Location Models for Two Different Applications

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Abstract

There has been a growing interest in location problems for their wide use in many areas, such as passive optical networks and logistics networks. However, as the papers appear in different literature, researchers usually do not take advantage of their mutual findings. We propose to bridge the gaps and therefore to propose efficient solutions schemes for two different applications.

In the first application, our research goal is to investigate the FTTX (Fiber-to-the Home/ Premises/Curb) passive optical network (PON) for the deployment of broadband access. We focus on designing the best possible architectures of FTTX hybrid PONs, which embraces both Time Division Multiplexing (TDM) and Wave Division Multiplexing (WDM) technology. A hybrid PON architecture is very efficient as it is not limited to any specific PON technology, rather it is flexible enough to deploy TDM/WDM technology depending on the type (i.e., unicast/multicast) and amount of traffic demand of the end-users. We investigate the optimized covering of a geographical area by a set of cost-effective hybrid PONs. We propose a novel network design optimization scheme for greenfield deployment of a set of hybrid PONs, in which all significant constraints are taken into account, e.g., type of traffic, attenuation, choice of splitting equipment.

In the second application, we revisit the p-center location problem in the context of disruption events. We propose an optimized covering in the geographical area for a given number of customers and suppliers, ensuring each customer is assigned a primary supplier and a different backup supplier unless the primary supplier has a so-called fortified facility. However, the budget for facility fortification is limited and only few facilities can be fortified.

We design an optimization model under the assumption of single event disruptions, and estimate accurately the required facility capacities while taking into account a sharing of the backup resources.

We evaluate our proposed models and algorithms by a comprehensive set of numerical experiments, with some comparisons in each of these two applications. Conclusions are drawn in the last chapter.

Acknowledgments

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I remember the first time when I came to Montreal, professor Jaumard told me: "John, studying abroad is a challenge, and a way to discover new challenges, and new ways to study, meet other students who are eager to study and learn more." I will always try my best, overcome new difficulties and challenge my limits no matter when and where.

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Abbreviations

AWGs Arrayed Waveguide Gratings

BISAN Broadband Internet Subscriber Access Network

BSP Best Set of PONs

CFLP Capacitated Facility Location Problem

CO Central Office

CPMP Capacitated p-median Problem

CPON Composite Passive Optical Network

DSL Digital Subscriber Line

EPON Ethernet Passive Optical Network

FLP Facility Location Problem

FTTC/FTTH/FTTB Fiber-to-the-Curb/Home/Building

ILP Integer Linear Programming

ISP Internet Service Provider

ITU The International Telecommunication Union

LARNet Local Access Router Network

LRPON Long Reach Passive Optical Network

OLT Optical Line Terminal

ONU Optical Network Unit

OSPON Optimized Set of PONs

P2MP Point to Multipoint

P2P Peer-to-peer

PON Passive Optical Network

RT Remote Terminal

SLAPONS Equipment Selection L/A in a set of PONs

TDM PON Time Division Multiplexed PON

UFLP Uncapacitated Facility Location Problem

UWB Ultra Wide Band

WDM PON Wavelength Division Multiplexed PON

WLAN Wireless Local Area Network

Chapter 1

Introduction

This chapter begins by laying out the general background of Passive Optical Networks, Logistics Networks and Facility Location Problems in Section 1.1 and motivation of this thesis in Section 1.2. Key contributions of this thesis are described in Section 1.3. Finally, organization of the thesis is described in Section 1.4.

1.1 General Background

1.1.1 Evolution of Passive Optical Networks

Internet has experienced a tremendous increase in network traffic, consumers are using a number of broadband applications that are emerging everyday. With the large increase in traffic demand, it requires new robust underlying infrastructure. Therefore optical networks play a crucial role in the development of Internet by offering a high speed infrastructure to cope with the rapid expansion of high bandwidth.

Optical fiber enabled technologies can be viewed as the best solution for access networks to face the challenges of the worldwide increasing broadband demand. Optical fiber has already been deployed in the backbone networks and it is the latest broadband access network technology embraced by the Internet Service Providers (ISPs) to provide Internet access to residential and business customers.

Optical access networks, often termed as FTTX (Fiber-to-the Home/Premises/Curb), are viewed as the last step for the future all-optical network revolution. It has two design patterns: point-to-point (P2P) or point-to-multi-point (P2MP). In a P2P architecture, a dedicated fiber runs from ISP's Central Office (CO) to each customer in which high installation and maintenance cost of each individual fiber is a major economic barrier [14]. P2MP architectures, offering an economically feasible solution compared to their P2P counterpart, may be either active or passive [39]. The active optical networks and passive optical networks are two important types of systems that make FTTX broadband connections possible. An active optical network consists of a remote curb switch close to the neighborhood, a single fiber from the CO to a switch, and a number of short branching fibers from the switch to each end user. However, the whole active optical network needs electric power for its working, thus will increase the operational and maintenance cost. A passive optical network shares fiber optic strands for portions of the network. Powered equipment is required only at the source and receiving ends of the signal. Therefore, ISPs prefer the passive architectures as its the most cost-effective and easy-maintenance solutions for optical access networks.

Telecommunications companies use PONs to provide triple-play services including TV, VoIP phone, and Internet service to subscribers. The benefit of PONs is much higher data rates that are indispensable to Internet services. The low cost of passive components means simpler systems with fewer optical components that fail or require maintenance. The primary disadvantage is its shorter range, commonly no more than 20 km or 12 miles. But in recent years, there is a growing interest on Long-Reach Passive Optical Network (LRPON) as it can enable broadband access for a large number of customers in the access/metro area, while decreasing capital and operational cost. PONs are growing in popularity as the demand for faster Internet service and more video grows [24].

1.1.2 Logistics Networks

The logistics networks are referred to the entire chain of distribution centers and transportation of goods or services from the supplier to the final customers or users. In modern logistics systems, on time production and delivery, resilience supplier and limitation on inventory are the main concerns of enterprises. Logistics networks are the management of the flow of things between a origin point and a consumption point in order to satisfy requirements of customers.

The resources managed in logistics networks can include physical items or abstract items. The complexity of logistics networks can be modeled, analyzed,and optimized by dedicated simulation software. The minimization of the use of resources is a common motivation in logistics networks for import and export [1].

1.1.3 Facility Location Problems

Location problems consist of clients and a set of potential sites where facilities can be located. The objective of facility location problems is to find a place to locate a facility in order to minimize the total setup cost and the total cost of transportation between clients and facilities [18]. In this thesis, I focus on the p-center/p-median problem among the location problem. The objective of p-median problem is to determine the locations of p facilities and their assigned clients in order to minimize the average distance between clients and facilities. In a p-center problem, p number of service facilities are allocated to a number of demand nodes such that the maximum distance between a demand node and its corresponding service facility is minimized [63]. All models mentioned can be considered with or without demands of clients and capacities of facilities. When the demands of clients and the capacities of facilities are included in the model, the case is often called a capacitated case. In a capacitated case, each client has a certain demand to meet and the facilities have capacity restrictions, i.e., the total demands of clients assigned to a facility cannot exceed that facility's capacity [27]. Both uncapacitated and capacitated facility location problems are known to be NP-hard. Facility location problems can be considered on the discrete or continuous space. When facilities can be located at any place in a region, the problem is a continuous location problem. When facilities can be placed only at specific locations, the problem is a discrete location problem.

The p-median problem was introduced by Hakimi (1964) and it arises naturally in locating plants/warehouses to serve other plants/warehouses or market areas [75]. The uncapacitated p-median problem can be solved in polynomial time for fixed values of p but this problem is NP-hard for variable values of p [27]. Methods for solving p-median problems are similar to methods for solving facility location problems such as variable neighborhood search, simulated annealing, greedy heuristic, etc [11].

P-center problems have been of interest since their first appearance in 1964 [31]. Most of

p-center problems are NP-hard [19], exact algorithm may not be able to solve such problems. The way to solve p-center problems is similar to that used for p-median problems. Adaptation of p-median problem heuristics to solve p-center problems has been done using tabu search and variable neighborhood methods [60]. The local search heuristics was introduced to solve capacitated p-center problems.

In this thesis, we focus more on facility location problem (FLP) among location problem, there are two main categories in facility location problem: single-objective facility location problem and location problem with cost component and distance component. In the single-objective there are three main categories: Minisum, Minimax, and covering problem. Minisum problem contains two kinds of problems, p-median problem and problem with mutual communication. Minimax problem contains two kinds of problems, p-center problem and problem with mutual communication.

1.2 Motivation of Thesis

We investigate two different applications of facility location problems in this thesis. As far as we know, there are not many papers related to facility location problems that use Column Generation approach. In the first application, our research goal is to investigate the FTTX passive optical network (PON) for the deployment of broadband access networks such that the opportunities of optical fiber enabled technologies as well as of passive switching equipment can be optimized. Indeed, the deployment of FTTX PON is the most OPEX (operational expenditure)-friendly scenario, because it allows for completely passive access networks through minimizing the usage of electric power in the network.

Previously, most FTTX PON architectures were designed based on the principle of either time division multiplexing (TDM) technology or wavelength division multiplexing (WDM) technology. Comprehensive comparison shows that TDM-PON own the advantage of rapid network deployment and low cost, but in the respect of bandwidth, scalability and expansion, it's not as good as WDM-PON. Therefore, with the development of passive optical devices and other technologies, WDM-PON will become the contender of current mainstream TDM-PON technology. Currently, TDM-PON and WDM-PON have their unique advantages and disadvantages in different application scenarios. Solely using a PON technology contains varying degrees of problems. Therefore, HPON (Hybrid TDM/WDM PON) seems to be the best solution because it is compatible with TDM and WDM technologies, and support TDM-PON smooth transition to WDM-PON [5].

Consequently, new questions arise in the context of the access network evolution with respect to how FTTX deployments can be supported in a cost-efficient manner when considering office consolidation strategies, and the impact on network architectures and related technologies. This motivated us to focus on designing the best possible architecture of FTTX PONs, specifically hybrid PONs, built on the principle of time/wavelength division multiplexing (TDM/WDM) technology.

In the second application of the thesis, our research goal is to investigate the optimal covering of a geographical area for a given number of customers and suppliers, in order to serve all the end users with a given primary supplier and a different backup supplier in logistics networks.

Because of todays globalized threats that comes in addition to, e.g., labor disruptions or failures resulting from harsh weather conditions, there has been a renewed interest in resilient facility location. The main application of interdiction or protection model for logistics networks is to solve real world problems when there is a disruption. Disruption is the result of an event that causes an unplanned, negative deviation from the expected delivery according to organization objectives. In the context of facility location, disruption affects facilities so that some users need to be directed to other facilities. They may also affect the routes between users and facilities. Designing a resilient logistics network in presence of disruptions has gained a lot of attention in recent years. Design of reliable logistics networks to avoid disruption can be accomplished by improving existing facilities and also setting up a backup facility when a facility is under disruption.This motivated us to focus on disruptions affecting facility locations and design a capacitated reliable facility location system in presence of disruption.

1.3 Thesis Contributions

In the first application of this thesis, we investigate the optimized covering of a geographical area by a set of cost-effective hybrid PONs. We also focus on the greenfield deployment of a single hybrid PON. Moreover, we investigate the maximum signal power loss experienced at end users' premises. We scrutinize the selection of the switching equipment. We also study the impact of multicast traffic of the deployment cost of hybrid PONs. Finally, we determine the best set of PON networks along with their cascading architecture, type and location of their switching equipment while satisfying the network design constraints such as the number of output ports of the switching equipment and maximum allowed signal power loss experienced at each end users' premises.

In the second application, we focus on minimizing the deployment cost with the optimal covering in the geographical area for a given number of customers and suppliers, in order to serve all the end users in a given primary supplier and a different backup supplier. As far as we know, we are the first one who consider capacitated reliable facility location under disruption. Moreover, we also satisfy the budget for the fortified supplier and select the primary supplier with less failure probability. We also determine to optimize the first phase model and imply the backup sharing in the next phase. Under the assumption that at most one supplier will fail at a time, and that we have the time to fix the failure before another one occurs, backup resources of the same users can be shared. So we avoid the overestimated capacity requirement and further optimize the model and the deployment cost in this logistics networks.

There are several papers published for this thesis:

[1] Brigitte Jaumard and Shibo Song, "Dimensioning hybrid PONs", *International Conference on Optical Networks Design and Modeling(ONDM)* , pp. 16-21, 2015.

[2] Brigitte Jaumard, Mostafa Badakhshian and Shibo Song, "Capacitated p-Median Facility Location Problem under Disruption", *The CORS/INFORMS International Conference*, June, 2015.

Submitted for publication in an international journal:

[3] Brigitte Jaumard, Shibo Song and Rejaul Chowdhury, ”Design and Dimensioning of Hybrid PONs”

[4] Brigitte Jaumard, Mostafa Badakhshian and Shibo Song, ”Capacitated Reliable Facility Location in Presence of Disruption: A Column Generation Approach”.

1.4 Organization of Thesis

The thesis is organized as follows. In Chapter 2, we introduce the general background of Facility Location Problem, Passive Optical Network and Logistics Networks. In Chapter 3, we summarize the literatures related to network planning and the placement of switching equipment in a hybrid PON, location problem with p-median/p-center problem and the location problem in logistics networks.

For the first application in Chapter 4, in Section 4.1, we provide a concise statement of the PON deployment problem and an outline of our proposed 3-phase scheme and 2-phase scheme. For the 3-phase scheme, the variables and two phases optimization process are illustrated in Section 4.2. The results obtained from these two phases are used to generate several potential hybrid PON hierarchies by fixing the location of the switching equipment. For the 2-phase scheme in section 4.3, the first phase we decide the placement of passive equipment and the cluster of ONUs in order to generate several potential hybrid PON hierarchies. In Section 4.4, it is the third phase of 3-phase scheme and second phase of 2-phase scheme. It takes care of the switching equipment selection with minimum network deployment cost for each potential hybrid PON hierarchy while satisfying the traffic request. Computational results and analysis of the first application are presented in Chapter 5 in order to validate and compare the proposed 3-phase/2-phase scheme.

For the second application in Chapter 6, in Section 6.1, we provide a concise statement of the reliable facility location model and an outline of our proposed capacitated reliable facility model and shared backup facility model. For the reliable facility location model in Section 6.2, the variables and column generation formulation is illustrated in Section 6.2.1,6.2.2. The results from reliable facility location model can be used for the shared backup facility model to reduce the extra backup resource which will show in section 6.3. Computational results and analysis of the second application are presented in Chapter 7.

Conclusions of the thesis and future work are drawn in chapter 8.

Chapter 2

Background

In this chapter, an overview of location problems is provided in Section 2.1. Then, PONs and its significant properties are described in Section 2.2. Finally, the details of location problems under disruption with logistics networks and its challenges are discussed in Section 2.3.

2.1 Facility Location Problem

2.1.1 Introduction

The facility location problem or facility location analysis, is a branch of operation research and computational geometry concerned with the optimal placement of facilities in order to minimize transportation costs while considering other factors like considering government policy or avoiding placing dangerous materials near uptown. The concept of classical location problem can be mapped onto the network dimensioning problem and networks equipment placement problem. For instance, we can consider the scenario of a supply chain of a complex logistics system which consists of two parts: production system and distribution system [26].

The development of a new facility is often an expensive and long-term investment. Before a facility can be purchased or constructed, enterprises identify good locations and allocate large amounts of capital. Therefore, facilities which are located today are expected to remain in operation for an extended time. Determining the best locations for new facilities is thus an

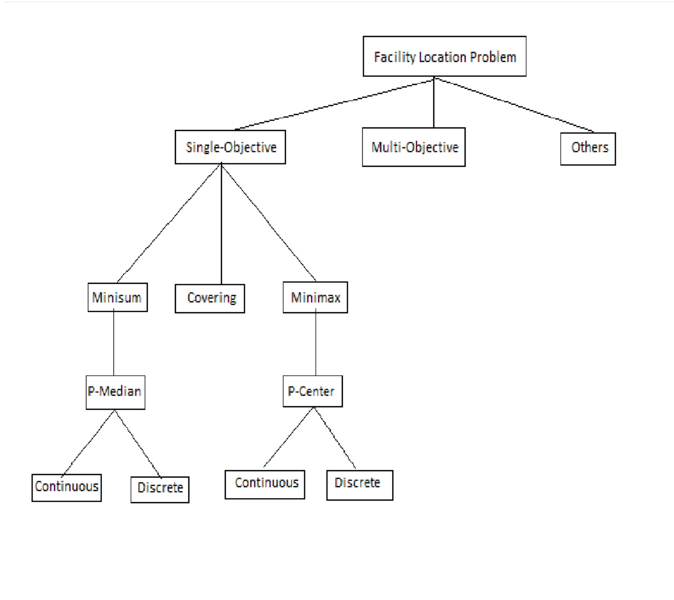


Fig. 2.1: Famility Tree for Facility Location Problem [63]

important strategic challenge [63]. Extensive literature has developed a broad-based interest in solving this challenge. Many mathematical models have been developed by operation research practitioners to represent a wide range of location problems. Several different objective functions have been formulated to make such models amenable to numerous applications.

The classical Single-Objective Facility Location Problem in Facility Location Problems can be categorized into three groups: (i) Minisum problems, (ii) Covering problems, and (iii) Minimax problems [63]. Minisum problems can be formulated as the minimization of the sum of weighted distance between the new facility and the other existing facilities. As mutual distance between demand points and facility locations decreases, facility accessibility and the location's effectiveness decreases. An extension of the Minisum problem is the p-median problem which can be defined as the determination of optimum locations of p facilities so that the average demand-weighted distance between demands and facilities is minimized.

Covering problems are intended to cover customers or demand nodes such that the distance between a customer and its closest facility is less than a specified distance. Two covering problems which illustrate the distinction are the location set covering problem and maximal covering problem. The location set covering problem can be formulated as to minimize the cost of facility locations such that a specified level of coverage is obtained. On the other hand, a maximal covering problem can be expressed as to maximize the number of demand nodes covered while weights all demand points equally without regarding to the size of the demand present.

The goal of Minimax problems is to minimize the maximum distance between the new facility and any existing facility. *P*-center problem is an extension of Minimax problems, it is also known as the minimax problem. "The p-Center problem consists of locating p facilities and assigning clients to them in order to minimize the maximum distance between a client and the facility to which he or she is allocated [61]." P-center problem have been widely investigated for solving different kinds of location problems. A *p*-center problem can be solved either heuristically or exactly. Solving such a problem exactly is a very difficult one as it is one of the best-known NP-hard location problems [63]. That's why, in most cases, it is solved heuristically.

In our research work, we adopt the concept of *p*-center/*p*-median problem and combine it with the concept of maximal covering problem. In our case, facilities can be located at any place in a region, so we consider a continuous version of the capacitated facility location problem and its application to the *p*-center/*p*-median problem. In the Continuous Facility Location problem, the value of a solution is the total cost to connect the clients to the respective facility plus the cost to open facilities. Therefore, the goal of my project is to minimize the the distance and the facility setup cost, in which the distance part consist of *p*-center and *p*-median problem. Also, based on the objective, we need to achieve the goals of selecting the facility location, and assignment of customer to facilities.

2.1.2 Uncapacitated Facility Location Problem

The Uncapacitated Facility Location Problem (UFLP) is one of the most widely studied discrete location problems, applications of this problem arise in a variety of settings [41]. It involves locating an undetermined facilities in order to minimize the sum of the (annualized) fixed setup costs and the variable costs of serving the market demand from these facilities [77]. In the uncapacitated facility location problem, each node i is associated with a facility cost f_i, which reflects the cost of opening a facility at this node. The problem is to open a subset of facilities so as to minimize the sum of facility costs and the service cost, which is defined to be the sum of distances from each node to its closest open facility. The p-median problem differs in that exactly p facilities be opened, and there is no facility cost, only service cost. The p-center problem differs from the p-median problem in that the service cost is defined to be the maximum distance (rather than the sum of distances) from any facility to its closest open facility. All of these optimization problems are NP-hard, and polynomial time approximation algorithms have been studied [56].

The Uncapacitated Facility Location Problem and its many variations have been widely studied and its applications arise in a variety of settings, i.e., *(i)* identifying multiple low-dimensional subspaces in high-dimensional data (Li, 2007; Lazic et al., 2009), *(ii)* computational biology (Dueck et al., 2008), *(iii)* self-configuration in wireless sensor networks (Frank & Romer, 2007) [41].

