Published for SISSA by Springer
Received: April 4, 2016
Accepted: May 29, 2016
Published: June 7, 2016

Electroproduction of tensor mesons in QCD

V.M. Braun,[a] N. Kivel,[b,1] M. Strohmaier[a] and A.A. Vladimirov[a]

[a] *Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany*
[b] *Helmholtz Institut Mainz, Johannes Gutenberg-Universität, D-55099 Mainz, Germany*

E-mail: vladimir.braun@ur.de, kivel@kph.uni-mainz.de, matthias.strohmaier@ur.de, aleksey.vladimirov@ur.de

Abstract: Due to multiple possible polarizations hard exclusive production of tensor mesons by virtual photons or in heavy meson decays offers interesting possibilities to study the helicity structure of the underlying short-distance process. Motivated by the first measurement of the transition form factor $\gamma^*\gamma \to f_2(1270)$ at large momentum transfers by the BELLE collaboration we present an improved QCD analysis of this reaction in the framework of collinear factorization including contributions of twist-three quark-antiquark-gluon operators and an estimate of soft end-point corrections using light-cone sum rules. The results appear to be in good agreement with the data, in particular the predicted scaling behavior is reproduced in all cases.

Keywords: QCD Phenomenology, NLO Computations

ArXiv ePrint: 1603.09154

[1] On leave of absence from St. Petersburg Nuclear Physics Institute, 188350, Gatchina, Russia.

doi:10.1007/JHEP06(2016)039

JHEP06(2016)039

Contents

1 Introduction

In recent years there has been increasing interest to hard exclusive production of tensor mesons $a_2(1320)$, $K_2^*(1430)$, $f_2(1270)$ and $f_2'(1520)$ by virtual photons or in heavy meson decays. In particular the possibility of three different polarizations of tensor mesons in weak B meson decays can shed light on the helicity structure of the underlying electroweak interactions. A different symmetry of the wave function and hence a different hierarchy of the leading contributions for the tensor mesons as compared to the vector mesons can lead to the situations that the color-allowed amplitude is suppressed and becomes comparable to the color-suppressed one. This feature can give an additional handle on penguin contributions. The early work was devoted mainly on the identification of the interesting decay modes and their basic theoretical description using various factorization techniques at the leading-order and the leading-twist level, see e.g. [1–6]. These studies are to a large extent exploratory. The physics potential of tensor meson production will depend on the accuracy of the theoretical description of such processes that can be achieved in QCD.

The recent study [7] of hard exclusive production of tensor mesons in single-tag two-photon processes is an important step forward in this context. This is a "gold-plated" reaction where the theoretical formalism can be tested and the relevant nonperturbative functions — tensor meson distribution amplitudes (DAs) — determined, or at least constrained. Our work aims to match this experimental progress with a development of the robust QCD framework for the study of the transition form factor $\gamma^*\gamma \to f_2(1270)$ in collinear factorization.

This reaction has already attracted some attention. Useful kinematic relations and estimates of the transition form factors for the mesons built of light and heavy quarks can be found in [8]. In ref. [9] it was pointed out that hard exclusive production of $f_2(1270)$ with helicity $\lambda = \pm 2$ is dominated by the gluon component in the meson wave function and can be used to determine gluon admixture in tensor mesons in a theoretically clean manner. In ref. [10] the helicity difference sum rule for the weighted integral of the $\gamma^*\gamma$ fusion cross section was derived and shown to provide constraints on the transition form factor in question. A phenomenological model for the tensor meson form factor can also be found in [11]. A related reaction $\gamma^*\gamma \to \pi\pi$ near the threshold has been discussed in [12–14].

Theory of the transition form factors goes back to the classical work on hard exclusive reactions in QCD [15–17]. The case of tensor mesons does not bring in complications of principle as compared to the pseudoscalar meson transition form factors that have been studied in great detail, but the tensor meson case is much less developed on a technical level. Our paper can be viewed as a major update of a earlier work [9] where the leading contributions to this process have been identified and calculated at the leading order. The new elements are:

- We introduce twist-three and twist-four DAs and calculate the corresponding contributions to the form factors;
- We calculate meson mass corrections terms in the higher-twist DAs and estimate the leading "genuine" three-particle contributions;
- We include the next-to-leading (NLO) corrections and calculate the charm-loop contribution for the helicity amplitude with $\lambda = \pm 2$ taking into account for the c-quark mass;
- We estimate quark-gluon coupling constants entering on the higher-twist level using QCD sum rules and the leading-twist gluon couplings using QCD sum rules and, alternatively, from the quarkonium decay $\Upsilon(1S) \to \gamma f_2$;
- We estimate the soft (end-point) correction for the leading, helicity-conserving amplitude.

The main conclusion from our study is that the experimental results on the $\gamma^*\gamma \to f_2(1270)$ transition form factors reported in ref. [7] appear to be in a very good agreement with the QCD scaling predictions starting already at moderate $Q^2 \simeq 5\,\mathrm{GeV}^2$. This is in contrast to the transition form factors for pseudoscalar π, η, η' mesons where large scaling violations have been observed [18–20]. The absolute normalization for all helicity form factors can be reproduced assuming a 10–15% lower value of the tensor meson coupling to the quark

JHEP06(2016)039

energy-momentum tensor as compared to the estimates existing in the literature, which is well within the uncertainty.

The presentation is organized as follows. Section 2 is introductory. It contains the definition of helicity amplitudes for the $\gamma^*\gamma \to f_2(1270)$ transition and the necessary kinematic relations. For the reader's convenience, the relation of our conventions to other definitions existing in the literature is explained in appendix A. Section 3 contains a detailed discussion of the leading-twist and higher-twist DAs of the tensor meson, which are the main nonperturbative input in the calculations. This section contains several new results. The relevant nonperturbative parameters are calculated in appendix D using QCD sum rules. In appendix E we estimate one of the leading-twist gluon couplings from the decay $\Upsilon(1S) \to \gamma f_2$. In section 4 we calculate the three existing helicity amplitudes in collinear factorization, including higher-twist and, partially, radiative corrections. In section 5 we discuss the power suppressed corrections $\sim 1/Q^2$ arising from the end-point regions. We explain how such corrections can be estimated using dispersion relations and duality and construct the light-cone sum rule for the largest, helicity conserving amplitude. In section 6 we compare our results to the experimental data [7] and summarize.

2 $f_2(1270)$ production in two-photon reactions

We consider the reaction

$$\gamma^*(q_1) + \gamma(q_2) \to f_2(P)\,, \qquad q_1^2 = -Q^2\,, \quad q_2^2 = 0\,, \quad P^2 = m^2 \tag{2.1}$$

with one real and one virtual photon, $P = q_1 + q_2$. Here and below $m = 1270\,\text{MeV}$ is the meson mass.

The transition amplitude can be related to the matrix element of the time-ordered product of two electromagnetic currents

$$T_{\mu\nu} = i\int d^4x\, e^{-iq_1x}\langle f_2(P,\lambda)|T\{j_\mu^{\text{em}}(x)j_\nu^{\text{em}}(0)\}|0\rangle\,, \tag{2.2}$$

where

$$j_\mu^{\text{em}}(x) = e_u\bar{u}(x)\gamma_\mu u(x) + e_d\bar{d}(x)\gamma_\mu d(x) + \dots\,.$$

The correlation function $T_{\mu\nu}$ can be decomposed in contributions of three Lorentz structures

$$T^{\mu\nu} = T_0^{\mu\nu} + T_1^{\mu\nu} + T_2^{\mu\nu}\,, \tag{2.3}$$

defined as

$$\begin{aligned}
T_0^{\mu\nu} &= e_{\alpha\beta}^{(\lambda)*}\ \left(-g_\perp^{\mu\nu}\right)(q_1-q_2)^\alpha(q_1-q_2)^\beta\frac{m^2}{(2q_1q_2)^2}T_0(Q^2)\,,\\
T_1^{\mu\nu} &= e_{\alpha\beta}^{(\lambda)*}\ \left(-g_\perp^{\alpha\nu}\right)(q_1-q_2)^\beta\left[q_1^\mu - q_2^\mu\frac{q_1^2}{(q_1q_2)}\right]\frac{m^2}{(2q_1q_2)^2}T_1(Q^2)\,,\\
T_2^{\mu\nu} &= e_{\alpha\beta}^{(\lambda)*}\left[g_\perp^{\alpha\mu}g_\perp^{\beta\nu} - \frac{1}{2}g_\perp^{\mu\nu}\frac{m^2}{(2q_1q_2)^2}(q_1-q_2)^\alpha(q_1-q_2)^\beta\right]T_2(Q^2)\,.
\end{aligned} \tag{2.4}$$

Here

$$g_{\perp}^{\mu\nu} = g^{\mu\nu} - \frac{1}{(q_1 q_2)}(q_1^\mu q_2^\nu + q_1^\nu q_2^\mu) + \frac{q_1^2}{(q_1 q_2)^2} q_2^\mu q_2^\nu\,, \qquad 2q_1 q_2 = m^2 + Q^2\,. \tag{2.5}$$

The polarization tensor $e^{(\lambda)}_{\alpha\beta}$ is symmetric and traceless, and satisfies the condition $e^{(\lambda)}_{\alpha\beta} P^\beta = 0$. Polarization sums can be calculated using

$$\sum_\lambda e^{(\lambda)}_{\mu\nu} e^{(\lambda)*}_{\rho\sigma} = \frac{1}{2} M_{\mu\rho} M_{\nu\sigma} + \frac{1}{2} M_{\mu\sigma} M_{\nu\rho} - \frac{1}{3} M_{\mu\nu} M_{\rho\sigma}\,, \tag{2.6}$$

where $M_{\mu\nu} = g_{\mu\nu} - P_\mu P_\nu / m^2$ and the normalization is such that $e^{(\lambda)}_{\mu\nu} e^{(\lambda')*}_{\mu\nu} = \delta_{\lambda\lambda'}$. The invariant form factors T_0, T_1 and T_2 correspond to the three possible helicity amplitudes

$$\begin{aligned} T_0: &\quad \gamma^*(\pm 1) + \gamma(\pm 1) \to f_2(0)\,, \\ T_1: &\quad \gamma^*(0) + \gamma(\pm 1) \to f_2(\mp 1)\,, \\ T_2: &\quad \gamma^*(\pm 1) + \gamma(\mp 1) \to f_2(\pm 2)\,. \end{aligned} \tag{2.7}$$

All three amplitudes (form factors) have mass dimension equal to one and scale as $T_k \sim Q^0$ (up to logarithms) in the $Q^2 \to \infty$ limit. The two-photon decay width of $f_2(1270)$ is given by [21]

$$\Gamma[f_2 \to \gamma\gamma] = \frac{\pi\alpha^2}{5m} \left(\frac{2}{3} |T_0(0)|^2 + |T_2(0)|^2 \right) = 3.03(40)\,\mathrm{keV}\,, \tag{2.8}$$

where $\alpha \simeq 1/137$ is the electromagnetic coupling constant. Assuming that $|T_2(0)| \gg |T_0(0)|$ we obtain

$$|T_2(0)| \simeq \sqrt{\frac{5m}{\pi\alpha^2} \Gamma[f_2 \to \gamma\gamma]} = 339(22)\,\mathrm{MeV}. \tag{2.9}$$

The relation of our definition of helicity form factors to the other existing in the literature definitions is given in appendix A.

3 Distribution amplitudes

In the standard classification the tensor $J^{PC} = 2^{++}$ $\mathrm{SU}(3)_f$ nonet is composed of $f_2(1270)$, $f_2'(1525)$, $a_2(1320)$ and $K_2^*(1430)$. Isoscalar tensor states $f_2(1270)$ and $f_2'(1525)$ have a dominant decay mode in two pions (or two kaons). The isovector $a_2(1320)$ decays only in three pions and is more difficult to observe in hard reactions. In the quark model these mesons are constructed from a constituent quark-antiquark pair in the P-wave and with the total spin equal to one. In QCD they can be represented by a set of Fock states in terms of quarks and gluons, that further reduce to DAs in the limit of small transverse separations.

In the exact SU(3)-flavor symmetry limit the $f_2(1270)$ meson is part of a flavor-octet, $f_2 = T_8$, and $f_2'(1525)$ is a flavor-singlet, $f_2' = T_1$. However, it is known empirically that the SU(3)-breaking corrections are large. Since $f_2(1270)$ and $f_2'(1525)$ decay predominantly in $\pi\pi$ and KK, it follows that they are close to the nonstrange and strange flavor eigenstates, respectively, with a small mixing angle, see [21, 22]. In this paper we assume ideal mixing at a low scale which we take to be $\mu_0 = 1\,\mathrm{GeV}$, for definiteness. In other words, we assume that $f_2(1270)$ at this scale is a pure nonstrange isospin singlet. This assumption can easily be relaxed when more precise data on the form factors become available. In what follows the notation $\bar{q}\ldots q$ refers to the SU(2)-flavor-singlet combination

$$\bar{q}\,q = \frac{1}{\sqrt{2}}\big[\bar{u}\,u + \bar{d}\,d\big]\,, \tag{3.1}$$

where u ans d are the usual “up” and “down” quark flavors.

