Charnes, A. and Cooper, M.J. "Chance-Constrained Programming." Management Science, Vol. 6, No.1 (October 1959): 73-80.

6. ________. "Deterministic Equivalents for Optimizing and Satisfying Under Chance Constraints." Operation Research, Vol.11, No. 1 (January 1963): 18-39.

7. ________. "Chance Constraints and Normal Deviates." Journal of American Statistical Association , Vol. 57, No. 297 (March 1962): 134-140.

8. Dantzig, G.B. "Linear Programming Under Uncertainty," Management Science, Vol. 1 (1955): 197-200.

9. Eisel, L.M. "Chance Constrained Reservoir Model," Water Resources Research, (April 1972): 339-347.

10. ________. "Comments on the Linear Decision Rule in Reservoir Management and Design." Water Resources Research, Vol. 6, No. 4 (1970): 1239-1241.

11. Fiering, M.B. "Queueing Theory and Simulation in Reservoir Design." Proceding of the American Society of Civil Engineers, Journal of Hydrolics Division, No. 6 (November 1961): 39-69.

12. Hall, W.A., and Buras, N. "The Dynamic Programming Approach to Water Resources Development," Journal Geophysical Research, Vol. 66, No. 2 (February 1961): 517-520.

13. Helm, J.C., Curry, G.L., and Clark, R.A., "Chance Constrained Model of System of Reservoirs," Proceding of the American Society of Civil Engineers, Journal of Hydrolics Division, Vol. 12 (December 1973): 2353.

14. Hillier, F.S. "Chance Constrained Programming With 0-1 or Bounded Continuous Decision Variables." Management Science, Vol. 14, No. 1 (September 1967): 34-57.

15. Langbein, W.B. "Queing Theory and Water Storage." Proceding of the American Society of Civil Engineers, Journal of Hydrolics Division, Vol. 7, No. 5 (October 1958): 1811-1824.

16. Louks, D. and Dorfman, P., "An Evaluation of Some Linear Decision Rules in Chance-Constrained Models for Reservoir Planning and Operation." Water Resources Research, Vol. 11, No. 6 (December 1975): 777-782.

17. ________. "Some Comments on Linear Decision Rules and Chance Constraints." Water Resources Research, Vol. 6, No. 2 (April 1970): 668-671.

18. Miller, B.L. and Wagner, H.M., "Chance Constrained Programming with Joint Constraints." Operation Research, Vol. 13 (1965): 930-945.

19. Nayak, S.C. and Arora, S.R. "Linear Decision Rule: A Note on Control Volume Being Constant." Water Resources Research, Vol. 10, No. 4 (August 1974): 637-642.

20. ________. "Optimal Capacities for a Multi-reservoir System Using the Linear Decision Rule." Water Resources Research, Vol. 7, No. 3 (June 1971): 485-498.

21. Prekopa, A. "On the Probability Distribution of the Optimum of a Random Linear Program." Society for Industrial and Applied Mathematics, Journal on Control and Optimization, Vol. 4, No. 1 (1966): 211-222.

22. Quirk, J., and Burness, H.S. "The Colorado River, Water Rights and Allocation." Environmental Quality Laboratory Publication, California Institute of Technology, December 1976.

23. Revelle, C., Joeres, E., and Kirby, W. "The Linear Decision Rule in Reservoir Management and Design: Development of the Stochastic Model." Water Resources Research, Vol. 5, No. 4 (1969): 767-777.

24. ________. "Linear Decision Rule in Reservoir Management and Design: Performance Optimization." Water Resources Research, Vol. 6, No. 4 (August 1970): 1033-1044.

25. Rossman, L.A. "Reliability-Constrained Dynamic Programming and Randomized Rules in Reservoir Management." Water Resources Research, Vol. 13, No. 2 (April 1977): 247-255.

26. Rudin, W. Principles of Mathematical Analysis. McGraw Hill, 1953.

27. Segupta, J.K. "Safety First Rules Under Chance-Constrained Linear Programming." Operation Research, Vol. 17, No. 1 (January 1969): 112-132.

28. ________. "A Generalisation of Some Distribution Aspects of Chance-Constrained Linear Programming." Operation Research, Vol. 2, No. 2 (June 1970): 287-303.

29. Sobel, M. "Reservoir Management Models." Water Resources Research, Vol. 11, No. 6 (December 1975): 767-776.

30. Young, G. "Finding Reservoir Operating Rules." Proceding of the American Society of Civil Engineers, Journal of Hydrolics Division, No. 6 (November 1967): 297-321.

31. Bellman, R. Dynamic Programming. Princeton University Press, 1957.

CHAPTER 2

ON ECONOMIC CONDITIONS AND CONGRESSIONAL ELECTIONS - A NOTE

2.1 Democratic Theory and Voter Rationality

The search for the impact of economic conditions on election outcomes is part of a wider search for issue voting in the electorate [22], [23].* It is a "treasure hunt" for a vital mechanism in classic democratic theories [12], [13]. The basic assumptions of these theories are the accountability of the legislator to the electorate and the latter's responsiveness to the former's performance and programs.[1] The classical democratic theorists[2] argue that there are necessary conditions for accountability to exist; these are the ability of a voter:

1. to evaluate the incumbent's performance, and
2. to signal his approval or disapproval in accordance with his evaluation by casting his vote for or against the incumbent.

The American Voter [21] states "Commentaries on Democracy often assume

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[1]W. Berns wrote in "A Critique of Berelson, Lazarfeld, and McPhee's Voting" that "probably the most decisive test of an electoral system is the quality of men elected to office .... and the quality of men chosen depends on the individual voters who choose them" [39] H. Mayo emphasizes "accountability"; thus, "the election from this viewpoint is a kind of accounting for stewardship." [15], p. 78.

[2]R. Dahl, Pluralist Democracy in the United States [13], p. 17.

*References are at the end of Chapter 3.

two basic facts about the electoral decision: first, that the public is generally in possession of sufficient information regarding the various policy alternatives of the moment to make a rational choice among them; that is, that it has clear goals and is able to assess what the actions of government shall mean for these goals; and second, that the election in fact presents the electorate with recognizable partisan alternatives through which it can express its policy preferences." Thus, accountability is not sustainable in a political system in which the voter is ignorant and disinterested. It is also seriously limited when the electorate's evaluations and signals consistently diverge.[3]

2.2 National Politicians and the Voter's "Rationality"

For accountability to exist, another condition is needed. This is the incumbent's belief that the electorate actually behaves in this manner when reaching a voting decision. There is evidence that national politicians believe economic issues matter in national elections and function on this basis. Thus, R.M. Nixon [16] writes in Six Crises,

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[3] Lazarfeld's et al. Voting disputes this statement by pointing out that "where the classical theory of Democracy is defective is in its concentration on the individual citizen. What are undervalued are certain collective properties that reside in the electorate as a whole..." [10]. H. Mayo was even more emphatic in rejecting the necessity of these conditions for the survival of Democratic theories. In Introduction to Democratic Theory he wrote, "There is nothing irrational about voting on a broad preference for a person, party or the general drift a candidate or party may take. It may appear irrational in an economic sense (with its prudent and specific calculation of costs and satisfaction, or ends and means) but the rationality of economic behavior is a very narrow type which, applied to other things in life (love, friendship, and even political policy), is as absurd and impossible as Bentham's 'felicific calculus'." [15], p. 77.

p. 303, "As the national economy began to turn upward in the winter and spring of 1959, the Administration's standing rose accordingly." Some political analysts take this concept further and attribute a premanent effect on the electorate to major economic programs. W. Leuchtenburg [5] writes in The New Deal p. 2, "The new deal produced an upheaval in American Politics .... A country which had been predominantly Republican changed to overwhelmingly Democratic."[4] Tufte [18] shows that since the depression, government macro-economic policy has insured that disposable personal income never falls during a Presidential election year. Moreover, he finds that the timing of various income expanding measures relates to the electoral cycle. Finally, several scholars [35] cite the belief of various chairmen of the Council of Economic Advisors, that an expanding economy has a favorable impact on electoral behavior towards the incumbent.

2.3 Political Science and the Voter's "Rationality"

The findings of Empirical political science during the last two decades contradict the notion that the voter is the classical political being portrayed in Democratic theories. Most of these studies [20] agree with the assessment in The American Voter, "The American electorate has generally low emotional involvement in politics, slight awareness of

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[4]Also, see Nixon in the White House, where Nixon wrote on the effect of different programs on the voters: "The Kennedy farm program provided for massive increases in the Federal Government control of Agriculture. Polls taken two weeks after the Kennedy farm speech and mine indicated a substantial shift in the farm vote, away from the Democratic candidate..." [17], p. 303.

public affairs, low ability of thinking in structured ideological terms and pervasive sense of attachment to one or the other of the two major parties." [21] The classical democratic theorists argue that a responsible party system cannot coexist with an apathetic electorate and democratic theory cannot be erected on the shaky foundation described in works such as The American Voter.

In more recent literature, political scientists develop a two-pronged attack on these findings.

1) In a reexamination of voting data, one group of political scientists seeks to reassert the existence and importance of issues in the voting decision and thus to restore the prospective element of the electorate decision. G. Pomper [22] criticizes the previous studies on two grounds. First, he claims that their experimental and questionnaire designs do not "specifically and substantially try to discover the latent ideologies of their subject." Second, he states that previous studies are time bound, in the sense that their findings will not generalize to elections not covered by their data. He notes that the election campaign of 1956, for example, could hardly stimulate ideological voting.[5] In a more direct approach, Boyd [23] claims to have established the importance of both urban unrest and the traditional issues which divide the two parties, such as the extent of the federal

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[5]Pomper's [22] examination of six issues: federal aid to education, government provision of medical care, government guarantee of full employment, federal enforcement of fair employment, and foreign aid, led him to conclude that:

- parties took differential stands on these issues
- over time, the electorate became more aware of party differences on these issues
- consensus had risen appreciably by 1968 on the important

government's role in ensuring social welfare.[6] Others [25] warn that the existence of policy voting cannot be established without examining the processes which are relevant to the electorate's perception of the proximity of his favored candidates position and his own. These processes include, in addition to the alleged "policy voting," "persuasion" and "projection." In the second process, the perception and evaluation are fixed, while the voter's position is being influenced by the position he believes is held by the candidate he favors. In the third process, the voter projects his own position on the candidate he favors.[7]

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aspects of party position vis-a-vis these issues. J. Kessel [26], however, did not share G. Pomper's reading of his data. He pointed out that the lack of precision in the wording of the questionnaire could result in capturing a covariation of two issues instead of isolating the particular influence of a certain issue. He also points out that the observable rise in the proportion of issue voting may be due to either a) the increase in the proportion of voters to whom those issues are important, or b) responses to shift in party's ideological stand by voters whose preferences remain the same.

[6]R. Boyd [23] tried to isolate the short-term effect of the issue on voting from the long-term commitment of party identification by use of "normal vote" technique [24]. Normal vote analysis provided an estimate of parties' expected proportion of the vote for any selected group of the electorate in a normal election. This estimate is based on the following factors:

1. The balance of party identifiers within the group.
2. The typical defection rates of party identifiers to the other party.
3. Turnout rate.

[7]S. Verba et al., in The Changing American Voter presents strong arguments supporting the existence of issue voting, but adds the following words of caution: "The data do not eliminate the alternative causal explanation: that people adopt issue positions to suit their already-selected voting choice." They argue with Brody and Page that "It is difficult if not impossible to choose between the two causal directions with the kinds of data available." [41], p. 173.

In short, the techniques used until now to study policy voting are notably flawed either methodologically or substantively, and the question of prospective issue voting is certainly not yet settled.[8]

2) The second group seeks to establish the existence of a modified form of rationality in the voting decision of the electorate; the rationality of "retrospective voting." The advocates of this hypothesis argue that the voter is faced with: a lack of information about the competing programs, a high level of noise (such as campaign rhetoric), and steeply rising information gathering and processing costs. Therefore, he will settle upon the use of specific cues in reaching his decision on participation and voting. They further argue that the electorate possesses one hard piece of information: the incumbent's past performance. It is this piece of information that the electorate will use consistently in making its voting decisions.[9]

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[8]This conviction is not shared by S. Verba et al. in The Changing American Voter. They assert that "the data suggest that the American public has been entering the electoral arena since 1964 with quite a different mental set than was the case in the last 1950's.... They have become more concerned with issues and less tied to their parties." p. 166. They point out that "the public responds to the political stimuli offered it. The political behavior of the electorate is not determined solely by psychological and sociological forces, but also by the issues of the day and by the way in which candidates present those issues." [41], p. 319.

[9]H. Mayo remains unconvinced. He points out that: "Disappointed politicians often talk of public ingratitude for past favors. But this is to confuse the personal reasons why the individual votes as he does with the fact that all election results are by their nature future-oriented. The authorization to form a government and to decide future policies is always given by the voters collectively and taken as such by the successful representatives. To the voters who are conscious of this, it is presumably not what they think a party has done, but what it will do, that counts most on election day. Such a diagnosis helps to explain why the so-called ingratitude for the past should have only a minor part in voting." [15], p. 80.

For example, Kramer [1] states that "a voter, even a rational, self-interested voter, may not find it practical or efficient to proceed by collecting information of various kinds -- and vote for the 'best' package offered. For example, there may be no relevant party platforms to compare, platforms may (indeed, usually do) concentrate on desired ends rather than specific policy proposals. Voters may not feel qualified to make a confident or a priori assessment of the relative merits of positions on subtle technical issues, or they may recognize that platforms are in no sense binding commitments. Other information, such as detailed legislative records of individual candidates, may be very costly to acquire and analyze -- these considerations suggest that a more relevant decision rule for voters would be based on readily available information such as the past performance of the incumbent party."[10]

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[10] The awareness of the retrospective element is not new. As far back as 1961, Key wrote [27], p. 473, "perhaps the public can express itself with greatest clarity when it speaks in disapprobation of the past policy or performance of administration" and that the principal role of the electorate is to appraise "past events, past performance and past elections", because "only infrequently is a new program or a new course of action advocated with such force and the attention it receives so widespread that the polling may be regarded as advance approval of a proposed course of action."

However, this realization remains at intuitive levels, coupled with an awareness of the difficulty of discerning which issues are involved in the retrospective voting process. On this, V.O. Key wrote in [27], p. 473, ".... The collective decision may not specify with minuteness the elements of policy or performance of which it disapproves and cannot indicate with precision the lines of policy that should be pursued, save that changes should be made." M. Fiorina [6] concludes that "political scientists are aware of retrospective voting, and at least some are convinced of its significance. But their studies provide little knowledge of the specifics of retrospective voting."

A variant of the "retrospective" model is the negative voting hypothesis. This postulates an asymmetric behavior in which voters penalize the incumbent if he errs, but do not reward him if he does not. The basis of this hypothesis can be found in The American Voter [21]; "The party division of the vote is most likely to be changed by a negative public reaction to the record of the party in power -- a majority party, once it is in office, will not continue to accrue electoral strength; it may preserve for a time its electoral majority, but the next marked change in the party vote will issue from a negative response of the electorate to some aspect of the party's conduct in office."

The theories of recent studies in the retrospective voting tradition concentrate on the effect of economic conditions on voting. This is because economic issues are, first, not time bound and have the nature of continuous recurrence, and second, they are believed to have a strong effect on the well-being of the individual voter. The idea is that a phenomenon which has such an effect on the micro-level will be the easiest to detect from aggregative data. Several studies have been made to investigate the covariation between economic conditions and both voting and turnout. These studies are classified according to the data base used in them.

Cross-Sectional Studies

The data in these studies are classified by counties or states over shorter series of elections.

Rees et al. [30] investigate the relationship between the

Republican Congressional vote, and state "insured unemployment as a percent of covered employment" as well as net income per farm in forty-one states during the seventy Congressional elections from 1946-58.

A simple tabulation was used to show that a small negative covariation exists between unemployment and Republican vote, but none with income. However, as Fiorina [7] notes, "political responsibility varied across the period considered, so one is uncertain about the meaning or significance of the previous results."

Poll Data Studies

These studies use poll data rather than election returns.

Clark [31] examines the correlation between the Gallup Poll monthly series data on Roosevelt's popularity and a national income series. He reports correlations of 0 - 0.52 in aggregate, and between 0.78 - .95 when the data are segmented according to different economic classes.

Longitudinal Studies

These studies analyze aggregate national election results over a long series of Congressional elections.

An early example of this group is L.H. Beam's study [28]. It examines the hypothesis that the public tends to vote for the retention of administrations that have been in power during prosperous times and to vote against the incumbent administration when depression has coincided with election time.

Using an index of general business activity, he compares changes in the index between October of the election year and October two years earlier, with changes in the House membership of the incumbent administration. Beam is able to show that in fifteen out of nineteen cases of decline of the indices, the incumbent has suffered a net loss in House membership. The effect has been particularly pronounced in mid-term elections, when losses have always been observed. In on-year elections, however, the phenomenon has been observed only in six out of ten.

He also concludes that "the President's personality and other factors partially offset the impact of economic conditions in Presidential years."

Recent contributions by Kramer [1], Stigler [3], and Arcelus and Meltzer [4] employ more sophisticated econometric techniques, but reach conflicting conclusions on this issue. Kramer poses two different models for mid-term and on-year Congressional elections to account for the possibility of "coattail" effects from the Presidential contest; the dependent variables are the Republican share of the vote in the first model and the President and Republican share of vote in the second. The same exogenous variables appear in both relations and include changes in employment, per capita real income, per capita money income and the consumer price index. A coattail effect variable is added to the on-year election model to account for the effect of the candidate and "campaign tactics" in the Presidential race. In the Congressional equation, it is assumed that a certain fraction of this presidential effect is carried over into the Congressional vote. (See Appendix A for a more thorough exposition.)

Kramer's findings are that unemployment fluctuations appear to have no impact on the aggregate vote, but changes in real income do. He also finds that the incumbency variable coefficient is "invariably small and insignificant."

He also finds that economic fluctuations affect the votes for Congressmen who belong to the President' party more than they do the vote for the President himself. This is puzzling because of Kramer's earlier argument in favor of associating incumbency with the control of the White House rather than Capitol Hill.[11]

Stigler [3] formulates a one-equation model to test Kramer's results. In this model the dependent variable is the percentage of total votes cast for Republicans,[12] or alternatively, the Republican share of total vote, and the explanatory variables are: incumbency, relative change in per capita real income (the sign is governed by

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[11]Kramer's [1] ambition to use his result for long-range quantitative predictive purposes -- in the sense that "10 percent decrease in per capita real personal income would cost the incumbent administration 4 - 5 percent of the congressional vote" -- is ill justified. This is so because of the presence of multicollinearity in the model (Stigler reports high correlation (-.78) between changes in unemployment and relative change in per capita real income), which makes the separation of their effect in the model difficult. Also, the long span of the period of study casts serious doubt on the real utility of the estimated values of the coefficients in any forecasting exercise. One cannot believe, for example, that the effect of incumbency or the variations in economic conditions have the same effect on the voter of 1900 as that of 1964.

[12]Stigler's formulations, where he uses changes in the Republican share of vote from preceding elections as dependent variable, suffer from a basic ambiguity. There are now four states of the World that Stigler is trying to represent by only one dichotomous variable; they are: whether or not the Republicans are incumbent at the t^{th} election, and whether or not they were in power at the $(t-1)^{th}$ election. The change in the Republican share of vote will depend on which of the four states occurs.

incumbency), and the price level (measured by the consumer price index).

He raises serious doubt about Kramer's findings by showing their sensitivity to changes in time periods covered. A shift from one year to a two year time base and whether certain years are included in the regression are also shown to affect the results.

Stigler suggests a new forecasting model where the voter develops "credibility" indices for the competing parties based on past income experience. He then uses these indices to guide his voting decision. (Each index is computed by discounting the early experience by some discount rate and summing over the relevant period.) He finds no significance for the weighted average of income experience, and also confirms Kramer's observation about the insignificance of the unemployment variable. His interpretation of his result, however, is controversial [32, 33]. For example, his claim that the voter perceives no significant differences between the position of the two parties runs against numerous studies [29], [33], and [8]. For example, Fiorina [6] presents evidence that suggests "employment, wage levels and farm policy are Democratic issues; government spending and taxation are Republican issues."[13]

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[13] In M. Fiorina's study [6], we find the following Table on p.12.

PRESIDENTIAL VOTE REPORTS BY SPECIFIC
CONSEQUENCES OF ELECTION

Consequence	1952	1956	1960
Employment	52	27	57
Wages	33	7	9
Depression	38	-27	14
Farm Policy	--	46	48
Group References	--	41	52

One cannot help disagreeing with Stigler's reading of his own results. In various regression coefficients, such as those of price terms and the real income term (in some models), are significant, and even employment terms are sometimes significant.

Arcelus and Meltzer (hereafter referred to as AM) examine the effect of economic conditions on both participation and votes in Congressional elections.

On the participation decision, AM distinguishes between three categories of voters: the habitual voters,[14] those who vote in a Presidential year because of relative decrease in information cost, and those who are induced to participate by the impact of economic conditions. The proportion of voters who participate in a given election is, then, given by the sum of the proportions of these categories. A certain proportion of the total number participating will vote Democratic. Similarly, certain proportions will vote Republican or third party,

PRESIDENTIAL VOTE REPORTS BY SPECIFIC CONSEQUENCES OF ELECTION (CONT'D)

Consequence	1952	1956	1960
Prices, Inflation	4	19	12
Government Spending, Taxes	-70	-28	-63

Entries are % Democratic - % Republican

[14]AM assume that voter participation fluctuates around relatively stable levels although inspection of data reveals that "the major changes have been gradual trends, persisting over a series of elections rather than the abrupt shifts posited by the Arcelus and Meltzer model through the inclusion of dummy variables. This is a serious misspecification in the long-term model which could lead to biased estimates of all coefficients on the various motivation factors which induce those voters who are effected by economic conditions to participate." [2].

such that the total partisan votes will balance the total number of voters participating in an election. AM conjecture that voters affected by economic conditions may be induced to vote because they either perceive differences between candidates or parties on a particular issue or they wish to reward a party or candidate for a policy or outcome and encourage continuance.[15] (For a more thorough exposition of their model, see Appendix B.)

The major finding of AM is that "aggregate variables affect the participation rate in Congressional elections but have little, if any, effect on the relative strengths of the two major parties. There is very little evidence that an incumbent President can effect the composition of the Congress by measures that have short-term effects on unemployment or real income -- the findings support the hypothesis that the principal fluctuation in the percentage of votes received in Congressional elections arises from changes in the participation rate and not from shifts between parties. The principal effect of Presidential incumbency is to increase participation. Democrats are more affected by Presidential elections and the incumbent's party affiliation than the Republicans."

