Essays in Information Transmission and Institution Design

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Abstract

Essays in Information Transmission and Institution Design

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This thesis consists of three essays in information transmission and institution design. Chapter 1 constructs a model of information transmission. The model was set up with an informed sender and two uninformed receivers, where the sender wants to convince the receivers to take a certain action. We analyze strategic information transmission model with two parameters, one is different levels of persuasive function of the channels; another is different degrees of connection between the receivers. We show that if persuasive function is a linear or convex function, the sender should invests all expenditure to one channel with higher level of persuasive function and higher degree of information transmission of the receiver; while if persuasive function is a concave function, three possible optimal behaviours of the sender are investing to one channel, both channels equally, or both channels unequally. Given two concave function examples, we show some decision rules for the sender's optimal expenditure allocation. Specifically, we show that it is not always to allocate expenditure in both channels equally in symmetric model; it is always to invest all expenditure to only one channel when another channel has very low level of persuasive function, or very low degree of connection between the receivers, and it is always to increase expenditure in one channel when the degree of information transmission of the corresponding receiver increases in asymmetric model.

Chapter 2 studies two scenarios in a formal analysis of scientists' effort provision in

research and dissemination. One is a simultaneous problem that the sender offers effort to send signal to two types of audiences, such as experts and public; another is a sequential problem that the sender offers effort in academic research, and then sends signal to one type of audiences to representation with effort in science popularization. We investigate how the scientist should divide their time or energy between academic research and science popularization to obtain maximum utility. Consider the same probability and different probability functions at two dimensional for each scenario. We show the optimal allocation of effort depends on the weight of payoff from academic research and science popularization, and the difference in two probability functions between two signals, or between signal and representation. Specifically, in scenario one, if there exist polarization in academic research and science popularization, we could prevent polarization by increasing the ratio of the weights of payoff from dissemination and research using incentives to guarantee the scientist keep the allocation of effort as before. In scenario two, the result shows that we should put equal effort on research and dissemination for scientific achievements transformation no matter how difference in two probability functions between signal and representation.

Chapter 3 constructs a simple model of direct democracy with supermajority rule and different preference intensities for two sides of a referendum: reform versus status quo. Two parties spend money and effort to mobilize their voters. We characterize the set of pure strategy Nash equilibria. We investigate the optimal majority rule that maximizes voters' welfare. Using an example, we show that if the preference intensity of the status quo side is relatively high, the higher preference intensity of the status quo side, the higher the optimal majority rule. While, if the preference intensity of status quo side is relatively low, the optimal majority rule decreases if the preference intensity of the status quo side increases. We also show that when the preference intensity of the status quo side is higher, or the easiness to mobilize voters on the status quo side is lower, the optimal majority rule is more likely to be supermajority.

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Contributions of Authors

Chapter 2: This paper is co-authored with Dr. Ming Li. He proposed the research idea. We both conceived the research Questions and collected the related literatures. Dr. Li developed the research design and methodology. I interpreted and proved the results, and wrote the manuscript. Dr. Li commented it, and I revised it.

Chapter 3: This paper is co-authored with Dr. Ming Li. We conceived the research idea, collected the related literatures, and developed the research design and methodology together. I interpreted and proved the results, and wrote the manuscript. Dr. Li. revised it critically for important intellectual content. He edited the paper and made corrections to the paper.

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Chapter 1
Strategic Information Transmission in Random Networks: Theory and Applications

-Abstract-

This paper constructs a model of information transmission. The model was set up with an informed sender and two uninformed receivers, where the sender wants to convince the receivers to take a certain action. We analyze strategic information transmission model with two parameters, one is different levels of persuasive function of the channels; another is different degrees of connection between the receivers. We show that if persuasive function is a linear or convex function, the sender should invests all expenditure to one channel with higher level of persuasive function and higher degree of information transmission of the receiver; while if persuasive function is a concave function, three possible optimal behaviours of the sender are investing to one channel, both channels equally, or both channels unequally. Given two concave function examples, we show some decision rules for the sender's optimal expenditure allocation. Specifically, we show that it is not always to allocate expenditure in both channels equally in symmetric model; it is always to invest all expenditure to only one channel when another channel has very low level of persuasive function, or very low degree of connection between the receivers, and it is always to increase expenditure in one channel when the degree of information transmission of the corresponding receiver increases in asymmetric model.

Keywords: Information Transmission, Persuasion, Campaign Advertising

1.1 Introduction

Information transmission plays an important role in economic and social life. At the same time, there are some important issues that have to be attention for the decision maker in information transmission, such as the choice of media, the selection of the channels or programs and so on. For launching a new product or service in the market, the most important is to choose the right channels to disseminate the information of new product for manufacturers in order to draw customer's attention and win customers' trust and favor. For scientific research, it is not only necessary for scientists to timely access to the results of others research, but also in time to publish their own research results across the right channels to gain more attention and responses. In addition, it is significant for politicians to spend effort on the following question: how to promote their claims better in the selection of the channels to obtain higher approval rating.

This paper regards information transmission as sender-receiver games. We investigate the optimal allocation of expenditure on media channels to make the sender to obtain a maximum persuasion power.

Take a presidential election as an example, lots of campaign advertising designed by political consultants influence political debate and voters through the media, which would be sent to larger groups of voters with some effort or expense. The reason is that the images and emotions evoked by campaign advertising could sway voters, which plays a key role in information transmission. This paper believes that a candidate risks missing the persuadable voters he needs to win an election if he spends money on the wrong network. How to choose targets wisely become important for political parties and candidates.

For the choice of media, as can be seen from Table 1.1 and Table 1.2, the total amount of campaign advertising spending is gradually increasing with the years, and the percent share of total political advertisement putting are gradually changing quietly. Specifically, political advertisers treat the Internet as a novelty. In 2008, digital advertising accounted for \$22 million, which was about 0.4 percent of the \$6.2 billion; while, digital accounted for \$1,000 million,

Table 1.1: Total Political Spend ($million)

	2008	2010	2012	2014	2016E
Broadcast	$4,320	$4,122	$5,640	$4,596	$6,055
Cable TV	$468	$493	$939	$719	$1,102
Radio	$553	$464	$819	$485	$827
Print	$644	$715	$874	$787	$848
OOH	$247	$573	$377	$635	$365
Digital	$22	$14	$159	$271	$1,000
Total	$6,254	$6,381	$8,809	$7,494	$10,197

Table 1.2: Percent Share

	2008	2010	2012	2014	2016E
Broadcast	69%	64.6%	64%	61.3%	59.4%
Cable TV	7.5%	7.7%	10.7%	9.6%	10.8%
Radio	8.8%	7.3%	9.3%	6.5%	8.1%
Print	10.3%	11.2%	9.9%	10.5%	8.3%
OOH	3.9%	9.0%	4.3%	8.5%	3.6%
Digital	0.4%	0.2%	1.8%	3.6%	9.8%
Total	100%	100%	100%	100%	100%

Source: Borrell and Associates, Kantar/CMAG, Numura estmates

which was about 10 percent of the $10.1 billion in 2016. It can be seen that digital is starting to eat into traditional media's share. Nomura says that the most of the spending that is moving to digital is going to end up with Alphabet Company and Facebook Company, which will get around $400 million and $350 million in political ads, respectively. Thus it seems very important and necessary to evaluate the effect of advertising expenditure in media.

Table 1.3: Top Recipients of Obama Campaign Online Media Spending in 2008

Media Company	Google	Yahoo	Centro	Advertising.com	Facebook	CNN.com	…
Estimated Amount Paid	$7,500,000	$1,500,000	$1,300,000	$947,000	$643,000	$461,000	…

Consider U.S. President Barack Obama campaign advertising strategy in 2008, Obama's presidential campaign was regarded as a classic of integrated marketing communications in new era, because he combined with video website, search engine advertising and other online methods to set up the relationship with voters by in-depth and interactive approach. He was ultimately successful in obtaining support and trust of the voters, and became the first black president of the United States.

ClickZ, the U.S. market research firm, released a statistical report in November 2008. The total cost of online advertising for the President-elect of the United States, Barack Obama, was over $16 million in 2008. From Table 1.3, it can be seen that the most of these funds went to Google, Yahoo and Facebook. Also, in October, the Obama campaign put a length of 30 minutes of campaign advertising in major commercial television stations, and the cost was $4 million. This shows that his precise campaign ads achieved good promotional effect. Precision is the direction of future development of advertising, which requires us to further improve the advertising algorithms, and to fractionize the market.

For the selection of channel or program, hundreds of channels vying for viewer attention, Figure 1.1 shows top cable networks for political Ads on the 2014 U.S. midterm elections in the television market. We already knew who campaigns are targeting and how they're trying to influence voters. Echelon Insights, a research and analytics firm, used cable TV data to produce the figure of political Ads allocation, which totally analyzed more than 2.6 billion TV spots. Obviously, the number of spots that Republicans bought is larger difference among top cable networks; while the number of spots that Democrats bought is almost equally among top cable networks. In addition, Fox News is the most popular channel with political ad buyers, on which

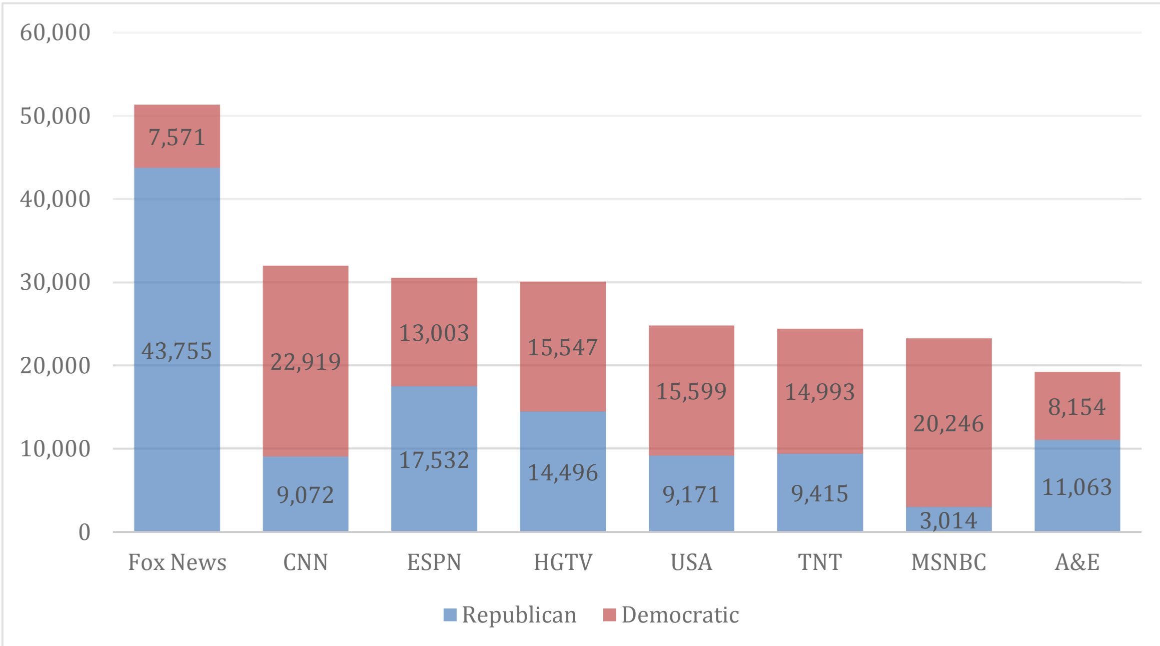


Figure 1.1: Top Cable Networks for Political Ads

Republicans alone bought more than 43,000 spots. Republicans also bought more than 10,000 spots on ESPN, HGTV and A&E. For Democrats, they bought almost 23,000 spots on CNN, which is more than twice the number of Republican ads. Democrats also purchased more than 10,000 spots on ESPN, HGTV, USA, TNT, and MSNBC.

In Networks, large numbers of media and hundreds of channels, it is important for candidates to put more effort on the questions that how to choose suitable media to transit information, and how to optimally allocate expenditure to different TV channels, different newspapers or different websites, and maximize the persuadable voters one needs to win an election. Invest equally, or selectively? Thus it is necessary for us to study strategy information transmission in random networks to enhance utility and value of information.

What we mainly focus on this paper is the optimal allocation of the sender's advertising expenditure on media channels in order to obtain a maximum persuasion power. In addition, what factors affect the sender's maximum persuasion power, and how? Specifically, we discuss different levels of persuasive function of the channels and different degrees of connection between the receivers. The purpose of this paper is to work out some decision rules for the optimal expenditure allocation under some special examples.

Our strategic information transmission model based on Shannon and Weaver model (1949), which is the best-known example of the "informational" approach to communication consisting of six elements: an information source, a transmitter, a channel, physical noise, a receiver, and a destination. In Shannon and Weaver Model, a speaker and a listener would be the source and the destination rather than the transmitter and the receiver. This paper proposes that the participants in the model are commonly humanized as the sender and the receiver. We introduce an information transmission model with participants as the sender and the receiver.

This paper focuses on a reduced-form approach to an information transmission model, which is constructed with one informed sender and two uninformed receivers, where the sender wants to convince receivers to take a certain action. The sender allocates expenditure to convey truthful information to the receivers by advertising in the channels, and the receivers update their beliefs rationally on the basis of the information that they obtained from own channel and other channel's receiver. In order to convincing the receiver's action, this paper introduces two parameters, which are the level of persuasive function of the channels and the degree of connection between the receivers.

One reason is that different channels have different levels of information transmission. For example, national channels and local channels have various degrees of impact for audiences in general. Specifically, the left party's channel has larger persuasion power for the left party's information, and smaller persuasion power for the right party's information; similarly for the right party's channel. Also, News channel has a larger probability to succeed in persuading political information than entertainment channel, while entertainment channel has a larger probability to succeed in persuading entertaining information than News channel. Thus it seems necessary for us to distinguish it by introducing a parameter. Another reason is for connection between receivers, each receiver has own degree of information transmission. For example, we are more likely to listen and follow what leaders and experts are saying. Thus they have higher degree of information transmission than general person.

The seminal paper on strategic information transmission by Crawford and Sobel (1982) is about a sender-receiver cheap talk game with soft information. Cheap-talk models address the question of how much information can be credibly transmitted when communication is direct and costless. Farrell and Gibbons (1989) develop the simplest model of cheap talk with two audiences. They show how costless, non-verifiable claims can affect the receivers' beliefs and how the incentives for truthful revelation to one receiver are affected by the presence of the other. Goltsman and Pavlov (2011) are closely related to Farrell and Gibbons (1989), who compare private and public communication in the cheap talk model and gave a considerable generalization. Caillaud and Tirole (2007) build one sender and multi-receivers model of persuasion, and explore how sponsors of ideas or projects should design their strategies to obtain favorable group decisions.

This paper considers information acquisition is costly. The sender invests expenditure for advertising in media channels, which has a slight different with cheap talk models above mentioned. Some recent literatures on costly information transmission are as follows. Li and Li (2013) assume information campaigns are costly and study two privately informed political candidates, while, this paper only focuses only one sender. Gentzkow and Kamenica (2014) analyze Bayesian persuasion with costly signals. The cost of a signal is proportional to the expected reduction in uncertainty relative to some fixed reference belief. Degan and Li (2017) assume the higher costs associated with higher precision and study a sender's optimal choice of precision. Here we analyze the sender's optimal allocation of expenditure in this paper.

Some literatures provide some lights on advertising. Bagwell (2007) discusses the persuasive, informative and complementary views of advertising to answer the question that why consumers respond to advertising. Specially, two mechanisms make advertisements informative. First, Coate (2004b) assumes that advertising contains hard information, and information that cannot be falsified. Advertising is directly informative, similar assumption is adopted in Ashworth (2003), Schultz (2007) and Tirole (1988). The second is indirectly informative advertising. Milgrom and Roberts (1986), Gerber (1996) and Prat (2002) show that equilibria with informative advertising exist, even though the advertisings have no direct information content.

This paper adopts persuasive function to represent persuasion power of the channels. We assume the sender has records that reveal own qualifications and the sender cannot lie about own records, i.e. the sender represents truthful information. The idea is similar to campaign advertising's rules from Coate (2004a, b), and others' work, such as Dixit and Norman (1978) and Becker and Murphy (1993). In addition, Santilli (1983) shows that persuasive advertising using rational means is moral as long as the product or service it represents is good or useful, and argues that advertisements which present information in a straightforward and truthful way are always moral no matter what they advertise.

Consider connection between receivers, the idea of it is from social proof in persuasion method; the reason is that we are influenced by others around us. Specifically in uncertain or ambiguous situation, we are likely to conform to what others do. In addition, persuasion is reciprocity because we dislike people who neglect to return a favor when offered a free service. We are also more likely to be persuaded by people we see as similar to ourselves. Here, we assume the receivers can persuade with each other.

The main purpose of this paper is to study the sender's optimal expenditure allocation. Chamberlin (1933) suggests that the purpose of advertising (persuasive or informative) and the extent of scale economies warrant greatest attention. So we focus on investigate persuasive function. If persuasive function is a linear or convex function, the sender should invests all expenditure to one channel with higher level of persuasive function and higher degree of information transmission for the receiver to reach a maximum power of persuasion; while if persuasive function is a concave function, behavior of the sender should depend on the levels of persuasive function of the channels and the degrees of connection between receivers. Thus our main results focus on concave function.

Given two particular examples, we mainly analyze them under symmetric environment with same levels of persuasive function of the channels and same levels of connection between receivers. Then we study other two asymmetric environments, one is with different levels of persuasive function of the channels and same levels of connection between receivers, and another is with different levels of connection between receivers and same levels of persuasive function of the channels. Some decision rules for the sender's optimal expenditure allocation are given in this paper.

Specifically, we show that it is not always to allocate expenditure in both channels equally in symmetric model; it is always to invest all expenditure to only one channel when another channel has very low level of persuasive function, or very low degree of connection between receivers, it is always to increase expenditure in one channel when the degree of information transmission of the corresponding receiver increases, and the sender is more likely to invest all expenditure to one channel when the degree of information transmission for another channel's receiver is lower enough in asymmetric model.

The rest of this paper is organized as follows. Section 2 sets up strategic information transmission model with two channels and studies some general results. Section 3 investigates two special examples of concave function in symmetric form model. Section 4 extends symmetric form to asymmetric form model for two examples given in section 3. Section 4 concludes. All the proofs of the results are in Appendix.

1.2 Model

Based on Shannon and Weaver (1949) model of communication, we adopt a reduced-form approach to construct strategic information transmission model. Consider strategic information transmission model with two channels, each channel has one loyal receiver, i.e. watching only in one channel. Receiver i is one type of audiences for the channel i, $i = 1,2$. Here we have three players, one informed sender and two uninformed receivers.

The informed sender transmits truthful information to both receivers by media separately. Assume information conveyed by the sender plays a positive role in society, the more information acquired is, the higher probability it is to persuade the receivers. For example, if a manufacture releases more information about nice characterizes of new product, then the consumer informed will be more likely to buy it; if an insurance company releases more information about details of insurance policies, then the people informed will be more likely to insure.

Introduce a functional form λ to measure the efficacy of information transmission technology of media, and then a persuasive function of media is denoted by $\lambda(\cdot) \in [0,1]$, which is continuous, increasing, twice differentiable, and satisfies $\lambda(0) = 0$. Also, different channels have different levels on information transmission because of its own characteristics in reality. For example, News channel has a larger probability to persuade political information than Sports channel. We introduce a parameter $\alpha_i \in [0,1]$ to represent the level of the channel i's persuasive function, then the persuasive functions of both channels are denoted by $\alpha_i\lambda(\cdot), i = 1,2$. Given total expenditure C, if the sender spends an amount C_i to the channel i, we have $C = \sum_{i=1}^{2} C_i$ and the persuasive function of the channel i is $\alpha_i\lambda(C_i), i = 1,2$. If $\alpha_1 = \alpha_2$, there exits symmetric level of both channels' persuasive function. If $\alpha_1 = \alpha_2 = 0$, it has no significant for both channels' persuasive functions; while if $\alpha_1 = \alpha_2 = 1$, there exits symmetric full level of both channels' persuasive functions.

Consider connection between both receivers, such as colleague, friend, relative, in other words, the receivers can exchange information with a certain degree. However, each receiver has own degree of information transmission. For example, a leader has higher degree of information transmission than the members. Thus we assume the parameter of transmitting information from receiver 1 to receiver 2 is denoted by $\rho_{12} \in [0,1]$, and from receiver 2 to receiver 1 is denoted

$$\text{Channel 1} \xrightarrow{\alpha_1\lambda(C_1)} \text{Receiver 1}$$

$$\text{Sender } (C) \xrightarrow{\text{Information}} \text{medium} \qquad\qquad \downarrow \rho_{12}, \uparrow \rho_{21}$$

$$\text{Channel 2} \xrightarrow{\alpha_2\lambda(C_2)} \text{Receiver 2}$$

Figure 1.2: Strategic Information Transmission Model with Two Channels

by $\rho_{21} \in [0,1], i, j = 1,2$. If $\rho_{12} = \rho_{21}$, there exists symmetric connection between both receivers. If $\rho_{12} = \rho_{21} = 0$, there does not exist connection between both receivers; while if $\rho_{12} = \rho_{21} = 1$, there exists full connection between both receivers.

In Figure 1.2, $\alpha_1\lambda\,(C_1), \alpha_2\lambda\,(C_2)$ are regarded as direct persuasion, which are the probabilities obtained from receivers' own channel; $\alpha_2\,\rho_{21}\lambda(C_2), \alpha_1\,\rho_{12}\lambda(C_1)$ are regarded as indirect persuasion, which are the probabilities obtained from the receivers in other channels; $\alpha_1\alpha_2\rho_{21}\lambda\,(C_1)\lambda\,(C_2), \alpha_1\alpha_2\rho_{12}\lambda\,(C_1)\lambda\,(C_2)$ are regarded as overlapped persuasion, which are overlapping pieces of information's probabilities obtained by one receiver from other channel's receiver. Receivers update their beliefs rationally from own channel and other channel's receiver, and take an action. The probability functions of the receivers persuaded are set as follows

$$Receiver\ 1: \pi_1(C_1, C_2) = \alpha_1\lambda(C_1) + \alpha_2\rho_{21}\lambda(C_2) - \alpha_1\alpha_2\rho_{21}\lambda(C_1)\lambda(C_2),$$

$$Receiver\ 2: \pi_2(C_1, C_2) = \alpha_2\lambda(C_2) + \alpha_1\rho_{12}\lambda(C_1) - \alpha_1\alpha_2\rho_{12}\lambda(C_1)\lambda(C_2),$$

$$where\ C = C_1 + C_2, \alpha_1, \alpha_2 \in [0,1], \rho_{12}, \rho_{21} \in [0,1].$$

The objective of the sender is to obtain optimal allocation of expenditure in media channels in order to reach a maximum persuasive power for convincing all receivers take a certain action.

Problem *The maximization problem for the sender in strategic information transmission model with two channels can be written as follows*

$$U(C_1) = MAX_{C_1}\alpha_1(1 + \rho_{12})\lambda(C_1) + \alpha_2(1 + \rho_{21})\lambda(C - C_1) - \alpha_1\alpha_2(\rho_{21} + \rho_{12})\lambda(C_1)\lambda(C - C_1)$$

$$where\ C_1 \in [0, C], \alpha_1, \alpha_2 \in [0,1], \rho_{12}, \rho_{21} \in [0,1].$$

For above Problem, since the maximization problem for the sender is equal to maximize the sum of the probability functions of both receivers persuaded, which can be written as

$$U(C_1, C_2) = MAX_{C_1,C_2}\pi_1(C_1, C_2) + \pi_2(C_1, C_2)$$

$$subject\ to\ C = C_1 + C_2.$$

Simplifying the probability functions of both receivers persuaded using the budget constraint $C_2 = C - C_1$, we have

$$Receiver\ 1 : \pi_1(C_1) = \alpha_1\lambda(C_1) + \alpha_2\rho_{21}\lambda(C - C_1) - \alpha_1\alpha_2\rho_{21}\lambda(C_1)\lambda(C - C_1),$$

$$Receiver\ 2 : \pi_2(C_1) = \alpha_2\lambda(C - C_1) + \alpha_1\rho_{12}\lambda(C_1) - \alpha_1\alpha_2\rho_{12}\lambda(C_1)\lambda(C - C_1).$$

Simplifying the objective function, we have

$$U(C_1) = MAX_{C_1}\pi_1(C_1, C - C_1) + \pi_2(C_1, C - C_1).$$

Substituting the probability functions of receivers persuaded, we have

$$U(C_1) = MAX_{C_1}\alpha_1(1 + \rho_{12})\lambda(C_1) + \alpha_2(1 + \rho_{21})\lambda(C - C_1) - \alpha_1\alpha_2(\rho_{21} + \rho_{12})\lambda(C_1)\lambda(C - C_1)$$

$$where\ C_1 \in [0, C], \alpha_1, \alpha_2 \in [0,1], \rho_{12}, \rho_{21} \in\ [0,1].$$

Property 1 *If the level of the channel's persuasive function or the parameter of information transmission between the receivers increases, the maximum persuasive power obtained by the sender will increase.*

Checking first derivative with respect to α_i, we have

$$\frac{dU(C_1)}{d\alpha_i} = \left(1 + \rho_{-i,i}\right)\lambda(C_i) - \alpha_{-i}\left(\rho_{-i,i} + \rho_{i,-i}\right)\lambda(C_i)\lambda(C - C_i)\ > 0.$$

Note that if the level of one channel's persuasion function increases, which means it is easier to persuade the receivers, the sender will obtain higher persuasive power. Then it is better for the sender to lead the channels to improve their own advertising technology level.

Checking first derivative with respect to $\rho_{i,-\,i}$, we have

$$\frac{dU(C_1)}{d\rho_{i,-i}} = \alpha_i\lambda(C_i) - \alpha_i\alpha_{-i}\lambda(C_i)\lambda(C - C_i) > 0.$$

Note that the parameter of information transmission between the receivers increases, which means there exists a positive interaction between both receivers. Then one receiver would be more likely to trust and take the same behavior of another receiver with larger probability, and the sender will obtain a larger persuasion power. Thus it is better for the sender to stimulate information transmission between the receivers by some activities, such as sharing feedback, debates and knowledge contests among receivers.

For the objective function of the sender, first derivative with respect C_1, we have

$$\frac{dU(C_1)}{dC_1} = \alpha_1(1+\rho_{12})\lambda'(C_1) - \alpha_2(1+\rho_{21})\lambda'(C-C_1) - \alpha_1\alpha_2(\rho_{21} + \rho_{12})[\lambda'(C_1)\lambda(C-C_1) - \lambda(C_1)\lambda'(C-C_1)].$$

Second derivative with respect C_1, we have

$$\frac{d^2U(C_1)}{d^2C_1} = \alpha_1(1+\rho_{12})\lambda''(C_1) + \alpha_2(1+\rho_{21})\lambda''(C-C_1) - \alpha_1\alpha_2(\rho_{21} + \rho_{12})[\lambda''(C_1)\lambda(C-C_1) - 2\lambda'(C_1)\lambda'(C-C_1) + \lambda(C_1)\lambda''(C-C_1)].$$

From the first derivative and second derivative, we notice that the result of the sender's maximization problem is related with the properties of the persuasive function $\lambda(\cdot)$.

In addition, as we mentioned above, the receivers can be persuaded by different types of persuasive function depend on information transmission technology of media, which is related with means of disseminating information, the modes of issuing information, the way of organizing information and so on. With the rapid development of information transmission technology, media has become more diverse in transmitting information. There are currently two classifications. One is electronic, such as telephone, radio, television and Internet. Another is non-electronic, like newspaper, magazines, textbook etc. Chamberlin (1933) suggests us to focus on the extent of scale economies. Then we simply classify the persuasive function of media by property of function. If the persuasive function is assumed to be linear or convex, it is constant or increasing return to scale; while, if the persuasive function is assumed to be concave, it is decreasing return to scale.

Proposition 1 *In strategic information transmission model with two channels, if* $\lambda(\cdot)$ *is a linear or convex function, the sender should invest all expenditure to one channel with higher level of persuasive function and higher degree of information transmission for the receiver to reach maximum persuasion power.*

Note that $\lambda(\cdot)$ is a linear or convex function if and only if $\lambda''(\cdot) \geq 0$. Checking second derivative of the objective function

$$\alpha_1(1+\rho_{12})\lambda''(C_1) + \alpha_2(1+\rho_{21})\lambda''(C-C_1) - \alpha_1\alpha_2(\rho_{21} + \rho_{12})[\lambda''(C_1)\lambda(C-C_1) - 2\lambda'(C_1)\lambda'(C-C_1) + \lambda(C_1)\lambda''(C-C_1)].$$

$$where\ C_1 \in [0,C], \alpha_1, \alpha_2, \rho_{12}, \rho_{21} \in [0,1], \lambda(\cdot) \in [0,1],\ \lambda'(\cdot) \geq 0,\ \lambda''(\cdot) \geq 0.$$

Rearrange above expression, we have that

$$\lambda''(C_1)[\alpha_1(1+\rho_{12}) - \alpha_1\alpha_2(\rho_{21}+\rho_{12})\lambda(C-C_1)] + 2\alpha_1\alpha_2(\rho_{21}+\rho_{12})\lambda'(C_1)\lambda'(C-C_1) + \lambda''(C-C_1)[\alpha_2(1+\rho_{21}) - \alpha_1\alpha_2(\rho_{21}+\rho_{12})\lambda(C_1)] \geq 0.$$

We find direct and indirect persuasion denoted by $[\alpha_1(1+\rho_{12})\lambda''(C_1) + \alpha_2(1+\rho_{21})\lambda''(C - C1$ dominate overlapped persuasion denoted by $\alpha_1\alpha_2(\rho_{21}+\rho_{12})[\lambda''(C_1)\lambda(C-C_1) - 2\lambda'(C_1)\lambda'(C-C_1) + \lambda(C_1)\lambda''(C-C_1)]$. The second derivative of the objective function is great or equal to zero, and the objective function is convex in C_1, so a critical value of C_1 is a global minimum value. Then we ignore first order condition. The objective function reaches maximum value when $C_1 = 0\ or\ C$. There exist two corner solutions, and then the sender should spend all expenditure C to one of the channels. The corner solutions $C_1 = 0\ or\ C$ are two local maximum points. According to Property 1, we know

$$\frac{dU(C_1)}{d\alpha_i} > 0, \frac{dU(C_1)}{d\rho_{i,-i}} > 0.$$

Then the sender should spend all expenditure C to one channel with higher level of persuasive function and higher degree of information transmission for the receiver to obtain a maximum persuasion power.

Proposition 2 *In strategic information transmission model with two channels, if* $\lambda\ (\cdot)$ *is a concave function, the optimal expenditure allocation for the sender depends on the levels of the channels' persuasive function and the degrees of information transmission between the receivers. Three optimal choices could be adopted, allocating expenditure to one channel, both channels equally, or both channels unequally.*

Since $\lambda\ (\cdot)$ is a concave function, it is decreasing return to scale, we have

$$\lambda'(C_1) \geq \lambda'(C - C_1)\ when\ C_1 \in [0, C/2],$$
$$\lambda'(C_1) \leq \lambda'(C - C_1)\ when\ C_1 \in [C/2, C].$$

Specially,

- $If\ \rho_{12} = \rho_{21} = 0, \alpha_1 = \alpha_2,$

$$C_1 \in [0, C/2], then \frac{dU(C_1)}{dC_1} = \alpha_1\lambda'(C_1) - \alpha_2\lambda'(C - C_1) \geq 0,$$
$$C_1 \in [C/2, C], then \frac{dU(C_1)}{dC_1} = \alpha_1\lambda'(C_1) - \alpha_2\lambda'(C - C_1) \leq 0,$$
$$\frac{d^2U(C_1)}{d^2C_1} = \alpha_1\lambda''(C_1) + \alpha_2\lambda''(C - C_1) < 0.$$

Thus $C_1 = C/2$ is a maximum point. The sender definitely allocates both channels equally.

- $If\ \rho_{12} = \rho_{21} = 1, \alpha_1 = \alpha_2 = 1,$

$$\frac{dU(C_1)}{dC_1} = 2\lambda'(C_1) - 2(C - C_1) - 2[\lambda'(C_1)\lambda(C - C_1) - \lambda(C_1)\lambda'(C - C_1)].$$

Consider the point $C_1 = 0$,

$$\left.\frac{dU(C_1)}{dC_1}\right|_{C_1=0} = 2\lambda'(0) - 2\lambda'(C) - 2\lambda'(0)\lambda(C).$$

When direct persuasion and indirect persuasion of both receivers denoted by $[2\lambda'(0) - 2\lambda'(C)]$ dominate overlapped persuasion denoted by $2\lambda'(0)\lambda(C)$, we have $\frac{dU(C_1)}{dC_1} > 0$, $C_1 = 0$ is a local minimum point; While, when direct persuasion and indirect persuasion of both receivers is dominated by overlapped persuasion, we have $\frac{dU(C_1)}{dC_1} < 0$, $C_1 = 0$ is a local maximum point. Thus the sender will not always allocate both channels equally.

In general, note that $\lambda(\cdot)$ is a concave function if and only if $\lambda''(\cdot) \geq 0$. Checking second derivative of the objective function

$$\frac{d^2U(C_1)}{d^2C_1} = \alpha_1(1+\rho_{12})\lambda''(C_1) + \alpha_2(1+\rho_{21})\lambda''(C-C_1) - \alpha_1\alpha_2(\rho_{21} + \rho_{12})[\lambda''(C_1)\lambda(C-C_1) - 2\lambda'(C_1)\lambda'(C-C_1) + \lambda(C_1)\lambda''(C-C_1)].$$

Because

$$\alpha_1(1+\rho_{12})\lambda''(C_1) + \alpha_2(1+\rho_{21})\lambda''(C-C_1) \leq 0,$$

$$\alpha_1\alpha_2(\rho_{21}+\rho_{12})[\lambda''(C_1)\lambda(C-C_1) - 2\lambda'(C_1)\lambda'(C-C_1) + \lambda(C_1)\lambda''(C-C_1)] \leq 0.$$

When $\alpha_1, \alpha_2, \rho_{21}, \rho_{12}$ are higher enough, overlapped persuasion dominates direct and indirect persuasion, and then the second order condition is great than 0. There may exist corner solution. The sender would likely to invest expenditure one of the channels. While, when $\alpha_1, \alpha_2, \rho_{21}, \rho_{12}$ is lower enough, overlapped persuasion is dominated by direct and indirect persuasion, the second order condition is less than 0. There exits internal solution. The sender would likely to invest expenditure to both channels.

1.3 Concave Function Examples in Symmetric Form Model

Although the sender conveys the truthful information, media has certain selectivity and tendencies for perception of information according to its own attitude, experience and so on. Thus there exists altered, omitted, or re-organized information during transmitting from the sender to media and then to receivers, resulting in information distortion. We introduce the parameter K to represent the information distortion in persuasive function, and assume higher K leads to lower persuasive function of media. Then the persuasive function is denoted by $\lambda(c, K)$, which is continuous, twice differentiable, and satisfies the properties

$$\frac{\partial\lambda(c,K)}{\partial K} < 0, \frac{\partial\lambda(c,K)}{\partial c} \geq 0, \lambda(0,K) = 0, K > 0.$$

Giving two concave functions, we consider symmetric strategic information transmission model with two channels. Assume we have same levels of both channels' persuasive functions, which are equal to 1; and same degrees of information transmission for both receivers, which are denoted by $\rho, \rho \in [0,1]$.

1.3.1 Example 1 Consider the following concave persuasive function of media

$$\lambda(c) = \frac{c}{c+K}, K \geq 0.$$

The maximization problem for the sender is written as

$$MAX_{C_1}(1+\rho)\left[\frac{C_1}{(C_1+K)} + \frac{(C-C_1)}{(C-C_1+K)}\right] - 2\rho\frac{C_1}{(C_1+K)}\frac{(C-C_1)}{(C-C_1+K)}$$

$$where\ C_1 \in [0,C], \rho \in [0,1], K \geq 0.$$

Theorem 1 *The optimal allocation of expenditure for the sender in Example 1 is allocating expenditure to both channels equally in symmetric strategic information transmission model.*

First derivative with respect to C_1, we have

$$\frac{dU(C_1)}{dC_1} = (1+\rho)\left[\frac{K}{(C_1+K)^2} - \frac{K}{(C-C_1+K)^2}\right] - 2\rho\left[\frac{K}{(C_1+K)^2}\frac{(C-C_1)}{(C-C_1+K)} - \frac{C_1}{(C_1+K)}\frac{K}{(C-C_1+K)^2}\right] = 0.$$

$$\frac{K}{(C_1+K)^2(C-C_1+K)^2}(C+2K-\rho K)(C-2C_1) = 0,$$

we obtain a critical value of $C_1 = C/2$.

Specially, consider the point $C_1 = 0$, we find the first order condition is always great than 0 at the point $C_1 = 0$, so $C_1 = 0$ is a local minimum point. Please see the following

$$\left.\frac{dU(C_1)}{dC_1}\right|_{C_1=0} = (1+\rho)\left[\frac{K}{K^2} - \frac{K}{(C+K)^2}\right] - 2\rho\frac{K}{K^2}\frac{C}{(C+K)}$$

$$= \frac{1}{K(C+K)^2}\left[(1+\rho)((C+K)^2 - K^2) - 2\rho C(C+K)\right]$$

$$= \frac{1}{K(C+K)^2}C(1+2K-\rho) > 0.$$

Second derivative with respect to C_1, we have

$$\frac{d^2U(C_1)}{d^2C_1} = (1+\rho)\left[\frac{-2K}{(C_1+K)^3} + \frac{-2K}{(C-C_1+K)^3}\right]$$

$$- 2\rho\left[\frac{-2K(C-C_1)}{(C_1+K)^3(C-C_1+K)} - 2\frac{KK}{(C_1+K)^2(C-C_1+K)^2}\right.$$

$$\left.+ \frac{-2KC_1}{(C_1+K)(C-C_1+K)^3}\right]$$

$$= \frac{-2K(C+2K-\rho C)}{(C_1+K)^3(C-C_1+K)^3}\left[(C_1+K)(C-C_1+K) + (C-2C_1)^2\right] < 0.$$

The second derivative of the objective function is less than zero, so the objective function is concave in C_1. Then a critical value of $C_1 = C/2$ is a global maximum value. It's an internal solution. The sender should invest expenditure C to both channels equally.