Let n^μ be an arbitrary light-like vector, $n^2 = 0$, and

$$p_\mu = P_\mu - \frac{1}{2}n_\mu\frac{m^2}{pn}\,, \qquad\qquad g^{\perp}_{\mu\nu} = g_{\mu\nu} - \frac{1}{pn}\big(n_\mu p_\nu + n_\nu p_\mu\big)\,. \tag{3.2}$$

We define the f_2-meson quark-antiquark light-cone DAs as matrix elements of nonlocal light-ray operators [9, 23]

$$\begin{aligned}
\langle f_2(P,\lambda)|\bar{q}(z_2n)\gamma_\mu q(z_1n)|0\rangle \;&=\; f_q m^2\frac{e^{(\lambda)*}_{nn}}{(pn)^2}p_\mu\int_0^1 du\, e^{iz^u_{12}(pn)}\,\phi_2(u,\mu)\\
&\quad + f_q m^2\frac{e^{(\lambda)*}_{\perp\mu n}}{pn}\int_0^1 du\, e^{iz^u_{12}(pn)}\,g_v(u,\mu)\\
&\quad - \frac{1}{2}n_\mu f_q m^4\frac{e^{(\lambda)*}_{nn}}{(pn)^3}\int_0^1 du\, e^{iz^u_{12}(pn)}\,g_4(u,\mu)\,,\\
\langle f_2(P,\lambda)|\bar{q}(z_2n)\gamma_\mu\gamma_5 q(z_1n)|0\rangle \;&=\; -if_q m^2\epsilon_{\mu\nu\alpha\beta}\frac{n^\nu p^\alpha}{pn}\frac{e^{(\lambda)*}_{\beta n}}{pn}\int_0^1 du\, e^{iz^u_{12}(pn)}\,g_a(u,\mu)\,,
\end{aligned} \tag{3.3}$$

where

$$e^{(\lambda)*}_{\mu n} \equiv e^{(\lambda)*}_{\mu\nu}n^\nu\,, \qquad e^{(\lambda)*}_{\perp\mu n} \equiv g^{\perp}_{\mu\nu}e^{(\lambda)*}_{\nu n} = e^{(\lambda)*}_{\mu n} - p_\mu\frac{e^{(\lambda)*}_{nn}}{(pn)} + \frac{1}{2}n_\mu e^{(\lambda)*}_{nn}\frac{m^2}{(pn)^2} \tag{3.4}$$

and we use a shorthand notation

$$z^u_{12} = \bar{u}z_1 + uz_2\,, \qquad \bar{u} = 1-u\,. \tag{3.5}$$

Note that

$$e^{(\lambda)*}_{pn} = -\frac{1}{2}e^{(\lambda)*}_{nn}\frac{m^2}{pn}\,. \tag{3.6}$$

In all expressions light-like Wilson lines between the quark fields are implied.

The DAs defined in (3.3) satisfy the following symmetry relations:

$$\phi_2(u) = -\phi_2(\bar{u})\,, \qquad g_v(u) = -g_v(\bar{u})\,, \qquad g_a(u) = +g_a(\bar{u})\,, \qquad g_4(u) = -g_4(\bar{u})\,. \tag{3.7}$$

and are normalized as

$$\int_0^1 du\,(2u-1)\phi_2(u) = \int_0^1 du\,(2u-1)g_v(u) = \int_0^1 du\,(2u-1)g_4(u) = 1\,. \tag{3.8}$$

The integral of the DA $g_a(u)$ vanishes

$$\int_0^1 du\, g_a(u) = 0\,, \tag{3.9}$$

and the first nonzero (second) moment, $\int_0^1 du\,(2u-1)^2 g_a(u)$, involves contributions of three-particle operators, see below.

The coupling f_q is defined as the matrix element of the local operator

$$\frac{1}{2}\langle f_2(P,\lambda)|\bar{q}\left[\gamma_\mu i \overleftrightarrow{D}_\nu + \gamma_\nu i \overleftrightarrow{D}_\mu\right] q|0\rangle = f_q m^2 e^{(\lambda)*}_{\mu\nu} \tag{3.10}$$

where $\overleftrightarrow{D}_\mu = \overrightarrow{D}_\mu - \overleftarrow{D}_\mu$ is the covariant derivative. This coupling is scale dependent and gets mixed with the gluon coupling and the similar coupling for strange quarks. In appendix B we summarize the scale dependence of all DA parameters introduced in this section.

The numerical value of f_q has been estimated in the past [23–25] (see also appendix D) using the QCD sum rule approach. Another possibility is to use the experimental result on the decay width $\Gamma(f_2 \to \pi\pi)$ and estimate f_q assuming that the matrix element of the energy-momentum tensor $\langle \pi^+\pi^-|\Theta_{\mu\nu}|0\rangle$ is saturated by the tensor meson [23–27]. These two estimates agree with each other surprisingly well, although this agreement should not be overrated as in both cases the non-resonant two-pion background is not taken into account. We use (cf. [23] and appendix D)

$$f_q = 101(10)\ \text{MeV} \tag{3.11}$$

(at the scale 1 GeV) as the default value for the present study. Note that the positive sign for this coupling is a phase convention, whereas the relative signs of the other matrix elements with respect to f_q are physical and can be determined by considering suitable correlation functions as explained in appendix D.

Using the definitions in (3.3) it is easy to derive the operator product expansion (OPE) of quark bilinears close to the light cone $x^2 \to 0$ (at the tree level):

$$\begin{aligned}
&\langle f_2(P,\lambda)|\bar{q}(x)\gamma_\mu q(-x)|0\rangle \\
&\qquad = f_q m^2 \frac{e^{(\lambda)*}_{xx}}{(Px)^2} P_\mu \int_0^1 du\, e^{i(2u-1)(Px)}\left[\phi_2(u) - g_v(u) + \frac{1}{4}x^2 m^2 \phi_4(u)\right] \\
&\qquad\quad + f_q m^2 \frac{e^{(\lambda)*}_{\mu x}}{Px}\int_0^1 du\, e^{i(2u-1)(Px)}\, g_v(u) \\
&\qquad\quad + \frac{1}{2} f_q m^4 x_\mu \frac{e^{(\lambda)*}_{xx}}{(Px)^3}\int_0^1 du\, e^{i(2u-1)(Px)}\left[2g_v(u) - \phi_2(u) - g_4(u)\right], \\
&\langle f_2(P,\lambda)|\bar{q}(x)\gamma_\mu\gamma_5 q(-x)|0\rangle \\
&\qquad = -i f_q m^2 \epsilon_{\mu\nu\alpha\beta}\frac{x^\nu P^\alpha}{Px}\frac{e^{(\lambda)*}_{\beta x}}{Px}\int_0^1 du\, e^{i(2u-1)(Px)}\, g_a(u)\,,
\end{aligned} \tag{3.12}$$

where $\phi_4(u)$ is another twist-four two-particle DA that can be expressed in terms of the other functions using QCD equations of motion (EOM), see below.

In addition we define three-particle twist-three DAs as

$$g_\perp^{\mu\mu'}\langle f_2(P,\lambda)|\bar q(z_3n)igG_{\mu'n}(z_2n)\not{n}q(z_1n)|0\rangle = f_q m^2(pn)e^{(\lambda)*}_{\perp\mu n}\int\mathcal{D}\alpha\, e^{ipn\sum\alpha_k z_k}\Phi_3(\alpha)\,,$$
$$g_\perp^{\mu\mu'}\langle f_2(P,\lambda)|\bar q(z_3n)g\widetilde G_{\mu'n}(z_2n)\not{n}\gamma_5 q(z_1n)|0\rangle = f_q m^2(pn)e^{(\lambda)*}_{\perp\mu n}\int\mathcal{D}\alpha\, e^{ipn\sum\alpha_k z_k}\widetilde\Phi_3(\alpha)\,. \tag{3.13}$$

The conformal expansion of the three-particle DAs reads [28, 29]

$$\Phi_3(\alpha) = 360\alpha_1\alpha_2^2\alpha_3\left[\zeta_3+\frac{1}{2}\omega_3(7\alpha_2-3)+\dots\right],$$
$$\widetilde\Phi_3(\alpha) = 360\alpha_1\alpha_2^2\alpha_3\left[0+\frac{1}{2}\widetilde\omega_3(\alpha_1-\alpha_3)+\dots\right]. \tag{3.14}$$

The two-particle DAs $g_a(u)$ and $g_v(u)$ have *collinear* twist three and contain contributions of *geometric* twist-two and twist-three operators.[1] The contributions of lower geometric twist are traditionally referred to as Wandzura-Wilczek (WW) contributions. They can be calculated in the terms of the leading-twist DA $\phi_2(u)$ as [9, 23]

$$g_v^{WW}(u) = \int_0^u dv\,\frac{\phi_2(v)}{\bar v} + \int_u^1 dv\,\frac{\phi_2(v)}{v}\,,$$
$$g_a^{WW}(u) = \int_0^u dv\,\frac{\phi_2(v)}{\bar v} - \int_u^1 dv\,\frac{\phi_2(v)}{v}\,. \tag{3.15}$$

Assuming for simplicity the asymptotic expression for the leading-twist quark DA

$$\phi_2^{as}(u) = 30u(1-u)(2u-1)\,, \tag{3.16}$$

one obtains

$$g_v^{WW}(u) = 3C_1^{1/2}(2u-1)+2C_3^{1/2}(2u-1)\,,$$
$$g_a^{WW}(u) = 5C_2^{1/2}(2u-1)\,, \tag{3.17}$$

where $C_n^{1/2}(x)$ are Legendre polynomials. The Legendre expansion can be motivated by the properties of these DAs under conformal transformations [28, 29]. The "genuine" geometric twist-three contributions can be related to the three-particle DAs using EOM, see appendix C. For the truncation in (3.14) one obtains

$$g_a(u) = g_a^{WW}(u) - 10\zeta_3 C_2^{1/2}(2u-1) + \frac{15}{8}(\omega_3-\widetilde\omega_3)C_4^{1/2}(2u-1)\,,$$
$$g_v(u) = g_v^{WW}(u) - \left[10\zeta_3-\frac{15}{8}(\omega_3-\widetilde\omega_3)\right]C_3^{1/2}(2u-1)\,. \tag{3.18}$$

[1] We remind that geometric twist is defined as "dimension minus spin" of the corresponding operators, whereas collinear twist is defined as "dimension minus spin projection on the light-ray direction". The collinear twist counting is closely related to the counting of powers of large "plus" components of meson momentum in the matrix elements and determines power suppression of the corresponding contributions at large Q^2, see e.g. ref. [29].

JHEP06(2016)039

The twist-three matrix elements can be estimated using QCD sum rules, see appendix D. We obtain (at the scale 1 GeV)

$$\zeta_3 = 0.15(8)\,, \qquad \omega_3 = -0.2(3)\,, \qquad \widetilde{\omega}_3 = 0.06(1)\,. \tag{3.19}$$

The DAs $\phi_4(u)$ and $g_4(u)$ have collinear twist four and receive contributions of the geometric twist-two, -three and -four operators. The Wandzura-Wilczek-type twist-two contributions assuming the asymptotic expression for $\phi_2(u)$ (3.16) have the form

$$\begin{aligned} \phi_4^{WW}(u) &= 100u^2(1-u)^2(2u-1)\,, \\ g_4^{WW}(u) &= 30u(1-u)(2u-1)\,. \end{aligned} \tag{3.20}$$

We expect that these contributions are the dominant source of the power-suppressed corrections $\sim 1/Q^2$ because of the large mass of the $f_2(1270)$ and will neglect "genuine" geometric twist-three and twist-four contributions. The derivation of the expressions in (3.20) proceeds similar to the case of the DAs of vector mesons considered in [28, 30, 31] so that we omit the details.