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[15] Arcelus and Meltzer offer no hypothesis as to the relative strengths of the three motivations in the electorate as a whole, or within each party advocate. In the absence of such information, their theoretical hypothesis does not lead to any clear predictions about the net effects of economic conditions on participation. In fact, this lack of integration extends to the theoretical underpinning of the AM model itself; that is, $R = PB - C + D$ suggested by Tullock [36], where R is the net utility of voting, B is the value of the victory of a particular candidate, P is the probability that the voter affects the outcome, C is cost of voting, and D the utility of other factors affecting voting. AM do not discuss in detail the relationship between the factors in this model and their subsequent modeling of participation.

AM's modeling of the two parties' vote is, however, unsatisfactory. The categorization of partisan vote into a proportion of habitual voters, a proportion of those affected by economic conditions (modified by incumbency), and proportion of the extra turnout in Presidential elections are not mutually exclusive and collectively exhaustive. Certainly, an important category is missing, the group of voters which Kramer and Goodman called "shift" voters who regularly vote, but switch from one party to the other in response to economic or other issues. The omission of this category is particularly surprising, given that the objective of the exercise is to establish the existence of this group. Kramer [2] adds a fourth block of voters to AM's model to account for the shift voters. He establishes that the shift effect is not identified in AM's model.

Moreover, AM's treatment of the various fractions of voters as being invariant with time seems at least dubious given the extended length of the study. The affect of time on information cost, shifts in demographic structure and the changing base of commodity production will undoubtedly affect these proportions.

On the whole, the AM results leave a great deal of ambiguity about the nature and magnitude of the possible effects of economic variables on turnout and election outcomes.

2.4 Survey Data Studies, the Micro-Approach

In all of these papers, aggregation causes an interpretive problem. A fundamental question remains to be answered. What is the linkage between the individual's response to his own financial fortune and his response to the overall changes in the economy?[16] Some studies point out the possibility of the voter being optimistic about his own well-being while remaining pessimistic about the general conditions of the economy [37]. The presentation of the previous papers, as has been shown, suffer from a number of logical and methodological shortcomings. Moreover, the results seem, on the whole, inconclusive.

M. Fiorina seeks firmer evidence by examining the effect of the individual's perception of his economic well-being on his voting decision [6]. Using data from 1952-1970 SRC election studies, he examines a number of related issues:[17]

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[16]Some attempt to provide such bases can be found in Ray C. Fair "The Effect of Economic Events on Votes for the President." [44].

[17]Fiorina used the following model:

$$Y = a + b(P) + c(PE) + \epsilon$$

where

$Y = 1$ if respondent voted Republican
$= 0$ otherwise

$P' = (P_1, P_2)$

where

$P_1 = 1$ if the respondent is Independent
$= 0$ otherwise

$P_2 = 1$ if respondent is Republican, 0 otherwise

while $(PE)' = [(PE)_1, (PE)_2, \ldots, (PE)_6]$ are interaction terms of party identification and economic conditions. For example

$(PE)_1 = 1$ if the respondent is Democrat and he perceived no change in his economic lot
$= 0$ otherwise

and

$(PE)_2 = 1$ if the respondent is Democrat and perceived an improvement in his economic condition
$= 0$ otherwise.

1) Do the American people perceive party differences on issues of economic policy?
2) If so, what is the substance of those perceptions?
3) Are those perceptions associated with variations in voting behavior?

On the first question, Fiorina satisfies himself in the affirmative; on the second, he finds some evidence that "employment, wage levels and farm policy are Democratic issues. Government spending and taxation are Republican issues." Both conclusions are based on examination of statistical evidence from cross tabulation. However, on the third question, the verdict is, for Congressional voting, that the findings are positive until 1960 and negative thereafter.

Fiorina's interpretation of his results are not based on firm ground due to the omission of important factors from the model. This alone could lead, under certain conditions, to misspecification and to erroneous readings of the significance of coefficients.

Appendix A

Kramer's Model

Kramer's dual model [1]:

$$Y_t^c = X_t B + U_t \text{ for mid-term elections} \quad (1)$$

and

$$Y_t^p = X_t B + U_t + v_t$$
$$Y_t^c = X_t B + U_t + \gamma v_t \quad (2)$$

he assumes the same B in (2) and (3), which leads to the conclusion that the expected share of the vote for the presidential candidate ($X_t B$) is the same as the expected vote for the Congressional candidates of the party. Such assumption is not substantiated by observations. The overwhelming victory of a Republican president in the 1972 election also witnessed the election of a strongly Democratic Congress. Moreover, if we examine the basic model of Kramer,

$$Y_t^p = v + \delta_t(\alpha^p + B_1^p \Delta_t) + u_t + v_t \quad (3)$$

$$Y_t^c = v + \delta_t(\alpha^c + B_1^c \Delta_t) + u_t + \gamma v_t \quad (4)$$

where (δ_t) is the incumbency advantage coefficient, we see that the assumption of equal B's in (2) and (3) implies that

$$\alpha^c = \alpha^p \text{ and } B_1^c = B_1^p \ .$$

This implies the unreasonable results that:

a) the incumbency advantage has the same effect in the Presidential and Congressional elections.

b) Voters hold Congressmen to the same degree of "acccountability" as the occupant of the White House for the coun-ry's economic conditions.

Although these assumptions were vital for the procedure of the estimation adopted by Kramer (see Kramer's treatment in his appendix), they are unnecessarily restrictive. To cope with the difficulty of the coattail variable, it is only necessary to reformulate the model as follows:

$$Y_t^c = X_t B + ku_t \quad \text{in Presidential elections}$$

$$Y_t^c = X_t B + u_t \quad \text{otherwise.}$$

A two-step GLS Theil procedure could then be used to estimate the model. [43]

Appendix B

Arcelus and Meltzer's Model

A) On the decision to participate, Arcelus and Meltzer distinguish between three categories of voters.

1) The Habitual Voter. Two major changes effect the proportion of habitual voters (H_t) during the period of study: extension of suffrage to women in 1920 and the "big" realignment of the 1932's. Thus,

$$H_t = v_0 + (\Delta_1 v_0)X_t 20 + (\Delta_2 v_0)X_t 32 + e_{1t}$$

where

$X20 = 1$ if $t \geq 1920$

$= 0$ otherwise

$X32 = 1$ if $t \geq 1932$

0 otherwise

and v_0 is the fraction of strictly "habitual" voters prior to 1920.

2) Those who vote in a Presidential election year because of the relative decrease in information cost. If we denote the incremental percentage of those voters by N_{1t}, then

$$N_{1t} = v_1 PR_t + \Delta v_1 \, (RI_t)(PR_t) + e_{2t}$$

where

$PR_t = 1$ if t is a Presidential Election year

$= 0$ otherwise

$RI_t = 1$ if incumbent is a Republican

$= 0$ otherwise .

Thus, the second term is an interaction term of incumbency and type of election.

3) Those who are induced to participate by the impact of economic conditions.

$$N_{2t} = a_1\dot{P}_t + a_2\ \dot{U}_t + a_3\ (\dot{\frac{C}{P}})_t + e_{3t}$$

where $\dot{P}$, $\dot{U}$, $\dot{C/P}$ are percentage changes in consumer prices, the unemployment rate and real compensation per man hour.

Thus, the percentage of voters participating in a Congressional election VP_t is given by

$$VP_t = V_0 + (\Delta_1 V_0)\ X_t 20 + (\Delta_2 V_0)\ X_t 32 + V_1 PR_t +$$

$$V_1 (RI)_t (PR)_t + aE_t + e_t$$

where

$$aE_t = a\ \dot{P}_t + a\ \dot{U}_t + a\ (\dot{\frac{C}{P}})_t$$

B) The choice of party candidate. If $VD_t = D_t \cdot VP_t$ is the proportion of those elegible to register who vote for the Democratic candidate in election year t and similarly $(VR)_t$ for Republican and third party vote $(VT)_t$, then

$$VP_t = VD_t + VR_t + VT_t$$

and

$$VD_t = d_0 + (\Delta_1 d_0) X_t 20 + (\Delta_2 d_0) X_t 32 + d_1\ PR_t +$$

$$(\Delta d_1)(PR_t) RI_t + d_2 P_t + d_3 \dot{U}_t + d_4\ (\frac{C}{P})_t +$$

$$(\Delta d_2)\ \dot{P}_t RI_t + d_3 \dot{U}_t\ RI_t + d_4\ (\dot{\frac{C}{P}})_t\ RI_t + e_{Dt} \quad .$$

Notice the inclusion of interaction terms between incumbency and economic variables, which makes this model more saturated than the previous ones.

CHAPTER 3

ON THE RETROSPECTIVE EFFECT OF ECONOMIC CONDITIONS IN CONGRESSIONAL ELECTIONS: A SIMULTANEOUS EQUATION MODEL APPROACH

3.1 Introduction

The various models developed by investigators to explain political choice have stressed either objective or "materialistic" factors, such as campaign expenditures or economic conditions [1] [6], or have stressed subjective or "nonmaterialistic" factors, such as salience of the candidates, party identification and some aspects of incumbency [8].

In a society where achievement and success are largely attributed to hard work and rational effort and where political structure results in political "education" which stresses the government responsibility and ability to influence the economic well-being of the country, it is hard to reject, a priori, the argument of the various investigators mentioned in Chapter 2. These arguments rationalized and presented some evidence as to the effect of the performance of the government in the economic arena on the voting decision, whether it is on the individual's level or on the level of the electorate as a whole. All things being equal, one would expect that a voter will choose to give his vote to the person whom he can recall by name in the voting booth rather than to a complete unknown. The catch words used here are "all things being equal"; the fact is that things are not equal as far as the individual is concerned. The question then becomes; "Under what conditions is the salience of the candidate transforms into a positive or negative vote?"

Incumbency, on the other hand, has enjoyed a more celebrated place in the literature of voting than salience (see Chapter 2). It has been hypothesized that incumbency is used to bribe certain effective sections of the electorate through pet projects, etc., or to buy salience through the acquisition and expenditure of campaign resources. It can also have a certain magic which expresses itself through the saying, "The devil we know is better than the devil we don't." The real world is not polarized into objective and subjective elements; there is a dialectical unity in the world which underlies socio-political phenomena. To explain the effect of the subjective elements, we have to look for the underlying objective factors, and vice versa. Models which capture this essence of the real world are the only models which answer such questions as; "How do these factors influence this phenomenon?", rather than, "What factors are involved?"

This paper examines the responsiveness of the participation and voting decisions of the voter to the performance of the incumbent president. This examination is conducted within a framework of the political phenomenon of simultaneity. Previous studies which were based on a single-equation estimation procedure suffer from conceptual as well as methodological shortcomings. A unification of the "objective" and "subjective" approaches in one framework will ascertain their effect on the individual's dual decisions on participation and voting. The emphasis, however, will be on the effect of economic conditions on turnout and on the electoral fortunes of the President's party in Congressional elections. Earlier studies by Kramer [1], [2], Stigler [3] and Arcelus and Metzler [4] use models in which the dependent variable is the parties' aggregate Congressional votes. Various macroeconomic indicators of performance such as inflation, employment and income serve as explanatory variables.

The contradictory findings and the numerous methodological and logical errors prompted M. Fiorina to seek confirmation of the phenomena at the individual voter's level [6]. Using SRC (1952-1974) data, he establishes that "a citizen's personal economic condition affects his Presidential vote, but for Congressional voting the findings are positive until 1960 and negative thereafter." He also observes that there is "no systematic relationship between a citizen's personal economic condition and his decision to vote or abstain."

The above mentioned models are single-equation formulations which ignore important determinants of voting behavior, making it subject to simultaneity as well as misspecification bias. This observation covers not only this study, but almost all studies in the field of voting. We could hardly stress the importance of simultaneity not only in the voting decision, but also in all political phenomena. On this, B. Page [38] writes, "Single or recursive equation models suffer from simultaneity bias, yet simultaneous equation models are exceedingly difficult to specify in a plausible fashion." What is surprising, though, is that little effort has been expended to locate those aspects of the problem where a simultaneous equation model can be formulated, and where some exogenous variables can be excluded from some equations on a sound theoretical basis, thereby facilitating identification and estimation of the model. In the particular setting of economic retrospective voting, there is also the possibility that economic factors, or for that matter, any other factor may operate directly and indirectly through some specific variable on the dual decision of participation and voting for the presidential party.

In general, the task is to take into consideration various simultaneity effects in order to answer a number of related questions.

1) What are the relative effects of objective factors, such as the individual's perception of his own economic well-being on both his decision to participate and his voting decision as opposed to informational factors, such as incumbency or saliency of the candidate?

2) What are the underlying influences behind informational factors? Are there objective factors driving individuals to seek information about the candidates? What are the secondary channels through which an informational factor may also exert its influence?

3) How do the effects of these variables vary over time? How do they vary between off-year and on-year Congressional elections?

4) What are the overall effects of incumbency?

In this paper a preliminary investigation of SRC (1952-1970) data is conducted to suggest the relevancy of various variables to the individual's dual decisions in participation and voting. Some testable hypotheses will be developed. A simultaneous equation model will then be formulated. Simultaneity is captured through the assumption that incumbency as well as economic conditions directly affect the voting decision, as well as the assumption that it is indirectly affected by the awareness of the Presidential party candidate. It will be established that there is no significant effect of economic conditions on the individual voter's decisions for all Congressional elections covered by the survey and for the pooled data. Other variables such as awareness of the President's party candidate and incumbency will show more significance in off-year elections than in on-year elections.

3.2 Preliminary Investigation of the Data

The task of this section is to probe the available data using simple statistical techniques to investigate the relevancy to the individual's voting decisions of various variables which are considered a priori as being relevant.[1] Also, the interaction between these variables will be examined. These findings will be used as motivation for the simultaneous equations model. Various indices are extracted from the raw data in the SRC surveys (1956-1970). These indices will then be used to make some tentative hypotheses and observations.

Three categories of party affiliations are considered:

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[1]This is a common procedure in political science, but discussion of relevancy should be based on theoretical ground, not statistical ground. However, these cross tabulation techniques, in certain circumstances, are sufficient to prove some points without going to elaborate regression models.

Democratic, Independent and Republican.[2] Also, three categories of respondents to the question regarding their perception of changes in economic conditions are considered: those who perceived "better" conditions, those who perceived the "same" conditions, and those who perceived "worse" conditions. For this purpose, use was made of the following question in the SRC survey. "During the last few years, has your financial situation been getting better, getting worse, or has it stayed the same?"

For the salience variable, use was made of a question in the SRC survey which asked the respondent to name the candidates for the House in his district. If the respondent could name the candidate, he was considered to be aware of him; otherwise not. The limitation of the data is mainly due to the availability of recognition data only for 1958, 1964, 1966, 1968, and 1970 elections.

The results are mainly reported in the appendix, and the tables are suitably labeled as such by appending the letter A to the number of the table to distinguish it from the summary tables in the main text. Since it will be necessary to make some observation regarding the relative effect of certain variables over time, the tables show the differential values of these variables rather than their absolute values. For example, if a test is to be made that recognition of the incumbent (Inc.) is increasing over time relative to that of the challenger, then

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[2]For the purpose of inference from pooled runs, it would have been better to code the PID variables as follows:

PID = 1 if respondent belongs to the president's party
= 0 otherwise

However, we opted for the three-way categorization D, R, and I to facilitate comparison with other works.

the relevant variable to observe over time is the differential recognition of the incumbent. That is (the percent recognizing the incumbent minus the percent recognizing the challenger). This has simplified the form and inference from the summary tables. The interest will be in the number of entries in the original table with positive or negative sign, the magnitude of the entries (how much positive or negative are they?) and the number of cases which show increasing (↑) or decreasing (↓) entries over time.

3.3 On Turnout and Economic Conditions

The question is whether the inference from the data support AM's contention that the main effect of the individual's perception of his well-being falls on his decision to participate. We can postulate two hypotheses in this regard.

a) The Apathy Hypothesis (A). This states that the probability of abstaining increases with betterment in the voter's economic conditions. Thus, it can be expected that a higher proportion of those in the "better" response category will be non-voters.

b) The Protest Hypothesis (R). This states that the probability of abstention increases with the worsening of the individual's economic condition. Thus, it can be expected that a higher proportion of those in the "worse" response category will be non-voters.

Let P_B be the proportion of abstention in the group who perceived betterment in their conditions and P_W be the proportion of abstention in the group who perceived worsening in their conditions. The weakest criterion to test the Apathy (A) hypothesis requires that $P_B \geq P_W$,[3] while for the support of the protest (R) hypothesis that $P_W \geq P_B$. To test the two hypotheses, the proportions of different party affiliates (PID), in various economic response categories, who abstained are calculated in Table 1A. For example, in 1956, 29.2 percent of the Democrats who perceived improvement in their economic conditions abstained. Tables 1 and 2 summarize the results. It is concluded that the protest hypothesis claims 75 percent of the cases for the Democrats, while the two hypotheses have equal strength in the case of the Republicans and Independents. As is clear from Table 1, no general conclusion can be drawn as to the relative strength of the various hypotheses except that the protest hypothesis seems to be strongest in the case of Democrats.

Table 1: Percent of Total Cases in

Support of Either Hypothesis A or R

Dem.		Rep.		Ind.	
A	R	A	R	A	R
25.0	75.0	37.5	37.5	50.0	50.0

Total number (n) of cases for each PID category = 8

Table 2 shows for different PID's the average proportion of non-voters during on-year and off-year elections. The table presents

[3]It is a weak criterion, because it does not insist on "complete" monotonicity, i.e., $P_B \geq P_S \geq P_W$, where P_S is the proportion of non-voter in the "same" economic response category.

evidence that mid-term Congressional elections have a higher total proportion of non-voters compared to on-year elections. For example, the average percentage of non-voters among the Democrats during mid-term elections is 47 percent compared to 30 percent during on-year elections. Thus, if we accept AM's contention that economic conditions mainly affect participation, the result may imply that the economic-condition[4] effect is relatively stronger in mid-term than in on-year elections. However, the lower turnout in off-year elections may be simply explained by the absence of the presidential race.

Table 2: Percent of Non-Voters

	on-year	off-year
Dem.	30 %	47 %
Rep.	22	38
Ind.	42	60
n=4		n=4

Table 1A provides, also, some evidence of increasing non-participation over time. This is true for all economic response categories in all party affiliations.

3.4 On Voting Decisions

The Effect of Economic Conditions

First, we seek some confirmation for the intuitive notion of a positive effect of economic conditions on the voting decisions. The effect of various other variables will then be investigated

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[4]Other explanations for this observation may be advanced. For example, the voter may simply believe that his vote matters less in mid-term than on-year elections; or, that information cost is higher in mid-term than on-year elections, due to the lower level of publicity of the former compared with the latter.

for different economic response categories, controlling for the presidential party's candidate and party affiliation (Table 2A). If the intuitive notion about the phenomena is correct, it is expected that a higher proportion of those who are in the "better" response category will vote for the candidates of the presidential party. Define P_{better} as the proportion of voters who favor the candidate of the presidential party among those in the "better" response category; P_{same} and P_{worse} are defined analogously. The weakest test of the model is whether $P_{better} \geq P_{worse}$, i.e., examining the two polar cases only.

Table 3 summarizes the evidence by showing the percentage of cases which support the hypothesis. For all party affiliates, the hypothesis passes the test with the support of more than 70% of the cases. The Republicans present the strongest support for the notion that the individual's perception of his well-being has a positive effect on his voting decision. In fact, only 1966 deviates from the general trend in this case.

Table 3: Percentage of Cases Which Support the Hypothesis

Democrats	Republicans	Independents
75%	87.5%	62.5%

Total no. of cases for each PID category = 8

The Effect of Incumbency

Kramer claimed that incumbency has no significant effect on voting once the effect of economic conditions is taken into account. Table 3A shows the incremental proportion in the presidential party candidates vote as a result of the incumbency of its candidates Thus, the more positive are the entries in the table, the firmer is the inference regarding the covariation of incumbency and vote.

Except for 1960 elections, entries are overwhelmingly positive (more than 80 percent of the 63 cases), indicating that incumbency affects the vote for candidates of the presidential party. The effect is especially strong in the latter parts of the period. Table 4 summarizes Table 3A. Consider that the first period includes 1956, 1958, and 1960; the second period 1964, 1966, 1968, and 1970.

Table 4: Proportion of Party Vote Due to Incumbency, Controlling for Presidential Party and Economic Conditions

	Average of Entries in 56, 58, 60	Average of Entries in 64, 66, 68, 70
Democrat	20.12	66.51
Republican	22.64	50.56
Independent	-13.1	59.77

Source; See Table 3A.

Table 4 provides some evidence to support the contention of increasing incumbency effect over time on the presidential party candidate's vote in congressional elections for all party identification categories. There is, also, some evidence that the effect of incumbency is strongest among the Democrats during the early part of the period while it is strongest among the Independents during the latter part of the period.

The Effect of Candidate Salience

Since Stokes and Miller's classic paper [20], in which they conclude that candidate salience has a positive effect on his vote, no one has challenged this proposition, except perhaps Ferejohn [8]. Thus, it is necessary to probe a bit further into the effect of candidate's salience on voting. Table 4A shows the differential vote of the presidential party candidate as a result of his recognition by the voter, controlling for PID and economic response categories. For example, during the 1958 election, of the Democrats (who perceive an improvement in their conditions and vote for the presidential party), the difference between those who know the incumbent and those who know the challenger is 17.1 percent of the electorate. The more positive are the entries of Table 4A, the firmer is the inference regarding the effect of salience on vote. The following observations can be made from the summary Table 5.

1) All entries are decidedly positive and reasonably large, indicating a possible favorable effect of salience on voting. The evidence on increasing effect of saliency over time is also

conclusive. 66 percent of cases support this observation.

2) No firm general observation can be made regarding differential impact of saliency on different categories of economic conditions and party identification. For example, it cannot be said of the people who vote for the presidential party candidates and perceive betterment in their lot that the Democrats are more likely to recognize those candidates.

For the moment, at least, it can be said that salience of the candidates is positively related to the voting decision of the individual.

Table 5: Differential Salience of the Presidential Party Candidate

	% positive entries in Table 4A	% of cases increasing over time
Differential Salience of The Presidential Party Candidate	87 n = 45	56 n = 45

(Summary of the Data in Table 4A)

Economic Conditions and Salience of the Presidential Party Candidate

It can be postulated for this relationship a variant of the negative voting hypothesis: "The Avenger Model." In this model, the individual voter is most likely to incur the cost of information by seeking the presidential party candidate's name if he is hurt economically by the latter's policies.

Table 5A tests the model by showing the covariation of the perception of economic conditions and salience for Republicans, controlling for incumbency. Specifically, it shows the salience of the presidential party in different economic response categories, controlling for incumbency. Table 5B shows the same effect on the challenger's party candidate.

