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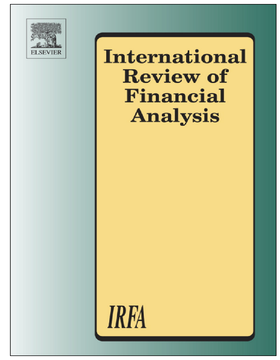

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Abstract

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JEL Classification: C14, C22, Q02, Q31

Keywords: ARFIMA, Fractional Integration, Long-Memory, Forecasting, Rare Earth Elements

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1. Introduction

Rare earth elements (REEs), the fifteen lanthanides plus scandium and yttrium, are critical components in many high- and green-technology products (Binnemans et al., 2013; Van Gosen et al., 2014; Apergis and Apergis, 2017). In addition to wide-ranging industrial applications, such as wind energy turbines and photovoltaic cells, REEs are key for consumer products ranging from mobile phones and CD, DVD, and hard disk drives, to hybrid and electric cars (Müller, Schweizer and Seiler, 2016). There are virtually no suitable substitutes for REEs. Combined with the heavy dependence on China as their main supplier (well over 90% of global REE supply originates in China; see Shih et al., 2012), this has induced massive price movements in the past, and will presumably do so in the future as well (see Figure 1).

While the volatility of commodities is comparable to that of stocks (see Proelss and Schweizer, 2008), there are hedging instruments available for commodities, such as futures and options markets, that can be used for active risk management purposes (Doran and Ronn, 2008). The lack of such instruments for REEs poses a serious challenge for (heavily) dependent industries (see Shen, 2014, for details about vague plans to establish REEs futures). Currently, trading of REEs between supply- and buy-side firms occurs over the counter (OTC).[1] Unlike other metals, physical settlement is the norm. Accordingly, the only methods user industries have to shield themselves from price movements are building strategic stockpiles of REEs, or entering into long-term contracts (Shih et al., 2012).

Therefore, REE price volatility and availability for consumer industries can ultimately have severe effects, as illustrated by the following examples:

[1] In 2014, the Baotou Rare Earth Products Exchange was established as a spot exchange in Baotou, China (see Bloomberg News, 2014, for further details). However, because the majority of trading remains OTC, this exchange has thus far played only a minor role in the industry (personal communication with Dr. Harald Elsner, Bundesanstalt für Geowissenschaften und Rohstoffe (BGR), i.e., Federal Institute for Geosciences and Natural Resources, 4 May 2017).

One major application of REEs is permanent magnets, which are needed to convert torque into electricity and vice versa. Permanent magnets are used in such essential industries as wind energy turbines,[2] electric and hybrid cars, and computer equipment, including DVD and hard disk drives. In 2011, in the U.S., many IT companies experienced huge reductions in their gross margins due to REE price increases in the July-September 2011 period. For example, Western Digital Corporation's decline reached 21%, while Seagate Technology's was 37% (Monahan, 2012). Moreover, while the sharp price increases in 2011 negatively affected industries that depend on REEs as inputs in their production processes, the subsequent downward trend also spelled disaster for Molycorp, one of the few major suppliers of REEs outside China. Molycorp was forced to file for Chapter 11 bankruptcy because its business could not sustain profitability at the drastically decreased prices (McCarty and Casey, 2015; Reuters, 2015). Thus, it is clear that companies active in REE markets (whether on the supply or demand side) need to reevaluate their risk management strategies for handling REE pricing risk.

Driven by the large REE price movements and their potentially severe impact, the German Bundesanstalt für Geowissenschaften und Rohstoffe (BGR) launched a volatility monitor in 2014 (BGR, 2014). Each month, this monitor reports the annualized standard deviations of monthly commodity returns, including several REEs. However, for practitioners and policy makers concerned with strategic metals and REEs, it is more valuable to have an ex ante predictor of future volatility rather than an ex post evaluation of it. This would allow REE-dependent companies to monitor volatility forecasts more effectively and proactively manage risk by, for example, stockpiling REEs.

However, determining a suitable predictor for future REE volatility is complex. In markets where options are available and actively traded, implied volatility is commonly agreed to

[2] See Shih et al. (2012) for an overview of the implications for the U.S. wind energy industry.

be suitable (Jiang and Tian, 2005; Xingguo, Shihua and Ye, 2016).[3] The most widely used method of modeling financial time series volatility is presumably a GARCH-type model (Bollerslev, 1986), where today's volatility depends on past realizations (typically short-term modeling). Nevertheless, GARCH-type models have been found less suitable for long-memory volatility, which is arguably the case for REEs. This is because of their distinct demand-side cyclicality and inelastic short-term supply side. Adjusting capacity in the mineral industry comes with considerable costs and ramp-up time due to long lags in exploration and capital formation (Barkoulas, Labys and Onochie, 1997; Labys, 2006).

Furthermore, REE supply is heavily concentrated in China, and has traditionally been controlled directly by the Chinese Ministry of Commerce, or indirectly by setting environmental standards (see Hayes-Labruto et al., 2013; Nieto, Guelly, and Kleit, 2013; Müller, Schweizer, and Seiler, 2016; Zhang, Kleit, and Nieto, 2017; Proelss, Schweizer, and Seiler, 2018; Mancheri et al., 2019). This clearly adds to the inelasticity of the supply side. Moreover, demand for minerals is generally inelastic, too. Because they are used as intermediate inputs, their cost constitutes only a small fraction of the overall price of the final goods. However, they are nevertheless essential to the end product.

REEs are no exception. If business cycles exhibit long memories, this is likely to influence commodity demand, and translate into the long-run dependence of commodity price series themselves (see Barkoulas, Labys and Onochie, 1997). Because supply and demand forces are especially important for spot markets, long-memory models seem especially useful here (see again Barkoulas, Labys and Onochie, 1997).

[3] See Brous, Ince and Popova, 2010, for opposing evidence. Moreover, Chernov (2007) shows that implied volatility inferred from at-the-money options is an inefficient and biased forecast of future realized volatility. Furthermore, Fernandez-Perez, Fuertes and Miffre (2016) show that idiosyncratic volatility seems to be negatively priced when using pricing models that do not account for backwardation and contango in commodity futures markets.

Our contributions are as follows: First, given the strategic importance of REEs, their extreme price movements, the lack of hedging instruments, and the potential of the long-memory of volatility, we formally test for fractional integration by applying Sowell's (1992) maximum likelihood estimator to the simplest ARFIMA$(0, d, 0)$ (autoregressive fractionally integrated moving average) baseline model. We find strong evidence for the presence of long-memory for both individual REEs and two REEs indices. This result is also supported by using the non-parametric log periodogram regression approach of Geweke and Porter-Hudak (1983) and Phillips (1999, 2007). When comparing the ARFIMA$(0, d, 0)$ baseline model with specifications that include additional autoregressive (AR) and moving average (MA) terms, the likelihood ratio test implies the baseline model is most suitable for describing REE volatility.

Second, we conduct extensive robustness checks, and find that our results remain robust for most REEs for various in- and out-of-sample periods and for most model specifications. Overall, our findings imply that ARFIMA$(0, d, 0)$-generated volatility forecasts for REEs that explicitly allow for long-memory characteristics generally *outperform* traditional ARMA forecasts, as well as forecasts generated by a GARCH(1,1) model.[4]

Third, we note that the outperformance of long-memory volatility models vis-à-vis short-memory models with regard to forecasting accuracy has been documented elsewhere (see, e.g., Chortareas, Jiang and Nankervis, 2011; Harris and Nguyen, 2013). Therefore, we go one step further. We aim to determine whether volatility forecasts based on the ARFIMA$(0, d, 0)$ baseline model convey material or economically meaningful information about Chinese publicly listed companies[5] in the REE market in the absence of, e.g., forward-looking implied volatilities, and

[4] Due to the absence of intraday data, we are not able to use advanced approaches for modelling long-memory such as realized volatilities, especially the heterogeneous autoregressive (HAR) model and its extensions (Andersen, Bollerslev and Diebold, 2007; Corsi, 2009 Bollerslev, Patton and Quaedvlieg, 2016).

[5] Because there is no electronic REE trading, we extrapolate from the direct development of a trading strategy based on the metals themselves. Our design allows us to show that information concerning expected REE volatility inferred from these metals is economically meaningful for companies active in this market. To ensure a clean research setup,

thus less consensus about expected REE volatility. If so, we expect *substantial* changes in forecasted volatility, versus actual REE volatility, to predict the direction of future price developments of REE companies. If the volatility forecasts of the ARFIMA$(0, d, 0)$ baseline model can predict future stock price movements, then an active trading strategy based on this prediction may be able to systematically outperform a passive buy-and-hold strategy on a return- and risk-adjusted basis.

We compare Sharpe ratios for an active trading strategy with those for a naïve buy-and-hold strategy. We find statistically significantly higher Sharpe ratios for most specifications of our active trading strategy (see Harris and Nguyen, 2013, for the suggestion to evaluate portfolios based on ARFIMA forecasts with the help of Sharpe ratios). This result is in line with Li, Nishimura and Men (2016) who find that, in the majority of cases, a trading strategy based on long-memory forecasts of NYMEX futures, and implemented with the help of binary options, will produce higher Sharpe ratios than those of a moving average or momentum strategy.[6] We interpret this result as strong support for the notion that volatility forecasts convey material information about Chinese companies in the REE industry, and are therefore of strong economic importance.

Fourth, our paper touches the fields of finance and natural resources, and thus contributes to the evolving literature in the field of resources finance (Lucey et al., 2018).

The remainder of this paper is structured as follows. Section 2 provides an overview of the methodology used to study long-memory processes. Section 3 describes our data in-depth. In section 4, we present the empirical results, and section 5 concludes.

– Please insert Figure 1 about here –

and avoid any influence from currency fluctuations, we use Chinese companies only, so that both the companies and the REEs are traded in renminbi.

[6] However, the trading strategy of Li, Nishimura and Men (2016) was not profitable after accounting for trading costs.

2. Long-Memory Processes

Long-memory processes are generally characterized by persistent and hyperbolically decaying autocorrelations, rather than by the exponentially decaying autocorrelations seen with autoregressive integrated moving average (ARIMA) processes. Technically speaking, these slowly decaying autocorrelations are due neither to processes with an order of integration of 1 ($I(1)$), nor 0 ($I(0)$). In other words, long-memory time series are neither stationary $I(0)$ nor unit root processes $I(1)$. Instead, Mandelbrot (1977) characterizes them as fractal, giving rise to the class of ARFIMA processes. The restriction of the differencing parameter d to integer values yields the ARIMA model. Accordingly, the ARFIMA class of models is a generalization of the latter.

Formally, an ARFIMA(p, d, q) model introduced by Granger and Joyeux (1980) and Hosking (1981) for some time series X_t (in our case, a measure of REE volatility) is given as:

$$\Phi(L)(1-L)^d(X_t-\mu)=\Theta(L)\epsilon_t, \quad \epsilon_t\sim iid(0,\sigma_\epsilon^2), \tag{1}$$

where L is the lag (or backshift) operator, μ is the time series mean, and d is the fractional differencing operator that is allowed to assume any real value. $(1-L)^d$ is then defined by:

$$(1-L)^d=\sum_{k=0}^{\infty}\frac{\Gamma(k-d)L^k}{\Gamma(-d)\Gamma(k+1)}, \tag{2}$$

where Γ is the gamma function.

