Multi-Criteria Decision Making under Uncertain Evaluations

Hesham Fahad A. Maghrabie

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Signed by the final examining committee:

____ Chair
Dr. Ketra Schmitt

____ External Examiner
Dr. George Adbul-Nour

____ External to Program
Dr. Onur Kuzgunkaya

____ Examiner
Dr. Amin Hammad

____ Examiner
Dr. Nizar Bouguila

____ Thesis Co-Supervisor
Dr. Andrea Schiffauerova

____ Thesis Co-Supervisor
Dr. Yvan Beauregard

Approved by ____
Dr. Chadi Assi, Graduate Program Director

Thursday, September 6, 2018 ____
Dr. Amir Asif, Dean
Faculty of Engineering and Computer Science

Abstract

Multi-Criteria Decision Making under Uncertain Evaluations

Hesham Fahad A. Maghrabie, Ph.D.
Concordia University, 2018

Multi-Criteria Decision Making (MCDM) is a branch of operation research that aims to empower decision makers (DMs) in complex decision problems, where merely depending on DMs judgment is insufficient. Conventional MCDM approaches assume that precise information is available to analyze decision problems. However, decision problems in many applications involve uncertain, imprecise, and subjective data.

This manuscripts-based thesis aims to address a number of challenges within the context of MCDM under uncertain evaluations, where the available data is relatively small and information is poor. The first manuscript is intended to handle decision problems, where interdependencies exist among evaluation criteria, while subjective and objective uncertainty are involved. To this end, a new hybrid MCDM methodology is introduced, in which grey systems theory is integrated with a distinctive combination of MCDM approaches. The emergent ability of the new methodology should improve the evaluation space in such a complex decision problem.

The overall evaluation of a MCDM problem is based on alternatives evaluations over the different criteria and the associated weights of each criterion. However, information on criteria weights might be unknown. In the second manuscripts, MCDM problems with completely unknown weight information is investigated, where evaluations are uncertain. At first, to estimate the unknown criteria weights a new optimization model is proposed, which combines the maximizing deviation method and the principles of grey systems theory. To evaluate potential alternatives under uncertain evaluations, the Preference Ranking Organization METHod for Enrichment Evaluations approach is extended using degrees of possibility.

In many decision areas, information is collected at different periods. Conventional MCDM approaches are not suitable to handle such a dynamic decision problem. Accordingly, the third manuscript aims to address dynamic MCDM (DMCDM) problems with uncertain evaluations over different periods, while information on criteria weights and the influence of different time periods are unknown. A new DMCDM is developed in which three phases are involved: (1) establish priorities among evaluation criteria over different periods; (2) estimate the weight of vectors of different time periods, where the variabilities in the influence of evaluation criteria over the different periods are considered; (3) assess potential alternatives.

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Dedication

To the soul of my father, Mr. Fahad Maghrabie; my mother Mrs. Manal Mattar; my soulmate, Dr. Hanadi Habibullah; and my lovely children, Yousif and Manal; for their boundless love, endless support, unlimited motivation, and continuous inspiration.

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List of Abbreviations

λ_{max}	Largest eigenvalue
AHP	Analytic Hierarchy Process
ANP	Analytical Network Process
BUM	Basic Unit-interval Monotonic
CI	Consistency Index
CR	Consistency Ratio
DFMCDM	Dynamic Fuzzy Multiple Criteria Decision Making
DGMCDM	Dynamic Grey Multi-Criteria Decision Making
DIFWA$^{\epsilon}$	Dynamic Intuitionistic Fuzzy Einstein Averaging
DIFWG$^{\epsilon}$	Dynamic Intuitionistic Fuzzy Einstein Geometric Averaging
DIFWG	Dynamic Intuitionistic Fuzzy Weighted Geometric
DMCDM	Dynamic Multi-Criteria Decision Making
DMs	Decision Makers
ELECTRE	ELimination and Choice Expressing REality

FAHP	Fuzzy Analytic Hierarchy Process
FEI	Front End Innovation
GANP	Grey Analytical Network process
GRA	Grey Relational Analysis
GWSM	Grey Weighted Sum Model
LINMAP	Linear Programming Technique for Multidimensional Analysis of Preference
MADA	Multi-Attribute Decision Analysis
MAUT	Multi-Attribute Utility Theory
MAVT	Multi-Attribute Value Theory
MCA	Multi-Criteria Analysis
MCDA	Multi-Criteria Decision Analysis
MCDM	Multi-Criteria Decision Making
PROMETHEE II	Preference Ranking Organization METHod for Enrichment Evaluation II
RI	Consistency Index of a Random-like Matrix
SMAA	Stochastic Multi-criteria Acceptability Analysis

SMART	Simple Multi-Attribute Rating Technique
TOPSIS	Technique for Order Preference by Similarity to Ideal Solution
UDIFWG	Uncertain Dynamic Intuitionistic Fuzzy Weighted Geometric
VIKOR	Vlsekriterijumska Optimizacijia I Kompromisno Resenje
WSM	Weighted Sum Method

Chapter 1

Introduction

1.1 Multi Criteria Decision Making

Decision making is an inherent character of human nature. The ultimate goal of any decision maker is to make the right decision. Sometimes the process of undertaking the right decision becomes a challenging task, especially when they encounter large amount of complex information. Therefore, tremendous efforts have been dedicated to enrich the decision making process by introducing multi-criteria analysis (MCA) approaches. The main role of MCA is to handle the associated difficulties with decision problems where the ability of decision makers (DMs) are deemed insufficient by itself to address the decision problems (Dodgson, Spackman, Pearman, & Phillips, 2009). MCA can be defined as structured approaches for DMs to have a thorough analysis over decision problems by analyzing and evaluating potential alternatives over a set of criteria (Antunes & Henriques, 2016; Kurka & Blackwood, 2013).

Many approaches of MCA are available in the literature. However, different reasons justify the existing of the different approaches (Dodgson et al., 2009):

- the dissimilarity among decisions in nature and purpose
- the available time for making a decision
- the nature of the data and its availability
- the analytical skills of the DMs
- the various type of administrative culture.

Among the popular approaches of MCA is the multi-criteria decision making (MCDM), also known as multi-criteria decision analysis (MCDA) and multi-attribute decision analysis (MADA) (Dodgson et al., 2009). The main role of MCDM is to aid DMs in establishing a coherent picture about complex decision problems, e.g. decision problems that incorporate monetary and non-monetary criteria (Kurka & Blackwood, 2013; Malczewski & Rinner, 2015; Roy, 2016; Wątróbski & Jankowski, 2016). Moreover, it simplifies the analysis of a decision problem by disaggregating the original problem into more manageable elements (Dodgson et al., 2009; Kurka & Blackwood, 2013).

A defined decision making problem can be categorized based on the problematic nature as follows (Greco, Ehrgott, & Figueira, 2016; Wątróbski & Jankowski, 2016):

1. Choice problematic: Finding the best alternative
2. Sorting problematic: Sorting alternatives into defined categories
3. Ranking problematic: Ranking alternatives from the best to the worst.

Despite the diversity of MCDM approaches, at the most primitive level, MCDM can be demonstrated by a set of alternatives, at least two evaluation criteria, and minimally one decision maker (Greco et al., 2016).

1.2 Uncertainty in MCDM

Conventional MCDM approaches presume that the required information for analyzing a decision problem is available and accurate. However, in many real life applications available information is subject to uncertainty, imprecision, and subjectivity; which would limit the applicability of conventional MCDM approaches (Banaeian, Mobli, Fahimnia, & Nielsen, 2018; Karsak & Dursun, 2015; Guangxu Li, Kou, & Peng, 2015; Małachowski, 2016). Consequently, different theories have been introduced to approximate ranges of evaluations using the related knowledge and the available information of the decision problem under consideration (Lin, Lee, & Ting, 2008).

The different approaches to address uncertainty have different use, which involves the way of establishing preferences within a decision problem and the way of representing possible outcomes (Durbach & Stewart, 2012). Therefore, differentiating among two types of uncertainty would be useful to properly handle the associated uncertainty in a decision problem. The two types of uncertainty are: (i) uncertainty associated with limited objective information, e.g., quantitative (interval scales) and stochastic (probability distribution) data, and (ii) uncertainty associated with subjective expert knowledge (i.e., ambiguous concepts and semantic meanings) (Malczewski & Rinner, 2015; Moretti, Öztürk, & Tsoukiàs, 2016).

1.3 MCDM with unknown weight information

Solving a MCDM requires information on alternatives evaluations over the different criteria and the associated weight of each criterion. Within the context of uncertain MCDM problems, decision makers may encounter decision problems with unknown criteria weights, as a result of different reasons such as time pressure, limited expertise, incomplete knowledge, and lack of information (Das, Dutta, & Guha, 2016; S. Zhang, Liu, & Zhai, 2011). Accordingly, the overall evaluations cannot be derived (Xu, 2015).

1.4 Dynamic multi-criteria decision problems

Conventional MCDM approaches have an implicit assumption in which decision problems are static overtime (single period) (Pruyt, 2007). However, in many real-life applications this assumption becomes inappropriate as decision information is provided at different periods such as multi-period investment and medical diagnosis (Eren & Kaynak, 2017; G. Wei, 2011). In such decision areas, the complexity of decision making process increases and requires the consideration of different evaluations over the different periods.

1.5 Research Motivation

The associated uncertainty with many real-life applications of MCDM problems complicates the decision-making process, in which it renders conventional MCDM approaches to be incapable of addressing the multi-criteria decision problems (Banaeian et al., 2018; Karsak & Dursun, 2015; Guangxu Li et al., 2015; Małachowski, 2016). Consequently, different hybrid methodologies have been introduced, in which MCDM approaches have been supplemented by different methods to handle the different types of uncertainty; such as probabilistic models, fuzzy set theory, and grey systems theory. However, the existing methodologies are limited when it comes to handle MCDM problems with a relatively small amount of data and poor information, where evaluation criteria are of different nature and interdependencies exist among them.

As mentioned earlier, information on criteria weights is critical to solve MCDM problems (Xu, 2015). However, in many decision problems criteria weights are unknown (Das et al., 2016; S. Zhang et al., 2011). Therefore, different approaches have been introduced to establish the unknown criteria weights, which are listed in Chapter 3. Nevertheless, the applicability of these approaches would be influenced when it comes to handle MCDM problems with small amount of data and poor information.

In many decision areas information at different periods should be considered, such as multi-period investment and medical diagnosis (G. Wei, 2011). Conventional MCDM approaches are static and cannot handle dynamic decision problems. Different hybrid approaches have been developed to overcome the shortcoming of the conventional approaches, which are mentioned in Chapter 4. However, the developed approaches lack of a proper procedure to address one or both of the following concerns: (1) Criteria weights establishment, where weight information is unknown; (2) Establishing priorities of different periods, where the influence on a decision problem of different evaluation criteria are changing over time.

In view of the aforementioned limitations, it is important to establish a decision model that consider the interdependencies among evaluation criteria within the context of MCDM problems with a relatively small amount of data and poor information. For decision problems with unknown criteria weights a better method is needed to establish unknown criteria weights, where information is poor and the available data is relatively small. In a multi-period MCDM where priority information of different periods is unknown, the changing in the influence of the different evaluation criteria over different periods should be considered in solving the dynamic decision problems.

1.6 Research Objectives

This manuscript-based thesis consists of three journal papers, each of which will be discussed in a separate chapter. The objectives of each paper are as follows:

- Paper 1: *Grey-based Multi-Criteria Decision Analysis Approach: Addressing Uncertainty at Complex Decision Problems*

The ultimate goal of this research is to optimize the evaluation space in complex MCDM that are subject to subjective and objective uncertainty over different types of interrelated criteria. To this end, the main objective of this paper are as follows: prioritize interrelated criteria of different nature, while uncertainty related aspects are present; evaluate different alternatives over complex MCDM problems under subjective and objective uncertainty.

- Paper 2: *Multi-Criteria Decision-Making Problems with Unknown Weight Information under Uncertain Evaluations*

The aim of this paper is to handle MCDM problems with small amount of data and poor information, where information of criteria weights is unknown. Therefore, the objectives of this paper are as follows: establish a new optimization model to estimate priorities among the evaluation criteria under such a decision problem; evaluate and rank potential alternatives, where uncertain information with respect to alternatives evaluations is provided.

- Paper 3: *A New Approach to Address Uncertain Dynamic Multi-Criteria Decision Problems with Unknown Weight Information*

This paper aims to address a dynamic MCDM, where evaluations are given over different periods, while information on criteria weights and the influence of different time periods are unknown. Consequently, the objectives of this paper are as follows: prioritize criteria weights over the different periods; establish weight vectors of different periods while considering the variabilities in the influence of different criteria over different periods.

1.7 Organization of the thesis

This manuscript-based thesis is divided into five chapters. Chapter 1 provides a brief background on related subject matters to introduce the motivations and objectives of this thesis. The objectives of this thesis have been carried-out through Chapters 2 to 4, as outlined below:

Chapter 2: *Grey-based Multi-Criteria Decision Analysis Approach: Addressing Uncertainty at Complex Decision Problem.* In this chapter, a new hybrid MCDM model is developed to better address complex decision problems with relatively small amount of data and poor information. The proposed methodology considers the interdependencies among the evaluation criteria of different clusters while establishing criteria weights. Moreover, it extends the approach of Preference Ranking Organization METHod for Enrichment Evaluation II (PROMETHEE II), in which it would be utilized to define optimal ranking among potential alternatives in complex decision problem under subjective and objective uncertainty.

Chapter 3: *Multi-Criteria Decision-Making Problems with Unknown Weight Information under Uncertain Evaluations*. This chapter introduces a new methodology to carry out the overall evaluation for MCDM problems with small amount of data and poor information, where information on criteria weight is unknown completely. A new optimization model is developed to establish criteria weights, while different scenarios are considered to generalize the model.

Chapter 4: *A New Approach to Address Uncertain Dynamic Multi-Criteria Decision Problems with Unknown Weight Information*. A new methodology is proposed in this chapter to account for the dynamic aspect of a MCDM, where evaluations from different periods are provided, while information on criteria weights and the influence of the different periods are unknown completely. The new approach pays attention to the variabilities in the influence of different evaluation criteria on a MCDM problem over different periods.

Finally, Chapter 5 provides a summary of this thesis including the concluding remarks, contributions and ideas for future research. The analytical framework of the thesis is illustrated in Figure 1.1.

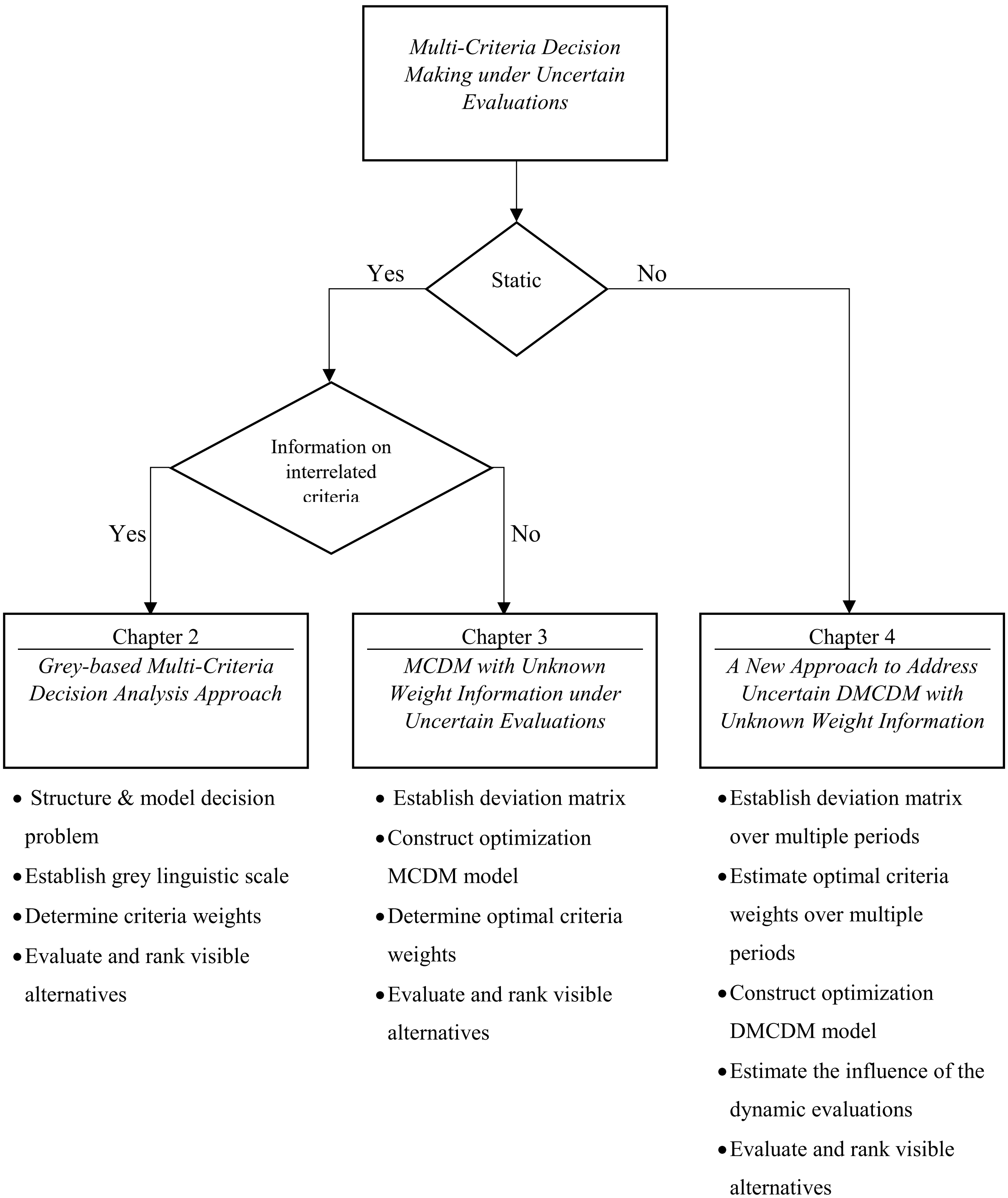

Figure 1.1: Thesis framework.

Chapter 2

Grey-based Multi-Criteria Decision Analysis Approach: Addressing Uncertainty at Complex Decision Problems

Abstract

In complex systems, decision makers encounter uncertainty from various sources. Grey systems theory is recommended to address uncertainty for decision problems with a relatively small amount of data (i.e., small samples) and poor information, which cannot be described by a probability distribution. However, the existing approaches do not consider the influence among criteria of different clusters. Accordingly, a new hybrid grey-based Multi-Criteria Decision Analysis (MCDA) approach is proposed to optimize the evaluation space in decision problems that are subject to subjective and objective uncertainty over different types of interrelated criteria. The four-phased methodology begins with the formulation of a decision problem through the analysis of the system of concern, its functionality, and substantial connections among criteria. The second phase involves the development of grey linguistic scales to handle uncertainty of human judgments. The third phase integrates the grey linguistic scale, concepts of grey systems theory, and principles of Analytical Network Process to prioritize the evaluation criteria. Finally, to evaluate and rank alternatives in such a complex setting, the Preference Ranking Organization METHod for Enrichment Evaluation II approach is extended using a grey linguistic scale to articulate subjective measures over qualitative criteria, grey numbers to account for objective uncertainty over quantitative criteria, grey operating rules to normalize evaluation measures, and the proposed approach of prioritizing the criteria to establish relative preferences. To demonstrate the viability of the methodology, a case study is presented, in which a strategic decision is made within the context of innovation. To validate the methodology, a comparative analysis is provided.

2.1 Introduction

Decision makers usually encounter large amount of complex information. The complexity of decision problems increases when different evaluation criteria of different nature (e.g., qualitative and quantitative), different scales, and different values (e.g., continuous, discrete, and linguistic) are involved.

Multi-Criteria Decision Analysis (MCDA) is therefore considered one of the most fruitful sub-disciplines of operations research. The main role of MCDA is to aid Decision Makers (DMs) in establishing a coherent picture about complex decision problems (Kurka & Blackwood, 2013). However, in many cases uncertainty-related aspects (i.e., uncertainty associated with limited objective information and uncertainty associated with subjective expert knowledge) are present. This adds to the complexity of analyzing the decision problems as the conventional MCDA approaches presume the availability of precise information (Kuang, Kilgour, & Hipel, 2015; Guo-dong Li, Yamaguchi, & Nagai, 2007).

Various methods have been proposed to deal with different types of uncertainty-related aspects. Grey systems theory is recommended for decision problems with a relatively small amount of data (i.e., small samples) and poor information, which cannot be described by a probability distribution (C. Li & Yuan, 2017; D.-C. Li, Chang, Chen, & Chen, 2012; S. Liu & Lin, 2006). Accordingly, different researchers have considered the grey systems theory to address uncertainty in decision problems, as presented in the next section. The existing approaches assumed that DMs are able to assign the weights of the evaluation criteria precisely, did not consider the interrelationships among evaluation criteria, or did not consider the relations among the criteria of different clusters, hence a better method is needed to address the existing research gaps.

The ultimate goal of this research is to enhance DMs abilities of handling multi-criteria decision problems under uncertainty. To this end, the main objective of this manuscript is to establish a structured methodology, which are able to carry on MCDA under uncertainty, by integrating the grey systems theory with a distinctive combination of MCDA techniques (i.e., Analytical Network Process (ANP) and Preference Ranking Organization METHod for

Enrichment Evaluation II (PROMETHEE II)). The hybrid methodology uses the grey systems theory as the key element for tackling uncertainty aspects; the principles of ANP to handle the complexity of the decision structure; the extended PROMETHEE II approach to evaluate feasible alternatives.

The contributions of this manuscript over other existing research works within the same context of MCDM problems can be summarized in the following points: (1) Establishing priorities among sub-criteria within a complex structure under uncertain subjective judgments using the combination of linguistic expression, grey systems theory, and the principles of ANP; (2) Extending PROMETHEE II, such that potential alternatives can be evaluated and ranked in such a complicated decision structure; (3) Improving the evaluation space in a complex decision problems under uncertainty by utilizing the emergent strengths of the integrated approach, which would enhance the evaluation of a DM.

This manuscript is organized as follows: first, a brief background on related subject matters is provided to identify the research problems and to establish the direction of the current research; next, the proposed methodology is discussed and explained; afterwards, a case study is presented to demonstrate the viability of the methodology; then, a comparative analysis with an existing approach is performed for the validation purpose; finally, the conclusion is put forward.

2.2 Background

2.2.1 Multi-criteria decision analysis

Despite the diversity of MCDA approaches, at the most primitive level, MCDA can be demonstrated by a set of alternatives, at least two evaluation criteria, and minimally one decision maker (Greco et al., 2016). Accordingly, MCDA can be described as a systematic methodology that helps in making decisions by evaluating a number of alternatives over a set of criteria according to the preferences of the involved decision maker(s).

There is no optimal MCDA's approach that would fit perfectly with every decision problem. Therefore, understanding a decision problem's nature is a critical step to identify the suitable approach for it (Jaini & Utyuzhnikov, 2017; Wątróbski & Jankowski, 2016). The various approaches of MCDA can be classified into three main categories (Belton & Stewart, 2002):

- **Value measurement models**: Approaches that belong to this category are value-focused, where the utility value of each alternative is being recognized based on its overall performance over the evaluation criteria. Among the most common approaches within this category are Analytic Hierarchy Process (AHP), Analytic Network Process (ANP), Multi-Attribute Value Theory (MAVT), Multi-Attribute Utility Theory (MAUT), Simple Multi-Attribute Rating Technique (SMART), Stochastic Multi-criteria Acceptability Analysis (SMAA), and Weighted Sum Method (WSM).
- **Goal, aspiration, or reference-level models**: In this set of approaches, alternatives are evaluated with respect to a targeted level of performance over a particular goal, aspiration, or reference levels, e.g., goal programming and heuristic algorithms. An example of this category is Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), Vlsekriterijumska Optimizacijia I Kompromisno Resenje (VIKOR), and Linear Programming Technique for Multidimensional Analysis of Preference (LINMAP).
- **Outranking methods**: A typical outranking approach performs pairwise comparisons between alternatives across a specified set of evaluation criteria. Subsequently, the resulting comparisons are aggregated and analyzed in accordance with the designated approach to favor one alternative over another. Outranking methods include ELimination and Choice Expressing REality (ELECTRE), and Preference Ranking Organization METHod for Enrichment Evaluations (PROMETHEE) family of methods.

These conventional approaches of MCDA have an implicit assumption, which presumes the availability and accuracy of information that is required for analyzing decision problems. However, in real world applications, DMs encounter uncertainty from various sources, such as limited human cognition, lack of understanding for interrelationships among decision criteria, and limited input data (Belton & Stewart, 2002; Małachowski, 2016).

