Ternary and Hybrid Event-based Particle Filtering for Distributed State Estimation in Cyber-Physical Systems

Somayeh Davar

A Thesis

in

The Department

of

Concordia Institute for Information System Engineering (CIISE)

Presented in Partial Fulfillment of the Requirements

for the Degree of

Master of Applied Science (Quality Systems Engineering) at

Concordia University

Montréal, Québec, Canada

September 2018

CONCORDIA UNIVERSITY

School of Graduate Studies

This is to certify that the thesis prepared

By: **Somayeh Davar**

Entitled: **Ternary and Hybrid Event-based Particle Filtering for Distributed State Estimation in Cyber-Physical Systems**

and submitted in partial fulfillment of the requirements for the degree of

Master of Applied Science (Quality Systems Engineering)

complies with the regulations of this University and meets the accepted standards with respect to originality and quality.

Signed by the Final Examining Committee:

______________________________ Chair
Dr. Jun Yan

______________________________ External Examiner
Dr. Wei-Ping Zhu

______________________________ Examiner
Dr. Abdessamad Ben Hamza

______________________________ Supervisor
Dr. Arash Mohammadi

Approved by ______________________________
Dr. Abdessamad Ben Hamza, Chair
Department of Concordia Institute for Information System Engineering (CIISE)

____________ 2018 ______________________________
Dr. Amir Asif, Dean
Faculty of Engineering and Computer Science

Abstract

Ternary and Hybrid Event-based Particle Filtering for Distributed State Estimation in Cyber-Physical Systems

Somayeh Davar

The thesis is motivated by recent advancements and developments in large, distributed, autonomous, and self-aware Cyber-Physical Systems (CPSs), which are emerging engineering systems with integrated processing, control, and communication capabilities. Efficient usage of available resources (communication, computation, bandwidth and energy) is a pre-requisite for productive operation of CPSs, where security, privacy, and/or power considerations limit the number of information transfers between neighbouring sensors. In this regard, the focus of the thesis is on information acquisition, state estimation, and learning in the context of CPSs by adopting an Event-based Estimation (EBE) strategy, where information transfer is performed only in occurrence of specific events identified via the localized triggering mechanisms. In particular, the thesis aims to address the following identified drawbacks of the existing EBE methodologies: (i) At one hand, while EBE using Gaussian-based approximations of the event-triggered posterior has been fairly investigated, application of non-linear, non-Gaussian filtering using particle filters is still in its infancy, and; (ii) On the other hand, the common assumption in the existing EBE strategies is having a binary (idle and event) decision process where during idle epochs, the sensor holds on to its local measurements while during the event epochs measurement communication happens. Although, binary event-based transfer of measurements potentially reduces the communication overhead, still communicating raw measurements during all the event instances could be very costly. To address the aforementioned shortcomings of existing EBE methodologies, first an intuitively pleasing event-based particle filtering (EBPF) framework is proposed for centralized, hierarchical, and distributed state estimation architectures. Furthermore, a novel ternary event-triggering framework, referred to as the TEB-PF, is proposed by introducing the ternary event-triggering (TET) mechanism coupled with a non-Gaussian fusion strategy that jointly incorporates hybrid measurements within the particle filtering framework. Instead of using a binary decision criteria, the proposed TET mechanism uses three local decision cases resulting in set-valued, quantized, and point-valued measurements. Due to joint utilization of quantized and set-valued measurements in addition to the point-valued ones, the proposed TEB-PF simultaneously reduces the communication overhead, in comparison to its binary triggering counterparts, while also improves the estimation accuracy especially in low communication rates.

Acknowledgments

To my life-coach, my eternal cheerleader, best supervisor ever, Dr. Arash: because I owe it all to you. Many Thanks!

I would like to express my sincere gratitude to my supervisor Dr.Arash Mohammadi for the continuous support of my master study and related researches, for his patience, motivation, and immense knowledge. The door to Dr. Arash's office was always open whenever I ran into a trouble spot or had a question about my research or in preparation of my papers. His guidance helped me in all the time of research and writing of this thesis. I could not have imagined having a better supervisor and mentor for my study. He consistently steered me in the right direction whenever he thought I needed it. He provided me with the opportunity to join his team at I-SIP LAB. Without his precious support, it would not be possible to conduct this research. I will miss our interesting discussions about our project, the sleepless nights we were working before the deadlines, and never forget the glance of his eyes when he finds a solution to one of our project. My forever interested, encouraging and always enthusiastic professor.

Besides my supervisor, I would like to thank my thesis committee: Prof. Abdessamad Ben Hamza, Prof. Wei – Ping Zhu, and Prof. Jun Yan, for their insightful comments and encouragement, and also for their great personality and friendly advice which incented me to widen my research from various perspectives. I am gratefully indebted to them for their very valuable comments on this thesis.

I am using this opportunity to express my very profound gratitude to my parents especially my mother for providing me with endless support, motivation, and encouragement throughout my years of study and in my life. I cannot express how I am grateful to my mother for all the sacrificed that she has made for me.

A warm and especial thanks to my husband, Ebrahim, for his unfailing support and continuous encouragement throughout my master study and through the process of researching and writing my thesis, who have provided me through moral and emotional support. This accomplishment would not have been possible without him.

And finally, last but by no means least, I thank my fellow lab mates for the stimulating discussions, and also for all the fun we have had in the last two years, it was great sharing the laboratory with all of you during the last two years.

Contents

List of Figures

List of Tables

Abbreviation

Abbreviation	**Description**
AN	Agent Networks
CPS	Cyber Physical System
EBE	Event-based Estimation
EBPF	Event-based Particle Filtering
ETDPF	Event Triggered Diffusion Particle Filtering
FC	Fusion Center
IE	Idle Epoch
KF	Kalman Filter
LQE	linear Quadratic Estimation
MC	Monte Carlo
MAC	Medium Access Control
MAP	Maximum A Posteriori
MSE	Mean Square Error
MMSE	Minimum Mean Square Error
MOPSO	Multi-Objective Particle Swarm Optimization
NCS	Network Control System
OBS	Optical Back Scatter
PF	Particle Filter
PSO	Particle Swarm Optimization
QE	Quantize Epoch

QM	Quantize Measurements
RM	Row Measurements
SN	Sensor Networks
SOD	Send-on-Delta
SMC	Sequential Monte Carlo
SLE	Smart Learning Environment
SIS	Sequential Importance Sampling
SIR	Sampling Importance Resampling
TC	Triggering Condition
TET	Ternary Event Triggering
TEBPF	Ternary Event-Based Particle Filtering
UWSN	Underwater Wireless Sensor Network
UAV	Unmanned Aerial Vehicle
V2X	Vehicle To Everything
WSN	Wireless Sensor Network

Chapter 1

Thesis Introduction and Overview

With recent advancements in sensor technologies and significant improvements in communication and networking solutions, the amount of sensory data collected via different modalities is exploding. Efficient and real-time processing of such large amounts of sensory signals play an indispensable role in the technological advancements of our daily lives. The main focus of this thesis is on information acquisition, state estimation, and learning in the context of cyber-physical systems (CPSs) [1] which are engineering systems with integrated computational and communication capabilities that interact with humans through the cyber space. Efficient usage of available resources (communication, computation, bandwidth and energy) is a pre-requisite for productive operation of CPSs. Besides, such systems typically consist of both wireless and wired sensor/agent networks with different capacity/reliability levels where the emphasis is on real-time operations while sensing and processing tasks are performed distributively. A significant challenge in making distributed monitoring in CPSs a reality is data collection from geographically distributed sensors in an adaptive and intelligent fashion. To accommodate these critical aspects of CPSs, the thesis adopts "event-based" signal processing methodology where in order to reduce the unnecessary processing and communication overhead and to preserve valuable resources in CPSs, each sensor transfers its measurements only in occurrence of specific events (asynchronously) identified using a local event-triggering mechanism. In summary, the main objective of the thesis is to develop innovative event-based signal processing methodologies for real-time processing and inference from large datasets in a computationally feasible manner.

1.1 Cyber-Physical Systems (CPS)

The CPSs are typically considered as "smart" systems interacting with humans through several mediums to expand the capabilities of the physical world through integration of communication, computation, and control. Intuitively speaking, the CPSs can be seen as (wireless) networked systems equipped with numerous distributed, linked, and autonomously operated nodes being monitored and controlled by computer-based algorithms within the cyber domain. In the other words, physical component, computational processes, software components, and networking technologies are deeply integrated into CPSs, each operating on a different scale.

At one hand, CPSs interacting with networks of physical and computational components including design and development of next-generation airplanes [2], fully autonomous vehicles [3, 4], and hybrid vehicles [5] to name a few. On the other hand, a branch of CPS referred to as human-in-the-loop CPSs [6] are being investigated where brain signals are used to control physical objects and provide futuristic smart services to potentially improve our quality of life from different aspects. In the near future, it is expected that CPSs will touch every aspect of our world and will create unmistakable and significant changes to our lives. The rapid advancement of the internet has changed and transformed our lives on how we interact with information. The interactions between the physical world and humans through CPSs will be leading to further extend boundaries of the information technology. We can not deny the vast impact of CPSs on the social structure, economy, and society market, as CPSs represent a new generation of systems which combine computing and communication efficiencies with the physical and engineered systems. However, all these intriguing benefits comes with a downside, i.e., the effect in the physical work is uncertain, therefore, we face several challenges (translating to various research opportunities) within the CPSs [7].

Recent scientific strides of distributed signal processing techniques for CPSs [8–12] rapidly becoming an inseparable part of our everyday lives. One example of these innovations is production of small-sized and cheap electronic devices (sensors) of different modalities being capable of measuring various physical characteristics including but not limited to, temperature, light intensity, optical backscatter (OBS), fluorescence, and seismic activities. These evolving sensor technologies are also capable of communicating within themselves and/or with other devices, therefore, they collect and store data for further processing if needed. These sensors can communicate their observations via wired as well as wireless means depending on the given scope of the application. The networked systems are constructed/designed by incorporation of typically a large number of such local processing nodes (sensors), resulting in creation of Sensor Networks (SN) and/or agent networks (AN) [13], Robotic Networks [14], Camera Networks [15], and Networks of Unmanned aerial vehicles (UAV) [16].

1.1.1 Advantages of Cyber Physical Systems

Advent of CPS introduces several advantages such as: (i) Making safe and more efficient systems; (ii) Allowing each agent (node) to work individually and collaborate locally to form a complex system with new capabilities, and; (iii) Reducing the cost of building and operating these systems. Furthermore, CPSs provide the following unique advantages:

(i) ***Human-Machine Interaction***: A CPS creates a model between humans and the underlying physical system. An important aspect of such a model is providing a venue for interaction, modeling, and measurement of situational human consciousness of the system and the environmental changes affecting different model parameters.

(ii) ***Quick Response Time***: A CPS will increase response time and early detection of a failure. Moreover, it facilitates proper utilization of resources such as bandwidth.

(iii) ***System Performance***: By integration of several sensors with the available cyber infrastructure, CPSs can provide better performance in terms of feedback and auto redesign. Consequently, they make systems more efficient and safer.

(iv) ***Deal with Uncertainty***: Certainty is the process of providing proof that a design is valid and trustworthy. CPSs can evolve and operate within a new and unreliable environment in an adaptive fashion, therefore, cope with the underlying uncertainties.

(v) ***Flexibility and Scalability***: One of the important aspects of the CPSs is potential integration of cloud infrastructures with SNs/ANs. A CPS can provide more facilities than wireless sensor networks (WSN) and cloud computing alone [17].

1.1.2 Applications of Cyber Physical Systems

As stated previously, CPSs have penetrated several applications of significant engineering importance including but not limited to the following categories:

(i) ***Smart Grid***: CPSs monitor the conditions and care for the stability of transmission and distribution networks that connect end-users to the smart grid. Generally, it will control network connectivity as well as operational aspects of power generation. This provides two-way communication between the electricity grid and consumers.

(ii) ***Smart Transportation Systems***: A road traffic-control CPS creates a physical cyber environmental system built across vast geographical area to manmade bridges across the sea/rivers, tunnels, high-risk sub-grade slope, and urban elevated bridges, to name a few. Such CPSs can improve operational efficiency and safety levels of the road traffic system via incorporation of a large number of advanced, distributed, and smart sensors.

(iii) ***Smart Learning Environments***: CPSs can be used in a smart learning environment (SLE) to collect sufficient information about the environment, transform measured data to information/knowledge, and ultimately provide quick and useful services for the students, staffs and the university. The SLE is posed to change the way people learn and work in universities [19].

(iv) ***Medical CPS***: Health monitoring, collecting and processing diagnostic information is another emerging domain for CPSs, referred to as the Medical CPS in brief. Integration of computations and control mechanisms with vast amount of medical data provides a fundamental prerequisite for development of future health-care systems.

(v) ***Green Buildings***: One of the major problems in today's world is the Greenhouse effect [20]. The old buildings consume 70% of the electricity and produce/generate the greenhouse gases which in turn increase the greenhouse effect. By incorporation of the CPS concept, zero net energy goal can be achieved.

(vi) ***Aviation CyberPhysical Systems***: Cyber-Physical Systems are used for Aeronautic applications such as flight test tools; Pilot-crew communications; Structure Health Monitoring; In-flight tests, and flight landing [21].

1.2 Event-based State Estimation in CPSs

The focus of this thesis is on state estimation in CPSs, where a single or several sensor nodes are scattered across the system collecting observations on the behavior of the underlying state of the physical phenomena under control. Sensor measurements are communicated to form an overall estimate of the state of the system. In particular, scenarios are considered where local nodes cooperatively estimate certain parameters (or states) of the surrounding environment based on their local measurements/observations. Cooperation is needed as local observations are individually insufficient for obtaining reliable estimates. Generally speaking, there are three main estimation architectures for cooperation in CPSs as explained below:

(1) ***Centralized Estimation***: Traditionally, state estimation is centralized where all the participating agents/nodes communicate their observations to the fusion centre (FC) which is a central processing unit, and responsible for performing a predefined task (e.g., tracking a target). Implementation of the centralized architecture is simple but is generally unscalable to add more sensor nodes to the system. The main disadvantage of a centralized architecture is the case where the FC breaks down (referred to as the single point of failure) resulting in the loss of all sensory information. An additional complexity arises with a change in the network topology requiring the routing tables to be redesigned adding to the complexity of this architecture.

(2) ***Hierarchical Estimation***: In a hierarchical architecture, a subset of sensor nodes is associated with a local processing node (local FC) to which local measurements from the associated sensor nodes are transferred. Instead of sending raw observations, local processing nodes communicate partial or fully processed data to the central FC. The overall performance of a hierarchical architecture still depends on the FC to combine the local processed data into a global state estimate.

(3) ***Distributed Estimation***: In a distributed architecture, there is no global FC, therefore, the sensors and the local processing nodes do not require global knowledge of the network topology. Furthermore, each local processing node collects data from the sensors in its communication range and exchanges data only with other local processing nodes in its local neighbourhood.

Such a distributed architecture offers the following three advantages over its counterparts:

(i) Each local node only knows the connections to its immediate neighboring nodes, therefore, global knowledge of the network topology is not needed locally.

(ii) Communication occurs on a node-to-node basis within local neighbourhoods.

(iii) Fusion occurs locally and the successful operation of the network is not dependent on implementation of a global FC.

Independent of the architecture used for cooperation within a CPS, there exist an urgent need to efficiently use valuable resources in CPSs. In other words, it is common in distributed estimation algorithms that agents have a limited bit budget for communication [22–25]. Traditionally, quantization has been viewed as a fundamental element in this regard for saving bandwidth to reduce the energy consumption which is related to the amount of data transmitted. At one extreme, harsh quantization is introduced for example in Reference [22] where only one bit is communicated (based on the sign of innovation). Emergence of CPS has further increased this urgency to reduce local bit budget of the sensor nodes resulting in a great surge of interest for development of ***Event-based Estimation (EBE) Methodologies*** [26–32]. Early research in this field [33, 34] was based on the send-on-delta (SOD) triggering where the transmission is performed only when the difference between the current measurement and the previously transmitted one is greater than a pre-defined threshold. Alternative triggering mechanisms were recently proposed such as innovation-based [35, 36], variance-based [37], KullbackLeibler divergence-based [38] and information-based [8].

This thesis aims to further advance the field of EBE. In particular, the following three CPS application domains are the main motivation behind the EBE research performed in this thesis:

(i) ***Underwater Wireless Sensor Networks (UWSN)***: First motivating practical application for reducing the number of communicated bits, is the UWSN technologies [39]. Underwater communications suffer from limited bandwidth due to the temporal and spatial variability of channels. A limited bandwidth leads to low bit rates, therefore, the data-efficiency of UWSNs can be improved by reducing the length of data packets transmitted from the sensor to the FC. In this domain, event-based estimation mechanisms improve data-efficiency by using a combination of different transmission scenarios.

(ii) ***Autonomous Driving***: Recent advances in the application of signal processing for environmental perception pave the way for development of fully automated vehicles [40]. Building a truly autonomous system, however, requires sophisticated self-assessment capabilities in order to be fail-operational (i.e., continue a safe operation) in all scenarios. Autonomous vehicles must therefore possess knowledge of (formal) boundaries of their actions to avoid catastrophic effects, which is achieved through distributed and collaborative state estimation. In order to meet the complex requirements of autonomous vehicles, communication overhead needs to be minimized, this is where event-based solutions can play a critical role.

(iii) ***Self-Awareness***: Self-awareness refers to a system's capability to recognize its own state, possible actions, and the result of these actions on the state of the overall system and on the environment. However, self-awareness mechanisms must be considered in combination with all other local and global processing tasks in order to have a coherent self-aware system. This can not be achieved without cooperation with the outer world (e.g., neighbouring agents or processing units) to avoid conflicting decisions or even catastrophic effects. EBE is, therefore, a critical component in making a local agent self-aware.

The thesis performs a fundamental assessment on the response of estimation algorithms (centralized, hierarchical, and distributed) receiving event sampled measurements. Specially, the focus is on the following research problems:

(i) It is common in EBE literature to consider a binary decision criteria, i.e., a sensor shares or holds on to its local measurements. In practical scenarios such a binary transmission mechanism will result in loss of communication resources;

(ii) In absence of an observation (idle mode), the FC can incorporate the information regarding the sensor's triggering mechanism as side information, which results in a hybrid estimation problem with point and set-valued observations. In such scenarios, the posterior will be non-Gaussian requiring nonlinear filtering approaches, however, application of such filtering methodologies in this area is still in its infancy, and;

(iii) Event-based estimation is mainly developed for single sensor scenarios. Hierarchical and distributed triggering mechanisms based on diffusive strategies have not yet been investigated thoroughly in this context.

To summarize, event-based signal processing for distributed state estimation scenarios in CPSs and utilization of advanced non-linear filtering in this context are still in their infancy. This thesis focuses on potential solutions to address this gap and the above mentioned challenges.

1.3 Thesis Contributions

Inspired by the stated issues of EBE in CPSs, I have made a number of contributions [41–45] during my thesis research work as briefly outlined below:

(1) **Event-Based State Estimation with Joint Point and Set-Valued Measurements**: Three EBE methodologies are proposed for centralized, hierarchical, and distributed estimation architectures as briefly described below:

- **Event-based Particle Filtering with Joint Point and Set-valued Measurements *(Centralized Architecture)*** [41]: An intuitively pleasing event-based particle filtering (EBPF) framework is proposed for state estimation in CPSs with power constraints at the sensor side. An open-loop estimation (i.e., no feedback communication is incorporated from the FC to the local sensor) and event-based architecture is considered. Point-valued measurements are incorporated in the estimation recursion via a conventional particle filter formulation, while set-valued measurements are incorporated by evaluating the probability that the unknown observation belongs to the event-triggering set based on the estimators particles which is then used to update the corresponding particle weights.
- **Multi-sensor EBE via Information-based Triggering *(Hierarchical Architecture)*** [42]: By considering the distributed resource management problem, a hierarchical EBE scenario is considered where the events are identified using the information-based triggering mechanism without incorporation of a feedback from the FC and/or implementation of a local filter at the sensor level. A multi-sensor triggering approach is proposed based on the projection of each local observation into the state-space which corresponds to the

achievable gain in the sensor's information state vector.

- **Event-Triggered Diffusion Particle Filter** ***(Distributed Architecture)*** [43]: The proposed EBE approaches are modified and extended to remove their dependency on the FC. In this regard, an event-triggered distributed state estimation via diffusion strategies (ET/DPF) is proposed, which is a systematic distributed state estimation algorithm that jointly incorporates point and set-valued measurements within the particle filtering framework.

(2) **Ternary EBE with Joint Point, Quantized, and Set-Valued Measurements** [44]: A novel ternary event-based particle filtering (TEB-PF) framework is proposed by introducing the ternary event-triggering (TET) mechanism coupled with a non-Gaussian fusion strategy that jointly incorporates point-valued, quantized, and set-valued measurements. The proposed TEB-PF simultaneously reduces the communication overhead, in comparison to its binary triggering counterparts, while also improves the estimation accuracy especially in low communication rates.

- **Designing Optimal Thresholds for TET Mechanism via Multi Objective Particle Swarm Optimizer** [45]: To complete our previous work on ternary EBE, a novel multi-objective approach is proposed for optimizing the threshold values used via the TET mechanism.

1.4 Thesis Organization

The rest of the thesis is organized as follows:

- Chapter 1 provides an introduction on the research area of this thesis followed by a brief overview of the reminder of the thesis.

- Chapter 2 provides an introduction to the EBE problem. The required background as the bases for the reminder of the thesis will also be presented in this chapter including a brief introduction on Kalman filtering (KF) and Particle filtering (PF).

- Chapter 3 considers the problem of event-based PF for centralized, hierarchical, and distributed architectures. Initially, the centralized approach is considered based on a single sensor scenario. Then, extension to multi-sensor hierarchal setting is developed. Finally, fully distributed scenario is discussed.

- Chapter 4 introduces our proposed Ternary EBE mechanism together with the optimal quantization-based target tracking scheme, and explains how particle swarm optimization is utilized to obtain the optimal thresholds for the TET mechanism.

- Chapter 5 concludes the thesis and explains some directions for future research work.

Chapter 2

Event-based State Estimation

Statistical signal processing is concerned with situations where the values of unknown parameters need to be evaluated from observations made under a state of uncertainty. The overall goal is to provide a rational framework for dealing with such situations. The Bayesian approach, the main theme of this thesis, is a well-known framework of formulating and dealing with such statistical estimation problems. The literature on Bayesian estimation is vast, therefore, in this chapter, I restrict myself to common approaches such as the KF, and Sequential Monte Carlo methods (the PF). Traditionally, these Bayesian approaches were developed for a centralized architecture (Fig. 2.1) based on time-driven transmission schemes (in contrary to event-based schemes) with a FC being responsible for collecting observations from across the CPS to compute the overall state estimates. Recent developments in hardware and advances in communication have paved the way for practical distributed implementations of the PF for an arbitrary deployed nonlinear AN/SN within the CPS. In this chapter, I start by formulating the EBE problem, which is the main theme of this thesis followed by introducing the basics of Bayesian recursive estimation. Finally, a brief presentation of the KF and PF as two Bayesian estimators for linear and non-linear systems, respectively, are included.