2.1.3 Capacitated Facility Location Problem

Capacitated Facility Location Problem (CFLP) is generalization of the Simple Plant Location Problem. It is also a variant of the Facility Location Problem(FLP), which includes capacities for the facilities [2]. With the inclusion of the capacities, an open facility with the least cost source for a demand node may not be able to serve any of the demand at that node. The capacities of the facilities and the demand at each of the demand nodes have been assumed to be deterministic parameters.

In Capacitated Facility Location Problem, different potential facility location have different fixed costs to locate a facility, we do not know the optimal number for facilities to open and demand they can serve. These factors are all restricting the selection of facility location. Thus makes the CFLP a complex problem that is difficult to solve. There are many literatures concerning the development of new algorithms for solving CFLP. Akinc and Khumawala (1977) first developed branch-and-bound procedures for this problem using linear programming relaxation. The cross-decomposition algorithm and the Lagrangean-based approach are among the most effective techniques that were subsequently devised for solving the CFLP [77]. More recently Greedy Heuristics, Tabu Search and Genetic Algorithms have been proposed to solve the CFLP. [70]

2.2 PON

2.2.1 Introduction

Passive Optical Network (PON) is the latest broadband access network technology embraced by the Internet Service Providers (ISPs) to provide Internet access to the residential and business customers. In a typical PON architecture, there is an optical line terminal (OLT) at the central office (CO) that connects the optical access network to the metro backbone, a number of optical network units (ONUs) at the end users' premises, and one or multiple passive switching equipment placed in a remote terminal (RT) between the OLT and the ONUs. In a typical PON, the presence of only passive elements from the OLT to the ONUs makes it relatively fault tolerant and decreases its operational and maintenance costs once the infrastructure has been laid down.

PONs are usually built following either time sharing principles known as time division multiplexed PON (TDM PON) or spectrum sharing principle recognized as wavelength division multiplexed PON (WDM PON) [76]. In a TDM PON, the RT consists of passive optical power splitters. In a WDM PON, the RT consists of arrayed waveguide gratings (AWGs). The characteristic of a splitter is different than that of an AWG as the former equipment splits the optical power whereas latter equipment multiplexes/de-multiplexes optical wavelengths.

Currently PONs are the most attractive technology to solve the "last one kilometer" question in access network and to achieve FTTX. The advantages of using PONs in access networks are numerous: *(i-a)* PONs can avoid electromagnetic interference from external devices and lightning effects, reducing the failure rate in lines and external equipments, improving system reliability. *(i-b)* PONs are easy to upgrade to higher bit rates or additional wavelengths. *(i-c)* Operating in the downstream as a broadcast network, PONs can provide triple-play services including TV, VoIP phone, and Internet service to subscribers. *(i-d)* PONs provide higher bandwidth due to deeper fiber penetration, offering gigabit per second solutions [40].

2.2.2 Overview of TDM/WDM PON Technology

In a time-division multiplex (TDM) PON, downstream traffic is handled by broadcasts from the OLT to all connected ONUs, a single wavelength channel is used along the downstream direction for broadcasting the same signal from the OLT to all ONUs by utilizing a passive optical power splitter. TDM PONs can be implemented either by a space division duplex approach in which two separate fibers are used for upstream and downstream communications or by a coarse WDM (CWDM) approach in which the upstream and downstream wavelengths are multiplexed on the same fiber. Power loss of the optical transmission medium decide the physical distance and splitting ratio in a TDM PON. In order to reduce the cost of an access network, power splitting allows the distribution of the cost from the OLT to ONUs and the reduction of the fiber distance in the field [14]. During downstream transmission, traffic is broadcast to all ONUs, each ONU inspects the headers, extracts the packets addressed to it, and discards the packets destined to other ONUs. While in the upstream direction an arbitration mechanism is required so that only a single ONU is allowed to transmit data at a given point in time because of the shared upstream channel [71]. In the special case, an ONU will fill the time slot with an idle signal if there is no packet to send. Although the broadcast transmission in the downstream and the arbitration mechanism transmissions in the upstream limit the bandwidth of each user, the resulting low transceiver cost plays a significant role to justify the trade off between the available bandwidth and economic feasibility [23].

With the standardization of TDM PONs, a cost-effective access technology based has been developed. However, upgrading optical access networks will still be a challenge when the demand for even higher data rates exceed the existing network capacity. TDM PON architecture is bandwidth limited as it has only one wavelength for downstream data and one for upstream data, thus limiting the average bandwidth per user to a few tens of megabits per second which can not fulfill the requirements of high capacity transmission [25]. These problems can be mitigated with wavelength-division multiplexing (WDM) PONs. "WDM technology has been considered an ideal solution to extend the capacity of optical networks without drastically changing the existing fiber infrastructure [5]."

In a traditional WDM PON, ONUs are assigned individual wavelengths which provides higher bandwidth to each ONU. Benefits of WDM technologies are manifold such as high performance, increased network capacity, flexibility with respect to network scalability. Besides, since ONUs are separated via physical wavelengths, aspects of network privacy/security and isolation of service should be accounted for [29]. However, it requires wavelength management of the optical transmitters and expensive wavelength specified light sources [43]. WDM PONs with broadband amplification, can also support enhanced distances in the range of 100 km which could play an crucial role in future metro access and backhaul convergence scenarios [21]. Although WDM PONs define an ideal solution for future optical network revolution, its commercial deployment will become more practical and viable in the near future as the bandwidth requirement is increasing and the cost of optical components is slowly decreasing [14].

No matter WDM PONs or TDM PONs, they all hold their unique advantages. Therefore several proposals have demonstrated the feasibility of combining WDM PONs and TDM PONs to finds a compromise between capacity and cost while offering centralized management and bandwidth allocation [10] [6]. That is Hybrid PON Technology.

2.2.3 Overview of Hybrid PON Technology

A hybrid PON can be built by combining the architectures of both TDM PON and WDM PON networks. On the physical layer of a hybrid PON, both TDM and WDM transmission (downstream and upstream) channels are utilized in the same PON [66]. A hybrid PON facilitates better bandwidth usage by adding a TDM layer on top of the WDM layer [57]. A hybrid PON architecture is very efficient as it is not limited to any specific PON technology, rather it is flexible enough to deploy TDM/WDM technology depending on the type (i.e unicast/multicast) and amount of traffic demand of the end-users. The advantages of a hybrid PON are two fold: *(i)* it can offer increased data rate to each user by employing WDM technology, *(ii)* it can provide flexible bandwidth utilization by employing TDM technology.

There are several variations of hybrid PON technologies, e.g., *(i-a)* A Hybrid WDM-TDM PON architecture with a RSOA-based colorless ONU where the downstream wavelength is remodulated to generate upstream signals (Payoux *et al.* 2006) [64]. *(i-b)* A Hybrid WDM/TDM PON serving 128 subscribers with wavelength-selection-free transmitters, which is presented by cascading 16 AWGs and 8 splitters (Shin *et al.* 2005) [69]. *(i-c)* A next-generation Hybrid WDM-TDM PON architecture which called Stanford University SUCCESS HPON. It based on a ring plus distribution trees topology, fast centralized tunable components, and novel scheduling algorithms (An *et al.* 2005) [5].

2.3 Location Problem under Disruption in Logistics Networks

2.3.1 Introduction to Logistics Networks

Logistics networks are the management of the flow of things between a origin point and a consumption point in order to satisfy requirements of customers. The resources managed in logistics can include physical items, such as food, materials, animals, equipment and liquids, as well as abstract items, such as time, information, particles, and energy. A logistic network with physical items usually involves the integration of information flow, material handling, production, packaging, inventory, transportation, warehousing, and often security. The minimization of the use of resources is a common motivation in logistics for import and export [1].

A variety of models and solution methodologies has been proposed and analysed in logistics networks design dates back to the 19th century. Research in logistic network design is mostly divided in two approaches: maximizing profits or minimizing environmental impact [62]. In both approaches the main questions are to determine the type, number, location and size of new facilities, as well as the divestment, displacement or downsizing of facilities. The general objective in business logistics is to minimize the total logistics cost which constrained by facility capacity and required customer service level. However, a number of actors will influence logistics costs and corresponding environmental impact. Suppliers, storage, consolidation, selling, incineration, consumers and transportation are the main players. Also, when designing the logistics network for special circumstances, different objectives may have to be considered.

Logistic network designing have two main part: *(i)* Facility Location Problem. *(ii)* Route

Location Problem. In this thesis, we concentrate more on the investigation of facility location problem. Facility location decisions must obviously be made when a logistic network is started from scratch. They are also required as a consequence of variations in the demand pattern or spatial distribution, or following modifications of materials, energy or labour cost. In particular, location decisions are often made when new products or services are launched, or outdated products are withdrawn from the market. In continue, we are looking at disruption in Facility Location Problem part.

2.3.2 Disruption in Decision-Making Environments

Disruption is an event, whether anticipated (e.g., a labour strike or hurricane) or unanticipated (e.g. a blackout or earthquake), which causes an unplanned, negative deviation from the expected delivery of products or services according to the organization's objectives [4]. Decisions about facility location are costly and difficult to reverse. The impact of decisions will remain for a long time. Costs, demands, travel time, and other inputs to classical facility location models may be under disruption. This has made the development of models for facility location under uncertainty a high priority for researchers in both the logistics and stochastic/robust optimization communities. Indeed, a large number of the approaches that have been proposed for optimization under disruption have been applied to facility location problems [74].

Decision-making environments can be divided into three categories: (*i*) certainty; (*ii*) risk; and (*iii*) uncertainty. In certainty situations, all parameters are deterministic and known, whereas risk and uncertainty situations both involve random disruption. In risk situations, there are uncertain parameters whose values are governed by probability distributions that are known by the decision maker. In uncertainty situations, parameters are uncertain, and furthermore, no information about probabilities is known.

Chapter 3

Literature Review

3.1 Introduction

Some of the studies on the logistics networks under disruption and the placement of switching equipment in PONs have exploited the resemblance with the location problem. In this chapter, we first present a literature review on the Network Planning and Placement of Equipment in PONs in Section 3.2 and explain how far the resemblance goes. While there are definitively some resemblance, there are also some differences such as the attenuation constraints which depend on the type of switching equipment and which limit the reach of the PON networks. Subsequently, we describe the Reliable Facility Location Models subject to Disruption in Logistics Networks in Section 3.3. After this, I will summarize the Location Problem and p-median/p-center Problem in order to see the similarities and differences in these two applications. This will appear in Section 3.4

3.2 Network Planning and Placement of Equipment in PONs

Several heuristics and ILP formulations have been reported for network planning problem in PONs.

Li and Shen [44] investigate the problem of network planning for PON deployment. The authors remark that heuristics are the most practical solution to solve this optimization problem as both subproblems are NP-complete. Two heuristic algorithms are considered in their study. The first one is the extension of the benchmark sectoring algorithm. The second heuristic is Recursive Allocation and Location Algorithm (RALA) which has been derived from Cooper's algorithm [17]. In this thesis, both sectoring and RALA schemes can not determine the optimal location of splitters such that the distance between an ONU and its associated splitter is minimized. Moreover, the authors do not investigate the compromise between one level networks with maximal signal splitting and two or more levels with reduced signal splitting.

Li and Shen [45] formulate a mathematical optimization model to minimize the deployment cost of a single-stage architecture based PON. Their proposed optimization model is non-linear. Moreover, the authors assume that the cost factor of a splitter has a linear relationship with the number of output ports of the corresponding splitter (which is not true in practice). As their proposed model is not tractable in practice, experiments are conducted with the heuristic proposed in [44].

Lee *et al.* [42] examine design problem for the deployment of PONs by analyzing the location-allocation problem of splitters. They formulate the single splitting problem (SSP) and the distributed splitting problem (DSP) in which SSP includes single-level splitters and DSP multi-level splitters. The optimality gaps (difference between lower and upper bounds) of their proposed formulation are quite large (up to 81%), so it is quite difficult to assess the quality of their solutions.

Later, Kim *et al.* [37] propose a relaxation of the objective function proposed in [42] and, with the help of valid inequalities and a local search heuristic, they reduce the optimality gap between the solutions of their LP and ILP formulations, and therefore obtained a better estimation of the quality of their solutions.

Hajduczenia *et al.* [30] implement clustering techniques to group the subscribers to be served with a separate PON network and then apply genetic algorithm to find the optimum path distribution. But their proposed technique does not specify how to find the optimal placement of ONUs and PSCs (splitters) in a given network environment.

Mitcsenkov *et al.* [59] propose a heuristic solution to address single stage TDM PON topology planning minimizing deployment cost along with operational aspects. They also propose an ILP to serve as a reference for smaller cases so that the performance of their heuristic can be compared with the optimal solution obtained by the ILP. But the proposed ILP does not optimize the location of the splitting nodes, it only connects the customers with the given splitting nodes. In the formulation of the ILP, the distance between the CO and the customers are not taken into account which is mandatory to compute the attenuation of a splitter.

Kokangul and Ari [38] develop optimization models for multi-hierarchy (two-stage) PON planning problem. First, they construct a large nonlinear mathematical model. Because of nonlinearity and NP-completeness, this model could not be solved. Then they propose a genetic algorithm (GA) based heuristic to solve the planning problem. Finally, they linearize the constructed nonlinear problem and obtain the optimal solution for a very small size problem instance. Exploiting GA and mathematical modeling, they optimize the positions of the primary and secondary nodes, the split levels of the nodes as well as assigning customers to secondary nodes and secondary nodes to primary nodes. Their proposed model has very limited capability as it considers only four possible primary node locations, twenty possible secondary node locations, and twenty-eight customers. In their proposed multi-hierarchy planning scheme, each secondary node can serve maximum eight customers and each primary node can serve maximum sixteen customers which implies that each PON can handle only sixteen customers. Moreover, the selection of split level of the primary and secondary nodes is also very much restricted.

Xiong *et al.* [79] propose a nonlinear ILP model for designing TDM PON networks. Their proposed model is formulated to determine the optimal number and locations of the OLT. Due to its nonlinearity, the proposed model can not be executed. Then they propose a partitioning algorithm with the same objective as the ILP model. But the objective function does not bear any significance for the designing of PON network. The authors consider single stage TDM PON in which different OLTs are situated in different locations and each OLT is connected to a single splitter that is connected to a number of subscribers in turn. But, in practice, the OLT is located at a single location, i.e., at the CO. Again, their proposed algorithm does not determine the location of the splitters. The authors do not take into account unicast/multicast traffic. They just consider total amount of traffic required by each ONU.

Roka [66] investigate the designing of next generation PON (NG-PON)networks using the hybrid PON (HPON) network configuration. He builds a simulation tool to select the environment for the HPON configuration and its capabilities. The tool is created in Matlab 7.0 and Visual C++ 6.0 which includes graphical interface to insert the input parameters of the HPON. This tool provides heuristic solution for single stage PON and determines the number of required splitters, AWGs, ordinary lasers, tunable lasers, receivers based on the number of total subscribers and the capacity of the hybrid network. But the author does not describe the algorithm of the simulation tool. His created tool, at best, can serve as an approximation model as it considers that all ONUs are located at equal distance from the OLT which is very unrealistic. Moreover, the simulation tool neither takes into account the unicast/multicast traffic while selecting the splitters/AWGs nor determines the optimized location of these switching equipment for the HPON.

Recently, significant amount of research activities have been noticed to investigate different aspects of hybrid PONs. Mahloo *et al.* [57] investigate the design of multi-stage hybrid PONs. They compute the capital expenditure (CAPEX) for different architectures of hybrid PONs. Their investigated architectures consist of an AWG in the remote terminal 1 (RT1) and a number of splitters in the remote terminal 2 (RT2). They experiment with different number of output ports for the AWG as well as varying number of output ports for the corresponding splitters. But the authors do not propose any generalized optimization model or heuristic solution to calculate CAPEX of a hybrid PON.

As a summary of all the studies reviewed in this section, we note that there is no study investigating the placement of both splitters and AWGs in a given hybrid PON network. In our research work, we plan to focus on the placement of splitters/AWGs based on the user density and required bandwidth of individual ONUs. We also propose to select the switching equipment depending on the type (unicast/multicast) of traffic demand. In our research, we consider two types of switching equipment: *(i)* splitters, *(ii)* arrayed waveguide gratings (AWGs). It should be noted that splitters are best suited for multicast traffic, whereas AWGs are appropriate for unicast traffic. Again, splitters are economically feasible switching equipment but they are badly susceptible to signal power loss with respect to the number of output ports. While selecting the switching equipment, unicast/multicast traffic together with the signal power loss experienced by the corresponding equipments play a vital role. A splitter may be selected to satisfy multicast requests with the condition that the maximum allowable signal power loss is satisfied. On the contrary, an AWG can be chosen either to serve unicast requests or to satisfy the signal power loss constraint. In this thesis, we investigate the maximum signal power loss experienced at the end users' premises. We scrutinize the selection of the switching equipment. We also study the impact of multicast traffic on the deployment cost of hybrid PONs.

Our proposed solution scheme will determine the optimal number of hybrid PONs to cover all ONUs (i.e., aggregated end users) in a neighborhood. The solution scheme will also determine the optimal coverage of each hybrid PON.

3.3 Reliable Facility Location Models subject to Disruption in Logistics Networks

In this section, we review the literatures related to facility location under disruption and the solution methods on reliable facility location under disruption.

We first summarize the papers related to classical facility location problem. The methods to solve this problem are reviewed by Snyder [72]. In that paper, they review the literature on stochastic and robust facility location models. They first illustrate variety of approaches for optimization under uncertainty that have appeared in the location problem, then they provided examples from the more general logistics literature for which examples in facility location are not available.

Louveaux *et al.* [55] investigate how the uncapacitated facility location problem can be transformed into a two-stage stochastic program with the uncertainty on demand or transportation. They present a dual-based procedure and indicate how the dual-descent and primal-dual adjustment previous procedures.

Averbakh *et al.* [8] consider single-facility location problems on a network with uncertain edge lengths which requires a robust (minmax regret) solution. Their paper indicates that robust single-facility location problems are strongly NP-hard, in contrast with the problems with only node weights uncertainty which are polynomially solvable.

Now we review the papers starting with those which used the concept of fortification, or equivalently interdiction. Losada *et al.* [53] introduce a stochastic interdiction problem for median systems in which the operational state of the system's disrupted elements is uncertain. In this paper, single and multiple disruptions are considered in the facility location problem.

Liberatore *et al.* [49] consider the problem of optimally protecting a capacitated median system with a limited amount of protective resources subject to disruptions. The model optimizes protection plans in different level of disruption as partial and complete. Their model is also a general framework for fortification problems in the context of location analysis, as it includes uncapacitated facilities and single-target disruptions as special cases.

Atoei *et al.* [7] propose a reliable capacitated supply chain network design model by considering random disruptions in both distribution centers and suppliers. Their proposed model determines the optimal location of distribution centers (DC) with the highest reliability, the best plan to assign customers to opened DCs. They introduce the assumption that a disrupted DC and a disrupted supplier may lose a portion of their capacities. Thererfore in their study, random disruption occurs at the location, capacity of the distribution centers and suppliers.

The objectives of facility location under disruption are to minimize the impact of disruption. It can be minimizing the worst-case impact of disruption or worse-case disruption scenario (e.g., Losada *et al.* [53]). It can be facility protection or fortification based on the first investment (Church and Scaparra [16]). Other objectives proposed by authors are minimizing the recovery of disrupted facilities (*e.g.*, Liberatore *et al.* [47]).

The most used method to solve the proposed models are Bender decomposition (Azad *et al.* [9],losada *et al.* [54]), Lagrangian relaxation (Snyder and Daskin [73]), pre−processing techniques based on the valid lower and upper bound, and heuristics methods(Liberatore *et al.* [48]).

We now review the papers dealing with the backup facility location. Lim *et al.* [50] considers for each user there is a primary facility and a backup facility or a layer of backup facilities. Also there is no capacity constraint for potential facilities. They also considered facilities selection which divided facilities as unreliable and another that is reliable. Different size of problem is considered by authors. Li *et al.* [46] solved the problem with size of 150 demand nodes and 30 to 50 supplier.