2.2.2 Handling uncertainty in MCDA

While the presence of uncertainty would limit the utilization of the MCDA approaches, several research works have proposed different hybrid approaches, in which MCDA techniques have been supplemented by uncertainty approaches. However, different uncertainty approaches have different use, which involves the way of establishing preferences within a decision problem and the way of representing possible outcomes (Durbach & Stewart, 2012). Therefore, differentiating among two types of uncertainty would be useful to properly address the associated uncertainty in a decision problem: (i) uncertainty associated with limited objective information, e.g., quantitative (interval scales) and stochastic (probability distribution) data, and (ii) uncertainty associated with subjective expert knowledge (i.e., ambiguous concepts and semantic meanings) (Malczewski & Rinner, 2015; Moretti et al., 2016). Different approaches have been proposed to handle different types of uncertainty:

Probabilistic models: A DM can assign probability distribution based on a relative experiences, beliefs, and available data to describe uncertain values (i.e., imperfect information) of a decision parameters (Mazarr, 2016). Consequently, comparisons can be established among feasible alternatives and probabilistic statements can be made to describe the probability of occurrence for each outcome, which can be achieved through different means (e.g., Monte Carlo simulation) (Malczewski & Rinner, 2015; Zhou, Wang, & Zhang, 2017).

Fuzzy set theory: Zadeh (1965) introduced this theory to handle the associated vagueness and imprecision with human judgments (i.e., ambiguous concepts and semantic meanings). Within the context of MCDA, fuzzy numbers are utilized to map linguistic expressions that would express human opinions using the concept of the membership function, such that by assigning a value between 0 and 1 the linguistic term can be stated more precisely, where 0 indicates no membership and 1 indicates full membership (Dincer, Hacioglu, Tatoglu, & Delen, 2016).

Grey systems theory: Ju-long (1982) introduced the grey systems theory as a methodology to handle data imprecision or insufficiency in a system. It is intended for problems that involve a relatively small amount of data and poor information, which cannot be described by a probability distribution. Thus, a better understanding for such a system can be achieved through partially known information using grey systems theory (S. Liu, Forrest, & Yang, 2015). Similar to fuzzy set theory, grey systems theory can handle associated vagueness with verbal statements (linguistic expressions) using grey numbers (Broekhuizen, Groothuis-Oudshoorn, Til, Hummel, & IJzerman, 2015), which is denoted by $\otimes$ (S. Liu & Lin, 2006).

As mentioned earlier, conventional MCDA approaches do not mimic the functional reality of the human cognitive system in decision problems. Therefore, various hybrid approaches have been proposed, in which uncertainty related aspects in MCDA are captured and represented using probabilistic models, fuzzy set theory, and grey systems theory.

Fuzzy set theory is widely utilized with MCDA under uncertainty (Broekhuizen et al., 2015). For instance, fuzzy AHP was used extensively in the literature: environmental impact assessment (Ruiz-Padillo, Ruiz, Torija, & Ramos-Ridao, 2016), investment decisions (Dincer et al., 2016), knowledge management (K.Patil & Kant, 2014); fuzzy ANP has been utilized to determine the most important factors for a hospital information system (Mehrbakhsh, Ahmadi, Ahani, Ravangard, & Ibrahim, 2016); fuzzy TOPSIS has been employed for the selection of renewable energy supply system (Şengül, Eren, Shiraz, Gezderd, & Şengül, 2015); fuzzy VIKOR has been used for evaluating and selecting green supplier development programs (Awasthi & Kannan, 2016); fuzzy PROMETHEE has been utilized to select the best waste treatment solution (Lolli et al., 2016). However, the Probabilistic approach is commonly used with SMAA to provide the specification of distributions (Groothuis-Oudshoorn, Broekhuizen, & van Til, 2017).

Although probabilistic models and fuzzy set theory are intended to investigate uncertain systems, grey systems theory is preferred when it comes to problems with a relatively small amount of data and poor information, which cannot be described by a probability distribution (C. Li & Yuan, 2017; D.-C. Li et al., 2012; S. Liu & Lin, 2006), due to its less restricted procedure that neither requires any robust membership function, nor a probability distribution (Memon, Lee, & Mari, 2015). Consequently, several research papers have proposed grey systems theory to supplement the deficiencies that exist in MCDA as a result of poor information. The rest of this section deliberates on existing methods to solve multi-criteria decision problems under uncertainty using grey systems theory, and the reasoning behind the proposed methodology.

Grey systems theory has been integrated with PROMETHEE II to evaluate performance of available alternatives on certain criteria where uncertainty aspects are involved (Kuang et al., 2015). However, the weights of evaluation criteria are assumed to be given by DMs precisely, which is hardly the case in complex decision problems under uncertainty. Some other works have tried to address this issue by integrating the grey systems theory with Analytic Hierarchy Process (AHP) to prioritize evaluation criteria and to evaluate potential alternatives under uncertainty (Jianbo, Suihuai, & Wen, 2016; Thakur & Ramesh, 2017). Also, Grey Relational Analysis (GRA), which is a branch of grey systems theory, has been combined with the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) approach to better rank feasible alternatives, using fuzzy analytic hierarchy process (FAHP) to evaluate the criteria weights (Celik, Erdogan, & Gumus, 2016; Sakthivel et al., 2014). Nevertheless, one of the underlying assumptions of AHP is the independency (Ishizaka & Labib, 2011), which implies that elements of a hierarchal structure are independent but in reality a complex system usually involves interactions and dependencies among the system's elements.

To tackle the problem of dependencies in a complex system, grey systems theory has been used with ANP. This combination has been proposed in different areas such as, green supplier development programs (Dou, Zhu, & Sarkis, 2014), R&D system development for a home appliances company (Tuzkaya & Yolver, 2015), and early evaluation model for storm tide risk (W. Zhang, Zhang, Fu, & Liu, 2009). However, the relations between sub-criteria of different clusters have not been considered. Accordingly, a better method is needed to bridge the existing research gaps.

In this manuscript, a new hybrid grey-based MCDA approach is proposed to enhance DMs abilities of handling multi-criteria decision problems under uncertainty. The proposed approach integrates the grey systems theory with a distinctive combination of MCDA (i.e., ANP and PROMETHEE II). The combination of the proposed methodology has been considered for the following reasons:

When it comes to performance evaluation of feasible alternatives, outranking approaches outperform other MCDA methodologies, as other methodologies are designed to enrich the dominance graph by reducing the number of incomparability and allocating an absolute utility to each alternative. Consequently, the original structure of a multi-criteria decision problem would be reduced to a single criterion problem for which an optimal solution exists (Maity & Chakraborty, 2015). In contrast, outranking methods preserve the structure of multi-criteria decision problems by considering the deviation between the evaluations of feasible alternatives over each criterion (Andreopoulou, Koliouska, Galariotis, & Zopounidis, 2017; Maity & Chakraborty, 2015; Segura & Maroto, 2017). Moreover, this category of MCDA can handle quantitative and qualitative criteria. Furthermore, it requires a relatively small amount of information from DMs (Malczewski & Rinner, 2015). Among the outranking methods, PROMETHEE is preferred due to its mathematical properties and simplicity (Brans & De Smet, 2016; Kilic, Zaim, & Delen, 2015; Malczewski & Rinner, 2015). Among the PROMETHEE family of methods, PROMETHEE II is preferred due to its ability of providing a complete ranking for available alternatives based on outranking relations (Sen, Datta, Patel, & Mahapatra, 2015). However, PROMETHEE II requires the weights of the evaluation criteria (Brans & De Smet, 2016; Segura & Maroto, 2017).

To estimate criteria weights, ANP is preferred over other MCDA approaches due to its superiority in addressing different types of interrelationships (e.g., interactions and interdependencies) within and between different evaluation clusters of a complex system (Hsu, 2015; Tuzkaya & Yolver, 2015).

While the presence of uncertainty would limit the utilization of the conventional approaches of MCDA, grey systems theory would perfectly bridge this limitation (Dou et al., 2014; Kuang et al., 2015). In particular, when it comes to address decision problems with a relatively small amount of data and poor information, which cannot be described by a probability distribution (D.-C. Li et al., 2012; S. Liu & Lin, 2006).

2.3 Grey-based MCDA methodology (G-ANP-PROMETHEE II)

The proposed decision analysis process (G-ANP-PROMETHEE II) is consisted of four phases: (1) structure and model the decision problem, (2) establish grey linguistic scales, (3) determine the weights of evaluation criteria, and (4) evaluate and rank feasible alternatives. The framework of the proposed methodology is illustrated in Figure 2.1. The procedural steps of G-ANP-

PROMETHEE II are explained in the following subsections.

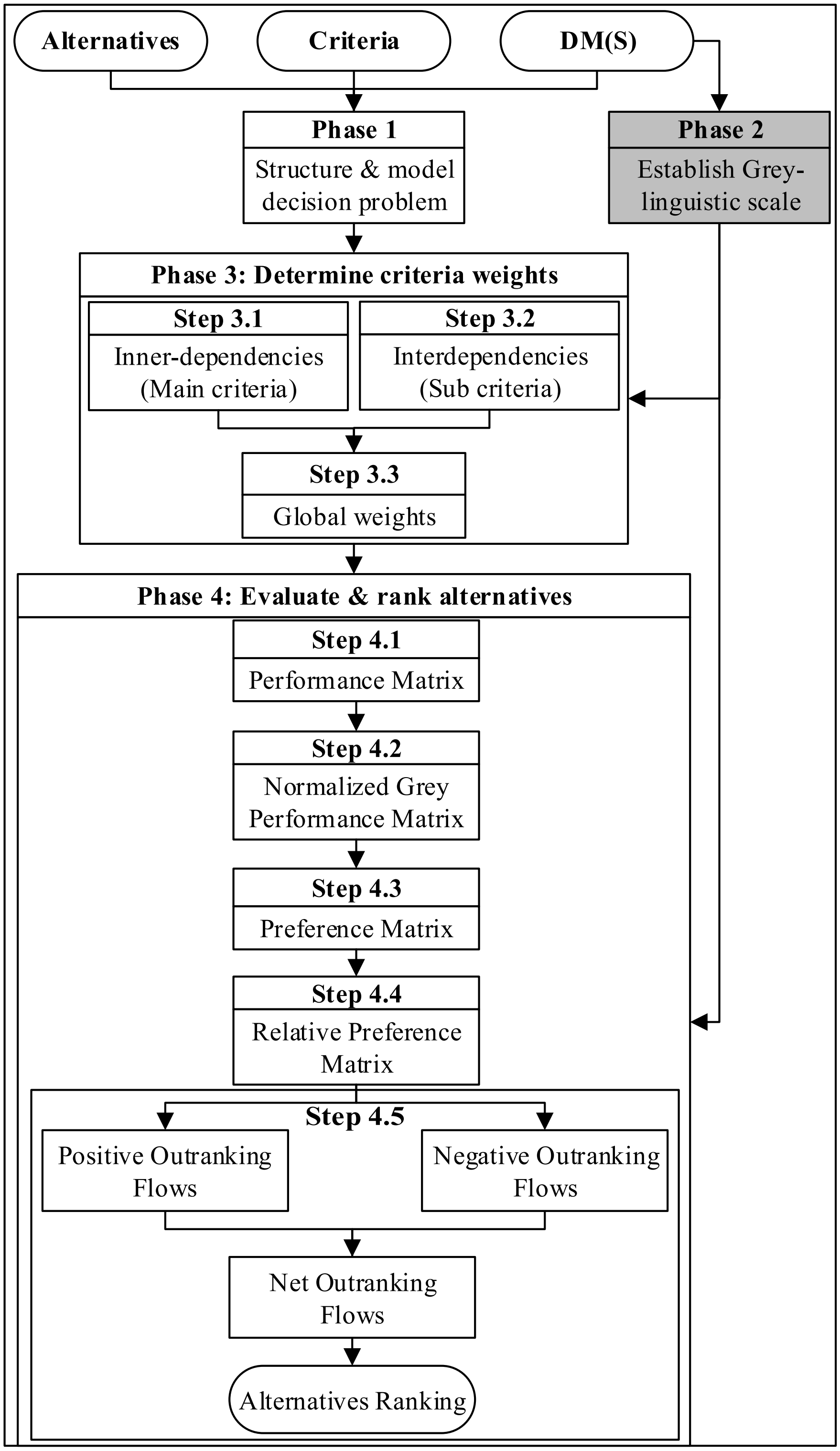

Figure 2.1: Analytic framework of G-ANP-PROMETHEE II.

2.3.1 Structure and model the decision problem

The first phase of the methodology is formulating the problem, which requires analysis for the system of concern, its functionality, and substantial connections (i.e., connections within and between various elements of the system; or between the system, relevant factors, and its environment). Accordingly, the network structure can be used to model the decision problem, as represented by Figure 2.2.

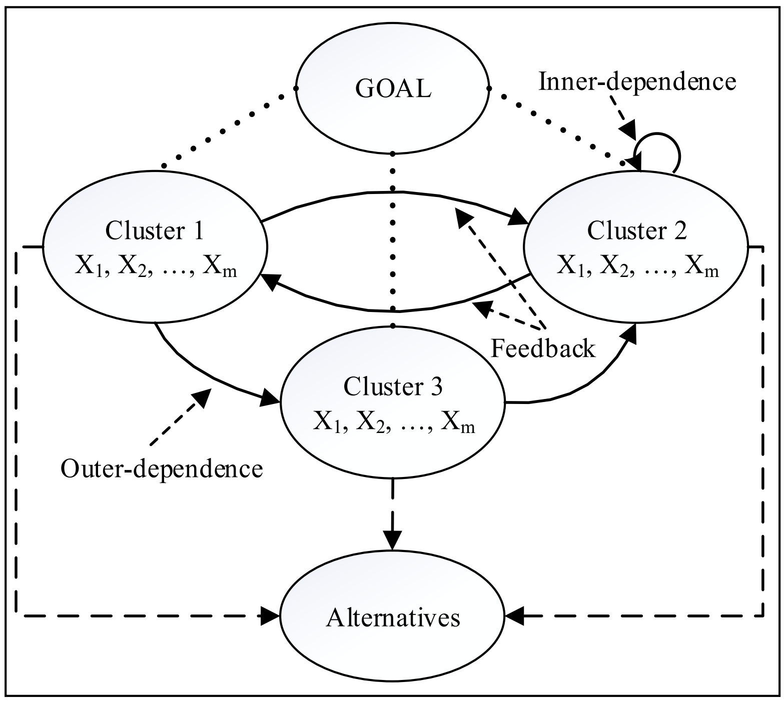

Figure 2.2: Network structure model (Görener, 2012).

2.3.2 Establish grey linguistic scales

Multi-criteria decision problems involve some uncertainty because they are unlikely to fully satisfy decision criteria. Also, it is difficult for DMs to precisely express preferences due to information limitations and the uncertainty of human judgment. Therefore, linguistic expressions are more often used in MCDA to articulate DMs' preferences between evaluation criteria, and to evaluate available alternatives over qualitative criteria (Kuang et al., 2015; Merigó, Palacios-Marqués, & Zeng, 2016).

In this research, the concepts of grey systems theory and linguistic expressions provide the basis for the proposed approach, in which linguistic expressions (e.g., low, medium, and high) are used to express DMs judgments and the grey systems theory is used to handle the associated vagueness with verbal statements through the operating rules of grey numbers.

To express preferences of DMs between evaluation criteria with respect to the system of concern, a grey linguistic scale of six levels is proposed, as illustrated in Table 2.1. The proposed scale has been established in accordance with the research of Ertay, Büyüközkan, Kahraman, and Ruan (2005).

Table 2.1: Pairwise preference scale.

Pairwise linguistic scale	Grey preference scale	Reciprocal grey preference scale
Just equal	[1,1]	[1,1]
Equally important	[1/2,3/2]	[2/3,2]
Weakly more important	[1,2]	[1/2,1]
Moderately more important	[3/2,5/2]	[2/5,2/3]
Strongly more important	[2,3]	[1/3,1/2]
Extremely more important	[5/2,7/2]	[2/7,2/5]

When it comes to the assessment of feasible alternatives over qualitative criteria, it is also expressed in linguistic values using a five-level scale as shown in Table 2.2.

2.3.3 Determine the weights of evaluation criteria

For a complex multi-criteria decision problem (i.e., decision problems that involves interrelationships between evaluation criteria), ANP provides a structured procedure to analyze such complexity in decision problems (Zaim et al., 2014). However, ANP cannot effectively address uncertainty-related issues (Nguyen, Dawal, Nukman, & Aoyama, 2014), which are usually present in real world applications. To overcome this limitation, linguistic expressions and concepts

Table 2.2: Performance evaluation scale over qualitative criteria.

Performance evaluation linguistic scale	Grey evaluation scale
Low (L)	[0,0.2]
Less than moderate (LM)	[0.2,0.4]
Moderate (M)	[0.4,0.6]
More than moderate (MM)	[0.6,0.8]
High (H)	[0.8,1.0]

of grey systems theory are integrated with ANP to establish the set of weights for evaluation criteria. The procedural steps are as follows: (1) determine inner-dependencies among main criteria, (2) examine interdependencies among sub-criteria, and (3) Estimate global weights for sub-criteria using the outputs of steps a and b.

2.3.3.1 Determine inner-dependencies among main criteria

The purpose of inner-dependencies evaluation is to detect the relative importance among various elements of the same level or cluster. This could be achieved by analyzing the influence of an evaluation criterion over other elements of the same level/cluster using linguistic expressions and relative grey numbers, as in Table 2.3, to articulate DMs' preferences between evaluation criteria.

2.3.3.1.1 Establish grey-based pairwise comparison matrices for main criteria

Definition 2.1 Let a set of criteria within a cluster be represented by $C = \{C_1, C_2, \dots, C_m\}$, where m is the number of criteria. Let a_{ij} indicate the existence of an influence relation of criterion C_i over C_j, where

$$a_{ij} = \begin{cases} 1 \leftrightarrow C_i \text{ influences } C_j \vee (\mathrm{i} = \mathrm{j}) \\ 0 \leftrightarrow C_i\ \underline{\boldsymbol{does\ not}} \text{ influence } C_j \end{cases}, \quad i,j = 1,2, \dots m \tag{2.1}$$

Definition 2.2 Let I_k represent a set of evaluation criteria that influence a criterion C_k, where $I_k \subset C$ and $C_k \notin I_k$. Let $R[\otimes]$ represent the set of grey numbers and $T_{k\otimes}$ denote a grey description function that describes the grey-based pairwise comparisons between elements of I_k with respect to C_k, such that

$$T_{k\otimes} : (I_k \times I_k) \rightarrow R[\otimes], \ \forall k = 1, 2, \dots m \tag{2.2}$$

Definition 2.3 Let $k_{ij}(\otimes) \in T_{k\otimes}$ denote a grey number that articulates DMs' verbal preference of C_i over C_j with respect to a control criterion C_k, where both C_i and C_j influence C_k and $k \neq i, j$. Thus,

$$\exists\, k_{ij}(\otimes) \leftrightarrow (a_{ik}, a_{jk} = 1), \ \forall\, T_{k\otimes}\,(C_i,\, C_j), \ where \tag{2.3}$$

$$k_{ij}(\otimes) = \begin{cases} k_{ji}(\otimes)^{-1} \leftrightarrow \ i \neq j \\ [1,1], \text{otherwise} \end{cases}, i, j, k = 1, 2 \dots, m$$

2.3.3.1.2 Estimate inner-dependence weights

To estimate the inner-dependence weights among evaluation criteria, the grey numbers at the grey-based pairwise comparisons matrices need to be transformed to white numbers (i.e., single values). To do this, the whitenization process should be performed.

The weight function of the whitenization process is decided based on the available information of the relative grey numbers (e.g., distribution information), knowledge, and experience of decision makers (S. Liu & Lin, 2006).

Definition 2.4 Assume that a whitenization function for a relative grey number $x(\otimes)$ is $f(x_i)$, then the whitenization value $x_i(\tilde{\otimes})$ can be defined as (Shi, Liu, & Sun, 2013):

$$x(\tilde{\otimes}) = x \ . \ f(x) \tag{2.4}$$

In many practical applications, the weight function of the whitenization process is unknown (S. Liu & Lin, 2006), which adds complexity to decision problems. Therefore, Liu and Lin (2006) proposed the equal weight mean whitenization function to establish the associated white values of interval grey numbers. In this research, the interval grey numbers are used and it is assumed that the weight function of the whitenization process is unknown due to the lack of information. Therefore, the equal weight mean whitenization function is considered for the whitenization process as follows:

Definition 2.5 Let $x(\otimes) \in [a, b]$ be a general interval grey number, where $a < b$ and the distribution information for the grey number is unknown. Let α denote the weight coefficient. The whitenization value $x(\tilde{\otimes})$ can be obtained using the equal weight mean whitenization, such that

$$x(\tilde{\otimes}) = \alpha a + (1 - \alpha)b, \text{ where } \alpha = \frac{1}{2} \tag{2.5}$$

Once the inner-dependence grey matrices have been transformed to fixed number matrices through the whitenization process, the inner-dependence weighs of the evaluation criteria can be estimated using the computation of the eigenvector method (R. Saaty, 2013). However, the resultant weights should be consistent relatively. In other words, if a criterion $C_a \succ C_b$, and $C_b \succ C_d$, the following can be inferred $C_a \succ C_d$; this is called "transitive law". To this end consistence test should be applied as follows:

Definition 2.6 Suppose the number of compared elements is m. Let a_{ij} denote the preference of C_i over $C_j, i, j = (1, 2 \ldots, m)$. Let s_j denote the sum of the corresponding column element a_{ij}. Let $w = (w_1, w_2, \ldots w_m)$ represent the eigenvector (priority vector). Let λ_{max} represent the largest eigenvalue. Let CI denote consistency index of a pairwise comparison matrix and RI represent the consistency index of a random-like matrix using the scale of T. L. Saaty (1996) (Table 2.3), in which RI represents the average of consistency indices of 500 randomly filled matrices of a similar size (Mu & Pereyra-Rojas, 2017). Let CR reflect a consistency ratio that compare CI versus RI, such that

$$CR = \frac{CI}{RI}, \text{ where} \tag{2.6}$$

$$CI = \frac{\lambda_{max} - m}{m - 1},$$

$$\lambda_{max} = \sum_{j=1}^{m} \sum_{i=1}^{m} \frac{a_{ij} w_j}{s_j}, i, j = (1, 2 \ldots, m)$$

Table 2.3: Consistency indices for a randomly generated matrix.

m	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

Using the values in Table 2.3, the estimated weight (priority) vector is considered acceptable for a consistency ratio of 0.10 or less (Mu & Pereyra-Rojas, 2017).

2.3.3.2 Examine interdependencies among sub-criteria

Different types of interdependencies exist in complex decision problems, as depicted in Figure 2.2. Therefore, these interdependencies should be considered for making better decisions.

To identify the relative importance among sub-criteria with respect to the system of concern, different types of interdependencies (i.e., inner-dependencies within each cluster and outer-dependencies between different clusters) should be identified using the network structure (Figure 2.2) or the influence matrix (Table 2.4), which is explained by **Definition 2.7**.

Definition 2.7 Let the set of sub-criteria for a criterion C_i be represented by $\{sc_{i1}, sc_{i2}, \dots, sc_{ir}\}$ and the set of sub-criteria for criterion C_j denoted by $\{sc_{j1}, sc_{j2}, \dots, sc_{jz}\}$, where $i, j = 1, 2 \dots, m$, and m is the number of the evaluation criteria. Let r represent the number of sub-criteria for C_i, such that $f = 1, 2 \dots, r$; and z denote the number of sub-criteria for C_j, where $h = 1, 2 \dots, z$. Let $C_i \times C_j$ denote the Cartesian product of two sets of evaluation criteria; and let B represent a collection of influence relations between the elements of the two sets, where $a_{if,jh} \in$ B represent the influence of $sc_{if} \in C_i$ over $sc_{jh} \in C_j$. Accordingly, an influence relation from C_i to C_j is written as

$$\exists \mathrm{B}(sc_{if}, sc_{jh}) = a_{if,jh}, \ \forall (C_i \times C_j) \rightarrow B, \ such \ that \qquad (2.7)$$

$$a_{if,jh} = \begin{cases} 1 \leftrightarrow sc_{if} \text{ influences } sc_{jh} \\ 0 \text{ if } sc_{if} \ \underline{\boldsymbol{does\ not}} \text{ influence } sc_{jh} \vee [(\mathrm{i} = \mathrm{j}) \wedge (\mathrm{f} = \mathrm{h})] \end{cases}$$

Table 2.4: Influence matrix.

		C_j			
		sc_{j1}	sc_{j2}	…	sc_{jz}
C_i	sc_{i1}	$a_{i1,j1}$	$a_{i1,j2}$	⋯	$a_{i1,jz}$
	sc_{i2}	$a_{i2,j1}$	$a_{i2,j2}$	⋯	$a_{i2,jz}$
	…	⋯	⋯	⋯	⋯
	sc_{ir}	$a_{ir,j1}$	$a_{i1,j2}$	⋯	$a_{ir,jz}$

Once all interdependencies among the sub-criteria have been identified, the grey-based pairwise comparisons should be utilized to examine the influences among sub-criteria. To examine the inner-dependence relations among sub-criteria of the same cluster, the same procedures for determining the inner-dependence weights among the main criteria (i.e., Definitions 2.2 and 2.3) are used. However, for the outer-dependence weights estimation, the following subsection describes the associated procedures.

