INTEGRATED TACTICAL PLANNING IN THE LUMBER SUPPLY CHAIN UNDER DEMAND AND SUPPLY UNCERTAINTY

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A thesis

in

The Department

of

Mechanical and Industrial Engineering

Presented in Partial Fulfillment of the Requirements

For the Degree of Doctor of Philosophy

Concordia University

Montréal, Québec, Canada

December 2015

Concordia University

School of Graduate Studies

This is to certify that the thesis prepared

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Entitled: **Integrated tactical planning in the lumber supply chain under demand and supply uncertainty**

and submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy (Industrial Engineering)

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Abstract

Integrated tactical planning in the lumber supply chain under demand and supply uncertainty

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Concordia University, 2016

Lumber supply chain includes forests as suppliers, sawmills as production sites, distribution centers, and different types of customers. In this industry, the raw materials are logs that are shipped from forest contractors to sawmills. Logs are then sawn to green/finished lumbers in sawmills and are distributed to the lumber market through different channels. Unlike a traditional manufacturing industry, the lumber industry is characterized by a divergent product structure with the highly heterogeneous nature of its raw material (logs). Moreover, predicting the exact amount of the product demand and the availability of logs in the forest is impossible in this industry. Thus, considering random demand and supply in the lumber supply chain planning is essential.

Integrated tactical planning in a supply chain incorporates the synchronized planning of procurement, production, distribution and sale activities in order to ensure that the customer demand is satisfied by the right product at the right time. Briefly, in this dissertation, we aim at developing integrated planning tools in lumber supply chains for making decisions in harvesting, material procurement, production, distribution, and sale activities in order to obtain a maximum robust profit and service level in the presence of uncertainty in the log supply and product demand. In order to gain the latter objectives, we can categorize this research into three phases. In the first phase, we investigate the integrated annual planning of harvesting, procurement, production, distribution, and sale activities in the lumber supply chain in a

deterministic context. The problem is formulated as a mixed integer programming (MIP) model. The proposed model is applied on a real-size case study, which leads to a large-scale MIP model that cannot be solved by commercial solvers in a reasonable time. Consequently, we propose a Lagrangian Relaxation based heuristic algorithm in order to solve the latter MIP model. While improving significantly the convergence, the proposed algorithm also guarantees the feasibility of the converged solution.

In the second phase, the uncertainty is incorporated in the lumber supply chain tactical planning problems. Thus, we propose a multi-stage stochastic mixed-integer programming (MS-MIP) model to address this problem. Due to the complexity of solving the latter MS-MIP model with commercial solvers or relevant solution methodologies in the literature, we develop a Hybrid Scenario Cluster Decomposition (HSCD) heuristic algorithm which is also amenable to parallelization. This algorithm decomposes the original scenario tree into a set of smaller sub-trees. Hence, the MS-MIP model is decomposed into smaller sub-models that are coordinated by Lagrangian terms in their objective functions. By embedding an ad-hoc heuristic and a Variable Fixing algorithm into the HSCD algorithm, we considerably improve its convergence and propose an implementable solution in a reasonable CPU time.

Finally, due to the computational complexity of multi-stage stochastic programming approach, we confine our formulation to the robust optimization method. Hence, at the third phase of this research, we propose a robust planning model formulated based on cardinality-constrained method. The latter provides some insights into the adjustment of the level of robustness of the proposed plan over the planning horizon and protection against uncertainty. An extensive set of experiments based on Monte-Carlo simulation is also conducted in order to better validate the proposed robust optimization approach applied on the harvesting planning in lumber supply chains.

Preface

This dissertation has been realized under the co-direction of Professor Masoumeh Kazemi Zanjani and Professor Mustapha Nourelfath, professors at the Mechanical and Industrial Engineering Department of Concordia University, and the Mechanical Engineering Department of Université Laval, respectively. It has been prepared in "Manuscript-based" format. This research was financially supported by NSERC Strategic Network on Value Chain Optimization (VCO).

The research includes three articles, co-authored by Professor Masoumeh Kazemi Zanjani and Professor Mustapha Nourelfath. All the contents that are represented in this thesis either have been accepted or submitted for publication in the form of journal articles. In all of the presented articles, I have acted as the principal researcher and performed the mathematical models development, coding the algorithms, analysis and validation of the results, as well as writing of the first drafts of the articles. Professor Masoumeh Kazemi Zanjani and Professor Mustapha Nourelfath have revised the articles to obtain the final version prior to submission.

The first article entitles *"The value of integrated tactical planning optimization in the lumber supply chain"*, co-authored by Professor Masoumeh Kazemi Zanjani and Professor Mustapha Nourelfath, was published in the *International Journal of Production Economics* in November 2015.

The second article entitled *"A hybrid scenario cluster decomposition algorithm* *for supply chain tactical planning under uncertainty"*, co-authored by Professor Masoumeh Kazemi Zanjani and Professor Mustapha Nourelfath, was accepted (with revision) to the *European Journal of Operational Research* in November 2015.

The third article entitled *"Addressing uncertainty in forest harvesting planning through cardinality-constrained approach"*, co-authored by Professor Masoumeh Kazemi Zanjani and Professor Mustapha Nourelfath, was submitted to the *International Journal of Production Research* in October 2015.

To my beloved Mom & Dad and my dear wife,

Haleh

for their love, endless support and encouragement

Acknowledgments

I would like to extend my sincerest gratitude to my thesis advisors, Masoumeh Kazemi Zanjani and Mustapha Nour El Fath, for their supports, valuable guidance and commitments during this thesis. My thesis and my personal development have benefited greatly from their insightful and endless supports. Their immense knowledge and inspiration helped me to overcome many difficulties during my research. I feel fortunate to have had the enlightening experience to work with these enthusiastic and talented people during my time at Concordia University. I want to also thank my thesis committee members for their precious time, brilliant comments and insightful suggestions. Their suggestions have helped to make this a better thesis.

I would like to express my deepest gratitude and appreciation to my brilliant wife, Haleh, and my adored parents for their ultimate supports, patience and encouragements. Their worthwhile support and love during my PhD thesis have encouraged me to do my best to achieve my goal. I do not know how I can express my appreciation to them for their ultimate supports and grace with the words. I only hope that I have made their support worthwhile, and compensate some parts of their fortitude.

Finally, I like to thank my amazing friends during this time in Montreal and previously in Iran who were sources of inspiration and friendship to me and made all the great memories for me during my life.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

In this chapter, first we provide the motivation of this thesis. Then, we describe the problem studied. The scope and objectives, as well as the thesis organization are provided at the end of this chapter.

1.1 Research motivation

In the divergent-type production systems, several products can be produced after processing a common material. Amongst different examples, we can refer to industries that process natural resources (such as forestry, oil & gas, etc.). The variable mix of products, in addition to the existence of by-products, make the coordination of production, procurement, distribution and sale planning in such supply chains more difficult. In lumber supply chain, in particular, different types of uncertainties, namely, uncertain supply and demand make the lumber supply chain planning even more complicated.

Because of these characteristics, lumber supply chain tactical planning represents a major challenge in this industry that cannot be addressed by commonly used spreadsheet solutions. Yet, more sophisticated approaches such as optimization models would be able to better capture the aforementioned features. On the other hand, forests in Canada belong to the public sector while sawmills and distribution channels are independent private companies. Hence, tactical planning in such value chain can be addressed via either a decoupled or an integrated approach.

A decoupled planning approach, which is the current practice in the lumber industry in Canada, is implemented via solving the decoupled harvesting, procurement, production, and sale/distribution models in a sequential manner. However, due to the fact that only rough estimations of upstream entities' capacities (e.g., harvesting capacity) are considered in models corresponding to downstream entities (e.g., sawmills), sub-optimal plans in terms of supply chain profit would be highly expected. For instance, smaller quantities of products can be promised to customers, which will directly impact the supply chain revenue. In contrary, promising sale amounts that are more than capacities of sawmills would lead to considerable quantities of backlogs and increased supply chain costs.

The purpose of an integrated model is to combine supply chain functions with the goal of increasing efficiency and better connecting demand with supply, which can both improve customer service and lower costs. Consequently, by proposing a mathematical model dealing with integrated planning in different entities of lumber supply chains as well as developing efficient solution approach to solve the resulting complex integrated mathematical model for real-size instances, some important challenges in the lumber supply chain tactical planning literature will be covered. To the best of our knowledge, less effort has been done in the literature in integrating tactical decisions in the lumber supply chain planning.

The plans proposed by deterministic approaches are not realistic and robust in the presence of future uncertain events [1]. In the integrated tactical planning in lumber supply chains, lack of knowledge in the availability of logs in the forest concludes that the right quality and quantity of raw materials are not considered in the procurement plan. This uncertain supply makes the output of the production process to be different from the planned production quantities. As a consequence, the customer demand cannot usually be satisfied compared to the promised service level. Finally, the unrealistic distribution decisions resulted by uncertain production quantities and demand can lead to increased transportation cost and negative environmental effects. This is the other motivation of this research in order to develop robust integrated planning tools that protects the plan against undesirable random variations in supply chain parameters. Thus, we aim for investigating the effectiveness of implementing stochastic programming and robust optimization approaches in obtaining a robust tactical plan in this value chain.

1.2 Problem description

The lumber supply chain investigated in this research is a network that includes forests as supply entities, sawmills as production units, distribution centers (DCs), sale departments, and customers. Forests are composed of blocks that contain different families and species of trees. Two main operations in the forests are harvesting and forwarding. Through the harvesting operations in forests, the trees are cut down, piled, and thinned in the harvesting blocks. The forwarding operations collect the piles and transfer them to the storage locations in the harvesting blocks or to the adjacent forest roads. Finally, the logs are transported to sawmills by trucks. In forests, we are dealing with harvesting planning where the harvesting schedule on different blocks over the planning horizon must be determined. Wood procurement planning that incorporates decisions on the quantity and timing of ordering logs is a complicated task, as a multitude of factors must be taken into account. It is even more complex in a multiform environment, where forest stands are composed of several tree species.

Sawmills purchase logs from the forest, and then transform them to lumbers as main products, and chips or sawdust as by-products. There are three main processes in sawmills: sawing, drying, and finishing. In the sawing process, the logs are cut into different sizes of rough lumbers by different cutting patterns. In the drying process, the lumber moisture contents are reduced in large dryers or in air. In the finishing process, the lumbers are surfaced, trimmed and sorted based on customer requirements. According to the demand, some logs are shipped to the distribution centers or directly to customers after the sawing process, while others are first sent to drying and finishing processes and then are shipped to distribution centers or customers. Product shipment to customers is carried out by a number of distribution companies that use different vehicle types. Sawmills are dealing with procurement and production planning during the planning horizon.

The supply chain serves contract-based customers (e.g., construction industry and furniture manufacturers) and non-contract based customers (e.g., pulp & paper industries or the spot market). Contract-based customers sign a contract at an agreed price and quantity for a given planning horizon. Although the contract demand must be satisfied, the enterprise reserves the right of postponing some parts of agreed quantities because of capacity shortage in the demand period. Unsatisfied demand may be served in a future period as the backlog. When there is surplus capacity in sawmills, the spot market is sought to absorb the remaining capacity. Distribution centers seek a distribution plan during the planning horizon while sale departments in the lumber supply chain are looking for realistic amounts of sale that can be promised to customers.

In the lumber supply chain, the availability of raw materials in the forest cannot be forecasted with certainty. Moreover, forecasting the exact amount of demand is also impossible. Hence, considering random log supply and lumber demand is essential for robust tactical planning in the lumber supply chain planning.

To the best of our knowledge, less attempt has been done in the literature on developing an integrated tactical planning model that leads to robust plans in the presence of uncertainties.

1.2.1 Scope and objectives

According to the existing gaps in the literature, our general objective is to develop a robust tactical planning tool for the lumber supply chain in the presence of uncertainties. The specific objectives of this dissertation is described as follows:

1. To propose a comprehensive literature review on integrating harvesting, wood procurement, production, distribution, and sale decisions in the lumber supply chain

2. To formulate a mathematical programming model to coordinate the harvesting, wood procurement, production, distribution, and sale planning in the lumber supply chain

3. To develop an efficient algorithm in order to solve the proposed large-scale integrated model in 2

4. To consider the random log supply and demand and model them into the proposed integrated model in 2 by the aid of stochastic and robust optimization optimization approaches

5. To develop efficient solution strategies that are able to find high quality solutions for the resulting stochastic programming and robust optimization models

6. To explore comprehensive test instances in order to validate the proposed models and methodologies under realistic circumstances in Canadian lumber supply chains

1.2.2 Organization of the thesis

This Section outlines the layout of this thesis. This thesis consists of five chapters and organizes as follows. Chapter 1 provides the motivations and a brief description of the problem investigated in this thesis. Chapter 2 is dedicated to developing an integrated model for the annual planning of harvesting, procurement, production, distribution, and sale activities in the lumber supply chain. A mixed integer programming (MIP) model as well as an efficient algorithm to solve it is developed in this chapter. Furthermore, in order to evaluate the value of integration , the comparison of the integrated model and decoupled planning models is also provided. In order to develop a more realistic plan in lumber supply chains, we take into account uncertain log supply and demand and propose a multi-stage stochastic mixed-integer programming (MS-MIP) model corresponding to the tactical planning in the lumber supply chain provided in Chapter 3. As the MS-MIP is a complex model with no special structure, we develop a Hybrid Scenario Cluster Decomposition (HSCD) heuristic in order to solve it. Afterwards, the efficiency of the HSCD algorithm is evaluated by a set of realistic-scale test cases in this chapter. Chapter 4 attempts to provide a robust optimization method in order to address stochasticity in lumber supply chains. Hence, we formulate a robust planning model based on cardinality-constrained method and adjust the level of robustness of the proposed plan. An extensive set of experiments based on Monte-Carlo simulation is also conducted in this chapter in order to validate the proposed robust optimization approach. Finally, Chapter 5 summarizes this dissertation by providing some concluding remarks and recommendations for future works.

Chapter 2

The value of integrated tactical planning optimization in the lumber supply chain

This chapter is dedicated to the article entitled *"The value of integrated tactical planning optimization in the lumber supply chain."* It was published in the *International Journal of Production Economics* in November 2015. The version presented in the thesis is identical to the final corrected version sent to the editor for publication.

Abstract

This study investigates the integrated annual planning of harvesting, procurement, production, distribution, and sale activities in the lumber supply chain. The problem is formulated as a mixed integer programming (MIP) model in which the binary variables correspond to the harvesting schedule over the planning horizon. The proposed model is applied on a real-size case study, which leads to a large-scale MIP model that cannot be solved by commercial solvers in a reasonable time. Consequently, we propose a heuristic algorithm which iteratively updates the search step-size of the sub-gradient method in the Lagrangian Relaxation algorithm through obtaining a new lower-bound on the objective function value based on the most recent upper-bound. While improving significantly the convergence, this heuristic also guarantees the feasibility of the converged solution. Furthermore, in order to measure the value of integration, we compare the integrated model with the decoupled planning models currently implemented in the lumber industry. It is observed that, depending on the number of decoupled models, 11%-84% profit improvement can be achieved by considering an integrated model. Finally, the advantage of the proposed heuristic algorithm in finding high quality plans in 51%-77% less CPU time comparing to a commercial solver and the classical Lagrangian Relaxation algorithm is demonstrated through a set of real-size test instances.

2.1 Introduction

2.1.1 Research motivation

Lumber supply chains incorporate forest, as the supplier, sawmills as the manufacturing entities, different distribution channels, as well as contract and non-contract-based customers. Unlike the manufacturing industry which has a convergent structure (i.e., assembly lines), the lumber supply chain (SC) is characterized by: (i) a divergent structure (i.e., logs are transformed into several products and by-products), (ii) the highly heterogeneous nature of its raw material, and (iii) different manufacturing processes [2]. Because of these characteristics, lumber supply chain tactical planning represents a major challenge in this industry that cannot be addressed by commonly used spreadsheet solutions. Yet, more sophisticated approaches such as optimization models would be able to better capture the aforementioned features. Furthermore, it is assumed that forests (supply entities) belong to the public sector, while sawmills and distribution channels are independent private companies. Hence, tactical planning in such value chain can be addressed via either a decoupled or an integrated approach.

A decoupled planning approach, which is the current practice in the lumber industry in Canada, is implemented via solving the decoupled harvesting, procurement, production, and sale/distribution models in a sequential manner. However, due to the fact that only rough estimations of upstream entities' capacities (e.g., harvesting capacity) are considered in models corresponding to downstream entities (e.g., sawmill production planning model), sub-optimal plans in terms of SC profit would be highly expected. For instance, smaller quantities of products can be promised to customers, which will directly impact the SC revenue. In contrary, promising sale amounts that are more than capacities of sawmills would lead to considerable quantities of backlogs and increased SC costs.

The purpose of an integrated model is to combine supply chain functions with the goal of increasing efficiency and better connecting demand with supply, which can both improve customer service and lower costs. To the best of our knowledge, less effort has been done in the literature in integrating tactical decisions in the lumber supply chain planning. The problem dealt with in this paper is focused on integrating tactical planning decisions in lumber supply chains. It can be stated by the following research questions:

(i) How to integrate all medium-term decisions that different entities of lumber supply chains are dealing with?

(ii) What are the benefits of the integrated model in comparison with decoupled models in lumber supply chains?

(iii) How to solve the resulting complex integrated mathematical model for real-size instances?

By answering the proposed research questions, some important challenges in the lumber supply chain tactical planning literature will be covered. In what follows, we first review the literature on lumber supply chain tactical planning; then, we summarize the contributions of the article.

2.1.2 Relevant literature

Tactical planning in a supply chain incorporates the synchronized planning of procurement, production, distribution and sale activities, in order to ensure that the customer demand is satisfied by the right product at the right time [3]. A systematic review on supply chain tactical planning models was provided in [4]. Comelli et al. [5] proposed an approach to evaluate financial benefits of supply chain tactical planning in terms of cash flow. An iterative procedure for optimizing production and inventory planning was proposed in [6]. A dynamic programming approach for production and inventory planning under random demand was proposed in [7].

Over the last twenty years, much research has been conducted into the partial integration of the functions in a SC due to the difficulty in their complete integration [8]. Integrated design and tactical planning in bio-mass value chains has been investigated in [9–11]. Moreover, SC tactical planning is also addressed in the framework of Sales & Operation planning (S&OP) in the literature. Recent studies consider S&OP as a synchronization mechanism that integrates the demand forecast with supply chain capabilities through coordination of marketing, manufacturing, purchasing, logistics, and financing decisions and activities [12, 13]. The relevant literature on the partial integrated planning in the forestry industry can be summarized as follows.

Harvesting planning is one of the most important decisions in the lumber supply chain. Two main operations in the forests are harvesting and forwarding. The main important tactical decisions in the forests are the harvesting area (block) selection and bucking over the planning horizon [14]. Wood procurement models can be tracked back to the early 1960s. Since that time, several models have been developed to address different aspects of wood procurement [15]. Some of these models have been designed for specific activities such as skidding or transportation [16, 17]. Beaudoin et al. [15] proposed a deterministic model for forest tactical planning. They also assessed the impact of uncertainty into their model and evaluated these uncertainties under alternative tactical scenarios by the aid of simulation. Other models tried to integrate several forest planning decisions in a single model, in order to capture possible synergies between them. As an instance, Burger and Jamnick [18] integrated harvesting, storage, and transportation decisions. Andalaft et al. [19] integrated harvesting and road-building decisions. Karlsson et al. [20] presented an optimization model for annual harvest planning. Their model includes transportation planning, road maintenance decisions, and control of storage in the forest and at terminals in mills. Bredstrom et al. [14] formulated a mixed-integer programming (MIP) model to integrate the assignment of machines and harvest teams to harvesting blocks. They proposed a two stage methodology such that the first one solves the assignment and the second one tries to schedule. Dems et al. [21] developed a MIP model for annual timber procurement planning with considering bucking decisions in order to minimize the operational costs such as harvesting, transportation, and inventory costs. In their proposed procurement planning model, they considered a multi-period, multi-product, multiple blocks and multi-mill setting. Chauhan et al. [22] proposed an integrated approach for harvesting, bucking, and transportation decisions. They assumed a multi-product and multi-mill setting in a single period planning horizon. To minimize the harvesting and transportation costs in the forest, they developed a heuristic algorithm based on the column generation approach. However, to the best of our knowledge, there is no attempt to coordinate the above mentioned decisions (i.e., harvesting schedule, raw material quantity, etc.) with production, distribution, and sale decisions in the lumber SC.

There are several contributions in the literature focused on lumber production planning. Among them, Maness et al. [23] proposed a MIP model to simultaneously determine the optimal bucking and sawing policies based on demand and final product prices. Singer et al. [24] presented a model for optimizing production planning decisions in the sawmill industry in Chile. They also demonstrated the benefit of collaboration in the SC. Kazemi Zanjani et al. [25, 26] proposed a two-stage stochastic programming model and two robust optimization models for sawmill production planning by considering the non-homogeneity of raw materials. Kazemi Zanjani et al. [27] proposed a multi-stage stochastic program for sawmill production planning under demand and yield uncertainty.

To summarize, the available research on lumber supply chain tactical planning only covers the decoupled or partially integrated models. In addition, majority of the existing MIP models are solved by the aid of commercial solvers such as CPLEX. However, solving the integrated tactical planning model in the lumber supply chain, which is a large-scale MIP model, by the aid of a commercial solver is expected to be very time-consuming for real-size instances.

2.1.3 Contribution and article outline

Based on the existing literature gaps in coordinating tactical decisions in lumber supply chains, we aim to integrate harvesting, procurement, production, distribution, and sale decisions in the lumber supply chain so as to maximize the total profit of the supply chain. Our integrated model considers all entities of the lumber supply chain; therefore it is more comprehensive than the existing models. Moreover, according to the current practice in the lumber industry, three decoupled models are formulated representing, respectively, harvesting/procurement, production, and sale/distribution decisions. In order to demonstrate the sub-optimality of the currently used tactical plans, obtained by solving the decoupled models in a sequential manner, we compare the SC profits. Our experimental results on a number of realistic test cases show a significant gap in the SC profit between our integrated model and the existing decoupled planning models. This evaluation of the profit gap can be exploited by our industrial partners to promote for more integration in the lumber SC. It allows the quantification of the acceptable effort to reach the integration. In fact, this value of integration can be interpreted as the maximum price that can be paid in order to facilitate such integration, for example by information sharing and collaboration mechanisms.

The integrated tactical planning problem is formulated as a MIP model with dozens of families of constraints corresponding to different entities of the lumber SC. Hence, solving this model for real-size instances in a reasonable time is another challenge which is covered in this paper. To solve this issue, we propose an enhanced Lagrangian Relaxation (LR) algorithm that addresses two key issues related to the classical LR method, namely slow convergence and infeasibility of the converged solution. The latter enhancement is achieved by proposing a heuristic algorithm which iteratively updates the search step-size of the sub-gradient method in the LR algorithm through obtaining an improved lower-bound on the objective function value. By the aid of several test cases, we demonstrate that the proposed heuristic for updating the lower-bound guides the sub-gradient algorithm in a way that a high quality feasible solution (i.e., with a very small optimality gap) can be obtained in a relatively small CPU time.

The proposed methodology can be summarized by the following steps:

Step 1 - Definition of the lumber supply chain network: This is based on the mapping of all the supply chain entities, in order to facilitate the development of the mathematical models.

Step 2 - Formulation of the integrated tactical planning optimization model: An integrated mathematical model is provided to simultaneously address the tactical planning decisions.

Step 3 - Formulation of the decoupled tactical planning optimization models: We develop decoupled mathematical models that correspond to the currently practiced tactical planning approach in industry.

Step 4 - Development of solution methods to solve the large sized integrated model: To solve the large mixed-integer programming formulated in Step 2, an efficient heuristic is developed.

Step 5 - Comparison of the models: The value of integration is evaluated by comparing the integrated tactical planning model with the decoupled models.

To summarize, the paper contribution is twofold. Not only a new integrated model is proposed and compared to several decoupled models, but also an efficient heuristic algorithm is developed in order to solve the resulting large-scale integrated model for real-size instances of an industrial case study.

The paper remainder is organized as follows. The mathematical models are presented in Section 2.2. The solution methodology is provided in Section 2.3. Finally, the numerical results and conclusions are presented in Sections 2.4 and 2.5, respectively.

2.2 Mathematical models

In this section, after explaining the lumber SC network, we formulate the integrated model and the decoupled models.

2.2.1 Defining the lumber supply chain network

The lumber supply chain entities are summarized as a network in Fig.1. This network includes forests as supplier entities, sawmills as production units, distribution centers (DCs), sale departments, and customers. Forests are composed of blocks that contain different families and species of trees. Through the harvesting operations in forests, the trees are cut down, piled, and thinned in the harvesting blocks. The forwarding operations collect the piles and transfer them to the storage locations in the harvesting blocks or to the adjacent forest roads. Finally, the logs are transported to sawmills by trucks. In forests (supply entities), we are dealing with harvesting planning where the harvesting schedule on different blocks over the planning horizon must be determined. The availability of each family of raw material, storage, and transportation capacity of each block are important parameters that should be considered in the harvesting planning. Furthermore, the maximum number of harvesting in each block and the maximum number of blocks in which harvesting can occur in each period in the planning horizon are other important factors that must be considered in the harvesting planning.