There are various studies on the reliable facility location problems in logistics networks. Lorena and Senne [51] adds capacity constraints on the facilities in the capacitated p-median problem (CPMP), but has not yet been studied in the context of disruptions. To our knowledge, facility location under uncertainty can be divided by to two main categories: *(i)* uncertainty regarding customer demand, and travel time(cost) between facility locations and customers, *(ii)* uncertainty pertaining to availability of facility locations.

The facility location problem under uncertainty for category (i) are well reviewed by Snyder [72]. In this case, problems are categorized into three main categories. *(i-a)*, when there are certainties on demand or travel time values. *(i-b)*, when there are uncertainties on demand or travel time whose values are governed by probability distributions that are known by the decision maker. *(i-c)*, when demand or travel time is uncertain, and furthermore, no information about probabilities is known. In the context of *(ii)*, authors have worked along two directions for the design of resilient facility location models: *(ii-a)* fortification of a subset of facilities subject to some number or budget constraints (e.g., Scaparra and Church [68]) and *(ii-b)* establishing some backup facilities (e.g., Lim *et al.* [50]).

There are not much papers related to facility location problems that use the column generation approach. In our research work, we plan to focus on minimizing the deployment cost with the optimal covering in the geographical area for a given number of customers and suppliers, in order to serve all the end users in a given primary supplier and a different backup supplier. As far as we know, we are the first one who consider capacitated reliable facility location using backup facility under disruption. Moreover, we also satisfy the budget for the fortified supplier and select the primary supplier with less failure probability. We also determine to optimize the first phase model and imply the backup sharing in the next phase. Under the assumption that at most one supplier will fail at a time, and that we have the time to fix the failure before another one occurs, backup resources of the same users can be shared. So we avoid the overestimated capacity requirement and further optimize the model and the deployment cost in this logistic network.

3.4 Location Problem with p-Median/p-Center Problem

The facility location problem and the route location problem are two directions in the network location problems. In this thesis, we focus on the facility location problem (FLP), and there are two main categories in FLP: (i) single-objective (ii) multi-objective. In the single-objective there are three main categories: Minisum, Minimax, and covering problem. The Minisum problem contains p-median problem and problem. Minimax problem contains p-center problem. The p-center problem can be considered as one of the locating center either to minimize a maximum loss or to provide good service. The p-median problem can be considered as minimizing the average distance between the customer points to their facility points in p opened facilities.

The p-Center problem was first formulated by Hakimi [3]. Subsequently, a number of solution procedures have been suggested.

Minieka [58] considered the unweighted case on a general network and showed that the problem can be reduced to a computationally finite one. As for the weighted case, Christofides and Viola [78] gave a solution procedure which relies on solving a sequence of r-cover problems with successively increasing values of r.

Kariv and Hakimi [35] showed that the p-Center problem on a general network is NP-hard. They also showed that the weighted case can be reduced to a computationally finite one. Based on this finiteness property they gave an algorithm whose complexity is
$O[|E|^p(n^{2p-1})(logn)/(p-1)]$.

Hsu and Nemhauser [34] showed that finding an approximate solution to the vertex restricted p-Center problem whose value is within either 100% or 50% respectively, of the optimal value is NP-hard.

A recent paper for analyzing p-Center algorithms given in [15] [78] [33] [35], shows how these algorithms fit into a comparative framework.

As for the p-Median problem, it arises naturally in locating plants/warehouses to serve other plants/warehouses or market areas. The problem is also motivated by ReVelle, Marks and Liebman [65] as an example of a public sector location model where vertices represent population centers and facilities represent post offices, schools, public buildings and the like: weights are typically proportional to the amount of "traffic" between medians and vertices.

Goldman [28] generalized the result to the case of a "two-stage" commodity. More specifically, one distinguishes a vertex as being a source or a destination. Hakimi and Maheshwari [32], in response to Goldman's conjecture, proved the vertex optimality result for the case of multiple commodities that go through multiple stages with the cost of transport from one stage to the next a concave nondecreasing function of the distance.

Wendell and Hurter [80] considered another form of the problem where the transportation cost functions are permitted to differ from edge to edge. The transport cost on any edge is a nondecreasing concave function of the distance. They proved that it is sufficient to consider the vertices of the network under such a structure. Furthermore, they obtained the conditions under which it is necessary for the solution to occur at the vertices.

Kariv and Hakimi [36] showed that the p-Median problem on a general network is NP-hard. For the case of tree networks, however, algorithm of polynomial complexity have been developed. Matula and Kolde suggested an $O(n^3p^2)$ algorithm for finding the p-Median of a tree network. For general networks, a number of solution procedures have been developed recently, all based on the vertex-optimality result. Their common aspect is that they all confine the search to vertex locations. The solution procedures based on mathematical programming relaxation and branch-and-bound techniques.

Lorena *et al.* [52] presented a column generation approach to capacitated p-median problems. In their paper, the new approaches integrate the traditional column generation to the Lagrangean/surrogate relaxation context to solve a capacitated p-median problem in real case. Their overall column generation process is accelerated, even when multiple pricing problem is observed.

As a summary of all the studies reviewed in this section, we note that there are not many papers using column generation approach to solve p-center/p-median problem. So in this thesis, I provide detailed column generation formulations for solving two different location models which can increase the processing speed.

Chapter 4

Optimization Models for PON

4.1 Multiple PON Deployment: Problem Statement and Optimization Process

4.1.1 Problem Statement

We propose to investigate the Multiple PON Design problem, defined as follows. It corresponds to the greenfield deployment of multiple TDM/WDM PONs in a given geographical area, i.e., determining a cost-effective set of PONs in order to serve the ONUs covering all the end users of the geographical area. It means partitioning the geographical area into a number of sub-areas where each sub-area is covered by a single PON. The network topology of each PON has to be determined with the intention of minimizing the overall network deployment cost based on the location of the OLT and the ONUs while granting all traffic demands. The network topology is characterized by the selection, location and cascading architecture of splitters or AWGs, which allocate a switching equipment to each group of ONUs in a PON. We allow flexible multi-stage cascading architectures for the various PONs. As shown in Figure 4.1, we assume all equipment to be distributed on two levels such that all ONUs are connected to either the first or the second level switching equipment and all 2nd level equipment are connected to the single 1st level equipment, which is itself connected to the OLT.

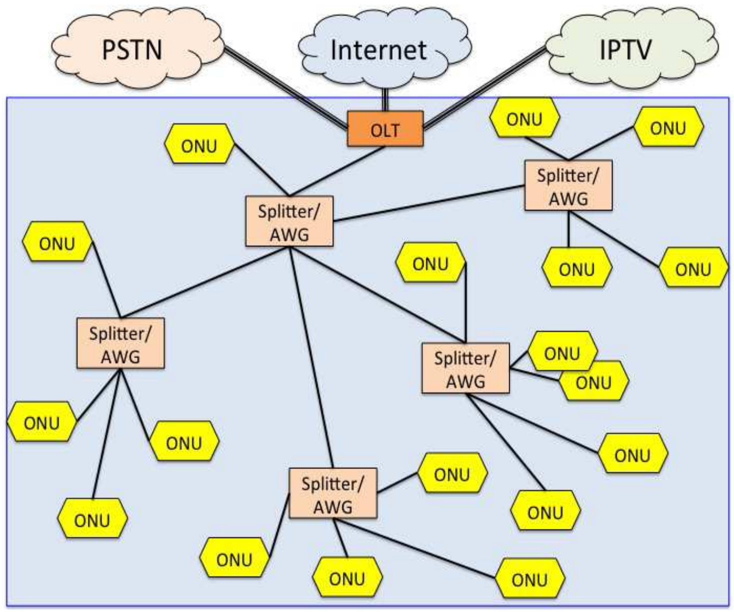


Fig. 4.1: Multi-stage PON

The input parameters of Multiple PON Design problem contain: *(i)* the location of the OLT and of the ONUs together with the distances (not necessarily geographical ones) of the ONUs to the OLT, *(ii)* the set of potential/candidate equipment locations together with their power loss depending on their number of output ports and their distances to the ONUs and the OLT, and *(iii)* the unicast/multicast traffic demand matrix (normalized values with respect to the transport capacity of the wavelengths). The output parameters comprise the most effective set of selected PONs with the locations, types and number of ports of the switching equipment (whether splitter or AWG) along with the PON cascading architecture. We limit to $\underline{N}_o$ and $\overline{N}_o$ the minimum and maximum number of ONUs that can be connected to a given passive switching equipment, respectively.

The overall objective corresponds to the minimization of the network deployment cost (i.e., initial infrastructure installation cost). Infrastructure installation cost is composed of the price of the switching equipment (splitter/AWG) and the optical fiber cables (including the cost of trenching and laying fibers). We denote by COST^{FT} the unit cost for trenching/laying fibers. There is no maintenance cost for the switching equipment as it is a passive one. We have not taken into account the installation and maintenance costs of the OLT as well as of the ONUs assuming that these equipment are already in place. We also assume that each ONU accommodates aggregated traffic requests of a number of end users.

Let $V = \{\text{OLT}\} \cup V^{\text{ONU}}$ be the set of nodes where $V^{\text{ONU}} = \{\text{ONU}_1, \text{ONU}_2, \ldots, \text{ONU}_n\}$. We consider that all ONUs are capable of transmitting and receiving single or multiple wavelengths.

A discrete set P of locations, indexed by p, such that: $P = \{p_{\text{OLT}}\} \cup P_{\text{ONU}} \cup P_{\text{EQ}}$, where : *(i)* p_{OLT}, the OLT location, is known, *(ii)* $P_{\text{ONU}} = \{p_{\text{ONU}_1}, p_{\text{ONU}_2}, \ldots, p_{\text{ONU}_n}\}$, the ONU locations are known as well, and *(iii)* P_{EQ} the set of potential locations for passive equipment.

The distance between ONU_i and potential equipment location p is denoted by d_{ip}, the distance between the OLT location and location p (for a first level equipment) by $d_{\text{OLT},p}$, and $d_{p,p'}$ is the distance between potential locations p and p'. Note that these distances do not necessarily correspond to geographical distances, but to the true length of the fibers (e.g., manhattan distances) in order to connect ONU_i and p, or p and p'.

Upstream traffic demand of ONU is represented by $T_{\text{ONU,OLT}}$ and downstream unicast and multicast traffic demands are represented by $T_{\text{OLT,ONU}}$ and $T_{\text{OLT},M}$, respectively, where $M \in \mathcal{M} \subseteq V^{\text{ONU}}$ represents a multicast destination set (generic notation) and $\mathcal{M}$ is the overall set of multicast destination sets. We assume traffic values to be normalized with respect to the transport capacity of a wavelength, i.e., a value of 1 means a bandwidth requirement equal to the transport capacity of a wavelength.

4.1.2 Outline of the 3-Phase Scheme

We first propose a 3-phase scheme, called OSP1 (Optimized Set of PONs), in order to solve the multiple PON design problem. In Phase I, a N_{I}-center based ILP model (namely,1st-Phase-N_{I}-ILP) as well as an alternative CG model (namely,1st-Phase-N_{I}-CG) is proposed to determine ONU clusters and the placement of the passive equipment (i.e., 2nd level equipment) for each cluster based on the geographical location and traffic demand of each ONU. In Phase II, a N_{II}-center based ILP model as well as an alternative CG model is formulated to determine the clustering of the 2nd level equipment and location of the 1st level passive equipment based on the locations of the 2nd level equipment selected during Phase I. Exploiting the results of Phases I and II, the covered region of each PON is determined and several potential PON hierarchies are generated. Each potential PON hierarchy, relies on an

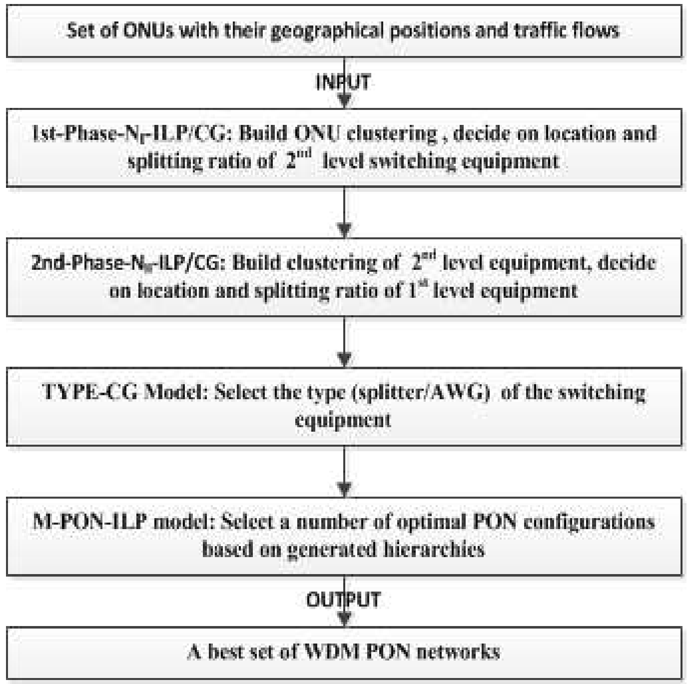


Fig. 4.2: BSP Scheme

ONU clustering, where each cluster confederates a set of ONUs connected to the same passive equipment, and where the clustering of the 2nd level equipment corresponds to passive equipment all connected to a single 1st level passive equipment. However, the type (splitter/AWG) of the passive equipment is not yet determined at this stage. Phases I and II of the BSP Scheme are illustrated in Figure 4.3,4.4,4.5. Phase III consists in selecting the best type of the passive equipment in each potential PON hierarchy, with the use of a column generation (CG) ILP model. An illustration of the 3 phases is presented in Chapter 5 with the case study used in the numerical experiments.

4.1.3 Outline of the 2-Phase Scheme

We next propose a 2-phase scheme, called OSP2 (Optimized Set of PONs), in which Phases I and II of Scheme OSP1 are merged in a single phase. Consequently, the first phase of Scheme OSP1 determines no more than N_c ONU clusters and the placement of the passive equipment (within a hierarchical framework with up to two levels) for each cluster based on the geographical location and traffic demand of each ONU. It therefore outputs a set of HPON hierarchies, with the covered region (i.e., subset of ONUs) for each HPON. The

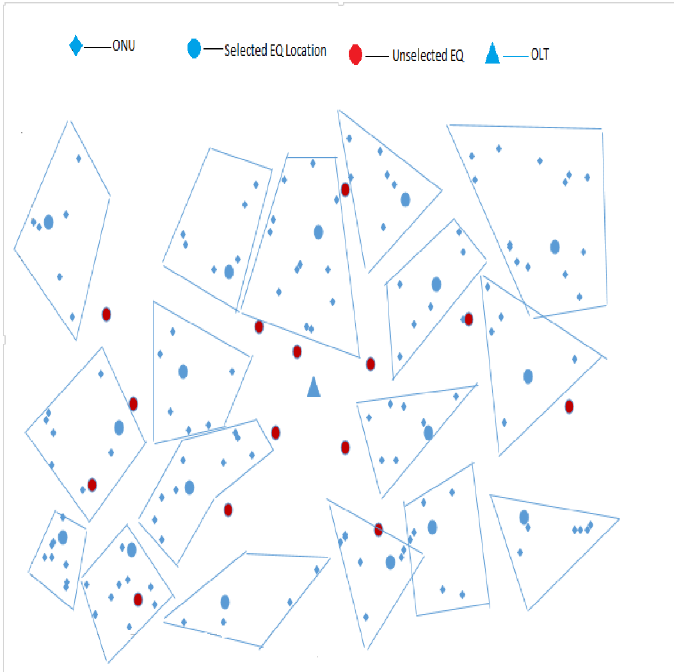


Fig. 4.3: Phase I: ONU Clustering (Illustration of Scheme OSP1 (3 Phases))

second phase is identical to Phase III of Scheme OSP1, i.e., selects the best passive switching equipment (minimum deployment cost) for each ONU cluster. Note that, in Scheme OSP2, we allow an ONU to be linked to a first level passive equipment, so that $N_c = N_{\mathrm{I}} + N_{\mathrm{II}}$ in order to later compare the performances of the two schemes.

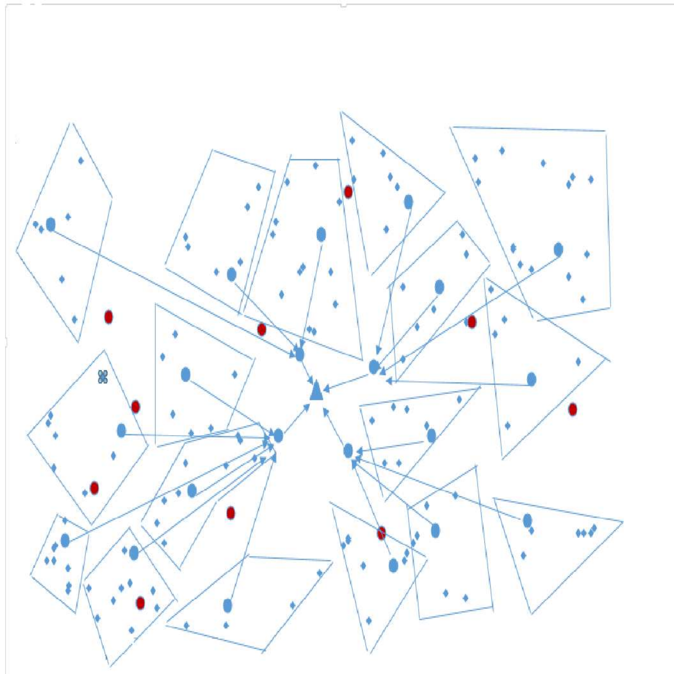


Fig. 4.4: Phase II: PON Definition (Illustration of Scheme OSP1 (3 Phases))

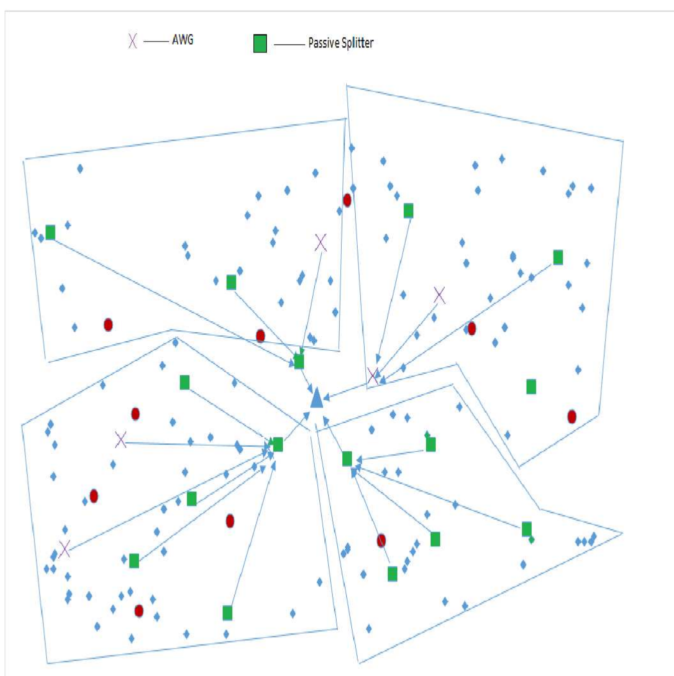


Fig. 4.5: Phase III: Equipment Selection (Illustration of Scheme OSP1 (3 Phases))

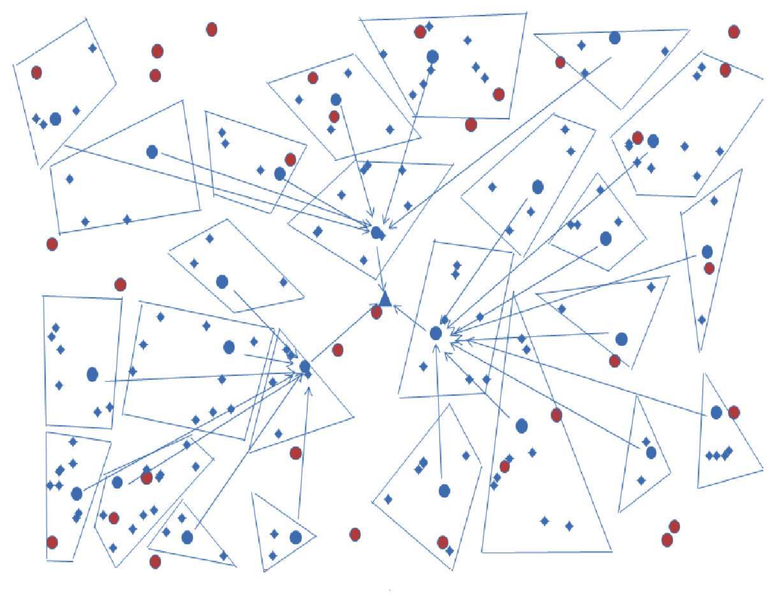


Fig. 4.6: Illustration of Scheme OSP2 (2 Phases)

4.2 Scheme OSP1

In this section, we successfully describe each of the optimization models associated with each of the three phases of Scheme OSP1. Each model corresponds to a clustering or a location problem, similar to either the classical p-center or p-median problems [67], with some variations in the objective and in the set of constraints.