For example, in 1958 there are 87.5 percent Republicans in the "better" category who recognize the presidential party candidate. The weakest test of the model requires that $P_{better} \leq P_{worse}$ for salience of the presidential party. The evidence in this table seems to indicate some covariation between salience of candidates and economic perception. Moreover, the support for the Avenger model is high, as 70 percent of the cases are in favor of it.

Salience of the Presidential Party Candidate and Incumbency

Several authors claim that salience varies positively with incumbency [11], [6]. To demonstrate the strong relationship between the salience of the presidential party candidate and incumbency, the differential of those who live in areas with an incumbent who belongs to the challenger party is calculated in Table 6A (controlling for economic responses and PIDs). For example, the table shows that in 1964, the proportion of the Democrats who recognize the presidential party candidate (in the "better" response category) and reside in areas with incumbents belonging to the presidential party, exceed that of those with similar characteristics who reside in areas with incumbents belonging to the challenger party. The larger the differential (the entries in Table 6A), the stronger is the inference regarding the effect of incumbency on the salience of the presidential party candidate. Table 6 summarizes the observations made from Table 6A.

Table 6: Differential Salience of the Presidential Party Candidate Due to Incumbency

	% Positive Entries	% Entries Greater than 80% differential
Democrat	100	80 n = 15
Republication	100	50 n = 15
Independent	100	80 n = 15

The data seem to indicate a stronger effect of incumbency on salience for the partisans than that for the Independents. Also, if the 1958 and 1964 elections are considered as a first period, and 1966, 1968 and 1970 as the second period, then only four out of nine cases indicate increasing effect over time.

3.5 The Model and Estimation Procedure

The information in the previous tables is certainly suggestive, but firm conclusions have to await further evidence which takes care of the simultaneity effect on one hand and insures the proper control of all relevant variables in the problem on the other. The evidence in the data provides a reasonable basis to establish the relevancy of the various factors to the individual voting decision. For example, it is shown, given the limitations of the data and the tabulation technique, that salience of candidates, incumbency, and to a lesser degree the individual's perception of his economic lot, are related to the voting decision. Moreover, it is shown that incumbency and economic perception are related to the salience of the candidate.

On participation, there is some support for the claim that the individual's perception of how well he has been faring economically is related to his decision to participate or abstain. It remains to formulate a model that captures the most critical relevant variables on one hand and takes into consideration the simultaneous nature of political phenomena on the other. The definitions of the variables in the model should facilitate the analysis of participation as well as voting decisions by using the same data base, whether for individual elections or in pooled form for all elections without redefining the variables. This kind of formulation will improve on the specification of previous models and reduce possible simultaneity bias. It will also exposit the primary and secondary influences of various variables on the individual's dual decision on participation and voting. A simultaneous two equations model is formulated. The first equation has the vote for presidential party candidates (or participation) as the dependent variable and the salience of the presidential party candidate, perception of economic conditions, incumbency, and party affiliation as explanatory variables. The second equation has the salience of the presidential party candidate as the dependent variable and incumbency, perception of economic conditions, education, and party identification as explanatory variables. A better specified model may be achieved by adding other variables, such as campaign expenditures and duration of incumbency in both equations. It may even be desirable to add a third equation for incumbency. While such modifications may affect some of the results obtained in this paper, it is proper to point out that the specification in this paper is dictated by both theoretical and practicla considerations posed by the limitation of the data.

The Model[5]

$$y = 1 \text{ if } \alpha_1 + b_1 \tilde{R}_1 + b_2 F + b_3 P + b_4 I_1 + \varepsilon_1 > 0 \qquad (1)$$

$$= 0 \text{ otherwise}$$

$$R = 1 \text{ if } \tilde{R}_1 = \alpha_2 + c_1 F + c_2 P + c_3 I_1 + c_4 E + \varepsilon_2 \geq K \qquad (2)$$

$$= 0 \text{ otherwise}$$

where:

y is either the participation variable or the voting variable; that is, y = 1 if participating[6] (or voting for the presidential

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[5]Another model with interactive terms between party ID and economic variables was formulated as follows:

$$y = 1 \text{ if } \alpha_1 + b_1 p + b_2 I_1 + b_3 (DB) + b_4, (DW) + b_5 (RB) + b_6 (PW) + b_7 (IB_1) + b_8 (IW) + b_9 R_1 + \varepsilon_1 > 0$$

$$= 0 \text{ otherwise}$$

$$R_1 = 1 \text{ if } \tilde{R}_1 = \alpha_2 + C_1 p + C_2 I_1 + C_3 E + C_4 (DB) + C_5 (DW) + C_6 (RB) + C_7 (RW) + C_8 (IB) + C_9 (IW) + \varepsilon_2 \geq K$$

$$= 0 \text{ otherwise}$$

where Y_1, P, I, R and E are defined as in the original model.

DB = 1 if the respondent is a Democrat who perceives a betterment of his economic condition

= 0 otherwise

DW = 1 if the respondent is a Democrat who perceives a worsening of his economic condition.

The other interaction terms for Republicans RB, RW and for Independents IB and IW are analogously defined. The results of this model are substantially the same as were obtained from the original model and are reported in Chapter 4 [45].

[6]For a different focus, see Campbell, et al. "Voting and Turnout" [40] or S. Verba, Participation in America [42], where the emphasis is on factors that determine the psychological involvement of the individual in politics.

party candidate)

= 0 otherwise.

R_1 is the salience of the presidential party candidate where

R_1 = 1 if recognize the presidential party candidate

= 0 otherwise.

F is the economic conditions variable

$$F' = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

where:

F_1 = 1 if the response if "better"

= 0 otherwise

F_2 = 1 if the response if "same"

= 0 otherwise

F_3 = 1 if the response is "worse"

= 0 otherwise.

P is the party identification variable

$$P' = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

where:

P_1 = 1 if the respondent is Democrat

= 0 otherwise

P_2 = 1 if the respondent is Republican

= 0 otherwise

P_3 = 1 if Independent

= 0 otherwise.

I is the incumbency variable

$I_1 = 1$ if the presidential party candidate is incumbent

$= 0$ otherwise.

E is the education variable

$E = 1$ if the respondent has college degree

$= 0$ otherwise.

Note that in every variable, one category is not included in the actual regression model to avoid singularity and over-identification. Also, the definitions of the dependent and independent variables facilitate pooling of data from several elections to nail down the effect of some crucial variables. As indicated, the data used are SRC (1956-1970) election data.

Although the salience variable, R, is observable as dichotomous, it will be assumed that R reflects an underlying continuous salience variable $\tilde{R}$ with a threshold level of k such that

$$\tilde{R} \geq k \Rightarrow R = 1$$

$$\tilde{R} < k \Rightarrow R = 0 \ . \qquad (3)$$

This assumption justifies the use of a two-step estimation procedure. Equations (2) and (3) define a standard probit model; coefficients of (2) can be estimated by maximum likelihood procedure. These estimated coefficients are used to construct $\tilde{R}$, which can be used as an instrument to replace R in (1). The rest of the estimation procedure proceeds analagously to the two-stage least square [47].

The two-stage probit technique is used in estimating the model for individual elections and for pooled runs. All tests of significance are conducted at 5 percent level of confidence.[7]

The Salience Equation

Table 7 shows the result of the first step of the procedure: the regression of the salience of the presidential party candidate. Note that c_1 is the coefficient of the "better" element of the vector of economic condition F, similarly c_1' is the coefficient of "worse" and c_2 is the coefficient of the Democrat, while c_2' is the coefficient of the Republican element of the vector of party identification P. From these tables we can draw the following conclusions.

1) Economic conditions have no significant effect upon salience of the presidential party candidate. Moreover, even the sign patterns are not consistent with a priori notions. The coefficients of the "better" and "worse" variables are overwhelmingly negative (more than 75 percent of the cases are negative), which does not support either a negative or positive hypothesis regarding the effect of economic conditions on the salience of the presidential party candidate, even on considerations of sign alone.

2) Education and incumbency are both very significant in determining the saliency of both candidates. All signs are positive except for the case of 1964 where the coefficient of incumbency is also insignificant.

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[7]The consistency of the estimates of the coefficients in this procedure has been established by Takeshi Amemiya [46]. However, he also established that the estimates of the standard errors in the second stage are not consistent. This makes the distribution of the ratio of the estimate to the standard error in probit, not exactly t. This effect has to be considered when interpreting the results in this chapter. The significance test will be supplemented with a likelihood ratio test whenever two or more variables are examined at the same time.

3) Party identification has no effect on the salience of the presidential party candidate during off-year elections. For on-year elections, it was only significant during the 1964 election. This finding is contrary to the hypothesis that one is more likely to know the presidential party candidate if the latter belongs to the same party.

The Voting Equation

Table 8 shows the result of the probit estimation of the voting regression equation (1) using the computed values of the salience variables from equation (2). Several comments can be made.

1) Economic perception has no significance or even sign stability on voting decisions. That is, the finding supports rejection of either a positive or negative hypothesis about the effect of the perception of economic conditions on the probability of voting for the presidential party candidate. A likelihood ratio test confirms this conclusion.

2) Recognition of the presidential party candidate has a more significant effect on voting decisions during mid-term Congressional elections than in on-year Congressional elections. However, when data are pooled for on-year elections, the coefficient of salience is unambiguously significant and positive. This finding is solid, considering the degrees of freedom attained by pooling the data.

3) The effect of incumbency is again more pronounced in off-year elections than in on-year elections (although for the latter there are only two cases -- that of 1964 and 1968 -- perhaps not enough to

Table 7: Salience of Presidential Party Candidate

Equation 2

	α_2	c_1 Better	c_1' Worse	c_2 Dem.	c_2' Rep.	c_3 Inc.	c_4 Ed.
	S.E.	S.E.	S.E.	S.E.	S.E.	S.E.	S.E.
58	-0.5*	0.1	-0.009	0.09	-0.04	0.4*	0.34*
	0.14	0.09	0.09	0.14	0.15	0.07	.09
64	-0.4*	-0.04	-0.08	0.5*	0.64*	-0.07	.28 *
	0.16	0.08	0.11	0.15	0.16	0.07	0.09
66	-0.7*	-0.14	-0.06	0.09	0.22	0.23*	0.5*
	0.14	0.09	0.1	0.12	0.14	0.09	0.09
68		-0.09	-0.14	0.16	0.22	0.4*	0.27*
	0.14	0.09	0.1	0.13	0.14	0.07	0.08
70	-0.7*	-0.15	0.003	0.03	0.02	0.43*	0.63*
	0.11	0.09	0.09	0.11	0.12	0.08	0.08
58 & 66	-0.6*	-0.08	-0.05	0.08	0.1	0.3*	0.5*
70	0.07	0.05	0.05	0.07	0.07	0.04	0.05

*Significant at .05 level

support such a strong judgment). However, the model suggests a significant effect of incumbency in off-year Congressional elections.[8]

Again, when data from on-year elections are pooled, incumbency becomes clearly significant. This is perhaps not surprising, and it has been pointed out by a number of scholars [6], [8]. These results show that both incumbency and salience of the candidate have strong, significant, and independent effects.

On Turnout

In this context, equation 1 is reinterpreted as a participation equation. The dependent variable is a dichotomous variable (y) which equals one if the individual participates and zero if he abstains. The two-stage probit estimation procedure is repeated with the regression of equation (2) first, then equation (1) using the computed values of the recognition variable from (1).

Table 9 shows the result of the second step of the procedure. Examination of this table does not support AM's hypothesis on the effect of economic conditions on turnout. Except for the 1958 elections, economic conditions fail to show significant effect on turnout. This may lead to the conclusion that economic conditions have some effect on turnout in the early part of the period examined. There are, however, some sign anomalies which cast doubt on this conclusion. For example, there is a negative significant coefficient for the "better" variable in the 1958 case.

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[8]In reference to the observation made in footnote 7, it is reassuring to note the absence of sign anomalies in these coefficients.

Table 8: Vote for Presidential Party Candidate

Equation 1

	α_1 Const.	b_1 Inc.	b_2 Better	b_2' Worse	b_3 Dem.	b_3' Rep.	b_4 Salience
	S.E.	S.E.	S.E.	S.E.	S.E.	S.E.	S.E.
58	-0.2	2.0*	-0.01	-0.07	-1.0*	0.98*	-0.31
	0.22	0.33	0.11	0.13	0.18	0.17	0.16
64	-0.43*	0.15	0.05	-0.02	0.18	0.17	0.16*
	0.2	0.33	0.08	0.12	0.22	0.27	0.08
66	-0.88*	0.45*	-0.01	0.06	0.7*	-0.4*	0.35*
	0.2	0.2	0.1	0.1	0.13	0.16	0.1
68	0.58*	0.51	-0.009	-0.15	0.62*	-0.64*	0.12
	0.2	0.34	0.09	0.12	0.15	0.18	0.17
70	-0.94*	0.56*	0.07	-0.02	-0.55*	0.89*	0.45*
	0.16	0.16	0.11	0.11	0.14	0.13	0.11
64 & 68	-0.5*	0.47*	0.06	-0.08	0.55*	-0.68*	0.21*
	0.12	0.15	0.06	0.08	0.11	0.13	0.06

The most interesting observation is the significant, negative effect of incumbency on turnout in comparision with a strongly positive influence of salience of the candidate. That is, incumbency increases the probability of the presidential party's candidate being known to the voter (i.e., increases his salience, which in turn affects positively the probability of turnout and his chances of receiving the vote), but the direct effect of incumbency on turnout is in the opposite direction. (See Figure 1)

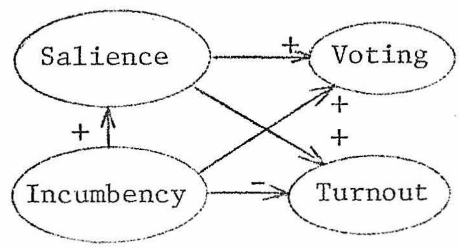

Figure 1

Thus, the boosting of the salience of the presidential party candidate by incumbency works in the opposite direction to its direct effect on turnout. Is there any "apathy" theory lurking in the background? Could the individual voter reason for himself that the incumbent is unbeatable and therefore there is no efficacy for his vote?

The answer could be yes, if the profile of the non-voter is postulated to be the same as that which emerges from Campbell et al. study [35].

> "The non-voter tends to be a person of lower involvement whose emotional investment in politics and its partisan decisions is on the average much less than that of the voter. As a result, we would expect the non-voter to be less stable in his partisan inclinations than the voter and more responsive to the massive political stimuli that produce shifts of popular attitude over time. And we have little doubt that for the non-voter a stimulus of great importance in this period, as in any other, was the fact of who was winning elections.... The power of partisan choice to motivate turnout is contingent on the individual feeling that his vote may count."

Table 9: Turnout

Coefficients / Years	α Const.	c_1 Dem.	c_1' Rep.	b_1 Worse	b_2 Better	b_3 Inc.	b_4 Recog.
58	0.68* / 0.20	0.14 / 0.13	0.29* / 0.13	-0.19* / 0.09	-0.28* / 0.09	-0.25* / 0.11	1.60* / 0.27
64	0.75* / 0.22	-0.26 / 0.23	-0.15 / 0.27	0.05 / 0.11	0.13 / 0.08	-0.14* / 0.08	1.5 / 0.34
66	0.53* / 0.18	-0.25* / 0.12	0.33* / 0.14	-0.05 / 0.09	-0.03 / 0.09	-0.31* / 0.09	0.81* / 0.18
68	0.79* / 0.20	0.14 / 0.13	0.20 / 0.16	0.12 / 0.11	0.02 / 0.09	-0.67* / 0.17	1.3* / 0.35
70	0.30* / 0.14	0.39* / 0.11	0.56* / 0.15	-0.06 / 0.08	0.01 / 0.09	-0.20* / 0.09	1.2* / 0.16
64 & 68	0.79* / 0.15	-0.03 / 0.12	0.06 / 0.14	0.10 / 0.08	0.06 / 0.06	-0.44* / 0.07	1.5* / 0.24

In other words, the would-be non-voter:

1) is most likely to have weak preferences,

2) is most likely to favor the incumbent.

If we add to this a further assumption that

3) he is most likely to believe that the incumbent would be the winner, which leads him to believe that his vote is of low efficacy,

then the would-be non-voter in Campbell et al. study will be a non-voter.

Conclusion

No significant effect of the individual's economic condition was found on either his voting or participation decisions. There is evidence to support the common belief that salience of the candidates and incumbency affect the individual's dual decision. In particular, incumbency affects the individual's dual decision both directly and indirectly through the awareness variable. The simultaneous equations formulation suggests that possible misspecification in AM's model is responsible for their results that, "The principle effect of incumbency is to increase participation." The direct effect of incumbency on participation appears to be negative, although it has a positive indirect effect through the salience variable. Finally, we find no effect of economic conditions on the awareness of the candidates by the individual.

Table 1A: Turnout

For Democrats

economic response \ year	*56	58	*60	62	*64	66	*68	70	
	–	–	(+)	–	–	–	–	+	
Better	29.2	41.0	25.0	42.4	28.3	45.6	31.3	57.8	↑
Same	30.2	40.6	24.9	46.2	32.1	40.7	35.5	48.4	↑
Worse	31.1	52.1	24.7	44.6	32.0	51.7	38.1	53.9	↑
Total % non-voters	30.1	43.9	24.9	44.5	30.3	45.5	36.4	52.8	↑

For Independents

economic response \ year	*56	58	*60	62	*64	66	*68	70	
	–	–	–	+	–	+	+	+	
Better	30.4	39.4	29.7	65.7	50.0	67.3	62.8	70.8	↑
Same	26.7	62.5	23.1	50.0	35.7	59.3	35.7	55.3	↑
Worse	43.8	70.4	53.8	45.0	57.9	64.5	55.2	63.6	↑
Total % non-voters	30.3	57.8	30.5	55.4	47.2	63.4	50.9	65.7	↑

For Republicans

economic response \ year	*56	58	*60	62	*64	66	*68	70	
	+	–	–	n	–	+	+	n	
Better	22.0	34.6	12.8	34.5	13.7	42.4	32.6	37.8	↑
Same	27.2	36.3	12.4	35.2	22.1	36.7	25.5	46.8	↑
Worse	19.2	50.5	14.3	34.4	20.5	28.8	24.7	37.9	↑
Total % non-voters	23.6	38.0	16.7	34.8	18.1	36.4	27.6	41.7	↑

*on-year elections
n does not support either hypothesis
+ supports "apathy" hypothesis
- supports "protest"
↑ increasing over time

Economic conditions and turnout, entries are % non-voters of particular party affiliation in various economic response categories.

Table 2A: Economic Conditions and the Plurality of the Presidential Party Candidates.
(Entries are the Presidential Party Plurality.)

DEMOCRAT

	*56	*58	*60	*62	64	66	*68	*70
Better	-48.1	-35.2	-58.6	44.5	50.3	32.8	36.6	-28.2
Same	-58.8	-55.5	-56.8	43.4	55.8	36.9	37.3	-38.9
Worse	-57.9	-44.1	-63.2	43.8	52.0	40.1	26.3	-31.6

INDEPENDENT

	*56	*58	*60	*62	64	66	*68	70
Better	33.9	12.0	16.2	17.1	7.2	8.2	0	-12.5
Same	10.0	-5.3	0	7.2	42.9	-7.4	7.1	-2.6
Worse	31.3	7.4	-30.8	15.0	31.5	9.7	-10.4	-6.0

REPUBLICAN

	*56	*58	*60	*62	*64	66	*68	*70
Better	64.1	44.8	62.4	-43.5	-37.9	-36.8	-46.6	40.6
Same	59.0	43.5	61.8	-44.3	-43.3	-42.0	-46.3	38.0
Worse	60.9	41.0	46.7	-51.5	-52.3	-34.0	-59.5	37.9

*Support the hypothesis.

Table 3A: Incumbency Effect on Vote

(For all PID's)

Better

	56	58	60	64	66	68	70	
Dem	10	68.6	-42.9	30	61.9	18.7	77.8	↑
Rep	30.9	41.6	10	20.4	30.0	87.5	10.5	↑
Ind	51.7	50	-25	8.3	92.3	42.8	50	↑

Worse

	56	58	60	64	66	68	70	
Dem	30	20	-9	46.7	84.2	17.3	90.9	↑
Rep	50.0	45.3	-17.7	16.7	84.2	10	7.0	<=>
Ind	-14	20	-100	-14	100	40.0	80	↑

Same

	56	58	60	64	66	68	70	
Dem	40.0	25.0	39.4	40.9	50.0	30.2	50.0	↑
Rep	30.4	26.5	-13.2	42.8	87.5	55.6	2.8	↑
Ind	4.0	33.4	-33.2	46.6	77.8	26.6	-12.5	↑

Entries are:

(Proportion who voted for the presidential party and reside in an area with incumbents belonging to the presidential party.) minus (Proportion who voted for the presidential party and reside in an area with a challenger party incumbent.)

↑ increasing over time

<=> No trend

↓ decreasing over time

Table 4A: Presidential Party Differential Vote and Salience Controlling for Different Party Affiliation/Economic Response Categories

	First Period		Second Period			
	58	64	66	68	70	
Dem/better	17.1	25.5	23.8	15.8	44.4	↑
Dem/same	14.3	29.7	17.3	35	25	↑
Dem/worse	60.0	28.0	31.6	26.9	45.5	↓
Ind/better	16.6	27.0	40	25	0	⇔
Ind/same	22.2	25.0	22.2	26.6	0	↓
Ind/worse	0	0	0	20	80	↑
Rep/better	23.2	11.4	46.2	14.3	18.9	↑
Rep/same	16.5	37.0	31.2	22.2	27.8	↑
Rep/worse	25.0	0	31.5	28.6	24.2	↑

Entries are:

(Proportion of a category of respondents who voted for the presidential party and recognise the presidential party's candidate.) minus (Proportion of the same category who voted for the presidential party and recognize the challenger party's candidate.)

Table 5A: Salience and Economic Conditions

economic response \ year	*58	*64	*66	68	*70
Better	87.5	81.3	80	83.3	63.6
Same	88.9	94.1	88.9	77.8	75.0
Worse	100	100	100	62.5	100

Entries are: proportion of Republicans recognizing the Incumbents of the presidential party.

Table 5B: Differential Salience of the Challenger's Party Incumbent Candidate Among the Republicans

	*58	*64	66	68	*70
Better	70	85.7	77.8	89.5	91.7
Same	93.3	94.7	87.5	94.7	100.0
Worse	100	100	70.0	62.5	100.0

Entries are as defined above for the challenger party.

*Supports the "Avenger Model."