Sawmills purchase logs from the forest, and then transform them to lumbers as main products, and chips or sawdust as by-products. There are three main processes in sawmills: sawing, drying, and finishing. In the sawing process, the logs are cut into different sizes of rough lumbers by different cutting patterns. In the drying process, the lumber moisture contents are reduced in large dryers or in air. In the finishing process, the lumbers are surfaced, trimmed and sorted based on customer requirements. According to the demand, some logs are shipped to the distribution centers or directly to customers after the sawing process, while others are first sent to drying and finishing processes and then are shipped to distribution centers or customers. Product shipment to customers is carried out by a number of distribution companies that use different vehicle types. Sawmills are dealing with procurement and production planning during the planning horizon.

The supply chain serves contract-based customers (e.g., construction industry and furniture manufacturers) and noncontract-based customers (e.g., pulp & paper industries or the spot market). Contract-based customers sign a contract at an agreed price and quantity for a given planning horizon. Although the contract demand must be satisfied, the enterprise reserves the right of postponing some parts of agreed quantities because of capacity shortage in the demand period. Unsatisfied demand may be served in a future period as the backlog. When there is surplus capacity in sawmills, the spot market is sought to absorb the remaining capacity. Distribution centers seek a distribution plan during the planning horizon while sale departments in the lumber supply chain are looking for realistic amounts of sale that can be promised to customers.

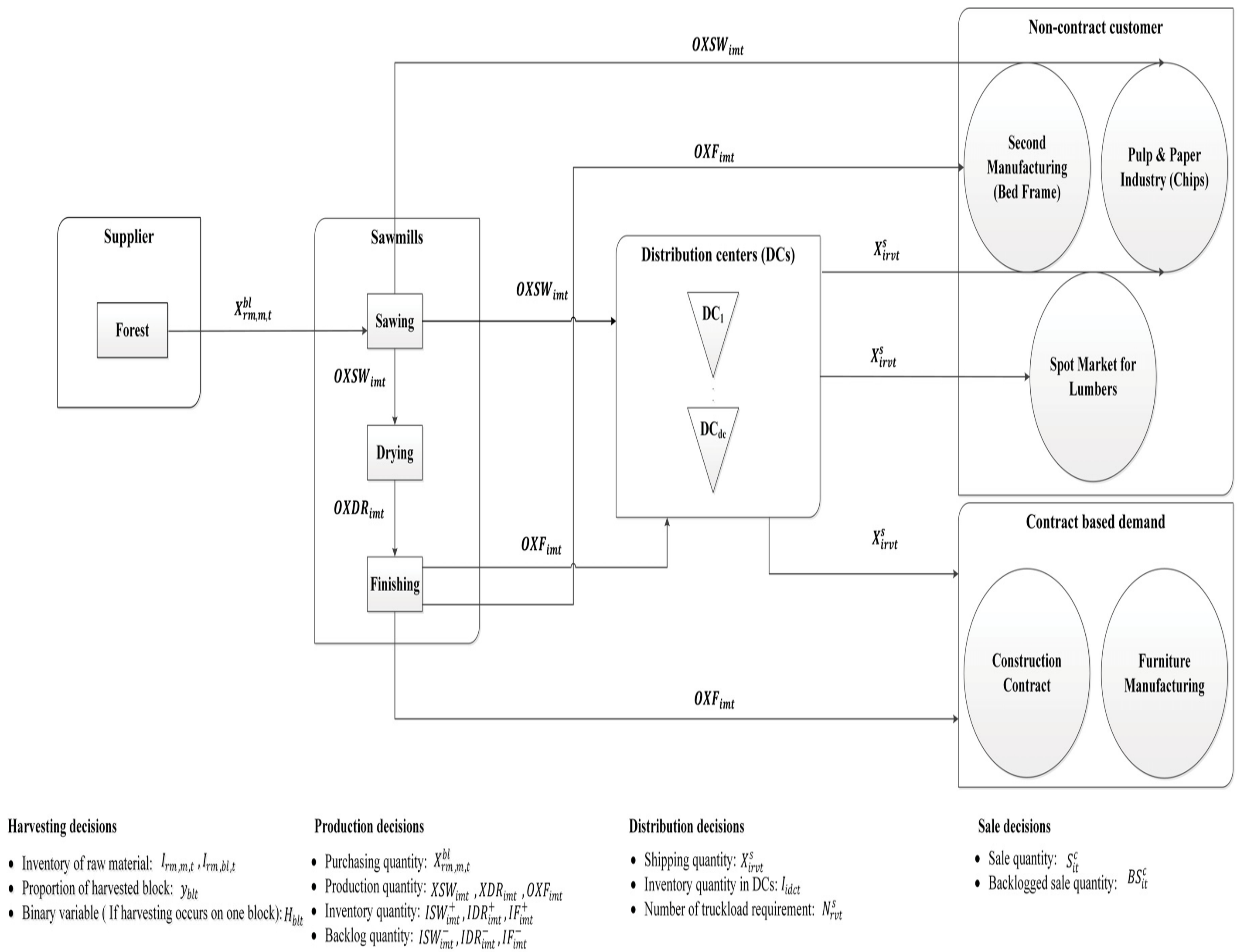

Figure 1: The lumber supply chain network

2.2.2 The integrated model

In this section, we provide a MIP model to simultaneously address harvesting, procurement, production, distribution and sale planning decisions in the lumber supply chain.

The objective is to maximize the global net profit by balancing the sale revenue and supply chain cost over a planning horizon T. The harvesting decisions involve the blocks where the harvesting should occur as well as the proportion of the harvested blocks in different periods of the planning horizon. The procurement decisions include the purchasing quantity of raw material from each block, and the inventory of raw materials in each block. Production decisions incorporate the quantity of lumbers that should be sawn, dried, and finished as well as inventory and backlog quantities of lumbers. Distribution decisions include the shipping quantity of products, the inventory quantity of products in each distribution center, the number of truckload requirement along with the type of vehicle and route. Finally, sale planning involves the amount of sale promised to customers as well as possible backlog quantity of products.

The indices, sets, parameters, and decision variables used in the proposed model are defined in the appendix.

The objective function of the proposed MIP model is defined in equation (1) and divided into several parts presented in (2)-(9), respectively representing the total cost of harvesting (including cost of building new access roads), stumpage, log transportation (from harvesting blocks to sawmills), log storage cost in harvesting blocks, various costs incurred by sawmills, and the final product distribution cost.

$$Max\ Z = R - (C_{harvesting} + C_{stumpage} + C_{transportation} + C_{storage}$$
$$+ C_{procurement} + C_{production} + C_{distribution}) \quad (1)$$

where:

$$R = \sum_{c \in C} \sum_{i \in I} \sum_{t \in T} b^c_{i,t} S^c_{i,t} \quad (2)$$

$$C_{harvesting} = \sum_{bl \in BL} \sum_{t \in T} c^H_{blt} y_{bl,t} (\sum_{rm \in RM} v_{rm,bl}) \quad (3)$$

$$C_{stumpage} = \sum_{rm \in RM} \sum_{bl \in BL} \sum_{t \in T} v_{rm,bl} f_{rm,bl,t} y_{bl,t} \quad (4)$$

$$C_{transportation} = \sum_{bl \in BL} \sum_{m \in M} \sum_{rm \in RM} \sum_{t \in T} c^T_{rm,bl,m,t} X^{bl}_{rm,m,t} \quad (5)$$

$$C_{storage} = \sum_{rm \in RM} \sum_{bl \in BL} \sum_{t \in T} c^S_{rm,bl,t} I_{rm,bl,t} \quad (6)$$

$$C_{procurement} = \sum_{bl \in BL} \sum_{rm \in RM} \sum_{m \in M} \sum_{t \in T} m^{bl}_{rm,t} X^{bl}_{rm,m,t}$$
$$+ \sum_{rm \in RM} \sum_{m \in M} \sum_{t \in T} h_{rm,m} I_{rm,m,t} \quad (7)$$

$$C_{production} = \sum_{m \in M} \sum_{i \in I} \sum_{t \in T} c_{im} (XSW_{i,m,t} + XDR_{i,m,t} + OXF_{i,m,t})$$
$$+ \sum_{m \in M} \sum_{i \in I_{SW}} \sum_{t \in T} h1_{im} ISW^+_{i,m,t}$$
$$+ \sum_{m \in M} \sum_{i \in I_{DR}} \sum_{t \in T} h2_{im} IDR^+_{i,m,t}$$
$$+ \sum_{m \in M} \sum_{i \in I_F} \sum_{t \in T} h3_{im} IF^+_{i,m,t}$$
$$+ \sum_{m \in M} \sum_{i \in I_{SW}} \sum_{t \in T} bo1_{im} ISW^-_{i,m,t}$$
$$+ \sum_{m \in M} \sum_{i \in I_{DR}} \sum_{t \in T} bo2_{im} IDR^-_{i,m,t}$$
$$+ \sum_{m \in M} \sum_{i \in I_F} \sum_{t \in T} bo3_{im} IF^-_{i,m,t} \quad (8)$$

$$C_{distribution} = \sum_{s\in S}\sum_{i\in I}\sum_{r\in R}\sum_{v\in V}\sum_{t\in T}(e^s_{irv}X^s_{i,r,v,t} + sh^s_{rv}N^s_{r,v,t})$$
$$+\sum_{s\in S}\sum_{i\in I}\sum_{dc\in DC}\sum_{r\in R_{m,dc}}\sum_{v\in V}\sum_{t\in T} tr_{idc}\ X^s_{i,r,v,t}$$
$$+\sum_{i\in I}\sum_{dc\in DC}\sum_{t\in T} h_{idc}I_{i,dc,t} \tag{9}$$

Equation (8) incorporates the cost sawmills incur to purchase logs from forest contractors, as well as the production, inventory, and backlog costs in sawing, drying, and finishing units of sawmills. Furthermore, equation (9) represents the total transportation/transshipment cost of end products (lumber and by-products) to customers in addition to inventory holding cost of products at distribution centers.

Harvesting constraints

$$\sum_{t\in T} y_{bl,t} \le 1 \quad \forall bl \tag{10}$$

$$y_{bl,t} \le H_{bl,t} \quad \forall bl, t \tag{11}$$

$$\sum_{t\in T} H_{bl,t} \le l_{bl} \quad \forall bl \tag{12}$$

$$\sum_{bl\in BL} H_{bl,t} \le n_t \quad \forall t \tag{13}$$

$$\sum_{bl\in BL} (y_{bl,t} \sum_{rm\in RM} v_{rm,bl}) \le b^H_t \quad \forall t \tag{14}$$

$$\sum_{rm\in RM}\sum_{m\in M}\sum_{bl\in BL} X^{bl}_{rm,m,t} \le b^T_t \quad \forall t \tag{15}$$

$$I_{rm,bl,T} = 0 \quad \forall rm, bl \tag{16}$$

$$I_{rm,bl,t} = I_{rm,bl,t-1} - \sum_{m\in M} X^{bl}_{rm,m,t} + v_{rm,bl}y_{bl,t} \quad \forall rm, bl, t \setminus 0 \tag{17}$$

Constraints (10)-(17) formulate various restrictions in the harvesting blocks in the forest. Constraint (10) ensures that the harvested proportion of a block does not exceed the availability of logs in that block. Constraint (11) describes that if harvesting occurs on a block, then we can ensure that raw materials from that block are available. Constraint (12) restricts the number of times each block can be harvested during the planning horizon; while (13) limits the number of blocks that can be harvested in each period. Constraints (14) and (15) correspond to total harvesting and transportation capacity of harvesting blocks, respectively. Constraint (16) represents the final inventory of raw materials in each block. Constraint (17) formulates the inventory balance of raw materials in each block which equals the initial inventory form previous period plus the harvested amount minus the total amount shipped to sawmills.

Procurement constraints

$$\sum_{bl \in BL} X^{bl}_{rm,m,t-L^{bl}_{rm}} + I_{rm,m,t-1} - I_{rm,m,t} = \sum_{i \in (I_{SW} \cup I'_{SW})} cu_{rm,i,m} XSW_{imt}$$

$$\forall rm, m, t = 1 + L^{bl}_{rm}, \ldots, T \tag{18}$$

$$I_{rm,m,t} - ss_{rm,m} \geq 0 \quad \forall rm, m, t \tag{19}$$

$$\sum_{rm \in RM} I_{rm,m,t} \leq KI_m \quad \forall m, t \tag{20}$$

$$\sum_{rm \in RM} \sum_{m \in M} X^{bl}_{rm,m,t} \leq KS^{bl}_t \quad \forall bl, t \tag{21}$$

$$\sum_{m \in M} \sum_{rm \in RM} \sum_{t \in T} X^{bl}_{rm,m,t} \geq qmin^{bl} \quad \forall bl \tag{22}$$

Constraints (18)-(22) formulate various purchasing conditions in sawmills. Constraint (18) represents the inventory balance of logs in sawmills, which is equal to the previous inventory level plus the quantity received from the harvesting blocks minus the quantity that will be consumed in sawmills. The raw material safety stock policies are stated in constraint (19) and the raw material inventory capacity constraint is provided in (20). Constraint (21) limits the maximum quantity of logs that can be purchased from the forest. Constraint (22) states that the material procured from a supplier must satisfy the contract minimum quantity commitment.

Production constraints

Sawing process

$$\sum_{m\in M}(XSW_{i,m,t}+ISW^{+}_{i,m,t-1}-ISW^{-}_{i,m,t-1}$$
$$-ISW^{+}_{i,m,t}+ISW^{-}_{i,m,t})\leq\sum_{c\in C}S^{c}_{i,t}\quad\forall i\in I'_{SW},\forall\, t\in T\tag{23}$$

$$\sum_{m\in M}ISW^{-}_{i,m,t}=\sum_{c\in C}BS^{c}_{i,t}\quad\forall i\in I'_{SW},\forall t\in T\tag{24}$$

$$\sum_{i\in(I_{SW}\cup I'_{SW})}p1_{imt}XSW_{i,m,t}\leq\ Ksw_{mt}\quad\forall m,t\tag{25}$$

$$\sum_{i\in I_{sw}}ISW^{+}_{i,m,t}+\sum_{i\in I'_{SW}}ISW^{+}_{i,m,t}\leq\ KIsw_{m}\quad\forall m,t\tag{26}$$

$$ISW^{-}_{i,m,0}=ISW^{-}_{i,m,T}=0\quad\forall i\in(I_{sw}\cup I'_{SW}),m\tag{27}$$

$$XSW_{i,m,t}+ISW^{+}_{i,m,t-1}-ISW^{-}_{i,m,t-1}-ISW^{+}_{i,m,t}$$
$$+ISW^{-}_{i,m,t}=OXSW_{i,m,t}\quad\forall i\in I_{SW},m,t\tag{28}$$

Drying process

$$\varnothing_{i'mt}OXSW_{i',m,t}=XDR_{i,m,t}\quad\forall\ i'\in I_{SW},i\in I_{DR},m,t\tag{29}$$

$$XDR_{i,m,t}+IDR^{+}_{i,m,t-1}-IDR^{-}_{i,m,t-1}-IDR^{+}_{i,m,t}$$
$$+IDR^{-}_{i,m,t}=OXDR_{i,m,t}\quad\forall i\in I_{DR},m,t\tag{30}$$

$$\sum_{i\in I_{DR}}p2_{imt}XDR_{i,m,t}\leq Kdr_{mt}\quad\forall m,t\tag{31}$$

$$\sum_{i\in I_{DR}}IDR^{+}_{i,m,t}\leq KIdr_{m}\quad\forall m,t\tag{32}$$

$$IDR^{-}_{i,m,0}=IDR^{-}_{i,m,T}=0\quad\forall i\in I_{DR},m\tag{33}$$

Finishing process

$$\rho_{i'mt} OXDR_{i',m,t} = OXF_{i,m,t} \quad \forall\, i' \in I_{DR}, i \in I_F, m, t \tag{34}$$

$$\sum_{m \in M} (OXF_{i,m,t} + IF^{+}_{i,m,t-1} - IF^{-}_{i,m,t-1} - IF^{+}_{i,m,t} + IF^{-}_{i,m,t}) \leq \sum_{c \in C} S^{c}_{i,t} \qquad \forall i \in I_F, t \tag{35}$$

$$\sum_{m \in M} IF^{-}_{i,m,t} = \sum_{c \in C} BS^{c}_{i,t} \quad \forall\, i \in I_F, t \tag{36}$$

$$\sum_{i \in I_F} p3_{imt} OXF_{i,m,t} \leq \ Kf_{mt} \quad \forall\, m, t \tag{37}$$

$$\sum_{i \in I_F} IF^{+}_{i,m,t} \leq \ KIf_{m} \quad \forall\, m, t \tag{38}$$

$$IF^{-}_{i,m,0} = IF^{-}_{i,m,T} = 0 \quad \forall\, i \in I_F, m \tag{39}$$

Constraints (23)-(39) describe various conditions in the sawing, drying, and finishing units of sawmills. Constraints (23) and (35) are the coupling constraints that link the production and sale decisions and determine the maximum net inventory level in the sawing and finishing units. The backlog quantities are converted into backlogged sale (BS^{c}_{it}) in (24) and (36), in order to be used in distribution constraint (40). Constraints (25), (31), and (37) formulate the production capacity constraints in sawing, drying, and finishing units. Constraints (26), (32), and (38) define the warehouse inventory capacity in sawing, drying, and finishing units. The beginning and ending backlog conditions in sawing, drying, and finishing units are described in constraints (27), (33), and (39), respectively. Constraint (28) is a flow balance that calculates the output quantity of products from the sawing unit. It indicates that the total quantity of green lumber that can be sent to the dying unit equals the initial net inventory plus the total quantity produced minus the net inventory that will be left. Constraint (29) ensures that the total amount of green lumber received from the sawing unit will be processed in the drying unit with a specific yield. Constraint (30) is a flow balance that calculates the quantity of dried lumber that will be sent to the finishing unit. Constraint (34) ensures that the total amount of dried lumbers received from the drying unit will be processed in the finishing unit by considering a specific yield.

Distribution constraints

$$\sum_{c\in C}(S^c_{i,t} + BS^c_{i,t-1} - BS^c_{i,t}) = \sum_{s\in S}\sum_{r\in(R_{m,c}\cup R_{dc,c})}\sum_{v\in V} X^s_{i,r,v,t} \quad \forall i,t \tag{40}$$

$$\sum_{m\in M}(XSW_{i,m,t} + ISW^+_{i,m,t-1} - ISW^+_{i,m,t}) = \sum_{s\in S}\sum_{r\in(R_{m,dc}\cup R_{m,c})}\sum_{v\in V} X^s_{i,r,v,t} \quad \forall i \in I'_{sw}, t \tag{41}$$

$$\sum_{m\in M}(OXF_{i,m,t} + IF^+_{i,m,t-1} - IF^+_{i,m,t}) = \sum_{s\in S}\sum_{r\in(R_{m,dc}\cup R_{m,c})}\sum_{v\in V} X^s_{i,r,v,t} \quad \forall\, i \in I_F, t \tag{42}$$

$$\sum_{s\in S}\sum_{r\in R_{m,dc}}\sum_{v\in V} X^s_{i,r,v,t} + I_{i,dc,t-1} - I_{i,dc,t} = \sum_{s\in S}\sum_{r\in R_{dc,c}}\sum_{v\in V} X^s_{i,r,v,t} \quad \forall i \in \left(I'_{SW}\cup I_F\right), dc, t \tag{43}$$

$$N^s_{r,v,t} \geq \sum_{i\in(I'_{SW}\cup I_F)} \frac{a_i X^s_{i,r,v,t}}{KV_v} \quad \forall s,r,v,t \tag{44}$$

$$\sum_{r\in R} N^s_{r,v,t} \leq KSH^s_v \quad \forall s,v,t \tag{45}$$

$$\sum_{s\in S}\sum_{r\in(R_{m,dc}\cup R_{m,c})}\sum_{v\in V} N^s_{r,v,t} \leq KD_m \quad \forall m,t \tag{46}$$

$$I_{i,dc,0} = 0 \quad \forall i, dc \tag{47}$$

Constraints (40)-(47) represent various limitations in the distribution centers. Constraint (40) links the sale and distribution decisions. It indicates that the total quantity of end products that will be shipped to customers equals the net sale amount after considering the backlogged sale. Constraints (41) and (42) link the production and distribution decisions. They, respectively, indicate that the total quantity of green or finished lumbers that can be shipped to customers equals the net inventory level of these products in the sawing and finishing units. Constraint (43) is the flow balance constraints at a distribution center. Constraint (44) calculates the number of truckload requirements for each vehicle type from each shipping supplier. Constraint (45) and (46) formulate the shipping supplier capacity and the mill dispatch capacity, respectively. Constraint (47) represents the initial inventory of each product in each distribution center.

Sale constraints:

$$S^c_{i,t} - BS^c_{i,t} \geq dmin^c_{it} \quad \forall c \in CC, i \in I_F, t \tag{48}$$

$$S^c_{i,t} \leq d^c_{i,t} \quad \forall c, i, t \tag{49}$$

$$BS^c_{i,t} \leq S^c_{i,t} \quad \forall c \in C, i, t \tag{50}$$

Finally, Constraints (48)-(50) formulate sale conditions. Constraints (48) and (49) describe the sale decisions for contract and non-contract-based demands. In the former case, a minimum amount of sale in the contract must be delivered. In both cases, the demand might be accepted and be served in future periods as the backlog (I^-_{imt}), or, might be rejected. In either case, the backlog amount (BS^c_{it}) should not be greater than the sale quantity (S^c_{it}) (50). Upon the satisfaction of the base amount (48), the company may continue serving the contract demand up to the capacity limit, or switch to serve non-contract demand, whichever is more profitable.

Domain constraints:

$$\begin{aligned}
&S^c_{i,t}, BS^c_{i,t}, OXSW_{i,m,t}, XSW_{i,m,t}, OXDR_{i,m,t}, XDR_{i,m,t}, OXF_{i,m,t},\\
&ISW^+_{i,m,t}, IDR^+_{i,m,t}, IF^+_{i,m,t}, ISW^-_{i,m,t}, IDR^-_{i,m,t}, IF^-_{i,m,t},\\
&X^s_{i,r,v,t}, I_{i,dc,t}, N^s_{r,v,t}, X^{bl}_{rm,m,t}, I_{rm,m,t}, y_{bl,t}, I_{rm,bl,t} \geq 0,\\
&H_{bl,t} \in \Big\{0,1\Big\} \forall bl, rm, c, i, m, s, r, v, dc, t
\end{aligned} \tag{51}$$

Constraint (51) represents the domain constraints.

2.2.3 Decoupled models

Two classes of decoupled models are considered. The first class considers three models: (i) sale & distribution; (ii) production; and (iii) harvesting & procurement. The second one considers two models: (i) sale & distribution & production; and (ii) harvesting & procurement. As mentioned earlier, adopting a decoupled planning approach would require solving the abovementioned decoupled models in sequential manner which might lead to infeasible plans in each echelon due to the lack of perfect information from the upstream echelon. Thus, it is necessary to add extra constraints in each (sub-)model in order to link (sub-)models to each other and to ensure the feasibility of each one. Moreover, the output of one (sub-)model acts as the input of another one. Note that these decoupling models are assumed to correspond to the currently practiced tactical plannig approach in industry. In practice, the planners are acting independently while taking into account the extra constraints described above. For instance, production planning in the production entities of the SC (i.e., sawmills) is carried out by considering a rough estimation of supply capacity of different harvesting blocks in the forest (supply entity). This estimation can be included as a supply capacity constraint in the production planning (sub-)model.

2.2.3.1 Sale & distribution model

The objective of this model is to maximize the total revenue from sale activities minus the distribution costs as follows:

$$Max\ Z = R - C_{Distribution} \tag{52}$$

The constraints of this model include constraints (40), and (43)-(51) in the integrated model, in addition to the following ones:

$$\sum_{i \in I_F} S^c_{it} \leq \sum_{m \in M} Kf_{mt} \quad \forall\, t \tag{53}$$

$$\sum_{i \in (I_{SW} \cup I'_{SW})} S^c_{it} \leq \sum_{m \in M} Ksw_{mt} \quad \forall t \tag{54}$$

Constraints (53) and (54) enforce the sale & distribution model to control the amount of the sale quantity of each product based on the production capacity of sawing and finishing units. These two constraints are added to this decoupled model in order to ensure the feasibility of promised sale amount to the customer.

2.2.3.2 Production model

The objective is to minimize the production, inventory, and backlog costs at sawing, drying and finishing units. Also, this model gets the sale and distribution decisions ($S^c_{it}, I_{i,dc,t}$) as parameters (input) from the sale & distribution (sub-)model (52)–(54).

$$Min\ Z = C_{Production} \tag{55}$$

The constraints of this model involve constraints (23) – (39) in the integrated model, in addition to the following ones:

$$\sum_{rm\in RM}\sum_{m\in M}\sum_{i\in(I_{SW}\cup I'_{SW})} cu_{rm,i,m}XSW_{i,m,t} \le b_t^T \quad \forall t \tag{56}$$

$$\sum_{rm\in RM}\sum_{m\in M}\sum_{i\in(I_{SW}\cup I'_{SW})} cu_{rm,i,m}\ XSW_{i,m,t} \le \sum_{bl\in BL} KS_t^{bl} \quad \forall t \tag{57}$$

$$\sum_{rm\in RM}\sum_{m\in M}\sum_{i\in(I_{SW}\cup I'_{SW})}\sum_{t\in T} cu_{rm,i,m}XSW_{i,m,t} \ge qmin^{bl} \quad \forall bl \tag{58}$$

$$\sum_{m\in M}(OXF_{i,m,t} + IF^{+}_{i,m,t-1} - IF^{+}_{i,m,t}) \ge \sum_{c\in CC} dmin^{c}_{it} \quad \forall i \in I_F, t \tag{59}$$

Constraints (56) and (57) enforce the production model to control the production amount based on the supply and transportation capacity of raw material in the forest. Constraints (58) and (59) ensure that the production amount satisfies the minimum purchase quantity of raw material from each block and minimum contract demand, respectively.