4.2.1 Phase I: ONU Clustering and Location of 2nd Level Switching Equipment

During Phase I, in accordance with the standard p-center model, ONUs and 2nd level passive equipment potential locations are considered as demand nodes and service facilities respectively. Therein, the value of p corresponds to the number N_{I} of ONU clusters, i.e., the number of 2nd level passive equipment. In Phase I, the location and number of required output ports of the 2nd level equipment are determined. By varying the value of N_{I}, we can obtain different clustering of ONUs. Our proposed model extends the standard N_{I}-center based ILP formulation as it not only minimizes the longest distance of an ONU to the OLT, while going through the 2nd level passive equipment, but also minimizes the PON deployment cost related to the second equipment level.

4.2.1.1 Master Model: Selection of the Best ONU Clusterings

Before setting the optimization model, we need to introduce the concept of configurations. Each configuration c is associated with a passive switching equipment, and is defined by a potential cluster of ONUs such that each ONU in the cluster is served by the switching equipment. We denote the overall set of configurations by $\mathcal{C}$ such that $\mathcal{C} = \bigcup_{p \in P_{\text{EQ}}} C^p$ where C^p represents a configuration related to a potential switching equipment located at p in which $p \in P_{\text{EQ}}$. A configuration $c \in \mathcal{C}$ is characterized by the following parameters:

- $y^c_{ip} \in [0, 1]$ represents an ONU-equipment association such that $y^c_{ip} = 1$ if an ONU_i is served by an equipment located at potential location p in configuration c and 0 otherwise, for $\text{ONU}_i \in V^{\text{ONU}}, p \in P_{\text{EQ}}$.

Variables:

- $z_c \in \{0, 1\}$ is a decision variable such that $z_c = 1$ if configuration c is selected, and 0 otherwise.

- δ^{I} = maximum distance between the OLT and an ONU.

The objective corresponds to the minimization of the fiber cost (trenching and laying) of the second level, and to the minimization ($N_{\text{I}}-$center criterion with a penalty term) of the maximum distance between an equipment and the farthest ONU assigned to it.

$$\min \text{COST}^{\text{FT}} \times \text{FIBER}_2(z) + \text{PENAL}_{\text{I}} \times \delta^{\text{I}} \tag{4.1}$$

where $\text{FIBER}_2(z)$ is the overall length of the deployed fiber in order to connect the 2nd level equipment to ONUs:

$$\text{FIBER}_2(z) = \sum_{\text{ONU} \in V^{\text{ONU}}} \sum_{p \in P^{\text{EQ}}} \sum_{c \in C^p} d^c_{\text{ONU},p} z_c.$$

Constraints

Computation of the longest distance between the OLT and an ONU:

$$\delta^{\text{I}} \geq \sum_{c \in C^p} (d_{p_0 p} + d_{pi}\, y^c_i\, z_c)\ \text{ONU}_i \in V^{\text{ONU}}, p \in P^{\text{EQ}}. \tag{4.2}$$

Each ONU must be assigned to one equipment:

$$\sum_{p \in P_{\text{EQ}}} \sum_{c \in C^p} y^c_i\, z_c = 1 \qquad \text{ONU}_i \in V^{\text{ONU}}. \tag{4.3}$$

ONUs will be grouped into N_{I} clusters, i.e., N_{I} passive switching equipment will be located at the second level:

$$\sum_{c \in \mathcal{C}} z_c \leq N_{\text{I}}. \tag{4.4}$$

Note that this constraint will be written as an equality constraint for the experiments described in Section 5.5.

As each configuration is associated with a single equipment location, at most one configuration corresponding to each equipment will be selected:

$$\sum_{c \in C^p} z_c \leq 1 \qquad p \in P_{\text{EQ}}. \tag{4.5}$$

4.2.1.2 Pricing Model: Generation of Promising ONU Clusters

Each pricing problem (PP_p) is defined for a given 2nd level potential equipment location p. Index c will be omitted in order to alleviate the presentation.

Variables

- y_i is a decision variable such that $y_i = 1$ if ONU_i is served by an equipment selected in configuration $c \in C^p$, 0 otherwise, for $\text{ONU}_i \in V^{\text{ONU}}$.

The reduced cost, i.e., the pricing problem objective can be written as follows:

$$\overline{\text{COST}}_c = \sum_{\text{ONU}_i \in V^{\text{ONU}}} d_{\text{ONU},p} + \sum_{\text{ONU}_i \in V^{\text{ONU}}} u_{ip}^{(4.2)} d_{pi}\, y_i - \sum_{\text{ONU}_i \in V^{\text{ONU}}} u_i^{(4.3)}\, y_i + u^{(4.4)} + \sum_{p \in P_{\text{EQ}}} u_p^{(4.5)} \quad (4.6)$$

where $u_{ip}^{(4.2)} \geq 0$, $u_i^{(4.3)} \gtreqless 0$, $u^{(4.4)} \geq 0$, and $u_p^{(4.5)} \geq 0$ are the dual values associated with constraints (4.2), (4.3), (4.4) and (4.5), respectively.

Constraints:

$$\underline{N}_O \leq \sum_{\text{ONU}_i \in V^{\text{ONU}}} y_i \leq \overline{N}_o \quad (4.7)$$

$$d_{p_0 p} + d_{pi} y_i \leq \text{ATT}^{\max} \qquad \text{ONU}_i \in V^{\text{ONU}}. \quad (4.8)$$

Constraints (4.7) ensures that the number of ONUs in a cluster remains within a minimum and a maximum number of ONUs. Constraints (4.8) make sure we do not exceed the attenuation threshold.

4.2.2 Phase II: Clustering of 2nd Level Equipment and Location of 1st Level Equipment

During Phase II, in accordance with the standard N_{II}-center model, 2nd level and 1st level passive equipment are considered as demand nodes and service facility respectively. All 2nd level equipment will be grouped into N_{II} clusters, each to be served by a 1st level equipment. In Phase II, the location and number of required output ports of the 1st level equipment are determined. Note that the value of N_{II} indicates the total number of 1st level equipment, i.e., the total number of PON hierarchies. By varying the value of N_{II}, we can obtain different sets of PON hierarchies. Our proposed model extends the standard N_{II}-center based ILP formulation as it minimizes the maximum distance between the OLT and one of the ONUs, together with the deployment cost associated with the first level equipment.

4.2.2.1 Master Model

In the optimization model of Phase II, a configuration c is associated with a potential location p for a first level equipment and is characterized by the set of 2nd level switching equipment that are connected to this latter equipment. More formally, a configuration c is defined by the following parameters:

- $y^c_{pp'} \in [0, 1]$ represents 2nd level-1st level equipment association such that $y^c_{pp'} = 1$ if 2nd level equipment p is served by a 1st level equipment located at site p' in configuration c and 0 otherwise, for $p \in P^{\text{L}_2}_{\text{EQ}}$, $p' \in P_{\text{EQ}}$. We denote by $P^{\text{L}_2}_{\text{EQ}}$ the set of second level equipment selected by Phase I.

We denote the overall set of configurations by $\mathcal{C}$ such that $\mathcal{C} = \bigcup_{P_{\text{EQ}}} C^p$ where C^p represents a configuration related to a potential switching equipment located at p, for $p \in P_{\text{EQ}}$.
Variables:

- $z_c \in \{0, 1\}$ is a decision variable such that $z_c = 1$ if configuration c is selected, and 0 otherwise.

The objective is defined in a similar way as for the objective of Phase I, i.e., the minimization of the fiber cost (trenching and laying) of the first level (including its connectivity with the second level), and to the minimization ($N_{\text{II}}-$center criterion with a penalty term) of the maximum distance between the OLT and an ONU. It is written as follows:

$$\min \ \textsc{cost}^{\text{FT}} \times \textsc{fiber}_1(y) + \ \textsc{penal}_{\text{II}} \times \sum_{c \in \mathcal{C}} \delta^{\text{II}}_c \, z_c \tag{4.9}$$

where $\textsc{fiber}_1(y)$ is the overall length of the deployed fiber from the OLT to the first level equipment, and from first to second level equipment:

$$\textsc{fiber}_1(y) = \sum_{p \in P_{\text{EQ}}} d_{\text{OLT},p} y_p + \sum_{p \in P_{\text{EQ}}, p' \in P^{\text{L}_2}_{\text{EQ}}} d_{pp'} y_{pp'}, \tag{4.10}$$

where $P_{\text{EQ}}^{\text{L}_2}$ is the set of locations of 2nd level equipment, as selected by Phase I, and δ_c^{II} is the distance between the OLT and the farthest ONU in configuration c.

Constraints:

Each 2nd level equipment must be served by one 1st level equipment:

$$\sum_{p' \in P_{\text{EQ}}} \sum_{c \in \mathcal{C}_{p'}} y_{pp'}^c \, z_c = 1 \qquad p \in P_{\text{EQ}}^{\text{L}_2}. \tag{4.11}$$

All 2nd level equipment will be grouped into N_{II} clusters, i.e., N_{II} 1st level equipment locations will be selected:

$$\sum_{c \in \mathcal{C}} z_c \leq N_{\text{II}}. \tag{4.12}$$

As each configuration is associated with a single 1st level equipment location, at most one configuration corresponding to each 1st level equipment must be selected:

$$\sum_{c \in \mathcal{C}_{p'}} z_c \leq 1 \qquad p' \in P_{\text{EQ}}. \tag{4.13}$$

4.2.2.2 Pricing Model

Each pricing problem ($\text{PP}_{p'}$) is defined for a given 1st level potential equipment location p'. Index c will be omitted in order to alleviate the presentation.

Variables:

- y_p is a decision variable such that $y_p = 1$ if 2nd level equipment located in p is served by the 1st level equipment equipment located in p' in configuration c and 0 otherwise where $p \in P_{\text{EQ}}^{L_2}$, $c \in \mathcal{C}_p$.

- δ^{II} is the cost of the configuration under construction.

Objective: The reduced cost, i.e., the pricing problem objective can be written as follows:

$$\overline{\text{COST}}_c = \delta^{\text{II}} - \sum_{p \in P_{\text{EQ}}^{\text{L}_2}} y_p \, u_p^{(4.11)} + u^{(4.12)} - \sum_{p' \in P_{\text{EQ}}} u_{p'}^{(4.13)} \tag{4.14}$$

where $u_p^{(4.11)} \gtrless 0$, $u^{(4.12)} \geq 0$ and $u_{p'}^{(4.13)} \geq 0$ are the dual values associated with constraints (4.11) , (4.12) and (4.13) respectively.

Constraints:

We compute the longest distance between an ONU and the OLT:

$$\delta^{\text{II}} \geq d_{p',p_{\text{OLT}}} + y_p\, d_{pp'} + d_{ip} y_p \qquad p' \in P_{\text{EQ}}, p \in P_{\text{EQ}}^{\text{L}_2}, \text{ONU}_i \in V^{\text{ONU}} \tag{4.15}$$

We need to guarantee that δ^{II} does not exceed the distance of maximum attenuation:

$$\delta^{\text{II}} \leq \text{ATT}^{\max}. \tag{4.16}$$

4.3 Scheme OSP2

4.3.1 Phase I: Model for ONU Clustering and Determining the Location of Passive Equipment based on the Clusters

The first phase of Scheme OSP2 corresponds to the merge of Phases I and II of Scheme OSP1. In that context, C^p is the set of configurations associated with equipment location p, and is characterized by the subset of ONUs assigned to an equipment located in p ($a^c_{\text{ONU}} = 1$ if ONU is assigned to equipment located at p in configuration $c \in C^p$, 0 otherwise). The difference with Phase I of Scheme OSP1 is that we do not limit the ONUs to be connected to the second level equipment, but allow them to be connected to first level equipment as well. Let $C = \bigcup_{p \in P_{\text{EQ}}} C^p$.

We have four set of variables:

$z_c \in \{0, 1\}$. $z_c = 1$ if c is selected in the optimal ILP solution, 0 otherwise.

$y_{pp'} \in \{0, 1\}$, for $p \prec p'$. $y_{pp'} = 1$ if the passive equipment located in $p \in P_{EQ}$ and $p' \in P_{EQ}$ are connected, 0 otherwise.

$y_p \in \{0, 1\}$. $y_p = 1$ if location $p \in P_{EQ}$ is selected for hosting a passive equipment, 0 otherwise.

δ = largest distance (going through the equipment) between OLT and one ONU.

4.3.2 Optimization Model

Objective. As in the objective of Phase II of Scheme OSP2, it is a compromise between the fiber cost (trenching and deployment) and the attenuation (quality of service) as expressed throughout the longest distance between the OLT and an ONU. The difference here is that we compute the overall fiber cost in one step. It is written as follows.

$$\min \qquad \text{COST}^{FT} \times \text{L\_FIB}(y,z) + \text{PENAL} \times \delta, \tag{4.17}$$

where $\text{L\_FIB}^{\text{LINK}}(y,z)$ is the overall length of the deployed fiber and is computed as follows:

$$\text{L\_FIB}(y,z) \quad = \quad \sum_{p \in P_{\text{EQ}}} d_{\text{OLT},p} \;\; y_p \;\; + \sum_{p,p' \in P_{\text{EQ}}:p \prec p'} d_{pp'} y_{pp'} \;\; + \sum_{c \in C_{p'}} \; \sum_{p' \in P_{\text{EQ}}} \; \sum_{\text{ONU}_i \in V^{\text{ONU}}} d^c_{ip'} z_c.$$

Constraints

$$\sum_{c \in C} z_c \leq N_c \tag{4.18}$$

$$\sum_{c \in C} a^c_{\text{ONU}} z_c = 1 \qquad \text{ONU} \in V^{\text{ONU}} \tag{4.19}$$

$$\sum_{c \in C^p} z_c = y_p \qquad p \in P_{\text{EQ}} \tag{4.20}$$

$$y_{pp'} \leq y_p \qquad p, p' \in P_{\text{EQ}} : p \prec p' \tag{4.21}$$

$$y_{pp'} \leq y_{p'} \qquad p, p' \in P_{\text{EQ}} : p \prec p' \tag{4.22}$$

$$y_p \leq \sum_{p' \in P_{\text{EQ}}:p<p'} y_{pp'} + \sum_{p' \in P_{\text{EQ}}:p'<p} y_{p'p} \qquad p \in P_{\text{EQ}} \tag{4.23}$$

$$\delta \geq d_{\text{OLT},p} y_p + d_{p,p'} y_{pp'} + \sum_{c \in C_{p'}} d^c_{ip'} z_c$$

$$p, p' \in P_{\text{EQ}} : p \prec p', \text{ONU}_i \in V^{\text{ONU}} \tag{4.24}$$

$$\delta \geq d_{\text{OLT},p} y_p + d_{p',p} y_{p'p} + \sum_{c \in C_{p'}} d^c_{ip'} z_c$$

$$p, p' \in P_{\text{EQ}} : p' \prec p, \text{ONU}_i \in V^{\text{ONU}} \tag{4.25}$$

$$\delta \geq d_{\text{OLT},p} y_p + \sum_{c \in C_p} d^c_{ip} z_c \quad p \in P_{\text{EQ}}, \text{ONU}_i \in V^{\text{ONU}} \tag{4.26}$$

$$0.2 \times \delta \leq \text{ATT}^{\max} \tag{4.27}$$

$$z_c \in \{0,1\} \quad c \in C \tag{4.28}$$

$$y_p \in \{0,1\} \quad p \in P_{\text{EQ}} \tag{4.29}$$

$$y_{pp'} \in \{0,1\} \quad p, p' \in P_{\text{EQ}} : p \prec p' \tag{4.30}$$

$$\delta \geq 0. \tag{4.31}$$

Constraints (4.18) enforce the limit on the number of ONU clusters. Constraints (4.19) guarantee that every ONU has to be embedded in exactly one selected configuration. For every potential equipment location, constraints (4.20) decide whether to select it or not ($y_p = 0$ or 1), and if location p is selected, we must choose exactly one configuration associated with it. Constraints (4.21) - (4.23) are the linearization constraints for the quadratic product $y_{pp'} = y_{p'} y_p$. Constraints (4.24) - (4.26), together with the minimization of δ in the objective, aim to compute the longest distance, and constraint (4.27) checks that we do not exceed the attenuation threshold: it is only, at this point, an estimation of not exceeding it as we do not yet take into account the switching equipment attenuation; it will be done in next phase, see Section 4.4.

It may happen that the model (4.17)-(4.31) has no solution, in such a case some ONUs need to be omitted because they are beyond the reach of the OLT with respect to the attenuation constraint, and the model can be easily modified so that we search for the feasible solution with the smaller number of omitted ONUs.

4.3.3 Pricing Problem

Each pricing problem PP(p) is solved for a given potential equipment location p. Let $u^{(4.18)} \geq 0, u^{(4.19)}_{\text{ONU}} \lesseqgtr 0, u^{(4.20)}_p \lesseqgtr 0, u^{(4.24)}_{p,p',\text{ONU}} \geq 0, u^{(4.25)}_{p,p',\text{ONU}} \geq 0, u^{(4.26)}_{p,\text{ONU}} \geq 0$ be the values of the dual variables associated with constraints (4.18), (4.19), (4.20), (4.24), (4.25), (4.26), respectively.

Moreover, let

$$u^{(4.24)}_{p',\text{ONU}} = \sum_{p \in P_{\text{EQ}} : p \prec p'} u^{(4.24)}_{p,p',\text{ONU}}$$
$$u^{(4.25)}_{p',\text{ONU}} = \sum_{p \in P_{\text{EQ}} : p' \prec p} u^{(4.25)}_{p,p',\text{ONU}}.$$

The pricing problem can now be expressed as follows:

$$\min \quad \overline{\text{COST}}(p) = u^{(4.18)} - u^{(4.20)}_p - \sum_{\text{ONU} \in V^{\text{ONU}}} u^{(4.19)}_{\text{ONU}} a_{\text{ONU}} - \sum_{\text{ONU} \in V^{\text{ONU}}} \left(u^{(4.24)}_{p,\text{ONU}} + u^{(4.25)}_{p,\text{ONU}} + u^{(4.26)}_{p,\text{ONU}} - 1 \right) d_{\text{ONU},p}\, a_{\text{ONU}}$$

subject to:

$$\underline{N}_o \leq \sum_{\text{ONU} \in V^{\text{ONU}}} a_{\text{ONU}} \leq \overline{N}_o \tag{4.32}$$

$$a_{\text{ONU}} \in \{0,1\} \qquad \text{ONU} \in V^{\text{ONU}}. \tag{4.33}$$

Observe that the relations between the variables of the pricing problem and the coefficients of the restricted master problem are as follows:

$$d^c_{\text{ONU},p} = d_{\text{ONU},p} \times a^c_{\text{ONU}}.$$

4.4 Selection of Switching Equipment

Phase III takes care of selecting the best switching equipment at each equipment location in the hierarchies output in Phase I & II of Scheme OSP1 and Phase I of Scheme OSP2. For a given PON hierarchy, let G be the set of clusters, indexed by g. We denote by g_0 the ONU cluster of level I, and $G^\star = G \setminus \{g_0\}$.

4.4.1 Optimization Model

The optimization model is again a decomposition one relying on equipment configurations (with a generic configuration denoted by c). In addition, each configuration is associated with one wavelength, and is characterized not only by the switching equipment at each equipment location, but also by the traffic carried on its associated wavelength. More formally, it is characterized by the following parameters:

$t^{c}_{\text{ONU,OLT}}, t^{c}_{\text{OLT,ONU}}, t^{M,c}_{\text{OLT,ONU}} \in [0,1]$: bandwidth amount of unicast upstream, unicast downstream, multicast downstream for ONU in multicast request M, carried out by configuration c, respectively

$a^{\text{A},c}_{g}, a^{\text{S},c}_{g} \in \{0,1\}$: indicators for an AWG or a splitter set at the equipment location governing the ONUs of cluster g in configuration c.