Finally, the leading-twist gluon DAs of $f_2(1270)$ can be defined as [9]

$$\begin{aligned} & g_\perp^{\mu\mu'} g_\perp^{\nu\nu'} \langle f_2(P,\lambda)| G^a_{n\mu'}(z_2 n) G^a_{n\nu'}(z_1 n')|0\rangle \\ &\quad = f_g^T \left[e^{(\lambda)}_{\perp\mu\nu}(pn)^2 - \frac{1}{2} g^\perp_{\mu\nu} m^2 e^{(\lambda)}_{nn}\right] \int_0^1 du\, e^{iz_{12}^u pn} \phi_g^T(u) \\ &\qquad - f_g^S m^2 g^\perp_{\mu\nu} e^{(\lambda)}_{nn} \int_0^1 du\, e^{iz_{12}^u pn}\phi_g^S(u)\,. \end{aligned} \tag{3.21}$$

The distribution amplitudes $\phi_g^T(u)$ and $\phi_g^S(u)$ are both symmetric to the interchange of $u \leftrightarrow \bar{u}$ and describe the momentum fraction distribution of the two gluons in the f_2-meson with the same and the opposite helicity, respectively. The asymptotic distributions at large scales are equal to

$$\phi_g^{T,\mathrm{as}}(u) = \phi_g^{S,\mathrm{as}}(u) = 30u^2(1-u)^2\,. \tag{3.22}$$

The normalization constants f_g^T and f_g^S are defined through the matrix element of the local two-gluon operator:

$$\begin{aligned} \langle f_2(P,\lambda)|G^a_{\alpha\beta}(0)G^a_{\mu\nu}(0)|0\rangle \,=\, & f_g^T \left\{ \left[(P_\alpha P_\mu - \frac{1}{2} m^2 g_{\alpha\mu})\, e^{(\lambda)}_{\beta\nu} - (\alpha\leftrightarrow\beta)\right] - (\mu\leftrightarrow\nu)\right\} \\ & - \frac{1}{2} f_g^S m^2 \left\{ \left[g_{\alpha\mu}\, e^{(\lambda)}_{\beta\nu} - (\alpha\leftrightarrow\beta)\right] - (\mu\leftrightarrow\nu)\right\}. \end{aligned} \tag{3.23}$$

The coupling f_g^S can be estimated from the radiative decay $\Upsilon(1S)\to\gamma f_2$, see appendix E. The result is consistent with the assumption that f_g^S is very small at hadronic scales and is generated mainly by the evolution. In the numerical analysis we use the value

$$f_g^S(1\,\mathrm{GeV}) = 0\,. \tag{3.24}$$

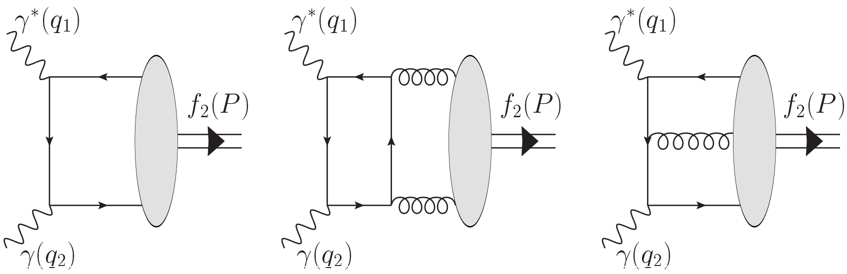

Figure 1. Leading contributions to the transition form factors $\gamma^*\gamma \to f_2(1270)$ in QCD. Adding crossing-symmetric diagrams is implied.

The coupling to a helicity-aligned gluon pair, f_g^T, is difficult to quantify. The calculation of the leading contributions to the relevant correlation functions suggests that the two couplings, f_g^S and f_g^T, have the same sign, see appendix D. In what follows we use

$$f_g^T(1\,\mathrm{GeV}) \approx 20\,\mathrm{MeV} \tag{3.25}$$

as a ballpark estimate.

As already mentioned, all couplings considered here are scale dependent. The relevant expressions are collected in appendix B.

4 QCD factorization

QCD description of the transition form factors in two-photon reactions is based on the analysis of singularities in the product of two electromagnetic currents in (2.2) in the limit $(x-y)^2 \to 0$. Typical Feynman diagrams contributing to the leading-order accuracy are shown in figure 1.

The leading contributions in the $Q^2 \to \infty$ limit have been calculated already in [9]. The form factor $T_0(Q^2)$ is of the leading twist and is dominated by the quark DA. In this case we include, in addition, NLO perturbative corrections to the leading twist contribution, which can be extracted from the corresponding expressions for the two-pion production in [13]. We also include the twist-four meson-mass correction m^2/Q^2 which is a new result.

The form factor $T_1(Q^2)$ is of twist-three. It receives the Wandzura-Wilczek-type contributions calculated in [9] and the "genuine" twist-three contributions of three-particle quark-antiquark gluon DAs (new result).

As already noticed in [9], the $T_2(Q^2)$ form factor is rather peculiar: the leading contribution at $Q^2 \to \infty$ comes in this case from the two-gluon DA with aligned helicity that we refer to as gluon transversity DA. However, this contribution is suppressed by the factor $\alpha_s/\pi \sim 0.1$ which is the standard perturbation theory factor for an extra loop, and also the two-gluon coupling to a "conventional" quark-antiquark meson is expected to be small as compared to the quark-antiquark coupling. By this reason the true QCD asymptotics for this form factor may be postponed to very large momentum transfers that are out of reach on the existing experimental facilities. The result for $T_2(Q^2)$ given below includes the leading term and the Wandzura-Wilczek-type higher-twist power correction that does not involve such small factors. We also calculate and add the leading-twist c-quark contribution.

With these new additions, the expressions for the form factors are

$$\begin{aligned} T_0 &= \langle f_q\rangle \int_0^1 \frac{du}{\bar u}\left[1+\frac{\alpha_s}{4\pi}\mathbb{C}_q(u)\right]\phi_2(u) - \frac{\alpha_s}{4\pi}\frac{2}{3}f_g^S\int_0^1 du\,\mathbb{C}_g^S(u)\,\phi_g^S(u) \\ &\quad + \frac{2m^2}{Q^2}\langle f_q\rangle \int_0^1 \frac{du}{\bar u}\left[u\ln u\,\phi_2(u) - \frac{1}{8\bar u}\phi_4(u)\right], \end{aligned} \tag{4.1}$$

$$\begin{aligned} T_1 &= 2\langle f_q\rangle \int_0^1 \frac{du}{\bar u}\Big[g_v(u) - g_a(u)\Big] \\ &= 4\langle f_q\rangle \int_0^1 \frac{du}{\bar u}\ln(u)\,\phi_2(u) + 2\langle f_q\rangle \int \mathcal{D}\alpha\,\mathbb{C}_\Phi(\alpha)\big[\Phi_3(\alpha)+\widetilde{\Phi}_3(\alpha)\big], \end{aligned} \tag{4.2}$$

$$T_2 = \frac{4m^2}{Q^2}\langle f_q\rangle \int_0^1 du\,\ln u\,g_v(u) + \frac{\alpha_s}{\pi}f_g^T\int_0^1\frac{du}{\bar u}\left[\frac{2}{3}+\frac{4}{9}\mathbb{C}_c(u)\right]\phi_g^T(u), \tag{4.3}$$

where the notation $\langle f_q\rangle$ stands for the sum of the light quark couplings weighted with the electromagnetic charges

$$\langle f_q\rangle = \frac{4}{9}f_u(\mu) + \frac{1}{9}f_d(\mu) + \frac{1}{9}f_s(\mu) = \frac{5\sqrt{2}}{18}f_q(\mu) + \frac{1}{9}f_s(\mu)\,. \tag{4.4}$$

The coefficient function of the three-particle DAs to T_1 is given by

$$\mathbb{C}_\Phi(\alpha) = \frac{1}{\alpha_2}\left[\frac{1}{\alpha_1\bar\alpha_1} + \frac{1}{\alpha_2}\left(\frac{\ln\alpha_1}{\bar\alpha_1} - \frac{\ln\bar\alpha_3}{\alpha_3}\right) + \frac{\ln\alpha_1}{\bar\alpha_1^2}\right], \tag{4.5}$$

and the NLO quark and gluon coefficient functions for T_0 read [13]

$$\mathbb{C}_q(u) = C_F\left[\ln^2\bar u + 3\ln u - 9\right], \qquad \mathbb{C}_g^S(u) = \frac{2\ln u}{u\bar u^2}\left[u\ln u - 2u - 2\right]. \tag{4.6}$$

The c-quark contribution to $T_2(Q^2)$ (this is a new result) is given by

$$\begin{aligned} \mathbb{C}_c(u) &= 1 + \frac{2m_c^2}{Q^2}\Bigg[-\frac{\beta}{u\bar u}\ln\left(\frac{\beta+1}{\beta-1}\right) + \frac{\beta_u}{\bar u}\ln\left(\frac{\beta_u+1}{\beta_u-1}\right) + \frac{\beta_{\bar u}}{u}\ln\left(\frac{\beta_{\bar u}+1}{\beta_{\bar u}-1}\right) \\ &\quad + \frac{1}{u\bar u}\left(\frac{1}{2}+\frac{m_c^2}{Q^2}\right)\left(\ln^2\left(\frac{\beta+1}{\beta-1}\right) - \ln^2\left(\frac{\beta_u+1}{\beta_u-1}\right) - \ln^2\left(\frac{\beta_{\bar u}+1}{\beta_{\bar u}-1}\right)\right)\Bigg], \end{aligned} \tag{4.7}$$

where

$$\beta_u = \sqrt{1+\frac{4m_c^2}{uQ^2}}\,, \qquad \beta \equiv \beta_1. \tag{4.8}$$

Here $m_c \simeq 1.4\,\text{GeV}$ is the c-quark mass. We did not calculate the corresponding contribution to $T_0(Q^2)$ because in this case it is a part of a $\mathcal{O}(\alpha_s)$ correction to the leading-order result $\mathcal{O}(1)$. It turns out (see below) that the c-quark contribution to T_2 is still strongly suppressed as compared to the light quarks in the Q^2 range of the Belle experiment, so that taking it into account for T_0 does not seem to be worth the effort at this stage in view of the other uncertainties.

We have checked the electromagnetic gauge invariance of our results by explicit calculation. Note that electromagnetic Ward identities relate the contributions of three-particle DAs, the last diagram in figure 1, to the higher-twist contributions in the first diagram encoded in the "genuine" twist-three contributions to the two-particle DAs. Such terms can be thought of as corresponding to gluon emission from the quark legs in the hard scattering amplitude. Thus it is not surprising that the twist-three form factor $T_1(Q^2)$ can be written in two equivalent representations as in (4.2): either contributions of the three-particle DAs can be eliminated in favor the two particle ones, or, vice verse, the "genuine" twist-three contributions to the two-particle DAs can be rewritten in terms of the three-particle DAs.

Evaluating (4.1), (4.2), (4.3) using the expressions for the DAs that are collected in the previous section we obtain

$$T_0 = 5\left(1-\frac{\alpha_s}{27\pi}\right)\langle f_q\rangle - \frac{215}{27}\frac{\alpha_s}{\pi}f_g^S - 5\frac{m^2}{Q^2}\langle f_q\rangle\,, \tag{4.9}$$

$$T_1 = \frac{10}{3}\langle f_q\rangle\left[1+4\zeta_3+\frac{9}{16}(\omega_3-\tilde{\omega}_3)\right], \tag{4.10}$$

$$T_2 = \frac{10}{3}\frac{m^2}{Q^2}\langle f_q\rangle\left[2-\zeta_3+\frac{3}{16}(\omega_3-\tilde{\omega}_3)\right] + \frac{5}{2}\frac{\alpha_s}{\pi}f_g^T\left[\frac{2}{3}+\frac{4}{9}\lambda(m_c^2/Q^2)\right], \tag{4.11}$$

where all nonperturbative parameters and the QCD coupling have to be taken at the hard scale $\mu \propto Q$. The function $\lambda(m_c^2/Q^2)$ takes into account suppression of the charm quark contribution in comparison to the light flavors; it is given by

$$\lambda(x) = 1 - 30x - 72x^2 + 24x(1+3x)\widehat{\beta}\ln\left(\frac{\widehat{\beta}+1}{\widehat{\beta}-1}\right) - 6x\left(1+6x+12x^2\right)\ln^2\left(\frac{\widehat{\beta}+1}{\widehat{\beta}-1}\right), \tag{4.12}$$

where $\widehat{\beta}=\sqrt{1+4x}$. The normalization is such that $\lambda(0)=1$. Note that $\lambda(0.1)\simeq 0.091$ so that the c-quark contribution at $Q^2\sim 20\,\text{GeV}^2$ is still suppressed by an order of magnitude as compared to the contributions of u, d, s quarks.

The expressions for the helicity form factors collected in this section present our main result.

5 Soft (end-point) contributions

Transition form factors with one real photon receive power corrections $\sim 1/Q^2$ coming from the region of large separation $(x-y)^2 \sim 1/\Lambda^2_{\text{QCD}}$ between the electromagnetic currents in (2.2). Such contributions are missing in the OPE and involve overlap integrals of the nonperturbative light-front wave functions at large transverse separations between the constituents and cannot be calculated in terms of DAs. They are revealed, nevertheless, as end-point divergences in the momentum fraction integrals in the framework of QCD factorization if one tries to extend it beyond the leading power accuracy. Such divergences are a clear indication that some extra contributions have to be added.

The technique that we adopt in what follows has been suggested originally [32] for the $\gamma^*\gamma\to\pi^0$, see [33, 34] for two recent state-of-the-art analysis. In this section we demonstrate how the same approach can be applied to the production of tensor mesons (cf. [35]). To this end we consider the simplest example: the form factor T_0 to the leading-order accuracy, leaving the NLO corrections to T_0 and the other two form factors, T_1 and T_2, for a future study. Our presentation follows closely the work [33] where further details and generalizations can be found.

The idea is to calculate the transition form factor for two large virtualities

$$q_1^2 = -Q^2\,, \qquad\qquad q_2^2 = -q^2\,, \qquad\qquad Q^2 \gg q^2\,,$$

using collinear factorization or, equivalently, OPE, and perform analytic continuation to the real photon limit $q^2 = 0$ using dispersion relations. In this way explicit evaluation of contributions of the end-point regions is avoided (since they do not contribute for sufficiently large q^2) and effectively replaced by certain assumptions on the physical spectral density in the q^2-channel.