2.2.3.3 Harvesting & procurement model

The objective is to minimize the harvesting cost, stumpage fee, storage and procurement cost in the forest. Also, this model receives the production quantities of lumber ($XSW_{imt}, OXSW_{imt}$) from the production (sub-)model (55) – (59) as the input.

$$Min\ Z = C_{harvesting} + C_{stumpage} + C_{transportation} + C_{storage} + C_{procurement} \tag{60}$$

Constraints of this model are the same as constraints (10)-(22) in the integrated model.

2.2.3.4 Production & sale & distribution model

The objective is to maximize the total revenue from sale activities minus the distribution and production costs.

$$Max\ Z = R - (C_{production} + C_{distribution}) \tag{61}$$

Constraints of this model include constraints (23)-(51) in the integrated model, in addition to constraints (56)-(58) from the production model.

2.3 Solution methodology

The proposed integrated model (1)-(51) is a mixed-integer programming model with dozens of families of constraints. Solving this model in a reasonable time is a challenge for realistic-scale problem instances. We propose an efficient heuristic within the framework of the LR algorithm, and we compare its performance in terms of solution quality and CPU time with a commercial solver (CLPLEX 12.3), and with the classical LR algorithm , where the classical sub-gradient method is exploited to solve Lagrangian sub-models [28]. In what follows, we provide a detailed description of our proposed heuristic algorithm.

It is worth mentioning that solving model (1)-(51) by the aid of classical LR algorithm suffers from two essential issues namely the slow convergence of the algorithm and the infeasibility of the converged solution. Consequently, the LR based heuristic algorithm in this paper proposes a procedure to iteratively update the search step-size of the sub-gradient method in the LR algorithm through obtaining a new lower-bound on the objective function value based on the most recent upper-bound. This heuristic improves the quality of lower-bound based on the improved upper-bound as we proceed in the sub-gradient algorithm. The improved lower-bound can be used to adjust the search step-size, which is expected to accelerate the convergence of the sub-gradient algorithm and to avoid the infeasibility of the converged solution.

2.3.1 Heuristic algorithm

It is worth mentioning that the complexity of the integrated model (1)-(51) is due to the existence of binary variables corresponding to harvesting schedule during the planning horizon, in addition to constraints (12) and (13) that formulate the harvesting constraints in the forest. In other words, the latter two constraints can be considered as complicating constraints in the sense that after relaxing them from model (1)-(51), the resulting model can be solved much faster by a commercial solver. However, the optimal solution of the relaxed model might be infeasible regarding the relaxed constraints. One way to tackle this difficulty is implementing the LR algorithm [28–33]. In LR algorithm, possible violations of relaxed constraints are penalized in the objective function of the relaxed model (i.e., the Lagrangian Relaxation model) by considering Lagrangian multipliers. In other words, in the integrated model (1)-(51), the violation of relaxed constraints (12)-(13) are incorporated into the objective function by introducing multipliers u_{bl}, and v_t. Thus, the Lagrangian Relaxation of model (1)-(51) (Lagrangian sub-model) can be stated as follows:

$$L_{IP}(u,v) = Maximize\left\{Z + \sum_{bl\in BL} u_{bl} * (l_{bl} - \sum_{t\in T} H_{blt}) + \sum_{t\in T} v_t * (n_t - \sum_{bl\in BL} H_{blt})\right\} \tag{62}$$

Subject to:

(10)-(11) and (14)-(51).

Since the above Lagrangian sub-model is convex and non-differentiable, the sub-gradient method is implemented to solve it. The summary of the sub-gradient method in classical LR algorithm for solving this model is summarized in algorithm (1).

The stopping criterion in algorithm (1) is considered as a predetermined number of iterations. The convergence of the sub-gradient method is heavily dependent on

Algorithm 1 Sub-gradient method in classical LR algorithm

Step 0 (initialization):
    Assign zero to u_{bl} and v_t
    Find an initial lower-bound (LB) (feasible solution) and assign ∞ to the upper-bound (UB)
    Let iteration counter k equal to 1
while the stopping criteria is not satisfied **do**
Step 1
    Solve the Lagrangian sub-model and determine $L_{IP}(u, v)$
Step 2
    If $L_{IP}(u, v) < UB)$ then $UB = L_{IP}(u, v)$
Step 3
    Update dual multipliers as follows:
        $u_{bl}^{k+1} = max\{u_{bl}^{k} - \epsilon_k * \frac{L_{IP}^{k}(u,v)-LB}{\|l_{bl}-\sum_{t \in T} H_{blt}\|^2} * (l_{bl} - \sum_{t \in T} H_{blt}), 0\}$
        $v_{t}^{k+1} = max\{v_{t}^{k} - \epsilon_k * \frac{L_{IP}^{k}(u,v)-LB}{\|n_t-\sum_{bl \in BL} H_{blt}\|^2} * (n_t - \sum_{bl \in BL} H_{blt}), 0\}$
        $k = k + 1$
end while

the step size. In the classical sub-gradient approach, the lower-bound is considered as a fixed amount which can hinder the convergence of the LR algorithm. Furthermore, the classical approach cannot guaranty the feasibility of converged solution. In order to solve these issues in the sub-gradient method, inspired from [28–33], we propose a heuristic to iteratively adjust the step size through updating the lower-bound. More precisely, we propose to improve the quality of the lower-bound (LB) based on the most recent upper-bound (UB) obtained at each iteration of the sub-gradient algorithm. The reason is that the quality of the UB is expected to be improved as we proceed in the sub-gradient algorithm. The following procedure is implemented in order to update the LB.

Lower-Bound Updating Heuristic

In each iteration of the sub-gradient method (algorithm (1)), we verify the obtained optimal solution in terms of its feasibility regarding the relaxed constraints (12)-(13). For this purpose, we calculate the slack variables corresponding to the relaxed constraints (12)-(13). If the slack variable is positive, it means that the corresponding constraint is satisfied. Hence, we suggest identifying binary variables with the value equal to zero (at optimal solution) in the non-violated constraints. Those variables are then fixed to zero in the original integrated MIP model in order to obtain a reduced MIP model that can be solved faster than the original one by a commercial solver. By solving the reduced model (1)–(51) by a commercial solver, we can obtain a new feasible solution (LB). The proposed lower-bound updating heuristic (LBUH) is summarized in algorithm (2).

Algorithm 2 Lower-Bound Updating Heuristic (LBUH)

Step 0:
  Calculate $\{slack_{bl} = (l_{bl} - \sum_{t \in T} H_{blt})\ \forall bl\}$ and $\{slack_t = (n_t - \sum_{bl \in BL} H_{blt})\ \forall t\}$ after solving Lagrangian problem in each iteration
if $slack_{bl} \geq 0$ **then**
  Step 1:
    Identify the binary variables which are equal to 0 in the optimal solution and fix them in the initial MIP model (1)-(51)
if $slack_t \geq 0$ **then**
  Step 2:
    Identify the binary variables which are equal to 0 in the optimal solution and fix them in the initial MIP model (1)–(51)
Step 3:
  Solve the reduced MIP model (1)-(51) resulted from steps (1) and (2) by a commercial solver to obtain new lower-bound (new LB)
if $(new\ LB) > (old\ LB)$ **then**
  Step 4:
    Lower-bound for the next iteration in the sub-gradient algorithm $(LB) = new\ LB$

Incorporating the LBUH into the sub-gradient method leads to an enhanced LR heuristic summarized in algorithm (3). This heuristic is proposed as an efficient algorithm for solving the integrated tactical planning model (1)-(51) for real-size instances.

Algorithm 3 Heuristic algorithm

Step 0 (initialization):
  (sub-gradient method (algorithm (1)))
while the stopping criteria is not satisfied **do**
Step 1
  If $L_{IP}(u,v) < UB$) then $UB = L_{IP}(u,v)$
  Update lower-bound (LB) based on the "lower-bound updating heuristic (LBUH)" (i.e., algorithm (2))
Step 2
  Update dual multipliers
  (sub-gradient method (algorithm (1)))
end while

2.4 Numerical results

2.4.1 Case study

The considered case study is characterized by a data set that sufficiently represents a realistic scale lumber SC in Canada. This SC consists of two sawmills producing 27 product families using 14 classes of logs. Products are shipped to 140 customers by 4 outbound shipping suppliers using 5 different vehicle types via 2 distribution centers and 20 routes. Also, we assumed that 50 harvesting blocks are available in the forest during the 12 month planning horizon. The supply capacity of each block per month is supposed to be 23,500 m^3. Adopted from Beaudoin et al. [15], the maximum number of periods (months) over which harvesting can occur in each block, the maximum number of blocks in which harvesting can occur per month are randomly selected from uniform distributions U(1-6) and U(10-12), respectively. The average volumes of each log class available in each block are also randomly generated [15]. The total harvesting capacity per month is supposed to be approximately 1,175,000 m^3. There are also further aspects that must be considered in harvesting planning such as weather conditions during the year, road maintenance, and crew scheduling. For example, it is not possible to transport the logs from some blocks to mills during winter, because the snow might close some roads. This would lead to road maintenance or substitution which will change the harvesting plan. The abovementioned aspects of harvesting were included implicitly in the transportation cost from the blocks to mills. For instance, if a road does not exist, the cost of building that road is included in the transportation cost. In sawmills, we supposed approximately 750,000 m^3 production capacity per month. Finally, the demand for each type of products is derived from [26]. The capacity of each vehicle type is randomly generated from the uniform distribution U(3-25).

This case study results in 280,000 continuous, 600 binary variables, and 280,000 constraints in the integrated tactical planning model. All algorithms were coded in C++ using CPLEX concert technology on a Dual-Core, 2.80GHz computer with 4.00 GB RAM.

2.4.2 The value of integration

In this section, we compare the integrated tactical planning model with the decoupled planning approach, in terms of the total revenue as well as the costs of harvesting, procurement, production and distribution. The results for the two classes of decoupled models are provided in tables (9) and (10). The decoupled class 1 (sale & distribution + production + harvesting & procurement) incorporates more (sub-)models comparing to class 2 (sale & distribution & production + harvesting & procurement). In tables (9) and (10), Δ denotes the difference between the revenue/cost of integrated and decoupled models. The negative value of Δ in the revenue and (costs) indicates that the total revenue (cost) of the decoupled model are greater (less) than the integrated one, respectively. As an instance, the negative value of Δ for revenue in table (9) implies that the revenue of the integrated model is less than that of the decoupled one. Also, the positive value of Δ for backlog cost in the same table indicates that the backlog cost of the integrated model is less than that of the decoupled model.

Table 1: Value of integrated planning versus decoupled class 1

Criteria	Integrated model	Decoupled models	Δ over class 1	Deviation over class 1
Actual revenue	435,359,356	709,780,241	-274,420,885	39%
Inventory cost at DCs	0	0	0	0%
Transshipment cost at DCs	3,787,256	5,097,326	1,310,070	26%
Inventory cost	27,086,403	29,822,645	2,736,242	9%
Backlog cost	86,258,429	479,162,072	392,903,643	82%
Production cost	28,125,487	26,322,034	-1,803,453	-7%
Harvesting cost	20,849,181	21,153,326	304,145	1%
Procurement cost	4,568,000	4,205,207	-362,793	-9%
Total profit	**264,684,600**	**144,017,631**	**120,666,969 (value of integration)**	84%

From table (9), it can be observed that the total revenue in the decoupled model is

Table 2: Value of integrated planning versus decoupled class 2

Criteria	Integrated model	Decoupled models	Δ over class 2	Deviation over class 2
Actual revenue	435,359,356	422,530,613	12,828,743	3%
Inventory cost at DCs	0	0	0	0%
Transshipment cost at DCs	3,787,256	3,594,670	-192,586	-5%
Inventory cost	27,086,403	10,810,574	-16,275,829	-151%
Backlog cost	86,258,429	115,758,944	29,500,515	25%
Production cost	28,125,487	26,829,579	-1,295,908	-5%
Harvesting cost	20,849,181	21,962,604	1,113,423	5%
Procurement cost	4,568,000	4,341,904	-226,096	-5%
Total profit	**264,684,600**	**239,232,338**	**25,452,262 (value of integration)**	**11%**

greater than the integrated one. This means that the delivered sale in the decoupled model is greater than the integrated one. In contrary, the inventory and backlog quantities in the decoupled models are much higher than the integrated one in order to satisfy the bigger amount of delivered sale in the decoupled models. The reason is that in the decoupled approach (class 1), sale & distribution decisions are not coordinated with the production planning model. In the same table, the production quantity and consequently the procurement quantity of logs in the integrated model are also greater than the decoupled one. Higher amounts of log procurement in the integrated model can be explained by the fact that the availability of logs in each block is different. Hence, the integrated model will satisfy the log requirement in sawmills from several blocks according to their log inventory and available harvesting quantity. Hence, the greater procurement quantity is not necessarily equivalent to the greater harvesting amount or higher log inventory in each block. Finally, we can observe that although the revenue in the decoupled models is greater than the integrated model, the latter made further modifications on sale decisions. In other words, while the overall revenue was reduced in the integrated model, the total inventory and backlog costs were reduced more significantly, resulting in a net profit improvement (84% improvement in the total profit). As expected, similar results can be observed in table (10). The only difference between the results presented in tables (9) and (10) is that the benefit of the integrated model in terms of total profit over the decoupled one in class 2 is more moderate comparing to class 1 (84% versus 11% improvement in profit). The reason is due to the fact that in class 2, production planning is integrated with sale and distribution planning. Hence, smaller quantities of inventory and backlog are obtained comparing to class 1 where production planning is decoupled from sale and distribution planning.

In summary, the results and the analysis above suggest that:

(i) The decoupling planning approach provides sub-optimal plans in terms of SC total profit comparing to the integrated planning model.

(ii) The abovementioned sub-optimality gets deteriorated as more decoupled (sub-)models are utilized (i.e., low coordination among SC entities). This is due to the lower level of sharing exact information among the upstream and downstream entities in the SC.

It is worth mentioning that the difference between the profit of the integrated model and the decoupled approach in each table represents the value of adopting an integrated planning approach versus a decoupled one. Furthermore, this value can be interpreted as the maximum price that the owner of the supply chain (for instance, a sawmill) is willing to pay in order to obtain a plan which is coordinated with all entities of the supply chain.

2.4.3 Results of implementing the heuristic solution algorithm

We provide here the results of implementing the heuristic algorithm proposed to solve the integrated tactical planning model for the case study. This algorithm is tested on a set of 10 large-scale test instances to compare the CPU time and the optimality gap with a commercial solver (CPLEX) and with the classical LR algorithm.

2.4.3.1 Implementing the heuristic algorithm on the case study

We first solved the integrated model and the different models of the decoupled approach by CPLEX. Table (3) provides the objective function value and CPU times of the abovementioned models. As it can be observed in this table, solving the large-scale MIP integrated model by a commercial solver is very time-consuming (more than $5h$). Hence, we applied the heuristic algorithm, described in 3.1, in order to reduce the CPU time while obtaining a feasible solution with a small optimality gap as shown in table (3) for the integrated model and the decoupled approach when using CPLEX.

Table 3: CPLEX results

Models	Objective function	CPU time (Sec)
Integrated model	264,684,600	17,097
Sale & Distribution model (Class 1)	704,682,915	78
Production model (Class 1)	535,306,751	141
Harvesting & Procurement model (Class 1)	25,358,533	4,368
Sale & Distribution & Production model (Class 2)	265,536,846	303
Harvesting & Procurement model (Class 2)	26,304,508	7,379

It is worth mentioning that in order to find the initial lower-bound in the heuristic algorithm (step 0 in algorithm (1)), we ran the original integrated model on CPLEX for 30 minutes and we considered the best feasible solution as the initial LB. Our experimental results indicate that this time limit is adequate in order to obtain an initial high quality lower-bound and increasing this time limit does not have a significant impact on accelerating the convergence of LR algorithm. Furthermore, the sub-gradient step-size (step 2 in algorithm (3)) was divided by 2 whenever no improvement in the upper-bound was observed.

Table (4) presents the results of implementing the heuristic algorithm on the case study. The result of applying the classical LR algorithm is also provided in this table. It should be noted that the classical LR algorithm converges in 11 iterations while the converged solution is not feasible regarding the relaxed harvesting constraints. As it can be observed in table (4), the heuristic algorithm provides a high quality feasible solution (0.022% optimality gap) in a significantly shorter CPU time ($1h$ versus $5h$). The results provided in table (4) confirm that the lower-bound updating heuristic (LBUH) (algorithm (2)) proposed to update the step-size in the sub gradient algorithm significantly accelerates the convergence of LR algorithm into a feasible solution (77% improvement in CPU time).

Table 4: Heuristic algorithm results

	Classical LR algorithm	Heuristic algorithm	CPLEX
Profit	264,653,000 (infeasible)	264,624,000	264,684,600
CPU time (Sec)	9,028	3,921	17,097

2.4.3.2 Validating the heuristic algorithm

In order to better validate the performance of the proposed heuristic algorithm, we implemented it on a set of 10 large-scale test instances as summarized in table (5). In this table, the "LB" and "UB" represents the best lower-bound and upper-bound of heuristic algorithm, respectively. Column "heuristic gap%" corresponds to the gap between the UB and LB calculated based on ($\frac{UB-LB}{UB} * 100$), while column "LR time(Sec)", "Heuristic time (Sec)", and "Total time (Sec)" represents the time spent by the commercial solver in the Lagrangian sub-problems, lower-bound updating heuristic (LBUH), and the total time of running the heuristic method, respectively. Moreover, we provided the "CPLEX results" and the "CPLEX time (Sec)" in table (5) which represent the optimal objective value and CPU time of different instances run with the commercial solver. Finally, the "Gap%" field represents the relative gap between the best feasible solution found by the heuristic algorithm and the optimal solution found by CPLEX, and is calculated as ($\frac{CPLEXresult-LB}{CPLEXresult} * 100$). As expected, we could not reach to the optimal solution by CPLEX in 5 hours ($> 5h$) in some instances. In such case, the "CPLEX results" and "Gap%" fields are represented by N.A. It is important to note that "LB", "UB", and "CPLEX results" are divided by 1000 in table (5).

As it can be observed in table (5), the proposed heuristic algorithm provides high quality feasible solutions with small (negligible) optimality gaps in a relatively small CPU time. The reason is that the proposed lower-bound updating heuristic (algorithm (2)) within the sub-gradient algorithm adjusts the search step-size based on an improved upper-bound as we proceed in the sub-gradient algorithm. Hence, not only the convergence of the algorithm is significantly reduced, but also the converged solution is always feasible, as was observed in all test instances in table (5).

Table 5: The results of implementing the heuristic algorithm in different test instances

Instances	LB	UB	heuristic Gap%	LR time (Sec)	Heuristic time (Sec)	Total time (Sec)	CPLEX time (Sec)	CPLEX results	Gap%
1	264,624	264,950	0.12%	2,275	1,646	3,921	17,097	264,684	0.022
2	429,115	429,439	0.08%	681	1,418	2,099	7,465	429,214	0.023
3	375,406	375,668	0.07%	2,151	1,554	3,705	7,616	375,444	0.01
4	524,317	524,361	0.01%	2,776	1,113	3,889	12,806	524,345	0.005
5	191,875	191,905	0.02%	1,424	805	2,229	>5h	N.A.	N.A.
6	352,000	353,153	0.33%	1,190	1,805	2,895	>5h	N.A.	N.A.
7	560,347	560,406	0.01%	633	1,644	2,277	>5h	N.A.	N.A.
8	244,833	244,973	0.06%	2,350	3,470	5,820	>5h	N.A.	N.A.
9	221,278	221,408	0.06%	3,605	2,321	5,926	>5h	N.A.	N.A.
10	47,972	48,116	0.3%	13,178	1,447	14,625	>5h	N.A.	N.A.

2.5 Conclusion

In this paper, we proposed a MIP model to address harvesting, procurement, production, distribution, and sale decisions in the lumber supply chain in an integrated scheme. We evaluated the value of adopting an integrated tactical planning approach versus a decoupled planning method where a set of models corresponding to different entities of supply chain are solved in a sequential manner. Our computational results on a realistic scale case study revealed the sub-optimality of the plan proposed by the decoupled planning approach in terms of the total profit of the supply chain. More precisely, we demonstrated that substantial cost/revenue improvement can be reached by using an integrated tactical planning model rather than a decoupled planning approach. Nonetheless, despite of the superiority of the integrated planning approach, several challenges are expected while trying to implement it in the lumber supply chain. The first issue is the fact that not all entities of this value chain, namely the forest, sawmills, and distribution channels, are owned by one company. In the existing non-integrated supply chain, each entity is seeking to maximize its own profit. In contrary, if stakeholders agree on a vertical collaboration (i.e., adopting an integrated planning approach), where they share information among each other in terms of profit margin, costs, and capacity, it is possible that some parties face with a major loss while others gain major profit. Hence, in order to make sure that the improved results of such integration are achieved, a mechanism must be devised so that all stakeholders are paid-off. Game-theoretical approaches could be adopted to achieve such a win-win situation for all parties. The value of integration can then be interpreted as the maximum price that can be paid in order to facilitate information sharing among entities of this supply chain. On the other hand, solving the integrated model for real-size instances is challenging due to the existence of binary variables corresponding to harvesting schedule. Hence, we proposed a heuristic algorithm in the framework of Lagrangian Relaxation algorithm where the performance of the sub-gradient algorithm was improved in terms of convergence and the feasibility of converged solution. The latter was obtained by updating iteratively the step-size of the sub-gradient algorithm through updating the lower-bound according to the most recent upper-bound. Our computational results on a set of large-scale test cases revealed the effectiveness of the proposed heuristic in obtaining high quality feasible solutions in a considerably reduced CPU time comparing to using a commercial solver, and the classical LR algorithm.

We are currently extending this work to take into account the randomness of demand and log supply into the proposed integrated model, and to solve the resulting large size stochastic formulations by developing efficient algorithms.

Acknowledgments

This work was financially supported by NSERC Strategic Network on Value Chain Optimization (VCO). The authors would like to thank Dr. Catalin Ristea, FPInnovations, for his collaboration in this project.

2.6 Appendix

The indices, parameters, and decision variables are summarized as follows.