2.1 Event-based State Estimation: Problem Formulation

The KF [46] is considered as the classical state estimation approach for state estimation in AN/SN systems due its simple and efficient sequential formulation. In a conventional KF-based

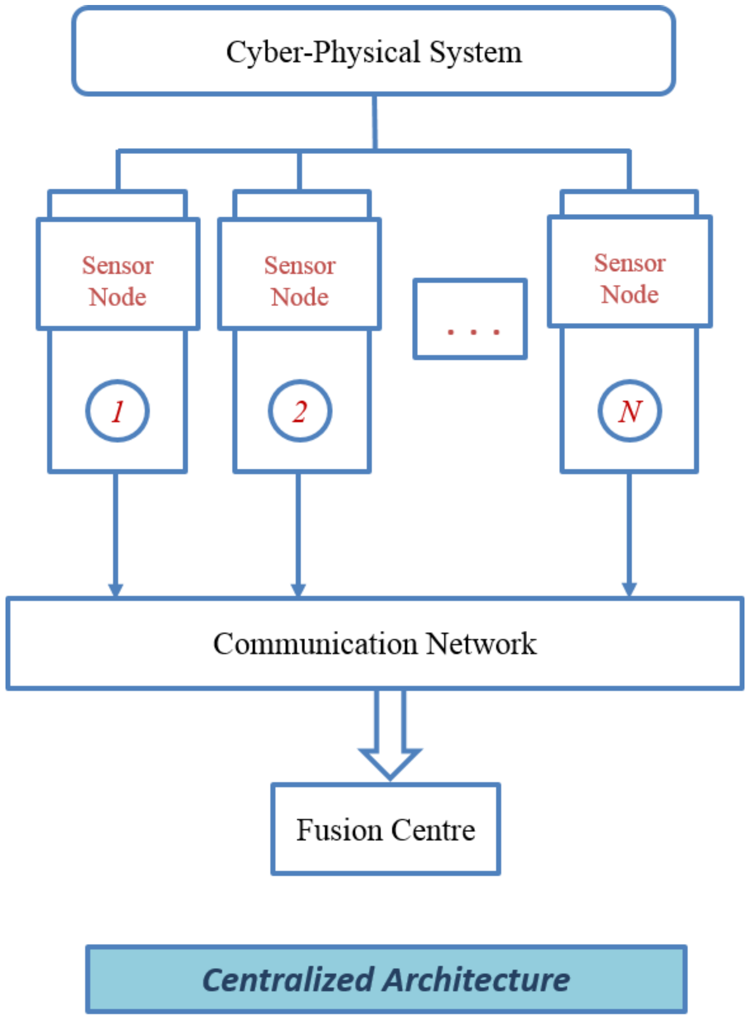


Figure 2.1: Block diagram of the centralized architecture.

estimation scheme, each sensor samples and communicates its local measurements to the FC periodically using equidistance samples (referred to as a time-driven strategy). Recent developments and advancements of sensor technologies, CPS [47], and network control systems (NCS) [48] render the above mentioned conventional approach to be impractical because of the following key reasons:

(i) Measurements contain different information contents over time, therefore, transmission of observations based on an equidistance sampling strategy could result in over-consumption of restricted local resources;

(ii) Sensor nodes have restricted power supplies, therefore, can not afford to periodically transfer information to the FC as communication is the main source of power consumption, and;

(iii) The channel bandwidth is limited, another barrier in implementing time-driven distributed estimation algorithms such as the conventional KF. In other words, limited channel bandwidth calls for alternative transmission and scheduling schemes [49–51] other than the conventional time-driven methodology.

These issues have resulted in a recent surge of interest in developing intelligent transmission, scheduling, and estimation schemes [26–31, 49–51] to reduce the communication overhead of sensors in order to increase their practical applicability by improving their energy efficiency.

Recent solution methodologies developed to reduce the aforementioned extra communication overhead, associated with distributed estimation, can be generally classified into:

(a) ***Offline scheduling schemes*** [52, 53], where the transmission schedule is designed in advance of employment.

(b) ***Event-based estimation (EBE) Methodologies*** [26–32], where communication of sensor information is only triggered once the system meets a specific condition, which is identified using a triggering mechanism at the sensor level based on real-time local observations.

While it is simpler to impalement algorithms belonging to the former category, a priori information regarding the physical system is required and their performance is typically unacceptable in practice, especially in hostile environments where the characteristics of the system constantly changes. This resulted in a recent surge of interest in designing/developing event-based implementations as they are capable of providing the possibility of maintaining the required estimation performance under strict communication constraints.

The event-based concept emerged by the seminal work of Astrom and Bernhardsson [54] where it was shown that Lebesgue sampling is superior for state estimation purposes in some dynamical systems. References [33,34] are among the early event-based methodologies and proposed the send-on-delta (SOD) triggering mechanism where the transmission is triggered only when the difference between the current measurement and the previously transmitted one is greater than a pre-defined threshold (delta). In such event-based estimation scenarios and in the absence of an observation (i.e., the triggering conditions are not satisfied) the estimator still has access to side information, i.e., the measurement belongs to the set characterized by the triggering mechanism. Incorporation of the side information from the event-triggering mechanism during non-event iterations results in a hybrid update strategy, i.e., state estimation with joint set-valued and point-valued measurements which is first considered in [32] and has recently been extended in Reference [26]. In this context the mechanism used to trigger an event at the sensor side dictates the nature of the posterior distribution at the remote estimator and consequently mandates the proper (possibly optimal) form of the estimation/fusion algorithm at the FC. In the conventional time-driven scenario with point-valued measurements the simple and efficient formulation of the KF comes from the Gaussianity of conditional posterior distribution. In the hybrid scenarios described above, however, due to joint incorporation of set and point valued measurements, the posterior distribution becomes non-Gaussian, therefore, the conventional KF is no longer applicable.

Next, we describe the state-space model of a CPS, which is used to develop the proposed EBE solutions.

2.1.1 Single-Sensor Architecture

We consider an estimation problem where the physical component of the CPSs is modeled with a set of n_x state variables

$$\boldsymbol{x} = [X^1, X^2, \dots, X^{n_x}]^T, \tag{2.1}$$

where n_x is the number of state variables, and T denotes matrix transposition. The evolution of states over time and the sensor observation model are represented by the following linear state-space model

State Model:

$$\boldsymbol{x}_k = \boldsymbol{F}_k \boldsymbol{x}_{k-1} + \boldsymbol{w}_k \tag{2.2}$$

Single-Sensor Observation Model:

$$z_k = \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{v}_k, \tag{2.3}$$

where k denotes iteration index; $\boldsymbol{x}_k \in \mathbb{R}^{n_x}$ denotes the state vector for iteration/time k; $z_k \in \mathbb{R}^{n_z}$ denotes the sensor's measurement; functions $\boldsymbol{F}_k$ and $\boldsymbol{H}_k$ represent the state and observation models, respectively, and; terms $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ represent the uncertainties in the state and observation models, respectively. It is assumed that $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are mutually uncorrelated white Gaussian noises with covariances $\boldsymbol{Q}_k > 0$, and $\boldsymbol{R}_k > 0$. In the centralized architecture, we consider a single-sensor architecture where the sensor communicates its observation z_k to the FC which recursively update the posterior distribution $P(\boldsymbol{x}_k|\boldsymbol{Z}_k)$ based on the collective set of observations $\boldsymbol{Z}_k = \{z_1, \ldots, z_k\}$ received up to and including the current iteration (k).

While the sensor has limited power sources, the FC has adequate power to perform complex estimation algorithms. In the event-based communication/fusion framework, after making each measurement the sensor decides on keeping or sending its observation to the remote estimator. The local decisions are governed by a binary triggering criteria denoted by γ_k which is defined as follows

$$\begin{cases} \gamma_k = 1: & \text{Event occurs, communication is triggered.} \\ \gamma_k = 0: & \text{Idle case, no communication.} \end{cases}$$

Based on the above triggering mechanism, the collective set of observations up to and including iteration k at the FC is defined as $\tilde{\boldsymbol{Z}}_k = \{\gamma_1 z_1, \ldots, \gamma_k z_k\}$. Based on the above definitions, the predicted state estimate (priory estimate) $\hat{\boldsymbol{x}}_{k|k-1}$ and its corresponding covariance matrix at iteration k are defined as

$$\hat{\boldsymbol{x}}_{k|k-1} \triangleq \mathbb{E}\{\boldsymbol{x}_k|\boldsymbol{Z}_{k-1}\} \tag{2.4}$$

$$\text{and } \boldsymbol{P}_{k|k-1} \triangleq \mathbb{E}\{(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k-1})(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k-1})^T|\boldsymbol{Z}_{k-1}\}, \tag{2.5}$$

where T denotes transpose operator. Similarly, the posteriori estimate $\hat{\boldsymbol{x}}_{k|k}$ and its corresponding error covariance matrix are defined as follows

$$\hat{\boldsymbol{x}}_{k|k} \triangleq \mathbb{E}\{\boldsymbol{x}_k|\boldsymbol{Z}_k\} \tag{2.6}$$

$$\text{and } \boldsymbol{P}_{k|k} \triangleq \mathbb{E}\{(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k})(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k})^T|\boldsymbol{Z}_k\}. \tag{2.7}$$

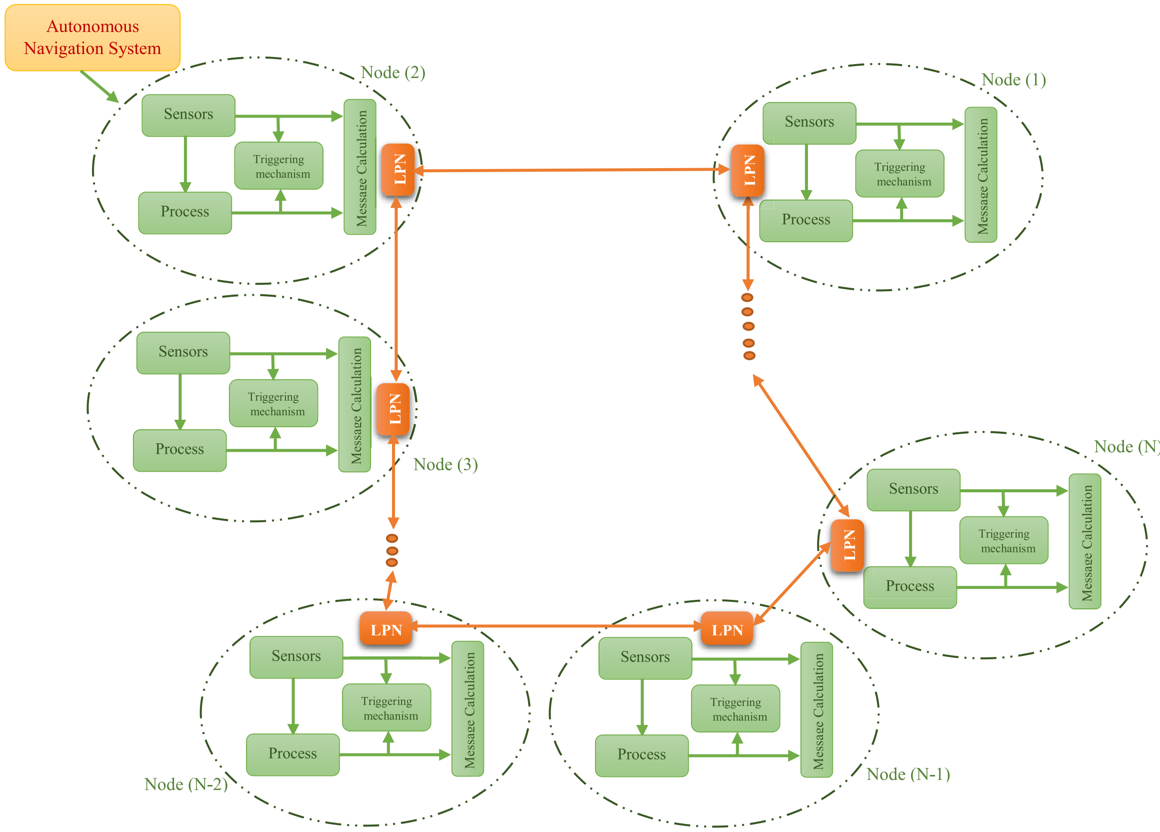


Figure 2.2: Block diagram of the multi-agent event-triggered distributed state estimation framework.

When the event-triggering condition is satisfied (i.e., $\gamma_k = 1$), the exact value of the sensor measurement z_k is known at the FC, referred to as "***point-valued observation information***". On the other hand, when the event-triggering condition is violated (i.e., $\gamma_k = 0$), some information contained in the event-triggering sets is known to the estimator instead, referred to as "***set-valued information***". The main issue here comes from the non-Gaussianity of the a posteriori distribution due to joint incorporation of point and set-valued measurements, i.e., $P(\boldsymbol{x}|\tilde{\boldsymbol{Z}}_k)$ no longer follows a Gaussian distribution.

2.1.2 Multi-Sensor Architecture

Consider an CPS comprising of N nodes[1] observing a set of n_x state variables $\boldsymbol{x}_k$. The set of neighboring nodes for node l for, $(1 \leq l \leq N)$, is denoted by $\aleph^{(l)}_{\text{fuse}}$. In the case that node l, for

[1]The term node here refers to a processing node or an agent with processing and observation capabilities.

example, is connected to all other nodes, $\aleph_{\text{fuse}}^{(l)} = N-1$. In such scenarios where more than one sensor is used for distributed estimation, Eq. (2.2) remains the same because the underlying physical system has not changed and only more sensors are utilized. On the other hand, Eq. (2.3) changes to represent multi-agent estimation problem, i.e.,

$$\text{Multi-Sensor Observation Model:} \quad \underbrace{\begin{bmatrix} z_k^{(1)} \\ \vdots \\ z_k^{(N)} \end{bmatrix}}_{\boldsymbol{z}_k} = \underbrace{\begin{bmatrix} \boldsymbol{h}_k^{(1)^T} \boldsymbol{x}_k \\ \vdots \\ \boldsymbol{h}_k^{(N)^T} \boldsymbol{x}_k \end{bmatrix}}_{\boldsymbol{H}_k \boldsymbol{x}_k} + \underbrace{\begin{bmatrix} v_k^{(1)} \\ \vdots \\ v_k^{(N)} \end{bmatrix}}_{\boldsymbol{v}_k}, \tag{2.8}$$

where $z_k^{(l)}$ denotes the local measurement made at node l, for $(1 \leq l \leq N)$, at time instant k. Terms $\{\boldsymbol{\xi}(\cdot), v^{(l)}(\cdot)\}$ are, respectively, the global and local possibly non-Gaussian uncertainties in the state and observation models. Matrix $\boldsymbol{H}_k$ represents the global observation dynamics.

The collective set of observations from sensor l up to and including iteration k is defined as

$$\boldsymbol{Z}_k^{(l)} = \{\gamma_1^{(l)} z_1^{(l)}, \ldots, \gamma_k^{(l)} z_k^{(l)}\}. \tag{2.9}$$

Finally, the overall collective set of observations up to and including iteration k is defined as follows

$$\boldsymbol{Z}_k = \{\boldsymbol{Z}_k^{(1)}, \ldots, \boldsymbol{Z}_k^{(N)}\}. \tag{2.10}$$

As sated previously in Chapter 1, in the autonomous and self-aware problem considered here, the network is not fully connected[2], besides, a local agent can not afford to communicate periodically with its neighbours of the FC. This could be due to bandwidth, security, privacy, and/or power considerations. Therefore, we consider an ET communication/fusion framework [8], where after making each measurement the sensor decides on keeping or sharing its measurements with its local neighbourhood. In an ET fusion architecture, local decisions at sensor node l is governed by a

[2]In a fully connected system (i.e., each agent has a direct connection to all the other agents), $\boldsymbol{z}_k^{(\aleph^{(l)})} = \boldsymbol{z}_k$.

binary triggering criteria denoted by $\gamma_k^{(l)}$ which is defined as follows

$$\begin{cases} \gamma_k^{(l)} = 1: & \text{Event occurs, Sensor } l \text{ communicates.} \\ \gamma_k^{(l)} = 0: & \text{Idle case, no communication from Sensor } l. \end{cases}$$

Later on and in Section 3.5.3, we consider a fully distributed estimation architecture (Fig. 2.2) where each agent shares its measurements within its local neighbourhood and recursively updates the posterior distribution based on the collective set of neighbourhood measurements $z_k^{(\aleph^{(l)})} \triangleq \{z_k^{(i)} : i \in \aleph^{(l)}\}$, where $\aleph^{(l)}$ denotes the set of agents connected to agent l.

Finally, in estimation problems where the state dynamics of the CPS and/or the sensor model are possibly non-linear/non-Gaussian, the overall state-space representation of the system is given by

$$\text{State Model:} \qquad \boldsymbol{x}_k = \boldsymbol{f}\big(\boldsymbol{x}_{k-1}, \boldsymbol{\xi}_k\big) \tag{2.11}$$

$$\text{Observation Model:} \quad \underbrace{\begin{bmatrix} z_k^{(1)} \\ \vdots \\ z_k^{(N)} \end{bmatrix}}_{z_k} = \underbrace{\begin{bmatrix} \boldsymbol{g}^{(1)}(\boldsymbol{x}_k) \\ \vdots \\ \boldsymbol{g}^{(N)}(\boldsymbol{x}_k) \end{bmatrix}}_{\boldsymbol{g}(\boldsymbol{x}_k)} + \underbrace{\begin{bmatrix} \boldsymbol{\zeta}_k^{(1)} \\ \vdots \\ \boldsymbol{\zeta}_k^{(N)} \end{bmatrix}}_{\boldsymbol{\zeta}_k}, \tag{2.12}$$

where $\boldsymbol{\xi}(\cdot)$ and $\boldsymbol{\zeta}(\cdot)$ are, respectively, the global uncertainties in the process and observation models. The state and observation functions $\boldsymbol{f}(\cdot)$ and $\boldsymbol{g}(\cdot)$ can possibly be non-linear, and vectors $\boldsymbol{\xi}(\cdot)$ and $\boldsymbol{\zeta}(\cdot)$ are not necessarily restricted to white Gaussian noise.

This completes the problem formulation. Next, I review the Bayesian recursive estimation framework which is used to develop the proposed EBE methodologies.

2.2 Bayesian Estimation

In sequential Bayesian estimation, the evolution of the state variables (e.g., 3-dimensional or 2-dimensional location of a target within the surveillance area) is modeled as a first-order Markov process, i.e., the current states only depends on its immediate previous values. In other words, due to the Markovian property, the value of the state $\boldsymbol{x}_k$ at time index k depends only on the value of the immediately proceeding state $\boldsymbol{x}_{k-1}$ and is independent of both the observations and states proceeding $(k-1)$, i.e.,

$$P\big(\boldsymbol{x}_k|\boldsymbol{x}_{0:k-1}, z_{1:\,k-1}\big) = P\big(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}\big). \tag{2.13}$$

Conditional independence is typically considered within the Bayesian framework such that given the current state values $\boldsymbol{x}_k$, the observation vector z_k is conditionally independent of the prior states variables, i.e.,

$$P\big(z_k|\boldsymbol{x}_{0:k}\big) = P\big(z_k|\boldsymbol{x}_k\big). \tag{2.14}$$

In the probabilistic form, the estimation problem formulated within the Bayesian framework is equivalent to determining the conditional filtering density $P(\boldsymbol{x}_k|z_{1:k},)$, i.e., the probability of the state variables for all time instances $k > 0$ given the received observations up to and including the current iteration. Using the Bayes' rule, the filtering density can be expressed as follows

$$P\big(\boldsymbol{x}_k|z_{1:k}\big) = \frac{\overbrace{P\big(z_k|\boldsymbol{x}_k\big)}^{\text{Likelihood}}\,\overbrace{P\big(\boldsymbol{x}_k|z_{1:k-1}\big)}^{\text{Predicted Density}}}{\underbrace{P\big(z_k|z_{1:k-1}\big)}_{\text{Normalization}}}. \tag{2.15}$$

The denominator $P(z_k|z_{1:k-1})$ in Eq. (2.15) is independent of the state variables and can be set as the normalizing constant, i.e., $P(z_k|z_{1:k-1}) = \alpha$. The second term $P(\boldsymbol{x}_k|z_{1:k-1})$ in the numerator of Eq. (2.15) can be expanded in terms of the *state transition model* $P(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$ and the filtering density $P(\boldsymbol{x}_{k-1}|z_{k-1})$ as follows

$$P\big(\boldsymbol{x}_k|z_{1:\,k-1}\big) = \int P(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}) \times P(\boldsymbol{x}_{k-1}|z_{1:\,k-1})d\boldsymbol{x}_{k-1}. \tag{2.16}$$

Eq. (2.15) is referred to as the *observation update step*, and Eq. (2.16) is referred to as the *prediction step*. In the Bayesian framework, Eqs. (2.15)-(2.16) define a recursive solution to compute the filtering density based on the following steps:

Step 1. *Prediction Update*: Given $P(\boldsymbol{x}_{k-1}|z_{1:\,k-1})$ compute $P(\boldsymbol{x}_k|z_{1:\,k-1})$.

Step 2. *Normalization Update*: Compute the normalization factor $P(z_k|z_{1:k-1})$.

Step 3. *Observation Update*: Using the sensor model $P(z_k|\boldsymbol{x}_k)$ compute $P(\boldsymbol{x}_k|z_{1:k})$.

One method, referred to as the maximum a posteriori (MAP) estimation, obtains the state estimate $\hat{\boldsymbol{x}}_k$ by determining the value of $\boldsymbol{x}_k$ that maximizes $P(\boldsymbol{x}_k|z_{1:\,k})$. In multisensor Bayesian estimation, several nodes make their own observations $z_k^{(l)}$. The conditional probability $P(z_k^{(l)}|\boldsymbol{x}_k)$ then serves the role of a sensor model and can be utilized in the distributed implementation of the Bayesian estimation algorithms. The multisensor form of Bayes' rule requires conditional independence, which results in the following global likelihood function

$$P\big(z_k|\boldsymbol{x}_k\big) = P\big(z_k^{(1)}, \ldots, z_k^{(N)}|\boldsymbol{x}_k\big) = \prod_{l=1}^{N} P\big(z_k^{(l)}|\boldsymbol{x}_k\big). \tag{2.17}$$

From Eq. (2.15), we have

$$P\big(\boldsymbol{x}_k|z_k^{(1)}, \ldots, z_k^{(N)}\big) = \alpha P\big(\boldsymbol{x}_k|z_{1:\,k-1}\big) \prod_{l=1}^{N} P\big(z_k^{(l)}|\boldsymbol{x}_k\big), \tag{2.18}$$

where $\alpha \triangleq P(z_k|z_{1:k-1})$ is the normalizing constant. Eq. (2.18) is known as the *independent likelihood pool*. This indicates that the filtering density of state variables $\boldsymbol{x}_k$ based on the observation of individual nodes is proportional to the multiplication of the prior density $P(\boldsymbol{x}_k|z_{1:\,k-1})$ with product of the individual likelihood functions $P(z_k^{(l)}|\boldsymbol{x}_k)$ for each sensor node. Next, I review the KF algorithm as a classical Bayesian estimator.

2.2.1 The Kalman Filter

The KF is an optimal estimator for a large class of problems and a very effective and useful estimator for an even larger class of practical applications of significant importance, for instance, a common application of the KF is for guidance, navigation, and control of vehicles, particularly aircraft and spacecraft [55]. Furthermore, the KF is a widely applied concept in time series analysis used in fields such as signal processing and econometrics. The KFs are one of the main topics in the field of robotic motion planning and control, and they are sometimes included in trajectory optimization. The KF also works for modeling the central nervous system's control of movement. Due to the time delay between issuing motor commands and receiving sensory feedback, use of the KF supports a realistic model for making estimates of the current state of the motor system and issuing updated commands.

Kalman filtering, also known as linear quadratic estimation (LQE), is based on Bayesian estimation framework described in Section 2.2 that uses a set of measurements observed by sensors over time (containing statistical noise and other inaccuracies) and produces estimates of the underlying state variables that tend to be more accurate than those based on measurements alone. This is achieved by estimating a joint probability distribution over the variables for each timeframe. Generally speaking, the KF works in a two-step process. In the prediction step, the KF produces estimates of the current state variables, along with their uncertainties without incorporation of the newly observed measurement. Once the outcome of the measurement (potentially corrupted with some amount of error, including random noise) is observed, the predicted estimates are updated using a weighted average, with more weight being given to estimates with higher certainty. The algorithm is recursive in nature and can run in real time, using only the present input measurements, the previously calculated state vector, and its uncertainty matrix (no additional past information is required).

The KF provides the optimum state estimates in the minimum mean square error (MMSE) sense, when the system and observation dynamics are linear, and the forcing terms and the observation noise are Gaussian. In such scenarios and by considering the state-space model given by Eqs. (2.2)-(2.3) and its statistical properties, the posterior follows a Gaussian distribution, i.e.,

$$P(\boldsymbol{x}_k|\boldsymbol{Z}_k) \sim \mathcal{N}(\hat{\boldsymbol{x}}_{k|k}, \boldsymbol{P}_{k|k}), \tag{2.19}$$

where $\hat{\boldsymbol{x}}_{k|k}$ and $\boldsymbol{P}_{k|k}$ are defined in Eqs. (2.6)-(2.7), respectively. The KF provides the optimal solution for this Gaussian case based on the following recursions

Prediction Step:

$$\hat{\boldsymbol{x}}_{k|k-1} = \boldsymbol{F}_k \hat{\boldsymbol{x}}_{k-1|k-1} \tag{2.20}$$

$$\boldsymbol{P}_{k|k-1} = \boldsymbol{F}_k \boldsymbol{P}_{k-1|k-1} \boldsymbol{F}_k^T + \boldsymbol{Q}_k. \tag{2.21}$$

Update Step:

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k \big(z_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k|k-1} \big) \tag{2.22}$$

$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^T \big(\boldsymbol{H}_k \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^T + \boldsymbol{R}_k \big)^{-1} \tag{2.23}$$

$$\boldsymbol{P}_{k|k} = \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_k \boldsymbol{H}_k^T \boldsymbol{P}_{k|k-1}. \tag{2.24}$$

This completes a brief introduction to the KFs for linear systems with Gaussian uncertainties. Next, I review particle filtering for nonlinear/non-Gaussian estimation problems.