1.3.2 Example 2 Consider the following concave persuasive function of media

$$\lambda(c) = \frac{ln\,(c+1)}{lnK}, K \geq C+1.$$

The maximization problem for the sender is written as

$$MAX_{C_1}(1+\rho)\left[\frac{ln(C_1+1)}{lnK} + \frac{ln(C-C_1+1)}{lnK}\right] - 2\rho\left[\frac{ln(C_1+1)}{lnK}\right]\left[\frac{ln(C-C_1+1)}{lnK}\right].$$

$$where\ C_1 \in [0,C], \rho \in [0,1], K \geq C+1$$

First derivative with respect to C_1,

$$\frac{dU(C_1)}{dC_1} = (1+\rho)\left[\frac{1}{(C_1+1)lnK} - \frac{1}{(C-C_1+1)lnK}\right]$$

$$- 2\rho\left[\frac{1}{(C_1+1)lnK}\frac{ln(C-C_1+1)}{lnK} - \frac{ln(C_1+1)}{lnK}\frac{1}{(C-C_1+1)lnK}\right]$$

- **Consider the point $C_1 = 0$**, checking the sign of $\left.\frac{dU(C_1)}{dC_1}\right|_{C_1=0}$

$$\left.\frac{dU(C_1)}{dC_1}\right|_{C_1=0} = (1+\rho)\left[\frac{1}{lnK} - \frac{1}{(C+1)lnK}\right] - 2\rho\frac{1}{lnK}\frac{ln(C+1)}{lnK}$$

$$= \frac{1}{lnKlnK(C+1)}[ClnK - \rho(2(C+1)\,ln(C+1) - ClnK)],$$

We have the results as follows

Lemma 1

1. $If\ \rho \in \left[0, \frac{ClnK}{2(C+1)\,ln(C+1)-ClnK}\right], K \in \left[C+1, (C+1)^{(C+1)/C}\right], then\ \left.\frac{dU(C_1)}{dC_1}\right|_{C_1=0} \geq 0.$

   $C_1 = 0\ is\ a\ local\ minimum\ point;$

2. $If\ \rho \in \left[\frac{ClnK}{2(C+1)\,ln(C+1)-ClnK}, 1\right], K \in \left[C+1, (C+1)^{(C+1)/C}\right], then\ \left.\frac{dU(C_1)}{dC_1}\right|_{C_1=0} \leq 0.$

   $C_1 = 0\ is\ a\ local\ maximum\ point.$

3. $Specially, if\ K \in \left((C+1)^{(C+1)/C}, +\infty\right), \frac{ClnK}{2(C+1)\,ln(C+1)-ClnK} > 1, then\ \left.\frac{dU(C_1)}{dC_1}\right|_{C_1=0} \geq 0.$

   The first order condition is always great than 0 regardless ρ $at\ the\ point\ C_1 = 0,$

   $C_1 = 0\ is\ always\ a\ local\ minimum\ point.$

We find the first order condition at the point $C_1 = 0$ is not always great than 0 like Example 1. When ρ is higher enough, the indirect and overlapped persuasion dominate direct persuasion, the first order condition is less than or equal to 0. Then the sender is more likely to invest one of the channels.

- **Consider the point $C_1 = C/2$**, checking first derivative with respect to C_1, we have

$$\left.\frac{dU(C_1)}{dC_1}\right|_{C_1=C/2} = (1+\rho)\left[\frac{1}{(C/2+1)lnK} - \frac{1}{(C-C/2+1)lnK}\right]$$
$$-2\rho\left[\frac{1}{(C/2+1)lnK}\frac{ln(C-C/2+1)}{lnK} - \frac{ln(C/2+1)}{lnK}\frac{1}{(C-C/2+1)lnK}\right] = 0$$

Then $C_1 = C/2$ is an extreme value point.

Checking second derivative with respect to $C_1 = C/2$,

$$\left.\frac{d^2U(C_1)}{d^2C_1}\right|_{C_1=C/2}$$

$$= \frac{(1+\rho)}{\mathrm{lnK}}\left[-\frac{1}{(\mathrm{C}/2+1)^2}-\frac{1}{(\mathrm{C}-\mathrm{C}/2+1)^2}\right]$$
$$-\frac{2\rho}{\mathrm{lnKlnK}}\left[-\frac{\ln(\mathrm{C}-\mathrm{C}/2+1)}{(\mathrm{C}/2+1)^2}-2\frac{1}{(\mathrm{C}/2+1)(\mathrm{C}-\mathrm{C}/2+1)}-\frac{\ln(\mathrm{C}/2+1)}{(\mathrm{C}-\mathrm{C}/2+1)^2}\right]$$
$$= \frac{-2}{\mathrm{lnKlnK}(\mathrm{C}/2+1)^2}\left[lnK-\rho\left(-lnK+2\ln\left((2/\mathrm{C}+1)e\right)\right)\right].$$

We have the results as follows

Lemma 2

1. $If\ \rho \in \left[0, \frac{lnK}{2\ln((2/\mathrm{C}+1)e)-lnK}\right], K \in [C+1, (2/\mathrm{C}+1)e], then\ \left.\frac{d^2U(C_1)}{d^2C_1}\right|_{C_1=2/C} \leq 0.$

   $C_1 = C/2$ is a local maximum point.

2. $If\ \rho \in \left[\frac{lnK}{2\ln\left(\left(\frac{2}{\mathrm{C}}+1\right)e\right)-lnK}, 1\right], K \in \left[C+1, \left(\frac{2}{\mathrm{C}}+1\right)e\right], then\ \left.\frac{d^2U(C_1)}{d^2C_1}\right|_{C_1=\frac{2}{C}} \geq 0.$

   $C_1 = C/2\ is\ a\ local\ minimum\ point.$

3. $Specially, if\ K \in \left((2/\mathrm{C}+1)e, +\infty\right), \frac{lnK}{2\ln((2/\mathrm{C}+1)e)-lnK} > 1, then\ \left.\frac{dU(C_1)}{dC_1}\right|_{C_1=0} \leq 0.$

   The second order condition is always less than 0 regardless $\rho\ at\ point\ C_1 = C/2,$

   $C_1 = C/2\ is\ always\ a\ local\ maximum\ point.$

We find the second order condition at the point $C_1 = C/2$ is not always less than 0 like Example 1. When ρ is higher enough, the indirect and overlapped persuasion is dominated by the direct persuasion, and then the second order condition is great than or equal to 0. The point $C_1 = C/2$ is a local minimum point. The sender should not invest expenditure to both channels equally.

Lemma 3 Let

$$\rho_1 = \frac{lnK}{2\,ln[(C/2+1)\,e] - lnK}, \rho_2 = \frac{ClnK}{2(C+1)\,ln(C+1) - ClnK},$$
$$K_1 = (C/2+1)e, K_2 = (C+1)^{(C+1)/C},$$

We have $\rho_1 < \rho_2, K_1 > K_2$.

The proof of Lemma 3 is provided in Appendix. It compares two critical values of the parameter of connection between the receivers and the degrees of information distortion of the channels. The critical value of ρ_1 happens when overlapped persuasion dominates the direct and indirect persuasion; while, the critical value of ρ_2 happens when overlapped persuasion is dominated by the direct and indirect persuasion.

From Lemma 1, Lemma 2, and Lemma 3, we find

$$If\ \rho \in \left(\frac{lnK}{2\,ln[(C/2+1)\,e] - lnK}, \frac{ClnK}{2(C+1)\,ln(C+1) - ClnK}\right),$$

$C_1 = 0\ and\ C_1 = C/2$ are both local minimum points. Then there exists a maximum point $C_1 \in (0, C/2)$.Then the first order condition has two solutions when $C_1 \in [0, C/2]$. The general results as follows

Simplifying first derivative, we have

$$\frac{dU(C_1)}{dC_1} = \frac{1}{(C_1+1)(C-C_1+1)lnK}\left\{(C-2C_1) + \rho\left\{(C-2C_1) - \frac{2}{lnK}[(C-C_1+1)\,ln(C-C_1+1) - (C_1+1)\,ln(C_1+1)]\right\}\right\}.$$

Definition 1

$(i)\ q(C_1) = \dfrac{1}{(C_1+1)(C-C_1+1)lnK},$

$(ii)\ f(C_1) = (C-2C_1)$

$$+ \rho\left\{(C-2C_1) - \frac{2}{lnK}[(C-C_1+1)\,ln(C-C_1+1) - (C_1+1)\,ln(C_1+1)]\right\}.$$

Note that (i) $q(C_1) > 0$ when $C_1 \in [0, C]$; (ii) the figure of $\mathrm{f(C_1)}$ is symmetric since $\mathrm{f(C_1)}$=$\mathrm{f(C - C_1)}$. Consider $\mathrm{C_1 \in [0, C/2]}$.

Lemma 4 *The first order condition of Example 2 has no more than two solutions when* $C_1 \in [0, C/2]$. *The sign of* $g(C_1)$ *changes no more than once. Three cases as follows:*

(1) $g(C_1)$ *always less than zero;*

(2) $g(C_1)$ *changes from less than zero to greater than zero;*

(3) $g(C_1)$ *always greater than zero;*

$$where\ g(C_1) = \frac{\partial f(C_1)}{\partial C_1} = -2 + 2\rho \frac{ln\,[(C - C_1 + 1)\,(C_1 + 1)] + 2 - lnK}{lnK}.$$

The proof of Lemma 4 is provided in Appendix. Since it is symmetric model, we just consider $C_1 \in [0, C/2]$. Because the sign of second order condition changes no more than once when $C_1 \in [0, C/2]$, the first order condition will have no more than two solutions. The graph of the object function has one maximum point and one local minimum point, or one maximum point and two local minimum points when $C_1 \in [0, C/2]$. We find $C_1 \in (0, C/2)$ is an internal solution of first order condition, $C_1 = C/2$ is an extreme value point of the objective function, and $C_1 = 0$ is a corner solution.

Lemma 5 *There exits two critical values of the parameter of connection between the receivers in Example 2.*

(1) The first critical point ρ_1 *happens at* $C_1 = C/2$, *SOC=0,*

$$\rho_1 = \frac{lnK}{2\,ln[(C/2 + 1)\,e] - lnK}.$$

(2) The second critical point ρ_2 *happens at* $C_1 = 0$, *FOC=0,*

$$\rho_2 = \frac{lnK}{2ln(C + 1)^{(C+1)/C} - lnK}.$$

If the graph of the objective function changes from one maximum point to two maximum points when $C_1 \in [0, C]$, there exits a critical point, and the critical point ρ_1 happens at $C_1 = C/2$, *SOC=0*. The degree of information distortion K_1 in the channel reaches maximum value when $\rho_1 = 1,\ K_1 = (C/2 + 1)e$. If the graph of the objective function changes from two maximum points to one minimum point when $C_1 \in [0, C]$, there exits a critical point, and the critical point ρ_2 happens at $C_1 = 0$, *FOC=0*. The degree of information distortion K_2 in the channel reaches maximum value when $\rho_2 = 1,\ K_2 = (C + 1)^{(C+1)/C}$.

Lemma 6 *Analysis of three cases about the graph of the objective function of Example 2.*

(1) Consider $g(C_1)$ *always less than zero when* $C_1 \in [0, C/2]$,

$$K \in (C + 1, (C/2 + 1)e), \rho \in \left[0, \frac{lnK}{2\ln[(C/2 + 1)\,e] - lnK}\right] \ or\ K \in \left((C/2 + 1)e, +\infty\right),$$

The objective function reaches maximum value at $C_1 = C/2$.

(2) Consider $g(C_1)$ *changes from less than zero to greater than zero when* $C_1 \in [0, C/2]$,

$$(i)\ K \in \left((C + 1)^{(C+1)/C}, (C/2 + 1)e\right), \rho \in \left(\frac{lnK}{2\,ln[(C/2 + 1)\,e] - lnK}, 1\right]$$

$$or$$

$$K \in \left(C + 1, (C + 1)^{(C+1)/C}\right), \rho \in \left(\frac{lnK}{2\,ln[(C/2 + 1)\,e] - lnK}, \frac{lnK}{2ln(C + 1)^{(C+1)/C} - lnK}\right)$$

The objective function reaches maximum value at

$$\rho = \frac{-(C - 2C_1)}{(C - 2C_1) - 2[(C - C_1 + 1)\,ln(C - C_1 + 1) - (C_1 + 1)\,ln(C_1 + 1)]/lnK};$$

$$(ii)\ K \in \left(C + 1, (C + 1)^{(C+1)/C}\right), \rho \in \left[\frac{lnK}{2ln(C + 1)^{(C+1)/C} - lnK}, 1\right],$$

The objective function reaches maximum value at $C_1 = 0$.

The proof of Lemma 6 is provided in Appendix. We ignore the result of the third case $g(C_1)$ always greater than zero when $C_1 \in [0, C/2]$ from Lemma 4, because the result of the third case is included in the result (ii) of the second case.

Theorem 2 Optimal Allocation of Expenditure in Example 2

Consider $C_1 \in [0, C]$, *we have*

(1) When $K \in (C+1, (C+1)^{(C+1)/C})$,

- *If* $\rho \in \left[0, \frac{lnK}{2\,ln[(C/2+1)e] - lnK}\right]$, *the objective function reaches maximum value at* $C_1 = C/2$.

  The sender should invest the expenditure to both channels equally.

- *If* $\rho \in \left(\frac{lnK}{2\,ln[(C/2+1)e] - lnK}, \frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}\right)$, *the objective function reaches maximum value at* C_1 *or* $(C - C_1)$ *satisfied the following expression:*

$$\rho = \frac{-(C-2C_1)}{(C-2C_1) - 2[(C-C_1+1)\,ln(C-C_1+1) - (C_1+1)\,ln(C_1+1)]/lnK}.$$

  The sender should invest the expenditure C_1 *or* $(C - C_1)$ *to channel 1.*

- *If* $\rho \in \left[\frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}, 1\right]$, *the objective function reaches maximum value at* $C_1 = 0$ *or C. The sender should invest the expenditure C to one of the channels.*

(2) When $K \in \left((C+1)^{(C+1)/C}, (C/2+1)e\right)$

- *If* $\rho \in \left[0, \frac{lnK}{2\,ln[(C/2+1)e] - lnK}\right]$, *objective function reaches maximum value at* $C_1 = C/2$.

  The sender should invest the expenditure C to both channels equally.

- *If* $\rho \in \left(\frac{lnK}{2\,ln[(C/2+1)e] - lnK}, 1\right]$, *the objective function reaches maximum value at* C_1 *and* $(C - C_1)$ *satisfied the following expression:*

$$\rho = \frac{-(C-2C_1)}{(C-2C_1) - 2[(C-C_1+1)\,ln(C-C_1+1) - (C_1+1)\,ln(C_1+1)]/lnK}.$$

  The sender should invest the expenditure C_1 *or* $(C - C_1)$ *to channel 1.*

(3) When $K \in \left((C/2+1)e, +\infty\right)$, *objective function reaches maximum value at* $C_1 = C/2$.

The sender should invest the expenditure to both channels equally.

Theorem 2 provides a specific example for proposition 2, and shows how the sender allocate the expenditure to both channels depending on the degree of connection between the receivers and the degree of information distortion of the channel.

Corollary 1 Decision Rules about Optimal Allocation of Expenditure in Example 2

(1) It would be better for the sender to invest expenditure to both channels equally: (i) when K is lower enough (close to C+1) and ρ *is lower enough (close to 0), or (ii) K is higher enough.*

(2) It would be better for the sender to invest expenditure to only one channel when K is lower enough (close to C+1) and ρ *is higher enough (close to 1).*

(3) It is better for the sender to invest expenditure to the channels unequally: (i) when K is lower enough and ρ *is intermediate value, or (ii) when K is intermediate value and* ρ *is higher enough (close to 1).*

Note that for the first situation of Corollary 1, the level of connection between the receivers is lower, which means receivers are independent; the degree of information distortion of the channel is higher, which means the information transmission technology of the channel is not good. Then when there are no good information transmission channels provided with independent receivers, it is rational for the sender to allocate expenditure to the channels equally under random situation. For the second situation, when there has positive interaction with the receivers and good information transmission channel, then one receiver could be more likely informed by another receiver. The sender should invest all expenditure to one channel.

Corollary 2 Optimal behavior of the sender as ρ increases in Example 2

(1) When K is low enough (close to C+1), if ρ *increases from zero to one, the graph of the objective function changes from one maximum point to two maximum points gradually, thus the sender should firstly invest expenditure to both channels equally, then invest more expenditure to one channel than another until only invest one channel.*

(2) When K is intermediate value, if ρ *increases from zero to one, the graph of the objective function changes from one maximum point to two maximum points gradually, thus the sender should firstly invest expenditure to both channels equally, then invest more expenditure to one*

channel than another gradually.

(3) When K is higher enough, the graph of the objective function only has one maximum point, thus it would be better for the sender to invest expenditure to both channels equally regardless ρ.

Note that it is not always for the sender to allocate the expenditure in both channels equally. If the information distortion of the channel is lower than a certain degree, the sender increases expenditure in one channel when ρ becomes higher than before. If the information distortion of the channel is higher than a certain degree, the send allocates expenditure in the channels equally.

Given some numerical examples for Corollary 2

Assume C=1.

(1) When $\mathrm{K} \in \left[\mathrm{C}+1, (\mathrm{C}+1)^{(\mathrm{C}+1)/\mathrm{C}}\right]$, we assume K=3.

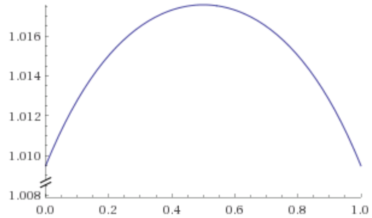
Figure 1.3 shows that the graphs of the objective function with different value of ρ. If ρ increases, C_1 has two solutions. One decreases from C/2 to 0, and another increases from C/2 to C. it would be better for the sender to invest from both channels equally to both channels unequally, and then from both channels unequally to only invest one channel.

(2) When $\mathrm{K} \in \left((\mathrm{C}+1)^{(\mathrm{C}+1)/\mathrm{C}}, (\mathrm{C}/2+1)e\right)$, we assume K=4.04.

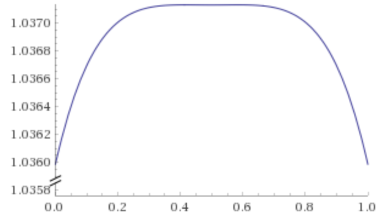
Figure 1.4 shows the graphs of the objective function with different value of ρ. If ρ increases, C_1 has two solutions. However, it is different from numerical 1, C_1 will not become corner solution as ρ increases. It would be better for the sender to invest from both channels equally to both channels unequally.

(3) When $\mathrm{K} \in \left((\mathrm{C}/2+1)\mathrm{e}, +\infty\right)$, we assume K=5.

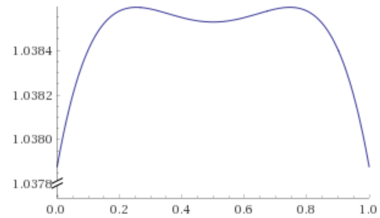
Figure 1.5 shows the graphs of the objective function with different value of ρ. It would be better for the sender to invest expenditure to both channels equally.



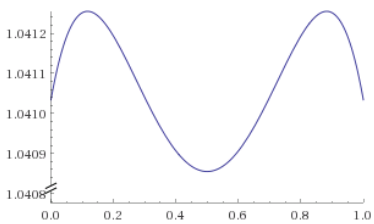
ρ=0.6



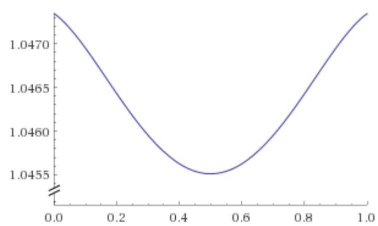
ρ=0.642



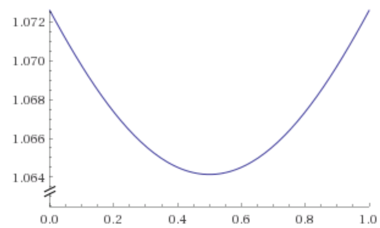
ρ=0.645



ρ=0.65

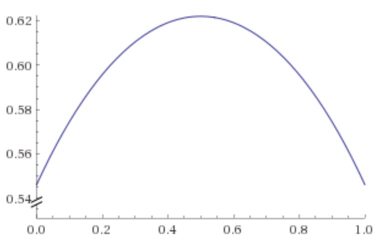


ρ=0.66

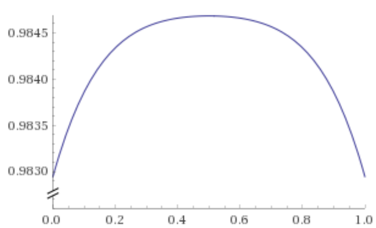


ρ=0.7

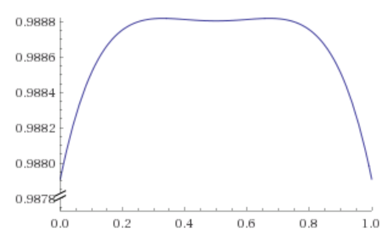
Figure 1.3: Numerical Examples of Example 2 (1)



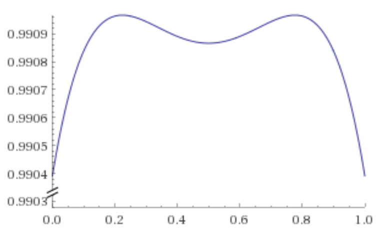
ρ=0.1



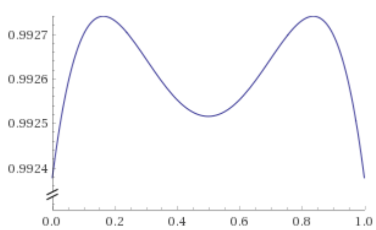
ρ=0.98



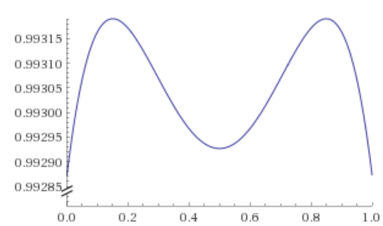
ρ=0.99



ρ=0.995

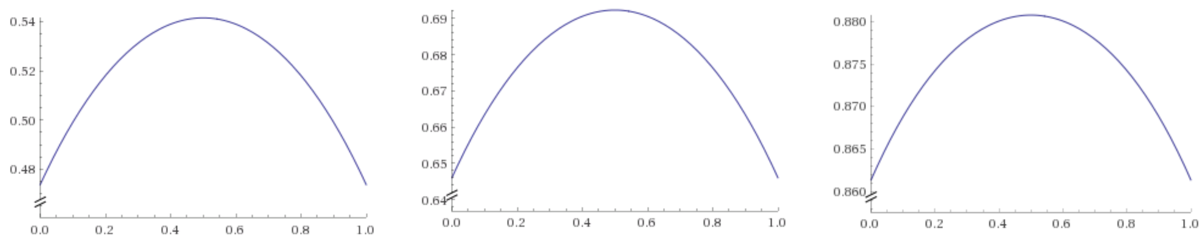


ρ=0.999



ρ=1

Figure 1.4: Numerical Examples of Example 2 (2)



ρ=0.1 ρ=0.5 ρ=1

Figure 1.5: Numerical Examples of Example 2 (3)

1.4 Concave Function Examples in Asymmetric Form Model

1.4.1 with different levels of the channel's persuasive function

Assume the levels of the channels' persuasive functions are different, the level of the channel 1 is denoted by $\alpha \in (0,1)$, and the level of the channel 2 is 1. Keep same degrees of connection between the receivers, which is denoted by $\rho \in [0,1]$.

1.4.1.1 Example 1

$$\lambda(c) = \frac{c}{c+K}, K > 0$$

The maximization problem for the sender is written as follows

$$MAX_{C_1}(1+\rho)\left[\frac{\alpha C_1}{(C_1+K)} + \frac{(C-C_1)}{(C-C_1+K)}\right] - 2\rho\alpha\frac{C_1}{(C_1+K)}\frac{(C-C_1)}{(C-C_1+K)},$$
$$where\ C_1 \in [0,C], \rho \in [0,1], \alpha \in (0,1), K > 0.$$

Theorem 3 Optimal Allocation of Expenditure in Example 1

(1) When $\alpha \in (0, K^2/(C+K)^2]$, *the objective function reaches maximum value point when* $C_1 = 0$. *It's a corner solution. Thus the sender should invest all expenditure to the channel 2.*

(2) When $\alpha \in (K^2/(C+K)^2, 1)$, *there exits an internal solution. The optimal of expenditure for the sender should satisfy the following condition*

$$\rho = \frac{\alpha(C-C_1+K)^2 - (C_1+K)^2}{\alpha C(C-2C_1) + (1-\alpha)(C_1+K)^2}.$$

The proof of Theorem 3 is provided in Appendix. If the sender only invest the expenditure to channel 2, we obtain a critical value of $\alpha = K^2/(C+K)^2$ by giving $\rho = 0, C_1 = 0$. Thus we find that the sender should only invest the expenditure to the channel 2 when the level of the channel's persuasive function α is lower than a certain value (close to 0).

Corollary 3 Decision Rules about Optimal Allocation of Expenditure in Example 1

When the level of persuasion function of the channel α is lower than a certain value (close to 1), the sender should invest all expenditure to channel 2; when the level of persuasion function of the channel α is higher than a certain value, the sender should invest expenditure to both channels, and expenditure invested in channel 1 increases from zero to C/2 when α increasing.

The proof of Corollary 3 is provided in Appendix. By the implicit function theorem, we find the sender should increase expenditure in channel 1 when α increases from $(K^2/(C+K)^2, 1)$. If $\alpha = 1$, it's symmetric information transmission model and the sender allocates expenditure to both channels equally. If $\alpha = 0$, it has no significant for the channel 1 and the sender should spend all expenditure to the channel 2.

1.4.1.2 Example 2

$$\lambda(c) = \frac{ln\ (c+1)}{lnK}, K > C+1.$$

The maximization problem for the sender is written as follows

$$U(C_1) = MAX_{C1}(1+\rho)\left[\alpha\frac{ln\ (C_1+1)}{lnK} + \frac{ln\ (C-C_1+1)}{lnK}\right] - 2\rho\alpha\left[\frac{ln(C_1+1)}{lnK}\right]\left[\frac{ln(C-C_1+1)}{lnK}\right],$$
$$where\ C_1 \in [0,C], \alpha \in (0,1), \rho \in [0,1], K > C+1.$$

First derivative with respect to C_1, we have

$$\frac{dU(C_1)}{dC_1} = (1+\rho)\left[\frac{\alpha}{(C_1+1)\ln K} - \frac{1}{(C-C_1+1)\ln K}\right] - 2\rho\alpha\left[\frac{1}{(C_1+1)\ln K}\frac{\ln(C-C_1+1)}{\ln K} - \frac{\ln(C_1+1)}{\ln K}\frac{1}{(C-C_1+1)\ln K}\right]$$

Specially, consider the point $C_1 = 0$,

$$\frac{\partial U(C_1)}{\partial C_1}|_{C_1=0} = \frac{1}{lnK}\left[(1+\rho)\left(\alpha - \frac{1}{C+1}\right) - 2\rho\alpha\frac{ln(C+1)}{lnK}\right] = \frac{1}{lnKlnK}\left[\big((1+\rho)lnK - 2\rho ln\,(C+1)\big)\alpha - \frac{(1+\rho)lnK}{C+1}\right]$$

Theorem 4 *Checking the sign of* $\frac{\partial U(C_1)}{\partial C_1}|_{C_1=0}$ *of Example 2 according to the partition of* α

1. $When\ \alpha \in (0,1), K \in \left(C+1, (C+1)^{\frac{C+1}{C}}\right), \rho \in \left(\frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}, 1\right),$

   $then\ \frac{\partial U(C_1)}{\partial C_1}|_{C_1=0} < 0, C_1 = 0\ is\ a\ local\ maximum\ point.$

2. $When\ \alpha \in \left(0, \frac{(1+\rho)\ln K}{(C+1)[(1+\rho)lnK - 2\rho\, ln(C+1)]}\right),$

   $K \in \left(C+1, (C+1)^{\frac{C+1}{C}}\right), \rho \in \left(0, \frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}\right)$ *or* $K \in \left((C+1)^{\frac{C+1}{C}}, +\infty\right),$

   $then$ $\frac{\partial U(C_1)}{\partial C_1}|_{C_1=0} < 0$, $C_1 = 0$ *is a local maximum point.*

3. $When\ \alpha \in \left(\frac{(1+\rho)\ln K}{(C+1)[(1+\rho)lnK - 2\rho\, ln(C+1)]}, 1\right),$

   $K \in \left(C+1, (C+1)^{\frac{C+1}{C}}\right), \rho \in \left(0, \frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}\right)$ *or* $K \in \left((C+1)^{\frac{C+1}{C}}, +\infty\right),$

   $then \frac{\partial U(C_1)}{\partial C_1}|_{C_1=0} > 0$, $C_1 = 0$ *is a local minimum point.*

The proof of Theorem 5 is provided in Appendix. We find $C_1 = 0$ is not always a local minimum point.

Lemma 7 The first order condition of Example 2 has no more than three solutions.

The proof of Lemma 7 is provided in Appendix. The graph of the object function has no more than two local maximum points. Since $\alpha \in (0,1)$, $C_1 \in (0, C/2)$ is an internal solution, and $C_1 = 0$ is corner solution of Example 2. We provide two optimal allocations to the sender, one is allocating less expenditure in channel 1 than channel 2, and another is allocating all expenditure in channel 2.

Corollary 4 Decision Rules about Optimal Allocation of Expenditure in Example 2

(1) Regardless the level of persuasive function of the channel α*, when K is lower enough (close to C+1) and* ρ *is higher enough (close to 1), it would be better for the sender to invest all expenditure C to channel 2.*

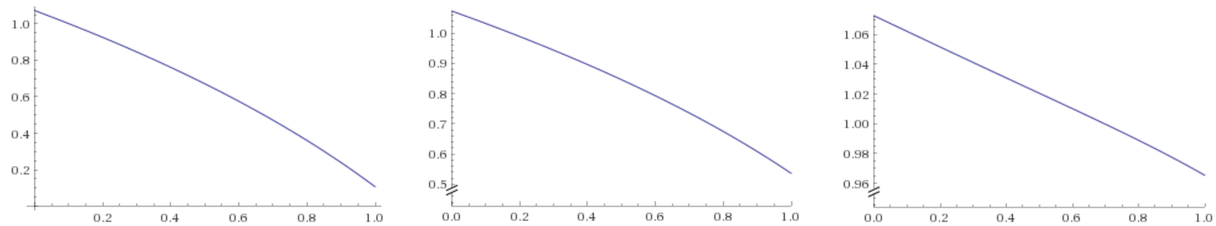
(2) When α *is lower enough (close to 0), K is higher enough, or K is lower enough (close to C+1) and* ρ *is lower enough (close to 0), it would be better for the sender to invest all expenditure C to channel 2.*

(3) When α *is higher enough (close to 1), K is higher enough, or K is lower enough (close to C+1) and* ρ *is lower enough (close to 0), it would be better for the sender to divide expenditure C to both channels, and allocate less expenditure in channel 1 than channel 2.*

Note that it is always for the sender to invest all expenditure to the channel when the level of persuasive function of this channel is lower enough; and the sender should invest expenditure to both channels, which happens only when the level of persuasive function of this channel is higher enough (close to 1).

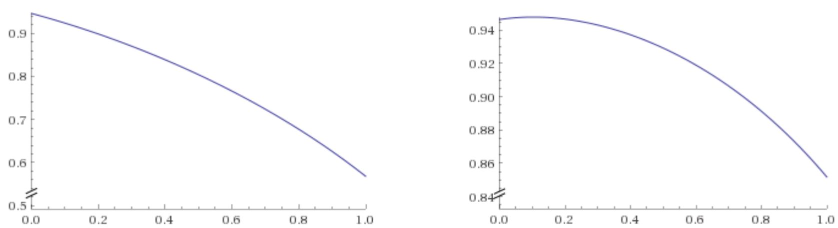
Given some numerical examples of Example 2 for Corollary 4

1) When $\mathrm{C} = 1, \mathrm{K} \in \left(\mathrm{C}+1, (\mathrm{C}+1)^{\frac{\mathrm{C}+1}{\mathrm{C}}}\right), \rho \in \left(\frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}, 1\right)$, assume $\mathrm{K} = 3, \rho = 0.7$. Figure 1.6 shows that the optimal allocation is $C_1 = 0$. Regardless the level of persuasive function of the channel α, when K is close to C+1 and ρ is higher enough (close to 1), it would



a=0.1 *a*=0.5 *a*=0.9

Figure 1.6: Numerical examples of Example 2 (4)



a=0.6 *a*=0.9

Figure 1.7: Numerical examples of Example 2 (5)

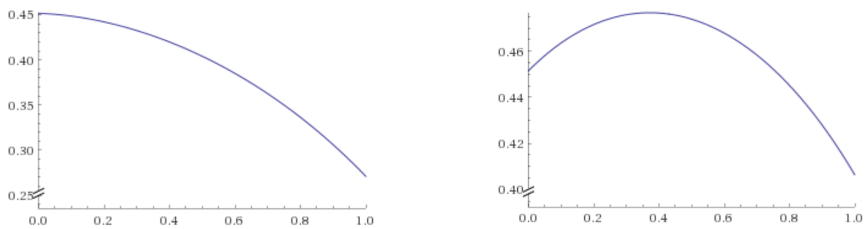
be better for the sender to invest all expenditure C to channel 2.

2) When $\mathrm{C} = 1, \mathrm{K} \in \left(\mathrm{C}+1, (\mathrm{C}+1)^{\frac{\mathrm{C}+1}{\mathrm{C}}}\right), \rho \in \left(0, \frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}\right)$, assume $\mathrm{K} = 3, \rho = 0.5$.

   Figure 1.7 shows that when K and ρ are lower enough, if α is lower enough, it would be better for the sender to invest expenditure C to channel 2; if α is higher enough, it would be better for the sender to invest expenditure C to both channels, and allocate less expenditure in channel 1 than channel 2.

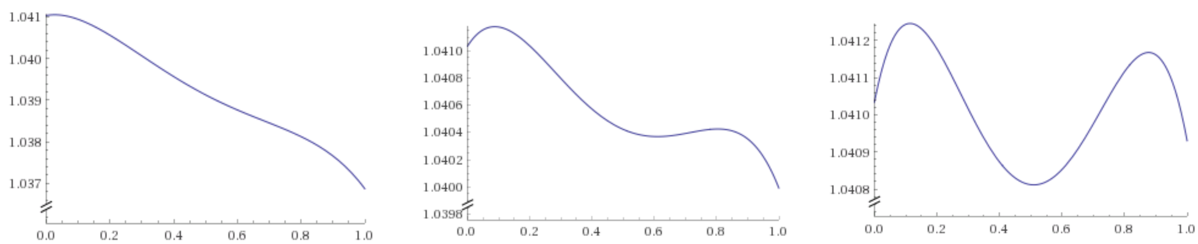
3) When $\mathrm{C} = 1, \mathrm{K} \in \left((\mathrm{C}+1)^{\frac{\mathrm{C}+1}{\mathrm{C}}}, +\infty\right)$, assume $\mathrm{K} = 10, \rho = 0.5$.

   Figure 1.8 shows that when K is higher enough, if α is lower enough, it would be better for the sender to invest all expenditure C to channel 2; if α is higher enough, it would be better for the sender to invest expenditure C to both channels, and allocate less expenditure in channel 1 than channel 2.



a=0.6 a=0.9

Figure 1.8: Numerical examples of Example 2 (6)



a=0.996 a=0.999 a=0.9999

Figure 1.9: Numerical examples of Example 2 (7)

4) When $\mathrm{C} = 1, \mathrm{K} \in \left(\mathrm{C}+1, (\mathrm{C}+1)^{\frac{\mathrm{C}+1}{\mathrm{C}}}\right), \rho \in \left(0, \frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}\right)$, assume $\mathrm{K} = 3, \rho = 0.65$

Figure 1.9 shows that when K and ρ are lower enough, and α is higher enough, it would be better for the sender to invest more expenditure to channel 1 as α is increasing.

1.4.2 with different degrees of information transmission for the receivers

Assume the parameter of information transmission from receiver 1 to receiver 2 is denoted by $\rho_{12} \in [0,1]$, and from receiver 2 to receiver 1 is denoted by $\rho_{21} \in [0,1]$, where $\rho_{12} \neq \rho_{21}$. Keep the levels of the channels' persuasive function constant, and assume it is equal to 1.