2.3.3.2.1 Estimate outer-dependence weights

Definition 2.8 Let $f_{jh\otimes}$ denote a grey description function that describes grey preference relations between elements of a criterion C_i over a sub-criterion $sc_{jh} \in C_j$, where $i \neq j$; and let $\otimes\, jh_{if,if*} \in f_{jh\otimes}$ represent a relative grey number that articulates DMs' verbal preference of $sc_{if} \in C_i$ in comparison to $sc_{if*} \in C_i$ with respect to sc_{jh}. Accordingly, the grey description function $f_{jh\otimes}$ is

$$f_{jh\otimes} : (C_i \times C_i) \rightarrow R[\otimes],\ such\ that \tag{2.8}$$

$$\exists jh_{if,if*}(\otimes) \leftrightarrow \left(a_{if,jh}, a_{if*,jh} = 1\right),\ \forall f_{jh\otimes}\left(sc_{if}, sc_{if*}\right),\ where$$

$$jh_{if,if*}(\otimes) = \begin{cases} jh_{if*,if}(\otimes)^{-1} \leftrightarrow f \neq f* \\ [1,1], \qquad \text{otherwise} \end{cases}$$

Once the levels of outer-dependence influences over the identified sub-criteria have been estimated using the grey-based pairwise comparisons approach, the outer-dependence weights over each sub-criterion can be established by applying eigenvector computations on the qualifying whitened values of the resultant grey numbers using Eq. (2.5). However, the consistence test should be applied using Eq. (2.6) to assure the consistency among the resultant weights. Consequently, the resultant interdependence matrices are the compositions of the unweighted supermatrix.

2.3.3.3 Estimate global weights of sub-criteria

The first step to estimate global weights of sub-criteria is to evaluate the relative importance among sub-criteria with respect to a final decision goal. To this end, the determined unweighted supermatrix is weighted using the computed inner-dependence weights of the main criteria.

Note: it is assumed that the self-influence of a main criterion is the highest, which represents one half of the total weight.

Definition 2.9 Let W_c denote inner-dependence weights matrix for main criteria, where $W_{ij} \in W_c$ indicate the influence of C_i over C_j; let w_{sc} represent the unweighted supermatrix, where $[w_{i.,j.}] \subset w_{sc}$ represent the interdependence unweighted matrix between the elements of C_i over the elements of C_j; and let Q_w denote weighted supermatrix, where $[Q_{i.,j.}] \subset Q_w$ represent the relevant interdependence weighted matrix of $[w_{i.,j.}]$. The function of the weighted supermatrix is

$$f\colon (W_c \times w_{sc}) \to Q_w \tag{2.9}$$

$$\exists [Q_{i.,j.}] \in Q_w,\ \forall f\left(W_{ij} \in W_c, [w_{i.,j.}] \in w_{sc}\right)$$

Once the weighted supermatrix has been calculated, it should be normalized to obtain synthesized results for the elements of the weighted supermatrix. To establish the normalized supermatrix, the linear normalization approach is utilized: elements of each column are divided by the column sum.

Subsequently, global weights of sub-criteria can be established by obtaining the limited supermatrix. To this end, the normalized supermatrix should be raised to powers (i.e., exponentiation) until it converges into a stable matrix, where the elements of each row converge (Hosseini, Tavakkoli-Moghaddam, Vahdani, Mousavi, & Kia, 2013). Thus, the overall priority across the identified sub-criteria can be established using the proposed Grey-based ANP (G-ANP) approach.

2.3.4 Evaluate and rank feasible alternatives

When it comes to performance evaluations and alternatives ranking with respect to the system of concern, the following procedural steps are used: (**1**) Assess alternatives performance over the evaluation criteria and establish performance matrix; (**2**) Normalize relative performance measures of feasible alternatives to establish a comparison ground; (**3**) Evaluate preferences between alternatives over each criterion by measuring the deviation between the evaluations of the alternatives; (**4**) Calculate the relative preferences between alternatives across the evaluation criteria; (**5**) Estimate the global preference of each alternative using the net outranking flow computations, and rank available alternatives accordingly.

2.3.4.1 Establish performance matrix

The system of concern involves different types of criteria (e.g., quantitative and qualitative), which require different assessment approaches. Moreover, the involvement of uncertainty adds to the complexity of the system. Accordingly, to establish the performance matrix for available alternatives within the context of the system at hand, each alternative should be evaluated over the sets of criteria. While the performance over quantitative criteria is represented in numerical values; the performance over qualitative criteria is articulated in linguistic expressions, in accordance with the judgments of the involved DMs.

Definition 2.10 Let the set of alternatives be represented by $A = \{A_1, A_2, \dots, A_n\}$, where n is the number of the feasible alternatives and $t = 1, 2 \dots, n$. Let $SC = \{sc_1, sc_2, \dots, sc_{im}\}$ denote the set of evaluation criteria, where "im" is the number of evaluation criteria, and $g = 1, 2 \dots, im$. Let $A \times SC$ be the Cartesian product of the set of alternatives and the set of criteria, and let $R\,[\otimes]$ denote the set of grey numbers. Let $y_{tg}(\otimes)$ represent the relative grey number that reflect the performance of an alternative A_t over an evaluation criterion sc_g; where sc_g is a qualitative criterion, or a quantitative criterion with uncertain data. Thus, the grey description function for the performance matrix based on the definition of Kuang et al. (2015) would be

$$f_{\otimes}: A \times SC \rightarrow R[\otimes], thus \tag{2.10}$$

$$\forall f_{\otimes}(A_t \in A, sc_g \in SC): y_{tg}(\otimes) \in R[\otimes]$$

Note that for performance assessment over qualitative criteria, $y_{tg}(\otimes)$ articulates DMs' verbal statements (i.e., linguistic expression) regarding the performance of A_t over the criterion sc_g; in this manuscript, the maps between linguistic expressions and grey numbers are identified in Table 2.2. However, to measure alternatives performance over quantitative criteria where uncertainty exists (e.g., imperfect numerical information), $y_{tg}(\otimes)$ would take its values from either a discrete set of values or an interval.

2.3.4.2 Normalize performance matrix

Once the performance matrix has been determined, consistency among performance measures should be established to draw proper comparisons. To this end, a normalization process is applied to adjust the performance matrix, wherein the following condition should be valid (Bai, Sarkis, Wei, & Koh, 2012)

$$[0,0] \leq y_{tg}(\otimes) \leq [1,1] \tag{2.11}$$

The normalization process is done in two steps: first, transform all non-grey values in the performance matrix into general grey numbers; second, normalize all the values.

Definition 2.11 Let y_{tg} denote a white number that represents the performance of alternative A_t on a quantitative criterion sc_g, the relative grey number of the white number (y_{tg}) is

$$y_{tg}(\otimes) = \left[\underline{y}_{tg}, \overline{y}_{tg}\right], \ where\ \underline{y}_{tg} = y_{tg} = \overline{y}_{tg} \tag{2.12}$$

Note that although some evaluations would be expressed by interval grey numbers, a normalized scale over the criteria is not guaranteed. To establish a normalized scale for the evaluations of feasible alternatives over different types of criteria, Algorithm 2.1, which is explained by **Definition 2.12**, is proposed.

$if\, g\ is\ increasing\ criterion$

$$y_{tg}(\tilde{\otimes}) = \frac{[y_{tg}(\otimes) - min(y_{kg})]}{[max(y_{kg}) - min(y_{kg})]}$$

$else\ if\ g\ is\ decreasing\ criterion$

$$y_{tg}(\tilde{\otimes}) = \frac{[max(y_{kg}) - y_{tg}(\otimes)]}{[max(y_{kg}) - min(y_{kg})]}$$

$else\ g\ is\ targeted\ criterion$

$$y_{tg}(\tilde{\otimes}) = 1 - \frac{|y_{tg}(\otimes) - y_g^*|}{Max\{max(y_{kg}), y_g^*\} - Min\{min(y_{kg}), y_g^*\}}$$

Algorithm 2.1: Normalize alternatives performance based on grey systems theory.

Definition 2.12 Let $y_{tg}(\otimes)$ represent a general grey number that reflects the performance of alternative A_t over a criterion sc_g; let $\min(y_{kg})$ and $\max(y_{kg})$ denote the lower and upper bounds of $y_{tg}(\otimes)$, respectively. Let y_g^* represent a given optimal performance value over a targeted criterion sc_g. Let $\otimes\, \tilde{y}_{tg}$ denote the relative normalized value of the general grey number $y_{tg}(\otimes)$, such that $y_{tg}(\tilde{\otimes})$ is determined based on criteria type, i.e., increasing, decreasing, and targeted.

2.3.4.3 Establish preference matrix

The differences of performance measures explain the preferences between feasible alternatives. Thus, the larger the difference, the larger the preference is. In order to establish the preference matrix, the deviation between the evaluations of the feasible alternatives on each criterion should be determined, based on the definition of Xu and Da (2002).

Definition 2.13 Let $y_{ag}(\tilde{\otimes}) = [\underline{\tilde{y}}_{ag}, \overline{\tilde{y}}_{ag}]$ and $y_{bg}(\tilde{\otimes}) = [\underline{\tilde{y}}_{bg}, \overline{\tilde{y}}_{bg}]$ represent general grey numbers that reflect the normalized performance values of alternatives A_a and A_b over sc_g, respectively. Let l_{ag} and l_{bg} denote the difference between the upper and lower limits of $y_{ag}(\tilde{\otimes})$ and $y_{bg}(\tilde{\otimes})$, respectively, such that

$$l_{ag} = \overline{\tilde{y}}_{ag} - \underline{\tilde{y}}_{ag} \tag{2.13}$$

$$l_{bg} = \overline{\tilde{y}}_{bg} - \underline{\tilde{y}}_{bg}$$

Definition 2.14 Let $\tilde{d}_g(A_a, A_b)$ denote the deviation between the performance of alternative A_a with respect to the performance of alternative A_b over sub-criterion sc_g, in which the function to obtain the deviation can be defined as

$$\tilde{d}_g(A_a, A_b) = \frac{\overline{\tilde{y}}_{ag} - \underline{\tilde{y}}_{bg}}{l_{ag} + l_{bg}} \tag{2.14}$$

Once the deviation between the evaluations of feasible alternatives over each of the evaluation criteria have been determined, preference degrees can be established as follows.

Definition 2.15 Let $\tilde{P}_g(A_a, A_b)$ represent the preference degree of alternative A_a over A_b with respect to sc_g. Let the degree of preference vary between 0 and 1, where 0 indicates no preference and 1 indicates full preference, such that (Kuang et al., 2015)

$$\tilde{P}_g(A_a, A_b) = \begin{cases} 0, & \tilde{d}_g(A_a, A_b) \leq 0 \\ \tilde{d}_g(A_a, A_b), & 0 < \tilde{d}_g(A_a, A_b) < 1 \\ 1, & \tilde{d}_g(A_a, A_b) \geq 1 \end{cases} \tag{2.15}$$

2.3.4.4 Determine relative preference matrix

To determine the overall preferences between alternatives with respect to the given system, the overall priority across the identified sets of the evaluation criteria (i.e., global weights) should be considered.

Definition 2.16 Let $\tilde{\pi}(A_a, A_b)$ denote the relative preference of alternative A_a over A_b across the set of evaluation criteria SC, where $SC = \{sc_1, sc_2, \ldots, sc_{im}\}$. Let the global weight of each criterion be represented by w_g, where $\sum_{g=1}^{im} w_g = 1, g = 1, 2 \ldots, im$, and im is the number of evaluation criteria. Accordingly, the relative preference of A_a over A_b can be calculated using the following function (Kuang et al., 2015).

$$\tilde{\pi}(A_a, A_b) = \sum_{g=1}^{im} w_g \, \tilde{P}_g(A_a, A_b)\, , where\ g = (1, 2, \ldots, im) \tag{2.16}$$

2.3.4.5 Estimate global preferences and rank available alternatives

Once the relative preferences have been determined for each pair of alternatives, the global preference among feasible alternatives can be estimated. To this end, the net outranking flow should be calculated using the outranking flows measures, which determine the superiority (i.e., positive outranking flow) and inferiority (i.e., negative outranking flow) levels of a given alternative over others.

Definition 2.17 Let $\tilde{\phi}^{+}(A_a)$ denote the positive outranking flow of alterative A_a, which indicates the preference of A_a over all other alternatives. Let $\tilde{\pi}\,(A_a, A_b)$ represent the extent to which alternative A_a is **preferred** over A_b. The positive outranking flow can be defined as (Kuang et al., 2015)

$$\tilde{\phi}^{+}(A_a) = \frac{1}{n-1}\sum_{b=1}^{n}\tilde{\pi}(A_a, A_b),\ a \neq b \tag{2.17}$$

Definition 2.18 Let $\tilde{\phi}^{-}(A_a)$ represent the negative outranking flow of alterative A_a, which indicates the preference of other alternatives over A_a. Let $\tilde{\pi}(A_b, A_a)$ represent the extent to which alternative A_a is **outranked** by A_b. The function to obtain $\tilde{\phi}^{-}(A_a)$ can be written as (Kuang et al., 2015)

$$\tilde{\phi}^{-}(A_a) = \frac{1}{n-1}\sum_{b=1}^{n}\tilde{\pi}(A_b, A_a),\ a \neq b \tag{2.18}$$

Definition 2.19 Let $\tilde{\phi}\,(A_a)$ denote the global preference (i.e., net outranking flow) of alternative A_a, which can be obtained by measuring the difference between $\tilde{\phi}^{+}(A_a)$ and $\tilde{\phi}^{-}(A_a)$. Thus, $\tilde{\phi}\,(A_a)$ can be determined as follows (Kuang et al., 2015):

$$\tilde{\phi}\,(A_a) = \tilde{\phi}^{+}\,(A_a) - \tilde{\phi}^{-}\,(A_a) \tag{2.19}$$

Once the net outranking flow has been estimated for all feasible alternatives, a complete ranking index can be established based on the values of global preferences, wherein the higher the value of $\tilde{\phi}\,(A_a)$, the better is the alternative. Thus, the best alternative is the one with the highest global preference value.

2.4 Case illustration of strategic decision making at the front end innovation for a small to medium-sized enterprise within the Canadian quaternary sector

The given company is suffering from a low level of formalization, when it comes to making strategic decisions with respect to innovation activities, which would jeopardize innovation success. Consequently, the company is looking for a more systematic approach to identify, characterize, evaluate, and respond better to potential opportunities for innovation. The proposed methodology would be implemented on the framework of the case to bridge deficiencies of the current process, thereby enhancing DMs' abilities in making strategic decisions.

In this case study, three different innovation projects have been reported for study. Abductive reasoning has been utilized to provide reasonable explanations about the different components of the problem at hand, and the existing interrelationships among evaluation criteria.

In order to establish a coherent understanding about the current practice of the Front End Innovation (FEI) within the firm, a triangulation technique has been used, which increases the validity of the data (Schweizer, 2015). The three different techniques for data collection process were semi-structured interviews (e.g., senior managers, systems engineers, and marketing personnel); on site observations to gain first-hand knowledge on innovation activities for the company; and reviewing the available archival data, including internal norms and strategies relevant to the innovation process.

2.4.1 Structure and model the decision problem

The first step of the proposed methodology is to formulate the decision problem. To this end, the different components of the decision problem (i.e., alternatives, criteria, and sub-criteria) should be identified. Afterwards, essential connections among the evaluation criteria would be modeled.

2.4.1.1 Identify feasible alternatives

As mentioned earlier, three innovation projects have been reported for the study, each of which aims to create a competitive advantage for the company. However, each project has different set of characteristics, which would make a difference in the evaluation process.

To differentiate between the potential alternatives, the type of the relative innovation strategy would be considered. Thus, the potential alternatives are alternative 1 (A_1): Radical Diversification; alternative 2 (A_2): Market Development; and alternative 3 (A_3): Product / Service Development.

2.4.1.2 Establish evaluation criteria

In this study, the evaluation aspects for the decision problem at hand have been established based on the knowledge acquired from the case study and by building on the literature of the relevant subject matter. Four main sets of criteria are proposed to evaluate feasible alternatives from different perspectives. Table 2.5 shows the main criteria and the associated sub-criteria with a brief description for each sub-criterion.

2.4.1.3 Establish various types of links and model the problem

To illustrate the different types of connections within the system of concern, a network structure model has been utilized to demonstrate the general framework of the existing interrelationships within and between the different evaluation clusters, as shown in Figure 2.3. However, the influence matrix has been used to give the detailed view of the interdependencies between the sub-criteria, as shown in Table 2.6; where 1 indicates the presence of influence relation between the associated pair of sub-criteria.

2.4.2 Establish grey linguistic scales

The system at hand is regarded as a complex system under uncertainty. The complexity of the decision problem could be handled by conventional MCDA approaches. However, the involved uncertainty, due to the nature of FEI (e.g., limited input data), would limit the outcomes of using MCDA solely. Therefore, the proposed methodology integrates grey systems theory with ANP and PROMETHEE II to overcome the uncertainty-related aspects as follows: firstly, grey systems theory would be utilized along with the principles of ANP to establish the set of weights for evaluation criteria with respect to the system of concern; secondly, grey systems theory would be integrated with PROMETHEE II to help measuring the performance of the alternatives over the evaluation criteria that involve uncertain evaluations.

Table 2.5: Evaluation criteria.

Evaluation Criteria	Description	Reference
Market (C1)		
Market insight (M1) (sc11)	Market related knowledge (e.g., ability to discover unfulfilled needs).	(Reid & Brentani, 2015)
Growth rate (M2) (sc12)	Potential increases in a market size (i.e., demand growth).	(Baker, Grinstein, & Harmancioglu, 2016)
Competitive degree (M3) (sc13)	Competition level indicator in a given market.	(Mendi & Costamagna, 2017)
Technology (C2)		
Sustainable competitive advantage (T1) (sc21)	Ability to sustain advantage(s) over competitors.	(Saeidi, Sofian, Saeidi, Saeidi, & Saaeidi, 2015)
Specification fuzziness (T2) (sc22)	Lack of clarity with respect to process functions, technical specifications, or technical requirements.	(Moos, Beimborn, Wagner, & Weitzel, 2013)
Financial (C3)		
Revenue stream (F1) (sc31)	Potential earning from a given investment.	(Gebauer, Worch, & Truffer, 2012)
Cost structure (F2) (sc32)	Delivery cost estimation.	(Onetti, Zucchella, Jones, & McDougall-
Potential sources of funding (F3) (sc33)	Potential sources of funding (e.g., R&D subsidy).	(Bronzini & Piselli, 2016)
Organizational (C4)		
Familiarity with targeted market (O1) (sc41)	Level of familiarity with a targeted market: new market, adjacent, or existing market.	(Tzokas, Ah, Akbar, & Al-dajani, 2015)
Current development capability (O2) (sc42)	e.g., technological capability, whether the targeted innovation project is fully applicable, require significant adaptation, or not applicable.	(Martín-de Castro, 2015)

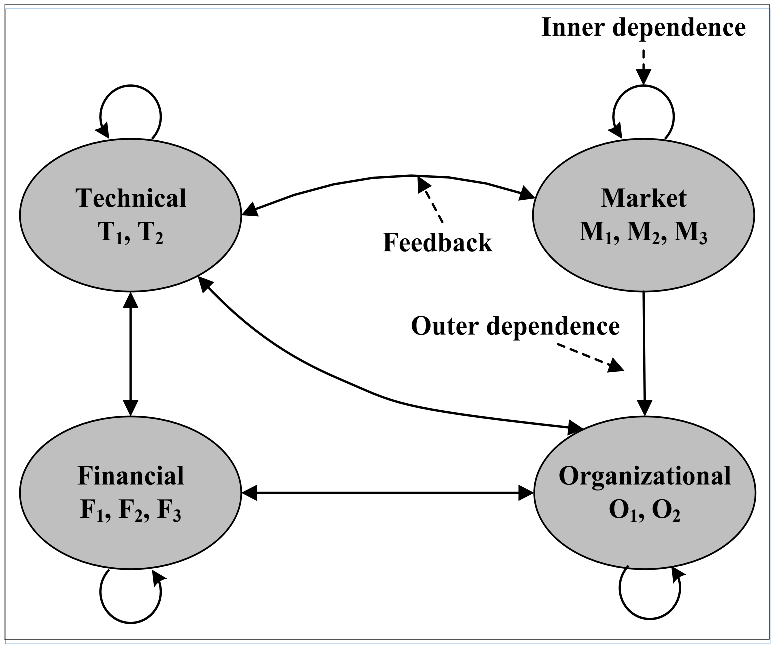

Figure 2.3: Network structure of the evaluation criteria within the context of FEI.

Table 2.6: Interdependencies between sub-criteria.

Sub criteria	M_1	M_2	M_3	T_1	T_2	F_1	F_2	F_3	O_1	O_2
M_1	0	1	1	1	1	1	1	1	1	1
M_2	1	0	1	1	1	1	1	1	1	1
M_3	1	1	0	1	1	1	1	1	1	1
T_1	0	1	1	0	1	1	1	1	1	1
T_2	1	1	1	1	0	1	1	1	1	1
F_1	1	1	1	1	1	0	1	1	1	1
F_2	1	1	1	1	1	1	0	1	1	1
F_3	1	1	1	1	1	1	1	0	1	1
O_1	1	1	1	1	1	1	1	1	0	1
O_2	1	1	1	1	1	1	1	1	1	0

In this case study, the type of the decision problem is considered as single participant-multiple criteria, which is the general case for MCDA. To express preferences of the involved DMs regarding the evaluation criteria, Table 2.1 has been utilized. When it comes to the performance evaluation stage, Table 2.2 has been applied to assess the performance of prospective projects over qualitative measures.

2.4.3 Establish the priority level across criteria

After identifying the different components of the system of concern and modeling substantial connections within the FEI, the next step is to establish the level of importance among the evaluation criteria by estimating the weights of each criterion. To this end, different types of interdependencies among evaluation criteria would be considered and analyzed using the proposed G-ANP approach.

2.4.3.1 Determine inner-dependence weights among main criteria

Interdependence weights among the main criteria (i.e., market, technology, financial, and organizational) are estimated by analyzing the influence of each criterion on other criteria, using the linguistic expressions and the relative grey numbers in Table 2.1 in accordance with Eq. (2.3) of the proposed methodology. Table 2.7 shows the grey-based pairwise comparison matrix between the main criteria with respect to Market factor, which represents the control criterion of this matrix.

Table 2.7: Inner-dependencies among main criteria with respect to Market.

Market	**Technical**	**Financial**	**Organizational**
Technical	[1, 1]	[1, 2]	[3/2, 5/2]
Financial	[1/2, 1]	[1, 1]	[1, 2]
Organizational	[2/5, 2/3]	[1/2, 1]	[1, 1]

Once all the inner-dependence relations among criteria have been established, the grey values would be transformed into fixed numbers (Table 2.8), using the whitenization process, Eq. (2.5), to estimate relative inner-dependencies through eigenvector computations.

Table 2.8: Inner-dependence weights matrix of the main criteria.

Main Criteria	Market	Technical	Financial	Organizational
Market	1	0.2268	0.5111	0.3527
Technical	0.4480	1	0.3067	0.4442
Financial	0.3232	0.4872	1	0.2031
Organizational	0.2289	0.2860	0.1821	1

2.4.3.2 Examine interdependence weights among sub-criteria

The complex interdependencies (i.e., inner-dependencies and outer-dependencies) among the identified sub-criteria have been examined according to section 2.3.3.2 of the proposed methodology. Note that Table 2.1 has been used to articulate DMs' preferences between the sub-criteria.

As mentioned in section 2.3.3.2, the estimated priority matrices, which demonstrate the level of influence among the identified sub-criteria, are the compositions of the unweighted supermatrix (Table 2.9). Note that the shaded areas of Table 2.9 represent the inner-dependence weights among sub-criteria of the same cluster.

2.4.3.3 Estimate global weights of sub-criteria

To estimate the relative importance of each sub-criterion with respect to the decision problem at hand, the constructed unweighted supermatrix has been weighted, as shown in Table 2.10, in accordance with Eq. (2.9) using the computed inner-dependence weights matrix of the main criteria (Table 2.8).

Accordingly, each element of the unweighted supermatrix (Table 2.9) is multiplied with the associated element in Table 2.8. For example, the influence of "competitive degree" (M_3), which is a sub-criterion of "Market" (C_1) cluster; on "sustainable competitive advantage" (F_1), which is a sub-criterion of "Technology" (C_2) cluster, is 0.2268. However, the associated inner-dependence weight in Table 2.8 is 0.4833. Consequently, the relevant value within the weighted supermatrix would be 0.1096.

Table 2.9: Supermatrix.