Other parameters are the unit cost of AWG and splitters for a given number of ports ($\lceil g \rceil$), denoted by $\text{COST}^{\text{AWG}}_{\lceil g \rceil}$ and $\text{COST}^{\text{S}}_{\lceil g \rceil}$, respectively, where $\lceil g \rceil$ is the closest upper rounding up to the cluster cardinality power of 2, as, in practice, the number of ports of such equipment is always a power of 2, i.e., belongs to $\{2, 4, 8, 16, 32, 64\}$, and so forth.

Variables

$z_c \in \{0,1\}$: decision variable such that $z_c = 1$ if configuration c is selected, 0 otherwise.

$y^{\text{S}}_{g} \in \{0,1\}$: decision variable such that $y^{\text{S}}_{g} = 1$ when a splitter connects the ONUs of cluster g.

$y^{\text{A}}_{g} \in \{0,1\}$: decision variable such that $y^{\text{A}}_{g} = 1$ when an AWG connects the ONUs of cluster g.

Objective: we minimize the switching equipment cost.

$$\text{COST}_{\text{III}}(z) = \sum_{c \in C} \sum_{g \in G} \left(\text{COST}^{\text{AWG}}_{\lceil g \rceil} a^{\text{A},c}_{g} + \text{COST}^{\text{S}}_{\lceil g \rceil} a^{\text{S},c}_{g} \right) z_c. \tag{4.34}$$

Constraints

Limit on the number of selected configurations

$$\sum_{c \in C} z_c \leq W \tag{4.35}$$

Constraints (4.35) express that the number of selected configurations cannot exceed the number of available wavelengths.

Traffic

$$\sum_{c \in C} t^c_{\text{ONU,OLT}} z_c \geq T_{\text{ONU,OLT}} \qquad \text{ONU} \in V^{\text{ONU}} \tag{4.36}$$

$$\sum_{c \in C} t^c_{\text{OLT,ONU}} \, z_c \geq T_{\text{OLT,ONU}} \qquad \text{ONU} \in V^{\text{ONU}} \tag{4.37}$$

$$\sum_{c \in C} t^{M,c}_{\text{OLT,ONU}} \, z_c \geq T_{\text{OLT},M} \quad \text{ONU} \in M, M \in \mathcal{M}. \tag{4.38}$$

Constraints (4.36) take care of the upstream (unicast) traffic, while constraints (4.37) and (4.38) take care of the downstream traffic. Unicast downstream is handled in (4.37) while multicast traffic is taken care in (4.38). For the latter set of constraints, since all ONUs of a multicast request are not necessarily served by the same configuration (i.e., wavelength), a constraint must be written for each ONU in a given multicast request.

The next set of constraints guarantee that the same switching equipment is selected in all selected configurations for a given cluster g:

$$y^{\text{S}}_g \leq \sum_{c \in C} a^{\text{S},c}_g z_c \qquad g \in G \tag{4.39}$$

$$y^{\text{A}}_g \leq \sum_{c \in C} a^{\text{A},c}_g z_c \qquad g \in G \tag{4.40}$$

$$\sum_{c \in C} a^{\text{S},c}_g z_c \leq W y^{\text{S}}_g \qquad g \in G \tag{4.41}$$

$$\sum_{c \in C} a^{\text{A},c}_g z_c \leq W y^{\text{A}}_g \qquad g \in G \tag{4.42}$$

$$y^{\text{S}}_g + y^{\text{A}}_g \leq 1 \qquad g \in G. \tag{4.43}$$

4.4.2 Solution Process

The solution process is similar to the previous optimization model, except that the pricing problem here outputs equipment configurations, instead of ONU configurations.

Variables

$\alpha_{\text{ONU}} \in \{0,1\}$ where $\alpha_{\text{ONU}} = 1$ if ONU is served by the configuration under construction.

$\beta_g \in \{0,1\}$ where $\beta_g = 1$ if all ONUs of cluster $g \in G$ are served by the configuration under construction.

$t_{\text{ONU,OLT}}, t_{\text{OLT,ONU}}, t^{M}_{\text{OLT,ONU}}, t_{\text{OLT},M}, a^{\text{S}}_{g}, a^{\text{A}}_{g}$, see the definition of their corresponding parameters in the optimization model in Section 4.4.1. Observe that $t^{M}_{\text{OLT,ONU}} = \alpha_{\text{ONU}}\, t_{\text{OLT},M}$.

The pricing problem, i.e., the generator of switching equipment configuration can be expressed as follows:

$$\begin{aligned}
\min \quad \overline{\text{COST}}(z) = \sum_{g \in G} \left(\text{COST}^{\text{AWG}}_{\lceil g \rceil} a^{\text{A}}_{g} + \text{COST}^{\text{S}}_{\lceil g \rceil} a^{\text{S}}_{g} \right) \\
- u^{(4.35)} - \sum_{\text{ONU} \in V^{\text{ONU}}} u^{(4.36)}_{\text{ONU}}\, t_{\text{ONU,OLT}} - \sum_{\text{ONU} \in V^{\text{ONU}}} u^{(4.37)}_{\text{ONU}}\, t_{\text{OLT,ONU}} \\
- \sum_{M \in \mathcal{M}} \sum_{\text{ONU} \in M} u^{(4.38)}_{\text{ONU},M}\, t^{M}_{\text{OLT},M} - \sum_{g \in G} u^{(4.39)}_{g} a^{\text{S}}_{g} \\
- \sum_{g \in G} u^{(4.40)}_{g} a^{\text{A}}_{g} + \sum_{g \in G} u^{(4.41)}_{g} a^{\text{S}}_{g} + \sum_{g \in G} u^{(4.42)}_{g} a^{\text{A}}_{g}, \quad (4.44)
\end{aligned}$$

where $u^{(4.35)}$ is the value of the dual variable associated with constraint (4.35), and $u^{(4.36)}_{\text{ONU}} \geq 0, u^{(4.37)}_{\text{ONU}} \geq 0, u^{(4.38)}_{\text{ONU},M} \geq 0$ are the values of the dual variables associated with constraints (4.36), (4.37), (4.38), respectively. In addition, $u^{(4.39)}_{g} \geq 0, u^{(4.40)}_{g} \geq 0, u^{(4.41)}_{g} \geq 0, u^{(4.42)}_{g} \geq 0$ are the dual values associated with constraints (4.39), (4.40), (4.41), (4.42), respectively.

Constraints

Equipment Selection. We must choose one equipment per cluster centre, either a splitter or an AWG.

$$a^{\text{A}}_{g} + a^{\text{S}}_{g} = 1 \qquad g \in G. \quad (4.45)$$

Traffic constraints. Since wavelength are bidirectional in HPONs, we need to check that the sum of both upstream and downstream traffic does not exceed the transport capacity of a wavelength, which is normalized to 1 in our study. On the other hand we need to check that we only account for the traffic that is provisioned on a given wavelength.

$$\sum_{\text{ONU} \in V^{\text{ONU}}} (t_{\text{OLT,ONU}} + t_{\text{ONU,OLT}}) + \sum_{M \in \mathcal{M}} t_{\text{OLT},M} \leq 1 \tag{4.46}$$

$$t_{\text{OLT,ONU}} \leq \alpha_{\text{ONU}} \qquad \text{ONU} \in V^{\text{ONU}} \tag{4.47}$$

$$t^M_{\text{OLT,ONU}} \leq \alpha_{\text{ONU}} \qquad \text{ONU} \in M, M \in \mathcal{M} \tag{4.48}$$

$$t^M_{\text{OLT,ONU}} \leq t_{\text{OLT},M} \qquad \text{ONU} \in M, M \in \mathcal{M} \tag{4.49}$$

$$\alpha_{\text{ONU}} + t_{\text{OLT},M} - 1 \leq t^M_{\text{OLT,ONU}} \qquad \text{ONU} \in M, M \in \mathcal{M} \tag{4.50}$$

$$t_{\text{ONU,OLT}} \leq \alpha_{\text{ONU}} \qquad \text{ONU} \in V^{\text{ONU}} \tag{4.51}$$

$$\sum_{\text{ONU} \in g} \alpha_{\text{ONU}} \leq a^{\text{A}}_g + \lceil g \rceil a^{\text{S}}_g \qquad g \in G^\star \tag{4.52}$$

$$\sum_{\text{ONU} \in g_0} \alpha_{\text{ONU}} + \sum_{g \in G^\star} \beta_g \leq a^{\text{A}}_{g_0} + \lceil g_0 \rceil a^{\text{S}}_{g_0} \tag{4.53}$$

$$\beta_g \geq \alpha_{\text{ONU}} \qquad \text{ONU} \in g, g \in G^\star. \tag{4.54}$$

Constraints (4.46) ensure that transport capacity is not exceeded. Throughout the α_{ONU} variables, which indicate which ONUs are served in the configuration under construction, we only account for the traffic of the served ONUs. For multicast traffic, as we do not assume that all destinations of a multicast are necessarily provisioned on the same wavelength, we make sure with the use of the variable $t^M_{\text{OLT,ONU}}$ to only consider the traffic of the served ONUs. Constraints (4.52) - (4.54) identify the served ONUs subject to the selection of the switching equipment. In the 3-phase Scheme OSP1, there is no ONU connected to the 1st level equipment, so there is no term $\sum_{\text{ONU} \in g_0} \alpha_{\text{ONU}}$ in constraints (4.53).

Attenuation Constraints. For each ONU, we have to make sure that the total signal loss is less than a given threshold value. In our numerical experiments, we use 20dB. The total signal P is given by:

$$P_p = P^{\text{FIBER}}_p + P^{\text{THROUGH}}_p + P^{\text{INSERTION}} + P^{\text{MARGIN}} \leq 20 \text{ dB} \tag{4.55}$$

where P^{FIBER}_p is the signal loss caused on the fiber to reach the ONU located at p, and is estimated as 0.2dB/km, P^{THROUGH}_p is the loss incurred by going through the equipment towards the ONU located at p, see Table 5.1 in Chapter 5, $P^{\text{INSERTION}} = 0.1$ dB is the insertion loss caused by all the nodes on the link, and $P^{\text{MARGIN}} = 1$ dB is the power margin.

It follows that:

$$P^{\text{THROUGH}}_{p_{\text{ONU}}} = \text{ATT}^{\text{A}} a^{\text{A}}_{g_0} + \text{ATT}^{S}_{\lceil g_0 \rceil} a^{\text{S}}_{g_0} + \text{ATT}^{\text{A}} a^{\text{A}}_{g} + \text{ATT}^{\text{S}}_{\lceil g \rceil} a^{\text{S}}_{g} \qquad \text{ONU} \in g, g \in G^{\star} \tag{4.56}$$

$$P^{\text{FIBER}}_{p_{\text{ONU}}} = 0.2(d_{\text{OLT}g_0} + d_{g_0 g} + d_{g\text{ONU}}) \qquad \text{ONU} \in g, g \in G^{\star} \tag{4.57}$$

$$P^{\text{THROUGH}}_{p_{\text{ONU}}} = \text{ATT}^{\text{A}} a^{\text{A}}_{g_0} + \text{ATT}^{S}_{\lceil g_0 \rceil} a^{\text{S}}_{g_0} \qquad \text{ONU} \in g_0 \tag{4.58}$$

$$P^{\text{FIBER}}_{p_{\text{ONU}}} = 0.2(d_{\text{OLT}g_0} + d_{g\text{ONU}}) \qquad \text{ONU} \in g_0. \tag{4.59}$$

Chapter 5

Computational Results and Analysis for PON

Both optimization schemes were implemented and tested on several data sets. We first describe the data sets in Section 5.1 and then the validation tests of both schemes in Section 5.2 and 5.3. We next investigate the goodput vs. throughtput in order to evaluate the wasted bandwidth in Section 5.4. Lastly, in Section 5.5, we look at the cost increase vs. the traffic increase, and determine the optimal size of the ONU clusters.

5.1 Data Sets

We implemented the two schemes described in the previous sections using the Optimization Programming Language (OPL) platform. Linear and integer linear programs were solved using the IBM CPLEX 12.6 package.

We conducted our experiments with randomly generated geographical locations of 128 and 512 ONUs and 60 candidate/potential locations for the placement of the passive equipment, respectively. Location of ONUs and potential equipment locations are randomly generated in a 40×20 km^2 rectangular grid such that the OLT is located at the center of the grid.

Table 5.1 contains the values used for the cost of the equipment (taken from [13]), as well as the attenuation parameters, which depend on the number of output ports for the splitters. For the costs related to optical fiber cables, we use the value of 7,160\$/km [13], assuming it includes the cost of trenching and laying the optical fiber cables.

For all experiments, we selected $\text{PENAL}_{\text{I}} = \text{PENAL}_{\text{II}} = \text{PENAL}$, therefore we only indicate the PENAL value in the following.

Table 5.1: Cost and Attenuation of Equipment

# output ports	Splitters cost ($)	Splitters att. (dB)	AWG cost ($)	AWG att. (dB)
2	800	3	950	3
4	900	6	1,100	
8	1,100	9	1,400	
16	1,500	12	2,000	
32	2,300	15	3,200	
64	3,700	18	5,600	

For each ONU, we randomly generate a normalized upstream unicast traffic within the range [0.05, 0.1]. Downstream traffic, both unicast and multicast, is randomly generated in [0.1, 0.4]. We consider 25 multicast traffic requests destined for different groups of ONUs. Each multicast request consists of 3 randomly generated ONU destinations unless otherwise specified.

5.2 Results of 3-Phase Scheme OSP1

For the numerical experiments with Scheme OSP1, at first, we solve the N_{I}-center ILP model of Phase I and obtain the grouping of ONUs to be served by a common 2nd level equipment. We experiment with two values of N_{I} (23 and 27), so that the same set of ONUs can be grouped into a different number of clusters in order to determine the most cost efficient ONU clustering.

We next solve the N_{II}-center ILP model (Phase II) in order to generate different PON hierarchies, with $N_{\text{II}} = 3$, implying that there are three 1st level equipment resulting in three different PON hierarchies to serve all the ONUs through $N_{\text{I}} = 23$ or 27 second level equipment.

The next phase of Scheme OSP1 is Phase III in order to: *(i)* select the type (splitter or AWG) of the passive equipment, *(ii)* provision the traffic flows, for each PON.

Table 5.2 summarizes the results derived from the solution of the optimization problem of Phase III. N_c^{PON} denotes the number of ONU clusters per PON while $\#\lambda$ indicates the number of required wavelengths per PON. The overall cost is indeed equal to:

$$\underbrace{\text{COST}^{\text{FT}} \times \text{FIBER}_2(z)}_{\text{see expression (4.1)}} + \underbrace{\text{COST}^{\text{FT}} \times \text{FIBER}_1(y)}_{\text{see expression (4.9)}} + \underbrace{\text{COST}_{\text{III}}(z)}_{\text{see expression (4.34)}} . \quad (5.1)$$

With $\text{PENAL} = 10^4$, we went on with different parameters (with different N_c and N_o number), changing one parameter at a time when going to one experiment to the next in order to further decrease the cost, from 4,251,169 to 3,681,927, i.e., 15 % reduction. Indeed, we successfully increase N_{I} from 23 to 27, then $\overline{N}_o$ from 10 to 15, and last N_{I} from 27 to 32. We can observe that not only the deployment cost is decreasing, but the attenuation improves (from 18.6 dB to 17.9 dB, i.e., about 1 dB difference), leading to a better quality of service for the users.

We next performed experiments with 512 ONUs, results are reported in Table 5.3. Again, we use $\text{PENAL} = 10^4$. We also manage to find parameter changes so that the cost decreases, going from 6,647,271 down to 6,348,281, i.e., a 5 % reduction, while improving the attenuation by 0.9 dB.

We report the distribution of the switching equipment for different parameter values in Table 5.4. We observe that, as expected, we usually find splitters at the upper level and a varying number of splitters and AWGs at the lower level. For the first level, this is explained by the overall amount of bandwidth that do not justify AWGs, and therefore the attenuation consideration take precedence in the equipment selection. For the second level, the unicast vs. multicast density make the decision whether to consider a splitter or an AWG in the cases where the attenuation is not critical due to either the distance or to the number of required ports.

From the results of the two schemes, we observe that a mix of splitters and AWGs are selected for most of the PON hierarchies whereas some PONs consist of only splitters. The selection of the switching equipment is made based on the best choice taking into account the cost, the traffic flows (a mix of unicast and multicast requests) and the attenuation constraints. While selecting the type of the equipment, the optimization model first takes into account the traffic flows (unicast/multicast) and favors splitters (as a splitter is less expensive compared to an AWG) in the case of multicast traffic requests to minimize the number of used wavelengths as long as the constraint for power signal attenuation is satisfied. But there does not always exist a feasible solution with the selection of splitters due to the signal attenuation constraint. Indeed, in a splitter, the attenuation increases significantly with the increase of the number of output ports. However, the attenuation caused by an AWG is low and independent of the number of the output ports. This is why splitters are not always selected as a 1st/2nd level equipment for a PON hierarchy, although the deployment cost of a PON hierarchy consisting of only splitters would be the most economical one.

We next analyze the transmission pattern of multicast traffic requests. In our experiment with the 1st set of 512 ONUs, we consider 25 multicast traffic requests in which each request consists of 3 randomly generated destination ONUs. We observe that only 15 requests are transmitted as multicast demand within a broadcast framework using splitters with the 3 ONU destinations served using the same wavelength. It is also noticed that 6 multicast requests are satisfied by transmitting 12 traffic streams in which each stream consists of either 1 or 2 destination. The remaining 5 multicast requests are served by 15 unicast traffic streams, i.e., each stream uses a different wavelength. Indeed, it all depends on the location of the ONU destinations, whether they are geographically distributed within a 45 degree cone.

5.3 Results of 2-Phase Scheme OSP2

In Scheme OSP2, at first, we solve the N_c-center ILP model of Phase I and obtain the grouping of ONUs to be served by a common 1st or 2nd level equipment. We experiment with different values of N_c (maximum number of clusters) and N_o (maximum number of ONUs per cluster) so that the same set of ONUs can be grouped into different set of clusters in order to make meaningful comparisons with the results of Scheme OSP1. The second phase of Scheme OSP2 is identical to Phase III of Scheme OSP1.

We selected $N_c = N_{\text{I}} + N_{\text{II}}$ in order to be able to compare the results (HPON cost) in Tables 5.2 and 5.5. The overall costs of Scheme OSP2 are lower than those of Scheme OSP1, with a reduction equal to 1.7 %, 6.4 %, 3.9 %, 5.2 %, 4.1 % for each of the five cases. In addition, the largest attenuation value is never larger ; it is sometimes equal (2 cases), sometimes smaller up to 0.2dB. Lastly, the number of required wavelengths in the cheapest solution of Scheme OSP2 is also smaller than for the comparable solution fo Scheme OSP1, i.e., 19, 16 and 19 wavelengths for the 3 PONs of Scheme OSP1 vs. 16, 18, and 14 wavelengths for 3 PONs of Scheme OSP2.

Table 5.6 provides the results for 512 ONUs. Reduction of the HPON deployment cost, going from Scheme OSP1 to Scheme OSP2 is as follows for each of the four cases: 4.5 %, 4.9 %, 3.5 %, and 4.9 %. Again, attenuation are smaller on average for Scheme OSP2.

Table 5.7 provides the distribution of splitters/AWGs in the HPONs. We observe that first level equipment are now always splitters, while distribution of AWGs and splitters follows a similar trend as for Scheme OSP1.

5.4 Goodput vs. Throughput

Throughput is the measurement of all data flowing through a link whether it is useful data or not, while goodput is focused on useful data only. In the context of HPONs where splitters and AWGs are selected as a compromise between the cost, the traffic requirement, and the signal attenuation, before looking at the gap between the throughput and the goodput, it is therefore of interest to investigate the amount of bandwidth that is unnecessarily carried, and that artificially increase the required transport capacity.

Splitters are better for multicast requests under the assumption that the ONU destinations belong to the same ONU cluster and AWGs better for unicast requests in terms of avoiding wasted bandwidth. However, ONU destinations are quite often not concentrated in the same ONU cluster.