2.2.2 The Particle Filter

As stated previously, for nonlinear systems with non-Gaussian uncertainties no analytic solution can be established in general and the KF, typically, provides poor approximations. Alternatively, one can utilize numerical Sequential Monte Carlo (SMC) solutions, also referred to as the Particle Filters (PFs) [10,12], as approximates to the Bayesian estimators. A PF is a recursive, Bayesian state estimator that uses discrete particles to approximate the non-Gaussian posterior distribution. Each particle represents a discrete state hypothesis. The set of all particles is used to help determining the final state estimate. To implement the PF, first system parameters such as the number of particles, the initial particle locations, and the statistical properties need to be specified. Besides, when a specific motion and sensor model is used, their associated parameters in the state transition function and measurement likelihood function need to be identified accordingly. Fig. 2.3 details the estimation workfellow of the PF and shows an example of how to run a PF in a loop to recursively update the state estimates.

More specifically, the PF is developed via the principle of sequential importance sampling

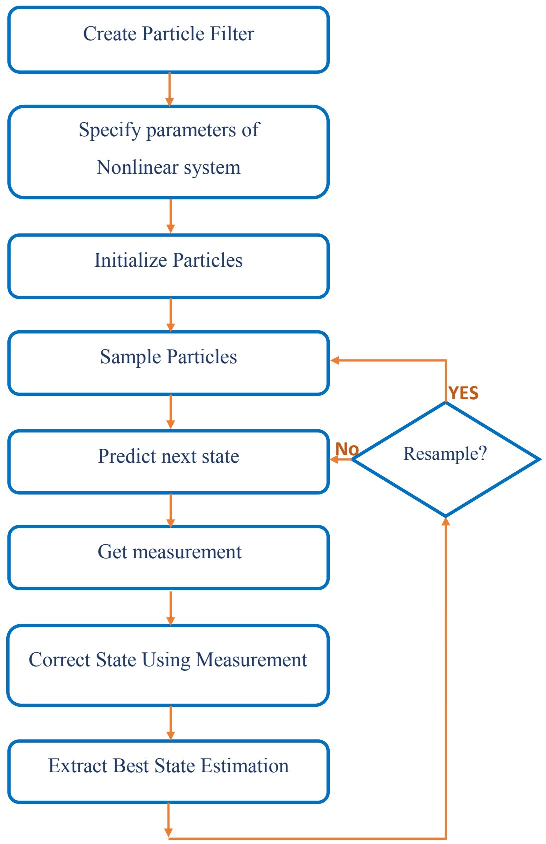


Figure 2.3: Steps for Particle Filter.

(SIS) [56], a suboptimal technique for implementing Bayesian estimator recursively (Eqs. (2.15)-(2.16)) through Monte Carlo simulations. Importance sampling is an approach to evaluate an integral, e.g.,

$$\mathbb{E}_{P(\boldsymbol{x}|z)}\{\boldsymbol{h}(\boldsymbol{x})\} = \int \boldsymbol{h}(\boldsymbol{x})P(\boldsymbol{x}|z)d\boldsymbol{x} \tag{2.25}$$

where $\mathbb{E}\{.\}$ denotes expectation. A numeric way to compute $\mathbb{E}\{h(\boldsymbol{x})\}$ is to draw n_p random samples $\mathbb{X}^{(i)}$, for $(1 \leq i \leq n_p)$, from the probability distribution $P(\boldsymbol{x}|z)$, evaluate the function $h(\boldsymbol{x})$ at these samples, and then compute their statistical mean as follows

$$\mathbb{E}\{\boldsymbol{h}(\boldsymbol{x})\} \approx \sum_{i=1}^{n_p} \boldsymbol{h}(\mathbb{X}^{(i)})P(\mathbb{X}^{(i)}|z). \tag{2.26}$$

In practice, however, the distribution $P(\boldsymbol{x}|z)$ is either unavailable, or, it is difficult to obtain particles from this distribution. Therefore, the particles are instead derived from a proposal distribution $q(\boldsymbol{x}|z)$. Eq. (2.25) can then be written as a function of the proposal distribution as follows

$$\mathbb{E}\{\boldsymbol{h}(\boldsymbol{x})\} = \int \boldsymbol{h}(\boldsymbol{x}) \underbrace{\frac{P(\boldsymbol{x}|z)}{q(\boldsymbol{x}|z)}}_{W} q(\boldsymbol{x}|z) d\boldsymbol{x}, \tag{2.27}$$

where W is called the weight function. Eq. (2.26), therefore, changes to

$$\mathbb{E}\{\boldsymbol{h}(\boldsymbol{x})\} \approx \sum_{i=1}^{n_p} \boldsymbol{h}(\mathbb{X}^{(i)}) W^{(i)} P(\mathbb{X}^{(i)}|z) \tag{2.28}$$

with weights $W^{(i)} = P(\mathbb{X}^{(i)}|z)/q(\mathbb{X}^{(i)}|z)$, for $(1 \leq i \leq n_p)$, associated to the vector particles $\mathbb{X}^{(i)}$.

Given the predictive particles $\mathbb{X}_{k-1}^{(i)}$ from the previous filtering iteration, the values of the particles $\mathbb{X}_k^{(i)}$ at time instant k are updated by generating random particles from the proposal distribution $q(\boldsymbol{x}_{0:\,k}|z_{1:\,k})$. For SIS, the proposal distribution is chosen such that it satisfies the following factorization

$$q\big(\boldsymbol{x}_{0:k}|z_{1:k}\big) = q\big(\boldsymbol{x}_{0:k-1}|z_{1:k-1}\big) q\big(\boldsymbol{x}_k|\boldsymbol{x}_{1:k-1}, z_{1:k}\big), \tag{2.29}$$

then one can obtain particles $\mathbb{X}_{0:\,k}^{(i)} \sim q\big(\boldsymbol{x}_{0:\,k}|z_{1:\,k}\big)$ by augmenting each of the existing samples $\mathbb{X}_{0:\,k-1}^{(i)} \sim q\big(\boldsymbol{x}_{0:\,k-1}|z_{1:\,k-1}\big)$ with the new particles generated as follows

$$\text{Prediction Step:} \qquad \mathbb{X}_k^{(i)} \sim q\big(\boldsymbol{x}_k|\boldsymbol{x}_{0:\,k-1}, z_{1:\,k}\big). \tag{2.30}$$

The next step is to update the weights as follows

$$\text{Observation Update Step:} \qquad W_k^{(i)} \propto W_{k-1}^{(i)} \frac{P\big(z_k|\mathbb{X}_k^{(i)}\big) P\big(\mathbb{X}_k^{(i)}|\mathbb{X}_{k-1}^{(i)}\big)}{q\big(\mathbb{X}_k^{(i)}|\mathbb{X}_{0:k-1}^{(i)}, z_{1:k}\big)}, \tag{2.31}$$

where notation $\propto$ stands for the proportional sign, which changes to an equality with the introduction of a constant. The accuracy of this importance sampling approximation depends on how close the proposal distribution is to the true posterior distribution. The optimal choice [57] for the proposal distribution that minimizes the variance of importance weights is the filtering density conditioned upon $\boldsymbol{x}_{0:k-1}$ and $z_{1:k}$, i.e.,

$$q\big(\boldsymbol{x}_k|\boldsymbol{x}_{0:k-1}, z_{1:k}\big) = P\big(\boldsymbol{x}_k|\boldsymbol{x}_{0:k-1}, z_{1:k}\big). \tag{2.32}$$

Because of the difficulty in sampling Eq. (2.32), a common choice [57] for the proposal distribution is the transition density, $P(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$, referred to as the sampling importance resampling (SIR) filter, where the weights are pointwise evaluation of the likelihood function at the particle values, i.e.,

$$W_k^{(i)} \propto W_{k-1}^{(i)} P\big(z_k|\mathbb{X}_k^{(i)}\big). \tag{2.33}$$

If the weights $W_k^{(i)}$ are all equal from the previous iteration, then $W_k^{(i)} \propto P(z_k|\mathbb{X}_k^{(i)})$. The likelihood function $P(z_k|\mathbb{X}_k^{(i)})$ is derived from the observation equation (Eq. (2.12)). Finally and if required, the overall filtering distribution of the state vector at iteration $k-1$ can be expressed in terms of the particles and their associated weights as

$$P(\boldsymbol{x}_{k-1}|z_{1:k-1}) \approx \sum_{i=1}^{n_p} W_{k-1}^{(i)} \delta\big(\boldsymbol{x}_{k-1} - \mathbb{X}_{k-1}^{(i)}\big), \tag{2.34}$$

where $\delta(\cdot)$ denotes the Dirac delta function. This completes the introduction of the PF.

2.3 Conclusion

In this Chapter, the problem of EBE in CPSs is formulated initially starting with single-sensor linear systems to multi-sensor hierarchical systems and finally to fully distributed systems. The basics of Bayesian estimation approaches were then reviewed as the required background material. The centralized Bayesian estimation framework were introduced together with a brief introduction to Kalman filtering and Particle filtering as two linear and non-linear Bayesian estimators.

Chapter 3

Event-based Particle Filtering: Centralized, Hierarchical, & Distributed

In this chapter and motivated by recent and rapid growth of CPSs and the critical necessity for preserving restricted communication resources, we develop a systematic and intuitively pleasing EBE framework which jointly incorporates point and set-valued measurements within the particle filtering framework. In this chapter, an open-loop event-based topology is considered, where each sensor transfers its measurements to the FC only in the occurrence of specific events (asynchronously). Events are identified using local triggering mechanisms without incorporation of a feedback from the FC and/or implementation of a local filter at the sensor level. First, in Section 3.1, by considering a single-sensor estimation architecture, we develop the event-based particle filter (EBPF) framework. The EBPF incorporates point-valued measurements in the estimation recursion via a conventional particle filter formulation, while set-valued measurements are incorporated by developing an observation update step similar in nature to quantized particle filtering approach. More specifically, in the absence of an observation (i.e., having a set-valued measurement), the proposed EBPF evaluates the probability that the unknown observation belongs to the event-triggering set based on its particles, which is then used to update the corresponding particle weights. The Single-Sensor EBPF is then extended to a hierarchal architecture in Section 3.2, where several remotely operated sensors communicate their measurements to the FC in an event-based fashion. Finally, in Section 3.3 a distributed architecture is considered without incorporation of a FC. For a distributed architecture, an event-triggered diffusive particle filter (ET/DPF) framework is proposed where a modified version of diffusive strategies [58] is utilized to achieve cooperation among distributed agents.

3.1 Single-Sensor Event-based Particle Filtering

In this section, an EBE framework is developed for state estimation problems where a remote sensor communicates its measurements to the FC in an event-based fashion, and the non-linear estimator, resided at the FC, jointly incorporates point and set-valued measurements to estimate the non-Gaussian posterior distribution. To overcome non-Gaussianity of the posterior distribution, some efforts have been recently considered specially by imposing a Gaussian assumption on the posterior distribution, e.g., using single Gaussian approximation [26, 27], Gaussian sum approximation [28], and non-linear filtering scenarios [29]. However, while Gaussian-based approximation of the event-based posterior has been investigated extensively, application of non-Gaussian filtering using particle filters [10–12] is still in its infancy. To the best of our knowledge, only very recently, EBE using non-Gaussian particle filter approximation is considered in [30] and [31], where in the latter simply the number of particles belonging to the triggering set is used to update particle weights, while the former uses stochastic triggering [59] which results in having a Gaussian posterior. This chapter will addresses this gap. In particular, we propose a systematic and intuitively pleasing mechanism to jointly incorporate point and set-valued measurements within the particle filter framework. More specifically, we capitalize on the fact that in particle filtering framework the observations's nature (being point or set-valued) will mainly affect the likelihood function which is used to update each particle's weight. In the presence of an observation (point-valued measurements), the likelihood function can exactly be evaluated for each particle. In the absence of an observation (set-valued measurement case), the probability of the unknown observations belongs to the event-triggering set will be evaluated by the proposed EBPF based on its particles which is then used to update the corresponding particle weights. Intuitively speaking, the proposed EBPF utilizes

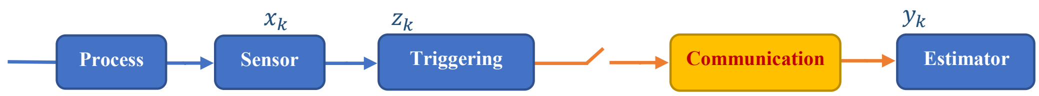


Figure 3.1: Block diagram of the open-loop event-based estimation architecture.

the set-valued information similar in nature to the way that particle filter utilizes quantized observations [60–62]. In the other words, point-valued measurements are incorporated in the estimation via a conventional particle filter while set-valued measurements are incorporated in the state estimates using a filter similar in nature to quantized particle filter.

3.1.1 Problem Formulation

The EBPF framework is developed by considering an estimation problem represented by the linear state-space model given by Eqs. (2.2)-(2.3). Furthermore, a single-sensor estimation architecture (Fig. 3.1) is considered where a sensor communicates its observation z_k to the FC which recursively update the posterior distribution $P(\boldsymbol{x}_k|\boldsymbol{Z}_k)$ based on the collective set of observations $\boldsymbol{Z}_k = \{z_1, \ldots, z_k\}$ received up to and including the current iteration (k). As stated previously, by considering the state-space model given by Eqs. (2.2)-(2.3) and its statistical properties, the posterior follows a Gaussian distribution, i.e. .

$$P(\boldsymbol{x}_k|\boldsymbol{Z}_k) \sim \mathcal{N}(\hat{\boldsymbol{x}}_{k|k}, \boldsymbol{P}_{k|k}) \tag{3.1}$$

$$\text{with} \quad \hat{\boldsymbol{x}}_{k|k} = \mathbb{E}\{\boldsymbol{x}_k|\boldsymbol{Z}_k\}, \tag{3.2}$$

$$\text{and} \quad \boldsymbol{P}_{k|k} = \mathbb{E}\{(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k})(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k}^T)\}. \tag{3.3}$$

The KF (Eqs. (2.20)-(2.24)) provides the optimal solution for this Gaussian case. While the sensor has limited power sources, the FC has adequate power to perform complex estimation algorithms. In the event-based communication/fusion framework, after making each measurement the sensor decides on keeping or sending its observation to the remote estimator. The local decisions are governed by a binary triggering criteria denoted by γ_k which is defined as follows

$$\begin{cases} \gamma_k = 1: & \text{Event occurs, communication is triggered.} \\ \gamma_k = 0: & \text{Idle case, no communication.} \end{cases}$$

Based on the above triggering mechanism, the collective set of observations up to and including iteration k at the FC is defined as $\tilde{\boldsymbol{Z}}_k = \{\gamma_1 z_1, \ldots, \gamma_k z_k\}$. When the event-triggering condition is satisfied (i.e., $\gamma_k = 1$), the exact value of the sensor measurement z_k is known at the FC, referred to as "point-valued observation information". On the other hand, when the event-triggering condition is violated (i.e., $\gamma_k = 0$), some information contained in the event-triggering sets is known to the estimator instead, referred to as "set-valued information". The main issue here comes from the non-Gaussianity of the a posteriori distribution due to joint incorporation of point and set-valued measurements, i.e., $P(\boldsymbol{x}|\tilde{\boldsymbol{Z}}_k)$ is no longer follows a Gaussian distribution. Next, we present the proposed EBPF implementation which systematically uses point and set-valued observation to perform the estimation task.

3.1.2 The EBPF Framework

Without loss of generality and for simplicity of the presentation, we consider the practical "Send-on-Delta (SOD)" triggering criteria/condition [33] for development of the EBPF. In an open-loop scenario, in order to decide whether or not to send new measurements, the sensor computes the distance between its current measurement and the previously transmitted measurement based on the following event-triggering schedule

$$\gamma_k = \begin{cases} 1, & \text{if } |z_k - z_{\tau_k}| \geq \Delta \\ 0, & \text{otherwise,} \end{cases} , \tag{3.4}$$

where τ_k denotes the time of last communication from the sensor to the FC, and Δ denotes the triggering threshold. Based on the above triggering mechanism, we define the hybrid observation vector as $\boldsymbol{Y}_k = \{\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_k\}$ where

$$\boldsymbol{y}_k = \begin{cases} z_k & \text{if } \gamma = 1 \\ \{z_k : z_k \in (z_{\tau_k} - \Delta, z_{\tau_k} + \Delta)\} & \text{if } \gamma = 0 \end{cases}.$$

As stated previously, the posterior distribution $P(\boldsymbol{x}_k|\boldsymbol{Y}_k)$ based on collective set of hybrid observations is no longer Gaussian, eliminating the application of linear filters such as the KF. In such a non-Gaussian scenario, the optimal Bayesian filtering recursion for iteration k is given by

$$P(\boldsymbol{x}_k|\boldsymbol{Y}_k) = \frac{P(\boldsymbol{y}_k|\boldsymbol{x}_k)P(\boldsymbol{x}_k|\boldsymbol{Y}_{k-1})}{P(\boldsymbol{y}_k|\boldsymbol{Y}_{k-1})}, \tag{3.5}$$

where

$$P(\boldsymbol{x}_k|\boldsymbol{Y}_{k-1}) = \int P(\boldsymbol{x}_{k-1}|\boldsymbol{Y}_{k-1}) f(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}) d\boldsymbol{x}_{k-1}. \tag{3.6}$$

In order to compute the non-Gaussian posterior distribution given by Eq. (3.5) jointly based on point and set-valued measurements, we develop the EBPF which approximates the filtering distribution $P(\boldsymbol{x}_k|\boldsymbol{Y}_k)$ using a set of samples (particles) $\{\mathbb{X}_k^{(i)}\}_{i=1}^{n_p}$ derived from a proposal distribution $q(\boldsymbol{x}_k|\boldsymbol{Y}_k)$ with normalized weights $W_k^{(i)} = \frac{P(\mathbb{X}_k^{(i)}|\boldsymbol{Y}_k)}{q(\mathbb{X}_k^{(i)}|\boldsymbol{Y}_k)}$ associated with the vector particles. Note that n_p denotes the number of particles used by the filter. The EBPF implements the filtering recursions by propagating the particles $\mathbb{X}_k^{(i)}$ and associated weights $W_k^{(i)}$, $(1 \leq i \leq n_p)$, as follows

$$\begin{aligned} \mathbb{X}_k^{(i)} &\sim q(\mathbb{X}_k^{(i)}|\mathbb{X}_{k-1}^{(i)}, \boldsymbol{Y}_{k-1}) & (3.7) \\ W_k^{(i)} &\propto W_{k-1}^{(i)} \frac{P(\boldsymbol{y}_k|\mathbb{X}_k^{(i)})P(\mathbb{X}_k^{(i)}|\mathbb{X}_{k-1}^{(i)})}{q(\mathbb{X}_k^{(i)}|\mathbb{X}_{k-1}^{(i)}, \boldsymbol{Y}_k)}. & (3.8) \end{aligned}$$

Consequently, the EBPF computes a particle-based approximation of the conditional posterior $p(\boldsymbol{x}_k|\boldsymbol{Y}_k)$ as follows

$$p(\boldsymbol{x}_k|\boldsymbol{Y}_k) = \sum_{i=1}^{n_p} W_k^{(i)} \delta\big(\boldsymbol{x}_k - \mathbb{X}_k^{(i)}\big). \tag{3.9}$$

The minimum mean square error (MMSE) estimate at iteration ($k \geq 1$) is defined as the expected value of the posterior distribution $P(\boldsymbol{x}_k|\boldsymbol{Y}_k)$, i.e.,

$$\hat{\boldsymbol{x}}_{k|k} \triangleq \mathbb{E}\{\boldsymbol{x}_k|\boldsymbol{Y}_k\}, \tag{3.10}$$

$$\text{and } \boldsymbol{P}_{k|k} \triangleq \mathbb{E}\{(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k})(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k})^T|\boldsymbol{Y}_k\}. \tag{3.11}$$

The EBPF computes the state estimate and its associated covariance matrix based on the particles as follows

$$\hat{\boldsymbol{x}}_{k|k} = \frac{1}{n_p}\sum_{i=1}^{n_p}\mathbb{X}_k^{(i)} \tag{3.12}$$

$$\text{and } \boldsymbol{P}_{k|k} = \frac{1}{n_p}\sum_{i=1}^{n_p}\big(\mathbb{X}_k^{(i)} - \hat{\boldsymbol{x}}_{k|k}\big)\big(\mathbb{X}_k^{(i)} - \hat{\boldsymbol{x}}_{k|k}\big)^T. \tag{3.13}$$

The required terms for computing Eqs. (3.7)-(3.13) at each iteration is the particle set $\{\mathbb{X}_k^{(i)}, W_k^{(i)}\}$ for which we need to define the proposal distribution and form $P(\boldsymbol{y}_k|\mathbb{X}_k^{(i)})$ to compute the weight equation. The EBPF generates n_p random particles from the transitional density, i.e., $\mathbb{X}_k^{(i)} \sim P(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$ which is considered as the conventional choice for the proposal distribution. Choice of the transitional density as the proposal results in the weight update equation (Eq. (3.8)) to become

$$W_k^{(i)} \propto W_{k-1}^{(i)}P(\boldsymbol{y}_k|\mathbb{X}_k^{(i)}). \tag{3.14}$$

The second step to implement the EBPF is to evaluate the weight update equation which depends on whether or not the current sensor measurement has been communicated.

(i) *Update based on Set-valued Measurements ($\gamma_k = 0$)*: In the absence of the sensor measurement and based on the triggering mechanism defined in Eq. (3.4), the estimator has the following side information

$$z_k \in (z_{\tau_k} - \Delta, z_{\tau_k} + \Delta), \tag{3.15}$$

where z_{τ_k} is the previously communicated observation. In this case, the likelihood function can be specified as follows

$$P\big(\boldsymbol{y}_k|\boldsymbol{x}_k, \gamma_k = 0\big) = P\big(z_{\tau_k} - \Delta \leq z_k \leq z_{\tau_k} + \Delta\big), \tag{3.16}$$

which by substituting for z_k from Eq. (2.3), we have

$$\begin{aligned} & P\big(\boldsymbol{y}_k|\boldsymbol{x}_k, \gamma_k = 0\big) \\ = \quad & P\big(z_{\tau_k} - \Delta \leq \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k \leq z_{\tau_k} + \Delta\big) \\ = \quad & P\big(\,[z_{\tau_k} - \Delta - \boldsymbol{H}_k\boldsymbol{x}_k] \leq \boldsymbol{v}_k \leq [z_{\tau_k} + \Delta - \boldsymbol{H}_k\boldsymbol{x}_k]\,\big). \end{aligned} \tag{3.17}$$

Note that in the third line of Eq. (3.17), we kept the noise in the middle and moved other terms to the sides in order to be able to compute the likelihood function based on the probability distribution of the noise. As the observation noise $\boldsymbol{v}_k$ has a zero-mean Gaussian distribution with variance $\boldsymbol{R}_k$, i.e., $z_k \sim \mathcal{N}(0, \boldsymbol{R}_k)$, the likelihood function $P(\boldsymbol{y}_k|\boldsymbol{x}_k, \gamma_k = 0)$ is given

$$\begin{aligned} & P(\boldsymbol{y}_k|\boldsymbol{x}_k, \gamma_k = 0) \\ & = \frac{1}{\sqrt{2\pi R_k}} \int_{z_\tau - \Delta - \boldsymbol{H}_k\boldsymbol{x}_k}^{z_\tau + \Delta - \boldsymbol{H}_k\boldsymbol{x}_k} \exp\left\{\frac{-t^2}{2R_k}\right\} dt \\ & = \underbrace{\Phi\left(\frac{z_\tau + \Delta - \boldsymbol{H}_k\boldsymbol{x}_k}{\sqrt{R_k}}\right) - \Phi\left(\frac{z_\tau - \Delta - \boldsymbol{H}_k\boldsymbol{x}_k}{\sqrt{R_k}}\right)}_{\boldsymbol{h}(\boldsymbol{x}_k)}, \end{aligned} \tag{3.18}$$

where $\Phi(\cdot)$ is the cumulative Gaussian distribution with zero mean and variance 1 as follows

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\big(-\frac{t^2}{2}\big) dt. \tag{3.19}$$

This completes the computation of the likelihood function in idle scenarios (no transmission).

(ii) *Update based on Point Measurements ($\gamma_k = 1$)*: In this case, the estimator receives the sensor measurement z_k, therefore, the hybrid likelihood function $P(\boldsymbol{y}_k|\boldsymbol{x}_k)$ reduces to the sensor

Algorithm 1 *EBPF* IMPLEMENTATION

Input: $\{\mathbb{X}_{k-1}^{(i)}, W_{k-1}^{(i)}\}_{i=1}^{n_p}$, γ_k, and $\boldsymbol{y}_k$.