Sub criteria	M_1	M_2	M_3	T_1	T_2	F_1	F_2	F_3	O_1	O_2
M_1	0	0.5857	0.3406	0.2289	0.2289	0.4442	0.2289	0.2860	0.3232	0.2860
M_2	0.6594	0	0.6594	0.4480	0.4480	0.3527	0.4480	0.4872	0.4480	0.4872
M_3	0.3406	0.4143	0	0.3232	0.3232	0.2031	0.3232	0.2268	0.2289	0.2268
T_1	0	0.4143	0.3406	0	1	0.5857	0.3406	0.5857	0.3406	0.3406
T_2	1	0.5857	0.6594	1	0	0.4143	0.6594	0.4143	0.6594	0.6594
F_1	0.4442	0.4480	0.4872	0.4872	0.4872	0	0.5857	0.6594	0.4872	0.4872
F_2	0.3527	0.3232	0.2860	0.2860	0.2860	0.4143	0	0.3406	0.2268	0.2860
F_3	0.2031	0.2289	0.2268	0.2268	0.2268	0.5857	0.4143	0	0.2860	0.2268
O_1	0.4143	0.4143	0.3810	0.4143	0.2899	0.3406	0.3406	0.4143	0	1
O_2	0.5857	0.5857	0.6190	0.5857	0.7101	0.6594	0.6594	0.5857	1	0

In order to determine the global weights, the elements of the weighted supermatrix results (Table 2.10**)** have been normalized to obtain synthesized results. Subsequently, the global weights of the sub-criteria can be obtained by raising the normalized supermatrix to powers until it converges into a stable matrix (i.e., the limited supermatrix). In this study, the limited supermatrix has been achieved at $\left[\tilde{Q}_w\right]^{15}$. As a result, the global weights of the sub-criteria are: **M_1** (0.0842), **M_2** (0.1095), **M_3** (0.0714), **T_1** (0.1282), **T_2** (0.1551), **F_1** (0.11), **F_2** (0.0745), **F_3** (0.0776), **O_1** (0.0865), and **O_2** (0.103).

2.4.4 Evaluate and rank feasible alternatives

Once the evaluation criteria have been analyzed, feasible alternatives can be ranked. The detailed procedure is explained in the following subsections.

Table 2.10: Weighted supermatrix.

Sub criteria	M_1	M_2	M_3	T_1	T_2	F_1	F_2	F_3	O_1	O_2
M_1	0	0.5857	0.3406	0.0519	0.0519	0.2270	0.1170	0.1462	0.1140	0.1009
M_2	0.6594	0	0.6594	0.1016	0.1016	0.1803	0.2290	0.2490	0.1580	0.1718
M_3	0.3406	0.4143	0	0.0733	0.0733	0.1038	0.1652	0.1159	0.0807	0.0800
T_1	0	0.1856	0.1526	0	1	0.1797	0.1045	0.1797	0.1513	0.1513
T_2	0.4480	0.2624	0.2954	1	0	0.1271	0.2023	0.1271	0.2929	0.2929
F_1	0.1435	0.1448	0.1575	0.2374	0.2374	0	0.5857	0.6594	0.0990	0.0990
F_2	0.1140	0.1044	0.0924	0.1394	0.1394	0.4143	0	0.3406	0.0461	0.0581
F_3	0.0656	0.0740	0.0733	0.1105	0.1105	0.5857	0.4143	0	0.0581	0.0461
O_1	0.0948	0.0948	0.0872	0.1185	0.0829	0.0620	0.0620	0.0755	0	1
O_2	0.1340	0.1340	0.1417	0.1675	0.2031	0.1201	0.1201	0.1067	1	0

2.4.4.1 Establish performance matrix

To evaluate feasible alternatives, the performance of each alternative should be assessed across the evaluation criteria. Different types of criteria are involved in the decision problem at hand, i.e., quantitative and qualitative criteria, in which the sub-elements of the financial cluster (i.e., revenue stream, cost structure, and potential sources of funding) are quantitative, while all others are qualitative.

Due to the presence of uncertainty within the context of FEI, grey numbers have been used to express the performance of feasible alternatives in which the performance over the quantitative sub-criteria has been estimated using intervals, while the performance over the qualitative sub-criteria has been evaluated based on DMs' verbal judgments using the linguistic expressions and the grey numbers in Table 2.2. Alternatives' performances over the qualitative and quantitative criteria are presented in Table 2.11 and Table 2.12, respectively. Note that the performance estimations over the quantitative criteria are in thousands.

Table 2.11: Evaluation of potential innovation projects on qualitative criteria.

Performance matrix	**Alternatives**		
Qualitative criteria	**A_1**	**A_2**	**A_3**
Market insight (M_1)	L	MM	M
Growth rate (M_2)	H	LM	MM
Competitive degree (M_3)	L	LM	M
Sustainable competitive advantage (T_1)	H	M	MM
Specification fuzziness (T_2)	MM	LM	M
Familiarity with targeted market (O_1)	L	MM	LM
Current development capability (O_2)	L	H	LM

Table 2.12: Evaluation of potential innovation projects on quantitative criteria.

Performance matrix	**Alternatives**		
Quantitative criteria	**A_1**	**A_2**	**A_3**
Revenue stream (F_1)	[150, 350]	[70, 200]	[85, 250]
Cost structure (F_2)	[45, 60]	[15, 25]	[35, 50]
Potential sources of funding (F_3)	[20, 40]	[5, 10]	[10, 25]

2.4.4.2 Normalized performance matrix

To assure consistency over the preference evaluation process, the resultant performance matrices have been normalized using Algorithm 2.1. The normalized performance matrix is shown in Table 2.13.

Table 2.13: Normalized Performance matrix.

Performance matrix	Alternatives		
Qualitative criteria	**A_1**	**A_2**	**A_3**
Market insight (M_1)	[0, 0.25]	[0.75, 1]	[0.5, 0.75]
Growth rate (M_2)	[0.75, 1]	[0, 0.25]	[0.5, 0.75]
Competitive degree (M_3)	[0.67, 1]	[0.33, 0.67]	[0, 0.33]
Sustainable competitive advantage (T_1)	[0.67, 1]	[0, 0.33]	[0.33, 0.67]
Specification fuzziness (T_2)	[0, 0.33]	[0.67, 1]	[0.33, 0.67]
Familiarity with targeted market (O_1)	[0, 25]	[0.75, 1]	[0.25, 0.5]
Current development capability (O_2)	[0, 2]	[0.8, 1]	[0.2, 0.4]
Revenue stream (F_1)	[0.286, 1]	[0, 0.464]	[0.054, 0.643]
Cost structure (F_2)	[0, 0.333]	[0.778, 1]	[0.222, 0.556]
Potential sources of funding (F_3)	[0.429, 1]	[0, 0.143]	[0.143, 0.571]

2.4.4.3 Establish preference matrix

To establish the preference degree between the prospective projects, the deviation between the evaluations of potential alternatives over each criterion has been evaluated using Eq. (2.14). Afterwards, the preference degree between the projects over each criterion has been estimated in accordance with Eq. (2.15). Accordingly, the resultant preferences of A1 over other alternatives are shown in Table 2.14.

2.4.4.4 Determine relative preference matrix

To determine the overall preferences between the prospective projects, the relative preferences between the projects should be determined by weighting the resultant preference measures (Table 2.14) using the global weights of the evaluation criteria in accordance with Eq. (2.16). Accordingly, the relative preference measures between alternatives are depicted in Table 2.15.

Table 2.14: Multi-criteria preference matrix of **A_1**.

Multi-criteria preference matrix		**Alternatives**		
Alternative	**Criteria**	**A_1**	**A_2**	**A_3**
A1	**Market insight (M_1)**	0.5	0	0
	Growth rate (M_2)	0.5	1	1
	Competitive degree (M_3)	0.5	1	1
	Sustainable competitive advantage (T_1)	0.5	1	1
	Specification fuzziness (T_2)	0.5	0	0
	Familiarity with targeted market (O_1)	0.5	0	0
	Current development capability (O_2)	0.5	0	0
	Revenue stream (F_1)	0.5	0.8485	0.726
	Cost structure (F_2)	0.5	0	0.1667
	Potential sources of funding (F_3)	0.5	1	0.8571

Table 2.15: Relative preference matrix.

Alternatives	**A_1**	**A_2**	**A_3**
A_1	0.5	0.48	0.4679
A_2	0.52	0.5	0.6176
A_3	0.5321	0.3824	0.5

2.4.4.5 Estimate global preferences and rank feasible alternatives

To prioritize the prospective projects, the global preference measures should be established in accordance with Eqs. (2.17), (2.18), and (2.19). Table 2.16 presents: outflows (positive outranking flow), inflows (negative outranking flow), and the net-flows (global preferences). Consequently, the three projects have been ranked based on the resultant net outranking flows, wherein the higher the value of the net-flow, the better is the alternative. Thus, **A_2** is the most preferred project and the ranking order of the prospective innovation projects according to the proposed methodology is **$A_2 \succ A_1 \succ A_3$**.

Table 2.16: Global preference matrix - Outranking flows computations.

Outranking flows	**Alternatives**		
	A_1	**A_2**	**A_3**
Outflow	0.7240	0.8188	0.7073
Inflow	0.7760	0.6812	0.7927
Net flow	-0.0521	0.1375	-0.0855

2.5 Comparative Analysis

To validate the proposed methodology, an existing case study in the literature is used. The case study is taken from the work of Kuo, Hsu, and Chen (2015), where a framework has been developed, by integrating fuzzy ANP and fuzzy TOPSIS approaches, to evaluate carbon performance of suppliers. Table 2.17 shows the grey paired comparison matrix between criteria (dimensions), and the estimated weights. The main criteria are: Organizational management (C1); Process management (C2); Procurement management (C3); R&D management (C4). Table 2.18 illustrates the estimated criteria weights using both methodologies, by looking at the results, the proposed methodology reflects similar priority order among the criteria.

Table 2.17: Grey paired comparison matrix between dimensions.

Main Criteria	**C1**	**C2**	**C3**	**C4**
C1	[1,1]	[1,3]	[3,5]	[6,8]
C2	[4,6]	[1,1]	[1,3]	[4,6]
C3	[0.2,0.333]	[0.333,1]	[1,1]	[2,4]
C4	[0.125,0.167]	[0.167,0.25]	[0.25,0.5]	[1,1]

Table 2.18: Criteria weights of the proposed methodology and the existing methodology.

Criteria	**Proposed methodology** **Grey ANP – Grey PROMETHEEII**	**Kuo et al., (2015)** **Fuzzy ANP - Fuzzy TOPSIS**
Organizational management (c_1)	0.413	0.494
Process management (c_2)	0.405	0.272
Procurement management (c_3)	0.134	0.141
R&D management (c_4)	0.047	0.099

Due to the shortage of provided data with respect to sub-criteria in the case study, the final weights of the sub-criteria, which is provided in the existing research, will be considered for evaluating the performance of potential suppliers (S_n, where n=1, 2, ...7) using the extended grey PROMETHEE II methodology. Table 2.19 demonstrates the sub-criteria and the associated weights. The grey decision matrix for suppliers' performance evaluations over the sub-criteria is demonstrated in Table 2.20.

By utilizing alternatives evaluation procedures of the proposed methodology and the sub-criteria weights in Table 2.19, the overall preferences between alternatives are reflected in Table 2.21. Accordingly, the global preferences among alternatives are demonstrated in Table 2.22, in which $\mathbf{S_1}$ is the most preferred supplier, and the ranking order of the potential suppliers is $\mathbf{S_1 \succ S_4 \succ S_2 \succ S_3 \succ S_7 \succ S_6 \succ S_5}$, which is similar to the ranking order of the exiting methodology.

Although both methodologies provide the same conclusion in this example, yet grey systems theory is more suitable for decision problems with a relatively small amount of data and poor information, which cannot be described by a probability distribution; as it offers simpler procedure, which does not require a robust membership function as in fuzzy theory (Memon et al., 2015). Moreover, PROMETHEE II is considered simple and easily comprehensible approach in comparison to other MCDM approaches, including TOPSIS (Maity & Chakraborty, 2015). Furthermore, it has been mentioned that the use of Euclidean distance in TOPSIS does not account for the correlation of attributes (Velasquez & Hester, 2013; P. Wang, Li, Wang, & Zhu, 2015), which influences the evaluation process of decision problems where correlations among criteria (attributes) exist.

Table 2.19: Sub-criteria weights.

Sub-criteria	Weights
Carbon governance (sc_1)	0.229
Carbon policy (sc_2)	0.188
Carbon reduction targets (sc_3)	0.174
GHG verification (ISO 14064) (sc_4)	0.003
Risk assessment for low carbon requirement (sc_5)	0.125
Training-related carbon management (sc_6)	0.073
Availability and use of low carbon technologies (sc_7)	0.015
Energy efficiency (sc_8)	0.014
Measures of carbon reduction (sc_9)	0.018
Availability of a carbon supplier selection system (sc_{10})	0.070
Requirement of low carbon purchasing (sc_{11})	0.052
Capability of low carbon design of product (sc_{12})	0.029
Inventory of carbon footprint of product (sc_{13})	0.009

Table 2.20: Grey decision matrix of supplier selection.

Sub criteria	S_1	S_2	S_3	S_4	S_5	S_6	S_7
sc_1	[6.35, 8.36]	[4.98, 7.03]	[4.70, 6.87]	[6.20, 8.25]	[3.56, 5.72]	[4.09, 6.35]	[3.56, 8.36]
sc_2	[6.73, 8.74]	[5.72, 7.79]	[5.10, 7.13]	[5.86, 7.89]	[3.56, 5.72]	[3.77, 5.86]	[4.44, 6.57]
sc_3	[5.10, 7.28]	[4.70, 6.87]	[4.98, 7.03]	[5.86, 7.89]	[3.56, 5.72]	[3.77, 5.86]	[5.72, 7.79]
sc_4	[6.20, 8.25]	[6.20, 8.25]	[5.86, 7.89]	[6.73, 8.74]	[4.98, 7.03]	[4.7, 6.73]	[5.27, 7.36]
sc_5	[6.00, 8.00]	[4.33, 6.35]	[4.09, 6.20]	[5.10, 7.13]	[3.28, 5.40]	[2.49, 4.59]	[3.10, 5.27]
sc_6	[4.09, 6.20]	[4.33, 6.49]	[3.28, 5.40]	[4.70, 6.87]	[3.28, 5.40]	[2.49, 4.59]	[3.10, 5.27]
sc_7	[6.00, 8.00]	[5.40, 7.45]	[4.44, 6.57]	[5.53, 7.55]	[3.56, 5.72]	[4.09, 6.20]	[4.70, 6.73]
sc_8	[6.20, 8.25]	[4.70, 6.73]	[6.00, 8.00]	[5.40, 7.45]	[3.28, 5.40]	[3.28, 4.44]	[4.33, 6.49]
sc_9	[6.20, 8.25]	[5.10, 7.13]	[5.53, 7.55]	[5.86, 7.89]	[4.33, 6.35]	[4.09, 6.20]	[5.53, 7.55]
sc_{10}	[4.09, 6.20]	[3.48, 5.53]	[3.10, 5.27]	[3.56, 5.72]	[2.00, 3.28]	[2.29, 3.56]	[2.63, 4.70]
sc_{11}	[4.33, 6.35]	[4.98, 7.03]	[3.48, 5.53]	[4.70, 6.73]	[3.03, 5.10]	[3.48, 5.53]	[4.00, 4.00]
sc_{12}	[5.10, 7.13]	[4.70, 6.87]	[5.53, 7.55]	[5.56, 7.89]	[3.56, 5.72]	[3.03, 5.10]	[5.53, 7.55]
sc_{13}	[6.57, 8.63]	[4.98, 7.18]	[5.53, 7.55]	[6.35, 8.36]	[3.56, 5.72]	[4.09, 6.20]	[5.10, 7.13]

Table 2.21: Relative preference matrix.

Alternatives	S_1	S_2	S_3	S_4	S_5	S_6	S_7
S_1	0.500	0.704	0.774	0.541	0.936	0.943	0.808
S_2	0.295	0.500	0.571	0.337	0.876	0.840	0.614
S_3	0.225	0.428	0.500	0.265	0.802	0.760	0.540
S_4	0.458	0.662	0.734	0.500	0.978	0.985	0.777
S_5	0.063	0.123	0.197	0.021	0.500	0.470	0.240
S_6	0.056	0.159	0.239	0.014	0.529	0.500	0.278
S_7	0.191	0.385	0.459	0.222	0.759	0.721	0.500

Table 2.22: Global preference matrix - Outranking flows computations.

Outranking flows	Alternatives						
	S_1	S_2	S_3	S_4	S_5	S_6	S_7
Outflow	0.868	0.672	0.587	0.849	0.269	0.296	0.540
Inflow	0.298	0.494	0.579	0.317	0.897	0.870	0.626
Net flow	0.569	0.179	0.008	0.532	-0.628	-0.574	-0.086

2.6 Conclusion

Although different methods are used to handle uncertainty-related aspects (i.e., subjective and objective uncertainty), grey systems theory is preferred when it comes to decision problems with a relatively small amount of data and poor information, which cannot be described by a probability distribution. Different researchers proposed grey systems theory to deal with uncertainty in decision problems. However, a number of shortcomings has been observed in the existing approaches with respect to the influence of the interdependencies among the evaluation criteria of different clusters on the evaluation process. As a result, a new hybrid grey-based MCDA approach is developed to better handle complex decision problems that are subject to different types of interrelated criteria (i.e., evaluation criteria with different nature, different scales, and different values) and different types of uncertainty-related aspects. The intended purpose of integrating the grey systems theory with a distinctive combinations of MCDA approaches (i.e., ANP and PROMETHEE II) is to optimize the evaluation space in such a complex system under uncertainty by utilizing the emergent strengths of the integrated approach: the mathematical ability and the associated simplicity of PROMETHEE II in providing a complete ranking of feasible alternatives over different types of criteria (i.e., quantitative and qualitative); the superiority of ANP in establishing priorities among evaluation criteria within complex systems; and the distinctive ability of the grey systems theory in handling problems with a relatively small amount of data and poor information, which cannot be described by a probability distribution.

The proposed methodology is capable of establishing priorities among complex interrelated criteria and account for the uncertainty of subjective judgments by combining linguistic expressions, grey systems theory, and principles of ANP. Furthermore, it extends the PROMETHEE II methodology to define optimal ranking among potential alternatives in such a complicated decision problem using a combination of linguistic expressions to articulate human judgments over subjective evaluations; grey systems theory to map linguistic expressions, to deal with subjective and objective uncertainty, and to normalize performance measures over different types of criteria; and the proposed G-ANP approach to establish relative preferences among alternatives over interrelated criteria. Future work is needed to extend the applicability of the proposed methodology for more complicated cases of MCDA; In particular, MCDA with multi-participants where consensus cannot be reached.

The viability and the effectiveness of the proposed methodology have been proven through an illustrative case study, in which the process of strategic decision making with respect to innovation activities was the target to improve.

Finally, to validate the proposed methodology, an existing case study has been used, and a comparative analysis with an existing hybrid approach (i.e., fuzzy ANP and fuzzy TOPSIS) has been established.

Chapter 3

Multi-Criteria Decision-Making Problems with Unknown Weight Information under Uncertain Evaluations

Abstract

Generally, the overall evaluation of a multi-criteria decision making (MCDM) problem is based on alternatives evaluations over a set of criteria and the weights of the criteria. However, in cases where the criteria weights are unknown, the overall evaluations cannot be derived. Therefore, several methods have been proposed to handle such MCDM problems. Nevertheless, there exist MCDM problems with small amount of data and poor information, which cannot be described by a probability distribution. In such MCDM problems, the applicability of existing approaches would be influenced. Accordingly, this manuscript investigates this type of MCDM problems with small amount of data and poor information, where information on criteria weights is unknown. To this end, a new hybrid MCDM is proposed; in which the unknown criteria weights are estimated using the maximizing deviation method with grey systems theory's principles. Consequently, potential alternatives are evaluated and ranked by integrating degrees of possibility and PROMETHEE II. To show the feasibility and practicability of the proposed methodology an example is provided and to validate the methodology, a comparative analysis with an existing approach is performed.

3.1 Introduction

Multi-Criteria Decision Making (MCDM) is widely recognized for dealing with the evaluation of alternatives with respect to multiple criteria. Different approaches of MCDM exist to handle different types of multi criteria decision problems. A critical step in the selection of the proper approach is to understand the nature of the decision problem (Wątróbski & Jankowski, 2016). Accordingly, these approaches can be classified into three main categories (Belton & Stewart, 2002):

(1) Value measurement models, where the utility value of each alternative is evaluated based on the overall performance over evaluation criteria. Examples of models within this category are: Analytic Hierarchy Process (AHP), Multi-Attribute Utility Theory (MAUT), and Weighted average approach.

(2) Goal, aspiration, or reference-level models, where alternatives are evaluated with respect to a targeted level of performance over a particular goal, aspiration, or reference levels. Among these models are: Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and Vlsekriterijumska Optimizacijia I Kompromisno Resenje (VIKOR).

(3) Outranking methods, where alternatives are ranked based on the aggregated comparisons results of their evaluations over a set of criteria. Outranking methods include ELimination and Choice Expressing REality (ELECTRE) and Preference Ranking Organization METHod for Enrichment Evaluations (PROMETHEE) family of methods.

Despite the ability of conventional MCDM approaches in handling complex decision problems (i.e., decision problems that involve different alternatives and multi-criteria), these approaches assume the accuracy of information. In real life situations, many decision problems involve uncertainty, imprecision, and subjectivity, which all add to the complexity of the decision process (Banaeian et al., 2018; Karsak & Dursun, 2015; Guangxu Li et al., 2015; Małachowski, 2016). Under such a decision environment, decision makers would need to approximate ranges of evaluations using their knowledge, cognition, and available information (Lin et al., 2008). Different theories have been proposed to approximate ranges of evaluations. Among the proposed theories is the grey systems theory, which was introduced by Ju-long (1982) to handle data imprecision or insufficiency using grey numbers, in which an exact value for a grey number is unknown but a range within which the value lies is known (Sifeng Liu & Lin, 2006). Grey systems theory is recommended to address uncertainty of decision problems with small amount of data and poor information, which cannot be described by a probability distribution (D.-C. Li et al., 2012; S. Liu & Lin, 2006).

Generally, the overall evaluation of a MCDM problem is based on alternatives evaluations over multi-criteria and the weights of the criteria. However, in cases where the criteria weights are completely unknown due to different reasons, e.g., time pressure, limited expertise, incomplete knowledge, and lack of information (Das et al., 2016; S. Zhang et al., 2011), the overall evaluations cannot be derived (Xu, 2015). Accordingly, the proper assessment of criteria weights is critical for MCDM problems and may influence the analysis of a decision problem (Das et al., 2016).

To handle decision problems with unknown criteria weights, the maximizing deviation method was introduced by Wang (1998). It is considered the most popular method in establishing objective weights (Gao & Liu, 2016). In the maximizing deviation method, larger deviations are being emphasized more than smaller ones. Thus, criteria which make larger deviations should be assigned higher weights in comparison to criteria which make smaller deviations.

3.2 Background

Several studies have integrated the maximizing deviation method to address MCDM problems with unknown weight information. For instance, Wei (2008) established an optimization model based on the maximizing deviation method and intuitionistic fuzzy set, which is an extension of fuzzy set that defines the degree of membership as well as the degree of non-membership for an element to a given set. To evaluate each alternative and provide a ranking index, the intuitionistic fuzzy weighted averaging operator was used. Zhang and Liu (2010) proposed a methodology to evaluate MCDM with unknown weight information, where evaluations are a mix of exact numbers, interval numbers, and linguistic fuzzy numbers. The proposed approach used Grey Relational Analysis approach (GRA), which is a branch of grey systems theory that can be used to rank alternatives by measuring the distance from a reference sequence (i.e., optimal sequence of the evaluations) (N. Li & Huiru, 2016; S. Zhang & Liu, 2011), based on the maximizing deviation method to solve such MCDM type of problems. Zhang et al. (2011) offered an extended approach of the combination of fuzzy theory and GRA, where the maximizing deviation method has been adapted using the principles of conventional GRA method to establish criteria weights. Zhang and Liu (2015) adapted the maximizing deviation method to estimate criteria weights using interval pythagorean fuzzy set, which is an extension of intuitionistic fuzzy set where the square sum of the membership degree and the non-membership degree should be equal to or less than one. To aggregate the evaluations, the weighted averaging method is adapted using interval pythagorean fuzzy set. Xu and Zhang (2013) proposed an approach based on TOPSIS and the maximizing deviation method to address MCDM problems, where the evaluations are expressed in hesitant fuzzy set, which has been introduced by Torra (2010) as an extension of fuzzy set where the membership of an element to a given set can be denoted by interval in order to deal with uncertain information. To estimate criteria weights, an optimization model has been proposed based on the maximizing deviation method using hesitant fuzzy information; where the deviaitions degrees over evaluation criteria would be measured using the principles of Euclidean distance, which is one of the most widely used distance measures (Chen, 2018) and it represents the squared distance between two data points (Dokmanic, Parhizkar, Ranieri, & Vetterli, 2015). Xu (2015) established an optimization model to determine criteria weights by adapting the maximization deviation method using the principles of interval numbers to measure deviations over evaluation criteria; to calculate the overall evaluation for each alternative, uncertain weighted averaging operator has been used, in which the weighted average approach was used with interval evaluations rather than exact inputs. Consequently, different alternatives have been compared using the possibility degrees approach, which represents the degree to which one alternative is being better or worse than another. Şahin and Liu (2016) developed an optimization model to establish criteria weights using the maximizing deviation method with neutrosophic set and interval neutrosophic set, where neutrosophic set and interval neutrosophic set use three parameters: truth membership degree, indeterminacy/neutrality membership degree, and falsity membership degree; to aggregate the evaluations for each alternative, weighted averaging approach has been adapted. Broumi, Ye, and Smarandache (2015) calculated criteria weights by the maximizing deviation method using the interval neutrosophic set; to evaluate and rank alternatives, TOPSIS method has been adapted. Chi and Liu (2013) extended TOPSIS using interval neutrosophic set to evaluate MCDM with unknown weight information based on the maximizing deviation method to establish criteria weights.