For our purposes it is sufficient to assume that the second photon is transversely polarized. Then there are no new Lorentz structures and the only difference is that the form factors depend on two virtualities. The starting point is that the form factor

$$\widehat{T}_0(Q^2, q^2) = \frac{1}{(2q_1q_2)^2}\, T_0(Q^2, q^2)\,, \qquad T_0(Q^2) \equiv (m^2 + Q^2)^2\, \widehat{T}_0(Q^2, q^2 = 0)\,, \tag{5.1}$$

satisfies an unsubtracted dispersion relation in the variable q^2 for fixed Q^2. Separating the contribution of the lowest-lying vector mesons ρ, ω one can write

$$\widehat{T}_0^{\gamma^*\gamma^*\to f_2}(Q^2, q^2) = \frac{\sqrt{2} f_\rho \widehat{T}_0^{\gamma^*\rho\to f_2}(Q^2)}{m_\rho^2 + q^2} + \frac{1}{\pi}\int_{s_0}^{\infty} ds\, \frac{\operatorname{Im} \widehat{T}_0^{\gamma^*\gamma^*\to f_2}(Q^2, -s)}{s + q^2}, \tag{5.2}$$

where s_0 is a certain effective threshold. Here we assumed that the ρ and ω contributions are combined in one resonance term and the zero-width approximation is adopted; $f_\rho \sim 200\,\text{MeV}$ is the usual vector meson decay constant. Since there are no massless states, the real photon limit is recovered by the simple substitution $q^2 \to 0$ in this equation.

If both virtualities are large, $Q^2, q^2 \gg \Lambda^2_{\text{QCD}}$, the same form factor can be calculated using OPE. Assume this calculation is done to some accuracy. The result is an analytic function that satisfies a similar dispersion relation

$$\widehat{T}_{0,\text{OPE}}^{\gamma^*\gamma^*\to f_2}(Q^2, q^2) = \frac{1}{\pi}\int_0^{\infty} ds\, \frac{\operatorname{Im} \widehat{T}_{0,\text{OPE}}^{\gamma^*\gamma^*\to f_2}(Q^2, -s)}{s + q^2}. \tag{5.3}$$

The basic assumption of the method is that the physical spectral density above the threshold s_0 coincides (if integrated with a smooth test function) with the spectral density calculated in OPE, in the very similar way as the total cross section of e^+e^--annihilation above the resonance region coincides with the QCD prediction,

$$\widehat{T}_{0,\text{OPE}}^{\gamma^*\gamma^*\to f_2}(Q^2, -s) \simeq \widehat{T}_0^{\gamma^*\gamma^*\to f_2}(Q^2, -s)\,, \qquad s > s_0\,. \tag{5.4}$$

We expect that OPE becomes exact as $q^2 \to \infty$ so that in this limit the calculation has to reproduce the "true" form factor. Equating the two representations in eqs. (5.2), (5.3) at

$q^2 \to \infty$ and subtracting the contributions of $s > s_0$ from the both sides one obtains

$$\sqrt{2} f_\rho \widehat{T}_0^{\gamma^*\rho\to f_2}(Q^2) = \frac{1}{\pi}\int_0^{s_0} ds\, \mathrm{Im}\, \widehat{T}_{0,\mathrm{OPE}}^{\gamma^*\gamma^*\to f_2}(Q^2,-s)\,. \tag{5.5}$$

This relation explains why s_0 is usually referred to as the interval of duality (in the vector channel). The (perturbative) QCD spectral density $\mathrm{Im}\, \widehat{T}_{0,\mathrm{OPE}}^{\gamma^*\gamma^*\to f_2}(Q^2,-s)$ is a smooth function, very different from the physical spectral density $\mathrm{Im}\, \widehat{T}_0^{\gamma^*\gamma^*\to f_2}(Q^2,-s) \sim \delta(s-m_\rho^2)$. Nevertheless, their integrals over a certain region of energies coincide. In this sense QCD description of correlation functions in the terms of quark and gluons is dual to the description in the terms of hadronic states.

In practical applications one uses a certain trick [36] which allows to reduce the sensitivity on the duality assumption in (5.4) and simultaneously suppress contributions of higher twists in the OPE. This is done going over to the Borel representation $1/(s+q^2) \to \exp[-s/M^2]$ the net effect being the appearance of an additional weight factor under the integral that suppresses the large invariant mass region:

$$\sqrt{2} f_\rho \widehat{T}_0^{\gamma^*\rho\to f_2}(Q^2) = \frac{1}{\pi}\int_0^{s_0} ds\, e^{-(s-m_\rho^2)/M^2}\, \mathrm{Im}\, \widehat{T}_{0,\mathrm{OPE}}^{\gamma^*\gamma^*\to f_2}(Q^2,-s)\,. \tag{5.6}$$

Varying the Borel parameter M^2 within a certain window, usually 1–2 GeV^2 one can test sensitivity of the results to the particular model of the spectral density.

With this refinement, substituting eq. (5.6) in (5.2) and using eq. (5.4) one obtains for $q^2 \to 0$ (cf. [32])

$$\begin{aligned}\widehat{T}_{0,\mathrm{LCSR}}^{\gamma^*\gamma\to f_2}(Q^2) = {}& \frac{1}{\pi}\int_0^{s_0} \frac{ds}{m_\rho^2}\, e^{(m_\rho^2-s)/M^2}\, \mathrm{Im}\widehat{T}_{0,\mathrm{OPE}}^{\gamma^*\gamma^*\to f_2}(Q^2,-s) \\ &+ \frac{1}{\pi}\int_{s_0}^{\infty} \frac{ds}{s} \mathrm{Im}\, \widehat{T}_{0,\mathrm{OPE}}^{\gamma^*\gamma^*\to f_2}(Q^2,-s)\,.\end{aligned} \tag{5.7}$$

The abbreviation LCSR stands for the Light-Cone Sum Rules [37], as this approach is usually referred to.

Adding and subtracting the contribution of $s < s_0$ in the second term,[2] one can rewrite the result as

$$\widehat{T}_{0,\mathrm{LCSR}}^{\gamma^*\gamma\to f_2}(Q^2) = \widehat{T}_{0,\mathrm{OPE}}^{\gamma^*\gamma\to f_2}(Q^2) + \widehat{T}_{0,\mathrm{soft}}^{\gamma^*\gamma\to f_2}(Q^2)\,, \tag{5.8}$$

where the first term is the original OPE expression which is possible but not necessary to write in the dispersion representation, and the second term is the correction of interest:

$$\widehat{T}_{0,\mathrm{soft}}^{\gamma^*\gamma\to f_2}(Q^2) = \frac{1}{\pi}\int_0^{s_0} \frac{ds}{s}\left[\frac{s}{m_\rho^2}\, e^{(m_\rho^2-s)/M^2} - 1\right] \mathrm{Im}\, \widehat{T}_{0,\mathrm{QCD}}^{\gamma^*\gamma^*\to f_2}(Q^2,-s)\,. \tag{5.9}$$

An attractive feature of this technique is that one does not need to evaluate the nonperturbative wave function overlap contributions explicitly: they are taken into account effectively via the modification of the spectral density.

[2]Such a rewriting is not always possible as the separation of the OPE result and the soft correction can suffer from end-point divergences, see [33].

As an illustration, consider the leading-twist QCD result at the leading order in strong coupling:

$$\widehat{T}_0(Q^2,q^2) = \langle f_q\rangle \int_0^1 du\, \frac{\widetilde{\phi}_2(u)}{[\bar{u}Q^2+u\bar{u}m^2+uq^2]^2}, \tag{5.10}$$

where

$$\widetilde{\phi}_2(u) = -\int_0^u dv\, \phi_2(v)\,, \qquad\qquad \widetilde{\phi}_2^{\rm as}(u) = 15u^2\bar{u}^2\,. \tag{5.11}$$

This expression can easily be brought to the form of a dispersion relation changing variables $u \to s = \frac{\bar{u}}{u}Q^2 + \bar{u}m^2$ and integrating by parts. In this way one obtains after some rewriting,

$$\begin{aligned}\widehat{T}^{\gamma^*\gamma\to f_2}_{0,\rm soft}(Q^2) &= -\langle f_q\rangle \int_{u_0}^1 du\, \widehat{\phi}_2(u)\left[\frac{1}{[\bar{u}Q^2+u\bar{u}m^2]^2} - \frac{e^{(m_\rho^2-s)/M^2}}{m_\rho^2u^2M^2}\right] \\ &\quad + \langle f_q\rangle \left[\frac{1}{m_\rho^2}\,e^{(m_\rho^2-s_0)/M^2} - \frac{1}{s_0}\right]\frac{\widehat{\phi}_2(u_0)}{u_0^2m^2+Q^2},\end{aligned} \tag{5.12}$$

where

$$u_0 = \frac{1}{2m^2}\left[\sqrt{(Q^2+s_0-m^2)^2+4m^2Q^2} - (Q^2+s_0-m^2)\right], \tag{5.13}$$

and, for comparison, to the same accuracy

$$\widehat{T}^{\gamma^*\gamma\to f_2}_{0,\rm OPE}(Q^2) = \langle f_q\rangle \int_0^1 du\, \frac{\widehat{\phi}_2(u)}{[\bar{u}Q^2+u\bar{u}m^2]^2}. \tag{5.14}$$

Note that $u_0 \to 1$ as $Q^2 \to \infty$ so that the integration region shrinks to the end-point $u \to 1$ and the correction is power suppressed $\sim 1/Q^2$ in this limit, as expected. Numerical results are presented in the next section.

6 Results and discussion

The effective form factor averaged over polarizations

$$T_{f_2}(Q^2) = \sqrt{\frac{2}{3}\left|\frac{T_0(Q^2)}{T_2(0)}\right|^2 + \frac{Q^2m^2}{(m^2+Q^2)^2}\left|\frac{T_1(Q^2)}{T_2(0)}\right|^2 + \left|\frac{T_2(Q^2)}{T_2(0)}\right|^2}\,, \tag{6.1}$$

is calculated using default values of the nonperturbative parameters and compared with the experimental data [7] in figure 2. We observe a perfect scaling behavior for $Q^2 \gtrsim 4$–$5\,\mathrm{GeV}^2$ as predicted by QCD, whereas the normalization is slightly off — about 1–1.5σ if systematic errors in the data are taken into account. This difference can easily be compensated by a 10–15% decrease of the value of the quark coupling f_q which serves as an overall normalization factor in the calculation, or, alternatively, by a moderate deviation of the leading twist DA $\phi_2(u)$ from its asymptotic form. For illustration we show in the same figure by short dashes the result of the QCD calculation with $f_q = 85\,\mathrm{MeV}$ at the scale 1 GeV.

JHEP06(2016)039

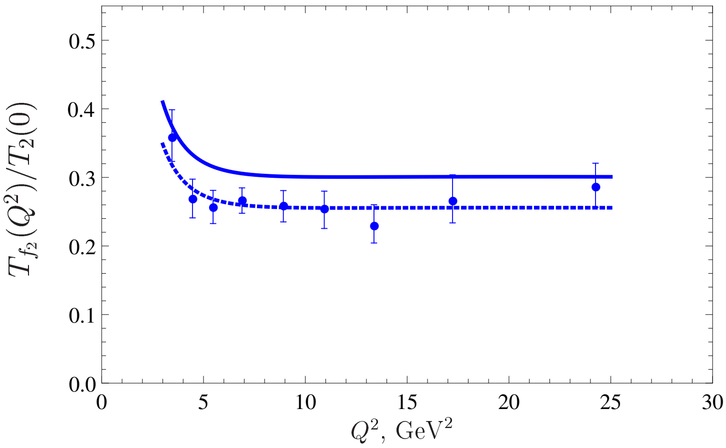

Figure 2. The effective form factor summed over polarizations normalized to $T_2(0) = 339\,\mathrm{MeV}$. The calculation using default values of the nonperturbative parameters is shown by the solid curve. The same calculation with the quark coupling f_q reduced by 15% is shown by short dashes. The experimental data are taken from ref. [7]. Only statistical errors are shown.

Such a 10–15% smaller coupling as compared to our default value $f_q = 101\,\mathrm{MeV}$ is certainly possible as the existing estimates are not reliable. A more precise number can eventually be obtained from lattice QCD, however, this calculation is rather complicated and will take time. It would be very interesting to measure the time-like transition form factor $e^+e^- \to f_2(1270)\gamma$ at large virtualities $q^2 \sim 100\,\mathrm{GeV}^2$ (cf. [38]) where the nonperturbative uncertainties are considerably reduced. This would give a direct measurement of the f_q-coupling.

Our results for the helicity-separated form factors $T_0(Q^2)$, $T_1(Q^2)$, $T_2(Q^2)$ are compared with the experimental data [7] in figure 3. All three form factors are described rather well, the QCD calculation being slightly above the data as we have already seen for the helicity-averaged form factor in figure 2. Note that our result for $T_1(Q^2)$ only includes the leading-power contribution at large Q^2 in contrast to $T_0(Q^2)$ and $T_2(Q^2)$ where we also calculated the $1/Q^2$ correction. Terms $\sim 1/Q^2$ in $T_1(Q^2)$ correspond to collinear-twist-five and soft contributions and are more difficult to estimate. They should be expected, however, to be negative and of the same order of magnitude as for $T_2(Q^2)$ so that the increase of the QCD curve for $T_1(Q^2)$ in figure 3 at smaller Q^2 will almost certainly be compensated by power corrections and is not a reason for concern. As expected, $T_1(Q^2)$ is also more sensitive to the twist-three quark-antiquark-gluon contributions as compared to the other two form factors, and the uncertainties in the corresponding parameters are not negligible, they are shown by the shaded area.