Output: $\{\mathbb{X}_{k}^{(i)}, W_{k}^{(i)}\}_{i=1}^{n_p}$, $\hat{\boldsymbol{x}}_{k|k}$ and $\boldsymbol{P}_{k|k}$.

EBPF updates its particle set, at iteration k, as follows:

S1. *Predictive Particle Generation*: Sample *predicted particle* from the proposal distribution i.e., $\mathbb{X}_k^{(i)} \sim P(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$.

S2. *Hybrid Likelihood Computation*:
- **If** $\gamma_k = 0$: Compute $P(\boldsymbol{y}_k|\mathbb{X}_k^{(i)})$ using Eq. (3.18).
- **If** $\gamma_k = 1$: Compute $P(\boldsymbol{y}_k|\mathbb{X}_k^{(i)})$ using Eq. (3.20).

S3. *Weight Update*: Compute the weights associated with $\mathbb{X}_k^{(i)}$ using Eq. (3.14).

S4. *State Estimates*: Approximate the state estimate and its corresponding error covariance $\boldsymbol{P}_{k|k}$ from $\{\mathbb{X}_{k}^{(i)}, W_{k}^{(i)}\}_{i=1}^{n_p}$ using Eqs. (3.12)-(3.13).

S5. *Resampling*: In case of degeneracy, particles using the replacement approach [63].

likelihood function $P(z_k|\boldsymbol{x}_k)$. Consequently, the hybrid likelihood function is given by

$$P(\boldsymbol{y}_k|\boldsymbol{x}_k, \gamma_k = 1) = P(z_k|\boldsymbol{x}_k) = \Phi\left(\frac{z_k - \boldsymbol{H}_k\boldsymbol{x}_k}{\sqrt{R_k}}\right). \tag{3.20}$$

This complete the presentation of the proposed EBPF. Algorithm 1 outlines the steps of the EBPF implementation.

3.2 Hierarchical Event-based State Estimation

The section proposes an event-triggered hierarchical state estimation framework (Fig. 3.2), based on an open-loop architecture for multi-sensor systems with restricted local resources. Different from Section 3.1, we focus on multi-sensor scenario with inclusion of a FC where the processing (being the estimation) is only performed at the central node. We consider a multi – sensor estimation topology [9, 10, 12] consisting of ($N > 1$) sensors and a FC which collects the information from all active sensors and performs the estimation task. We propose an efficient way to design the local triggering mechanism without using local/global state estimates via a feedback from the FC or implementing a local KF at the sensor level. We propose to use the information-state contribution from each local observation for joint triggering and estimation update purposes. Intuitively speaking, the information-state contribution is the projection of the observation on to the state space

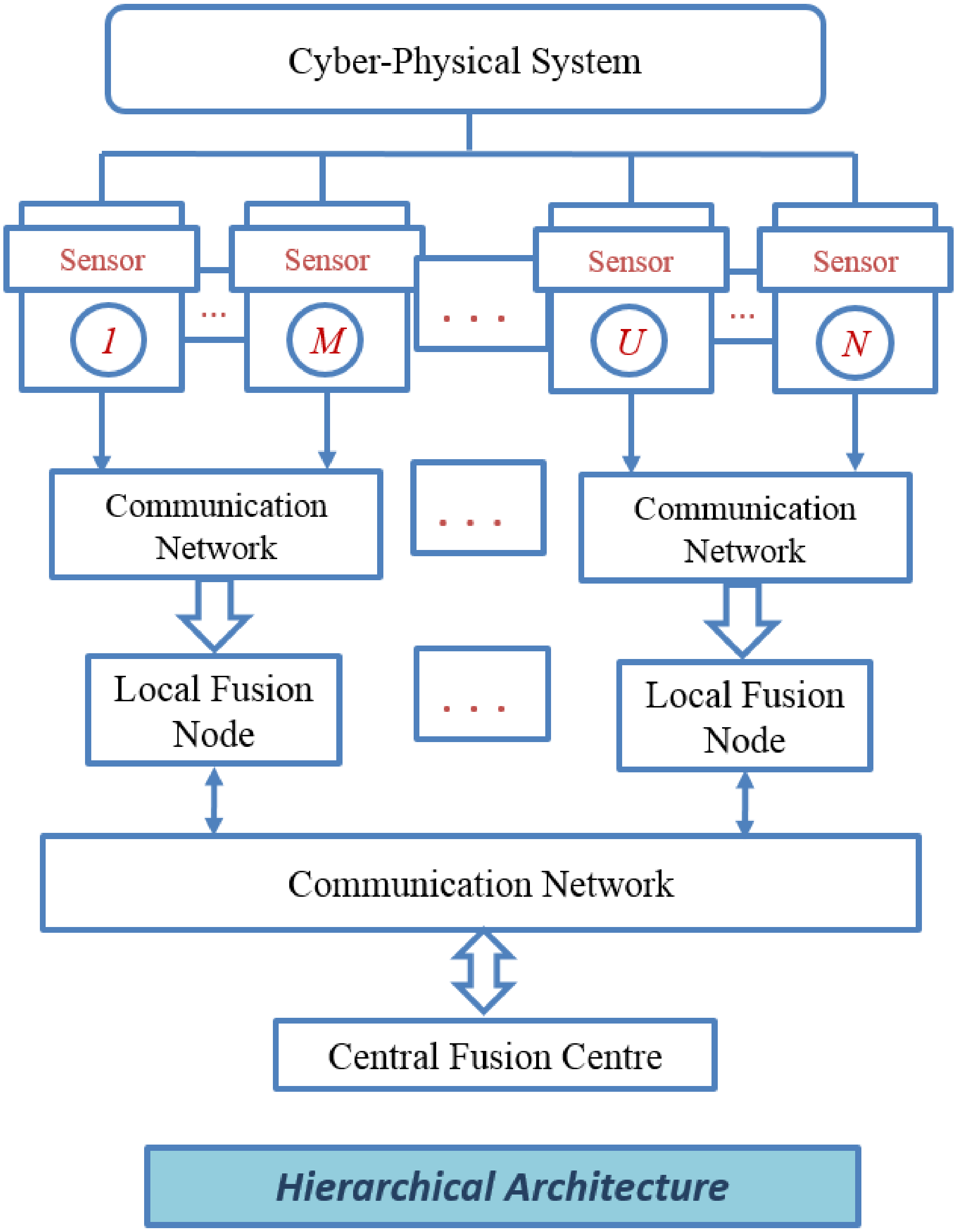


Figure 3.2: Block diagram of the hierarchical architecture.

which includes the effects of the observation model parameters. The information fusion at the FC is designed based on the received information contributions instead of the conventional measurement-based fusion. The estimator uses a modified observation model to directly incorporate/fuse received information contribution of each sensor in an adaptive fashion. The proposed event-triggered estimation algorithm implemented at the FC is capable of fusing multi-sensor measurements in an efficient fashion with reduced communication overhead. Next, we present information-based event-triggering approach.

3.2.1 Information-based Event-Triggering

The proposed multi-sensor and event-triggered estimation framework developed here uses the information-state contribution from a local observation and its associated information matrix for joint triggering and fusion purposes. The proposed framework is developed based on ideas from the information form of the KF, also referred to as inverse-covariance filter [64], which propagates the information state vector, $\boldsymbol{P}_{k|k}^{-1}\hat{\boldsymbol{x}}_{k|k}$, and the Fisher information matrix, $\boldsymbol{P}_{k|k}^{-1}$, instead of the state estimate $\hat{\boldsymbol{x}}_{k|k}$ and its corresponding error covariance matrix $\boldsymbol{P}_{k|k}$. Information form of the KF reduces the computational complexity of update step of the KF especially for multi-sensor systems. The update equations for the information filter are given by

$$\boldsymbol{Y}_{k|k} \triangleq \boldsymbol{P}_{k|k}^{-1} = \boldsymbol{P}_{k|k-1}^{-1} + \boldsymbol{I}_k \tag{3.21}$$

$$\hat{\boldsymbol{y}}_{k|k} \triangleq \boldsymbol{P}_{k|k}^{-1}\hat{\boldsymbol{x}}_{k|k} = \boldsymbol{P}_{k|k-1}^{-1}\hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{i}_k, \tag{3.22}$$

where, each observation contributes $\boldsymbol{i}_k^{(l)}$ to the information state and $\boldsymbol{I}_k^{(l)}$ to the Fisher matrix, respectively, defined as

$$\boldsymbol{i}_k^{(l)} = [\boldsymbol{H}_k^{(l)}]^T[\boldsymbol{R}_k^{(l)}]^{-1}z_k^{(l)} \tag{3.23}$$

$$\text{and } \boldsymbol{I}_k^{(l)} = [\boldsymbol{H}_k^{(l)}]^T[\boldsymbol{R}_k^{(l)}]^{-1}\boldsymbol{H}_k^{(l)}. \tag{3.24}$$

The prediction equations of the information filter are expressed in terms of $\hat{\boldsymbol{y}}_{k|k-1} \triangleq \boldsymbol{P}_{k|k-1}^{-1}\hat{\boldsymbol{x}}_{k|k-1}$ and $\boldsymbol{Y}_{k|k-1} \triangleq \boldsymbol{P}_{k|k-1}^{-1}$ as follows

$$\hat{\boldsymbol{y}}_{k|k-1} = (\boldsymbol{I}_{n_x \times n_x} - \boldsymbol{\Omega}_k)\boldsymbol{F}_k^{-T}\hat{\boldsymbol{y}}_{k-1|k-1} \tag{3.25}$$

$$\boldsymbol{Y}_{k|k-1} = \boldsymbol{M}_k - \boldsymbol{\Omega}_k\boldsymbol{\Sigma}_k\boldsymbol{\Omega}_k^T, \tag{3.26}$$

with $\boldsymbol{I}_{n_x \times n_x}$ denote a $(n_x \times n_x)$ identity matrix,

$$\boldsymbol{M}_k = \boldsymbol{F}_k^{-T}\boldsymbol{Y}_{k-1|k-1}\boldsymbol{F}_k^{-1}, \tag{3.27}$$

$$\boldsymbol{\Sigma}_k = \boldsymbol{M}_k + \boldsymbol{Q}^{-1}, \tag{3.28}$$

$$\text{and } \boldsymbol{\Omega}_k = \boldsymbol{M}_k\boldsymbol{\Sigma}_k^{-1}. \tag{3.29}$$

The main advantage of the information filter over the KF is the relative simplicity of its update stage for multi-sensor architectures. For N-sensor network, Eqs. (3.21)-(3.22) are reduced to

$$\hat{\boldsymbol{y}}_{k|k} = \hat{\boldsymbol{y}}_{k|k-1} + \sum_{l=1}^{N}[\boldsymbol{H}_k^{(l)}]^T \boldsymbol{R}_k^{(l)^{-1}} z_k^{(l)}, \tag{3.30}$$

$$\text{and} \quad \boldsymbol{Y}_{k|k} = \boldsymbol{Y}_{k|k-1} + \sum_{l=1}^{N}\boldsymbol{H}_k^{(l)}\boldsymbol{R}_k^{(l)^{-1}}[\boldsymbol{H}_k^{(l)}]^T, \tag{3.31}$$

In other words, the global information vector $\boldsymbol{i}_k$ and its associated information matrix $\boldsymbol{I}_k$ can be expressed in terms of their localized counterparts as

$$\boldsymbol{i}_k = \sum_{l=1}^{N}\boldsymbol{i}_k^{(l)} \tag{3.32}$$

$$\text{and } \boldsymbol{I}_k = \sum_{l=1}^{N}\boldsymbol{I}_k^{(l)}. \tag{3.33}$$

From theoretical point of view for a linear Gaussian system, both the covariance-based KF and information-based KF provide the optimum estimator but from implementation point of view they are not the same. We design the information-based triggering mechanism based on the above quantities as $\boldsymbol{i}_k$ represents the new information content of a local measurement, while $\boldsymbol{I}_k$ provides the expected information gain from making an observation based on a given sensor model.

Based on the above discussion, we define a modified form of local observation as follows

$$\tilde{z}_k^{(l)} = \boldsymbol{I}_k^{(l)}\boldsymbol{x}_k + [\boldsymbol{H}_k^{(l)}]^T[\boldsymbol{R}_k^{(l)}]^{-1}\boldsymbol{v}_k^{(l)}, \tag{3.34}$$

where the equality is obtained by substituting $z_k^{(l)}$ from the observation model. The transformed observation noise $\tilde{\boldsymbol{v}}_k^{(l)}$ is zero mean with its covariance matrix equal to the local information matrix $\boldsymbol{I}_k^{(l)}$, i.e.,

$$\tilde{\boldsymbol{v}}_k^{(l)} \sim \mathcal{N}\big(0, \boldsymbol{I}_k^{(l)}\big). \tag{3.35}$$

We note that similar to the information filter, communicating the information-form of the observations $\tilde{z}_k$ results in a simplified multi-sensor fusion at the remote estimator, i.e., the above fusion can be extended to multi-sensor ($N > 1$) scenario by using the following replacements

$$\tilde{z}_k = \sum_{l=1}^{N} \tilde{z}_k^{(l)} \tag{3.36}$$

$$\text{and } \boldsymbol{I}_k = \sum_{l=1}^{N} \boldsymbol{I}_k^{(l)}, \tag{3.37}$$

where $\tilde{z}_k^{(l)}$ and $\boldsymbol{I}_k^{(l)}$ are, respectively, the local information vector and information matrix corresponding to sensor l, for $(1 \leq l \leq N)$. After receiving new information $\tilde{z}_k^{(l)}$ form a local sensor, the remote estimator only needs to sum it up with the previously received modified observations.

Based on the above developments, we design the multi-sensor triggering criteria-condition for an open-loop scenario based on the above information-based modified observations. Because of using the information form of local observations, neither a feedback from the remote estimator nor a local KF co-located with the sensor is required in order to compute the triggering condition. In an open-loop scenario, Sensor l, for $(1 \leq l \leq N)$, computes the information-based observation $\tilde{z}_k^{(l)} \in \mathbb{R}^{n_x}$ (which is the local information state vector $\boldsymbol{i}_k^{(l)}$), and the transformed observation model $\boldsymbol{I}_k^{(l)} \in \mathbb{R}^{n_z \times n_z}$ (which is the information matrix $\boldsymbol{I}_k^{(l)}$ and also is the covariance matrix of the transformed measurement noise $\tilde{\boldsymbol{v}}_k^{(l)}$). Based on the information form of local observation and following [59], we consider the following stochastic event-triggering scheduler

$$\gamma_k^{(l)} = \begin{cases} 0, & \xi_k^{(l)} \leq \varphi(z_k^{(l)}, \boldsymbol{R}_k^{(l)}, \boldsymbol{H}_k^{(l)}) \\ 1, & \xi_k^{(l)} > \varphi(z_k^{(l)}, \boldsymbol{R}_k^{(l)}, \boldsymbol{H}_k^{(l)}), \end{cases}$$

where $\xi_k^{(l)}$ is uniformly distributed between $[0, 1]$, and function $\varphi(z_k^{(l)}, \boldsymbol{R}_k^{(l)}, \boldsymbol{H}_k^{(l)})$ is defined as

$$\varphi(z_k^{(l)}, \boldsymbol{R}_k^{(l)}, \boldsymbol{H}_k^{(l)}) \triangleq \exp\Big(-\frac{1}{2}[\tilde{z}_k^{(l)}]^T \boldsymbol{\mathcal{Y}}^{(l)} \tilde{z}_k^{(l)}\Big), \tag{3.38}$$

where matrices $\boldsymbol{\mathcal{Y}}^{(l)}$ is pre-defined [59]. The reason for using stochastic triggering at this stage is that it keeps the Gaussian property of the posterior distribution. Based on the above triggering scheduler, the FC updates its estimates as follows

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k \left[\sum_{l=1}^{N} \Big\{\gamma_k^{(l)} \tilde{z}_k^{(l)} - \big((1-\gamma_k^{(l)})\boldsymbol{I}^{(l)}(t) + \gamma_k^{(l)} \boldsymbol{I}_k^{(l)}\big)\hat{\boldsymbol{x}}_{k|k-1}\Big\}\right] \tag{3.39}$$

$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1}\Big(\sum_{l=1}^{N}\{(1-\gamma_k^{(l)})[\boldsymbol{I}^{(l)}(t)]^T + \gamma_k^{(l)}[\boldsymbol{I}_k^{(l)}]^T\}\Big)\boldsymbol{S}_k^{-1} \tag{3.40}$$

$$\boldsymbol{P}_{k|k} = \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_k\Big(\sum_{l=1}^{N}\{(1-\gamma_k^{(l)})[\boldsymbol{I}^{(l)}(t)]^T + \gamma_k^{(l)}[\boldsymbol{I}_k^{(l)}]^T\}\Big)\boldsymbol{P}_{k|k-1}. \tag{3.41}$$

$$\begin{aligned}\boldsymbol{S}_k = {} & \Big(\sum_{l=1}^{N}\{(1-\gamma_k^{(l)})\boldsymbol{I}^{(l)}(t) + \gamma_k^{(l)}\boldsymbol{I}_k^{(l)}\}\Big)\boldsymbol{P}_{k|k-1}\Big(\sum_{l=1}^{N}\{(1-\gamma_k^{(l)})[\boldsymbol{I}^{(l)}(t)]^T + \gamma_k^{(l)}[\boldsymbol{I}_k^{(l)}]^T\}\Big) \\ & + \sum_{l=1}^{N}\Big\{(1-\gamma_k^{(l)})[\boldsymbol{I}^{(l)}(t)]^T + \gamma_k^{(l)}[\boldsymbol{I}_k^{(l)}]^T + (1-\gamma_k^{(l)})[\boldsymbol{\mathcal{Y}}^{(l)}]^{-1}\Big\}.\end{aligned} \tag{3.42}$$

Note that, term $\boldsymbol{I}^{(l)}(t)$ is the previously known value of the information matrix corresponding to the latest event iteration ($t < k$), i.e., $\gamma_t^{(l)} = 1$. This completes the presentation of the hierarchical EBE algorithm.

3.3 Distributed Event-based Particle Filtering

In this section, we couple the EBE frameworks developed in Sections 3.1 and 3.2 and propose an event-triggered distributed state estimation via diffusion strategies (ET/DPF) without inclusion of a FC. Developments of this section is motivated by recent advancements and developments in large, distributed, autonomous, and self-aware systems such as autonomous vehicles and vehicle-to-everything (V2X) technologies, where bandwidth, security, privacy, and/or power considerations limit the number of information transfers between neighbouring agents. In the ET/DPF framework and in the absence of a measurement form a neighbouring node (i.e., having a set-valued measurement), each local agent/node evaluates the probability that the unknown measurement belongs to the event-triggering set based on its particles which is then used to update the corresponding particle weights. More specifically, in the proposed ET/DPF framework, each agent communicates its sensor measurements only to its neighbouring nodes (no long distance or broadcast communication) without inclusion of a FC, and only in an ET fashion (Fig. 2.2). Diffusion strategies [11, 65] are used to fuse the ET information in a distributed fashion as these strategies are robust to changes in the underlying network topology and outperform [58] consensus approaches for distributed estimation in autonomous AN/SN systems. As stated previously, when the event-triggering condition is satisfied (i.e., $\gamma_k^{(l)} = 1$), the exact value of the sensor measurement z_k is known at all its neighbouring nodes, referred to as "point-valued observation information". On the other hand, when the ET condition is violated (i.e., $\gamma_k^{(l)} = 0$), some information contained in the ET sets is known to the neighbouring nodes instead, referred to as "set-valued information". The main issue here comes from the non-Gaussianity of the a posteriori distribution due to joint incorporation of point and set-valued measurements, i.e., the posterior distribution no longer follows a Gaussian distribution. Next, we present the proposed ET/DPF implementation which systematically uses point and set-valued observation to approximate this non-Gaussian ET posterior.

3.3.1 The ET/DPF Framework

In the proposed ET/DPF, each agent implements a localized filter to compute an intermediate local estimate based on the ET measurements limited to its immediate neighbourhood. Local agents then cooperate distributively in an ET fashion to improve the accuracy of their intermediate localized state estimates. Below, we explain these steps in more details.

Local Filtering Step

The local filter at Node l computes an intermediate state estimate of the entire state vector $\boldsymbol{x}_k$ by running one localized Gaussian particle filter. In computing the localized state estimates, communication is limited to the local neighbourhoods and ET measurements. Similar to Section 3.2, SOD triggering criteria/condition is used. In order to decide whether or not to send new measurements, Sensor l, for $(1 \leq l \leq N)$, computes the distance between its current local measurement and the previously transmitted measurement based on the following ET schedule

$$\gamma_k^{(l)} = \begin{cases} 1, & \text{if } |z_k^{(l)} - z_{\tau_k}^{(l)}| \geq \Delta^{(l)} \\ 0, & \text{otherwise,} \end{cases} , \tag{3.43}$$

where $\tau_k^{(l)}$ denotes the time of last communication from sensor l, and $\Delta^{(l)}$ denotes its local triggering threshold. Based on the above triggering mechanism, we define the following local hybrid observation

$$\tilde{z}_k^{(l)} = \begin{cases} z_k^{(l)} & \text{if } \gamma^{(l)} = 1 \\ \{z_k^{(l)} : z_k^{(l)} \in (z_{\tau_k^{(l)}}^{(l)} - \Delta^{(l)}, z_{\tau_k^{(l)}}^{(l)} + \Delta^{(l)})\} & \text{if } \gamma^{(l)} = 0 \end{cases}$$

The collective set of ET measurements at node l is denoted by

$$\boldsymbol{y}_k^{(l)} = \{\tilde{z}_k^{(i)} : i \in \aleph^{(l)}\}, \tag{3.44}$$

and over time defined as $\boldsymbol{Y}_k^{(l)} = \{\boldsymbol{y}_1^{(l)}, \ldots, \boldsymbol{y}_k^{(l)}\}$. The local posterior distribution $P(\boldsymbol{x}_k|\boldsymbol{Y}_k^{(l)})$ based on collective set of hybrid observations is no longer Gaussian, eliminating the application of linear filters such as the KF. In such a non-Gaussian scenario, the optimal Bayesian filtering recursion based on local information is given by

$$P(\boldsymbol{x}_k|\boldsymbol{Y}_k^{(l)}) = \frac{P(\boldsymbol{y}_k|\boldsymbol{x}_k)P(\boldsymbol{x}_k|\boldsymbol{Y}_{k-1}^{(l)})}{P(\boldsymbol{y}_k|\boldsymbol{Y}_{k-1}^{(l)})}, \tag{3.45}$$

$$P(\boldsymbol{x}_k|\boldsymbol{Y}_{k-1}^{(l)}) = \int P(\boldsymbol{x}_{k-1}|\boldsymbol{Y}_{k-1}^{(l)}) f(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}) d\boldsymbol{x}_{k-1}. \tag{3.46}$$

In order to compute the non-Gaussian posterior distribution given by Eq. (3.45) jointly based on point and set-valued measurements, each localized filter approximates the filtering distribution $P(\boldsymbol{x}_k|\boldsymbol{Y}_k^{(l)})$ using a set of particles $\{\mathbb{X}_{k_{\boldsymbol{i}}}^{(l)}\}_{i=1}^{n_p}$ derived from a proposal distribution $q(\boldsymbol{x}_k|\boldsymbol{Y}_k^{(l)})$, and computes their associated weights $W_{k_i}^{(l)}$. The ET/DPF implements the filtering recursions by propagating the particles $\mathbb{X}_{k_i}^{(l)}$ and associated weights $W_{k_i}^{(l)}$, ($1 \leq i \leq n_p$), as

$$\mathbb{X}_{k_i}^{(l)} \sim q(\mathbb{X}_{k_i}^{(l)}|\mathbb{X}_{k-1_i}^{(l)}, \boldsymbol{Y}_k^{(l)}) \tag{3.47}$$

$$W_{k_i}^{(l)} \propto W_{k-1_i}^{(l)} \frac{P(\boldsymbol{y}_k^{(l)}|\mathbb{X}_{k_i}^{(l)})P(\mathbb{X}_{k_i}^{(l)}|\mathbb{X}_{k-1_i}^{(l)})}{q(\mathbb{X}_{k_i}^{(l)}|\mathbb{X}_{k-1_i}^{(l)}, \boldsymbol{Y}_k^{(l)})}. \tag{3.48}$$

Consequently, the local filter at Agent l computes a particle-based approximation of the local ET conditional posterior as follows

$$P(\boldsymbol{x}_k|\boldsymbol{Y}_k^{(l)}) = \sum_{i=1}^{n_p} W_{k_i}^{(l)} \delta\big(\boldsymbol{x}_k - \mathbb{X}_{k_i}^{(l)}\big). \tag{3.49}$$

The local intermediate state estimate denoted by $\boldsymbol{\psi}_k^{(l)}$ at iteration (k) is defined as the expected value of the posterior distribution, i.e.,

$$\boldsymbol{\psi}_k^{(l)} = \mathbb{E}\{\boldsymbol{x}_k|\boldsymbol{Y}_k^{(l)}\} = \int \boldsymbol{x}_k p(\boldsymbol{x}_k|\boldsymbol{Y}_k^{(l)}) \boldsymbol{d}\boldsymbol{x}_k \approx \sum_{i=1}^{n_p} W_{k_i}^{(l)} \mathbb{X}_{k_i}^{(l)}. \tag{3.50}$$

Node l fuses its local intermediate state estimate $\boldsymbol{\psi}_k^{(l)}$ with those of its neighbouring nodes using diffusive strategies to form its updated local state estimate, denoted by $\hat{\boldsymbol{x}}_k^{(l)}$. Assume all local filters are at steady-state at the end of iteration $(k-1)$, i.e., node l, has computed $\hat{\boldsymbol{x}}_k^{(l)}$ and its corresponding error covariance $\boldsymbol{P}_k^{(l)}$. At iteration k, the local filtering step is then completed at each node l, ($1 \leq l \leq N$) based on the following sub-steps:

Sub-Step L1. *Observation Collection*: Node l collects observations made in its neighbourhood to form $\boldsymbol{y}_k^{(l)}$, i.e., the collection of ET measurements available in the local neighbourhood $\aleph^{(l)}$ of node l.