With regard to the fractional differencing parameter, the larger d is, the higher the degree of long-memory. More precisely, if $-0.5<d<0$, the process will exhibit intermediate memory, i.e., long-range negative dependence, and will be said to be anti-persistent (Mandelbrot, 1977). If $d=0$, the process will exhibit only short-memory, and will correspond to the standard ARMA model. For $d=0.5$, the process is discrete-time $1/f$ noise (Mandelbrot, 1967). For $0.5<d<1$, the process is mean-reverting. If $d = 1$, the process is integrated of order 1, i.e., a unit root process, and corresponds to the standard ARIMA model (Baillie, 1996).

Several procedures have been proposed to estimate the degree of long-memory and fractional integration, such as log periodogram regressions (Geweke and Porter-Hudak, 1983; Phillips, 1999, 2007), and maximum likelihood estimation (Sowell, 1992). Phillips (1999, 2007) extends the original non-parametric approach of Geweke and Porter-Hudak (1983) to the unit root null. However, Sowell (1992) proposes a maximum likelihood estimator of ARFIMA(p, d, q) models that explicitly allows for handling short-run dependence via AR and MA parameters. Thus, we can directly compare them with other parametric approaches, such as traditional short-memory ARMA and GARCH-type models (Bollerslev, 1986). We base our results on the maximum likelihood procedure, and use the semi-parametric approaches in a robustness check.

3. Data

3.1 REE Data

All REE (price) data come from the Asian Metal database, the leading data provider in the REE field, and cover the January 28, 2005-February 28, 2015 period. The Asian Metal database is used by major research institutions such as the Federal Institute for Geosciences and Natural Resources (Bundesanstalt für Geowissenschaften und Rohstoffe, BGR), which is the central geoscientific authority providing advice to the German federal government on geo-relevant questions.

The Chinese government has essentially established a dual pricing system: Companies based in China pay the domestic price, while buyers outside China pay the significantly higher FOB (freight on board) price, i.e., the export price (see Müller, Schweizer and Seiler, 2016, and Proelss, Schweizer and Seiler, 2018, for further details on the dual pricing structure in the REE market, and Charlier and Guillou, 2014, for details on the dual pricing structure of various other raw materials). For our analysis, we focus exclusively on listed Chinese companies and the domestic China prices (RMB/kg), so as to avoid any influence of exchange rate effects. Moreover, an exclusive focus on companies in one particular country eliminates confounding effects due to institutional factors, such as different legal systems or differences in corporate governance. This is especially important given the unique features and dynamic nature of the Chinese capital market.

In order to ensure sufficient observations in the (in-sample) estimation, as well as during the forecast (out-of-sample) period, we restrict our analysis to the four largest REE oxides in terms of market share (Goonan, 2011). Together, they account for about 90% of overall usage: cerium (42,220 metric tons/32.94%), lanthanum (38,665 metric tons/30.16%), neodymium (22,868 metric tons/17.84%), and yttrium (11,610 metric tons/9.06%). We also mandate that data be available in Asian Metal since at least January 2005.

Based on these usage statistics, we construct an equally weighted index (EWI), as the mean of weekly returns, and a value-weighted index (VWI). Moreover, we follow Ghysels, Santa-Clara and Valkanov (2006) and Forsberg and Ghysels (2007), and use absolute REE returns to proxy for volatility.[7] This choice is motivated by the fact that the influence of outliers is reduced with absolute returns, while squaring only amplifies that impact (Brous, Ince and Popova, 2010). For a recent application of absolute returns as a volatility proxy, see Fernandez (2010).

3.2 Chinese Stock Price Data

[7] Alternatively, we could use range-based volatility estimators, which are more accurate than squared returns due to less noise (Molnár, 2012). Because the Asian Metal database does not report open or closing prices, we are not able to use alternative volatility proxies such as that of Garman and Klass (1980).

We obtain the stock price data used in the trading strategy for all Chinese firms listed on the Shanghai, Shenzhen, and Growth Enterprise Market (GEM) stock exchanges from the China Stock Market Trading Database (CSMAR). CSMAR is the leading data provider for Chinese stock price information. The data series in this study covers the same sample period, January 28, 2005-February 28, 2015. Note that our volatility forecast model requires 300 weeks of continuing REE data to be calibrated. Thus, the earliest date our trading models can begin is June 3, 2011 (= January 28, 2005 + 300 weeks).[8]

If an REE or a company's stock is not traded for more than twelve consecutive days, we set the return to "Not Available" in order to avoid high price/return jumps, which could influence the results. Because of lower REE liquidity during the 2005-2007 period, and several stock market holidays (such as, e.g., lunar year celebrations), an average trading year in China for the 2005-2015 period contains forty-eight trading weeks.

4. Results

Due to the previously discussed features of the REE market (distinct cyclicality, inelastic short-term supply and demand, and governmental regulations), we expect REE volatility to exhibit long-memory, i.e., fractional integration. In subsection 4.1, we provide first evidence by evaluating the descriptive statistics of REE absolute returns and their autocorrelation structure. Next, we formally test for fractional integration (see Sowell, 1992) for the entire sample period, and compare the baseline model against more complex ARFIMA(p, d, q) specifications and a standard GARCH model in an in-sample setting. In subsection 4.2, we investigate the out-of-sample forecasting performance of our ARFIMA$(0, d, 0)$ baseline model against alternative parameterizations. Subsection 4.3 gives a short summary of our most important findings. To test

[8] In a robustness check, we also test our trading strategy by using a period beginning in January 2008 that includes both a bull and a bear market. We then estimate a volatility forecast model with a shorter calibration period of 100 weeks of REE volatility (see Figure A3 and Tables A2, A4, and A6 in the online appendix).

the economic significance of our baseline model forecasts, we provide results for a simple trading strategy based on a rolling one-step-ahead out-of-sample volatility forecast in subsection 4.4.

4.1 Long-Memory of Volatility

Figure 1, panel A, shows the price development of the four REE oxides; panel B shows the two REE indices over time, as well as the development of the absolute returns, which we use as our volatility measure (Ghysels, Santa-Clara and Valkanov, 2006; Forsberg and Ghysels, 2007). REEs exhibit phases of strong and persistent volatility clusters, especially around the price run-up in 2011, indicating the importance of an ARFIMA(p, d, q) model in modeling REE volatility. No structural breaks are apparent in the absolute return series.[9]

Table 1 presents the related descriptive statistics. Our volatility measure is right-skewed and leptokurtic for both the four individual REEs and the two indices. This finding of deviations from the normal distribution due to skewness and leptokurtosis indicates non-linear dynamics (Fang, Lai and Lai, 1994), another hint of potential long-memory. The Ljung-Box (1978) test up to lag 20 and the magnitudes of the autocorrelation coefficients up to lag 5 exhibit a statistically significant serial correlation structure for the individual REEs and the REE indices. Accordingly, a simple short-memory ARFIMA($p, 0, q$) model specification would make it necessary to incorporate many AR terms, because autocorrelations at even higher lags are statistically significant. However, including a higher number of AR terms can increase the risk of "over-parameterization." It also violates the principle of parsimony (McLeod, 1993), which is particularly important when it comes to time series forecasting (Ledolter and Abraham, 1981). Hence, these rather simple metrics indicate that a long-memory model could be better suited to capture the slowly decaying autocorrelation structure.

– Please insert Table 1 about here –

[9] See Caporale and Gil-Alana (2013) for the use of absolute returns as a volatility proxy for the USD/GBP spot exchange rate, and visual inspection of the time series plots with regard to structural breaks. Note that, because we complement our analysis by using rolling estimates, we allow for the presence of structural breaks (Fernandez, 2010).

This first impression of potential long-term dependence is formally supported by the findings of Sowell's (1992) long-memory test: The degree of fractional integration (d) of the volatility series falls into the [0.2,0.4] interval, and is statistically significant (see columns "(0,d,0)" in Table 2).[10] Adding AR and MA terms to explicitly capture short-term influences to the ARFIMA$(0, d, 0)$ baseline model does not change our findings (see all columns "(1,d,0)," "(2,d,0)," "(1,d,1)," and "(2,d,1)" for rows "d" in Table 2). For all parameterizations, we find a statistically and economically significant degree of long-memory, except for neodymium, yttrium, and lanthanum when including two autoregressive terms. Moreover, comparing the baseline model with higher-order parameterizations using a likelihood ratio test (see rows "LR" in Table 2),[11] we find that adding AR or MA terms does not result in statistically significant improvements for either single REEs or for the REE indices. This finding is further underlined by the AIC (Akaike, 1974) and BIC (Schwarz, 1978) information criteria. The only exception is yttrium, for which short-memory AR and MA components should be included.

The different behavior of yttrium might be explained by the fact that it is a heavy rare earth element. This differentiation between light (cerium group, elements with atomic numbers 57-63) and heavy (yttrium group, elements with atomic numbers 64-71 plus yttrium) REEs is important, as light REEs are more abundant and concentrated and usually account for 80% to 99% of a given mineral deposit (see Humphries, 2010). Yttrium is extracted primarily from

[10] To check for robustness, we also apply the non-parametric estimators of Geweke and Porter-Hudak (1983) and Phillips (1999, 2007) using periodogram ordinates in the spectral regressions equal to $T^{0.5}$ (Diebold and Rudebusch, 1989, and Cheung, 1993). The results are similar, except for neodymium ($d_{GPH} = 0.5940$, $d_{Phil} = 0.5964$) and the value-weighted index ($d_{GPH} = 0.5761$, $d_{Phil} = 0.5696$). Moreover, for cerium and lanthanum, the semi-parametric estimators $d_{GPH}^{Cerium} = 0.2634$, $d_{GPH}^{Lanthanum} = 0.2240$, and $d_{Phil}^{Lanthanum} = 0.2662$ are economically meaningful, but not statistically significant (see Table A1 in the online appendix). In unreported results, we test the stability of the non-parametric estimators of Geweke and Porter-Hudak (1983) and Phillips (1999, 2007) by using a range of periodogram ordinates in the spectral regressions, from $T^{0.4}$ to $T^{0.8}$, with a step size of 0.05. The results indicate that the non-parametric tests provide results very similar to Sowell's (1992) maximum likelihood approach for $T^{0.6}$ and higher. The table is available from the authors upon request.

[11] The test statistic of the likelihood ratio is defined as $LR = -2(L_1 - L_0)$, where L_0 and L_1 are the log-likelihoods of the full and constrained model. The LR statistic is approximately χ^2 distributed.

Xenotime (Hedrick, 2004, and Hurst, 2010), which is only available in a few mines and at relatively low concentration. This also holds for planned mineral mining projects (TMR, 2015).

Overall, the availability and the difficulties of the yttrium supply will most likely also impact yttrium's price formation, making it more sensitive to supply-side news. Therefore, it seems plausible that the demand side will cause a long-memory effect, as is the case for the other REEs, too. However, the difference in the supply side may also cause overlying short-term effects.[12] Our intermediary result for the entire sample period is that the baseline model – ARFIMA$(0, d, 0)$ – is the most suitable for single REEs and the related indices, except for yttrium.

– Please insert Table 2 about here –

Next, we examine the stability of the long-memory parameter over time using various subperiods (see, e.g., Cajueiro and Tabak, 2005; Tabak and Cajueiro, 2007; Cajueiro, Gogas and Tabak, 2009; Hull and McGroarty, 2014;, and Auer, 2016, for time-varying long-memory effects). We estimate d and the AR and MA terms on a 300 (T)-week rolling basis (moving forward on a weekly basis until the end of the observation period) for different ARFIMA(p, d, q) parameterizations (see Figure A1 in the online appendix for more details about the rolling window). More precisely, for the first subperiod, we use data from $t = 1, \ldots, T = 300$, with a window width T, to estimate the ARFIMA(p, d, q) models. Setting T equal to 300 (weeks) is arguably a good compromise. We therefore have a sufficiently long in-sample estimation period for the long-memory coefficient (d), and a sufficient number of out-of-sample weeks to meaningfully evaluate the out-of-sample forecasting performance in subsection 4.2. This window is moved ahead one time period (one week), and the models are re-estimated using data from $t =$

[12] This view is supported by Apergis and Apergis (2017) using a cointegration approach to analyze the impact of REE prices on renewable energy consumption. They find that neodymium and yttrium have the highest adjustment coefficients, so shocks do not last long and instead return to equilibrium quickly, which is similar to what we observe.