1.4.2.1 Example 1

$$\lambda(c) = \frac{c}{c+K}, K > 0$$

The maximization problem for the sender is written as follows

$$MAX_{C1}(1+\rho_{12})\frac{C_1}{(C_1+K)}+(1+\rho_{21})\frac{(C-C_1)}{(C-C_1+K)}-(\rho_{21}+\rho_{12})\frac{C_1}{(C_1+K)}\frac{(C-C_1)}{(C-C_1+K)},$$

$$where\ C_1 \in [0,C], \rho_{12}, \rho_{21} \in\ [0,1], \rho_{12} \neq \rho_{21}, K > 0.$$

Theorem 5 Optimal Allocation of Expenditure in Example 1

There exists an internal solution. The optimal of expenditure for the sender should satisfy the following condition

$$\rho_{12} = \frac{\rho_{21}[(C-C_1)^2+CK+K^2]+(C+2K)(2C_1-C)}{{C_1}^2+K^2+CK}.$$

The proof of Theorem 5 is provided in Appendix. Specially, if $\rho_{21}=0$, $\rho_{12}=\frac{(C+2K)(2C_1-C)}{{C_1}^2+K^2+CK}$. We find that the sender should increase expenditure in channel 1 from C/2 to C when ρ_{12} increases. If $\rho_{12}=0$, $\rho_{21}=\frac{(C+2K)(C-2C_1)}{(C-C_1)^2+CK+K^2}$. We find the sender should decrease expenditure in channel 1 from C/2 to 0 when ρ_{21} increases.

Corollary 5 Decision Rules about Optimal Allocation of Expenditure in Example 1

It would be better for the sender to invest more expenditure to channel 1 than channel 2 when ρ_{12} *is larger than* ρ_{21}. *In addition, the sender should increase expenditure in channel 1 when* ρ_{12} *increases or* ρ_{21} *decreases.*

The proof of Corollary 2 is provided in Appendix. By the implicit function theorem, we find the sender should increase expenditure in channel 1 when ρ_{12} increases or ρ_{21} decreases. If $\rho_{12}=\rho_{21}, C_1=C/2$. It’s symmetric information transmission model and the sender allocates expenditure to both channels equally.

1.4.2.2 Example 2

$$\lambda(c)=\frac{ln\ (c+1)}{lnK},K>C+1.$$

The maximization problem for the sender is written as

$$U(C_1)=MAX_{C_1}\alpha_1(1+\rho_{12})\frac{\ln\ (\mathrm{C}_1+1)}{\ln\mathrm{K}}+\alpha_2(1+\rho_{21})\frac{\ln\ (\mathrm{C}-\mathrm{C}_1+1)}{\ln\mathrm{K}}-\alpha_1\alpha_2(\rho_{21}+\rho_{12})\frac{\ln\ (\mathrm{C}_1+1)}{\ln\mathrm{K}}\frac{\ln\ (\mathrm{C}-\mathrm{C}_1+1)}{\ln\mathrm{K}}$$

$$where\ C_1\in[0,C],\alpha_1,\alpha_2\in[0,1],\rho_{12},\rho_{21}\in\ [0,1],K>C+1.$$

First derivative with respect to C_1,

$$\frac{dU(C_1)}{dC_1}=(1+\rho_{12})\frac{1}{(\mathrm{C}_1+1)\ln\mathrm{K}}-(1+\rho_{21})\frac{1}{(\mathrm{C}-\mathrm{C}_1+1)\ln\mathrm{K}}-(\rho_{21}+\rho_{12})\left[\frac{1}{(\mathrm{C}_1+1)\ln\mathrm{K}}\frac{\ln\ (\mathrm{C}-\mathrm{C}_1+1)}{\ln\mathrm{K}}-\frac{\ln\ (\mathrm{C}_1+1)}{\ln\mathrm{K}}\frac{1}{(\mathrm{C}-\mathrm{C}_1+1)\ln\mathrm{K}}\right]$$

Specially, consider point $C_1=0$, checking the sign of $\left.\frac{dU(C_1)}{dC_1}\right|_{C_1=0}$

$$\left.\frac{dU(C_1)}{dC_1}\right|_{C_1=0}=(1+\rho_{12})\frac{1}{\ln\mathrm{K}}-(1+\rho_{21})\frac{1}{(\mathrm{C}+1)\ln\mathrm{K}}-(\rho_{21}+\rho_{12})\frac{1}{\ln\mathrm{K}}\frac{\ln(\mathrm{C}+1)}{\ln\mathrm{K}})$$

$$=\frac{1}{(\mathrm{C}+1)\ln\mathrm{K}}\{(1+\rho_{12})(\mathrm{C}+1)-(1+\rho_{21})-(\rho_{21}+\rho_{12})(\mathrm{C}+1)\frac{\ln(\mathrm{C}+1)}{\ln\mathrm{K}}\}$$

$$=\frac{1}{(\mathrm{C}+1)\ln\mathrm{K}}\left\{C+\rho_{12}(\mathrm{C}+1)\left[1-\frac{\ln(\mathrm{C}+1)}{\ln\mathrm{K}}\right]-\rho_{21}\left[1+(\mathrm{C}+1)\frac{\ln(\mathrm{C}+1)}{\ln\mathrm{K}}\right]\right\}$$

We have the results as follows

$$If\ \rho_{12}\in\left[0,\frac{\rho_{21}[\ln\mathrm{K}+(\mathrm{C}+1)\ln(\mathrm{C}+1)]-\mathrm{C}\ln\mathrm{K}}{(\mathrm{C}+1)[\ln\mathrm{K}-\ln(\mathrm{C}+1)]}\right],we\ get\ \left.\frac{dU(C_1)}{dC_1}\right|_{C_1=0}\leq 0,$$

$$then\ C_1=0\ is\ a\ local\ maximum\ point;$$

$$If\ \rho_{12}\in\left[\frac{\rho_{21}[\ln\mathrm{K}+(\mathrm{C}+1)\ln(\mathrm{C}+1)]-\mathrm{C}\ln\mathrm{K}}{(\mathrm{C}+1)[\ln\mathrm{K}-\ln(\mathrm{C}+1)]},1\right],we\ get\ \left.\frac{dU(C_1)}{dC_1}\right|_{C_1=0}\geq 0,$$

$$then\ C_1=0\ is\ a\ local\ minimum\ point.$$

We find the first order condition is not always great than 0. When ρ_{12} is lower enough, the persuasion of receiver 1 is dominated by the persuasion of the receiver 2, and the first order condition is less than 0. Then the sender is more likely to invest one of the channels.

Theorem 6 Optimal Allocation of Expenditure in Example 2

If there exists an internal solution, the optimal of expenditure for the sender should satisfy the following condition

$$\rho_{12} = \frac{\rho_{21}[(C_1+1)\ln K + (C-C_1+1)\ln(C-C_1+1) - (C_1+1)\ln(C_1+1)] - (C-2C_1)\ln K}{[(C-C_1+1)\ln K - (C-C_1+1)\ln(C-C_1+1) + (C_1+1)\ln(C_1+1))]}.$$

The proof of Theorem 6 is provided in Appendix. Specially

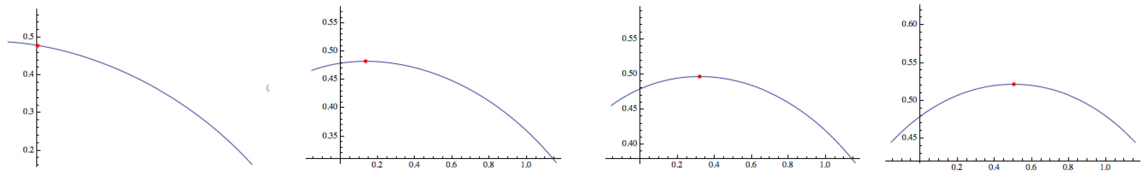
1. If $\rho_{21}=0$, $\rho_{12}=\frac{-(C-2C_1)\ln K}{[(C-C_1+1)\ln K-(C-C_1+1)\ln(C-C_1+1)+(C_1+1)\ln(C_1+1))]}$. We find that the sender should increase expenditure in channel 1 from C/2 to C when ρ_{12} increases.
2. If $\rho_{12}=0$, $\rho_{21}=\frac{(C-2C_1)\ln K}{[(C_1+1)\ln K+(C-C_1+1)\ln(C-C_1+1)-(C_1+1)\ln(C_1+1)]}$. We find the sender should decrease expenditure in channel 1 from C/2 to 0 when ρ_{21} increases.
3. If $\rho_{12}=\rho_{21}$, it's symmetric information transmission model.

Corollary 6 Decision Rules about Optimal Allocation of Expenditure in Example 2

If ρ_{12} or ρ_{21} is lower enough, the sender is more likely to invest all expenditure to anther channel. *It would be better for the sender to increase expenditure in channel 1 as* ρ_{12} *increases; while it would be better for the sender to decrease expenditure in channel 1 as* ρ_{21} *increases. It would be better for the sender to invest more expenditure to channel 1 than channel 2 when* ρ_{12} *is larger than* ρ_{21}.

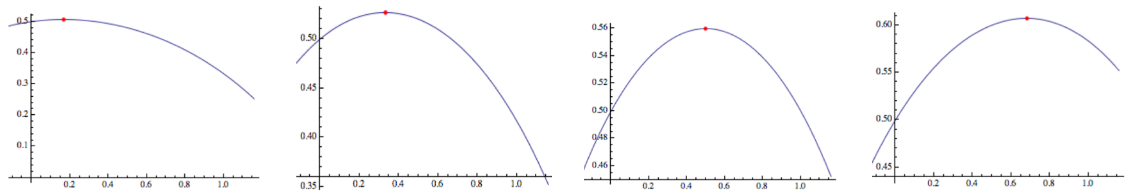
Given some numerical examples for Corollary 6

Assume $C=1, K=18$



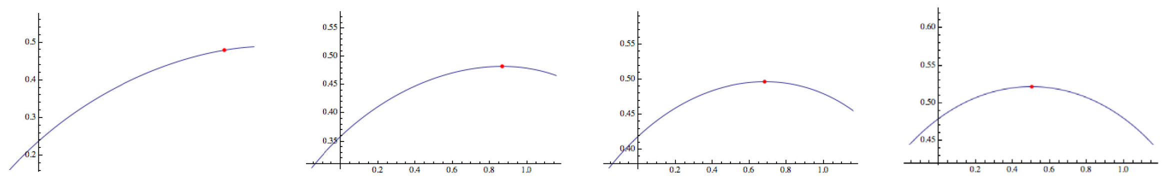
$\rho_{12} = 0$ $\rho_{12} = 0.5$ $\rho_{12} = 0.75$ $\rho_{12} = 1$

Figure 1.10: Numerical examples of Example 2 (8)



$\rho_{12} = 0$ $\rho_{12} = 0.25$ $\rho_{12} = 0.5$ $\rho_{12} = 0.75$

Figure 1.11: Numerical examples of Example 2 (9)



$\rho_{21} = 0$ $\rho_{21} = 0.5$ $\rho_{21} = 0.75$ $\rho_{21} = 1$

Figure 1.12: Numerical examples of Example 2 (10)

(1) When $\rho_{21} = 1$, Figure 1.10 shows that it would be better for the sender to increase expenditure to channel 1 from 0 to C/2 as the level of information transmission from receiver 1 to receiver 2 (ρ_{12}) is increasing from 0 to 1.

(2) When $\rho_{21} = 0.5$, Figure 1.11 shows that it would be better for the sender to increase expenditure to channel 1 as the level of information transmission from receiver 1 to receiver 2 (ρ_{12}) is increasing.

(3) When $\rho_{12} = 1$, Figure 1.12 shows that it would be better for the sender to decrease expenditure to channel 1 from C to C/2 as the level of information transmission from receiver 1 to receiver 2 (ρ_{21}) is increasing from 0 to 1.

1.5 Conclusion

This paper constructs a model of information transmission. We mainly analyze a symmetric strategic information transmission model by introducing two parameters. One is the level of persuasive function of the channel; another is the degree of connection between the receivers. The model was constructed with an informed sender and two uninformed receivers, where the sender wants to convince the receivers to take a certain action. General results suggest that the sender obtain higher maximum persuasive power with higher level of the channel's persuasive function and higher degree of connection between the receivers.

Considering the persuasive function of the channel, we prove that if persuasive function is a liner or convex function, the sender should invests all expenditure to one channel with higher level of persuasive function and higher degree of information transmission for the receiver. However, if persuasive function is a concave function, optimal behavior of the sender is uncertain, which depends on the level of persuasive function of the channel, the degree of connection between the receiver, and the degree of information distortion in the channel. The optimal allocation could be allocate one channel, both channels equally or both channels unequally. Thus our results focus on concave function.

This paper solved the sender's expenditure allocation problem. Our analysis mainly focused on the concave function of the channel under three specific environments. For concave function example 1, the results are regular for us. The sender should invest expenditure to both channels equally in symmetric model, invest expenditure more in one channel with higher level of persuasive function, and invest expenditure more in one channel with higher degree of information transmission for the receiver in asymmetric model.

However, for concave function example 2, the results are irregular for us. In the symmetric strategic information transmission model, it would be better for the sender to invest expenditure to both channels equally when the degree of information distortion and level of information transmission are lower enough, or the degree of information distortion is higher enough. Also, it would be better for the sender to invest expenditure to only one channel when the degree of information distortion is lower enough and the parameter of information transmission between receivers is higher enough. Others, it would be better for the sender to invest expenditure to two channels unequally. Thus we find it is not always for the sender to allocate the expenditure in the channels equally in the symmetric model.

In asymmetric strategic information transmission model with different levels of persuasive function of the channels, it would be better for the sender to invest expenditure to channel 2 when the degree of information distortion and degree of information transmission are higher enough regardless the level of persuasive function, or the higher degree of information distortion with lower level of persuasive function, or the lower degree of information distortion and lower degree of information transmission with lower level of persuasive function. Others, it would be better for sender to allocate expenditure to two channels, and an amount of expenditure to channel 1 is less than channel 2. Thus we find that it is always for the sender to invest all expenditure to channel 2 with lower level of persuasive function of the channel in asymmetric model with different levels of persuasive function of the channels.

In asymmetric strategic information transmission model with different parameter of information transmission, the sender is more likely to invest all expenditure to one channel when the degree of information transmission for another channel's receiver is lower enough. It would be better for the sender to increase expenditure in channel 1 as the degree of information transmission for the receiver 1 ρ_{12} increases; while the result for ρ_{21} is contrary to the result for ρ_{12}.

This paper is conducive to obtain more receivers to participation and support. In addition, it provides the optimal allocation of expenditure for the sender. Limitation of this paper is only consider two-channel problem, thus we could use general persuasive function instead of specific persuasive function to analyze the problem with multi-channel in the future.

1.6 Appendix

Proof of Lemma 3 Comparing two critical values of the parameter of connection between the receivers and the degrees of information distortion of the channels.

$$\rho_1 = \frac{lnK}{2\,ln[(C/2+1)\,e] - lnK}, \rho_2 = \frac{ClnK}{2(C+1)\,ln(C+1) - ClnK},$$

$$K_1 = (C/2+1)e, K_2 = (C+1)^{(C+1)/C},$$

Let

$$Q(C) = \frac{K_2}{K_1} = \frac{(C+1)^{(C+1)/C}}{(C/2+1)e},$$

We have

$$\lim_{C\to 0}\frac{(C+1)^{(C+1)/C}}{(C/2+1)e} = 1, \lim_{C\to\infty}\frac{(C+1)^{(C+1)/C}}{(C/2+1)e} = \frac{2}{e},$$

$$Q'(C) = -\frac{2(C+1)^{(C+1)/C}\big((C+2)\,ln(C+1) - 2C\big)}{eC^2(C+2)^2}.$$

Let

$$h(C) = (C+2)\,ln(C+1) - 2C,$$

We have

$$f(C) = h'(C) = ln(C+1) - \frac{C}{C+1},$$

$$g(C) = f'(C) = \frac{C}{(C+1)^2}.$$

$Since\ g(0) = 0\ and\ g(C) = \frac{C}{(C+1)^2} > 0, we\ have$

$$f(C) = \int_0^C g(c)dc > 0.$$

$Since\ f(0) = 0, and\ f(C) = ln(C+1) - \frac{C}{C+1} > 0, we\ have$

$$h(C) = \int_0^C f(c)dc > 0.$$

$Since\ h(C) > 0, Q'(C) = -\frac{2(C+1)^{(C+1)/C}\big((C+2)\,ln(C+1) - 2C\big)}{eC^2(C+2)^2} < 0,$

$$Q(C) = \frac{K_2}{K_1} = \frac{(C+1)^{(C+1)/C}}{(C/2+1)e} \in \left(\frac{2}{e}, 1\right),$$

$$K_2 < K_1.$$

$$\frac{\rho_2}{\rho_1} = \frac{\dfrac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}}{\dfrac{lnK}{2\,ln(C/2+1)\,e - lnK}} = \frac{2\,ln(C/2+1)\,e - lnK}{2\,ln(C+1)^{(C+1)/C} - lnK}.$$

Since $(C/2+1)e > (C+1)^{\frac{C+1}{C}}$, *we have*

$$ln(C/2+1)\,e > ln(C+1)^{(C+1)/C},$$

$$\frac{\rho_2}{\rho_1} = \frac{2\,ln(C/2+1)\,e - lnK}{2\,ln(C+1)^{(C+1)/C} - lnK} > 1,$$

$$\rho_1 < \rho_2.$$

$\boxdot$

Proof of Lemma 4 Checking the first order condition of Example 2 has no more than two solutions when $C_1 \in [0, C/2]$.

First derivative with respect to C_1,

$$\frac{dU(C_1)}{dC_1} = (1+\rho)\left[\frac{1}{(C_1+1)lnK} - \frac{1}{(C-C_1+1)lnK}\right] - 2\rho\left[\frac{1}{(C_1+1)lnK}\frac{ln(C-C_1+1)}{lnK} - \frac{ln(C_1+1)}{lnK}\frac{1}{(C-C_1+1)lnK}\right].$$

Simplifying first derivative, we have

$$\frac{dU(C_1)}{dC_1} = \frac{1}{(C_1+1)(C-C_1+1)lnK}\left\{(C-2C_1) + \rho\left\{(C-2C_1) - \frac{2}{lnK}[(C-C_1+1)\,ln(C-C_1+1) - (C_1+1)\,ln(C_1+1)]\right\}\right\}.$$

Definition 1

$(i)\ q(C_1) = \dfrac{1}{(C_1+1)(C-C_1+1)lnK}$,

$(ii)\ f(C_1) = (C-2C_1) + \rho\left\{(C-2C_1) - \dfrac{2}{lnK}[(C-C_1+1)\,ln(C-C_1+1) - (C_1+1)\,ln(C_1+1)]\right\}$.

Note that (i) $q(C_1) > 0$ when $C_1 \in [0, C]$; (ii) the figure of $f(C_1)$ is symmetric since $f(C_1) = f(C - C_1)$. Consider $C_1 \in [0, C/2]$.

$$g(C_1) = \frac{\partial f(C_1)}{\partial C_1} = -2 + 2\rho \frac{ln\,[(C - C_1 + 1)\,(C_1 + 1)] + 2 - lnK}{lnK}.$$

$$h(C_1) = \frac{\partial g(C_1)}{\partial C_1} = \frac{2\rho}{lnK}\left[\frac{-(C_1 + 1) + (C - C_1 + 1)}{(C_1 + 1)(C - C_1 + 1)}\right] = \frac{2\rho(C - 2C_1)}{(C_1 + 1)(C - C_1 + 1)lnK}.$$

Then we have

$$h(C_1) \geq 0 \; when \; C_1 \in [0, C/2].$$

Thus the sign of $g(C_1)$ changes no more than once, and $f(C_1) = 0$ has no more than two solutions. Since we have $q(C_1) > 0 \; when \; C_1 \in [0, C/2]$, $q(C_1)f(C_1) = 0$ has no more than two solutions. Thus the first order condition of Example 2 has no more than two solutions when $C_1 \in [0, C/2]$.

□

Proof of Lemma 6 Analyzing three cases about the graph of the objective function in Example 2

(1) Consider $g(C_1)$ always less than zero when $C_1 \in [0, C/2]$.

The following conditions must be satisfied

$$K \in (C + 1, (C/2 + 1)e), \rho \in \left(0, \frac{\ln K}{2\ln[(C/2 + 1)\,e] - \ln K}\right] \text{ or } K \in \left((C/2 + 1)e, +\infty\right).$$

In this case, $f(C_1)$ will decrease when C_1 increases, and $f(C/2) = 0$, so the sign of $f(C_1)$ changes from positive to zero when C_1 increases. Since $q(C_1) > 0$, we get the first derivative $q(C_1)f(C_1)$ changes from positive to zero. Then the objective function reaches maximum value at $C_1 = C/2$ when $C_1 \in [0, C/2]$.

(2) Consider $g(C_1)$ changes from less than zero to greater than zero when $C_1 \in [0, C/2]$.

We have following three situations:

(i) $K \in \left(C + 1, (C + 1)^{\frac{C+1}{C}}\right), \rho \in \left(\frac{\ln K}{2\ln[(C/2 + 1)\,e] - \ln K}, \frac{\ln K}{2\ln(C + 1)^{(C+1)/C} - \ln K}\right)$ or

$$K \in \left((C + 1)^{\frac{C+1}{C}}, (C/2 + 1)e\right), \rho \in \left(\frac{\ln K}{2\ln[(C/2 + 1)\,e] - \ln K}, 1\right).$$

In above situation, the sign of $f(C_1)$ changes from positive to zero, changes from zero to negative, and changes from negative to zero when C_1 increases. Since $q(C_1) > 0$, then the first derivative $q(C_1)f(C_1)$ changes from positive to zero, changes from zero to negative, then changes from negative to zero. Then when $C_1 \in [0, C/2]$, the objective function reaches maximum value at

$$\rho = \frac{-(C - 2C_1)}{(C - 2C_1) - \frac{2[(C - C_1 + 1)\ln(C - C_1 + 1) - (C_1 + 1)\ln(C_1 + 1)]}{\ln K}}.$$

(ii) $K \in \left(C + 1, (C + 1)^{\frac{C+1}{C}}\right), \rho = \frac{\ln K}{2\ln(C + 1)^{(C+1)/C} - \ln K}.$

In above situation, the sign of $f(C_1)$ changes from zero to negative, and then changes from negative to zero when C_1 increases. Since $q(C_1) > 0$, then the first derivative $q(C_1)f(C_1)$ changes from zero to negative, then changes from negative to zero. Then the objective function reaches maximum value at $C_1 = 0$ when $C_1 \in [0, C/2]$.

(iii) $K \in \left(C + 1, (C + 1)^{\frac{C+1}{C}}\right), \rho \in \left(\frac{\ln K}{2\ln(C + 1)^{(C+1)/C} - \ln K}, 1\right).$

In above situation, the sign of $f(C_1)$ changes from negative to zero. Since $q(C_1) > 0$ when C_1 increases, then the first derivative $q(C_1)f(C_1)$ changes from negative to zero. Then the objective function reaches maximum value at $C_1 = 0$ when $C_1 \in [0, C/2]$.

(3) Consider $g(C_1)$ always greater than zero when $C_1 \in [0, C/2]$

The following condition must be satisfied

$$C \in (0, e^2 - 1), K \in \left(C + 1, e\sqrt{(C + 1)}\right), \rho \in \left[\frac{\ln K}{\ln(C + 1) + 2 - \ln K}, 1\right]$$

In this case, the sign of $f(C_1)$ changes from negative to zero when C_1 increases. Since $q(C_1) > 0$, then the first derivative $q(C_1)f(C_1)$ changes from negative to zero. Then the objective function reaches maximum value at $C_1 = 0$ when $C_1 \in [0, C/2]$.

We show that the third result (3) is included in the second result (2)-(ii) as follows

For the condition $(2) -$ (ii)

$$C_1 \in \left[0, \frac{C}{2}\right], K \in \left(C+1, (C+1)^{\frac{C+1}{C}}\right), \rho \in \left[\frac{\ln K}{2\ln(C+1)^{(C+1)/C} - \ln K}, 1\right],$$

We have

$$(C+1)^{\frac{C+1}{C}} \text{ will increase when C increase;}$$

$$\frac{\ln K}{2\ln(C+1)^{(C+1)/C} - \ln K} \text{ will decrease when C increases.}$$

At $C = e^2 - 1$, we have

$$(C+1)^{\frac{C+1}{C}} = (e^2)^{\frac{e^2}{e^2-1}}$$

$$\frac{\ln K}{2\ln(C+1)^{\frac{(C+1)}{C}} - \ln K} = \frac{\ln K}{2\ln(e^2)^{\frac{e^2}{e^2-1}} - \ln K} - - - (A)$$

For the condition of the third result in Lemma 4

$$C \in (0, e^2 - 1), K \in \left(C+1, e\sqrt{(C+1)}\right), \rho \in \left[\frac{\ln K}{\ln(C+1) + 2 - \ln K}, 1\right]$$

We have

$$e\sqrt{(C+1)} \text{ will increase when C increase;}$$

$$\frac{\ln K}{\ln(C+1) + 2 - \ln K} \text{ will decrease when C increases.}$$

At $C = e^2 - 1$, we have

$$e\sqrt{(C+1)} = e^2, \frac{\ln K}{\ln(C+1) + 2 - \ln K} = \frac{\ln K}{\ln e^2 + 2 - \ln K} - - - (B)$$

Compare (A) and (B), we get

$$\text{When } C \in (0, e^2 - 1), \left(C+1, e\sqrt{(C+1)}\right) \in \left(C+1, (C+1)^{\frac{C+1}{C}}\right);$$

$$\left[\frac{\ln K}{\ln(C+1) + 2 - \ln K}, 1\right] \in \left[\frac{\ln K}{2\ln(C+1)^{(C+1)/C} - \ln K}, 1\right].$$

Thus the third result (3) is included in the second result (2)-(ii), we can ignore the result (3).

$\boxdot$

Proof of Theorem 3 Checking optimal allocation of expenditure in Example 1 in asymmetric information transmission with different levels of the channel's persuasive function

$$MAX_{C_1}(1+\rho)\left[\frac{\alpha C_1}{(C_1+K)}+\frac{(C-C_1)}{(C-C_1+K)}\right]-2\rho\alpha\frac{C_1}{(C_1+K)}\frac{(C-C_1)}{(C-C_1+K)},$$
$$where\ C_1\in[0,C],\rho\in\ [0,1],\alpha\in(0,1),K>0.$$

First order condition with respect to C_1

$$(1+\rho)\left[\frac{\alpha K}{(C_1+K)^2}-\frac{K}{(C-C_1+K)^2}\right]$$
$$-2\rho\alpha\left[\frac{K}{(C_1+K)^2}\frac{(C-C1)}{(C-C_1+K)}-\frac{C_1}{(C_1+K)}\frac{K}{(C-C_1+K)^2}\right]=0.$$
$$\frac{(1+\rho)[\alpha(C-C_1+K)^2-(C_1+K)^2]\ -2\rho\alpha(C-2C_1)(C+K)}{(C_1+K)^2(C-C_1+K)^2}=0.$$

We have

$$\rho=\frac{\alpha(C-C_1+K)^2-(C_1+K)^2}{2\alpha(C-2C_1)(C+K)-\alpha(C-C_1+K)^2+(C_1+K)^2}$$
$$=\frac{\alpha(C-C_1+K)^2-(C_1+K)^2}{\alpha C(C-2C_1)+(1-\alpha)(C_1+K)^2}.$$

Second derivative with respect to C_1

$$(1+\mathrm{\rho})\left[\frac{-2\mathrm{\alpha K}}{(\mathrm{C_1+K})^3}+\frac{-2\mathrm{K}}{(\mathrm{C-C_1+K})^3}\right]$$
$$-2\mathrm{\rho\alpha}\left[\frac{-2\mathrm{K}}{(\mathrm{C_1+K})^3}\frac{(\mathrm{C-C_1})}{(\mathrm{C-C_1+K})}-2\frac{\mathrm{K}}{(\mathrm{C_1+K})^2}\frac{\mathrm{K}}{(\mathrm{C-C_1+K})^2}\right.$$
$$\left.+\frac{\mathrm{C_1}}{(\mathrm{C_1+K})}\frac{-2\mathrm{K}}{(\mathrm{C-C_1+K})^3}\right]$$
$$=\frac{-2\mathrm{K}}{(\mathrm{C_1+K})^3(\mathrm{C-C_1+K})^3}\{(1+\mathrm{\rho})[(\mathrm{C_1+K})^3+\mathrm{\alpha}(\mathrm{C-C_1+K})^3]$$
$$-2\mathrm{\rho\alpha}[(\mathrm{C-C_1})(\mathrm{C-C_1+K})^2+\mathrm{K}(\mathrm{C_1+K})(\mathrm{C-C_1+K})+\mathrm{C_1}(\mathrm{C_1+K})^2]\}<0.$$

The second derivative of the objective function is less than zero, so a critical value of $\mathrm{C_1}$ is a global maximum value.

Solve first order condition at $\rho=0,C_1=0$, we obtain a critical value of α,

$$\alpha(C-0+K)^2-(0+K)^2=0$$
$$\alpha=\frac{K^2}{(C+K)^2}.$$

(1) When $\alpha\in\ (0,\mathrm{K}^2/(\mathrm{C+K})^2]$, the objective function reaches maximum value point when $\mathrm{C_1}=0$. It's a corner solution. Thus the sender should invest all expenditure to the channel 2.

(2) When $\alpha \in (K^2/(C+K)^2, 1)$, there exits an internal solution. The optimal of expenditure for the sender should satisfy the following condition

$$\rho = \frac{\alpha(C - C_1 + K)^2 - (C_1 + K)^2}{\alpha C(C - 2C_1) + (1 - \alpha)(C_1 + K)^2}.$$

$\boxdot$

Proof of Corollary 3 Checking decision rules about optimal allocation of expenditure in Example 1 in asymmetric information transmission with different levels of the channel's persuasive function

$$\rho = \frac{\alpha(C - C_1 + K)^2 - (C_1 + K)^2}{2\alpha(C - 2C_1)(C + K) - \alpha(C - C_1 + K)^2 + (C_1 + K)^2}$$

By the implicit function theorem

$$f(\alpha, C_1) = \rho[2\alpha(C - 2C_1)(C + K) - \alpha(C - C_1 + K)^2 + (C_1 + K)^2] - [\alpha(C - C_1 + K)^2 - (C_1 + K)^2] = 0$$

$$\frac{\partial f(\alpha, C_1)}{\partial \alpha} = \rho\left[2(C - 2C_1)(C + K) - 2 * 2\alpha(C + K)\frac{\partial C_1}{\partial \alpha} - (C - C_1 + K)^2 + 2\alpha(C - C_1 + K)\frac{\partial C_1}{\partial \alpha} + 2(C_1 + K)\frac{\partial C_1}{\partial \alpha}\right] - \left[(C - C_1 + K)^2 - 2\alpha(C - C_1 + K)\frac{\partial C_1}{\partial \alpha} - 2(C_1 + K)\frac{\partial C_1}{\partial \alpha}\right] = 0$$

$$\frac{\partial C_1}{\partial \alpha} = \frac{2\rho(C - 2C_1)(C + K) - (1 + \rho)(C - C_1 + K)^2}{\{2\rho * 2\alpha(C + K) - (1 + \rho)[2\alpha(C - C_1 + K) + 2(C_1 + K)]\}} > 0$$

The sender should increase expenditure in channel 1 when α increases from $(K^2/(C+K)^2, 1)$.

$\boxdot$

Proof of Theorem 4 Checking the sign of $\frac{\partial U(C_1)}{\partial C_1}|_{C_1=0}$ of Example 2 in asymmetric information transmission with different levels of the channel's persuasive function

First derivate at $C_1 = 0$

$$\frac{\partial U(C_1)}{\partial C_1}|_{C_1=0} = \frac{1}{lnK}\left[(1+\rho)\left(\alpha\ -\frac{1}{C+1}\right) - 2\rho\alpha\frac{ln(C+1)}{lnK}\right]$$
$$= \frac{1}{lnKlnK}\left[\big((1+\rho)lnK - 2\rho ln\,(C+1)\big)\alpha - \frac{(1+\rho)lnK}{C+1}\right]$$

(i) Assume

$$\frac{\partial U(C_1)}{\partial C_1}|_{C_1=0} = \frac{1}{lnKlnK}\left[\big((1+\rho)lnK - 2\rho ln\,(C+1)\big)\alpha - \frac{(1+\rho)lnK}{C+1}\right] < 0$$

Then

$$\alpha < \frac{(1+\rho)\ln K}{(C+1)[(1+\rho)lnK - 2\rho\, ln(C+1)]}$$

1. $If\ (C+1)[(1+\rho)\ lnK - 2\rho\, ln(C+1)] > (1+\rho)\ln K, we\ have$

(1) $when\ K \in \left(C+1, (C+1)^{\frac{2(C+1)}{C}}\right), \rho < \dfrac{lnK}{2ln(C+1)^{(C+1)/C} - lnK},$

- $K \in \left(C+1, (C+1)^{\frac{C+1}{C}}\right), \rho \in \left(0, \frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}\right);$
- $K \in \left((C+1)^{\frac{C+1}{C}}, (C+1)^{\frac{2(C+1)}{C}}\right), \rho < 1 < \frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}, regardless\ of\ \rho.$

(2) $When\ K \in \left((C+1)^{\frac{2(C+1)}{C}}, +\infty\right), \rho > 0 > \dfrac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}, regardless\ of\ \rho.$

2. $If\ (C+1)[(1+\rho)\ lnK - 2\rho\, ln(C+1)] < (1+\rho)\ln K, regardless\ of\ \alpha, we\ have$

(1) $When\ K \in \left(C+1, (C+1)^{\frac{2(C+1)}{C}}\right), \rho > \dfrac{lnK}{2ln(C+1)^{(C+1)/C} - lnK},$

- $K \in \left(C+1, (C+1)^{\frac{C+1}{C}}\right), \rho \in \left(\frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}, 1\right).$
- $K \in \left((C+1)^{\frac{C+1}{C}}, (C+1)^{\frac{2(C+1)}{C}}\right), \rho > \frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK} > 1, no\ solution.$

(2) $When\ K \in \left((C+1)^{\frac{2(C+1)}{C}}, +\infty\right), \rho < \dfrac{lnK}{2ln(C+1)^{(C+1)/C} - lnK} < 0, no\ solution.$

Results of (i) situation as follows

1. $When\ \alpha \in (0,1), K \in \left(C+1, (C+1)^{\frac{C+1}{C}}\right), \rho \in \left(\dfrac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}, 1\right),$

   $then\ \dfrac{\partial U(C_1)}{\partial C_1}|_{C_1=0} < 0.$

2. $When\ \alpha \in \left(0, \dfrac{(1+\rho)\ln K}{(C+1)[(1+\rho)lnK - 2\rho\ln(C+1)]}\right),$

- $K \in \left(C+1, (C+1)^{\frac{C+1}{C}}\right), \rho \in \left(0, \frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}\right), then\ \frac{\partial U(C_1)}{\partial C_1}|_{C_1=0} < 0.$

- $K \in \left((C+1)^{\frac{C+1}{C}}, +\infty\right), then\ \frac{\partial U(C_1)}{\partial C_1}|_{C_1=0} < 0.$

(ii) Assume

$$\frac{\partial U(C_1)}{\partial C_1}|_{C_1=0} = \frac{1}{lnKlnK}\left[\big((1+\rho)lnK - 2\rho ln\ (C+1)\big)\alpha - \frac{(1+\rho)lnK}{C+1}\right] > 0,$$

Then

$$\alpha > \frac{(1+\rho)\ln K}{(C+1)[(1+\rho)lnK - 2\rho\ ln(C+1)]}$$

1. $If\ (C+1)[(1+\rho)lnK - 2\rho\ ln(C+1)] > (1+\rho)\ln K,$

(1) $When\ K \in \left(C+1, (C+1)^{\frac{2(C+1)}{C}}\right), \rho < \dfrac{lnK}{2ln(C+1)^{(C+1)/C} - lnK},$

- $K \in \left(C+1, (C+1)^{\frac{C+1}{C}}\right), \rho \in \left(0, \frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}\right).$
- $K \in \left((C+1)^{\frac{C+1}{C}}, (C+1)^{\frac{2(C+1)}{C}}\right), \rho < 1 < \frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}, regardless\ of\ \rho.$

(2) $When\ K \in \left((C+1)^{\frac{2(C+1)}{C}}, +\infty\right), \rho > 0 > \dfrac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}, regardless\ of\ \rho.$

2. If $(C+1)[(1+\rho)lnK - 2\rho\ ln(C+1)] < (1+\rho)\ln K$, no solution.

Result of (ii) situation as follows

$when\ \alpha \in \left(\dfrac{(1+\rho)\ln K}{(C+1)[(1+\rho)lnK - 2\rho\ ln(C+1)]}, 1\right),$

- $K \in \left(C+1, (C+1)^{\frac{C+1}{C}}\right), \rho \in \left(0, \frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}\right), then\ \frac{\partial U(C_1)}{\partial C_1}|_{C_1=0} > 0.$
- $K \in \left((C+1)^{\frac{C+1}{C}}, +\infty\right), then\ \frac{\partial U(C_1)}{\partial C_1}|_{C_1=0} > 0.$

Conclude above results (i) and (ii), we have

1. $When\ \alpha \in (0,1), K \in \left(C+1, (C+1)^{\frac{C+1}{C}}\right), \rho \in \left(\dfrac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}, 1\right),$

   $then\ \dfrac{\partial U(C_1)}{\partial C_1}|_{C_1=0} < 0, C_1 = 0\ is\ a\ local\ maximum\ point.$

2. $When\ \alpha \in \left(0, \dfrac{(1+\rho)\ln K}{(C+1)[(1+\rho)lnK - 2\rho\ ln(C+1)]}\right),$

   $K \in \left(C+1, (C+1)^{\frac{C+1}{C}}\right), \rho \in \left(0, \frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}\right)\ or\ K \in \left((C+1)^{\frac{C+1}{C}}, +\infty\right),$

then $\frac{\partial U(C_1)}{\partial C_1}|_{C_1=0} < 0,\ C_1 = 0$ *is a local maximum point.*

3. $When\ \alpha \in \left(\frac{(1+\rho)\ln K}{(C+1)[(1+\rho)lnK - 2\rho\, ln(C+1)]}, 1\right),$

$K \in \left(C+1, (C+1)^{\frac{C+1}{C}}\right), \rho \in \left(0, \frac{lnK}{2ln(C+1)^{(C+1)/C} - lnK}\right)$ *or* $K \in \left((C+1)^{\frac{C+1}{C}}, +\infty\right),$

then $\frac{\partial U(C_1)}{\partial C_1}|_{C_1=0} > 0,\ C_1 = 0$ *is a local minimum point.*

⊡

Proof of Lemma 7 The first order condition of Example 2 has no more than three solutions in asymmetric information transmission with different levels of the channel's persuasive function.