Table 6A: Incumbency on Salience

"Better"

	58	64	66	68	70	
Dem	100	88.4	85.3	82.2	66.6	↓
Rep	84.4	62.5	70	66.6	36.3	↓
Ind	100	25	50	100	80	↑

"Same"

	58	64	66	68	70	
Dem	66.1	85.3	100	91.2	57.1	↑
Rep	86.6	88.2	77.8	61.1	56.3	↓
Ind	100	100	100	25	0	↓

"Worse"

	58	64	66	68	70	
Dem	100	91.6	100	87.5	100	↑
Rep	100	100	100	25	100	↓
Ind	94	100	100	100	100	↑

Entries are: of the different PID

(Proportion who recognize the presidential party's candidate and reside in an area with incumbents belonging to the presidential party. minus (Proportion who recognize the presidential party's candidate and reside in an area with a challenger party incumbent.)

REFERENCES

1. Kramer, G. "Short-Term Fluctuations in U.S. Voting Behavior 1896-1964." American Political Science Review 65 (March 1971): 131-143.

2. Goodman, S., Kramer, G. "Comments on Arcelus and Meltzer, 'The Effect of Aggregate Economic Conditions on Congressional Elections.'" American Political Science Review 69 (1975): 1255-1265.

3. Stigler, G. "General Economic Conditions and National Elections." American Economic Review 63 (May 1973): 160-167.

4. Arcelus and Meltzer. "The Effect of Aggregate Economic Variables on Congressional Elections." American Political Science Review. 69 (December 1975): 1232-1259.

5. Leuchtenburg, W. The New Deal, Documentary History. University of Carolina Press, 1968.

6. Fiorina, M. "Economic Retrospective Voting in American National Elections: A Micro-analysis." American Journal of Political Science Vol. 22, No. 2 (May 1978): 426-443.

7. Fiorina, M. "Retrospective Voting in American National Elections." Research proposal, National Science Foundation.

8. Ferejohn, J. "On the Decline of Competition in Congressional Elections." _American Political Science Review_ 71 (March 1977): 166-175.

9. Lazerfeld, P., Berelson, B. and Gaudet, H. _The People's Choice_. Columbia University Press, 1968.

10. Lazerfeld, P., Berelson, B. and McPhee. _Voting_. University of Chicago Press, 1968.

11. Mayhew, D. "Congressional Elections: The Case of the Vanishing Marginals." _Polity_ 6 (Spring 1974): 295-317.

12. Dahl, R. _Democratic Theory_. The University of Chicago Press, 1973.

13. Dahl, R. _Pluralistic Democracy in the U.S_. New York: Rand-McNally, 1967.

14. MacPherson, B. _Democratic Theory_. Oxford University Press, 1975.

15. Mayo, H. _An Introduction to Democratic Theory_. Oxford University Press, 1960.

16. Nixon, Richard. Six Crises. New York: Doubleday, 1962.

17. Evans, R. Nixon in the White House: The Frustration of Power. New York: Random House, 1971.

18. Tufte, E. The Political Manipulation of the Economy: Influence of the Electoral Cycle on Macroeconomic Performance and Policy. Princeton University, August 1974.

19. Wright, G. "The Political Economy of the New Deal: An Econometric Analysis." Review of Economics and Statistics 56 (1974): 30-38.

20. Miller and Stokes. Elections and Political Order. John Wiley and Son, 1966.

21. Campbell, A., Converse, P., Miller, W. and Stokes, D. The American Voter. New York: John Wiley and Son, 1964.

22. Pomper, G. "From Confusion to Clarity: Issues and American Voters, 1956-1968." American Political Science Review 66 (June 1972): 415-428.

23. Boyd, R. "Popular Control of Public Policy: A Normal Vote Analysis of the 1968 Election." <u>American Political Science Review</u> 66 (June 1972): 429-449.

24. Converse, P. "The Concept of Normal Vote." In <u>Elections and Political Order</u>, pp. 9-39. Stokes, et al. New York: Wiley and Sons, 1966.

25. Page, B. and Brody, R. "Comment: The Assessment of Policy Voting." <u>American Political Science Review</u> 66 (June 1972): 450-458.

26. Kessel, J. "Comment: the Issue in Issue Voting." <u>American Political Science Review</u> 66 (June 1972): 459-465.

27. Key, V.O. <u>Public Opinion and American Democracy</u>. New York: Knopf, 1961.

28. Beam, J.H. <u>Ballot Behavior</u>. Washington, D.C.: American Council on Public Affairs, 1940.

29. Tufte, E. "Determinants of the Outcome of Mid-Term Congressional Elections." <u>American Political Science Review</u> 69 (September 1975): 812-826.

30. Rees, A. and Kaufman, H. "The Effect of Economic Conditions on Congressional Elections '46-'58." Review of Economics and Statistics 44 (1962).

31. Clark, W. "Economic Aspects of a President's Popularity." Ph.D. dissertation, University of Pennsylvania, 1943.

32. Okun, A. "Comments on Stigler's Paper." American Economic Review Papers and Proceedings 63 (May 1973): 172-177.

33. McCracken, P. "The Practice of Political Economy." American Economic Review 63 (May 1973): 168-177.

34. The Gallup Poll Index, August 1971.

35. Bailey, M. "The Welfare Cost of Inflationary Finance." Journal of Political Economy 64 (1956).

36. Tullock, Gordon. Towards a Mathematics of Politics. Ann Arbor: University of Michigan Press, 1968.

37. SRC - Burkhard Strumpel - ASA, August 1972.

38. Page, B. "Elections and Social Choice." Discussion paper presented at Conference on Social Choice Theory, Key Biscayne, Florida, December 15-20, 1975.

39. Berns, W. "A Critique of Berelson, Lazarfeld and McPhee's Voting." In Essays on the Scientific Study of Politics. Edited by H. Storing. Holt, Rinehart and Winston, 1962.

40. Campbell, Converse, Miller & Stokes. "Voting and Turnout." Quoted in Public Opinion and Public Policy. Edited by Luttbeg. The Dorsey Press, 1968.

41. Verba, Sidney, Nie, N. and Petrocik, J. The Changing American Voter. Harvard University Press, 1976.

42. Verba, Sidney, Nie, N. Participation in America. New York: Harper and Row, 1972.

43. Theil, H. Principles of Econometrics. New York: Wiley & Sons, 1971.

44. Fair, Ray C. "The Effect of Economic Events on Votes for the President." Cowles Foundation Discussion Paper No. 420, January 1976.

45. Al-Adhadh, Naim., "The Decline of Competition in Congressional Elections -- Mayhew May Still be Right." Pasadena: California Institute of Technology Social Science Working Paper No. 19, November 1977.

46. Amemiya, Takeshi. "The Estimation of A Simultaneous-Equation Generalised Probit Model." Technical Report No. 237, Institute for Mathematical Studies In the Social Sciences, Stanford University, May 1977.

47. Nelson, Forrest and Lawrence Olson. "Specification and Estimation of a Simultaneous-Equation Model with Limited Dependent Variables." Social Science Working Paper, No. 149, California Institute of Technology, Humanities and Social Science, January 1977.

CHAPTER 4

THE DECLINE OF COMPETITION IN CONGRESSIONAL ELECTIONS: MAYHEW MAY STILL BE RIGHT!

4.1 Introduction

A number of authors have presented evidence that since 1950, there has been a noticeable decline in the proportion of competitive Congressional districts. For example, Tufte [4] shows that there has been a decrease in the "swing ratio", that is, the likelihood that a 1 percent shift in votes will cause a change in the outcome of an election. Kostroski [8] and Erikson [5] confirm the casual observations by showing that there has been a substantial increase in the incumbency advantage in postwar Congressional elections.

Three competing theories have been advanced to explain this phenomenon. The first is advanced by Tufte [4], who explains it by the incumbent manipulation of the redistricting schemes. He argues that "reapportionment rulings have given incumbents new opportunities to construct secure districts for themselves." The second theory is due to Burnham [7]. This one attributes the causes to a basic change in the behavior of the electorate. He points out that Tufte's observation regarding the drop in swing ratio may be due to the decreasing salience of party identification in the voting decision of the individual. The third theory, advanced by Mayhew [2], attributes the causes to the more effective use of the institutional advantages of incumbency by the incumbents. He argues that increasing use of the resources of the incumbency office, such as the franking privilege and publicity by the incumbent increased his salience,[1] which in turn increased his share of the aggregate vote.

Ferejohn [3] finds himself in substantial agreement with the theory of basic change in electorate behavior. He successfully presents evidence against the theory of the incumbent manipulation of redistricting schemes by showing that the phenomenon of declining competitiveness has occurred both in the states that have been redistricted and in those that have not. He also argues against the theory of institutional advantage of incumbency.

This paper will show that both the theories of basic change in electorate behavior and institutional advantage of incumbency may account for the change in voting behavior. It will first establish the relevancy of incumbency and candidate's salience to the individual's voting decision in a framework which recognizes the potential effect of other variables, such as economic conditions. It will then establish and explore the interactive nature of incumbency and salience. The various findings of this preliminary analysis will be used to motivate the form and variables of a simultaneous equations model of electoral competition. This procedure is necessary in order to avoid ad hoc inclusion of variables and to reduce the possibility of bias due to simultaneity and misspecification.

---

[1]Candidate's salience means his recognition by the voter. Recognition and salience will be used interchangeably in this paper.

The most general formulation of the model establishes that both incumbency and salience have positive and significant effects on voting. It further establishes that incumbency also works through the salience variable in influencing the voting decision of the individual.

4.2 On The Theory of Institutional Advantages of Incumbency

In a discussion of the kinds of activities in which congressmen find it electorally advantageous to engage, Mayhew identifies "advertising" as an effective activity in winning votes. His definition of "advertising" is simple: "It is any effort to disseminate one's name among constituents to create a favorable image, but in messages having little or no issue content." [2] Mayhew essentially agrees with Stokes and Miller's assertion [9], that "Recognition carries a positive valence; to be perceived at all is to be perceived favorably." He further points out that incumbents engaging in "standard routines," such as frequent visits to constituency, nonpolitical speeches, and correspondence with constituents, will be better known than their challengers. The incumbent can afford to engage in these "advertising" activities, because the public largely foots the bill, while challengers must meet their own expenses.

Thus Mayhew's model is simple: incumbency means greater control of electoral resources, which in turn produces higher salience of incumbents, which leads to greater incumbency voting. See Figure 1.

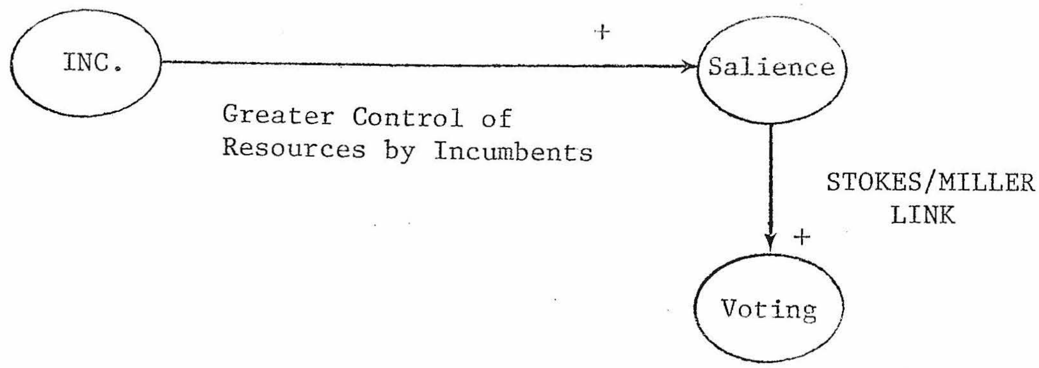

Figure 1

4.3 Ferejohn on The Theory of Institutional Advantages of Incumbency

Ferejohn [3] casts doubt on the theory of Institutional Advantages of Incumbency. For this theory to be correct, it must be true that: 1) there should be an overall increase in the level of recognition of the incumbent; 2) the relative level of recognition of incumbents versus challengers should also show an increase; 3) Increased level (or relative level) of recognition translates behaviorally into an increased level of incumbency voting.... However, he establishes that: 1) the level of incumbency voting increases over time; 2) this increase is not accompanied by increasing salience of the incumbent over time; 3) increasing salience of both the incumbent and the challenger may decrease the probability of voting in their favor.

Essentially, he doubts that the Stokes/Miller link is always positive; he presents evidence that the link might sometimes be negative [3]. He also denies the positive link between incumbency and salience. Ferejohn's model is shown in Figure 2.

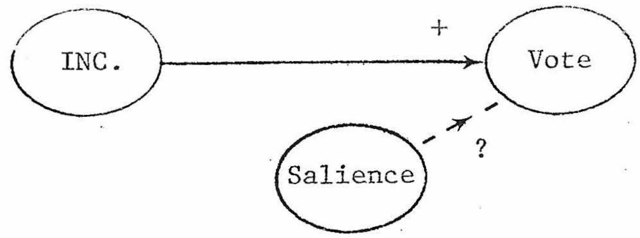

Figure 2

4.4 Preliminary Investigation of the Data

The task of this section is to probe the available data using simple statistical techniques to suggest the relevancy to the individual's voting decision of various variables which are considered a priori as being relevant. It will also exposit the interaction between these variables. These findings will be used as motivation for the simultaneous equations model. Various indices will be extracted from the raw data in the SRC surveys (1956-1970). These indices will then be used to make some tentative hypotheses and observations.

Three categories of party affiliations will be considered: Democratic, Independent and Republican. Also considered will be three categories of respondents to the question regarding their perception of changes in economic conditions: those who perceived "better" conditions, those who perceived the "same" conditions, and those who perceived "worsened" conditions. For this, use will be made of the following question in the SRC survey: "During the last few years, has your financial situation been getting better, getting worse, or stayed the same?"

For the salience variable, use will be made of a question in the SRC survey that asked the respondent to name the candidates for the House in his district. If the respondent could name the candidate, he was considered to recognise him; otherwise not. The limitation of our data is mainly due to the availability of recognition data only for the 1958, 1964, 1966, 1968, and 1970 elections.

The results are mainly reported in the Appendix, and the Appendix tables have the labels A and B following the table number to distinguish them from the summary tables in the main text. Since it will be necessary to make some observations regarding the relative effect of certain variables over time, the differential values of these variables will be shown in the tables rather than their absolute values. For example, if a test is to be made that recognition of the incumbent is increasing over time relative to that of the challenger, then the relevant variable to observe over time is the differential recognition of the incumbent: the percent recognizing the incumbent minus the percent recognizing the challenger. This will simplify the form and inference from the summary tables. Of interest will be the number of entries in the original table with a positive or negative sign, the magnitude of the entries (how positive or negative they are), and the number of cases that show increasing or decreasing entries over time.

The Effect of Incumbency

Ferejohn [3] demonstrates the influence of incumbency on the voting decision, contradicting an earlier finding by Kramer [10]. Kramer's model, however, eliminates some spurious effects by controlling for economic conditions and presidential coattails.

Table 1A investigates the effect on the proportion of the Democratic vote of Democratic incumbency rather than Republican (Table

1B exhibits the same effect on the proportion of the Republican vote), controlling for different economic responses.

Entries in Table 1A are Democratic incumbency advantage, and are given by: proportion of people who voted Democratic in Democratic incumbent district, minus proportion of people who voted Democratic in Republican incumbent district. Thus, positive entries imply positive effect of Democratic incumbency on the Democratic vote. Moreover, the higher these entries are, the greater is the inferred effect of incumbency on the share of votes. The evidence is summarized in Table 1.

Table 1
Vote Differentials Due to Incumbency:
Summary of the Entries in Tables 1A and 1B

	Proportion of Positive Differentials in Tables	Proportion of Cases Supporting Increasing Effect of Inc.
Democratic Inc. 1A	89	100
Republican Inc. 1B	72	50
	n = 42	n = 6

Several observations may be made from Table 1:

a) In general, the data support the contention of positive and increasing effect of incumbency on voting. Only 17 percent of all the cases show a negative entry. This agrees with Ferejohn's finding [3].

b) The Democratic share of votes is more sensitive to Democratic incumbency than is the Republican share to Republican incumbency. This is shown by a higher proportion of large entries in Table 1A than 1B (there are 17 percent more entries which are greater than 40 percent in Democratic incumbency). Moreover, the former shows increasing effect of incumbency on voting. More than 80 percent of cases in this category support this observation, while the picture for the Republican share of votes is not clear enough to reach a conclusion.

There is other interesting information which may be obtained from the raw data in Tables 1A and 1B:

c) The people who perceive worsening economic conditions (whether they vote Democratic or Republican) are the least likely to be influenced by incumbency. For example, in the case of Republican incumbency, 67 percent of the negative entries fall in the "worse" category.

d) Republicans seem to be the least affected by their own incumbency.

Thus, it seems legitimate to include incumbency as a relevant variable in any further analysis of the question under investigation. This finding agrees, in general, with Ferejohn's.

The Effect of Candidate Salience

In this section, a search will be made for evidence in support of the Stokes/Miller observation: "to be perceived at all, is to be perceived favorably." The differential salience of the other party's candidate is calculated for those who voted for the other party candidate, controlling for party identification (PID) and economic response. See Table 2A. There is a similar table for those who voted for their own party candidate (Table 2B). Thus, entries in Tables 2 are: the percent of those who voted for candidate X and recognized him, minus the percent of those who voted for X and recognized the other candidate. Hence, the more positive the entries in the table, the firmer the inference regarding the positive effect of salience on the candidate's vote.

All entries are positive and reasonably large, indicating some positive correlation between salience and vote. Moreover, if 1958, 1964, and 1966 are regarded as the first period, and 1968 and 1970 as the second period, some weak inference can be drawn regarding the effect of salience over time; more than 50 percent of cases show increasing effect over time. This inference stays the same, whatever definitions are adopted for the first period and the second period of analysis. Moreover, the data in Tables 2A and 2B show the relationship between salience and voting to be more strongly positive for the other party's candidate than for the candidate of the voter's party: 75 percent of the cases in the former category show an increasing effect, while only 50 percent of the cases show such a trend for the latter category. Table 2 summarizes these findings.

These tentative results indicate, to a certain extent, the existence of the Stokes/Miller link between salience and vote. Thus, for the moment at least, it can be concluded that the salience of candidates is positively related to the voting decision of the individual voter.

Table 2

	Percent of Positive Entries	Percent of Cases Increasing Over Time
Differential Salience of Own Party In Its Vote	100	50
Differential Salience of Other Party In Its Vote	100	75

Salience and Incumbency -- Another Dimension

Most of the models dealing with the effect of salience on voting consider recognition of the candidates by the voter as an exogenous phenomenon beyond the rational calculus of the voter. Very little effort has been expended to discover the underlying process behind the quest of the individual voter for knowledge of the candidate's name. Knowledge and retention of this piece of information is not costless, hence there must be a process through which this cost is defrayed or compensated. Investigating this process helps to avoid simultaneity bias in specifications of models for voting. It also promises to enrich our knowledge of how various variables interact to effect the voting decision. Mayhew's [1] explanation of the principal source of decline in the number of competitive seats in Congress, may be viewed as a model of "defrayed cost." See Figure 3.

The Elements of the "Defrayed Cost" Model

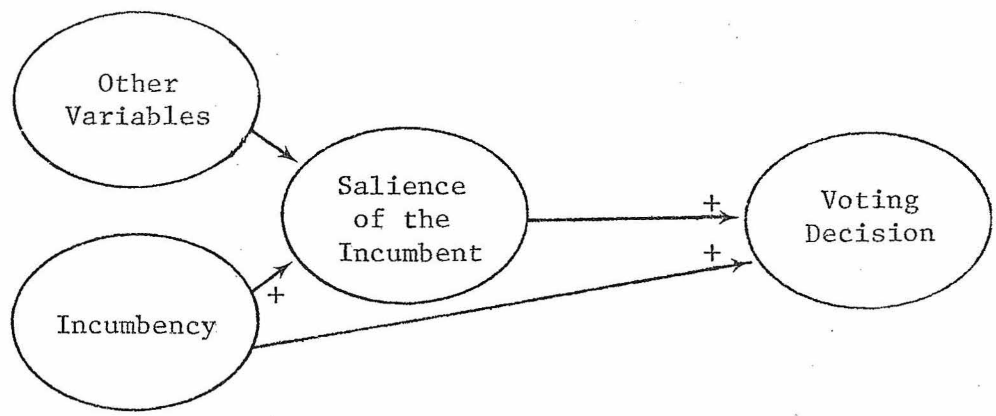

Figure 3

Consider "salience" as a consumable political good with a positive "perceived" price that could be consumed in any quantity. This assumption is valid if salience is considered as a continuum, starting from a mere recollection of the candidate's name to a comprehensive knowledge of his personality, achievements and background. However, only one level of recognition is observable, due to various institutional and experimental design considerations. Let g be the quantity of knowledge which an individual possesses about the candidate and k be the quantity of knowledge which corresponds to knowing the "name" only. Then

$$\text{if } g \geq k \text{ we observe } 1,$$

$$\text{and if } g \leq k \text{ we observe } 0.$$

Assume a neoclassical utility function (the assumption of diminishing marginal utility of information in this case is highly plausible) and a positive perceived price for information. Then, an individual's maximization of this utility subject to his budget constraint will determine whether he will purchase this political commodity and how much he will consume. The demand equation for g will be

$$D_g = D_g(P_1, P_2 \ldots P_g, \ldots P_h, I)$$

where I is the "income" of the individual. Thus, the greater the income, the higher the demand for this political good (unless an inferior good is considered; in this case, the opposite is true). Also, in the tradition of general equilibrium analysis, assuming all goods are gross substitutes, the lower the price of g, the greater is D_g [12]. Thus, salience should rise if the incumbent lowers the cost of information to the individual voter by making use of media and publicizing his Congressional activities.

Does the data capture this covariation of incumbency and awareness? Table 3A shows the incremental percentage of those who recognize the Democratic candidate and reside in an area with Democratic incumbents over those who reside in an area with Republican incumbents, controlling for economic responses. For example, in 1958, the percentage of the Democrats in the "better" category who recognize the Democratic candidate and reside in a district with a Democratic incumbent exceed those in the same category who reside in a district with a Republican incumbent by 89.7 percent. Table 3A is summarized in Table 3.

Table 3

Differential Salience of the Democratic Candidate Due to His Incumbency

Percent of Positive Entries (n = 15)	Percent of Entries Greater Than 80 Percent (n = 15)	Percent of Cases Increasing Over Time (n = 3)
100	60	100

The following tentative observations may be made:

1) All entries are positive and exceptionally large, which confirms strong covariation between incumbency and salience.

2) There is some evidence to indicate an increasing effect of incumbency on salience over time, at least for the "better" response category.