$2, \ldots, T+1$ until $t = 301, \ldots, T+189$, which is the end of our time series at $t = 489$. This gives us a total of 189 subsamples, with different in-sample periods for estimating the models as well as different out-of-sample periods.[13]

A first visual inspection of the fractional integration term d over time for our ARFIMA$(0, d, 0)$ baseline model in Figure 2 shows no severe fluctuations or jumps. For all REEs and REE indices, the fractional integration term falls in the [0.2,0.5] interval for the ARFIMA$(0, d, 0)$ specification. However, although the fractional differencing parameter is quite stable over time for the ARFIMA$(0, d, 0)$ baseline model and the ARFIMA$(0, d, 1)$ model, adding an additional AR term results in large fluctuations in d for all REEs and the two related indices (see the ARFIMA$(1, d, 1)$ model in Figure 2).[14] We observe similar behavior for lanthanum for the ARFIMA$(1, d, 0)$ model in later periods ($t > 140$). Thus, it seems that including additional AR and MA terms does not necessarily cause model improvements but might instead result in parameter instability.

– Please insert Figure 2 about here –

To more formally test the stability of the fractional integration term d, Table 3 replicates Table 2 for five subperiods (t =1, 48, 95, 142, and 189) instead of the entire period. Overall, and in line with the visual inspection in Figure 2, we find that fractional integration for the baseline model is statistically significant for all REEs and indices for all shown subperiods, with only minimal variations. Similarly to Figure 2, it seems that the ARFIMA$(1, d, 1)$ model is not stable over time. This results in an insignificant fractional integration term that also changes sign in the

[13] In unreported results, we find that our results for the degree of fractional integration and for the in-sample performance of our baseline model are not distorted when we use a 100 (T)-week rolling basis instead of the 300. However, we do find, on average, that the baseline model for the 100 (T)-week rolling basis is less dominant than for the 300-day observation period, because it seems that a longer observation period is needed for long-term memory models. Most importantly, the out of-sample performance remains intact for the 100-day estimation window. See Figure A2 in the online appendix. Additional tables are available from the authors upon request.

[14] Note that ARFIMA$(1, d, 1)$ cannot be estimated for lanthanum, neodymium, or the VWI for some windows, because the roots of the MA polynomial are approaching the unit circle. This further supports our argument that, for REE data, a simpler model is preferable to a more complex model.

later subperiods $t > 142$ covering 300 subsequent weeks, which translates to early January 2008 through the end of our sample and includes the most severe REE price jumps in history. These sharp price increases occurred when the Chinese Ministry of Commerce, also known as MOFCOM, heavily intervened in the REE market by setting export quotas and only allowing certain companies (so-called qualified export enterprises) to export REEs (see Müller, Schweizer and Seiler, 2016, for an overview).

Given such politically motivated interventions in the REE market, which presumably caused some short- to mid-term disruptions, we further extend the comparison to different ARMA (ARFIMA$(p, 0, q)$) specifications and a GARCH model to capture whether this effect is present in the data. Neither model type has a long-memory component, but both can account for relatively short-term influences. In detail, as per Gallant, Hsu and Tauchen (1999) and Alizadeh, Brandt and Diebold (2002), we consider ARFIMA(1,0,0) and ARFIMA(2,0,1) parameterizations (see also Pong et al., 2004), and the presumably most widely used GARCH(1,1) model (see Bollerslev, 1986; Bollerslev, Chou and Kroner, 1992; Engle, 2001).

To effectively judge the model choice, we calculate the AIC and BIC for each of the 189 different rolling 300-week windows for all models and specifications. We then calculate the percentage to which the ARFIMA$(0, d, 0)$ baseline model outperforms the other models with regard to the information criteria (see "$\%_{AIC}$" and "$\%_{BIC}$" in the three bottom rows in Table 3).

To summarize, the ARFIMA$(0, d, 0)$ baseline model is equal to or better than the more complex ARFIMA(p, d, q) specifications in at least 97% (86%) of all window iterations measured by BIC (AIC) for cerium, neodymium, and the two REE indices, and in 80% (31%) for lanthanum. For yttrium, we continue to find that the ARFIMA$(0, d, 0)$ specification is equal to or better than ARFIMA$(1, d, 0)$ and ARFIMA$(0, d, 1)$ in at least 50% of the iterations measured by

BIC, but not for ARFIMA$(1, d, 1)$. Similarly to our prior findings, the baseline model is subpar for modeling yttrium's volatility structure.

We next compare the ARFIMA$(0, d, 0)$ baseline model with the two pure short-memory models, ARFIMA$(1,0,1)$ and ARFIMA$(2,0,1)$. We find a similar picture for lanthanum, neodymium, and the two REE indices, with information criteria at least equal to or lower in 88% (70%) or more window iterations. However, for cerium and yttrium, the picture is somewhat mixed, because the two information criteria return some opposing results. We find that, based on the information criteria, the baseline model is almost always superior to the GARCH$(1,1)$ model for the REE indices. However, the GARCH$(1,1)$ model appears to work better for individual REEs (cerium, lanthanum, and yttrium).

Note that the information criteria provide a somewhat mechanical model selection rule, and do not offer any guidance about the economic meaningfulness of the various specifications. Therefore, we examine the out-of-sample forecasting performance of the different models and specifications in the next subsection. Even when a model or specific model specification works well in-sample, the results can be dramatically different out-of-sample. However, high out-of-sample accuracy is vital when using volatility forecasts for the active trading strategy (see subsection 4.4).

– Please insert Table 3 about here –

4.2 Forecasting Performance

To test the out-of-sample performance for the different models and parameterizations, we first use a parameter set estimated over the first 300 weeks (in-sample period) for the different ARFIMA(p, d, q) model specifications, as well as for the GARCH(1,1) model. Given these estimated parameters, we predict the volatility for the subsequent week ($t = 301$) until the end of our observation period ($t = 489$), and we obtain 189 volatility forecasts for the out-of-sample period. To quantify the forecasting performance of the different models, we regress the volatility forecasts on the ex post observed volatility, and calculate the coefficient of determination (R^2) and the root mean squared error (RMSE). Note that the RMSE is considered a loss function, while the R^2 captures the information content of the respective forecasting method (see also Pong et al., 2004).

To assess the robustness of our results, we roll the window one week forward, and repeat these steps. Therefore, the in-sample period still includes 300 weeks, but it begins and ends one week later, and we re-estimate all models and parameterizations based on the new in-sample period. The first out-of-sample volatility forecast is then $t = 302$, but the ending period remains $t = 489$, reducing the number of volatility forecasts to 188. We roll the in-sample period forward on a weekly basis until week 95 (about two years), in order to ensure a sufficient number of weeks in the out-of-sample period for a meaningful estimation of the R^2 and RMSE (see Figure A1 in the online appendix for more details about the in- and out-of-sample periods for the rolling window).

The resulting R^2s for all REEs, their related indices, and the model specifications for the different rolling windows are displayed in Figure 3, which is essentially a visualization of Table 4. For the sake of brevity, we only visualize the R^2s, not the RMSEs.[15] Table 4 shows the resulting R^2s for the various starting points ($t = 1,\ 25, 48, 71, 95$), and compares the ARFIMA($0, d, 0$) baseline model to all other specifications. The last two lines of Table 4 indicate how many times (of the ninety-five different starting points) the baseline model is equal to or outperforms the other model specifications with respect to R^2 and RMSE. Figure 3 shows that the

[15] Note that visualizing the RMSEs is not useful for determining performance differences between the models over the various subperiods. The absolute differences are very low in comparison to the changes in RMSE between the subperiods (this is due primarily to the shortening of the out-of-sample period).

R^2s for all REEs and related indices are generally among or the highest for the ARFIMA$(0, d, 0)$ baseline model, except for cerium.

These out-of-sample results are strongly similar to the in-sample ones, with two main differences: Our baseline model was not suitable in-sample for yttrium, but it exhibits good out-of-sample performance. The opposite is true for cerium.[16] The forecasting performance of the ARFIMA$(0, d, 0)$ baseline model is better than or equal to that of all the other models in at least 60% (64%) of all iterations for the individual REEs as measured by R^2 (RMSE). At 75% (85%), it is exceptionally good for the REE indices. Only toward the end of our dataset do the volatility forecasts from the GARCH model exhibit better performance for neodymium and the value-weighted index. It seems the GARCH model is better at adjusting to extreme short-term changes in volatility, being present in this time period, than the ARFIMA$(0, d, 0)$ baseline model.

– Please insert Figure 3 and Table 4 about here –

4.3 Summary of Results

To summarize, we find that for the baseline model all REEs and the respective indices show a long-memory effect (see column "Long-Memory" in Table 5). Overall, the ARFIMA$(0, d, 0)$ baseline model appears to be the best choice for the ARFIMA-type models for all REEs and both indices, except for yttrium (see columns "ARFIMA$(0, d, 0)$" in Table 5). The in-sample fit reveals that the baseline model is the best fit for the REE indices, but the GARCH(1,1) model is best for three of the four REEs (see column "In-Sample" in Table 5). Most importantly for forecasting, we find that the ARFIMA$(0, d, 0)$ baseline model exhibits the best out-of-sample performance overall for the individual REEs (except cerium), and especially

[16] We find that our RMSEs are somewhat lower than those of Pong et al. (2004), which we suspect is attributable to the fact that 1) they annualize the data before running their regressions, 2) they use higher-frequency data (sampled every five minutes, and every thirty minutes in robustness checks) than we do, and 3) the foreign exchange market is fundamentally distinct from the commodity market in general, and the REE market in particular.

for the two indices (see column "Out-of-Sample" in Table 5). Therefore, we are confident that the baseline model provides the best choice for generating robust volatility forecasts. We use these forecasts in the next subsection for the active trading strategy.

– Please insert Table 5 about here –

4.4 Trading Strategy

4.4.1 Methodology

To measure the economic importance of the volatility forecasts produced by our baseline model, we set up an active trading strategy for Chinese companies operating in the REE industry. We then compare it with a passive buy-and-hold strategy by analyzing the difference in Sharpe ratios. The active trading strategy is based on the idea that, if these forecasts are *good* predictors of future REE volatility, and, if REE fluctuations serve as material information for companies in the REE industry, we can indirectly forecast the direction of stock price movements. To measure the effectiveness, i.e., the economic importance, of our volatility forecasts, we compare the risk-adjusted performance of the active trading strategy against the passive buy-and-hold alternative using Opdyke's (2008) Sharpe ratio test.

Because there are no available tradable financial products based on REEs, we first need to identify Chinese companies active in the REE market on which we can base the trading strategy. Nevertheless, we also provide evidence based on a non-investable paper portfolio that invests directly in REEs themselves as a robustness check. We use the following statistical method to identify those companies active in the REE industry.