Although several research papers have tried to address MCDM problems with unknown weight information using different types of information (e.g., fuzzy set theory, intuitionistic fuzzy set, hesitant fuzzy set, interval fuzzy set, neutrosophic set), there exist decision problems with small amount of data and poor information, which cannot be described by a probability distribution. In such decision problems, grey systems theory is recommended (D.-C. Li et al., 2012; S. Liu & Lin, 2006) due to its less restricted procedure that neither requires any robust membership function, nor a probability distribution (Memon et al., 2015). Therefore, the aim of this manuscript is to contribute to the current literature by addressing MCDM problems with small amount of data and poor information where criteria weights are unknown. To this end, a new hybrid MCDM method is proposed, which uses the principles of grey systems theory, maximizing deviation method, possibility degrees, and PROMETHEE II.

This manuscript is organized as follow: section 3.3 establishes the proposed methodology to handle uncertain evaluations MCDM with unknown criteria weight information; section 3.4 reflects the feasibility of the methodology using an illustrative example; section 3.5 validates the methodology by providing a comparative analysis with an existing approach; Finally, section 3.6 provides the concluding remarks.

3.3 Decision making method of interval grey MCDM with unknown weight information

3.3.1 Problem description

Definition 3.1 Suppose that the set of feasible alternatives of a multi-criteria decision making under uncertainty is $A = \{A_1, A_2, \dots, A_n\}$, where n is the number of the feasible alternatives and $i = 1, 2, \dots, n$. The set of evaluation criteria is denoted by $C = \{c_1, c_2, \dots, c_m\}$, where m is the number of criteria, and $j = 1, 2, \dots, m$. Let the weight of criterion c_j be represented by w_j, where $w_j \in [0,1]$ and $\sum_{j=1}^{m} w_j = 1$; but information on criteria weights is unknown.

Definition 3.2 Let $A \times C$ be the Cartesian product of the set of alternatives and the set of criteria, and $R[\otimes]$ denote the set of interval grey numbers, where the evaluation of A_i $(i = 1, 2, \dots, n)$ over c_j $(j = 1, 2, \dots, m)$ is represented by interval grey number $y_{ij}(\otimes) \in \left[\underline{y}_{ij}, \overline{y}_{ij}\right]$, where $0 \leq \underline{y}_{ij} \leq \overline{y}_{ij}$. Accordingly, the grey description function for the decision matrix $(f_{\otimes})$ is defined as follows:

$$f_{\otimes}: A \times C \to R[\otimes], thus \tag{3.1}$$

$$\forall f_{\otimes}(A_i \in A, c_j \in C): y_{ij}(\otimes) \in R[\otimes]$$

3.3.2 Normalize decision matrix

For consistency in the decision matrix such that all the evaluations would be grey based with $[0,0] \leq y_{ijk}(\otimes) \leq [1,1]$ (Bai et al., 2012), a normalization procedure of two steps is used: first, turn all non-grey values (e.g., crisp number, which is a single precise number) in the decision matrix into interval grey numbers according to Definition 3.3; second, normalize all the values using Algorithm 3.1, which is explained by Definition 3.4.

Definition 3.3 Let y_{ij} denote a white number that represents the evaluation of alternative A_i $(i = 1, 2, \ldots, n)$ over criterion c_j $(j = 1, 2, \ldots, m)$, the relative grey number $(y_{ij}(\otimes))$ of the given white number (y_{ij}) is

$$y_{ij}(\otimes) = \left[\underline{y}_{ij}, \overline{y}_{ij}\right], \text{ where } \underline{y}_{ij} = y_{ij} = \overline{y}_{ij} \tag{3.2}$$

Note that although some evaluations would be expressed by interval grey numbers, a normalized scale over the given criteria is not guaranteed. Therefore, all the interval grey values should be normalized.

Definition 3.4 Let $y_{ij}(\otimes)$ represent a general grey number that reflects the evaluation of alternative A_i $(i = 1, 2 \ldots, n)$ over criterion c_j $(j = 1, 2 \ldots, m)$; let $\min(y_{ij})$ and $\max(y_{ij})$ denote the lower and upper bounds among all $y_{ij}(\otimes)$, respectively. Let y_j^* represent a given optimal evaluation over a targeted criterion c_j. Let $\tilde{y}_{ij}(\otimes) \in \tilde{R}[\otimes]$ denote the relative normalized value of the general grey number $y_{ij}(\otimes) \in R[\otimes]$.

$$
\begin{array}{l}
if\, j \text{ is increasing criterion} \\
\qquad y_{ij}(\tilde{\otimes}) = \dfrac{[y_{ij}(\otimes) - min(y_{ij})]}{[max(y_{ij}) - min(y_{ij})]} \\
else\ if\ j \text{ is decreasing criterion} \\
\qquad y_{ij}(\tilde{\otimes}) = \dfrac{[max(y_{ij}) - y_{ij}(\otimes)]}{[max(y_{ij}) - min(y_{ij})]} \\
else\ j \text{ is targeted criterion} \\
\qquad y_{ij}(\tilde{\otimes}) = 1 - \dfrac{|y_{ij}(\otimes) - y_j^*|}{Max\{max(y_{ij}), y_j^*\} - Min\{min(y_{ij}), y_j^*\}}
\end{array}
$$

Algorithm 3.1: Normalize alternatives evaluations based on grey systems theory.

3.3.3 Establish criteria weights

Since the criteria weights are unknown for the considered type of MCDM problems, the overall evaluation of the MCDM problem cannot be derived directly from the uncertain evaluations. To determine the criteria weights, an optimization model based on the maximizing deviation method is established, in which the evaluations are assumed to be interval grey numbers, as follows:

3.3.3.1 Construct deviation matrix

As mentioned earlier, the maximizing deviation method emphasizes on the deviation degrees between evaluations; in which the larger the deviation, the higher the importance of the associated criterion. Accordingly, deviations between normalized interval grey evaluations can be established as follows:

Definition 3.5 Assume $\tilde{a}(\otimes) = [\underline{\tilde{a}}, \overline{\tilde{a}}]$ and $\tilde{b}(\otimes) = [\underline{\tilde{b}}, \overline{\tilde{b}}]$ are two normalized interval grey numbers. Let the deviation degree between $\tilde{a}(\otimes)$ and $\tilde{b}(\otimes)$ be denoted by $d(\tilde{a}(\otimes), \tilde{b}(\otimes))$.

Theorem 3.1 The deviation between $\tilde{a}(\otimes)$ and $\tilde{b}(\otimes)$, where none of them fully preferred over the other, is

$$d(\tilde{a}(\otimes), \tilde{b}(\otimes)) = |\tilde{a}(\otimes) - \tilde{b}(\otimes)| = |\tilde{b}(\otimes) - \tilde{a}(\otimes)| = \left|\left(\underline{\tilde{a}} - \overline{\tilde{b}}\right) + \left(\overline{\tilde{a}} - \underline{\tilde{b}}\right)\right| \tag{3.3}$$

Proof 3.1 Let $\tilde{a}(\otimes)$ and $\tilde{b}(\otimes)$ denote two normalized grey numbers in one dimensional space, such that

$\overline{\tilde{a}} \succcurlyeq \overline{\tilde{b}} \succ \underline{\tilde{a}} \succcurlyeq \underline{\tilde{b}}$. Let the distance function between $\tilde{a}(\otimes)$ and $\tilde{b}(\otimes)$ be denoted by $d(\tilde{a}(\otimes), \tilde{b}(\otimes))$ and be represented by Manhattan distance, which is a distance traveled to get from one data point to another (Ang, Lee, Ooi, & Ooi, 2017), such that

$$d(\tilde{a}(\otimes), \tilde{b}(\otimes)) = \|\tilde{a}(\otimes) - \tilde{b}(\otimes)\| = |(\underline{\tilde{a}} - \underline{\tilde{b}})| + \left|\left(\overline{\tilde{a}} - \overline{\tilde{b}}\right)\right| \tag{3.4}$$

Considering the ordering of the normalized grey numbers ($\overline{\tilde{a}} \succcurlyeq \overline{\tilde{b}} \succ \underline{\tilde{a}} \succcurlyeq \underline{\tilde{b}}$), Eq. (3.4) would be

$$d(\tilde{a}(\otimes), \tilde{b}(\otimes)) = \tilde{a}(\otimes) - \tilde{b}(\otimes) = (\underline{\tilde{a}} - \underline{\tilde{b}}) + \left(\overline{\tilde{a}} - \overline{\tilde{b}}\right) \tag{3.5}$$

By applying basic arithmetic operations, Eq. (3.5) can be written as

$$d(\tilde{a}(\otimes), \tilde{b}(\otimes)) = \tilde{a}(\otimes) - \tilde{b}(\otimes) = \left(\underline{\tilde{a}} - \overline{\tilde{b}}\right) + (\overline{\tilde{a}} - \underline{\tilde{b}}) \tag{3.6}$$

To generalize Eq. (3.6) for $\tilde{a}(\otimes)$ and $\tilde{b}(\otimes)$, where full preference relation does not exist, the absolute value of the total deviations should be considered, such that

$$d(\tilde{a}(\otimes), \tilde{b}(\otimes)) = |\tilde{a}(\otimes) - \tilde{b}(\otimes)| = |\tilde{b}(\otimes) - \tilde{a}(\otimes)| = \left|\left(\underline{\tilde{a}} - \overline{\tilde{b}}\right) + (\overline{\tilde{a}} - \underline{\tilde{b}})\right|$$

Theorem 3.2 If $\tilde{a}(\otimes)$ has a full preference over $\tilde{b}(\otimes)$, such that $\underline{\tilde{a}} \succcurlyeq \overline{\tilde{b}}$, the deviation between $\tilde{a}(\otimes)$ and $\tilde{b}(\otimes)$ is

$$d(\tilde{a}(\otimes), \tilde{b}(\otimes)) = \overline{\tilde{a}} - \underline{\tilde{b}} \tag{3.7}$$

Proof 3.2 The distance between two numbers a and b on a number line is given by the absolute value of their difference, that is $|(a - b)| = |(b - a)|$ (Cohen, Lee, & Sklar, 2011), by the same token the difference between two interval grey numbers $\tilde{a}(\otimes)$ and $\tilde{b}(\otimes)$ on a number line, where $\overline{\tilde{a}} \succ \underline{\tilde{a}} \succcurlyeq \overline{\tilde{b}} \succ \underline{\tilde{b}}$, can be given as follows:

$$d(\tilde{a}(\otimes),\tilde{b}(\otimes)) = \left|(\overline{\tilde{a}} - \underline{\tilde{a}})\right| + \left|\left(\underline{\tilde{a}} - \overline{\tilde{b}}\right)\right| + \left|\left(\overline{\tilde{b}} - \underline{\tilde{b}}\right)\right| \tag{3.8}$$

Considering the ordering of the normalized grey numbers ($\overline{\tilde{a}} \succ \underline{\tilde{a}} \succcurlyeq \overline{\tilde{b}} \succ \underline{\tilde{b}}$), Eq. (3.8) would be

$$d(\tilde{a}(\otimes),\tilde{b}(\otimes)) = (\overline{\tilde{a}} - \underline{\tilde{a}}) + \left(\underline{\tilde{a}} - \overline{\tilde{b}}\right) + \left(\overline{\tilde{b}} - \underline{\tilde{b}}\right) \tag{3.9}$$

By applying basic arithmetic operations, Eq. (3.9) can be written as

$$d(\tilde{a}(\otimes),\tilde{b}(\otimes)) = (\overline{\tilde{a}} - \underline{\tilde{b}}) \tag{3.10}$$

Theorem 3.3 If a normalized grey number has a partial preference over another such that,

$\overline{\tilde{a}} \succ \overline{\tilde{b}} \succ \underline{\tilde{b}} \succ \underline{\tilde{a}}$ or $\overline{\tilde{b}} \succ \overline{\tilde{a}} \succ \underline{\tilde{a}} \succ \underline{\tilde{b}}$; the deviation between them would be

$$d(\tilde{a}(\otimes),\tilde{b}(\otimes)) = \left|\left(\underline{\tilde{a}} + \overline{\tilde{b}}\right) - (\overline{\tilde{a}} + \underline{\tilde{b}})\right| \tag{3.11}$$

Proof 3.3 Let $\tilde{a}(\otimes)$ and $\tilde{b}(\otimes)$ denote two normalized grey numbers in one dimensional space, such that

$\overline{\tilde{a}} \succ \overline{\tilde{b}} \succ \underline{\tilde{b}} \succ \underline{\tilde{a}}$. Let the distance between the $\tilde{a}(\otimes)$ and $\tilde{b}(\otimes)$ be denoted by $d(\tilde{a}(\otimes),\tilde{b}(\otimes))$ and be represented by the Manhattan distance function (Ang et al., 2017),

$$\|\tilde{a}(\otimes) - \tilde{b}(\otimes)\| = \left|(\underline{\tilde{a}} - \underline{\tilde{b}})\right| + \left|\left(\overline{\tilde{a}} - \overline{\tilde{b}}\right)\right| \tag{3.12}$$

Considering the ordering of the normalized grey numbers $\overline{\tilde{a}} \succ \overline{\tilde{b}} \succ \underline{\tilde{b}} \succ \underline{\tilde{a}}$, Eq. (3.12) can be represented as

$$\tilde{a}(\otimes) - \tilde{b}(\otimes) = \left(\overline{\tilde{a}} - \overline{\tilde{b}}\right) + (\underline{\tilde{b}} - \underline{\tilde{a}}) \tag{3.13}$$

By applying basic arithmetic operations, Eq. (3.13) can be written as

$$\tilde{a}(\otimes) - \tilde{b}(\otimes) = (\overline{\tilde{a}} + \underline{\tilde{b}}) - \left(\underline{\tilde{a}} + \overline{\tilde{b}}\right) \tag{3.14}$$

To generalize Eq. (3.14) for $\tilde{a}(\otimes)$ and $\tilde{b}(\otimes)$, where partial preference exists, the absolute value should be considered, such that

$$d\big(\tilde{a}(\otimes),\tilde{b}(\otimes)\big) = \left|\tilde{a}(\otimes)-\tilde{b}(\otimes)\right| = \left|\tilde{b}(\otimes)-\tilde{a}(\otimes)\right| = \left|\left(\underline{\tilde{a}}+\overline{\tilde{b}}\right)-\left(\overline{\tilde{a}}+\underline{\tilde{b}}\right)\right|$$

The larger the value of $d\big(\tilde{a}(\otimes),\tilde{b}(\otimes)\big)$, the greater the deviation degree between $\tilde{a}(\otimes)$ and $\tilde{b}(\otimes)$. Thus,

$$d\big(\tilde{a}(\otimes),\tilde{b}(\otimes)\big) = 0 \text{ if and only if } \tilde{a}(\otimes) = \tilde{b}(\otimes) \tag{3.15}$$

Definition 3.6 Let the deviation degree between the normalized grey evaluations of two alternatives, i.e. A_i and A_k $(i,k = 1,2\ldots,n)$, over c_j $(j = 1,2\ldots,m)$ be denoted by $d\big(\tilde{y}_{ij}(\otimes),\tilde{y}_{kj}(\otimes)\big)$. Based on Eqs. (3.3), (3.7), (3.11), and (3.15), let Algorithm 3.2 represent all possible evaluations of $d\big(\tilde{y}_{ij}(\otimes),\tilde{y}_{kj}(\otimes)\big)$:

$$\begin{array}{l}
if\ \tilde{y}_{ij}(\otimes) = \tilde{y}_{kj}(\otimes) \\
\qquad\qquad d\big(\tilde{y}_{ij}(\otimes),\tilde{y}_{kj}(\otimes)\big) = 0 \\
else\ if\ \tilde{y}_{ij}(\otimes) \neq \tilde{y}_{kj}(\otimes) \wedge \left(\underline{\tilde{y}}_{ij} \geqslant \overline{\tilde{y}}_{kj}\right) \\
\qquad\qquad d\big(\tilde{y}_{ij}(\otimes),\tilde{y}_{kj}(\otimes)\big) = \left(\overline{\tilde{y}}_{ij}-\underline{\tilde{y}}_{kj}\right) \\
else\ if\ \overline{\tilde{y}}_{ij} \succ \overline{\tilde{y}}_{kj} \succ \underline{\tilde{y}}_{kj} \succ \underline{\tilde{y}}_{ij} \\
\quad d\big(\tilde{y}_{ij}(\otimes),\tilde{y}_{kj}(\otimes)\big) = \left|\left(\underline{\tilde{y}}_{ij}+\overline{\tilde{y}}_{kj}\right)-\left(\overline{\tilde{y}}_{ij}+\underline{\tilde{y}}_{kj}\right)\right| \\
else \\
\quad d\big(\tilde{y}_{ij}(\otimes),\tilde{y}_{kj}(\otimes)\big) = \left|\left(\underline{\tilde{y}}_{ij}-\overline{\tilde{y}}_{kj}\right)+\left(\overline{\tilde{y}}_{ij}-\underline{\tilde{y}}_{kj}\right)\right|
\end{array}$$

Algorithm 3.2: Deviation evaluations between normalized interval grey numbers.

3.3.3.2 Estimate criteria weights

Definition 3.7 Let $D_{ij}(w)$ denote the overall deviation between alternative A_i and other alternatives over criterion c_j. Let w_j denote the associated criterion weight such that,

$$D_{ij}(w) = \sum_{\substack{k=1,\\k\neq i}}^{n} d\big(\tilde{y}_{ij}(\otimes)\,,\tilde{y}_{kj}(\otimes)\,\big)w_j\,,\;\; i,k = 1,2,\dots,n,\;\; j = 1,2\dots,m \tag{3.16}$$

Definition 3.8 Let $D_j(w)$ denote the global deviation between alternatives over a criterion c_j. Then,

$$D_j(w) = \sum_{i=1}^{n} D_{ij}(w) = \sum_{i=1}^{n}\sum_{\substack{k=1,\\k\neq i}}^{n} d\big(\tilde{y}_{ij}(\otimes)\,,\tilde{y}_{kj}(\otimes)\,\big)w_j\,, j = 1,2\dots,m \tag{3.17}$$

Definition 3.9 Let $D(w)$ denote the total deviations over all the criteria, such that the total deviation function would be expressed as follows:

$$D(w) = \sum_{j=1}^{m} D_j(w) = \sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{\substack{k=1,\\k\neq i}}^{n} d\big(\tilde{y}_{ij}(\otimes)\,,\tilde{y}_{kj}(\otimes)\,\big)w_j \tag{3.18}$$

Theorem 3.4 In a grey MCDM with completely unknown criteria weights, the optimal solution to maximize the decision space can be obtained using $w_j{}^* = (w_1^*, w_2^*, \dots, w_m^*)$, where

$$w_j^* = \frac{\sum_{i=1}^{n}\sum_{\substack{k=1,\\k\neq i}}^{n} d\big(\tilde{y}_{ij}(\otimes),\tilde{y}_{kj}(\otimes)\,\big)}{\sqrt{\sum_{j=1}^{m}\left[\sum_{i=1}^{n}\sum_{\substack{k=1,\\k\neq i}}^{n} d\big(\tilde{y}_{ij}(\otimes),\tilde{y}_{kj}(\otimes)\,\big)\right]^2}},\;\; j = 1,2\dots,m \tag{3.19}$$

In order to satisfy the normalization constraint condition, the weight of criterion c_j can be expressed as

$$w_j = \frac{w_j^*}{\sum_{j=1}^{m} w_j^*} = \frac{\sum_{i=1}^{n}\sum_{\substack{k=1,\\k\neq i}}^{n} d\big(\tilde{y}_{ij}(\otimes)\,,\tilde{y}_{kj}(\otimes)\,\big)}{\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{\substack{k=1,\\k\neq i}}^{n} d\big(\tilde{y}_{ij}(\otimes),\tilde{y}_{kj}(\otimes)\,\big)},\;\; j = 1,2\dots,m \tag{3.20}$$

Proof 3.4 For unknown weights of criteria, the maximizing deviation method (Y. M. Wang, 1998) can be adapted, in which the criteria weights are being estimated based on the deviation degrees between evaluations. In other words, the larger the deviation, the higher the importance of the associated criterion. To this end, the following optimization decision making model is established

$$max\, D(w) = \sum_{j=1}^{m} D_j(w) = \sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{\substack{k=1,\\k\neq i}}^{n} d\big(\tilde{y}_{ij}(\otimes), \tilde{y}_{kj}(\otimes)\,\big)w_j{}^* \tag{3.21}$$

$$s.t.\sum_{j=1}^{m}\left(w_j{}^*\right)^2 = 1\,, w_j{}^* \geq 0,\ j = 1,2\dots,m \tag{3.22}$$

Note that $\sum_{j=1}^{m}\left(w_j{}^*\right)^2 = 1$ is used as a constraint to accentuate the higher deviations by reflecting a wider range, which would maximize the evaluation space.

The model can be solved using Lagrange function (Z. Xu & Cai, 2012):

$$L\left(w_j{}^*, \lambda\right) = D(w) - \frac{1}{2}\lambda\left(\sum_{j=1}^{m}\left(w_j{}^*\right)^2 - 1\right), \text{ where} \tag{3.23}$$

λ is the Lagrange multiplier. Thus,

$$L\left(w_j{}^*, \lambda\right) = \sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{\substack{k=1,\\ k\neq i}}^{n} d\left(\tilde{y}_{ij}(\otimes), \tilde{y}_{kj}(\otimes)\right)w_j{}^* - \frac{1}{2}\lambda\left(\sum_{j=1}^{m}\left(w_j{}^*\right)^2 - 1\right) \tag{3.24}$$

By calculating the partial derivative of Eq. (3.24) with respect to $w_j{}^*(j = 1, 2\dots, m)$ and λ, while setting these partial derivatives to zero, the following equations are obtained:

$$\frac{\partial L(w_j{}^*,\lambda)}{\partial w_j{}^*} = \sum_{i=1}^{n}\sum_{\substack{k=1,\\ k\neq i}}^{n} d\left(\tilde{y}_{ij}(\otimes), \tilde{y}_{kj}(\otimes)\right) - \lambda w_j{}^* = 0 \tag{3.25}$$

$$\frac{\partial L(w_j{}^*,\lambda)}{\partial \lambda} = \sum_{j=1}^{m}\left(w_j{}^*\right)^2 - 1 = 0 \tag{3.26}$$

From Eq. (3.25), we can obtain

$$w_j^* = \frac{\sum_{i=1}^{n}\sum_{\substack{k=1,\\ k\neq i}}^{n} d\left(\tilde{y}_{ij}(\otimes),\tilde{y}_{kj}(\otimes)\right)}{\lambda} \tag{3.27}$$

Since $\sum_{j=1}^{m}\left(w_j{}^*\right)^2 = 1$, the value of λ can be obtained using Eq. (3.27) as follows:

$$\sum_{j=1}^{m}\left(\frac{\sum_{i=1}^{n}\sum_{\substack{k=1,\\ k\neq i}}^{n} d\left(\tilde{y}_{ij}(\otimes),\tilde{y}_{kj}(\otimes)\right)}{\lambda}\right)^2 = 1 \tag{3.28}$$

By simplifying Eq. (3.28)

$$\lambda = \sqrt{\sum_{j=1}^{m}\left[\sum_{i=1}^{n}\sum_{\substack{k=1,\\k\neq i}}^{n} d\big(\tilde{y}_{ij}(\otimes), \tilde{y}_{kj}(\otimes)\big)\right]^2} \tag{3.29}$$

Thus, the optimal weight value of a given criterion would be obtained as follows:

$$w_j^* = \frac{\sum_{i=1}^{n}\sum_{\substack{k=1,\\k\neq i}}^{n} d\big(\tilde{y}_{ij}(\otimes), \tilde{y}_{kj}(\otimes)\big)}{\sqrt{\sum_{j=1}^{m}\left[\sum_{i=1}^{n}\sum_{\substack{k=1,\\k\neq i}}^{n} d\big(\tilde{y}_{ij}(\otimes), \tilde{y}_{kj}(\otimes)\big)\right]^2}},\ j = 1,2 \ldots, m$$

3.3.4 Evaluate and rank feasible alternatives

After establishing the weights of the criteria for a grey MCDM problem with unknown weight information, the MCDM problem would be evaluated using the established criteria weights, possibility degrees, and PROMETHEE II. The detailed process is described in the following subsections.

3.3.4.1 Establish preferences

In order to establish preferences among the evaluation measures over each criterion, the possibility degree formula is used based on the definition of Xu and Da (2002).