First derivative with respect to C_1, we have

$$\frac{\partial U(\mathrm{C}_1)}{\partial \mathrm{C}_1} = (1+\rho)\left[\frac{\alpha}{(\mathrm{C}_1+1)\ln\mathrm{K}} - \frac{1}{(\mathrm{C}-\mathrm{C}_1+1)\ln\mathrm{K}}\right] - 2\rho\alpha\left[\frac{1}{(\mathrm{C}_1+1)\ln\mathrm{K}}\frac{\ln(\mathrm{C}-\mathrm{C}_1+1)}{\ln\mathrm{K}} - \frac{\ln(\mathrm{C}_1+1)}{\ln\mathrm{K}}\frac{1}{(\mathrm{C}-\mathrm{C}_1+1)\ln\mathrm{K}}\right].$$

Simplifying first derivative, we have

$$= \frac{1}{(C_1+1)(C-C_1+1)lnK}\Big\{(1+\rho)[\alpha(\mathrm{C}-\mathrm{C}_1+1) - (\mathrm{C}_1+1)] - \frac{2\rho\alpha}{\ln\mathrm{K}}[(\mathrm{C}-\mathrm{C}_1+1)\ln(\mathrm{C}-\mathrm{C}_1+1) - (C_1+1)\,ln(C_1+1)]\Big\}.$$

Definition 2

$(i)\ v(C_1) = \frac{1}{(C_1+1)(C-C_1+1)lnK}.$

$$(ii)\ w(C_1) = (1+\rho)[\alpha(C-C_1+1) - (C_1+1)] - \frac{2\rho\alpha}{lnK}[(C-C_1+1)\,ln(C-C_1+1) - (C_1+1)\,ln(C_1+1)].$$

$$g(C_1) = \frac{\partial w(C_1)}{\partial C_1} = -(1+\rho)(\alpha+1) + \frac{2\rho\alpha}{lnK}[ln(C_1+1) + ln(C-C_1+1) + 2]$$
$$= -(1+\rho) - \alpha\left[(1+\rho) - \frac{2\rho}{lnK}[\ln[(C_1+1)(C-C_1+1)] + 2]\right].$$

$$h(C_1) = \frac{\partial g(C_1)}{\partial C_1} = \frac{2\alpha\rho}{lnK}\left[\frac{-(C_1+1) + (C-C_1+1)}{(C_1+1)(C-C_1+1)}\right] = \frac{2\alpha\rho(C-2C_1)}{(C_1+1)(C-C_1+1)lnK}.$$

Then we have

$$h(C_1) \geq 0 \; when \; C_1 \in \left[0, \frac{C}{2}\right]; h(C_1) \leq 0 \; when \; C_1 \in \left[\frac{C}{2}, C\right].$$

Thus the sign of $g(C_1)$ changes no more than three times when $C_1 \in [0, C]$, and $w(C_1) = 0$ has no more than three solutions. Since we have $v(C_1) > 0$, the first order condition of example 2 has no more than three solutions when $C_1 \in [0, C]$.

⊡

Proof of Theorem 5 Checking optimal allocation of expenditure in example 1 in asymmetric information transmission with different degrees of information transmission for the receivers

$$MAX_{C1}(1+\rho_{12})\frac{C_1}{(C_1+K)} + (1+\rho_{21})\frac{(C-C_1)}{(C-C_1+K)} - (\rho_{21}+\rho_{12})\frac{C_1}{(C_1+K)}\frac{(C-C_1)}{(C-C_1+K)}$$

$$Where \; C_1 \in [0, C], \rho_{12}, \rho_{21} \in [0,1], \rho_{12} \neq \rho_{21}, K > 0.$$

First order condition with respect to C_1

$$(1+\rho_{12})\frac{K}{(C_1+K)^2} - (1+\rho_{21})\frac{K}{(C-C_1+K)^2} - (\rho_{21} + \rho_{12})\left[\frac{K}{(C_1+K)^2}\frac{(C-C_1)}{(C-C_1+K)} - \frac{C_1}{(C_1+K)}\frac{K}{(C-C_1+K)^2}\right] = 0,$$

$$K\frac{(1+\rho_{12})(C-C_1+K)^2 - (1+\rho_{21})(C_1+K)^2 - (\rho_{21}+\rho_{12})(C-2C_1)(C+K)}{(C_1+K)^2(C-C_1+K)^2} = 0.$$

We have

$$\rho_{12} = \frac{\rho_{21}[(C-C_1)^2 + CK + K^2] + (C+2K)(2C_1 - C)}{{C_1}^2 + K^2 + CK}.$$

Second derivative with respect to C_1

$$(1+\rho_{12})\frac{-2K}{(C_1+K)^3} + (1+\rho_{21})\frac{-2K}{(C-C_1+K)^3} - (\rho_{21} + \rho_{12})\left[\frac{-2K}{(C_1+K)^3}\frac{(C-C_1)}{(C-C_1+K)} - 2\frac{K}{(C_1+K)^2}\frac{K}{(C-C_1+K)^2} + \frac{C_1}{(C_1+K)}\frac{-2K}{(C-C_1+K)^3}\right]$$

$$\frac{-2K}{(C_1+K)^3(C-C_1+K)^3}\{(1+\rho_{12})(C_1+K)^3 + (1+\rho_{21})(C-C_1+K)^3 - (\rho_{21} + \rho_{12})[(C-C_1)(C-C_1+K)^2 + K(C_1+K)(C-C_1+K) + C_1(C_1+K)^2]\} < 0$$

The second derivative of the objective function is less than zero, so a critical value of C_1 is a maximum value.

$\boxdot$

Proof of Corollary 5 Checking decision rules about optimal allocation of expenditure in Example 1 in asymmetric information transmission with different degrees of information transmission for the receivers

$$\rho_{12} = \frac{\rho_{21}[(C - C_1)^2 + CK + K^2] + (C + 2K)(2C_1 - C)}{{C_1}^2 + K^2 + CK}.$$

By the implicit function theorem

1. $f(\rho_{12}, C_1) = \rho_{12}\left({C_1}^2 + K^2 + CK\right) - \rho_{21}[(C - C_1)^2 + CK + K^2] - (C + 2K)(2C_1 - C) = 0,$

$$\frac{\partial f(\rho_{12}, C_1)}{\partial \rho_{12}} = \left({C_1}^2 + K^2 + CK\right) + 2\rho_{12}C_1\frac{\partial C_1}{\partial \rho_{12}} + 2\rho_{21}(C - C_1)\frac{\partial C_1}{\partial \rho_{12}} - 2(C + 2K)\frac{\partial C_1}{\partial \rho_{12}} = 0,$$

$$\frac{\partial C_1}{\partial \rho_{12}} = \frac{-\left({C_1}^2 + K^2 + CK\right)}{2\rho_{12}C_1 + 2\rho_{21}(C - C_1) - 2(C + 2K)} > 0.$$

2. $f(\rho_{21}, C_1) = \rho_{12}\left({C_1}^2 + K^2 + CK\right) - \rho_{21}[(C - C_1)^2 + CK + K^2] - (C + 2K)(2C_1 - C) = 0,$

$$\frac{\partial f(\rho_{12}, C_1)}{\partial \rho_{21}} = 2\rho_{12}C_1\frac{\partial C_1}{\partial \rho_{21}} - [(C - C_1)^2 + CK + K^2] + 2\rho_{21}(C - C_1)\frac{\partial C_1}{\partial \rho_{21}} - 2(C + 2K)\frac{\partial C_1}{\partial \rho_{21}} = 0,$$

$$\frac{\partial C_1}{\partial \rho_{21}} = \frac{(C - C_1)^2 + CK + K^2}{2\rho_{12}C_1 + 2\rho_{21}(C - C_1) - 2(C + 2K)} < 0.$$

The sender should increase expenditure in channel 1 when ρ_{12} increases or ρ_{21} decreases.

$\boxdot$

Proof of Theorem 6 Checking optimal allocation of expenditure in example 2 in asymmetric information transmission with different degrees of information transmission for the receivers

$$MAX_{C_1}(1 + \rho_{12})\frac{ln\,(C_1 + 1)}{lnK} + (1 + \rho_{21})\frac{\ln(C - C_1 + 1)}{lnK} - (\rho_{21} + \rho_{12})\frac{ln(C_1 + 1)}{lnK}\frac{ln(C - C_1 + 1)}{lnK},$$

$$where\ C_1 \in [0, C], \rho_{12}, \rho_{21} \in\ [0,1], \rho_{12} \neq \rho_{21}, K > C + 1.$$

First order condition with respect to C_1,

$$\frac{(1+\rho_{12})}{(C_1+1)\ln K}-\frac{(1+\rho_{21})}{(C-C_1+1)\ln K}-(\rho_{21}+\rho_{12})\left[\frac{\ln(C-C_1+1)}{(C_1+1)\ln K\ln K}-\frac{\ln(C_1+1)}{\ln K(C-C_1+1)\ln K}\right]=0.$$

$$\frac{1}{(C_1+1)\ln K(C-C_1+1)\ln K}\{(1+\rho_{12})(C-C_1+1)\ln K-(1+\rho_{21})(C_1+1)\ln K-(\rho_{21}$$
$$+\rho_{12})[(C-C_1+1)\ln(C-C_1+1)-(C_1+1)\ln(C_1+1)]\}=0$$

$$(1+\rho_{12})(C-C_1+1)\ln K-(1+\rho_{21})(C_1+1)\ln K-(\rho_{21}$$
$$+\rho_{12})[(C-C_1+1)\ln(C-C_1+1)-(C_1+1)\ln(C_1+1))]=0$$

We have

$$\rho_{12}=\frac{\rho_{21}[(C_1+1)\ln K+(C-C_1+1)\ln(C-C_1+1)-(C_1+1)\ln(C_1+1)]-(C-2C_1)\ln K}{[(C-C_1+1)\ln K-(C-C_1+1)\ln(C-C_1+1)+(C_1+1)\ln(C_1+1))]}.$$

$\boxdot$

Chapter 2
A formal analysis of scientists' effort provision in research and dissemination

Abstract

This paper studies two scenarios in a formal analysis of scientists' effort provision in research and dissemination. One is a simultaneous problem that the sender offers effort to send a signal to two types of audiences, such as experts and public; another is a sequential problem that the sender offers effort in academic research, and then sends signal to one type of audiences to representation with effort in science popularization. We investigate how the scientist should divide their time or energy between academic research and science popularization to obtain maximum utility. Consider the same probability and different probability functions at two dimensional for each scenario. We show the optimal allocation of effort depends on the weight of payoff from academic research and science popularization, and the difference in two probability functions between two signals, or between signal and representation. Specifically, in scenario one, if there exist polarization in academic research and science popularization, we could prevent polarization by increasing the ratio of the weights of payoff from dissemination and research using incentives to guarantee the scientist keep the allocation of effort as before. In scenario two, the result shows that we should put equal effort on research and dissemination for scientific achievements transformation no matter how difference in two probability functions between signal and representation.

Keywords: strategic communication, sender-receiver game, signal, representation, information provision

2.1 Introduction

Besides academic research, science popularization is especially important in social progress. However, the public and society have recently been expressing dissatisfaction with the lack of effort from scientists in science popularization. Despite public interest in science, multiple nations report that their populations are lacking in basic factual knowledge about science, which is documented by National Science Board (2012). Some within academia even suggest that popularization should be a secondary activity. There will be questions about the academic's reputation and motivations. Bentley and Kyvik (2011) also suggest that only a minority of academic staff undertakes popular science publishing by a survey of popular science publishing across 13 countries. The main reason is that scientists, especially young scientists, care more about academic research that is more important to the advancement of their career. Some scientists are ashamed of science popularization, and believe that if one is regarded as "science popularization personality", her image will be harmed and her career advancement prospects will be diminished. Thus, scientists would like to communicate with experts and scholars in the arena of internal scientific communication, rather than the general public in the public arena.

While Jensen, Rouquier, Kreimer, and Croissant (2008) find that scientists' dissemination activities have almost no impact on their careers, Weigold (2001) believes that scientists have a basic responsibility to interact with the public. Moreover Bentley and Kyvik (2011) show that the positive relationship between academic research and science popularization is consistent across all countries and academic fields. Their data suggests that scientist with popular publications in science popularization have higher levels of scientific publishing and academic rank. Thus, scientists have responsibility to become the backbone power in science communication and science popularization.

This paper focuses on a scientist's tradeoff between academic research and science popularization. We believe that allocation of time conflict is inevitable. The purpose of this paper is to solve the problem that how to divide their time or energy between academic research and science popularization for a scientist.

We consider two types of audiences for scientists. For academic research, audience is mainly experts and scholars; also, audience of science popularization is general public. In addition, since audiences have different preference, we introduce preference bias measuring how nearly receivers' interests coincide, which is similar to the bias in the model of Crawford and Sobel (1982). Goltsman and Pavlov (2011) show that the sender prefers communicating by private messages if the receivers' average bias is high, and by public messages if the receivers' average bias is low.

This paper constructs two scenarios in a formal analysis of scientists' effort provision in research and dissemination. One scenario is that the sender offers effort to send signal to two types of audiences simultaneously. This scenario is similar to horizontal communication in Hori (2006), who considers the state of nature is a linear function of the agents' information: $\theta = \delta_A \alpha + \delta_B \beta$. Agents' information transmissions are independent. Here is another similar model, Farrell and Gibbons (1989) develop the simplest possible model of cheap talk with two audiences, and show that cheap talk may influence bargaining with asymmetric information.

Another scenario is that the sender offers effort in academic research, and then sends signal to one type of audiences to representation with effort in science popularization. Our model here is close to the model constructed in McGee and Yang (2013). They focus on two senders with complementary information, and show that the senders' information transmissions exhibit strategic complementarity. The receiver's ideal action is multiplicative in the realized states:$y^*(\theta_1, \theta_2) = \theta_1 \theta_2$. While we introduce the probability of the signal and representation for the sequential effort from the sender in research and dissemination.

We form the sender's maximum problem and the receiver's maximum payoff problem depends on Vroom's expectancy theory by giving the utility function of the sender and receivers, which is related with the model of strategic communication developed by Crawford and Sobel (1982). They show that the sender sends a possibly noisy signal to a receiver who takes an action that determines the welfare of both. While in our model, communication is not a cheap talk, because the scientist offers time or effort in research and dissemination. We represent the probability of the signal and representation by effort.

Some recent literatures deal with costly information transmission, including works by Degan and Li (2017), Li and Li (2013), Gentzkow and Kamenica (2014). Li and Li (2013) study two privately informed political candidates. They assume the information campaigns are costly, and a higher level of campaign, whether positive or negative, costs more than a lower level one. Gentzkow and Kamenica (2014) analyze Bayesian persuasion with costly signals. The cost of a signal is proportional to the expected reduction in uncertainty relative to the some fixed reference belief. Degan and Li (2017) study a sender's optimal choice of precision. They assume the higher costs associated with higher precision.

Because the probability of the signal for different type of audiences is different, and the probability of the signal and representation is different in general, so we consider the same probability and different probability functions at two dimensional for each scenario.

We investigate how the scientist should divide their time or energy between academic research and science popularization by the sender and receiver's maximum problem. In scenario one, we show the optimal allocation of effort not only depends on the weights of payoff from two receivers in academic research and science popularization, but also depend on the difference in two probability functions between two signals. While, we show that the effort should be allocated equally in research and dissemination in scenario two.

This paper explains the question that how to allocate the effort for the scientist when there exists polarization in two types of audiences in scenario one. In addition, we show that which is important in transforming scientific achievements to productive force in scenario two.

The rest of the paper is organized as follows. Section 2.2 sets up a model with simultaneous efforts offered by sender under the same and different probability of signals for two types of audiences. We deal with sender's maximum problem and receiver's maximum problem, and obtain some results about time or effort allocation. Section 2.3 sets up a model with sequential efforts offered by sender under the same and different probability for signal and representation. Section 2.4 concludes.

2.2 Scenario One: Simultaneous Efforts Offered by the Sender

We set up a model with one informed sender and two uninformed receivers. It's a simultaneous problem with additive method for the scientist to offer efforts simultaneously in academic research and science popularization.

The sender faces two groups of audiences. One group consists of experts, and another group consists of public audiences. We assume that all experts are identical, and regard its group as the receiver 1; and assume all public audiences are identical, and regard its group as the receiver 2. The sender offers efforts of information provision to them separately, which is denoted by $e_i, i = 1,2,\ e_1 + e_2 \leq 1$.

The state of the world, $\theta \in \{0,1\}$, is chosen by nature, which is observed by the sender but not the receiver. The probability of $\theta = 1$ is defined by $p \in (0,1)$, then the probability of $\theta = 0$ is $1 - p$. After observing the state of world, the sender sends signal to two receivers separately, which is denoted by $s_i \in \{0,1\}, i = 1,2$. After observing signal, the receiver i make a decision $y_i, i = 1,2$, which can be observed by the sender. The receiver i makes decision $y_i(0)$ if observing signal 0, and $y_i(1)$ if observing signal 1. In addition, the receivers could be biased and unbiased, which is denoted by $\beta_i, i = 1, 2$. The bias is common knowledge.

The utility function for the sender is

$$U(\theta, y_1, y_2) = -[\alpha_1(y_1 - \theta)^2 + \alpha_2(y_2 - \theta)^2],$$

where α_i is the weight of payoff from receiver i.

The utility function for receiver i is

$$V(\theta, y_i, \beta_i) = -(y_i - \theta - \beta_i)^2, i = 1, 2.$$

2.2.1 With the Same Probability Function of the Signals for Two Receivers

For simplicity, we assume the probability function of the signal for two types of receivers is the same. Given the probability function of the signal $f(e_i)$: $\mathbb{R}_+ \in \left[\frac{1}{2}, 1\right]$, it is continuous, twice differentiable for $e_i > 0$, strictly increasing, and strictly concave, and satisfices $f(0) = 1/2$.

From Table 2.1, we have the conditional probability of the sender as follows

$$P(s_i = 1|\theta = 0) = P(s_i = 0|\theta = 1) = 1 - f(e_i),$$

$$P(s_i = 1|\theta = 1) = P(s_i = 0|\theta = 0) = f(e_i), i = 1,2.$$

Using Bayes' Rule, we have the conditional probability of the receiver as follows

$$P(\theta = 0|s_i = 1) = \frac{P(\theta = 0, s_i = 1)}{P(\theta = 0, s_i = 1) + P(\theta = 1, s_i = 1)}$$

$$= \frac{P(s_i = 1|\theta = 0)P(\theta = 0)}{P(s_i = 1|\theta = 0)P(\theta = 0) + P(s_i = 1|\theta = 1)P(\theta = 1)} = \frac{(1-p)(1-f(e_i))}{(1-p)(1-f(e_i)) + pf(e_i)},$$

$$P(\theta = 1|s_i = 1) = \frac{P(\theta = 1, s_i = 1)}{P(\theta = 0, s_i = 1) + P(\theta = 1, s_i = 1)}$$

$$= \frac{P(s_i = 1|\theta = 1)P(\theta = 1)}{P(s_i = 1|\theta = 0)P(\theta = 0) + P(s_i = 1|\theta = 1)P(\theta = 1)} = \frac{pf(e_i)}{(1-p)(1-f(e_i)) + pf(e_i)},$$

$$P(\theta = 0|s_i = 0) = \frac{P(\theta = 0, s_i = 0)}{P(\theta = 0, s_i = 0) + P(\theta = 1, s_i = 0)}$$

$$= \frac{P(s_i = 0|\theta = 0)P(\theta = 0)}{P(s_i = 0|\theta = 0)P(\theta = 0) + P(s_i = 0|\theta = 1)P(\theta = 1)} = \frac{(1-p)f(e_i)}{(1-p)f(e_i) + p(1-f(e_i))},$$

$$P(\theta = 1|s_i = 0) = \frac{P(\theta = 1, s_i = 0)}{P(\theta = 0, s_i = 0) + P(\theta = 1, s_i = 0)}$$

$$= \frac{P(s_i = 0|\theta = 1)P(\theta = 1)}{P(s_i = 0|\theta = 0)P(\theta = 0) + P(s_i = 0|\theta = 1)P(\theta = 1)} = \frac{p\big(1-f(e_i)\big)}{(1-p)f(e_i) + p\big(1-f(e_i)\big)},$$

$i = 1,2.$

Table 2.1 The Matrix with the Probability Function of the Signal s_i

		Signal s_i	
		0	1
State	0	$f(e_i)$	$1-f(e_i)$
θ	1	$1-f(e_i)$	$f(e_i)$

Lemma 1 Rule of the conditional probability of the receiver $i \in \{1,2\}$ as follows

$$P(\theta=1|s_i=1) \geq P(\theta=1|s_i=0), P(\theta=0|s_i=1) \leq P(\theta=0|s_i=0).$$

If p<1/2, we have $P(\theta=0|s_i=0) \geq P(\theta=0|s_i=1) \geq P(\theta=1|s_i=1) \geq P(\theta=1|s_i=0)$.

Proof of lemma 1 is provided in Appendix.

Receiver Problem 1 *The maximization problem for the receiver with the same probability function of the signals for two receivers in scenario one can be written as*

$$\max_{y_i} EV(\theta, y_i(s_i), \beta_i), i = 1,2.$$

For receiver $i \in \{1,2\}$, we have

$$\max_{y_i} EV(\theta, y_i(s_i), \beta_i) = \max_{y_i} P(\theta=0|s_i)[-(y_i-\beta_i)^2] + P(\theta=1|s_i)[-(y_i-1-\beta_i)^2]$$

Proposition 1 *The receiver i obtains the maximum expectation utility with the same probability function of the signals for two receivers in scenario one when*

$$y_i(s_i=1) = \beta_i + P(\theta=1|s_i=1) = \beta_i + \frac{pf(e_i)}{(1-p)(1-f(e_i)) + pf(e_i)}$$

Or

$$y_i(s_i=0) = \beta_i + P(\theta=1|s_i=0) = \beta_i + \frac{p(1-f(e_i))}{(1-p)f(e_i) + p(1-f(e_i))}, i = 1,2.$$

The maximum expectation utility of the receiver $i \in \{1,2\}$ *is*

$$\max_{y_i} EV(\theta, y_i(s_i = 1), \beta_i) = -P(\theta = 1|s_i = 1)P(\theta = 0|s_i = 1),$$

$$\max_{y_i} EV(\theta, y_i(s_i = 0), \beta_i) = -P(\theta = 1|s_i = 0)P(\theta = 0|s_i = 0).$$

Proof of Proposition 1 is provided in Appendix. Note that the best respond decision of the receiver is related with the corresponding probability of the signal when the state of the world is one and own bias. For two signals 0 and 1, the best respond decision of the receiver for two signals are complemented with each other regardless the receiver's own bias. The maximum expectation utility of the receiver is the negative product of two corresponding probability of the state of the world given a signal.

Sender Problem 1 *The maximization problem for the sender with the same probability function of the signals for two receivers in scenario one can be written as*

$$MAX_{e_1, e_2}\, EU(\theta, y_1, y_2) = MAX_{e_1, e_2} - E[\alpha_1(y_1 - \theta)^2 + \alpha_2(y_2 - \theta)^2],$$
$$where\ e_1 + e_2 \leq 1.$$

$$\begin{aligned}
&MAX_{e_1, e_2} - E[\alpha_1(y_1 - \theta)^2 + \alpha_2(y_2 - \theta)^2] \\
&= MAX_{e_1, e_2} - \{\alpha_1[P(s_1 = 0|\theta = 0)(y_1(0) - 0)^2 + P(s_1 = 1|\theta = 0)(y_1(1) - 0)^2] \\
&\qquad + \alpha_2[P(s_2 = 0|\theta = 0)(y_2(0) - 0)^2 + P(s_2 = 1|\theta = 0)(y_2(1) - 0)^2]\}P(\theta = 0) \\
&\qquad - \{\alpha_1[P(s_1 = 0|\theta = 1)(y_1(0) - 1)^2 + P(s_1 = 1|\theta = 1)(y_1(1) - 1)^2] \\
&\qquad + \alpha_2[P(s_2 = 0|\theta = 1)(y_2(0) - 1)^2 + P(s_2 = 1|\theta = 1)(y_2(1) - 1)^2]\}P(\theta = 1)
\end{aligned}$$

Lemma 2 *The expectation utility of the sender will increase when the sender increases effort in either receiver with the same probability function of the signals for two receivers in scenario one. The maximum expectation utility of the sender as follows:*

$MAX_{e_1, e_2} EU(\theta, y_1, y_2)$

$$= -\alpha_1 \left\{ {\beta_1}^2 + \frac{p(1-p)f(e_1)\big(1-f(e_1)\big)}{[f(e_1) - 2pf(e_1) + p][1 - p - f(e_1) + 2pf(e_1)]} \right\}$$

$$- \alpha_2 \left\{ {\beta_2}^2 + \frac{p(1-p)f(e_2)\big(1-f(e_2)\big)}{[f(e_2) - 2pf(e_2) + p][1 - p - f(e_2) + 2pf(e_2)]} \right\}$$

$$= -\alpha_1 \left[{\beta_1}^2 + \sqrt{P(\theta = 0|s_1 = 1)P(\theta = 1|s_1 = 1)P(\theta = 0|s_1 = 0)P(\theta = 1|s_1 = 0)} \right]$$

$$-\alpha_2 \left[{\beta_2}^2 + \sqrt{P(\theta = 0|s_2 = 1)P(\theta = 1|s_2 = 1)P(\theta = 0|s_2 = 0)P(\theta = 1|s_2 = 0)} \right].$$

The Proof of lemma 2 is provided in Appendix. Note that the sender could obtain more expectation utility by increasing effort in either receiver. So the sender should use up all effort supply in order to obtain a maximum expectation utility. The maximum expectation utility of the sender is the weighted sum of both receivers' square root of all conditional probability with bias square.

Proposition 2 *Given a certain amount of the sender's total effort t, t is less and equal to one, the optimal allocation of efforts in the receiver one and two for the sender to obtain a maximum expectation utility with the same probability function of the signals for two receivers in scenario one satisfies the following equation,*

$$\frac{[1 - 2f(e_1)]f'(e_1)}{[1 - 2f(e_2)]f'(e_2)} \frac{\{[f(e_2) - 2pf(e_2) + p][1 - p - f(e_2) + 2pf(e_2)]\}^2}{\{[f(e_1) - 2pf(e_1) + p][1 - p - f(e_1) + 2pf(e_1)]\}^2} = \frac{\alpha_2}{\alpha_1},$$

$$if\ 0 \le e_1 + e_2 \le t \le 1.$$

Then

$$\frac{f'(e_1)[1 - 2f(e_1)][p(1-p) + (1-2p)^2 f(t - e_1)(1 - f(t - e_1))]^2}{f'(t - e_1)[1 - 2f(t - e_1)][p(1-p) + (1-2p)^2 f(e_1)(1 - f(e_1))]^2} = \frac{\alpha_2}{\alpha_1}.$$

Proof of Proposition 2 is provided in Appendix. The result shows that how to offer effort into academic research and science popularization for the sender to obtain the maximum expectation utility depend on the weight of payoff from two receivers and the probability of the state of the world.

Corollary 1 *Given a certain amount of the sender's total effort t, t is less and equal to one, if we increase the ratio of the weight of payoff between the receivers* α_2/α_1*, the sender would like to decrease effort on the receiver one to obtain maximum expectation utility with the same probability function of the signals for two receivers in scenario one. Specifically,*

(1) If $\alpha_2/\alpha_1 = 1,\ 0 < e_1 + e_2 \le t \le 1$*, we have* $0 < e_1 = e_2 \le t/2$*,*

(2) If $\alpha_2/\alpha_1 > 1, 0 < e_1 + e_2 \le t \le 1,$ *we have* $0 < e_1 < t/2 < e_2 \le t$*,*

(3) If $\alpha_2/\alpha_1 < 1, 0 < e_1 + e_2 \le t \le 1,$ *we have* $0 < e_2 < t/2 < e_1 \le t$.

Proof of Corollary 1 is provided in Appendix. Note that if the ratio of the weight of payoff from general public and the weight of payoff from the experts increase, the scientist would incline to spend more effort in science popularization in order to obtain the maximum expectation utility.

If the weight of payoff from general public in the public arena were equal to that from the experts in the arena of internal scientific communication, the scientist would like to offer the same effort on and science popularization. If the weight of payoff from general public were more than that from experts, the scientist would like to offer more effort on science popularization. If the weight of payoff from general public were less than that from experts, the scientist would like to offer more effort on academic research.

For the whole society, some local places are weak in academic research; some local places are weak in science popularization. We could adjust the weights of payoff from general public and the experts by incentives to change the proportion of effort offered by the scientist in academic research and science popularization when the behavior of the scientist is inconsistent with the goal of the society.

2.2.2 With the Different Probability Function of the Signals for Two Receivers

In general, the probability function of the signal for different type of the receivers is different. We assume the probability function of the signal for the receiver 1 is $f_1(e_1): \mathbb{R}_+ \in \left[\frac{1}{2}, 1\right]$, and the probability function of the signal for the receiver 2 is $f_2(e_2) = \frac{1}{2} + \lambda \left[f_1(e_2) - \frac{1}{2}\right]: \mathbb{R}_+ \in \left[\frac{1}{2}, 1\right], \lambda \in [0, 1]$. Here $f_i(e)$ is continuous, twice differentiable for $e > 0$, strictly increasing, and strictly concave, and satisfices $f_i(0) = \frac{1}{2}$.

From Table 2.2 and 2.3, we have the conditional probability of the sender as follows

$$P(s_1 = 1|\theta = 0) = P(s_1 = 0|\theta = 1) = 1 - f_1(e_1),$$

$$P(s_1 = 1|\theta = 1) = P(s_1 = 0|\theta = 0) = f_1(e_1),$$

$$P(s_2 = 0|\theta = 0) = P(s_2 = 1|\theta = 1) = \frac{1}{2} + \lambda \left[f_1(e_2) - \frac{1}{2}\right],$$

$$P(s_2 = 0|\theta = 1) = P(s_2 = 1|\theta = 0) = \frac{1}{2} - \lambda \left[f_1(e_2) - \frac{1}{2}\right].$$

Using Bayes' Rule, we have the conditional probability of the receiver as follows

$$P(\theta = 0|s_1 = 1) = \frac{P(s_1 = 1|\theta = 0)P(\theta = 0)}{P(s_1 = 1|\theta = 0)P(\theta = 0) + P(s_1 = 1|\theta = 1)P(\theta = 1)}$$
$$= \frac{(1 - p)(1 - f_1(e_1))}{(1 - p)(1 - f_1(e_1)) + pf_1(e_1)},$$

$$P(\theta = 1|s_1 = 1) = \frac{P(s_1 = 1|\theta = 1)P(\theta = 1)}{P(s_1 = 1|\theta = 0)P(\theta = 0) + P(s_1 = 1|\theta = 1)P(\theta = 1)}$$
$$= \frac{pf_1(e_1)}{(1 - p)(1 - f_1(e_1)) + pf_1(e_1)},$$

$$P(\theta = 0|s_1 = 0) = \frac{P(s_1 = 0|\theta = 0)P(\theta = 0)}{P(s_1 = 0|\theta = 0)P(\theta = 0) + P(s_1 = 0|\theta = 1)P(\theta = 1)}$$
$$= \frac{(1 - p)f_1(e_1)}{(1 - p)f_1(e_1) + p(1 - f_1(e_1))},$$

Table 2.2 The Matrix with the Probability of the Signal s_1

		Signal s_1	
		0	1
State θ	0	$f_1(e_1)$	$1-f_1(e_1)$
	1	$1-f_1(e_1)$	$f_1(e_1)$

Table 2.3 The Matrix with the Probability of the Signal s_2

		Signal s_2	
		0	1
State θ	0	$\frac{1}{2}+\lambda\left[f_1(e_2)-\frac{1}{2}\right]$	$\frac{1}{2}-\lambda\left[f_1(e_2)-\frac{1}{2}\right]$
	1	$\frac{1}{2}-\lambda\left[f_1(e_2)-\frac{1}{2}\right]$	$\frac{1}{2}+\lambda\left[f_1(e_2)-\frac{1}{2}\right]$

$$P(\theta=1|s_1=0)=\frac{P(s_1=0|\theta=1)P(\theta=1)}{P(s_1=0|\theta=0)P(\theta=0)+P(s_1=0|\theta=1)P(\theta=1)}$$
$$=\frac{p(1-f_1(e_1))}{(1-p)f_1(e_1)+p(1-f_1(e_1))},$$

$$P(\theta=0|s_2=1)=\frac{P(s_2=1|\theta=0)P(\theta=0)}{P(s_2=1|\theta=0)P(\theta=0)+P(s_2=1|\theta=1)P(\theta=1)}$$
$$=\frac{(1-p)\left\{\frac{1}{2}-\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right\}}{(1-p)\left\{\frac{1}{2}-\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right\}+p\left\{\frac{1}{2}+\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right\}}=\frac{(1-p)\left\{\frac{1}{2}-\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right\}}{\frac{1}{2}-\lambda(1-2p)\left[f_1(e_2)-\frac{1}{2}\right]},$$

$$P(\theta=1|s_2=1)=\frac{P(s_2=1|\theta=1)P(\theta=1)}{P(s_2=1|\theta=0)P(\theta=0)+P(s_2=1|\theta=1)P(\theta=1)}$$
$$=\frac{p\left[\frac{1}{2}+\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right]}{(1-p)\left\{\frac{1}{2}-\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right\}+p\left\{\frac{1}{2}+\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right\}}=\frac{p\left\{\frac{1}{2}+\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right\}}{\frac{1}{2}-\lambda(1-2p)\left[f_1(e_2)-\frac{1}{2}\right]},$$

$$P(\theta=0|s_2=0)=\frac{P(s_2=0|\theta=0)P(\theta=0)}{P(s_2=0|\theta=0)P(\theta=0)+P(s_2=0|\theta=1)P(\theta=1)}$$

$$=\frac{(1-p)\left\{\frac{1}{2}+\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right\}}{(1-p)\left\{\frac{1}{2}+\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right\}+p\left\{\frac{1}{2}-\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right\}}=\frac{(1-p)\left\{\frac{1}{2}+\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right\}}{\frac{1}{2}+\lambda(1-2p)\left[f_1(e_2)-\frac{1}{2}\right]},$$

$$P(\theta=1|s_2=0)=\frac{P(s_2=0|\theta=1)P(\theta=1)}{P(s_2=0|\theta=0)P(\theta=0)+P(s_2=0|\theta=1)P(\theta=1)}$$

$$=\frac{p\left\{\frac{1}{2}-\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right\}}{(1-p)\left\{\frac{1}{2}+\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right\}+p\left\{\frac{1}{2}-\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right\}}=\frac{p\left\{\frac{1}{2}-\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right\}}{\frac{1}{2}+\lambda(1-2p)\left[f_1(e_2)-\frac{1}{2}\right]}.$$

Receiver Problem 2 *The maximization problem for the receiver with the different probability function of the signals for two receivers in scenario one can be written as*

$$\max_{y_i} EV(\theta, y_i(s_i), \beta_i)\,, i=1,2.$$

Proposition 3

(1) The receiver 1 obtains the maximum expectation utility with the different probability function of the signals for two receivers in scenario one when

$$y_1(s_1=1)=\beta_1+P(\theta=1|s_1=1)=\beta_1+\frac{pf_1(e_1)}{(1-p)\big(1-f_1(e_1)\big)+pf_1(e_1)}$$

Or

$$y_1(s_1=0)=\beta_1+P(\theta=1|s_1=0)=\beta_1+\frac{p(1-f(e_1))}{(1-p)f_1(e_1)+p(1-f_1(e_1))}.$$

The maximum expectation utility of the receiver 1 is

$$\max_{y_1} EV(\theta, y_1(s_1=1), \beta_1)=-P(\theta=1|s_1=1)P(\theta=0|s_1=1),$$

$$\max_{y_1} EV(\theta, y_1(s_1=0), \beta_1)=-P(\theta=1|s_1=0)P(\theta=0|s_1=0).$$

(2) The receiver 2 obtains the maximum expectation utility with the different probability function of the signals for two receivers in scenario one when

$$y_2(s_2=1)=\beta_2+P(\theta=1|s_2=1)=\beta_2+\frac{p\left\{\frac{1}{2}+\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right\}}{\frac{1}{2}-\lambda(1-2p)\left[f_1(e_2)-\frac{1}{2}\right]}$$

Or

$$y_2(s_2=0)=\beta_2+P(\theta=1|s_2=0)=\beta_2+\frac{p\left\{\frac{1}{2}-\lambda\left[f_1(e_2)-\frac{1}{2}\right]\right\}}{\frac{1}{2}+\lambda(1-2p)\left[f_1(e_2)-\frac{1}{2}\right]}.$$

The maximum expectation utility of the receiver 2 is

$$\max_{y_2} EV(\theta,y_2(s_2=1),\beta_2)=-P(\theta=1|s_2=1)P(\theta=0|s_2=1),$$

$$\max_{y_2} EV(\theta,y_2(s_2=0),\beta_2)=-P(\theta=1|s_2=0)P(\theta=0|s_2=0).$$

Proof of Proposition 3 is provided in Appendix.