First, we consider only three of the six industries classified under Industry Code A by the CSMAR database, because REE companies are most likely present in Public Utilities, Conglomerates, and Industry, but not in Finance, Properties, or Commerce. Second, to calculate reliable correlations, we require that each company be listed and have weekly returns for more than 70% of each year's total trading weeks prior to the subsequent trading year (2007-2014). Third, we require a minimum availability of stock returns, so we exclude stocks with weekly return data for less than 280 trading weeks for the overall trading period (2008-2015). 1,158 companies with A shares meet these conditions, and can theoretically be chosen by the active trading strategy. There is no single best way to identify companies in the REE industry based on a statistical method, so we tried a few alternatives. To identify which of the 1,158 companies are most sensitive to changes in REE returns, we calculate the Pearson correlation for each company i with its respective two REE indices (EWI and VWI) for each year from 2007 through 2014, as follows:

$$\rho(R_{i,Y}, R_{REE,Y})_Y = \frac{E[(R_{i,Y} - \mu_{i,Y})(R_{REE,Y} - \mu_{REE,Y})]}{\sigma_{i,Y} \cdot \sigma_{REE,Y}}, \tag{3}$$

where $R_{i,Y}$ is company i's vector of weekly returns for year Y; $\mu_{i,Y}$ is the mean return of company i in year Y; and $\sigma_{i,Y}$ is the standard deviation for company i in year Y, where Y ranges from 2007 to 2013. $R_{REE,Y}$, $\mu_{REE,Y}$, and $\sigma_{REE,Y}$ denote the same quantities for the respective REE index (VWI and EWI). Our rationale is that the higher the $\rho(i, REE)_Y$, the higher the exposure to the REE industry, because a larger proportion of a company's stock price variations can be explained by REE price movements. We calculate $\rho(i, REE)_Y$ for each year separately, which allows us to consider different companies each year for our active trading strategy.

To assess the robustness of our results, we vary several *elevating screws*, e.g., the number of possible REE companies included in the trading strategies, and the selection of REE companies based on either positive or absolute correlations as measured by either the VWI or the EWI REE index. First, we select the three, five, eight, or ten companies with the highest positive $\rho^+(i, REE)_Y$ according to Equation (3), as well as the three, five, eight, or ten companies with the highest absolute $|\rho(i, REE)_Y|$ for each year during the 2007-2014 period, using the VWI or EWI

REE index as a reference. We keep the REEs identified as the highest positive and absolute correlations within a respective year the same, but we repeat the exercise at the beginning of each subsequent year (rolling strategy).[17] This implies that, within one year, both the active trading and the buy-and-hold strategies can only select REE companies identified in the previous year.

We are left with sixteen possible combinations = 4 (3, 5, 8, and 10 selected REE companies) × 2 (VWI or EWI REE index) × 2 ($\rho^{+}(i, REE)_{Y}$ or $|\rho(i, REE)_{Y}|$). Table 6 reports the mean, minimum, and maximum correlations aggregated for all years for the three, five, eight, and ten companies, as well as for the two REE indices. We manually check three company profiles identified as having the highest correlations with the REE indices, and find that all operate directly in the REE industry or hold stakes in REE companies (see Table A8 in the online appendix). Therefore, we are convinced this procedure is suitable for identifying companies with REE exposure. If we mistakenly include a company that does not have REE exposure, and if our reasoning that REE volatility forecasts can predict future stock prices of REE companies is correct, this would work against the statistical significance in our tests, and would render our results conservative.

– Please insert Table 6 about here –

For our trading strategy, we use the rolling one-step-ahead volatility forecasts based on the ARFIMA$(0, d, 0)$ baseline model for the two REE indices, with an estimation period of 300 (T) weeks as described in subsection 4.2. This translates to a start date of June 2011 for the trading strategy.[18] The active trading strategy is then based on the divergence between the most recent observed volatility and the one-step-ahead volatility forecast: If this discrepancy contains

[17] For example, REE companies identified on the basis of the correlation in 2007 are used for the trading strategies in 2008.

[18] In a robustness check, we reduce the rolling window estimation period to 100 (T) weeks. Accordingly, the start date for the trading strategy is January 2008.

material information for the identified REE companies, we expect the active trading strategy to generate risk-adjusted returns that will exceed those of a buy-and-hold strategy.

More precisely, the active trading strategy is based on the following principle: An increase (decrease) in REE volatility is, on average, accompanied by a decrease (increase) in stock prices. This rationale builds on Duffee (1995), who establishes a relationship between firm-level volatility and returns, but recent research describes an increasing integration of commodity and stock markets. As a result, the correlation and volatility spillovers across different markets increased over time (see Silvennoinen and Thorp, 2013).[19] Note that ample evidence of volatility spillovers exists between, e.g., commodity and financial markets. As we are interested in the spillover of REE volatility changes to the stock returns of companies active in the REE market, our study is more in line with Antonakakis et al. (2018), who examine volatility spillovers of oil prices to stock prices of a sample of major oil and gas companies.

Let $|r_{REE,t}|$ denote the (ex post) observed volatility of the respective REE index at time t, and $|\hat{r}_{REE,t+1|t}|$ be the one-step-ahead out-of-sample forecast from time t to $t+1$. To generate a trading signal, we use a straightforward and intuitive trading rule: If the volatility forecast *considerably* (as measured by ς) exceeds (falls below) the most recent observed volatility, the REE companies are sold (bought) if their correlations to the REE index are positive (negative). Otherwise, no trading will take place. ς denotes the sensitivity to the signal, and is set to 5%, 10%, and 15%, respectively. We formalize this trading rule as follows:

[19] Examples of volatility spillovers include the oil and stock markets as a whole (see, for example, Malik and Hammoudeh, 2007; Arouri, Jouini and Nguyen, 2012; Bouri, 2015; Boldanov, Degiannakis and Filis, 2016), as well as spillovers between oil and different stock market sectors (see Malik and Ewing, 2009; Sadorsky, 2012). Beyond the oil market, recent studies have covered volatility transmission between energy, agricultural, and precious metal commodities and equity markets (Mensi et al., 2013), precious metal ETFs and global equity markets (Lau et al., 2017), commodity and currency markets (Antonakakis and Kizys, 2015; Yip, Brooks and Do, 2017), and commodity and CDS markets (Bouri, de Boyrie and Pavlova, 2017).

Trigger	Positively Correlated REE Companies	Negatively Correlated REE Companies
$\lvert\hat{r}_{REE,t+1\|t}\rvert > \lvert r_{REE,t}\rvert \cdot (1+\varsigma)$	Sell	Buy
$\lvert\hat{r}_{REE,t+1\|t}\rvert < \lvert r_{REE,t}\rvert \cdot (1+\varsigma)$	Buy	Sell
Otherwise	No Investment	No Investment

4.4.2 Results

The trading results for the forty-eight strategies (= 16 (company, index, and correlation combinations) × 3 (5%, 10%, and 15% trading signal sensitivities)) are reported in Table 7; some are graphed in Figure 4. The benchmark for comparing the trading strategy results is a naively diversified long buy-and-hold portfolio. Figure 4 clearly shows that the active trading strategy outperforms the buy-and-hold strategy (regardless of whether three, five, eight, or ten REE companies are combined in a portfolio) for both REE indices used for a given sensitivity (ς) of 5%. This is the first (visual) evidence that the volatility forecasts for the REE indices convey material information for REE companies, which can be exploited by using an active trading strategy.

Table 7 explores whether these results are generalizable for all forty-eight strategies, and tests whether the outperformance (on a risk-adjusted return basis) is significantly greater than that for the buy-and-hold portfolio return. We find that our trading strategy always results in significantly higher Sharpe ratios when selecting the REE companies with the highest absolute correlations ($|\rho(i, REE)_Y|$) to the respective REE indices. This represents strong support for our argument that REE volatility forecasts convey material information for firms active in this industry. This result is similar when REE companies are chosen based on positive correlations, $\rho^+(i, REE)_Y$, but the active trading strategy does not outperform based on all possible combinations.

The statistically insignificant results are most likely attributable to the combinations used when choosing ten REE companies, and can be explained as follows. First, by definition, the correlations decrease with a higher number of companies included in the portfolios. This implies that the link to sensitivity to REE prices is likewise decreasing, which works against our strategy. Second, because we only consider positive correlations, we virtually exclude REE user companies. These companies would arguably react differently to REE price changes because they are expected to react negatively to REE price increases. Thus, intuitively, we expect to observe a positive correlation between the level of absolute correlations of REE companies with respective REE indices and trading strategy outperformance.

On a related note, as the number of REE companies in the portfolio increases from five to ten, the average (absolute) correlation, and thus the outperformance of the trading strategy, decreases (compared to the buy-and-hold strategy). This result makes intuitive sense, because the *weaker* the link between REE price fluctuations and stock price reactions (measured by correlation), the lower the informational content of the REE volatility forecasts. This translates into lower expected total returns and lower Sharpe ratios.[20]

– Please insert Table 7 and Figure 4 about here –

4.4.3 Robustness Checks

To test the stability of our results, we perform the following series of robustness tests: 1) we determine whether our results are sensitive to the estimation period, 2) we measure the influence of direct trading costs on the results, 3) we estimate the sensitivity of our results when considering direct and indirect trading costs, and 4) we apply our trading strategy directly to REEs.

[20] Note that the results for the three REE companies in our portfolio can be less stable, despite having a higher (absolute) correlation to the respective REE index. This can occur simply because one "outlier" in a three-stock portfolio technically has a higher weight, and thus a greater influence, on portfolio performance.

First, when changing the rolling window of the estimation period to 100 weeks (including the 2008 bear market), the results are very similar. This indicates that the length of the estimation period, and whether it includes both bull and bear markets, has no obvious impact on the profitability of the trading strategy (see Table A2 and Figure A3 in the online appendix).

Second, to check robustness with regard to the transaction costs of the various active trading portfolios, we multiply the number of trades (# p.a.) times the transaction costs of 0.169% (i.e., handling fee of 0.0487%, plus management fee of 0.02%, plus stamp duty of 0.1%, based on the latest fees at the Shanghai and Shenzhen stock exchanges). We find that statistical significance is not affected for the vast majority of specifications (see Table A3 in the online appendix). By “not affected,” we mean that, for a given specification of the trading strategy (e.g., the number of REE companies traded in the strategy (#) and the sensitivity to changes in volatility (ς)), the Δ Sharpe ratio remains significant at the same level. We find nearly identical results for the rolling window of the 100-week estimation period specification (see Table A4 in the online appendix).

Third, the actual transaction costs charged by the stock exchanges account for only the direct trading costs. To measure the impact of total trading costs, we must also consider indirect costs, such as bid-ask spreads. These costs are more difficult to measure because they generally depend on variables such as trading volume, liquidity and trading time. Thus, because it is impossible to fairly estimate indirect trading costs, we calculate an annual return buffer instead.

In detail, we calculate the maximum return deduction per year so that the trading strategy remains statistically significant at the same level (and, if applicable, at lower levels of significance). This allows us to form our own opinion about total (relative) trading costs, and to gauge whether the outperformance of the active trading strategy would remain statistically significant at a certain level. When considering the three REE companies in the active trading portfolio, and when using the EWI REE index for the volatility forecasts (see panel A of Table A5 in the online appendix), we find that the annual return buffers are fairly large (7%-17%). We argue that these buffers are sufficient to absorb even outsize trading costs, and will still result in a statistically significant outperformance of the active trading strategy. However, we find that the return buffers tend to be lower 1) when using the VWI REE index, 2) when a higher number of REE companies are included in the portfolio, and 3) if companies are selected based on positive correlations only. This reflects our previous observation from Table 7. Results for the rolling window of the 100-week estimation period are comparable (see Table A6 in the online appendix).

Note that our goal in assessing the active trading strategy is not to show monetizable trading profits after including trading costs. Rather, we aim to show that our REE volatility forecasts convey economically meaningful material information that can be used by the REE companies to proactively manage uncertainty in the REE market. This is evidenced by the statistically significant outperformance of the active trading strategy without trading costs.