Definition 3.10 Let $l_{\tilde{a}(\otimes)j}$ and $l_{\tilde{b}(\otimes)j}$ denote the difference between the upper and lower limits for two normalized grey numbers $\tilde{a}(\otimes)$ and $\tilde{b}(\otimes)$, respectively, such that

$$l_{\tilde{a}(\otimes)j} = \overline{\tilde{a}}_j - \underline{\tilde{a}}_j \tag{3.30}$$

$$l_{\tilde{b}(\otimes)j} = \overline{\tilde{b}}_j - \underline{\tilde{b}}_j \tag{3.31}$$

Definition 3.11 Let $\tilde{p}_j\big(\tilde{a}(\otimes) \succ \tilde{b}(\otimes)\big)$ denote the degree of preference of $\tilde{a}(\otimes)$ over $\tilde{b}(\otimes)$ with respect to c_j. Using possibility degrees, $\tilde{p}_j\big(\tilde{a}(\otimes) \succ \tilde{b}(\otimes)\big)$ would be determined as

$$\tilde{p}_j\big(\tilde{a}(\otimes) \succ \tilde{b}(\otimes)\big) = \frac{\overline{\tilde{a}}_j - \underline{\tilde{b}}_j}{l_{\tilde{a}(\otimes)j} + l_{\tilde{b}(\otimes)j}} \tag{3.32}$$

Theorem 3.5 For any two unequal grey numbers (e.g., $\tilde{a}(\otimes) = [\underline{\tilde{a}}, \overline{\tilde{a}}]$ and $\tilde{b}(\otimes) = [\underline{\tilde{b}}, \overline{\tilde{b}}]$) the sum of preferences $\tilde{p}_j\big(\tilde{a}(\otimes), \tilde{b}(\otimes)\big)$ is

$$\tilde{p}_j\big(\tilde{a}(\otimes), \tilde{b}(\otimes)\big) = \tilde{p}_j\big(\tilde{a}(\otimes) \succ \tilde{b}(\otimes)\big) + \tilde{p}_j\big(\tilde{b}(\otimes) \succ \tilde{a}(\otimes)\big) = 1 \tag{3.33}$$

Proof 3.5 Let $\tilde{p}_j\big(\tilde{a}(\otimes) \succ \tilde{b}(\otimes)\big) = \frac{\overline{\tilde{a}}_j - \underline{\tilde{b}}_j}{l_{\tilde{a}(\otimes)j} + l_{\tilde{b}(\otimes)j}}$, $\tilde{p}_j\big(\tilde{b}(\otimes) \succ \tilde{a}(\otimes)\big) = \frac{\overline{\tilde{b}}_j - \underline{\tilde{a}}_j}{l_{\tilde{a}(\otimes)j} + l_{\tilde{b}(\otimes)j}}$ and $\tilde{a}(\otimes) \neq \tilde{b}(\otimes)$. By taking the sum of preferences, the following equation is obtained:

$$\frac{\overline{\tilde{a}}_j - \underline{\tilde{b}}_j}{l_{\tilde{a}(\otimes)j} + l_{\tilde{b}(\otimes)j}} + \frac{\overline{\tilde{b}}_j - \underline{\tilde{a}}_j}{l_{\tilde{a}(\otimes)j} + l_{\tilde{b}(\otimes)j}} = \frac{(\overline{\tilde{a}}_j - \underline{\tilde{b}}_j) + (\overline{\tilde{b}}_j - \underline{\tilde{a}}_j)}{(\overline{\tilde{a}}_j - \underline{\tilde{a}}_j) + (\overline{\tilde{b}}_j - \underline{\tilde{b}}_j)} = 1 \tag{3.34}$$

Based on Eqs. (3.32) and (3.33), $\tilde{p}_j\big(\tilde{a}(\otimes) \succcurlyeq \tilde{b}(\otimes)\big)$ can be represented as

$$\tilde{p}_j\big(\tilde{a}(\otimes) \succcurlyeq \tilde{b}(\otimes)\big) = \begin{cases} 0 & \text{if } \tilde{p}_j\big(\tilde{a}(\otimes) \succ \tilde{b}(\otimes)\big) \leq 0 \\ \frac{\overline{\tilde{a}}_j - \underline{\tilde{b}}_j}{l_{\tilde{a}(\otimes)j} + l_{\tilde{b}(\otimes)j}} & \text{if } 0 < \tilde{p}_j\big(\tilde{a}(\otimes) \succ \tilde{b}(\otimes)\big) < 1 \\ 1 & \text{if } \tilde{p}_j\big(\tilde{a}(\otimes) \succ \tilde{b}(\otimes)\big) \geq 1 \end{cases} \tag{3.35}$$

3.3.4.2 Determine relative preferences

The relative preferences between alternatives can be determined using the calculated degrees of preferences between the alternatives and the associated criteria weights.

Definition 3.12 Let $\tilde{p}_j(A_a \succ A_b)$ represent the degree of preference of alternative A_a over A_b with respect to criterion c_j. Let the associated normalized optimal weight of c_j be represented by w_j, where $\sum_{j=1}^{m} w_j = 1, j = 1, 2 \ldots, m$. Let $\tilde{\pi}(A_a \succ A_b)$ denote the relative preference of alternative A_a over A_b across the set of evaluation criteria $C = \{c_1, c_2 \ldots, c_m\}$. Accordingly, $\tilde{\pi}(A_a \succ A_b)$ is

$$\tilde{\pi}(A_a \succ A_b) = \sum_{j=1}^{m} w_j\, \tilde{p}_j(A_a \succ A_b),\ j = (1, 2, \ldots, m) \tag{3.36}$$

3.3.4.3 Estimate global preferences and rank available alternatives

Using the resultant relative preferences matrix, the global preference of a given alternative over others can be determined using the outranking flows measures of PROMETHEE II.

Definition 3.13 For a grey MCDM problem, let $\tilde{\phi}^+(A_a)$ denote the extent to which alterative A_a is preferred over all other alternatives (i.e., positive outranking flow of A_a). Let $\tilde{\pi}\ (A_a \succ A_b)$ indicate the relative preference of A_a over A_b. Using the preference indication measures of A_a over other alternatives, the function of $\tilde{\phi}^+(A_a)$ would be

$$\tilde{\phi}^+(A_a) = \frac{1}{n-1}\sum_{b=1}^{n} \tilde{\pi}(A_a \succ A_b) \tag{3.37}$$

Definition 3.14 For a grey MCDM problem, let $\tilde{\phi}^-(A_a)$ denote the extent to which alterative A_a is outranked by all other alternatives (i.e., negative outranking flow of A_a). Let $\tilde{\pi}\ (A_b \succ A_a)$ indicate the degree by which alternative A_a is outranked by A_b. Using $\tilde{\pi}\ (A_b \succ A_a)$, the function of $\tilde{\phi}^+(A_a)$ would be

$$\tilde{\phi}^-(A_a) = \frac{1}{n-1}\sum_{b=1}^{n} \tilde{\pi}(A_b \succ A_a) \tag{3.38}$$

Definition 3.15 For a grey MCDM problem, let $\tilde{\phi}\ (A_a)$ denote the global preference (i.e., net outranking flow) of alternative A_a. Using the measures of positive and negative outranking flows, $\tilde{\phi}\ (A_a)$ can be obtained, such that

$$\tilde{\phi}\ (A_a) = \tilde{\phi}^+\ (A_a) - \tilde{\phi}^-\ (A_a) \tag{3.39}$$

Once the net outranking flow has been estimated for all feasible alternatives, a complete ranking index can be established based on the values of global preferences, wherein the higher the value of $\tilde{\phi}\ (A_a)$, the better is the alternative. Thus, the best alternative is the one with the highest global preference value.

3.4 Illustrative example

To demonstrate the feasibility and practicability of the proposed methodology, an illustrative example is given. Consider a problem that a factory plans to install a fire protection system for the new expansion; four subcontractors are considered for the project and represented by A_i ($i = 1,2,3,4$); each of which should be evaluated over five different criteria: $c_1 -$ reliability, $c_2 -$ reaction ability, $c_3 -$ control distance, $c_4 -$ impact on the fire, and $c_5 -$ cost. All the criteria, except c_5, are increasing criteria (benefit type criteria). Each of c_j ($j = 1,2,3,4,5$) has an associated weight $w_j \in [0,1]$, in which $\sum_{j=1}^{5} w_j = 1$, where the criteria weights are unknown. The evaluations of the alternatives are listed in Table 3.1.

Table 3.1: Uncertain decision matrix

Performance matrix	**Alternatives**			
Criteria	**A1**	**A2**	**A3**	**A4**
Reaction ability (c1) m	[160, 180]	[200, 300]	[150, 250]	[180, 200]
Reliability (c2) %	[0.6, 0.8]	[0.7, 0.8]	[0.6, 0.9]	[0.5, 0.7]
Control distance (c3) m	[180, 190]	[150, 170]	[160, 180]	[170, 200]
Impact on the fire (c4) %	[0.7, 0.8]	[0.5, 0.7]	[0.6, 0.9]	[0.4, 0.6]
Cost (c5) $	[15000, 16000]	[27000, 29000]	[24000, 26000]	[15000, 17000]

The proposed methodology is utilized to solve the decision problem as follows:

Step 1 Normalize the grey numbers of the uncertain decision matrix using Algorithm 3.1. The grey normalized values of the uncertain decision matrix is listed in Table 3.2.

Step 2 Estimate criteria weight vector $w = (w_1, w_2, w_3, w_4, w_5)$ using Eq. (3.20):

$$w = (0.2036, 0.1636, 0.2073, 0.2073, 0.2182)$$

Step 3 Evaluate preferences between alternatives using Eq. (3.35). The preference matrices are reflected in Tables 3.3, 3.4, 3.5, and 3.6 for A_1 to A_4, respectively.

Step 4 Determine relative preferences (Table 3.7) among alternatives by considering the calculated degrees of preferences between the alternatives and the estimated weights using Eq. (3.36).

Table 3.2: Normalized uncertain decision matrix

Performance matrix		**Alternatives**		
Criteria	**A_1**	**A_2**	**A_3**	**A_4**
Reaction ability (c_1)	[0.067, 0.2]	[0.333, 1]	[0, 0.667]	[0.2, 0.333]
Reliability (c_2)	[0.6, 0.8]	[0.7, 0.8]	[0.6, 0.9]	[0.5, 0.7]
Control distance (c_3)	[0.6, 0.8]	[0, 0.4]	[0.2, 0.6]	[0.4, 1]
Impact on the fire (c_4)	[0.7, 0.8]	[0.5, 0.7]	[0.6, 0.9]	[0.4, 0.6]
Cost (c_5)	[0.923, 1]	[0, 0.143]	[0.214, 0.357]	[0.857, 1]

Table 3.3: Preference matrix of **A_1**

Multi-criteria preference matrix		**Alternatives**		
Base alternative	**Criteria**	**A_2**	**A_3**	**A_4**
A_1	**Reaction ability (c_1)**	0	0.25	0
	Reliability (c_2)	0.3333	0.4	0.75
	Control distance (c_3)	1	1	0.5
	Impact on the fire (c_4)	1	0.5	1
	Cost (c_5)	1	1	0.6667

Step 5 Establish global preferences among alternatives (Table 3.8) using Eqs. (3.37), (3.38), and (3.39).

Step 6 Rank the alternatives according to the net outranking flow values in descending order:

$$A_1 \succ A_3 \succ A_4 \succ A_2$$

Thus, alternative A_1 is the best selection for the given uncertain MCDM problem.

Table 3.4: Preference matrix of **A_2**

Multi-criteria preference matrix		**Alternatives**		
Base alternative	**Criteria**	**A_1**	**A_3**	**A_4**
A_2	**Reaction ability (c_1)**	1	0.75	1
	Reliability (c_2)	0.6667	0.5	1
	Control distance (c_3)	0	0.25	0
	Impact on the fire (c_4)	0	0.2	0.75
	Cost (c_5)	0	0	0

Table 3.5: Preference matrix of **A_3**

Multi-criteria preference matrix		**Alternatives**		
Base alternative	**Criteria**	**A_1**	**A_2**	**A_4**
A_3	**Reaction ability (c_1)**	0.75	0.25	0.5833
	Reliability (c_2)	0.6	0.5	0.8
	Control distance (c_3)	0	0.75	0.2
	Impact on the fire (c_4)	0.5	0.8	1
	Cost (c_5)	0	1	0

Table 3.6: Preference matrix of **A_4**

Multi-criteria preference matrix		Alternatives		
Base alternative	**Criteria**	**A_1**	**A_2**	**A_3**
A_4	**Reaction ability (c_1)**	1	0	0.4167
	Reliability (c_2)	0.25	0	0.2
	Control distance (c_3)	0.5	1	0.8
	Impact on the fire (c_4)	0	0.25	0
	Cost (c_5)	0.3333	1	1

Table 3.7: Relative preference matrix

Alternatives	A_1	A_2	A_3	A_4
A_1	0.5	0.6873	0.6455	0.5791
A_2	0.3127	0.5	0.3278	0.5227
A_3	0.3545	0.6722	0.5	0.4984
A_4	0.4209	0.4773	0.5016	0.5

Table 3.8: Outranking flows computations

Outranking flows	Alternatives			
	A_1	**A_2**	**A_3**	**A_4**
Outflow	0.8039	0.5544	0.6751	0.6333
Inflow	0.5294	0.7789	0.6583	0.7001
Net flow	0.2745	-0.2245	0.0168	-0.0668

3.5 Comparative analysis

To validate the proposed methodology, an existing example in the literature is used. The example was adopted from the work of Xu (2015), which addressed MCDM problems with unknown weight information based on the deviation degrees of interval numbers.

In order to compare the proposed methodology in this manuscript and the original approach of Xu (2015) with respect to criteria weights estimation, four different approaches have been considered as shown in Table 3.9. The approaches are as follows: (1) the proposed methodology of this manuscript; (2) Combination A: used the normalization approach of Xu (2015) and the deviation function of this manuscript; (3) Combination B: used the proposed normalization approach of this manuscript and the deviation function of Xu (2015) (i.e., Manhattan distance); (4) the approach of Xu (2015). The different normalization approaches were considered to highlight the influence of normalization procedure on the final outcomes.

Table 3.9: Criteria weights using different approaches

Criteria	Proposed	Combination A	Combination B	Xu (2015)
Fire attack ability (c_1)	0.2021	0.2215	0.1980	0.2189
Reaction ability (c_2)	0.2068	0.2755	0.1504	0.2182
Maneuverability (c_3)	0.1736	0.1376	0.2032	0.1725
Survival ability (c_4)	0.2068	0.2135	0.2082	0.2143
Cost (c_5)	0.2108	0.1518	0.2402	0.1761

From Table 3.9, it is obvious that different normalization approaches resulted in different criteria weights, regardless of the employed deviation function. However, Maneuverability (c_3) scored the lowest weight in three approaches out of the four, i.e., the proposed methodology, Combination A, and the approach of Xu (2015).

When it comes to deviation degrees consideration, by using the proposed deviation function in this manuscript, regardless of the normalization approach, Reaction ability (c_2) scored the highest weight and Maneuverability (c_3) scored the lowest weight.

To compare the final ranking order of the proposed methodology with the results of Xu (2015), criteria weights of the proposed approach are used. Consequently, the ranking order of the given decision problem based on the proposed methodology would be

$$A_1 \succ A_4 \succ A_3 \succ A_2$$

with A_1 as the best option, which is the best option while using the approach of Xu (2015) as well. However, the ranking order is different for the rest of alternatives due to procedure differences in addressing the problem, which can be summarized in the following: (1) Normalization approach; (2) Deviation function to estimate criteria weights; (3) Alternatives evaluation and ranking methodology.

It is noteworthy to mention that during the analysis of different possible scenarios for the deviation between two interval numbers, one scenario was identified where the deviation function of Xu (2015) (i.e. Manhattan distance) fails to address. The scenario can be denoted by the empty intersection scenario; which indicates that a given two data points have no points in common. However, the proposed approach in this manuscript considers this scenario. As a result, **Theorem 3.6** is introduced.

Theorem 3.6 For a given two interval numbers $a = [\underline{a}, \overline{a}]$ and $b = [\underline{b}, \overline{b}]$, where the intersection between the two data points is empty (i.e. $\underline{a} \succ \overline{b}$ or $\underline{b} \succ \overline{a}$), the deviation between the two data points cannot be determined using the Manhattan distance.

Proof 3.6 The distance between two numbers a and b on a number line is given by the absolute value of their difference, that is $|(a - b)| = |(b - a)|$ (Cohen et al., 2011), by the same token the difference between two interval numbers $a = [\underline{a}, \overline{a}]$ and $b = [\underline{b}, \overline{b}]$ on a number line, where $\overline{a} \succ \underline{a} \succ \overline{b} \succ \underline{b}$, can be given as follows:

$$d(a,b) = \left|\left(\overline{a} - \underline{a}\right)\right| + \left|\left(\underline{a} - \overline{b}\right)\right| + \left|\left(\overline{b} - \underline{b}\right)\right| \tag{3.40}$$

Considering the ordering of the normalized grey numbers ($\overline{a} \succ \underline{a} \succ \overline{b} \succ \underline{b}$), Eq. (3.40) would be

$$d(a,b) = \left(\overline{\tilde{a}} - \underline{\tilde{a}}\right) + \left(\underline{\tilde{a}} - \overline{\tilde{b}}\right) + \left(\overline{\tilde{b}} - \underline{\tilde{b}}\right) \tag{3.41}$$

By applying basic arithmetic operations, Eq. (3.41) can be written as

$$d(a,b) = \left(\overline{\tilde{a}} - \underline{\tilde{b}}\right) \neq \left|\left(\underline{\tilde{a}} - \underline{\tilde{b}}\right)\right| + \left|\left(\overline{\tilde{a}} - \overline{\tilde{b}}\right)\right| \tag{3.42}$$

Accordingly, the deviation function by Xu (2015) fails to address the deviation between two interval numbers in which they have no data points in common. In contrast, the proposed deviation algorithm (i.e., Algorithm 3.2) in this thesis considers this case.

3.6 Conclusion

Although MCDM problems with unknown weight information have been addressed in the literature using different approaches and different types of information (e.g., fuzzy set theory, intuitionistic fuzzy set, hesitant fuzzy set, interval fuzzy set, neutrosophic set), there exist MCDM problems with small amount of data and poor information, which cannot be described by a probability distribution. In such MCDM problems, the applicability of existing approaches would be influenced. To overcome this limitation, this manuscript investigates MCDM problems with small amount of data and poor information, where information on criteria weights is completely unknown, and proposes to use grey systems theory due to its less restricted procedure that neither requires any robust membership function, nor a probability distribution. To this end, a new hybrid MCDM methodology is proposed. At first, the unknown criteria weights should be estimated. Accordingly, a new optimization model is established based on the integration of grey system theory's principles and the maximizing deviation method. By solving the optimization model, a function to determine the optimal criteria weights is obtained. Consequently, potential alternatives are evaluated by integrating degrees of possibility and PROMETHEE II, where the resultant weights are used to evaluate possible alternatives. Once the evaluation process is completed, the rank index for potential alternatives is provided based on the values of global preferences, wherein the higher the value, the better is the alternative.

To determine the feasibility of the proposed methodology, an illustrative example is given; whereas to validate the methodology, a comparative analysis with an existing approach is provided.

The contributions of this manuscript over existing research works can be summarized in the following points: (1) Determine the optimal criteria weights for MCDM problems with small amount of data and poor information where information on criteria weights is unknown, by establishing a new optimization model which integrates the principles of the grey system theory and the maximizing deviation method; (2) Extend the PROMETHEE II approach such that to evaluate and rank potential alternatives within uncertain MCDM with unknown weights information; (3) Generalize the determination procedure of deviation degrees to account for various scenarios of deviation between interval numbers.

Chapter 4

A New Approach to Address Uncertain Dynamic Multi-Criteria Decision Problems with Unknown Weight Information

Abstract

Conventional Multi-Criteria Decision Making (MCDM) approaches are not suitable to address the dynamics over time. Therefore, different hybrid approaches have been introduced to deal with Dynamic MCDM (DMCDM) problems. Two main concerns with existing approaches were identified: (1) Criteria weights establishment within the context of DMCDM with unknown weight information; (2) Weight vector establishment of different periods where the influences of the evaluation criteria are changing over time. To overcome the shortcomings of existing approaches, this manuscript proposed a new hybrid methodology to handle DMCDM with small amount of data and poor information, which cannot be described by a probability distribution, where information on criteria weights and the influence of different time periods are unknown. The proposed methodology estimates the unknown criteria weights using the maximizing deviation method, where the deviation degrees calculations adopted the principles of grey systems theory. When it comes to weight vector establishments of different periods, a new optimization model is introduced where the influence of different evaluation criteria on decision problems are changing over different periods. To evaluate and rank potential alternatives of DMCDM problems, the PROMETHEE II approach is extended using the optimized weights and the possibility degrees.

4.1 Introduction

Multi-Criteria Decision Making (MCDM) is a branch of operation research that is intended to aid decision makers in establishing coherent preferences in complex decision problems (Roy, 2016; Wątróbski & Jankowski, 2016). Different methods of MCDM have been proposed, each of which has its own characteristics in evaluating decision problems (Mardani et al., 2015). Therefore, understanding the background differences among these methods is essential to properly analyze a decision problem. To differentiate between the MCDM methods, the following classification is provided (Belton & Stewart, 2002):

(1) Value measurement methods: this group of methods are value focused, in which the overall evaluation over different criteria is used to select between alternatives; among this category of methods are Analytic Hierarchy Process (AHP), Multi-Attribute Utility Theory (MAUT), and Weighted Average (WA) approach.

(2) Goal, aspiration, or reference-level methods: this group of methods are optimal value focused where alternatives are evaluated with respect to a particular goal, aspiration, or reference levels; examples of methods within this category are: Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and Vlsekriterijumska Optimizacijia I Kompromisno Resenje (VIKOR).

(3) Outranking methods: this group of methods are pairwise comparisons focused where alternatives are compared over each criterion and the aggregated comparisons would be used to rank the alternatives; the most prominent methods within this category are: ELimination and Choice Expressing REality (ELECTRE) and Preference Ranking Organization METHod for Enrichment Evaluations (PROMETHEE) family of methods.

Although MCDM conventional approaches can be used to establish a better ground for decision makers in complex decision problems, they are deemed inappropriate to handle decision problems that involve uncertain, imprecise, and subjective data; as they presume the availability of precise information.

The complexity of decision problems increases in many real life applications, where information from different periods (e.g., multi-period investment, medical diagnosis) should be considered in the decision process (G. Wei, 2011). Conventional MCDM approaches are not appropriate to understand the dynamics over time, as they are clearly static (Pruyt, 2007).

4.2 Background

To account for the dynamic aspects in MCDM problems, different hybrid approaches have been proposed. For instance, Lin et al. (2008) developed a DMCDM method using a hybrid approach, in which TOPSIS was used as the main structure, while the combination of grey concept (which was introduced by Ju-long (1982) to handle data imprecision or insufficiency using interval number where its exact value is unknown but a range within which the value lies is known (Sifeng Liu & Lin, 2006)) and Minkowski distance function (which is a generalization function of the most widely used distance measures (i.e., Euclidean distance and the Manhattan distance) between two data points (Chen, 2018)) to handle uncertain information and the multi period evaluations. Park, Cho, and Kwun (2013) proposed two aggregation operators called Dynamic Intuitionistic Fuzzy Weighted Geometric (DIFWG) operator and Uncertain Dynamic Intuitionistic Fuzzy Weighted Geometric (UDIFWG) operator to solve MCDM problems under dynamic intuitionistic fuzzy environment using VIKOR principles; in which the evaluations are expressed in different periods using intuitionistic fuzzy set, which is an extension of Zadeh's fuzzy set whose basic component is only a membership function for an element to a given set, in which the intuitionistic fuzzy set defines the degree of membership as well as the degree of non-membership. Gümüş and Bali (2017) introduced two methods based on Einstein operational laws on intuitionistic fuzzy sets, which are Dynamic Intuitionistic Fuzzy Einstein Averaging (DIFWA$^{\epsilon}$) operator and Dynamic Intuitionistic Fuzzy Einstein Geometric Averaging (DIFWG$^{\epsilon}$) operator, to solve multi-periods MCDM problems within intuitionistic fuzzy environment. Esangbedo and Che (2016) proposed a Grey Weighted Sum Model (GWSM) to handle uncertainty over multi-periods multi-criteria decision problems, in which interval grey numbers are used to express varying performances over multi-periods, while the concept of WSM is used to aggregate the results of different alternatives. Wei and Lin (2008) proposed a new concept of dynamic uncertain multiplicative linguistic preference relations to solve the multi-periods MCDM problems, where the criteria values are given in uncertain multiplicative linguistic relations collected in different time periods.

A common concern of the above-mentioned approaches is related to the weight vector establishment for different periods. The weights of different periods are assumed to be given or simply ignored. Moreover, information on criteria weights is assumed to be known.

To account for the weights of different time periods in MCDM problems, different studies used the Basic Unit-interval Monotonic (BUM) function (Yager, 1998, 2004), which is one of the most common weight determination methods of different time periods. For instance, Peng and Wang (2014) presented some dynamic hesitant fuzzy aggregation operators, based on the weighted averaging and the weighted geometric approaches, to tackle multi-period decision making problems where all decision information from different period is provided in hesitant fuzzy information (which has been introduced by Torra (2010) as an extension of fuzzy set where the membership of an element to a given set can be denoted by an interval in order to deal with uncertain information). However, two linguistic approaches were introduced based on BUM function to estimate the weight vector of the different periods: one approach depends on the degree of emphasis between the initial and the final period; while the other focuses on the values of the argument variables at each period, which is determined based on the score function of Xia and Xu (2011). Similarly, G. Li et al. (2015) proposed a dynamic fuzzy multiple criteria decision making (DFMCDM) method based on TOPSIS, where the time weight is calculated using BUM function.