Sender Problem 2 *The maximization problem for the sender with the different probability function of the signals for two receivers in scenario one can be written as*

$$MAX_{e_1,e_2}\, EU(\theta,y_1,y_2)=MAX_{e_1,e_2}-E[\alpha_1(y_1-\theta)^2+\alpha_2(y_2-\theta)^2],$$
$$where\ e_1+e_2\le 1.$$

Lemma 3 *The expectation utility of the sender will increase when the sender increases the effort in either receiver with the different probability function of the signals for two receivers in scenario one. And the maximum expectation utility of the sender increases with the increase of* λ. *The maximum expectation utility of the sender as follows:*

$$MAX_{e_1,e_2}\, EU(\theta,y_1,y_2)$$
$$=-\alpha_1\left[{\beta_1}^2+\frac{p(1-p)f_1(e_1)(f_1(e_1)-1)}{p(p-1)+(1-2p)^2f_1(e_1)(f_1(e_1)-1)}\right]$$
$$-\alpha_2\left[{\beta_2}^2+\frac{p(1-p)\left[\frac{1}{4}-\lambda^2\left[f_1(e_2)-\frac{1}{2}\right]^2\right]}{\frac{1}{4}-(1-2p)^2\lambda^2\left[f_1(e_2)-\frac{1}{2}\right]^2}\right]$$

$$= -\alpha_1\left[{\beta_1}^2 + \sqrt{P(\theta = 0|s_1 = 1)P(\theta = 1|s_1 = 1)P(\theta = 0|s_1 = 0)P(\theta = 1|s_1 = 0)}\right]$$

$$-\alpha_2\left[{\beta_2}^2 + \sqrt{P(\theta = 0|s_2 = 1)P(\theta = 1|s_2 = 1)P(\theta = 0|s_2 = 0)P(\theta = 1|s_2 = 0)}\right].$$

The Proof of Lemma 3 is provided in Appendix. The result shows that the expectation utility of the sender will increase when the sender increases the effort on academic research or science popularization. The sender would obtain more expectation utility when the probability functions of the signal for different type of the receivers are closer.

Proposition 4 *Given a certain amount of the sender's total effort t, t is less and equal to one, the optimal allocation of effort in receiver one and two for the sender to obtain a maximum expectation utility with the different probability function of the signals for two receivers in scenario one satisfies the following equation,*

$$\frac{{f_1}'(e_1)[1-2f_1(e_1)]\left[\frac{1}{4}-(1-2p)^2\lambda^2\left[f_1(e_2)-\frac{1}{2}\right]^2\right]^2}{\lambda^2{f_1}'(e_2)[1-2f_1(e_2)]\left[\frac{1}{4}-(1-2p)^2\left[f_1(e_1)-\frac{1}{2}\right]^2\right]^2} = \frac{\alpha_2}{\alpha_1}, if\ 0 < e_1 + e_2 \le t \le 1.$$

Then

$$\frac{{f_1}'(e_1)[1-2f_1(e_1)]\left[\frac{1}{4}-(1-2p)^2\lambda^2\left[f_1(t-e_1)-\frac{1}{2}\right]^2\right]^2}{\lambda^2{f_1}'(t-e_1)[1-2f_1(t-e_1)]\left[\frac{1}{4}-(1-2p)^2\left[f_1(e_1)-\frac{1}{2}\right]^2\right]^2} = \frac{\alpha_2}{\alpha_1}.$$

The proof of Proposition 4 is provided in Appendix. The result shows that how to offer effort into academic research and science popularization to obtain the maximum expectation utility, which is not only depend on the weights of payoff from two receivers in academic research and science popularization and the probability of the state of the world, but depend on the difference between the probability function of the signals for two types of audiences.

Corollary 2 *Given a certain amount of the sender's total effort t, t is less and equal to one, if we increase the ratio of the weight of payoff between receivers* α_2/α_1, *the sender would like to decrease effort in the receiver one to obtain maximum expectation utility; or if we decrease the difference between the probability function of the signals for two receivers, the sender would like to decrease effort in receiver one to obtain maximum expectation utility with the different probability function of the signals for two receivers in scenario one. Specifically,*

(1) $\alpha_2/\alpha_1 = 1$ *if and only if* $[1 - 2f_1(e_1)]/[1 - 2f_1(t - e_1)] = \lambda$,

(2) $\alpha_2/\alpha_1 > 1$ *if and only if* $[1 - 2f_1(e_1)]/[1 - 2f_1(t - e_1)] > \lambda$,

(3) $\alpha_2/\alpha_1 < 1$ *if and only if* $[1 - 2f_1(e_1)]/[1 - 2f_1(t - e_1)] < \lambda, where\ \lambda \in (0,1)$.

Proof of Corollary 2 is provided in Appendix. Note that if the ratio of the weight of payoff from the public and the weight of payoff from the experts increase or the difference between the probability function of the signals for two receivers is smaller than before, the scientist would like to put more effort on science popularization. Moreover if there were a big difference between the probability functions of the signals for two types of audience, which means λ is very small, the scientist would like to increase effort in academic research to obtain the maximum expectation utility, or we could increase the ratio of the weight of payoff from two receivers.

Corollary 3 *Given a certain amount of the sender's total effort t, t is less and equal to one, if the difference between the probability functions of the signals for two receivers is smaller than before, we could increase the ratio of the weight of payoff between two receivers* α_2/α_1 *to make the sender to obtain maximum expectation utility as before with the different probability function of the signals for two receivers in scenario one.*

Note that if there exists a big difference between the probability functions of the signals for two receivers unusually, i.e. there exists polarization, we could make the sender to keep the allocation of effort as before by increasing the ratio of the weights of payoff from dissemination and research, such as using incentives.

2.3 Scenario Two: Sequential Efforts offered by the Sender

We set up a model with one informed sender and one uninformed receiver. It is a sequential problem with multiplicative method for the scientist to offered efforts sequentially in academic research and science popularization.

The sender faces a group of public audiences. We assume all public audiences are identical, and regard its group as one receiver. The sender firstly offers effort of information provision to academic research, which is denoted by e_1; then offers effort of information provision to science popularization, which is denoted by e_2; $e_1 + e_2 \leq 1$.

The state of world $\theta \in \{0,1\}$, is chosen by nature, which is observed by the sender but not the receiver. The probability of $\theta = 1$ is defined by $p \in (0,1)$, then the probability of $\theta = 0$ is $1 - p$. After observing the state of world θ, the sender sends signal to the receiver, where signal $s \in \{0,1\}$ with effort e_1 offered in academic research. After observing signal s, the receiver gives a representation r, where representation $r \in \{0,1\}$ with the sender's effort e_2 offered in science popularization. At the same time, the receiver makes a decision y_2 which can be observed by the sender. The receiver makes a decision $y_2(0)$ if representing 0, and $y_2(1)$ if representing 1. The receiver's bias is denoted by β_2. The bias is common knowledge.

The utility function for the sender is

$$U(\theta, y_2) = -\alpha(y_2 - \theta)^2,$$

Where α is the weight of payoff from the receiver

The utility function for the receiver

$$V(\theta, y_2, \beta_2) = -(y_2 - \theta - \beta_2)^2.$$

2.3.1 With the Same Probability Function of the Signal and Representation

For simplicity, we assume the probability function of the signal and representation is the same. Given the probability function of the signal and representation $f(e_i)$: $\mathbb{R}_+ \in \left[\frac{1}{2}, 1\right], i = 1,2$, it is continuous, twice differentiable for $e_i > 0$, strictly increasing, and strictly concave, and satisfices $f(0) = \frac{1}{2}$.

From Table 2.4 and 2.5, we have the conditional probability of the sender as follows

$$P(r=0|\theta=1) = P(r=1|\theta=0) = f(e_1)\big(1-f(e_2)\big) + f(e_2)\big(1-f(e_1)\big)$$
$$= f(e_1) + f(e_2) - 2f(e_1)f(e_2),$$
$$P(r=0|\theta=0) = P(r=1|\theta=1) = f(e_1)f(e_2) + \big(1-f(e_1)\big)\big(1-f(e_2)\big)$$
$$= 1 - f(e_1) - f(e_2) + 2f(e_1)f(e_2).$$

Using Bayes' Rule, we have the conditional probability of the receiver as follows

$$P(\theta=0|r=1) = \frac{P(r=1|\theta=0)P(\theta=0)}{P(r=1|\theta=0)P(\theta=0) + P(r=1|\theta=1)P(\theta=1)}$$
$$= \frac{(1-p)\big[f(e_1)\big(1-f(e_2)\big) + f(e_2)\big(1-f(e_1)\big)\big]}{(1-p)\big[f(e_1)\big(1-f(e_2)\big) + f(e_2)\big(1-f(e_1)\big)\big] + p\big[f(e_1)f(e_2) + \big(1-f(e_1)\big)\big(1-f(e_2)\big)\big]}$$
$$= \frac{(1-p)(f(e_1) + f(e_2) - 2f(e_1)f(e_2))}{p + (1-2p)(f(e_1) + f(e_2) - 2f(e_1)f(e_2))},$$

$$P(\theta=1|r=1) = \frac{P(r=1|\theta=1)P(\theta=1)}{P(r=1|\theta=0)P(\theta=0) + P(r=1|\theta=1)P(\theta=1)}$$
$$= \frac{p\big[f(e_1)f(e_2) + \big(1-f(e_1)\big)\big(1-f(e_2)\big)\big]}{(1-p)\big[f(e_1)\big(1-f(e_2)\big) + f(e_2)\big(1-f(e_1)\big)\big] + p\big[f(e_1)f(e_2) + \big(1-f(e_1)\big)\big(1-f(e_2)\big)\big]}$$
$$= \frac{p(1 - f(e_1) - f(e_2) + 2f(e_1)f(e_2))}{p + (1-2p)(f(e_1) + f(e_2) - 2f(e_1)f(e_2))},$$

Table 2.4 The Matrix with the Probability of the Signal for the Sender

		Signal s	
		0	1
State θ	0	$f(e_1)$	$1-f(e_1)$
	1	$1-f(e_1)$	$f(e_1)$

Table 2.5 The Matrix with the Probability of the Representation for the Receiver

		Representation r	
		0	1
Signal s	0	$f(e_2)$	$1-f(e_2)$
	1	$1-f(e_2)$	$f(e_2)$

$$P(\theta=0|r=0)=\frac{P(r=0|\theta=0)P(\theta=0)}{P(r=0|\theta=0)P(\theta=0)+P(r=0|\theta=1)P(\theta=1)}$$

$$=\frac{(1-p)\big[f(e_1)f(e_2)+\big(1-f(e_1)\big)\big(1-f(e_2)\big)\big]}{(1-p)\big[f(e_1)f(e_2)+\big(1-f(e_1)\big)\big(1-f(e_2)\big)\big]+p\big[f(e_1)\big(1-f(e_2)\big)+f(e_2)\big(1-f(e_1)\big)\big]}$$

$$=\frac{(1-p)(1-f(e_1)-f(e_2)+2f(e_1)f(e_2))}{1-p-(1-2p)(f(e_1)+f(e_2)-2f(e_1)f(e_2))},$$

$$P(\theta=1|r=0)=\frac{P(r=0|\theta=1)P(\theta=1)}{P(r=0|\theta=0)P(\theta=0)+P(r=0|\theta=1)P(\theta=1)}$$

$$=\frac{p\big[f(e_1)\big(1-f(e_2)\big)+f(e_2)\big(1-f(e_1)\big)\big]}{(1-p)\big[f(e_1)f(e_2)+\big(1-f(e_1)\big)\big(1-f(e_2)\big)\big]+p\big[f(e_1)\big(1-f(e_2)\big)+f(e_2)\big(1-f(e_1)\big)\big])}$$

$$=\frac{p(f(e_1)+f(e_2)-2f(e_1)f(e_2))}{1-p-(1-2p)(f(e_1)+f(e_2)-2f(e_1)f(e_2))}.$$

Receiver Problem 3 *The maximization problem for the receiver with the same probability function of the signal and representation in scenario two can be written as*

$$\max_{y_2} EV(\theta, y_2(r), \beta_2)$$

Proposition 5 *The receiver obtains the maximum expectation utility with the same probability function of the signal and representation in scenario two when*

$$y_2(r=1) = \beta_2 + P(\theta = 1|r = 1) = \beta_2 + \frac{p(1 - f(e_1) - f(e_2) + 2f(e_1)f(e_2))}{p + (1 - 2p)(f(e_1) + f(e_2) - 2f(e_1)f(e_2))}$$

Or

$$y_2(r=0) = \beta_2 + P(\theta = 1|r = 0) = \beta_2 + \frac{p(f(e_1) + f(e_2) - 2f(e_1)f(e_2))}{1 - p - (1 - 2p)(f(e_1) + f(e_2) - 2f(e_1)f(e_2))}$$

The maximum expectation utility of the receiver is

$$\max_{y_2} EV(\theta, y_2(r=1), \beta_2) = -P(\theta = 1|r = 1)P(\theta = 0|r = 1),$$

$$\max_{y_2} EV(\theta, y_2(r=0), \beta_2) = -P(\theta = 1|r = 0)P(\theta = 0|r = 0).$$

Proof of Proposition 5 is provided in Appendix.

Sender Problem 3 *The maximization problem for the sender with the same probability function of the signal and representation in scenario two can be written as*

$$MAX_{e_1, e_2} EU(\theta, y_2) = MAX_{e_1, e_2} - E[\alpha(y_2 - \theta)^2],$$

$$where\ e_1 + e_2 \leq 1.$$

The maximum expectation utility of the sender

$$MAX_{e_1, e_2} - E[\alpha(y_2 - \theta)^2]$$

$$= MAX_{e_1, e_2} - \alpha\{[P(r = 0|\theta = 0)(y_2(0) - 0)^2 + P(r = 1|\theta = 0)(y_2(1) - 0)^2]P(\theta = 0)$$

$$+ [P(r = 0|\theta = 1)(y_2(0) - 1)^2 + P(r = 1|\theta = 1)(y_2(1) - 1)^2]P(\theta = 1)\}.$$

Lemma 4 *The expectation utility of the sender will increase when the sender increases the effort in research or dissemination with the same probability function of the signal and representation in scenario two. The maximum expectation utility of the sender as follows:*

$$MAX_{e_1,e_2}\, EU(\theta, y_2)$$
$$= -\alpha \left\{ {\beta_2}^2 + \frac{-p(1-p)\big(f(e_1)+f(e_2)-2f(e_1)f(e_2)\big)\big(1-f(e_1)-f(e_2)+2f(e_1)f(e_2)\big)}{\big[p+(1-2p)\big(f(e_1)+f(e_2)-2f(e_1)f(e_2)\big)\big]\big[1-p-(1-2p)\big(f(e_1)+f(e_2)-2f(e_1)f(e_2)\big)\big]} \right\}$$
$$= -\alpha \left[{\beta_2}^2 + \sqrt{P(\theta=0|r=1)P(\theta=1|r=1)P(\theta=0|r=0)P(\theta=1|r=0)}\right].$$

The Proof of lemma 4 is provided in Appendix. Note that the sender could obtain more expectation utility by increasing effort in research or dissemination.

Proposition 6 *Given a certain amount of the sender's total effort t, t is less and equal to one, the optimal allocation of effort in research or dissemination for the sender to obtain maximum expectation utility with the same probability function of the signal and representation in scenario two satisfies the following equation*

$$\frac{f'(e_1)}{[1-2f(e_1)]} = \frac{f'(e_2)}{[1-2f(e_2)]},$$
$$i.e.\, e_1 = e_2 = t/2 \leq 1/2.$$

Proof of Proposition 6 is provided in Appendix. The result shows that effort in science popularization is just as important to scientist's growth as academic research no matter what the receiver's action is.

2.3.2 With the Different Probability Function of the Signal and Representation

In general, the probability function of the signal and representation is different. We assume the probability function of the signal is $f_1(e_1)\colon \mathbb{R}_+ \in \left[\frac{1}{2}, 1\right]$, and the probability function of the representation is $f_2(e_2) = 1/2 + \gamma[f_1(e_2) - 1/2]\colon \mathbb{R}_+ \in \left[\frac{1}{2}, 1\right], \gamma \in [0,1]$. $f_i(e)$ is continuous, twice differentiable for $e > 0$, strictly increasing, strictly concave, and satisfices $f_i(0) = 1/2$.

From Table 2.6 and 2.7, we have the conditional probability of the sender as follows

$$P(r=1|\theta=0) = P(r=0|\theta=1)$$
$$= f_1(e_1)\left[\frac{1}{2} - \gamma\left[f_1(e_2) - \frac{1}{2}\right]\right] + \left[\frac{1}{2} + \gamma\left[f_1(e_2) - \frac{1}{2}\right]\right]\left(1 - f_1(e_1)\right)$$
$$= \left[\frac{1}{2} + \gamma\left[f_1(e_2) - \frac{1}{2}\right]\right] - 2\gamma\left[f_1(e_2) - \frac{1}{2}\right] f_1(e_1) = \frac{1}{2} + \gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)],$$

$$P(r=0|\theta=0) = P(r=1|\theta=1)$$
$$= f_1(e_1)\left[\frac{1}{2} + \gamma\left[f_1(e_2) - \frac{1}{2}\right]\right] + \left(1 - f_1(e_1)\right)\left[\frac{1}{2} - \gamma\left[f_1(e_2) - \frac{1}{2}\right]\right]$$
$$= \left[\frac{1}{2} - \gamma\left[f_1(e_2) - \frac{1}{2}\right]\right] + 2\gamma\left[f_1(e_2) - \frac{1}{2}\right] f_1(e_1) = \frac{1}{2} - \gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)],$$

Using Bayes' Rule, we have the conditional probability of the receiver as follows

$$P(\theta=0|r=1) = \frac{P(r=1|\theta=0)P(\theta=0)}{P(r=1|\theta=0)P(\theta=0) + P(r=1|\theta=1)P(\theta=1)}$$
$$= \frac{(1-p)\left[\frac{1}{2} + \gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right]}{(1-p)\left[\frac{1}{2} + \gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right] + p\left[\frac{1}{2} - \gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right]}$$
$$= \frac{(1-p)\left[\frac{1}{2} + \gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right]}{\frac{1}{2} + (1-2p)\gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]},$$

Table 2.6 The Matrix with the Probability of the Signal for the Sender

		Signal s	
		0	1
State θ	0	$f_1(e_1)$	$1-f_1(e_1)$
	1	$1-f_1(e_1)$	$f_1(e_1)$

Table 2.7 The Matrix with the Probability of the Representation for the Receiver

		Signal s_2	
		0	1
State θ	0	$\frac{1}{2}+\lambda\left[f_1(e_2)-\frac{1}{2}\right]$	$\frac{1}{2}-\lambda\left[f_1(e_2)-\frac{1}{2}\right]$
	1	$\frac{1}{2}-\lambda\left[f_1(e_2)-\frac{1}{2}\right]$	$\frac{1}{2}+\lambda\left[f_1(e_2)-\frac{1}{2}\right]$

$$P(\theta=1|r=1)=\frac{P(r=1|\theta=1)P(\theta=1)}{P(r=1|\theta=0)P(\theta=0)+P(r=1|\theta=1)P(\theta=1)}$$

$$=\frac{p\left[\frac{1}{2}-\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]\right]}{(1-p)\left[\frac{1}{2}+\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]\right]+p\left[\frac{1}{2}-\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]\right]}$$

$$=\frac{p\left[\frac{1}{2}-\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]\right]}{\frac{1}{2}+(1-2p)\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]},$$

$$P(\theta=0|r=0)=\frac{P(r=0|\theta=0)P(\theta=0)}{P(r=0|\theta=0)P(\theta=0)+P(r=0|\theta=1)P(\theta=1)}$$

$$=\frac{(1-p)\left[\frac{1}{2}-\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]\right]}{(1-p)\left[\frac{1}{2}-\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]\right]+p\left[\frac{1}{2}+\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]\right]}$$

$$= \frac{(1-p)\left[\frac{1}{2} - \gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right]}{\frac{1}{2} - (1-2p)\gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]}$$

$$P(\theta = 1|r = 0) = \frac{P(r = 0|\theta = 1)P(\theta = 1)}{P(r = 0|\theta = 0)P(\theta = 0) + P(r = 0|\theta = 1)P(\theta = 1)}$$

$$= \frac{p\left[\frac{1}{2} + \gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right]}{(1-p)\left[\frac{1}{2} - \gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right] + p\left[\frac{1}{2} + \gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right]}$$

$$= \frac{p\left[\frac{1}{2} + \gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right]}{\frac{1}{2} - (1-2p)\gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]}.$$

Receiver Problem 4 *The maximization problem for the receiver with the different probability function of the signal and representation can be written as*

$$\max_{y_2} EV(\theta, y_2(r), \beta_2).$$

Proposition 7 *The receiver obtains the maximum expectation utility with the different probability function of the signal and representation in scenario two when*

$$y_2(r = 1) = \beta_2 + P(\theta = 1|r = 1) = \beta_2 + \frac{p\left[\frac{1}{2} - \gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right]}{\frac{1}{2} + (1-2p)\gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]}$$

Or

$$y_2(r = 0) = \beta_2 + P(\theta = 1|r = 0) = \beta_2 + \frac{p\left[\frac{1}{2} + \gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right]}{\frac{1}{2} - (1-2p)\gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]}.$$

The maximum expectation utility of the receiver is

$$\max_{y_2} EV(\theta, y_2(r=1), \beta_2) = -P(\theta=1|r=1)P(\theta=0|r=1),$$

$$\max_{y_2} EV(\theta, y_2(r=0), \beta_2) = -P(\theta=1|r=0)P(\theta=0|r=0).$$

Proof of Proposition 7 is provided in Appendix.

Sender Problem 4 *The maximization problem for the sender with the different probability function of the signal and representation can be written as*

$$MAX_{e_1, e_2}\, EU(\theta, y_2) = MAX_{e_1, e_2} - E[\alpha(y_2-\theta)^2],$$
$$where\ e_1 + e_2 \leq 1.$$

Lemma 5 *The expectation utility of the sender will increase when the sender increases effort in research or dissemination with the different probability function of the signal and representation in scenario two. And the maximum expectation utility of the sender increases with the increase of* γ. *The maximum expectation utility of the sender as follows:*

$MAX_{e_1, e_2}\, EU(\theta, y_2)$

$$= -\alpha\left\{{\beta_2}^2 + \frac{p(1-p)\left[\frac{1}{4}-\left[\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]\right]^2\right]}{\frac{1}{4}-\left[(1-2p)\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]\right]^2}\right\}$$

$$= -\alpha\left[{\beta_2}^2 + \sqrt{P(\theta=0|r=1)P(\theta=1|r=1)P(\theta=0|r=0)P(\theta=1|r=0)}\right].$$

The Proof of Lemma 5 is provided in Appendix. The result shows that the expectation utility of the sender will increase when the sender increases the effort on academic research or science popularization. The sender would obtain more expectation utility when the probability functions of the signal and representation are closer.

Proposition 8 *Given a certain amount of the sender's total effort t, t is less and equal to one, the optimal allocation of effort in research or dissemination for the sender to obtain maximum expectation utility with the different probability function of the signal and representation in scenario two:*

$$\frac{f_1'(e_1)}{[1-2f_1(e_1)]} = \frac{f_1'(e_2)}{[1-2f_1(e_2)]},$$
$$i.e. e_1 = e_2 = t/2 \leq 1/2.$$

Proof of Proposition 8 is provided in Appendix. The result shows that effort offered by the sender in science popularization is just as important to scientist's growth as academic research no matter what the receiver's action is.

2.4 Conclusion

This paper studies a formal analysis of scientists' effort provision in research and dissemination. We consider two scenarios. One scenario is a simultaneous problem with additive method. The sender sends signal to two types of audiences simultaneously. We investigate how the scientists should divide their time or energy between academic research and science popularization to obtain maximum utility. In general, the probability of the signal for different type of audiences is different, so we consider two situations. One situation is under the same probability function of the signals for two types of audiences for a special case, while another situation is under the different probability function of the signals for two types of audiences.

Under the same probability function situation, the results shows that if the ratio between the weight payoff from the public and the weight payoff from experts were increasing, scientist would like to decease effort in academic research to obtain the maximum expectation utility. In addition, if the behavior of the scientist is inconsistent with the goal of the society, we could adjust the weights of payoff from general public and the experts by incentives to change the proportion of effort offered by the scientist in academic research and science popularization.

Under the different probability function situation, if there exists a big difference between the probability functions of the signals for two types of audience, the scientist would like to increase effort in academic research to obtain the maximum expectation utility. In fact, we could make the sender to keep the allocation of effort as before by increasing the ratio of the weights of payoff from dissemination and research. In other words, if there exists polarization in academic research and science popularization, we could prevent polarization by increasing the ratio of the weights of payoff from dissemination and research using incentives.

Another scenario is a sequential problem with multiplicative method. The sender offers effort in academic research, and then sends signal to one type of audiences to representation with effort in science popularization. In general, the probability of the signal and representation is different, so we consider two situations. One situation is under the same probability of the signal and representation for a special case, while another situation is under the different probability of the signal and representation.

No matter the same probability function situation or the different probability function situation, the result shows that the effort in science popularization is always just as important to scientist's growth as academic research no matter what the receiver's action is. Take scientific achievements transformation as an example; we should put equal effort on research and dissemination.

In general, the best respond decision of the receiver is related with the corresponding probability of the signal and own bias no matter which situation. For two signals, 0 and 1, the best respond decisions of the receiver for two signals are complemented with each other regardless the receiver's own bias. The maximum expectation utility of the receiver is the negative product of two corresponding probability of the signal. For the sender, he could obtain more expectation utility by increasing effort no matter which situation. Thus the sender should use up all effort to obtain a maximum expectation utility. And the maximum expectation utility of the sender is the negative weighted sum of receivers' square root of all conditional probability with their bias square.

2.6 Appendix

Proof of Proposition 1

(1) Consider $s_i = 1$,

$\max_{y_i} EV(\theta, y_i(s_i = 1), \beta_i)$

$$= \max_{y_i} P(\theta = 0|s_i = 1)[-(y_i - 0 - \beta_i)^2] + P(\theta = 1|s_i = 1)[-(y_i - 1 - \beta_i)^2].$$

First derivative

$$\frac{dEV(\theta, y_i(s_i = 1), \beta_i)}{dy_i} = -2P(\theta = 0|s_i = 1)(y_i - \beta_i) - 2P(\theta = 1|s_i = 1)(y_i - 1 - \beta_i)$$

$$= -2\{[P(\theta = 0|s_i = 1) + P(\theta = 1|s_i = 1)](y_i - \beta_i) - P(\theta = 1|s_i = 1)\}$$

$$= -2[y_i - \beta_i - P(\theta = 1|s_i = 1)].$$

We obtain a critical value

$$y_i(s_i = 1) = \beta_i + P(\theta = 1|s_i = 1) = \beta_i + \frac{pf(e_i)}{(1-p)(1-f(e_i)) + pf(e_i)}.$$

Second derivative

$$\frac{d^2EV(\theta, y_i(s_i = 1), \beta_i)}{d^2y_i} = -2 < 0.$$

Thus

$$y_i(s_i = 1) = \beta_i + \frac{pf(e_i)}{(1-p)(1-f(e_i)) + pf(e_i)}$$

is a maximum point value point.

$\max_{y_i} EV(\theta, y_i(s_i = 1), \beta_i)$

$= \max_{y_i} P(\theta = 0|s_i = 1)[-(y_i - 0 - \beta_i)^2] + P(\theta = 1|s_i = 1)[-(y_i - 1 - \beta_i)^2]$

$= P(\theta = 0|s_i = 1)[-(\beta_i + P(\theta = 1|s_i = 1) - 0 - \beta_i)^2]$

$$+ P(\theta = 1|s_i = 1)[-(\beta_i + P(\theta = 1|s_i = 1) - 1 - \beta_i)^2]$$

$$= P(\theta = 0|s_i = 1)[-P(\theta = 1|s_i = 1)^2] + P(\theta = 1|s_i = 1)[-(P(\theta = 1|s_i = 1) - 1)^2]$$

$$= -\{P(\theta = 0|s_i = 1)P(\theta = 1|s_i = 1)^2$$

$$+ P(\theta = 1|s_i = 1)[P(\theta = 1|s_i = 1)^2 - 2P(\theta = 1|s_i = 1) + 1]\}$$

$$= -\{[P(\theta = 0|s_i = 1) + P(\theta = 1|s_i = 1)]P(\theta = 1|s_i = 1)^2 - 2P(\theta = 1|s_i = 1)^2$$

$$+ P(\theta = 1|s_i = 1)\}$$

$$= P(\theta = 1|s_i = 1)^2 - P(\theta = 1|s_i = 1)$$

$$= -P(\theta = 1|s_i = 1)[1 - P(\theta = 1|s_i = 1)]$$

$$= -P(\theta = 1|s_i = 1)P(\theta = 0|s_i = 1).$$

(2) Consider $s_i = 0$,

$$\max_{y_i} EV(\theta, y_i(s_i = 0), \beta_i)$$

$$= \max_{y_i} P(\theta = 0|s_i = 0)[-(y_i - 0 - \beta_i)^2] + P(\theta = 1|s_i = 0)[-(y_i - 1 - \beta_i)^2].$$

First derivative

$$\frac{dEV(\theta, y_i(s_i = 0), \beta_i)}{dy_i} = -2P(\theta = 0|s_i = 0)(y_i - \beta_i) - 2P(\theta = 1|s_i = 0)(y_i - 1 - \beta_i)$$

$$= -2\{[P(\theta = 0|s_i = 0) + P(\theta = 1|s_i = 0)](y_i - \beta_i) - P(\theta = 1|s_i = 0)\}$$

$$= -2[y_i - \beta_i - P(\theta = 1|s_i = 0)].$$

We obtain a critical value

$$y_i(s_i = 0) = \beta_i + P(\theta = 1|s_i = 0) = \beta_i + \frac{p(1 - f(e_i))}{(1 - p)f(e_i) + p(1 - f(e_i))}.$$

Second derivative

$$\frac{d^2EV(\theta, y_i(s_i = 0), \beta_i)}{d^2y_i} = -2 < 0,$$

Thus

$$y_i(s_i = 0) = \beta_i + \frac{p(1 - f(e_i))}{(1 - p)f(e_i) + p(1 - f(e_i))}$$

is a maximum point value point.

$$\max_{y_i} EV(\theta, y_i(s_i = 0), \beta_i)$$

$$= \max_{y_i} P(\theta = 0|s_i = 0)[-(y_i - 0 - \beta_i)^2] + P(\theta = 1|s_i = 0)[-(y_i - 1 - \beta_i)^2]$$

$$= P(\theta = 0|s_i = 0)[-(\beta_i + P(\theta = 1|s_i = 0) - 0 - \beta_i)^2]$$

$$+ P(\theta = 1|s_i = 0)[-(\beta_i + P(\theta = 1|s_i = 0) - 1 - \beta_i)^2]$$

$$= -P(\theta = 1|s_i = 0)P(\theta = 0|s_i = 0)$$

$\boxdot$

Proof of Lemma 1

1. $P(\theta = 1|s_i = 1) - P(\theta = 1|s_i = 0)$

$$= \frac{pf(e_i)}{(1-p)\big(1-f(e_i)\big) + pf(e_i)} - \frac{p\big(1-f(e_i)\big)}{(1-p)f(e_i) + p\big(1-f(e_i)\big)}$$

$$= \frac{-p(1-p)(1-2f(e_i))}{\big[(1-p)\big(1-f(e_i)\big) + pf(e_i)\big](1-p)f(e_i) + p\big(1-f(e_i)\big)} \geq 0.$$

2. $P(\theta = 0|s_i = 1) - P(\theta = 0|s_i = 0)$

$$= \frac{(1-p)(1-f(e_i))}{(1-p)(1-f(e_i)) + pf(e_i)} - \frac{(1-p)f(e_i)}{(1-p)f(e_i) + p(1-f(e_i))}$$

$$= \frac{p(1-p)(1-2f(e_i))}{\big[(1-p)\big(1-f(e_i)\big) + pf(e_i)\big]\big[(1-p)f(e_i) + p\big(1-f(e_i)\big)\big]} \leq 0.$$

3. $P(\theta = 1|s_i = 0) - P(\theta = 0|s_i = 0)$

$$= \frac{p\big(1-f(e_i)\big)}{(1-p)f(e_i) + p\big(1-f(e_i)\big)} - \frac{(1-p)f(e_i)}{(1-p)f(e_i) + p(1-f(e_i))}$$

$$= \frac{p - f(e_i)}{(1-p)f(e_i) + p\big(1-f(e_i)\big)}$$

$If\ p < \frac{1}{2}, P(\theta = 1|s_i = 0) - P(\theta = 0|s_i = 0) \leq 0.$

4. $P(\theta = 0|s_i = 1) - P(\theta = 1|s_i = 1)$

$$= \frac{(1-p)(1-f(e_i))}{(1-p)(1-f(e_i))+pf(e_i)} - \frac{pf(e_i)}{(1-p)\big(1-f(e_i)\big)+pf(e_i)}$$

$$= \frac{1-p-f(e_i)}{\big[(1-p)\big(1-f(e_i)\big)+pf(e_i)\big]}$$

$If\ p < \frac{1}{2}, P(\theta = 0|s_i = 1) - P(\theta = 1|s_i = 1) \geq 0.$

$\boxdot$

Proof of Lemma 2

$$MAX_{e_1, e_2} - E[\alpha_1(y_1-\theta)^2 + \alpha_2(y_2-\theta)^2]$$

$$\begin{aligned}
&= MAX_{e_1, e_2} - \{\alpha_1[P(s_1=0|\theta=0)(y_1(0)-0)^2 + P(s_1=1|\theta=0)(y_1(1)-0)^2] \\
&\qquad + \alpha_2[P(s_2=0|\theta=0)(y_2(0)-0)^2 + P(s_2=1|\theta=0)(y_2(1)-0)^2]\}P(\theta=0) \\
&\qquad - \{\alpha_1[P(s_1=0|\theta=1)(y_1(0)-1)^2 + P(s_1=1|\theta=1)(y_1(1)-1)^2] \\
&\qquad + \alpha_2[P(s_2=0|\theta=1)(y_2(0)-1)^2 + P(s_2=1|\theta=1)(y_2(1)-1)^2]\}P(\theta=1)
\end{aligned}$$

$$\begin{aligned}
&= -\alpha_1\{{\beta_1}^2 + \frac{p(1-p)f(e_1)\big(1-f(e_1)\big)}{[f(e_1)-2pf(e_1)+p][1-p-f(e_1)+2pf(e_1)]}\} \\
&\qquad - \alpha_2\left\{{\beta_2}^2 + \frac{p(1-p)f(e_2)\big(1-f(e_2)\big)}{[f(e_2)-2pf(e_2)+p][1-p-f(e_2)+2pf(e_2)]}\right\}
\end{aligned}$$

$$\begin{aligned}
&= -\alpha_1\left[{\beta_1}^2 + \sqrt{P(\theta=0|s_1=1)P(\theta=1|s_1=1)P(\theta=0|s_1=0)P(\theta=1|s_1=0)}\right] \\
&\qquad -\alpha_2\left[{\beta_2}^2 + \sqrt{P(\theta=0|s_2=1)P(\theta=1|s_2=1)P(\theta=0|s_2=0)P(\theta=1|s_2=0)}\right]
\end{aligned}$$

First derivative with respect to e_1

$$\frac{\partial - E[\alpha_1(y_1-\theta)^2 + \alpha_2(y_2-\theta)^2]}{\partial e_1} = -\alpha_1 p(1-p)[1-2f(e_1)]f'(e_1)$$

$$\left\{\frac{[f(e_1)-2pf(e_1)+p][1-p-f(e_1)+2pf(e_1)] - f(e_1)\big(1-f(e_1)\big)(1-2p)^2}{\{[f(e_1)-2pf(e_1)+p][1-p-f(e_1)+2pf(e_1)]\}^2}\right\}$$

$$= \frac{-\alpha_1[p(1-p)]^2[1-2f(e_1)]f'(e_1)}{\{[f(e_1)-2pf(e_1)+p][1-p-f(e_1)+2pf(e_1)]\}^2} \geq 0$$

First derivative with respect to e_2

$$\frac{\partial - E[\alpha_1(y_1-\theta)^2 + \alpha_2(y_2-\theta)^2]}{\partial e_2} = \frac{-\alpha_2[p(1-p)]^2[1-2f(e_2)]f'(e_2)}{\{[f(e_2)-2pf(e_2)+p][1-p-f(e_2)+2pf(e_2)]\}^2} \geq 0$$

$\boxdot$

Proof of Proposition 2

$$MAX_{e_1,e_2} - E[\alpha_1(y_1-\theta)^2 + \alpha_2(y_2-\theta)^2],$$

$$where\ e_1 + e_2 \leq 1.$$

The Lagrangian can be written

$$L(e_1,e_2;\delta) = -\alpha_1\left\{{\beta_1}^2 + \frac{p(1-p)f(e_1)\big(1-f(e_1)\big)}{[f(e_1)-2pf(e_1)+p][1-p-f(e_1)+2pf(e_1)]}\right\}$$

$$-\alpha_2\left\{{\beta_2}^2 + \frac{p(1-p)f(e_2)\big(1-f(e_2)\big)}{[f(e_2)-2pf(e_2)+p][1-p-f(e_2)+2pf(e_2)]}\right\} + \delta(1-e_1-e_2)$$

Kuhn-Trucker conditions are

$$\delta \geq 0, 1-e_1-e_2 \geq 0 \text{ with CS}$$

$$\frac{\partial L(e_1,e_2;\delta)}{\partial e_1} = \frac{-\alpha_1[p(1-p)]^2[1-2f(e_1)]f'(e_1)}{\{[f(e_1)-2pf(e_1)+p][1-p-f(e_1)+2pf(e_1)]\}^2} - \delta = 0$$

$$\frac{\partial L(e_1,e_2;\delta)}{\partial e_2} = \frac{-\alpha_2[p(1-p)]^2[1-2f(e_2)]f'(e_2)}{\{[f(e_2)-2pf(e_2)+p][1-p-f(e_2)+2pf(e_2)]\}^2} - \delta = 0$$

Combining above two equations, we have

$$\frac{-\alpha_1[p(1-p)]^2[1-2f(e_1)]f'(e_1)}{\{[f(e_1)-2pf(e_1)+p][1-p-f(e_1)+2pf(e_1)]\}^2} = \frac{-\alpha_2[p(1-p)]^2[1-2f(e_2)]f'(e_2)}{\{[f(e_2)-2pf(e_2)+p][1-p-f(e_2)+2pf(e_2)]\}^2}$$

$$\frac{[1-2f(e_1)]f'(e_1)}{[1-2f(e_2)]f'(e_2)}\frac{\{[f(e_2)-2pf(e_2)+p][1-p-f(e_2)+2pf(e_2)]\}^2}{\{[f(e_1)-2pf(e_1)+p][1-p-f(e_1)+2pf(e_1)]\}^2} = \frac{\alpha_2}{\alpha_1}$$

If $e_1 + e_2 = t \leq 1,\ e_2 = t - e_1$

$$\frac{f'(e_1)[1-2f(e_1)][p(1-p)+(1-2p)^2f(t-e_1)(1-f(t-e_1))]^2}{f'(t-e_1)[1-2f(t-e_1)][p(1-p)+(1-2p)^2f(e_1)(1-f(e_1))]^2} = \frac{\alpha_2}{\alpha_1}$$

$\boxdot$

Proof of Corollary 1

$$\frac{f'(e_1)}{f'(t-e_1)}\frac{[1-2f(e_1)][p(1-p)+(1-2p)^2f(t-e_1)(1-f(t-e_1))]^2}{[1-2f(t-e_1)][p(1-p)+(1-2p)^2f(e_1)(1-f(e_1))]^2}=\frac{\alpha_2}{\alpha_1}$$

Let A=$[1-2f(e_1)]\big[p(1-p)+(1-2p)^2f(t-e_1)\big(1-f(t-e_1)\big)\big]^2$, we have

$$\frac{\partial A}{\partial e_1}=-2f'(e_1)\big[p(1-p)+(1-2p)^2f(t-e_1)\big(1-f(t-e_1)\big)\big]^2$$
$$+2[1-2f(e_1)]\big[p(1-p)+(1-2p)^2f(t-e_1)\big(1-f(t-e_1)\big)\big](1-2p)^2\big(-1$$
$$+2f(t-e_1)\big)f'(t-e_1)<0,$$

$$\frac{\partial f'(e_1)A}{\partial e_1}=f''(e_1)A+f'(e_1)\frac{\partial A}{\partial e_1}<0.$$

Let B=$[1-2f(t-e_1)]\big[p(1-p)+(1-2p)^2f(e_1)\big(1-f(e_1)\big)\big]^2$

$$\frac{\partial B}{\partial e_1}=2f'(t-e_1)\big[p(1-p)+(1-2p)^2f(e_1)\big(1-f(e_1)\big)\big]^2$$
$$+2[1-2f(t-e_1)]\big[p(1-p)+(1-2p)^2f(e_1)\big(1-f(e_1)\big)\big](1-2p)^2\big(1$$
$$-2f(e_1)\big)f'(e_1)>0,$$

$$\frac{\partial f'(t-e_1)B}{\partial e_1}=-f''(t-e_1)B+f'(t-e_1)\frac{\partial B}{\partial e_1}>0.$$

Then

$$\frac{\partial\frac{f'(e_1)A}{f'(t-e_1)B}}{\partial e_1}=\frac{\frac{\partial f'(e_1)A}{\partial e_1}f'(t-e_1)B-f'(e_1)A\frac{\partial f'(t-e_1)B}{\partial e_1}}{[f'(t-e_1)B]^2}<0.$$

Thus e_1 should be decreased for the sender to obtain the maximum expectation utility as α_2/α_1 increases. We have

(1) If $\alpha_2/\alpha_1=1$, $0<e_1+e_2\le t\le 1$, we have $0<e_1=e_2\le t/2$. Specifically, when $e_1+e_2=1$, we have $e_1=e_2=1/2$.