Sets

M: Set of manufacturing mills

I_{SW}: Set of products produced by sawing process that are transferred to drying unit (such as lumbers)

I'_{SW}: Set of products produced by sawing process (such as chips and green lumbers)

I_{DR}: Set of products produced by drying process

I_F: Set of products produced by sawing, drying and finishing processes (such as finished product)

I: Set of end products ($I = I'_{SW} \cup I_F$)

T: Set of time periods

C: Set of customers

CC: Set of contract customers

NC: Set of non-contract customers

RM: Set of raw materials

DC: Set of distribution centers

V: Set of vehicles

R: Set of all routes

S: Set of outbound shipping suppliers

$R_{m,dc}$: Set of routes from mills to distribution centers

$R_{dc,c}$: Set of routes from distribution centers to customers

$R_{m,c}$: Set of routes from mills to customer directly

BL: Set of harvesting blocks

Parameters

b_{it}^{c}: The price of product i for customer c during period t

d_{it}^{c}: The forecasted demand of product i from customer c during period t

$dmin_{it}^{c}$: The minimum demand of product i from customer c during period t

Ksw_{mt}: Production capacity of mill m in period t at sawing unit

Kdr_{mt}: Production capacity of mill m in period t at drying unit

Kf_{mt}: Production capacity of mill m in period t at finishing unit

$p1_{imt}$: Capacity consumption for producing product i at mill m in sawing unit during period t

$p2_{imt}$: Capacity consumption for producing product i at mill m in drying unit during period t

$p3_{imt}$: Capacity consumption for producing product i at mill m in finishing unit during period t

$h1_{im}$: Inventory cost of product i at sawing unit of mill m

$h2_{im}$: Inventory cost of product i at drying unit of mill m

$h3_{im}$: Inventory cost of product i at finishing unit of mill m

$bo1_{im}$: Backlog cost of product i at sawing unit of mill m

$bo2_{im}$: Backlog cost of product i at drying unit of mill m

$bo3_{im}$: Backlog cost of product i at finishing unit of mill m

$KIsw_m$: Warehouse inventory capacity of mill m at sawing unit

$KIdr_m$: Warehouse inventory capacity of mill m at drying unit

KIf_m: Warehouse inventory capacity of mill m at finishing unit

KD_m: Expedition capacity of mill m

c_{im}: Unit production cost to produce product i at mill m

ρ_{imt}: Average yield of product i processed at finishing unit of mill m in period t

ϕ_{imt}: Average yield of product i processed at drying unit of mill m in period t

sh^s_{rv}: Shipping fixed cost of supplier s on route r using vehicle type v

e^s_{irv}: Shipping variable cost of supplier s for product i on route r using vehicle type v

h_{idc}: Inventory holding cost for unit quantity of product i at distribution center dc

a_i: Vehicle capacity absorption coefficient per unit of product i

tr_{idc}: Transshipment cost of product i through distribution center dc

KSH^s_v: Shipping capacity of supplier s with vehicle v

KV_v: Capacity of vehicle type v

$cu_{rm,i,m}$: Consumption of raw material rm for producing unit quantity of product i at mill m

$KI_{m,t}$: Inventory capacity at mill m during period t

KS^{bl}_t: Supply capacity of block bl in period t

$qmin^{bl}$: Minimum contract purchase quantity from block bl

$ss_{rm,m}$: Safety stock of raw material rm at mill m

$m^{bl}_{rm,t}$: Unit purchase cost of raw material rm from block m in period t

$h_{rm,m}$: Inventory holding cost of raw material rm at mill m

L_{rm}^{bl}: Lead time of procuring raw material rm from block bl

c_{blt}^{H}: Unit cost to harvest block bl during period t

$c_{rm,bl,t}^{S}$: Unit cost to store raw material rm in block bl during period t

$c_{rm,bl,m,t}^{T}$: Unit cost to transport raw material rm from block bl to mill m during period t

$f_{rm,bl,t}$: Stumpage fee for raw material rm in block bl during period t

l_{bl}: Maximum number of periods over which harvesting can occur in block bl

n_t: Maximum number of blocks in which harvesting can occur during period t

b_t^H: The total harvesting capacity in period t

b_t^T: The total transportation capacity in period t

$v_{rm,bl}$: Volume of available raw material rm in block bl

ϵ_k: Step size modifier in iteration k in algorithm (1)

Decision variables

$X_{rm,m,t}^{bl}$: Purchasing quantity of raw material rm from block bl in period t

$I_{rm,m,t}$: Inventory of raw material rm at mill m at the end of period t

$I_{rm,bl,t}$: Inventory of raw material rm in block bl at the end of period t

$y_{bl,t}$: Proportion of harvested block bl in period t

$H_{bl,t}$: Binary variable that takes 1 if harvesting occurs in block bl during time period t and 0 otherwise

$OXSW_{imt}$: Quantity of product i that should be transferred from sawing to drying unit of mill m in period t

XSW_{imt}: Quantity of product i which should be sawn at sawing unit of mill m in period t

XDR_{imt}: Quantity of product i which should be processed at drying unit of mill m in period t

$OXDR_{imt}$: Quantity of product i which should be transferred from drying to finishing unit of mill m in period t

OXF_{imt}: Quantity of product i which should be transferred from finishing unit of mill m in period t

ISW^{+}_{imt}: Inventory quantity of product i at sawing unit of mill m in period t

IDR^{+}_{imt}: Inventory quantity of product i at drying unit of mill m in period t

IF^{+}_{imt}: Inventory quantity of product i at finishing unit of mill m in period t

ISW^{-}_{imt}: Backlog quantity of product i at sawing unit of mill m in period t

IDR^{-}_{imt}: Backlog quantity of product i at drying unit of mill m in period t

IF^{-}_{imt}: Backlog quantity of product i at finishing unit of mill m in period t

X^{s}_{irvt}: Shipping quantity of product i with shipping supplier s on route r with vehicle v in period t

N^{s}_{rvt}: Number of truckload requirement from shipping supplier s on route r with vehicle v in period t

$I_{i,dc,t}$: Inventory quantity of product i in distribution center dc at the end of period t

S^{c}_{it}: Sale quantity of product i to customer c in period t

BS^{c}_{it}: Backlogged sale quantity of product i to customer c in period t

Chapter 3

A hybrid scenario cluster decomposition algorithm for supply chain tactical planning under uncertainty

The article entitled *"A hybrid scenario cluster decomposition algorithm for supply chain tactical planning under uncertainty"* is included in this chapter. It was accepted (with revision) to the *European Journal of Operational Research* in November 2015. The titles, figures, tables, algorithms and mathematical formulations have been revised to the keep the coherence through the thesis.

Abstract

We propose a Hybrid Scenario Cluster Decomposition (HSCD) heuristic for solving a large-scale multi-stage stochastic mixed-integer programming (MS-MIP) model corresponding to a supply chain tactical planning problem. The HSCD algorithm decomposes the original scenario tree into smaller sub-trees that share a certain number of predecessor nodes. Then, the MS-MIP model is decomposed into smaller scenario-cluster multi-stage stochastic sub-models coordinated by Lagrangian terms in their objective functions, in order to compensate the lack of non-anticipativity corresponding to common ancestor nodes of sub-trees. The sub-gradient algorithm is then implemented in order to guide the scenario-cluster sub-models into an implementable solution. Moreover, a Variable Fixing Heuristic is embedded into the sub-gradient algorithm in order to accelerate its convergence. Along with the possibility of parallelization, the HSCD algorithm provides the possibility of embedding various heuristics for solving scenario-cluster sub-models. The algorithm is specialized to lumber supply chain tactical planning under demand and supply uncertainty. An ad-hoc heuristic, based on Lagrangian Relaxation, is proposed to solve each scenario-cluster sub-model. Our experimental results on a set of realistic-scale test cases reveal the efficiency of HSCD in terms of solution quality and computation time.

3.1 Introduction

Large-scale Multi-stage Stochastic Mixed Integer Programming (MS-MIP) models usually arise in multi-period planning models under uncertain parameters with dynamic and non-stationary behavior over the planning horizon. Supply chain planning (e.g., [35, 36]), and production planning ([27, 37, 38]) under uncertainty are few examples among others.

Such models are among the most intractable ones due to the fact that the number of complicating binary and/or integer variables in the deterministic MIP model increases exponentially once the uncertainty is modeled as a scenario tree in a multi-stage setting. The latter is a viable way of capturing the evolution of all information trajectories over time. A variety of algorithms for solving multi-stage stochastic MIP models have been proposed (e.g., see [39]). Branch-and-Price ([40]) and Branch-and-Fix Coordination methods ([35, 36], [41, 42]) are two of prevalent methods in the literature for solving MS-MIP models. Nonetheless, such algorithms are designated for special structured models such as lot-sizing and batch-sizing problems or pure 0-1 integer programming models. This makes them less suitable for general large-scale MS-MIP models with no particular structure similar to the supply chain tactical planning model investigated in this article.

Scenario decomposition strategies (e.g., see [43, 44]) are among the most efficient approaches to solve large-scale multi-stage stochastic programs. Progressive Hedging Algorithm (PHA) [43] is one of the scenario decomposition techniques that has been successfully applied as a heuristic to solve multi-stage stochastic MIP models. The main idea behind this algorithm is to decompose the original multi-stage stochastic program into deterministic scenario sub-models. Such subproblems are then coordinated by Lagrangian penalty terms in their objective function in order to obtain an implementable solution. Løkketangen and Woodruff [45] proposed a heuristic algorithm based on PHA and Tabu Search. Haugen et al. [46] cast the PHA in a meta-heuristic algorithm where the generated sub-problems for each scenario are solved heuristically. Despite several advantages of Scenario Decomposition (SD) algorithms in solving stochastic programs, such approaches suffer from critical issues non-convergence or unacceptably long run-times in the context of large-scale

MS-MIP models. Recently, Watson and Woodruff [47] proposed algorithmic innovations to address several critical issues of PHA in the context of large-scale two-stage discrete optimization problems. In an attempt to speed up scenario decomposition algorithms in the context of large-scale MS-MIP models, the idea of scenario partitioning (clustering) in scenario trees has been proposed by several authors. The idea is to decompose the initial scenario tree into smaller sub-trees that share a certain number of ancestor nodes. The multi-stage stochastic model is then decomposed into scenario cluster sub-models which are coordinated by Lagrangian penalty terms in their objective function in order to compensate the lack of non-anticipativity. Escudero et al. [41] embedded the idea of scenario partitioning with the Branch-and-Fix Coordination method to solve large-scale 0-1 multi-stage stochastic models. Escudero et al. [48] proposed a cluster Lagrangian decomposition algorithm for solving MS-MIP model while implementing four approaches for updating Lagrangian multipliers. Carpentier et al. [49] proposed a heuristic for scenario partitioning of large scenario trees within the PHA. Escudero et al. [48, 50] demonstrated that adopting the sub-gradient algorithm to coordinate scenario cluster sub-models would result considerably higher convergence comparing to the PHA. It is noteworthy that in the PHA, an implementable solution in each node of the scenario tree is considered as the average of solutions of the set of scenario cluster sub-models that are indistinguishable at that node. In contrary, the sub-gradient method directly imposes the implementability condition in each node through a pair-wise comparison between the solutions of the set of indistinguishable scenario clusters at that node.

In this study, *we extend the idea of scenario clustering of Escudero et al. [48] based on an accelerated sub-gradient method*. More precisely, we propose a scenario cluster decomposition (SCD) based on the sub-gradient method. Along with this contribution, there are also three other main contributions in the present paper. First, with the goal of reducing the number of iterations, *we embed a variable-fixing heuristic within the sub-gradient algorithm*. This algorithm fixes the value of binary variables in the common nodes of scenario cluster sub-models obtained at each iteration of SCD algorithm to zero or one in the next iteration according to a consensus rule among the solution of indistinguishable scenario cluster sub-models. Second, *the accelerated SCD algorithm is specialized to tactical supply and procurement planning in the lumber supply chain under demand and supply uncertainty*. To the best of our knowledge, due to high computational complexity, this problem has never been addressed in the literature. In this problem, scenario-cluster sub-models are MS-MIP models that are hard to solve. Hence, our third contribution is focused on *proposing an ad-hoc heuristic to solve such sub-models*. This algorithm is a Lagrangian Relaxation-based heuristic enhanced through updating the sub-gradient step-size.

Hence, the proposed algorithm in this study is a Hybrid Scenario Cluster Decomposition (HSCD) heuristic applicable to large-scale MS-MIP models with a particular application in supply chain tactical planning. Along with the possibility of parallelization, the main advantage of the HSCD heuristic is accelerating the sub-gradient algorithm applied to coordinate scenario cluster sub-models into an implementable solution. Furthermore, it provides the possibility of embedding proper heuristics for solving scenario cluster sub-models depending on their special structure. Hence, significant improvement in terms of the convergence of the sub-gradient algorithm within the HSCD algorithm can be expected.

Our experimental results on a set of real-size test cases in a Canadian lumber supply chain indicate the high quality of the solutions obtained by the HSCD heuristic in addition to significant CPU time reduction comparing to a commercial solver and the SCD algorithm proposed in [42] and [48].

This article is organized as follows. In Section 3.2, a brief description of multi-stage stochastic programming is provided. The details of the HSCD algorithm is presented in Section 3.3. Section 3.4 summarizes the specialization of the HSCD algorithm to tactical supply and procurement planning in the lumber supply chain. Numerical results and concluding remarks are respectively provided in Sections 3.5 and 3.6. Finally, the multi-stage programming model for harvesting and procurement tactical planning in the lumber supply chain that is another contribution of this study is presented in the Appendix.

3.2 Multi-stage stochastic mixed-integer programs

Let us consider the following multi-period deterministic mixed integer model [1]:

$$
\begin{aligned}
& Z = \min \sum_{t \in \tau} [a_t x_t] \\
& \text{Subject to:} \\
& A_1 x_1 \leq c_1 \\
& A'_t x_{t-1} + A_t x_t \leq c_t \qquad \forall t \in \tau \setminus \{1\} \\
& x_t \geq 0 \quad \forall t \in \tau
\end{aligned}
\tag{63}
$$

where $\tau = \{1, 2, 3, ..., T\}$ is the set of periods such that $T = |\tau|$; x_t is the vector of decision variables including binary, integer, and continuous variables. a_t is the vector of objective function coefficients, A_t and A'_t are the constraints matrices, and c_t is the right-hand-side vector in period t. For the sake of simplicity, we only present constraints with variables linking consecutive periods. These types of models are very useful in practice and usually arise in multi-period planning models under uncertain parameters with dynamic and non-stationary behavior over the planning horizon. Supply chain planning, lot-sizing, and batch-sizing problems are few examples among others.

Without loss of generality, we assume that the uncertainty may affect parameters associated with technological coefficients in the constraints, as well as the right-hand-side vector. Furthermore, we consider that uncertain parameters have a dynamic behavior over time, hence they can be modeled as a scenario tree. A scenario tree, represented by $Tree$ in this paper, is a computationally viable way of discretising the underlying dynamic stochastic data over time. Figure 2 represents a four-stage scenario tree. Each stage in a scenario tree denotes the stage of the time when new information is available to the decision maker. The root node in a scenario tree represents the current state while the other nodes represent scenarios in other stages. Moreover, a scenario ω is a specific path from the root to the leaf of the tree. A probability ($pr(n)$) is associated to each node of the scenario tree indicating the likelihood of the corresponding node.

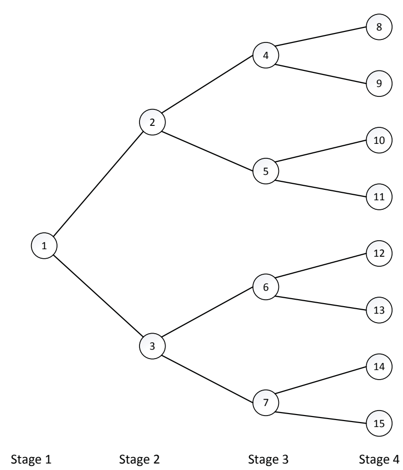

Figure 2: A four-stage scenario tree

It is possible to reformulate model (63) in the stochastic context based on a given scenario tree that leads to the Deterministic Equivalent Model (DEM). In order to obtain the multi-stage stochastic formulation, each decision variable in model (63) can be defined for each scenario, leading to the scenario reformulation model. However, the latter will not yield an implementable solution, as the decision maker cannot foresee the unknown parameters. The implementability or non-anticipativity condition (NAC) indicates that the decision variables corresponding to each node of the scenario tree at stage t are identical for any pair of indistinguishable scenarios at that stage. There are at least two ways to impose NAC in multi-stage stochastic programs leading to split and compact variable formulations. In the split formulation (model (64)), we need to enforce non-anticipativity by adding extra constraints explicitly.

$$
\begin{aligned}
& Z = \min \sum_{\omega \in \Omega} \sum_{t \in \tau} w^{\omega} [a_t x_t^{\omega}] \\
& \text{Subject to:} \\
& A_1^{\omega} x_1^{\omega} \leq c_1^{\omega} \quad \forall \omega \in \Omega \\
& A'^{\omega}_t x_{t-1}^{\omega} + A_t^{\omega} x_t^{\omega} \leq c_t^{\omega} \quad \forall t \in \tau \setminus \{1\}, \omega \in \Omega \\
& x_t^{\omega} = x_t^{\omega'} \quad \forall t \in \tau \setminus \{T\}, \omega, \omega' \in B_t^{\omega}, \omega \neq \omega' \\
& x_t^{\omega} \geq 0 \quad \forall t \in \tau, \omega \in \Omega
\end{aligned} \tag{64}
$$

where Ω represents the set of scenarios, w^{ω} represents the probability attributed to scenario ω, and B_t^{ω} is bundle of scenarios that are indistinguishable from ω at stage t. A_t^{ω} and A'^{ω}_t are the constraints matrices and c_t^{ω} is the right-hand-side vector for scenario ω in period t. Moreover, the non-anticipativity condition is explicitly formulated as $x_t^{\omega} = x_t^{\omega'}$.

In contrast, in the compact formulation, the NAC is implicitly considered in DEM through defining decision variable for the nodes of the scenario tree. Model (65) is the compact formulation of model (63) in the stochastic context.

$$Z = \min \sum_{n \in Tree} \sum_{t \in \tau} pr\left(n\right)\left[a_t x_t\left(n\right)\right]$$

Subject to:

$$A_1 x_1 \leq c_1$$

$$A'_t\left(n\right) x_{t-1}\left(a(n)\right) + A_t\left(n\right) x_t\left(n\right) \leq c_t\left(n\right) \qquad \forall t \in \tau \setminus \{1\}, n \in Tree$$

$$x_t\left(n\right) \geq 0 \quad \forall t \in \tau, n \in Tree \tag{65}$$

where $x_t(n)$ represents the vector of decision variables at node n in stage t in the scenario tree. $A_t(n)$ and $A'_t(n)$ are the constraints matrices and $c_t(n)$ is the right-hand-side vector at node n in stage t. The immediate predecessor of node n in the scenario tree is denoted as $a(n)$. Moreover, the probability of each node, $pr(n)$, is calculated as $\sum_{\omega \in B_t^\omega} w^\omega$.

Comparing multi-stage stochastic models (64) and (65) with the deterministic model (63) clearly indicates that the number of decision variables grows exponentially as the number of stages and branches in the scenario tree increases. This would make the model computationally intractable, particularly in cases that all or part of decisions are binary/integer. This is the main motivation behind developing efficient solution algorithms for solving this class of problems.

3.3 Hybrid Scenario Cluster Decomposition (HSCD) algorithm

The main idea behind the HSCD algorithm is to embed efficient heuristics within the scenario cluster decomposition (SCD) scheme in order to accelerate its convergence. Hence, we extend the method of Escudero et al. [48] in MS-MIP models based on an accelerated sub-gradient method.

The acceleration is based on a Variable Fixing Heuristic (VFH). Furthermore, since each scenario cluster sub-model is a multi-stage stochastic program that might be hard to solve in real-size instances, we propose to use an efficient ad-hoc heuristic to solve them. The procedure described above is summarized in Algorithm 4 and will be elaborated in what follows.

Algorithm 4 HSCD heuristic

Step 1. Scenario Cluster Decomposition (SCD) algorithm
- Step 1.1. Partition the scenario tree into a number of sub-trees after choosing the break stage [42]
- Step 1.2. Formulate scenario cluster sub-models in a compact form after adding NAC violation terms in their objective function

Step 2. Sub-gradient method (to obtain an implementable solution for scenario cluster sub-models)
- Step 2.1. Ad-hoc heuristic (to solve each scenario-cluster sub-model)
- Step 2.2. VFH heuristic (to speed-up the sub-gradient algorithm)

3.3.1 Scenario Cluster Decomposition (SCD) algorithm

Scenario clustering in multi-stage stochastic programming is equivalent to breaking down the original scenario tree in a given stage (i.e., break stage) and obtain a set of scenario cluster sub-trees. For instance, in the scenario tree depicted in Figure 2, if the break stage is chosen as stage 2, four scenario cluster sub-trees are obtained, as depicted in Figure 3. As can be observed in Figure 3, two of the latter sub-trees share node 2 and the other two share node 3, and all share node 1 in the original scenario tree. In the SCD algorithm proposed in this article, after clustering the original scenario tree, we formulate a multi-stage stochastic model for each scenario sub-tree (cluster) using the compact formulation by relaxing the NACs corresponding to common nodes. Scenario cluster sub-models are then coordinated by Lagrangian penalty terms in their objective function in order to compensate the lack of non-anticipativity. Finally, scenario cluster sub-models are solved into an implementable solution by the aid of sub-gradient algorithm. The advantages of using a direct sub-gradient method for coordinating scenario cluster sub-models rather than the PHA in terms of convergence has been demonstrated in Escudero et al. [50] for two-stage stochastic programs.

In order to select the break stage in the SCD algorithm, the trade-off between the size and the number of scenario cluster sub-models must be taken into consideration. On the one hand, as the number of scenario cluster sub-trees increases more sub-problems in the sub-gradient algorithm should be solved which would slow down its convergence. On the other hand, if the chosen number of scenario clusters is small, each scenario cluster sub-model becomes a large-scale MS-MIP which might be hard to solve. Finally, it should be noted that the convergence of scenario decomposition algorithms to an optimal solution has been proved in the literature ([43]) for linear MSP problems. However, such algorithms can be considered as heuristics in the context of integer or MIP problems (see, e.g., [45]). Same can be expected when applying the SCD algorithm to MS-MIP models. In what follows, we provide more details on partitioning the original scenario tree into sub-trees as well as the general formulation of scenario-cluster sub-models within the SCD framework.

3.3.1.1 Partitioning the scenario tree into scenario cluster sub-trees

As mentioned earlier, the first step of the SCD algorithm is choosing the break stage (t^*). Hence, in a symmetric scenario tree, the original scenario tree is decomposed to $\hat{p} = |\Omega|/l$ scenario cluster sub-trees, where Ω indicates the scenario set and l denotes the number of scenarios in each sub-tree. It should be noted that the abovementioned sub-trees share common nodes that belong to stages $t = 1, 2, ..., t^*$ in the initial scenario tree. Let us denote N^p as the set of nodes belonging to sub-tree p and η^n as sub-tree sets that have node n in common. N_1 and N_2 are sets of nodes belonging to stages that are not after (i.e., $t = 1, 2, ..., t^*$) and after the break stage (i.e., $t = t^*+1, t^*+2, ..., T$). Furthermore, $N_1^p = N_1 \cap N^p$ and $N_2^p = N_2 \cap N^p$ are sets of nodes in sub-tree p belonging to N_1 and N_2, respectively. For example, consider the four-stage scenario tree in Figure 2 that is broken down into four scenario cluster sub-trees in Figure 3. In this figure, the breaking stage is at $t^* = 2$, and there are four scenario cluster sub-trees. $\eta^1 = \{sub-tree\ 1, sub-tree\ 2, sub-tree\ 3, sub-tree\ 4\}, \eta^2 = \{sub-tree\ 1, sub-tree\ 2\}, \eta^3 = \{sub-tree\ 3, sub-tree\ 4\}$ show that node 1 is included in sub-trees 1, 2, 3 and 4; node 2 is included in sub-trees 1 and 2; and node 3 is included in sub-trees 3 and 4. Also, $N^1 = \{1, 2, 4, 8, 9\}$, $N^2 = \{1, 2, 5, 10, 11\}$, $N^3 = \{1, 3, 6, 12, 13\}$, and $N^4 = \{1, 3, 7, 14, 15\}$. Moreover, $N_1 = \{1, 2, 3\}$, $N_2 = \{4, 5, ..., 15\}$, $N_1^1 = \{1, 2\}$, and $N_2^1 = \{4, 8, 9\}$.

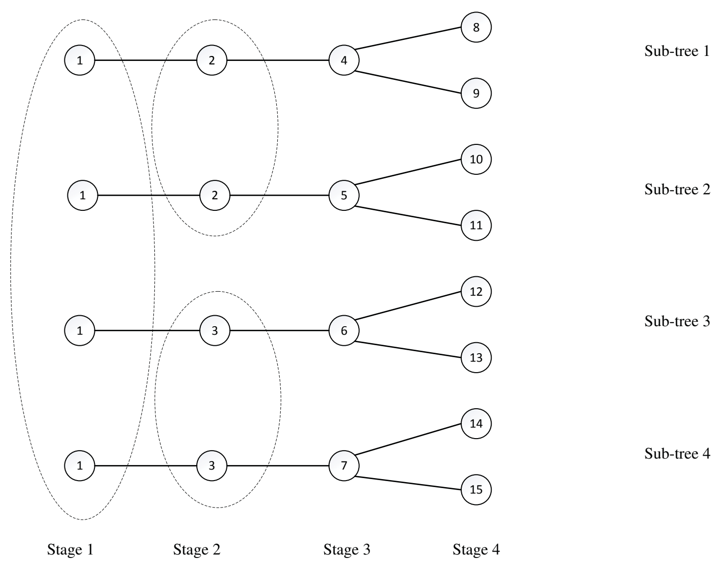

Figure 3: A decomposed scenario tree to four clusters

3.3.1.2 Scenario cluster sub-model formulation

Let us consider the compact formulation of the DEM of a multi-stage stochastic problem (model (65)). After breaking down the scenario tree into $\hat{p}$ sub-trees, we decompose model (65) into $\hat{p}$ sub-models accordingly. Thus, the multi-stage stochastic MIP sub-model for each scenario cluster p can be expressed by the following compact formulation (model (66)).

$$
\begin{aligned}
& Z^p = \min \sum_{n \in N_1^p} \sum_{t \in \tau} w^p(n)[a_t x_t^p(n)] \\
& + \sum_{n \in N_2^p} \sum_{t \in \tau} pr(n)\,[a_t x_t^p(n)] \\
& \text{Subject to:} \\
& A_1 x_1 \leq c_1 \\
& A_t^{'p}(n)\, x_{t-1}^p(a(n)) + A_t^p(n)\, x_t^p(n) \leq c_t^p(n) \quad \forall t \in \tau \setminus \{1\},\ n \in N^p \\
& x_t^p(n) \geq 0 \quad \forall t \in \tau, n \in N^p
\end{aligned} \tag{66}
$$

where $w^p(n) = \sum_{\omega \in \Omega_p} w^\omega$ is the weight (probability) of nodes before the break stage ($n \in N_1$) in sub-tree p; w^ω is likelihood of scenario ω; and Ω_p is the set of scenarios belonging to sub-tree p. Moreover, the $\hat{p}$ sub-problems (66) should be linked with each other by the NAC as follows:

$$
x_t^p(n) - x_t^{p'}(n) = 0 \quad \forall p, p' \in \eta^n : p \neq p', t \in \tau, n \in N_1 \tag{67}
$$

Considering the same example in Figure 3, the explicit NAC formulated in (67) can be expressed as follows:

$$
\begin{aligned}
& x_1^1(1) = x_1^2(1) = x_1^3(1) = x_1^4(1) \\
& x_2^1(2) = x_2^2(2) \\
& x_2^3(3) = x_2^4(3)
\end{aligned} \tag{68}
$$