Within the context of DMCDM problems under uncertain environment, existing methods to obtain the weights of different time periods, such as BUM function, would address the influence of different periods independently from the evaluation criteria. In other word, the variabilities in nature of different decision criteria are ignored, which impose a falsify assumption with regards to the dynamic aspect of this type of MCDM problems. Furthermore, the overall evaluation of a MCDM problem is based on alternatives evaluations over the multi-criteria and the weights of the criteria. The above-mentioned approaches assume that information about criteria weights is known. However, in cases where the criteria weights are unknown due to different reasons, e.g., time pressure, limited expertise, incomplete knowledge, and lack of information (Das et al., 2016; S. Zhang et al., 2011), the overall evaluations cannot be derived (Xu, 2015). Accordingly, the proper assessment of criteria weights is critical for MCDM problems and may influence the analysis of a decision problem (Das et al., 2016).

In such decision settings, it is useful to utilize the available information, the knowledge, and the expertise of the decision makers to approximate evaluation ranges (Lin et al., 2008). To approximate evaluation ranges, Gao and Liu (2016) mentioned that maximizing deviation method is the most popular approach to establish objective weights for solving MCDM problems with unknown weight information. The maximizing deviation method introduced by Wang (1998) where larger deviations are being emphasized more than smaller ones.

Although several research papers have tried to address MCDM problems with unknown criteria weights using different types of information (e.g., fuzzy set theory, intuitionistic fuzzy set, and hesitant fuzzy set), there exist decision problems with small amount of data and poor information, which cannot be described by a probability distribution. In such decision problems, grey systems theory is recommended (D.-C. Li et al., 2012; S. Liu & Lin, 2006) due to its less restricted procedure that neither requires any robust membership function, nor a probability distribution (Memon et al., 2015).

Building on the above arguments, the aim of this manuscript is to contribute to the current literature by addressing DMCDM problems with small amount of data and poor information where information on criteria weights and the weight vector of different time periods are unknown. To this end, a new hybrid DMCDM methods is proposed using the principles of grey systems theory, the maximizing deviation method, possibility degrees, and PROMETHEE II.

This manuscript is organized as follows: section 4.3 establishes the proposed approach to handle Dynamic Grey Multi-Criteria Decision Making (DGMCDM) problems; section 4.4 reflects the feasibility and the practicability of the proposed approach using an adopted case; section 4.5 provides a comparative analysis with existing approaches to validate the proposed approach; finally, section 4.6 provides the concluding remarks.

4.3 Decision making method of DGMCDM with unknown weight information

4.3.1 Problem description

Definition 4.1 Let $A = \{A_1, A_2 \ldots, A_n\}$ denote the set of available alternatives of a multi-periods-multi-criteria decision making, where n is the number of the alternatives and $i = 1, 2 \ldots, n$. Let $C = \{c_1, c_2 \ldots, c_m\}$ represent the set of evaluation criteria, where m is the number of criteria and $j = 1, 2 \ldots, m$. Let $S(t) = \{t_1, t_2 \ldots, t_p\}$ indicate the set of time periods, where p is the number of different periods and $k = 1, 2 \ldots, p$.

Definition 4.2 Let $A \times C \times S(t)$ be the Cartesian product of the set of alternatives, the set of criteria, and the set of periods. Let R [$\otimes$] denote the set of interval grey numbers, in which each element of R [$\otimes$] represent an evaluation of A_i over c_j within t_k and denoted by an interval grey number $y_{ijk}(\otimes) \in \left[\underline{y}_{ijk}, \overline{y}_{ijk}\right]$, where $0 \leq \underline{y}_{ijk} \leq \overline{y}_{ijk}$. Accordingly, the grey description function for a decision matrix ($f_{\otimes}$) within a DGMCDM problem is defined as follows:

$$f_{\otimes}: A \times C \times S(t) \rightarrow R[\otimes], thus \tag{4.1}$$

$$\forall f_{\otimes}(A_i \in A, c_j \in C, t_k \in S(t)): y_{ijk}(\otimes) \in R[\otimes]$$

Definition 4.3 Assume that the influence of $c_j \in C$ $(j = 1, 2 \dots, m)$ on a decision problem is changing over t_k $(k = 1, 2 \dots, p)$. Let the weight of c_j over t_k be denoted by w_{jk}, where $w_{jk} \in [0,1]$ and $\sum_{j=1}^{m} w_{jk} = 1$; but the information on criteria weights is unknown.

Definition 4.4 Let $\theta_j = \{\theta_j(t_1), \theta_j(t_2) \dots, \theta_j(t_p)\}$ $(j = 1, 2 \dots, m)$ denote the weight vector of $S(t) = \{t_1, t_2 \dots, t_p\}$ with respect to c_j, where $\theta_j(t_k) \in [0,1]$ and $\sum_{k=1}^{p} \theta_j(t_k) = 1$ $(k = 1, 2 \dots, p)$; but the information on θ_j is unknown.

4.3.2 Normalize DGMCDM

For consistency in the evaluations over each period such that all the evaluations would be grey based with $[0,0] \leq y_{ijk}(\otimes) \leq [1,1]$ (Bai et al., 2012), a normalization procedure of two steps is used over each decision matrix: first, turn all non-grey values (e.g., crisp number, which is a single precise number) in the decision matrix into interval grey numbers according to Definition 4.5; second, normalize all the values using Algorithm 4.1, which is explained by Definition 4.6.

Definition 4.5 Let y_{ij} denote a white number that represents the evaluation of alternative A_i $(i = 1, 2 \dots, n)$ over criterion c_j $(j = 1, 2 \dots, m)$ within a given period t_k $(k = 1, 2 \dots, p)$ such that, the relative grey number ($y_{ijk}(\otimes)$) of the given white number (y_{ijk}) is

$$y_{ijk}(\otimes) = \left[\underline{y}_{ijk}, \overline{y}_{ijk}\right], \text{ where } \underline{y}_{ijk} = y_{ijk} = \overline{y}_{ijk} \tag{4.2}$$

Note that although some evaluations would be expressed by interval grey numbers, a normalized scale over the given criteria is not guaranteed. Therefore, all the interval grey values should be normalized.

Definition 4.6 Let $y_{ijk}(\otimes)$ represent an interval grey number that reflects the evaluation of an alternative A_i $(i = 1, 2 \dots, n)$ over a criterion c_j $(j = 1, 2 \dots, m)$ within a given period t_k $(k = 1, 2 \dots, p)$. Let $\min(y_{ij})^k$ and $\max(y_{ij})^k$ denote the lower and upper bounds among all $y_{ij}(\otimes)$ over t_k , respectively. Let $(y_{jk}^*)^k$ represent a given optimal evaluation over a targeted criterion c_j within a given period t_k. Let $\tilde{y}_{ijk}(\otimes) \in \tilde{R}[\otimes]$ denote the relative normalized value of the interval grey number $y_{ijk}(\otimes) \in R[\otimes]$.

$if\, j\ is\ increasing\ criterion$

$$y_{ijk}(\tilde{\otimes}) = \frac{[y_{ijk}(\otimes) - min(y_{ij})^k]}{[max(y_{ij})^k - min(y_{ij})^k]}$$

$else\ if\ j\ is\ decreasing\ criterion$

$$y_{ijk}(\tilde{\otimes}) = \frac{[max(y_{ij})^k - y_{ijk}(\otimes)]}{[max(y_{ij})^k - min(y_{ij})^k]}$$

$else\ j\ is\ targeted\ criterion$

$$y_{ijk}(\tilde{\otimes}) = 1 - \frac{\left|y_{ijk}(\otimes) - (y_j^*)^k\right|}{Max\{max(y_{ij})^k, (y_j^*)^k\} - Min\{min(y_{ij})^k, (y_j^*)^k\}}$$

Algorithm 4.1 Normalizing algorithm in DGMCDM.

4.3.3 Establish criteria weights

Since the criteria weight information in this manuscript is assumed to be completely unknown, proper assessment of criteria weights is required to carry out the overall evaluations in MCDM. To this end, a combination of grey systems theory and the maximizing deviation method is proposed in the following subsections, in which the total deviation between alternatives over each period would be maximized.

4.3.3.1 Construct deviation matrix

As mentioned earlier, the maximizing deviation method considers criteria with larger deviations more than criteria with smaller deviations. Accordingly, deviation measures between normalized interval grey evaluations over a given period can be established using Algorithm 4.2, which is explained by Definition 4.7.

Definition 4.7 Let the deviation measure between the normalized grey evaluations of two alternatives over a criterion c_j within a period t_k be denoted by $d\big(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\big)$, where $i, h = 1, 2 \dots, n; j = 1, 2 \dots, m$ and $k = 1, 2 \dots, p$.

$$
\begin{aligned}
& if\ \tilde{y}_{ijk}(\otimes) = \tilde{y}_{hjk}(\otimes) \\
& \qquad d\big(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\big) = 0 \\
& else\ if\ \tilde{y}_{ijk}(\otimes) \neq \tilde{y}_{hjk}(\otimes) \wedge \left(\underline{\tilde{y}}_{ijk} \geqslant \overline{\tilde{y}}_{hjk}\right) \\
& \qquad d\big(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\big) = \left(\overline{\tilde{y}}_{ijk} - \underline{\tilde{y}}_{hjk}\right) \\
& else\ if\ \overline{\tilde{y}}_{ijk} \succ \overline{\tilde{y}}_{hjk} \succ \underline{\tilde{y}}_{hjk} \succ \underline{\tilde{y}}_{ijk} \\
& \qquad d\big(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\big) = \left|\left(\underline{\tilde{y}}_{ijk} + \overline{\tilde{y}}_{hjk}\right) - \left(\overline{\tilde{y}}_{ijk} + \underline{\tilde{y}}_{hjk}\right)\right| \\
& else \\
& \qquad d\big(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\big) = \left|\left(\underline{\tilde{y}}_{ijk} - \overline{\tilde{y}}_{hjk}\right) + \left(\overline{\tilde{y}}_{ijk} - \underline{\tilde{y}}_{hjk}\right)\right|
\end{aligned}
$$

Algorithm 4.2: Deviation measure algorithm in DGMCDM.

4.3.3.2 Estimate criteria weights

Definition 4.8 Let $D_{ijk}(w)$ denote the overall deviation between alternative A_i and other alternatives over a criterion c_j within a period t_k. Let w_{jk} denote the associated criterion weight within t_k such that,

$$D_{ijk}(w) = \sum_{\substack{h=1,\\ h\neq i}}^{n} d\big(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\big) w_{jk}, \tag{4.3}$$

$$i = 1, 2 \dots, n; \quad j = 1, 2 \dots, m; \quad k = 1, 2 \dots, p$$

Definition 4.9 Let $D_{jk}(w)$ denote the global deviation between alternatives over c_j within t_k, such that

$$D_{jk}(w) = \sum_{i=1}^{n} D_{ijk}(w) = \sum_{i=1}^{n} \sum_{\substack{h=1,\\ h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\right) w_{jk}, \tag{4.4}$$

$$j = 1, 2 \dots, m;\ k = 1, 2 \dots, p$$

Definition 4.10 Let $D_k(w)$ denote the total deviations over all the criteria within t_k, in which the total deviation function over t_k is expressed as follows:

$$D_k(w) = \sum_{j=1}^{m} D_{jk}(w) = \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{\substack{h=1,\\ h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\right) w_{jk}, \tag{4.5}$$

$$k = 1, 2 \dots, p$$

Theorem 4.1 In DGMCDM problems with unknown criteria weights, the optimal criteria weights $w_{jk}^* = (w_{1k}^*, w_{2k}^* \dots, w_{mk}^*)$ which maximize the total deviation between alternatives over each period t_k can be obtained as follows:

$$w_{jk}^* = \frac{\sum_{i=1}^{n} \sum_{\substack{h=1,\\ h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\right)}{\sqrt{\sum_{j=1}^{m} \left[\sum_{i=1}^{n} \sum_{\substack{h=1,\\ h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\right)\right]^2}}, \tag{4.6}$$

$$j = 1,2 \dots, m;\ k = 1, 2 \dots, p$$

In order to satisfy the normalization constraint condition, the weight of criteria c_j within a period t_k can be determined as follows:

$$w_{jk} = \frac{w_{jk}^*}{\sum_{j=1}^{m} w_{jk}^*} = \frac{\sum_{i=1}^{n} \sum_{\substack{h=1,\\ h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\right)}{\sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{\substack{h=1,\\ h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\right)} \tag{4.7}$$

Proof 4.1 For unknown criteria weights, the maximizing deviation method (Y. M. Wang, 1998) can be adapted, in which the criteria weights over a period t_k are being estimated based on the deviation degrees between the evaluations within t_k. To this end, the following optimization model is established

$$maxD_k(w) = \sum_{j=1}^{m} D_{jk}(w) = \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{\substack{h=1, \\ h \neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\right) w_{jk}^{*} \quad (4.8)$$

$$s.t. \sum_{j=1}^{m} \left(w_{jk}^{*}\right)^2 = 1, w_{jk}^{*} \geq 0, \quad (4.9)$$

$$j = 1,2 \dots, m; \ k = 1, 2 \dots, p$$

Note that $\sum_{j=1}^{m} \left(w_{jk}^{*}\right)^2 = 1$ is used as a constraint to accentuate the larger deviations within each time period. Thus, to ensure that criteria which make larger deviations would be assigned higher weights in comparison to criteria which make smaller deviations.

The model can be solved using Lagrange function (Z. Xu & Cai, 2012):

$$L\left(w_{jk}^{*}, \lambda\right) = D_k(w) - \frac{1}{2}\lambda \left(\sum_{j=1}^{m} \left(w_{jk}^{*}\right)^2 - 1\right), \text{where} \quad (4.10)$$

λ is the Lagrange multiplier. Thus,

$$L\left(w_{jk}^{*}, \lambda\right) = \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{\substack{h=1, \\ h \neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\right) w_{jk}^{*} - \frac{1}{2}\lambda \left(\sum_{j=1}^{m} \left(w_{jk}^{*}\right)^2 - 1\right) \quad (4.11)$$

By calculating the partial derivative of Eq. (4.11) with respect to w_{jk}^{*} $(j = 1, 2 \dots, m, k = 1, 2 \dots, p)$ and λ, while setting these partial derivatives to zero, the following equations are obtained:

$$\frac{\partial L\left(w_{jk}^{*}, \lambda\right)}{\partial w_{jk}^{*}} = \sum_{i=1}^{n} \sum_{\substack{h=1, \\ h \neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\right) - \lambda w_{jk}^{*} = 0 \quad (4.12)$$

$$\frac{\partial L\left(w_{jk}^{*}, \lambda\right)}{\partial \lambda} = \sum_{j=1}^{m} \left(w_{jk}^{*}\right)^2 - 1 = 0 \quad (4.13)$$

From Eq. (4.12), we can obtain

$$w_{jk}^{*} = \frac{\sum_{i=1}^{n} \sum_{\substack{h=1, \\ h \neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\right)}{\lambda} \quad (4.14)$$

Since $\sum_{j=1}^{m}\left(w_{jk}^{*}\right)^{2} = 1$, the value of λ can be obtained using Eq. (4.14) as follows:

$$\sum_{j=1}^{m}\left(\frac{\sum_{i=1}^{n}\sum_{\substack{h=1,\\ h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes),\tilde{y}_{hjk}(\otimes)\right)}{\lambda}\right)^{2} = 1 \tag{4.15}$$

By simplifying Eq. (4.15)

$$\lambda = \sqrt{\sum_{j=1}^{m}\left[\sum_{i=1}^{n}\sum_{\substack{h=1,\\ h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes),\tilde{y}_{hjk}(\otimes)\right)\right]^{2}} \tag{4.16}$$

Thus, the optimal solution of the model is

$$w_{jk}^{*} = \frac{\sum_{i=1}^{n}\sum_{\substack{h=1,\\ h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes),\tilde{y}_{hjk}(\otimes)\right)}{\sqrt{\sum_{j=1}^{m}\left[\sum_{i=1}^{n}\sum_{\substack{h=1,\\ h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes),\tilde{y}_{hjk}(\otimes)\right)\right]^{2}}},\ j = 1,2\ldots,m;\ k = 1,2\ldots,p$$

4.3.4 Estimate the influence of dynamic evaluations

Since weight information of different time periods is unknown, the overall evaluation over a DGMCDM problem cannot be obtained. As a result, the weight of the different time periods with respect to each criterion should be established. To this end, a new optimization model is proposed, in which the evaluation space over a DGMCDM problem would be maximized.

Theorem 4.2 In a DGMCDM where the weight information of different time periods is unknown, the optimal solution to maximize the evaluation space over the dynamic decision problem can be obtained using $\theta_j = \{\theta_j(t_1),\theta_j(t_2)\ldots,\theta_j(t_p)\}$, where

$$\theta_j(t_k)^{*} = \frac{\dfrac{\sum_{i=1}^{n}\sum_{\substack{h=1,\\ h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes),\tilde{y}_{hjk}(\otimes)\right)}{\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{\substack{h=1,\\ h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes),\tilde{y}_{hjk}(\otimes)\right)}}{\sqrt{\sum_{k=1}^{p}\left[\dfrac{\sum_{i=1}^{n}\sum_{\substack{h=1,\\ h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes),\tilde{y}_{hjk}(\otimes)\right)}{\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{\substack{h=1,\\ h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes),\tilde{y}_{hjk}(\otimes)\right)}\right]^{2}}},\ j = 1,2\ldots,m;\ k = 1,2\ldots,p \tag{4.17}$$

In order to satisfy the normalization constraint condition, the optimal weight of period t_k with respect to c_j can be expressed as

$$\theta_j\,(t_k) = \frac{\theta_j\,(t_k)^*}{\sum_{k=1}^{p}\theta_j\,(t_k)^*} = \frac{\dfrac{\sum_{i=1}^{n}\sum_{\substack{h=1,\\h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes),\tilde{y}_{hjk}(\otimes)\right)}{\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{\substack{h=1,\\h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes),\tilde{y}_{hjk}(\otimes)\right)}}{\sum_{k=1}^{p}\left[\dfrac{\sum_{i=1}^{n}\sum_{\substack{h=1,\\h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes),\tilde{y}_{hjk}(\otimes)\right)}{\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{\substack{h=1,\\h\neq i}}^{n} d\left(\tilde{y}_{ijk}(\otimes),\tilde{y}_{hjk}(\otimes)\right)}\right]} \tag{4.18}$$

Proof 4.2 In DMCDM problems, if the influence of $c_j \in C$ is the same over $t_k \in S(t)$, for all $j = 1, 2 \ldots, m$ and $k = 1, 2 \ldots, p$, then

$$w_{jk} = \sum_{k=1}^{p}\theta_j\,(t_k)w_{jk}, \text{ for all } j = 1, 2 \ldots, m \text{ and } k = 1, 2 \ldots, p \tag{4.19}$$

However, Eq. (4.19) does not generally hold, i.e., the influence of c_j is changing over t_k and thus, there is always a difference in the value of w_{jk}. Consequently, a general deviation function $F\left(e_{jk}\right)$ is introduced:

$$F\left(e_{jk}\right) = \left(\sum_{k=1}^{p}\theta_j\,(t_k)^* w_{jk} - w_{jk}\right), \text{ for all } j = 1, 2 \ldots, m \text{ and } k = 1, 2 \ldots, p \tag{4.20}$$

Accordingly, to estimate the weight vector $\theta_j = \left\{\theta_j\,(t_1), \theta_j\,(t_2) \ldots, \theta_j\,(t_p)\right\}$ for c_j, the following optimization model is constructed by adapting the maximizing deviation method:

$$max\, F\left(e_{jk}\right) = \left(\sum_{k=1}^{p}\theta_j\,(t_k)^* w_{jk} - w_{jk}\right) \tag{4.21}$$

$$s.t. \sum_{k=1}^{p}\left(\theta_j\,(t_k)^*\right)^2 = 1\,, \theta_j\,(t_k) \in [0,1],\; j = 1,2 \ldots, m,\; k = 1, 2 \ldots, p \tag{4.22}$$

Note that $\sum_{k=1}^{p}\left(\theta_j\,(t_k)^*\right)^2 = 1$ is used as a constraint to stress the larger deviations by reflecting a wider range, which would maximize the evaluation space.

The model can be solved using Lagrange function:

$$L\left(\theta_j\,(t_k)^*, \lambda\right) = F\left(e_{jk}\right) - \tfrac{1}{2}\lambda\left(\sum_{k=1}^{p}\theta_j\,(t_k)^2 - 1\right), \text{ where} \tag{4.23}$$

λ is the Lagrange multiplier. Thus,

$$L\left(\theta_j\,(t_k)^*, \lambda\right) = \left(\sum_{k=1}^{p} \theta_j\,(t_k)^* w_{jk} - w_{jk}\right) - \frac{1}{2}\lambda\left(\sum_{k=1}^{p}\left(\theta_j\,(t_k)^*\right)^2 - 1\right) \tag{4.24}$$

By calculating the partial derivative of Eq. (4.24) with respect to $\theta_j\,(t_k)^*$ $(j = 1, 2 \dots, m, k = 1, 2 \dots, p)$ and λ, while setting these partial derivatives to zero, the following equations are obtained:

$$\frac{\partial L\left(\theta_j\,(t_k)^*, \lambda\right)}{\partial \theta_j\,(t_k)^*} = w_{jk} - \lambda\theta_j\,(t_k)^* = 0 \tag{4.25}$$

$$\frac{\partial L\left(\theta_j\,(t_k)^*, \lambda\right)}{\partial \lambda} = \sum_{k=1}^{p}\sum_{k=1}^{p}\left(\theta_j\,(t_k)^*\right)^2 - 1 = 0 \tag{4.26}$$

From Eq. (4.25), we can obtain

$$\theta_j\,(t_k)^* = \frac{w_{jk}}{\lambda} \tag{4.27}$$

Since $\sum_{k=1}^{p}\left(\theta_j\,(t_k)^*\right)^2 = 1$, the value of λ can be obtained using Eq. (4.27) as follows:

$$\sum_{k=1}^{p}\left(\frac{w_{jk}}{\lambda}\right)^2 = 1 \tag{4.28}$$

By simplifying Eq. (4.28)

$$\lambda = \sqrt{\sum_{k=1}^{p}\left[w_{jk}\right]^2} \tag{4.29}$$

Thus, the optimal solution of the model is

$$\theta_j\,(t_k)^* = \frac{w_{jk}}{\sqrt{\sum_{k=1}^{p}\left[w_{jk}\right]^2}} = \frac{\dfrac{\sum_{i=1}^{n}\sum_{h=1,\, h\neq i}^{n} d\left(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\right)}{\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{h=1,\, h\neq i}^{n} d\left(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\right)}}{\sqrt{\sum_{k=1}^{p}\left[\dfrac{\sum_{i=1}^{n}\sum_{h=1,\, h\neq i}^{n} d\left(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\right)}{\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{h=1,\, h\neq i}^{n} d\left(\tilde{y}_{ijk}(\otimes), \tilde{y}_{hjk}(\otimes)\right)}\right]^2}}$$

4.3.5 Evaluate and rank feasible alternatives

Once criteria weights are established and the dynamism over the multi-criteria decision problem has been considered, the DGMCDM problem would be evaluated by extending the PROMETHEE II approach using the optimized weights and the possibility degrees. The detailed procedure of evaluating and ranking decision alternatives is provided in the rest of this section.

4.3.5.1 Establish preferences

To establish preferences among available alternatives over each criterion within a given period t_k, the possibility degree formula is used based on the definition of Xu and Da (2002).

Definition 4.11 Let $l_{\tilde{a}_{jk}(\otimes)}$ and $l_{\tilde{b}_{jk}(\otimes)}$ denote the difference between the upper and lower limits of two normalized grey numbers $\tilde{a}(\otimes)$ and $\tilde{b}(\otimes)$, respectively, over a criterion c_j within a period t_k such that,

$$l_{\tilde{a}_{jk}(\otimes)} = \overline{\tilde{a}}_{jk} - \underline{\tilde{a}}_{jk} \tag{4.30}$$

$$l_{\tilde{b}_{jk}(\otimes)} = \overline{\tilde{b}}_{jk} - \underline{\tilde{b}}_{jk}$$

Definition 4.12 Let $\tilde{p}_{jk}\big(\tilde{a}(\otimes) \succ \tilde{b}(\otimes)\big)$ denote the preference degree of $\tilde{a}(\otimes)$ over $\tilde{b}(\otimes)$ with respect to c_j within t_k, in which $\tilde{p}_{jk}\big(\tilde{a}(\otimes) \succ \tilde{b}(\otimes)\big)$ would be determined using the possibility degree formula as follows:

$$\tilde{p}_{jk}\big(\tilde{a}(\otimes) \succ \tilde{b}(\otimes)\big) = \frac{\overline{\tilde{a}}_j - \underline{\tilde{b}}_j}{l_{\tilde{a}(\otimes)j} + l_{\tilde{b}(\otimes)j}} \tag{4.31}$$

Definition 4.13 Let the sum of preferences for any two unequal grey numbers (e.g., $\tilde{a}(\otimes) = \left[\underline{\tilde{a}}, \overline{\tilde{a}}\right]$ and $\tilde{b}(\otimes) = \left[\underline{\tilde{b}}, \overline{\tilde{b}}\right]$) equal to 1, such that

$$\tilde{p}_{jk}\big(\tilde{a}(\otimes), \tilde{b}(\otimes)\big) = \tilde{p}_{jk}\big(\tilde{a}(\otimes) \succ \tilde{b}(\otimes)\big) + \tilde{p}_{jk}\big(\tilde{b}(\otimes) \succ \tilde{a}(\otimes)\big) = 1 \tag{4.32}$$

Based on Eqs. (4.31) and (4.32), $\tilde{p}_{jk}\big(\tilde{a}(\otimes) \succ \tilde{b}(\otimes)\big)$ would be determined as follows:

$$\tilde{p}_{jk}\big(\tilde{a}(\otimes) \succ \tilde{b}(\otimes)\big) = \begin{cases} 0 & \text{if } \tilde{p}_{jk}\big(\tilde{a}(\otimes) \succ \tilde{b}(\otimes)\big) \leq 0 \\ \frac{\overline{\tilde{a}}_{jk} - \underline{\tilde{b}}_{jk}}{l_{\tilde{a}_{jk}(\otimes)} + l_{\tilde{b}_{jk}(\otimes)}} & \text{if } 0 < \tilde{p}_{jk}\big(\tilde{a}(\otimes) \succ \tilde{b}(\otimes)\big) < 1 \\ 1 & \text{if } \tilde{p}_{jk}\big(\tilde{a}(\otimes) \succ \tilde{b}(\otimes)\big) \geq 1 \end{cases} \quad (4.33)$$

4.3.5.2 Determine relative preferences

In order to determine the relative preferences between alternatives in DGMCDM problems based on the degrees of preferences between the alternatives, the criteria weights, and the associated time period weight; the following definition is proposed.