In order to compare the goodput and the throughput, we only consider the downlink requests. For each ONU, we compute the fraction of bandwidth that is destined to it, and

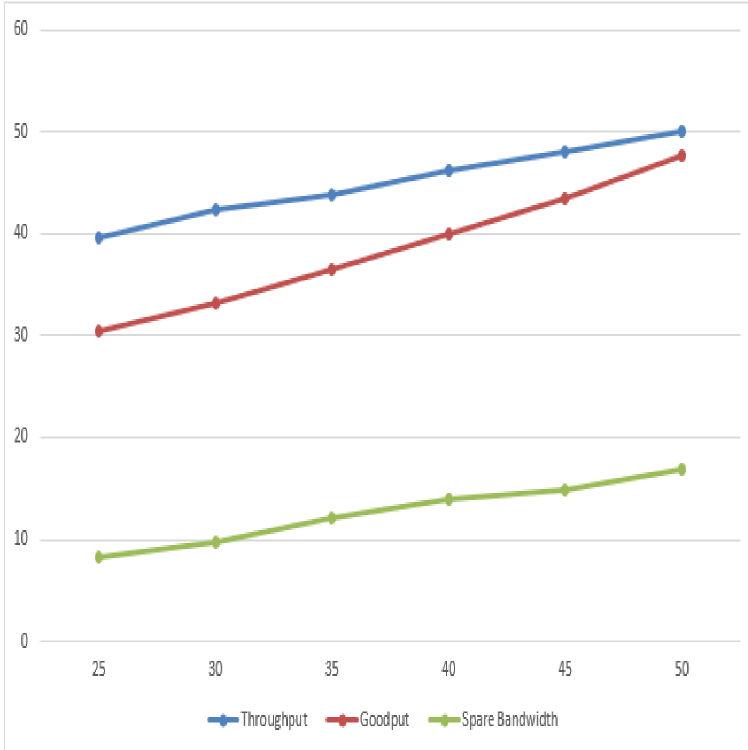


Fig. 5.1: 2-Phase Scheme (Throughput vs. Goodput - Multicast with 3 ONUs)

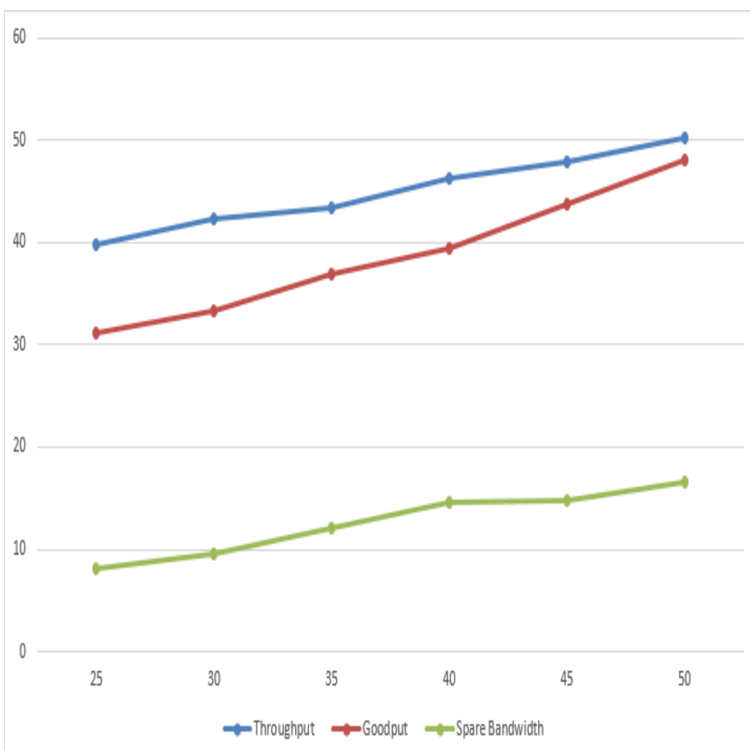
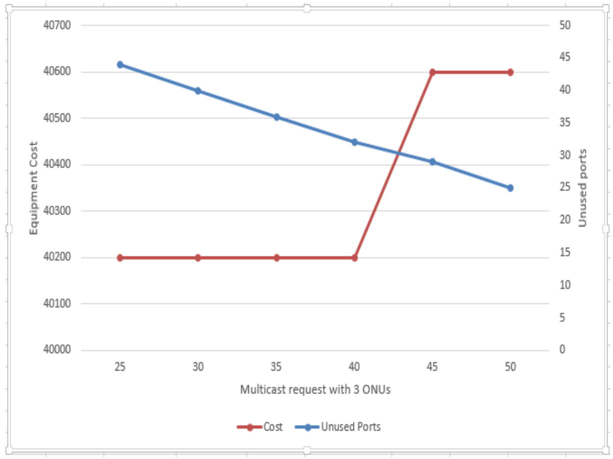


Fig. 5.2: 3-Phase Scheme (Throughput vs. Goodput - Multicast with 3 ONUs)

Fig. 5.3: Cost vs. Number of Used Ports Following a Traffic Increase



the remaining one divides between the spare bandwidth and the goodput, i.e., the bandwidth that is transported unnecessarily to an ONU while the latter is not the or one of the bandwidth destinations.

Results are reported in Figures 5.2 and 5.1 for Schemes OSP1 and OSP2, respectively. We observe that results are very similar for both schemes. The goodput value is quite large in comparison to the throughput, due to the use of splitters. Indeed, it is 46% of the transported bandwidth. Spare bandwidth is reasonable taking into account that it was computed considering only the used ports. However, remember that switching equipment come with only standard numbers of ports, i.e., 2, 4, 8, 16, and so on. Consequently, spare bandwidth could reach quite large values in some cases, depending on the number of unused ports, if we do not restrict its computation to the used ports.

In Figure 5.3, we look at the cost evolution (left vertical axis) vs. the number of used ports (right vertical axis), following an increase of the traffic (horizontal axis). While the number of used ports increases linearly with the traffic, the cost increase follows a staircase behavior due to the standard values of the number of ports for the switching equipment.

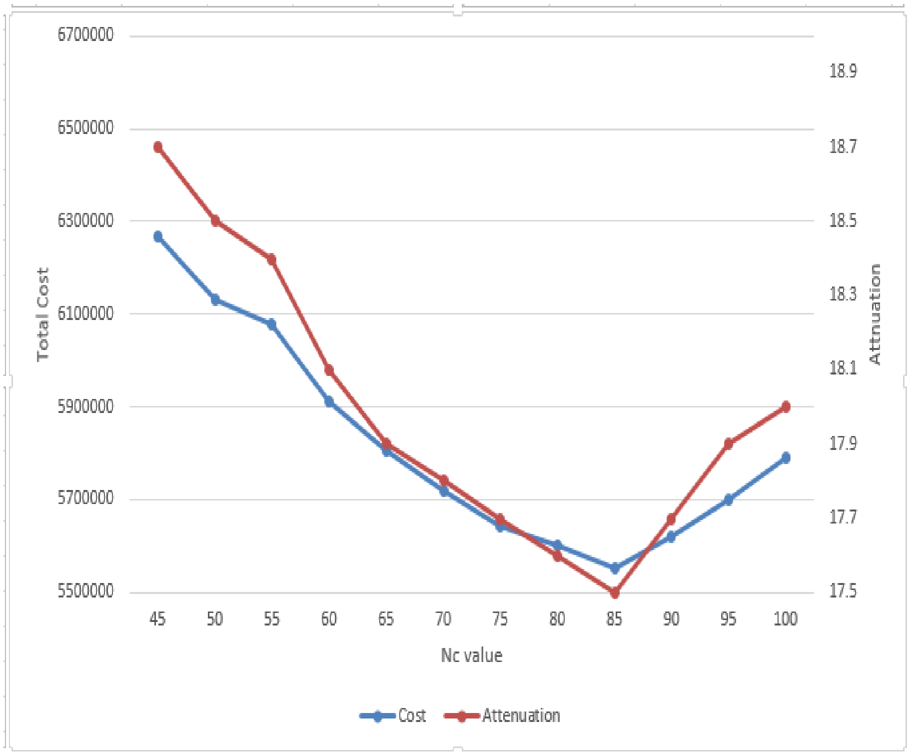


Fig. 5.4: 512 ONUs for 2-Phase Scheme (Deployment Cost and Signal Attenuation vs. N_c)

5.5 Cost Evaluation vs. Traffic Increase

In this last section, we investigate the cost evolution vs. the N_c value (maximum number of ONU clusters) and the attenuation.

In Figure 5.4, 5.5, N_c is on the horizontal axis, while the cost is on the left vertical axis, and the attenuation on the right vertical axis. We note that the overall deployment cost decreases as N_c increases until it reaches a certain value and then increases again. The explanation is that when we select a small N_c value, each cluster consists of a large number of ONUs which requires a switching equipment with a large number of output ports. As the attenuation of a splitter with a high splitting ratio is much higher compared to an AWG, the optimization scheme selects AWGs for most of clusters resulting in a higher deployment cost. On the contrary, when we select a larger N_c value, less ONUs are aggregated in each cluster, thereby switching equipment requires a smaller number of output ports to connect the ONUs, which results in minimized deployment cost due to the selection of the splitters.

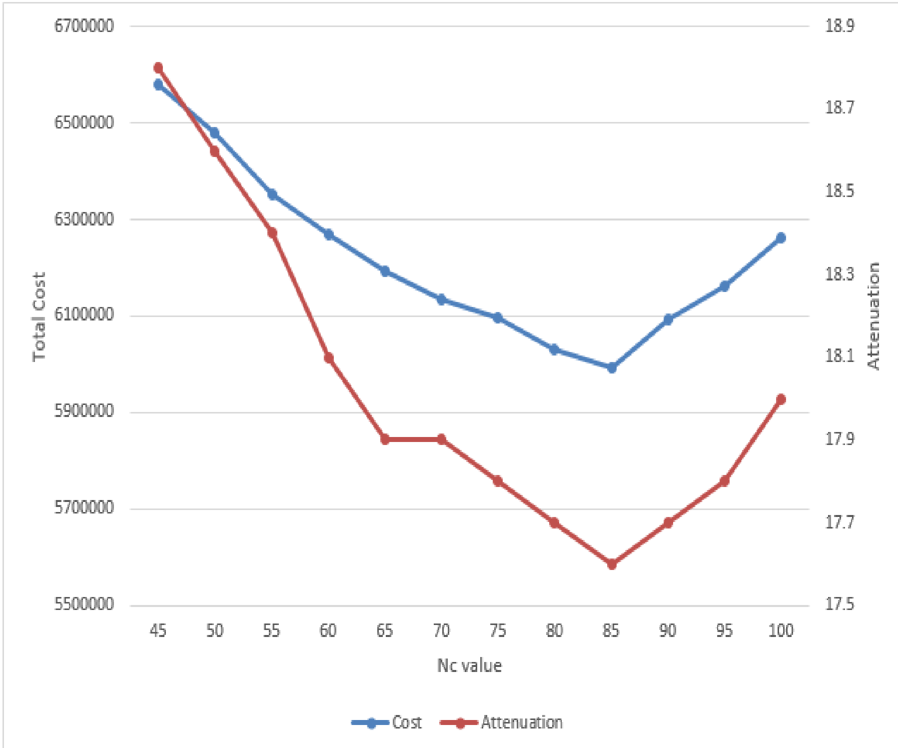


Fig. 5.5: 512 ONUs for 3-Phase Scheme (Deployment Cost and Signal Attenuation vs. N_c)

In addition, when N_c increases, it is easier to find a shorter routing for each ONU to OLT, so that distances are reduced, and more freedom is available with respect to the choice of a splitter vs. an AWG. As observed in Tables 5.2 and 5.5, the trenching and laying cost is dominant in comparison with the equipment cost, and therefore influence a PON layout with shortest possible routing (distances) when N_c increases. However, beyond a given N_c value, the situation is reversed. As the size of ONU clusters shrinks, and the number of clusters increases, routing distances increase again. In addition, although not a dominant cost, the equipment cost increases as well.

Figure 5.4, 5.5 also presents the amount of maximum signal attenuation experienced when N_c varies. It follows the same behavior as the deployment cost, first a decreasing and then an increasing behavior. Again, it can explained by the fact that attenuation decreases when N_c increases as *(i)* the distances decrease when N_c increases because the size of the ONU clusters decreases and therefore gives more flexibility for the cluster connectivity to the OLT, *(ii)* the number of required ports decreases and therefore, especially for the splitters, the attenuation due to the switching equipment decreases. However, beyond a N_c threshold, distances will increase again as explained above for the cost behavior, as well as the number of switching equipment, resulting in an increase of the attenuation.

Such an investigation is very useful in order to identify the best N_{I} and N_{II} for Scheme OSP1, or N_c values for Scheme OSP2 in order to get the minimum deployment cost.

Table 5.2: Cost Comparison of Different PONs - Scheme OSP1 - 128 ONUs

PONs	N_c^{PON}	EQ cost	FT (%)	overall cost	served ONUs	$\#\lambda$
PENAL $= 1$, $N_{\text{I}} = 23, N_{\text{II}} = 3, N_o = 10, \delta^\star = 28.2$ km (18.7 dB)						
PON1	8	1.3	98.7	1,072,066	41	16
PON2	10	1.3	98.7	1,716,832	45	18
PON3	8	0.8	99.2	1,026,454	42	16
Overall cost				3,815,352		
PENAL$= 10^4$, $N_{\text{I}} = 23, N_{\text{II}} = 3, N_o = 10, \delta^\star = 27.6$ km (18.6 dB)						
PON4	8	1.3	98.7	772,030	41	16
PON5	10	1.1	98.9	1,392,081	47	18
PON6	8	0.9	99.1	1,820,074	40	16
Overall cost				4,251,169		
PENAL$= 10^4$, $N_{\text{I}} = 27, N_{\text{II}} = 3, N_o = 10, \delta^\star = 26.4$ km (18.4 dB)						
PON7	12	0.9	99.1	1,669,634	51	19
PON8	10	1.3	98.7	1,405,583	46	17
PON9	8	0.6	99.4	820,566	31	15
Overall cost				3,895,738		
PENAL$= 10^4$, $N_{\text{I}} = 27, N_{\text{II}} = 3, N_o = 15, \delta^\star = 24.5$ km (18.0 dB)						
PON10	12	0.9	99.1	1,615,583	48	19
PON11	11	1.4	98.6	1,443,595	43	16
PON12	7	0.7	99.3	788,781	37	15
Overall cost				3,847,959		
PENAL$= 10^4$, $N_{\text{I}} = 32, N_{\text{II}} = 3, N_o = 15, \delta^\star = 23.9$ km (17.9 dB)						
PON13	13	1.7	98.3	1,455,689	43	19
PON14	8	1.2	98.8	649,826	37	16
PON15	14	0.9	99.1	1,562,908	48	19
Overall cost				3,681,927		

Table 5.3: Cost Comparison of Different PONs - Scheme OSP1 - 512 ONUs

N_{I}	N_{II}	N_c	Overall cost	Attenuation $\delta^\star$ (km)
		45	6,647,271	28.9 (18.9 dB)
		50	6,560,310	27.4 (18.6 dB)
		55	6,414,836	26.0 (18.3 dB)
		60	6,348,281	24.6 (18.0 dB)

Table 5.4: Equipment distribution - Scheme OSP1 - 128/512 ONUs

# ONUs	N_c	$\underline{N}_o$	$\overline{N}_o$	# splitters		# AWGs	
				Level 1	Level 2	Level 1	Level 2
128	26	3	10	3	16	0	7
	30	3	10	3	19	0	8
	30	3	15	3	21	0	6
	35	3	15	3	23	0	9
512	45	12	30	4	24	0	17
	50	12	30	4	29	0	17
	55	12	30	3	32	1	19
	60	12	30	4	33	0	23

Table 5.5: Cost Comparison of Different PONs - Scheme OSP2 - 128 ONUs

PONs	N_c^{PON}	EQ cost	FT (%)	overall cost	served ONUs	$\#\lambda$
PENAL= 1 $N_c = 26, N_o = 10, \delta^\star = 28.1$ km (18.7 dB)						
PON1	7	1.2	98.8	723,482	34	14
PON2	8	0.8	99.2	1,308,026	40	16
PON3	11	0.8	99.2	1,721,731	54	19
overall cost				3,753,239		
PENAL= 10,000, $N_c = 26, N_o = 10, \delta^\star = 27.6$ km (18.6 dB)						
PON4	7	1.3	98.7	772,030	33	14
PON5	8	0.9	99.1	1,394,596	40	16
PON6	11	0.8	99.2	1,828,178	55	19
overall cost				3,994,804		
PENAL= 10,000, $N_c = 30, N_o = 10, \delta^\star = 26.2$ km (18.3 dB)						
PON7	12	0.9	99.1	1,741,264	51	18
PON8	11	1.1	98.9	1,267,778	41	16
PON9	7	1.3	98.7	740,424	36	15
overall cost				3,749,466		
PENAL= 10,000, $N_c = 30, N_o = 15, \delta^\star = 24.4$ km (17.8 dB)						
PON10	12	0.9	99.1	1,690,639	52	18
PON11	11	1.2	98.8	1,242,814	43	16
PON12	7	1.4	98.6	722,612	33	14
overall cost				3,656,065		
PENAL= 10,000, $N_c = 35, N_o = 15, \delta^\star = 23.7$ km (17.8 dB)						
PON13	12	0.9	99.1	1,201,824	42	16
PON14	11	1.2	98.8	1,639,878	50	18
PON15	7	1.4	98.6	694,997	36	14
overall cost				3,536,699		

Table 5.6: Cost Comparison of Different Hierarchies - Scheme OSP2 - 512 ONUs

N_c	Overall cost	Attenuation $\delta^\star$ (km)
45	6,358,384	28.6 (18.8 dB)
50	6,254,805	27.3 (18.6 dB)
55	6,199,171	26.6 (18.4 dB)
60	6,052,245	24.4 (17.9 dB)

Table 5.7: Equipment distribution - 128 ONUs/512 ONUs

# ONUs	N_c	$\underline{N}_o$	$\overline{N}_o$	# splitters		# AWGs	
				Level 1	Level 2	Level 1	Level 2
128	26	3	10	3	17	0	6
	30	3	10	3	20	0	7
	30	3	15	3	20	0	7
	35	3	15	3	23	0	9
512	45	12	30	4	24	0	17
	50	12	30	4	30	0	16
	55	12	30	4	33	0	18
	60	12	30	4	37	0	19

Chapter 6

Reliable Facility Location

In this chapter, we discuss a second application related to the facility location problem under disruption.

6.1 Problem Statement

Let I be the set of customers, J the set of potential locations for the facilities, and p the maximum number of facilities to be opened. Each customer $i \in I$ has demand D_i, and each potential location $j \in J$ has a capacity (available resources) $\textsc{cap}_j$. A limited budget is available in order to fortify some facilities, which are considered as non failable. In order to be able to overcome disruptions, each customer is assigned a primary supplier and a different backup supplier, unless the primary facility is fortified. Let $\textsc{cost}_{ij}$ be the transport cost (transport cost here means the transport distance) between facility location $j \in J$ and customer $i \in I$ (with the convention $\textsc{cost}_{ij} = 0$ for all $i \in I$). For a given facility location j with a set I_j of assigned customers, the transportation cost related to j can be written as follows:

$$\textsc{cost}_j = \textsc{cost}_j^W + \textsc{cost}_j^B \quad ; \quad \textsc{cost}_j^W = \sum_{i \in I_j^{\text{W}}} d_{ij} \quad ; \quad \textsc{cost}_j^B = \sum_{i \in I_j^{\text{B}}} d_{ij},$$

where I_j^{W} (resp. I_j^{B}) is the set of primary (resp. backup) customers assigned to facility location j.

Under the assumption of a single facility failure (we have time to recover from a facility failure before a new one occurs), the aim is to optimize the facility location and the usage of the fortification budget in order to reduce the required amount of facility resources/capacities. In the selection of the primary vs. backup vs. fortified facility locations, we will take into account the failure probability $0 \leq q_j \leq 1$ of each facility location j.

The setup cost s_j is a fixed cost required to implement facility fortification (the costs of contract negotiation, overhead, personnel training, etc.). The variable fortification cost varies with the amount of reliability improvement of the facility (the cost of acquiring and installing the units of protective measures, the cost of procurement and storage of backup inventory, and the cost of hiring extra workforce, etc). We define r_j as the cost associated with the unit reduction in the failure probability of facility j. The total available fortification budget is equal to B.