(2) If $\alpha_2/\alpha_1>1, 0<e_1+e_2\le t\le 1$, we have $0<e_1<t/2<e_2\le t$.

(3) If $\alpha_2/\alpha_1<1, 0<e_1+e_2\le t\le 1$, we have $0<e_2<t/2<e_1\le t$.

⊡

Proof of Proposition 3

(1) Consider $s_1 = 1$,

$$\max_{y_1} EV(\theta, y_1(s_1 = 1), \beta_1)$$

$$= \max_{y_1} P(\theta = 0|s_1 = 1)[-(y_1 - 0 - \beta_1)^2] + P(\theta = 1|s_1 = 1)[-(y_1 - 1 - \beta_1)^2].$$

First derivative

$$\frac{dEV(\theta, y_1(s_1 = 1), \beta_1)}{dy_1} = -2P(\theta = 0|s_1 = 1)(y_1 - \beta_1) - 2P(\theta = 1|s_1 = 1)(y_1 - 1 - \beta_1)$$

$$= -2\{[P(\theta = 0|s_1 = 1) + P(\theta = 1|s_1 = 1)](y_1 - \beta_1) - P(\theta = 1|s_1 = 1)\}$$

$$= -2[y_1 - \beta_1 - P(\theta = 1|s_1 = 1)].$$

We obtain a critical value

$$y_1(s_1 = 1) = \beta_1 + P(\theta = 1|s_1 = 1) = \beta_1 + \frac{pf(e_1)}{(1-p)(1-f(e_1)) + pf(e_1)}.$$

Second derivative

$$\frac{d^2EV(\theta, y_1(s_1 = 1), \beta_1)}{d^2y_1} = -2 < 0.$$

Thus

$$y_1(s_1 = 1) = \beta_1 + P(\theta = 1|s_1 = 1) = \beta_1 + \frac{pf(e_1)}{(1-p)(1-f(e_1)) + pf(e_1)}$$

is a maximum point value point.

$$\max_{y_1} EV(\theta, y_1(s_1 = 1), \beta_1)$$

$$= \max_{y_1} P(\theta = 0|s_1 = 1)[-(y_1 - \beta_1)^2] + P(\theta = 1|s_1 = 1)[-(y_1 - 1 - \beta_1)^2]$$

$$= P(\theta = 0|s_1 = 1)[-P(\theta = 1|s_1 = 1)^2] + P(\theta = 1|s_1 = 1)[-[P(\theta = 1|s_1 = 1) - 1]^2]$$

$$= -\{P(\theta = 0|s_1 = 1)P(\theta = 1|s_1 = 1)^2$$

$$+ P(\theta = 1|s_1 = 1)[P(\theta = 1|s_1 = 1)^2 - 2P(\theta = 1|s_1 = 1) + 1]\}$$

$$= -\{P(\theta = 1|s_1 = 1)^2 + P(\theta = 1|s_1 = 1)[1 - 2P(\theta = 1|s_1 = 1)]\}$$

$$= -P(\theta = 1|s_1 = 1)[1 - P(\theta = 1|s_1 = 1)]$$

$$= -P(\theta = 1|s_1 = 1)P(\theta = 0|s_1 = 1)$$

(2) Consider $s_1 = 0$,

$$\max_{y_1} EV(\theta, y_1(s_1 = 0), \beta_1)$$

$$= \max_{y_1} P(\theta = 0|s_1 = 0)[-(y_1 - 0 - \beta_1)^2] + P(\theta = 1|s_1 = 0)[-(y_1 - 1 - \beta_1)^2].$$

First derivative

$$\frac{dEV(\theta, y_1(s_1 = 0), \beta_1)}{dy_1} = -2P(\theta = 0|s_1 = 0)(y_1 - \beta_1) - 2P(\theta = 1|s_1 = 0)(y_1 - 1 - \beta_1)$$

$$= -2\{[P(\theta = 0|s_1 = 0) + P(\theta = 1|s_1 = 0)](y_1 - \beta_1) - P(\theta = 1|s_1 = 0)\}$$

$$= -2[y_1 - \beta_1 - P(\theta = 1|s_1 = 0)].$$

We obtain a critical value

$$y_1(s_1 = 0) = \beta_1 + P(\theta = 1|s_1 = 0) = \beta_1 + \frac{p(1 - f(e_1))}{(1 - p)f(e_1) + p(1 - f(e_1))}.$$

Second derivative

$$\frac{d^2EV(\theta, y_1(s_1 = 0), \beta_1)}{d^2y_1} = -2 < 0,$$

Thus

$$y_1(s_1 = 0) = \beta_1 + P(\theta = 1|s_1 = 0) = \beta_1 + \frac{p(1 - f(e_1))}{(1 - p)f(e_1) + p(1 - f(e_1))}.$$

is a maximum point value point.

$$\max_{y_1} EV(\theta, y_1(s_1 = 0), \beta_1)$$

$$= \max_{y_1} P(\theta = 0|s_1 = 0)[-(y_1 - 0 - \beta_1)^2] + P(\theta = 1|s_1 = 0)[-(y_1 - 1 - \beta_1)^2]$$

$$= P(\theta = 0|s_1 = 0)[-(\beta_1 + P(\theta = 1|s_1 = 0) - 0 - \beta_1)^2]$$

$$+ P(\theta = 1|s_1 = 0)[-(\beta_1 + P(\theta = 1|s_1 = 0) - 1 - \beta_1)^2]$$

$$= -P(\theta = 1|s_1 = 0)P(\theta = 0|s_1 = 0)$$

(3) Consider $s_2 = 1$,

$$\max_{y_2} EV(\theta, y_2(s_2 = 1), \beta_2)$$

$$= \max_{y_1} P(\theta = 0|s_2 = 1)[-(y_2 - 0 - \beta_2)^2] + P(\theta = 1|s_2 = 1)[-(y_2 - 1 - \beta_2)^2].$$

First derivative

$$\frac{dEV(\theta, y_2(s_2 = 1), \beta_2)}{dy_2} = -2P(\theta = 0|s_2 = 1)(y_2 - \beta_2) - 2P(\theta = 1|s_2 = 1)(y_2 - 1 - \beta_2)$$

$$= -2\{[P(\theta = 0|s_2 = 1) + P(\theta = 1|s_2 = 1)](y_2 - \beta_2) - P(\theta = 1|s_2 = 1)\}$$

$$= -2[y_2 - \beta_2 - P(\theta = 1|s_2 = 1)].$$

We obtain a critical value

$$y_2(s_2 = 1) = \beta_2 + P(\theta = 1|s_2 = 1) = \beta_2 + \frac{p\left\{\frac{1}{2} + \lambda\left[f_1(e_2) - \frac{1}{2}\right]\right\}}{\frac{1}{2} - \lambda(1 - 2p)\left[f_1(e_2) - \frac{1}{2}\right]}.$$

Second derivative

$$\frac{d^2EV(\theta, y_2(s_2 = 1), \beta_2)}{d^2y_2} = -2 < 0$$

Thus

$$y_2(s_2 = 1) = \beta_2 + P(\theta = 1|s_2 = 1) = \beta_2 + \frac{p\left\{\frac{1}{2} + \lambda\left[f_1(e_2) - \frac{1}{2}\right]\right\}}{\frac{1}{2} - \lambda(1 - 2p)\left[f_1(e_2) - \frac{1}{2}\right]}$$

is a maximum point value point.

$$\max_{y_2} EV(\theta, y_2(s_2 = 1), \beta_2)$$

$$= \max_{y_2} P(\theta = 0|s_2 = 1)[-(y_2 - \beta_2)^2] + P(\theta = 1|s_2 = 1)[-(y_2 - 1 - \beta_2)^2]$$

$$= P(\theta = 0|s_2 = 1)[-P(\theta = 1|s_2 = 1)^2] + P(\theta = 1|s_2 = 1)[-[P(\theta = 1|s_2 = 1) - 1]^2]$$

$$= -\{P(\theta = 0|s_2 = 1)P(\theta = 1|s_2 = 1)^2$$

$$+ P(\theta = 1|s_2 = 1)[P(\theta = 1|s_2 = 1)^2 - 2P(\theta = 1|s_2 = 1) + 1]\}$$

$$= -P(\theta = 1|s_2 = 1)P(\theta = 0|s_2 = 1)$$

(4) Consider $s_2 = 0$

$$\max_{y_2} EV(\theta, y_2(s_2 = 0), \beta_2)$$

$$= \max_{y_2} P(\theta = 0|s_2 = 0)[-(y_2 - 0 - \beta_2)^2] + P(\theta = 1|s_2 = 0)[-(y_2 - 1 - \beta_2)^2].$$

First derivative

$$\begin{aligned}\frac{dEV(\theta, y_2(s_2 = 0), \beta_2)}{dy_2} &= -2P(\theta = 0|s_2 = 0)(y_2 - \beta_2) - 2P(\theta = 1|s_2 = 0)(y_2 - 1 - \beta_2) \\ &= -2\{[P(\theta = 0|s_2 = 0) + P(\theta = 1|s_2 = 0)](y_2 - \beta_2) - P(\theta = 1|s_2 = 0)\} \\ &= -2[y_2 - \beta_2 - P(\theta = 1|s_2 = 0)].\end{aligned}$$

We obtain a critical value

$$y_2(s_2 = 0) = \beta_2 + P(\theta = 1|s_2 = 0) = \beta_2 + \frac{p\left\{\frac{1}{2} - \lambda\left[f_1(e_2) - \frac{1}{2}\right]\right\}}{\frac{1}{2} + \lambda(1 - 2p)\left[f_1(e_2) - \frac{1}{2}\right]}.$$

Second derivative

$$\frac{d^2EV(\theta, y_2(s_2 = 0), \beta_2)}{d^2y_2} = -2 < 0,$$

Thus

$$y_2(s_2 = 0) = \beta_2 + P(\theta = 1|s_2 = 0) = \beta_2 + \frac{p\left\{\frac{1}{2} - \lambda\left[f_1(e_2) - \frac{1}{2}\right]\right\}}{\frac{1}{2} + \lambda(1 - 2p)\left[f_1(e_2) - \frac{1}{2}\right]}$$

is a maximum point value point.

$$\max_{y_2} EV(\theta, y_2(s_2 = 0), \beta_2)$$

$$= \max_{y_2} P(\theta = 0|s_2 = 0)[-(y_2 - 0 - \beta_2)^2] + P(\theta = 1|s_2 = 0)[-(y_2 - 1 - \beta_2)^2]$$

$$\begin{aligned}&= P(\theta = 0|s_2 = 0)[-(\beta_2 + P(\theta = 1|s_2 = 0) - 0 - \beta_2)^2] \\ &\qquad + P(\theta = 1|s_2 = 0)[-(\beta_2 + P(\theta = 1|s_2 = 0) - 1 - \beta_2)^2]\end{aligned}$$

$$= -P(\theta = 1|s_2 = 0)P(\theta = 0|s_2 = 0)$$

$\boxdot$

Lemma 3

$$MAX_{e_1,e_2} - E[\alpha_1(y_1-\theta)^2 + \alpha_2(y_2-\theta)^2]$$

$$= MAX_{e_1,e_2} - \alpha_1\{[P(s_1=0|\theta=0)y_1(0)^2 + P(s_1=1|\theta=0)y_1(1)^2]P(\theta=0)$$

$$+ [P(s_1=0|\theta=1)(y_1(0)-1)^2 + P(s_1=1|\theta=1)(y_1(1)-1)^2]P(\theta=1)\}$$

$$- \alpha_2\{[P(s_2=0|\theta=0)y_2(0)^2 + P(s_2=1|\theta=0)y_2(1)^2]P(\theta=0)$$

$$+ [P(s_2=0|\theta=1)(y_2(0)-1)^2 + P(s_2=1|\theta=1)(y_2(1)-1)^2]P(\theta=1)\}$$

$$= -\alpha_1\left[{\beta_1}^2 + \frac{p(1-p)f_1(e_1)(f_1(e_1)-1)}{p(p-1)+(1-2p)^2 f_1(e_1)(f_1(e_1)-1)}\right]$$

$$- \alpha_2\left[{\beta_2}^2 + \frac{p(1-p)\left[\frac{1}{4}-\lambda^2\left[f_1(e_2)-\frac{1}{2}\right]^2\right]}{\frac{1}{4}-(1-2p)^2\lambda^2\left[f_1(e_2)-\frac{1}{2}\right]^2}\right]$$

$$= -\alpha_1\left[{\beta_1}^2 + \sqrt{P(\theta=0|s_1=1)P(\theta=1|s_1=1)P(\theta=0|s_1=0)P(\theta=1|s_1=0)}\right]$$

$$-\alpha_2\left[{\beta_2}^2 + \sqrt{P(\theta=0|s_2=1)P(\theta=1|s_2=1)P(\theta=0|s_2=0)P(\theta=1|s_2=0)}\right].$$

First derivative with respect to e_1

$$\frac{\partial - E[\alpha_1(y_1-\theta)^2 + \alpha_2(y_2-\theta)^2]}{\partial e_1} = -\alpha_1 \frac{-p^2(1-p)^2[2f_1(e_1)-1]{f_1}'(e_1)}{[p(p-1)+(1-2p)^2 f_1(e_1)(f_1(e_1)-1)]^2} > 0$$

First derivative with respect to e_2

$$\frac{\partial - E[\alpha_1(y_1-\theta)^2 + \alpha_2(y_2-\theta)^2]}{\partial e_2} = -\alpha_2 \frac{-p^2(1-p)^2\lambda^2[2f_1(e_2)-1]{f_1}'(e_2)}{\left[\frac{1}{4}-(1-2p)^2\lambda^2\left[f_1(e_2)-\frac{1}{2}\right]^2\right]^2} > 0$$

First derivative with respect to λ

$$\frac{\partial p(1-p)\left[\frac{1}{4}-\lambda^2\left[f_1(e_2)-\frac{1}{2}\right]^2\right]}{\partial\lambda} = -2p(1-p)\left[f_1(e_2)-\frac{1}{2}\right]^2\lambda < 0$$

$$\frac{\partial\left[\frac{1}{4}-(1-2p)^2\lambda^2\left[f_1(e_2)-\frac{1}{2}\right]^2\right]}{\partial\lambda} = -2(1-2p)^2\left[f_1(e_2)-\frac{1}{2}\right]^2\lambda < 0$$

$$\frac{\partial\, MAX_{e_1,e_2}\, EU(\theta,y_1,y_2)}{\partial\lambda}$$

$$= -\alpha_2\left\{\frac{-2p(1-p)\left[f_1(e_2)-\frac{1}{2}\right]^2\lambda\left[\frac{1}{4}-(1-2p)^2\lambda^2\left[f_1(e_2)-\frac{1}{2}\right]^2\right]}{\left[-2(1-2p)^2\left[f_1(e_2)-\frac{1}{2}\right]^2\lambda\right]^2}\right.$$

$$\left.+\frac{2(1-2p)^2\left[f_1(e_2)-\frac{1}{2}\right]^2\lambda p(1-p)\left[\frac{1}{4}-\lambda^2\left[f_1(e_2)-\frac{1}{2}\right]^2\right]}{\left[-2(1-2p)^2\left[f_1(e_2)-\frac{1}{2}\right]^2\lambda\right]^2}\right\}$$

$$= -\alpha_2\frac{-2[p(1-p)]^2\left[f_1(e_2)-\frac{1}{2}\right]^2\lambda}{\left[-2(1-2p)^2\left[f_1(e_2)-\frac{1}{2}\right]^2\lambda\right]^2} > 0$$

$\boxdot$

Proof of Proposition 4

$$MAX_{e_1,e_2}\, EU(\theta,y_1,y_2) = MAX_{e_1,e_2} - E[\alpha_1(y_1-\theta)^2+\alpha_2(y_2-\theta)^2], where\ e_1+e_2\le 1.$$

The Lagrangian can be written

$L(e_1,e_2;\delta) = -E[\alpha_1(y_1-\theta)^2+\alpha_2(y_2-\theta)^2]+\delta(1-e_1-e_2)$

Kuhn-Trucker conditions are

$$\delta\ge 0, 1-e_1-e_2\ge 0 \text{ with CS}$$

$$\frac{\partial L(e_1,e_2;\delta)}{\partial e_1} = \frac{\partial - E[\alpha_1(y_1-\theta)^2+\alpha_2(y_2-\theta)^2]+\delta(1-e_1-e_2)}{\partial e_1} =$$

$$= -\alpha_1\frac{-p^2(1-p)^2[2f_1(e_1)-1]f_1{}'(e_1)}{[p(p-1)+(1-2p)^2 f_1(e_1)(f_1(e_1)-1)]^2}-\delta = 0$$

$$\frac{\partial L(e_1,e_2;\delta)}{\partial e_2} = \frac{\partial - E[\alpha_1(y_1-\theta)^2+\alpha_2(y_2-\theta)^2]+\delta(1-e_1-e_2)}{\partial e_2}$$

$$= -\alpha_2\frac{-p^2(1-p)^2\lambda^2[2f_1(e_2)-1]f_1{}'(e_2)}{\left[\frac{1}{4}-(1-2p)^2\lambda^2\left[f_1(e_2)-\frac{1}{2}\right]^2\right]^2}-\delta = 0$$

Combining above two equations, we have

$$\frac{\alpha_1[2f_1(e_1)-1]f_1{}'(e_1)}{[p(p-1)+(1-2p)^2 f_1(e_1)(f_1(e_1)-1)]^2} = \frac{\alpha_2\lambda^2[2f_1(e_2)-1]f_1{}'(e_2)}{\left[\frac{1}{4}-(1-2p)^2\lambda^2\left[f_1(e_2)-\frac{1}{2}\right]^2\right]^2}$$

$$\frac{\alpha_1[(2f_1(e_1)-1)]f_1{}'(e_1)}{\left[\frac{1}{4}-(1-2p)^2\left[f_1(e_1)-\frac{1}{2}\right]^2\right]^2}=\frac{\alpha_2\lambda^2[2f_1(e_2)-1]f_1{}'(e_2)}{\left[\frac{1}{4}-(1-2p)^2\lambda^2\left[f_1(e_2)-\frac{1}{2}\right]^2\right]^2}$$

$$\frac{f_1{}'(e_1)[1-2f_1(e_1)]\left[\frac{1}{4}-(1-2p)^2\lambda^2\left[f_1(e_2)-\frac{1}{2}\right]^2\right]^2}{\lambda^2 f_1{}'(e_2)[1-2f_1(e_2)]\left[\frac{1}{4}-(1-2p)^2\left[f_1(e_1)-\frac{1}{2}\right]^2\right]^2}=\frac{\alpha_2}{\alpha_1}$$

If $e_1+e_2=t,\ e_2=t-e_1$

$$\frac{f_1{}'(e_1)[1-2f_1(e_1)]\left[\frac{1}{4}-(1-2p)^2\lambda^2\left[f_1(t-e_1)-\frac{1}{2}\right]^2\right]^2}{\lambda^2 f_1{}'(t-e_1)[1-2f_1(t-e_1)]\left[\frac{1}{4}-(1-2p)^2\left[f_1(e_1)-\frac{1}{2}\right]^2\right]^2}=\frac{\alpha_2}{\alpha_1}$$

$\boxdot$

Proof of Corollary 2

$$\frac{f_1{}'(e_1)[1-2f_1(e_1)]\left[\frac{1}{4}-(1-2p)^2\lambda^2\left[f_1(t-e_1)-\frac{1}{2}\right]^2\right]^2}{\lambda^2 f_1{}'(t-e_1)[1-2f_1(t-e_1)]\left[\frac{1}{4}-(1-2p)^2\left[f_1(e_1)-\frac{1}{2}\right]^2\right]^2}=\frac{\alpha_2}{\alpha_1}$$

First order with respect to e_1

Let A=$[1-2f_1(e_1)]\left[\frac{1}{4}-(1-2p)^2\lambda^2\left[f_1(t-e_1)-\frac{1}{2}\right]^2\right]^2$, we have

$$\frac{\partial A}{\partial e_1}=-2f_1'(e_1)\left[\frac{1}{4}-(1-2p)^2\lambda^2\left[f_1(t-e_1)-\frac{1}{2}\right]^2\right]^2$$

$$+4[1-2f_1(e_1)]\left[\frac{1}{4}-(1-2p)^2\lambda^2\left[f_1(t-e_1)-\frac{1}{2}\right]^2\right](1-2p)^2\lambda^2\left[f_1(t-e_1)\right.$$

$$\left.-\frac{1}{2}\right]f'(t-e_1)<0,$$

$$\frac{\partial f'(e_1)A}{\partial e_1}=f''(e_1)A+f'(e_1)\frac{\partial A}{\partial e_1}<0.$$

Let B=$[1-2f_1(t-e_1)]\left[\frac{1}{4}-(1-2p)^2\left[f_1(e_1)-\frac{1}{2}\right]^2\right]^2$

$$\frac{\partial B}{\partial e_1} = 2f'(t-e_1)\left[\frac{1}{4} - (1-2p)^2\left[f_1(e_1) - \frac{1}{2}\right]^2\right]^2$$

$$- 4[1-2f_1(t-e_1)]\left[\frac{1}{4} + (1-2p)^2\left[f_1(e_1) - \frac{1}{2}\right]^2\right](1-2p)^2\left[f_1(e_1)\right.$$

$$\left. - \frac{1}{2}\right] f'(e_1) > 0,$$

$$\frac{\partial f'(t-e_1)B}{\partial e_1} = -f''(t-e_1)B + f'(t-e_1)\frac{\partial B}{\partial e_1} > 0.$$

Then

$$\frac{\partial \dfrac{f'(e_1)A}{\lambda^2 f'(t-e_1)B}}{\partial e_1} = \frac{\dfrac{\partial f'(e_1)A}{\partial e_1} f'(t-e_1)B - f'(e_1)A\dfrac{\partial \lambda^2 f'(t-e_1)B}{\partial e_1}}{[f'(t-e_1)B]^2} < 0.$$

First order with respect to λ, we have

$$\frac{\partial f'(e_1)A}{\partial \lambda} = -4\left[\frac{1}{4} - (1-2p)^2\lambda^2\left[f_1(1-e_1) - \frac{1}{2}\right]^2\right](1-2p)^2\left[f_1(1-e_1) - \frac{1}{2}\right]^2 \lambda < 0.$$

$$\frac{\partial \lambda^2 f'(t-e_1)B}{\partial \lambda} = 2\lambda f'(t-e_1)B > 0.$$

Then

$$\frac{\partial \dfrac{f'(e_1)A}{\lambda^2 f'(t-e_1)B}}{\partial \lambda} = \frac{\dfrac{\partial f'(e_1)A}{\partial \lambda} f'(t-e_1)B - f'(e_1)A\dfrac{\partial \lambda^2 f'(t-e_1)B}{\partial \lambda}}{[\lambda^2 f'(t-e_1)B]^2} < 0.$$

Thus e_1 should be decreased by the sender to obtain maximum expectation utility as α_2/α_1 increases or λ increases.

Specially,

$$\frac{f_1'(e_1)[1-2f_1(e_1)]\left[\frac{1}{4} - (1-2p)^2\lambda^2\left[f_1(t-e_1) - \frac{1}{2}\right]^2\right]^2}{\lambda f_1'(t-e_1)\lambda[1-2f_1(t-e_1)]\left[\frac{1}{4} - (1-2p)^2\left[f_1(e_1) - \frac{1}{2}\right]^2\right]^2} = \frac{\alpha_2}{\alpha_1}$$

We have

(1) $\alpha_2/\alpha_1 = 1$ if and only if $[1-2f_1(e_1)]/[1-2f_1(t-e_1)] = \lambda$.

(2) $\alpha_2/\alpha_1 > 1$ if and only if $[1-2f_1(e_1)]/[1-2f_1(t-e_1)] > \lambda$.

(3) $\alpha_2/\alpha_1 < 1$ if and only if $[1-2f_1(e_1)]/[1-2f_1(t-e_1)] < \lambda$.

$\boxdot$

Proof of Proposition 5

(1) Consider $r = 1$

$\max_{y_2} EV(\theta, y_2(r=1), \beta_2)$

$$= \max_{y_2} P(\theta = 0|r = 1)[-(y_2 - 0 - \beta_2)^2] + P(\theta = 1|r = 1)[-(y_2 - 1 - \beta_2)^2]$$

First derivative

$$\frac{dEV(\theta, y_2(r=1), \beta_2)}{dy_2} = -2P(\theta = 0|r = 1)(y_2 - 0 - \beta_2) - 2P(\theta = 1|r = 1)(y_2 - 1 - \beta_2)$$

$$= -2[y_2 - \beta_2 - P(\theta = 1|r = 1)].$$

We obtain a critical value

$$y_2(r=1) = \beta_2 + P(\theta = 1|r = 1) = \beta_2 + \frac{p(1 - f(e_1) - f(e_2) + 2f(e_1)f(e_2))}{p + (1-2p)(f(e_1) + f(e_2) - 2f(e_1)f(e_2))}.$$

Second derivative

$$\frac{d^2EV(\theta, y_2(r=1), \beta_2)}{d^2y_2} = -2 < 0$$

Thus

$$y_2(r=1) = \beta_2 + P(\theta = 1|r = 1) = \beta_2 + \frac{p(1 - f(e_1) - f(e_2) + 2f(e_1)f(e_2))}{p + (1-2p)(f(e_1) + f(e_2) - 2f(e_1)f(e_2))}$$

is a maximum point value point.

$\max_{y_2} EV(\theta, y_2(r=1), \beta_2)$

$= \max_{y_2} P(\theta = 0|r = 1)[-(y_2 - 0 - \beta_2)^2] + P(\theta = 1|r = 1)[-(y_2 - 1 - \beta_2)^2]$

$= -P(\theta = 0|r = 1)P(\theta = 1|r = 1)^2 - P(\theta = 1|r = 1)(P(\theta = 1|r = 1) - 1)^2$

$= -P(\theta = 0|r = 1)P(\theta = 1|r = 1)^2$

$$- P(\theta = 1|r = 1)([P(\theta = 1|r = 1)]^2 - 2P(\theta = 1|r = 1) + 1)$$

$= -P(\theta = 1|r = 1)\big(1 - P(\theta = 1|r = 1)\big)$

$= -P(\theta = 1|r = 1)P(\theta = 0|r = 1)$

(2) Consider $r = 0$

$\max_{y_2} EV(\theta, y_2(r = 0), \beta_2)$

$$= \max_{y_2} P(\theta = 0|r = 0)[-(y_2 - 0 - \beta_2)^2] + P(\theta = 1|r = 0)[-(y_2 - 1 - \beta_2)^2]$$

First derivative

$$\frac{dEV(\theta, y_2(r = 1), \beta_2)}{dy_2} = -2P(\theta = 0|r = 0)(y_2 - 0 - \beta_2) - 2P(\theta = 1|r = 0)(y_2 - 1 - \beta_2)$$

$$= -2[y_2 - \beta_2 - P(\theta = 1|r = 0)].$$

We obtain a critical value

$$y_2(r = 0) = \beta_2 + P(\theta = 1|r = 0) = \beta_2 + \frac{p(f(e_1) + f(e_2) - 2f(e_1)f(e_2))}{1 - p - (1 - 2p)(f(e_1) + f(e_2) - 2f(e_1)f(e_2))}.$$

Second derivative

$$\frac{d^2EV(\theta, y_2(r = 0), \beta_2)}{d^2y_2} = -2 < 0.$$

Thus

$$y_2(r = 0) = \beta_2 + P(\theta = 1|r = 0) = \beta_2 + \frac{p(f(e_1) + f(e_2) - 2f(e_1)f(e_2))}{1 - p - (1 - 2p)(f(e_1) + f(e_2) - 2f(e_1)f(e_2))}$$

is a maximum point value point.

$\max_{y_2} EV(\theta, y_2(r = 0), \beta_2)$

$= \max_{y_2} P(\theta = 0|r = 0)[-(y_2 - 0 - \beta_2)^2] + P(\theta = 1|r = 0)[-(y_2 - 1 - \beta_2)^2]$

$= -P(\theta = 0|r = 0)P(\theta = 1|r = 0)^2 - P(\theta = 1|r = 0)(P(\theta = 1|r = 0) - 1)^2$

$= -P(\theta = 0|r = 0)P(\theta = 1|r = 0)^2$

$$- P(\theta = 1|r = 0)([P(\theta = 1|r = 1)]^2 - 2P(\theta = 1|r = 0) + 1)$$

$= -P(\theta = 1|r = 0)\big(1 - P(\theta = 1|r = 0)\big)$

$= -P(\theta = 1|r = 0)P(\theta = 0|r = 0)$

$\boxdot$

Proof of Lemma 4

$MAX_{e_1, e_2} - E[\alpha(y_2-\theta)^2]$

$= MAX_{e_1, e_2} - \alpha\{[P(r=0|\theta=0)(y_2(0)-0)^2 + P(r=1|\theta=0)(y_2(1)-0)^2]P(\theta=0)$

$+ [P(r=0|\theta=1)(y_2(0)-1)^2 + P(r=1|\theta=1)(y_2(1)-1)^2]P(\theta=1)\}$

$$= -\alpha\left\{{\beta_2}^2 + \frac{-p(1-p)\big(f(e_1)+f(e_2)-2f(e_1)f(e_2)\big)\big(1-f(e_1)-f(e_2)+2f(e_1)f(e_2)\big)}{\big[p+(1-2p)\big(f(e_1)+f(e_2)-2f(e_1)f(e_2)\big)\big]\big[1-p-(1-2p)\big(f(e_1)+f(e_2)-2f(e_1)f(e_2)\big)\big]}\right\}$$

$$= -\alpha\left[{\beta_2}^2 + \sqrt{P(\theta=0|r=1)P(\theta=1|r=1)P(\theta=0|r=0)P(\theta=1|r=0)}\right]$$

First derivative with respect to e_1, we have

$$\frac{\partial MAX_{e_1, e_2} - E[\alpha(y_2-\theta)^2]}{\partial e_1}$$

$$= \frac{\partial - \alpha \dfrac{-p(1-p)(f(e_1)+f(e_2)-2f(e_1)f(e_2))(1-f(e_1)-f(e_2)+2f(e_1)f(e_2))}{[p+(1-2p)(f(e_1)+f(e_2)-2f(e_1)f(e_2))][1-p-(1-2p)(f(e_1)+f(e_2)-2f(e_1)f(e_2))]}}{\partial e_1}$$

$$= \frac{\alpha p(1-p)(1-2p)^2\big(1-2f(e_1)-2f(e_2)+4f(e_1)f(e_2)\big)[1-2f(e_2)]f'(e_1)}{[p+(1-2p)(f(e_1)+f(e_2)-2f(e_1)f(e_2))]^2[1-p-(1-2p)(f(e_1)+f(e_2)-2f(e_1)f(e_2))]^2}$$

$> 0.$

First derivative with respect to e_2, we have

$$\frac{\partial MAX_{e_1, e_2} - E[\alpha(y_2-\theta)^2]}{\partial e_1}$$

$$= \frac{\partial - \alpha \dfrac{-p(1-p)(f(e_1)+f(e_2)-2f(e_1)f(e_2))(1-f(e_1)-f(e_2)+2f(e_1)f(e_2))}{[p+(1-2p)(f(e_1)+f(e_2)-2f(e_1)f(e_2))][1-p-(1-2p)(f(e_1)+f(e_2)-2f(e_1)f(e_2))]}}{\partial e_1}$$

$$= \frac{\alpha p(1-p)(1-2p)^2\big(1-2f(e_1)-2f(e_2)+4f(e_1)f(e_2)\big)[1-2f(e_1)]f'(e_2)}{[p+(1-2p)(f(e_1)+f(e_2)-2f(e_1)f(e_2))]^2[1-p-(1-2p)(f(e_1)+f(e_2)-2f(e_1)f(e_2))]^2}$$

$> 0.$

$\boxdot$

Proof of Proposition 6

$$MAX_{e_1, e_2} - E[\alpha(y_2 - \theta)^2]$$
$$where\ e_1 + e_2 \leq 1.$$

The Lagrangian can be written

$$L(e_1, e_2; \delta) = -E[\alpha(y_2 - \theta)^2] + \delta(1 - e_1 - e_2).$$

Kuhn-Trucker conditions are

$$\delta \geq 0, 1 - e_1 - e_2 \geq 0 \text{ with CS}$$

$$\frac{\partial L(e_1, e_2; \delta)}{\partial e_1} = \frac{\partial - E[\alpha(y_2 - \theta)^2] + \delta(1 - e_1 - e_2)}{\partial e_1}$$
$$= \frac{\alpha p(1-p)(1-2p)^2\big(1 - 2f(e_1) - 2f(e_2) + 4f(e_1)f(e_2)\big)[1 - 2f(e_2)]f'(e_1)}{[p + (1-2p)(f(e_1) + f(e_2) - 2f(e_1)f(e_2))]^2[1 - p - (1-2p)(f(e_1) + f(e_2) - 2f(e_1)f(e_2))]^2}$$
$$- \delta = 0$$

$$\frac{\partial L(e_1, e_2; \delta)}{\partial e_2} = \frac{\partial - E[\alpha(y_2 - \theta)^2] + \delta(1 - e_1 - e_2)}{\partial e_2}$$
$$= \frac{\alpha p(1-p)(1-2p)^2\big(1 - 2f(e_1) - 2f(e_2) + 4f(e_1)f(e_2)\big)[1 - 2f(e_1)]f'(e_2)}{[p + (1-2p)(f(e_1) + f(e_2) - 2f(e_1)f(e_2))]^2[1 - p - (1-2p)(f(e_1) + f(e_2) - 2f(e_1)f(e_2))]^2}$$
$$- \delta = 0$$

Combining above two equations, we have

$$[1 - 2f(e_2)]f'(e_1) = [1 - 2f(e_1)]f'(e_2)$$
$$\frac{f'(e_1)}{[1 - 2f(e_1)]} = \frac{f'(e_2)}{[1 - 2f(e_2)]}$$

Since $f'(\cdot)/[1 - 2f(\cdot)]$ is an increasing function.