It is also possible to consider the NAC (67) as a set of inequalities:

$$x_t^p(n) - x_t^{p+1}(n) \leq 0 \quad \forall p = \underline{p}_{\eta^n}, ..., (\bar{p}_{\eta^n}) - 1, t \in \tau, n \in N_1 \tag{69}$$

$$x_t^{\bar{p}_{\eta^n}}(n) - x_t^{\underline{p}_{\eta^n}}(n) \leq 0 \quad \forall t \in \tau, n \in N_1 \tag{70}$$

where $\underline{p}_{\eta^n}$ and $\bar{p}_{\eta^n}$ are the first and the last ordered sub-trees belonging to η^n(i.e., $\min\{p|p \in \eta^n\}$ and $\max\{p|p \in \eta^n\}$, respectively). At each stage belonging to $\tau = \{1, 2, ..., t^*(break\ stage)\}$, η^n represents the set of scenario sub-trees that have node $n \in N_1$ in common. For instance, in the decomposed scenario sub-trees in Figure 3, at stage 2, η^2 represents the scenario sub-trees 1 and 2; and η^3 represents the scenario sub-trees 3 and 4. Finally, the original MS-MIP model can be formulated as a splitting-compact variable representation model over the set of sub-trees [42] as follows:

$$Z_{MS-MIP} = \min \sum_{p=1}^{\hat{p}} \sum_{n \in N_1^p} \sum_{t \in \tau} w^p(n)[a_t x_t^p(n)] + \sum_{p=1}^{\hat{p}} \sum_{n \in N_2^p} \sum_{t \in \tau} pr(n)[a_t x_t^p(n)]$$

Subject to:

$$A_1 x_1 \leq c_1$$

$$A'^p_t(n)\, x_{t-1}^p(a(n)) + A_t^p(n)\, x_t^p(n) \leq c_t^p(n) \quad \forall p \in \eta^n, t \in \tau \setminus \{1\}, n \in \{N_1, N_2\}$$

$$x_t^p(n) - x_t^{p+1}(n) \leq 0 \quad \forall p = \underline{p}_{\eta^n}, ..., (\bar{p}_{\eta^n}) - 1, t \in \tau \setminus \{T\}, n \in N_1$$

$$x_t^{\bar{p}_{\eta^n}}(n) - x_t^{\underline{p}_{\eta^n}}(n) \leq 0 \quad \forall t \in \tau \setminus \{T\}, n \in N_1$$

$$x_t^p(n) \geq 0 \quad \forall t \in \tau, n \in Tree, p \in \eta^n \tag{71}$$

By relaxing the NACs (the last two set of constraints) in model (71), the latter is decomposable into $\hat{p}$ sub-problems corresponding to each scenario cluster sub-tree p. In order to compensate the lack of non-anticipativity in such sub-models, Lagrangian penalty terms ($\mu_t^p(n)$) can be added to their objective function and the sub-gradient algorithm can be implemented to solve the Lagrangian models. Model (72) is the

Lagrangian relaxation of model (71) after relaxing the NACs.

$$Z_{SCD}(\mu,\hat{p}) = \min \sum_{p=1}^{\hat{p}} \sum_{n\in N_1^p} \sum_{t\in\tau} w^p(n)\left[a_t x_t^p(n)\right] + \sum_{p=1}^{\hat{p}} \sum_{n\in N_2^p} \sum_{t\in\tau} pr(n)\left[a_t x_t^p(n)\right]$$

$$+ \sum_{p=\underline{p}_{\eta^n}}^{(\bar{p}_{\eta^n})-1} \sum_{n\in N_1} \sum_{t\in\tau} \mu_t^p(n)(x_t^p(n) - x_t^{p+1}(n)) + \sum_{n\in N_1} \sum_{t\in\tau} \mu_t^{\bar{p}_{\eta^n}}(n)(x_t^{\bar{p}_{\eta^n}}(n) - x_t^{\underline{p}_{\eta^n}}(n))$$

Subject to:

$$A_1 x_1 \le c_1$$

$$A'^{p}_{t}(n)\, x_{t-1}^p(a(n)) + A_t^p(n)\, x_t^p(n) \le c_t^p(n) \quad \forall p\in\eta^n, t\in\tau\setminus\{1\}, n\in\{N_1, N_2\}$$

$$x_t^p(n) \ge 0 \quad \forall t\in\tau, n\in Tree, p\in\eta^n \tag{72}$$

As we mentioned before, model (72) can be decomposed into $\hat{p}$ smaller sub-problems (model (74)), and its objective function can be obtained as the sum of $Z_{SCD}^p(\mu)$ values corresponding to each sub-tree as follows:

$$Z_{SCD}(\mu,\hat{p}) = \sum_{p=1}^{\hat{p}} Z_{SCD}^p(\mu) \tag{73}$$

where $Z_{SCD}^p(\mu)$ is the objective value of the pth scenario sub-tree. For $p = \underline{p}_{(\eta^n)} + 1, \ldots, \bar{p}_{\eta^n}$, the scenario cluster sub-model can be expressed (in compact formulation) as follows:

$$Z^p_{SCD}(\mu) = \min \sum_{n\in N^p_1} \sum_{t\in\tau} w^p(n)\left[a_t x^p_t(n)\right]$$

$$+ \sum_{n\in N^p_2} \sum_{t\in\tau} pr(n)\left[a_t x^p_t(n)\right] + \sum_{n\in N_1} \sum_{t\in\tau} (\mu^p_t(n) - \mu^{p-1}_t(n))(x^p_t(n))$$

Subject to:

$$A_1 x_1 \leq c_1$$

$$A'^p_t(n)\, x^p_{t-1}(a(n)) + A^p_t(n)\, x^p_t(n) \leq c^p_t(n) \quad \forall t \in \tau \setminus \{1\}, n \in \{N_1, N_2\}$$

$$x^p_t(n) \geq 0 \quad \forall t \in \tau, n \in N^p \tag{74}$$

It should be noted that for $p = \underline{p}_{\eta^n}$, the term $\sum_{n\in N_1}\sum_{t\in\tau}(\mu^p_t(n) - \mu^{p-1}_t(n))(x^p_t(n))$ should be replaced by $\sum_{n\in N_1}\sum_{t\in\tau}(\mu^{\underline{p}_{\eta^n}}_t(n) - \mu^{\bar{p}_{\eta^n}}_t(n))$ $(x^{\underline{p}_{\eta^n}}_t(n))$ in the objective function. Finally, scenario cluster sub-models (74) are co-ordinated to an implementable solution by the aid of the sub-gradient algorithm. In what follows, we provide the details of sub-gradient algorithm and heuristics embedded in it to speed-up its convergence.

3.3.2 Scenario-cluster decomposition algorithm

The SCD algorithm is summarized in Algorithm 5. After initializing the Lagrangian multipliers $(\mu^p_t(n))$, each scenario cluster sub-model is solved by the aid of an ad-hoc heuristic. Once, the solutions of all sub-trees are obtained, the violation of corresponding NACs are verified and subsequently the sub-gradient vector $(s^k_t(n))$ is calculated. Next, the Lagrangian multipliers are updated. This procedure is repeated for a number of iterations until the NAC are satisfied within a given error threshold. In Algorithm 5, $\bar{Z}_{HSCD}$ is an upper-bound on the objective function of each scenario cluster sub-model and α_k is the step modifier. The stopping criterion in this algorithm requires that Z_{HSCD} does not improve after two consecutive iterations.

Algorithm 5 SCD algorithm (sub-gradient based)

Step 0 (initialization):

Assign zero to Lagrangian multipliers vector ($\mu^0 = 0$) and solve the $\hat{p}$ sub-problems (74) to obtain x^0 and $Z_{HSCD}\left(\mu^0, \hat{p}\right)$ based on equation (73)

Let iteration counter k equal to 1

while the stopping criteria is not satisfied **do**

Step 1 ($\forall n \in N_1$,$t \in \tau$)

Compute the sub-gradient vector at node n for the set of sub-trees in η^n

$$s_t^k(n) = \begin{pmatrix} x_t^{k,\underline{p}_{\eta^n}}(n) - x_t^{k,(\underline{p}_{\eta^n})+1}(n) \\ . \\ . \\ . \\ x_t^{k,((\bar{p}_{\eta^n})-1)}(n) - x_t^{k,\bar{p}_{\eta^n}}(n) \\ x_t^{k,\bar{p}_{\eta^n}}(n) - x_t^{k,\underline{p}_{\eta^n}}(n) \end{pmatrix}$$

Step 2

Update Lagrangian multipliers as follows:

$$\mu_t^{k+1}(n) = \mu_t^k(n) + \alpha_k . \frac{\left(\bar{Z}_{HSCD} - Z_{HSCD}\left(\mu^k, \hat{p}\right)\right)}{\left\| s_t^k(n) \right\|^2} . s_t^k(n)$$

Step 3

Solve the $\hat{p}$ sub-problems (74) with μ^{k+1} to obtain x^{k+1} and $Z_{HSCD}\left(\mu^{k+1}, \hat{p}\right)$

Step 4

Set $k \leftarrow k{+}1$

end while

3.3.2.1 Variable Fixing Heuristic (VFH) algorithm

As mentioned earlier, in order to accelerate the convergence of sub-gradient algorithm within the SCD algorithm, we propose a Variable Fixing Heuristic. The idea is to reduce the number of binary variables in each scenario cluster sub-model (74) in order to reduce the CPU time required to solve them. The details of this heuristic algorithm are elaborated in Algorithm 6. In this algorithm, $x_t^{p\mathbb{B}}(n)$ represents the vector of binary decision variables.

At each iteration of the SCD algorithm, the VFH algorithm verifies the value of binary variables corresponding to all nodes in each stage before the break stage ($n \in N_1$), obtained in the previous iteration over all scenario cluster sub-models (74).

Algorithm 6 VFH algorithm

Step 1.

Solve MS-MIP sub-problem for all scenario sub-trees (model (74)) in SCD algorithm (Algorithm 5) and obtain the vector of binary decision variables $x_t^{p\mathbb{B}}(n)$ at all nodes before the break stage ($\forall n \in N_1$)

for ($\forall n \in N_1$) **do**

Step 2. Over all sub-trees (p) that share node n, count the number of sub-trees where a given binary variable takes 1 or 0.

Step 3. Update $Counter = \sum_{p\in\eta^n} x_t^{p\mathbb{B}}(n)$

Step 4. Consensus rule:

Step 4.1. Fix $x_t^{p\mathbb{B}}(n)$ in the next iteration in SCD as follows:

$$x_t^{p\mathbb{B}}(n) = \begin{cases} 1\ ,\ Counter > \frac{|\eta^n|}{2} \\ \\ 0\ ,\ Counter < \frac{|\eta^n|}{2} \end{cases}$$

end for

Next, such binary variables are fixed to 0 or 1 according to a consensus rule among all sub-trees. For instance, consider the case of 3 scenario sub-trees, where at a given iteration k, the value of a given binary variable at node n in a given stage before the break stage is equal to 1 in two of sub-trees and equal to 0 in the third one. According to the consensus rule given in Algorithm 6, we fix that binary variables at node n to 1 at iteration $k + 1$ in all scenario sub-trees that share node n.

3.4 Application of HSCD heuristic to tactical supply and procurement planning in the lumber supply chain

In this section, we aim at specializing the proposed HSCD algorithm described in Section 3.3 to a tactical supply and procurement planning in the lumber supply chain (SC) under supply and demand uncertainty. In what follows, we first describe the problem and the uncertainty involved in it. Furthermore, we provide the details of the application of the HSCD algorithm to solve this problem.

3.4.1 Tactical supply and procurement planning in the lumber supply chain

In the lumber supply chain, the raw materials are logs that are shipped from forest contractors to sawmills. Logs are then sawn to green/finished lumbers in sawmills and are distributed to the lumber market through different channels. Supply and procurement planning in the lumber SC determines the optimal harvesting and procurement decisions over a planning horizon with the goal of minimizing cost while satisfying the log demand in sawmills. More precisely, we are looking for the harvesting schedule, i.e. the selection of harvesting blocks and the quantity of harvesting in each period by considering several harvesting constraints as well as log procurement decisions in sawmills. This problem can be formulated as a MIP model in the deterministic context [51].

In lumber SC, forecasting the exact amount of demand for log type rm at sawmill m in period t ($d_{rm,m,t}$) is almost impossible. Furthermore, the availability of log type rm in harvesting block bl ($v_{rm,bl}$) is also uncertain. Thus, considering random demand and supply in harvesting and lumber procurement tactical planning is essential. The uncertain log demand can be modeled as a scenario tree similar to the one depicted in Figure 2. The nodes at each stage of the scenario tree constitute the states of the demand that can be distinguished by the information available up to that stage. On the other hand, the uncertainty in forecasting the availability of logs in each block can be modeled as a time independent random variable over the planning horizon; hence it can be modeled as a scenario set. According to the abovementioned uncertainty modeling for the log demand and supply, the Harvesting/Procurement (HP) tactical planning problem can be formulated as multi-stage mixed-integer stochastic program with recourse (model (79)-(93) in the appendix).

It should be noted that model (79)-(93) is a compact formulation, where the decision variables have been defined for each node of the scenario tree. Also, the stages in models (79)-(93) incorporate a number of time periods, and t_n denotes the set of all time periods corresponding to node n. Due to the large size of tactical planning model in a real-size case study, we applied the HSCD heuristic described in Section 3.3 as a solution method.

3.4.2 Solving HP tactical planning model by the aid of HSCD heuristic

The HP problem is a multi-stage-stochastic mixed-integer programming problem. After decomposing the original scenario tree into a set of sub-trees (step 1.1. of Algorithm 4), we formulate the HP problem as an MS-MIP model for each scenario sub-tree (step 1.2) by considering the NACs corresponding to the implementability of $H_{bl,t}(n)$ (harvesting schedule), $Y_{bl,t}(n)$ (portion of harvesting), $X^{bl}_{rm,m,t}(n)$ (procurement quantity) in model (79)-(93) as follows:

$$H^{p}_{bl,t}(n) - H^{p'}_{bl,t}(n) = 0 \quad \forall p, p' \in \eta^n : p \neq p', bl \in BL, t \in t_n, n \in N_1 \tag{75}$$

$$Y^{p}_{bl,t}(n) - Y^{p'}_{bl,t}(n) = 0 \quad \forall p, p' \in \eta^n : p \neq p', bl \in BL, t \in t_n, n \in N_1 \tag{76}$$

$$X^{bl,p}_{rm,m,t}(n) - X^{bl,p'}_{rm,m,t}(n) = 0 \quad \forall p, p' \in \eta^n : p \neq p', bl \in BL, rm \in RM,$$
$$m \in M, t \in t_n, n \in N_1 \tag{77}$$

Next, we implement the sub-gradient algorithm to solve the scenario cluster sub-models into an implementable solution. Since, such sub-models are MS-MIP problems that are hard to solve for large-scale test instances, steps 2.1 and 2.2 in Algorithm 4 are applied as follows:

Step 2.1 (Algorithm 4): A Lagrangian Relaxation-based Heuristic (LRH) for solving scenario cluster HP sub-models

It is worth mentioning that the complexity of the integrated model (79)-(93) is due to the existence of binary variables corresponding to harvesting schedule during the planning horizon, in addition to constraints (84) and (85) that formulate the harvesting constraints in the forest. In other words, the latter two constraints can be considered as complicating constraints in the sense that after relaxing them from model (79)-(93), the resulting model can be solved much faster by a commercial solver. Hence, we propose a Lagrangian Relaxation (LR) algorithm where the aforementioned constraints are relaxed and their violation is penalized in the objective function by the aid of Lagrangian multipliers. Next, the sub-gradient method can be used to solve the Lagrangian model. The summary of LRH algorithm for model (79)-(93) is summarized in Algorithm 7. The stopping criterion in the LRH is the same as SCD algorithm (Algorithm 5), and ϵ_h is the step modifier. The Lagrangian Relaxation of model (79)-(93) (Lagrangian model) can be stated as model (78) where $u_{bl}(n)$ and $v_t(n)$ are the Lagrangian multipliers corresponding to constraints (84) and (85), respectively. $A(n)$ is the set of ancestors of nodes that belong to the last stage (N_T).

$$L^h_{IP}(u,v) = Minimize \Bigg\{ Z_{HP} - \sum_{n' \in N_T} \sum_{bl \in BL} u_{bl}(n') * (l_{bl} - \sum_{t \in t_n} \sum_{n \in A(n')} H_{bl,t}(n)) -$$

$$\sum_{n \in Tree} \sum_{t \in t_n} v_t(n) * (n_t - \sum_{bl \in BL} H_{bl,t}(n)) \Bigg\} \quad (78)$$

Subject to:

(79)-(83) and (86)-(93).

Note that solving model (78) by a classical sub-gradient algorithm suffers from two essential issues namely the slow convergence of the algorithm and the infeasibility of the converged solution. Consequently, we propose a Lagrangian Relaxation Heuristic (LRH) where we embed a heuristic algorithm which iteratively updates the search step-size of the sub-gradient algorithm. This is achieved through obtaining a new upper-bound on the objective function value based on the most recent lower-bound. The idea is to improve the quality of upper-bound based on the improved lower-bound as we proceed in the sub-gradient algorithm. The improved upper-bound can be used to adjust the search step-size (step 3 in Algorithm 7) which is expected to accelerate the convergence of the sub-gradient algorithm and to avoid the infeasibility of the converged solution. Algorithm 8 summarizes the upper-bound updating algorithm. Updating the upper-bound is performed by solving a reduced MS-MIP model obtained after fixing a certain number of binary decision variables to zero according to the following criteria. In each iteration of the LRH, the solution of the Lagrangian model is verified in terms of feasibility of the relaxed binary constraints. Next, in the non-violated constraints, we identify those binary variables with the value equal to zero at the optimal solution. Those variables are then fixed to zero in each HP scenario cluster sub-model in order to obtain a feasible solution and a new upper-bound on the optimal objective function value.

Algorithm 7 LRH algorithm

Step 0 (initialization):

Assign zero to $u_{bl}(n')(n' \in N_T)$ and $v_t(n)(n \in Tree)$

Find an initial upper-bound (UB) (feasible solution) and assign $-\infty$ to the lower-bound (LB)

Let iteration counter k equal to 1

while the stopping criteria is not satisfied **do**

Step 1

Solve the Lagrangian problem (78) and determine the objective function value of $L^k_{IP}(u,v)$

Step 2

If $(L^k_{IP}(u,v) > LB)$ then $LB = L^k_{IP}(u,v)$

Update the upper-bound (UB) based on the "**Upper-bound updating algorithm**"

Step 3

Update Lagrangian multipliers as follows:

$$u^{k+1}_{bl}(n') = max\{u^k_{bl}(n') - \epsilon_k * \frac{L^k_{IP}(u,v)-LB}{\left\|l_{bl}-\sum_{t\in t_n}\sum_{n\in A(n')} H_{bl,t}(n)\right\|^2} * (l_{bl} - \sum_{t\in t_n}\sum_{n\in A(n')} H_{bl,t}(n)), 0\}$$

$$v^{k+1}_t(n) = max\{v^k_t(n) - \epsilon_k * \frac{L^k_{IP}(u,v)-LB}{\left\|n_t-\sum_{bl\in BL} H_{bl,t}(n)\right\|^2} * (n_t - \sum_{bl\in BL} H_{bl,t}(n)), 0\}$$

$k = k + 1$

end while

Step 2.2 (Algorithm 4) (VFH)

In each iteration of Algorithm 5 (SCD), we apply VFH heuristic (Algorithm 6) in order to reduce the number of iterations. For this purpose, we should find the value of $\sum_{p\in\eta^n} H^p_{bl,t}(n)$ in all nodes belonging to the stages before the break stage $(n \in N_1)$ and fix the value of $H^p_{bl,t}(n)$ to 0 or 1 in the next iteration of SCD algorithm based on the given criterion in Algorithm 6.

3.5 Numerical results

In this section, we first provide details on generating a testbed of problem instances along with algorithmic implementations. Then we provide the results of applying the

Algorithm 8 Upper-bound updating algorithm

Step 0 (initialization):
  Calculate *slack variable* corresponding to binary constraints (84) and (85)
if *slack variable* ≥ 0 **then**
  Step 1:
    Identify the binary variables which are equal to 0 and fix them to zero in each scenario cluster sub-model (74) in order to obtain a reduced sub-model
  Step 2:
    Solve each reduced scenario cluster sub-model (74) (by a commercial solver) to obtain a new upper-bound ($new\ UB$)
  Step 3:
  if ($new\ UB$) < ($old\ UB$) **then**
      Update upper-bound for the next iteration in LRH algorithm
(UB) = $new\ UB$
  end if
end if

HSCD heuristic proposed in this article to those test instances, where we compare the CPU time and the quality of solutions obtained by this algorithm with a commercial solver (CPLEX v12.5) and a 3-cluster SCD algorithm that provides optimal solution to the MS-MIP HP model.

3.5.1 Testbed data and implementation details

We consider 3 classes of tactical supply and procurement (HP) tactical planning problems (model (79-93)) that differ in terms of the number stages. In the first class (problem instances P1-P4), each stage incorporates 6 month (periods) over the 12-month planning horizon, leading to a 3-stage MS-MIP model; while in the second and third classes (P5-P8 and P9-P12 instances), each stage encompasses, respectively 4 and 3 months, leading to 4 and 5-stage MS-MIP models. In each class of test instances, we consider two types of lumber supply chains that differ in terms of the number of harvesting blocks available during the planning horizon and the amount of log demand. Finally, within each class and supply chain size, two variants of problems have been investigated that differ in terms of the variability level of the random log demand. More specifically, we consider two normal distributions for the log demand with equal mean-values and variances equal to 5% and 20% of the mean-values, respectively. The above-mentioned approach leads to 12 problem instances as summarized in Table 6. Its headings are as follows: *Instance*, member of the testbed we have experimented with; *nc*, number of constraints; *ncv*, number of continuous variables; *nbv*, number of 0-1 variables; *nel*, number of nonzero coefficients in the constraint matrix; dens, constraint matrix density (in %); $|\Omega|$, number of scenarios; $|Tree|$, number of nodes in the scenario tree; T, number of stages; *Variability%*, variance of random log demand as a percentage of mean-value; and |BL|, number of harvesting blocks. In all test instances, the supply capacity of each block in the forest per month is supposed to be 2,350 m^3. As in Beaudoin et al. [15], the maximum number of periods (months) over which harvesting can occur in each block in the forest as well as the maximum number of blocks in which harvesting can occur per month are randomly generated from uniform distributions in the following intervals: U[10, 12] and U[1, 6], respectively. The average volumes of each log class available in each block are also randomly generated based on Beaudoin et al. [15]. The total harvesting capacity per month is supposed to be approximately 175,000 m^3. At each stage in the scenario tree corresponding to each test instance, we consider a normal distribution for the log demand which is approximated by a 3-point discrete distribution (i.e., high, average, and low demand). Furthermore, in each node of the demand scenario tree, 3 scenarios have been considered for the yield of different blocks in the forest according to the yield data provided in Beaudoin et al. [15]. The above-mentioned data sets have been validated by our industrial partner to correspond to the reality.

All algorithms in this paper were coded in C++ using CPLEX v12.5 concert technology on a Dual-Core CPU 3.40GHz computer with 16.00 GB RAM.

Table 6: Testbed problem dimensions

Instance	nc	ncv	nbv	nel	dens	$\|\Omega\|$	$\|Tree\|$	T	Variability%	|BL|
P1	233,715	126,847	1,800	669,762	0.0022	9	13	3	5%	25
P2	233,715	126,847	1,800	669,762	0.0022	9	13	3	20%	25
P3	459,950	247,257	3,600	1,321,236	0.0011	9	13	3	5%	50
P4	459,950	247,257	3,600	1,321,236	0.0011	9	13	3	20%	50
P5	522,774	276,382	3,900	1,448,136	0.0009	27	40	4	5%	25
P6	522,774	276,382	3,900	1,448,136	0.0009	27	40	4	20%	25
P7	1,029,746	538,929	7,800	2,855,628	0.0005	27	40	4	5%	50
P8	1,029,746	538,929	7,800	2,855,628	0.0005	27	40	4	20%	50
P9	1,237,935	641,899	9,000	3,332,394	0.0004	81	121	5	5%	25
P10	1,237,935	641,899	9,000	3,332,394	0.0004	81	121	5	20%	25
P11	2,439,950	1,234,089	18,000	6,570,036	0.0002	81	121	5	5%	50
P12	2,439,950	1,234,089	18,000	6,570,036	0.0002	81	121	5	20%	50

3.5.2 Application of the HSCD heuristic on test instances

In this section, we first provide the main results of our computational experiments on implementing the HSCD heuristic (Algorithm 4) on the 12 HP tactical planning instances, described in Table 6, where we compare the solution and CPU time with a commercial solver (CPLEX v12.5). Next, we verify the importance of embedding LRH and VFH algorithms into SCD algorithm in terms of improvement in CPU time and the quality of the solution.