Sub-Step L2. *Local State Estimation*: Node l computes the local state estimate $\boldsymbol{\psi}_k^{(l)}$ by generating n_p particles from the transitional density $p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$ and computes the mean $\bar{\boldsymbol{\mu}}_k^{(l)}$ and covariance $\bar{\boldsymbol{\Sigma}}_k^{(l)}$ of its predictive particles as $\bar{\boldsymbol{\mu}}_k^{(l)} = 1/n_p \sum_{i=1}^{n_p} \mathbb{X}_{k_i}^{(l)}$ and $\bar{\boldsymbol{\Sigma}}_k^{(l)} = 1/n_p \sum_{i=1}^{n_p} \big(\bar{\boldsymbol{\mu}}_k^{(l)} - \mathbb{X}_{k_i}^{(l)}\big)\big(\bar{\boldsymbol{\mu}}_k^{(l)} - \mathbb{X}_{k_i}^{(l)}\big)^T$. Node l then updates the corresponding weights of its predictive particles as follows

$$\tilde{W}^{(l)}_{k_i} = \frac{p\big(\boldsymbol{y}^{(l)}_k|\mathbb{X}^{(l)}_{k_i}\big)\overbrace{\mathcal{N}[\mathbb{X}^{(l)}_{k_i};\bar{\boldsymbol{\mu}}^{(l)}_k,\bar{\boldsymbol{\Sigma}}^{(l)}_k]}^{p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})}}{\pi\big(\mathbb{X}^{(l)}_{k_i}|\boldsymbol{Y}^{(l)}_k\big)}, \tag{3.51}$$

and normalize them as $W^{(l)}_{k_i} = \tilde{W}^{(l)}_{k_i} / \sum_{i=1}^{n_p}\tilde{W}^{(l)}_{k_i}$. In Eq. (3.51), $\mathcal{N}[\cdot]$ denotes the Gaussian distribution with mean and covariance specified within its parenthesis. Further, agent l updates its local intermediate state estimate and its corresponding covariance as

$$\boldsymbol{\psi}^{(l)}_k = \sum_{i=1}^{n_p} W^{(l)}_{k_i}\mathbb{X}^{(l)}_{k_i} \tag{3.52}$$

$$\text{and } \boldsymbol{P}^{(l)}_k = \sum_{i=1}^{n_p} W^{(l)}_{k_i}\big(\boldsymbol{\psi}^{(l)}_k - \mathbb{X}^{(l)}_{k_i}\big)\big(\boldsymbol{\psi}^{(l)}_k - \mathbb{X}^{(l)}_{k_i}\big)^T. \tag{3.53}$$

Consequently, the localized filtering density at node l is approximated with a single Gaussian as follows

$$P(\boldsymbol{x}_k|\boldsymbol{Y}^{(l)}_k) = \mathcal{N}\big(\boldsymbol{x}_k;\boldsymbol{\psi}^{(l)}_k,\boldsymbol{P}^{(l)}_k\big). \tag{3.54}$$

The final step to implement localized filters within the ET/DPF framework is to evaluate the ET likelihood, $P\big(\boldsymbol{y}^{(l)}_k|\mathbb{X}^{(l)}_{k_i}\big)$. For this purpose, we make the common assumption that measurements are uncorrelated, i.e.,

$$P(\boldsymbol{y}^{(l)}_k|\boldsymbol{x}_k) = \prod_{j\in\aleph^{(l)}} P(y^{(j)}_k|\boldsymbol{x}_k). \tag{3.55}$$

Therefore, Agent l computes the likelihood function for each of its neighbouring nodes based on one of the following two approaches.

(i) *Update based on Set-valued Measurements ($\gamma^{(j)}_k = 0$)*: In the absence of the sensor measurement from agent $j \in \aleph^{(l)}$, and based on the triggering mechanism defined in Eq. (3.4), the estimator at Node l has the following side information

$$z^{(j)}_k \in (z^{(j)}_{\tau_k} - \Delta^{(j)}, z^{(j)}_{\tau_k} + \Delta^{(j)}), \tag{3.56}$$

where $z_{\tau_k}^{(j)}$ is the previously communicated observation from node j. In this case, the likelihood function can be specified as follows

$$P\big(y_k^{(j)}|\boldsymbol{x}_k, \gamma_k^{(j)} = 0\big) = P\big(z_{\tau_k}^{(j)} - \Delta^{(j)} \leq z_k^{(j)} \leq z_{\tau_k}^{(j)} + \Delta^{(j)}\big), \tag{3.57}$$

which by substituting from Eq. (2.3), we have

$$\begin{aligned} &P\big(y_k^{(j)}|\boldsymbol{x}_k, \gamma_k^{(j)} = 0\big) \\ &\quad = P\big(z_{\tau_k}^{(j)} - \Delta^{(j)} \leq \boldsymbol{h}_k^{(j)^T}\boldsymbol{x}_k + v_k^{(j)} \leq z_{\tau_k}^{(j)} + \Delta^{(j)}\big) \\ &\quad = P\Big(\Big[z_{\tau_k}^{(j)} - \Delta^{(j)} - \boldsymbol{h}_k^{(j)^T}\boldsymbol{x}_k\Big] \leq v_k^{(j)} \leq \Big[z_{\tau_k}^{(j)} + \Delta^{(j)} - \boldsymbol{h}_k^{(j)^T}\boldsymbol{x}_k\Big] \Big). \end{aligned} \tag{3.58}$$

Note that in the third line of Eq. (3.58), we kept the noise in the middle and moved other terms to the sides in order to be able to compute the likelihood function based on the probability distribution of the noise. As the observation noise $v_k^{(j)}$ has a zero-mean Gaussian distribution with variance $R_k^{(j)}$, the likelihood function reduces to

$$\begin{aligned} &P(y_k^{(j)}|\boldsymbol{x}_k, \gamma_k^{(j)} = 0) \\ &\quad = \frac{1}{\sqrt{2\pi R_k^{(j)}}} \int_{z_\tau^{(j)} - \Delta^{(j)} - \boldsymbol{h}_k^{(j)^T}\boldsymbol{x}_k}^{z_\tau^{(j)} + \Delta^{(j)} - \boldsymbol{h}_k^{(j)^T}\boldsymbol{x}_k} \exp\left\{\frac{-t^2}{2R_k^{(j)}}\right\} dt \\ &\quad = \underbrace{\Phi\left(\frac{z_\tau^{(j)} + \Delta^{(j)} - \boldsymbol{h}_k^{(j)^T}\boldsymbol{x}_k}{\sqrt{R_k^{(j)}}}\right) - \Phi\left(\frac{z_\tau^{(j)} - \Delta^{(j)} - \boldsymbol{h}_k^{(j)^T}\boldsymbol{x}_k}{\sqrt{R_k^{(j)}}}\right)}_{\boldsymbol{h}^{(j)^T}(\boldsymbol{x}_k)}, \end{aligned} \tag{3.59}$$

where $\Phi(\cdot)$ is the cumulative Gaussian distribution with zero mean and variance 1. This completes the computation of the likelihood function in idle scenarios (no transmission).

(ii) *Update based on Point Measurements* ($\gamma_k^{(j)} = 1$): In this case, the estimator receives the sensor measurement $z_k^{(j)}$, therefore, the hybrid likelihood function $P(y_k^{(j)}|\boldsymbol{x}_k)$ reduces to the sensor likelihood function $P(z_k^{(j)}|\boldsymbol{x}_k)$. Consequently, the hybrid likelihood function is

$$P(y_k^{(j)}|\boldsymbol{x}_k, \gamma_k^{(j)} = 1) = P(z_k^{(j)}|\boldsymbol{x}_k) = \Phi\left(\frac{z_k^{(j)} - \boldsymbol{h}_k^{(j)^T}\boldsymbol{x}_k}{\sqrt{R_k^{(j)}}}\right). \tag{3.60}$$

This complete the presentation of the localized filters of the ET/DPF. Next, we present the diffusive fusion step where each node updates its local state estimates by collaborating with its neighbouring nodes.

Diffusion Step

The second step is based on local collaboration, where Node l, $(1 \leq l \leq N)$, fuses its local intermediate estimate $\boldsymbol{\psi}_k^{(l)}$ with that of its neighbouring nodes as follows

$$\hat{\boldsymbol{x}}_k^{(l)} = \sum_{j \in \aleph^{(l)}} \underbrace{\gamma_k^{(j)} \times \alpha_k^{(j,l)}}_{\beta_k^{(j,l)}} \times \boldsymbol{\psi}_k^{(j)}, \tag{3.61}$$

such that if we collect the nonnegative weights $\beta_k^{(j,i)}$ into a $N \times N$ matrix $\boldsymbol{A}_k$, the weights $\beta_k^{(j,l)}$ satisfy the following properties: (i) $\beta_k^{(j,l)} \geq 0$; (ii) $\boldsymbol{A}_k^T \mathbf{1} = \mathbf{1}$, and; (iii) $\beta_k^{(j,l)} = 0$ if $j \notin \aleph^{(l)}$ or $\gamma_k^{(j)} = 0$. Term $\mathbf{1}$ is a vector of size N with all entries equal to one. These conditions imply that the weights on the links arriving at a single node add up to one, which is equivalent to saying that the matrix is left-stochastic. Moreover, if two nodes are not connected or an event is not triggered, then their corresponding entry is zero. The ET diffusive matrix $\boldsymbol{A}_k$ can be designed using covariance intersection [66] or updated adaptively as explained in [67]. A simple approach for choosing the ET diffusion matrix is to assign a weigh to each node according to the cardinality of its neighbourhood by considering the triggering variables. Through diffusive fusion, the filter implemented at node l, $(1 \leq l \leq N)$, forms a Gaussian approximation of the posterior distribution as

$$p(\boldsymbol{x}_k|z_k) = \mathcal{N}\big(\boldsymbol{x}_k; \hat{\boldsymbol{x}}_k^{(l)}, \boldsymbol{P}_k^{(l)}\big). \tag{3.62}$$

Note that, $\boldsymbol{P}_k^{(l)}$ in Eq. (3.62) is not a representative of the covariance of the diffusive state estimate $\hat{\boldsymbol{x}}_k^{(l)}$ as the diffusion update is not performed on the covariance matrices.

3.4 Simulations

In this section, simulation experiments are developed to evaluate the performance of different event-based estimation frameworks.

3.4.1 Evaluation of the Single-Sensor EBPF

In this sub-section, simulation experiments are developed to evaluate the performance of the proposed EBPF (Section 3.1). Following the recent literature on event-based estimation [50], a target tracking problem is considered where observations from a sensor are used to sequentially estimate the state of the target denoted by $\boldsymbol{x}_k$ consisting of its position and speed. Target's dynamic is given by

$$\boldsymbol{x}_k = \begin{bmatrix} 0.8 & 1 \\ 0 & 0.95 \end{bmatrix} \boldsymbol{x}_{k-1} + \boldsymbol{w}_k, \tag{3.63}$$

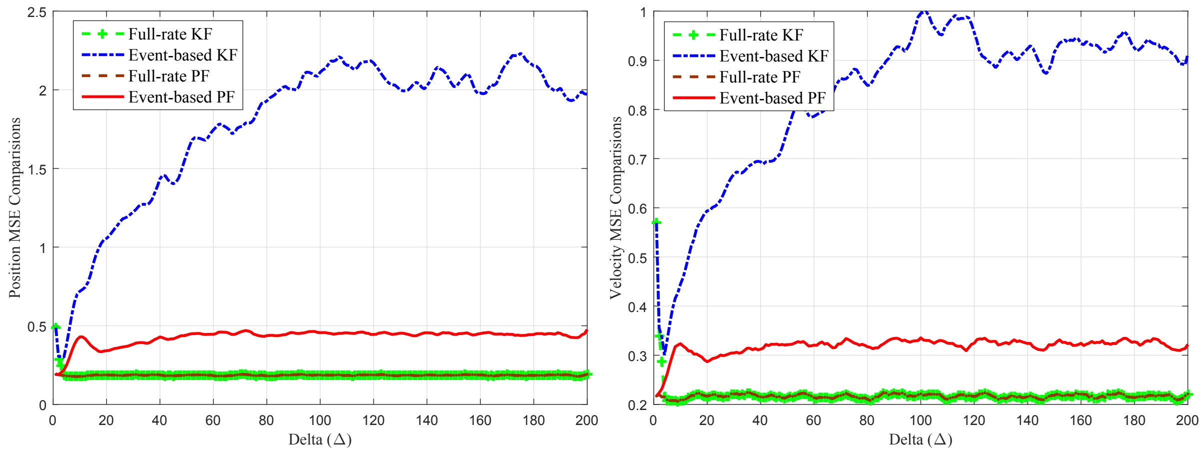
where

$$\boldsymbol{w}_k \sim \mathcal{N}\left(\boldsymbol{0}, \boldsymbol{Q} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}\right). \tag{3.64}$$

The sensor periodically measures the position and speed of the target based on the following observation model

$$z_k = \begin{bmatrix} 0.7 & 0.6 \end{bmatrix} \boldsymbol{x}_k + v_k. \tag{3.65}$$

In this experiment, the observation noise variance is $\sigma_v^2 = 0.01$. The following results are computed over Monte-Carlo (MC) simulations of 1000 runs. The object's position and speed used in each simulation run changes randomly to provide a fair experimental benchmark.



(a) (b)

Figure 3.3: The MSE comparison when $\Delta = 1.2$. (a) Position MSE. (b) Velocity MSE.

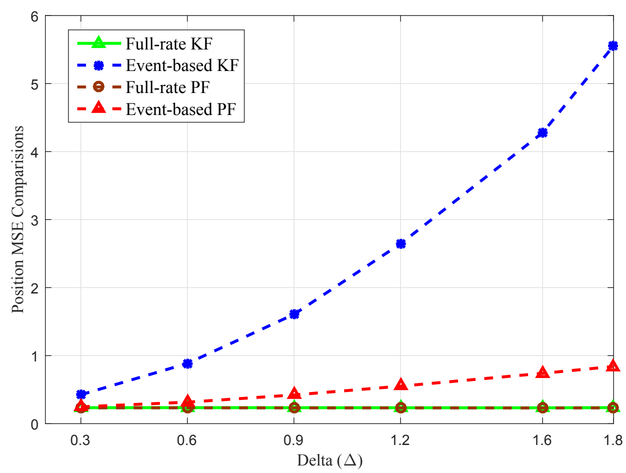


Figure 3.4: Position MSE comparison over different values of Δ.

Furthermore, the following four estimators are implemented and compared for accuracy: (i) The full-rate estimation based on KF where the sensor communicates its observation to the remote estimator every iteration; (ii) The full-rate estimation based on particle filter; (iii) Open-loop and event-based KF, where SOD triggering is used, and; (iii) The proposed open-loop and event-based estimation algorithm, where the triggering decisions at the sensor level are made based on SOD mechanism and the fusion is performed by jointly incorporating set-valued and point-valued measurements based on the proposed EBPF.

Fig. 3.3, illustrates the estimated mean-square errors (MSE) obtained from the four implemented filters. In this experiment, the value of Δ is set equal to 1.2. In the low communication rate scenario, it is observed that the proposed EBPF algorithm provides acceptable results and closely follows its full-rate counterparts and shows significant improvements in comparison to its KF-based counterpart. Fig. 3.4, shows the position MSE plots over varying values of Δ which in turn results in varying values of the communication rate. It is observed that the proposed EBPF algorithm provides acceptable results in very low communication rates (high values of Δ) and closely follows its full-rate counterparts in high communication rates. Besides, when the communication rate increases (i.e., small values for Δ), the proposed event-based methodology approaches the full-rate estimator. Finally, it is observed that the proposed EBPF provides significantly superior results in comparison to its KF-based counterpart.

3.4.2 Evaluation of the Hierarchical EBE

In this sub-section, simulation experiments are developed to evaluate the performance of the proposed multi-sensor and information-based triggering mechanism (Section 3.2). Following the recent literature on event-based estimation, a target tracking problem is considered where $N = 20$ sensors are used to sequentially estimate the state of the target denoted by $\boldsymbol{x}_k$ consisting of its position and speed. Target's dynamic is given by

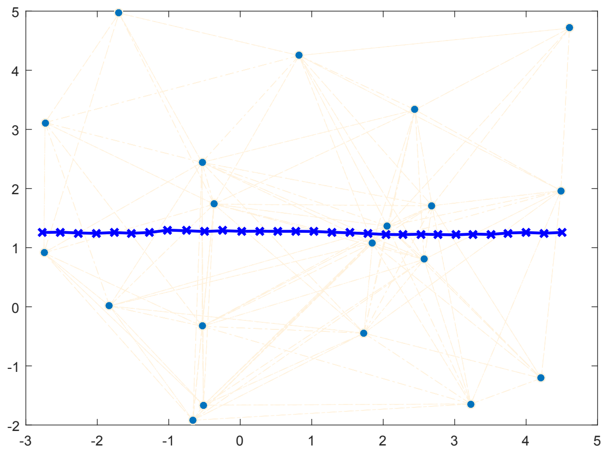
$$\boldsymbol{x}_k = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix} \boldsymbol{x}_{k-1} + \begin{bmatrix} \tau^2/2 \\ \tau \end{bmatrix} w_k, \tag{3.22}$$

where sampling time is selected as $\tau = 0.3$ second, and the variance of the target acceleration is set to 0.5. Fig. 3.5 (a) depicts the postilion of sensors and the target's track. Each sensor periodically measures the position and speed of the target based on the following observation model

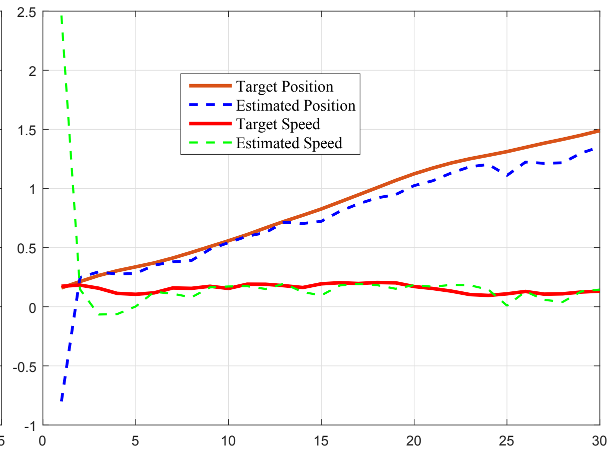
$$z_k^{(l)} = \begin{bmatrix} 0.7 & 0.31 \end{bmatrix} \boldsymbol{x}_k + v_k^{(l)}. \tag{3.23}$$

In this experiment, the observation noise is considered to be state dependent such that the noise variance $\sigma^2_{v_k}$ depends on the distance r_k between the observer and target as follows

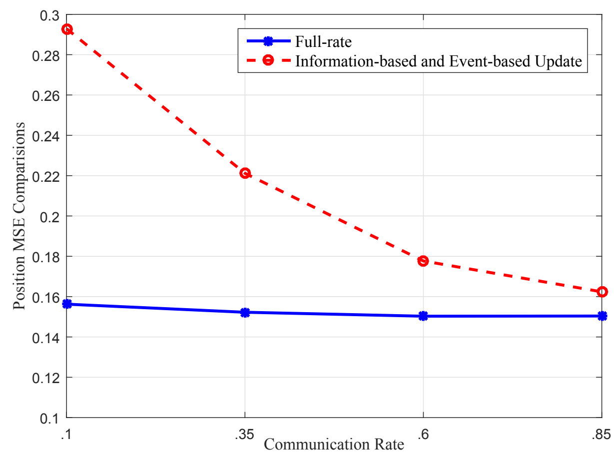
$$\sigma^2_{v_k} = .001 r_k^2 + 0.25 r_k + 0.0905. \tag{3.24}$$



(a)



(b)



(c)

Figure 3.5: (a) Target's track and location of sensor nodes. (b) Position and velocity estimates over time computed based on the proposed estimation algorithm when the communication rate between the sensors and the FC averaged over all sensors and the MC runs is 0.35. (c) Monte Carlo simulations of 100 runs. Position MSE plots over varying communication rates corresponding to the full-rate KF and the proposed information-based triggering mechanism.

The following results are computed over MC simulations of 100 runs. The object's position and speed used in each simulation run changes randomly to provide a fair experimental benchmark. Fig. 3.5 (b) illustrates the estimated target position and velocity when the proposed open-loop and event-based estimation developed in Section 3.2.1 is used to estimate the targets' state and the triggering decisions at the sensor level are made based on the information vector. In this experiment, the communication rate between the sensors and the FC averaged over all sensors and the MC runs is 0.35. It is observed that the information-based triggering mechanism provides acceptable results

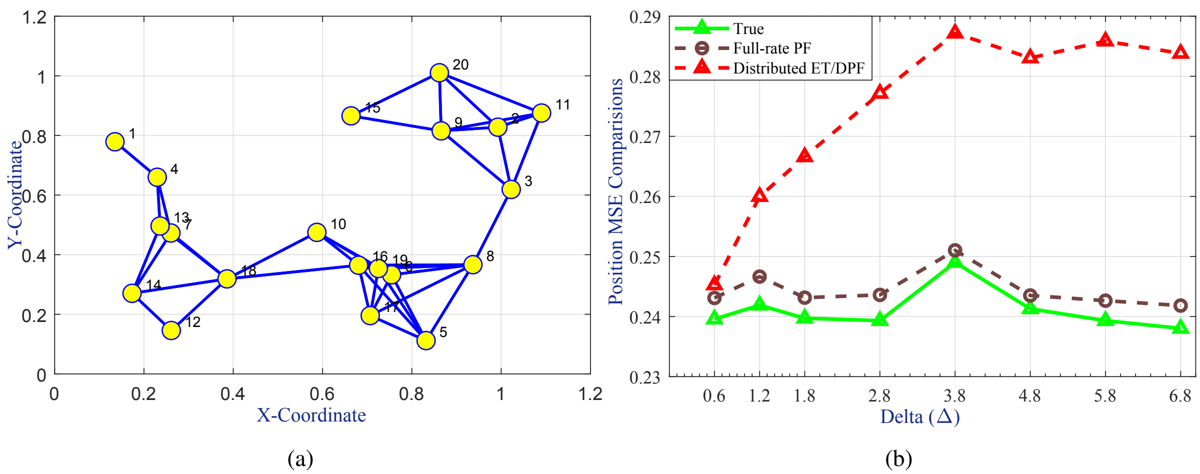


Figure 3.6: (a) Sensor placements. (b) Agent networks and connections. (c) Position MSE comparison over different values of $\Delta^{(l)}$.

in such a low communication rate scenario. Fig. 3.5 (c), shows the position mean-square error (MSE) plots over varying values of the communication rate based on the proposed event-triggered estimation algorithm and its full-rate counterpart. It is observed that the proposed information-based triggering algorithm closely follows its full-rate counterpart. As the communication rate increases to one, the proposed event-based methodology approaches the full-rate estimator.

3.4.3 Evaluation of the Distributed EBPF

In this sub-section, simulation experiments are developed, as proof-of-concept, to evaluate the performance of the proposed ET/DPF (Section 3.3). Following the recent literature on ET estimation [50], a tracking problem is considered where observations from an agent network of $N = 20$ nodes is used to sequentially estimate the state of the target denoted by $\boldsymbol{x}_k$ consisting of its position and speed. Sensors are distributed randomly in a square region and each sensor communicates with its neighbours within a connectivity radius of $\sqrt{2\log(N)/N}$ units. Target's dynamic and measurement models are given by Eqs. (3.63)-(3.65). The following results are computed over MC simulations of 100 runs. The object's position and speed used in each simulation run changes randomly to provide a fair experimental benchmark. Furthermore, the following three estimators are implemented and compared for accuracy: (i) The full-rate diffusion-based KF where each sensor communicates its measurements to its neighbouring nodes every iteration; (iii) Event-based diffusive KF, where SOD triggering is used, and; (iii) The proposed ET/DPF algorithm, where the triggering decisions at the sensor level are made based on SOD mechanism and the fusion is performed distributively using diffusive strategies by jointly incorporating set-valued and point-valued measurements.