Thus we have

$$e_1 = e_2 \leq t/2 \leq 1/2$$

$\boxdot$

Proof of Proposition 7

(1) Consider $r = 1$,

$$\max_{y_2} EV(\theta, y_2(r=1), \beta_2)$$

$$= \max_{y_2} P(\theta = 0|r = 1)[-(y_2 - 0 - \beta_2)^2] + P(\theta = 1|r = 1)[-(y_2 - 1 - \beta_2)^2]$$

First derivative

$$\frac{dEV(\theta, y_2(r=1), \beta_2)}{dy_2} = -2P(\theta = 0|r = 1)(y_2 - \beta_2) - 2P(\theta = 1|r = 1)(y_2 - 1 - \beta_2)$$

$$= -2[y_2 - \beta_2 - P(\theta = 1|r = 1)].$$

We obtain a critical value

$$y_2(r=1) = \beta_2 + P(\theta = 1|r = 1) = \beta_2 + \frac{p\left[\frac{1}{2} - \gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right]}{\frac{1}{2} + (1-2p)\gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]}.$$

Second derivative

$$\frac{d^2EV(\theta, y_2(r=1), \beta_2)}{d^2y_2} = -2 < 0$$

Thus

$$y_2(r=1) = \beta_2 + P(\theta = 1|r = 1) = \beta_2 + \frac{p\left[\frac{1}{2} - \gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right]}{\frac{1}{2} + (1-2p)\gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]}$$

is a maximum point value point.

$$\max_{y_2} EV(\theta, y_2(r=1), \beta_2)$$

$$= \max_{y_2} P(\theta = 0|r = 1)[-(y_2 - 0 - \beta_2)^2] + P(\theta = 1|r = 1)[-(y_2 - 1 - \beta_2)^2]$$

$$= -P(\theta = 0|r = 1)P(\theta = 1|r = 1)^2 - P(\theta = 1|r = 1)(P(\theta = 1|r = 1) - 1)^2$$

$$= -P(\theta = 0|r = 1)P(\theta = 1|r = 1)^2$$

$$- P(\theta = 1|r = 1)([P(\theta = 1|r = 1)]^2 - 2P(\theta = 1|r = 1) + 1)$$

$$= -P(\theta = 1|r = 1)\big(1 - P(\theta = 1|r = 1)\big)$$

$$= -P(\theta = 1|r = 1)P(\theta = 0|r = 1)$$

(2) Consider $r = 0$,

$$\max_{y_2} EV(\theta, y_2(r=0), \beta_2)$$

$$= \max_{y_2} P(\theta = 0|r = 0)[-(y_2 - 0 - \beta_2)^2] + P(\theta = 1|r = 0)[-(y_2 - 1 - \beta_2)^2]$$

First derivative

$$\frac{dEV(\theta, y_2(r=1), \beta_2)}{dy_2} = -2P(\theta = 0|r = 0)(y_2 - 0 - \beta_2) - 2P(\theta = 1|r = 0)(y_2 - 1 - \beta_2)$$
$$= -2[y_2 - \beta_2 - P(\theta = 1|r = 0)].$$

We obtain a critical value

$$y_2(r=0) = \beta_2 + P(\theta = 1|r = 0) = \beta_2 + \frac{p\left[\frac{1}{2} + \gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right]}{\frac{1}{2} - (1-2p)\gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]}.$$

Second derivative

$$\frac{d^2EV(\theta, y_2(r=0), \beta_2)}{d^2y_2} = -2 < 0.$$

Thus

$$y_2(r=0) = \beta_2 + P(\theta = 1|r = 0) = \beta_2 + \frac{p\left[\frac{1}{2} + \gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right]}{\frac{1}{2} - (1-2p)\gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]}$$

is a maximum point value point.

$$\max_{y_2} EV(\theta, y_2(r=0), \beta_2)$$

$$= \max_{y_2} P(\theta = 0|r = 0)[-(y_2 - 0 - \beta_2)^2] + P(\theta = 1|r = 0)[-(y_2 - 1 - \beta_2)^2]$$

$$= -P(\theta = 0|r = 0)P(\theta = 1|r = 0)^2 - P(\theta = 1|r = 0)(P(\theta = 1|r = 0) - 1)^2$$

$$= -P(\theta = 0|r = 0)P(\theta = 1|r = 0)^2$$
$$- P(\theta = 1|r = 0)([P(\theta = 1|r = 1)]^2 - 2P(\theta = 1|r = 0) + 1)$$

$$= -P(\theta = 1|r = 0)\big(1 - P(\theta = 1|r = 0)\big)$$

$$= -P(\theta = 1|r = 0)P(\theta = 0|r = 0)$$

$\boxdot$

Proof of Lemma 5

$$MAX_{e_1,e_2} - E[\alpha(y_2-\theta)^2]$$

$$= MAX_{e_1,e_2} - \alpha\{[P(r=0|\theta=0)(y_2(0)-0)^2 + P(r=1|\theta=0)(y_2(1)-0)^2]P(\theta=0)$$
$$+ [P(r=0|\theta=1)(y_2(0)-1)^2 + P(r=1|\theta=1)(y_2(1)-1)^2]P(\theta=1)\}$$

$$= -\alpha\left\{\beta_2\right.$$
$$\left. + \frac{p(1-p)\left[\frac{1}{2}+\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]\right]\left[\frac{1}{2}-\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]\right]}{\left[\frac{1}{2}+(1-2p)\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]\right]\left[\frac{1}{2}-(1-2p)\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]\right]}\right\}$$

$$= -\alpha\left\{\beta_2 + \frac{p(1-p)\left[\frac{1}{4}-\left[\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]\right]^2\right]}{\frac{1}{4}-\left[(1-2p)\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]\right]^2}\right\}$$

$$= -\alpha\left[{\beta_2}^2 + \sqrt{P(\theta=0|r=1)P(\theta=1|r=1)P(\theta=0|r=0)P(\theta=1|r=0)}\right].$$

First derivative with respect to e_1, we have

$$\frac{\partial MAX_{e_1,e_2} - E[\alpha(y_2-\theta)^2]}{\partial e_1} = -\frac{4\alpha[p(1-p)]^2\gamma^2\left[f_1(e_2)-\frac{1}{2}\right]^2[1-2f_1(e_1)]f_1{}'(e_1)}{\left[\frac{1}{4}-\left[(1-2p)\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]\right]^2\right]^2} > 0.$$

First derivative with respect to e_2

$$\frac{\partial MAX_{e_1,e_2} - E[\alpha(y_2-\theta)^2]}{\partial e_2} = \frac{2\alpha[p(1-p)]^2\gamma^2[1-2f_1(e_1)]^2\left[f_1(e_2)-\frac{1}{2}\right]f_1'(e_2)}{\left[\frac{1}{4}-\left[(1-2p)\gamma\left[f_1(e_2)-\frac{1}{2}\right][1-2f_1(e_1)]\right]^2\right]^2} > 0.$$

First derivative with respect to λ

Let $M = \left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]$

$$\frac{\partial p(1-p)\left[\frac{1}{4} - \left[\gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right]^2\right]}{\partial\lambda} = \frac{\partial p(1-p)\left[\frac{1}{4} - [\gamma M]^2\right]}{\partial\lambda} = -2p(1-p)M^2\gamma,$$

$$\frac{\partial \frac{1}{4} - \left[(1-2p)\gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right]^2}{\partial\lambda} = \frac{\partial\left\{\frac{1}{4} - [(1-2p)\gamma M]^2\right\}}{\partial\lambda} = -2(1-2p)^2M^2\gamma$$

$$\frac{\partial MAX_{e_1,e_2} - E[\alpha(y_2-\theta)^2]}{\partial\lambda}$$

$$= -\alpha\frac{\frac{\partial p(1-p)\left[\frac{1}{4} - [\gamma M]^2\right]}{\partial\lambda}\left\{\frac{1}{4} - [(1-2p)\gamma M]^2\right\} - \frac{\partial\left\{\frac{1}{4} - [(1-2p)\gamma M]^2\right\}}{\partial\lambda}p(1-p)\left[\frac{1}{4} - [\gamma M]^2\right]}{\left\{\frac{1}{4} - [(1-2p)\gamma M]^2\right\}^2}$$

$$= -\alpha\frac{-2p(1-p)M^2\gamma\left\{\frac{1}{4} - [(1-2p)\gamma M]^2\right\} + 2(1-2p)^2M^2\gamma p(1-p)\left[\frac{1}{4} - [\gamma M]^2\right]}{\left\{\frac{1}{4} - [(1-2p)\gamma M]^2\right\}^2}$$

$$= -\alpha\frac{p(1-p)M^2\gamma\{-2\left\{\frac{1}{4} - [(1-2p)\gamma M]^2\right\} + 2(1-2p)^2\left[\frac{1}{4} - [\gamma M]^2\right]\}}{\left\{\frac{1}{4} - [(1-2p)\gamma M]^2\right\}^2}$$

$$= -\alpha\frac{-2[p(1-p)]^2M^2\gamma}{\left\{\frac{1}{4} - [(1-2p)\gamma M]^2\right\}^2} > 0$$

$\boxdot$

Proof of Proposition 8

$$MAX_{e_1,e_2} - E[\alpha(y_2-\theta)^2]$$

$$where\ e_1 + e_2 \leq 1.$$

The Lagrangian can be written

$$L(e_1, e_2; \delta) = -E[\alpha(y_2-\theta)^2] + \delta(1 - e_1 - e_2).$$

Kuhn-Trucker conditions are

$$\delta \geq 0, 1 - e_1 - e_2 \geq 0 \text{ with CS}$$

$$\frac{\partial L(e_1, e_2; \delta)}{\partial e_1} = \frac{\partial - E[\alpha(y_2 - \theta)^2] + \delta(1 - e_1 - e_2)}{\partial e_1}$$

$$= -\frac{4\alpha[p(1-p)]^2\gamma^2\left[f_1(e_2) - \frac{1}{2}\right]^2[1 - 2f_1(e_1)]f_1{}'(e_1)}{\left[\frac{1}{4} - \left[(1-2p)\gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right]^2\right]^2} - \delta = 0$$

$$\frac{\partial L(e_1, e_2; \delta)}{\partial e_2} = \frac{\partial - E[\alpha(y_2 - \theta)^2] + \delta(1 - e_1 - e_2)}{\partial e_2}$$

$$= \frac{2\alpha[p(1-p)]^2\gamma^2[1 - 2f_1(e_1)]^2\left[f_1(e_2) - \frac{1}{2}\right]f_1'(e_2)}{\left[\frac{1}{4} - \left[(1-2p)\gamma\left[f_1(e_2) - \frac{1}{2}\right][1 - 2f_1(e_1)]\right]^2\right]^2} - \delta = 0$$

Combining above two equations, we have

$$-4\left[f_1(e_2) - \frac{1}{2}\right]^2[1 - 2f_1(e_1)]f_1{}'(e_1) = 2[1 - 2f_1(e_1)]^2\left[f_1(e_2) - \frac{1}{2}\right]f_1'(e_2)$$

$$-[2f_1(e_2) - 1]f_1'(e_1) = [1 - 2f_1(e_1)]f_1'(e_2)$$

$$\frac{f_1{}'(e_1)}{[1 - 2f_1(e_1)]} = \frac{f_1{}'(e_2)}{[1 - 2f_1(e_2)]}$$

Since $f'(\cdot)/[1 - 2f(\cdot)]$ are increasing function.

Thus we have

$$e_1 = e_2 = t/2 \leq 1/2$$

$\boxdot$

Chapter 3
Optimal Majority Rule in Referenda

-Abstract-

We construct a simple model of direct democracy with supermajority rule and different preference intensities for two sides of a referendum: reform versus status quo. Two parties spend money and effort to mobilize their voters. We characterize the set of pure strategy Nash equilibria. We investigate the optimal majority rule that maximizes voters' welfare. Using an example, we show that if the preference intensity of the status quo side is relatively high, the higher preference intensity of the status quo side, the higher the optimal majority rule. While, if the preference intensity of status quo side is relatively low, the optimal majority rule decreases if the preference intensity of the status quo side increases. We also show that when the preference intensity of the status quo side is higher, or the easiness to mobilize voters on the status quo side is lower, the optimal majority rule is more likely to be supermajority.

Keywords: Referendum, Majority rule, Equilibrium, Welfare, Maximization

3.1 Introduction

A referendum is a mechanism through which all citizens collectively make a decision on major issues. Recent examples include the British vote on EU membership in 2016 (commonly known as “Brexit”) and the Scottish vote on UK membership in 2014. Since the Second World War, most of the world's countries are moving towards a greater use of referenda, and paying more attention to designs and implementation of referenda. Referenda have become an established mechanism for decision-making in democratic systems.

Despite its wide use, the adoption of referenda as a decision mechanism is often controversial. Outcomes of referenda are sometimes far from voters’ expectations. Two recent instances are the shock Brexit vote and the failure of the Italian constitutional reforms. If not properly designed and managed, referenda have the potential of causing division and instability in society.

Take the Brexit referendum as an example. In 2016, the United Kingdom held a referendum on June 23 to decide whether the UK should leave or remain in the European Union. The outcome was that 17.4 million people, or 51.9 per cent of the participants, voted to leave the EU while 48.1 per cent, or 16.1 million people, voted to stay. Leave won by 51.9% to 48.1%. The referendum turnout was 72.21%, with more than 30 million people voting. Leave scored a narrow victory over Remain. On the day after the referendum, Prime Minister Cameron announced his resignation. More than 4 million people signed a petition calling for a second EU referendum to be held. The petition reads: “we the undersigned call upon HM Government to implement a rule that if the remain or leave vote is less than 60% based a turnout less than 75% there should be another referendum.” However, the British government formally rejected the petition on July 9, 2016. The Harvard economist Kenneth Rogoff considered the Brexit vote as a democratic failure: “The real lunacy of the United Kingdom’s vote to leave the European Union was not that British leaders dared to ask their populace to weigh the benefits of membership against the immigration pressures it presents. Rather, it was the absurdly low bar for exit,

TABLE 3.1 Majority Provisions on Referenda in Established Western Democracies*

Australia	Geographical requirement: majority of votes and majority of states
Austria	Simple majority
Belgium	No provisions for referendums
Canada	Under debate
Denmark	Registered voter requirement: 30% of voters, 40% of voters on constitutional changes
France	Simple majority
Finland	Simple majority
Germany	No provisions for referendums
Iceland	Simple majority
Ireland	Simple majority
Italy	Turnout requirement: 50% of the registered voters
Luxembourg	Simple majority
Malta	Simple majority
Netherlands	Simple majority
Switzerland	Geographical requirement: simple majority and majority of cantons
United Kingdom	Simple majority (40% of registered voters in 1979)
USA	No provisions for nationwide referendums

***A Comparative Study of Referendums* Qvortrup 2005 P.171

requiring only a simple majority. Given voter turnout of 70%, this meant that the leave campaign won with only 36% of eligible voters backing it."

Referenda are often criticized for two reasons. The first reason is that many countries adopt the simple majority rule. Qvortrup (2005, p171) provides a list of majority provisions in established Western Democracies. See TABLE 3.1 for a list of the threshold requirements for referenda held in established Western democracies.

TABLE 3.2 Referenda with Close Results

Year	Referendum	Outcome
1992	The Danish Maastricht referendum	49.3-50.7 per cent
1992	The French Maastricht Treaty referendum	51-49 per cent
1995	The Quebec Secession referendum	49.4-50.6 per cent
1995	The Irish Divorce referendum	50.3-49.7 per cent
2014	Scottish independence referendum	44.7-55.3 per cent
2016	The Brexit referendum	51.9-48.1 per cent

The second reason is that referenda often produce close outcomes. "A close result threatens the legitimacy of the outcome" (Qvortrup 2005, 174). Former Canadian Prime Minister Jean Chretien once said that 50 percent plus one vote could split a country. There are a handful of examples of referendums showed on TABLE 3.2 that were decided by a whisker.

The above two problems make it worthwhile to investigate the determining factors of the outcome of referenda and consider how referenda should be optimally designed. As is well known, some restrictions are relatively uncontroversial in a democratic system, such as the encouragement of voting, the location of the polling station, and the size of the ballots. However, there are some controversial restrictions imposed in the referendum process in order to ensure a fair outcome, such as limits on campaign expenditures, disclosure laws, super-majority requirements, and higher signature requirements. Take for example super-majority provisions, known from the UK and Italy, which led to the defeat of several laws in Italy and to the defeat of the devolution proposal for Scotland in the UK.

Another important factor to consider is the true meaning of democracy. The famous commentator on American democracy, Alexis de Tocqueville, talked a great deal in his book *Democracy in America* about the tyranny of the majority. This is when majority rule - the basis of democracy - ends up perverting democracy by forcing injustice on the minority. For example, Proposition 8, a California ballot proposition was passed in 2008 to amend the California constitution and ban gay marriages, which was later ruled unconstitutional by the Federal courts, because it was viewed to purported to *re-remove* rights from a disfavored class only, with no rational basis.

Joseph Stiglitz mentioned, “On both sides of the Channel, politics should be directed at understanding the underlying sources of anger. Also how, in a democracy, the political establishment could have done so little to address the concerns of so many citizens, and figuring out how to do that now: to create within each country, and through cross-border arrangements, a new, more democratic Europe, which sees its goal as improving the wellbeing of ordinary citizens.”

This paper mainly focuses the following question: How to obtain the optimal majority rule that maximizes voters’ welfare, in order to reflect the superiority of the referendum and make the results of the referendum convincing? We show that it is necessary to strengthen research on the supermajority requirement of referendum and the optimization expectation payoff of all voters. In addition, the outcome of the referendum must be a clear expression of a will by a clear majority of participants. The Government should stipulate a specific percentage below which the result of a referendum becomes legally void.

This paper mainly analyzes the effect of the optimal majority rule in referenda. Our model is similar with a group turnout model developed by Herrera and Mattozzi (2010) based on Snyder (1989) and Shachar and Nalebuff (1999), where two opposed parties spend effort to mobilize their supporters to the polls, while facing aggregate uncertainty on the voters’ preferences. Herrera and Mattozzi (2010) mainly analyze how a participation requirement affects the distribution of voting outcomes. We updated their model by introducing new factors, such as supermajority rule and different intensities preference of two sides in referenda: reform versus status quo, and ignoring participation requirement.

The main results of this paper are as following. First, we introduce a majority rule which affect the probability that alternative reform is selected, and different intensities of preference of two sides of a referendum: reform versus status quo. By characterizing the set of pure strategy Nash equilibriums, we show that the parties could stipulate upper bound of campaign funds to spend, which depend on the payoff of parties receive if their preferred alternative is chosen. The higher the payoffs of parties receive, the higher upper bound of campaign fund to spend is stipulated.

Second, we maximize the welfare of all voters to obtain the threshold value of the proportion of voters support policy reform, and get a general formula of the optimal majority rule in referenda.

Third, using an example, we obtain the special optimal majority rule. With the method of comparative analysis, we find there is a critical value of the preference intensity of the status quo side. The optimal majority rule is inversely proportional to the preference intensity of the status quo side as the preference intensity of the status quo low enough, while the optimal majority rule is proportional to the preference intensity of the status quo as the preference intensity of the status quo side high enough.

Also, there exists a critical value of the ratio of the easiness to mobilize of reform to status quo. The optimal majority rule is proportional to payoff of reform party receives as the ratio of the easiness to mobilize of reform to status quo low enough, while the optimal majority rule is inversely proportional to as payoff of reform party receives as the ratio of the easiness to mobilize of reform to status quo high enough.

Under the analysis of supermajority requirement, we obtain one necessary and sufficient condition and some judgement rules. For example, we show that when it is easier for the reform side to mobilize, then the optimal majority rule is more likely to be supermajority.

In our paper, the intuition “High turnout is a result of increased effort that parties exert” is supported by the empirical work of Gerald H. Kramer (1971) and Peter W. Wielhouwer and Brad Lockerbie (1994), who show that respondents who were contacted by the parties are more likely to participate. Thus efforts must be made to ensure that the electorate is as wide and as inclusive as possible. Above intuition also is supported by the theoretical work by Shachar and Nalebuff (1999) show that participation rate and political parties' efforts are a positive function of predicted closeness and a negative function of the voting population size.

Our maximization problem is similar to welfare analysis from Osborne and Turner (2010), who analyze welfare of all agents. In their paper they study how a referendum or a cost benefit analysis leads to higher welfare, and show that the outcome of a cost benefit analysis is superior when individuals have diverse preferences but similar information, whereas the outcome of a referendum is superior when individuals have similar preferences but different degrees of uncertainty. While we investigate the optimal majority rule that maximizes voters' welfare. We show that if the intensity preference of the status quo side becomes higher, the optimal majority rule should not always be stipulate higher by government.

The rest of the paper is organized as follows. Section 2 is model. We focus on pure strategy Nash equilibrium, optimal analysis of party campaign funding allocation problem, and the voters' expectation optimization problem. We also analyze the specific optimal majority rule M and supermajority requirement through an example. Section 3 is conclusion. We summarize results of this paper, give some impacts on the future, and show shortcoming and further research direction.

3.2 Model

3.2.1 Model Setup

Consider a direct democracy model, the individuals legally eligible to vote in a society. Each individual can only choose between two alternatives: r (reform) and s (status quo). Government selects a majority rule $M \in (0,1]$, defining the fraction of voters that must approve a policy proposed to replace the status quo. The unanimity rule, for example, is defined by $M = 1$, while $M = 1/2$ is the simple majority rule.

There are two exogenously given parties supporting policies r and s, and a continuum of voters of measure 1, of which a proportion $\tilde{r} \in [0,1]$ supports policy r, and the remaining supports policy s. Here $\tilde{r}$ is a random variable with uniform distribution. Each voter has a personal cost of voting $c \in [0,1]$ that is also drawn from a uniform distribution.

Parties decide simultaneously the amount of campaign funds to spend (equivalently, the amount of effort to exert) to mobilize voters in order to win the referendum. The parties' objective functions are

$$\pi_R(x_r, x_s) = BP - x_r$$
$$\pi_s(x_r, x_s) = \beta B(1 - P) - x_s$$

Where P is the (endogenous) probability that alternative r is selected. $x_r \ and \ x_s$ are spending of parties r and s, respectively. Consider different payoffs that parties receive, B>0 is the payoff of reform party receives, and $\beta B > 0$ is the payoff of status quo party receives where β is the preference intensity of party and voters in favor of status quo.

Voters decide whether to vote in referendum or not depending a benefit from the party and cost of voting. We assume that voters receive a benefit from voting their preferred policy that is strictly concave in parties' mobilization efforts. In particular, if a party spends x, the benefit to a voter who supports that party's policy is $\rho_i(x): \mathbb{R}_+ \to [0,1], i = 1, 2$ is continuous, twice differentiable for $x > 0$, strictly increasing, and strictly concave, and satisfies the properties

$$\lim_{x \to 0} x\rho_i'(x) = 0, \lim_{x \to 0} x\rho_i''(x) = 0, \lim_{x \to \infty} \rho_i'(x) = 0, \lim_{x \to \infty} \rho_i(x) = 1$$

This specification is equivalent to having parties' expenditures affect individual cost of voting. For the sake of simplicity, we will assume that $\rho_i(0) = 0$.

Let us denote $\rho_1(x)$ is the benefit to a voter who supports policy r, and $\rho_2(x)$ is the benefit to a voter who supports policy s. For a given level of spending x_r, a voter who support policy r and has a voting cost equal to c votes for alternative r if and only if $\rho_1(x_r) \geq c$. Because this holds for fraction $\rho_1(x_r)$ of the voters supporting policy r, the vote share for that policy (as a fraction of the total population) is $v_R = \tilde{r}\rho_1(x_r)$. Likewise, the vote share for policy s is $v_S = (1-\tilde{r})\rho_2(x_s)$.

The probability that alternative r is selected is:

$$P = \Pr\left(\frac{v_R}{v_R + v_S} \geq M\right) = \Pr\left(\frac{\tilde{r}\rho_1(x_r)}{\tilde{r}\rho_1(x_r) + (1-\tilde{r})\rho_2(x_s)} \geq M\right)$$
$$= \Pr\left(\tilde{r} \geq \frac{\rho_2(x_s)}{\left(\frac{1}{M}-1\right)\rho_1(x_r) + \rho_2(x_s)}\right) = 1 - \frac{\rho_2(x_s)}{\left(\frac{1}{M}-1\right)\rho_1(x_r) + \rho_2(x_s)}$$

Note that P is represented as a function of $\rho_1(x_r)$ and $\rho_2(x_s)$ for any given M, and is continuous in its arguments on the whole space $\left(\rho_1(x_r), \rho_2(x_s)\right) \in [0,1]^2$.

Characteristics of P (M): The probability $P(M)$ is decreasing in M; the value of P is determined as follows.

$$P(M) = 1 - \frac{\rho_2(x_s)}{\left(\frac{1}{M}-1\right)\rho_1(x_r) + \rho_2(x_s)} \begin{cases} = 0 & M = 1 \\ < \frac{1}{2} & \frac{\rho_1(x_r)}{\rho_1(x_r) + \rho_2(x_s)} < M < 1 \\ = \frac{1}{2} & M = \frac{\rho_1(x_r)}{\rho_1(x_r) + \rho_2(x_s)} \\ > \frac{1}{2} & 0 < M < \frac{\rho_1(x_r)}{\rho_1(x_r) + \rho_2(x_s)} \\ \to 1 & M \to 0 \end{cases}$$

The values of P are showed in FIGURE 3.1

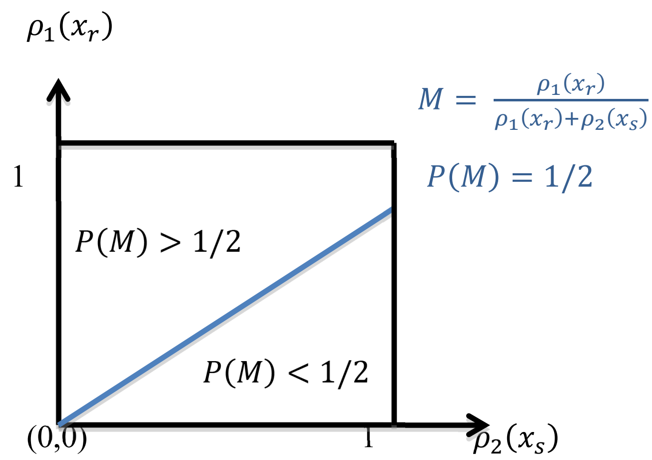


FIGURE 3.1 Probability of approval

Note that P is continuous in its arguments on the whole space $\left(\rho_1(x_r), \rho_2(x_s)\right) \in [0,1]^2$. If M=1, that is, there is a unanimity rule, the probability of the reform policy is selected is zero. If $M \to 0,$ that is, there is no majority rule, the probability of the reform policy is selected close to 1. As M increases, the line rotates around the origin counter clockwise, and on the line the probability that the reform policy is selected is equal to 1/2, and below the line the probability that the reform policy is selected is below 1/2, and above the line the probability that the reform policy is selected is above 1/2.

3.2.2 Equilibrium Characterization

Considering Parties Behavior, we start by focusing on pure-strategy Nash equilibrium.

Proposition 1 (Nash Equilibrium) *A pure-strategy Nash equilibrium of parties is the positive spending profile* $C \equiv (\widehat{x_r}, \widehat{x_s})$*, which satisfies the following two equations*

$$\rho_2(x_s)\rho_1'(x_r) = \beta\rho_2'(x_s)\rho_1(x_r)$$

$$B\left(\frac{1}{M} - 1\right)\rho_2(x_s)\rho_1{}'(x_r) = [\left(\frac{1}{M} - 1\right)\rho_1(x_r) + \rho_2(x_s)]^2$$

The proof of Proposition 1 is provided in Appendix.

Corollary 1 $When\ M \rightarrow 1\ or\ 0, then\ corner\ solution\ (x_s(M), x_r(M)) \rightarrow (0,0)$

The proof of Corollary 1 is provided in Appendix. Corollary 1 means if the majority rule close to one or zero, the best respond of the reform party and status quo party have corner solution (0,0).

Corollary 2 *There exists a majority rule M to obtain maximum value of spending profile* $C \equiv (x_r{}^*, x_s{}^*)$ *in the pure-strategy Nash equilibrium, where*

$$\frac{\rho_1'(x_r{}^*)}{\rho_1(x_r{}^*)} = \frac{\beta\rho_2'(x_s{}^*)}{\rho_2(x_s{}^*)} = \frac{4}{B}$$

The proof of Corollary 2 is provided in Appendix. Corollary 2 represents that the party sides have a unique best respond spending profile with each other for any M from zero to one. Among those unique profiles, we have a maximum value of spending profile given a specific value M.

Corollary 3 $h_i(x) = \rho_i'(x)/\rho_i(x)$ *is a decreasing function in x.*

The proof of Corollary 3 is provided in Appendix. The parties could stipulate upper bound of campaign funds to spend depend on the payoff of parties receive if their preferred alternative is chosen. The higher payoff of reform party receive, the high upper bound of campaign funds to spend is stipulated.

3.2.3 Welfare Maximization

Considering voter's behavior, we measure welfare by summing the expectation payoff of all voters.

Problem. *The maximization problem for the expectation payoff of all voters*

$$U(M) = \max_{M} E_{\tilde{r} \geq K}[\tilde{r}] + \beta E_{\tilde{r} \leq K}[1 - \tilde{r}]$$

$$where\ K = \frac{\rho_2(x_s)}{\left(\frac{1}{M} - 1\right)\rho_1(x_r) + \rho_2(x_s)}$$

Note that $E_{\tilde{r} \geq K}[\tilde{r}]$ represents the expectation payoff of voters in favor of policy r when policy r is chosen, and $\beta E_{\tilde{r} \leq K}[1 - \tilde{r}]$ represents the expectation payoff of voters in favor of policy s when policy s is chosen. Here K is a threshold point of the proportion of voters support policy r; parameter β is the preference intensity of the voters in favor of status quo. Clearly, it shows that the higher β, the higher expectation payoff of voters in favor of status quo, and the higher expectation payoff of all voters. Moreover, if $0 < \beta < 1$, i.e. the preference intensity of the voters in favor of status quo is lower than the voters in favor of reform, then the expectation payoff of voters in favor of reform is higher than the voters in favor of status quo; if $\beta > 1$, then expectation payoff of voters in favor of reform is lower than the voters in favor of status quo.

Proposition 2 (Social Welfare) *The expectation payoff of all voters' reaches the maximum value if and only if the threshold value* $K = \beta/(\beta + 1)$**,** *i.e. the optimal majority rule satisfies the following equation*

$$M = \frac{\beta}{1 + \frac{\rho_2(x_s)}{\rho_1(x_r)}}$$

And the maximum value of the expectation payoff of all voters $U(M)$ *is equal to* $(\beta^2 + \beta + 1)/[2(\beta + 1)]$.

Proof

Solving problem

$$E_{\tilde{r}\geq K}[\tilde{r}] + \beta E_{\tilde{r}\leq K}[1-\tilde{r}] = \int_K^1 r\,dr + \int_0^K \beta(1-r)\,dr = \frac{r^2}{2}\Big|_K^1 + \beta\left(r - \frac{r^2}{2}\right)\Big|_0^K$$

$$= \frac{1}{2} - \frac{K^2}{2} + \beta\left(K - \frac{K^2}{2}\right) = \frac{1}{2} + \beta K - \frac{(\beta+1)}{2}K^2$$

$$= -\frac{(\beta+1)}{2}\left(K - \frac{\beta}{\beta+1}\right)^2 + \frac{\beta^2+\beta+1}{2(\beta+1)}$$

Thus when $K = \beta/(\beta+1)$, $E_{\tilde{r}\geq K}[\tilde{r}] + \beta E_{\tilde{r}\leq K}[1-\tilde{r}]$ reaches the maximum value $(\beta^2+\beta+1)/2(\beta+1)$.

$$M = \frac{\rho_2(x_s)}{\left(\frac{1}{M}-1\right)\rho_1(x_r) + \rho_2(x_s)} = \frac{\beta}{\beta+1}$$

Then

$$M = \frac{\beta\rho_1(x_r)}{\rho_1(x_r)+\rho_2(x_s)} = \frac{\beta}{1+\frac{\rho_2(x_s)}{\rho_1(x_r)}}$$

$\boxdot$

Note that if the preference intensity of the status quo side β becomes higher, we need the higher threshold value K, and gain the higher value of the expectation payoff of all voters $U(M)$. Specifically, if the preference intensity of the status quo side $\beta = 0$, it means there is no benefit to voters in favor of status quo, so the threshold value K=0 and a majority rule M=0 which means the party in favor of status quo will not spend campaign funds to mobilize voter. Depend on the Pure-strategy Nash equilibrium, the best respond of party reform to party status quo is to also not spend campaign funds to mobilize voter. Thus the pure-strategy Nash equilibrium of parties is the positive spending profile $C = (0,0)$ as $\beta = 0$ while the expectation payoff of all voters $U(M) = 1/2$. Please recall corollary 1.

2.4. An Example

Consider the case in which $\rho_1(x_r) = 1 - e^{-\alpha_1 x_r}$ and $\rho_2(x_s) = 1 - e^{-\alpha_2 x_s}$, where α_1, α_2

represents the easiness to mobilize, i.e. the larger α, the easier to mobilize. As we have proved in the model, a pure-strategy Nash equilibrium of parties is the positive spending profile $C \equiv (\widehat{x_r}, \widehat{x_s})$, which satisfies the following two equations

$$\alpha_1(1 - e^{-\alpha_2 x_s})\, e^{-\alpha_1 x_r} = \beta \alpha_2 e^{-\alpha_2 x_s}(1 - e^{-\alpha_1 x_r})$$
$$B\left(\frac{1}{M} - 1\right)(1 - e^{-\alpha_2 x_s})\alpha_1 e^{-\alpha_1 x_r} = \left[\left(\frac{1}{M} - 1\right)(1 - e^{-\alpha_1 x_r}) + (1 - e^{-\alpha_2 x_s})\right]^2$$

Here we introduce parameter α to represent benefit to voter. For example, we assume most young people more prefer reform, while most old people more prefer status quo. Because most of young people living in city have job and have to work, so they have not much time left in the voting queue. It seems that it is difficulty to mobilize young people than old people. Thus base the same effort of party, the mobilize level α is different between status quo and reform. It is suitable for us to introduce α here.

Proposition 3 (Optimal Majority Rule) *The expectation payoff of all voters reaches the maximum value if and only if the optimal majority rule*

$$M^* = \frac{B\beta^2\alpha_1\alpha_2 + (\beta+1)^2\alpha_1}{B\beta\,(\beta+1)\alpha_1\alpha_2 + (\beta+1)^2\alpha_1 + (\beta+1)^2\,\alpha_2}$$

And the positive spending profile of parties $C \equiv (x_r^*, x_s^*)$, *where*

$$e^{-\alpha_1 x_r^*} = \frac{(\beta+1)^2}{\beta B\alpha_1 + (\beta+1)^2}, e^{-\alpha_2 x_s^*} = \frac{(\beta+1)^2}{B\beta^2\alpha_2 + (\beta+1)^2}, x_r^* > 0, x_s^* > 0$$
$$i.e.\, \rho_1(x_r) = \frac{\beta B\alpha_1}{\beta B\alpha_1 + (\beta+1)^2}, \rho_2(x_s) = \frac{B\beta^2\alpha_2}{B\beta^2\alpha_2 + (\beta+1)^2}$$

The proof of Proposition 3 is provided in Appendix.

Comparative analysis for the optimal majority rule M

$$M = \frac{B\beta^2\alpha_1\alpha_2 + (\beta+1)^2\alpha_1}{B\beta(\beta+1)\alpha_1\alpha_2 + (\beta+1)^2\alpha_1 + (\beta+1)^2\,\alpha_2}$$

Corollary 4 *The optimal majority rule*

$$M = \frac{B\beta^2\beta\alpha_2 + (\beta+1)^2\beta}{B\beta(\beta+1)\beta\alpha_2 + (\beta+1)^3} = \frac{\beta}{\beta+1}$$

as $\alpha_1 = \beta\alpha_2$*, which has no relation to parameter B, and equal to threshold value K.*

Note that the optimal majority rule M increases when the preference intensity of the status quo side increases if the ratio of easiness to mobilize of two sides equal to β.