It is noteworthy that CPLEX v12.5 could find a feasible solution with 18% optimality gap after $15h$ CPU time for the smallest test instances (P1 and P2) while considering a compact representation. Hence, this commercial solver was disregarded as an efficient tool for solving the 12 test instances. Alternatively, by decomposing the original scenario tree into 3 clusters, the SCD algorithm (Algorithm 5) (solving scenario cluster sub-models by CPLEX v12.5) can obtain the optimal solution of the MS-MIP HP model (79-93). The reason is the lack of NACs in stage one (break stage). Table 7 represents the comparison between the results of HSCD algorithm with a 3-cluster SCD algorithm applied to solve the HP model for 12 test instances described in Table 6. $4h$ has been considered as the time limit to run both algorithms. The results are provided for two different values of break-stage including $t^* = 1$ and $t^* = 2$ that results, respectively, 3 and 9 scenario clusters in both algorithms. The headings of Table 7 are as follows: *Instance*, member of testbed; *Cost (HSCD)*, the best objective function value obtained by HSCD; *HSCD time*, time it takes for the HSCD algorithm to converge; *Optimal cost*, the optimal objective function value (cost) obtained by the 3-cluster SCD algorithm; and *Gap %*, the relative gap between the optimal objective value of the HSCD and SCD algorithms (relative optimality gap). In this table, N.A. indicates that the algorithm does not converge in $4h$. All cost values in this table are divided by 1000 and CPU times are indicated in seconds. Furthermore, as indicated in Section 3.3, the stopping criterion for the HSCD heuristic requires that Z_{HSCD} does not improve in two consecutive iterations. It should be noted that the HSCD heuristic is a Lagrangian-Relaxation based algorithm that provides a Lower Bound (LB) on the objective function value of the original MS-MIP HP model. Hence, it is expected that the bound obtained for break-stage $t^* = 1$ is smaller than the one for $t^* \geq 2$ (as also indicated in [48]). As it can be observed in this table, for $t^* = 1$, SCD cannot provide an optimal solution for the last 4 test instances within $4h$ CPU time. In contrary both SCD and HSCD algorithms provide an optimal solution for the first 8 instances (0 optimality gap between the LB obtained by HSCD and the SCD algorithm). In contrary, for $t^* = 2$ (9 scenario clusters), HSCD converges in less than $3h$ for the largest test instances. As expected, the quality of LB obtained by the HSCD heuristic is worse than $t^* = 1$ although the optimality gap is less than 1% (0.68%) in average over the first 8 test instances. Furthermore, all non-anticipativity constraints are satisfied over all test instances. Since the quality of the LB obtained by considering $t^* > 2$ was not better than $t^* = 2$, the results are not provided in the article.

Next, in Table 8, for the same test instances, we compare the performance of the HSCD heuristic with the following three algorithms: *i)* the SCD algorithm (Algorithm 5), where scenario cluster sub-models are solved by CPLEX v12.5, indicated as "SC"; *ii)* the SCD algorithm, where scenario cluster sub-models are solved by the aid of LRH

Table 7: HSCD algorithm results with different break stages

Instance	Cost (HSCD)	HSCD time	Optimal cost	Gap%
Three clusters ($t^* = 1$)				
P1	13,796	114	13,796	0
P2	13,878	129	13,878	0
P3	26,729	125	26,729	0
P4	26,783	101	26,783	0
P5	13,744	12,014	13,744	0
P6	13,803	7,196	13,803	0
P7	26,406	1,559	26,406	0
P8	26,489	1,290	26,489	0
P9	N.A.	N.A.	N.A.	N.A.
P10	N.A.	N.A.	N.A.	N.A.
P11	N.A.	N.A.	N.A.	N.A.
P12	N.A.	N.A.	N.A.	N.A.
Nine clusters ($t^* = 2$)				
P1	13,706	39	13,796	0.65%
P2	13,702	38	13,878	1.26%
P3	26,580	115	26,729	0.56%
P4	26,680	112	26,783	0.38%
P5	13,663	380	13,744	0.58%
P6	13,732	316	13,803	0.51%
P7	26,227	375	26,406	0.68%
P8	26,268	389	26,489	0.83%
P9	14,624	10,084	N.A.	N.A.
P10	14,674	7,829	N.A.	N.A.
P11	27,636	8,391	N.A.	N.A.
P12	27,946	6,562	N.A.	N.A.

(Algorithm 7), indicated as "SCD-LRH"; and *iii)* the combination of SCD-LHR and VFH (Algorithm 6), indicated as"HSCD". In Table 8 the headings are defined as follows: *Instance*, member of testbed; *Algorithm*, type of algorithm used to solve each test instance; *cost (LB)*, the converged LB in each algorithm; *# iteration*, number of iterations that each algorithm requires to converge according to the stopping criterion previously defined; and *CPU time*, the convergence time of each algorithm (seconds). It should be noted that the results provided in Table 8 correspond to $t^* = 2$ (9 scenario clusters) since the last 4 instances did not converge by the above-mentioned algorithms within $4h$ CPU time. Also, all algorithms are run until a converged LB is obtained. As it can be observed in this table, embedding the LRH algorithm in the SCD algorithm considerably improves the CPU time (average of 40% over all test instances) with a negligible degradation in the objective function value. Furthermore, embedding VFH heuristic in addition to LRH and the SCD algorithm (i.e. HSCD heuristic) improves the CPU time by 60% on average with a slight degradation in the objective function value.

Table 8: Comparison of HSCD, SCD-LRH and SCD algorithms ($t^* = 2$)

Instance	Algorithm	Cost (LB)	# Iteration	CPU time
P1	SCD	13,706	3	621
	SCD-LRH	13,706	3	430
	HSCD (SCD-LRH-VFH)	13,706	3	39
P2	SCD	13,701	5	1,600
	SCD-LRH	13,702	2	410
	HSCD (SCD-LRH-VFH)	13,702	2	38
P3	SCD	26,579	6	1,420
	SCD-LRH	26,580	4	1,102
	HSCD (SCD-LRH-VFH)	26,580	4	115
P4	SCD	26,574	7	1,420
	SCD-LRH	26,579	5	1,100
	HSCD (SCD-LRH-VFH)	26,680	4	112
P5	SCD	13,653	5	2,093
	SCD-LRH	13,661	4	1,418
	HSCD (SCD-LRH-VFH)	13,663	3	380
P6	SCD	13,727	5	2,113
	SCD-LRH	13,733	3	1,337
	HSCD (SCD-LRH-VFH)	13,734	3	316
P7	SCD	26,226	4	2,145
	SCD-LRH	26,227	3	1,327
	HSCD (SCD-LRH-VFH)	26,227	3	375
P8	SCD	26,261	3	1,084
	SCD-LRH	26,268	2	1,057
	HSCD (SCD-LRH-VFH)	26,268	2	389
P9	SCD	14,104	8	12,830
	SCD-LRH	14,614	5	8,920
	HSCD (SCD-LRH-VFH)	14,624	5	10,084
P10	SCD	14,251	9	12,560
	SCD-LRH	14,626	6	9,130
	HSCD (SCD-LRH-VFH)	14,674	3	7,829
P11	SCD	27,628	5	62,648
	SCD-LRH	27,635	3	10,844
	HSCD (SCD-LRH-VFH)	27,636	3	8,391
P12	SCD	27,934	6	45,500
	SCD-LRH	27,935	4	7,604
	HSCD (SCD-LRH-VFH)	27,946	2	6,562

To conclude, the results provided in Tables 7 and 8 demonstrate the high quality of LB that can be obtained by the HSCD algorithm in a reasonable amount of time while CPLEX v12.5 is not able to find a high quality feasible solution within $15h$ CPU time for the smallest test instance. Furthermore, embedding the VFH into SCD algorithm in addition to solving scenario cluster sub-models by the aid of LRH significantly improves the convergence of SCD algorithm with a negligible degradation of the converged LB.

3.6 Concluding remarks

In this study, we proposed a new algorithmic procedure to solve multi-stage stochastic mixed-integer programming models applicable to supply chain tactical planning problems. This algorithm is based on the idea of on scenario clustering in multi-stage stochastic programs. The Hybrid Scenario Cluster Decomposition scheme proposed in this article is an accelerated scenario cluster decomposition method that decomposes the MS-MIP model into smaller MS-MIP sub-models after breaking down the initial scenario tree into a set of smaller sub-trees. The scenario sub-models are formulated in a compact format and are coordinated by Lagrangian penalty terms in order to compensate the lack of non-anticipativity corresponding to the nodes of scenario sub-trees that are common in the initial scenario tree. The scenario tree decomposition framework described above is expected to converge faster than classical scenario decomposition methods, due to smaller number of relaxed non-anticipativity constraints in MS-MIP sub-models. In contrary, each sub-model is an MS-MIP model on its own that can be challenging to solve in realistic-size instances. Consequently, with the goal of accelerating the SCD algorithm, we proposed to solve scenario cluster sub-models by the aid of an ad-hoc heuristic (a Lagrangian based heuristic). Furthermore, we embedded a Variable Fixing Heuristic within the SCD algorithm in order to speed up its convergence. Another contribution of this article is the specialization of the above-mentioned algorithm to supply and procurement tactical planning in the lumber SC under demand and supply uncertainty. Our experimental results on a set of realistic-size cases revealed that the HSCD algorithm proposed in this article can find high quality solutions in a reasonable CPU time while CPLEX fails to find a high quality feasible solution within $15h$ CPU time for the smallest test instance. Furthermore, it was observed that embedding the above-mentioned accelerating heuristics into the

SCD scheme has substantially reduced the CPU time with a negligible degradation of the solution. Two of the main advantages of the HSCD algorithm are: *i*) amenability to parallelization; and *ii*) possibility of embedding specialized algorithms for solving scenario cluster sub-models depending on their particular structure.

Since many industries are faced with several types of perturbations in their business environment, adopting a stochastic optimization approach in their decision models seems to be inevitable for robust decision making. More specially, when the industry is dealing with sequential decisions over time such as supply chain tactical planning or dynamic supply chain design problems, multi-stage stochastic programming is one of the most promising methods in order to obtain robust decisions in the presence of future uncertainties. In contrast, such models are featured as intractable ones for real-size problem instances. While the ability to come up with robust plans in a relatively short amount of time is one of the main competitive advantages of an industry, the new HSCD algorithmic procedure proposed in this paper is an attempt to reduce the challenge of solving such problems. The high quality of the plans proposed by our algorithm while overcoming the computational complexity of multi-stage stochastic MIP models, demonstrated through our industrial case study, can motivate the lumber industry to move toward adopting more robust decision making tools rather than current deterministic ones.

Future research would focus on the implementation of the HSCD algorithm on parallel machines in order to reduce the CPU time. Furthermore, this algorithm can be applied to other supply chain tactical planning problems that incorporate uncertain parameters with a dynamic behavior over time. Finally, other efficient heuristic algorithms can be embedded within the HSCD scheme in order to efficiently solve scenario cluster sub-models.

Acknowledgments

The authors would like to thank the anonymous reviewers for their invaluable comments on the initial version of the manuscript. This work was financially supported by Natural Science and Engineering Council of Canada (NSERC) Strategic Network on Value Chain Optimization (VCO) (Grant no.387200) and collaboration with FPInnovations, Canada.

3.7 Multi-stage stochastic mixed-integer programming model for Harvesting and Procurement (HP) tactical planning in the lumber supply chain

In this appendix, the multi-stage stochastic model representing the HP model is provided. The description of constraints and notations are followed by the model.

HP model (Compact formulation)

$$
\begin{aligned}
Min\ Z_{HP} = & \sum_{n \in Tree} p(n) \sum_{sc \in SC} p_{sc} \sum_{bl \in BL} \sum_{t \in t_n} c^{H}_{blt} y_{bl,t}(n) (\sum_{rm \in RM} v_{rm,bl,sc}) \\
& + \sum_{n \in Tree} p(n) \sum_{sc \in SC} p_{sc} \sum_{rm \in RM} \sum_{bl \in BL} \sum_{t \in t_n} v_{rm,bl,sc} f_{rm,bl,t} y_{bl,t}(n) \\
& + \sum_{n \in Tree} p(n) \sum_{bl \in BL} \sum_{m \in M} \sum_{rm \in RM} \sum_{t \in t_n} c^{T}_{rm,bl,m,t} X^{bl}_{rm,m,t}(n) \\
& + \sum_{n \in Tree} p(n) \sum_{sc \in SC} p_{sc} \sum_{rm \in RM} \sum_{bl \in BL} \sum_{t \in t_n} c^{S}_{rm,bl,t} I_{rm,bl,t,sc}(n) \\
& + (\sum_{n \in Tree} p(n) \sum_{bl \in BL} \sum_{rm \in RM} \sum_{m \in M} \sum_{t \in t_n} m^{bl}_{rm,t} X^{bl}_{rm,m,t}(n) \\
& + \sum_{n \in Tree} p(n) \sum_{sc \in SC} p_{sc} \sum_{rm \in RM} \sum_{m \in M} \sum_{t \in t_n} h_{rm,m} I_{rm,m,t,sc}(n)
\end{aligned} \tag{79}
$$

Subject to:

Harvesting constraints

Flow balance constraints

$$I_{rm,bl,T,sc}(n) = 0 \quad \forall rm, bl, sc, n \in Tree \tag{80}$$

$$I_{rm,bl,t,sc}(n) = I_{rm,bl,t-1,sc}(n') - \sum_{m \in M} X^{bl}_{rm,m,t}(n) + v_{rm,bl,sc} y_{bl,t}(n) \quad \forall rm, bl, sc,$$

$$t \in t_n, n \in Tree, n' = \begin{cases} a(n) & t-1 \notin t_n \\ n & t-1 \in t_n \end{cases} \tag{81}$$

Capacity Constraints

$$\sum_{t \in t_n} \sum_{n \in A(n')} y_{bl,t}(n) \leq 1 \quad \forall bl, \forall n' \in N_T \tag{82}$$

$$y_{bl,t}(n) \leq H_{bl,t}(n) \quad \forall bl, t \in t_n, n \in Tree \tag{83}$$

$$\sum_{t \in t_n} \sum_{n \in A(n')} H_{bl,t}(n) \leq l_{bl} \quad \forall bl, \forall n' \in N_T \tag{84}$$

$$\sum_{bl \in BL} H_{bl,t}(n) \leq n_t \quad \forall t \in t_n, n \in Tree \tag{85}$$

$$\sum_{bl \in BL} (y_{bl,t}(n) \sum_{rm \in RM} v_{rm,bl,sc}) \leq b^H_t \quad \forall sc, t \in t_n, n \in Tree \tag{86}$$

$$\sum_{rm \in RM} \sum_{m \in M} \sum_{bl \in BL} X^{bl}_{rm,m,t}(n) \leq b^T_t \quad \forall t \in t_n, n \in Tree \tag{87}$$

Procurement constraints

Flow balance constraints

$$\sum_{bl \in BL} X^{bl}_{rm,m,t-L^{bl}_{rm}}(n) + I_{rm,m,t-1,sc}(n') - I_{rm,m,t,sc}(n) = d_{rm,m,t}(n)$$

$$\forall rm, m, sc, t \in t_n, t = 1 + L^{bl}_{rm}, \ldots, T, n \in Tree, n' = \begin{cases} a(n) & t-1 \notin t_n \\ n & t-1 \in t_n \end{cases} \tag{88}$$

Capacity Constraints

$$I_{rm,m,t,sc}(n) - ss_{rm,m} \geq 0 \quad \forall rm, m, sc, t \in t_n, n \in Tree \tag{89}$$

$$\sum_{rm \in RM} I_{rm,m,t,sc}(n) \leq KI_{m,t} \quad \forall m, sc, t \in t_n, n \in Tree \tag{90}$$

$$\sum_{rm \in RM} \sum_{m \in M} X^{bl}_{rm,m,t}(n) \leq KS^{bl}_t \quad \forall bl, t \in t_n, n \in Tree \tag{91}$$

$$\sum_{m \in M} \sum_{rm \in RM} \sum_{t \in t_n} \sum_{n \in Tree} X^{bl}_{rm,m,t}(n) \geq qmin^{bl} \quad \forall bl \tag{92}$$

$$X^{bl}_{rm,m,t}(n), I_{rm,m,t,sc}(n), y_{bl,t}(n), I_{rm,bl,t,sc}(n) \geq 0, H_{bl,t}(n) \in \Big\{0,1\Big\} \forall m, bl, rm, sc, t \in t_n \tag{93}$$

Model (79)-(93) (HP model) is a multi-stage-stochastic programming MIP trying to minimize harvesting, stumpage, transportation, storage and procurement costs. Constraint (80) represents the final inventory of raw materials in each harvesting block. Constraint (81) formulates the inventory balance of raw materials in each block. Constraint (82) ensures that the harvested proportion of a block does not exceed the availability of logs in that block. Constraint (83) describes that if harvesting occurs in a block, then we can ensure that raw materials from that block are available. Constraints (84) and (85) correspond to the maximum number of harvesting and maximum number of blocks in which harvesting can occur, respectively. Constraints (86) and (87) correspond to the harvesting and transportation capacity from each block to each mill, respectively. Constraint (88) formulates the inventory balance of raw materials at each mill. The raw material safety stock policies are stated in constraint (89) and the raw material inventory capacity constraint is provided in constraint (90). Constraint (91) describes the raw material supply capacity. Constraint (92) states that the material procured from a supplier must satisfy the contract quantity commitment.

Chapter 4

Forest harvesting planning under uncertainty: a cardinality-constrained approach

The forth chapter consists the article entitled *"Forest harvesting planning under uncertainty: a cardinality-constrained approach"* was submitted to the *International Journal of Production Research* in October 2015. The titles, figures, tables, algorithms and mathematical formulations have been revised to the keep the coherence through the thesis.

Abstract

Harvesting planning is a key tactical decision in lumber supply chains. Harvesting areas in the forests are divided into different blocks with different types and quantities of raw materials (logs). Predicting the availability of raw materials in each block along with log demand is impossible in this industry. Hence, incorporating uncertainty into the harvesting planning problem is essential in order to obtain robust plans that do not drastically fluctuate in the presence of future perturbations in the forest and log market. In this paper, we propose a robust harvesting planning model formulated based on cardinality-constrained method. The latter provides some insights into the adjustment of the level of robustness of the harvesting plan over the planning horizon and protection against uncertainty. An extensive set of experiments based on Monte-Carlo simulation is also conducted in order to better validate the proposed robust optimization approach.

4.1 Introduction

Lumber supply chains (SC) incorporate forest, as the supplier, sawmills as the manufacturing entities, different distribution channels, as well as contract and non-contract-based customers. Harvesting planning in the lumber SC incorporates the optimal harvesting schedule, i.e. the selection of harvesting blocks and the quantity of harvesting in each period by considering several harvesting constraints with the goal of minimizing costs of harvesting and log storage while satisfying sawmills demand [52].

Harvesting planning has been addressed with the other tactical decisions in the lumber SC in a deterministic context (e.g., see [14–22, 52, 53]). In harvesting planning, forecasting the exact amount of log demand in sawmills is almost impossible. Furthermore, the availability of logs in each block of the forest is also uncertain.

These uncertainties affect the amount of log inventory and harvesting capacity in the forest as well. Thus, considering random log demand and supply in harvesting tactical planning is crucial in order to obtain plans that do not drastically fluctuate in the presence of forest and log market perturbations.

Robust optimization is one of the predominant approaches for addressing optimization problems with uncertain data where there is not enough information about their probability distributions. The classical approach to robust optimization is to search for an optimal solution so that the solution will satisfy all possible outcomes of uncertain parameters. On the other hand, such an approach might lead to a conservative solution that is overprotected against uncertainty. In order to find a trade-off between the cost of robustness and protection of the solution against uncertainty, Bertsimas and Sim [54] proposed the cardinality-constrained robust optimization approach. This method constitutes the main methodology in this article for addressing uncertainty in the harvesting planning problem. Nonetheless, the structure of constraints and objective function terms in this problem makes the application of this approach less trivial.

To the best of our knowledge, harvesting tactical planning under supply and demand uncertainty has been less investigated in the literature. Scenario analysis is the only approach that has been applied in order to study the impact of uncertain log availability on this problem [15]. This article contributes to the literature through proposing a robust optimization model based on cardinality constrained approach which provides the possibility of finding a trade-off between the protection of the harvesting plan against uncertainties and the cost of such protection. It is noteworthy that the uncertain log supply and demand affects simultaneously constraints' coefficients and right-hand-sides as well as objective function coefficients. Furthermore, due to dynamic behavior of demand over the planning horizon in addition to the complex structure of the objective function and constraints, formulating the robust counterpart of this problem is not straightforward. This will distinguish the model from the existing robust optimization models in the literature applied to supply chain tactical planning. Another contribution of this study revolves around conducting an extensive set of experimental results with the goal of analyzing the degree of robustness of the proposed harvesting plan in addition to the extra cost incurred to obtain such protection against uncertainty under realistic circumstances. This has been achieved through using the existing theoretical bounds in addition to Monte-Carlo simulation approach.

The remainder of the paper is organized as follows. The related literature is reviewed in Section 4.2. The cardinality-constrained optimization approach is described in Section 4.3. The robust harvesting planning model is provided in Section 4.4. The numerical results are presented in Section 4.5. Finally, the conclusions and future works are presented in Sections 4.6.

4.2 Literature review

In this section, we first provide a review on various robust optimization approaches. Next, we focus on the review of articles relevant to the application of this approach on supply chain planning problems.

The robust optimization approach has been categorized into static and dynamic models. In the static robust optimization approach, the decision maker must choose a strategy before the exact values of uncertain parameters are revealed. In other words, all decision variables are "here and now". The objective is typically to minimize the worst-case cost. The first step in the static robust optimization approach was taken by Soyster [55]. In this study, he proposed a linear optimization model to construct a solution that is feasible for all outcomes of uncertain data that belong to a convex set. This approach was further developed by several authors (e.g., see [56–58], [59], [60], [54], and [61]).

In order to reduce the degree of conservatism of Soyster [55], Bertsimas and Sim [54] proposed the cardinality-constrained approach for solving linear mathematical model with uncertain coefficient matrix. By assuming interval uncertainty, their approach provides a robust solution whose level of conservatism can be flexibly adjusted. They define a predetermined budget of uncertainty for every constraint in order to provide an optimal solution that guarantees feasibility for all admissible data realization of uncertain parameters at a given probability (confidence level).

When planning is dynamic (e.g., multi-period planning problems), it is reasonable to expect that better solutions can be found as we can dynamically adjust the planning when more information is known. This is called a dynamic robust solution. Ben-Tal [58] introduced a computationally tractable robust formulation for the special case where the future decision variables can be expressed as affine functions of the uncertainty set. This method, however, has no flexibility in elaborations with uncertainty sets, since a minor adjustment could change the robust counterpart into an intractable formulation. Bertsimas and Caramanis [62] approached a more general method where the uncertainty set may be a general polytope. In the solution approach, they use the partitioning of the uncertainty set and find a static robust solution for each partition. In a later stage, without uncertainty, at least one of the static solutions fulfills the now realized parameters, and the best static solution is selected for implementation. The difficulty with this approach is to select a well performing partitioning so that the static robust solutions are reasonable while at the same time keeping the number of partitions low for the sake of efficiency. Bertsimas and Thiele [61] presented the robust optimization method for inventory management under demand uncertainty over a multi-period planning horizon. By assuming an interval uncertainty for demand, they developed the robust counterpart for the inventory control problem in dynamic settings. They also proposed a method for eliminating state (inventory) variables, so all decisions in the model are "here and now". Furthermore, since in this problem the inventory balance constraints depend on periods in the planning horizon, the uncertainty set will depend on time periods as well. They modeled the uncertainty as the cumulative demand up to each time period. This motivates defining a sequence of budgets of uncertainty for each period, rather than using a single predetermined budget explained in the static case. The main advantage of this approach is its applicability to a wide range of supply chain tactical planning problems similar to the one investigated in this article.

Adida and Perakis [63] introduced a robust optimization model to dynamic pricing and inventory control. They proposed a linear function for a time-dependent budget of uncertainty such that it avoids very conservative values for the budget of uncertainty and control the level of conservatism. There are more papers in the literature focused on dynamic robust optimization where the budget of uncertainty for each period is generated randomly (e.g., see [64] and [65]). Alvarez and Vera [66] presented the application of robust optimization to a production planning problem. They consider an equal amount of budget of uncertainty for each period to represent the grade of robustness to each constraint. Alem and Morabito [67] explored a robust optimization model to integrate lot sizing and cutting stock model in furniture industry under cost and demand uncertainty. They control the level of uncertainty with a predetermined budget of uncertainty in a dynamic setting.

Bredstrom et al. [68] proposed a rolling horizon method based on the robust optimization approach for tactical planning in supply chains under demand uncertainty. In their approach, the uncertainties are described as an arbitrary polytope and formulated as explicit constraints. Carlsson et al. [69] applied the above-mentioned robust optimization approach to handle the uncertainty in distribution and inventory planning in the pulp production context.

Wu [70] developed a scenario-based robust optimization method to solve uncertain production loading problem in the global supply chain management environment. This approach is a goal programming method that balances the trade-off between solution robustness and model robustness in the context of two-stage stochastic programs. Similar approach has been applied by other authors in different supply chain planning problems (e.g., see [71], [72] and [73]). Kanyalkar and Adil [71] developed a robust optimization model by considering random demand in order to integrate multi-site procurement, production, and distribution decisions in a supply chain. Kazemi Zanjani et al. [72] presented two robust optimization models with different variability measures to address multi-period sawmill production planning by considering the uncertainty in quality of raw materials. They also proposed an efficient solution algorithm to solve this model for large instances in [38]. Khakdaman et al. [73] proposed a robust tactical plan for the hybrid Make-to-Stock-Make-to-Order manufacturing system by considering demand, process and supply uncertainties.