Fourth, we apply our trading strategy for listed companies to the REE industry, which is clearly an indirect approach to using the REEs themselves.[21] As previously mentioned, REEs are not readily investable. Nevertheless, if our REE volatility forecasts convey material information about future price developments, we expect the economic importance of REEs themselves to be higher. We therefore replicate the previous procedure and apply it directly to the REE indices. In line with our intuition, we find that the active trading strategy largely outperforms the buy-and-hold strategy on a risk-adjusted basis, irrespective of the REE index, the estimation period, or any sensitivity to changes in volatility (see Table A7 in the online appendix).

[21] The significant outperformance could be driven by systematic risk factors such as Fama and French's Small-Minus-Big or High-Minus-Low factors. We could certainly control for both factors in the Chinese market. But we do not expect any impact on our results, because the active trading strategy and the buy-and-hold strategy have identical shares in the portfolio ("matched portfolio" comparison). Therefore, both are exposed equally to such systematic risk factors.

To summarize, we find compelling evidence that the out-of-sample volatility forecasts generated by the long-memory ARFIMA$(0, d, 0)$ baseline model convey material information that predicts the stock prices of Chinese REE companies and REE indices. The price movements are not only predictable to a certain extent, but they can also be measured by using an active trading strategy. We interpret these results as strong evidence that REE volatility forecasts are of strong economic importance.

5. Conclusion

REEs have garnered increasing attention lately due to their importance for various high- and green-technology applications, China's monopoly over this key industry, and the resultant dramatic price increases instituted by China in recent years, which sparked a series of U.S. Congressional reports (Humphries, 2010; Morrison and Tang, 2012; Bailey Grasso, 2013). Because there are currently no effective methods available to hedge against REE price volatility (such as derivatives), industries that depend on REEs for production require a reliable estimate of volatility. The creation of a volatility monitor is a first step in this direction.

However, we suggest complementing observed volatility with reliable volatility forecasts. In this paper, we show that the time series of REE volatilities exhibit long-memory, and we find that a simple ARFIMA$(0, d, 0)$ model is generally the most accurate method by which to describe the volatility of REEs and their related indices. The use of different observation periods, estimation windows, and out-of-sample analyses do not alter these results: REE volatility remains highly persistent throughout time.

Moreover, we find that, in the absence of, e.g., forward-looking implied volatilities and thus a weaker consensus about expected REE volatility, a volatility forecast based on the ARFIMA$(0, d, 0)$ baseline model can foreshadow, or predict, future price developments of REE companies. Using this information in an active trading strategy generates statistically higher risk-adjusted returns (as measured by the Sharpe ratio) than a buy-and-hold strategy. We interpret this as solid support for the notion that our REE volatility forecasts convey economically meaningful material information that can be used by companies in the REE market to proactively manage uncertainty.

Our findings provide some instructive insights that should be of interest to practitioners, policy makers, and academics concerned with the dynamics of the REE market. For example, the volatility distributions seem to be non-normal stable Paretian, rather than Gaussian (Fama, 1963; Mandelbrot, 1972; Greene and Fielitz, 1977). Hence, periods of extreme volatility are more common than under the normal distribution. Ultimately, this finding has some key implications. If options markets for REEs are established, ensuring an adequate option pricing framework would be paramount. Current pricing models for commodity options assume either deterministic (Back, Prokopczuk and Rudolf, 2013) or stochastic (Arsimendi et al., 2016) seasonal volatility. In the case of REEs, volatility does not seem to exhibit a seasonal component. Rather, it is non-periodic and exhibits "non-sinusoidal" cycles (Mandelbrot, 1971, p. 228), which need to be adequately incorporated into the pricing framework.

Furthermore, given the extreme price movements of REEs and their importance for the entire high-tech industry, more research on their dynamics is warranted. Future research should consider analyzing the impact of macroeconomic factors, e.g., interest rates, exchange rates, and the business cycle, on the dynamics of REE prices in order to foster a fuller understanding of this important market. Related research from other commodity markets such as gold and oil may provide some additional guidance (see, for example, Faff and Chan, 1998; Tufano, 1998; Boyer and Filion, 2007; Batten, Ciner and Lucey, 2010; Baur, 2014; Haugom et al., 2014).

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Figure 1: Time Series Plot of Rare Earth Element Prices

This figure illustrates the evolution of weekly REE prices (cerium, lanthanum, neodymium, and yttrium) (panel A) and the two REE indices (EWI and VWI) (panel B), as well as the related volatility (measured by absolute returns). Prices (RMB/kg) come from the Asian Metal database, and cover the January 28, 2005-February 28, 2015 period. In panel A, the prices of cerium, lanthanum, and neodymium are plotted on the Y1 axis, and the price of yttrium on the Y2 axis.

Panel A **Panel B**

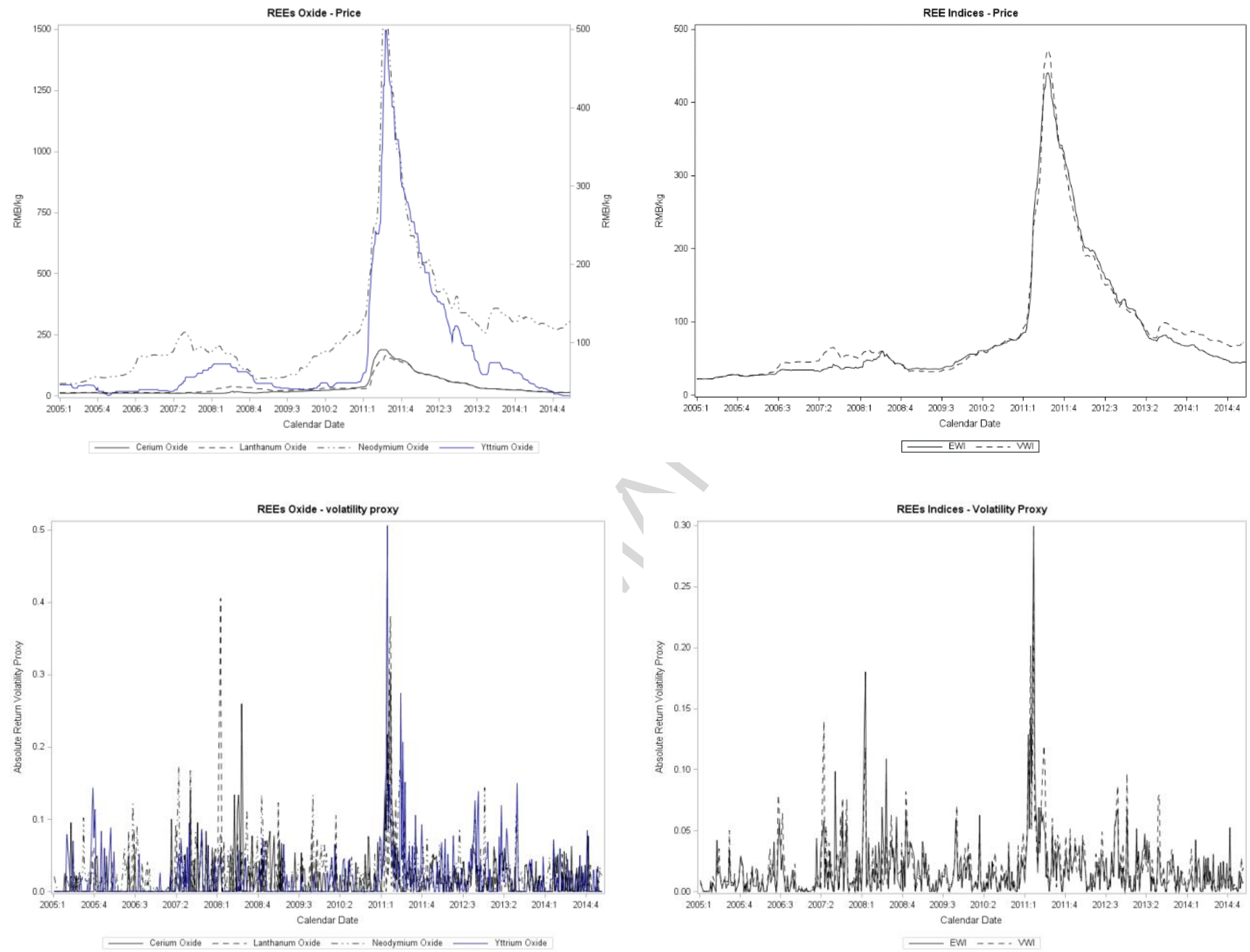

Figure 2: Stability of the Fractional Differencing Parameter for the Volatility Series

This figure shows the development of the fractional differencing parameter d obtained using maximum likelihood estimation (Sowell, 1992) with different specifications of an ARFIMA(p, d, q) model on the absolute returns for the four REEs (cerium, lanthanum, neodymium, and yttrium) and the two REE indices (EWI and VWI). We use a rolling window of 300 weeks to estimate the degree of long-memory.

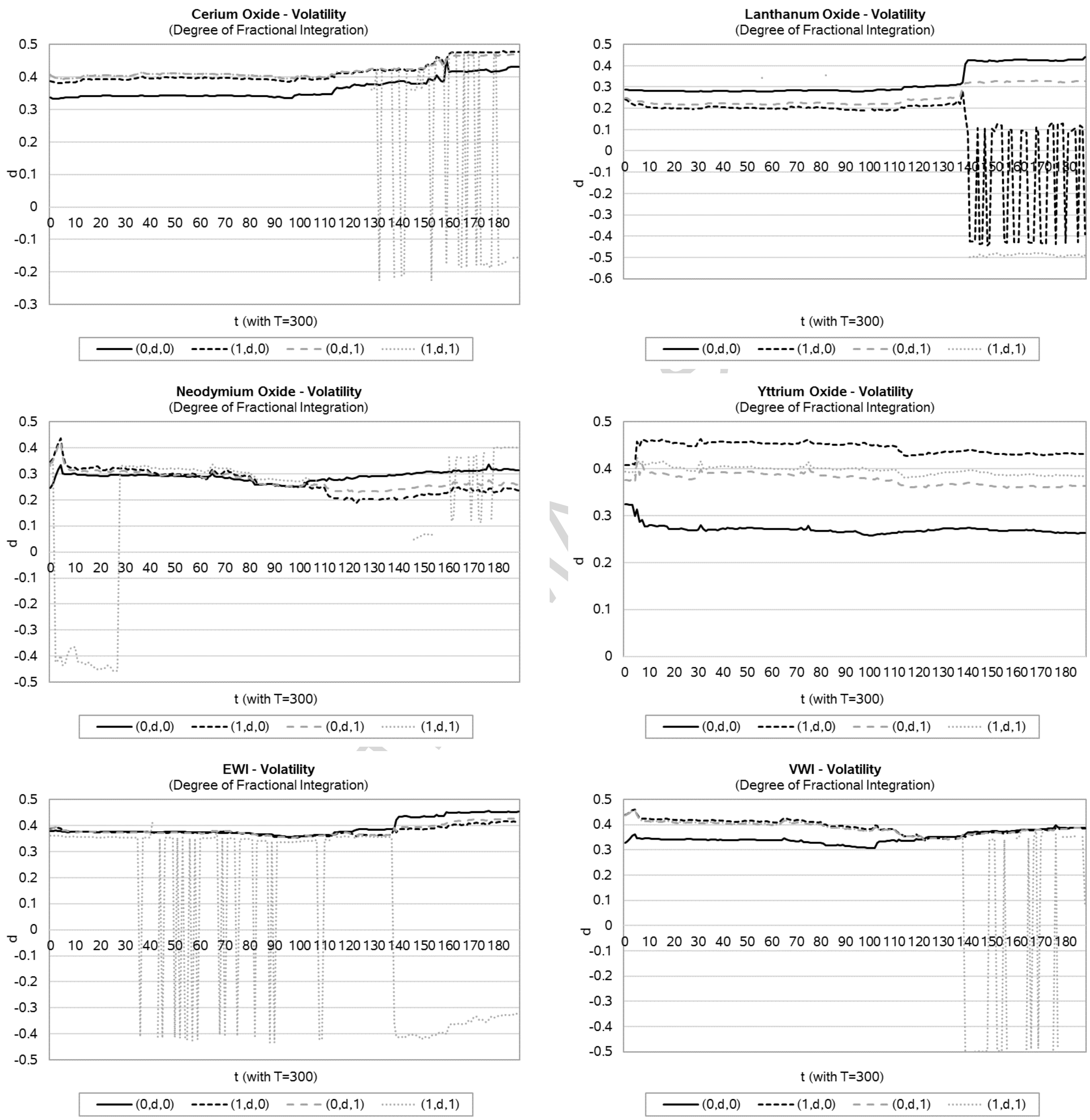

Figure 3: Out-of-Sample Forecasting Performance for the Volatility Series
This figure shows the developments of the coefficient of determination (R^2) from regressing the one-step-ahead volatility forecast generated from different specifications of an ARFIMA(p, d, q) model and the GARCH(1,1) model on the absolute returns for the four REEs (cerium, lanthanum, neodymium, and yttrium) and the two REE indices (EWI and VWI). We use a rolling window of 300 weeks to estimate the degree of long-memory.