As discussed in [9], the form factor $T_2(Q^2)$ at asymptotically large Q^2 is dominated by the two-gluon contribution with aligned helicity that we refer to as gluon transversity DA. This contribution is suppressed, however, by the factor $\alpha_s/\pi \sim 0.1$ which is the standard penalty for an extra loop. Also the two-gluon coupling to a "conventional" quark-antiquark meson is unlikely to be large as compared to the quark-antiquark coupling. By this reason, $T_2(Q^2)$ at realistic Q^2 is still dominated by the Wandzura-Wilczek-type higher-twist power correction that does not involve such small factors: the shaded area in the plot for $T_2(Q^2)$

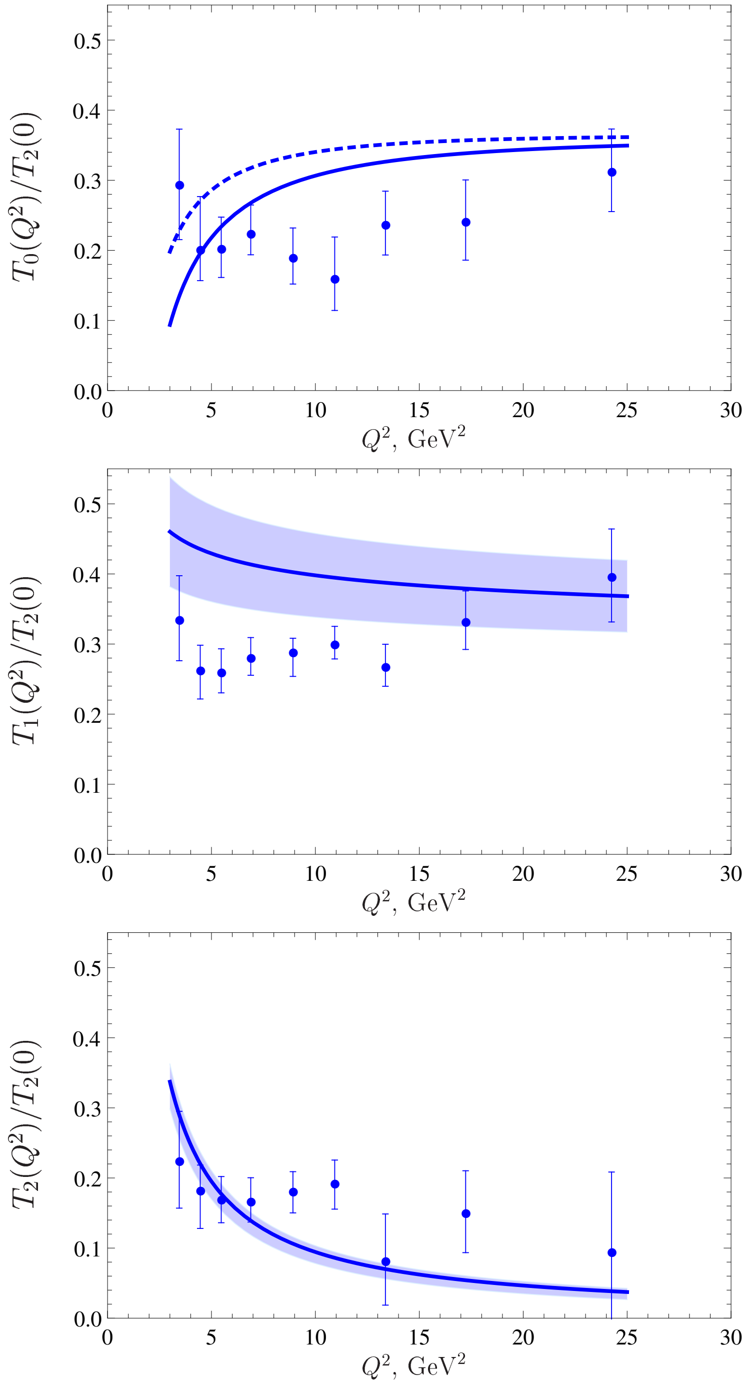

Figure 3. The form factors $T_0(Q^2)$, $T_1(Q^2)$, $T_2(Q^2)$ (from top to bottom) normalized to $T_2(0) = 339\,\mathrm{MeV}$. The result for $T_0(Q^2)$ shown by the solid line includes the estimate of soft end-point contributions using light-cone sum rules. The result without the soft correction is shown by dashes. The error band for $T_1(Q^2)$ (shaded area) corresponds to variation of the twist-three parameters in the range specified in (3.19), whereas for $T_2(Q^2)$ we also include variation of the tensor gluon coupling f_g^T in the range $\pm 50\,\mathrm{MeV}$. The experimental data are taken from ref. [7]. Only statistical errors are shown.

JHEP06(2016)039

includes variation of the tensor gluon coupling f_g^T in a rather broad range, $\pm 50\,\text{MeV}$, but the effect is barely visible. Our result does not mean that studies of the T_2 form factor at large Q^2 are not interesting. On the contrary, a broad resonance structure in the two-pion channel with a scaling behavior $T_2 \sim Q^0$ would be a clear signature of a tensor gluonium state.

To summarize, the main conclusion from our study is that the experimental results on the $\gamma^*\gamma \to f_2(1270)$ transition form factors reported in ref. [7] appear to be in a very good agreement with QCD scaling predictions starting already at moderate $Q^2 \simeq 5\,\text{GeV}^2$. The absolute normalization for all helicity form factors can be reproduced assuming a 10–15% lower value of the tensor meson coupling to the quark energy-momentum tensor as compared to the estimates existing in the literature, which is well within the uncertainty. These findings are in contrast to the transition form factors to pseudoscalar π, η, η' mesons where large scaling violations have been observed [18–20]. If confirmed by future higher-statistics measurements that can come from BELLE II, perfect scaling behavior can be an indication that higher-twist and soft corrections are less of an issue for tensor as compared to pseudoscalar mesons. This can be interesting in context of the studies of heavy meson decays [1–6] where the effective hard scale is not very large and estimates of preasymptotic corrections are difficult. In turn, the QCD description implemented in our analysis can still be improved in many ways, e.g., taking into account deviation from ideal SU(3)-flavor mixing at hadronic scales, two-loop scale dependence of the couplings, higher-twist and end-point corrections to $T_1(Q^2)$, more elaborate models for the DAs, etc. The corresponding studies will become necessary if the accuracy of the experimental data is increased.

Acknowledgments

N.K. is grateful to M. Vanderhaeghen for useful discussions. The work by M.S. is supported by a stipend through the F+E (Research and Development) grant by the GSI and the Helmholz Graduate School (HGS-HIRe), project number RSCHÄF1416.

A Other conventions

The experimental results in ref. [7] are presented for a different set of transition form factors $F_i(Q^2)$ suggested in [8]. The form factors $T_i(Q^2)$ defined in (2.4) are more convenient for the QCD study but in order to compare our results with the data we need to establish the precise correspondence between these two descriptions.

In ref. [8], the cross section $\sigma_{\lambda_1\lambda_2}$ for the production of $f_2(1270)$ by photons with helicities λ_1 and λ_2 is written as

$$\sigma_{\lambda_1\lambda_2} = \delta(s-m^2)8\pi^2\frac{5\Gamma_{\gamma\gamma}}{m}\, f_{\lambda_1\lambda_2}(Q^2), \tag{A.1}$$

where $s = (q_1+q_2)^2$ and $\Gamma_{\gamma\gamma}$ denotes the two-photon decay width (2.8). The form factors are defined in terms of the helicity cross sections as [8]

$$F_0(Q^2) = \sqrt{\frac{f_{TT}^{\pm\pm}(Q^2)}{(1+Q^2/m^2)}}, \quad F_1(Q^2) = \sqrt{\frac{f_{LT}(Q^2)}{(1+Q^2/m^2)}}, \quad F_2(Q^2) = \sqrt{\frac{f_{TT}^{\pm\mp}(Q^2)}{(1+Q^2/m^2)}}. \tag{A.2}$$

Calculation of the helicity cross sections (A.1) in terms of the Lorentz covariant amplitudes similar to T_i was done in ref. [10], see appendix C3. Using the expressions presented there we obtain

$$\sigma_{TT}^{\pm\pm} = \delta(s-m^2)\, 8\pi^2 \frac{5\Gamma_{\gamma\gamma}}{m} \left\{ \frac{\Gamma_{\gamma\gamma}^{\Lambda=0}}{\Gamma_{\gamma\gamma}} \left(1+\frac{Q^2}{m^2}\right)^{-1} \left|\frac{T_0(Q^2)}{T_0(0)}\right|^2 \right\}, \tag{A.3}$$

$$\sigma_{LT} = \delta(s-m^2)\, 8\pi^2 \frac{5\Gamma_{\gamma\gamma}}{m} \left\{ \frac{\pi\alpha^2}{5m\Gamma_{\gamma\gamma}} \frac{Q^2/m^2}{(1+Q^2/m^2)^3} \left|T_1(Q^2)\right|^2 \right\}, \tag{A.4}$$

$$\sigma_{TT}^{\pm\mp} = \delta(s-m^2)\, 8\pi^2 \frac{5\Gamma_{\gamma\gamma}}{m} \left\{ \frac{\Gamma_{\gamma\gamma}^{\Lambda=2}}{\Gamma_{\gamma\gamma}} \left(1+\frac{Q^2}{m^2}\right)^{-1} \left|\frac{T_2(Q^2)}{T_2(0)}\right|^2 \right\}, \tag{A.5}$$

where $\Gamma_{\gamma\gamma}^{\Lambda}$ stands for the two-photon decay width of $f_2(1270)$ with the polarization Λ:

$$\Gamma_{\gamma\gamma}^{\Lambda=2} = \frac{\pi\alpha^2}{5m} \left|T_2(0)\right|^2, \qquad \Gamma_{\gamma\gamma}^{\Lambda=0} = \frac{\pi\alpha^2}{5m}\frac{2}{3} \left|T_0(0)\right|^2. \tag{A.6}$$

Using these expressions and the definitions in (A.2) one finds

$$F_0(Q^2) = \sqrt{\frac{\Gamma_{\gamma\gamma}^{\Lambda=0}}{\Gamma_{\gamma\gamma}}} \left(1+\frac{Q^2}{m^2}\right)^{-1} \left|\frac{T_0(Q^2)}{T_0(0)}\right|, \tag{A.7}$$

$$F_1(Q^2) = \sqrt{\frac{\pi\alpha^2}{5m\Gamma_{\gamma\gamma}}} \frac{\sqrt{Q^2/m^2}}{(1+Q^2/m^2)^2} \left|T_1(Q^2)\right|, \tag{A.8}$$

$$F_2(Q^2) = \sqrt{\frac{\Gamma_{\gamma\gamma}^{\Lambda=2}}{\Gamma_{\gamma\gamma}}} \left(1+\frac{Q^2}{m^2}\right)^{-1} \left|\frac{T_2(Q^2)}{T_2(0)}\right|. \tag{A.9}$$

Experimentally the ratio of the decay widths with $\Lambda = 0$ and $\Lambda = 2$ is small [39]:

$$\frac{\Gamma_{\gamma\gamma}^{\Lambda=0}}{\Gamma_{\gamma\gamma}^{\Lambda=2}} \simeq (3.7 \pm 0.3) \times 10^{-2}. \tag{A.10}$$

Hence the expressions in (A.7)–(A.9) can be simplified neglecting the contribution of $\Gamma_{\gamma\gamma}^{\Lambda=0}$ in the full decay width:

$$F_0(Q^2) \simeq \sqrt{\frac{2}{3}} \left(1+\frac{Q^2}{m^2}\right)^{-1} \left|\frac{T_0(Q^2)}{T_2(0)}\right|, \tag{A.11}$$

$$F_1(Q^2) \simeq \frac{\sqrt{Q^2/m^2}}{(1+Q^2/m^2)^2} \left|\frac{T_1(Q^2)}{T_2(0)}\right|, \tag{A.12}$$

$$F_2(Q^2) \simeq \left(1+\frac{Q^2}{m^2}\right)^{-1} \left|\frac{T_2(Q^2)}{T_2(0)}\right|. \tag{A.13}$$

We use these simplified relations in order to present the data [7] in terms of the T_i form factors that are more suitable for comparison with QCD predictions.

The effective form factor $F_{f_2}(Q^2)$ is defined in [7] as

$$F_{f_2}(Q^2) = \sqrt{F_0^2(Q^2) + F_1^2(Q^2) + F_2^2(Q^2)}\,. \tag{A.14}$$

It is written in our notation as

$$(1+Q^2/m^2)F_{f_2}(Q^2) = \sqrt{\frac{2}{3}\left|\frac{T_0(Q^2)}{T_2(0)}\right|^2 + \frac{Q^2m^2}{(m^2+Q^2)^2}\left|\frac{T_1(Q^2)}{T_2(0)}\right|^2 + \left|\frac{T_2(Q^2)}{T_2(0)}\right|^2}. \tag{A.15}$$

For completeness we quote the phenomenological ansatz for the form factors F_i suggested in [8]:

$$F_0 = (1+Q^2/m^2)^{-2}\frac{1}{\sqrt{6}}\frac{Q^2}{m^2}, \quad F_1 = (1+Q^2/m^2)^{-2}\frac{Q}{m}, \quad F_2 = (1+Q^2/m^2)^{-2}. \tag{A.16}$$

Note that the asymptotic behavior for the FF F_2 is different from the QCD result, see eq. (4.3), because the contribution of the gluon transversity distribution has not been taken into account. More model predictions can be found in refs. [10, 11].