4.3 Cardinality-constrained robust optimization approach

In this paper, we use the robust optimization approach developed by Bertsimas and Sim [54] for linear programming problems. We present here the idea and a summary of this method. For more details, the reader is referred to [54]. Let consider the following Linear Programming (LP) model:

$$Max\ Z = cx$$
$$\tilde{a}x \leq b$$
$$l \leq x \leq u \tag{94}$$

where some parameters of the coefficient matrix ($\tilde{a}_{ij}$) are uncertain. In addition, each uncertain parameter ($\tilde{a}_{ij}$) takes a value according to a symmetric distribution with the mean equal to the nominal value ($\bar{a}_{ij}$) in the interval $[\bar{a}_{ij} - \hat{a}_{ij}, \bar{a}_{ij} + \hat{a}_{ij}]$. Furthermore, we define a parameter Γ_i, budget of uncertainty, for every constraint i. This parameter is not necessarily integer and takes a value in the interval $[0, |J_i|]$, where J_i is the set of uncertain parameters in the ith constraint. A linear robust counterpart can be obtained to protect against all cases that $\lfloor \Gamma_i \rfloor$ coefficients of set J_i are permitted to change to their worst-case value, and one coefficient (a_{it_i}) can change by a fraction of its worst-case value (i.e., $(\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{a}_{ij}$). In order to guarantee feasibility of constraints affected by uncertainty, a protection function, denoted as $\beta(x, \Gamma_i)$, can be added to the left-side of every constraint i as follows:

$$\beta(x, \Gamma_i) = \max_{\{\, S_i \cup t_i | S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i \,\}} \left\{ \sum_{j \in J_i} \hat{a}_{ij} |x_j| + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{it_i} |x_{t_i}| \right\} \tag{95}$$

The protection function is a an optimization model by itself that tries to select a subset of uncertain coefficients in each constraint to take their worst-case value such that the highest increase in the left-side of constraint is achieved. Therefore, model (94) can be rewritten as model (96):

$$
\begin{aligned}
& Max\ Z = cx \\
& s.t. : \sum_j \bar{a}_{ij} x_j + \max_{\{\, S_i \cup t_i | S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i \,\}} \left\{ \sum_{j \in J_i} \hat{a}_{ij} y_j + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{it_i} y_{t_i} \right\} \leq b_i \quad \forall i \\
& - y_j \leq x_j \leq y_j \quad \forall j \\
& l_j \leq x_j \leq u_j \quad \forall j \\
& y_j \geq 0 \quad \forall j
\end{aligned} \tag{96}
$$

In order to avoid the above nonlinear protection function, it can be replaced by its dual counterpart. Hence, model (96) can be formulated into its robust counterpart as follows:

$$
\begin{aligned}
& Max\ Z = cx \\
& s.t. : \sum_j \bar{a}_{ij} x_j + z_i \Gamma_i + \sum_{j \in J_i} p_{ij} \leq b_i \quad \forall i \\
& z_i + p_{ij} \geq \hat{a}_{ij} y_j \quad \forall i, j \in J_i \\
& - y_j \leq x_j \leq y_j \quad \forall j \\
& l_j \leq x_j \leq u_j \quad \forall j \\
& y_j \geq 0 \quad \forall j \\
& z_i \geq 0 \quad \forall i \\
& p_{ij} \geq 0 \quad \forall i, j \in J_i
\end{aligned} \tag{97}
$$

It is noteworthy that the robust counterpart model (97) is a linear programming model that can be efficiently solved. Finally, the cardinality-constrained approach, described above, provides an effective method to determine probability bounds for the constraint violation. The probability bound that the ith constraint is violated can be approximated as follows:

$$pr(\sum_j \tilde{a}_{ij} x_j^* > b_i) \leq 1 - \phi(\frac{\Gamma_i - 1}{\sqrt{|J_i|}}) \tag{98}$$

where $\phi(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha} \exp(-\frac{y^2}{2}) dy$ is the cumulative standard normal distribution function, and x_j^* is the optimal solution of the robust optimization problem.

To summarize, in order to obtain the robust counterpart of an uncertain LP model similar to (94), the decision-maker needs to set a budget of uncertainty and add accordingly a protection term in the left-side of the constraints affected by uncertainty. Finally, replacing the latter protection function by its dual counterpart will provide a linear robust counterpart that is computationally tractable.

4.4 Robust harvesting planning model

In this section, we first present the deterministic mathematical model of tactical harvesting planning; then we develop its robust counterpart.

4.4.1 Deterministic harvesting planning model

Notations

Sets

RM: Set of raw materials

BL: Set of harvesting blocks

T: Set of time periods

M: Set of manufacturing mills

Parameters

$d_{rm,t}^{bl}$: Forecasted demand of raw material rm in period t at harvesting block bl

c^H_{blt}: Unit cost to harvest block bl during period t

$c^S_{rm,bl,t}$: Unit cost to store raw material rm in block bl during period t

$f_{rm,bl,t}$: Stumpage fee for raw material rm in block bl during period t

l_{bl}: Maximum number of periods over which harvesting can occur in block bl

n_t: Maximum number of blocks in which harvesting can occur during period t

b^H_t: The total harvesting capacity in period t

$v_{rm,bl}$: Volume of available raw material rm in harvesting block bl

Decision variables

$I_{rm,bl,t}$: Inventory of raw material rm in harvesting block bl at the end of period t

$y_{bl,t}$: Proportion of harvested block bl in period t

$H_{bl,t}$: Binary variable that takes 1 if harvesting occurs in block bl during time period t and 0 otherwise

The tactical harvesting planning (HP) model tries to minimize the harvesting, inventory, and stumpage costs. The harvesting decisions involve the blocks where the harvesting should occur ($H_{bl,t}$) as well as the proportion of the harvested blocks in different periods of the planning horizon ($y_{bl,t}$). The inventory of each raw material in each block in different periods ($I_{rm,bl,t}$) is the other decision in the harvesting model. Constraint (100) formulates the inventory balance of raw materials in each block. Constraint (101) represents the final inventory of raw materials in each block. Constraint (102) ensures that the harvested proportion of a block do not exceed the availability of logs in that block. Constraint (103) describes that if harvesting occurs in a block, then we can ensure that raw materials from that block are available. Constraints (104) and (105) correspond to the maximum number of harvesting and maximum number of blocks in which harvesting can occur, respectively. Constraints

(106) limits the harvesting capacity of the blocks in each period.

$$
\begin{aligned}
Min\ Z = & \sum_{bl \in BL} \sum_{t \in T} c^{H}_{blt} y_{bl,t} (\sum_{rm \in RM} v_{rm,bl}) \\
+ & \sum_{rm \in RM} \sum_{bl \in BL} \sum_{t \in T} \tilde{v}_{rm,bl} f_{rm,bl,t} y_{bl,t} \\
+ & \sum_{rm \in RM} \sum_{bl \in BL} \sum_{t \in T} c^{S}_{rm,bl,t} I_{rm,bl,t} \qquad (99)
\end{aligned}
$$

Subject to:

$$I_{rm,bl,t} = I_{rm,bl,t-1} - d^{bl}_{rm,t} + \tilde{v}_{rm,bl} y_{bl,t} \quad \forall rm, bl, t \qquad (100)$$

$$I_{rm,bl,T} = 0 \quad \forall rm, bl \qquad (101)$$

$$\sum_{t \in T} y_{bl,t} \leq 1 \quad \forall bl \qquad (102)$$

$$y_{bl,t} \leq H_{bl,t} \quad \forall bl, t \qquad (103)$$

$$\sum_{t \in T} H_{bl,t} \leq l_{bl} \quad \forall bl \qquad (104)$$

$$\sum_{bl \in BL} H_{bl,t} \leq n_t \quad \forall t \qquad (105)$$

$$\sum_{bl \in BL} (y_{bl,t} \sum_{rm \in RM} \tilde{v}_{rm,bl}) \leq b^{H}_{t} \quad \forall t \qquad (106)$$

$$y_{bl,t}, I_{rm,bl,t} \geq 0, H_{bl,t} \in \left\{0, 1\right\} \forall bl, rm, t \qquad (107)$$

4.4.2 Robust harvesting planning model

The HP model is affected by two uncertain parameters namely random log demand and supply. Notice that these uncertain parameters affect constraints coefficients and right-hand-sides as well as objective function coefficients. In this study, we model random demand and supply as uncertain intervals.

Let's denote $\tilde{d}^{bl}_{rm,t}$ as uncertain log demand with the nominal value of $\bar{d}^{bl}_{rm,t}$, and $\tilde{v}_{rm,bl}$ as uncertain log supply with a nominal value of $\bar{v}_{rm,bl}$. The uncertain log demand is assumed symmetric and time-dependent. This random variable ($\tilde{d}^{bl}_{rm,t}$) takes values in the interval $[\bar{d}^{bl}_{rm,t} - \hat{d}^{bl}_{rm,t}, \bar{d}^{bl}_{rm,t} + \hat{d}^{bl}_{rm,t}]$. Then, the scale deviation $z^{bl}_{rm,t}$ (belonging to $[-1, 1]$) of $\tilde{d}^{bl}_{rm,t}$ from its nominal value is defined as $z^{bl}_{rm,t} = (\tilde{d}^{bl}_{rm,t} - \bar{d}^{bl}_{rm,t})/\hat{d}^{bl}_{rm,t}$. Thus, we can also write $\tilde{d}^{bl}_{rm,t} = \bar{d}^{bl}_{rm,t} + \hat{d}^{bl}_{rm,t} z^{bl}_{rm,t}$.

Furthermore, $\tilde{v}_{rm,bl}$ is assumed time independent and takes values in the interval $[\bar{v}_{rm,bl} - \hat{v}_{rm,bl}, \bar{v}_{rm,bl} + \hat{v}_{rm,bl}]$. We consider the scale deviation of $\tilde{v}_{rm,bl}$ from its nominal value as $w_{rm,bl} = (\tilde{v}_{rm,bl} - \bar{v}_{rm,bl})/\hat{v}_{rm,bl}$ that belongs to $[-1, 1]$. Similarly, the random supply might be rewritten as $\tilde{v}_{rm,bl} = \bar{v}_{rm,bl} + w_{rm,bl}\hat{v}_{rm,bl}$.

As was illustrated in Section 4.3, in order to construct the robust counterpart of model (99)-(107), we should follow two steps. At first, we should formulate the protection function for the constraints affected by uncertain parameters as an optimization problem. Next, by incorporating the dual of the aforementioned protection function into each constraint, their robust counterpart will be extracted. In what follows, we provide the robust counterpart of uncertain constraints and objective function terms, respectively.

4.4.2.1 Robust counterpart of uncertain constraints

The first constraint in the HP model formulates the inventory balance of raw materials in each block and includes both uncertain parameters (supply and demand). The main decision variable in this constraint is the proportion of harvested block ($y_{bl,t}$) while the quantity of the inventory in each block is the state variable ($I_{rm,bl,t}$). In the static robust optimization approach, all decision variables are "here and now" and there is not possibility for recourse actions. Inspired by Bertsimas and Thiele [61], it is possible to remove the state variables and cumulate the effects of uncertain parameters by rewriting constraint (100) as the following closed-form equation which models the evolution of the inventory over time.

$$I_{rm,bl,t} = I_{rm,bl,0} + \sum_{s=1}^{t} (\tilde{v}_{rm,bl} y_{bl,s} - \tilde{d}_{rm,s}^{bl}) \quad \forall rm, bl, t \tag{108}$$

As the inventory quantity (state variable) also exists in the objective function, it is possible to consider the total amount of the storage cost as a constraint such as (109) by using the closed-form equation (108) and substitute term $c^S_{rm,bl,t} I_{rm,bl,t}$ in the objective function by $HH_{rm,bl,t}$ which is defined as the storage cost at the end of period t.

$$c^S_{rm,bl,t}(I_{rm,bl,0} + \sum_{s=1}^{t} (\tilde{v}_{rm,bl} y_{bl,s} - \tilde{d}_{rm,s}^{bl})) \leq HH_{rm,bl,t} \quad \forall rm, bl, t \tag{109}$$

Hence, Constraint (100) in model (99)-(107) is replaced by (109) that includes only "here-and-now" decisions and the uncertain demand and supply affect both its coefficients and right-hand-side vector. Next, we are looking for the robust counterpart of constraint (109) which is equivalent to minimizing the inventory cost over all realizations of uncertain demand and log supply. In other words, we aim for minimizing the maximum amount of the right-hand side of constraint (109) over the set of all admissible realization of uncertain log demand and supply. To do this, we should find a feasible solution by considering the worst-cases for uncertain parameters. Intuitively, the maximum storage cost occurs when the log availability and demand, respectively, reach their maximum and minimum values in the given uncertain intervals. In other words, the worst-case for uncertain supply ($\tilde{v}_{rm,bl}$) which is a time-independent parameter is $\bar{v}_{rm,bl} + \hat{v}_{rm,bl}$, and the worst-case for uncertain demand ($\tilde{d}^{bl}_{rm,t}$), the time-dependent parameter, is calculated as follows. In reality, it is unlikely that all uncertain demand parameters from period 1 to a given period t change to their worst-case value, thus we assume a predetermined number of log demand parameters in constraint (109) in each block can change to their worst-case value ($\Gamma^{bl}_{rm,t} \in [0,t]$). With this parameter, the decision maker considers a trade-off between the level of protection of the constraint satisfaction against the degree of conservatism of the solution. Accordingly, the following protection function can be defined for constraint (109).

$$\begin{aligned}
&Maximize \sum_{s=1}^{t} \hat{d}^{bl}_{rm,s} z^{bl}_{rm,s} \\
&s.t. \quad \sum_{s=1}^{t} z^{bl}_{rm,s} \leq \Gamma^{bl}_{rm,t} \\
&0 \leq z^{bl}_{rm,s} \leq 1 \quad \forall s \leq t
\end{aligned} \tag{110}$$

where $z^{bl}_{rm,s}$ denotes the scaled deviation of uncertain demand from its nominal value. The dual counterpart of this protection function is the following optimization problem.

$$\begin{aligned}
&Minimize \quad \lambda^{bl}_{rm,t} \Gamma^{bl}_{rm,t} + \sum_{s=1}^{t} \theta_{rm,bl,t,s} \\
&s.t. \quad \lambda^{bl}_{rm,t} + \theta_{rm,bl,t,s} \geq \hat{d}^{bl}_{rm,s} \quad \forall rm, bl, t, \forall s \leq t \\
&\lambda^{bl}_{rm,t}, \theta_{rm,bl,t,s} \geq 0 \quad \forall rm, bl, t, \forall s \leq t
\end{aligned} \tag{111}$$

where $\lambda^{bl}_{rm,t}$ and $\theta_{rm,bl,t,s}$ are the dual variables corresponding to the constraints of protection function (110). By substitution of protection function (110) by (111) and adding this protection function to (109), the robust counterpart of this constraint is obtained. Moreover, because the inventory amount at the end of each period ($I_{rm,bl,0} + \sum_{s=1}^{t}(\tilde{v}_{rm,bl} y_{bl,s} - \tilde{d}^{bl}_{rm,s})$) is always greater or equal to zero, constraint (114) should also be added to the robust counterpart which guarantees the positive amount of inventory when the minimum amount of log supply and maximum amount of log demand in the given uncertain intervals is realized. Hence, model (112)-(115)

represent the robust counterpart of constraint (100).

$$HH_{rm,bl,t} \geq c^S_{rm,bl,t}(I_{rm,bl,0} + \sum_{s=1}^{t}((\bar{v}_{rm,bl} + \hat{v}_{rm,bl})y_{bl,s} - \bar{d}^{bl}_{rm,s}) + \lambda^{bl}_{rm,t}\Gamma^{bl}_{rm,t} + \sum_{s=1}^{t}\theta_{rm,bl,t,s})$$

$$\forall rm, bl, t \tag{112}$$

$$\lambda^{bl}_{rm,t} + \theta_{rm,bl,t,s} \geq \hat{d}^{bl}_{rm,s} \quad \forall rm, bl, t, \forall s \leq t \tag{113}$$

$$(I_{rm,bl,0} + \sum_{s=1}^{t}((\bar{v}_{rm,bl} - \hat{v}_{rm,bl})y_{bl,s} - \bar{d}^{bl}_{rm,s}) - \lambda^{bl}_{rm,t}\Gamma^{bl}_{rm,t} - \sum_{s=1}^{t}\theta_{rm,bl,t,s}) \geq 0 \quad \forall rm, bl, t \tag{114}$$

$$\lambda^{bl}_{rm,t}, \theta_{rm,bl,t,s} \geq 0 \quad \forall rm, bl, t, \forall s \leq t \tag{115}$$

Constraint (106) in model (99)-(107) is another constraint faced with uncertain supply ($\tilde{v}_{rm,bl}$). This constraint indicates that the total quantity that might be harvested must not exceed the harvesting capacity in each period. Thus, its robust counterpart is equivalent to satisfying the capacity constraint in case the total harvesting quantity is maximized as a result of maximum amount of available logs in each block. Accordingly, the following protection function (116) might be developed for a given t by the budget of uncertainty Γ^v_{bl} which indicates the maximum number of uncertain supply parameters in each block that can take their worst-case value. In (116), $y^*_{bl,t}$ is the optimal solution of model (99)-(107), and $w^1_{rm,bl}$ is the scaled deviation from the nominal value of $\tilde{v}_{rm,bl}$.

$$Maximize \sum_{bl \in BL}(y^*_{bl,t} \sum_{rm \in RM} \hat{v}_{rm,bl} w^1_{rm,bl})$$

$$s.t. \quad \sum_{rm \in RM} w^1_{rm,bl} \leq \Gamma^v_{bl} \quad \forall bl$$

$$0 \leq w^1_{rm,bl} \leq 1 \quad \forall rm, bl \tag{116}$$

For the above protection function, we define q^v_{bl} and $\mu^v_{rm,bl}$ as the dual variables corresponding to its constraints. By applying the same approach explained earlier, we can substitute the dual of protection function in constraint (106) in order to find its robust counterpart (constraints (117)-(119)).

$$\sum_{bl\in BL}(y_{bl,t}\sum_{rm\in RM}\bar{v}_{rm,bl}) + \sum_{bl\in BL}\Gamma^{v}_{bl}q^{v}_{bl} + \sum_{rm\in RM}\sum_{bl\in BL}\mu^{v}_{rm,bl} \leq b^{H}_{t} \quad \forall t \tag{117}$$

$$q^{v}_{bl} + \mu^{v}_{rm,bl} \geq \sum_{t\in T} y_{bl,t}\hat{v}_{rm,bl} \quad \forall rm, bl \tag{118}$$

$$q^{v}_{bl}, \mu^{v}_{rm,bl} \geq 0 \quad \forall rm, bl \tag{119}$$

4.4.2.2 Robust counterpart of uncertain terms in the objective function

As can be observed in model (99)-(107), the first two terms in the objective function contain uncertain log availability. In order to obtain their robust counterpart, first, we consider them as constraints. Hence, we substitute $(\sum_{bl\in BL}\sum_{t\in T} c^{H}_{blt} y_{bl,t}(\sum_{rm\in RM}\tilde{v}_{rm,bl})$ by π_1, and $\sum_{rm\in RM}\sum_{bl\in BL}\sum_{t\in T}\tilde{v}_{rm,bl} f_{rm,bl,t} y_{bl,t})$ by π_2 in the objective function and consider the following constraints in the harvesting planning model.

$$\sum_{bl\in BL}\sum_{t\in T} c^{H}_{blt} y_{bl,t}(\sum_{rm\in RM}\tilde{v}_{rm,bl}) \leq \pi_1 \tag{120}$$

$$\sum_{rm\in RM}\sum_{bl\in BL}\sum_{t\in T}\tilde{v}_{rm,bl} f_{rm,bl,t} y_{bl,t} \leq \pi_2 \tag{121}$$

Recall from 4.4.1 that constraints (120) and (121) try to minimize the harvesting and stumpage cost, respectively. Hence, their robust counterpart is equivalent to finding a harvesting plan such that the harvesting and stumpage costs over all realizations of uncertain log supply within the given interval is minimized. Moreover, it is unlikely that all uncertain supply parameters change simultaneously to their worst-case value; thus we consider the budgets of uncertainty $\Gamma^{\pi_1}_{bl}$ and Γ^{π_2} for the above-mentioned constraints. Intuitively the worst-case for both constraints occur when the supply quantities ($\tilde{v}_{rm,bl}$) take their maximum value. As a consequence, we propose the following protection functions ((122) and (123)) for constraints (120) and (121). Notice that in (122) and (123), $y^*_{bl,t}$ is considered as the optimal solution of model (99)-(107), and $w^2_{rm,bl}$ and $w^3_{rm,bl}$ are the scaled deviations from the nominal value of $\tilde{v}_{rm,bl}$.

$$\begin{aligned}
&Maximize \sum_{bl\in BL}\sum_{t\in T} c^H_{blt} y^*_{bl,t} (\sum_{rm\in RM} \hat{v}_{rm,bl} w^2_{rm,bl}) \\
&s.t. \quad \sum_{rm\in RM} w^2_{rm,bl} \leq \Gamma^{\pi_1}_{bl} \quad \forall bl \\
&0 \leq w^2_{rm,bl} \leq 1 \quad \forall rm, bl
\end{aligned} \tag{122}$$

$$\begin{aligned}
&Maximize \sum_{rm\in RM}\sum_{bl\in BL}\sum_{t\in T} \hat{v}_{rm,bl} f_{rm,bl,t} y^*_{bl,t} w^3_{rm,bl} \\
&s.t. \quad \sum_{rm\in RM}\sum_{bl\in BL} w^3_{rm,bl} \leq \Gamma^{\pi_2} = \sum_{bl\in BL} \Gamma^{\pi_1}_{bl} \\
&0 \leq w^3_{rm,bl} \leq 1 \quad \forall rm, bl
\end{aligned} \tag{123}$$

Afterwards, by considering $g^{\pi_1}_{bl}$, g^{π_2}, $\varepsilon^{\pi}_{rm,bl}$, and $\nu^{\pi}_{rm,bl}$ as the dual variables of the related protection functions (122) and (123), the dual of protection functions are added to constraints (120)-(121) in order to formulate their robust counterparts (constraints (124)-(128)).

$$\sum_{bl\in BL}\sum_{t\in T} c^H_{blt} y_{bl,t} (\sum_{rm\in RM} \bar{v}_{rm,bl}) + \sum_{bl\in BL} \Gamma^{\pi_1}_{bl} g^{\pi_1}_{bl} + \sum_{rm\in RM}\sum_{bl\in BL} \varepsilon^{\pi}_{rm,bl} \leq \pi_1 \tag{124}$$

$$\sum_{rm\in RM}\sum_{bl\in BL}\sum_{t\in T} \bar{v}_{rm,bl} f_{rm,bl,t} y_{bl,t} + \Gamma^{\pi_2} g^{\pi_2} + \sum_{rm\in RM}\sum_{bl\in BL} \nu^{\pi}_{rm,bl} \leq \pi_2 \tag{125}$$

$$g^{\pi_1}_{bl} + \varepsilon^{\pi}_{rm,bl} \geq \sum_{t\in T} c^H_{blt} y_{bl,t} \hat{v}_{rm,bl} \quad \forall rm, bl \tag{126}$$

$$g^{\pi_2} + \nu^{\pi}_{rm,bl} \geq \sum_{t\in T} \hat{v}_{rm,bl} f_{rm,bl,t} y_{bl,t} \quad \forall rm, bl, t \tag{127}$$

$$g^{\pi_1}_{bl}, g^{\pi_2}, \varepsilon^{\pi}_{rm,bl}, \nu^{\pi}_{rm,bl} \geq 0 \quad \forall rm, bl \tag{128}$$

Finally, the robust counterpart of the HP model, denoted as R-HP can be formulated as follows:

$$Min\ Z_{R-HP} = \pi_1 + \pi_2 + \sum_{rm\in RM}\sum_{bl\in BL}\sum_{t\in T} HH_{rm,bl,t} \tag{129}$$

Subject to:

$$\text{Constraints} \quad (101)-(105); (107); (112)-(115); (117)-(119); (124)-(128) \tag{130}$$

The R-HP model is a mixed-integer program including more decision variables and constraints compared to the deterministic HP model. In contrary, this model protects the harvesting plan against the uncertainty in log demand and supply.

4.5 Numerical results

In this section, we validate the proposed robust harvesting planning (R-HP) model through a set of realistic-size instances from a lumber supply chain in Canada. The purpose of the numerical experiments is three-fold.

Our first goal is to analyze the trade-off between the level and the cost of robustness. More precisely, we are interested in investigating how increasing the degree of robustness (budget of uncertainty) affects the feasibility and optimality of the nominal (deterministic) solution. This is important for the decision maker in order to set a budget of uncertainty so that the harvesting plan obtained by the R-HP model is feasible in the presence of future uncertainties while it is not too costly (i.e., overprotected against uncertainty). This goal is achieved by the aid of theoretical probability bounds and through comparison between the solution obtained by the R-HP model and the nominal one for different levels of budget of uncertainty.

Our second goal is to verify the feasibility and the degree of conservatism of the robust optimal solution in the presence of randomly generated uncertain demand and supply parameters. Monte-Carlo simulation is employed for this purpose. This is a more realistic approach for evaluating the impact of the budget of uncertainty that will be adopted by the decision maker on the feasibility and the cost of the plan.

Finally, we are interested in studying the structural changes in the optimal nominal solution after implementing the robust optimization approach. This is due the fact that decision makers do not prefer drastic changes in plans resulting from adopting new planning approaches compared to their current practice.

The above-mentioned analyses are carried out on two sets of test problems that are distinguished by the level of variability of uncertain parameters. In what follows, the results are provided in three sub-sections corresponding to considering uncertainty in log supply, log demand, and both log supply and demand.

4.5.1 Case data implementation details

In the harvesting planning case study, we assume that 50 harvesting blocks are available in the forest during the 12 month planning horizon. The supply capacity of each block per month is supposed to be 2350 m^3. Adopted from Beaudoin et al. [15], the maximum number of periods (months) over which harvesting can occur in each block, and the maximum number of blocks in which harvesting can occur per month are randomly selected from uniform distributions U(1-6) and U(10-12), respectively. The total harvesting capacity per month is supposed to be approximately 117,500 m^3.

We define γ as the level of variability of uncertain log supply and demand quantities comparing to their nominal values and consider two classes of test problems corresponding to $\gamma = 5\%$ and 20%. This will result $\hat{v}_{rm,bl} = \gamma \bar{v}_{rm,bl}$ and $\hat{d}^{bl}_{rm,t} = \gamma \bar{d}^{bl}_{rm,t}$ in R-HP model provided in 4.4.2.