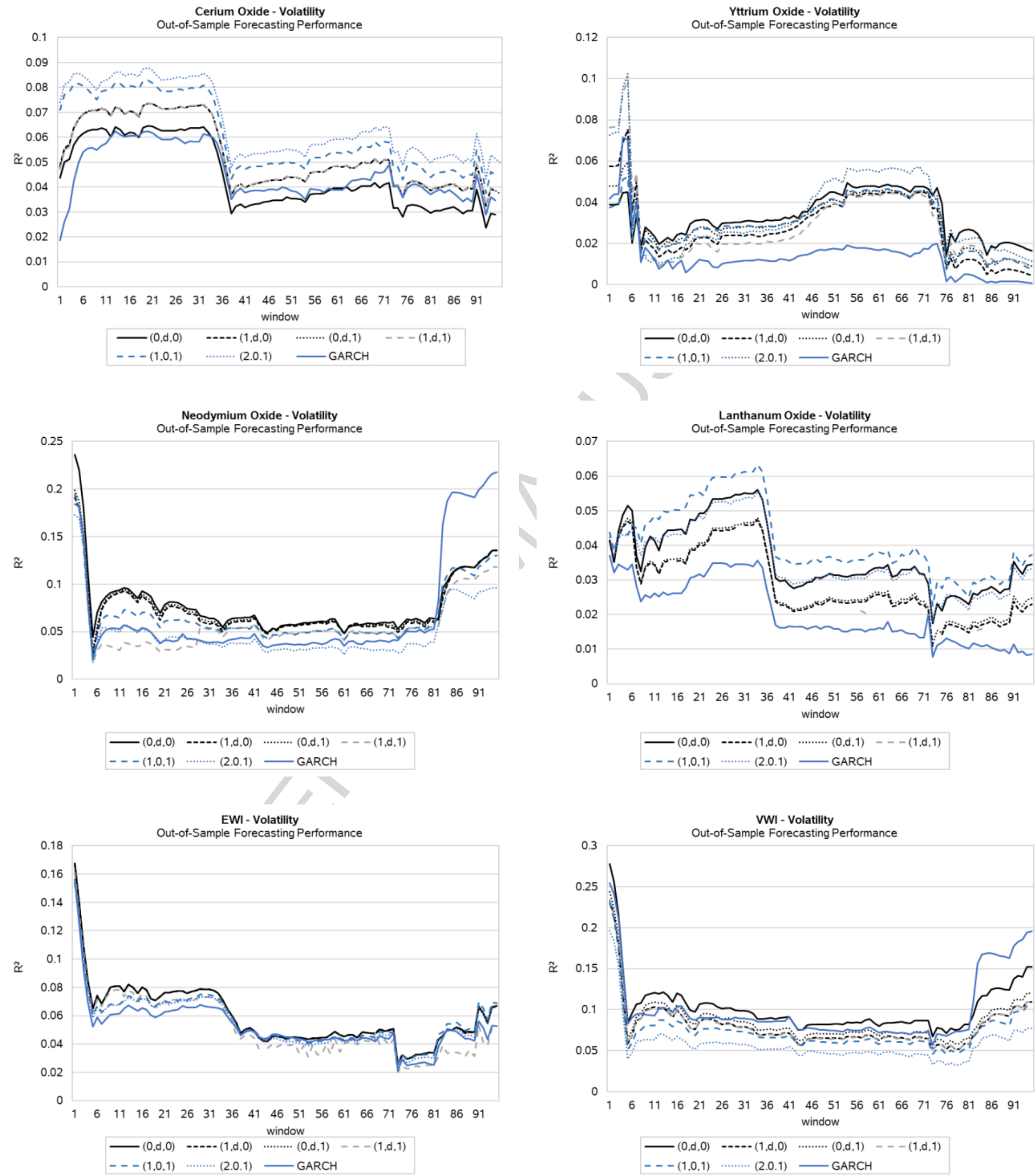

Figure 4: Trading Strategy Results for T = 300 Model

This figure shows the developments of the buy-and-hold strategy versus the trading strategy based on the ARFIMA $(0, d, 0)$ baseline model with T = 300 (weeks). For the sake of comparability, we standardize weekly trading returns into a price index beginning with 100 points. "Buy and hold" refers to the index development based on buy-and-hold returns; "trading strategy" refers to that based on trading strategy returns. (+) means REE companies were selected based on $\rho^{+}(i, REE)_{Y}$, and (+/-) means REE companies were selected based on $|\rho(i, REE)_{Y}|$. The trading strategy includes three, five, eight, or ten REE companies, and covers the June 3, 2011-February 28, 2015 period.

(continued)

Figure 4: Trading Strategy Results for T = 300 model – *continued*

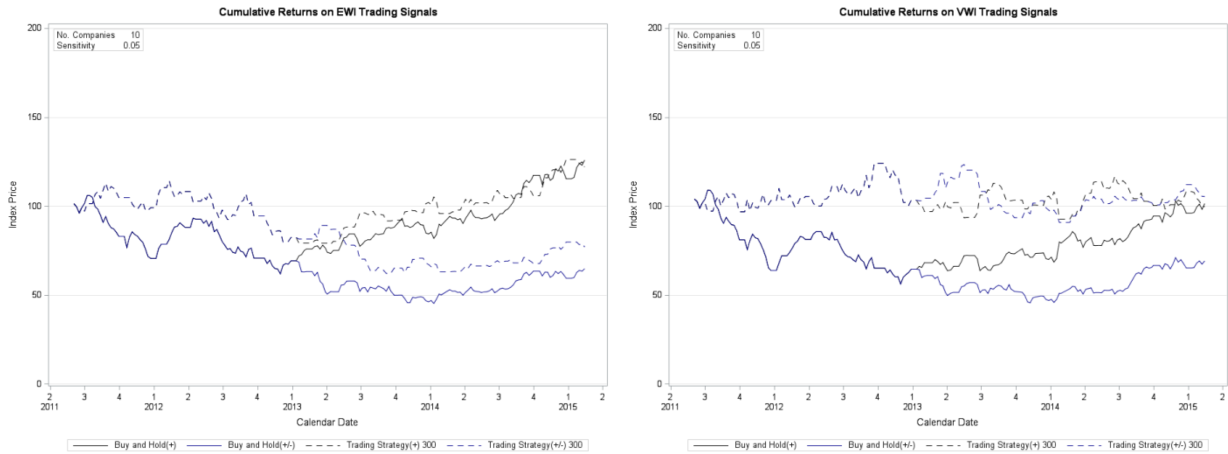

Table 1: Descriptive Statistics

This table shows the descriptive statistics of volatility (measured by absolute returns) for the four REEs (cerium, lanthanum, neodymium, and yttrium) and the two REE indices (EWI and VWI) using the complete sample of 489 weeks. The Ljung-Box test statistics (Ljung and Box, 1978) show substantial serial correlation in our volatility proxy. The autocorrelations from one to five lags are all statistically significant at least at a 10% confidence level.

	Cerium	**Lanthanum**	**Neodymium**	**Yttrium**	**EWI**	**VWI**
Mean	0.0167	0.0153	0.0217	0.0175	0.0171	0.0184
Std. Dev.	0.0333	0.0377	0.0305	0.0391	0.0255	0.0248
Skewness	3.889	6.002	2.583	5.721	4.982	3.639
Kurtosis	25.045	53.118	11.712	57.917	43.181	23.869
Minimum	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Maximum	0.3038	0.4055	0.2139	0.5062	0.2994	0.2373
Autocorrelation						
Lag(1)	0.408	0.337	0.376	0.295	0.467	0.442
Lag(2)	0.353	0.245	0.274	0.378	0.394	0.374
Lag(3)	0.386	0.143	0.211	0.164	0.347	0.344
Lag(4)	0.293	0.150	0.099	0.135	0.268	0.198
Lag(5)	0.209	0.142	0.135	0.087	0.276	0.240
Ljung-Box Test						
Q(20)	316.149	128.752	217.400	232.837	397.590	405.516

Table 2: Results of Long-Memory Tests

This table shows the fractional differencing parameter d, as well as the AR and MA terms obtained using Sowell's (1992) maximum likelihood estimator of various ARFIMA(p, d, q) specifications for REE volatility (measured by absolute returns) for the four individual elements (cerium, lanthanum, neodymium, and yttrium) and the two REE indices (EWI and VWI). It also shows the results of likelihood ratio tests (LR) of the different parameterizations against the ARFIMA$(0, d, 0)$ baseline model. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively. LR: Likelihood ratio test statistic. AIC: Akaike (1974) information criterion. BIC: Schwarz (1978) information criterion.