The conclusion [3] that there is no evidence of an increasing effect of incumbency on salience is not substantiated. Hence, a major link in the theory of the institutional advantage of incumbency remains unbroken. Ferejohn's contrary conclusion follows from a table which shows no increase in the proportion of total voters who recognize the incumbent. The variable that should have been considered is the proportion who voted for the incumbent, not the proportion of total registered voters. When considering total registered voters, Ferejohn's conclusion is not surprising, since a higher proportion of those who don't vote cannot recall the name of the incumbent [13]. Admittedly, however, these initial results, like Ferejohn's, are based on a small number of cases.

A preliminary model is formulated to test the relationship between salience of the Democratic candidate and incumbency. The model is of the form:

$$RD = \alpha + \alpha_1 Ed + \alpha_2 D + \alpha_3 R + \alpha_4 ID + u$$

where

$Ed = 1$	if respondent has college degree
$= 0$	otherwise
$D = 1$	if respondent is a Democrat
$= 0$	otherwise
$R = 1$	if respondent is a Republican
$= 0$	otherwise
$ID = 1$	if the incumbent is a Democrat
$= 0$	otherwise.

A probit estimation procedure is used. The results are reported in Table 4. Education and incumbency are significant in all years, while the party identification variable is significant in only one case, which corresponds to the respondent being Republican in the 1970 election. The important item to notice, however, is that incumbency increases its significance and influence over time. This is a crucial link in the theory of institutional advantage of incumbency, as Ferejohn rightly observes.

Table 4

Equation for the Recognition of the Democratic Candidate

	Const.	Education	Democrat	Republican	Inc. Dem.
1958	-0.35 * (0.19)	0.28 * (0.11)	0.25 (0.20)	-0.03 (0.20)	0.19** (0.10)
1964	-0.13 (0.21)	0.30 * (0.10)	0.15 (0.21)	0.02 (0.22)	0.21* (0.09)
1966	-0.28 (0.21)	0.32 * (0.12)	-0.02 (0.20)	-0.10 (0.21)	0.45* (0.11)
1968	-0.25 (0.18)	0.20 * (0.10)	0.03 (0.2)	-0.03 (0.20)	0.72* (0.01)
1970	-0.40* (0.20)	0.44 * (0.11)	0.02 (0.20)	-0.43* (0.19)	0.38* (0.11)

* significant at 5 percent

** significant at 10 percent

Hence, there does exist some support for the "defrayed cost" model in this data. Incumbents provide information about their activities at a low cost to the electorate. This low cost information impinges on the individual voter randomly, affecting the later acquisition of this information, which translates itself into higher incumbency voting.

Economic Condition and Salience

Another possible model is to postulate that the salience of the Congressional candidates increases if the individual voter is economically worse off, once control is made for incumbency. This model will be termed "the Avenger." That is, the individual voter is most likely to incur the cost of information by seeking the candidates' names if he is hurt economically by the President's policies. Ferejohn finds that, of those who vote for their own party, a lower proportion are aware of the candidate's name. This may be due to the presence of a lower proportion of those whose conditions are worsened within that group.

Table 4A tests this model by showing the covariation of the perception of economic conditions and salience. Specifically, it shows the salience of the incumbent Congressman within each economic response catagory in two cases:

a) When the incumbent Congressman belongs to the President's party.

b) When the incumbent Congressman belongs to the challenger's party.

This will establish the presidential effect, if there is any. For example, in 1958 there are 87.5 percent Republicans in the "better" category who recognize the incumbents from the presidential party. Define P_{better} as the salience of candidate X in the "better" response category, and P_{worse} as the salience in the "worse" response category.

The model is supported if $P_{worse} \geq P_{better}$ for salience of the Congressional candidates. The evidence in Table 4A seems to indicate some covariation between salience of candidates and economic perception.

The support for the model is also satisfactory, since 70 percent of all the cases support the model. The support for the model is the case of the presidential party candidates is overwhelmingly stronger than the case of the challengers.

Thus, economic conditions should be included as a relevant variable in any further investigation of the salience variable. There is some evidence in support of both the "defrayed cost model" and "the Avenger" model.

Next, some of Ferejohn's evidence will be examined. His conclusion is that "controlling for incumbency status, in four of ten comparisons, increased recognition of his own party candidate actually decreased the probability of voting for him!" This conclusion, however, does not agree with the results of his earlier model without interaction terms between salience and incumbency [3]. For example, the regression results of his model show that the recognition of the Democratic candidate is positively significant for all elections, and that for the Republican candidate is positively significant for all elections except that of 1966. His model also shows that incumbency is positively significant in all the cases except that of 1958. See Table 5A. Ferejohn suggests estimating a more saturated version of his model by including interaction variables between incumbency and salience. Hence, the following model is estimated.

$$Y = \alpha + b_1P + b_2I + b_3R + b_4RI + u$$

Y is the voting variable

where Y = 1 if vote Democrat

= 0 otherwise.

P is the party identification variable

$$P = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

where P_1 = 1 if respondent is Democrat

= 0 otherwise

P_2 = 1 if respondent is Republican

= 0 otherwise.

I is the incumbency variable

I = 1 if incumbent is Democrat

= 0 otherwise.

R is the salience variable

R = 1 if recognize Democrat candidate

= 0 otherwise.

R I is the salience/incumbency interaction variable.

Table 5 shows the results of this regression.

Table 5

Vote for the Democratic Candidate A Saturated Version of Ferejohn's Model

	CONST.	RI	P_1	P_2	I	R
58	-0.57*	-0.04	1.50*	-1.0*	0.47*	0.36*
	(0.21)	(0.25)	(0.21)	(0.21)	(0.18)	(0.15)
64	0.25	0.11	0.52*	-1.3*	0.53*	0.08
	(0.23)	(0.21)	(0.23)	(0.23)	(0.16)	(0.14)
66	-0.65*	0.47*	1.0*	-0.91*	-.66*	-0.02
	(0.23)	(0.26)	(0.21)	(0.22)	(0.18)	(0.20)
68	-0.62*	-0.21	0.86*	-0.85*	0.60*	0.46*
	(0.19)	(0.22)	(0.19)	(0.20)	(0.17)	(0.14)
70	-0.74*	-0.26	1.20*	-1.30*	1.30*	0.36*
	(0.22)	(0.28)	(0.21)	(0.22)	(0.20)	(0.18)

The model supports Ferejohn's observation that the incumbency variable exhibits a significant and increasing effect on vote. However, the model also shows that the recognition variables have a similar trend in the later part of the period. To establish the significance of the recognition terms, a likelihood ratio test is conducted and the null hypothesis is rejected at 5 percent for all elections. However, this model is so riddled with multicollinearity that some interaction terms are bound to lose their sign stability and that all coefficients of the model are suspect.

The lesson learned from the previous exercise is that a single equation formulation that has both recognition and incumbency variables as independent variables is not suitable for answering this inquiry on two grounds: first, the true model is susceptible to multicollinearity; second and more seriously, it is established that the model is misspecified under the most general assumptions regarding the interaction of salience and incumbency.

Finally, the model formulated by Ferejohn does not disprove that an adequate distributional shift in party identifiers may account for the observable change in the pattern of voting. His model was of the form:

$$\text{Vote} = \delta(\text{PID}, \text{Rec}, \text{PID}\cdot\text{Rec}) + u.$$

It follows from the evidence presented in this paper that recognition is driven in part by incumbency and that incumbency is a significant factor in explaining the voting behavior. However, Ferejohn's model relegates incumbency to the error terms while keeping the recognition variable as an explanatory variable. This renders the model misspecified and casts doubts on the interpretation and significance of the variables in the model.

4.5 The Model and Estimation Procedure

The information in the previous tables are certainly suggestive, but firm conclusions have to await further evidence which takes care of the simultaneity effect, on one hand, and insures proper control of all relevant variables in the problem, on the other. The evidence in the data provides a reasonable basis to establish relevancy of the various factors to the individual voting decision. For example, it has been shown, given the limitations of data and of tabulation technique, that the salience of candidates, incumbency, and to a lesser degree, the individual's perception of his economic lot are related to the voting decision. Moreover, it has been shown that incumbency and economic perception are related to the salience of the candidate.

It remains to formulate a model that captures the most critical relevant variables on the one hand and takes into consideration the simultaneous nature of the political phenomena on the other. This kind of formulation improves on the specification of previous models and reduces simultaneity bias; it will also exposit the primary and secondary influences of various variables on the individual's vote. A two-equation model will be formulated. The first equation will have the voting for presidential party candidates as a dependent variable, and the salience of the presidential party candidate, perception of economic conditions, incumbency, and party affiliation as explanatory variables. The second equation will have the salience of the presidential party candidate as dependent variable, and incumbency, interaction between perception of economic conditions and party ID, education, and party identification as explanatory variables.

However, Mayhew's interpretation of the powers of the incumbency office may be restrictive. Such powers include, in addition to the advantages of the label of incumbency and use of the franking privileges, the opportunities to do more services for the constituencies. In this case, the opportunities given to Congressmen by the incumbency office to render services to their constituencies increases the more the Congressmen remain in office. The model, as it stands, measures the overall effect of the powers of the incumbency office, viewed from this wider interpretation of these powers. However, the observable rise in the effect of incumbency on salience may be due not only to the increasing power of the incumbency office, but also to the increasing efficiency of long-time incumbents in using these powers. In this case, it may be advantageous to include two variables in the salience equation: the dichotomous variable, I, measures the power of the incumbency office, and a continuous variable, I_t, measures the accumulated learning of incumbents. This modification may affect some of the results reported in this paper, and shed further light on the effect of incumbency in the electoral process, but we leave it to future studies. Nevertheless, this paper demonstrates that even with this self-imposed limitation on the structure of the model different results are obtained.

The Model

$$Y_1 = 1 \text{ if } \alpha_1 + b_1 P + b_2 I_1 + b_3(D\beta) + b_4(DW) + b_5(R\beta) + b_6(RW) + b_7(I\beta) + b_8(IW) + b_9(R_1) + \varepsilon_1 > 0 \quad (1)$$
$$= 0 \text{ otherwise}$$

$$R_1 = 1 \text{ if } \tilde{R} = \alpha_2 + c_1 P + c_2 I_1 + c_3 E + c_4(D\beta) + c_5(DW) + c_6(R\beta) + c_7(RW) + c_8(I\beta) + c_9(IW) \geq K \quad (2)$$
$$= 0 \text{ otherwise}$$

and a similar equation for R_2

where:

y is the voting variable

i.e. y = 1 if voting for the presidential party candidate

= 0 otherwise.

R is the salience variable

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

where: R_1 = 1 if recognize the Presidential party candidate

= 0 otherwise.

F is the economic response variable

$$F' = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

where: F_1 = 1 if the response is "better"

= 0 otherwise

F_2 = 1 if the response is "same"

= 0 otherwise

F_3 = 1 if the response is "worse"

= 0 otherwise.

P is the party identification variable

$$P = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

where: P_1 = 1 if the respondent is Democrat

= 0 otherwise

P_2 = 1 if the respondent is Republican

= 0 otherwise

P_3 = 1 if Independent

= 0 otherwise.

I is the incumbency variable

I_1 = 1 if the Presidential party candidate is incumbent

= 0 otherwise.

E is the education variable

E = 1 if the respondent has college degree

= 0 otherwise.

The interaction terms are

Dβ = 1 if the voter is Democrat and perceived "better" conditions

= 0 otherwise

DW = 1 if Democrat and perceived "worsened" conditions

= 0 otherwise

Rβ = 1 if Republican and perceived "better" conditions

= 0 otherwise

RW = 1 if Republican and perceived "worsened" conditions

= 0 otherwise

Iβ = 1 if Independent and perceived "better" conditions

= 0 otherwise

and $\in_1$, $\in_2$ are random components.

In every variable, one category is not included in the actual regression model to avoid overidentification. Notice that the formulation of the model allows pooling of data from several elections to nail down the effect of some crucial variables. As has been indicated, the data used is SRC (1956-1970) election data.

Although the salience variable R is observable as dichotomous, it will be assumed to reflect a continuous variable $\tilde{R}$, with a threshold k such that

$$\tilde{R} > k \Rightarrow R = 1,$$

$$\tilde{R} < k \Rightarrow R = 0. \qquad (3)$$

This assumption will facilitate using a two-step probit estimation procedure. Equations (2) and (3) define a standard probit model; coefficients of (2) can be estimated by maximum likelihood procedure. These estimated coefficients are used to construct $\tilde{R}$, which can be used as an instrument to replace R in (1). The rest of the estimating procedure proceeds analagously to the two-stage least square [15].

A two-stage probit technique is used in estimating the model for individual elections and for pooled runs. All tests of significance are conducted at the 5 percent level of confidence.

A word of caution has to be added here. It has been shown that in the second stage of such procedures, the standard errors of the coefficients are not consistent [14]. This makes the distribution of the ratio of the coefficients to their standard errors not t exactly. Therefore, the conclusion of significance derived from the inspection of these ratios has to be taken with this fact in mind.

Results

The Salience Equation. Table 6 shows the result of the regression of the first equation. The following observations may be made:

1. Incumbency is positive and significant in all elections (except that of 1964 where it also picks up the wrong sign). Pooling of data establishes this observation firmly. There is some evidence that incumbency increased in influence towards the end of the period considered. This is a crucial step in Mayhew's argument, which seems to be supported by these findings.

2. Except for two cases, that of "Dembet" in 1970 and "Indworse" in 1958, the coefficients of economic conditions are not significant and do not possess sign stability in the equation for the salience of the presidential party candidate.

3. Party identification does not have any effect on salience.

4. Education is significantly positive in all elections and in pooled runs.

A modified equation for the salience of the challenger's party candidate, where no interaction terms are included, is run. Table 7 shows the results of this regression. The above findings are firmly supported.

Table 6

Salience of the Presidential Party Candidate

	CONST. α_1	DEM b_1	REP b_1'	INC b_2	DEM BET b_3	DEM WOR b_4	REP BET b_5	REP WOR b_6	IND BET b_7	IND WR b_8	ED b_9
	S.E	S.E	S.E	S.E	S.E	S.E	S.E	S.E	S.E	S.E	S.E
58	-0.62*	0.15	0.16	0.35*	0.18	0.07	0.3	-0.07	0.09	-0.51*	0.35*
	0.18	0.19	0.2	0.07	0.1	0.1	0.29	0.31	0.12	0.17	0.08
64	-0.14	0.17	0.35	-0.08	-0.002	-0.01	-0.48	-0.52	-0.05	-0.13	0.28*
	0.26	0.26	0.27	0.07	0.1	0.13	0.33	0.4	0.14	0.22	0.09
66	-0.6*	-0.06	0.1	0.23*	-0.1	-0.01	-0.35	-0.29	-0.15	-0.07	0.50*
	0.19	0.2	0.2	0.09	0.12	0.13	0.27	0.3	0.16	0.16	0.10
68	-0.36	0.004	0.1	0.42*	-0.06	-0.05	-0.26	-0.40	-0.1	-0.2	0.27*
	0.2	0.2	0.22	0.08	0.11	0.14	0.28	0.32	0.14	0.17	0.09
70	-0.75*	0.21	-0.05	0.44*	-0.36*	-0.13	0.04	0.07	0.10	0.19	0.63*
	0.16	0.18	0.19	0.08	0.13	0.12	0.25	0.23	0.16	0.16	0.09
64 & 68	-0.26	0.08	0.20	0.18*	-0.007	-0.04	-0.04	-0.17	-0.36	-0.44	0.27*
	0.16	0.16	0.17	0.05	0.07	0.1	0.1	0.13	0.21	0.25	0.06

Table 7

Equation for the Recognition
of the Challenger's Party

	CONST.	EDUCATION	DEMOCRAT	REPUBLICAN	INC. REP.
1958	-0.75* (0.20)	0.18** (0.11)	0.17 (0.20)	0.42* (0.21)	0.80* (0.09)
1964	-0.64* (0.22)	0.40* (0.10)	-0.11 (0.22)	0.23 (0.22)	0.92* (0.09)
1966	-0.41* (0.19)	0.29* (0.12)	-0.04 (0.20)	0.25 (0.21)	0.36* (0.12)
1968	0.14 (0.18)	0.48* (0.10)	-0.31 (0.19)	-0.18 (0.19)	0.19* (0.09)
1970	-0.38* (0.19)	0.60* (0.12)	-0.40* (0.19)	-0.24 (0.20)	0.79* (0.11)

The Voting Equation

Table 8 shows the results of regressing the first equation of the model using the computed values of the salience variable from the first step of the estimation procedure. The following observations may be made.

1. Except in 1958, where it also picks up the wrong sign, incumbency is positive and significant. Pooling data establishes this observation firmly. There is also some evidence in support of Ferejohn's assertion that the incumbency effect is greater during off-year elections than in on-year elections [3].

2. Except for the 1964 election, salience has a positive, significant effect on voting. Pooling the data, however, seems to indicate that the effect is primarily during off-year elections. This is perhaps due to the "drowning" of the effect of salience by the presidential coat-tail effect. Moreover, there is some evidence that salience exhibits an increasing effect on voting. This finding, and the previous one that indicates that incumbency exerts increasing influence on salience, strengthen Mayhew's argument.[2]

3. The pattern of signs for the economic conditions/party affiliation interaction terms is confusing and does not support any positive or negative hypotheses about their effect on voting.

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[2] In reference to the observation, made earlier, regarding the lack of consistency of the standard errors of the coefficients in the second stage, it is reassuring to note the absence of sign anomalies in these coefficients. Moreover, the fact that what is of interest here is trends over time rather than individual significance of coefficients.

Table 8
Voting Equation

	α_2	DEM c_1	REP c_1'	INC b_3	DEM BET b_4	DEM WOR b_5	REP BET b_6	REP WOR b_7	IND BET b_8	IND WR b_9	REG b_{10}
	S.E	S.E	S.E	S.E	S.E	S.E	S.E	S.E	S.E	S.E	S.E
58	-0.13	-1.4*	0.74*	-0.12	+0.44*	-0.17	0.003	0.27	-0.29*	0.64*	0.9*
	0.27	0.27	0.23	0.13	0.2	0.26	0.32	0.36	0.13	0.21	0.29
64	-0.19	0.36	-0.94*	0.21*	-0.009	-0.02	-0.4	0.16	0.30	-0.09	0.15
	0.26	0.27	0.3	0.08	0.1	0.14	0.36	0.42	0.16	0.27	0.33
66	-0.1*	0.97*	-0.31	0.36*	-0.09	-0.11	0.34	0.34	0.04	0.36	0.43*
	0.24	0.22	0.25	0.1	0.12	0.13	0.3	0.33	0.2	0.20	0.19
68	0.45*	0.44*	-0.71*	0.13	0.09	-0.11	-0.33	-0.33	0.16	-0.27	0.49*
	0.22	0.21	0.23	0.17	0.11	0.13	0.32	0.37	0.19	0.24	0.24
70	-0.66*	-0.91*	0.61*	0.45*	0.25	0.02	-0.69*	-0.38	+0.05	0.15	0.58*
	0.2	0.22	0.20	0.11	0.2	0.19	0.33	0.26	0.15	0.16	0.15
64 & 68	-0.40*	0.42*	-0.79*	0.24*	0.06	-0.06	0.14	-0.22	-0.33	-0.26	0.24
	0.17	0.16	0.18	0.07	0.07	0.09	0.12	0.17	0.24	0.27	0.24

On examining the evidence presented in these tables, the following observations may be made from Tables 8 and 9:

1. There is a slight increase in the coefficient of incumbency in the equation for votes of candidates of the presidential party.

2. There is also a slight increase in the coefficient of saliency of the candidate of the presidential party in the vote equations.

3. Almost all of these coefficients are significant and positive (Table 9).

4. There is a slight increase in the coefficient of incumbency in the saliency equation for presidential party candidate equation.

The second and third observations do not support Ferejohn's [3] contention of Mayhew's theory. The link between the increasing significance of incumbency voting and the increasing salience of the incumbent must be broken in order to sustain objections to Mahyew's theory.

However, the fourth observation shows that incumbency is increasing in significance even when salience is controlled. This means that the Ferejohn-Burnham theory of basic change in the electorate behavior may also be right.

It is the conclusion of this paper that Mayhew's theory is not defeated. It must therefore await further evidence to either substantiate it or to discard it. Further, it is found that both the theories of institutional advantage of incumbency and basic change in electorate behavior account for a significant part of the decline of competition in Congressional elections.

A more direct examination of the problem posed by Mayhew is still desirable. Such examination involves the inclusion of campaign expenditure and duration of incumbency in both equations of the model. A better specified model may even involve adding a third equation for incumbency. While such modifications may affect some of the results reported in this paper, it is proper to point out that the specification in this paper is dictated by both theoretical and practical considerations posed by the availability of data.

APPENDIX

Table 1A

Democratic Incumbency Advantage

The Case of "Better" Response

PID \ YEAR	*56	58	*60	62	*64	66	*68	70
DEMOCRAT	17.8	22.3	54.9	-	29.9	61.9	18.7	22.3 ↑
INDEPENDENT	55.0	25.0	20.0	-	8.3	92.3	42.8	100.0 ↑
REPUBLICAN	10.0	-13.7	63.3	-	20.4	30.0	87.5	100.0 ↑

The Case of "Worse" Response

PID \ YEAR	*56	58	*60	62	*64	66	*68	70
DEMOCRAT	19.0	28.7	51.6	-	46.7	84.2	17.3	40.7 ↑
INDEPENDENT	100.0	-33.4	40.0	-	-14.2	100.0	40.0	42.8 ↓
REPUBLICAN	-50.0	25.0	20.0	-	16.7	84.2	10.0	28.5 ↑

Entries are: Proportion of people who voted Democratic in Democratic incumbent district- proportion of people who voted Democratic in Republican incumbent district.

* on-year Congressional elections; ↑ increasing over time

Table 1B

Republican Incumbency Advantage

"Better"

PID \ YEAR	*56	58	*60	62	*64	66	*68	70
DEMOCRAT	10.0	68.6	-42.9	-	37.0	23.0	25.0	77.8 ↑
INDEPENDENT	51.7	50.0	-25.0	-	44.5	33.4	87.5	50.0 ↔
REPUBLICAN	30.9	41.6	10.0	-	3.5	-16.9	29.8	10.5 ↓

"Worse"

PID \ YEAR	*56	58	*60	62	*64	66	*68	70
DEMOCRAT	30.0	20.0	-9.0	-	-20.0	42.8	38.1	90.0 ↑
INDEPENDENT	-14.2	20.0	-100.0	-	100.0	-50.0	25.0	80.0 ↑
REPUBLICAN	50.0	45.3	-17.7	-	24.1	-12.8	-10.0	6.9 ↓

Entries are: Proportion of people who voted Republican in Republican incumbent district -proportion of people who voted Republican in Democratic incumbent district.