In this thesis, we propose a first exact mathematical model that combines fortification and backup facilities (Section 6.2). We also investigate the resource saving if we allow resource sharing in the event of a disruption in Section 6.3.

6.2 Capacitated Reliable Facility Problem

We propose a new decomposition model which relies on facility configurations, where each configuration is associated with a potential facility location, and a potential set of users for which the facility is either a primary or a backup supplier. More precisely, let j be a potential facility location. A configuration c associated with j is formally defined by two sets of parameters: $a_i^{\text{W},c}, a_i^{\text{B},c} \in \{0,1\}$ such that $a_i^{\text{W},c} = 1$ (resp. $a_i^{\text{B},c} = 1$) if customer i uses the facility of configuration c as a primary (resp. backup) facility location, 0 otherwise.

We will use three sets of decision variables:

$z_c = 1$ if configuration c is selected in the optimal solution, 0 otherwise.

$x_j = 1$ if facility location is selected and fortified, 0 otherwise.

$y_j = 1$ if facility location j is open and is used as a primary (y_j^W) or a backup (y_j^B) supplier, 0 otherwise.

We can now state the mathematical model, called Model DFL, for the capacitated location problem under disruption, assuming a fortification budget and backup suppliers.

6.2.1 Master Problem

For a given configuration $c \in C_j$, let

$$\text{COST}^c = \text{COST}^{\text{W},c} + \text{COST}^{\text{B},c} \quad \text{with} \quad \text{COST}^{\text{W},c} = \sum_{i \in I} a_i^{\text{W},c} d_{ij} \; ; \; \text{COST}^{\text{B},c} = \sum_{i \in I} a_i^{\text{B},c} d_{ij}.$$

COST^c represents the total travel cost in each configuration including working cost and backup cost.

[DFL] $\quad \min \quad \sum_{c \in C} \text{COST}_c \, z_c$

subject to:

$$\sum_{c \in C_j} z_c = y_j \qquad j \in J \qquad (6.1)$$

$$x_j \leq y_j \qquad j \in J \qquad (6.2)$$

$$\sum_{j \in J} y_j \leq p \qquad (6.3)$$

$$\sum_{c \in C} a_i^{\text{W},c} z_c = 1 \qquad i \in I \qquad (6.4)$$

$$\sum_{c \in C_j} a_i^{\text{W},c} z_c + \sum_{c \in C} a_i^{\text{B},c} z_c \leq 2 - x_j \qquad i \in I, j \in J \qquad (6.5)$$

$$\sum_{c \in C} a_i^{\text{B},c} z_c \geq \sum_{c \in C_j} a_i^{\text{W},c} z_c - x_j \qquad i \in I, j \in J \qquad (6.6)$$

$$\sum_{j \in J} (s_j + r_j q_j) x_j \leq B \qquad (6.7)$$

$$y_j \in \{0, 1\} \qquad j \in J \qquad (6.8)$$

$$x_j \in \{0, 1\} \qquad j \in J \qquad (6.9)$$

$$z_c \in \{0, 1\} \qquad c \in C. \qquad (6.10)$$

Constraints (6.1) checks whether location j is open for a facility that will be used as a primary or a backup supplier. If $y_j = 0$, no facility is open in location j. If $y_j = 1$, one facility is opened in location j, and we make sure to select exactly one primary/backup facility configuration in location j. Constraints (6.2) ensure that the values of x_j and y_j are consistent, i.e., that $x_j = 0$ if $y_j = 0$ (no facility in location j). Constraint (6.3) sets the limit on the number of facilities. Constraint (6.4) guarantee that each user i is assigned to a primary supplier. Constraints (6.5) and (6.6) guarantee that each user i is assigned to a backup supplier, if its primary supplier is not a fortified facility. We now justify those last constraints in detail. Indeed, consider (6.5) for the facility location $j^{\text{W}}(i)$ that is the primary supplier of user i. Then, $\sum\limits_{c \in C_{j^{\text{W}}(i)}} a_i^{\text{W},c} z_c = 1$. Consequently, if $x_{j^{\text{W}}(i)} = 1$, then $\sum\limits_{c \in C} a_i^{\text{B},c} z_c = 0$. On the other hand, if $x_{j^{\text{W}}(i)} = 0$, then according to constraint (6.6), then if $\sum\limits_{c \in C} a_i^{\text{B},c} z_c = 1$, meaning that user i needs a backup facility. Note that, due to the constraints in the pricing problem, a given facility location cannot be used both as a primary and a backup supplier. Constraints (6.7) enforce the budget constraint on the selection of fortified facilities.

In order to favor solutions such that *(i)* the primary facility has a lower probability failure than the backup facility if no fortification occurs for a given primary facility, and *(ii)* fortification is done for facility locations with a higher failure probability, we consider the following modified model in which two penalized terms are introduced in the objective:

$$[\text{DFL\_P}] \qquad \min \sum_{c \in C} \text{COST}_c \, z_c + \text{PENAL}_1 \times \sum_{j \in J} q_j \left(\sum_{c \in C_j} a_i^{\text{W},c} z_c \right) - \text{PENAL}_2 \times \sum_{j \in J} q_j x_j .$$

6.2.2 Pricing Problem

We now write the pricing problem (PP_j) for potential facility location j.

Let $u_j^{(6.1)} \lessgtr 0$, $u_i^{(6.4)} \lessgtr 0$, $u_{ij}^{(6.5)} \leq 0$, and $u_{ij}^{(6.6)} \geq 0$, be the values of dual variables associated with constraints (6.1), (6.4), (6.5), (6.6), respectively.

$$\min \overline{\text{COST}}_j^{\text{PP}^{\text{NO\_SHARE}}} = \text{COST}_j - u_j^{(6.1)} - \sum_{i \in I} u_i^{(6.4)} a_i^{\text{W}} - \sum_{i \in I} u_{ij}^{(6.5)} (a_i^{\text{B}} + a_i^{\text{W}}) - \sum_{j' \in J : j' \neq j} \sum_{i \in I} u_{ij'}^{(6.5)} a_i^{\text{B}}$$
$$- \sum_{i \in I} u_{ij}^{(6.6)} (a_i^{\text{B}} - a_i^{\text{W}}) - \sum_{j' \in J : j' \neq j} \sum_{i \in I} u_{ij'}^{(6.6)} a_i^{\text{B}}$$

subject to:

$$\sum_{i \in I} D_i \, (a_i^{\text{W}} + a_i^{\text{B}}) \leq \text{CAP}_j \tag{6.11}$$

$$a_i^{\text{W}} + a_i^{\text{B}} \leq 1 \qquad i \in I \tag{6.12}$$

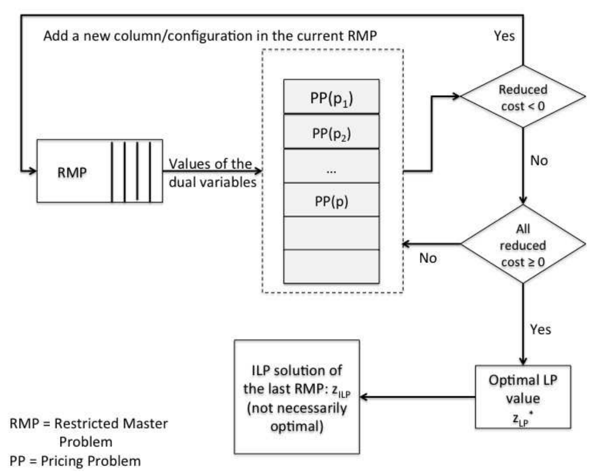
$$a_i^{\text{W}}, a_i^{\text{B}} \in \{0, 1\} \qquad i \in I. \tag{6.13}$$

Constraint (6.11) enforce the differentiated facility location capacity constraints. Note that we have an overestimation of the capacity under the assumption of a single facility location. Because for the backup cost, under the normal condition, it will not use all of its capacity, so there is an overestimation of the capacity.

6.2.3 Solution Process

The flowchart of the column generation solution is depicted in Figure 6.1. Note that there are different pricing problems, and a good strategy is to solve them in a round robin order. In other words, memorize the last pricing problem that was solved, and next time a pricing problem has to be solved, solve the next one (with respect to the last pricing problem we solved) in the order in which we have stored the pricing problems.

Fig. 6.1: Column generation

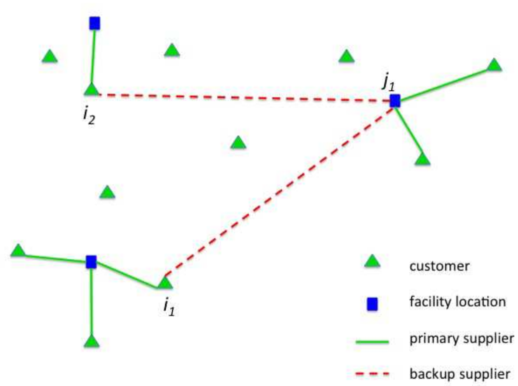


The next section is the shared backup facility problem. Under the assumption that at most one supplier will fail at a time, and that we have the time to fix the failure of a failure before another one occurs

6.3 Shared Backup Facility Problem

In the previous model, capacity requirement is overestimated as it does not take into account that under the assumption of single facility failure, backup resource can be shared. Let us have a look to the example depicted in Figure 6.2. Therein, users i_1 and i_2 have different primary suppliers, while they share the same backup supplier (j_1). Under the assumption that at most one supplier will fail at a time, and that we have the time to fix the failure of a failure before another one occurs, backup resources of i_1 and i_2 can be shared, i.e., instead of requiring $D_1 + D_2$ with respect to facility location j_1, $\max\{D_1, D_2\}$ suffice.

Fig. 6.2: Sharing Backup Resources



We will next explain how to modify the optimization models of Section 6.2 in order to take into account the sharing of the backup resources.

6.3.1 Shared protection assuming the primary/working elements are known

Variables and Parameters

$z_c \in \{0, 1\}$. $z_c = 1$ if configuration c is selected in the optimal solution, 0 otherwise.

$x_j \in \{0, 1\}$. $x_j = 1$ if facility location is selected and fortified, 0 otherwise.

$y_j \in \{0, 1\}$. $y_j = 1$ if facility location j is open and is used as a primary or a backup supplier, 0 otherwise.

Q_j^{B} Backup capacity for facility J

Before formulating this model, we have tried to solve the backup resources sharing problem with both primary facilities and backup facilities are unknown. However, we find the mathematical model is not scalable and hard to be solved. So we assume here that location and user assignment to the primary facilities are known, through the coefficients a^{W}_{ij}: equal to 1 if user i is assigned to a primary facility located in j, 0 otherwise. Moreover,

$$Q^{\text{W}}_j = \sum_{i \in I} a^{\text{W}}_{ij}\, D_i.$$

Base on the solution of DFL, we can do a precalculation: for a^{W}_{ij}=1 then y_j=1 according to connected user to j as the primary working path. Therefore, we can observe the number of y_j that are selected, then we can know the occupied facilities in p of constraints (6.17). The new input variable for this model is the working path connected between user and facility, the working/backup capacity and capacity limit for each facility. The output is to get the shared backup path connection. For a given facility location j, let C_j be the set of facility location j configurations, where $c \in C_j$ is characterized by the set of users assigned to a facility in location j with $a^{\text{B},c}_i \in \{0, 1\}$. $a^{\text{B},c}_i = 1$ if customer i uses the facility of configuration c as a backup facility location, 0 otherwise.

The objective corresponds to the minimization of the backup cost (total backup distance) and increase the failure probability in the fortified facility.

$$\min \sum_{c \in C} \text{COST}^B_c z_c - \text{PENAL}_2 \times \sum_{j \in J} q_j x_j \tag{6.14}$$

subject to:

$$\sum_{c \in C_j} z_c = y_j \qquad j \in J \tag{6.15}$$

$$x_j \leq y_j \qquad j \in J \tag{6.16}$$

$$\sum_{j \in J} y_j \leq p \tag{6.17}$$

$$\sum_{c \in C} a^{\text{B},c}_i z_c + x_j \geq a^{\text{W}}_{ij} i \in I, j \in J \tag{6.18}$$

$$\sum_{c \in C} a_i^{\mathrm{B},c} z_c + x_j \leq 2 - a_{ij}^{\mathrm{W}} \qquad i \in I, j \in J \tag{6.19}$$

$$\sum_{j \in J} (s_j + r_j q_j) x_j \leq B \tag{6.20}$$

$$Q_j^{\mathrm{W}} + Q_j^{\mathrm{B}} \leq \mathrm{CAP}_j \qquad j \in J \tag{6.21}$$

$$\sum_{i \in I} D_i a_{ij}^{\mathrm{W}} \sum_{c \in C_{j'}} a_i^{\mathrm{B},c} z_c \leq Q_j^{\mathrm{B}} \qquad j, j' \in J : j \neq j' \tag{6.22}$$

$$Q_j^{\mathrm{B}} \geq 0 \qquad j \in J \tag{6.23}$$

$$y_j \in \{0,1\} \qquad j \in J \tag{6.24}$$

$$x_j \in \{0,1\} \qquad j \in J \tag{6.25}$$

$$z_c \in \{0,1\} \qquad c \in C. \tag{6.26}$$

Constraints (6.15) checks whether location j is open for a facility that will be used as a primary or a backup supplier. If $y_j = 0$, no facility is open in location j. If $y_j^w = 1$, one facility is opened in location j, and we make sure to select exactly one primary/backup facility configuration in location j. Constraints (6.16) ensure that the values of x_j and y_j are consistent, i.e., that $x_j = 0$ if $y_j = 0$ (no facility in location j). Constraint (6.17) sets the limit on the number of facilities. Constraints (6.18) and (6.19) guarantee that each user i is assigned to a backup supplier, if its primary supplier is not a fortified facility. We now justify those last constraints in detail. Indeed, consider (6.19) for the facility location $j^{\mathrm{W}}(i)$ that is the primary supplier of user i. Then, $\sum\limits_{c \in C_{j^{\mathrm{W}}(i)}} a_i^{\mathrm{W},c} z_c = 1$. Consequently, if $x_{j^{\mathrm{W}}(i)} = 1$, then $\sum\limits_{c \in C} a_i^{\mathrm{B},c} z_c = 0$. On the other hand, if $x_{j^{\mathrm{W}}(i)} = 0$, then according to constraint (6.18), then if $\sum\limits_{c \in C} a_i^{\mathrm{B},c} z_c = 1$, meaning that user i needs a backup facility. Note that, due to the constraints in the pricing problem, a given facility location cannot be used both as a primary and a backup supplier. Constraints (6.20) is the fortification budget constraint for the selection of fortified facilities. Constraints (6.21) ensure that the sum of working path and backup path capacity of a facility can not exceed its capacity limit. Constraints (6.22) indicates the backup path capacity of a facility is greater or equal to another associated backup path capacity and its original total working demand. Constraints (6.23) force the backup path capacity of a facility be greater or equal to 0.

6.3.2 Pricing Problem

The pricing problem is modified as follows. We now write the pricing problem (PP_j) for potential facility location j.

Let

$u_j^{(6.15)} \lesseqgtr 0$, $u_{ij}^{(6.18)} \geq 0$, $u_{ij}^{(6.19)} \leq 0$, and $u_{ij}^{(6.22)} \leq 0$

be the values of dual variables associated with constraints

(6.15), (6.18), (6.19) and (6.22)

respectively.

$$\min \overline{\text{COST}_j^{\text{SHARED}}} = \text{COST}_j - u_j^{(6.15)} - \sum_{i \in I} u_{ij}^{(6.18)} a_i^{\text{B}} - \sum_{i \in I} u_{ij}^{(6.19)} a_i^{\text{B}}$$
$$- \sum_{i \in I} \Big(\sum_{j' \in J: j' \neq j} u_{ij'}^{(6.19)} a_i^{\text{B}} \Big) - \sum_{i \in I} D_i a_{ij}^{\text{W}} \sum_{j' \in J: j' \neq j} u_{jj'}^{(6.22)} a_i^{\text{B}}$$

subject to:

$$a_{ij}^{\text{W}} + a_i^{\text{B}} \leq 1 \qquad i \in I \tag{6.27}$$

$$a_i^{\text{B}} \in \{0, 1\} \qquad i \in I. \tag{6.28}$$

Constraints (6.27) ensure that a customer can not have a facility act both as primary facility and backup facility at the same time.

Chapter 7

Computational Results and Analysis for Reliable Facility Location

7.1 Data Sets

Models and algorithms proposed in the previous sections were tested on two data sets taken from [73], which can be found in the online appendix of [20], with 49 and 88 users, with $m = n$, together with their demand and distance values. We assume that the transportation cost is proportional to the Euclidean distances. We generated the location capacity values as follows. Let $\overline{D} = \sum_{i \in I} D_i/p$ be the average demand per facility location (under the assumption there are p facilities and load is balanced among the facilities), where the demand values are taken from [73] p=5, 10, 20. Then, for each potential facility location j, we computed the Q_j capacity value as a randomly generated value in the interval $[2\overline{D}, 2.2\overline{D}]$. The fixed cost s_j based on [73] are drown from $U \sim [500, 1500]$ and rounded to nearest integer. Following [50], the hardening cost is set as follows: $r_j = 0.2 \times s_j$. q_j the probability of failure is generated randomly by uniform distribution $U \sim [0, 0.05]$.

PENAL$_1$ and PENAL$_2$ depend on the problem size and cost are changing and are indicated later for each experiments.

The budget for fortification is calculated as follows:

$$\overline{B} = \sum_{j \in J} (s_j + r_j q_j).$$

The fortification budget depends on the percentage of open facilities we decide to fortify (f is the percentage of facilities we decide to fortify). In this case, p is an upper bound on the percentage of resource saving number of facilities to open and the budget is:

$$B = \frac{\overline{B}}{f \times p}.$$

We denote it by $RS\%$. It is calculated as follows:

$$RS\% = (1 - (Q^{\text{B}}/Q^{\text{W}})) \times 100$$

where Q^{B} *and* Q^{W} are as defined in section 6.3.1.

7.2 Generation of Initial Solutions

We provide an initial set of columns in order to raise the solution of the proposed model.

In model DFL, each column is associated to a facility location, together with its set of assigned users, for which it is either a primary or a backup facility. First, we order the facility locations in the increasing order of their q_j values. Taking into account the budget constraint, fortify as many locations as possible, considering the facility location in the increasing order of their fortification probabilities. For the fortified facility locations, assign as many primary users as possible taking into account the facility capacity constraints. If some users are still without a primary supplier, assign them in priority with the remaining unfortified facility location with the smallest q_j.

In Model DFL_S, the results for primary allocation (user primary assignment $a_i^{\text{B},c}$) are deduced from the solution of DFL and denoted by a_{ij}^{W}. In this case, the primary assignment is given and we are looking for the optimal allocation for backup facility if the primary one is not fortified, and for the selection of the fortified facilities. To build an initial solution, we rank the selected facilities for primary supplier based on their failure probability. Then, we fortify facilities with highest probabilities until the budget constraint is exhausted. We assign a back_up supplier for those users with no fortified primary facilities.

7.3 Algorithm Performances

7.3.1 Model DFL: No Resource Sharing

We first report on the computational times and the accuracies of the solutions. Results are summarized in Table 7.1 for DFL. The first column is number of potential facility location (N). In the second column, p is an upper bound on the the number of facilities which can be opened. $z^{\star}_{\text{LP}}$ is linear optimal solution for the DFL+ problem. In Model DFL, $\tilde{z}^{\text{W}}_{\text{ILP}}$ is the integer optimal solution for primary user allocation and $\tilde{z}^{\text{B}}_{\text{ILP}}$ the integer optimal solution for backup user allocation. The gap calculated based on the formula: $\frac{(\tilde{z}^{\text{W}}_{\text{ILP}}+\tilde{z}^{\text{B}}_{\text{ILP}})-z^{\star}_{\text{LP}}}{(\tilde{z}^{\text{W}}_{\text{ILP}}+\tilde{z}^{\text{B}}_{\text{ILP}})}\%$. Through the columns 7 to 9 in the Table 7.1, the number of initial generated, and selected columns are shown. the CPU usage (time to solve) is shown in last column.