A realization of the sensor placement is shown in Fig. 3.6 (a). Fig. 3.6 (b), shows the position MSE plots over varying values of $\Delta^{(l)}$ without inclusion of the KF-based curve for better clarity. It is observed that the proposed ET/DPF algorithm provides acceptable results in very low communication rates (high values of $\Delta^{(l)}$) and closely follows its full-rate counterparts in high communication rates. Besides, when the communication rate increases (i.e., small values for $\Delta^{(l)}$), the proposed event-based methodology approaches the full-rate estimator.

3.5 Conclusion

In this section we proposed an EBPF framework for distributed state estimation in systems with communication/power constraints at the sensor side. An event-based and open-loop estimation architecture (i.e., no feedback communication is incorporated from the FC to local sensors) is considered.

3.5.1 Concluding Remarks on Single-Sensor EBPF

In Section 3.1, a centralized architecture is considered based on a single remote sensor. Local sensor uses practical SOD event triggering mechanism resulting in availability of side information at the FC in the absence of an observation. Utilization of this side information results in estimation with joint set-valued and point-valued measurements which consequently translates in to a non-Gaussian state estimation problem. The proposed EBPF is a systematic and intuitively pleasing non-Gaussian estimation algorithm which jointly incorporates point and set-valued measurements within the particle filter framework by capitalizing on the fact that particle filters only require new measurements to evaluate the likelihood function during the weight update step. In presence of an observation (point-valued measurement), the likelihood function can exactly be evaluated for each particle. In the absence of an observation, the likelihood becomes the probability that the observation belongs to the triggering set which is derived in the thesis to utilize set-valued measurements in the proposed EBPF framework.

3.5.2 Concluding Remarks on Hierarchical EBE

In Section 3.2, a hierarchical architecture is considered and a multi-sensor and information-based event-triggering estimation framework is proposed. In the proposed multi-sensor and information-based framework, a triggering mechanism is developed based on the projection of local observations into the state-space which in turn is a measure of the achievable gain in the local information state vector. Incorporation of the modified measurement model at the sensor level results in an event-based information (inverse-covariance) filter at the FC. The event-triggered information filter implemented at the FC is capable of fusing multi-sensor measurements in an adaptive and efficient manner with reduced communication overhead.

3.5.3 Concluding Remarks on Distributed EBE

In Section 3.3, we proposed an event-triggered particle filter (ET/DPF) framework for distributed state estimation in autonomous agent-sensor systems without incorporation of a FC. Each sensor uses practical SOD event triggering mechanism resulting in availability of side information at its neighbouring nodes in the absence of an observation. Utilization of this side information results in estimation with joint set-valued and point-valued measurements which consequently translates in to a non-Gaussian state estimation problem. The proposed ET/DPF uses diffusion strategies for distributed implementations.

Chapter 4

Ternary Event-based State Estimation

The chapter proposes a novel ternary event-based particle filtering (TEB-PF) framework by introducing the ternary event-triggering (TET) mechanism coupled with a non-Gaussian fusion strategy that jointly incorporates point-valued, quantized, and set-valued measurements. In contrary to the existing binary event-triggering solutions, the TEB-PF is a distributed state estimation architecture where the remote sensor communicates its measurements to the estimator, resided at the FC, in a ternary event-based fashion, i.e., holds on to its observation during idle epochs, transfers quantized ones during the transitional epochs, and; only communicates raw observations during event epochs. Due to joint utilization of quantized and set-valued measurements in addition to the point-valued ones, the proposed TEB-PF simultaneously reduces the communication overhead, in comparison to its binary triggering counterparts, while also improves the estimation accuracy especially in low communication rates.

4.1 Problem Formulation

As stated previously, in the EBE approaches, measurements from a sensor to the estimator are communicated only in occurrence of specific events identified based on a local triggering mechanism implemented at the sensor level. The basic motivation behind development of EBE algorithms is to reduce the communication overhead by avoiding periodic transfer of measurements. In this chapter, we focus on an alternative solution with the goal to simultaneously address the following three potential shortcomings of recently developed EBE strategies:

(i) It is commonly assumed that during the event epochs, sensor communicates its raw measurement. Although, EB transfer of measurements potentially reduces the communication overhead, still communicating raw measurements during all the event instances could be very costly;

(ii) Another common assumption in the EBE strategies is having a binary (idle and event) decision process where during idle epochs, the sensor holds on to its local measurements while during the event epochs measurement communication happens, and;

(iii) The EB posterior distribution is inherently non-Gaussian, it is expected that stochastic triggers (proposed in part to cope with this non-Gaussian posterior) under-perform in comparison to their deterministic counterparts as the side-information at the estimator side is not used efficiently [8].

The chapter addresses these drawbacks and makes the following main contributions:

- A novel ternary event-triggering (TET) mechanism is proposed that instead of using a binary decision criteria, uses three local decision cases resulting in set-valued, quantized, and point-valued measurements, and;

- A systematic fusion mechanism is proposed to jointly incorporate ternary hybrid measurements within the particle filtering framework.

The TEB-PF framework is developed by considering an estimation problem represented by the linear state-space model given by Eqs. (2.2)-(2.3). When sensor's observation z_k is communicated in an EB fashion (i.e., not at all iterations $k > 1$ and based on the senor's triggering methodology), the posterior distribution $Pr(\boldsymbol{x}_k|\{z_1, \ldots, z_k\})$ becomes non-Gaussian. It is a common assumption in the EBE literature (except our recent work in Chapter 3) to consider that the remote estimator knows whether or not the received signal at its communication channel at each time is signal bearing or not. We consider this case, referred to as the supervised EBE [8], because the main focus of this chapter is on introduction of the TET mechanism and its particle-based fusion. Similar to the EBPF

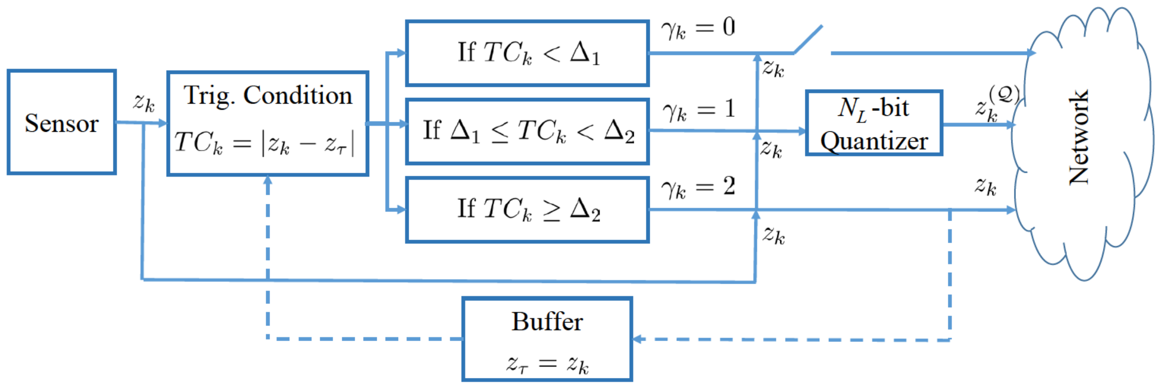


Figure 4.1: The TET mechanism.

and ET/DPF, here we consider the well-known and widely used supervised triggering criteria based on the SOD concept, which is a deterministic scheduler where the transmission is triggered when the difference between the new measurement and the previously transmitted one becomes greater than a pre-defined threshold (denoted by Δ). The decision variable in the SOD approach is binary in the sense that it translates to two modes (idle or event) and results in availability of set and point-valued measurements. To the best of our knowledge, this is the case in all of the recently proposed EBE algorithms [8, 26, 27, 30, 35–38, 59]. Next, we present the proposed TET mechanism, which is an alternative and intuitively pleasing counterpart to such binary event-triggering schedulers. As will be shown in the next section, incorporation of the TET mechanism, results in having access to three decision levels, referred to as *idle* (Case 1), *transitional* (Case 2), and *event* (Case 3) epochs which in turn provides three different information contents (point, quantized, and set-valued measurements).

4.2 The TEB-PF Framework

The TET mechanism, and its EB particle filtering fusion are, respectively, described below in Sub-sections 4.2.1 and 4.2.2.

4.2.1 Ternary Event-Triggering (TET) Mechanism

As stated previously, the TET mechanism is a deterministic and ternary scheduler (i.e., there are three decision levels instead of conventional binary decisions) and is similar in nature to the SOD concept. More specifically, in the TET mechanism, the sensor computes the distance between each of its current observations and the previously transmitted ones, and decides based on the following ternary criteria

$$\gamma_k = \begin{cases} 0, & \text{if } |z_k - z_\tau| < \Delta_1 \\ 1, & \text{if } \Delta_1 \leq |z_k - z_\tau| < \Delta_2 \\ 2, & \text{if } |z_k - z_\tau| \geq \Delta_2 \end{cases}, \tag{4.1}$$

where z_τ denotes the observation value of the last transfer from the sensor to the FC, and Δ_1 and Δ_2 denote the two triggering thresholds identifying the ternary levels. The block diagram in Fig. 4.1 illustrates the TET mechanism and transmission process of the sensor's observation. At each iteration, the TET first computes the triggering condition (TC) and then compares it with two thresholds, resulting in three possible cases:

Case 1: Observation is discarded;

Case 2: Observation is quantized and then communicated to the FC, and;

Case 3: The TET communicates raw observation and also updates the buffer based on the current sensor measurement (i.e., z_τ is updated with the current observation value).

We define the resulting hybrid observation as follows

$$\boldsymbol{y}_k = \begin{cases} z_k & \text{if } \gamma_k = 2 \\ z_k^{(\mathcal{Q})} \wedge \Big\{ z_k : z_k \in (z_\tau + \Delta_1, z_\tau + \Delta_2) \oplus \\ \qquad z_k \in (z_\tau - \Delta_2, z_\tau - \Delta_1) \Big\} & \text{if } \gamma_k = 1 \\ \{ z_k : z_k \in (z_\tau - \Delta_1, z_\tau + \Delta_1) \} & \text{if } \gamma_k = 0 \end{cases} \tag{4.2}$$

where symbols $\wedge$, $\oplus$, and $\in$ denote logical "and", "xor", and "set membership", respectively. Eq. (4.2) represents the information that is available at the FC at iteration (k) depending on the triggering mechanism adopted at the sensor level. For example, when $\gamma_k = 0$, the FC does not receive any data at its communication channel, which means that the sensor has not transmitted at this particular iteration. Absence of data at the FC results in a side information, i.e., the value of the current sensor measurement belongs to the set $(z_\tau - \Delta_1, z_\tau + \Delta_1)$, this is referred to as set-valued information. In other words, the FC does not know/use the exact value of z_k when $\gamma_k \in \{0, 1\}$. The TET mechanism results in the availability of the following hybrid measurements at the FC:

1. *Set-Valued Information ($\gamma_k = 0$)*: As stated previously, in this case the FC does not know the exact value of the current sensor observation but instead knows the set to which it belongs based on the previously received observation (z_τ).

2. *Joint Quantized and Set-Valued Information ($\gamma_k = 1$)*: Similar to the previous case, during the transitional epochs, remote estimator does not know the exact value of current observation (z_k) but instead now has access to its quantized version ($z_k^{(\mathcal{Q})}$) together with the set over which the quantization is performed. In this case, not only quantized information $z_k^{(\mathcal{Q})}$ is available at the FC, but additional set-valued information is also available, i.e., the observation z_k belongs to either of the following two sets $(z_\tau + \Delta_1, z_\tau + \Delta_2)$ or $(z_\tau - \Delta_2, z_\tau - \Delta_1)$. In this case, the quantization is performed around the previously communicated observation from the sensor node to the FC (denoted by z_τ), which is already available at the FC, therefore, the interesting point here is that there is no need for extra communication in this regard for transferring the quantization point to the FC.

3. *Point-Valued Information ($\gamma_k = 2$)*: The event-triggering condition is satisfied and the exact value of the current sensor observation (z_k) is known at the FC. The FC uses this exact value instead of a substitute quantity to both update its buffer (similar to the triggering mechanism), and to complete the update from prediction to estimation in the particle filter.

Such a TET mechanism has the potential to reduce the event epochs (reduce the communication of raw measurements) which in turn reduces the communication overhead (achieved by using quantized measurements) without compromising the accuracy. A very interesting and intuitively pleasing feature of the proposed TET is availability of joint quantized and set-valued measurements which can provide improved accuracy with lower number of quantization levels. As stated previously, here we focused on supervised EBE [8], one difficulty may arise when an unsupervised EBE is considered. In this case, differentiating between the three event-triggered scenarios without extra information could be more complicated than the case we considered in [8].

4.2.2 Event-based Particle Filters with TET

The posterior distribution $Pr(\boldsymbol{x}_k|\boldsymbol{Y}_k)$ based on point, quantized, and set-valued measurements ($\boldsymbol{Y}_k = \{\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_k\}$) is no longer Gaussian, therefore, linear filters such as the KF are not applicable. The proposed TEB-PF approximates $Pr(\boldsymbol{x}_k|\boldsymbol{Y}_k)$ using a set of n_p particles $\{\mathbb{X}_k^{(i)}\}_{i=1}^{n_p}$ derived from the transitional density, i.e., $\mathbb{X}_k^{(i)} \sim Pr(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$ which is the conventional proposal distribution. This choice of the proposal results in the following weight update $W_k^{(i)} \propto W_{k-1}^{(i)} Pr(\boldsymbol{y}_k|\mathbb{X}_k^{(i)})$, which is the conventional weight update equation within the context of particle filtering [10, 63]. The key step to implement the TEB-PF is to evaluate the weight update equation separately based on quantized, set-valued, and point-valued measurements. We capitalize on the fact that in particle filtering [61,62], the observations' nature (being point-valued, quantized, or set-valued) will mainly affect the likelihood which is used to update each particle's weight. In the presence of an observation (event case), the likelihood can exactly be evaluated for each particle. In the absence of an observation (idle case), we evaluate the probability that the unknown measurement belongs to the event-triggering set. In the transitional case, the TEB-PF combines quantized information with set-valued information to evaluate the probability that the quantized observation belongs to the event set.

Joint Update based on Set-valued and Quantized Measurements ($\gamma_k = 1$)

In this case, the sensor communicates a quantized version $z_k^{(\mathcal{Q})}$ to the remote estimator based on the following model

$$z_k^{(\mathcal{Q})} = \mathcal{Q}\big(\boldsymbol{h}_k^T \boldsymbol{x}_k + v_k\big), \tag{4.3}$$

where $\mathcal{Q}(\cdot)$ is the quantization operator. The quantizer $\mathcal{Q}(\cdot)$ is a nonlinear mapping from observation z_k to the quantized observation $z_k^{(\mathcal{Q})}$. We consider an N_L-bit quantization scheme, where a quantized observation can take any discrete values from 0 to $2^{N_L} - 1$. We define $L = 2^{N_L}$, therefore, the set of quantization thresholds is denoted by $\boldsymbol{q} = \{q_0, q_1, \ldots, q_L\}$. In a general scenario without having access to side information, the first and last thresholds are set to $q_0 = -\infty$ and $q_L = \infty$. The advantage of such a quantization approach is that we only need to compute the quantization thresholds once, which can be done in an offline fashion. However, to have an acceptable and high resolution, typically, a higher order quantization is required. On the other hand, the proposed TEB-PF has access to both the quantized measurement ($z_k^{(\mathcal{Q})}$) and extra side-information (i.e., $z_k \in (z_\tau + \Delta_1, z_\tau + \Delta_2)$ or $z_k \in (z_\tau - \Delta_2, z_\tau - \Delta_1)$), therefore, higher resolutions can be achieved with low order quantization as now only the range between Δ_1 and Δ_2 needs to be quantized and not the whole observation space. In this scenario, a quantized observation takes the following values

$$z_k^{(\mathcal{Q})} = \begin{cases} 0 & \text{if} \quad z_\tau - \Delta_2 < z_k \leq q_1 \\ \vdots & \qquad \vdots \\ L/2 - 1 & \text{if} \quad q_{L/2-1} < z_k < z_\tau - \Delta_1. \\ L/2 & \text{if} \quad q_{L/2} = z_\tau + \Delta_1 < z_k \leq q_{L/2+1} \\ \vdots & \qquad \vdots \\ L - 1 & \text{if} \quad q_{L-1} < z_k \leq z_\tau + \Delta_2. \end{cases} \tag{4.4}$$

where $\{z_\tau - \Delta_2, z_\tau - \Delta_1, z_\tau + \Delta_2, q_1, \ldots, q_{L-1}\}$ is the set of quantization thresholds. The discrete probability density for $q_i \leq z_k \leq q_{i+1}$ is given by

$$\begin{aligned} Pr\big(z_k^{(\mathcal{Q})} = q_i | \boldsymbol{x}(k)\big) &= Pr\big(q_i < \big[\boldsymbol{h}_k^T \boldsymbol{x}_k + v_k\big] \leq q_{i+1}\big) \\ &= Pr\big(\, \big[q_i - \boldsymbol{h}_k^T \boldsymbol{x}_k\big] < v_k \leq \big[q_{i+1} - \boldsymbol{h}_k^T \boldsymbol{x}_k\big]\, \big), \end{aligned} \tag{4.5}$$

where superscript T denotes the transpose operator. Eq. (4.5) represents the likelihood function for the received quantized observation $z_k^{(\mathcal{Q})}$, which is the probability that the quantized observation $z_k^{(\mathcal{Q})}$ takes a specific value (q_i in this case) given the state vector at iteration k. Note that, except the boundary levels, $z_k^{(\mathcal{Q})} = q_i$ means that $q_i < z_k \leq q_{i+1}$ which is used to further expand Eq. (4.5). In the second line of Eq. (4.5), we kept the noise in the middle and moved other terms to the sides to compute the likelihood based on the probability distribution of the noise. Considering that the observation noise model (Eq. (2.3)) is Gaussian, the likelihood in Eq. (4.5) reduces to

$$Pr(\boldsymbol{y}_k|\boldsymbol{x}_k,\gamma_k=1)=\Phi\left(\frac{q_i-\boldsymbol{h}_k^T\boldsymbol{x}_k}{\sqrt{R_k}}\right)-\Phi\left(\frac{q_{i+1}-\boldsymbol{h}_k^T\boldsymbol{x}_k}{\sqrt{R_k}}\right), \tag{4.6}$$

where $\Phi(\cdot)$ is cumulative Gaussian distribution with zero mean and variance 1, and R_k denotes observation noise variance.

Update based on Set-valued Measurements ($\gamma_k = 0$)

In the absence of the sensor measurement and based on the triggering mechanism defined in Eq. (4.1), the estimator has only access to the side information that observation belongs to $(z_\tau - \Delta_1, z_\tau + \Delta_1)$. The likelihood, therefore, is specified as

$$Pr\big(\boldsymbol{y}_k|\boldsymbol{x}_k,\gamma_k=0\big)=Pr\big(z_\tau-\Delta_1\leq z_k\leq z_\tau+\Delta_1\big), \tag{4.7}$$

which by substituting for z_k from Eq. (2.3), we have

$$\begin{aligned} Pr\big(\boldsymbol{y}_k|\boldsymbol{x}_k,\gamma_k=0\big) &= Pr\big(z_\tau-\Delta\leq \boldsymbol{h}_k^T\boldsymbol{x}_k+v_k\leq z_\tau+\Delta\big)\\ &= Pr\big(\,\big[z_\tau-\Delta-\boldsymbol{h}_k^T\boldsymbol{x}_k\big]\leq v_k\leq\big[z_\tau+\Delta-\boldsymbol{h}_k^T\boldsymbol{x}_k\big]\,\big). \end{aligned} \tag{4.8}$$

The likelihood, therefore, is given by

$$Pr(\boldsymbol{y}_k|\boldsymbol{x}_k,\gamma_k=0)=\Phi\left(\frac{z_\tau+\Delta_1-\boldsymbol{h}_k^T\boldsymbol{x}_k}{\sqrt{R_k}}\right)-\Phi\left(\frac{z_\tau-\Delta_1-\boldsymbol{h}_k^T\boldsymbol{x}_k}{\sqrt{R_k}}\right). \tag{4.9}$$

Update based on Point Measurements ($\gamma_k = 2$):

The FC receives the actual sensor observation z_k, therefore, the hybrid likelihood function $P(\boldsymbol{y}_k|\boldsymbol{x}_k)$ reduces to the sensor likelihood function $Pr(\boldsymbol{y}_k|\boldsymbol{x}_k,\gamma_k=2)=\Phi(\frac{z_k-\boldsymbol{h}_k^T\boldsymbol{x}_k}{\sqrt{R_k}})$.

4.2.3 Concluding Remarks on the TET Mechanism

Algorithm 2 outlines the TEB-PF implementation. Following [68], the computational complexity of the TEB-PF in Algorithm 2, which is implemented at the FC, is of O($n_x n_p$). It is common in distributed estimation algorithms that agents have a limited bit budget for communication [22–24]. Quantization has been viewed as a fundamental element in this regard for saving bandwidth to reduce the energy consumption which is related to the amount of data transmitted. At one extreme, harsh quantization is introduced for example in Reference [22] an interesting distributed KF-based estimation algorithm is developed where only one bit is communicated (based on the sign of innovation). It is also common to determine the encoding/quantization of sensor measurement by the information available at the encoder/observer at each time [25] as is the case in the proposed TET-PF. We can analyze the achievable benefits of the proposed TET-PF via the reduction in the energy consumption, which is related to the amount of data (bits) transmitted. Local energy consumption is mainly associated with the required energy by the sensor for transmitting the (quantized) measurements to the FC. For clarification purposes, let us assume that the channel between the sensor and the FC experiences a path loss proportional to the transmission distance between the sensor and the FC. Then the consumed energy of the sensor at time step k is given by [69]

$$E_k = \beta\big(2^{b_k - 1}\big) \tag{4.10}$$

where b_k-bit message at iteration k is transmitted, $\beta = \rho d^{\alpha} \ln(2/P_b)$ is the energy density, in which d is the distance between the sensor and FC; ρ is a constant depending on the noise profile, and P_b is the target bit error rate. Therefore, reducing number of communicated bits (which is achieved via the TET mechanism) will lead to energy (resource) savings.

Regarding the two thresholds used within the TET mechanism, these coefficients determine the communication rate/bandwidth requirements of the overall EBE algorithm. One of the main parameters that describe the SOD concept is the mean communication rate of messages transferred from the sensor node to the remote estimator, which is a function of the two thresholds. Unfortunately, the mean communication rate cannot be evaluated analytically. In the next section, we consider an optimization framework to find the two required thresholds. Later on, Monte Carlo simulations will be used to evaluate/compare the communication rate of the overall system.

Bandwidth partitioning among ternary communications can be implemented through modification of the medium access control (MAC) layer. To accommodate intermittent node activities of

Algorithm 2 TEB-PF IMPLEMENTATION

Input: $\{\mathbb{X}_{k-1}^{(i)}, W_{k-1}^{(i)}\}_{i=1}^{n_p}$, γ_k, and $\boldsymbol{y}_k$.

Output: $\{\mathbb{X}_{k}^{(i)}, W_{k}^{(i)}\}_{i=1}^{n_p}$, $\hat{\boldsymbol{x}}_{k|k}$ and $\boldsymbol{P}_{k|k}$.

At iteration k, TEB-PF updates its particle set as follows:

S1. *Predictive Particle Generation*: $\mathbb{X}_k^{(i)} \sim P(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$.

S2. *Hybrid Likelihood Computation*:

- **If** $\gamma_k = 0$: Compute $Pr(\boldsymbol{y}_k|\mathbb{X}_k^{(i)}, \gamma_k = 0)$ using Eq. (4.9).
- **If** $\gamma_k = 1$: Compute $Pr(\boldsymbol{y}_k|\mathbb{X}_k^{(i)}, \gamma_k = 1)$ using Eq. (4.6).
- **If** $\gamma_k = 2$: Evaluate $\Phi(\frac{z_k - \boldsymbol{h}_k^T \mathbb{X}_k^{(i)}}{\sqrt{R_k}})$.