B Scale dependence

In this appendix we summarize the scale dependence and mixing under renormalization to the leading one-loop accuracy for all relevant parameters. In what follows

$$L = \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}, \qquad \beta_0 = \frac{11}{3}N_c - \frac{2}{3}n_f\,. \tag{B.1}$$

As already mentioned in the main text, for simplicity, we make use of the decoupling scheme, or fixed flavor number scheme (FFNS), such that the DAs only involve the three light flavors and the charm c-quark contributions are included in the coefficient function. Going over to the variable flavor number scheme (VFNS) is straightforward but has very limited numerical impact so that we do not implement it in this study.

For definiteness we also assume ideal quark mixing at a low normalization point $\mu_0 = 1\,\mathrm{GeV}$, $f_2 \sim (u\bar{u}+d\bar{d})/\sqrt{2}$. Thus all matrix elements involving strange quark vanish at this scale, but appear at higher scales because of the evolution. Staying within the fixed three-flavor scheme we decompose the SU(2)-flavor singlet coupling f_q in the SU(3)-flavor singlet and octet parts that have different scale dependence:

$$f_{(8)} = \frac{1}{\sqrt{6}}\big(f_u+f_d-2f_s\big)\,, \qquad f_{(1)} = \frac{1}{\sqrt{3}}\big(f_u+f_d+f_s\big)\,, \tag{B.2}$$

where $f_{u,d,s}$ are the couplings for the separate flavors. Thus

$$\begin{aligned} f_q(\mu) &= \sqrt{\frac{1}{3}}f_{(8)}(\mu) + \sqrt{\frac{2}{3}}f_{(1)}(\mu)\,, \\ f_s(\mu) &= -\sqrt{\frac{2}{3}}f_{(8)}(\mu) + \sqrt{\frac{1}{3}}f_{(1)}(\mu)\,. \end{aligned} \tag{B.3}$$

Ideal mixing at the reference scale implies

$$f_{(8)}(\mu_0) = \sqrt{\frac{1}{3}}f_q(\mu_0)\,, \qquad f_{(1)}(\mu_0) = \sqrt{\frac{2}{3}}f_q(\mu_0)\,, \qquad f_s(\mu_0) = 0\,. \tag{B.4}$$

The relevant renormalization group equation reads [40–42]

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right)\begin{pmatrix}f_{(8)}\\ f_{(1)}\\ f^s_g\end{pmatrix}=\frac{\alpha_s}{2\pi}\begin{pmatrix}\frac{8}{3}C_F & 0 & 0\\ 0 & \frac{8}{3}C_F & -\frac{4}{3}\sqrt{n_f}\\ 0 & -\frac{4}{3}\sqrt{n_f}C_F & \frac{2}{3}n_f\end{pmatrix}\begin{pmatrix}f_{(8)}\\ f_{(1)}\\ f^s_g\end{pmatrix}, \tag{B.5}$$

where from one finds

$$\begin{aligned}
f_{(8)}(\mu) &= L^{(\frac{8}{3}C_F)/\beta_0}f_{(8)}(\mu_0)\,,\\
f_{(1)}(\mu) &= f_{(1)}(\mu_0)+\left[L^{(\frac{8}{3}C_F+\frac{2}{3}n_f)/\beta_0}-1\right]\left[\frac{4C_F}{4C_F+n_f}f_{(1)}(\mu_0)-\frac{2\sqrt{n_f}}{4C_F+n_f}f^s_g(\mu_0)\right],\\
f^s_g(\mu) &= f^s_g(\mu_0)-\left[L^{(\frac{8}{3}C_F+\frac{2}{3}n_f)/\beta_0}-1\right]\left[\frac{2C_F\sqrt{n_f}}{4C_F+n_f}f_{(1)}(\mu_0)-\frac{n_f}{4C_F+n_f}f^s_g(\mu_0)\right],\\
f^T_g(\mu) &= L^{(\frac{7}{3}C_A+\frac{2}{3}n_f)/\beta_0}f^T_g(\mu_0)\,.
\end{aligned} \tag{B.6}$$

The last expression is based on the calculation of the relevant anomalous dimension by Hoodbhoy and Ji [43]. Note that the following combination of the quark and gluon couplings is scale-independent:

$$\sqrt{n_f}f_{(1)}(\mu)+2f^s_g(\mu)=\sqrt{n_f}f_{(1)}(\mu_0)+2f^s_g(\mu_0)\,, \tag{B.7}$$

as it corresponds to the matrix element of a conserved current: the traceless part of the QCD energy-momentum tensor.

The scale dependence of the flavor-nonsinglet twist-three couplings ζ_3, ω_3 and $\widetilde{\omega}_3$ can be found, e.g., in [28, 44]. Since the twist-three gluon DAs are completely unknown, using flavor-singlet evolution equations is not justified, and also the numerical difference between flavor-singlet and flavor-nonsinglet evolution is negligible as compared with the errors on the parameters. Staying with the flavor-nonsinglet evolution one obtains

$$\zeta_3(\mu)=L^{3(C_A-C_F)/\beta_0}\zeta_3(\mu_0)\,. \tag{B.8}$$

The remaining couplings ω_3 and $\widetilde{\omega}_3$ mix with each other:

$$\begin{pmatrix}\widetilde{\omega}_3\\ \omega_3\end{pmatrix}(\mu)=L^{\Gamma/\beta_0}\begin{pmatrix}\widetilde{\omega}_3\\ \omega_3\end{pmatrix}(\mu_0) \tag{B.9}$$

with the anomalous dimension matrix

$$\Gamma=\begin{pmatrix}\frac{13}{6}C_A-\frac{1}{12}C_F & \frac{7}{2}C_A-\frac{21}{4}C_F\\ \frac{1}{6}C_A-\frac{1}{4}C_F & \frac{25}{6}C_A-\frac{29}{12}C_F\end{pmatrix}=\begin{pmatrix}\frac{115}{18} & \frac{7}{2}\\ \frac{1}{6} & \frac{167}{18}\end{pmatrix}. \tag{B.10}$$

C Equations of motion

Two-particle meson DAs of higher twist can be expressed in terms of the three-particle DAs using QCD equations of motion (EOM). The case at hand is very similar to the vector meson DAs considered in [28], see section 4.1 and 4.2, so that we simply quote the result:

$$
\begin{aligned}
g_v(u) &= \int_0^u dv\,\frac{\Omega(v)}{\bar v} + \int_u^1 dv\,\frac{\Omega(v)}{v} - \frac{d}{du}\int_0^{\bar u} d\alpha_1 \int_0^u d\alpha_3 \frac{1}{\alpha_2}\Phi_3(\alpha_1,\alpha_2,\alpha_3) \\
&\quad + \int_0^{\bar u} d\alpha_1 \int_0^u d\alpha_3 \frac{1}{\alpha_2}\left(\frac{d}{d\alpha_3}+\frac{d}{d\alpha_1}\right)\widetilde\Phi_3(\alpha_1,\alpha_2,\alpha_3)\,, \\
g_a(u) &= \int_0^u dv \frac{\Omega(v)}{\bar v} - \int_u^1 dv \frac{\Omega(v)}{v}\,,
\end{aligned}
\tag{C.1}
$$

where

$$
\begin{aligned}
\Omega(u) = \phi_2(u) &- \frac{1}{2}\frac{d}{du}\int_0^{\bar u} d\alpha_1 \int_0^u d\alpha_3 \frac{1}{\alpha_2}\left(\alpha_1\frac{d}{d\alpha_1}+\alpha_3\frac{d}{d\alpha_3}\right)\Phi_3(\alpha_1,\alpha_2,\alpha_3) \\
&- \frac{1}{2}\frac{d}{du}\int_0^{\bar u} d\alpha_1 \int_0^u d\alpha_3 \frac{1}{\alpha_2}\left(\alpha_1\frac{d}{d\alpha_1}-\alpha_3\frac{d}{d\alpha_3}\right)\widetilde\Phi_3(\alpha_1,\alpha_2,\alpha_3)
\end{aligned}
\tag{C.2}
$$

and we tacitly assume $\alpha_2 = 1-\alpha_1-\alpha_3$.

The conformal expansion of the two- and three-particle DAs takes the form

$$
\begin{aligned}
\phi_2(u) &= 6u\bar u \sum_{n=1,3,\ldots}^{\infty} a_n C_n^{3/2}(2u-1)\,, \\
\Phi_3(\alpha_1,\alpha_2,\alpha_3) &= 360\alpha_1\alpha_2^2\alpha_3 \sum_{k,l=0}^{\infty}\omega_{kl}J_{kl}(\alpha_1,\alpha_3)\,, \\
\widetilde\Phi_3(\alpha_1,\alpha_2,\alpha_3) &= 360\alpha_1\alpha_2^2\alpha_3 \sum_{k,l=0}^{\infty}\widetilde\omega_{kl}J_{kl}(\alpha_1,\alpha_3)\,,
\end{aligned}
\tag{C.3}
$$

where $C_n^{3/2}(2u-1)$ are Gegenbauer polynomials and $J_{kl}(\alpha_1,\alpha_3)$ are Appell polynomials of two variables that are orthogonal (for different $k+l$) with the weight function $\alpha_1\alpha_2^2\alpha_3$. The summation index is related to the conformal spin: $j = n+2$ for the two-particle distribution and $j = k+l+7/2$ for the three-particle ones. Converting to the notation used in the main text, eqs. (3.14) and (3.16), one obtains

$$
a_1 \equiv \frac{5}{3}\,, \qquad \zeta_3 \equiv \omega_{00}\,, \qquad \omega_3 \equiv \omega_{10} = \omega_{01}\,, \qquad \widetilde\omega_3 \equiv 3\,\widetilde\omega_{10} = -3\,\widetilde\omega_{01}\,. \tag{C.4}
$$

Plugging the conformal expansion (C.3) in (C.1) it is straightforward to derive the general expansion of $g_a(u)$ and $g_v(u)$ in orthogonal polynomials, which reduces to the result given in (3.18) for the particular truncation corresponding to eq. (3.14).

D QCD sum rules

The twist-three quark-gluon couplings can be estimated from the tensor meson contribution to the correlation functions of

$$
J_{\mu\nu}(x) = \frac{1}{2}\bar q(x)\left[\gamma_\mu i \overleftrightarrow{D}_\nu + \gamma_\nu i \overleftrightarrow{D}_\mu\right] q(x)\,, \qquad q\bar q \equiv (u\bar u + d\bar d)/\sqrt{2}\,, \tag{D.1}
$$

JHEP06(2016)039

$\overleftrightarrow{D} = \overrightarrow{D} - \overleftarrow{D}$, and the quark-gluon light-ray operators that enter the definition of the corresponding DAs,

$$\begin{aligned}
\mathcal{G}_\alpha(z_1, z_2, z_3; x) &= \bar{q}(z_3 n + x) ig G_{\alpha n}(z_2 n + x) \not{n} q(z_1 n + x)\,, \\
\widetilde{\mathcal{G}}_\alpha(z_1, z_2, z_3; x) &= \bar{q}(z_3 n + x) g \widetilde{G}_{\alpha n}(z_2 n + x) \not{n} \gamma_5 q(z_1 n + x)\,.
\end{aligned} \tag{D.2}$$

In particular we consider the following correlation functions:

$$\begin{aligned}
T_{\alpha nn,\bar{n}\bar{n}} &= i \int d^4x\, e^{iqx} \langle 0|T\{J_{\bar{n}\bar{n}}(x) \mathcal{G}_\alpha(z_1, z_2, z_3; 0)\}|0\rangle \\
&= \Big[\bar{n}_\alpha(qn) - q_\alpha(n\bar{n})\Big](n\bar{n}) \int \mathcal{D}\alpha\, e^{iqn\sum z_k\alpha_k}\, T(q^2; \alpha) + \mathcal{O}(n_\alpha)
\end{aligned} \tag{D.3}$$

$$\begin{aligned}
\widetilde{T}_{\alpha nn,\bar{n}\bar{n}} &= i \int d^4x\, e^{iqx} \langle 0|T\{J_{\bar{n}\bar{n}}(x) \widetilde{\mathcal{G}}_\alpha(z_1, z_2, z_3; 0)\}|0\rangle \\
&= \Big[\bar{n}_\alpha(qn) - q_\alpha(n\bar{n})\Big](n\bar{n}) \int \mathcal{D}\alpha\, e^{iqn\sum z_k\alpha_k}\, \widetilde{T}(q^2; \alpha) + \mathcal{O}(n_\alpha)\,,
\end{aligned} \tag{D.4}$$

where it is assumed that the auxiliary light-like vectors are chosen such that

$$(q\bar{n}) = 0\,, \qquad (qn) \neq 0\,. \tag{D.5}$$

We obtain

$$\begin{aligned}
T(q^2; \alpha) &= \frac{\alpha_s}{2\pi^3} \frac{\Gamma[2-d]}{[-q^2]^{2-d}} \alpha_1\alpha_2\alpha_3 \left[\frac{1-2\alpha_1}{1-\alpha_1} + \frac{1-2\alpha_3}{1-\alpha_3} + 4\right] + \frac{\langle g^2G^2\rangle}{12\pi^2} \frac{\Gamma[2-\frac{d}{2}]}{[-q^2]^{2-\frac{d}{2}}} \alpha_1\alpha_3\delta(\alpha_2) \\
&\quad + \frac{2}{9} g^2 \langle\bar{q}q\rangle^2 \frac{1}{-q^2} \delta(\alpha_1)\delta(\alpha_3)\,, \\
\widetilde{T}(q^2; \alpha) &= \frac{\alpha_s}{2\pi^3} \frac{\Gamma[2-d]}{[-q^2]^{2-d}} \alpha_1\alpha_2\alpha_3 \left[\frac{1-2\alpha_1}{1-\alpha_1} - \frac{1-2\alpha_3}{1-\alpha_3}\right] + 0 \cdot \langle g^2G^2\rangle + 0 \cdot \langle\bar{q}q\rangle^2\,,
\end{aligned} \tag{D.6}$$

where $\langle g^2G^2\rangle$ is the gluon condensate and $\langle\bar{q}q\rangle$ is the quark condensate and we used the usual factorization approximation for the vacuum expectation values of the four-fermion operators. Note that the correlation function $\widetilde{T}(q^2; \alpha)$ does not receive nonperturbative corrections (to this power accuracy in the OPE and to the leading order in the strong coupling).