	ARFIMA(p, d, q)					ARFIMA(p, d, q)				
Parameter	**(0,d,0)**	**(1,d,0)**	**(2,d,0)**	**(1,d,1)**	**(2,d,1)**	**(0,d,0)**	**(1,d,0)**	**(2,d,0)**	**(1,d,1)**	**(2,d,1)**
			Cerium					*Lanthanum*		
d	0.32***	0.39***	0.45***	0.41***	0.44***	0.27***	0.22***	0.15	0.25***	0.22***
AR(1)		-0.11	-0.17**	-0.03	-0.77***		0.08	0.15	-0.48	-0.52
AR(2)			-0.07		-0.17***			0.06		0.04
MA(1)				-0.10	0.62**				0.52	0.60
LR	NA	2.30	3.38	2.47	5.96	NA	0.87	1.59	0.68	0.89
AIC	-2,060	-2,060	-2,060	-2,050	-2,060	-1,880	-1,880	-1,880	-1,880	-1,870
BIC	-2,040	-2,040	-2,030	-2,030	-2,030	-1,870	-1,860	-1,860	-1,860	-1,850
			Neodymium					*Yttrium*		
d	0.29***	0.27***	0.16	0.29***	0.13	0.25***	0.41***	0.16	0.35***	0.16
AR(1)		0.03	0.14	-0.34	0.40		-0.24***	0.03	-0.67***	-0.01
AR(2)			0.11*		0.08			0.25***		0.25***
MA(1)				0.34	-0.23				0.50***	0.04
LR	NA	0.12	3.12	0.00	3.77	NA	10.01***	24.95***	18.57***	25.01***
AIC	-2,110	-2,100	-2,100	-2,100	-2,100	-1,850	-1,860	-1,880	-1,870	-1,870
BIC	-2,090	-2,090	-2,080	-2,080	-2,080	-1,840	-1,850	-1,850	-1,850	-1,850
			EWI					*VWI*		
d	0.36***	0.38***	0.39***	0.38***	0.41***	0.33***	0.38***	0.33***	0.38***	0.41***
AR(1)		-0.02	-0.03	0.00	-0.78***		-0.07	-0.02	-0.15	-0.91***
AR(2)			-0.01		-0.09			0.05		-0.14**
MA(1)				-0.02	0.73***				0.08	0.80***
LR	NA	0.09	0.11	0.09	2.70	NA	0.90	1.39	0.96	4.50
AIC	-2,350	-2,350	-2,340	-2,340	-2,350	-2,360	-2,360	-2,360	-2,360	-2,360
BIC	-2,340	-2,330	-2,320	-2,320	-2,320	-2,350	-2,340	-2,340	-2,330	-2,330

Table 3: In-Sample Parameter Estimation for the Volatility Series

This table shows the results for the presence of long-memory in REE volatility (measured by absolute returns) using Sowell's (1992) maximum likelihood estimation under various specifications of an ARFIMA(p, d, q) model and two ARFIMA($p, 0, q$) models for the four REEs (cerium, lanthanum, neodymium, and yttrium) and the two REE indices (EWI and VWI). We use a rolling window of 300 weeks to estimate the degree of long-memory; for clarity, we report one iteration each year (see Figure A1 in the online appendix for more details about the rolling window). Other iterations are available from the authors upon request. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively. $\%_{AIC}$ ($\%_{BIC}$) denotes the relative number of rolling periods when the ARFIMA($0, d, 0$) is equal to or better than the respective model measured by AIC (BIC). AIC: Akaike (1974) information criterion. BIC: Schwarz (1978) information criterion.

		ARFIMA(p, d, q)						ARFIMA(p, d, q)					
t	**Parameter**	**(0,d,0)**	**(1,d,0)**	**(0,d,1)**	**(1,d,1)**	**(1,0,1)**	**(2,0,1)**	**(0,d,0)**	**(1,d,0)**	**(0,d,1)**	**(1,d,1)**	**(1,0,1)**	**(2,0,1)**
				Cerium						*Lanthanum*			
1	d	0.336***	0.387***	0.404***	0.407***			0.288***	0.242***	0.249***			
1	AR(1)		-0.089		0.046	0.872***	0.843***		0.069			0.723***	0.904***
1	AR(2)						0.022						-0.089
1	MA(1)			-0.113	-0.161	-0.58***	-0.559***			0.06		-0.433***	-0.601**
48	d	0.343***	0.398***	0.411***	0.41***			0.28***	0.199**	0.219***			
48	AR(1)		-0.091		-0.006	0.859***	0.793***		0.115			0.701***	0.882***
48	AR(2)						0.051						-0.087
48	MA(1)			-0.106	-0.1	-0.554***	-0.509***			0.092		-0.412***	-0.579**
95	d	0.336***	0.386***	0.395***	0.394***			0.279***	0.191*	0.217***			
95	AR(1)		-0.08		-0.015	0.843***	0.775***		0.124			0.69***	0.843***
95	AR(2)						0.051						-0.072
95	MA(1)			-0.09	-0.075	-0.542***	-0.495***			0.092		-0.4***	-0.543*
142	d	0.386***	0.423***	0.427***	-0.21			0.426***	-0.426***	0.321***	-0.5***		
142	AR(1)		-0.061		0.896***	0.848***	0.778***		0.898***		0.92***	0.647***	-0.333***
142	AR(2)						0.054						0.528***
142	MA(1)			-0.065	-0.353**	-0.496***	-0.446***			0.16**	0.058	-0.151**	0.85***
189	d	0.431***	0.477***	0.47***	-0.155			0.442***	-0.391***	0.326***	-0.491***		
189	AR(1)		-0.109		0.897***	0.859***	0.581***		0.888***		0.918***	0.65***	0.073
189	AR(2)						0.221***						0.331**
189	MA(1)			-0.078	-0.353***	-0.454***	-0.241**			0.184**	0.078	-0.127**	0.448*
	Model	**GARCH**	**(1,d,0)**	**(0,d,1)**	**(1,d,1)**	**(1,0,1)**	**(2,0,1)**	**GARCH**	**(1,d,0)**	**(0,d,1)**	**(1,d,1)**	**(1,0,1)**	**(2,0,1)**
	$\%_{AIC}$	6%	100%	100%	88%	28%	34%	1%	83%	95%	31%	70%	78%
	$\%_{BIC}$	16%	100%	100%	97%	41%	85%	1%	89%	100%	80%	88%	90%

(continued)

Table 3: In-Sample Parameter Estimation for Volatility Series – *continued*

t	Parameter	ARFIMA(p,d,q) (0,d,0)	(1,d,0)	(0,d,1)	(1,d,1)	(1,0,1)	(2,0,1)
		Neodymium					
1	d	0.246***	0.345***	0.338***	0.331***		
1	AR(1)		-0.153*		-0.257	0.83***	0.64***
1	AR(2)						0.114
1	MA(1)			-0.133	0.118	-0.608***	-0.455***
48	d	0.292***	0.298***	0.295***	0.32***		
48	AR(1)		-0.008		-0.377	0.721***	0.422***
48	AR(2)						0.164*
48	MA(1)			-0.005	0.332	-0.412***	-0.143
95	d	0.253***	0.252**	0.253***	0.274***		
95	AR(1)		0.001		-0.373	0.693***	0.419**
95	AR(2)						0.137
95	MA(1)			0.001	0.339	-0.423***	-0.175
142	d	0.297***		0.241***			
142	AR(1)					0.657***	0.535**
142	AR(2)						0.058
142	MA(1)			0.084		-0.325***	-0.21
189	d	0.314***	0.237***	0.259***	0.399***		
189	AR(1)		0.117		0.784***	0.678***	0.573***
189	AR(2)						0.054
189	MA(1)			0.087	-0.855***	-0.318***	-0.219
	Model	**GARCH**	**(1,d,0)**	**(0,d,1)**	**(1,d,1)**	**(1,0,1)**	**(2,0,1)**
	$\%_{AIC}$	100%	100%	100%	98%	90%	89%
	$\%_{BIC}$	100%	100%	100%	99%	100%	100%

t	Parameter	ARFIMA(p,d,q) (0,d,0)	(1,d,0)	(0,d,1)	(1,d,1)	(1,0,1)	(2,0,1)
		Yttrium					
1	d	0.325***	0.408***	0.375***	0.394***		
1	AR(1)		-0.126		-0.575***	0.801***	0.341
1	AR(2)						0.264**
1	MA(1)			-0.073	0.454*	-0.484***	-0.056
48	d	0.274***	0.456***	0.392***	0.404***		
48	AR(1)		-0.3***		-0.717***	0.856***	0.149
48	AR(2)						0.411***
48	MA(1)			-0.173**	0.495***	-0.598***	0.061
95	d	0.267***	0.453***	0.384***	0.401***		
95	AR(1)		-0.297***		-0.674***	0.839***	0.182
95	AR(2)						0.386***
95	MA(1)			-0.168**	0.447***	-0.582***	0.012
142	d	0.274***	0.439***	0.37***	0.396***		
142	AR(1)		-0.256***		-0.663***	0.818***	0.151
142	AR(2)						0.379***
142	MA(1)			-0.137*	0.459***	-0.549***	0.067
189	d	0.262***	0.432***	0.363***	0.385***		
189	AR(1)		-0.26***		-0.654***	0.818***	0.162
189	AR(2)						0.364***
189	MA(1)			-0.143*	0.45***	-0.562***	0.041
	Model	**GARCH**	**(1,d,0)**	**(0,d,1)**	**(1,d,1)**	**(1,0,1)**	**(2,0,1)**
	$\%_{AIC}$	0%	14%	62%	2%	43%	0%
	$\%_{BIC}$	0%	50%	100%	41%	98%	4%

(continued)

Table 3: In-Sample Parameter Estimation for Volatility Series – *continued*

		ARFIMA(p,d,q)						**ARFIMA(p,d,q)**					
t	**Parameter**	**(0,d,0)**	**(1,d,0)**	**(0,d,1)**	**(1,d,1)**	**(1,0,1)**	**(2,0,1)**	**(0,d,0)**	**(1,d,0)**	**(0,d,1)**	**(1,d,1)**	**(1,0,1)**	**(2,0,1)**
		EWI						*VWI*					
1	d	0.379***	0.389***	0.39***	0.36***			0.328***	0.44***	0.438***	0.438***		
1	AR(1)		-0.018		-0.748***	0.85***	0.931***		-0.196***		-0.211	0.872***	0.66***
1	AR(2)						-0.057						0.156***
1	MA(1)			-0.02	0.798***	-0.504***	-0.571***			-0.174**	0.018	-0.582***	-0.426***
48	d	0.376***	0.372***	0.372***	0.354***			0.341***	0.414***	0.403***	0.406***		
48	AR(1)		0.006		-0.736***	0.838***	0.923***		-0.119		-0.224	0.841***	0.644***
48	AR(2)						-0.059						0.132**
48	MA(1)			0.006	0.791***	-0.491***	-0.562***			-0.095	0.112	-0.52***	-0.354***
95	d	0.358***	0.355***	0.355***	0.336***			0.312***	0.389***	0.381***	0.382***		
95	AR(1)		0.004		-0.748***	0.825***	0.896***		-0.123		-0.198	0.835***	0.645***
95	AR(2)						-0.047						0.122**
95	MA(1)			0.004	0.801***	-0.495***	-0.554***			-0.104	0.082	-0.544***	-0.383***
142	d	0.435***	0.387***	0.395***	-0.411**			0.369***	0.362***	0.363***			
142	AR(1)		0.079		0.937***	0.818***	0.953***		0.01			0.812***	0.863***
142	AR(2)						-0.094						-0.032
142	MA(1)			0.07	-0.089	-0.388***	-0.509***			0.009		-0.45***	-0.497***
189	d	0.454***	0.417***	0.429***	-0.31			0.388***	0.385***	0.386***	0.075		
189	AR(1)		0.069		0.925***	0.825***	0.833***		0.005		0.792***	0.827***	0.832***
189	AR(2)						-0.006						-0.004
189	MA(1)			0.05	-0.153	-0.36***	-0.368***			0.004	-0.476**	-0.44***	-0.445***
	Model	**GARCH**	**(1,d,0)**	**(0,d,1)**	**(1,d,1)**	**(1,0,1)**	**(2,0,1)**	**GARCH**	**(1,d,0)**	**(0,d,1)**	**(1,d,1)**	**(1,0,1)**	**(2,0,1)**
	$\%_{AIC}$	99%	100%	100%	86%	72%	85%	100%	98%	99%	90%	74%	81%
	$\%_{BIC}$	98%	100%	100%	99%	97%	100%	99%	100%	100%	100%	96%	100%

Table 4: Comparison of Out-of-Sample Forecasting Performance

This table presents a comparison of the forecasting performance measured by the coefficient of determination (R^2) for our out-of-sample forecasts for absolute returns using various specifications of ARFIMA(p, d, q) models and the GARCH(1,1) model for the four REEs (cerium, lanthanum, neodymium, and yttrium) and the two REE indices (EWI and VWI). We use a rolling window of 300 weeks to estimate the parameters (in-sample). Thus, the number of out-of-sample observations to estimate R^2 declines from 190 (over about four years) for t = 1 by one observation for each rolling window. To ensure we are left with a sufficient number of observations, we estimate out-of-sample performance until t = 95, which leaves us with a minimum of ninety-six observations (over two years) for our R^2 estimation. For clarity, we report two iterations each year (approximately forty-seven weeks). Other iterations, as well as adjusted R^2s and $RMSE$ estimates, are available from the authors upon request. Prices (RMB/kg) come from the Asian Metal database, and cover the January 2005-February 2015 period. $\%_{R^2}$ ($\%_{RMSE}$) denotes the relative number of rolling periods when the ARFIMA$(0, d, 0)$ is equal to or better than the respective model measured by R^2 ($RMSE$).