S3. *Weight Update*: Update the weights.

the event-triggered traffic, the link layer medium access coordination is typically required to be in the form of a random access protocol. Generally speaking, the TET mechanism correlate well to the event-triggered nature of such protocols, where the sensor determines to transmit in an autonomous fashion. The TET mechanism can be used, e.g., with different variations/modifications of the carrier-sense multiple access (CSMA/CA), which has been effectively used for event-based estimation/control. As a final note, we would also like to mention one motivating practical application for reducing the number of communicated bits, which is the underwater wireless sensor network (UWSN) technologies [39]. Underwater communications suffer from limited bandwidth due to the temporal and spatial variability of channels. A limited bandwidth leads to low bit rates, therefore, the data-efficiency of UWSNs can be improved by reducing the length of data packets transmitted from the sensor to the FC. In this case, similarly, the proposed TET mechanism improves data-efficiency by using a combination of quantized, raw and no transmission scenarios.

4.3 Optimized Ternary Event-based Estimation

In Section 4.2, we proposed an alternative solution, the TET mechanism, with the goal of simultaneously addressing the aforementioned issues. In Section 4.2, however, we used a deterministic triggering mechanism (the SOD) via thresholding, instead of using a stochastic triggering approach, which reduces the non-Gaussian posterior to its Gaussian counterpart. The main rational behind this choice is that through non-linear/non-Gaussian filtering [10] we can provide a better approximation of the event-based posterior in comparison to a Gaussian approximation that stochastic triggering

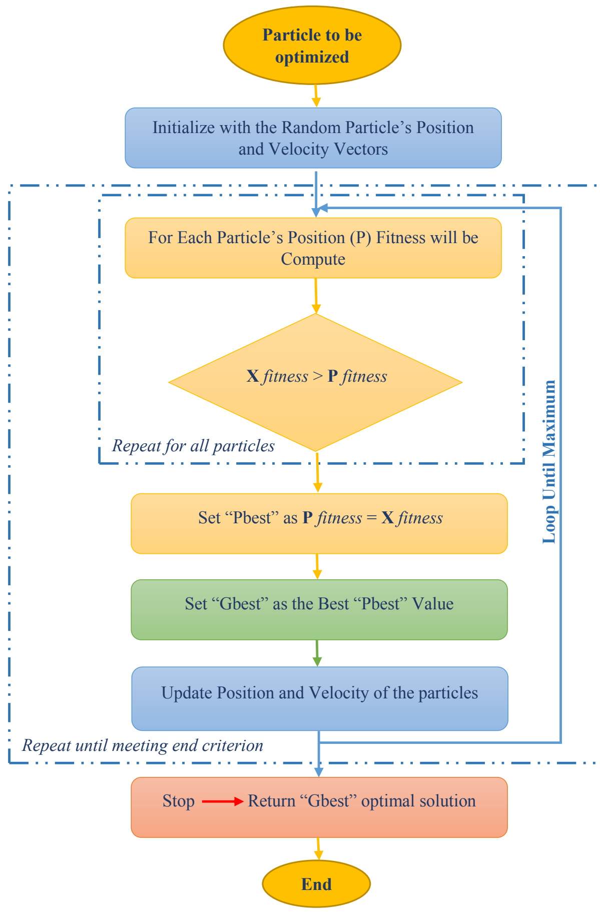


Figure 4.2: General steps of the PSO.

provides. However, the main problem with a deterministic triggering method (such as the SOD) is its dependence on a predefined threshold. *The motivation behind this section is development of a multi objective approach, referred to as the TEB-PSO, to optimize the two threshold values used by our recently proposed TET mechanism.* In other words, the goal in this section is to reduce the communication overhead by using optimized values for the thresholds using multi objective particle swarm optimization (MOPSO) technique. The MOPSO is designed to maximize the transfer rate of quantize measurements (QM) while tries to transfer minimum periodic measurements within the idle epochs, which preserves restricted power resources.

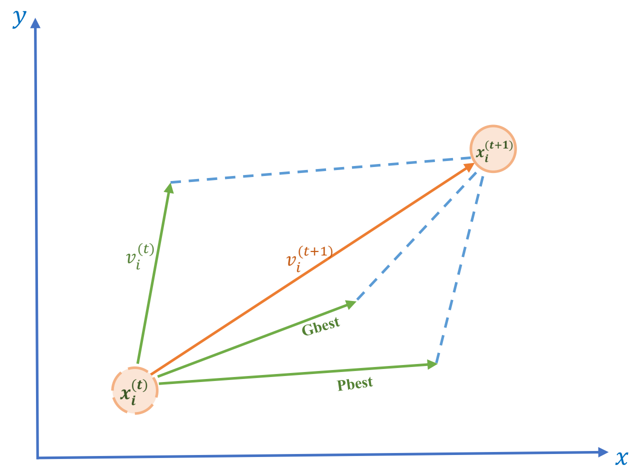


Figure 4.3: Concept of modifying the positions of particles in PSO.

4.3.1 Multi Objective Particle Swarm Optimization

Nowadays, there are various challenges in solving engineering problems in CPSs. One of the most important characteristics of such real practical problems, which makes them challenging, is their multi-objective nature. Multi objective optimization refers to finding the optimum solution for problems where more than one objective is of paramount importance and one needs to satisfy all the objectives simultaneously. There is no single solution for such problems, therefore, a set of optimum solutions representing the best trade-offs between the underlying multiple objectives are the answer to these problems.

In this subsection, we develop a multi-objective approach for optimizing the threshold values in the TET mechanism within event-based estimation architecture. In particular, the MOPSO is employed as the optimization technique considering three objectives, i.e., the maximization of the rate of communicating quantized measurements together with the minimization of the number of idle and event epochs. In addition, the optimization process is subject to three constraints in order to guarantee the feasibility of the overall structure. The proposed method, referred to as the TEB-PSO, is capable of identifying a set of optimal values for the two thresholds within the TET to reduce the communication overhead. The simulation results confirm the effectiveness of the proposed method with the TET mechanism.

The PSO approach [70, 71] is an evolutionary computation technique inspired by the social behavior of bird flocking and was proposed initially by Kennedy and Eberhart [72]. The PSO uses a number of particles (candidate solutions) which fly around in the search space to find the best solution. Meanwhile, each particle traces the best location (best solution) in its path. In other words, particles consider their own best solutions as well as the best solution the swarm has obtained so far. Each particle in the PSO should consider the current position, the current velocity, the distance to its personal best solution, denoted by P_{Best}, and the distance to the global best solution, denoted by G_{Best}, to modify its position. The PSO is mathematically modeled as follows

$$\begin{aligned} \boldsymbol{v}^{(i)}(t+1) &= w\boldsymbol{v}^{(i)}(t) + c_1 r_1\big(P^{(i)}_{\text{Best}} - \boldsymbol{x}^{(i)}(t)\big) \\ &+ c_2 r_2\big(G^{(i)}_{\text{Best}} - \boldsymbol{x}^{(i)}(t)\big) \qquad (4.11) \\ \boldsymbol{x}^{(i)}(t+1) &= \boldsymbol{x}^{(i)}(t) + \boldsymbol{v}^{(i)}(t+1), \qquad (4.12) \end{aligned}$$

where $\boldsymbol{v}^{(i)}(t+1)$ is the velocity of particle i, for $(1 \leq i \leq N_{sp})$ at iteration t. Term w is a weighting function, c_j and r_j, for $(1 \leq j \leq 2)$, represent, respectively, an acceleration coefficient, and a random number uniformly distributed between 0 and 1. Furthermore, $\boldsymbol{x}^{(i)}(t)$ is the current position of particle i at iteration t, $P^{(i)}_{\text{Best}}$ is the best solution that the i^{th} particle has obtained so far, and G_{Best} indicates the best solution the swarm has obtained so far. The concepts of position updating are illustrated in Fig. 4.2. The first part (i.e., $w\boldsymbol{v}^{(i)}(t)$) on the right hand side (RHS) of Eq. (2.2), provides exploration ability for PSO, while the second and third parts (i.e., $c_1r_1(P^{(i)}_{\text{Best}} - \boldsymbol{x}^{(i)}(t))$ and $c_2r_2(G^{(i)}_{\text{Best}} - \boldsymbol{x}^{(i)}(t))$) represent private thinking and collaboration of particles, respectively. The PSO starts with randomly placing the particles in the problem space and over the course of the iterations, the velocities of particles are calculated using Eq. (2.2). After defining the velocities, the position of particles can be calculated using Eq. (2.3). The process of changing particles' positions continues until a predefined completion criterion is satisfied.

As stated previously, we consider an open-loop state estimation problem where a remote sensor communicates its measurements to the FC, only in occurrence of specific events. We define a set of thresholds, acting as our variable range, i.e.,

$$\begin{aligned} \Delta_L &= \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6 \\ \text{and } \Delta_W &= \Delta_7, \Delta_8, \Delta_9, \Delta_{10}, \Delta_{11}, \Delta_{12}, \end{aligned}$$

where the range of each Δ_k is defined between **(0.01 and 10)**. By considering these thresholds, the goal of the TEB-PSO is to maximize the number of quantized measurement, which constitute the joint and set-valued measurement component of the hybrid observation set, while minimizing the point-valued and set-valued measurements [44]. We would like to point out that the solution to the problem at hand needs to satisfy some constraints for prevention of band-mixing in the set of the threshold, which is describe as follow

$$\gamma_k = \begin{cases} 1: & \Delta_L(k) > \Delta_L(k-1) \text{ and } \Delta_W(k) > \Delta_W(k-1) \\ 2: & \Delta_W(k) > \Delta_L(k) \end{cases} .$$

According to the first constraint the amounts of $\Delta_L(k)$ should be greater than $\Delta_L(k-1)$ and the amounts of $\Delta_W(k)$ should be greater than $\Delta_W(k-1)$. In the second constraint, the set of thresholds within the whole set of $\Delta_W(k)$ should be greater than $\Delta_L(k)$.

Now, our objective function is receiving these values of $\Delta_L(k-1)$, $\Delta_W(k-1)$ as an input and adopt our TEB-PSO framework in order to maximize the quantized measurement and minimize the point-valued and set-valued measurements. By sending more quantized measurement, we reduce the amount of communication rate which in turn reduces the communication rate, as well as, the total cost. The multi-objective PSO is used to solve this problem.

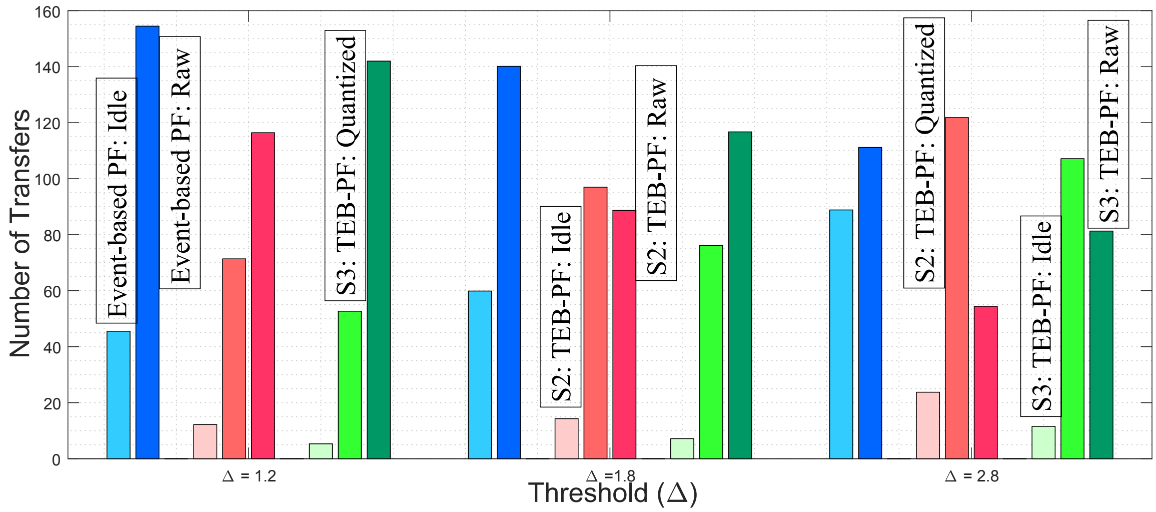
4.4 Simulations

In the following two subsections, we evaluate the performance of the TEB-PF and the TEB-PSO frameworks, respectively.

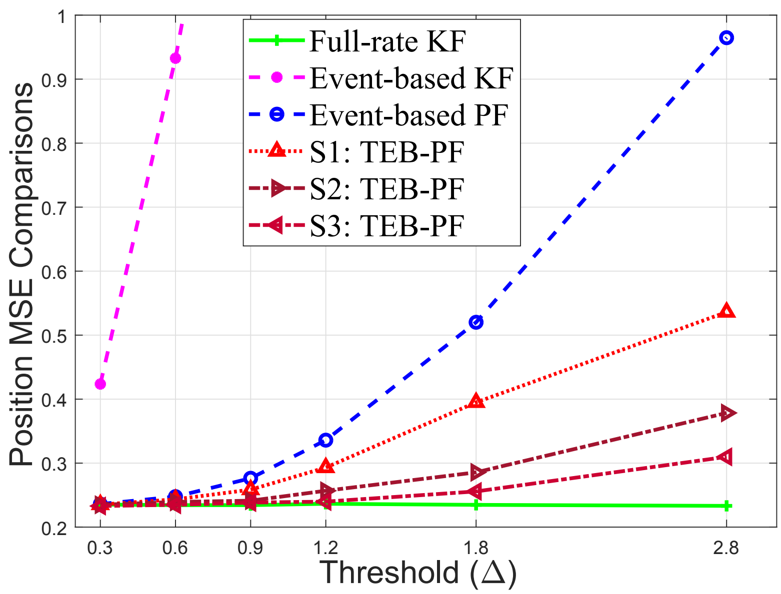
4.4.1 Evaluation of the TEB-PF Framework

Following the recent literature on the EBE [50], a target tracking problem is considered to evaluate performance of the proposed TEB-PF framework. Target's dynamic and measurement models are given by Eqs. (3.63)-(3.65). The following results are computed over Monte-Carlo (MC) simulations of 100 runs with $n_p = 500$ particles. Four filters are compared: (i) Full-rate KF; (ii) Event-based KF, which disregards the set-valued information; (iii) Binary event-based PF, and; (iv) The proposed TEB-PF. The results are computed over varying communication rates. The mean communication rate depends on the values of the two thresholds Δ_1 and Δ_2 defined in Eq. (4.1). To provide fair comparison with the conventional SOD approaches, we fixed the value of Δ (used by Filters (ii)-(iii)) and consider three scenarios for Filter (iv): *Scenario 1*: $\Delta_1 = \Delta/2$, $\Delta_2 = 3\Delta$; *Scenario 2*: $\Delta_1 = \Delta/4$, $\Delta_2 = 2\Delta$, and; *Scenario 3*: $\Delta_1 = \Delta/8$, $\Delta_2 = 1.5\Delta$.

Intuitively speaking, when Δ_1 becomes smaller than Δ, the TEB-PF sends more quantized observations instead of staying idle. On the other hand, when Δ_2 becomes larger than Δ, the TEB-PF sends fewer raw data. The two thresholds determine the bandwidth requirements of the TEB-PF. Fig. 4.4(a) illustrates the position mean-square errors (MSE) over varying values of Δ which in turn results in varying values of the communication rate. We note that, in general, analytical evaluation of the communication rate cannot be achieved. It is observed that the TEB-PF has the potential to significantly reduce the overall estimation error especially in low communication rates. Fig. 4.4(b) illustrates mean communication rate comparison between the proposed TEB-PF (Filter (iv), Scenarios 2 and 3) and Filter (iii) where binary decision criteria is used. It is observed that the TEB-PF not only reduces the MSE in low communication rates but also reduces the total number of communicated raw observations. We analyze the achievable benefits of the TEB-PF via the reduction in the energy consumption [69], which is related to the amount of transmitted bits. Having a limited bit budget to reduce the energy consumption [22–24] is common for agents in distributed estimation, e.g., at one extreme, harsh quantization is introduced [22] where only one bit is communicated (based on the sign of innovation). It is also common to perform encoding/quantization of sensor measurement at each time [25] as is the case in the proposed TEB-PF. One motivating practical application for reducing the number of communicated bits is the underwater wireless sensor network



(a)



(b)

Figure 4.4: (a) The Position MSE comparison over varying communication rate obtained from the four implemented filters (two scenarios for TEB-PF).(b) Distribution of the decision variable (γ_k) among the ternary levels of Filter (iv), and among the binary levels of Filter (iii).

(UWSN) technologies [39], which extensively suffers from limited bandwidth. Table 4.1 provides comparison between the total number of communicated bits by the proposed TEB-PF framework (Filter (iv), Scenarios 1-3) and Filter (iii). A 4-bit quantizer (i.e., $L = 16$) is considered together with requiring 32 bits (4 Bytes) per raw observation. For instance, when $\Delta = 2.8$, the TEB-PF reduces the total number of communicated bits by factors of 2.5, 1.6, and 1.2, for Scenarios 1-3,

Table 4.1: Comparison between total numbers of communicated bits.

Items	$\Delta = 0.3$	$\Delta = 0.6$	$\Delta = 0.9$	$\Delta = 1.2$	$\Delta = 1.8$	$\Delta = 2.8$
Binary PF- # Bits	754.7	719.4	681.8	638.5	558.9	445.1
S1: TEB-PF- # Bits	678.7	586.1	493.3	415.4	282.1	174.1
S2: TEB-PF- # Bits	726.4	654.4	585.2	501.4	403.3	278.7
S3: TEB-PF- # Bits	746.8	688.1	638.6	594.3	504.9	378.8

respectively. This is while the overall MSE is reduced by factors of 1.8, 2.5 and 3, respectively for Scenarios 1-3. Results corroborate the effectiveness of the TET mechanism.

4.4.2 Evaluation of the TEB-PSO Framework

In this section, we perform different simulations to evaluate the performance of the proposed TEB-PSO framework. Following the recent literature on EBE [50], a target tracking problem is considered where position and speed of the target are given by Eqs. (3.63)-(3.65). The following results are computed over Monte-Carlo (MC) simulations of 100 runs with $n_p = 200$ particles. The following four filters are compared in the TET part: (i) Full-rate KF; (ii) Open-loop and event-based KF; (iii) Open-loop and event-based PF with binary decision variable, and; (iv) The TEB-PSO algorithm. For the TEB-PSO, we consider $C1 = 1.4962$, $C2 = 1.4962$, and the number of particles within the swarm is consider to be 60. Our multi-objective problem consists of 3 objectives and the maximum number of iterations for this simulation is set to 251. The results reported below are computed over varying communication rates.

We consider a matrix consisting of a set of Δ_1 and Δ_2, each of which include a set of numbers that can be chosen randomly from 0.001 to 10 ($0.01 =< \Delta <= 10$). One of the constraints which is satisfied by the proposed TEB-PSO is that the values in the second set of the Δs should be greater that first set of the Δs. Another constraint is all of the numbers in each set should be sorted ascending. So we have the first 6 numbers as the set of Δ_1, and the second 6 numbers as the set of Δ_2. As mentioned previously, there is no single solution for such a multiobjective problem, therefore, we will face the set of optimal solutions not just one solution. It is worth mentioning here that all the members of the set of optimal solutions are considered as acceptable designs. The Proposed TEB-PSO algorithm will find the optimal set of solutions (optimal threshold

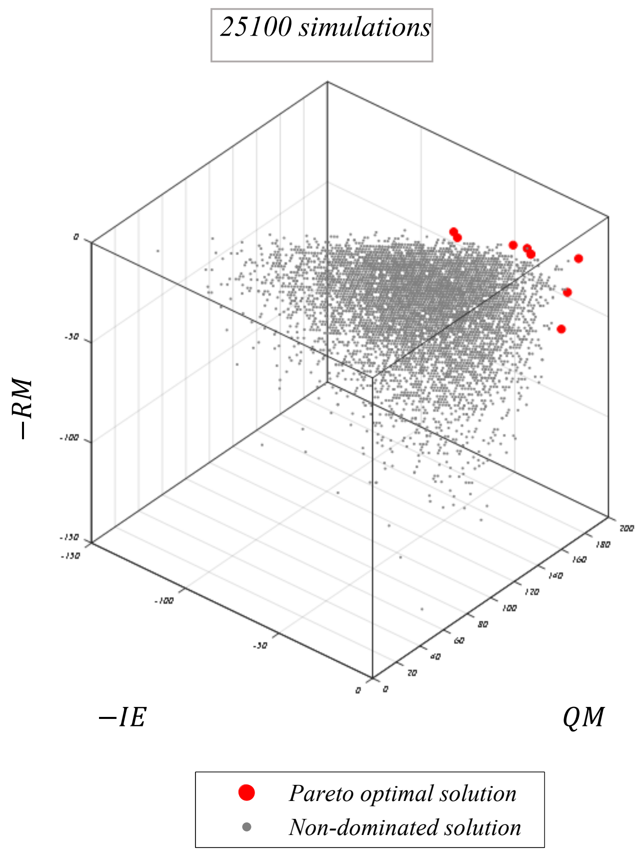


Figure 4.5: Search history of MOPSO with some highlighted designs

values). Fig. 4.5 shows the search history of the TEB-PSO where the marked red points are the optimum solutions. The position mean squared error (MSE) comparison over varying of the (Δ) among the ternary levels of the TEB-PSO can be inferred from Fig. 4.6. It is observed that our proposed model has the potential to significantly reduce the (Δ) rates in comparison with the other filters. The Position MSE comparison over varying communication rate has shown in the Fig. 4.7. As can be seen, it is apparent that the proposed approach results in lower communication rate and can outperform its counterparts. Table 4.2 illustrates the comparison between the Point, Quantized, and Set-valued Measurements over varying values of Δ which is shown the results in nine best optimum solutions. It is observed that the proposed approach has the potential to significantly reduce the overall communication rates by transferring more quantized measurement and reducing

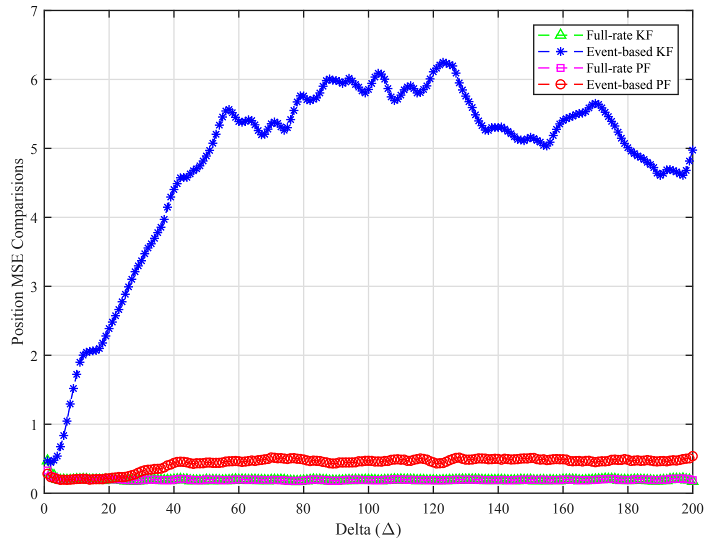


Figure 4.6: The Position MSE comparison over varying of the (∆) among the ternary levels of the TEB-PSO.

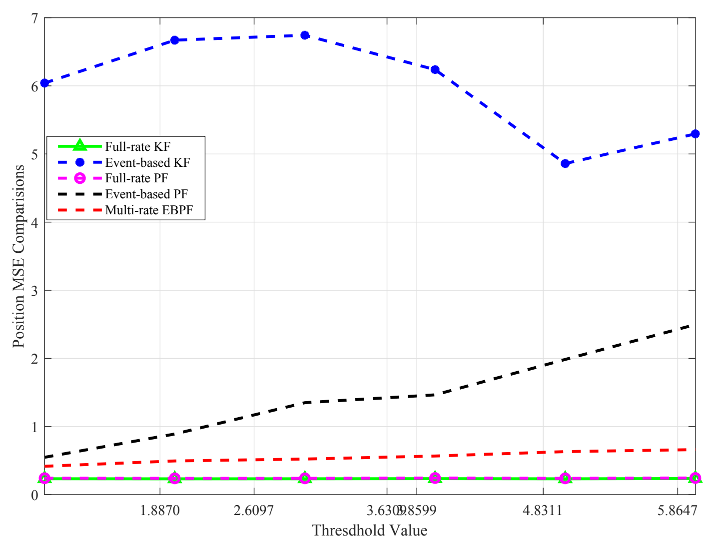


Figure 4.7: The Position MSE comparison over varying communication rate obtained from the proposed TEB-PSO

Table 4.2: Comparison between total number of transmitted measurements in Idle (IE), transient (QE), & row (RM) Epochs.

Items	1	2	3	4	5	6	7	8	9
IE	20	3	46	22	48	4	0	27	20
QM	166	170	145	166	145	181	160	162	166
RM	14	27	9	12	7	15	40	11	14

the transfer of measurement during the event epochs.