* on-year Congressional elections

Table 2A

	1ST PERIOD			2ND PERIOD		
	58	64	66	68	70	
DEMOCRAT/BETTER	17.1	10.0	19.2	17.1	44.4	↑
DEMOCRAT/WORSE	60.0	20.0	42.9	23.8	45.5	↓
REPUBLICAN/BETTER	15.0	11.5	46.2	14.3	62.5	↑
REPUBLICAN/WORSE	25	0	31.5	28.6	14.3	↑

Entries are: among the people who voted for the other party candidate (percent recognize other party candidate - percent recognize own party candidate).

Table 2B

	1ST PERIOD			2ND PERIOD		
	58	64	66	68	70	
DEMOCRAT/BETTER	22.9	25.5	23.8	15.8	4.5	↓
DEMOCRAT/WORSE	22.7	28.0	31.6	26.9	38.2	↑
REPUBLICAN/BETTER	23.2	2.8	8.5	21.1	19.0	↑
REPUBLICAN/WORSE	25.0	13.8	10.9	5.0	24.0	↓

Entries are: among the people who voted for their own party candidate (percent recognize own party candidate - percent recognize other party candidate).

Table 3A

Salience and Incumbency (in Democratic Candidacy) Effect on PID's Controlling for Economic Conditions

"Better"

PID \ year	58	64	66	68	70
Democrat	89.7	88.4	85.3	82.2	100
Independent	60.0	25.0	50.0	100	100
Republican	100	62.5	70.0	66.6	100

Entries are:

(proportion recognizing the Democratic candidates in an area with Democratic incumbents) - (proportion recognizing the Democratic candidate in an area with Republican incumbents)

Table 4A: Salience and Economic Conditions

ECONOMIC RESPONSE \ YEAR	*58	*64	*66	68	*70
BETTER	87.5	81.3	80	83.3	63.6
SAME	88.9	94.1	88.9	77.8	75.0
WORSE	100	100	100	62.5	100

Entries are: proportion of Republicans recognizing the Incumbents of the presidential party.

Differential Salience of the Challenger's Party

Incumbent Candidate Among the Republicans

ECONOMIC RESPONSE \ YEAR	*58	*64	66	68	*70
BETTER	70	85.7	77.8	89.5	91.7
SAME	93.3	94.7	87.5	94.7	100.0
WORSE	100	100	70.0	62.5	100.0

Entries are as defined above for the challenger party.

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*Supports the "Avenger Model."

Table 5A

Ferejohn's Model, Reestimated Using Probit

	CONST.	DEMOCRAT	REPUBLICAN	RI	RD	RR
1958	-0.12 (0.21)	1.5* (0.21)	-0.98* (0.22)	-0.28* (0.13)	0.81* (0.17)	-0.72* (0.18)
1964	0.78* (0.23)	0.52* (0.23)	-1.3* (0.23)	-0.40* (0.11)	0.44* (0.13)	-0.53* (0.14)
1966	0.15 (0.21)	0.99* (0.22)	-0.91* (0.22)	-0.73* (0.16)	0.74* (0.17)	-0.81* (0.17)
1968	-0.61* (0.2)	0.82* (0.19)	-0.89* (0.2)	0.33* (0.11)	0.83* (0.15)	-0.70* (0.14)
1970	0.49* (0.21)	1.2 * (0.21)	-1.3* (0.22)	-0.81* (0.16)	0.73* (0.18)	-0.99* (0.19)

REFERENCES

1. Mayhew, David. "Congressional Elections: The Case of the Vanishing Marginals." Polity 6 (Spring 1974): 295-317.

2. Mayhew, David. Congress, The Electoral Connection. New Haven: Yale University Press, 1974.

3. Ferejohn, John. "On the Decline of Competition in Congressional Elections." American Political Science Review 71 (March 1977): 166-175.

4. Tufte, Edward. "The Relationship Between Seats and Votes in Two Party Systems." American Political Science Review 67 (June 1973): 540-554.

5. Erikson, Robert. "The Advantage of Incumbency in Congressional Elections." Polity 3 (Spring 1971): 395-405.

6. Erikson, Robert. "Malapportionment, Gerrymandering, and Party Fortunes in Congressional Elections." American Political Science Review 66 (December 1972): 1234-1335.

7. Burnham, Walter. "Communication." American Political Science Review 68 (March 1974): 210.

8. Kostroski, Warren Lee. "Party and Incumbency in Postwar Senate Elections." American Political Science Review 67 (December 1973): 1213-1234.

9. Stokes, Donald and Miller, Warren. "Party, Government and the Saliency of Congress." In Elections and the Political Order, edited by Angus Campbell, Philip Converse, et al., New York: John Wiley and Sons, 1966.

10. Kramer, G. "Short Term Fluctuation in U.S. Voting Behavior 1896-1964." American Political Science Review 64 (1971): 131-143.

11. Dhrymes, Phoebus. Econometric, Statistical Foundations and Applications. New York: Springer-Verlag, 1974.

12. Quirk, J. and Saposnik, R. Introduction to General Equilibrium Theory and Welfare Economics. New York: McGraw Hill, 1968.

13. Al-Adhadh, Naim. "On the Retrospective Effect of Economic Conditions in Congressional Elections: A Simultaneous Equation Model Approach." Social Science Working Paper No. 179, California Institute of Technology, 1977.

14. Amimiya, Takeshi. "The Estimation of a Simultaneous-Equation Generalized Probit Model." Technical Report No. 237, Institute for Mathematical Studies in the Social Sciences, Stanford University, May 1977.

15. Nelson, Forest and Lawrence Olson. "Specification and Estimation of A Simultaneous-Equation Model with Limited Dependent Variables." Social Science Working Paper, No. 149, California Institute of Technology, Humanities and Social Science, January 1977.

CHAPTER 5

LEFTIST IDEOLOGICAL SHIFTS IN ARAB CONTEMPORARY POLITICS: A SPATIAL THEORY APPROACH

5.1 Introduction

In this paper, a formal model of the political motivation behind the choice of Marxism/Leninism ideology by a number of non-communist leftist parties in the Arab world is presented. This conversion to Marxism has occurred in many political parties throughout the Arab world, including the Popular Front for the Liberation of Palestine (P.F.L.P.) and Democratic Front for the Liberation of Palestine (D.F.L.P.) in the Palestinian Arab movement, the National Front for the Liberation of South Yemen (N.F.L.Y.), the Popular Front for the Liberation of Oman (P.F.L.O.) [16], the Popular Front for the Liberation of Western Sahara, formerly Spanish Sahara (POLISARIO), as well as scores of lesser political parties and movements in Lebanon and the Arab east. All these parties, together with the Arab Baath Socialist Party, the Union of Popular Forces in Morocco, the original Arab Socialist Union of Egypt (which has significantly changed since Nasser's death) may be identified as leftist according to the criteria presented in the next section.

Consider the P.F.L.P. as an example of the group that shifted to Marxism-Leninism. The P.F.L.P. grew out of the Arab Nationalist movement, whose transformation to Marxism-Leninism (M-L) is striking [9, 14, 15]. The initial structure and ideas of this movement were not left of the center by any local or external standards.[1] Most of the cadre and leadership were people with property who did not reject their background [14]. The movement's goals (unity, liberation and retribution) were broad enough to make everybody from the extreme right to the extreme left eligible for membership [9, 14, 15]. Yet, despite the party's early nationalist heritage and its initial hostility towards the Communists and Baathists -- indeed, toward all socialist ideas [15] -- it took less than six years [9] for the party's leadership and cadre to transform to the Marxist-Leninist ideology.

To distinguish their position from the traditional Arab Communists, they point out the Communists do not comprehend that "the analysis which Marxism gave to a particular situation at certain times cannot be the same for the new situation which has been developed from the previous one." [9] Thus, the P.F.L.P. political strategy stress their convictions that:

"1. The Modern Capitalism is different from what it was during Marx's days.
2. The class structure in underdeveloped societies are different from that in an Industrial society which Marx tackled.
3. Nationalism, which was used by the European Bourgeoisie during the Eighteenth and Nineteenth Centuries for conquest and exploitations, has acquired a revolutionary, progressive content in the third world. It became the framework of their struggle against Imperialism, which is the highest stage of Capitalism."

These are the basic arguments with which the P.F.L.P. hopes to distinguish their position from the Arab Communists. They summarize the distinction as follows [9]:

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[1]On this the P.F.L.P. strategy [9] states that "the political organization of the P.F.L.P. is a continuation of the Arab nationalist movement which has a petit-bourgeoise structure and content."

"The Arab Communist parties were only committed to Marxism-Leninism superficially, hence were incapable of leading the revolution in our country. They either understood the theory in a static, lifeless manner, or implemented it mechanically without due analysis to its local contact."[2]

In order to relate their views to traditional Marxism, the P.F.L.P. points out that "what is essential and immortal in Marxism is the dialectical way in which all problems are analyzed -- the dialectics of continuous change and movement...." Hence, the P.F.L.P. agrees with the Marxist method of analysis, but tries to escape some of the historic predictions and specifics of the theory.

The adoption by the P.F.L.P. of Marxism-Leninism as the core of their ideology was not an isolated event. In June 1968, the N.F.L.Y., the movement which shared in the struggle against the British in Aden, shook off the historical leadership of Al-Shaabi to adopt Marxism-Leninism [16].

A series of questions arise. Did these parties shift positions because Marxist-Leninist ideology captured the imagination of the Arab masses? How could the strong leadership of N.F.L.Y. lose their ideological grip on the rank and file of their party so swiftly, and after that leadership had achieved a resounding victory over the British?

One fact is certain: the standard bearers of Marxism-Leninism in the Arab world, the Communist parties, did not make any significant in-road in contemporary Arab politics [7, 9, 13], nor did they at any time have a chance of building a stable presence inside any Arab

[2]The researchers failed to find any adequate exposition of the P.F.L.P.'s new theoretical contributions beyond what is already mentioned. For example, the role of nationalism and the "organic" relationship between class and national struggle has been developed by the Baathists a quarter of a century before the 1967 P.F.L.P.'s Congress.

government even though they were the oldest organized party of the left.[3] An explanation of the shift towards Marxism-Leninism by the P.F.L.P. and others must be sought outside the example set by the Arab Communists.

5.2 On Ideology in Arab Context

In this section, the importance of ideology and what it means in contemporary Arab politics will be pointed out.

In his book on revolutionary Arab ideology [1], Dr. Elias Farah[4] gave the following interpretation to the concept of ideology:

> "The ideology is a system of ideas with a concrete purpose; it is a collection of ideas about the world, life, and society which together form the basis of collective action.... Thus, we can distinguish between religious, nationalistic, democratic, bourgeoisie, socialist and communist ideologies.... The ideology is a social condition for any renaissance, that is because it helps to describe the total picture of society and in particular specify the relationship between what is special and temporary in the society and what is general for all humanity."

More critically, he observes that:

a) Modern societies are ideological.

b) Ideology is a "must" to guide change.

c) No revolutionary party is without an ideological commitment.

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[3]The P.F.L.P. states that "the traditional Communist parties in the Arab world failed because of their structure and mistaken strategies...." P. 130-140 [9].

[4]Dr. E. Farah is a member of the National Command of Arab Baath Socialist Party. The extract is from the Arabic text. (5th ed., 1975, pages 10, 11.)

It is on these sets of ideas that "almost" all the Arab parties on the left agree, and it is on these observations a serious study of ideological choice in Arab context should be based. On the same theme, the P.F.L.P. political and organizational strategy [9] states:

> "The foundation of a revolutionary party is a revolutionary ideology; without it, the party cannot be but a group moving instinctively or under the influence of events. The revolutionary ideology is that which deals with all aspects of man and society in the relevant time."

These convictions are shared by a large number of Arab intellectuals outside the organized parties.[5] Indeed, the consensus that no serious party could be without an elaborate and complete ideological structure is shared even by many parties on the right such as the Moslem Brotherhood and El-Tahrir. In their writings, the leaders of these parties go to great lengths to show that their ideology, Islam, answers unambiguously all the questions that modern ideologies try to answer [22, 23]. In fact, the conviction that a well developed ideology is an absolute necessity for any meaningful social, economic and cultural development can be traced to the heritage of Islam. Islam is more than a complete set of religious and ethical doctrines; it is an ideology, because it concretely outlines the shape and content of the Islamic state [21-27].[6]

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[5]See Klofis Makhsood, The Crises of Arab Left, Arabic text, First edition [17], Self Criticism After Defeat, S.J. Al-A'Adhin, Criticism of Religious Ideology [34]. Arabic Texts, First editions, and N. Alwash in [36].

[6]See, for example, M. Aflag, On Memory of the Arab Prophet, and The Baath and Heritage [5].

Some revolutionary Arabs credit the early successes of Islam to the comprehensiveness and completeness of its ideological structure. [33, 34]. Because of the completeness of his traditional ideology, Islam, the active Arab individual demands a similar level of completeness before considering any new, competing alternative.

5.3 The Left in Arab Context

A particular party will be identified as leftists if the following subjective and objective factors are present:[7]

1) Subjectively, if the relevant party identifies itself publicly as being leftist.
2) Objectively, if:
   a) other Arab parties recognize it as leftist,
   b) there is an international recognition of this claim,
   c) the party manifesto limits, to various degrees, private ownership and the role of the market.

The most important objective criteria for identification is (c). This factor actually is a composite one, for it involves a measure of attitude toward religion and the organization of the state.

Islam, the dominant religion of the Arab world, is not only a set of rules to regulate a relationship with God, but also a system of government. It describes the role of an Islamic government, the basic tax system, economic classes, and the relationship between

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[7]See, for example, On Revolutionary Arab Politics Before and After Defeat. [3]

individuals and government.[8] Hence, a party position for public ownership which deviates from the widest interpretations of the teaching of Islam reflects a rejection of the role of religion in guiding the state.

Thus, it seems that (c) encompasses the two most important objectives of the Arab left: limited private ownership, and restriction of the role of religion in state affairs.

5.4 Conventional Explanations for Ideological Choices

It is difficult to disentagle, in the plethora of articles and papers dealing with Arab politics, those elements which deal primarily with ideological choice. Some terms have been so much abused as to cause much confusion. Such terms as "radicals" have been associated exclusively with the "leftists." Radicalism in the Arab world is associated with issues, not ideologies. It is possible for an individual to be radical on a political issue without being in the slightest way socialistic, and vise versa. Arab Communists have been the least radical on the Arab-Israel issue, and Fatah, the largest Palestinian Arab movement, has no particular allegiance to socialist ideals. Such confusion about terms leads to serious errors in analysis of Arab politics.

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[8]See, for example, Our Philosophy, M. El-Sader, first edition, Arabic text [21]; Mr. Yousif, The Contrived Gap Between Science and Religion [24], p. 1-10; Said Kotib, Islam and the Problems of Civilization [22], p. 164-168; Mr. Khallaf-Allah, "The Koran and Socialism," in Il-Katib, July 1966; D. Saab H., Islam Facing the Challenges of Modern Life, [23], p. 20-25; Y. Al-Milaiji, Consultation in Islam Versus Its Role in the Western Democracies and Marxism [26]; and A. Al-Khatib, The Financial Policies in Islam [25].

Therefore, rather than documenting the traditional scholarly literature dealing with Arab ideological choices, two conventional explanations will be outlined and an informational version of our theory will be presented.

Leadership Transformation Theory

This is the oldest and most traditional explanation of ideological shifts by third world parties. Leaders are sovereign. They perceive the "common good" of society in a particular way at certain stages of their lives and experiences and adopt a certain ideology. Their ideological position changes to the left or the right through further personal experiences.[9] According to this theory, George Hahbash and the group leadership in the P.F.L.P. would have had to undergo an ideological transformation from the right to the left, then have influenced the cadre and supporters to follow suit.

The support for this theory, however, is not limited to the traditionalists. The revolutionary faction[10] of the P.F.L.P. asserts that [14] "the basic lesson from the Cuban experience is that when the elements in control of the leadership of a particular petit-bourgeoise

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[9]Almost all articles dealing with "Nasser" ideology fall in this category.

[10]This faction had seceded from the P.F.L.P. in March, 1972.

movement have the ability of transformation to Marxism-Leninism, then the possibility of transformation of the whole movement is there."[11] They go on to assert that "the existence of Marxist-Leninist elements in a petit-bourgeoise organization is not the important factor in transforming the whole organization, but it's the position and effectivity of those elements in the leadership that are vital."

The reasons that are usually given for an ideological shift by a leader, such as personal experience, leadership ego, or other psychological factors, are unconvincing. The theory leaves unanswered the question of why the followers went along with the leader.

Outside Pressures

This theory states that the parties have no independent ideological choice. Parties adopt various positions on political issues (e.g., the Palestinians, oil, West-East orientation, etc.) as a result of forces largely outside the control of the political party. Thus, the more external force applied on the parties to be "radical" on these issues, the greater will be the shift to the left. For example, a conflict between a less desirable Western policy and a more desirable Soviet policy in the Arab world would make Hahbash a staunch Marxist-Leninist, and thus transform the whole P.F.L.P. Another example is the

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[11]See [4], p. 29. In fact, this faction even denies the sincerity of the P.F.L.P. adoption of Marxism/Leninism. They state in [14], p. 34, that "the official adoption of Marxism/Leninism does not mean that the P.F.L.P. possesses the necessary conditions for the transformation. For history taught us -- as Lenin mentioned -- that when the enemies of Marxism fail to fight it on an open ground, they are apt to fight it from inside by adopting it superficially. All those who lived inside the P.F.L.P. and are acquainted with its leadership cannot but doubt the earnestness of the transformation."

account of Nasser's policies related by M. Haiykel in his book, Abdul Nasser and the World. He portrays Nasser's drift to the left as a reaction to the West's unwillingness to finance the high dam. This theory asserts that the ideological shift is a choice, but it is a forced one.

A variant of this theory is what can be termed "the conspiracy theory." Here, international forces conspire to form and aid various factions in the party and force a change in leadership by either deposing them physically or usurping their real power over the faction loyal to their ideology.[12] It cannot be denied that super-power policies have appreciable effect on creating issues and affecting the position which various Arab parties assume,[13] but the political and ideological conflicts in the Arab world are driven mainly by Arab aspirations. The final results of outside efforts can be very different from what was originally intended.[14]

The 1967 document, in which the P.F.L.P. declares the adoption of Marxism-Leninism, criticizes the general Soviet policy in the Middle East, particularly in the case of Palestine. Later, the Movement

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[12]Laqueur, N., "Russia Enters the Middle East." Foreign Affairs, January 1969: [31].

[13]Foreign Affairs, July 1957, "Strategy of the Middle East," [32], p. 661.

[14]The Soviet policy is seen by the rejection front (the Baath and the P.F.L.P.) as being too moderate towards Israel. Essentially, Soviet policy has no squabble with the legitimacy of its existence, only with its "imperialist connection" and its ultra-religious/nationalist character. In fact, the Soviets offered their own guarantee to the security of 1967 borders of Israel.

criticizes Chinese policy in the Arabian Gulf.[15] These observations discredit the one-directional coupling theory:

Choice of Issue Position ⟶ Choice of Ideology

The conspiracy version can only explain short-term phenomena. such as temporary shifts of position on certain issues, but not a radical, long-term ideological shift. The P.F.L.P. leadership has not been changed by outside power, but has itself undergone an ideological shift.

5.5 A Theory of the Leftward Shift

The ritual of applying lessons from European class conflicts to the economic, social, national, and religious conflicts in the Arab world has a rational basis. Those who are successful in projecting the appropriate images can hope to obtain useful international socialist or communist support in the conflicts with their opponents, or at least to diffuse leftist support of their opponents. On the far left, the Marxist-Leninist ideology, which has been developed and refined over decades of conflicts with democrats, socialists, anarchists, and capitalists, offers a politicized individual an apparently coherent and modern world view as a substitute for an old religious theology which no longer is satisfying. In developing Arab countries, many people raised in a traditional society become frustrated and anxious in the process of adapting to modern urban society. Similar psychological

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[15] The Chinese, in an effort to outbid the Soviet policy in the Middle East, recognized what they called the "legitimate" interest of the Shah in the Arabian Gulf, according to the official version of the talks between the Chinese prime minister the Shah in Tehran, 1974.

pressures are faced by Europeans who change from a static, traditional social structure to a more dynamic, technological, and urban society. It is here argued that many politically active Arabs prefer a political party that seems to clearly define its economic and social consequences to a party that might offer them greater benefits, but with risks stemming from non-clarity. In other words, risk-averse individuals participating in a risky political struggle prefer a less desirable but certain outcome to a more desirable uncertain outcome. If, in addition, the Marxist-Leninist positions appear to be more certain than other leftist party positions in Arab politics, then it will be demonstrated that the risk aversion assumption causes the leftward shift already discussed.

In particular, it can be theorized that a rational actor (party) would maximize political support, measured by enrollment in the party. However, it can be argued that maximizing membership is only a derivative objective and that the real objective is assuming power. For example, infiltrating the Army and police, assassination, and factional elitist politics have been a central part of the struggle for power on many occasions.

While this argument cannot be disregarded, it is true that all these methods of assuming power are the final act of a long process of popular education and psychological preparation of the masses against the incumbent regime by the organized revolutionary parties. The highest form of this preparation is enrollment in the revolutionary party itself. All Arab leftist parties argue that no stable revolutionary change of government can be affected without wide and solid support.[16] The P.F.L.P. political and organizational strategy states: "Working with the masses, caring about its problems and helping solve these problems, organizing and leading the masses is our first and paramount task. The masses are the reason behind our existence. It is the only way to mobilize the revolutionary potential to achieve our goals."[17] It goes on to say that "any gap between us and the masses is a dangerous signal to our own existence."

On the same theme, M. Aflag writes, "The masses are the greatest force and the only insurance of achieving the goals of Arab revolution."[18] He also asserts that, "The masses are the final reference, the masses are now, more than any time before, the fermentor of revolution and history peacemaker."

These convictions are not merely ideological, they are pragmatic. The continuous failure of military regimes in the Arab world, starting with El-Zaim and Shishakely in Syria, and continuing with Kassim in Iraq, Nasser in Egypt and Jadid in Syria, are obvious examples for changes which fail to have the prerequisite of a revolutionary, solid grass roots support among the Arab masses. Thus, the choice

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[16]See, for example, analysis for the collapse of the Baathist's 8th of February revolution in Iraq 1963, in M. Aflag, The Point of Commencement [6].

[17]See [9], page 101.

[18]See [6], page 183.

of party enrollment as an objective for a rational leftist party is reasonable in terms of the nature of Arab society.