As it is shown the model in larger size problem behave better than in small size one and the gap is less. But the time and the number of generated columns is more in large size problem because there are more option to generate column and select.

The optimal solution (transportation cost) either LP or ILP is decreasing when p is increasing both 49 and 88 nodes. We analyze the set of columns and distinguish the initial set of columns when I denotes their number, the set of generated columns (# I is Initial Columns, # G is Generated Columns, # S is Selected Columns, # F is Fortified Facilities). The meaning of $z_{1_{\text{LP}}}$ and $z_{2_{\text{LP}}}$ I will explain it in Section 7.4 Penalty Analysis.

7.3.2 Model DFL_S: Resource Sharing

The results for model DFL_Sare summarized in Table 7.2. In this case we are looking into backup allocation based on the primary assignment. In comparison between $\tilde{z}_{\text{ILP}}$ in Table 7.2 and $\tilde{z}^{\text{B}}_{\text{ILP}}$ in Table 7.1 shows that by increasing p the optimal solution will decrease in both test cases and also the gap is increased in the second model (DFL_S).

Comparison in terms of required resources is done in Section 7.6.

We next investigate the resource saving when sharing the backup resources. Results are summarized in Table 7.3. Again, in the first two columns, we provide the number of nodes and p value is shown in first and second columns. In Columns 3, Q^{W} is the total primary

		DFL_P			DFL+					# columns			CPU
N	p	COST	$z_{1_{\text{LP}}}$	$z_{2_{\text{LP}}}$	$\tilde{z}_{\text{ILP}}$	$z^{\star}_{\text{LP}}$	$\tilde{z}^{\text{W}}_{\text{ILP}}$	$\tilde{z}^{\text{B}}_{\text{ILP}}$	gap (%)	I	G	S	sec.
49	5	198.6	7.8	2.1	203.8	200.1	141.6	62.2	1.8%	5	394	5	1103
49	10	173.8	5.4	2.7	178.3	174.6	140.6	37.7	2.1%	10	556	10	1589
49	20	119.0	5.8	4.6	123.2	119.3	94.4	28.7	3.2%	20	689	20	1804
88	5	630.9	27.3	4.4	643.9	636.5	489.7	154.2	1.1%	5	512	5	2537
88	10	564.8	21.6	7.6	581.7	578.8	504.9	76.8	0.5%	10	637	10	3203
88	20	529.7	19.8	11.3	537.7	532.3	492.5	45.2	1.0%	20	809	17	3872

$\text{PENAL}_1 = 0.1, \text{PENAL}_2 = 1.3$

Table 7.1: Computational Times and Solution Accuracies (Model DFL)

capacity which is needed to support all users as primary supplier. The results in Columns 4 to 7 is related to first model (DFL) as follows: In Column 4, Q^{B} is the total backup capacity which is needed to support those users need to be supported as backup. In column 5, provide $RS\%$ and the number of fortified facilities is shown in Column 6. Total needed capacity is shown in Column 7 for model DFL. It is the same for model DFL_S in columns 8 to 11.

We solved the experimental test cases with same amount of fortification and the same budget of fortification. The results show that in small size test cases the model DFL, and model DFL_S, are performing almost same while in case of 49 nodes and $p = 10$ the model DFL_S, perform better than model DFL, and there are more resource saving.

The results show that in large size test cases the model DFL_S, performs better than model DFL. the percentage of resource saving is increasing with increasing of p value. The resource saving is significant in case of 88 nodes and $p = 20$. the total needed backup capacity is 258.2 and the resource saving percentage is 96.86% in case of model DFL_S, which in case of model DFL, it is 1,901.1 and 76.86% mean while the number of fortified facilities is the same and it is equal 9.

Nodes	p	COST	$z^{\star}_{2_{\text{LP}}}$	$z^{\star}_{\text{LP}}$	$\bar{z}^{\text{B}}_{\text{ILP}}$	gap (%)	# columns			CPU
							I	G	S	(sec.)
49	5	62.2	0	62.2	62.2	0.0%	5	0	0	0
49	10	27.9	3.1	24.8	26.3	5.8%	10	72	4	394
49	20	22.8	3.7	19.1	19.9	3.7%	20	83	5	433
88	5	154.2	0	154.2	154.2	0.0%	5	0	0	0
88	10	40.7	0.4	40.3	41.3	3.5%	10	102	3	539
88	20	28.2	0.9	27.3	28.6	4.6%	20	87	2	435

Table 7.2: Computational Times and Solution Accuracies (Model DFL_S)

7.4 Penalty Analysis

In this section we calculate the optimal solution once without penalties and once including penalties. In the equation 7.1, the objective function include three elements. The first element is calculating the transportation cost (distances) based on the selected configurations. The second elements in this objective function is for favoring selecting those facilities as primary with lower failure probability. The third element is for prioritizing of fortification of those facilities with higher failure probability. The second and third elements of objective function are normalized using PENAL_1 and PENAL_2.

$$\min \sum_{c \in C} \text{COST}_c z_c + \text{PENAL}_1 \underbrace{\Big(\sum_{j \in J} q_j \Big(\sum_{c \in C_j} a_i^{\text{W},c} z_c\Big)\Big)}_{z_{1_{\text{LP}}}} - \text{PENAL}_2 \underbrace{\Big(\sum_{j \in J} q_j x_j\Big)}_{z_{2_{\text{LP}}}}. \tag{7.1}$$

To analyze the effect of the penalties on the objective, we remove z_1, z_2 and add them to constraints (like changing objective 7.2 to objective 7.3 plus constraints 7.4, 7.5. Constraints 7.4, 7.5 have the same effect like z_1, z_2 in the objective function). In order to generate constraints 7.4, 7.5, we first need to know the upper bound and lower bound for z_1 and z_2 respectively. To calculate upper bound for this constraints, we have two terms related to PENAL_1 and PENAL_2 and we first consider them equal zero and then we consider a random

N	p	Q^{W}	DFL Q^{B}	$RS\%$	#F	Q	DFL_S Q^{B}	$RS\%$	#F	Q
49	5	247,051.3	96,409.7	60.9%	3	343,461.0	96,409.7	60.9%	3	343,461
49	10	247,051.3	115,399.9	53.3%	6	362,451.2	60,947.6	75.3%	6	308,026
49	20	247,051.3	30,185.8	87.8%	13	277,237.1	31,413.7	87.3%	13	278l,465
88	5	8,213.9	3,143.5	61.3%	3	11,357.4	3,143.5	61.3%	3	11,357.4
88	10	8,213.9	2,238.4	72.8%	6	10,452.3	1083.4	86.8%	6	9297.3
88	20	8,213.9	1,901.1	76.9%	9	10,115.0	258.2	96.8%	9	8472.1

Table 7.3: Resource Saving

value for them like 0.1 and 1.3. For instance, model DFL+ is similar to constraints 7.3, when we change the objective function from equation 7.3 to 7.2 and remove constraints 7.4, 7.5, then we can calculate the value of z_1, z_2.

$$\min \; g + \text{PENAL}_1 * z_1 + \text{PENAL}_2 * z_2 \tag{7.2}$$

$$\min \; g \tag{7.3}$$

subject to

$$\underline{z}_2 \leq z_2 \tag{7.4}$$

$$\overline{z}_1 \geq z_1 \tag{7.5}$$

We also did the penalty analysis with different PENAL value of model DFL in Table 7.4.

7.5 No Fortification Budget

In this section, we consider a situation that there is no budget for the enterprise to fortify the facilities. In this case we need both primary and backup facility supplier for each user.

P_1	P_2	Nodes	p	DFL_P		DFL+			DFL_P	DFL+
				$z^{\star}_{\text{LP}}$	$\tilde{z}_{\text{ILP}}$	$z^{\star}_{\text{LP}}$	$\tilde{z}^{\text{W}}_{\text{ILP}}$	$\tilde{z}^{\text{B}}_{\text{ILP}}$	gap(%)	gap/z(%)
0.1	1.3	49	5	204.3	208.1	200.1	141.6	62.2	1.8%	1.7%
		49	10	176.5	180.9	174.6	140.6	37.7	2.1%	2.4%
		49	20	120.2	123.1	119.3	94.4	28.7	3.2%	2.3%
0.7	6.5	49	5	218.9	221.1	201.6	141.6	62.2	1.8%	0.99%
		49	10	191.3	195.6	173.8	140.6	37.7	2.5%	2.1%
		49	20	139.0	141.6	121.7	94.4	28.7	1.2%	1.8%
7	65	49	5	295.3	309.3	258.7	196.5	76.2	5.1%	4.5%
		49	10	255.7	264.2	219.9	178.7	58.6	7.3%	3.2%
		49	20	187.9	197.8	152.3	121.8	41.4	6.6%	5.0%
7	0	49	5	360.2	371.1	241.9	182.2	69.2	3.7%	2.9%
		49	10	323.8	336.8	205.7	164.3	52.6	5.1%	3.8%
		49	20	255.4	262.3	140.4	108.6	35.4	2.5%	2.6%
0	65	49	5	132.7	143.5	226.1	161.4	73.8	3.8%	7.5%
		49	10	97.1	110.6	189.4	153.6	48.1	5.4%	12.2%
		49	20	47.7	50.4	137.4	109.5	34.3	4.5%	5.3%

Table 7.4: Results under different Penalty value ($PENAL_1$, $PENAL_2$)

In Table 7.5 and Table 7.6, we calculated the optimal solution and needed capacity for two different models DFL and DFL_S. Indeed by setting fortification budget equal zero we should have a backup supplier for each user.

From the result, we can see when B = 0, the model requires more time to get the optimal results, meanwhile it generate more columns than the model which have the fortification budget. For the backup cost in the result, we can see a dramatically increase compared to the previous model (with budget), because the model need to ensure each user have a working path and a backup path under the no budget situation.

Nodes	p	$z^{\star}_{\text{LP}}$	$\tilde{z}^{\text{W}}_{\text{ILP}}$	$\tilde{z}^{\text{B}}_{\text{ILP}}$	gap (%)	# I	# G	# S	CPU times (sec.)
49	5	272.159	139.445	135.706	1.01%	5	769	5	2631
49	10	235.878	113.523	126.475	1.59%	10	924	10	3371
49	20	166.616	91.182	80.920	2.97%	20	1120	20	4204
88	5	925.752	531.223	402.241	0.81%	5	972	5	3966
88	10	880.378	501.272	386.952	0.78%	10	1267	10	4839
88	20	787.145	486.117	306.289	0.67%	20	1576	20	6034

Table 7.5: Computational Times and Solution Accuracies (Model DFL) when *B=0*

7.6 Travel Cost under Different Assumptions

In this section, we have the comparison of different results in Table 7.7. As we can see, the result in B = 0 is greater than the result with fortification budget in no sharing case because no fortification result need a full backup path to protect working path. Thus it increases the total cost. The results with fortification budget in no sharing case is greater or equal to the results with fortification budget in sharing case because we can do the backup sharing as we mentioned before.

For the p = 5 in 49, 88 nodes, the backup cost is same in both sharing and no sharing case. Because we find in the result, there are 3 fortified facilities, that means the left 2 facilities share the working and backup path for the given users. So the backup cost did not reduce. Moreover, we can observe that in these three situations, the overall cost are all reduced with the increase of p value.

7.7 Different Objective Functions

In this section we analyze the impact of different objective functions in the model DFL. In the sequel we propose different objective function and we analyze the different solution based on the different objectives.

Nodes	p	Q^{W}	DFL			DFL_S		
			Q^{B}	RS%	Q	Q^{B}	RS%	Q
49	5	247,051.3	247,051.3	0%	494,102.6	112,804.5	54.3%	359,855.8
49	10	247,051.3	247,051.3	0%	494,102.6	81,461.3	67 %	328,512.6
49	20	247,051.3	247,051.3	0%	494,102.6	59,739.2	75.8%	306,790.5
88	5	8,213.9	8,213.9	0%	16,427.8	3,982.6	51.5%	12,196.5
88	10	8,213.9	8,213.9	0%	16,427.8	2,801.4	65.9%	11,015.3
88	20	8,213.9	8,213.9	0%	16,427.8	2,103.9	74.4%	10,317.8

Table 7.6: No Facility Fortification Resource Saving

7.7.1 Objective 1

Minimization of transportation cost

The objective function is based on the model in Section 6.2. There in the objective is considered based on the distance or transportation cost which we call it $Z^{\overline{Obj1}}$ in this section.

$$Z^{\overline{Obj1}} = \sum_{c \in C} \mathrm{COST}_c z_c \tag{7.6}$$

where

$$\mathrm{COST}_{c_j} = \sum_{i \in I} (a_i^{\mathrm{W}} + a_i^{\mathrm{B}}) d_{ij}.$$

7.7.2 Objective 2

Minimization of resource usage

The objective function in this section is based on capacity usage which we call it $Z^{\overline{Obj2}}$.

$$Z^{\overline{Obj2}} = \sum_{j \in J} (Q_j^{\mathrm{W}} + Q_j^{\mathrm{B}}) \tag{7.7}$$

		sharing			no sharing					
		Forti. budget			Forti. budget			$B=0$		
Nodes	p	$\tilde{z}^{\text{W}}_{\text{ILP}}$	$\tilde{z}^{\text{B}}_{\text{ILP}}$	$z^{\star}_{\text{ILP}}$	$\tilde{z}^{\text{W}}_{\text{ILP}}$	$\tilde{z}^{\text{B}}_{\text{ILP}}$	$z^{\star}_{\text{ILP}}$	$\tilde{z}^{\text{W}}_{\text{ILP}}$	$\tilde{z}^{\text{B}}_{\text{ILP}}$	$z^{\star}_{\text{ILP}}$
49	5	141.6	62.2	203.8	141.6	62.2	203.8	139.4	135.7	275.1
49	10	140.6	26.3	166.9	140.6	37.7	178.3	113.5	126.5	240.0
49	20	94.4	19.9	114.3	94.4	28.7	123.2	91.2	80.9	172.1
88	5	489.7	154.2	643.9	489.7	154.2	643.9	531.2	402.2	933.4
88	10	504.9	41.3	546.2	504.9	76.8	581.7	501.3	386.9	888.2
88	20	492.5	28.6	521.1	492.5	45.2	537.7	486.1	306.3	792.4

Table 7.7: Comparison of Result With/Without Fortification Cost

Where

$$Q^{\text{W}}_j = \sum_{c\in C_j} \sum_{i\in I} D_i a^{\text{W},c}_i z_c$$

$$Q^{\text{B}}_j = \sum_{c\in C_j} \sum_{i\in I} D_i a^{\text{B},c}_i z_c$$

The pricing problem is modified as follows. We now write the pricing problem (PP$_j$) for potential facility location j. Let $u^{(6.15)}_j \lessgtr 0$, $u^{(6.18)}_{ij} \geq 0$, $u^{(6.19)}_{ij} \leq 0$, and $u^{(6.22)}_{ij} \leq 0$ be the values of dual variables associated with constraints (6.15), (6.18), (6.19) and (6.22) respectively.

$$\min \overline{\text{COST}^{\text{SHARED}}_j} = \text{COST}_j - u^{(6.15)}_j - \sum_{i\in I} u^{(6.18)}_{ij} a^{\text{B}}_i - \sum_{i\in I} u^{(6.19)}_{ij} a^{\text{B}}_i - \sum_{i\in I} \Big(\sum_{j'\in J: j'\neq j} u^{(6.19)}_{ij'} a^{\text{B}}_i \Big)$$
$$- \sum_{i\in I} D_i a^{\text{W}}_{ij} \sum_{j'\in J: j'\neq j} u^{(6.22)}_{jj'} a^{\text{B}}_i$$

subject to:

$$a_{ij}^{\mathrm{W}} + a_i^{\mathrm{B}} \leq 1 \qquad i \in I \tag{7.8}$$

$$a_i^{\mathrm{B}} \in \{0,1\} \qquad i \in I. \tag{7.9}$$

Where

$$\mathrm{COST}_j = \sum_{c\in C_j} \sum_{i\in I} D_i a_i^{\mathrm{W},c} z_c + \sum_{c\in C_j} \sum_{i\in I} D_i a_i^{\mathrm{B},c} z_c.$$

We summarized the different solutions for 49/88 nodes and $p = 10$ in Table 7.8.

Node		$Z^{\overline{Obj1}}$	$Z^{\overline{Obj2}}$
49	$dist^{\mathrm{W}}$ primary distance	140.6	152.3
	$dist^{\mathrm{B}}$ backup distance	37.7	74.8
	Q^{W} primary resource	247,051.3	247,051.3
	Q^{B} backup resource	115,399.9	87,524.9
88	$dist^{\mathrm{W}}$primary distance	504.897	537.124
	$dist^{\mathrm{B}}$ backup distance	76.834	110.507
	Q^{W} primary resource	8,213.9	8,213.9
	Q^{B} backup resource	2,238.4	1,655.5

Table 7.8: Different objective function solutions for 49 and 88 nodes test case

From Table 7.8, we observe that results influenced by different objective functions, the impact of total cost and total resource are interrelated. The two objective mainly focus on the backup cost/resource minimization, because all of the user must have a working facility, so we can see for the working cost/resource, there are not much different. We can observe that the total cost in $Obj1$ is nearly 21.6% reduction to $Obj2$. As for the resource usage, $Obj2$ have 7.69% reduction compared to $Obj1$, which are as we expected.

Chapter 8

Conclusions and Future Work

In this thesis, we discussed two different location problems, and analyze the implementation and results. Conclusions and future work of PON application and Reliable Facility Location application are presented in Section 6.1, 6.2 respectively.

8.1 PON application

In the first application of the thesis, we have incorporated both physical and optical layer constraints while devising our proposed large scale optimization scheme for hybrid PONs. We propose a novel planning scheme for the deployment of a set of hybrid PONs which optimizes the selection, location and allocation of the passive switching equipment of each PON while provisioning the unicast/multicast traffic demand of individual ONUs in a given geographical area. Our proposed scheme also determines the covered region of each PON optimally. It relies on a simple ILP mathematical model as well as an ILP with a CG model in which both of the models are formulated exploiting the principle of p-center/p-median based optimization scheme.

The proposed scheme can optimize the design of a set of hybrid PONs covering a given geographic area as well as the selection of the best cascading architecture (1/2/mixed-stage) for each selected PON. It minimizes the overall network deployment cost based on the location of the OLT and the ONUs while granting all traffic demands. The scheme emphasizes on the optimum placement of equipment in a hybrid PON infrastructure due to the critical dependency between the network performances and a proper deployment of its equipment, which, in turn depends on the locations of the users. We have also accomplished the sensitivity analysis. We investigate how the value of N_c of our proposed scheme affects the solution. Besides, we have studied the impact of N_c values on the signal power attenuation experienced and the total cost. We have also explored the effect of multicast traffic requests on the deployment cost. 2-Step Model is the most comprehensive scheme, for designing, dimensioning and planning of multiple networks in terms of minimum deployment cost. It is a quite powerful scheme as it can handle data instances with up to several thousands ONUs. On the basis of the computational results, the proposed scheme leads to an efficient automated tool for network design, planning, and performance evaluation which can be beneficial for the network designers.

In future work, we plan to expand the proposed optimization schemes in order to consider long reach (hybrid) PONs [22] as well as the placement of Base-Band Units (BBUs) [12] within the framework of an integrated broadband access network.

8.2 Reliable Facility Location application

In the second application of the thesis, we have presented a new integer-linear programming model for identifying optimal fortification strategies of supply systems in the event of intentional attacks. We also optimize the selection, location and allocation of the facility while satisfy all the covering of all the users in the geographical area with the lower total cost.

Also, we include backup sharing as well as a more accurate estimate of the required resource taking into account that not all failures occur at the same time,and that we have the time to fix the failure of a failure before another one occurs, backup resources of the same users can be shared. So we avoid the overestimated capacity requirement and deeply optimize the model and the deployment cost in this logistic network.

Moreover, for the total cost, we consider three component: distance, capacity and budget. At first we optimize them separately, and then we discuss a combination that will give the best solution to reduce the total cost. We also did the PENALTY value analysis, to see the effect of the penalties on objective and different variables. We have tested this model on two different geographical data sets, using Cplex studio. On the basis of the computational results, the proposed scheme leads to an efficient model to design the logistic network.

In this model, the facility fortification is a binary variable. However, in the real world it's impossible for a fortified facility with totally no disruption. So for future work we suggest to consider partial fortification and also partial disruption with different probability.

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