4.5 Conclusion

In this chapter, we proposed a ternary event-based particle filtering (TEB-PF) framework where a novel ternary event-triggering (TET) mechanism is proposed coupled with a non-Gaussian fusion strategy that jointly incorporates point, quantized, and set-valued measurements. Furthermore, in order to compute the optimum threshold values for the TET mechanism, we developed a multi objective approach (MOPSO) which results in an overall systematic and intuitively pleasing non-Gaussian distributed event-based estimation algorithm. The proposed framework is applicable to any (TET) mechanism. This method does not require human involvement to provide an initial threshold value to start the optimization. Instead, there is the possibility for this strategy to add set of threshold value in order to achieve a wide range of optimal designs. Finally, the proposed multi-objective framework opens up an effective way which uses hybrid set of information in an intelligent fashion resulting in simultaneous reduction in both MSE and communication overhead.

Chapter 5

Summary and Future Research Directions

The chapter concludes the thesis with a list of important contributions made in the dissertation and some proposed directions for future work.

5.1 Summary of Thesis Contributions

The research work performed in this thesis is motivated by recent advancements and developments in large, distributed, autonomous, and self-aware Cyber-Physical Systems (CPSs) such as autonomous vehicles and vehicle-to-everything (V2X) technologies, where bandwidth, security, privacy, and/or power considerations limit the number of information transfers between neighbouring agents. In short, there exists a critical necessity for preserving restricted communication resources in such CPS application domains. The focus of the thesis is on addressing the identified drawbacks of the existing open-loop (i.e., no feedback communication is incorporated from the FC to local sensors) and event-based estimation (EBE) strategies (i.e., communication is only performed in occurrence of specific events identified via the localized triggering mechanism at the sensor side). In this regard, the thesis made a number of contributions [41–45] as briefly outlined below:

(1) ***Event-Based State Estimation***: In the thesis, three EBE frameworks for Centralized, Hierarchical, and Distributed architectures are proposed as briefly outlined below:

- ***Event-based Particle Filtering with Joint Point & Set-valued Measurements (Centralized Architecture)*** [41]: A centralized state estimation architecture (distributed in the sense that the sensor is not co-located with the remote estimator) is considered where a remote sensor communicates its measurements to the FC in an event-based fashion. Referred to as the event-based particle filter (EBPF), point-valued measurements are incorporated in the estimation recursion via a conventional particle filter formulation, while set-valued measurements are incorporated by developing an observation update step similar in nature to quantized particle filtering approach. More specifically, in the absence of an observation (i.e., having a set-valued measurement), the proposed EBPF evaluates the probability that the unknown observation belongs to the event-triggering set based on its particles which is then used to update the corresponding particle weights. The simulation results show that the proposed EBPF outperforms its counterparts, and confirms the effectiveness of the proposed hybrid estimation algorithm.

  Pros and Cons: The proposed EBPF is a systematic and intuitively pleasing distributed state estimation algorithm, which jointly incorporates point and set-valued measurements within the particle filtering framework. The EBPF considers a single-sensor scenario and depends on a remote estimator (FC) to form the global state estimates. Besides, binary event triggering (the common existing approach) is utilized.

- ***Multi-Sensor EBE with an Information-Based Triggering Mechanism (Hierarchical Architecture)*** [42]: A multi-sensor and open-loop estimation algorithm with an information-based triggering mechanism is proposed based on a hierarchical architecture. In the open-loop topology considered, each sensor transfers its measurements to the FC only in occurrence of specific events (asynchronously). Events are identified using the information-based triggering mechanism without incorporation of a feedback from the FC and/or implementation of a local filter at the sensor level. We propose a multi-sensor triggering approach based on the projection of each local observation into the state-space which corresponds to the achievable gain in the sensor's information state vector. The simulation results show that the proposed multi-sensor information-based triggering mechanism closely follows its full-rate estimation counterpart.

Pros and Cons: The proposed information-based multi-sensor triggering approach is developed based on the projection of each local observation into the state-space which corresponds to the achievable gain in the sensor's information state vector. Therefore, it is expected to have a more accurate representation of the estimator's performance and, consequently, a better criteria for sensor selection. Although, in contrary to the EBPF, multi-sensors are incorporated, still the overall performance of the algorithm depends on the remote estimator implemented at the FC. Besides, the multi-sensor settings came with the cost of using Gaussian approximation for the event-based posterior distribution (a drawback in comparison to the EBPF).

- ***Event-Triggered Diffusion Particle Filter (ET/DPF) (Distributed Architecture)*** [43]: An event-triggered distributed state estimation via diffusion strategies is proposed referred to as the ET/DPF. Each sensor uses a deterministic SOD event triggering mechanism resulting in availability of side information at its neighbouring nodes in the absence of an observation. Utilization of this side information results in distributed estimation with joint set-valued and point-valued measurements, which consequently translates in to a non-Gaussian state estimation problem. The proposed ET/DPF is an intuitively pleasing non-Gaussian estimation framework within the particle filter framework and uses diffusion strategies for distributed implementations. Through proof-of-concept simulations, it is shown that the proposed ET/DPF outperforms its counterparts.

  Pros and Cons: The proposed ET/DPF is a systematic distributed state estimation algorithm without the need for the FC that jointly incorporates point and set-valued measurements within the particle filtering framework. The ET/DPF is multi-sensor and does not require implementation of a FC, which addresses the drawbacks of the previous two algorithms. However, similar to the previous two algorithms and in par with existing literature a binary triggering approach is used where sensors either share or hold on to their local measurements.

(2) ***Ternary-Event-based State Estimation***: The thesis addresses the identified issue of using a binary triggering mechanism within the EBE approaches. The proposed solution is outlined below:

- ***Ternary-Event-Triggering Mechanism (TET) with Joint Point, Quantized, and Set-Valued Measurements*** [44]: We proposed a novel ternary event-based particle filtering (TEB-PF) framework by introducing the ternary event-triggering (TET) mechanism coupled with a non-Gaussian fusion strategy that jointly incorporates point-valued, quantized, and set-valued measurements. Due to joint utilization of quantized and set-valued measurements in addition to the point-valued ones, the proposed TEB-PF simultaneously reduces the communication overhead, in comparison to its binary triggering counterparts, while also improves the estimation accuracy especially in low communication rates.

  Pros and Cons: The proposed TEB-PF is a systematic and intuitively pleasing non-Gaussian distributed estimation algorithm which jointly uses such hybrid set of information in an intelligent fashion resulting in simultaneous reduction in both MSE and communication overhead. The performance of the proposed TET mechanism depends on the values of two pre-defined thresholds, which need to be computed in advance.

- ***Designing Optimal Thresholds for TET Mechanism via Multi Objective Particle Swarm Optimizer*** [45]: To complete our previous work on ternary EBE, we designed a multi-objective approach for optimizing the threshold values used via the TET mechanism. In order to achieve the optimum threshold values in a TET mechanism the Multi-Objective Particle Swarm Optimization (MOPSO) is utilized. The proposed method is capable of finding an optimal set of threshold values to reduce the communication overhead. The developed optimization framework is comprehensive and is able to find a significantly wide range of optimal threshold for a TET mechanism.

5.2 Future Research

Below, potential future research directions are discussed to further improve the proposed contributions made throughout this thesis:

- **EBE with Joint Point, and Set-Valued Measurements**: The proposed centralized, hierarchical and distributed EBPF approaches can be further improved along the following directions:

  (1) The fusion rule in the proposed multi-sensor EBE with an information-based triggering mechanism is based on Kalman filtering. Extensions to incorporate particle filtering can be a direction for future research.

  (2) In the proposed EBPF framework, we used the SOD triggering mechanism, utilization of other triggering mechanisms can be an interesting future research direction.

- **Ternary EBE with Joint Point, Quantized, and Set-Valued Measurements**: The proposed TET mechanism can be further improved along the following directions:

  (1) In the current form, only two thresholds are considered resulting in a three level (hence ternary) triggering. One can extend the proposed solution by considered more decision levels, i.e., development of a multi-level triggering mechanism. In such a multi-level triggering, e.g., the number of quantization bins used within each interval can be increased as the difference between the current observation and the previously transmitted one increases.

  (2) The objective function used to find the optimum values of the threshold values can be further improved. For instance, implementation of an optimization algorithm based on energy consumption can be considered.

  (3) The proposed TET mechanism is developed for a single remote sensor, extension to other distributed architectures and multi-sensor scenarios is another direction for the future research.

Bibliography

[1] R. Baheti, and H. Gill, "Cyber-physical Systems," *The impact of control technology, IEEE Control Systems Society*, vol. 12, No. 1, pp. 161–166, 2011.

[2] J. Baek, E. Hableel, Y. J. Byon, D. S. Wong, K. Jang and H. Yeo, "How to Protect ADS-B: Confidentiality Framework and Efficient Realization Based on Staged Identity-Based Encryption," *IEEE Transactions on Intelligent Transportation Systems*, vol. 18, no. 3, pp. 690-700, 2017.

[3] J. Levinson, J. Askeland, J. Becker, J. Dolson, D. Held, S. Kammel, J. Z. Kolter, D. Langer, O. Pink, V. Pratt, M. Sokolsky, G. Stanek, D. Stavens, A. Teichman, M. Werling, and S. Thrun, "Towards Fully Autonomous Driving: Systems and Algorithms," *2011 IEEE Intelligent Vehicles Symposium (IV)*, Baden-Baden, Germany, pp. 163-168, 2011.

[4] G. Fasano, D. Accardo, A. Moccia, C. Carbone, U. Ciniglio, F. Corraro, and S. Luongo, "Multi-Sensor-Based Fully Autonomous Non-Cooperative Collision Avoidance System for Unmanned Air Vehicles," *Journal of Aerospace Computing, Information, and Communication* , vol.5, pp.338-360, 2008.

[5] D. F. Opila, X. Wang, R. McGee, R.B.Gillespie, J. A. Cook, J. W. Grizzle, "An Energy Management Controller to Optimally Trade off Fuel Economy and Drivability for Hybrid Vehicles," *IEEE Transactions on Control Systems Technology* , vol. 20, No. 6, pp. 1490, 2012.

[6] D. S. Nunes, P. Zhang and J. Silva, "A Survey on Human-in-the-Loop Applications Towards an Internet of All," *IEEE Communications Surveys & Tutorials*, vol. 17, no. 2, pp. 944-965, 2015.

[7] S.Rho, A. V. Vasilakos, W.Chen, "Cyber Physical Systems Technologies and Applications," *Future Generation Computer Systems*, vol. 56, pp. 436-437, 2016.

[8] A. Mohammadi and K. N. Plataniotis, "Event-Based Estimation With Information-Based Triggering and Adaptive Update," *IEEE Trans. Signal Process.*, vol. 65, no. 18, pp. 4924-4939, 2017.

[9] A. Mohammadi and K. N. Plataniotis, "Distributed Widely Linear Multiple-Model Adaptive Estimation," *IEEE Trans. Signal Process.*, vol. 1, no. 3, pp. 164-179, 2015.

[10] A. Mohammadi, A. Asif, "Distributed Consensus + Innovation Particle Filtering for Bearing/Range Tracking With Communication Constraints," *IEEE Trans. Signal Process.*, vol. 63, no. 3, pp. 620-635, 2015.

[11] A. Mohammad and A. Asif, "Diffusive Particle Filtering for Distributed Multisensor Estimation," *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pp. 3801-3805, 2016.

[12] A. Mohammadi and A. Asif, "Distributed Particle Fillter Implementation with Intermittent/Irregular Consensus Convergence," *IEEE Trans. Signal Process.*, vol. 61, pp. 2572-2587, 2013.

[13] Y. Huang, W. Liang, H.B Yu, and Y. Xiao, "Target Tracking based on a Distributed Particle Filter in Underwater Sensor Networks," *Wireless Communications and Mobile Computing*, vol. 8, No. 8, pp. 1023–1033, 2008.

[14] A. Simonetto, T. Keviczky, R. Babuska, "Distributed Non-linear Estimation for Robot Localization using Weighted Consensus," *In IEEE Inter. Con. on Robotics and Automation(ICRA)*, pp. 3026-3031, 2010.

[15] H. Aghajan and A. Cavallaro, "Multi-Camera Networks," *Principles and Applications*, New York: Academic, 2009.

[16] G.G. Rigatos, "Distributed Particle Filtering over Sensor Networks for Autonomous Navigation of UAVs," *Advanced Strategies for Robot Manipulators*, SciYo Publications, Croatia, 2010.

[17] J.C. Eidson, E. A. Lee, S. Matic, S. A. Seshia and J. Zou, "Distributed Real-Time Software for Cyber Physical Systems," *Proceedings of the IEEE (special issue on CPS)* , Vol 100, No. 1, pp. 45-59, 2012.

[18] A. Mohammadi and K. N. Plataniotis, "Non-Circular Attacks on Phasor Measurement Units for State Estimation in Smart Grid," *IEEE Journal of Selected Topics in Signal Processing* , Vol. 17, , pp. 583 - 585, 2013.

[19] C.U. Lei, K. Wan and K.L. Man, "Developing a Smart Learning Environment in Universities Via Cyber-Physical Systems," *Procedia Computer Science Journal*, Vol. 17, , pp. 583 - 585, 2013.

[20] R. Rajkumar, I. Lee, L. Sha and J. Stankovic, "Cyber-physical Systems: the Next Computing Revolution," *Proceedings of the 47th Design Automation Conference (ACM)*, pp. 731-736, 2010.

[21] K. Sampigethaya and R. Poovendran, “Aviation CyberPhysical Systems: Foundations for Future Aircraft and Air Transport,” *Proceedings of the IEEE*, vol. 101, no. 8, pp. 1834-1855, 2013.

[22] A. Ribeiro, G.B. Giannakis and S.I. Roumeliotis, “SOI-KF: Distributed Kalman Filtering With Low-Cost Communications Using the Sign of Innovations,” *IEEE Trans. Signal Processing*, vol. 54, no. 12, pp. 4782-4795, 2006.

[23] K. You, L. Xie, S. Sun, and W. Xiao, “Multiple-Level Quantized Innovation Kalman Filter,” *IFAC Proceedings Volumes*, vol. 41, no. 2, 2008.

[24] J.H. Park and S. Han, “Improved Remote Estimation Over a Capacity-Limited and Lossy Channel,” *IEEE Control Systems Letters*, vol. 1, no. 2, pp. 406-411, 2017.

[25] R.T. Sukhavasi and B. Hassibi, “The Kalman-Like Particle Filter: Optimal Estimation With Quantized Innovations/Measurements,” *IEEE Trans Signal Process.*, vol. 61, no. 1, pp. 131-136, 2013.

[26] D. Shi, T. Chen and L. Shi, “On Set-Valued Kalman Filtering and Its Application to Event-Based State Estimation,” *IEEE Transactions on Automatic Control*, vol. 60, no. 5, pp. 1275-1290, 2015.

[27] D. Shi, T. Chen, and L. Shi, “An Event-Triggered Approach to State Estimation with Multiple point- and set-valued Measurements,” *Automatica*, vol. 50, no. 6, pp. 16411648, 2014.

[28] J. Sijs, M. Lazar, “Event based State Estimation with Time Synchronous Updates,” *IEEE Trans. Autom. Control*, vol. 57, pp. 2650-2655, 2012.

[29] M.G. Cea, G.C. Goodwin, “Event based Sampling in Nonlinear Filtering,” *Control Eng. Practice*, vol. 20, no. 10, pp. 963-971, 2012.

[30] N. Sadeghzadeh-Nokhodberiz, M. Davoodi and N. Meskin, “Stochastic Event-Triggered Particle Filtering for State Estimation,” *Int. Conf. Event-based Control, Comm., & Sig. Process.*, Krakow, pp. 1-4, 2016.

[31] M.A. Sid and S. Chitraganti, “Nonlinear Event-based State Estimation using Particle Filtering Approach,” *Int. Conf. Model., Ident. & Control*, pp. 874-879, 2016.

[32] J. Sijs, B. Noack, and U. Hanebeck, “Event-based State Estimation with Negative Information,” *16th International Conference on Information Fusion*, pp. 2192-2199, 2013.

[33] M. Miskowicz, “Send-on-delta Concept: An Event-based Data Reporting Strategy,” *Sensors*, vol. 6, no. 1, pp. 49-63, 2006.

[34] V.H. Nguyen, Y.S. Suh, "Improving Estimation Performance in Networked Control Systems Applying the Send-on-Delta Transmission Method," *Sensors*, vol. 7, no. 10, pp. 2128-2138, 2007.

[35] J. Wu, Q. Jia, K. Johansson, L. Shi, "Event-based Sensor Data Scheduling: Trade-off Between Communication Rate and Estimation Quality," *IEEE Trans. Autom. Control*, vol. 58, no. 4, pp. 1041-1045, 2013.

[36] D. Shi, T. Chen, L. Shi, "Event-triggered Maximum Likelihood State Estimation," *Automatica*, vol. 50, no. 1, pp. 247-254, 2014.

[37] S. Trimpe and R. D'Andrea, "Event-Based State Estimation with Variance-Based Triggering," *IEEE Trans. Automatic Control*, vol. 59, no. 12, pp. 3266,-281, 2014.

[38] J.W. Marck, J. Sijs, "Relevant Sampling Applied to Event-Based State-Estimation," *Int. Conf. Sensor Technol. & Appl.*, pp. 618-624, 2010.

[39] S. Zhang , H. Chen, M. Liu, and Q. Zhang, "Optimal Quantization Scheme for Data-Efficient Target Tracking via UWSNs using Quantized Measurements," *Sensors* vol. 17, No. 11, 2017.

[40] J. Schlatow, M. Moostl, R. Ernst, M. Nolte, I. Jatzkowski, M. Maurer, C. Herber and A. Herkersdorf, "Self-awareness in Autonomous Automotive Systems," *Design, Automation Test in Europe Conference Exhibition (DATE)* pp. 1050-1055, 2017.

[41] S. Davar and A. Mohammadi, "Event-based Particle Filtering with Point and Set-valued Measurements," *25th European Signal Processing Conference (EUSIPCO)*, pp. 211-215, 2017.

[42] S. Davar and A. Mohammadi, "Multi-sensor and Information-Based Event Triggered Distributed Estimation," *13th International Conference on Distributed Computing in Sensor Systems (DCOSS)*, pp. 130-133, 2017.

[43] S. Davar, A. Mohammadi, and K. N. Plataniotis, "Event-Triggered Particle Filtering Via Diffusion Strategies For Distributed Estimation In Autonomous Systems," *2018 International Conference on Acoustics, Speech, and Signal (ICASP)*, 2018, In Press.

[44] A. Mohammadi, S. Davar and K. N. Plataniotis, "Ternary-Event-Based State Estimation with Joint Point, Quantized, and Set-Valued Measurements," *IEEE Signal Processing Letters*, vol. 25, no. 5, pp. 665-669, 2018.

[45] S. Davar and A. Mohammadi, "Designing Optimal Thresholds for Ternary Event-Based State Estimation via Multi Objective Particle Swarm Optimizer," Accepted in *IEEE International Midwest Symposium on Circuits and Systems (MWSCAS)*, 2018.

[46] R.E. Kalman, "A New Approach to Linear Filtering and Prediction Problems," *J. Basic. Eng.*, vol. 82, no. 1, pp. 35-45, 1960.

[47] S.K. Khaitan and J.D. McCalley, "Design Techniques and Applications of Cyber-physical Systems: A Survey," *IEEE Systems Journal*, vol. 9, no. 2, pp. 350-365, 2015.

[48] J.P. Hespanha, P. Naghshtabrizi, Y. Xu, "A Survey of Recent Results in Networked Control Systems," *Proc. IEEE*, vol. 95, pp. 138-162, 2007.

[49] L. Canzian and M. van der Schaar, "Timely Event Detection by Networked Learners," *IEEE Trans. Signal Process.*, vol. 63, no. 5, pp. 1282-1296, 2015.

[50] D. Han, K. You, L. Xie, J. Wu, L. Shi, "Optimal Parameter Estimation Under Controlled Communication Over Sensor Networks," *IEEE Trans. Signal Process.*, vol. 63, no. 24, pp. 6473-6485, 2015.

[51] K. You and L. Xie, "Kalman Filtering with Scheduled Measurements," *IEEE Trans. Signal Process.*, vol. 61, no. 6, pp. 1520-1530, March 2013.

[52] C. Yang and L. Shi, "Deterministic Sensor Data Scheduling under Limited Communication Resource," *IEEE Trans. Signal Process.*, vol. 59, no. 10, pp. 5050-5056, 2011.

[53] L. Shi, H. Zhang, "Scheduling Two Gauss-Markov Systems: An Optimal Solution for Remote State Estimation under Bandwidth Constraint," *IEEE Trans. Signal Proc.*, vol. 60, no. 4, pp. 2038-2042, 2012.

[54] K. J. Astrom and B. M. Bernhardsson, "Comparison of Riemann and Lebesgue Sampling for First Order Stochastic Systems," *IEEE Conf. on Decision and Control*, vol. 2, pp. 2011-2016, 2002.

[55] Y. Bar-Shalom, X.R. Li, T. and T. Kirubarajan "Estimation with Applications to Tracking and Navigation: Theory, Algorithms, and Software," *New York: John Wiley & Sons*, 2004.

[56] M. S. Arulampalam and S. Maskell and N. Gordon and T. Clapp, "A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking," *IEEE Transactions on Signal Processing*, Vol 50, No. 2, PP. 174-188, 2002.

[57] R. Van der Merwe, A. Doucet, N. de Freitas and E. Wan, "The Unscented Particle Filter," Tech. Rep. CUED/F-INFENG/TR 380, Cambridge University, 2000.

[58] S.Y. Tu and A.H. Sayed, "Diffusion Strategies Outperform Consensus Strategies for Distributed Estimation Over Adaptive Networks," *IEEE Transactions on Signal Processing*, vol. 60, no. 12, pp. 6217-6234, 2012.

[59] D. Han, Y. Mo, J. Wu, S. Weerakkody, B. Sinopoli, L. Shi, "Stochastic Event-Triggered Sensor Schedule for Remote State Estimation," *IEEE Trans. Automatic Control*, vol. 60, no. 10, pp. 2661-2675, Oct. 2015.

[60] A. Mohammadi, A. Asif, X. Zhong and A. B. Premkumar, "Decentralized Bayesian Estimation with Quantized Observations: Theoretical Performance Bounds," *IEEE International Conference on Distributed Computing in Sensor Systems*, Cambridge, MA, 2013, pp. 149-156.

[61] Y.H. Ruan, P. Willett, and A. Marrs, "Fusion of Quantized Measurements via Particle Filtering," *Proceedings of the 2003 Aerospace Conference*, pp. 1967-1978, 2003.

[62] R. Karlsson and F. Gustafsson, "Particle Filtering for Quantized Sensor Information," *European Signal Processing Conference*, 2005.

[63] T. Li, M. Bolic and P. M. Djuric, "Resampling Methods for Particle Filtering: Classification, implementation, and strategies," *IEEE Signal Processing Magazine*, vol. 32, no. 3, pp. 70-86, May 2015.

[64] P.S. Maybeck, "Stochastic Models, Estimation, and Control," *New York: Academic*, 1979.

[65] A.H. Sayed, "Adaptive Networks," *Proceedings of the IEEE*, vol. 102, no. 4, pp. 460-497, 2014.

[66] J. Hu, L. Xie, and C. Zhang, "Diffusion Kalman Filtering Based on Covariance Intersection," *IEEE Transactions on Signal Processing*, vol. 60, no. 2, pp. 891-902, 2012.

[67] F. Cattivelli and A.H. Sayed, "Diffusion Distributed Kalman Filtering with Adaptive Weights," *Asilomar Conference on Signals, Systems and Computers*, pp. 908-912, 2009.

[68] R. Karlsson, T. Schon ,and F. Gustafsson, "Complexity Analysis of the Marginalized Particle Filter," *IEEE Trans. Signal Processing*, vol. 53, no. 11, pp. 4408-4411, 2005.

[69] S. Cui, A. J. Goldsmith, and A. Bahai, "Energy-Constrained Modulation Optimization," *IEEE Trans. Wireless Communications*, vol. 4, no. 5, pp. 2349-2360, 2005.

[70] W.B. Du, W. Ying, G. Yan, Y.B. Zhu and X.B. Cao, "Heterogeneous Strategy Particle Swarm Optimization," *IEEE Trans. Circuits Syst. II, Exp. Brief*, vol. 64, no. 4, pp. 467-471, 2017.

[71] T. Kaur and D. Kumar, "Particle Swarm Optimization-Based Unequal and Fault Tolerant Clustering Protocol for Wireless Sensor Networks," *IEEE Sensors Journal*, vol. 18, no. 11, pp. 4614-4622, 2018.

[72] E. Russell and K. James, "A New Optimizer using Particle Swarm Theory," *International Symposium on Micro Machine and Human Science*, 1995, pp. 39-43.