It is also postulated that individuals have specific preferences for ideological positions. These preferences reflect class and family background, formal and informal education, and social association. The individual will enroll or support the party that:

1) offers an ideological structure nearest to personal position,

2) offers the most complete ideological structure.

Parties are hampered from ascertaining the distribution of the preferences among the population by sampling or polling due to obvious political and institutional constraint, even when they assume power. Also, for a long time after a revolutionary change, the psychological drag of the previous regimes will distort individual preferences.

Yet, in spite of their incomplete knowledge of the distribution of preferences in the population, leftist parties have to take positions on a variety of ideological and political issues as they face the challenges of day-to-day political events. In a way, this exercise can be viewed as a trial-and-error procedure that increases their knowledge about the underlying distribution of preferences. The results will be evaluated through their effects on enrollment.

Thus, in this theory, it is essential for any viable party to have as complete an ideological structure as possible, in order for such an ideology to be able to answer not only questions pertinent to the organization of society, but philosophical and ethical questions of the type which Islam, the traditional ideology, sought to answer.[19] These are precisely the questions to which Marxism-Leninism give central importance. Therefore, the rational party in this model seeks to maximize its enrollment while facing the following decisional cost structure.

1) A penalty cost of incompleteness of its ideological structure.
2) Information and organizational cost of developing, articulating and communicating its ideology.

The choice is conducted within the following informational environment:

1) The true preference distribution of the population is uncertain.
2) Marxism-Leninism is an apparently tried and well-articulated ideology that is followed by millions of people, so that acquisition of this ideology by a particular party entails negligible informational and organizational cost.
3) In the Arab world, the position of Marxism-Leninism (x^*) is to the left of the preferred positions of most people on the

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[19]Such questions as why and how society changes, thought and matter dialectics.

ideological axis.[20] See Figure 1.

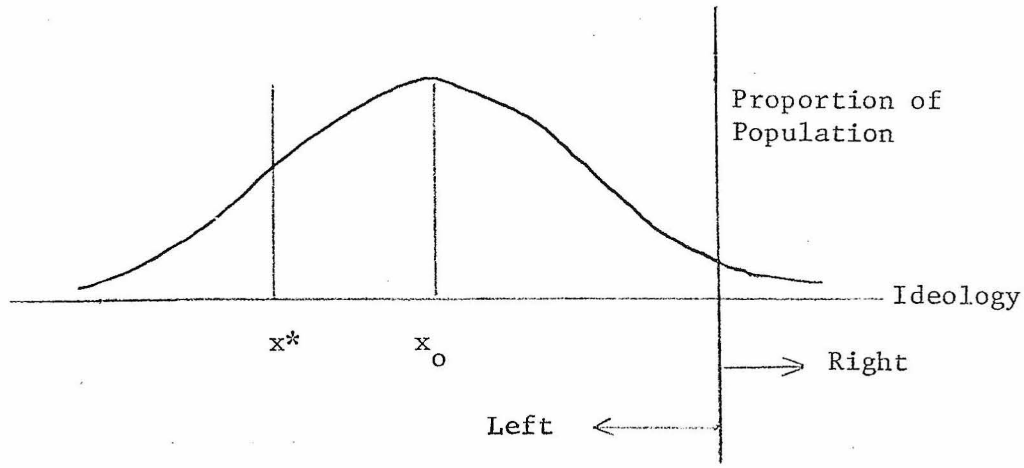

Figure 1

The model which is developed in the next section shows how a rational party will move toward Marxism-Leninism in its own ideology.[21]

The document published by the National Leadership of the Baath Party [7] seems to support this theory. It states:

> "The Baath always believed that scientific social discoveries come only through effort to analyze the situation and discover facts.... This attitude has always exposed those who adopted other revolutionary ideologies, particularly the Marxist-Leninist, as ready-made framework because of laziness and incapability."[22]

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[20]On this, the P.F.L.P. strategy [9] states that "The present popular ideology among the masses are to the right of our position." p. 19.

[21]Another explanation that can be loosely termed rational choice of the main phenomenon addressed by this paper, is that advanced by R. Wesson in, Why Marxism? The Continuing Success of a Failed Theory. N.Y. 1976. The gist of his argument is that Marxism is adopted because it can be made to serve the needs of diverse protest groups and radical movements. However, his analysis is mainly concerned with Western groups and movements.

[22]There is an interesting footnote on the same page. It states: "The behavior of those who adopted Marxism-Leninism without due analysis to the local conditions and the socio-historical context of the current stage can be termed scientific surrender.... This means that such

5.6 The Model

The spatial model about to be introduced is an extension of the unidimensional spatial model of party competition exposited by Downs.[23] Suppose that all Arab political elites conceptualize the spectrum of left parties as points on a single ideological dimension. Let θ_{ij} denote the position of party j as perceived by individual i (i = 1, ..., N and j = 1, ..., p), where N denotes the number of elites and p denotes the number of parties on the left.[24] In order to develop specific results which illuminate the phenomenon under discussion, it

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adoption is a result of cultural and ideological bankruptcy." On the same theme in [2], p. 7, Dr. Farah wrote, "There is an essential difference between adopting an ideology as a result of deep belief and conviction and adopting it because of competition for followers and supporters.... It is said that a great deal of socialist ideals are adopted because of the latter factor, not because of deep conviction or belief."

[23]The unidimensional spatial model was introduced into the political science literature by Downs [39]. The social choice theory for the unidimensional case was developed by Black [40]. Davis and Hinich [41] extended and developed spatial models for two party electoral competition; see Davis, Hinich and Ordeshook [42].

[24]Allowing each citizen to have a different perception of each party position is an extension from traditional spatial models. Shepsle [43] introduced candidate uncertainty into spatial models, but he assumed that the uncertainty term for party j in his model does not depend on the policy positions of the jth party. We differ from Shepsle by assuming a relationship between the perceptual uncertainty and the party position, and by using a different type of multiparty competition.

is necessary to use a special form for individual utility functions.

Suppose that all individuals have quadratic utility functions for party positions, i.e., let

$$u(\theta_{ij}, x_i) = c_i - a_{ik} (\theta_{ij} - x_i)^2$$

represent the utility which individual i receives if the position is adopted. The term x_i is individual i's ideal point; c_i is a constant which will play no role in our results; and a_{ik} is a positive constant which is a function of the positions of the other parties, but is independent of θ_{ij}; e.g., when p = 3, a_{i1} depends only on θ_{i2} and θ_{i3}, and a_{i3} depends only on θ_{i1} and θ_{i2}. The quadratic utility function is the simplest model that incorporates risk aversion.[25]

In keeping with the game theoretic orientation of spatial theory, suppose that the parties are willing to alter their positions in order to increase their political power and hence their chances of implementing their program. In terms of this model, suppose that the average perceived position of the jth party $\theta_j = N^{-1}\Sigma_{i=1}^{N} \theta_{ij}$ is determined by the positions that the party advocates. The party can alter its average position in the population by changing its platform, but there will still be a diversity of individual perceptions.

As an important additional assumption, suppose that parties do not coordinate their platforms. Occasionally, several parties form a coalition against others, but the competition for supporters is fierce and very non-cooperative. This is in fact a correct observation about leftist parties all over the world.

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[25] See pp. 75-76 of Riker and Ordeshook [44], and Shepsle [43].

The parties choose their positions in our model to maximize their political support among the effective part of the population, and for this reason need to know the preferences of the cadre. The language used by Downs and in the formal theories of voting, conveys the impression that the results require the assumption that the candidates (or parties) know the distribution of ideal points; but for the median voter result for two candidates, each candidate need only know the median ideal point in order to guarantee at least a tie. For elections with more than two candidates, however, the form of the ideal point distribution determines the play of the game. This perfect information assumption for candidates has been made to facilitate the theorizing, but it is obvious that it is impossible to know the preferences of a large, heterogeneous political body, and it is difficult to even obtain a precise estimate of the median position using the type of survey data and methods that are available in the West. In order to model the uncertainty about preferences, assume that the population of N political individuals is a random sample from an infinite population whose statistical parameters are imperfectly known by the parties. This trick is commonly used by statisticians. For example, the average position of party j, $\theta_j = N^{-1} \Sigma_{i=1}^{N} \theta_{ij}$ is a random variable. By the Central Limit Theorem (assuming finite variance), the difference between θ_j for a given group and μ_j, the mean of the infinite population, is approximately $1/\sqrt{N}$ for large N. Thus, μ_j can be estimated within an accuracy of $1/\sqrt{N^*}$ by taking a random sample of the N individuals who themselves are a representative sample from the unobservable infinite population.[26]

Suppose that the amount of time, money, and energy which individual i contributes is proportional to $u(\theta_{ij}, x_i)$, and that the party's political power is the sum of the resources contributed by the cadre to that party. Then party j maximizes the expected value of its political power by choosing a position θ_j which maximizes the average utility in the population.[27] Let $\varepsilon_{ij} = \theta_{ij} - \theta_j$ denote idiosyncratic perceptual error of the jth party's position, and let σ_j^2 denote the variance of ε_{ij} in the population. By straightforward algebraic manipulation, the average individual utility for party j is

$$\overline{c} - \overline{a}_{ik} (\theta_j^2 + \sigma_j^2) + \overline{2a_{ik}x_i}\,\theta_j - \overline{a_{ik}\,x_i^2}\,, \qquad (2)$$

where the overbars denote mean values.

As the special feature of this model, assume that σ_j^2 is a non-decreasing function of θ_j, i.e., the slope of $d\sigma_j^2/d\theta_j$ is positive, and

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[26]When N is large, the difference between θ_j and μ_j is negligible. For a formal statement of the relationship between sample statistics and population parameters, the reader should consult any standard mathematical statistics books; e.g., Sections 7.5 and 7.6 of Modd and Graybill [46].

[27]This is due to the fact that the average of the sum of the utilities is the sum of the average utility. To be formally correct, the terms "expected" and "average" refer to mean values in the infinite population, but when N is large, there is no practical distinction between these mean values and averages over the set of N individuals. Thus, θ_j is used in this paper instead of μ_j in order to reduce the notational complexity as much as possible. We will also use x to denote the population mean ideal point. These distinctions will be confusing for readers who regularly use sample averages as if they were true population parameters, but a note is made here of the distinction as a modest concession to mathematical rigor.

thus the variance of the perceptual error decreases as θ_j moves to the left. Recall that the Marxist-Leninist position was assumed to anchor the left of the ideology dimensions; thus, the assumption on σ_j^2 implies that the M/L position has the smallest variance.

As an additional modest assumption, suppose that σ_j^2 is a strictly convex function of θ_j, i.e., its second derivative is positive. It is then clear from (2) that the average utility, and thus the expected power of party j, is a concave function of θ_j ($j = 1, \ldots, p$). As long as $\theta_1, \ldots, \theta_p$ are restricted to some common interval, the concavity of the party objective functions (their expected power) implies that there exists a Nash equilibrium, which we denote $\theta_1^*, \ldots, \theta_p^*$.[28] This means that if party j chooses $\theta_j \neq \theta_j^*$, but all the other parties choose their equilibrium positions, then party j's utility is less than if it had chosen θ_j^*, i.e., once the parties are at the equilibrium, there is no incentive for one party to make a unilateral move.[29]

The property of an equilibrium that we wish to exploit is as follows: any equilibrium set $\theta_j^*, \ldots, \theta_p^*$ of party positions must satisfy the first order conditions

$$\theta_j^* + \frac{d\sigma_j^2}{d\theta_j} = \frac{\overline{a_{ik}x_i}}{\overline{a_{ik}}} \qquad (3)$$

---

[28]See Hinich, Ledyard, and Ordeshook [45]. The non-cooperative assumption is important here.

[29]There may be many equilibria. In order to obtain a unique equilibrium, we would have to restrict the form of a_{ik}. For example, there is a unique equilibrium if a_{ik} is independent of the party positions.

for each $j = 1, \ldots, p$. In order to simplify the exposition, restrict attention to the special case when $d\sigma_j^2/d\theta_j = \beta$, a constant, and the a_{ik} are independent of the other party positions. Then (3) becomes

$$\theta_j = \alpha_k - \beta \tag{4}$$

where

$$\alpha_k = \frac{\overline{a_{ik}x_i}}{\overline{a_{ik}}}$$

is independent of the party position. Consequently, the equilibrium defined by (4) is unique. As β, the slope of the relation between σ_j^2 and θ_j, increases the party's move to the left. If in the course of party competition the variance of the other parties increases relative to the variance of the Marxist-Leninist position, then β increases. The other parties then move to the left in order to increase their power. See Appendix for a clarifying example.

The interpretation of this result in the Arab context is this. Suppose that the variance of the party positions in the population is the same for all parties at the initial stages of political socialization of the proto-elites. As these politically active individuals become more aware of the ideological positions of the parties, suppose that the Marxist-Leninist position appears to be more certain than any other left position. Moreover, suppose that the variance of a party decreases as it adopts Marxist-Leninist policies, but the relative variance increases as individuals compare the parties. In terms of this model, this means that β increases over time. If, on the other hand, the variances of the parties decrease (or increase at the same rate) then

β remains constant. It is here argued that competition between Arab left parties results in a leftward shift toward the Marxist-Leninist position as a result of a perceived increase in the variance of the other parties relative to the Marxist-Leninist position by the elites. This increase is due in part to the increasing articulation of the Marxist-Leninist ideology made possible by a variety of relevant new Communist experiences, such as the Cuban, Vietnamese, and Western Communist parties, coupled with the increasing resources devoted by the Chinese and the Russians to clarify the ideological issues through which they project their struggle. On the other hand, only meagre resources are devoted by the third world countries to classify the ideology of their societies in the ever-increasing complexity of the modern world.

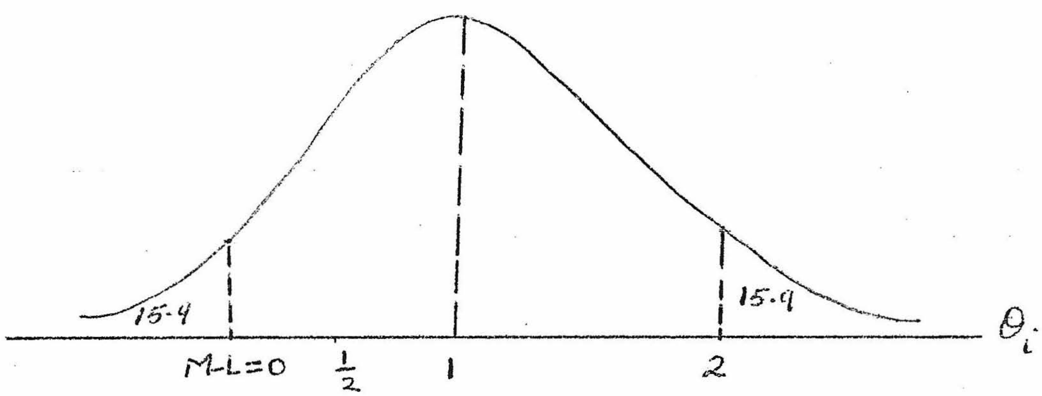
Appendix

In order to illuminate the formal reasoning behind our result, consider the following example. Suppose that all politically active individuals in a village have identical ideal positions that they are unable to articulate. Assume that this position is to the right of the M-L position, and involves compromises between M-L interpretations and traditional positions on social and economic issues. Since the origin and unit of the space are arbitrary, set the M-L position at zero and let $x = 1$ be the ideal position of the politicized villagers.

Assume that two parties are competing for support in the village. One party adopts the M-L position, while the other articulates a program that it hopes to be perceived at the village position. Due to the ambiguities and confusions inherent in a compromise between Marxism and traditional concepts of property rights and the like, each villager perceives the party to be at $\theta_i = 1 + \varepsilon_i$. The term ε_i is an idiosyncratic distortion of the party's ideological position. Assume that the variation of the ε_i in the village is modeled by a normal distribution with mean zero and variance one. As can easily be seen from a table of the normal distribution 15.9 percent of the village perceive the "compromise" party to be to the left of the M-L = 0 (Figure 2). These individuals prefer the M-L party since it is closer to their ideal ($x=1$). Only 68.2 percent prefer the "compromise."

Suppose the "compromise" party moves to the left and adopts the position $\theta = 1/2$. Assume that as a result of moving towards the M-L position, the standard deviation of ε_i is reduced to the value 1/4.

Then only 2.3 percent of the village perceive the party to be to the left of zero. Another 2.3 percent perceive the party to be to the right of $\theta + \frac{2}{4} = 1$, and thus they prefer the M-L party. Most (95.4 percent) prefer the party at θ = 1/2 to the M-L party.



Normal Distribution of Perceived Positions

of Leftist Party.

Figure 2

REFERENCES

1. Farah, E. The Development of Arab Revolutionary Ideology. 5th ed. Beirut: Institute for Arab Studies & Publications, 1975,

2. _______. Ideologies of the New Left in the World. 1st ed. Baghdad: General Institute of Journalism, 1971.

3. _______. On Revolutionary Arab Politics Before and After Defeat. Baghdad: Dar Al-Hurriah, 1972.

4. Aflag, M. Baath and Socialism. 1st ed. Beirut: Institute for Arab Studies and Publications, 1973.

5. _______. In Memory of the Arab Prophet. Baghdad: Dar Il-Thawara, 1970.

6. _______. The Point of Commencement. 5th ed. Baghdad: Dar Il-Thawara, 1975.

7. Arab Baath Socialist Party. Some Ideological Bases of the Arab Socialist Party. Baghdad: Dar Al-Hurriah, 1976.

8. Arab Baath Socialist Party. <u>On Revolutionary Arab Ideology</u>. Beirut: Institute for Arab Studies and Publications, 1975.

9. Popular Front for the Liberation of Palestine. <u>Political and Organizational Strategy</u>. 2nd ed. Amman: Central Information Office, P.F.L.P., 1970.

10. _______. <u>Tasks of the New Stage</u>. 2nd ed. Amman: Central Information Office, P.F.L.P., 1975.

11. The Arab Worker Party in Oman. <u>The General Outline of the Democratic Revolution in Oman</u>. AWPO Publication, 1970.

12. _______. <u>The Political Report and the Nationalist Movement</u>. AWPO Publications, 1970.

13. Democratic Front for the Liberation of Palestine. <u>The Political Program</u>. D.F.L.P. Central Information Office, 1975.

14. The "Revolutionary" Popular Front for the Liberation of Palestine. <u>How to Understand the Crises of the P.F.L.P</u>. Central Information Office, 1972.

15. Al-Kubaisi, B. <u>The Arab Nationalist Movement</u>. 1st ed. Arab Text, 1969.

16. Hawatmah, N. The Crises of Revolution in South Yemen. 1st ed. Dar-El-Talea, 1968.

17. Makhsood, K. The Crises of Arab Left. Dar-El Ilm, 1963.

18. Al-Razzaz, M. The Crises of Arab Left. 1st ed. Dar-El-Talea, 1973.

19. _______. Unity: Is There a Way Towards It? 1st ed. Institute for Arab Studies, 1971.

20. _______. Why Socialism Now? 1st ed. Dar-El-Talea, 1973.

21. El-Sader, M. Our Philosophy. 1st ed., 1959.

22. Sayed Kotib. Islam and the Problems of Civilization. Aisa El-Bahi and Company, 1962.

23. Saab, H. Islam Facing the Challenges of Modern Life. Dar-El-A'Adeb, 1965.

24. Yousif, M. The Contrived Gap Between Science and Religion. Dar El-Hayat, 1966.

33. Al-Adhim, S. Self-Criticism After Defeat. 2nd ed. Dar-El-Taliea, 1969.

34. _______. Criticism of the Religious Ideology. 2nd ed. Dar-El-Taliea, 1972.

35. _______. Criticism of the Ideology of the Palestinian Resistance. 1st ed. Dar-El-Taliea, 1972.

36. Al-Wash, N. Reflections. Arabic Studies, July 1968.

37. Wesson, R. Why Marxism? New York: 1968.

38. Schumpeter, J. History of Economic Analysis. Oxford University Press: 1954.

39. Downs, A. An Economic Theory of Democracy. New York: Harper and Row, 1957.

40. Black, D. The Theory of Committees and Elections. Cambridge University Press: 1958.

41. Davis and Hinich. "A Mathematical Model of Policy Formation in a Democratic Society." In Mathematical Applications in Political Science II. Edited by J.L. Bernal. Dallas: SMU Press for the Arnold Foundation, 1966.

25. Al-Khatib, A. The Financial Policies in Islam. 2nd ed. Dar-El-Maarif, 1975.

26. Al-Milaij, Y. Consultation in Islam Versus Its Role in the Democracies and Marxism. El-Exandiria: Academic/Cultural Institute, 1973.

27. Khallaf-Allah, M. "The Koran and Socialism." Il-Katib. July 1966.

28. Ramadhan, A. "El-Wafed Between the Right and the Left." Il-Katib. June 1973.

29. Badeau, J. "Islam and the Modern Middle East." Foreign Affairs. October 1959.

30. Ajami, F. "Between Cairo and Damascus." Foreign Affairs. April 1976.

31. Lagueuer, N. "Russia Enters the Middle East." Foreign Affairs. January 1969.

32. _______. "Strategy of the Middle East." Foreign Affairs. July 1957.

42. Davis, Hinich and Ordeshook. "An Expository Development of a Mathematical Model of the Electoral Process." APSR 64, (1964): 426-448.

43. Shepsle, K. "The Strategy of Ambiguity: Uncertainty in Electoral Competition." APSR.

44. Riker and Ordeshook. Positive Political Theory. New York: Prentice-Hall, 1973.

45. Hinich, Ledyard and Ordeshook. "The Existence of Equilibrium Under Majority Rule." Journal of Economic Theory 4 (1972): 144-153.

46. Modd and Graybill. An Introduction to Linear Statistical Models. Vol. 1. New York: McGraw-Hill, 1961.

47. Byrness, J.D.L. BuoRgabisim, An Approach to the Politics of Modernization Within the Focus of the Decision-Makers. Thesis, Maryland: 1967.

CHAPTER 6

AN ECONOMIC MODEL OF PRODUCTION AND INVESTMENT FOR PETROLEUM RESERVOIR - A NOTE ON KULLER AND CUMMING'S MODEL

6.1 Introduction

Since the classical paper by Davidson [1], there have been many models which illustrate the role of user's costs in oil production. However, a paper by Kuller and Cumming offers the most comprehensive treatment of user costs by introducing the following assumptions:

1. Total recovery, as well as annual production rates from natural drive, depends not only on cumulative production, but also on the rate at which production has taken place.
2. The recoverable stock, as well as the production rate, depends on the time path of investment as well as on cumulative investment (i.e., the capital stock).