The contribution of $f_2(1270)$ to these correlation functions is

$$g^\perp_{\alpha\alpha'} T_{\alpha' nn,\bar{n}\bar{n}} = -q^\perp_\alpha (n\bar{n})^2 \frac{|f_q|^2 m^4}{m^2 - q^2} \int \mathcal{D}\alpha\, e^{iqn\sum\alpha_k z_k} \Phi_3(\alpha) + \ldots \tag{D.7}$$

and similar for $\widetilde{T}_{\alpha' nn,\bar{n}\bar{n}}$, so that taking moments and applying the Borel transformation one ends up with the sum rules

$$\begin{aligned}
|f_q|^2 m^4\, e^{-m^2/M^2} &= \frac{3}{40\pi^2} \int_0^{s_0} s^2 ds\, e^{-s/M^2} - \frac{2}{9} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \int_0^{s_0} ds\, e^{-s/M^2} + \frac{16\pi\alpha_s}{9} \langle\bar{q}q\rangle^2\,, \\
|f_q|^2 m^4 e^{-m^2/M^2} \zeta_3 &= \frac{7\alpha_s}{720\pi^3} \int_0^{s_0} s^2 ds\, e^{-s/M^2} + \frac{1}{18} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \int_0^{s_0} ds\, e^{-s/M^2} + \frac{8\pi\alpha_s}{9} \langle\bar{q}q\rangle^2, \\
|f_q|^2 m^4 e^{-m^2/M^2} \frac{3}{4}\omega_3 &= -\frac{7\alpha_s}{1440\pi^3} \int_0^{s_0} s^2 ds\, e^{-s/M^2} - \frac{1}{6} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \int_0^{s_0} ds\, e^{-s/M^2} + \frac{32\pi\alpha_s}{9} \langle\bar{q}q\rangle^2, \\
|f_q|^2 m^4 e^{-m^2/M^2} \frac{1}{28} \widetilde{\omega}_3 &= \frac{\alpha_s}{1440\pi^3} \int_0^{s_0} s^2 ds\, e^{-s/M^2}
\end{aligned} \tag{D.8}$$

where, for completeness, we added in the first line the sum rule for the coupling $|f_q|^2$ derived in [24, 25] and reanalyzed more recently in [23]. Using the value $s_0 = 2.53\,\mathrm{GeV}^2$ [23] and the interval $1.0 < M^2 < 1.4\,\mathrm{GeV}^2$ for the Borel parameter we obtain from this sum rule for the standard values of the gluon $\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle = 0.012\,\mathrm{GeV}^4$ and quark $\langle \bar{q} q \rangle = (-240\,\mathrm{MeV})^3$ condensates

$$f_q(\mu = 1\,\mathrm{GeV}) = 101(10)\ \mathrm{MeV}\,. \tag{D.9}$$

The quoted error corresponds to a 50% uncertainty in the gluon condensate, other uncertainties are much smaller. The quark-gluon couplings $\zeta_3, \omega_3, \widetilde{\omega}_3$ can best be estimated by taking the ratios of the corresponding sum rules to the sum rule for $|f_q|^2$. Using the same values of input parameters we obtain

$$\zeta_3 = 0.15(8)\,, \qquad \omega_3 = -0.2(3) \qquad \widetilde{\omega}_3 = 0.06(1)\,. \tag{D.10}$$

The given values correspond to the scale 1 GeV. Note that the uncertainty in ω_3 is very large because of the cancellations between gluon and quartic condensates. For $\widetilde{\omega}_3$ the leading nonperturbative corrections vanish and the perturbative contribution is very small. It is tempting to conclude that $\widetilde{\omega}_3$ is much smaller than ζ_3 and ω_3, but the number given above should be viewed with caution as the sum rule for this coupling is likely to be dominated by uncalculated higher-order corrections and/or condensates of higher dimension.

Estimates of gluon couplings are notoriously very difficult, see e.g. [45]. A limited insight can be obtained by considering the correlation function

$$\begin{aligned}\mathbb{G}_{\mu\nu} &= i \int d^4x\, e^{iqx} \langle 0 | T\{ G^a_{n\mu}(x) G^a_{n\nu}(x) G^b_{\bar{n}\xi}(0) G^b_{\bar{n}\xi}(0)\} | 0 \rangle \\ &= \left(q_\mu q_\nu - \frac{1}{2} q^2 g_{\mu\nu} \right) (n\bar{n})^2 \mathbb{G}_1(q^2) + \frac{1}{2} g_{\mu\nu} (n\bar{n})^2 \mathbb{G}_2(q^2) + \ldots\end{aligned} \tag{D.11}$$

where the ellipses stand for the structures $\sim n_\mu, \bar{n}_\mu, n_\nu, \bar{n}_\nu$ and, as above, we assumed that $(\bar{n}q) = 0$. Since tensor 2^{++} gluonium (glueball) states are expected to be rather heavy, see e.g. [46], by choosing a sufficiently low interval of duality in these invariant functions one can constrain the contribution of $f_2(1270)$. The leading contributions to the invariant functions $\mathbb{G}_1(q^2)$ and $\mathbb{G}_2(q^2)$ are, retaining singular terms only (cf. [45]),

$$\mathbb{G}_1(Q^2) \;=\; \frac{\langle G^2 \rangle}{3q^2}\,, \qquad \mathbb{G}_2(Q^2) \;=\; \frac{1}{5\pi^2} \frac{\Gamma[-\frac{d}{2}]}{[-q^2]^{-\frac{d}{2}}} + 0 \cdot \langle G^2 \rangle\,, \tag{D.12}$$

and the contribution of the tensor $f_2(1270)$ meson is

$$\mathbb{G}_1(Q^2) \;=\; -\frac{f_g^S f_g^T m^2}{m^2 - q^2} + \ldots\,, \qquad \mathbb{G}_2(Q^2) \;=\; \frac{|f_g^S|^2 m^4}{m^2 - q^2} + \ldots\,, \tag{D.13}$$

respectively. Thus

$$|f_g^S|^2 m^4 e^{-m^2/M^2} \;\approx\; \frac{1}{10\pi^2} \int_0^{s_0} s^2 ds\, e^{-s/M^2}\,, \qquad f_g^S f_g^T m^2 e^{-m^2/M^2} \;\approx\; \frac{1}{3} \langle G^2 \rangle\,. \tag{D.14}$$

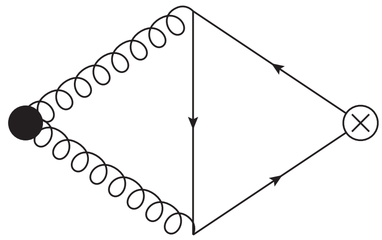

Figure 4. The leading contribution to the correlation function in eq. (D.15).

Taken at face value, these sum rules suggest that both couplings are of the order of 100 MeV (which should be viewed as an estimate from above), and have the same sign.

A somewhat better estimate can be obtained by considering the correlation function

$$\begin{aligned}\mathbb{H}_{\mu\nu} &= i\int d^4x\, e^{iqx}\langle 0|T\{G^a_{n\mu}(x)G^a_{n\nu}(x)J_{\bar n\bar n}(0)\}|0\rangle \\ &= (q_\mu q_\nu - \frac{1}{2}q^2 g_{\mu\nu})(n\bar n)^2\mathbb{H}_1(q^2) + \frac{1}{2}g_{\mu\nu}(n\bar n)^2\mathbb{H}_2(q^2) + \ldots\end{aligned} \tag{D.15}$$

Assuming $(q\bar n) = 0$, the contribution of $f_2(1270)$ to this correlator is

$$\mathbb{H}_1(Q^2) = \frac{f_q f_g^T m^2}{m^2-q^2} + \ldots\,, \qquad \mathbb{H}_2(Q^2) = -\frac{f_q f_g^S m^4}{m^2-q^2} + \ldots\,. \tag{D.16}$$

The leading contribution in QCD is given by the Feynman diagram shown in figure 4. We obtain

$$\begin{aligned}\mathbb{H}_1 &= \sqrt{2}\frac{\alpha_s}{(4\pi)^3}q^2 C_A C_F\left[\frac{8}{9}\ln\left(\frac{\mu^2}{-q^2}\right) + \frac{139}{54}\right] + \ldots \\ \mathbb{H}_2 &= -\sqrt{2}\frac{\alpha_s}{(4\pi)^3}q^4 C_A C_F\left[\frac{8}{15}\ln^2\left(\frac{\mu^2}{-q^2}\right) + \frac{598}{225}\ln\left(\frac{\mu^2}{-q^2}\right) + \frac{5627}{1500}\right] + \ldots\end{aligned} \tag{D.17}$$

where from one obtains the sum rules

$$\begin{aligned}f_q f_g^T m^2 e^{-m^2/M^2} &\approx \frac{8\sqrt{2}}{9}\frac{\alpha_s}{(4\pi)^3}C_A C_F\int_0^{s_0} ds\, s e^{-s/M^2}, \\ f_q f_g^S m^4 e^{-m^2/M^2} &\approx \frac{\sqrt{2}\alpha_s}{(4\pi)^3}C_A C_F\int_0^{s_0} ds\, s^2 e^{-s/M^2}\left[\frac{16}{15}\ln\frac{\mu^2}{s} + \frac{598}{225}\right].\end{aligned} \tag{D.18}$$

Dividing these expressions by the sum rule for $|f_q|^2$ we obtain for the same values of parameters

$$f_g^T/f_q = 0.25 - 0.29\,, \qquad f_g^S/f_q = 0.53 - 0.58\,. \tag{D.19}$$

Again, it appears that the two gluon couplings have the same sign. The accuracy of this calculation is very difficult to quantify, we view the numbers in (D.19) as order-of-magnitude estimates only.

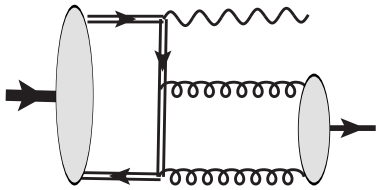

Figure 5. The leading contribution to the radiative decay $\Upsilon(1S) \to \gamma f_2(1270)$.

E f_g^S from the radiative decay $\Upsilon(1S) \to \gamma f_2$

The scalar gluon coupling f_g^S can be estimated from the bottomonium decay $\Upsilon(1S) \to \gamma f_2(1270)$. The dominant contribution comes from the two-quark $Q\bar{Q}$ component of the bottomonium wave function; the contribution of higher Fock states is suppressed by the small relative velocity of the heavy quarks. To the leading-order accuracy the decay amplitude is described by the diagram in figure 5. The corresponding calculation was already done in refs. [47–49]. The result reads

$$A\,[\Upsilon(1S) \to \gamma\, f_2] = (\epsilon_\gamma^* \cdot \epsilon_\Upsilon)\sqrt{2M_\Upsilon}\sqrt{\frac{3}{2\pi}}\frac{R_{10}(0)}{m_b^4} 2\pi\alpha_s e e_b\, e_{nn}^{(\lambda)*}\, f_g^S m_f^2 \frac{1}{4}\int_0^1 \frac{du}{u\bar{u}}\,\phi_g^S(u)\,, \quad \text{(E.1)}$$

where ϵ_γ^* and ϵ_Υ are the polarization vectors of the photon and heavy meson, respectively, m_b is the b-quark (pole) mass and $R_{10}(0)$ denotes the radial wave function of $\Upsilon(1S)$ at the origin. Potentially there could be also a contribution of the transverse DA $\phi_g^T(t)$, but the corresponding terms cancel to the leading-order accuracy.