Definition 4.14 Let $\tilde{p}_{jk}(A_a \succ A_b)$ represent the preference degree of alternative A_a over A_b with repsect to criterion c_j within period t_k. Let the weight of c_j within t_k be represented by w_{jk}, where $\sum_{j=1}^{m} w_{jk} = 1, j = 1, 2 \ldots, m$. Let the weight of the associated period t_k with respect to c_j be donated by $\theta_j\,(t_k)$, where $\sum_{k=1}^{p}\, \theta_j\,(t_k) = 1\,, k = 1, 2 \ldots, p$. Let $\tilde{\pi}_k(A_a \succ A_b)$ denote the relative preference of alternative A_a over A_b across the set of evaluation criteria $C = \{c_1, c_2 \ldots, c_m\}$ within t_k. Accordingly, $\tilde{\pi}_k(A_a \succ A_b)$ would be determined using the cumulative preference degrees of A_a over A_b with respect to the evaluation criteria within t_k such that,

$$\tilde{\pi}_k(A_a \succ A_b) = \textstyle\sum_{j=1}^{m} w_{jk}\, \theta_j\,(t_k)\, \tilde{p}_{jk}(A_a \succ A_b)\,,\ j = (1, 2 \ldots, m) \quad (4.34)$$

4.3.5.3 Estimate global preferences and rank available alternatives

The global preference of a given alternative over others in DGMCDM problems can be estimated using the resultant relative preferences matrices as follows:

Definition 4.15 For a DGMCDM problem, let $\tilde{\phi}^{+}{}_{k}(A_a)$ denote the positive outranking flow of A_a within t_k, which represents the extent to which an alterative A_a is preferred over all others within t_k. Let $\tilde{\pi}_k(A_a \succ A_b)$ indicate the relative preference of A_a over A_b within t_k. The function of $\tilde{\phi}^{+}{}_{k}(A_a)$ would be determined using the relative preference measures of A_a over other alternatives within t_k such that,

$$\tilde{\phi}^{+}{}_{k}(A_a) = \frac{1}{n-1} \sum_{b=1}^{n} \tilde{\pi}_k(A_a \succ A_b) \quad (4.35)$$

Definition 4.16 For a DGMCDM problem, let $\tilde{\phi}^{-}{}_{k}(A_a)$ denote the negative outranking flow of A_a within t_k, which represents the extent to which an alterative A_a is outranked by all other alternatives within t_k. Let $\tilde{\pi}_k(A_b \succ A_a)$ indicate the degree by which an alternative A_a is outranked by A_b within t_k. The function of $\tilde{\phi}^{-}{}_{k}(A_a)$ would be determined using $\tilde{\pi}_k(A_b \succ A_a)$ within t_k such that,

$$\tilde{\phi}^{-}{}_{k}(A_a) = \frac{1}{n-1}\sum_{b=1}^{n}\tilde{\pi}_k(A_b \succ A_a) \tag{4.36}$$

Definition 4.17 For a DGMCDM problem, let $\tilde{\phi}\,(A_a)$ denote the global preference (i.e., net outranking flow) of alternative A_a over multi-periods, in which the function of $\tilde{\phi}\,(A_a)$ is based on the net measures of positive and negative outranking flows over $S(t)$ such that,

$$\tilde{\phi}\,(A_a) = \sum_{k=1}^{p}\left(\tilde{\phi}^{+}{}_{k}(A_a) - \tilde{\phi}^{-}{}_{k}(A_a)\right) \tag{4.37}$$

After estimating the net outranking flow for each alternative, the ranking index would be established using the resulted global preferences over the multi-periods decision problem, where the higher the value of $\tilde{\phi}\,(A_a)$, the better is the associated alternative.

4.4 Illustrative example

To demonstrate the feasibility and practicability of the proposed methodology, a practical example in the literature was used. The example was adopted from the work of Peng and Wang (2014), which intended to address multi-period decision making problems using dynamic interval-valued hesitant fuzzy aggregation operators.

A given company is going through a subcontractor selection process, in which the performance of each subcontractor is examined over different periods. Four subcontractors are considered, namely $A_1, A_2, A_3,$ and A_4, and three evaluation periods, i.e., $T_1, T_2,$ and T_3. The performance of each subcontractor is evaluated by the following criteria: Reliability (C_1), which is evaluated by the reputation, records and financial condition; Schedule-control ability (C_2), which is measured by the subcontractors' mobilization and efficiency; Management ability (C_3), which is intended to assess the quality, safety, and environmental management level of each subcontractor. The weight information with regard to the evaluation criteria and the different periods are completely unknown. Moreover, the evaluations are subjective and uncertain. Therefore, the evaluation information is represented by interval grey numbers over each period as represented in Table 4.1.

Table 4.1: Uncertain decision matrix at each period

Performance matrix (T_1)				
Criteria	**Alternatives**			
	A_1	**A_2**	**A_3**	**A_4**
Reliability (C_1)	[0.3, 0.8]	[0.3, 0.5]	[0.3, 0.4]	[0.5, 0.8]
Schedule-control ability (C_2)	[0.3, 0.6]	[0.5, 0.8]	[0.2, 0.7]	[0.3, 0.5]
Management ability (C_3)	[0.3, 0.6]	[0.4, 0.9]	[0.4, 0.8]	[0.3, 0.4]
Performance matrix (T_2)				
Reliability (C_1)	[0.2, 0.9]	[0.2, 0.5]	[0.2, 0.8]	[0.6, 0.7]
Schedule-control ability (C_2)	[0.8, 0.9]	[0.2, 0.8]	[0.6, 0.7]	[0.5, 0.6]
Management ability (C_3)	[0.5, 0.9]	[0.3, 0.8]	[0.7, 0.9]	[0.3, 0.7]
Performance matrix (T_3)				
Reliability (C_1)	[0.5, 0.7]	[0.4, 0.8]	[0.2, 0.5]	[0.2, 0.7]
Schedule-control ability (C_2)	[0.4, 0.9]	[0.7, 0.8]	[0.5, 0.7]	[0.2, 0.9]
Management ability (C_3)	[0.5, 0.6]	[0.6, 0.7]	[0.3, 0.6]	[0.4, 0.6]

In order to overcome the decision problem at hand, the proposed methodology is applied as follows:

Step 1 Normalize the grey evaluations of the uncertain decision matrices using Algorithm 4.1. The grey normalized values of the uncertain decision matrix are listed in Table 4.2.

Table 4.2: Normalized uncertain decision matrix at each period

Criteria	Alternatives			
	A1	**A2**	**A3**	**A4**
Performance matrix (T_1)				
Reliability (C_1)	[0, 1]	[0, 0.4]	[0, 0.2]	[0.4, 1]
Schedule-control ability (C_2)	[0.167, 0.667]	[0.5, 1]	[0, 0.833]	[0.167, 0.5]
Management ability (C_3)	[0, 0.5]	[0.167, 1]	[0.167, 0.833]	[0, 0.167]
Performance matrix (T_2)				
Reliability (C_1)	[0, 1]	[0, 0.429]	[0, 0.857]	[0.571, 0.714]
Schedule-control ability (C_2)	[0.857, 1]	[0, 0.857]	[0.571, 0.714]	[0.429, 0.571]
Management ability (C_3)	[0.333, 1]	[0, 0.833]	[0.667, 1]	[0, 0.667]
Performance matrix (T_3)				
Reliability (C_1)	[0.5, 0.833]	[0.333, 1]	[0, 0.5]	[0, 0.833]
Schedule-control ability (C_2)	[0, 1]	[0.6, 0.8]	[0.2, 0.6]	[0.6, 1]
Management ability (C_3)	[0.5, 0.75]	[0.75, 1]	[0, 0.75]	[0.25, 0.75]

Step 2 Estimate criteria weights at each period using Eq. (4.7). Thus, the criteria weigh vector over each period $w_{t_k} = (w_{1k}, w_{2k}, w_{3k})$, $k = 1, 2,$ and 3, would be:

$$w_{t_1} = (0.4099, 0.2950, 0.2950)$$

$$w_{t_2} = (0.3221, 0.3490, 0.3289)$$

$$w_{t_3} = (0.3273, 0.3535, 0.3191)$$

Step 3 Estimate the weight of each period with respect to each criterion using Eq. (4.18). Thus, the weight vector of each period with respect to each criterion $\theta_j = \{\theta_j(t_1), \theta_j(t_2), \theta_j(t_3)\}, j = 1, 2,$ and 3, would be:

$$\theta_1 = (0.3869, 0.3041, 0.3090)$$

$$\theta_2 = (0.2958, 0.3499, 0.3544)$$

$$\theta_3 = (0.3129, 0.3487, 0.3384)$$

Step 4 Evaluate the preferences between the alternatives using Eq. (4.33). Take alternative A_1 over period T_1 for example, Table 4.3 shows the preference degrees of A_1 over other alternatives within period T_1.

Table 4.3: Preference matrix of $\mathbf{A_1}$ over $\mathbf{T_1}$

Multi-criteria preference matrix		**Alternatives**		
Base alternative	**Criteria**	$\mathbf{A_2}$	$\mathbf{A_3}$	$\mathbf{A_4}$
$\mathbf{A_1}$	**Reliability** $(\boldsymbol{C_1})$	0.714	0.833	0.375
	Schedule-control ability $(\boldsymbol{C_2})$	0.167	0.500	0.600
	Management ability $(\boldsymbol{C_3})$	0.250	0.286	0.750

Step 5 Determine relative preferences by considering the calculated degrees of preferences between the alternatives, the criteria weights, and the associated time period weight; using Eq. (4.34). For illustration, Table 4.4 reflects the relative preference measures of alternative A_1 over other alternatives within period T_1.

Step 6 Establish global preferences among alternatives using Eqs. (4.35), (4.36), and (4.37):

$$\tilde{\phi}(A_1) = 0.1829, \tilde{\phi}(A_2) = 0.0834, \tilde{\phi}(A_3) = -0.2024, \tilde{\phi}(A_4) = -0.0639$$

Step 7 Rank the alternatives and select the best one(s) in accordance with the net outranking flow values:

$$A_1 \succ A_2 \succ A_4 \succ A_3$$

Thus, alternative A_1 is the appropriate subcontractor for the uncertain DMCDM problem.

Table 4.4: Relative preference matrix of **A_1** over **T_1**

Multi-criteria preference matrix		Alternatives		
Base alternative	**Criteria**	**A_2**	**A_3**	**A_4**
A_1	**Reliability (C_1)**	0.113	0.132	0.059
	Schedule-control ability (C_2)	0.015	0.044	0.052
	Management ability (C_3)	0.023	0.026	0.069

4.5 Comparative analysis

By comparing the obtained ranking order for the adopted subcontractor selection case using the proposed methodology in this manuscript with the original results of Peng and Wang (2014) using dynamic interval-valued hesitant fuzzy aggregation operators, similar ranking order was obtained.

Furthermore, Peng and Wang (2014) proposed another approach where dynamic hesitant fuzzy aggregation operators were used to handle uncertain DMCDM problems. Likewise, the same example was analyzed using hesitant fuzzy information instead of interval hesitant fuzzy information. Once again, the proposed methodology in this manuscript provided the same ranking order.

The advantages of the proposed methodology over the work of Peng and Wang (2014) can be summarized as follows: (1) the original work presumed that decision makers are able to assign criteria weights precisely, which is hardly the case in practical applications especially with decision problems under uncertainty. In contrast, the proposed methodology is able to overcome a more complicated DMCDM problems, in which the criteria weights are completely unknown using the available information on alternatives evaluations; (2) the original work adapted the concept of the BUM function for obtaining the weight vector of different periods, which does so independently from the evaluation criteria. In contrast, the proposed methodology estimates the weight vector of different time periods considering the changing in the influence of each evaluation criterion over different periods. Thus, the dynamic aspects of different criteria are considered in the proposed methodology, which have not been considered in BUM or other existing methods for obtaining the weight vector of time periods.

4.6 Conclusion

Conventional MCDM approaches are not suitable to address the dynamics over time. Therefore, different hybrid approaches have been developed to deal with DMCDM problems. Different issues with the existing approaches have been identified in this research: (1) Criteria weights: the overall evaluation of MCDM problems is based on alternatives evaluations and the criteria weights, existing approaches in DMCDM problems assumes that information about criteria weights is known, static over different periods, and/or can be assigned by decision makers. However, there are cases where information on criteria weights is completely unknown due to different reasons (e.g., time pressure, limited expertise, incomplete knowledge, and lack of information), which would be an obstacle for the overall evaluations; (2) Weight vector establishment of different time periods: within the context of DMCDM, the existing approaches which estimate weight vector of different time periods does not consider the variabilities in the influence of different criteria over different periods.

To overcome the shortcomings of the existing approaches, this manuscript is provided. It investigates DMCDM problems with small amount of data and poor information, which cannot be described by a probability distribution, where information on criteria weights and the influence of different time periods are unknown. To this end, a new hybrid DMCDM methodology is proposed. At first, consistency among evaluations over each period should be established. After that, the unknown criteria weights should be estimated over each period; thus, an optimization model was established based on the integration of grey system theory's principles and the maximizing deviation method. By solving the optimization model, a function to determine the optimal criteria weights was obtained. Subsequently, the weight vector of different time periods with respect to each evaluation criterion would be established using a new optimization model where the evaluation space over a DMCDM problem would be maximized. Consequently, the DGMCDM problem would be evaluated using a combination of the possibility degrees and PROMETHEE II. Once the evaluation process is completed, the rank index for potential alternatives would be provided based on the values of global preferences; in which the higher the value, the better is the alternative.

To determine the feasibility and practicability of the proposed methodology, an existing case in the literature was adopted; whereas to validate the methodology, a comparative analysis with existing approaches was provided.

The advantages of this manuscript over other existing research works can be summarized in the following points: (1) Estimate unknown criteria weights within the context of DMCDM problems with small amount of data and poor information, which cannot be described by a probability distribution, using the maximizing deviation method where the deviation degrees functions are based on the principles of grey systems theory; (2) Establish the unknown weight vector of different time periods by introducing a new optimization model, which considers the changing in the influence of the different evaluation criteria over different periods; (3) Extend the PROMETHE II approach to evaluate and rank potential alternatives within the context of DMCDM problems using the optimized weights and the possibility degrees.

Chapter 5

Conclusions, Contributions and Future Research

5.1 Concluding remarks

MCDM is well recognized branch of operation research that intended to aid DMs in complex decision problems, where different types of criteria of different nature should be considered. A variety of MCDM approaches exist to address decision problems of different nature. However, the various approaches of MCDA can be classified into three main categories: value measurement models; goal, aspiration, or reference-level models; and outranking models.

The conventional approaches of MCDM have an implicit assumption, which presumes the availability and accuracy of information. However, in many real life applications available information is subject to uncertainty, imprecision, and subjectivity; which renders the conventional approaches insufficient to handle the associated decision problem.

Although different hybrid methodologies have been introduced to handle the lack of accurate information in a MCDM, there exist decision problems with small amount of data and poor information in which the applicability of existing approaches would be influenced. Accordingly, this manuscript-based thesis is intended to address decision problems that involve uncertain evaluations due to lack of information. The significance of this thesis is highlighted in the followings: (1) It focuses on MCDM problems under uncertain evaluations with small amount of data and poor information. (2) It assesses different alternatives over complex interrelated decision structure, which are subject to subjective and objective uncertain evaluations. (3) It provides a systematic approach to carry out the overall evaluation for MCDM problems with small amount of data and poor information, where information on criteria weight is unknown completely. (4) It addresses DMCDM problems with small amount of data and poor information where information on criteria weights and the weight vector of different time periods are unknown. (5) It considers the fluctuations in the influence of different evaluation criteria on a decision problem over different time periods. (6) It maximizes the evaluation space over MCDM and DMCDM with small amount of data and poor information.

The conceptual and methodological contributions of this thesis to the current literature are presented in the following subsections.

5.2 Conceptual contributions

This research proposed a new systematic methodology to address the limitation of conventional MCDM approaches when uncertainty related aspects are present in a decision problem. Different types of uncertainty related aspects are considered in the proposed approach: (i) uncertainty associated with limited objective information, e.g., quantitative (interval scales), and (ii) uncertainty associated with subjective expert knowledge (i.e., ambiguous concepts and semantic meanings).

Although there exist different hybrid approaches to address different types of uncertainty, the applicability of the existing approaches are limited when it comes to handle MCDM problems with a relatively small amount of data and poor information, where evaluation criteria are of different nature and interdependencies exist among them. This type of MCDM problem has been tackled in the first manuscript of this thesis by considering interactions and interdependencies within and between different evaluation clusters of a complex system, using linguistic expressions to articulate DMs' preferences, and employing range of values instead of exact values to map linguistic expressions.

One of the major contributions of this research is to optimize the evaluation space over uncertain MCDM problems, where the influence of each criterion over the decision problem is unknown completely as a result of different reasons, such as time pressure, limited expertise, incomplete knowledge and lack of information. The second manuscript of this thesis proposes a new approach to tackle this type of MCDM problems. The concept behind the proposed approach is to emphasize criteria that reflect larger deviations among alternatives. Thus, criteria which make larger deviations should be assigned higher weights in comparison to criteria which make smaller deviations.

Another contribution of this research is to handle MCDM problems within a dynamic context, in which uncertain evaluations from different periods are provided, while information on the influence of different criteria and the different periods over a decision problem are unknown completely. Although there exist a number of approach to address dynamic MCDM problems, this thesis presents in the third manuscript the first attempt that focuses on the variabilities in the influence of the evaluation criteria throughout the different periods.

5.3 Methodological contributions

This manuscript-based thesis is intended to address a number of shortcomings in the existing approaches associated with the modeling and analysis of MCDM with a relatively small amount of data and poor information.

The first challenge was to optimize the evaluation space in decision problems that are subject to subjective and objective uncertainty over different types of interrelated criteria (i.e., evaluation criteria with different nature, different scales, and different values). To this end, the first manuscript proposes a four-phased methodology, in which grey systems theory is integrated with a distinctive combination of MCDM approaches (i.e., ANP and PROMETHEE II) as follows: the grey systems theory is utilized to articulate the associated uncertainty with subjective and objective evaluations; the principles of ANP is used to handle the complexity of the decision structure; the PROMETHEE II approach is extended to evaluate and rank potential alternatives over interrelated criteria under uncertain evaluations. The emergent strengths of the integrated approach should improve the evaluation space in such a complex decision problem under uncertainty. To validate the proposed methodology, a comparative analysis is put forward with the work of Kuo, Hsu, and Chen (2015), which used fuzzy theory to address the uncertain related aspects. Although both approaches reflect the same ranking order of available alternatives, yet the proposed methodology is suitable to address decision problems with a relatively small amount of data and poor information since it uses grey systems theory, which is preferred over fuzzy theory in such a decision problem context where available information cannot be used to draw a robust membership function that is required to carry out fuzzy theory’s calculations.

In the second manuscript, a more complicated scenario of MCDM problems within the context of small amount of data and poor information was considered in which information on criteria weights was completely unknown. The first phase was concerned with criteria weights estimation. Accordingly, a new optimization model was introduced using the maximizing deviation method and the principles of grey systems theory, where various scenarios of deviation were considered to generalize the model. In the light of the proposed optimization model, a new function was obtained to determine the optimal criteria weights in such MCDM problems. In the second phase, potential alternatives were evaluated and ranked based on the integration of PROMETHEE II and degrees of possibility, where the resultant criteria weights from the first phase were used as inputs for the evaluation process. In order to validate the proposed methodology, a comparative analysis with the work of Xu (2015) is carried out. The conclusion of the analysis indicates that three factors influence alternatives ranking order, i.e., normalization approach, deviation function, and alternative evaluation and ranking methodology. Moreover, a wider deviation area is covered under the proposed methodology, which promotes the use of it over the work of Xu (2015).

In the third manuscript, the dynamic aspect of MCDM is investigated, in which evaluations over different criteria change throughout the time while information on criteria weights and the influence of different time periods is completely unknown. Although a number of approaches have been introduced to address DMCDM, a number of shortcomings in the existing approaches are identified in terms of criteria weights evaluations and the weight vector establishment of different time periods. Consequently, a new hybrid DMCDM methodology is developed. The proposed methodology targets three research problems: (1) establish priorities among evaluation criteria over different time periods; (2) estimate weight vectors of different time periods considering the variabilities in the influence of the evaluation criteria over the different periods to maximize the evaluation space over a DMCDM; (3) evaluate and rank potential alternatives over DMCDM problems with a relatively small amount of data and poor information. To validate the proposed methodology, a comparative analysis with existing approaches (Peng & Wang, 2014) is established. Although, the proposed methodology arrive to the same conclusion of Peng and Wang (2014) with respect to alternatives ranking order, the proposed methodology surpass the work of Peng and Wang (2014) in a number of issues: (1) providing a systematic approach to establish criteria weights, where information on criteria weights is unknown; (2) considering the fluctuations in the influence of different evaluation criteria on a decision problem over different time periods, while estimating weight vectors of different time periods to maximize the evaluation space in a dynamic decision problem.

5.4 Ideas for future research

The increased complexity of many decision problems makes it essential to look after specialized knowledge, this impose the need of involving a number of DMs to build upon their various knowledge and experience. In this thesis, it is assumed that consensus can be reached among DMs or there is only one decision maker. However, conflicts in interests among decision makers may raise; hence, the proposed methodologies can be enhanced by incorporating a conflict resolution technique in future works. Moreover, the criticality of a DM can be considered in terms of the level of related knowledge and expertise to the given decision problem, also the level of authority among DMs. To this end, interactions among the involved DMs can be simulated to provide valuable insights about the influence of a DM and the degree of involvement in a given decision.

The learning curve within a dynamic decision environment can be considered as a factor in order to improve perceptions of DMs, and thereby enhance their judgments.

Finally, real-world applications within the context of MCDM can be analyzed using the proposed methodologies to refine and modify the methodologies. Also, the real data of these applications can be simulated to demonstrate the feasibility of the proposed approaches; for instance, uncertain evaluations and weights information can be generated from the corresponding distributions, then the approximated values are used in the proposed functions. After a number of iterations, criteria weights, alternatives ranking order and confidence factor can be obtained.

5.5 Related publications

During the course of work on this thesis, the following list of papers have been integrated into the writing of it:

5.3.1 Journal papers

Maghrabie, H., Schiffauerova, A. & Beauregard, Y. "*Grey-based Multi-Criteria Decision Analysis Approach: Addressing Uncertainty at Complex Decision Problems*", *Technological Forecasting & Social Change*. Submitted on October 2017.

Maghrabie, H., Beauregard, Y. & Schiffauerova, A., "*Multi-Criteria Decision-Making Problems with Unknown Weight Information under Uncertain Evaluations*", *Computers & Industrial Engineering*. Submitted on November 2017.

Maghrabie, H., Beauregard, Y. & Schiffauerova, A., "*A New Approach to Address Uncertain Dynamic Multi-Criteria Decision Problems with Unknown Weight Information*", *Expert Systems with Applications*. Submitted on February 2018.

5.3.2 Conference papers

Maghrabie, H., Schiffauerova, A., & Beauregard, Y. (2016). *The use of Grey Systems Theory in Complex Decision Problems under Uncertainty*. Presented at MOSIM annual meeting 2016, Montreal, QC, Canada.

Maghrabie, H., Schiffauerova, A., & Beauregard, Y. (2016). *Grey based Multi Criteria Decision Analysis to Address Multi Criteria Decision Problems under Uncertainty*. Presented at GDN annual meeting 2016, Bellingham, WA, United States.

Maghrabie, H., Schiffauerova, A., & Beauregard, Y. (2015). *Improving the Front End Innovation - Systems Engineering Approach*. Presented at INFORMS annual meeting 2015, Philadelphia, PA, United States.

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