	ARFIMA(p, d, q)						GARCH	ARFIMA(p, d, q)						GARCH
t	**(0,d,0)**	**(1,d,0)**	**(0,d,1)**	**(1,d,1)**	**(1,0,1)**	**(2,0,1)**	**(1,1)**	**(0,d,0)**	**(1,d,0)**	**(0,d,1)**	**(1,d,1)**	**(1,0,1)**	**(2,0,1)**	**(1,1)**
	Cerium							*Lanthanum*						
1	0.044	0.049	0.048	0.047	0.071	0.074	0.019	0.042	0.041	0.041	n/a	0.044	0.042	0.037
25	0.063	0.071	0.071	0.071	0.079	0.083	0.059	0.053	0.044	0.045	0.042	0.060	0.053	0.035
48	0.035	0.043	0.043	0.043	0.049	0.054	0.039	0.031	0.023	0.023	n/a	0.035	0.030	0.016
71	0.041	0.051	0.051	0.051	0.058	0.064	0.046	0.032	0.023	0.024	n/a	0.037	0.031	0.013
95	0.029	0.039	0.039	0.039	0.045	0.051	0.035	0.035	0.023	0.025	n/a	0.037	0.033	0.009
$\%_{R^2}$		0%	0%	0%	0%	0%	37%		97%	100%	89%	4%	69%	100%
$\%_{RMSE}$		17%	18%	17%	0%	0%	69%		97%	100%	89%	53%	96%	100%
	Neodymium							*Yttrium*						
1	0.236	0.191	0.199	0.194	0.194	0.173	0.184	0.039	0.057	0.048	0.076	0.037	0.073	0.042
25	0.078	0.074	0.076	0.035	0.064	0.045	0.048	0.027	0.020	0.024	0.016	0.026	0.022	0.008
48	0.057	0.056	0.057	0.048	0.049	0.032	0.037	0.042	0.037	0.039	0.035	0.039	0.046	0.017
71	0.059	0.054	0.057	0.048	0.048	0.030	0.039	0.048	0.045	0.046	0.043	0.046	0.054	0.017
95	0.135	0.136	0.136	0.118	0.130	0.096	0.218	0.016	0.005	0.009	0.009	0.007	0.011	0.001
$\%_{R^2}$		93%	93%	100%	95%	100%	85%		93%	93%	93%	96%	60%	93%
$\%_{RMSE}$		97%	98%	100%	97%	100%	85%		93%	93%	93%	98%	64%	94%
	EWI							*VWI*						
1	0.167	0.168	0.168	0.161	0.153	0.154	0.156	0.278	0.234	0.244	0.234	0.230	0.196	0.255
25	0.078	0.077	0.077	0.072	0.072	0.071	0.067	0.103	0.085	0.091	0.084	0.077	0.061	0.091
48	0.044	0.044	0.044	0.039	0.043	0.041	0.045	0.082	0.065	0.071	0.064	0.060	0.047	0.075
71	0.050	0.050	0.050	0.039	0.048	0.046	0.044	0.086	0.065	0.072	0.064	0.061	0.045	0.072
95	0.067	0.066	0.066	0.053	0.069	0.066	0.053	0.152	0.109	0.120	0.109	0.105	0.077	0.196
$\%_{R^2}$		75%	74%	98%	77%	100%	91%		100%	100%	100%	100%	100%	83%
$\%_{RMSE}$		100%	100%	99%	96%	100%	100%		100%	100%	100%	100%	100%	85%

Table 5: Summary of Results

This table provides an overview of Tables 2, 3, and 4 for the four individual elements (cerium, lanthanum, neodymium, and yttrium) and the two REE indices (EWI and VWI). "Long-Memory" indicates whether the fractional differencing parameter d obtained using Sowell's (1992) maximum likelihood estimator is different from 0 at least at the 10% level for volatility. "ARFIMA$(0, d, 0)$ +AR/+MA" indicates whether adding AR or MA terms to the baseline model (ARFIMA$(0, d, 0)$) results in an improvement. "Preferred Model" indicates the best in-sample (based on Akaike (1974) and Schwarz (1978) information criterion) and out-of-sample (based on R^2 and RMSE) performing model/model specification.

REEs and REE Indices	Long-Memory	ARFIMA$(0, d, 0)$ +AR	ARFIMA$(0, d, 0)$ +MA	Preferred Model In-Sample	Preferred Model Out-of-Sample
Cerium	Yes	No	No	GARCH	Indeterminate
Lanthanum	Yes	No	No	GARCH	ARFIMA$(0, d, 0)$
Neodymium	Yes	No	No	ARFIMA$(0, d, 0)$	ARFIMA$(0, d, 0)$
Yttrium	Yes	Yes	Yes	GARCH	ARFIMA$(0, d, 0)$
EWI	Yes	No	No	ARFIMA$(0, d, 0)$	ARFIMA$(0, d, 0)$
VWI	Yes	No	No	ARFIMA$(0, d, 0)$	ARFIMA$(0, d, 0)$

Table 6: Pearson Correlation Coefficients of REE Companies for Trading Strategy

This table gives the mean, minimum, and maximum of the annually calculated Pearson correlations for the weekly returns of REE companies. Companies are selected for the trading strategy based on their highest positive and absolute (positive as well as negative) correlations with the REE equally weighted index (EWI) and the usage-weighted index (VWI), respectively, for the January 2007-December 2014 period. # refers to the number of REE companies considered and ultimately used to calculate the mean, minimum, and maximum, and employed in the trading strategy.

Correlation	REE Index	#	Mean	Minimum	Maximum
$\rho^+_{i,REER,Y}$	EWI	3	0.398	0.177	0.546
		5	0.381	0.170	
		8	0.360	0.134	
		10	0.350	0.122	
	VWI	3	0.373	0.230	0.503
		5	0.357	0.185	
		8	0.340	0.152	
		10	0.331	0.144	
$\lvert\rho_{i,REER,Y}\rvert$	EWI	3	0.494	0.406	0.756
		5	0.476	0.389	
		8	0.457	0.355	
		10	0.446	0.342	
	VWI	3	0.477	0.365	0.712
		5	0.462	0.335	
		8	0.442	0.325	
		10	0.432	0.314	

Table 7: Results of Trading Strategy versus Buy-and-Hold Strategy for T = 300 Model

This table shows the results for the volatility trading strategies based on the one-step-ahead ARFIMA$(0, d, 0)$ volatility forecasts using absolute returns of the REE equally weighted (EWI) and usage-weighted (VWI) indices, respectively. Panel A gives the trading results for REE companies selected based on the highest positive $\rho^+(i, REE)_Y$ and highest absolute correlation $|\rho(i, REE)_Y|$ of REE companies with the EWI REE index. Panel B gives the trading results for the REE companies selected based on correlations with the VWI REE index. # refers to the number of REE companies traded in the strategy. ς refers to the sensitivity to changes in volatility to obtain a trading signal. Total return denotes the cumulative returns of the buy-and-hold and trading strategies, respectively, for the June 2011-February 2015 period. Sharpe ratio refers to the annualized Sharpe ratio for the buy-and-hold and trading strategies, respectively, with a risk-free rate equal to 0%. Δ Sharpe ratio refers to the difference between the respective trading strategy's Sharpe ratio minus the buy-and-hold Sharpe ratio of the buy-and-hold portfolio. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A		**Buy-and-Hold**		**Trading on $\rho^+(i, REE)_Y$**			**Buy-and-Hold**		**Trading on $\lvert\rho(i, REE)_Y\rvert$**		
#	**ς**	***Total Return***	***Sharpe Ratio***	***Total Return***	***Sharpe Ratio***	***Δ Sharpe Ratio (Trading - Buy and Hold)***	***Total Return***	***Sharpe Ratio***	***Total Return***	***Sharpe Ratio***	***Δ Sharpe Ratio (Trading - Buy and Hold)***
						EWI					
	5%			82.78%	0.606	0.337***			31.28%	0.267	0.681***
3	10%	33.14%	0.269	80.47%	0.606	0.336***	-36.50%	-0.414	37.35%	0.315	0.729***
	15%			96.61%	0.703	0.434***			37.79%	0.323	0.737***
	5%			68.82%	0.645	0.426***			4.58%	0.051	0.503***
5	10%	21.98%	0.219	56.56%	0.563	0.344***	-36.16%	-0.452	7.31%	0.081	0.533***
	15%			71.51%	0.694	0.475***			7.21%	0.082	0.533***
	5%			34.71%	0.425	0.134			-12.22%	-0.171	0.243**
8	10%	25.42%	0.292	26.79%	0.349	0.057	-29.78%	-0.414	-9.10%	-0.127	0.286***
	15%			35.06%	0.453	0.161*			-6.56%	-0.093	0.321***
	5%			21.97%	0.306	-0.012			-22.97%	-0.376	0.164*
10	10%	25.71%	0.318	17.10%	0.251	-0.067	-35.31%	-0.540	-18.82%	-0.305	0.234**
	15%			22.82%	0.335	0.017			-17.46%	-0.289	0.25**
Panel B		**Buy-and-Hold**		**Trading on $\rho^+(i, REE)_Y$**			**Buy-and-Hold**		**Trading on $\lvert\rho(i, REE)_Y\rvert$**		
						VWI					
	5%			0.39%	0.004	0.084			-7.59%	-0.088	0.53***
3	10%	-7.15%	-0.079	8.91%	0.098	0.178*	-45.46%	-0.618	-17.23%	-0.218	0.400***
	15%			2.86%	0.033	0.113			-13.73%	-0.174	0.444***
	5%			29.11%	0.281	0.279**			15.52%	0.155	0.696***
5	10%	0.25%	0.003	37.23%	0.357	0.354***	-42.22%	-0.541	2.90%	0.031	0.572***
	15%			29.44%	0.296	0.293**			1.61%	0.018	0.559***
	5%			16.12%	0.180	0.228**			9.29%	0.111	0.728***
8	10%	-4.18%	-0.049	19.73%	0.223	0.272**	-42.26%	-0.617	-4.43%	-0.059	0.558***
	15%			17.80%	0.208	0.257**			-2.73%	-0.037	0.581***
	5%			-0.35%	-0.005	0.021			5.36%	0.071	0.526***
10	10%	-2.07%	-0.026	1.54%	0.020	0.046	-32.09%	-0.454	-8.38%	-0.125	0.329***
	15%			4.45%	0.058	0.084			-3.12%	-0.046	0.408***

Highlights

- We document the existence of long-memory effects in the volatility of rare earth elements (REEs)
- A comparison of the suitability of short-memory models (ARMA), long-memory models (ARFIMA), and a GARCH model to describe the volatility of REEs reveals that a simple ARFIMA$(0, d, 0)$ model shows generally superior accuracy
- Results hold for in- and out-of-sample, and are robust for various subsamples and estimation windows
- Volatility forecasts produced by the ARFIMA$(0, d, 0)$ model convey material forward-looking information for companies in the REEs industry