Corollary 5 *There exists one critical value of* β*, when*

$$\beta \in \left[\frac{-\alpha_2 + \sqrt{B{\alpha_1}^2\alpha_2 + 2\alpha_1\alpha_2 + {\alpha_1}^2 + {\alpha_2}^2}}{(B\alpha_1\alpha_2 + \alpha_1 + 2\alpha_2)}, +\infty\right]$$

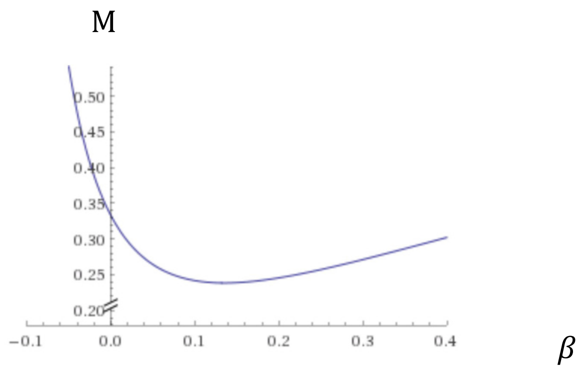
The optimal majority rule M increases as β *increases; when*

$$\beta \in \left[0, \frac{-\alpha_2 + \sqrt{B{\alpha_1}^2\alpha_2 + 2\alpha_1\alpha_2 + {\alpha_1}^2 + {\alpha_2}^2}}{B\alpha_1\alpha_2 + \alpha_1 + 2\alpha_2}\right]$$

The optimal majority rule M decreases as β *increases.*

The proof of Corollary 4 is provided in Appendix. Note that if the intensity preference of status quo β is close to zero, the optimal majority rule M decreases if β increases a little. Also if the preference intensity of the status quo side β is high enough, the optimal majority rule M would higher as the preference intensity of the status quo side β become higher.

FIGURE 3.2 A Numerical Example 1



Numerical Example 1 Given $\alpha_1 = 1, \alpha_2 = 2, B = 10,$ then

$$M = \frac{20\beta^2 + (\beta + 1)^2}{20\beta(\beta + 1) + 3(\beta + 1)^2}$$

Please see FIGURE 2. Note that the change of the majority rule M as β increases from zero to infinity. It shows that the optimal majority rule decreases firstly, then increases as β increases.

Corollary 6 *There exists one critical value of* α_1/α_2, *the optimal majority rule M increases as B increases when* $\alpha_1/\alpha_2 < \beta$; *the optimal majority rule M decreases as B increases when* $\alpha_1/\alpha_2 > \beta$.

The proof of Corollary 6 is provided in Appendix. Not saying that higher payoff of the party reform received, the lower optimal majority rule M should be stipulated. If the ratio of easiness to mobilize of two sides greater than β, the higher optimal majority rule M should be stipulated as B becomes higher.

Proposition 4 (Supermajority Requirement) *Referendum must satisfy supermajority requirement if and only if*

$$\frac{1}{\alpha_1}-\frac{(1/s-1)}{\alpha_2}<\frac{B\beta\,((1/s-1)\,\beta-1)}{(\beta+1)^2}$$

Where s is a supermajority rule with s > 1/2, α_1 *is the easiness to mobilize of reform, and* α_2 *is the easiness to mobilize of status quo party.*

The proof of Proposition 4 is provided in Appendix.

Corollary 7 (Judgment rule for supermajority requirement) *Referendum must satisfy supermajority requirement if any of the following conditions is standing.*

(1) $\frac{\alpha_1}{\alpha_2}>\frac{1}{(1/s-1)},\beta>\frac{1}{(1/s-1)}$

(2) $\frac{1}{\alpha_1}-\frac{(1/s-1)}{\alpha_2}=t<-\frac{Bs}{4}$

(3) $\frac{1}{\alpha_1}-\frac{(1/s-1)}{\alpha_2}=t\in\left(-\frac{Bs}{4},0\right),\beta\in\ (0,\frac{-\sqrt{Bs}\sqrt{Bs+4t}-Bs-2st}{2(Bs-B+st)}]\ \cup\ [\frac{\sqrt{Bs}\sqrt{Bs+4t}-Bs-2st}{2(Bs-B+st)},+\infty)$

(4) $\frac{1}{\alpha_1}-\frac{(1/s-1)}{\alpha_2}=t\in(0,+\infty),\beta\in\ [\frac{\sqrt{Bs}\sqrt{Bs+4t}-Bs-2st}{2(Bs-B+st)},+\infty]$

The proof of Corollary 7 is provided in Appendix. Note that if the easiness to mobilize of status quo α_2 is low enough, the optimal majority rule is more likely to be supermajority. If the intensity preference of the status quo side β is high enough, the optimal majority rule is more likely to be supermajority.

Numerical Example 2 (Simple majority requirement) Referendum must have simple majority requirement if and only if

$$\frac{1}{\alpha_1}-\frac{1}{\alpha_2}<\frac{B\beta\,(\beta-1)}{(\beta+1)^2}$$

The proof of Numerical Example 2 is provided in Appendix.

Judgment rule for simple majority requirement of numerical example 2 Referendum must satisfy simple majority requirement if any of the following conditions is standing.

(1) $\frac{\alpha_1}{\alpha_2} > 1, \beta > 1$

(2) $\frac{1}{\alpha_1} - \frac{1}{\alpha_2} < -\frac{B}{8}$

(3) $\frac{1}{\alpha_1} - \frac{1}{\alpha_2} = t \in \left(-\frac{B}{8}, 0\right), \beta \in \ (0, \frac{\sqrt{B}-\sqrt{B+8t}}{3\sqrt{B}+\sqrt{B+8t}}] \ \cup \ [\frac{\sqrt{B}+\sqrt{B+8t}}{3\sqrt{B}-\sqrt{B+8t}}, +\infty]$

(4) $\frac{1}{\alpha_1} - \frac{1}{\alpha_2} = t \in (0, +\infty), \beta \in \ [\frac{\sqrt{B}+\sqrt{B+8t}}{3\sqrt{B}-\sqrt{B+8t}}, +\infty]$

3.3 Conclusions

This paper constructed a simple direct democracy model with majority rule and preference intensity of the status quo side. Focus on pure strategy Nash equilibrium theory, we show that the parties could stipulate upper bound of campaign fund to spend depend on the payoff of parties receive if their preferred alternative is chosen. Also if the majority rule close to zero or one, then the spending profile of two parties have corner solution, which is close to (0,0).

We construct a maximization problem about voter's expectation payoff, and investigate the optimal majority rule. Using an example, we show the specific optimal majority rule and the specific spending profile of parties.

From the optimal majority rule, we show that if the preference intensity of the status quo side β is high enough, the optimal majority rule M would higher as the preference intensity of the status quo side β become higher. While if the preference intensity of the status quo side β is low enough, then the optimal majority rule M decreases if β increases. In addition, the optimal majority rule M increases when the preference intensity of the status quo side β increases if the ratio of easiness to mobilize of two sides equal to β. For payoff of the party reform received B, not saying that higher payoff of the party reform received, the lower optimal majority rule M should be stipulated, because the ratio of easiness to mobilize of two sides greater than β, the higher optimal majority rule M should be stipulated as B becomes higher.

From the supermajority requirement, we find that when the preference intensity of status quo side β is high enough, or the easiness to mobilize of status quo α_2 is low enough, the optimal majority rule M is more likely to be supermajority.

It is conducive to the parties to rationally use of the referendum funds to gain maximum benefit. It is conducive to the voters to achieve the optimal expectation. It is conducive to government to stipulate a specific percentage (optimal majority rule), which if the result of a referendum below, then the result of a referendum becomes legally void. It is conducive to make the outcome of the referendum to better reflect the expression of public opinion.

In this study, the participation quorum requirement is not considered. If participation rate is less than a certain threshold, it may distort the public opinion from the outcome of the referendum. So in the future study, we could combine quorum requirement and majority rule together into the model, to do further analysis, make the outcome of referendum to better reflect voters' optimal expectation.

3.4 Appendix

Proof of Proposition 1

The parties' objective functions are

$$\begin{cases} \pi_r(x_r, x_s) = BP - x_r \\ \pi_s(x_r, x_s) = \beta B(1-P) - x_s \end{cases}$$

Then

$$\begin{cases} \pi_r(x_r, x_s) = B\left(1 - \dfrac{\rho_2(x_s)}{\left(\frac{1}{M}-1\right)\rho_1(x_r) + \rho_2(x_s)}\right) - x_r \\ \pi_s(x_r, x_s) = \beta B \dfrac{\rho_2(x_s)}{\left(\frac{1}{M}-1\right)\rho_1(x_r) + \rho_2(x_s)} - x_s \end{cases}$$

Thus

$$\widehat{x_r} = arg\max\left(B\left(1 - \frac{\rho_2(\widehat{x_s})}{\left(\frac{1}{M}-1\right)\rho_1(\widehat{x_r}) + \rho_2(\widehat{x_s})}\right) - \widehat{x_r}\right)$$

$$\widehat{x_s} = arg\,max\left(\beta B\frac{\rho_2(\widehat{x_s})}{\left(\frac{1}{M}-1\right)\rho_1(\widehat{x_r}) + \rho_2(\widehat{x_s})} - \widehat{x_s}\right)$$

First order condition

$$\begin{cases} B\dfrac{\left(\frac{1}{M}-1\right)\rho_2(x_s)\rho_1{}'(x_r)}{[\left(\frac{1}{M}-1\right)\rho_1(x_r) + \rho_2(x_s)]^2} - 1 = 0 \\ \beta B\dfrac{\left(\frac{1}{M}-1\right)\rho_2'(x_s)\rho_1(x_r)}{[\left(\frac{1}{M}-1\right)\rho_1(x_r) + \rho_2(x_s)]^2} - 1 = 0 \end{cases}$$

Then

$$B\left(\frac{1}{M}-1\right)\rho_2(x_s)\rho_1{}'(x_r) = [\left(\frac{1}{M}-1\right)\rho_1(x_r) + \rho_2(x_s)]^2$$

$$\beta B\left(\frac{1}{M}-1\right)\rho_2{}'(x_s)\rho_1(x_r) = [\left(\frac{1}{M}-1\right)\rho_1(x_r) + \rho_2(x_s)]^2$$

Thus

$$\rho_2(x_s)\rho_1'(x_r) = \beta\rho_2'(x_s)\rho_1(x_r)$$

$\boxdot$

Proof of Corollary 1

When $M \to 1$, we have

$$B\left(\frac{1}{M}-1\right)\rho_2(x_s)\rho_1'(x_r) = \left[\left(\frac{1}{M}-1\right)\rho_1(x_r) + \rho_2(x_s)\right]^2$$

$$\rho_2(x_s) \to 0$$

Put into the following equation

$$\rho_2(x_s)\rho_1'(x_r) = \beta\rho_2'(x_s)\rho_1(x_r)$$

$$0 \leftarrow \beta\rho_2'(x_s)\rho_1(x_r)$$

$$\rho_1(x_r) \to 0$$

Then

$$(x_s(M), x_r(M)) \to (0,0)$$

When $M \to 0$, we have

$$\lim_{M\to 0} \pi_s(x_r, x_s) = \lim_{M\to 0} \beta B \frac{\rho_2(x_s)}{\left(\frac{1}{M}-1\right)\rho_1(x_r)+\rho_2(x_s)} - x_s = -x_s$$

We have

$$Max \lim_{M\to 0} \pi_s(x_r, x_s) = 0, where\ x_s(M) = 0$$

$$\lim_{M\to 0} \pi_r(x_r, x_s) = \lim_{M\to 0} B\left(1 - \frac{\rho_2(x_s)}{\left(\frac{1}{M}-1\right)\rho_1(x_r)+\rho_2(x_s)}\right) - x_r = B - x_r$$

We have

$$Max \lim_{M\to 0} \pi_r(x_r, x_s) = B, where\ x_r(M) = 0$$

$\odot$

Proof of Corollary 2

By the Implicit Function Theorem

$$f(x_s, x_r, M) = \rho_2(x_s)\rho_1'(x_r) - \beta\rho_2'(x_s)\rho_1(x_r) = 0$$

$$g(x_s, x_r, M) = B\left(\frac{1}{M}-1\right)\rho_2(x_s)\rho_1'(x_r) - \left[\left(\frac{1}{M}-1\right)\rho_1(x_r)+\rho_2(x_s)\right]^2 = 0$$

$$\begin{bmatrix} \frac{\partial x_s{}^*}{\partial M} \\ \frac{\partial x_r{}^*}{\partial M} \end{bmatrix} = -\begin{bmatrix} \frac{\partial f}{\partial x_s} & \frac{\partial f}{\partial x_r} \\ \frac{\partial g}{\partial x_s} & \frac{\partial g}{\partial x_r} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial f}{\partial M} \\ \frac{\partial g}{\partial M} \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial x_s{}^*}{\partial M} \\ \frac{\partial x_r{}^*}{\partial M} \end{bmatrix} = -\frac{1}{\frac{\partial f}{\partial x_s}\frac{\partial g}{\partial x_r} - \frac{\partial f}{\partial x_r}\frac{\partial g}{\partial x_s}} \begin{bmatrix} \frac{\partial g}{\partial x_r} & -\frac{\partial f}{\partial x_r} \\ -\frac{\partial g}{\partial x_s} & \frac{\partial f}{\partial x_s} \end{bmatrix} \begin{bmatrix} \frac{\partial f}{\partial M} \\ \frac{\partial g}{\partial M} \end{bmatrix}$$

$$= -\frac{1}{\frac{\partial f}{\partial x_s}\frac{\partial g}{\partial x_r} - \frac{\partial f}{\partial x_r}\frac{\partial g}{\partial x_s}} \begin{bmatrix} \frac{\partial g}{\partial x_r}\frac{\partial f}{\partial M} - \frac{\partial f}{\partial x_r}\frac{\partial g}{\partial M} \\ -\frac{\partial g}{\partial x_s}\frac{\partial f}{\partial M} + \frac{\partial f}{\partial x_s}\frac{\partial g}{\partial M} \end{bmatrix} = \begin{bmatrix} \frac{-\frac{\partial g}{\partial x_r}\frac{\partial f}{\partial M} + \frac{\partial f}{\partial x_r}\frac{\partial g}{\partial M}}{\frac{\partial f}{\partial x_s}\frac{\partial g}{\partial x_r} - \frac{\partial f}{\partial x_r}\frac{\partial g}{\partial x_s}} \\ \frac{\frac{\partial g}{\partial x_s}\frac{\partial f}{\partial M} - \frac{\partial f}{\partial x_s}\frac{\partial g}{\partial M}}{\frac{\partial f}{\partial x_s}\frac{\partial g}{\partial x_r} - \frac{\partial f}{\partial x_r}\frac{\partial g}{\partial x_s}} \end{bmatrix}$$

$$Since\ \frac{\partial f}{\partial M} = 0, then$$

$$\begin{bmatrix} \dfrac{\partial x_s{}^*}{\partial M} \\ \dfrac{\partial x_r{}^*}{\partial M} \end{bmatrix} = \begin{bmatrix} \dfrac{\dfrac{\partial f}{\partial x_r}\dfrac{\partial g}{\partial M}}{\dfrac{\partial f}{\partial x_s}\dfrac{\partial g}{\partial x_r} - \dfrac{\partial f}{\partial x_r}\dfrac{\partial g}{\partial x_s}} \\ \dfrac{-\dfrac{\partial f}{\partial x_s}\dfrac{\partial g}{\partial M}}{\dfrac{\partial f}{\partial x_s}\dfrac{\partial g}{\partial x_r} - \dfrac{\partial f}{\partial x_r}\dfrac{\partial g}{\partial x_s}} \end{bmatrix}$$

Where

$$\frac{\partial f}{\partial x_s} = \rho_2'(x_s)\rho_1'(x_r) - \beta\rho_2''(x_s)\rho_1(x_r) > 0$$

$$\frac{\partial f}{\partial x_r} = \rho_2(x_s)\rho_1''(x_r) - \beta\rho_2'(x_s)\rho_1'(x_r) < 0$$

$$\frac{\partial g}{\partial M} = -B\rho_2(x_s)\rho_1'(x_r)\frac{1}{M^2} + 2\left[\left(\frac{1}{M} - 1\right)\rho_1(x_r) + \rho_2(x_s)\right]\rho_1(x_r)\frac{1}{M^2}$$

There exists an extreme value of spending profile $C \equiv (x_r{}^*, x_s{}^*)$ when $\frac{\partial g}{\partial M} = 0$, thus

$$B\rho_2(x_s)\rho_1'(x_r) = 2\left[\left(\frac{1}{M} - 1\right)\rho_1(x_r) + \rho_2(x_s)\right]\rho_1(x_r)$$

Depend on the following two equations

$$B\left(\frac{1}{M} - 1\right)\rho_2(x_s)\rho_1{}'(x_r) = [\left(\frac{1}{M} - 1\right)\rho_1(x_r) + \rho_2(x_s)]^2$$

$$B\rho_2(x_s)\rho_1'(x_r) = 2\left[\left(\frac{1}{M} - 1\right)\rho_1(x_r) + \rho_2(x_s)\right]\rho_1(x_r)$$

We have

$$[\left(\frac{1}{M} - 1\right)\rho_1(x_r) + \rho_2(x_s)]^2 = 2\left(\frac{1}{M} - 1\right)\left[\left(\frac{1}{M} - 1\right)\rho_1(x_r) + \rho_2(x_s)\right]\rho_1(x_r)$$

$$\left(\frac{1}{M} - 1\right)\rho_1(x_{\square}) + \rho_2(x_s) = 2\left(\frac{1}{M} - 1\right)\rho_1(x_r)$$

$$\frac{\rho_1(x_r)}{\rho_2(x_s)} = \frac{M}{1 - M}$$

Put above equation into the following equation

$$B\left(\frac{1}{M} - 1\right)\rho_2(x_s)\rho_1'(x_r) = \left[\left(\frac{1}{M} - 1\right)\rho_1(x_r) + \rho_2(x_s)\right]^2$$

$$B\left(\frac{1}{M} - 1\right)\rho_2(x_s)\rho_1'(x_r) = \left[\left(\frac{1}{M} - 1\right)\frac{M}{1 - M}\rho_2(x_s) + \rho_2(x_s)\right]^2$$

$$\rho_1'(x_r) = \frac{4\rho_2(x_s)}{B\left(\frac{1}{M} - 1\right)}$$

Put above equation into the following equation

$$\rho_2(x_s)\rho_1'(x_r) = \beta\rho_2'(x_s)\rho_1(x_r)$$

$$\rho_2(x_s)\frac{4\rho_2(x_s)}{B\left(\frac{1}{M} - 1\right)} = \beta\rho_2'(x_s)\rho_1(x_r)$$

$$\frac{\rho_2'(x_s)}{\rho_2(x_s)} = \frac{4\rho_2(x_s)}{\beta B\left(\frac{1}{M} - 1\right)\rho_1(x_r)} = \frac{4}{\beta B\left(\frac{1}{M} - 1\right)\frac{M}{1 - M}} = \frac{4}{\beta B}$$

$$\frac{\rho_1'(x_r)}{\rho_1(x_r)} = \frac{\beta\rho_2'(x_s)}{\rho_2(x_s)} = \frac{4}{B}$$

$\boxdot$

Proof of Corollary 3

$$h_i'(x) = \frac{\rho_i''(x)\rho_i(x) - \rho_i'(x)\rho_i'(x)}{[\rho_i(x)]^2} < 0$$

$\boxdot$

Proof of Proposition 3

Put

$$\rho_1(x_r) = 1 - e^{-\alpha_1 x_r} = 1 - P, \rho_2(x_s) = 1 - e^{-\alpha_2 x_s} = 1 - Q$$

Into

$$\frac{\rho_1(x_r)}{\rho_2(x_s)} = \frac{M}{\beta\,(1 - M)}$$

Then

$$\frac{1 - P}{1 - Q} = \frac{M}{\beta\,(1 - M)}$$

Combine the following equations

$$\begin{cases} (1 - Q)\,\alpha_1 P = \beta\alpha_2 Q(1 - P) \\ \dfrac{1 - P}{1 - Q} = \dfrac{M}{\beta\,(1 - M)} \end{cases}$$

$$P = \frac{\alpha_2[\beta(1-M)-M]}{(\beta\alpha_2-\alpha_1)(1-M)}, \quad Q = \frac{\alpha_1[\beta(1-M)-M]}{(\beta\alpha_2-\alpha_1)M}$$

Put P, Q into the following equation

$$B\left(\frac{1}{M}-1\right)(1-Q)\alpha_1 P = \left[\left(\frac{1}{M}-1\right)(1-P)+(1-Q)\right]^2$$

Then

$$B\left(\frac{1}{M}-1\right)\left(1-\frac{\alpha_1[\beta(1-M)-M]}{(\beta\alpha_2-\alpha_1)M}\right)\alpha_1\frac{\alpha_2[\beta(1-M)-M]}{(\beta\alpha_2-\alpha_1)(1-M)}$$
$$= \left[\left(\frac{1}{M}-1\right)\left(1-\frac{\alpha_2[\beta(1-M)-M]}{(\beta\alpha_2-\alpha_1)(1-M)}\right)+\left(1-\frac{\alpha_1[\beta(1-M)-M]}{(\beta\alpha_2-\alpha_1)M}\right)\right]^2$$

$$B\frac{(1-M)}{M}\frac{\beta\big(\alpha_2 M-\alpha_1(1-M)\big)}{(\beta\alpha_2-\alpha_1)M}\frac{\alpha_1\alpha_2[\beta(1-M)-M]}{(\beta\alpha_2-\alpha_1)(1-M)}$$
$$= \left[\frac{(1-M)}{M}\frac{\big(\alpha_2 M-\alpha_1(1-M)\big)}{(\beta\alpha_2-\alpha_1)(1-M)}+\frac{\beta\big(\alpha_2 M-\alpha_1(1-M)\big)}{(\beta\alpha_2-\alpha_1)M}\right]^2$$

$$B\frac{\beta(\alpha_2 M-\alpha_1(1-M))\alpha_1\alpha_2[\beta(1-M)-M]}{M(\beta\alpha_2-\alpha_1)M(\beta\alpha_2-\alpha_1)} = \left[\frac{(\alpha_2 M-\alpha_1(1-M))(1+\beta)}{M(\beta\alpha_2-\alpha_1)}\right]^2$$

$$B\beta\alpha_1\alpha_2[\beta(1-M)-M] = (\alpha_2 M-\alpha_1(1-M))(1+\beta)^2$$

$$M = \frac{B\beta^2\alpha_1\alpha_2+(\beta+1)^2\alpha_1}{B\beta(\beta+1)\alpha_1\alpha_2+(\beta+1)^2\alpha_1+(\beta+1)^2\,\alpha_2}$$

Put

$$M = \frac{B\beta^2\alpha_1\alpha_2+(\beta+1)^2\alpha_1}{B\beta\,(\beta+1)\alpha_1\alpha_2+(\beta+1)^2\alpha_1+(\beta+1)^2\,\alpha_2}$$

Into the result

$$P = \frac{\alpha_2[\beta(1-M)-M]}{(\beta\alpha_2-\alpha_1)(1-M)}, Q = \frac{\alpha_1[\beta(1-M)-M]}{(\beta\alpha_2-\alpha_1)M}$$

Then

$$e^{-\alpha_1 x_r} = P = \frac{(\beta+1)^2}{B\beta\alpha_1+(\beta+1)^2}, \quad e^{-\alpha_2 x_s} = Q = \frac{(\beta+1)^2}{B\beta^2\alpha_2+(\beta+1)^2}$$
$$\rho_1(x_r) = 1-e^{-\alpha_1 x_r} = 1-\frac{(\beta+1)^2}{\beta B\alpha_1+(\beta+1)^2} = \frac{B\beta\alpha_1}{B\beta\alpha_1+(\beta+1)^2}$$
$$\rho_2(x_s) = 1-e^{-\alpha_2 x_s} = 1-\frac{(\beta+1)^2}{B\beta^2\alpha_2+(\beta+1)^2} = \frac{B\beta^2\alpha_2}{B\beta^2\alpha_2+(\beta+1)^2}$$

⊡

Proof of Corollary 5

First derivate for β

$$\frac{\partial M}{\partial \beta}=\frac{B\alpha_1\alpha_2(B\beta^2\alpha_1\alpha_2+(\beta^2-1)\alpha_1+2\beta\,(\beta+1)\alpha_2)}{(B\beta(\beta+1)\alpha_1\alpha_2+(\beta+1)^2\alpha_1+(\beta+1)^2\,\alpha_2)^2}=0$$

Since $B\beta^2\alpha_1\alpha_2+(\beta^2-1)\alpha_1+2\beta\,(\beta+1)\alpha_2=(B\alpha_1\alpha_2+\alpha_1+2\alpha_2)\beta^2+2\alpha_2\beta-\alpha_1$, then

$$\beta=\frac{-\alpha_2\pm\sqrt{B{\alpha_1}^2\alpha_2+2\alpha_1\alpha_2+{\alpha_1}^2+{\alpha_2}^2}}{B\alpha_1\alpha_2+\alpha_1+2\alpha_2}$$

When $\beta\in[\,(-\alpha_2+\sqrt{B{\alpha_1}^2\alpha_2+2\alpha_1\alpha_2+{\alpha_1}^2+{\alpha_2}^2})/(B\alpha_1\alpha_2+\alpha_1+2\alpha_2)\,,+\infty]$, the optimal majority rule M increases as β increases;

When $\beta\in[0,(-\alpha_2+\sqrt{B{\alpha_1}^2\alpha_2+2\alpha_1\alpha_2+{\alpha_1}^2+{\alpha_2}^2})/(B\alpha_1\alpha_2+\alpha_1+2\alpha_2)]$, the optimal majority rule M decreases as β increases.

⊡

Proof of Corollary 6

First derivate for B

$$\frac{\partial M}{\partial B}=\frac{\beta\alpha_1\alpha_2(\beta\alpha_2-\alpha_1)}{(B\beta\alpha_1\alpha_2+(\beta+1)(\alpha_1+\alpha_2))^2}=0$$

Thus the optimal majority rule M increases as B increases when $\alpha_1/\alpha_2<\beta$; the optimal majority rule M decreases as B increases when $\alpha_1/\alpha_2>\beta$.

⊡

Proof of Proposition 4

$$M=\frac{B\beta^2\alpha_1\alpha_2+(\beta+1)^2\alpha_1}{B\beta(\beta+1)\alpha_1\alpha_2+(\beta+1)^2\alpha_1+(\beta+1)^2\,\alpha_2}>s$$

$$\frac{B\beta^2\alpha_1\alpha_2+(\beta+1)^2\alpha_1}{s}>B\beta(\beta+1)\alpha_1\alpha_2+(\beta+1)^2\alpha_1+(\beta+1)^2\,\alpha_2$$

$$B\beta((1/s-1)\beta-1)\alpha_1\alpha_2>(\beta+1)^2\,\alpha_2-(1/s-1)(\beta+1)^2\alpha_1$$

$$B\beta\big((1/s-1)\beta-1\big)\alpha_1\alpha_2 > (\beta+1)^2(\alpha_2-(1/s-1)\alpha_1)$$
$$\frac{B\beta((1/s-1)\beta-1)}{(\beta+1)^2} > \frac{(\alpha_2-(1/s-1)\alpha_1)}{\alpha_1\alpha_2}$$
$$\frac{1}{\alpha_1}-\frac{(1/s-1)}{\alpha_2} < \frac{B\beta((1/s-1)\beta-1)}{(\beta+1)^2}$$

$\boxdot$

Proof of Corollary 7

First order condition

$$\frac{\partial \frac{B\beta((1/s-1)\beta-1)}{(\beta+1)^2}}{\partial\beta} = B\frac{\left(2(\frac{1}{s}-1)\beta-1\right)(\beta+1)-2\beta((\frac{1}{s}-1)\beta-1)}{(\beta+1)^3} = \frac{B((\frac{2}{s}-1)\beta-1)}{(\beta+1)^3}$$

Critical value of β

$$\beta = \frac{1}{(1/s-1)}$$

Second order condition

$$\frac{\partial^2 \frac{B\beta\ ((1/s-1)\ \beta-1)}{(\beta+1)^2}}{\partial\beta\partial\beta} = \frac{B\ (\frac{2}{s}-2\frac{2}{s}\beta+2\beta+2)}{(\beta+1)^4} > 0\ if\ s > 1/2$$

Thus the shape of $\frac{B\beta\ ((1/s-1)\ \beta-1)}{(\beta+1)^2}$ is convex.

$$min\frac{B\beta((1/s-1)\beta-1)}{(\beta+1)^2} = -\frac{Bs}{4}$$

Let

$$\frac{1}{\alpha_1}-\frac{(1/s-1)}{\alpha_2} = t$$

If

$$\frac{B\beta\ ((1/s-1)\ \beta-1)}{(\beta+1)^2} = t$$

Then

$$\beta_1 = \frac{\sqrt{Bs}\sqrt{Bs+4t}-Bs-2st}{2(Bs-B+st)}, \beta_2 = \frac{-\sqrt{Bs}\sqrt{Bs+4t}-Bs-2st}{2(Bs-B+st)}$$

The graph of inequality is showed on FIGURE 3.

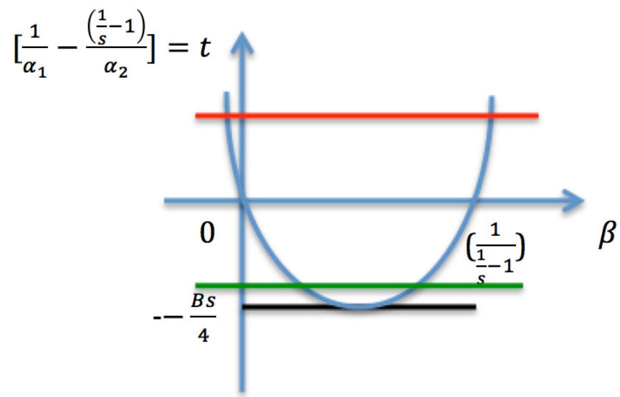


Figure 3.3 Sufficient and necessary condition of Corollary 7

(1). If

$$\frac{1}{\alpha_1}-\frac{(1/s-1)}{\alpha_2}=t=0$$

Above the blue line

$$\beta>\frac{1}{1/s-1}$$

We have necessary and sufficient condition is established, thus referendum must satisfy supermajority requirement.

(2). If

$$\frac{1}{\alpha_1}-\frac{(1/s-1)}{\alpha_2}=t<-\frac{Bs}{4}$$

Regardless of β, we have necessary and sufficient condition is established, thus referendum must satisfy supermajority requirement.

(3). If

$$\frac{1}{\alpha_1}-\frac{(1/s-1)}{\alpha_2}=t\in\left(-\frac{Bs}{4},0\right)$$

Above green line

$$\beta\in\left(0,\frac{-\sqrt{Bs}\sqrt{Bs+4t}-Bs-2st}{2(Bs-B+st)}\right]\cup\left[\frac{\sqrt{Bs}\sqrt{Bs+4t}-Bs-2st}{2(Bs-B+st)},+\infty\right)$$

We have necessary and sufficient condition is established, referendum must satisfy supermajority requirement.

(4). If

$$\frac{1}{\alpha_1} - \frac{(1/s - 1)}{\alpha_2} = t \in (0, +\infty)$$

Above red line

$$\beta \in \left[\frac{\sqrt{Bs}\sqrt{Bs+4t} - Bs - 2st}{2(Bs - B + st)}, +\infty\right]$$

We have necessary and sufficient condition is established, referendum must satisfy supermajority requirement.

$\boxdot$

Proof of Numerical example 2

Proof

$$M = \frac{B\beta^2\alpha_1\alpha_2 + (\beta+1)^2\alpha_1}{B\beta(\beta+1)\alpha_1\alpha_2 + (\beta+1)^2\alpha_1 + (\beta+1)^2\,\alpha_2} > \frac{1}{2}$$

$$2B\beta^2\alpha_1\alpha_2 + 2(\beta+1)^2\alpha_1 > B\beta(\beta+1)\alpha_1\alpha_2 + (\beta+1)^2\alpha_1 + (\beta+1)^2\,\alpha_2$$

$$B\beta(\beta-1)\alpha_1\alpha_2 > (\beta+1)^2\,\alpha_2 - (\beta+1)^2\alpha_1$$

$$B\beta(\beta-1)\alpha_1\alpha_2 > (\beta+1)^2(\alpha_2 - \alpha_1)$$

$$\frac{B\beta(\beta-1)}{(\beta+1)^2} > \frac{(\alpha_2 - \alpha_1)}{\alpha_1\alpha_2}$$

$$\frac{1}{\alpha_1} - \frac{1}{\alpha_2} < \frac{B\beta(\beta-1)}{(\beta+1)^2}$$

$\boxdot$

References

Ashworth, S. (2006). Campaign Finance and Voter Welfare with Entrenched Incumbents. *American Political Science Review* 100(1), 55-68.

Bagwell, K. (2007). The Economic Analysis of Advertising. *Handbook of Industrial Organization* 3, 1701-1844.

Becker, G.S., & and Murphy, K.S. (1993). A Simple Theory of Advertising as Good or Bad. *Quarterly Journal of Economics* 108.

Bentley P., & Kyvik, S. (2011). Academic staff and public communication: A survey of popular science publishing across 13 countries. *Public Underst Sci,* 20, 48–63.

Caillaud, B., & Tirole, J. (2007). How to Persuade a Group. *The American Economic Review*, Vol.97, No.5, 1877-1900.

Chamberlin, E. (1933). The Theory of Monopolistic Competition. Harvard Univ. Press, Cambridge, MA.

Coate, S. (2004a). Pareto-Improving Campaign Finance Policy. *The American Economic Review*, 94(3), 628-655.

Coate, S. (2004b). Political Competition with Campaign Contributions and Informative Advertising. Journal of the European Economic association, 2(5), 772-804.

Crawford, V.P., & Sobel, J. (1982). Strategic Information Transmission, *Econometrica*. 50(6), 1431-1451

Degan, A., & Li, M. (2017). Persuasion with costly precision. Working paper.

Dixit, A., & Norman, V. (1978). Advertising and Welfare. *Bell Journal of Economics* 9(1),1-17.

Farrell J., & Gibbons, R. (1989). Cheap Talk with Two Audiences. *The American Economic Review*, 79(5), 1214-1223.

Gentzkow M., & Kamenica, E. (2014). Costly Persuasion. *American Economic Review*, 104(5), 457- 462.

Gerber, E. R. (1996). Legislative Response to the Threat of Popular Initatives. *American Journal of Political Science*, 40(1), 99-128.

Goltsman, M., & Pavlov, G. (2011). How to talk to multiple audiences. *Games and Economic Behavior*, 72, 100-122.

Hagenbach J., & Koessler, F. (2010). Strategic Communication Networks. *Review of Economic Studies*, 77, 1072-1099.

Herrera, H.， & Mattozzi, A. (2010). Quorum and Turnout in Referenda. *Journal of the European Economic Association*, 8 (4), 838-871.

Hori, K. (2006). Organizing information transmission with multiple agents. Working paper.

Jackson M., & Wolinsky. A. 1996. A Strategic Model of Social and Economic Network. *Journal of Economic Theory*, 71, 44-74.

Jensen P., Rouquier J.B., Kreimer P., & Croissant Y. (2008). Scientists who engage with society perform better academically. *Sci Public Policy,* 35, 527–541.

Kamenica E., & Gentzkow, M. (2011). Bayesian Persuasion. *American Economic Association*, 101(6), 2590-2615.

Kolotilin A., Li, M., Mylovanov T., & Zapechelnyuk A. (2015). Persuasion of a Privately informed Receiver. Working Paper

Kramer, G.H. (1971). The Effect of Precinct Level Canvassing on Voter Behavior. *Public Opinion Quarterly,* 34, 560-572

Li, H., & Li, W. (2013). Misinformation. *International Economic Review*, 54(1), 253-277.

McGee, A., & Yang, H. (2013). Cheap talk with two senders and complementary information. *Games and Economic Behavior,* 79, 181-191.

Milgrom, P., & Roberts, J. (1986). Price and Advertising Signals of Product Quality. *Journal of Political Economy* 94, 796-821.

National Science Board. (2012). *Science and engineering indicators.* Retrieved from: http://www.nsf.gov/statistics/seind12/pdf/c07.pdf Accessed 29 December 2012.

Petty, R. E., & Cacioppo, J. T. (1984). The Elaboration Likelihood Model of Persuasion. Advances in Consumer Research, 11(1), 673-675.

Oliveira, J. M., & Pagano, A.S. (2006). The research article and the science popularization article: a probabilistic functional grammar perspective on direct discourse representation. *Discourse Studies*, 8(5), 627-646.

Osborne J. M., & Matthew, A.T. (2010). Cost Benefit Analyses versus Referenda. *Journal of Political Economy*, 118(1), 156-187.

Prat A. (2002a). Campaign Spending with Office-Seeking Politicians, Rational Voters, and Multiple Lobbies. *Journal of Economic Theory*, 103, 162-189.

Prat A. (2002b). Campaign Advertising and Voter Welfare. *Review of Economic Studies*, 69(4), 999-1017.

Qvortrup, M. (2005) . A Comparative Study of Referendums. Manchester University Press

Santilli, P. C. (1983). The informative and Persuasive Functions of Advertising: A Moral Appraisal. *Journal of Business Ethics* 2, 27-33.

Shannon, C.E., & Weaver W. (1949), The Mathematical Theory of Communication. Urbana, IL: University of Illinois Press.

Shachar, Ron, & Barry Nalebuff (1999). Follow the Leader: Theory and Evidence on Political Participation. *American Economic Review*, 57, 637-660.

Schultz, C. (2007). Strategic Campaigns and Redistributive Politics. *The Economic Journal* 117(522), 936-963.

Smith D.A. (1994). Strategic Transmission of Costly Information. *Econometrica*, 62(4), 955-963.

Snyder, J. M. (1989). Election Goals and Allocation of Campaign Resources. *Econometrica*, 89, 525-547

Tanner, N. (2013). Optimal Delegation Under Uncertain Bias.

Tirole, J. (1988). The Theory of Industrial Organization. Cambridge, MA: MIT Press.

Weigold M.F. (2001). Communicating science: A review of the literature. *Sci Commun,* 23, 164–193

Wielhouwer, P.W. & Lockerbie B. (1994). Party Contacting and Political Participation, 1952-1990. *American Journal of Political Science*, 38(1), 211-229.