While considering uncertainty in log supply, we assume that $\Gamma^{\pi_1}_{bl}$ and Γ^{v}_{bl} vary from 0 to $|RM| = 14$ (the worst-case). Also, Γ^{π_2} vary from 0 to $|RM||BL| = 700$ since $\Gamma^{\pi_2} = \sum_{bl \in BL} \Gamma^{\pi_1}_{bl}$.

When dealing with uncertainty in log demand, we aim at considering time-dependent budgets of uncertainty inspired by Adida and Perakis [63]. Hence, the following three scenarios for $\Gamma^{bl}_{rm,t}$ based on a linear function of t are investigated including: (1) $\Gamma^{bl}_{rm,t} = 0.5 + 0.2t$; (2) $\Gamma^{bl}_{rm,t} = 0.5 + 0.4t$; and (3) $\Gamma^{bl}_{rm,t} = t$. It is important to note that $\Gamma^{bl}_{rm,t} \geq t$ means the worst-case is obtained. The last category of results is conducted based on the combination of budgets of uncertainty described for uncertain supply and demand. Furthermore, Z^{WC} provided in our experimental results denotes the optimal nominal objective value when the deterministic (nominal) model is solved by considering the worst-case bound in the given uncertain interval. This measure is used to verify whether or not the solution of R-HP model is over-protected against uncertainty.

Finally, the Monte-Carlo simulation is based on generating random scenarios for uncertain log supply and demand from their corresponding uniform distribution, and verifying the feasibility and the actual objective function value of the robust solution for each scenario. In other words, for each scenario, the optimal solution of the R-HP model is plugged into the deterministic model where the uncertain parameters are substituted by the simulated value. Afterwards, the feasibility and the actual objective function value of the robust solution are verified.

The robust model in this paper is coded in C++ using CPLEX concert technology on a Core i7 CPU 3.40GHz computer with 8.00 GB RAM.

4.5.2 Results for uncertainty in log supply parameters

As previously mentioned,the uncertain supply parameter affects harvesting and stumpage costs in the objective function of R-HP model. Moreover, this parameter affects constraint (106) related to the harvesting capacity. Hence, it can be expected that uncertain supply affects on the feasibility and optimality of the R-HP model's solution.

The trade-off between robustness (i.e., the budget of uncertainty) and the cost of robustness is estimated by calculating the cost deviation $(Z^R_{R-HP} - Z^N_{R-HP})/Z^N_{R-HP}$, where Z^R_{R-HP} and Z^N_{R-HP} are the robust and nominal optimal objective values, respectively.

Another important issue is the analysis of the robust solution in terms of feasibility. As explained earlier, when the budget of uncertainty achieves its maximum value, the robust solutions are always feasible. In contrast, the feasibility condition cannot be guaranteed by considering smaller values for the budget of uncertainty. Recall from Section 3 that it is possible to provide probability bounds for the constraint violation in such cases. Based on equation (98), the probabilistic bounds of constraint violation depend only on the number of coefficients subject to uncertainty (i.e., $|J|$) and the budget of uncertainty.

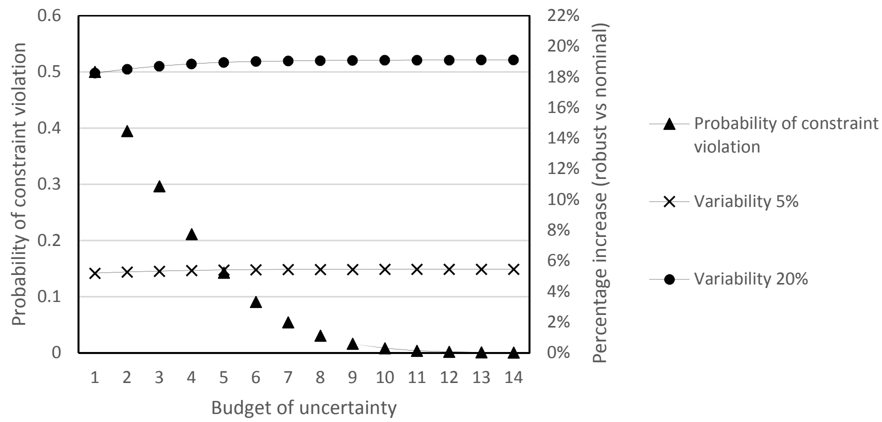

Figure 4: Robust optimal values vs nominal and the probability bounds for constraint violation for random supply

Figure (4) represents the percentage increase in the objective function value versus the nominal one for two levels of supply variability and different values of budget of uncertainty in constraints (106) and (120)-(121). As expected, when robustness is enforced to the model, the cost is increased in order to envisage worst-case values for a certain number of log availability parameters in the objective function (constraints (120)-(121)) and in constraint (106). The latter increase in the robust objective function value is the effect of considering such worst-case supply values on harvesting and stumpage costs. It is worth noting that such worst-cases occur when log availability parameters ($\tilde{v}_{rm,bl}$) take their highest value. In such cases , more logs are harvested and consequently the corresponding harvesting and stumpage costs are increased. Additionally, we can conclude from Figure (4), when the variability level of uncertain parameters is small (i.e., $\gamma = 5\%$), the impact of imposing robustness on the objective function is less significant in comparison with higher level of variability. Moreover, the objective function is increased by increasing the budget of uncertainty. When the budget of uncertainty is increased, it indicates that the number of uncertain parameters that take their worst-case value are increased which in turn, is expected to increase the cost.

As is observable in Figure (4), the violation probability decreases and tends to reach zero as we increase the budget of uncertainty to its maximum value. This probability is near zero when the budget of uncertainty is greater than 8. In other words, the probability violation in this figure is stable in near 50% of budget of uncertainty. By increasing the budget of uncertainty, the number of uncertain parameters that take their worst-case value in the constraint (106) is increased. Thus, the R-HP model tries to find a feasible solution to satisfy the harvesting capacity in constraint (106) for such worst-case scenarios. Consequently, the violation probability of constraint (106) is reduced.

Furthermore, we compare the structural changes in the solutions of robust and nominal problems. By considering the cumulative portion of harvested blocks ($\sum_{bl,t} y_{bl,t}$), no significant structural changes in the solutions of robust model compared to the nominal problem is observed. Moreover, the changes in the $\sum_{bl,t} y_{bl,t}$ for lower amount of budget of uncertainty is more, but this dispersion is not significant.

For the Monte-Carlo simulation results, we randomly generate 500 random log supply quantities based on different variability levels for each value of budget of uncertainty. Consequently, we solved $500 * 14 * 2 = 14,000$ deterministic models in order to better analyze the feasibility and the level of conservatism of the robust solution. At the first test, we check the feasibility of the robust solution in the nominal problem with simulated random log supplies. These results are depicted in Figure (5) for the two classes of test instances.

As is shown in Figure (5), when the budget of uncertainty is greater than 3, the number of infeasible instances equal to zero. Thus, by considering $\Gamma \geq 3$, it is possible to guarantee the feasibility of solutions for random log supplies, and it is not necessary to increase the value of Γ and enforce more costs.

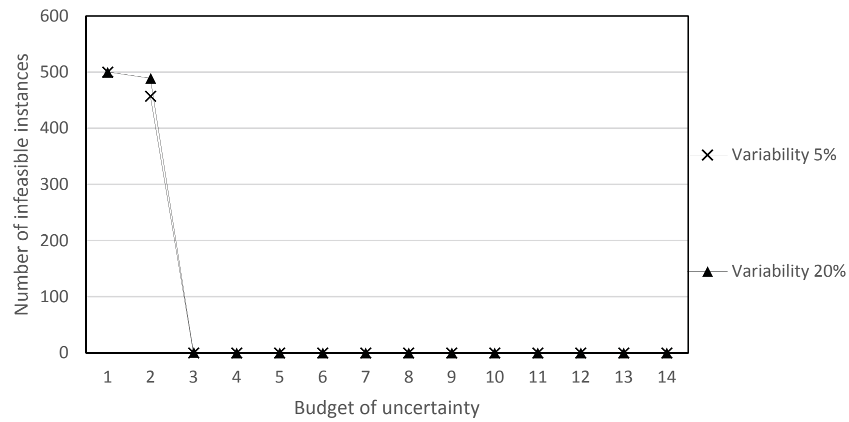

Figure 5: Feasibility of robust solutions with simulated log supplies

Additionally, we calculate the fraction of simulated instances where the actual cost is lower than the optimal objective value of the optimal value of robust model. We observe that in all cases the optimal objective function of robust problem is worse than the simulated ones. As a consequence, the robust problem overestimates the actual cost in comparison with the simulated instances.

Finally, we compare the robust problem with the worst-case deterministic model (WC). If $(Z^{WC} - Z^{R}_{R-HP}) \geq 0$, it is concluded that the robust problem proposes a solution with lower cost. Noted that the Z^{R}_{R-HP} are related to the average of the robust objective values obtained based on different values of budget of uncertainty. The latter analysis is presented in Table (9).

Table 9: Comparison of Z^{WC} and Z^{R}_{R-HP} with uncertain log supply

	$\gamma = 5\%$	$\gamma = 20\%$
Z^{WC}	21,081,400	23,809,300
Z^{R}_{R-HP}	21,070,193	23,773,686

As $(Z^{WC} - Z^{R}_{R-HP}) \geq 0$ for both variability levels in Table (9), it is concluded that the robust problem outperforms the worst-case deterministic problem.

4.5.3 Results for uncertainty in log demand parameters

As mentioned earlier, uncertain demand affects the feasibility of log inventory levels as well as the inventory holding cost. Table (10) shows the percentage increase of the robust objective function value compared to the nominal case for the three scenarios corresponding to the time-dependent budget of uncertainty, described in 4.5.1. As expected, the optimal robust costs and the level of conservatism increase as the uncertainty variability level increases. Also, no significant difference between different scenarios for modeling the budget of uncertainty is observed. It is noteworthy that the increased objective function value is the result of augmented log inventory holding cost obtained in the R-HP model. Recall from section 4.4.2.1 that the robust counterpart of constraint (109) is equivalent to the case where some log demand parameters take their smallest value. This, as a consequence, will increase the inventory holding cost.

Table 10: Percentage increase in the cost of R-HP model compared to the nominal model

	$\gamma = 5\%$	$\gamma = 20\%$
Scenario 1 ($\Gamma^{bl}_{rm,t} = 0.5 + 0.2t$)	20.70%	83.01%
Scenario 2 ($\Gamma^{bl}_{rm,t} = 0.5 + 0.4t$)	21.45%	86.06%
Scenario 3 ($\Gamma^{bl}_{rm,t} = t$)	21.47%	86.17%

Again, we calculate the probability bounds for constraints (109) and (114) violation for different scenarios of budget of uncertainty in different periods in the planning horizon. The results of the latter probability bound are summarized in Figure (6). Recall from 4.4.2.1, constraint (109) might be violated when all log demand parameters ($\tilde{d}^{bl}_{rm,t}$) take their minimum values in their uncertainty intervals. In contrast, constraint (114) is expected to be violated when all demand parameters take their maximum values in the uncertainty interval. The curve of probability bounds in Figure (6) reaches to zero when we face with the worst-case scenario which is shown by scenario 3. As scenarios 1 and 2 do not reflect the worst-case for demand, their curves do not reach to zero and decreased convexly by the time. Furthermore, when the budget of uncertainty in scenario 3 is greater and equal to 7, the probability of violation reaches to zero. In other words, the probability violation in this figure is stable in near 50% of budget of uncertainty. On the other hand, as we proceed over the planning horizon, the budget of uncertainty is increased in constraints (112) and (114). Consequently, the violation probability of constraints are reduced.

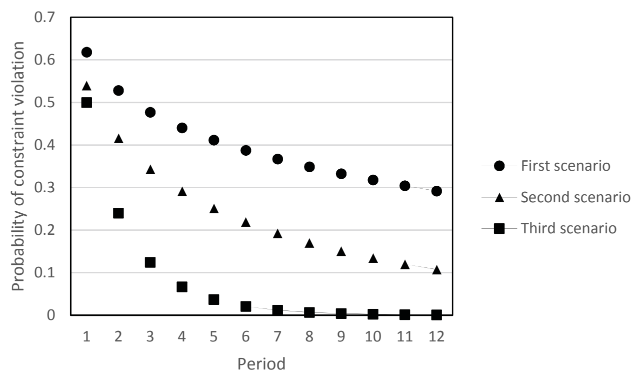

Figure 6: The probability bounds for violation of constraints (109) and (114) in R-HP model

In the presence of uncertainty in log demand, we compare structural changes in solution of robust and nominal problems. Since the robust model tries to find a feasible solution in the worst-case perspective, demand is always assumed as its highest value (in order to satisfy constraint (114)), so the robust model increases the portion of harvested blocks. Again, this comparison indicates that there is not a significant change in ($\sum_{bl,t} y_{bl,t}$) compared to nominal solution although the difference is more than the previous case where the log supply was considered as an uncertain parameter. Moreover, the behavior to the structural changes increases by increasing the budget of uncertainty as well as increasing the variability level.

For the simulation tests, we randomly generate 500 random demand parameters for each variability level and each budget of uncertainty scenario (totally 3000 instances). First, the feasibility of the robust solution in the nominal problem with randomly generated log demand is verified. It should be noted that the uncertain demand affects 8,400 constraints (constraint (114)) in R-HP model. The simulation results reveal that the number of infeasible constraints are negligible. Table (11) provides the percentage of infeasible constraints for different scenarios of budget of uncertainty and demand variability levels.

Table 11: The percentage of infeasible constraints in nominal model

	$\gamma = 5\%$	$\gamma = 20\%$
Scenario 1 $(0.2t + 0.5)$	0.02%	0.03%
Scenario 2 $(0.4t + 0.5)$	0.003%	0.005%
Scenario 3 (t)	0%	0%

As the third scenario ($\Gamma^{bl}_{rm,t} = t$) considers the worst-case of uncertain demand in the robust model, the number of infeasible constraints in the nominal problem with simulated demand equals zero. Moreover, as the variability level is increased, the dispersion of simulated demand and the number of infeasibile constraints are increased.

Next, we compare the objective functions of simulated instances by the robust one for feasible instances. The results indicate that the objective functions of all simulated instances are better than the corresponding objective value in the robust problem. Finally, we can conclude that the robust problem presents a more conservative solution in comparison with the simulated instances.

Finally, the comparison of the robust model with the worst-case deterministic one (WC) is presented in Table (12). Noted that the Z^{R}_{R-HP} objective values are related to the second scenario ($\Gamma^{bl}_{rm,t} = 0.4t + 0.5$) in the robust model.

Table (12) indicates that the robust problem outperforms the worst-case deterministic problem in both uncertainty levels.

Table 12: Comparison of Z^{WC} and Z^{R}_{R-HP} with uncertain log demand

	$\gamma = 5\%$	$\gamma = 20\%$
Z^{WC}	24,305,800	37,308,800
Z^{R}_{R-HP}	24,279,000	37,195,100

4.5.4 Results for uncertainty in log supply and demand parameters

In this case, we test all generated instances in the previous sections for some budget of uncertainty and variability levels for uncertain log supply and demand. In Figures (7)-(9), γ_1 and γ_2 indicate the variability levels for supply and demand, respectively. These figures show the percentage increase in the objective function of the robust model in comparison with the nominal one, where $(\gamma_1, \gamma_2) = \{(5\%, 5\%), (5\%, 20\%), (20\%, 5\%)\}$. Based on the latter set, we considered 3 cases for variability level of supply and demand, 3 scenarios to generate budget of uncertainty corresponding to demand ($\Gamma^{bl}_{rm,t}$), and 14 cases for Γ^{π}_{bl} or Γ^{v}_{bl} that totally tests $14 * 3 * 3 = 126$ problems.

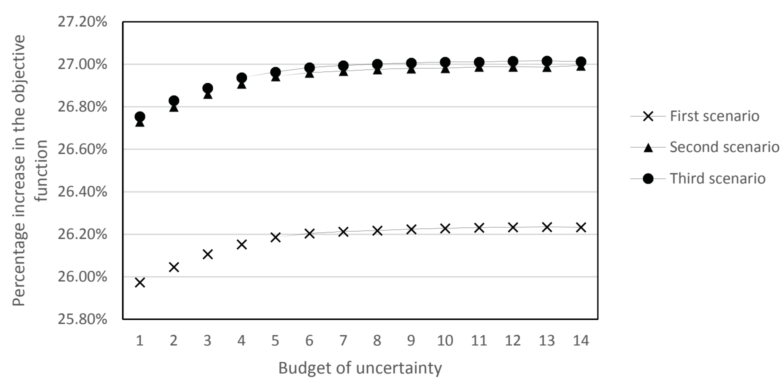

Figure 7: Percentage increase in the objective function where $(\gamma_1, \gamma_2) = (5\%, 5\%)$

Similarly, the elaborated increase in the objective function with considering uncertainty in both supply and demand is the consequence of increasing harvesting,

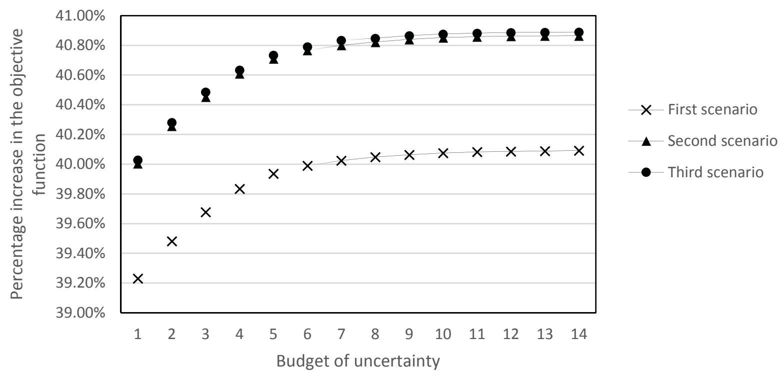

Figure 8: Percentage increase in the objective function where $(\gamma_1, \gamma_2) = (20\%, 5\%)$

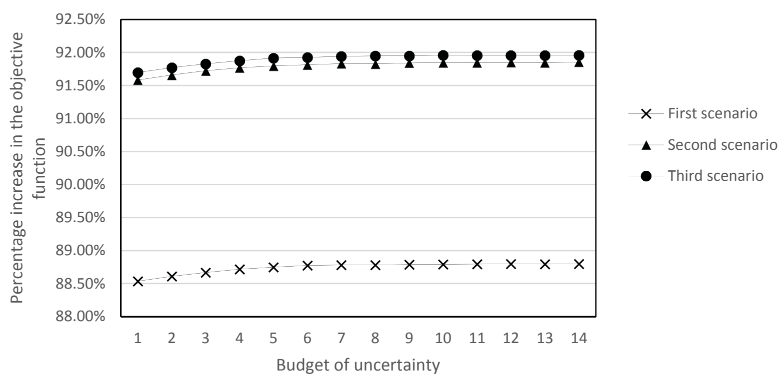

Figure 9: Percentage increase in the objective function where $(\gamma_1, \gamma_2) = (5\%, 20\%)$

stumpage and inventory holding costs in the forest in order to satisfy R-HP constraints for all admissible amount of supply and demand. Moreover, it is concluded that when the variability level of uncertain parameters are smaller, the impact of imposing robustness is less in comparison with higher level of variability.

The comparison between Figure (4), Table (10) and Figures (7)-(9) indicates that the objective function is more sensitive to the uncertain demand.

4.6 Conclusions

In this paper, we proposed a robust optimization model based on cardinality-constrained approach to address the forest harvesting planning under log supply and demand uncertainty. The latter provides the possibility of adjusting the level of robustness of the solution in terms of feasibility in the presence of uncertainty against the cost of such a robust solution. We also conducted an extensive set of experiments to compare the quality of the robust solution in terms of feasibility and cost. This has been realized by the aid of theoretical bounds and Monte-Carlo simulation. Our experimental results revealed the high quality of robust solutions in terms of feasibility with an acceptable overestimation of the cost. The above-mentioned results provide enough insight to the decision maker in order to choose the right budget of uncertainty such that a feasible plan in the presence of future uncertainties at a reasonable cost is obtained. Furthermore, compared to the plan proposed by the nominal (deterministic) model, no significant structural change in the total amount of harvesting in different blocks was observed. This is a desirable feature given the fact that the decisions makers do not prefer fluctuations in the harvesting plan while facing with uncertainties. According to the experimental results, it can be concluded that the proposed robust planning tool is essential for forest supply chain in order to survive against market perturbations and log growth variations.

Future research would focus on the implementation of the proposed robust approach in integration with other tactical decisions in the lumber supply chain such as sawmills production planning where non-homogeneous and random characteristics of log supply and demand might result in more random parameters.

Acknowledgments

This work was financially supported by NSERC Strategic Network on Value Chain Optimization (VCO).

Chapter 5

Conclusion and Future Work

In this thesis, we investigated in integrated tactical planning in the lumber supply chain while considering uncertain supply and demand. At first, we focused on the deterministic context. Then, we considered uncertainty into the tactical decisions in the lumber supply chain and proposed stochastic programming and robust optimization models in order to handle the latter uncertainties. To solve the models in each section, we developed efficient algorithms and evaluated them based on data sets that sufficiently represent realistic-scale lumber supply chains in Canada. The experimental results in each section, showed the high performance of the proposed mathematical models and solution algorithms in order to find high quality plans in a considerably small CPU time.

The remainder of this chapter is dedicated to elaborate the concluding remarks of this research provided in each chapter. Then, several avenues for future work following this dissertation are presented.

5.1 Concluding Remarks

In the second chapter, we proposed a mixed-integer programming (MIP) model to address harvesting, procurement, production, distribution, and sale decisions in the lumber supply chain in an integrated scheme. The benefit of the integrated model was evaluated by comparing the total profit/cost of the integrated model and the decoupled planning models. Our experimental results revealed that substantial improvement can be obtained by using an integrated model rather than the decoupled models. On the other hand, solving the integrated MIP model for large-scale instances was an issue. Hence, in order to overcome the complexity of the integrated model for real-size instances, we proposed a heuristic algorithm in the framework of Lagrangian Relaxation algorithm where the performance of the sub-gradient algorithm was improved in terms of convergence and the feasibility of converged solution. Our computational results on a set of large-scale test cases revealed the effectiveness of the proposed heuristic in obtaining high quality feasible solutions in a considerably reduced CPU time comparing to using a commercial solver, and the classical Lagrangian Relaxation algorithm.

In Chapter 3, we proposed a multi-stage stochastic mixed-integer programming (MS-MIP) model to incorporate uncertainties into the lumber supply chain tactical planning model. As the proposed model was a large-scale MS-MIP model with no special structure, it could not be solved by commercial solvers or relevant approached in the literature. Hence, we proposed a new algorithmic procedure entitled as the Hybrid Scenario Cluster Decomposition (HSCD) algorithm. The HSCD scheme proposed in this research is an accelerated scenario cluster decomposition method that decomposes the original MS-MIP model into smaller MS-MIP sub-models corresponding to decomposed scenario sub-trees in the original scenario tree. As solving each sub-problem in real instances would be another issue, we proposed an ad-hoc heuristic (a Lagrangian based heuristic) and a Variable Fixing Heuristic (VFH) in order to speed up the convergence of the HSCD algorithm. Finally, we evaluated the performance of the HSCD algorithm by conducting a set of large-scale test instances. The numerical results revealed that the HSCD algorithm can overcome the computational complexity of MS-MIP models, and find high quality solutions in a reasonable CPU time.

Due to the computational complexity of multi-stage stochastic programming approach, in Chapter 4, a robust optimization model based on cardinality-constrained approach was proposed. This approach provided the possibility of adjusting the level of robustness of the harvesting plan over the planning horizon in terms of feasibility in the presence of uncertainty against the cost of such robust solution. An extensive set of experiments through Monte-Carlo simulation was also conducted in order to evaluate the quality of the robust solution in terms of feasibility and cost. The numerical results revealed the high quality of robust solutions in terms of feasibility with a negligible increase in the cost. Moreover, by comparing the proposed plan in the deterministic and robust model indicated that there is no significant structural change in the total amount of decision variables.

5.2 Future research directions

There are several avenues for future research directions following this thesis. The perspectives driven from this dissertation mostly revolve around the following directions.

- It would be interesting to investigate various coordination mechanisms in the lumber supply chain in order to facilitate the implementation of the proposed integrated tactical planning tool in this industry. Game-theoretical approaches could be adopted to achieve a win-win situation for all parties of the lumber supply chain. The value of integration can then be interpreted as the maximum price that can be paid in order to facilitate information sharing among entities of this supply chain.

- It is of interest to implement the proposed approach in Chapter 3, Hybrid Scenario Cluster Decomposition (HSCD) algorithm, on parallel machines in order to reduce the CPU time. Moreover, it is possible to embed other efficient heuristic algorithms within the HSCD scheme in order to efficiently solve scenario cluster sub-models.

  Furthermore, the HSCD algorithm can be applied to other supply chain tactical planning problems that incorporate uncertain parameters with a dynamic behavior over time. Finally, further research can also be focused on considering robustness terms into the objective function of the multi-stage stochastic model in Chapter 3 and controlling the variability of the recourse cost under various scenarios.

- The cardinality-constrained approach developed in Chapter 4 can be applied to other supply chain tactical planning problems that incorporate uncertainties.

- The uncertain parameters studied in the thesis are random log supply and lumber demand. Other random parameters such as sawmills production yield can be taken into account in the proposed stochastic and robust optimization models and applying efficient algorithms to solve them would be interesting in order to further investigations.

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