In their model, n firms are exploiting a given petroleum reservoir under centralized management which maximizes the expected profit function, Π, over a known planning horizon T, subject to constraints reflecting the above two assumptions and non-negativity. They identify four user cost elements: stock user costs, boundary user costs, user costs of capital consumption, and production user costs. Their policy prescriptions are simple: 1) produce at a rate which equates marginal net income to firm J and the user cost association with firm J's production, and 2) equate the marginal cost of investment (to firm J for capital-type k) with the marginal present value of the reservoir-wide benefits associated with such investment. The latter includes not only direct impacts on the marginal productivity of J's capital and J's future variable and boundary costs, but also external impacts on other firms' variable and boundary costs as well as on the recoverable stock [2].

This note will extend the results of Kuller and Cumming by introducing an additional source of randomness in the planning model, that which pertains to the planning period.

6.2 The Effect of Random Planning Horizon

One element in the decision matrix of the oil producer is uncertainty about the arrival date, T*, of the "backstop" technology that will replace hydrocarbon fuels as the principal source of energy. This uncertainty introduces another element into user cost and modifies the production decision of the producer. Assume that the central management of a field believes that T* is randomly distributed on the range $[0,\overline{T}]$. To facilitate comparison of these results with those obtained by Kuller and Cumming, assume further that their T corresponds to the expected value of T* in this framework.

Let K_t be the capital stock at period t; $R_t = (r_1, r_2, \ldots, r_t)$ is the history of production; $V_t = (v_1, v_2, \ldots, v_t)$ is the history of investment; C_t is the generalized cost function of period t. Then, let

$C_t = C_t(R_t, V_t, K_t)$

$$\frac{\partial C_t}{\partial r_\tau} \geq 0, \quad \frac{\partial C_t}{\partial v_t} \geq 0, \quad \frac{\partial C_t}{\partial v_\tau} \leq 0, \quad \frac{\partial C_t}{\partial K_t} \leq 0$$

$$\tau \neq t$$

r_{Jt} = the volume of petroleum extracted by firm J, J = 1, ..., n during period t

R_t = annual production rate by all firms during all periods, i.e., $R_t = (r_{11}, r_{21}, \ldots, r_{n1}, \ldots, r_{1n-1}, \ldots, r_{nt-1}, r_{1t}, \ldots, r_{nt})$

V_{Jkt} = gross investment by firm J in capital component k, k = 1, ..., q, during period t

V_t = gross investment for all capital components by all firms during the periods 1, ..., t

K_{Jkt} = firms J's stock of capital components k at the beginning of period t

$K_{Jt} = (K_{J1t}, \ldots, K_{Jqt})$

D_{Jkt} = net depreciation of firm J's stock of capital component k during period t

x = the recoverable stock

F_{Jt} = an upper (physical) bound on firm J's capacity to produce petroleum during period t

C_{Jt} = firm J's cost function during period t

β_t = a discount factor, $(1 + r)^{-t}$ where r is the appropriate discount rate

P_t = unit price of petroleum during period t

where[1]

$$\frac{\partial D_{Jkt}}{\partial v_{Jkt}} \leq 0, \quad \frac{\partial D_{Jkt}}{\partial r_{JT}} \geq 0, \quad \frac{\partial D_{Jkt}}{\partial K_{Jkt}} \geq 0$$

$$\frac{\partial F_{Jt}}{\partial r_{i\tau}} \leq 0, \quad \frac{\partial F_{Jt}}{\partial v_{i\tau}} \geq 0, \quad \frac{\partial F_{Jt}}{\partial k_{Jt}} \geq 0$$

$$\frac{\partial x}{\partial r_{i\tau}} \leq 0, \quad \frac{\partial x}{\partial v_{i\tau}} \geq 0$$

$\tau = 1, \ldots, t$;

$i, J = 1, 2, \ldots, n$;

$k = 1, \ldots, q$

$1 \leq t \leq T$.

6.3 Chance Constrained Formulation

The problem will be formulated as a chance constrained optimizing decision [4]. In particular, the constraint relating to the total recoverable stock becomes of the form

$$\text{Probability} \left\{ x(R_{T*}, V_{T*}) - \sum_{\tau=1}^{T} \sum_{J=1}^{n} r_{J\tau} \geq 0 \right\} = 1 .$$

---

[1]This "all or nothing" situation for the lifetime of the oil industry is unrealistic, since it is known that oil will command a positive price long after the emergence of the backstop technology.

And the problem is then:

$$\text{Max } E \ \{ \sum_{t=1}^{T^*} \sum_{J=1}^{n} [P_t r_{Jt} - C_{Jt}(R_t, V_t, K_{Jt})] \beta_t \}$$

subject to

$$P \{- \sum_{\tau=1}^{T^*} \sum_{J=1}^{n} r_{J\tau} + x(R_{T^*}, V_{T^*}) \geq 0\} = 1$$

$$K_{Jk,t+1} = K_{Jkt} - D_{Jkt}(r_{Jt}, v_{Jkt}, K_{Jkt})$$

$$r_{Jt} \leq F_{Jt}(R_t, V_t, K_{Jt})$$

$$r_{Jt} \geq 0, \ v_{Jkt} \geq 0 \qquad \forall \ J, k \text{ and } t, \ 0 < t < T^* .$$

Let T^* obey a probability mass function, γ_t, defined on $[0,\overline{T}]$ such that

$$\gamma_t > 0 \text{ for } 0 \leq t \leq \overline{T}, \ \gamma_t = 0 \quad t \notin [0, \overline{T}]$$

and

$$\sum_{t=0}^{\overline{T}} \gamma_t = 1.$$

Define the probability that the "backstop" technology does not emerge in the period 0 to t by Φ_t, i.e., the probability that T is in the range t to $\overline{T}$ is

$$\Phi_t = \sum_{\tau=t}^{\overline{T}} \gamma_t \quad .$$

Let $\overline{R}$ be the production plan for the entire period 0 to $\overline{T}$.

Thus,

$$E\ \{\pi(\overline{R})\} = \sum_{t=1}^{\overline{T}} \gamma_t \sum_{\tau=1}^{t} \sum_{J=1}^{n} [P_\tau r_{J\tau} - C_{J\tau}(R_\tau,V_\tau,K_{J\tau})]\ \beta_\tau$$

or, changing the order of summation:

$$E\ \{\pi(\overline{R})\} = \sum_{J=1}^{n} \sum_{t=1}^{\overline{T}} \Phi_t\ \beta_t [P_t r_{Jt} - C_{Jt}\ (R_t,V_t,K_{Jt})].$$

Let

$$S(T^*) = x(R_{T^*},V_{T^*}) - \sum_{J=1}^{n} \sum_{t=1}^{T^*} r_{Jt}\ .$$

Then the problem becomes:

$$\text{Max} \sum_{J=1}^{n} \sum_{t=1}^{\overline{T}} \Phi_t\ \beta_t [P_t r_{Jt} - C_{Jt}(R_t,V_t,K_{Jt})]$$

$$p(S(T^*) \geq 0) = 1$$

$$K_{Jk,t+1} = K_{Jkt} - D_{Jkt}\ (r_{Jt},v_{Jkt},K_{Jkt})$$

$$r_{Jt} \leq F_{Jt}\ (R_t,V_t,K_{Jt})$$

$$r_{Jt} \geq 0,\ v_{Jkt} \geq 0 \quad \forall J,k \text{ and } t\ .$$

But $p(S(T^*) \geq 0) = 1$, under the assumption that $\gamma_t > 0$ for $0 < t < \overline{T}$, is equivalent [3] (up to a set of γ_t-measure zero) to

$S(t) \geq 0$ for all t. Thus, the Langrangian for the problem is:

$$L = \sum_{J=1}^{n} \sum_{t=1}^{\overline{T}} \Phi_t \beta_t [P_t r_{Jt} - c_{Jt}\ (R_t,V_t,K_{Jt})]$$

$$- \sum_{t=1}^{\overline{T}} \sum_{J=1}^{n} [\sum_{k=1}^{q} \Delta_{Jk,t+1}\ \beta_{t+1}\ \{K_{Jk,t+1} - K_{Jkt} + D_{Jkt}\ (r_{Jt},v_{Jkt},K_{Jkt})\}$$

$$- \psi_{Jt}\beta_t \{r_{Jt} - F_{Jt}(R_t, V_t, K_{Jt})\}$$

$$-(\lambda\beta)_t \{\sum_{\tau=1}^{t} \sum_{J=1}^{n} r_{J\tau} - x(R_\tau, V_\tau)\}$$

$$+ \zeta_{Jt}\beta_t u_{Jt} + \sum_{J=1}^{n} \sum_{k=1}^{q} \sigma_{Jkt}\beta_t v_{Jkt}] .$$

6.4 Characteristics of Optimum Production Rates

From the Langrangian expression:

$$\frac{\partial L}{\partial r_{Jt}} = (P_t \Phi_t \beta_t - \sum_{\tau=t}^{\bar{T}} \sum_{i=1}^{n} \frac{\partial c_{i\tau}}{\partial r_{Jt}} \beta_\tau \Phi_t)$$

$$- \sum_{k=1}^{q} \Delta_{Jk,t+1} \beta_{t+1} \frac{\partial D_{Jkt}}{\partial r_{Jt}} - \psi_{Jt}\beta_t + \sum_{\tau=1}^{\bar{T}} \sum_{i=1}^{n} \psi_{i\tau}\beta_\tau \frac{\partial F_{i\tau}}{\partial r_{Jt}}$$

$$- \sum_{t=1}^{\bar{T}} (\lambda\beta)_t (1 - \frac{\partial x}{\partial r_{Jt}}) + \zeta_{Jt}\beta_t = 0$$

or,

$$(P_t - \frac{\partial c_{Jt}}{\partial r_{Jt}}) \Phi_t \beta_t = (\lambda\beta)_{\bar{T}} (1 - \frac{\partial x}{\partial r_{Jt}}) + \psi_{Jt}\beta_t$$

$$+ \sum_{\tau=t}^{\bar{T}} \sum_{i=1}^{n} \psi_{i\tau} \left|\frac{F_{i\tau}}{r_{Jt}}\right| \beta_\tau + \sum_{k=1}^{q} \Delta_{Jk,t+1}\beta_{t+1} \frac{\partial D_{Jkt}}{\partial r_{Jt}}$$

$$+ \sum_{\tau=t+1}^{\bar{T}} \frac{\partial c_{Jt}}{\partial r_{Jt}} \beta_\tau \Phi_i + \sum_{\tau=t}^{\bar{T}} \sum_{\substack{i=1 \\ i \neq J}}^{n} \frac{\partial c_{i\tau}}{\partial r_{Jt}} \beta_\tau \Phi_\tau$$

$$+ \sum_{\tau=1}^{\bar{T}-1} (\lambda\beta)_\tau (1 - \frac{\partial x}{\partial r_{i\tau}}) \quad \forall i, J = 1, \ldots, n; \quad 1 \leq t \leq \bar{T} .$$

6.5 Elements of User Costs

As in Kuller and Cumming, the following user cost components can be identified.

Stock User Costs for Firm J

λ measures the increase in net incomes from the reservoir associated with an incremental change in the endogenously determined stock; the stock user cost for firm J in period t is given by

$$\lambda\beta_{\bar{T}}\ (1 - \frac{\partial x}{\partial r_{Jt}}) .$$

Boundary User Costs

Since $\Psi_{i\tau}$ measures the increase in net incomes which would result from an incremental relaxation of the restriction, the boundary user cost is given by

$$\Psi_{Jt}\beta_t - \sum_{\tau=t}^{\bar{T}} \sum_{i=1}^{n} \Psi_{i\tau} \left|\frac{\partial F_{ii}}{\partial r_{Jt}}\right| \beta_\tau .$$

User Costs of Capital Consumption

The multiplier $\Delta_{Jk,t+1}$ associated with the capital equation measures the marginal productivity of capital type k used by firm J in all future periods t+1, t+2, ..., T; the user costs of capital consumption is given by

$$\sum_{k=1}^{q} \Delta_{Jk,t+1}\beta_{t+1} \frac{\partial D_{Jkt}}{\partial r_{Jt}} .$$

Production User Costs

These user costs reflect the stock value of oil and gas to the firm, contributing to output as natural forces of production, and are given by

$$\sum_{\tau=t+1}^{\overline{T}} \frac{\partial C_{ii}}{\partial r_{Jt}} \beta_\tau + \sum_{\tau=t}^{\overline{T}} \sum_{i=1}^{n} \frac{\partial C_{i\tau}}{\partial r_{Jt}} \beta_\tau$$

$$i,J=1,\ldots,n;\ 1 \leq t \leq \overline{T}\ .$$

However, a new user cost element is now introduced by the randomness of the planning horizon. This element will be termed "the boundary-time cost." It is equal to

$$\sum_{\tau=1}^{\overline{T}-1} (\lambda\beta_\tau)(1 - \frac{\partial x}{\partial r_{i\tau}}) \ .$$

6.6 The Effect on the Optimal Production Rates

Comparing these first order conditions with those of Kuller and Cumming, the following can be noticed:

1) The net marginal benefit of producing one extra unit is decreased by a factor $\Phi_t(<1)$. This decrease causes the net marginal benefit curve to shift downward.

2) The effect of time-horizon uncertainty on marginal cost is indeterminate, and depends on the relative magnitudes of changes of opposite directions in the terms of the first order conditions equation. In comparison with the corresponding terms in Kuller and Cumming, the term

$$\sum_{\tau=1}^{\overline{T}} \sum_{i=1}^{n} \psi_{i\tau} \left|\frac{\partial F_{i\tau}}{\partial r_{Jt}}\right| \beta_t$$

is greater, because of the additional uncertainty.

The terms $(\lambda\beta)_{\overline{T}}\,(1 - \frac{\partial x}{\partial r_{Jt}})$ are smaller and the terms

$$\sum_{\tau=t+1}^{\overline{T}} \frac{\partial c_{i\tau}}{\partial r_{Jt}} R_\tau \Phi_\tau \quad \text{and} \quad \sum_{\tau=t}^{\overline{T}} \sum_{\substack{i=1 \\ i \neq J}}^{n} \frac{\partial c_{i\tau}}{\partial r_{Jt}} \beta_\tau \Phi_\tau$$

may increase or decrease depending on whether the extra terms in the summation which correspond to $\tau = T^*, T^* + 1, \ldots \overline{T}$ balance the reduction in each term of the summation caused by the weighting factor Φ_τ.

On the whole, if $\sum_{\tau=1}^{\overline{T}-1} (\lambda\beta)_\tau (1 - \frac{\partial x}{\partial r_i})$, the boundary time user cost, is sufficiently large, then the marginal user cost increases in comparison with that obtained from Kuller's and Cumming's formulation. This means, that a reduction in marginal benefit causes a reduction in production rate. In other cases, the effect on the production rate is ambiguous, since it depends on the shape and relative shifts in the marginal cost and marginal benefit curves.[2]

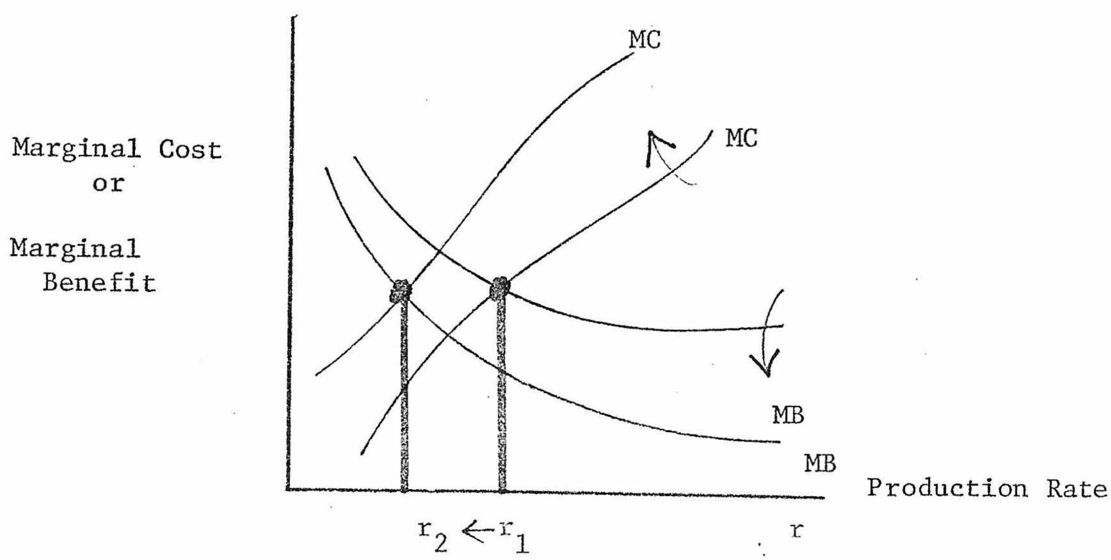


Figure 1

[2]In comparing the effect of the introduction of the boundary time user cost on the production decision with that obtained from Kuller's and Cumming's formulation, it is here assumed that their T corresponds to the expected value of T* in this formulation. Thus, $\overline{T} > T$.

6.7 Characteristics of Optimum Investment Rates

From the Langrangian expression:

$$\frac{\partial C_{Jt}}{\partial v_{Jkt}} \beta_t \Phi_t = - \Delta_{Jk,\ t+1} \beta_{t+1} \frac{\partial D_{Jkt}}{\partial v_{Jkt}} + \lambda\beta_{\bar{T}} \frac{\partial x}{\partial v_{Jkt}}$$

$$+ \sum_{\tau=t}^{\bar{T}} \sum_{i=1}^{n} \psi_{i\tau} \beta_\tau \frac{\partial F_{i\tau}}{\partial v_{Jkt}} - \sum_{\substack{i=1 \\ i \neq J}}^{n} \frac{\partial C_{it}}{\partial v_{Jkt}} \Phi_t \beta_t$$

$$- \sum_{\tau=t+1}^{\bar{T}} \sum_{i=1}^{n} \frac{\partial C_{i\tau}}{\partial v_{Jkt}} \Phi_\tau \beta_\tau + \sum_{\tau=1}^{\bar{T}-1} \lambda\beta_\tau \frac{\partial x}{\partial v_{Jkt}}$$

$$i, J = 1, - - - n;$$

$$k = 1, - - - q$$

$$1 \leq t \leq \bar{T}.$$

These first order conditions state that the optimal level of firm J's investment in capital-type k during any t, $1 \leq t \leq \bar{T}$ is given by equating the present value of the marginal costs of such investment, adjust for the uncertainty of the planning horizon, to the aggregate benefits of the reservoir associated with such investment. The interpretation of the terms in the above expression follows closely that given by Kuller and Cummings's [2]. Comparing with their results note that the discounted marginal cost of the investment is reduced by a factor of $\Phi < 1$ and that the aggregate benefit to the reservoir as a whole has a new term as a result of the inclusion of uncertainty in the planning horizon. However, even if D, F, x and C are the same functions as those considered by Kuller and Cummings, the effect on the aggregate benefit of the reservoir is ambiguous. Only, if

$$\sum_{\tau=1}^{\bar{T}-1} \lambda\beta_{\tau} \frac{\partial x}{\partial v_{Jkt}}$$ is large enough to swamp all the changes in the

other terms on the right hand side of the first order conditions that the aggregate benefit increases at all levels of investments for all capital components. In this case the optimal investment level increases unambiguously.

See Figure 2.

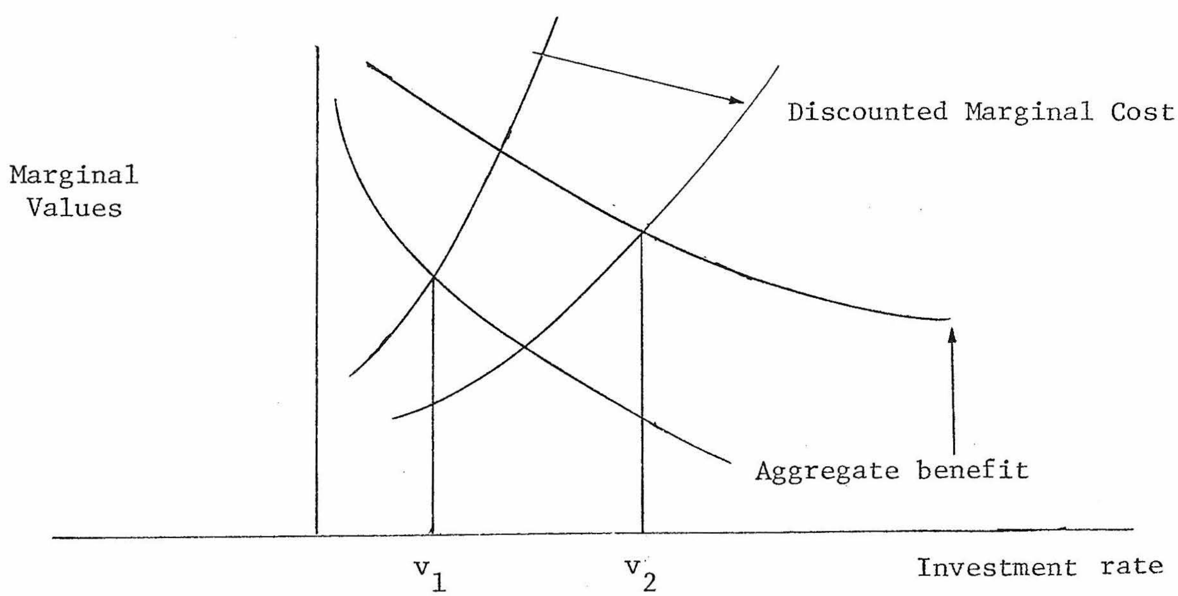


Figure 2

This note captures the effect of only one aspect of uncertainty, that which is related to the time of the emergence of the backstop technology. Other sources of uncertainty remain unexamined, such as uncertainty related to the price path and particularly the uncertainty regarding the prevailing price of the emerging alternatives. Moreover, a more realistic treatment should deal with the situation where:

a) the oil commands a positive price after the emergence of the backstop technology;

b) the strategic aspects provide the oil producers with a strategy of delaying the emergence of the alternative technologies.

The preceding analysis demonstrates that the theory of crude oil production is affected by incorporating the type of uncertainty considered in this note.

REFERENCES

1. Davidson, P. "Public Policy Problems of the Domestic Crude Oil Industry." American Economic Review 63 (1963): 85-108.

2. Kuller and Cumming. "An Economic Model of Production and Investment for Petroleum Reservoir." American Economic Review 64 (March 1974): 66.

3. Yaari, M. "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer." RES, 1965, pp. 138-150.

4. Charnes, A. and W. Cooper. "Chance Constrained Programming." Management Sciences 6 (1959).