In order to avoid the dependence on the nonperturbative parameter $R_{10}(0)$ it is convenient to consider the ratio

$$\frac{Br[\Upsilon(1S) \to \gamma\ f_2]}{Br[\Upsilon(1S) \to e^+e^-]} = \frac{64\pi}{3}\frac{\alpha_s^2(4m_b^2)}{\alpha}\left(1 - \frac{m^2}{M_\Upsilon^2}\right)\frac{\left[f_g^S\, I_g^S\right]^2}{m_b^2}\,, \quad \text{(E.2)}$$

where this dependence cancels. Here we used the notation I_g^S for the integral

$$I_g^S(\mu) = \frac{1}{4}\int_0^1 \frac{du}{u\bar{u}}\,\phi_g^S(u,\mu)\,. \quad \text{(E.3)}$$

For the asymptotic DA $\phi_g^S(u,\mu) = 30u^2(1-u)^2$ one obtains $I_g^S = \frac{5}{4}$. The branching fractions on the l.h.s. of eq. (E.2) are known, see [21]:

$$\begin{aligned} Br[\Upsilon(1S) \to \gamma\ f_2] &= (1.01 \pm 0.09) \times 10^{-4}, \\ Br[\Upsilon(1S) \to e^+e^-] &= (2.38 \pm 0.11) \times 10^{-2}\,. \end{aligned} \quad \text{(E.4)}$$

Using $m_b \simeq 4.8\text{GeV}$, $\alpha_s(4m_b^2) = 0.176$ and $\alpha \simeq 1/137$ we obtain

$$|f_g^S\, I_g^S|(\mu^2 = 4m_b^2) = (18.6 \pm 1.9)\,,\text{MeV}\,, \quad \text{(E.5)}$$

where from, for the asymptotic DA, one finds

$$f_g^S(\mu^2 = 4m_b^2) = (14.9 \pm 0.8)\,\text{MeV}. \quad \text{(E.6)}$$

Here we tacitly assumed that this coupling is positive (with respect to f_q), as suggested by the QCD sum rule analysis in appendix D. The given error bar reflects experimental uncertainties only. The theoretical uncertainties are much larger so that we estimate the overall accuracy of the value in (E.6) as 30–50%.

This result appears to support an intuitive picture that the gluon coupling of "ordinary" quark-antiquark mesons is very small at hadronic scales and is generated entirely by the evolution. Indeed, assuming $f_q(1\,\text{GeV}) = 101(10)\,\text{MeV}$ and $f_g^S(1\,\text{GeV}){=}0$ and using the expressions collected in appendix B one finds

$$f_g^S(\mu^2 = 4m_b^2) = (25 \pm 3)\,\text{MeV}. \tag{E.7}$$

This number is in a reasonable agreement with the above extraction from the bottomonium radiative decay having in mind the theoretical uncertainties.

References

[1] W. Wang, *B to tensor meson form factors in the perturbative QCD approach*, Phys. Rev. **D 83** (2011) 014008 [arXiv:1008.5326] [INSPIRE].

[2] K.-C. Yang, *B to light tensor meson form factors derived from light-cone sum rules*, Phys. Lett. **B 695** (2011) 444 [arXiv:1010.2944] [INSPIRE].

[3] H.-Y. Cheng and K.-C. Yang, *Charmless hadronic B decays into a tensor meson*, Phys. Rev. **D 83** (2011) 034001 [arXiv:1010.3309] [INSPIRE].

[4] R.-H. Li, C.-D. Lu and W. Wang, *Branching ratios, forward-backward asymmetries and angular distributions of $B \to K_2^* l^+ l^-$ in the standard model and new physics scenarios*, Phys. Rev. **D 83** (2011) 034034 [arXiv:1012.2129] [INSPIRE].

[5] C.-D. Lu and W. Wang, *Analysis of $B \to K_J^*(\to K\pi)\mu^+\mu^-$ in the higher kaon resonance region*, Phys. Rev. **D 85** (2012) 034014 [arXiv:1111.1513] [INSPIRE].

[6] Z.-T. Zou, X. Yu and C.-D. Lu, *The $B_c \to D^{(*)}T$ decays in perturbative QCD approach*, Phys. Rev. **D 87** (2013) 074027 [arXiv:1208.4252] [INSPIRE].

[7] Belle collaboration, M. Masuda et al., *Study of π^0 pair production in single-tag two-photon collisions*, Phys. Rev. **D 93** (2016) 032003 [arXiv:1508.06757] [INSPIRE].

[8] G.A. Schuler, F.A. Berends and R. van Gulik, *Meson photon transition form-factors and resonance cross-sections in e^+e^- collisions*, Nucl. Phys. **B 523** (1998) 423 [hep-ph/9710462] [INSPIRE].

[9] V.M. Braun and N. Kivel, *Hard exclusive production of tensor mesons*, Phys. Lett. **B 501** (2001) 48 [hep-ph/0012220] [INSPIRE].

[10] V. Pascalutsa, V. Pauk and M. Vanderhaeghen, *Light-by-light scattering sum rules constraining meson transition form factors*, Phys. Rev. **D 85** (2012) 116001 [arXiv:1204.0740] [INSPIRE].

[11] N.N. Achasov, A.V. Kiselev and G.N. Shestakov, *Study of the $f_2(1270)$ and $a_2(1320)$ resonances in $\gamma^*(Q^2)\gamma$ collisions*, JETP Lett. **102** (2015) 571 [arXiv:1509.09150] [INSPIRE].

JHEP06(2016)039

[12] M. Diehl, T. Gousset, B. Pire and O. Teryaev, *Probing partonic structure in $\gamma^*\gamma \to \pi\pi$ near threshold*, Phys. Rev. Lett. **81** (1998) 1782 [hep-ph/9805380] [INSPIRE].

[13] N. Kivel, L. Mankiewicz and M.V. Polyakov, *NLO corrections and contribution of a tensor gluon operator to the process $\gamma^*\gamma \to \pi\pi$*, Phys. Lett. **B 467** (1999) 263 [hep-ph/9908334] [INSPIRE].

[14] M. Diehl, T. Gousset and B. Pire, *Exclusive production of pion pairs in $\gamma^*\gamma$ collisions at large Q^2*, Phys. Rev. **D 62** (2000) 073014 [hep-ph/0003233] [INSPIRE].

[15] V.L. Chernyak and A.R. Zhitnitsky, *Asymptotic behavior of hadron form-factors in quark model* (in Russian), *JETP Lett.* **25** (1977) 510 [*Pisma Zh. Eksp. Teor. Fiz.* 544 **25** (1977)]. [INSPIRE].

[16] A.V. Efremov and A.V. Radyushkin, *Factorization and asymptotical behavior of pion form-factor in QCD*, Phys. Lett. **B 94** (1980) 245 [INSPIRE].

[17] G.P. Lepage and S.J. Brodsky, *Exclusive processes in perturbative quantum chromodynamics*, Phys. Rev. **D 22** (1980) 2157 [INSPIRE].

[18] BaBar collaboration, B. Aubert et al., *Measurement of the $\gamma\gamma* \to \pi^0$ transition form factor*, Phys. Rev. **D 80** (2009) 052002 [arXiv:0905.4778] [INSPIRE].

[19] BaBar collaboration, P. del Amo Sanchez et al., *Measurement of the $\gamma\gamma^* \to \eta$ and $\gamma\gamma* \to \eta'$ transition form factors*, Phys. Rev. **D 84** (2011) 052001 [arXiv:1101.1142] [INSPIRE].

[20] Belle collaboration, S. Uehara et al., *Measurement of $\gamma\gamma^* \to \pi^0$ transition form factor at Belle*, Phys. Rev. **D 86** (2012) 092007 [arXiv:1205.3249] [INSPIRE].

[21] Particle Data Group collaboration, K.A. Olive et al., *Review of particle physics*, Chin. Phys. **C 38** (2014) 090001 [INSPIRE].

[22] D.-M. Li, H. Yu and Q.-X. Shen, *Properties of the tensor mesons $f_2(1270)$ and $f_2'(1525)$*, J. Phys. **G 27** (2001) 807 [hep-ph/0010342] [INSPIRE].

[23] H.-Y. Cheng, Y. Koike and K.-C. Yang, *Two-parton light-cone distribution amplitudes of tensor mesons*, Phys. Rev. **D 82** (2010) 054019 [arXiv:1007.3541] [INSPIRE].

[24] T.M. Aliev and M.A. Shifman, *Old tensor mesons in QCD sum rules*, Phys. Lett. **B 112** (1982) 401 [INSPIRE].

[25] T.M. Aliev and M.A. Shifman, *QCD sum rules and tensor mesons* (in Russian), *Sov. J. Nucl. Phys.* **36** (1982) 891 [*Yad. Fiz.* **36** (1982) 1532] [INSPIRE].

[26] H. Terazawa, *f_2 dominance of the energy momentum tensor*, Phys. Lett. **B 246** (1990) 503 [INSPIRE].

[27] M. Suzuki, *Tensor meson dominance: phenomenology of the f_2 meson*, Phys. Rev. **D 47** (1993) 1043 [INSPIRE].

[28] P. Ball, V.M. Braun, Y. Koike and K. Tanaka, *Higher twist distribution amplitudes of vector mesons in QCD: formalism and twist-three distributions*, Nucl. Phys. **B 529** (1998) 323 [hep-ph/9802299] [INSPIRE].

[29] V.M. Braun, G.P. Korchemsky and D. Mueller, *The uses of conformal symmetry in QCD*, Prog. Part. Nucl. Phys. **51** (2003) 311 [hep-ph/0306057] [INSPIRE].

[30] P. Ball, V.M. Braun and A. Lenz, *Twist-4 distribution amplitudes of the K^* and ϕ mesons in QCD*, JHEP **08** (2007) 090 [arXiv:0707.1201] [INSPIRE].

JHEP06(2016)039

[31] P. Ball and V.M. Braun, *Higher twist distribution amplitudes of vector mesons in QCD: twist-4 distributions and meson mass corrections*, Nucl. Phys. **B 543** (1999) 201 [hep-ph/9810475] [INSPIRE].

[32] A. Khodjamirian, *Form-factors of $\gamma^*\rho \to \pi$ and $\gamma^*\gamma \to \pi^0$ transitions and light cone sum rules*, Eur. Phys. J. **C 6** (1999) 477 [hep-ph/9712451] [INSPIRE].

[33] S.S. Agaev, V.M. Braun, N. Offen and F.A. Porkert, *Light cone sum rules for the π^0-γ^*-γ form factor revisited*, Phys. Rev. **D 83** (2011) 054020 [arXiv:1012.4671] [INSPIRE].

[34] A.P. Bakulev, S.V. Mikhailov, A.V. Pimikov and N.G. Stefanis, *Pion-photon transition: the new QCD frontier*, Phys. Rev. **D 84** (2011) 034014 [arXiv:1105.2753] [INSPIRE].

[35] N. Kivel and L. Mankiewicz, *Power corrections to the process $\gamma\gamma^* \to \pi\pi$ in the light cone sum rules approach*, Eur. Phys. J. **C 18** (2000) 107 [hep-ph/0008168] [INSPIRE].

[36] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *QCD and resonance physics. Theoretical foundations*, Nucl. Phys. **B 147** (1979) 385 [INSPIRE].

[37] I.I. Balitsky, V.M. Braun and A.V. Kolesnichenko, *Radiative decay $\Sigma^+ \to p\gamma$ in quantum chromodynamics*, Nucl. Phys. **B 312** (1989) 509 [INSPIRE].

[38] BaBar collaboration, B. Aubert et al., *Measurement of the eta and eta-prime transition form-factors at $Q^2 = 112\,GeV^2$*, Phys. Rev. **D 74** (2006) 012002 [hep-ex/0605018] [INSPIRE].

[39] Belle collaboration, S. Uehara et al., *High-statistics measurement of neutral pion-pair production in two-photon collisions*, Phys. Rev. **D 78** (2008) 052004 [arXiv:0805.3387] [INSPIRE].

[40] D.J. Gross and F. Wilczek, *Asymptotically free gauge theories. 1*, Phys. Rev. **D 8** (1973) 3633 [INSPIRE].

[41] H. Georgi and H.D. Politzer, *Electroproduction scaling in an asymptotically free theory of strong interactions*, Phys. Rev. **D 9** (1974) 416 [INSPIRE].

[42] D.J. Gross and F. Wilczek, *Asymptotically free gauge theories. 2*, Phys. Rev. **D 9** (1974) 980 [INSPIRE].

[43] P. Hoodbhoy and X.-D. Ji, *Helicity flip off forward parton distributions of the nucleon*, Phys. Rev. **D 58** (1998) 054006 [hep-ph/9801369] [INSPIRE].

[44] P. Ball and G.W. Jones, *Twist-3 distribution amplitudes of K^* and ϕ mesons*, JHEP **03** (2007) 069 [hep-ph/0702100] [INSPIRE].

[45] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Are all hadrons alike?* Nucl. Phys. **B 191** (1981) 301 [INSPIRE].

[46] E. Gregory et al., *Towards the glueball spectrum from unquenched lattice QCD*, JHEP **10** (2012) 170 [arXiv:1208.1858] [INSPIRE].

[47] V.N. Baier and A.G. Grozin, *Certain exclusive processes in QCD taking into account two-gluon states*, Sov. J. Nucl. Phys. **35** (1982) 596 [*Yad. Fiz.* **35** (1982) 1021] [INSPIRE].

[48] V.N. Baier and A.G. Grozin, *Gluonic contributions to the exclusive amplitudes*, Z. Phys. **C 29** (1985) 161 [INSPIRE].

[49] S. Fleming, C. Lee and A.K. Leibovich, *Exclusive radiative decays of Υ in SCET*, Phys. Rev. **D 71** (2005) 074002 [hep-ph/0411180] [INSPIRE].

JHEP06(2016)039